Computer Algebra Independent Integration Tests

Summer 2023 edition

3-Logarithms/63-3.4-u-a+b-log-c-d+e-x^m-^n-^p

Nasser M. Abbasi

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [641]. This is test number [63].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (641)	0.00 (0)
Mathematica	94.23 (604)	5.77 (37)
Maple	65.21 (418)	34.79 (223)
Fricas	61.31 (393)	38.69 (248)
Giac	54.76 (351)	45.24 (290)
Maxima	52.73 (338)	47.27 (303)
Mupad	50.86 (326)	49.14 (315)
Sympy	30.73 (197)	69.27 (444)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

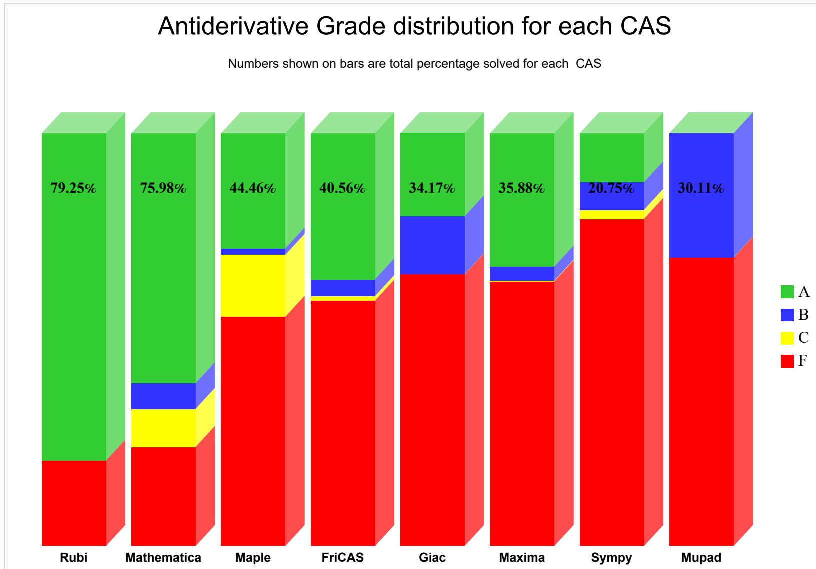
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

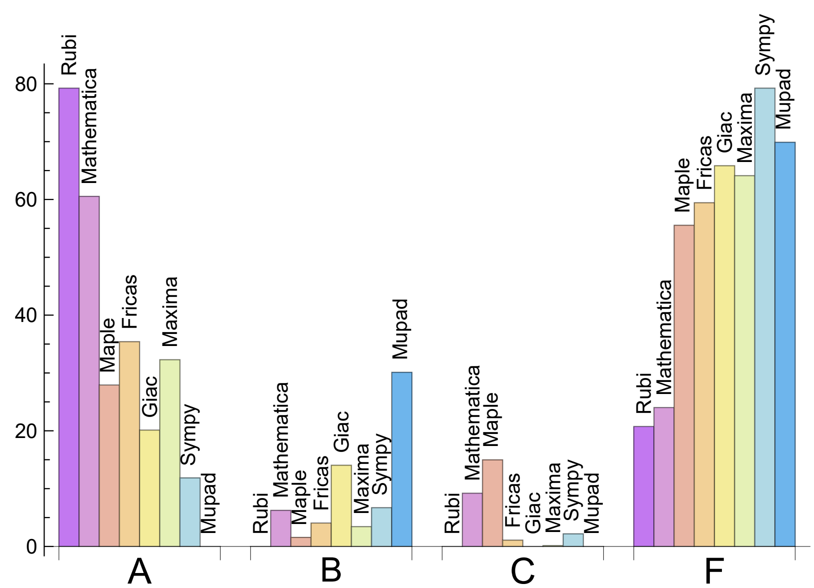
System	% A grade	% B grade	% C grade	% F grade
Rubi	79.251	0.000	0.000	20.749
Mathematica	60.530	6.240	9.204	24.025
Fricas	35.413	4.056	1.092	59.438
Maxima	32.293	3.432	0.156	64.119
Maple	27.925	1.560	14.977	55.538
Giac	20.125	14.041	0.000	65.835
Sympy	11.856	6.708	2.184	79.251
Mupad	0.000	30.109	0.000	69.891

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	37	100.00	0.00	0.00
Maple	223	100.00	0.00	0.00
Fricas	248	100.00	0.00	0.00
Maxima	303	73.60	0.33	26.07
Giac	290	99.66	0.00	0.34
Mupad	315	0.00	100.00	0.00
Sympy	444	40.77	55.18	4.05

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.23
Maxima	0.28
Fricas	0.40
Giac	0.52
Mathematica	0.72
Maple	1.19
Mupad	1.79
Sympy	20.87

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	130.90	1.04	48.00	1.00
Maxima	157.80	1.72	98.00	1.00
Sympy	172.43	1.96	83.00	1.08
Maple	195.92	1.45	86.00	1.00
Giac	224.79	1.53	85.00	1.10
Rubi	226.62	1.00	121.00	1.00
Mathematica	266.60	3.55	114.50	1.00
Fricas	297.37	1.76	84.00	1.18

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

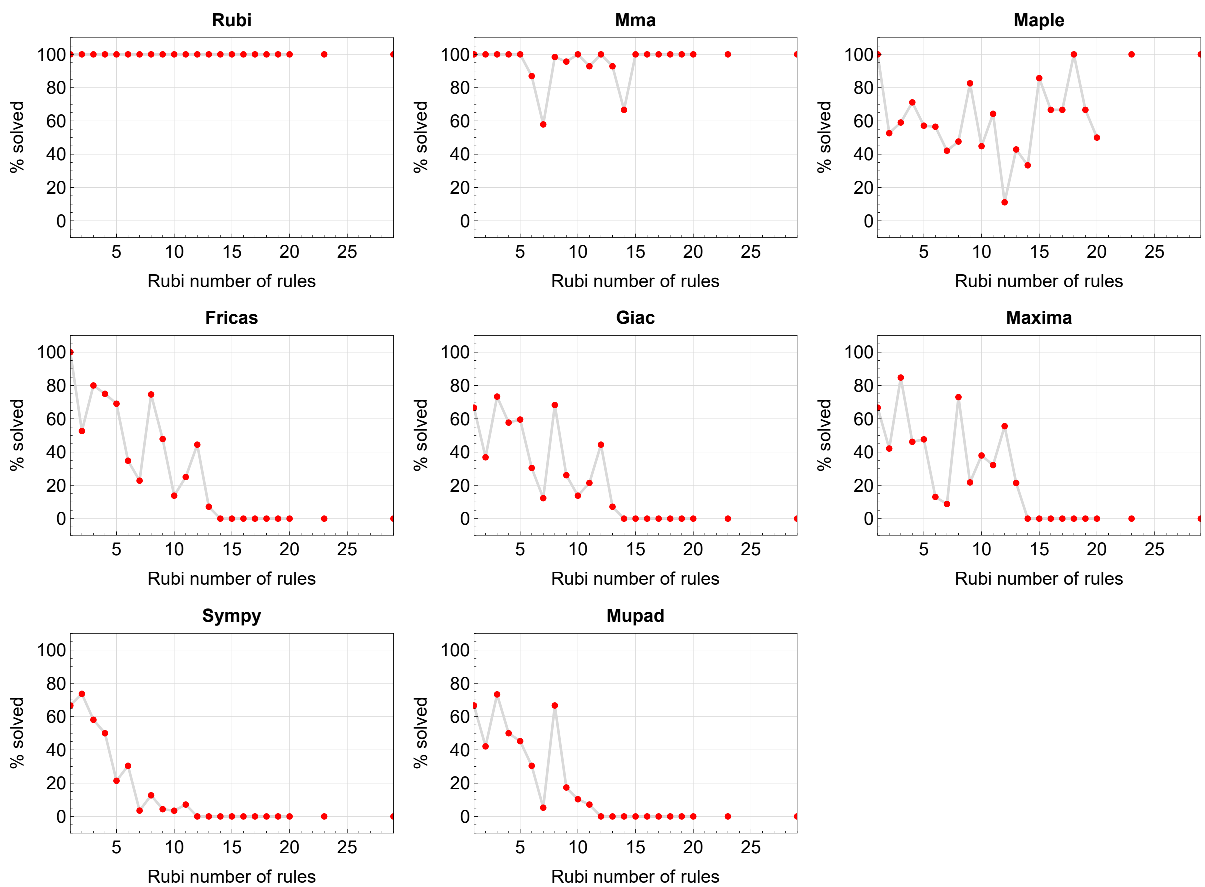


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

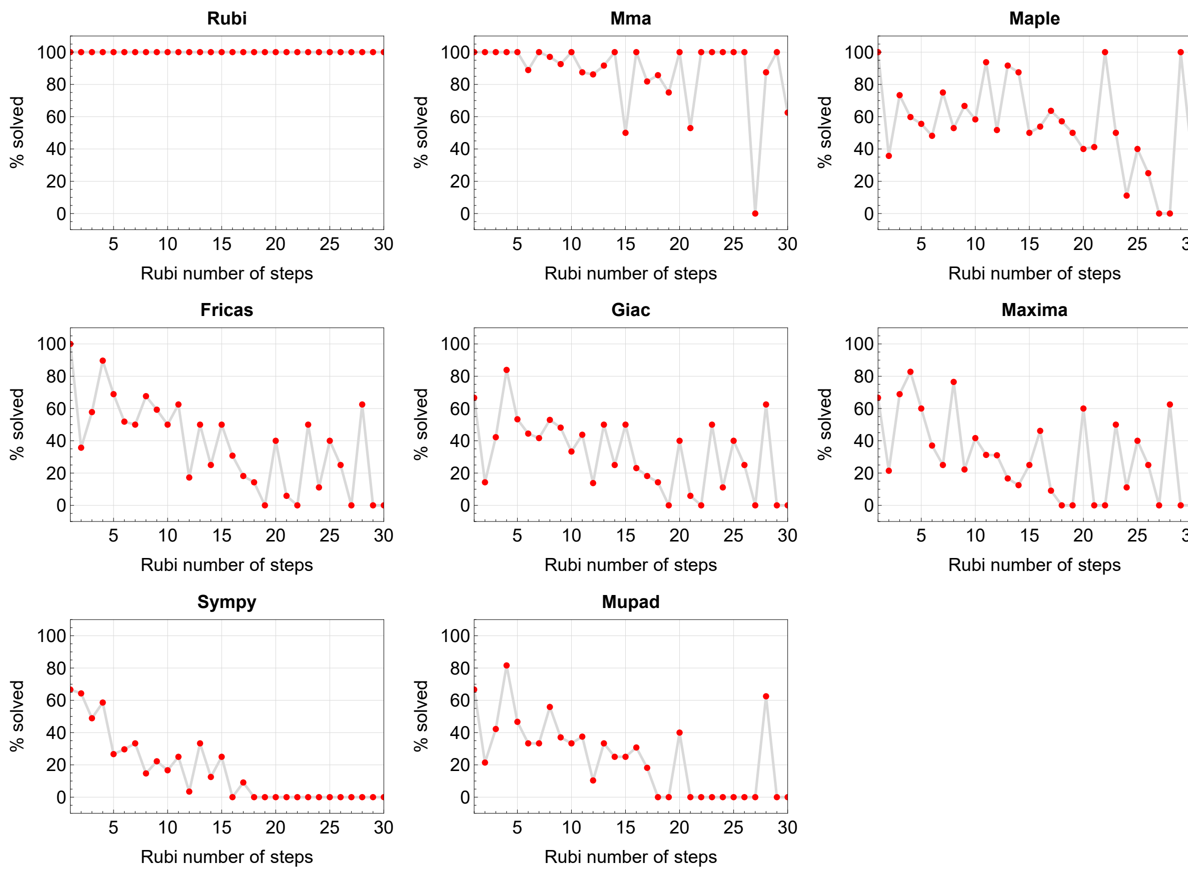


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

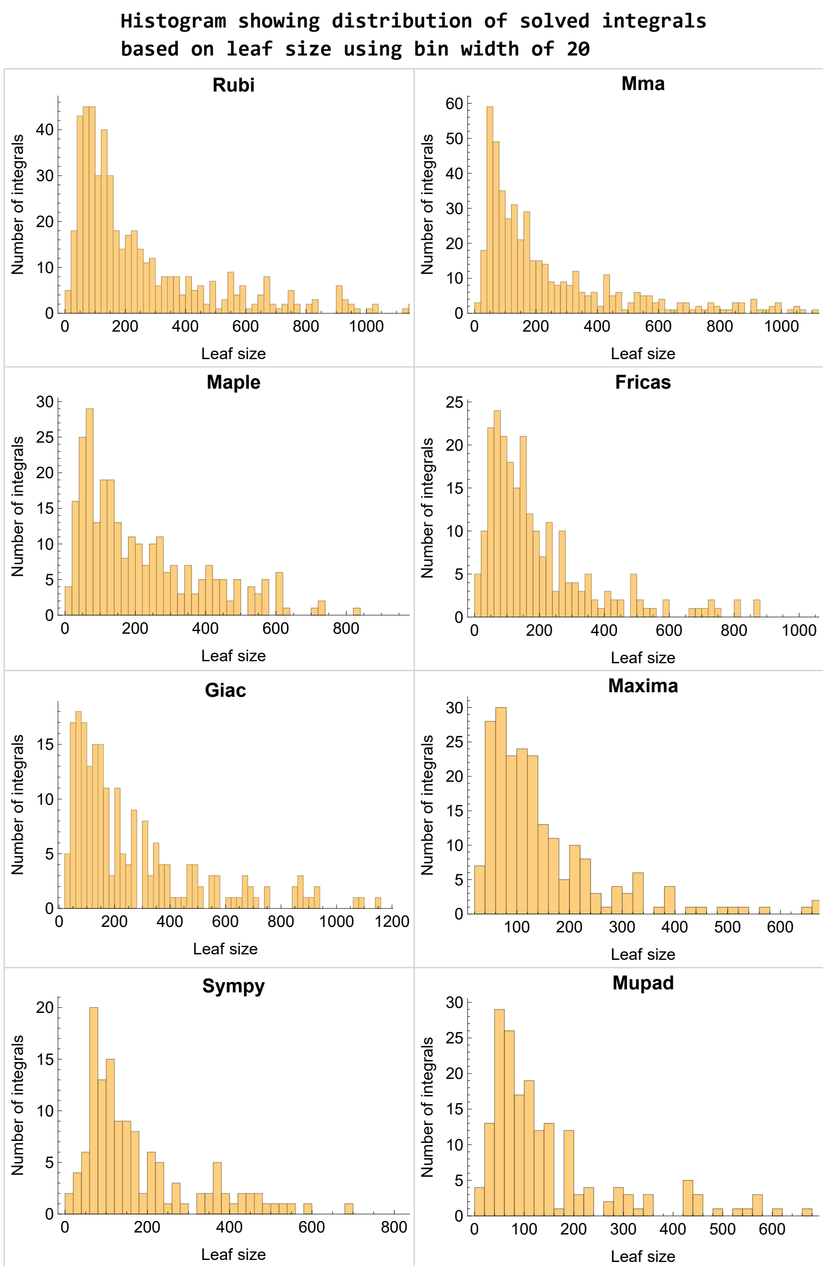


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

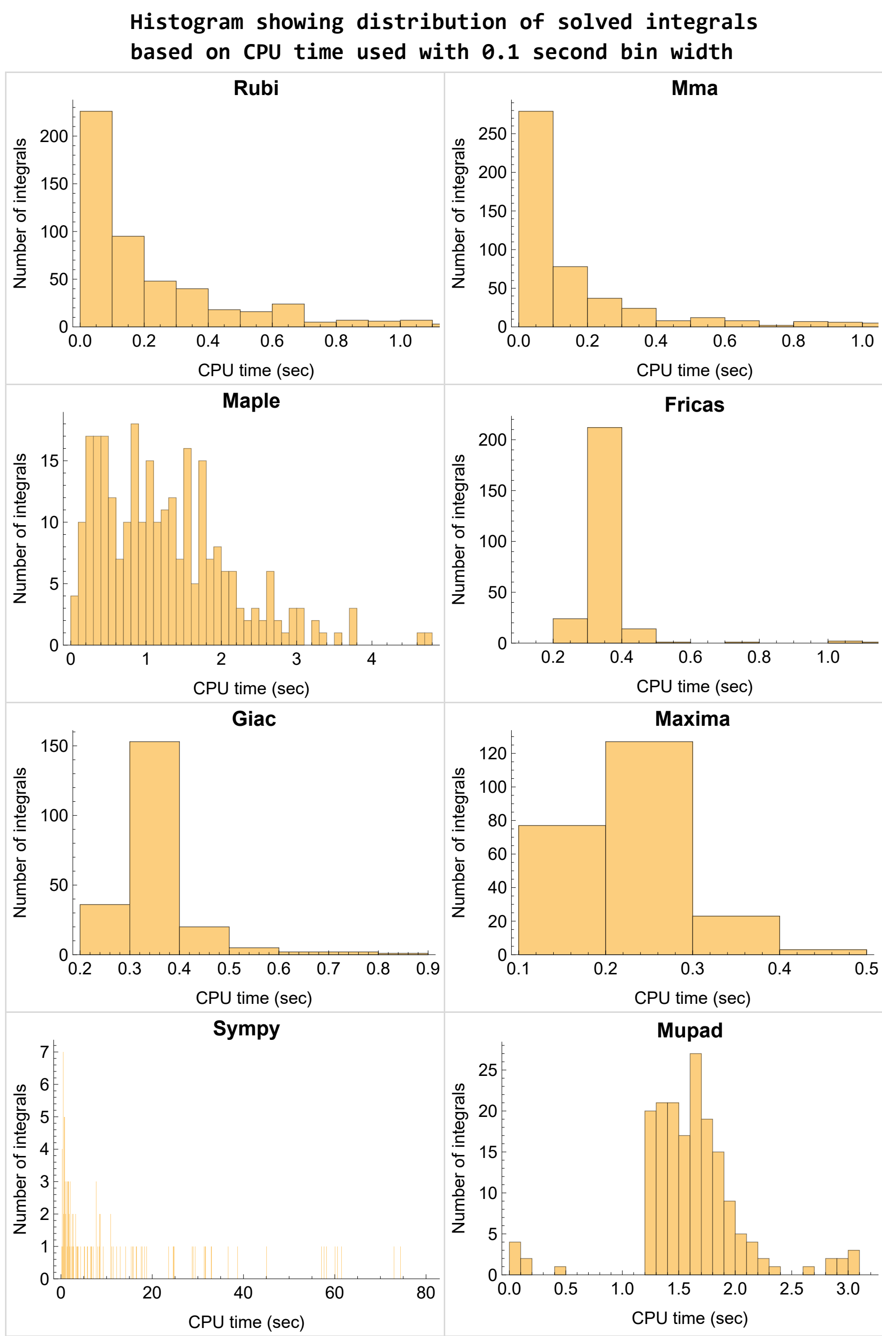


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

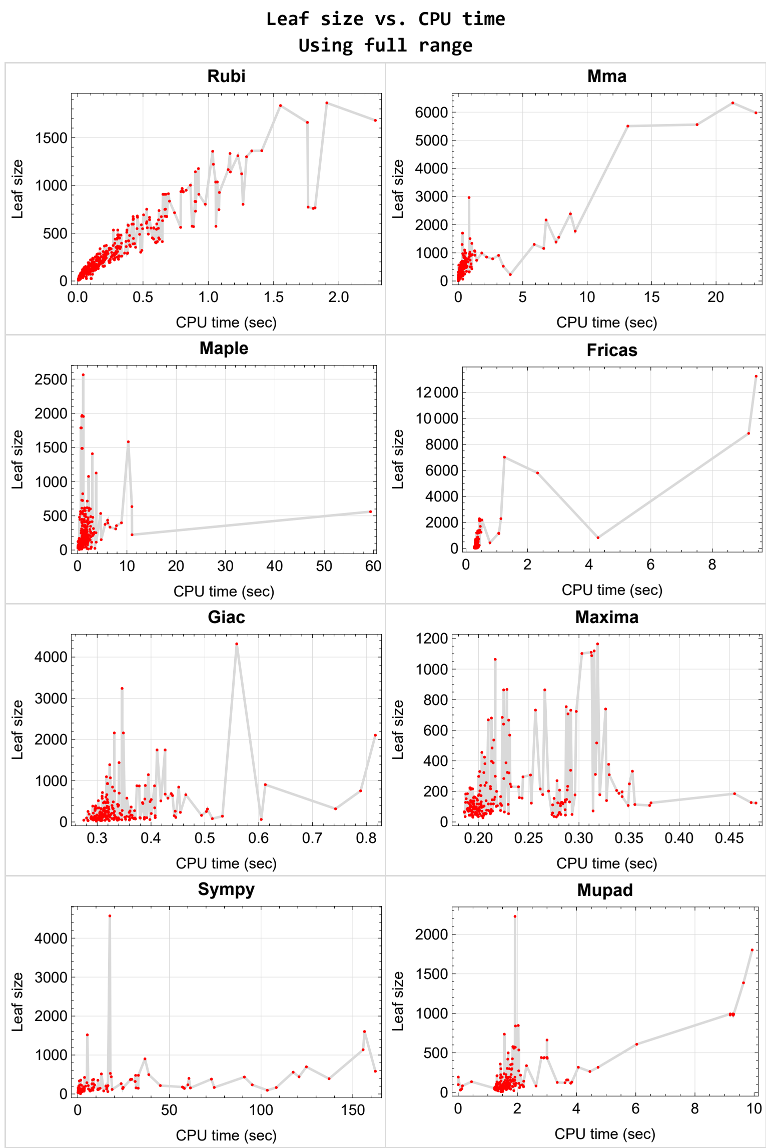


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{98, 99, 100, 101, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 211, 216, 217, 218, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 381, 382, 383, 384, 385, 386, 387, 388, 398, 485, 486, 487, 488, 528, 529, 530, 531, 536, 537, 542, 543, 544, 545, 546, 550, 551, 552, 560, 561, 566, 567, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 588, 589, 590, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 625, 626, 627, 635, 640, 641\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{98, 99, 100, 101, 158, 159, 277, 298, 299, 485, 486, 487, 488, 528, 530, 531\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{\}$

Mathematica $\{168, 292, 358, 376\}$

Maple $\{67, 73, 81, 82, 83, 84, 85, 87, 88, 89, 90, 102, 103, 109, 110, 116, 117, 132, 133, 135, 136, 137, 138, 139, 140, 148, 149, 150, 166, 173, 174, 175, 238, 239, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 271, 273, 274, 291, 293, 294, 295, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348,$

349, 351, 352, 357, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 619, 620, 624}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

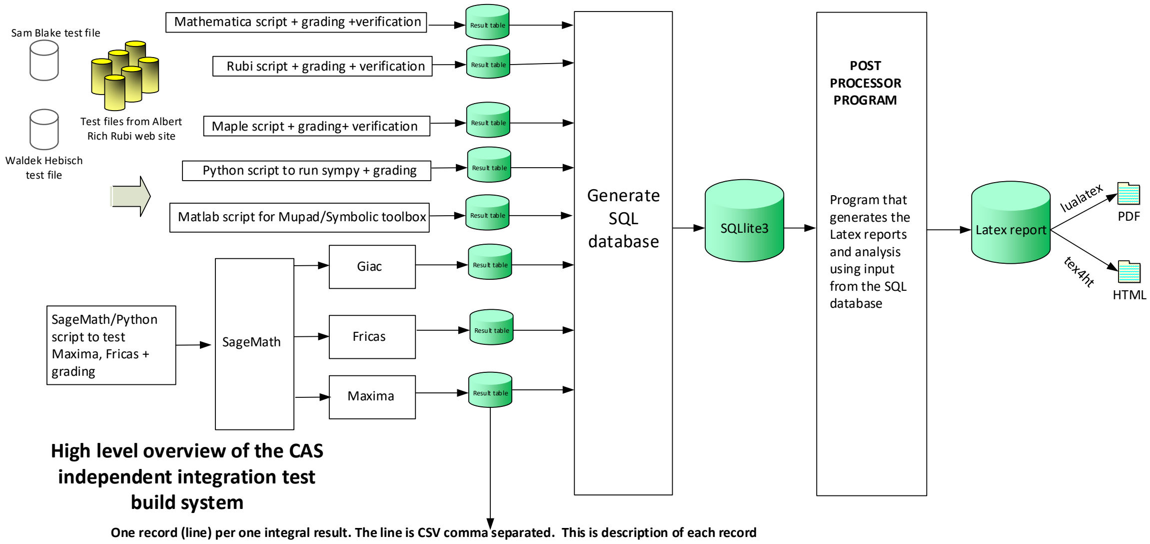
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 102, 103, 109, 110, 116, 117, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 148, 149, 150, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 288, 289, 290, 291, 292, 293, 294, 295, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 389, 390, 391, 392, 393, 394, 395, 396, 397, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 13, 15, 16, 18, 19, 21, 22, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 37, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 91, 92, 93, 97, 102, 103, 109, 110, 116, 117, 124, 126, 128, 129, 130, 132, 134, 135, 136, 138, 139, 140, 148, 149, 150, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 194, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 235, 236, 237, 240, 241, 242, 243, 244, 245, 246, 248, 249, 250, 251, 252, 253, 257, 258, 261, 262, 263, 264, 265, 268, 269, 270, 271, 272, 273, 274, 288, 289, 290, 291, 292, 293, 294, 295, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 371, 372, 375, 378, 379, 380, 389, 391, 392, 393, 394, 395, 396, 397, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 412, 413, 414, 415, 416, 417, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 454, 455, 456, 457, 458, 459, 461, 462, 463, 464, 465, 466, 467, 469, 471, 472, 474, 475, 476, 477, 478, 481, 482, 484, 485, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 506, 507, 508, 510, 512, 513, 514, 515, 522, 523, 527, 534, 535, 541, 547, 548, 549, 559, 585, 586, 587, 606, 607, 608, 611, 612, 613, 616, 617, 618, 619, 624, 628, 629, 630, 631, 632, 633, 638, 639 }

B grade { 45, 80, 94, 95, 96, 98, 99, 101, 158, 159, 174, 175, 277, 298, 299, 376, 390, 411, 418, 419, 432, 453, 460, 473, 483, 486, 487, 488, 500, 516, 517, 528, 530, 531, 620, 621, 622, 623, 636, 637 }

C grade { 9, 11, 14, 17, 20, 23, 24, 36, 38, 89, 90, 100, 131, 133, 137, 191, 192, 193, 196, 197, 233, 234, 238, 239, 247, 254, 255, 256, 259, 260, 266, 267, 321, 322, 335, 336, 337, 347, 433, 434, 435, 468, 470, 479, 480, 501, 502, 509, 511, 518, 519, 520, 521, 524, 609, 610, 614, 615, 634 }

F normal fail { 123, 125, 127, 370, 373, 374, 377, 436, 437, 438, 503, 504, 505, 525, 526, 532, 533, 538, 539, 540, 553, 554, 555, 556, 557, 558, 562, 563, 564, 565, 568, 569, 575, 576, 591, 592, 593 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 77, 78, 79, 91, 92, 93, 123, 124, 125, 126, 127, 128, 129, 130, 169, 172, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 268, 269, 270, 288, 289, 290, 310, 311, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 350, 358, 359, 378, 379, 380, 389, 390, 392, 393, 395, 396, 397, 403, 424, 426, 445, 466, 492, 630, 631, 632, 639 }

B grade { 6, 41, 170, 171, 176, 313, 391, 394, 511, 633 }

C grade { 19, 67, 73, 81, 82, 83, 84, 85, 87, 88, 89, 90, 102, 103, 109, 110, 116, 117, 132, 133, 135, 136, 137, 138, 139, 140, 148, 149, 150, 166, 173, 174, 175, 195, 233, 234, 235, 236, 237, 238, 239, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 271, 273, 274, 291, 293, 294, 295, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 357, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 619, 620, 624, 629, 634 }

F normal fail { 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 74, 75, 76, 80, 86, 94, 95, 96, 97, 131, 134, 160, 163, 164, 165, 167, 168, 206, 207, 208, 209, 210, 212, 213, 214, 215, 264, 265, 266, 267, 272, 292, 353, 354, 355, 356, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 489, 490, 491, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 621, 622, 623, 628, 636, 637, 638 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 64, 65, 66, 67, 68, 69, 73, 77, 78, 79, 91, 92, 93, 102, 103, 109, 110, 116, 117, 123, 124, 125, 126, 127, 128, 129, 130, 138, 139, 140, 148, 149, 150, 163, 164, 165, 167, 168, 169, 172, 173, 177, 178, 179, 181, 184, 185, 186, 187, 189, 194, 198, 199, 200, 202, 268, 269, 270, 288, 289, 290, 310, 311, 312, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 329, 330, 331, 332, 333, 334, 335, 336, 337, 350, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 378, 379, 380, 389, 390, 391, 395, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 415, 416, 421, 422, 423, 424, 426, 427, 428, 433, 434, 435, 440, 441, 442, 443, 444, 445, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 463, 464, 465, 466, 468, 469, 470, 471, 472, 481, 482, 489, 490, 491, 492, 494, 495, 496, 501, 502, 507, 508, 509, 510, 513, 514, 515, 519, 520, 527, 619, 620, 623, 624, 630, 631, 632, 639 }

B grade { 170, 171, 176, 182, 183, 190, 203, 204, 392, 417, 439, 506, 511, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 621, 622, 633 }

C grade { 191, 192, 193, 196, 197, 629, 634 }

F normal fail { 6, 19, 31, 41, 50, 55, 56, 57, 58, 59, 60, 61, 62, 63, 70, 71, 72, 74, 75, 76, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 131, 132, 133, 134, 135, 136, 137, 160, 166, 174, 175, 180, 188, 195, 201, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 271, 272, 273, 274, 291, 292, 293, 294, 295, 313, 314, 326, 327, 328, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 370, 371, 372, 373, 374, 375, 376, 377, 393, 394, 396, 397, 399, 404, 411, 412, 413, 414, 418, 419, 420, 425, 429, 430, 431, 432, 436, 437, 438, 446, 453, 454, 455, 460, 461, 462, 467, 473, 474, 475, 476, 477, 478, 479, 480, 483, 484, 493, 497, 498, 499, 500, 503, 504, 505, 512, 516, 517, 518, 521, 522, 523, 524, 525, 526, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 616, 617, 618, 628, 636, 637, 638 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 46, 47, 48, 49, 51, 52, 53, 54, 64, 65, 66, 68, 69, 77, 78, 79, 80, 81, 82, 83, 91, 92, 93, 95, 96, 97, 129, 130, 132, 163, 164, 165, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 223, 224, 225, 243, 244, 245, 246, 264, 310, 311, 312, 315, 316, 317, 323, 324, 325, 329, 330, 331, 341, 342, 350, 351, 352, 358, 371, 372, 375, 376, 393, 396, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 415, 416, 417, 421, 422, 423, 424, 426, 427, 428, 433, 434, 435, 440, 441, 442, 443, 444, 445, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 463, 465, 469, 471, 472, 481, 482, 489, 490, 491, 492, 494, 495, 496, 501, 502, 506, 507, 508, 510, 514, 519, 520, 527, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 631, 632, 639 }

B grade { 6, 19, 31, 41, 45, 50, 94, 180, 222, 340, 378, 379, 390, 391, 394, 404, 425, 439, 446, 467, 493, 512 }

C grade { 263 }

F normal fail { 55, 56, 57, 58, 59, 60, 61, 62, 63, 67, 70, 71, 72, 73, 74, 75, 76, 84, 85, 86, 87, 88, 89, 90, 102, 103, 109, 110, 116, 117, 123, 124, 125, 126, 127, 128, 131, 138, 139, 140, 148, 149, 150, 160, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 188, 195, 206, 207, 208, 209, 210, 212, 213, 214, 215, 219, 220, 221, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 265, 266, 267, 271, 291, 313, 314, 326, 327, 328, 338, 339, 345, 348, 349, 357, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 373, 374, 377, 380, 389, 392, 395, 397, 399, 411, 412, 413, 414, 418, 419, 420, 429, 430, 431, 432, 436, 437, 438, 453, 454, 455, 460, 461, 462, 473, 474, 475, 483, 484, 497, 498, 499, 500, 503, 504, 505, 516, 517, 518, 524, 525, 526, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 616, 617, 618, 619, 620, 621, 622, 623, 624, 628, 629, 634, 636, 637, 638 }

F(-1) timedout fail { 600 }

F(-2) exception fail { 100, 133, 134, 135, 136, 137, 268, 269, 270, 272, 273, 274, 276, 277, 279, 288, 289, 290, 292, 293, 294, 295, 297, 298, 299, 301, 318, 319, 320, 321, 322, 332, 333, 334, 335, 336, 337, 343, 344, 346, 347, 353, 354, 355, 356, 381, 382, 383, 384, 385, 386, 387, 388, 464, 466, 468, 470, 476, 477, 478, 479, 480, 485, 486, 487, 488, 509, 511, 513, 515, 521, 522, 523, 528, 529, 530, 531, 630, 633 }

Giac

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 11, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 36, 37, 38, 39, 40, 42, 43, 44, 54, 77, 78, 79, 92, 93, 102, 103, 110, 123, 124, 125, 126, 127, 128, 129, 130, 138, 139, 140, 150, 178, 179, 181, 182, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 196, 197, 202, 268, 269, 270, 288, 289, 290, 312, 318, 319, 320, 321, 322, 332, 333, 334, 335, 336, 337, 350, 403, 410, 421, 423, 424, 433, 434, 452, 464, 465, 466, 468, 469, 470, 472, 482, 489, 490, 491, 492, 494, 495, 496, 501, 508, 509, 510, 511, 513, 514, 515, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 629, 630, 631 }

B grade { 10, 12, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 45, 46, 47, 48, 49, 51, 52, 53, 91, 109, 116, 117, 148, 149, 176, 177, 183, 198, 199, 200, 203, 204, 310, 311, 315, 316, 317, 323, 324, 325, 329, 330, 331, 390, 393, 400, 401, 402, 405, 406, 407, 408, 409, 415, 416, 417, 422, 426, 427, 428, 435, 439, 440, 441, 442, 443, 444, 445, 447, 448, 449, 450, 451, 456, 457, 458, 459, 463, 471, 481, 502, 506, 507, 632, 633, 634, 639 }

C grade { }

F normal fail { 6, 19, 41, 50, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 131, 132, 133, 134, 135, 136, 137, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 180, 188, 195, 201, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 271, 272, 273, 274, 291, 292, 293, 294, 295, 313, 314, 326, 327, 328, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 389, 391, 392, 394, 395, 396, 397, 399, 404, 411, 412, 413, 414, 418, 419, 420, 425, 429, 430, 431, 432, 436, 437, 438, 446, 453, 454, 455, 460, 461, 462, 467, 473, 474, 475, 476, 477, 478, 479, 480, 483, 484, 493, 497, 498, 499, 500, 503, 504, 505, 512, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 616, 617, 618, 619, 620, 621, 622, 623, 624, 628, 636, 637, 638 }

F(-1) timedout fail { }

F(-2) exception fail { 211 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 77, 78, 79, 91, 92, 93, 124, 126, 128, 129, 130, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 202, 203, 204, 268, 269, 270, 288, 289, 290, 310, 311, 312, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 329, 330, 331, 332, 333, 334, 335, 336, 337, 350, 379, 390, 391, 393, 394, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 415, 416, 417, 421, 422, 423, 424, 426, 427, 428, 433, 434, 435, 439, 440, 441, 442, 443, 444, 445, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 463, 465, 466, 469, 471, 472, 481, 482, 489, 490, 491, 492, 494, 495, 496, 501, 502, 506, 507, 508, 510, 511, 514, 519, 520, 527, 629, 630, 631, 632, 633, 634, 639 }

C grade { }

F normal fail { }

F(-1) timedout fail { 6, 19, 31, 41, 50, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 102, 103, 109, 110, 116, 117, 123, 125, 127, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 148, 149, 150, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 180, 188, 195, 201, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 271, 272, 273, 274, 291, 292, 293, 294, 295, 313, 314, 326, 327, 328, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 380, 389, 392, 395, 396, 397, 399, 404, 411, 412, 413, 414, 418, 419, 420, 425, 429, 430, 431, 432, 436, 437, 438, 446, 453, 454, 455, 460, 461, 462, 464, 467, 468, 470, 473, 474, 475, 476, 477, 478, 479, 480, 483, 484, 493, 497, 498, 499, 500, 503, 504, 505, 509, 512, 513, 515, 516, 517, 518, 521, 522, 523, 524, 525, 526, 532, 533, 534, 535, 538, 539, 540, 541, 547, 548, 549, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 568, 569, 575, 576, 585, 586, 587, 591, 592, 593, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 628, 636, 637, 638 }

F(-2) exception fail { }

Sympy

A grade $\{$ 1, 2, 4, 8, 10, 12, 13, 16, 17, 18, 20, 22, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 37, 39, 43, 46, 47, 48, 49, 56, 58, 59, 77, 78, 79, 91, 92, 93, 124, 126, 128, 129, 130, 160, 178, 179, 191, 192, 193, 194, 200, 268, 289, 290, 311, 312, 332, 400, 401, 402, 403, 421, 422, 423, 424, 442, 443, 444, 445, 465, 466, 490, 491, 492, 511, 639 $\}$

B grade $\{$ 3, 5, 7, 9, 11, 36, 38, 40, 42, 44, 51, 52, 54, 176, 177, 181, 182, 183, 184, 185, 186, 187, 198, 199, 202, 269, 270, 315, 318, 319, 320, 321, 322, 325, 333, 334, 335, 336, 405, 426, 630, 631, 632 $\}$

C grade $\{$ 45, 70, 71, 72, 74, 75, 76, 169, 170, 171, 212, 213, 214, 215 $\}$

F normal fail $\{$ 6, 19, 31, 41, 50, 57, 61, 62, 63, 64, 65, 66, 67, 73, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 102, 103, 109, 110, 116, 117, 123, 125, 127, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 148, 149, 150, 163, 164, 165, 166, 167, 168, 172, 173, 174, 175, 180, 188, 201, 205, 209, 219, 220, 221, 222, 223, 225, 228, 229, 241, 242, 243, 244, 246, 250, 262, 263, 271, 273, 274, 293, 294, 295, 313, 314, 326, 327, 328, 339, 340, 344, 345, 346, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 390, 391, 392, 393, 394, 395, 396, 397, 399, 404, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 425, 429, 430, 431, 432, 433, 434, 436, 437, 438, 439, 440, 446, 450, 451, 452, 453, 454, 455, 457, 458, 459, 460, 461, 462, 467, 473, 477, 483, 493, 498, 499, 500, 501, 503, 504, 505, 506, 535, 559, 620, 622, 624, 628, 636, 637, 638 $\}$

F(-1) timedout fail $\{$ 14, 15, 21, 23, 24, 53, 55, 60, 189, 190, 195, 196, 197, 203, 204, 206, 207, 210, 211, 218, 224, 226, 227, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 245, 247, 248, 249, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 264, 265, 266, 267, 272, 276, 279, 287, 288, 291, 292, 296, 297, 300, 301, 305, 309, 310, 316, 317, 323, 324, 329, 330, 331, 337, 338, 341, 342, 343, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 370, 371, 373, 374, 375, 376, 377, 380, 381, 383, 384, 385, 388, 406, 407, 427, 428, 435, 441, 447, 448, 449, 456, 463, 464, 468, 469, 470, 471, 472, 474, 475, 476, 478, 479, 480, 481, 482, 484, 485, 487, 488, 489, 494, 496, 497, 502, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 609, 610, 614, 615, 616, 617, 618, 621, 626, 627, 629, 633, 634 $\}$

F(-2) exception fail $\{$ 68, 69, 208, 372, 378, 379, 382, 387, 389, 495, 606, 607, 608, 611, 612, 613, 619, 623 $\}$

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	74	71	72	188	156	71	62
N.S.	1	1.00	0.92	0.89	0.90	2.35	1.95	0.89	0.78
time (sec)	N/A	0.029	0.032	0.652	0.314	0.318	31.797	0.308	1.234

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	57	55	57	65	97	51
N.S.	1	1.00	1.00	0.97	0.93	0.97	1.10	1.64	0.86
time (sec)	N/A	0.033	0.012	0.382	0.197	0.317	0.812	0.293	1.228

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	62	60	59	152	141	59	50
N.S.	1	1.00	0.94	0.91	0.89	2.30	2.14	0.89	0.76
time (sec)	N/A	0.025	0.017	0.428	0.280	0.341	7.766	0.296	1.239

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	34	37	44	40	51	43	39
N.S.	1	1.00	0.97	1.06	1.26	1.14	1.46	1.23	1.11
time (sec)	N/A	0.018	0.007	0.452	0.205	0.311	0.293	0.275	1.295

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	46	45	107	100	41	37
N.S.	1	1.00	1.00	1.02	1.00	2.38	2.22	0.91	0.82
time (sec)	N/A	0.013	0.009	0.279	0.287	0.330	2.010	0.282	1.233

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	43	110	80	0	0	0	0
N.S.	1	1.00	0.98	2.50	1.82	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.007	0.227	0.199	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	37	36	105	258	40	36
N.S.	1	1.00	1.00	0.84	0.82	2.39	5.86	0.91	0.82
time (sec)	N/A	0.013	0.008	0.305	0.275	0.311	7.755	0.315	1.246

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	45	45	42	44	43	65	58	41
N.S.	1	1.18	1.18	1.11	1.16	1.13	1.71	1.53	1.08
time (sec)	N/A	0.025	0.004	0.246	0.191	0.303	0.900	0.297	1.245

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	49	52	49	135	496	58	46
N.S.	1	1.00	0.82	0.87	0.82	2.25	8.27	0.97	0.77
time (sec)	N/A	0.019	0.004	0.549	0.293	0.302	38.768	0.398	1.230

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	56	54	54	58	83	132	56
N.S.	1	1.00	0.88	0.84	0.84	0.91	1.30	2.06	0.88
time (sec)	N/A	0.034	0.028	0.416	0.205	0.304	2.630	0.305	1.240

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	49	61	62	170	583	71	61
N.S.	1	1.00	0.66	0.82	0.84	2.30	7.88	0.96	0.82
time (sec)	N/A	0.023	0.004	0.941	0.281	0.321	162.331	0.296	1.323

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	75	66	69	71	97	191	68
N.S.	1	1.00	0.96	0.85	0.88	0.91	1.24	2.45	0.87
time (sec)	N/A	0.039	0.021	0.769	0.200	0.298	6.674	0.285	1.267

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	57	55	57	65	97	51
N.S.	1	1.00	1.00	0.97	0.93	0.97	1.10	1.64	0.86
time (sec)	N/A	0.033	0.012	0.784	0.198	0.285	2.421	0.280	1.288

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	69	139	147	161	0	162	157
N.S.	1	1.00	0.43	0.87	0.92	1.01	0.00	1.02	0.99
time (sec)	N/A	0.091	0.004	0.870	0.288	0.301	0.000	0.332	3.705

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	147	136	144	144	0	160	129
N.S.	1	1.00	0.94	0.87	0.92	0.92	0.00	1.02	0.82
time (sec)	N/A	0.085	0.040	0.665	0.286	0.283	0.000	0.365	3.839

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	34	37	44	40	51	43	39
N.S.	1	1.00	0.97	1.06	1.26	1.14	1.46	1.23	1.11
time (sec)	N/A	0.021	0.008	0.513	0.198	0.271	0.636	0.338	1.295

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	53	128	131	150	178	150	121
N.S.	1	1.00	0.36	0.87	0.89	1.02	1.21	1.02	0.82
time (sec)	N/A	0.059	0.004	0.497	0.291	0.317	57.146	0.316	3.609

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	129	122	125	110	165	143	134
N.S.	1	1.00	0.97	0.92	0.94	0.83	1.24	1.08	1.01
time (sec)	N/A	0.054	0.027	0.388	0.279	0.331	24.860	0.297	1.675

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	43	55	80	0	0	0	0
N.S.	1	1.00	0.98	1.25	1.82	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.006	0.322	0.199	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	47	113	119	126	165	137	149
N.S.	1	1.00	0.35	0.85	0.89	0.95	1.24	1.03	1.12
time (sec)	N/A	0.049	0.004	0.437	0.284	0.354	108.289	0.295	1.911

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	134	113	120	150	0	138	115
N.S.	1	1.00	0.96	0.81	0.86	1.08	0.00	0.99	0.83
time (sec)	N/A	0.048	0.024	0.494	0.282	0.331	0.000	0.296	3.803

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	42	44	43	65	58	41
N.S.	1	1.00	1.00	0.93	0.98	0.96	1.44	1.29	0.91
time (sec)	N/A	0.026	0.004	0.405	0.194	0.321	1.802	0.293	1.339

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	49	128	127	138	0	153	125
N.S.	1	1.00	0.32	0.85	0.84	0.91	0.00	1.01	0.83
time (sec)	N/A	0.060	0.004	0.934	0.284	0.357	0.000	0.308	3.346

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	49	128	128	172	0	149	156
N.S.	1	1.00	0.32	0.85	0.85	1.14	0.00	0.99	1.03
time (sec)	N/A	0.060	0.004	1.310	0.283	0.342	0.000	0.319	3.663

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	56	54	54	58	83	132	56
N.S.	1	1.00	0.88	0.84	0.84	0.91	1.30	2.06	0.88
time (sec)	N/A	0.034	0.027	1.084	0.212	0.340	7.053	0.304	1.324

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	85	74	74	89	100	308	77
N.S.	1	1.00	0.96	0.83	0.83	1.00	1.12	3.46	0.87
time (sec)	N/A	0.035	0.037	0.398	0.192	0.295	2.513	0.322	1.277

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	74	63	64	77	87	257	65
N.S.	1	1.00	0.99	0.84	0.85	1.03	1.16	3.43	0.87
time (sec)	N/A	0.027	0.023	0.161	0.197	0.330	1.375	0.332	1.258

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	62	52	51	64	73	210	53
N.S.	1	1.00	1.02	0.85	0.84	1.05	1.20	3.44	0.87
time (sec)	N/A	0.021	0.019	0.114	0.206	0.316	0.841	0.327	1.313

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	40	41	40	50	60	152	41
N.S.	1	1.00	0.85	0.87	0.85	1.06	1.28	3.23	0.87
time (sec)	N/A	0.014	0.014	0.098	0.203	0.363	0.519	0.311	1.293

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	37	28	27	33	36	96	27
N.S.	1	1.00	1.37	1.04	1.00	1.22	1.33	3.56	1.00
time (sec)	N/A	0.006	0.003	0.072	0.204	0.360	0.290	0.337	1.287

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	41	67	83	0	0	152	0
N.S.	1	1.00	1.02	1.68	2.08	0.00	0.00	3.80	0.00
time (sec)	N/A	0.024	0.004	0.117	0.199	0.000	0.000	0.408	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	37	50	36	37	63	40
N.S.	1	1.00	1.00	1.23	1.67	1.20	1.23	2.10	1.33
time (sec)	N/A	0.015	0.006	0.269	0.193	0.309	0.464	0.306	1.772

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	63	63	55	61	150	53
N.S.	1	1.00	1.00	1.07	1.07	0.93	1.03	2.54	0.90
time (sec)	N/A	0.023	0.012	0.140	0.186	0.312	0.800	0.308	1.420

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	75	74	66	75	234	65
N.S.	1	1.00	1.00	1.03	1.01	0.90	1.03	3.21	0.89
time (sec)	N/A	0.033	0.016	0.185	0.189	0.340	1.033	0.320	1.475

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	85	85	79	88	317	78
N.S.	1	1.00	1.00	0.98	0.98	0.91	1.01	3.64	0.90
time (sec)	N/A	0.039	0.018	0.247	0.196	0.332	1.523	0.308	1.506

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	49	60	59	178	148	75	56
N.S.	1	1.00	0.68	0.83	0.82	2.47	2.06	1.04	0.78
time (sec)	N/A	0.026	0.006	0.670	0.274	0.309	32.991	0.306	1.366

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	56	46	44	56	66	59	45
N.S.	1	1.00	1.10	0.90	0.86	1.10	1.29	1.16	0.88
time (sec)	N/A	0.024	0.015	0.253	0.192	0.287	1.281	0.331	1.322

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	47	49	48	141	133	63	44
N.S.	1	1.00	0.81	0.84	0.83	2.43	2.29	1.09	0.76
time (sec)	N/A	0.018	0.003	0.265	0.273	0.301	11.200	0.299	1.335

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	45	34	33	42	53	47	33
N.S.	1	1.00	1.22	0.92	0.89	1.14	1.43	1.27	0.89
time (sec)	N/A	0.010	0.004	0.125	0.194	0.324	0.593	0.293	1.339

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	43	34	33	107	95	42	33
N.S.	1	1.00	1.05	0.83	0.80	2.61	2.32	1.02	0.80
time (sec)	N/A	0.010	0.007	0.128	0.277	0.345	3.582	0.299	0.097

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	45	126	89	0	0	0	0
N.S.	1	1.00	1.02	2.86	2.02	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.004	0.098	0.189	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	52	52	49	119	97	54	42
N.S.	1	1.00	1.04	1.04	0.98	2.38	1.94	1.08	0.84
time (sec)	N/A	0.019	0.011	0.196	0.282	0.306	8.310	0.304	1.439

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	34	37	54	41	53	57	47
N.S.	1	1.00	0.97	1.06	1.54	1.17	1.51	1.63	1.34
time (sec)	N/A	0.020	0.007	0.410	0.208	0.269	0.802	0.298	1.387

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	70	61	62	154	138	73	55
N.S.	1	1.00	1.03	0.90	0.91	2.26	2.03	1.07	0.81
time (sec)	N/A	0.024	0.017	0.434	0.281	0.353	24.503	0.371	1.338

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	34	9	35	11	8	110	8
N.S.	1	1.00	4.25	1.12	4.38	1.38	1.00	13.75	1.00
time (sec)	N/A	0.005	0.006	0.286	0.200	0.317	1.517	0.446	1.445

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	133	121	120	129	146	339	121
N.S.	1	1.00	0.87	0.79	0.78	0.84	0.95	2.22	0.79
time (sec)	N/A	0.081	0.084	0.519	0.195	0.324	12.254	0.301	1.382

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	112	99	98	105	119	255	97
N.S.	1	1.00	0.91	0.80	0.80	0.85	0.97	2.07	0.79
time (sec)	N/A	0.057	0.038	0.395	0.204	0.333	3.255	0.283	1.331

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	88	77	76	80	92	171	73
N.S.	1	1.00	0.95	0.83	0.82	0.86	0.99	1.84	0.78
time (sec)	N/A	0.040	0.025	0.357	0.210	0.301	1.026	0.345	1.404

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	52	50	51	61	97	47
N.S.	1	1.00	1.00	0.98	0.94	0.96	1.15	1.83	0.89
time (sec)	N/A	0.020	0.018	0.254	0.196	0.295	0.507	0.294	0.125

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	47	58	79	0	0	0	0
N.S.	1	1.00	1.02	1.26	1.72	0.00	0.00	0.00	0.00
time (sec)	N/A	0.028	0.004	0.358	0.200	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	55	54	53	55	352	132	49
N.S.	1	1.00	0.87	0.86	0.84	0.87	5.59	2.10	0.78
time (sec)	N/A	0.031	0.029	0.252	0.201	0.325	10.997	0.308	1.749

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	90	77	76	84	435	232	72
N.S.	1	1.00	0.90	0.77	0.76	0.84	4.35	2.32	0.72
time (sec)	N/A	0.040	0.032	0.230	0.201	0.343	90.875	0.305	1.571

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	114	99	98	109	0	324	97
N.S.	1	1.00	0.88	0.76	0.75	0.84	0.00	2.49	0.75
time (sec)	N/A	0.053	0.042	0.259	0.197	0.341	0.000	0.307	1.746

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	33	32	31	28	156	31	33
N.S.	1	1.00	1.03	1.00	0.97	0.88	4.88	0.97	1.03
time (sec)	N/A	0.012	0.009	0.210	0.194	0.297	0.240	0.283	1.458

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	70	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	0.021	0.000	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	70	0	0	0	377	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	4.65	0.00	0.00
time (sec)	N/A	0.029	0.021	0.000	0.000	0.000	29.453	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	56	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.021	0.021	0.000	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	0	231	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	3.45	0.00	0.00
time (sec)	N/A	0.028	0.014	0.000	0.000	0.000	9.209	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	76	0	0	0	366	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	4.46	0.00	0.00
time (sec)	N/A	0.037	0.022	0.000	0.000	0.000	28.763	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	76	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	0.022	0.000	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	76	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.034	0.026	0.000	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	77	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	0.028	0.000	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	77	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	0.027	0.000	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	92	0	115	112	0	0	0
N.S.	1	1.00	0.65	0.00	0.82	0.79	0.00	0.00	0.00
time (sec)	N/A	0.050	0.057	0.000	0.229	0.291	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	112	112	74	0	95	92	0	0	0
N.S.	1	1.00	0.66	0.00	0.85	0.82	0.00	0.00	0.00
time (sec)	N/A	0.049	0.032	0.000	0.203	0.304	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	48	0	70	57	0	0	0
N.S.	1	1.00	0.70	0.00	1.01	0.83	0.00	0.00	0.00
time (sec)	N/A	0.028	0.024	0.000	0.203	0.316	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	50	50	46	182	0	63	0	0	0
N.S.	1	1.00	0.92	3.64	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.032	0.007	1.278	0.000	0.311	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	57	0	71	75	0	0	0
N.S.	1	1.00	0.71	0.00	0.89	0.94	0.00	0.00	0.00
time (sec)	N/A	0.025	0.014	0.000	0.214	0.310	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	76	0	99	104	0	0	0
N.S.	1	1.00	0.63	0.00	0.82	0.87	0.00	0.00	0.00
time (sec)	N/A	0.037	0.034	0.000	0.196	0.302	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	61	0	0	0	128	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	1.97	0.00	0.00
time (sec)	N/A	0.019	0.024	0.000	0.000	0.000	5.885	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	61	0	0	0	128	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	1.97	0.00	0.00
time (sec)	N/A	0.019	0.024	0.000	0.000	0.000	2.884	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	52	0	0	0	76	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	1.41	0.00	0.00
time (sec)	N/A	0.014	0.022	0.000	0.000	0.000	1.515	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	44	44	43	170	0	60	0	0	0
N.S.	1	1.00	0.98	3.86	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.028	0.001	1.089	0.000	0.342	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	59	0	0	0	73	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	1.11	0.00	0.00
time (sec)	N/A	0.023	0.027	0.000	0.000	0.000	3.458	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	62	0	0	0	78	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	1.08	0.00	0.00
time (sec)	N/A	0.023	0.022	0.000	0.000	0.000	6.767	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	62	0	0	0	78	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	1.11	0.00	0.00
time (sec)	N/A	0.022	0.021	0.000	0.000	0.000	14.169	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	127	190	145	189	182	370	126
N.S.	1	1.00	0.59	0.88	0.67	0.88	0.85	1.72	0.59
time (sec)	N/A	0.195	0.067	0.910	0.213	0.312	3.223	0.296	1.406

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	105	151	120	148	139	216	100
N.S.	1	1.00	0.72	1.04	0.83	1.02	0.96	1.49	0.69
time (sec)	N/A	0.111	0.040	1.333	0.202	0.358	1.272	0.315	1.424

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	63	105	97	96	90	96	70
N.S.	1	1.00	1.03	1.72	1.59	1.57	1.48	1.57	1.15
time (sec)	N/A	0.037	0.007	0.651	0.195	0.311	0.481	0.296	1.298

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	163	0	118	0	0	0	0
N.S.	1	1.00	2.26	0.00	1.64	0.00	0.00	0.00	0.00
time (sec)	N/A	0.076	0.083	0.000	0.210	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	80	80	93	481	118	0	0	0	0
N.S.	1	1.00	1.16	6.01	1.48	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	0.019	0.383	0.202	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	129	129	141	554	142	0	0	0	0
N.S.	1	1.00	1.09	4.29	1.10	0.00	0.00	0.00	0.00
time (sec)	N/A	0.157	0.057	0.641	0.199	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	193	193	190	607	173	0	0	0	0
N.S.	1	1.00	0.98	3.15	0.90	0.00	0.00	0.00	0.00
time (sec)	N/A	0.241	0.057	1.303	0.200	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	336	336	248	612	0	0	0	0	0
N.S.	1	1.00	0.74	1.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.256	0.129	0.864	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	294	294	223	565	0	0	0	0	0
N.S.	1	1.00	0.76	1.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.212	0.087	0.591	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	237	237	193	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.169	0.062	0.000	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	190	190	173	446	0	0	0	0	0
N.S.	1	1.00	0.91	2.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.114	0.042	0.490	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	254	254	207	522	0	0	0	0	0
N.S.	1	1.00	0.81	2.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.202	0.066	0.556	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	296	296	294	568	0	0	0	0	0
N.S.	1	1.00	0.99	1.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.216	0.124	1.129	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	338	338	353	619	0	0	0	0	0
N.S.	1	1.00	1.04	1.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.252	0.150	1.924	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	178	289	239	359	289	662	187
N.S.	1	1.00	0.53	0.87	0.72	1.07	0.87	1.98	0.56
time (sec)	N/A	0.245	0.138	1.589	0.202	0.308	5.000	0.318	1.429

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	146	223	203	275	223	385	144
N.S.	1	1.00	0.69	1.06	0.96	1.30	1.06	1.82	0.68
time (sec)	N/A	0.143	0.083	11.034	0.201	0.325	2.075	0.292	1.449

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	87	151	164	176	143	169	103
N.S.	1	1.00	0.94	1.62	1.76	1.89	1.54	1.82	1.11
time (sec)	N/A	0.046	0.010	1.124	0.196	0.352	0.791	0.300	1.301

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	279	0	217	0	0	0	0
N.S.	1	1.00	2.63	0.00	2.05	0.00	0.00	0.00	0.00
time (sec)	N/A	0.104	0.106	0.000	0.212	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	302	0	202	0	0	0	0
N.S.	1	1.00	2.54	0.00	1.70	0.00	0.00	0.00	0.00
time (sec)	N/A	0.091	0.245	0.000	0.270	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	219	219	477	0	270	0	0	0	0
N.S.	1	1.00	2.18	0.00	1.23	0.00	0.00	0.00	0.00
time (sec)	N/A	0.248	0.280	0.000	0.278	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	352	352	571	0	338	0	0	0	0
N.S.	1	1.00	1.62	0.00	0.96	0.00	0.00	0.00	0.00
time (sec)	N/A	0.443	0.342	0.000	0.292	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	909	18	123	20	17	20	20
N.S.	1	1.00	50.50	1.00	6.83	1.11	0.94	1.11	1.11
time (sec)	N/A	0.532	3.127	0.039	1.121	0.297	3.881	0.379	1.275

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	789	14	111	16	14	16	16
N.S.	1	1.00	56.36	1.00	7.93	1.14	1.00	1.14	1.14
time (sec)	N/A	0.281	2.667	0.035	0.850	0.324	1.871	0.353	1.305

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	C	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	505	18	0	20	17	20	20
N.S.	1	1.00	28.06	1.00	0.00	1.11	0.94	1.11	1.11
time (sec)	N/A	0.031	1.004	0.042	0.000	0.298	2.953	0.348	1.363

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	851	18	117	20	17	20	20
N.S.	1	1.00	47.28	1.00	6.50	1.11	0.94	1.11	1.11
time (sec)	N/A	0.223	2.195	0.040	0.958	0.291	4.762	0.348	1.392

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	107	107	96	547	0	68	0	69	0
N.S.	1	1.00	0.90	5.11	0.00	0.64	0.00	0.64	0.00
time (sec)	N/A	0.100	0.093	0.878	0.000	0.290	0.000	0.289	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	51	51	51	272	0	29	0	31	0
N.S.	1	1.00	1.00	5.33	0.00	0.57	0.00	0.61	0.00
time (sec)	N/A	0.035	0.042	1.951	0.000	0.315	0.000	0.307	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	15	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.83	1.11	1.11
time (sec)	N/A	0.011	0.131	0.017	0.234	0.333	1.873	0.373	1.301

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.11
time (sec)	N/A	0.012	0.246	0.018	0.241	0.295	4.348	0.282	1.318

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	15	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.83	1.11	1.11
time (sec)	N/A	0.012	0.211	0.039	0.238	0.294	1.569	0.360	1.400

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	14	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.14	1.00	1.14	1.14
time (sec)	N/A	0.003	0.009	0.032	0.227	0.315	1.119	0.289	1.313

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.11
time (sec)	N/A	0.012	0.268	0.028	0.243	0.283	3.217	0.307	1.276

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	138	138	157	1487	0	141	0	313	0
N.S.	1	1.00	1.14	10.78	0.00	1.02	0.00	2.27	0.00
time (sec)	N/A	0.132	0.075	0.846	0.000	0.314	0.000	0.296	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	83	83	97	421	0	78	0	141	0
N.S.	1	1.00	1.17	5.07	0.00	0.94	0.00	1.70	0.00
time (sec)	N/A	0.048	0.034	1.772	0.000	0.321	0.000	0.303	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	69	20	17	20	20
N.S.	1	1.00	1.11	1.00	3.83	1.11	0.94	1.11	1.11
time (sec)	N/A	0.011	0.149	0.016	0.254	0.299	3.113	0.331	1.317

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	78	20	19	20	20
N.S.	1	1.00	1.11	1.00	4.33	1.11	1.06	1.11	1.11
time (sec)	N/A	0.011	0.825	0.017	0.238	0.321	5.814	0.310	1.353

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	66	20	17	20	20
N.S.	1	1.00	1.11	1.00	3.67	1.11	0.94	1.11	1.11
time (sec)	N/A	0.011	0.229	0.004	0.239	0.322	2.080	0.306	1.328

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	73	16	15	16	16
N.S.	1	1.00	1.14	1.00	5.21	1.14	1.07	1.14	1.14
time (sec)	N/A	0.003	0.177	0.006	0.233	0.334	1.890	0.320	1.299

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	79	20	19	20	20
N.S.	1	1.00	1.11	1.00	4.39	1.11	1.06	1.11	1.11
time (sec)	N/A	0.011	0.683	0.005	0.236	0.347	4.333	0.298	1.356

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	204	204	185	1969	0	270	0	874	0
N.S.	1	1.00	0.91	9.65	0.00	1.32	0.00	4.28	0.00
time (sec)	N/A	0.182	0.101	0.899	0.000	0.342	0.000	0.378	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	114	114	113	716	0	157	0	406	0
N.S.	1	1.00	0.99	6.28	0.00	1.38	0.00	3.56	0.00
time (sec)	N/A	0.060	0.037	1.954	0.000	0.329	0.000	0.340	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	161	20	17	20	20
N.S.	1	1.00	1.11	1.00	8.94	1.11	0.94	1.11	1.11
time (sec)	N/A	0.011	0.244	0.017	0.236	0.337	4.278	0.306	1.269

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	184	20	19	20	20
N.S.	1	1.00	1.11	1.00	10.22	1.11	1.06	1.11	1.11
time (sec)	N/A	0.011	1.924	0.032	0.233	0.314	7.812	0.309	1.306

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	180	20	17	20	20
N.S.	1	1.00	1.11	1.00	10.00	1.11	0.94	1.11	1.11
time (sec)	N/A	0.011	0.332	0.003	0.244	0.353	3.172	0.302	1.280

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	164	16	15	16	16
N.S.	1	1.00	1.14	1.00	11.71	1.14	1.07	1.14	1.14
time (sec)	N/A	0.003	0.232	0.021	0.233	0.298	2.667	0.302	1.216

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	187	20	19	20	20
N.S.	1	1.00	1.11	1.00	10.39	1.11	1.06	1.11	1.11
time (sec)	N/A	0.011	1.274	0.003	0.240	0.291	5.814	0.300	1.238

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	A	F	A	F	A	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	0	43	0	54	0	44	0
N.S.	1	1.00	0.00	0.96	0.00	1.20	0.00	0.98	0.00
time (sec)	N/A	0.064	0.000	1.716	0.000	0.338	0.000	0.304	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	23	0	19	27	20	18
N.S.	1	1.00	1.00	1.15	0.00	0.95	1.35	1.00	0.90
time (sec)	N/A	0.018	0.011	0.405	0.000	0.273	0.696	0.298	1.349

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	A	F	A	F	A	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	0	74	0	99	0	100	0
N.S.	1	1.00	0.00	1.04	0.00	1.39	0.00	1.41	0.00
time (sec)	N/A	0.095	0.000	2.012	0.000	0.278	0.000	0.302	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	43	48	0	55	49	47	46
N.S.	1	1.00	0.91	1.02	0.00	1.17	1.04	1.00	0.98
time (sec)	N/A	0.029	0.017	0.395	0.000	0.301	0.717	0.360	1.390

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	A	F	A	F	A	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	0	105	0	142	0	151	0
N.S.	1	1.00	0.00	0.83	0.00	1.12	0.00	1.19	0.00
time (sec)	N/A	0.115	0.000	2.056	0.000	0.270	0.000	0.306	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	55	70	0	79	70	71	74
N.S.	1	1.00	0.75	0.96	0.00	1.08	0.96	0.97	1.01
time (sec)	N/A	0.041	0.018	0.362	0.000	0.280	0.728	0.331	1.476

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	105	151	120	148	136	216	100
N.S.	1	1.00	0.70	1.01	0.80	0.99	0.91	1.44	0.67
time (sec)	N/A	0.106	0.045	4.786	0.222	0.311	3.666	0.308	1.285

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	63	101	97	96	100	96	71
N.S.	1	1.00	0.95	1.53	1.47	1.45	1.52	1.45	1.08
time (sec)	N/A	0.039	0.008	1.050	0.198	0.341	0.995	0.293	1.291

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	2965	0	0	0	0	0	0
N.S.	1	1.00	38.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.084	0.836	0.000	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	86	86	99	411	118	0	0	0	0
N.S.	1	1.00	1.15	4.78	1.37	0.00	0.00	0.00	0.00
time (sec)	N/A	0.073	0.022	0.817	0.194	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	1294	1300	1041	1957	0	0	0	0	0
N.S.	1	1.00	0.80	1.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.293	0.519	0.832	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1304	1310	1101	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.227	0.494	0.000	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	1137	1143	972	1787	0	0	0	0	0
N.S.	1	1.01	0.85	1.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.901	0.345	0.643	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	1170	1176	766	1787	0	0	0	0	0
N.S.	1	1.01	0.65	1.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.924	0.639	0.766	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	1328	1334	912	1954	0	0	0	0	0
N.S.	1	1.00	0.69	1.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.166	1.014	1.174	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	164	164	146	823	0	116	0	108	0
N.S.	1	1.00	0.89	5.02	0.00	0.71	0.00	0.66	0.00
time (sec)	N/A	0.159	0.148	1.033	0.000	0.270	0.000	0.307	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	107	107	96	547	0	68	0	69	0
N.S.	1	1.00	0.90	5.11	0.00	0.64	0.00	0.64	0.00
time (sec)	N/A	0.101	0.075	0.889	0.000	0.273	0.000	0.305	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	51	51	51	272	0	29	0	31	0
N.S.	1	1.00	1.00	5.33	0.00	0.57	0.00	0.61	0.00
time (sec)	N/A	0.042	0.038	1.790	0.000	0.306	0.000	0.322	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	15	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.83	1.11	1.11
time (sec)	N/A	0.011	0.125	0.020	0.257	0.281	9.113	0.307	1.205

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.11
time (sec)	N/A	0.011	0.225	0.022	0.265	0.316	32.744	0.305	1.218

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	15	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.83	1.11	1.11
time (sec)	N/A	0.012	0.172	0.042	0.258	0.317	10.184	0.304	1.189

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.12
time (sec)	N/A	0.007	0.157	0.032	0.259	0.286	4.801	0.403	1.207

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	14	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.14	1.00	1.14	1.14
time (sec)	N/A	0.003	0.010	0.033	0.255	0.296	4.788	0.299	1.173

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.11
time (sec)	N/A	0.011	0.238	0.030	0.259	0.296	18.027	0.299	1.203

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.11
time (sec)	N/A	0.011	0.256	0.031	0.263	0.331	24.277	0.286	1.245

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	195	195	290	2564	0	211	0	487	0
N.S.	1	1.00	1.49	13.15	0.00	1.08	0.00	2.50	0.00
time (sec)	N/A	0.248	0.120	1.135	0.000	0.339	0.000	0.330	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	141	141	157	1487	0	141	0	313	0
N.S.	1	1.00	1.11	10.55	0.00	1.00	0.00	2.22	0.00
time (sec)	N/A	0.137	0.076	0.957	0.000	0.304	0.000	0.313	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	83	83	97	421	0	78	0	141	0
N.S.	1	1.00	1.17	5.07	0.00	0.94	0.00	1.70	0.00
time (sec)	N/A	0.065	0.030	1.841	0.000	0.293	0.000	0.315	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	69	20	17	20	20
N.S.	1	1.00	1.11	1.00	3.83	1.11	0.94	1.11	1.11
time (sec)	N/A	0.016	0.140	0.017	0.265	0.290	16.374	0.289	1.330

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	78	20	19	20	20
N.S.	1	1.00	1.11	1.00	4.33	1.11	1.06	1.11	1.11
time (sec)	N/A	0.015	0.807	0.016	0.276	0.338	39.464	0.297	1.305

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	66	20	17	20	20
N.S.	1	1.00	1.11	1.00	3.67	1.11	0.94	1.11	1.11
time (sec)	N/A	0.012	0.207	0.003	0.263	0.311	15.323	0.278	1.240

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	74	18	15	18	18
N.S.	1	1.00	1.12	1.00	4.62	1.12	0.94	1.12	1.12
time (sec)	N/A	0.007	0.284	0.003	0.256	0.335	9.074	0.351	1.222

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	77	16	15	16	16
N.S.	1	1.00	1.14	1.00	5.50	1.14	1.07	1.14	1.14
time (sec)	N/A	0.003	0.173	0.004	0.274	0.290	9.046	0.293	1.184

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	79	20	19	20	20
N.S.	1	1.00	1.11	1.00	4.39	1.11	1.06	1.11	1.11
time (sec)	N/A	0.011	0.706	0.003	0.268	0.307	22.473	0.297	1.228

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	80	20	19	20	20
N.S.	1	1.00	1.11	1.00	4.44	1.11	1.06	1.11	1.11
time (sec)	N/A	0.011	0.668	0.006	0.268	0.277	30.190	0.279	1.230

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	994	20	171	22	19	22	22
N.S.	1	1.00	49.70	1.00	8.55	1.10	0.95	1.10	1.10
time (sec)	N/A	0.071	1.822	0.043	0.411	0.295	82.577	0.365	1.259

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	466	20	126	22	19	22	22
N.S.	1	1.00	23.30	1.00	6.30	1.10	0.95	1.10	1.10
time (sec)	N/A	0.055	0.528	0.040	0.387	0.342	46.406	0.335	1.185

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	70	0	0	0	377	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	4.65	0.00	0.00
time (sec)	N/A	0.028	0.015	0.000	0.000	0.000	28.947	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	1.10
time (sec)	N/A	0.011	0.394	0.033	0.266	0.331	12.991	0.311	1.188

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	97	22	19	22	22
N.S.	1	1.00	1.10	1.00	4.85	1.10	0.95	1.10	1.10
time (sec)	N/A	0.012	0.915	0.007	0.293	0.339	29.330	0.298	1.245

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	372	372	171	0	239	266	0	0	0
N.S.	1	1.00	0.46	0.00	0.64	0.72	0.00	0.00	0.00
time (sec)	N/A	0.219	0.172	0.000	0.206	0.363	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	255	255	140	0	200	204	0	0	0
N.S.	1	1.00	0.55	0.00	0.78	0.80	0.00	0.00	0.00
time (sec)	N/A	0.140	0.144	0.000	0.218	0.328	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	74	0	146	121	0	0	0
N.S.	1	1.00	0.73	0.00	1.45	1.20	0.00	0.00	0.00
time (sec)	N/A	0.057	0.020	0.000	0.215	0.342	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	88	88	168	614	0	0	0	0	0
N.S.	1	1.00	1.91	6.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.079	0.100	2.296	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	124	124	150	0	0	197	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	1.59	0.00	0.00	0.00
time (sec)	N/A	0.077	0.066	0.000	0.000	0.365	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	200	200	288	0	0	279	0	0	0
N.S.	1	1.00	1.44	0.00	0.00	1.40	0.00	0.00	0.00
time (sec)	N/A	0.183	0.248	0.000	0.000	0.329	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	0	12	14	0	0
N.S.	1	1.00	1.00	1.08	0.00	0.92	1.08	0.00	0.00
time (sec)	N/A	0.006	0.006	0.846	0.000	0.295	1.464	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	40	0	40	100	0	0
N.S.	1	1.00	1.00	1.90	0.00	1.90	4.76	0.00	0.00
time (sec)	N/A	0.019	0.003	0.845	0.000	0.328	1.651	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	41	0	41	100	0	0
N.S.	1	1.00	1.00	1.95	0.00	1.95	4.76	0.00	0.00
time (sec)	N/A	0.021	0.004	0.876	0.000	0.316	1.633	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	39	38	0	54	0	0	0
N.S.	1	1.00	0.95	0.93	0.00	1.32	0.00	0.00	0.00
time (sec)	N/A	0.027	0.009	1.247	0.000	0.318	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	44	44	43	170	0	60	0	0	0
N.S.	1	1.00	0.98	3.86	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.027	0.004	0.320	0.000	0.337	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	79	79	164	578	0	0	0	0	0
N.S.	1	1.00	2.08	7.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.068	0.036	2.293	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	113	113	270	1409	0	0	0	0	0
N.S.	1	1.00	2.39	12.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.096	0.105	2.991	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	185	284	214	269	335	573	208
N.S.	1	1.00	1.32	2.03	1.53	1.92	2.39	4.09	1.49
time (sec)	N/A	0.054	0.135	1.364	0.192	0.338	0.932	0.309	1.309

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	121	184	136	172	202	316	131
N.S.	1	1.00	1.08	1.64	1.21	1.54	1.80	2.82	1.17
time (sec)	N/A	0.037	0.073	0.940	0.193	0.308	0.574	0.297	1.309

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	78	77	74	91	105	136	68
N.S.	1	1.00	0.93	0.92	0.88	1.08	1.25	1.62	0.81
time (sec)	N/A	0.023	0.032	0.368	0.191	0.344	0.363	0.311	1.254

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	32	35	32	36	39	29
N.S.	1	1.00	1.00	1.33	1.46	1.33	1.50	1.62	1.21
time (sec)	N/A	0.007	0.005	0.319	0.187	0.334	0.140	0.299	0.076

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	57	98	118	0	0	0	0
N.S.	1	1.00	0.98	1.69	2.03	0.00	0.00	0.00	0.00
time (sec)	N/A	0.034	0.004	1.684	0.198	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	62	66	65	80	236	81	70
N.S.	1	1.00	0.91	0.97	0.96	1.18	3.47	1.19	1.03
time (sec)	N/A	0.020	0.032	1.003	0.211	0.323	1.474	0.305	2.208

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	80	88	120	236	1518	185	96
N.S.	1	1.00	0.76	0.84	1.14	2.25	14.46	1.76	0.91
time (sec)	N/A	0.041	0.051	1.230	0.191	0.324	5.246	0.306	1.645

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	105	111	232	443	4571	365	145
N.S.	1	1.00	0.79	0.83	1.74	3.33	34.37	2.74	1.09
time (sec)	N/A	0.054	0.075	1.781	0.212	0.389	17.519	0.321	1.858

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	249	244	177	498	527	218	222
N.S.	1	1.00	1.40	1.37	0.99	2.80	2.96	1.22	1.25
time (sec)	N/A	0.110	0.507	1.787	0.296	0.368	17.796	0.331	1.438

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	211	187	131	320	374	152	263
N.S.	1	1.00	1.50	1.33	0.93	2.27	2.65	1.08	1.87
time (sec)	N/A	0.085	0.312	1.217	0.286	0.357	8.663	0.443	4.450

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	83	78	80	198	199	85	81
N.S.	1	1.00	0.84	0.79	0.81	2.00	2.01	0.86	0.82
time (sec)	N/A	0.053	0.020	0.485	0.286	0.337	4.322	0.327	2.196

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	46	45	107	100	41	37
N.S.	1	1.00	1.00	1.02	1.00	2.38	2.22	0.91	0.82
time (sec)	N/A	0.016	0.006	0.093	0.279	0.307	2.086	0.309	1.302

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	201	196	0	0	0	0	0
N.S.	1	1.00	1.00	0.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.187	0.058	1.187	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	137	99	108	261	0	121	337
N.S.	1	1.00	1.15	0.83	0.91	2.19	0.00	1.02	2.83
time (sec)	N/A	0.065	0.046	1.289	0.370	0.300	0.000	0.334	2.307

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	217	147	206	744	0	278	272
N.S.	1	1.00	1.25	0.84	1.18	4.28	0.00	1.60	1.56
time (sec)	N/A	0.099	0.358	2.019	0.338	0.358	0.000	0.296	2.044

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	286	398	332	8840	265	381	536
N.S.	1	1.00	0.89	1.24	1.04	27.62	0.83	1.19	1.68
time (sec)	N/A	0.491	0.321	2.322	0.353	9.178	23.691	0.339	2.048

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	231	336	249	5799	173	274	358
N.S.	1	1.00	0.92	1.34	1.00	23.20	0.69	1.10	1.43
time (sec)	N/A	0.304	0.190	1.522	0.350	2.323	15.874	0.369	1.421

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	204	247	187	2284	112	212	210
N.S.	1	1.00	0.89	1.08	0.82	9.97	0.49	0.93	0.92
time (sec)	N/A	0.211	0.046	0.707	0.340	1.132	10.937	0.342	1.314

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	129	122	125	110	165	143	134
N.S.	1	1.00	0.97	0.92	0.94	0.83	1.24	1.08	1.01
time (sec)	N/A	0.056	0.018	0.126	0.372	0.344	24.713	0.309	0.450

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	313	101	0	0	0	0	0
N.S.	1	1.00	1.02	0.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	0.104	1.302	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	254	277	311	7010	0	363	736
N.S.	1	1.00	0.87	0.95	1.07	24.01	0.00	1.24	2.52
time (sec)	N/A	0.359	0.214	1.734	0.316	1.247	0.000	0.363	1.559

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	325	347	517	13236	0	660	2227
N.S.	1	1.00	0.83	0.89	1.32	33.85	0.00	1.69	5.70
time (sec)	N/A	0.436	0.392	2.622	0.318	9.421	0.000	0.465	1.920

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	114	244	166	239	355	847	184
N.S.	1	1.00	0.82	1.76	1.19	1.72	2.55	6.09	1.32
time (sec)	N/A	0.082	0.099	1.042	0.222	0.302	1.656	0.328	1.431

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	86	168	102	153	216	490	111
N.S.	1	1.00	0.84	1.65	1.00	1.50	2.12	4.80	1.09
time (sec)	N/A	0.060	0.053	0.775	0.218	0.322	0.947	0.319	1.305

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	85	65	55	80	112	233	57
N.S.	1	1.00	1.09	0.83	0.71	1.03	1.44	2.99	0.73
time (sec)	N/A	0.039	0.020	0.154	0.207	0.333	0.542	0.454	1.331

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	114	137	159	0	0	0	0
N.S.	1	1.00	1.01	1.21	1.41	0.00	0.00	0.00	0.00
time (sec)	N/A	0.104	0.020	1.584	0.241	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	86	85	148	425	145	85
N.S.	1	1.00	1.00	1.06	1.05	1.83	5.25	1.79	1.05
time (sec)	N/A	0.052	0.041	0.891	0.196	0.380	3.722	0.303	1.525

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	113	120	160	428	0	470	217
N.S.	1	1.00	0.89	0.94	1.26	3.37	0.00	3.70	1.71
time (sec)	N/A	0.081	0.117	1.327	0.197	0.776	0.000	0.320	2.106

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	164	165	299	818	0	1097	662
N.S.	1	1.00	0.94	0.94	1.71	4.67	0.00	6.27	3.78
time (sec)	N/A	0.120	0.169	2.181	0.217	4.286	0.000	0.317	3.001

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	115	114	82	0	0	0	0
N.S.	1	1.00	1.10	1.09	0.78	0.00	0.00	0.00	0.00
time (sec)	N/A	0.116	0.037	2.842	0.213	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	301	301	239	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.482	0.419	0.000	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	205	205	176	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.160	0.153	0.000	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	77	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.032	0.037	0.000	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	123	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.068	0.049	0.000	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	257	257	211	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.320	0.313	0.000	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	100	22	0	0	22
N.S.	1	1.00	1.10	1.00	5.00	1.10	0.00	0.00	1.10
time (sec)	N/A	0.007	0.382	0.210	0.320	0.294	0.000	0.000	1.556

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	234	234	224	0	0	0	515	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	2.20	0.00	0.00
time (sec)	N/A	0.154	0.299	0.000	0.000	0.000	12.912	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	181	181	178	0	0	0	360	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	1.99	0.00	0.00
time (sec)	N/A	0.117	0.157	0.000	0.000	0.000	8.093	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	130	0	0	0	214	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	1.62	0.00	0.00
time (sec)	N/A	0.087	0.075	0.000	0.000	0.000	5.776	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	52	0	0	0	76	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	1.41	0.00	0.00
time (sec)	N/A	0.012	0.016	0.000	0.000	0.000	1.459	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	1.10
time (sec)	N/A	0.008	1.543	0.194	0.342	0.288	3.048	0.340	1.315

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	105	33	19	22	22
N.S.	1	1.00	1.10	1.00	5.25	1.65	0.95	1.10	1.10
time (sec)	N/A	0.009	0.364	0.214	0.351	0.309	43.215	0.324	1.411

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	174	44	0	22	22
N.S.	1	1.00	1.10	1.00	8.70	2.20	0.00	1.10	1.10
time (sec)	N/A	0.008	0.365	0.224	0.359	0.300	0.000	0.316	1.504

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	183	297	0	0	0	0	0
N.S.	1	1.00	0.73	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.155	0.119	1.842	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	127	213	0	0	0	0	0
N.S.	1	1.00	0.80	1.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.107	0.057	1.776	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	79	156	0	0	0	0	0
N.S.	1	1.00	0.87	1.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.074	0.025	1.714	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	57	98	118	0	0	0	0
N.S.	1	1.00	0.98	1.69	2.03	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	0.001	1.546	0.212	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	98	156	123	0	0	0	0
N.S.	1	1.00	1.01	1.61	1.27	0.00	0.00	0.00	0.00
time (sec)	N/A	0.082	0.016	0.757	0.253	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	147	201	156	0	0	0	0
N.S.	1	1.00	1.01	1.38	1.07	0.00	0.00	0.00	0.00
time (sec)	N/A	0.118	0.027	0.908	0.243	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	194	260	216	0	0	0	0
N.S.	1	1.00	0.85	1.15	0.95	0.00	0.00	0.00	0.00
time (sec)	N/A	0.144	0.108	1.092	0.261	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	370	412	0	0	0	0	0
N.S.	1	1.00	0.94	1.05	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.284	0.197	1.446	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	310	348	0	0	0	0	0
N.S.	1	1.00	0.99	1.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.220	0.086	1.295	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	238	263	0	0	0	0	0
N.S.	1	1.00	0.93	1.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.179	0.076	1.224	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	201	196	0	0	0	0	0
N.S.	1	1.00	1.00	0.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.123	0.001	1.014	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	230	311	0	0	0	0	0
N.S.	1	1.00	0.93	1.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.199	0.054	0.809	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	282	354	0	0	0	0	0
N.S.	1	1.00	0.92	1.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.227	0.133	0.941	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	347	410	0	0	0	0	0
N.S.	1	1.00	0.94	1.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	0.128	1.395	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	692	692	581	302	0	0	0	0	0
N.S.	1	1.00	0.84	0.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.504	0.267	1.780	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	643	643	504	250	0	0	0	0	0
N.S.	1	1.00	0.78	0.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.424	0.260	1.529	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	457	457	430	206	0	0	0	0	0
N.S.	1	1.00	0.94	0.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.310	0.169	1.371	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	313	101	0	0	0	0	0
N.S.	1	1.00	1.02	0.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.195	0.001	1.220	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	352	352	327	170	0	0	0	0	0
N.S.	1	1.00	0.93	0.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.293	0.079	1.045	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	510	510	395	292	0	0	0	0	0
N.S.	1	1.00	0.77	0.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.376	0.099	1.191	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	674	674	542	421	0	0	0	0	0
N.S.	1	1.00	0.80	0.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.424	0.255	1.921	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	308	301	0	0	0	0	0
N.S.	1	1.00	1.04	1.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	0.102	1.837	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	229	240	0	0	0	0	0
N.S.	1	1.00	1.05	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.171	0.058	1.706	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	165	188	0	0	0	0	0
N.S.	1	1.00	1.09	1.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.149	0.035	1.601	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	114	137	159	0	0	0	0
N.S.	1	1.00	1.01	1.21	1.41	0.00	0.00	0.00	0.00
time (sec)	N/A	0.100	0.001	1.509	0.195	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	161	209	179	0	0	0	0
N.S.	1	1.00	1.01	1.31	1.13	0.00	0.00	0.00	0.00
time (sec)	N/A	0.161	0.032	0.918	0.264	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	200	264	230	0	0	0	0
N.S.	1	1.00	1.01	1.33	1.16	0.00	0.00	0.00	0.00
time (sec)	N/A	0.195	0.049	1.084	0.240	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	263	345	307	0	0	0	0
N.S.	1	1.00	0.92	1.20	1.07	0.00	0.00	0.00	0.00
time (sec)	N/A	0.232	0.151	1.313	0.252	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	421	421	404	411	0	0	0	0	0
N.S.	1	1.00	0.96	0.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.389	0.164	1.816	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	349	362	0	0	0	0	0
N.S.	1	1.00	0.99	1.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.307	0.106	1.516	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	271	296	0	0	0	0	0
N.S.	1	1.00	0.93	1.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.280	0.110	1.362	0.000	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	242	234	0	0	0	0	0
N.S.	1	1.00	1.00	0.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.218	0.038	1.296	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	289	362	0	0	0	0	0
N.S.	1	1.00	1.01	1.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.295	0.062	1.068	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	327	418	0	0	0	0	0
N.S.	1	1.00	0.92	1.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	0.128	1.339	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	414	384	488	0	0	0	0	0
N.S.	1	1.00	0.93	1.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.369	0.174	1.712	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	714	714	536	301	0	0	0	0	0
N.S.	1	1.00	0.75	0.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.742	0.212	2.471	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	666	666	483	269	0	0	0	0	0
N.S.	1	1.00	0.73	0.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.550	0.132	1.950	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	488	488	403	238	0	0	0	0	0
N.S.	1	1.00	0.83	0.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.474	0.107	1.645	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	344	344	350	142	0	0	0	0	0
N.S.	1	1.00	1.02	0.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.322	0.072	1.566	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	388	388	395	220	0	0	0	0	0
N.S.	1	1.00	1.02	0.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.423	0.088	1.548	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	557	557	463	345	0	0	0	0	0
N.S.	1	1.00	0.83	0.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.533	0.104	1.970	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	737	737	535	489	0	0	0	0	0
N.S.	1	1.00	0.73	0.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.623	0.125	2.659	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	749	749	867	272	0	0	0	0	0
N.S.	1	1.00	1.16	0.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.667	0.503	1.451	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	533	533	564	449	0	0	0	0	0
N.S.	1	1.00	1.06	0.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.377	0.243	1.337	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	229	229	178	392	309	0	0	0	0
N.S.	1	1.00	0.78	1.71	1.35	0.00	0.00	0.00	0.00
time (sec)	N/A	0.156	0.075	0.918	0.330	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	360	360	373	0	377	0	0	0	0
N.S.	1	1.00	1.04	0.00	1.05	0.00	0.00	0.00	0.00
time (sec)	N/A	0.318	0.162	0.000	0.330	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	597	597	706	0	0	0	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.605	0.289	0.000	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	541	541	422	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.603	0.206	0.000	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	561	561	912	0	0	0	0	0	0
N.S.	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.788	0.396	0.000	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	338	215	258	0	596	697	268	298
N.S.	1	1.00	0.64	0.76	0.00	1.76	2.06	0.79	0.88
time (sec)	N/A	0.171	0.165	3.392	0.000	0.300	124.716	0.319	1.440

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	151	167	0	404	478	174	193
N.S.	1	1.00	0.68	0.76	0.00	1.83	2.16	0.79	0.87
time (sec)	N/A	0.105	0.072	1.699	0.000	0.299	33.097	0.309	1.488

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	89	0	220	260	94	97
N.S.	1	1.00	1.00	0.76	0.00	1.88	2.22	0.80	0.83
time (sec)	N/A	0.057	0.026	0.565	0.000	0.312	8.497	0.380	1.521

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	533	533	564	449	0	0	0	0	0
N.S.	1	1.00	1.06	0.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.307	0.071	0.878	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	751	751	877	0	0	0	0	0	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.672	0.870	0.000	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	945	945	435	1077	0	0	0	0	0
N.S.	1	1.00	0.46	1.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.801	0.327	2.216	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	548	548	281	729	0	0	0	0	0
N.S.	1	1.00	0.51	1.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.454	0.175	0.832	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	20	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.83	1.08	1.08
time (sec)	N/A	0.017	2.408	0.270	1.096	0.323	10.868	0.327	1.380

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	0	37	0	26	26
N.S.	1	1.00	1.08	1.00	0.00	1.54	0.00	1.08	1.08
time (sec)	N/A	0.016	6.688	0.237	0.000	0.308	0.000	0.352	1.365

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	1772	22	0	24	20	24	24
N.S.	1	1.00	80.55	1.00	0.00	1.09	0.91	1.09	1.09
time (sec)	N/A	0.917	9.071	0.067	0.000	0.312	10.492	0.378	1.405

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	20	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.83	1.08	1.08
time (sec)	N/A	0.018	3.983	0.238	2.026	0.297	17.201	0.374	1.360

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	0	37	0	26	26
N.S.	1	1.00	1.08	1.00	0.00	1.54	0.00	1.08	1.08
time (sec)	N/A	0.016	6.573	0.267	0.000	0.307	0.000	0.356	1.337

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	35	20	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.46	0.83	1.08	1.08
time (sec)	N/A	0.016	0.260	0.200	0.293	0.293	8.551	0.296	1.347

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	19	24	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.86	1.09	1.09
time (sec)	N/A	0.009	0.155	0.047	0.297	0.290	4.485	0.481	1.396

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	20	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.83	1.08	1.08
time (sec)	N/A	0.019	0.451	0.239	0.271	0.308	15.323	0.338	1.322

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	37	22	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.54	0.92	1.08	1.08
time (sec)	N/A	0.017	1.100	0.247	0.260	0.299	167.169	0.309	1.323

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	148	35	22	26	26
N.S.	1	1.00	1.08	1.00	6.17	1.46	0.92	1.08	1.08
time (sec)	N/A	0.015	0.602	0.004	0.320	0.309	11.740	0.312	1.491

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	101	24	20	24	24
N.S.	1	1.00	1.09	1.00	4.59	1.09	0.91	1.09	1.09
time (sec)	N/A	0.009	0.325	0.004	0.330	0.301	7.321	0.306	1.421

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	157	26	22	26	26
N.S.	1	1.00	1.08	1.00	6.54	1.08	0.92	1.08	1.08
time (sec)	N/A	0.017	2.116	0.023	0.287	0.302	22.769	0.359	1.386

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	214	37	0	26	26
N.S.	1	1.00	1.08	1.00	8.92	1.54	0.00	1.08	1.08
time (sec)	N/A	0.016	4.871	0.003	0.284	0.301	0.000	0.335	1.402

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	258	309	0	708	0	325	316
N.S.	1	1.00	0.70	0.84	0.00	1.93	0.00	0.89	0.86
time (sec)	N/A	0.205	0.161	7.719	0.000	0.311	0.000	0.330	4.726

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	178	195	0	454	440	204	317
N.S.	1	1.00	0.77	0.84	0.00	1.97	1.90	0.88	1.37
time (sec)	N/A	0.120	0.127	2.723	0.000	0.302	120.703	0.309	4.059

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	110	104	0	250	214	98	94
N.S.	1	1.00	1.00	0.95	0.00	2.27	1.95	0.89	0.85
time (sec)	N/A	0.064	0.034	0.714	0.000	0.303	16.504	0.317	2.164

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	1165	1165	990	577	0	0	0	0	0
N.S.	1	1.00	0.85	0.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.152	0.574	1.541	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1861	1863	2168	0	0	0	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.908	6.810	0.000	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	1221	1221	780	1584	0	0	0	0	0
N.S.	1	1.00	0.64	1.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.039	0.760	10.279	0.000	0.000	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	835	835	475	1127	0	0	0	0	0
N.S.	1	1.00	0.57	1.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.703	0.339	3.751	0.000	0.000	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	395	395	398	724	0	0	0	0	0
N.S.	1	1.00	1.01	1.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.336	0.083	1.016	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	0	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.08
time (sec)	N/A	0.019	17.721	0.796	1.127	0.304	0.000	0.331	1.448

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	0	37	0	26	26
N.S.	1	1.00	1.08	1.00	0.00	1.54	0.00	1.08	1.08
time (sec)	N/A	0.017	26.202	0.832	0.000	0.306	0.000	0.342	1.434

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	2385	24	0	35	22	26	26
N.S.	1	1.00	99.38	1.00	0.00	1.46	0.92	1.08	1.08
time (sec)	N/A	1.657	8.700	0.191	0.000	0.294	32.813	0.399	1.566

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	1051	22	0	24	20	24	24
N.S.	1	1.00	47.77	1.00	0.00	1.09	0.91	1.09	1.09
time (sec)	N/A	0.485	1.295	0.063	0.000	0.280	10.498	0.348	1.576

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	0	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.08
time (sec)	N/A	0.018	27.151	0.908	1.610	0.311	0.000	0.341	1.541

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	0	37	0	26	26
N.S.	1	1.00	1.08	1.00	0.00	1.54	0.00	1.08	1.08
time (sec)	N/A	0.017	29.941	0.807	0.000	0.330	0.000	0.367	1.549

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	35	20	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.46	0.83	1.08	1.08
time (sec)	N/A	0.017	0.180	0.181	0.295	0.303	14.233	0.321	1.614

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	19	24	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.86	1.09	1.09
time (sec)	N/A	0.009	0.142	0.050	0.299	0.281	5.656	0.303	1.441

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	20	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.83	1.08	1.08
time (sec)	N/A	0.019	7.204	0.217	0.264	0.301	101.482	0.292	1.684

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	37	0	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.54	0.00	1.08	1.08
time (sec)	N/A	0.017	9.493	0.240	0.266	0.321	0.000	0.488	1.498

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	152	35	22	26	26
N.S.	1	1.00	1.08	1.00	6.33	1.46	0.92	1.08	1.08
time (sec)	N/A	0.017	0.419	0.006	0.316	0.295	18.464	0.296	1.456

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	104	24	20	24	24
N.S.	1	1.00	1.09	1.00	4.73	1.09	0.91	1.09	1.09
time (sec)	N/A	0.009	0.268	0.003	0.313	0.337	8.645	0.291	1.579

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	159	26	22	26	26
N.S.	1	1.00	1.08	1.00	6.62	1.08	0.92	1.08	1.08
time (sec)	N/A	0.018	9.468	0.004	0.279	0.341	139.765	0.323	1.499

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	215	37	0	26	26
N.S.	1	1.00	1.08	1.00	8.96	1.54	0.00	1.08	1.08
time (sec)	N/A	0.024	10.862	0.026	0.298	0.348	0.000	0.352	1.531

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	170	140	132	152	0	399	127
N.S.	1	1.00	1.20	0.99	0.93	1.07	0.00	2.81	0.89
time (sec)	N/A	0.155	0.034	1.589	0.192	0.344	0.000	0.328	1.550

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	140	117	108	128	156	269	103
N.S.	1	1.00	1.18	0.98	0.91	1.08	1.31	2.26	0.87
time (sec)	N/A	0.122	0.022	1.069	0.192	0.337	61.460	0.312	1.505

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	98	127	99	99	126	143	78
N.S.	1	1.00	1.04	1.35	1.05	1.05	1.34	1.52	0.83
time (sec)	N/A	0.063	0.023	0.848	0.193	0.323	15.771	0.316	1.486

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	80	157	0	0	0	0	0
N.S.	1	1.00	0.98	1.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.076	0.018	0.313	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	92	155	0	0	0	0	0
N.S.	1	1.00	0.99	1.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.089	0.022	0.434	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	105	88	77	97	167	209	85
N.S.	1	1.00	1.13	0.95	0.83	1.04	1.80	2.25	0.91
time (sec)	N/A	0.091	0.031	0.481	0.199	0.306	74.453	0.309	1.687

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	134	108	104	129	0	316	113
N.S.	1	1.00	1.07	0.86	0.83	1.03	0.00	2.53	0.90
time (sec)	N/A	0.108	0.050	0.887	0.205	0.316	0.000	0.329	1.724

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	162	131	132	155	0	379	134
N.S.	1	1.00	1.09	0.89	0.89	1.05	0.00	2.56	0.91
time (sec)	N/A	0.126	0.071	1.517	0.203	0.331	0.000	0.318	1.685

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	118	118	0	300	320	121	126
N.S.	1	1.00	0.77	0.77	0.00	1.95	2.08	0.79	0.82
time (sec)	N/A	0.082	0.044	1.195	0.000	0.336	31.345	0.297	1.651

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	89	0	220	260	94	97
N.S.	1	1.00	1.00	0.76	0.00	1.88	2.22	0.80	0.83
time (sec)	N/A	0.053	0.021	0.294	0.000	0.355	7.798	0.288	0.002

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	93	62	73	0	199	204	68	83
N.S.	1	1.29	0.86	1.01	0.00	2.76	2.83	0.94	1.15
time (sec)	N/A	0.051	0.037	0.666	0.000	0.347	15.371	0.320	1.615

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	96	77	0	191	901	88	65
N.S.	1	1.00	0.89	0.71	0.00	1.77	8.34	0.81	0.60
time (sec)	N/A	0.063	0.029	0.536	0.000	0.342	36.668	0.308	1.708

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	101	96	0	259	1134	117	88
N.S.	1	1.00	0.72	0.69	0.00	1.85	8.10	0.84	0.63
time (sec)	N/A	0.076	0.008	1.145	0.000	0.330	155.655	0.306	1.630

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	205	253	223	262	0	753	224
N.S.	1	1.00	0.82	1.01	0.89	1.04	0.00	3.00	0.89
time (sec)	N/A	0.304	0.125	3.722	0.191	0.308	0.000	0.332	1.615

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	257	211	185	224	0	544	184
N.S.	1	1.00	1.22	1.00	0.88	1.07	0.00	2.59	0.88
time (sec)	N/A	0.239	0.058	2.470	0.189	0.301	0.000	0.319	1.645

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	135	206	152	180	235	340	142
N.S.	1	1.00	1.09	1.66	1.23	1.45	1.90	2.74	1.15
time (sec)	N/A	0.095	0.075	1.809	0.188	0.322	60.076	0.304	1.643

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	121	209	0	0	0	0	0
N.S.	1	1.00	0.79	1.37	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.138	0.059	1.148	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	133	211	0	0	0	0	0
N.S.	1	1.00	0.99	1.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.131	0.052	1.719	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	152	208	0	0	0	0	0
N.S.	1	1.00	0.88	1.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.146	0.076	1.774	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	141	159	137	183	0	464	151
N.S.	1	1.00	1.08	1.22	1.05	1.41	0.00	3.57	1.16
time (sec)	N/A	0.141	0.081	1.425	0.194	0.322	0.000	0.316	1.657

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	184	193	183	230	0	604	190
N.S.	1	1.00	0.85	0.89	0.85	1.06	0.00	2.80	0.88
time (sec)	N/A	0.196	0.106	2.033	0.187	0.341	0.000	0.316	1.688

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	215	230	223	268	0	699	225
N.S.	1	1.00	0.85	0.91	0.88	1.06	0.00	2.76	0.89
time (sec)	N/A	0.215	0.134	3.249	0.192	0.344	0.000	0.341	1.789

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	188	211	0	492	559	215	235
N.S.	1	1.00	0.68	0.76	0.00	1.77	2.01	0.77	0.85
time (sec)	N/A	0.147	0.096	3.053	0.000	0.336	117.508	0.293	1.549

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	151	167	0	404	478	174	193
N.S.	1	1.00	0.68	0.76	0.00	1.83	2.16	0.79	0.87
time (sec)	N/A	0.098	0.044	1.486	0.000	0.322	31.580	0.312	0.003

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	112	127	0	366	400	136	180
N.S.	1	1.00	0.63	0.71	0.00	2.06	2.25	0.76	1.01
time (sec)	N/A	0.099	0.096	1.803	0.000	0.341	60.654	0.321	1.673

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	113	125	0	350	381	135	108
N.S.	1	1.00	0.67	0.74	0.00	2.07	2.25	0.80	0.64
time (sec)	N/A	0.092	0.090	2.151	0.000	0.326	72.934	0.318	1.594

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	156	134	0	351	1603	163	115
N.S.	1	1.00	0.78	0.67	0.00	1.76	8.02	0.82	0.58
time (sec)	N/A	0.113	0.047	1.902	0.000	0.337	156.417	0.315	1.608

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	161	167	0	429	0	207	149
N.S.	1	1.00	0.64	0.66	0.00	1.70	0.00	0.82	0.59
time (sec)	N/A	0.134	0.024	2.388	0.000	0.344	0.000	0.343	1.660

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	188	188	143	410	0	0	0	0	0
N.S.	1	1.00	0.76	2.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.190	0.075	2.483	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	112	112	91	358	0	0	0	0	0
N.S.	1	1.00	0.81	3.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.131	0.030	1.500	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	70	70	64	301	138	0	0	0	0
N.S.	1	1.00	0.91	4.30	1.97	0.00	0.00	0.00	0.00
time (sec)	N/A	0.068	0.007	1.744	0.195	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	119	119	92	420	140	0	0	0	0
N.S.	1	1.00	0.77	3.53	1.18	0.00	0.00	0.00	0.00
time (sec)	N/A	0.139	0.028	1.232	0.328	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	176	176	168	471	178	0	0	0	0
N.S.	1	1.00	0.95	2.68	1.01	0.00	0.00	0.00	0.00
time (sec)	N/A	0.174	0.044	2.137	0.321	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	667	667	697	616	0	0	0	0	0
N.S.	1	1.00	1.04	0.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.458	0.436	1.448	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	585	585	619	525	0	0	0	0	0
N.S.	1	1.00	1.06	0.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.385	0.241	1.153	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	533	533	564	449	0	0	0	0	0
N.S.	1	1.00	1.06	0.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.274	0.041	0.860	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	581	581	613	526	0	0	0	0	0
N.S.	1	1.00	1.06	0.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.388	0.210	1.579	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	651	651	670	602	0	0	0	0	0
N.S.	1	1.00	1.03	0.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.427	0.318	2.598	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	199	199	166	442	0	0	0	0	0
N.S.	1	1.00	0.83	2.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.195	0.176	2.089	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	155	155	131	381	0	0	0	0	0
N.S.	1	1.00	0.85	2.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.160	0.065	2.083	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	73	75	74	91	0	126	80
N.S.	1	1.00	0.88	0.90	0.89	1.10	0.00	1.52	0.96
time (sec)	N/A	0.050	0.036	1.529	0.193	0.339	0.000	0.323	2.629

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	201	201	170	492	197	0	0	0	0
N.S.	1	1.00	0.85	2.45	0.98	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	0.069	1.565	0.343	0.000	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	251	251	224	535	295	0	0	0	0
N.S.	1	1.00	0.89	2.13	1.18	0.00	0.00	0.00	0.00
time (sec)	N/A	0.258	0.104	4.671	0.244	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	802	802	915	0	0	0	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.266	1.281	0.000	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	746	746	850	0	0	0	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.082	0.827	0.000	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	751	751	877	0	0	0	0	0	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.528	0.436	0.000	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	803	803	939	0	0	0	0	0	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.977	1.078	0.000	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	163	163	128	264	0	0	0	0	0
N.S.	1	1.00	0.79	1.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.091	0.039	1.250	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	159	194	208	0	0	0	0
N.S.	1	1.00	0.67	0.81	0.87	0.00	0.00	0.00	0.00
time (sec)	N/A	0.194	0.083	1.043	0.281	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	335	282	0	0	0	0	0
N.S.	1	1.00	1.54	1.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.171	0.097	0.727	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	144	144	118	267	0	148	0	0	0
N.S.	1	1.00	0.82	1.85	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.111	0.121	2.649	0.000	0.417	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	124	124	100	249	0	133	0	0	0
N.S.	1	1.00	0.81	2.01	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.095	0.076	2.666	0.000	0.380	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	83	83	68	223	0	100	0	0	0
N.S.	1	1.00	0.82	2.69	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.075	0.040	2.161	0.000	0.392	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	97	97	90	242	0	114	0	0	0
N.S.	1	1.00	0.93	2.49	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.098	0.062	2.902	0.000	0.344	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	126	126	104	267	0	150	0	0	0
N.S.	1	1.00	0.83	2.12	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.111	0.127	2.638	0.000	0.339	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	327	327	209	436	0	291	0	0	0
N.S.	1	1.00	0.64	1.33	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.242	0.203	6.079	0.000	0.377	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	254	254	171	375	0	242	0	0	0
N.S.	1	1.00	0.67	1.48	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.182	0.167	5.569	0.000	0.346	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	176	176	124	314	0	192	0	0	0
N.S.	1	1.00	0.70	1.78	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.128	0.112	3.212	0.000	0.349	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	193	193	160	334	0	210	0	0	0
N.S.	1	1.00	0.83	1.73	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.166	0.209	6.569	0.000	0.357	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	257	257	188	396	0	265	0	0	0
N.S.	1	1.00	0.73	1.54	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.208	0.369	6.129	0.000	0.376	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	266	266	0	478	0	0	0	0	0
N.S.	1	1.00	0.00	1.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.282	0.000	3.084	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	121	121	92	323	154	0	0	0	0
N.S.	1	1.00	0.76	2.67	1.27	0.00	0.00	0.00	0.00
time (sec)	N/A	0.137	0.059	1.721	0.275	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	70	70	64	243	112	0	0	0	0
N.S.	1	1.00	0.91	3.47	1.60	0.00	0.00	0.00	0.00
time (sec)	N/A	0.113	0.019	3.036	0.274	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	221	221	0	398	0	0	0	0	0
N.S.	1	1.00	0.00	1.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.281	0.000	8.892	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	419	419	0	635	0	0	0	0	0
N.S.	1	1.00	0.00	1.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.391	0.000	11.003	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	204	204	171	414	233	0	0	0	0
N.S.	1	1.00	0.84	2.03	1.14	0.00	0.00	0.00	0.00
time (sec)	N/A	0.182	0.123	2.928	0.289	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	156	156	433	356	209	0	0	0	0
N.S.	1	1.00	2.78	2.28	1.34	0.00	0.00	0.00	0.00
time (sec)	N/A	0.205	1.054	7.877	0.288	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	No	TBD	TBD	TBD	TBD	TBD
size	377	377	0	561	0	0	0	0	0
N.S.	1	1.00	0.00	1.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.398	0.000	59.384	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	26	23	106	25	0	0	0
N.S.	1	1.00	1.04	0.92	4.24	1.00	0.00	0.00	0.00
time (sec)	N/A	0.104	0.057	1.874	0.282	0.315	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	26	23	109	25	0	0	21
N.S.	1	1.00	1.04	0.92	4.36	1.00	0.00	0.00	0.84
time (sec)	N/A	0.067	0.002	2.688	0.203	0.304	0.000	0.000	1.881

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	34	24	0	30	0	0	0
N.S.	1	1.00	1.31	0.92	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	0.102	0.063	3.506	0.000	0.316	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	0	42	0	31	31
N.S.	1	1.00	1.07	1.00	0.00	1.45	0.00	1.07	1.07
time (sec)	N/A	0.054	0.250	0.283	0.000	0.305	0.000	3.770	1.514

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	0	40	0	29	29
N.S.	1	1.00	1.07	1.00	0.00	1.48	0.00	1.07	1.07
time (sec)	N/A	0.049	0.185	0.256	0.000	0.307	0.000	2.751	1.515

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	0	47	0	31	31
N.S.	1	1.00	1.07	1.00	0.00	1.62	0.00	1.07	1.07
time (sec)	N/A	0.068	0.296	0.275	0.000	0.333	0.000	2.496	1.469

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	31	0	49	0	33	33
N.S.	1	1.00	1.07	1.07	0.00	1.69	0.00	1.14	1.14
time (sec)	N/A	0.072	0.261	0.280	0.000	0.334	0.000	2.495	1.506

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	0	31	0	31	31
N.S.	1	1.00	1.07	1.00	0.00	1.07	0.00	1.07	1.07
time (sec)	N/A	0.055	0.973	0.396	0.000	0.329	0.000	2.988	1.595

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	29	27	0	29	22	29	29
N.S.	1	1.00	1.07	1.00	0.00	1.07	0.81	1.07	1.07
time (sec)	N/A	0.052	0.816	0.412	0.000	0.340	9.211	2.467	1.525

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	0	32	0	31	31
N.S.	1	1.00	1.07	1.00	0.00	1.10	0.00	1.07	1.07
time (sec)	N/A	0.079	0.949	0.592	0.000	0.337	0.000	2.522	1.590

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	29	31	31	0	36	0	33	33
N.S.	1	1.00	1.07	1.07	0.00	1.24	0.00	1.14	1.14
time (sec)	N/A	0.070	0.210	0.638	0.000	0.343	0.000	3.047	1.628

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	75	66	0	76	0	0	0
N.S.	1	1.00	1.09	0.96	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.081	0.044	1.030	0.000	0.333	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	12	34	9	59	11	0	68	8
N.S.	1	1.50	4.25	1.12	7.38	1.38	0.00	8.50	1.00
time (sec)	N/A	0.008	0.004	0.418	0.193	0.300	0.000	0.304	1.446

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	76	69	15	0	0	10
N.S.	1	1.00	1.00	6.33	5.75	1.25	0.00	0.00	0.83
time (sec)	N/A	0.012	0.005	0.200	0.194	0.307	0.000	0.000	1.504

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	0	61	0	0	0
N.S.	1	1.00	1.00	1.07	0.00	4.36	0.00	0.00	0.00
time (sec)	N/A	0.013	0.005	2.164	0.000	0.297	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	36	34	67	0	0	204	37
N.S.	1	1.00	1.03	0.97	1.91	0.00	0.00	5.83	1.06
time (sec)	N/A	0.040	0.005	0.780	0.186	0.000	0.000	0.400	1.525

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	40	108	77	0	0	0	33
N.S.	1	1.00	1.03	2.77	1.97	0.00	0.00	0.00	0.85
time (sec)	N/A	0.032	0.005	0.222	0.190	0.000	0.000	0.000	1.628

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	44	44	0	67	0	0	0
N.S.	1	1.00	0.94	0.94	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.033	0.014	2.785	0.000	0.307	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	115	114	124	0	0	0	0
N.S.	1	1.00	1.10	1.09	1.18	0.00	0.00	0.00	0.00
time (sec)	N/A	0.117	0.031	3.770	0.194	0.000	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	228	230	0	0	0	0	0
N.S.	1	1.00	1.00	1.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.249	0.081	1.690	0.000	0.000	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	14	24	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.64	1.09	1.09
time (sec)	N/A	0.018	0.367	0.286	0.263	0.297	23.042	0.337	1.433

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	82	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.034	0.037	0.000	0.000	0.000	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	166	166	157	0	128	148	155	348	137
N.S.	1	1.00	0.95	0.00	0.77	0.89	0.93	2.10	0.83
time (sec)	N/A	0.086	0.087	0.000	0.187	0.319	8.467	0.318	1.803

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	129	0	106	122	128	264	111
N.S.	1	1.00	0.96	0.00	0.79	0.91	0.96	1.97	0.83
time (sec)	N/A	0.066	0.065	0.000	0.219	0.313	2.692	0.316	1.722

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	102	101	0	84	95	100	180	85
N.S.	1	1.00	0.99	0.00	0.82	0.93	0.98	1.76	0.83
time (sec)	N/A	0.048	0.045	0.000	0.191	0.300	1.231	0.325	1.745

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	53	57	65	66	102	52
N.S.	1	1.00	1.00	0.88	0.95	1.08	1.10	1.70	0.87
time (sec)	N/A	0.028	0.021	0.500	0.207	0.388	0.527	0.293	1.708

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	53	0	108	0	0	0	0
N.S.	1	1.00	1.04	0.00	2.12	0.00	0.00	0.00	0.00
time (sec)	N/A	0.046	0.004	0.000	0.349	0.000	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	69	0	61	65	442	144	58
N.S.	1	1.00	0.99	0.00	0.87	0.93	6.31	2.06	0.83
time (sec)	N/A	0.047	0.032	0.000	0.212	0.339	18.305	0.310	2.071

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	100	0	84	97	0	249	83
N.S.	1	1.00	0.92	0.00	0.77	0.89	0.00	2.28	0.76
time (sec)	N/A	0.052	0.027	0.000	0.188	0.353	0.000	0.306	1.931

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	128	0	106	124	0	343	110
N.S.	1	1.00	0.91	0.00	0.75	0.88	0.00	2.43	0.78
time (sec)	N/A	0.074	0.086	0.000	0.192	0.348	0.000	0.407	1.882

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	480	480	311	0	324	487	0	930	434
N.S.	1	1.00	0.65	0.00	0.68	1.01	0.00	1.94	0.90
time (sec)	N/A	0.332	0.250	0.000	0.229	0.375	0.000	0.319	2.889

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	342	342	223	0	257	357	0	624	420
N.S.	1	1.00	0.65	0.00	0.75	1.04	0.00	1.82	1.23
time (sec)	N/A	0.241	0.137	0.000	0.208	0.357	0.000	0.309	1.674

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	195	150	0	179	225	0	340	186
N.S.	1	1.00	0.77	0.00	0.92	1.15	0.00	1.74	0.95
time (sec)	N/A	0.117	0.060	0.000	0.201	0.327	0.000	0.313	1.627

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	195	0	0	0	0	0	0
N.S.	1	1.00	2.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.083	0.095	0.000	0.000	0.000	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	155	189	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.193	0.113	0.000	0.000	0.000	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	293	293	353	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.383	0.228	0.000	0.000	0.000	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	408	408	437	0	0	0	0	0	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.601	0.281	0.000	0.000	0.000	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	907	907	577	0	666	1197	0	2160	976
N.S.	1	1.00	0.64	0.00	0.73	1.32	0.00	2.38	1.08
time (sec)	N/A	0.675	0.326	0.000	0.230	0.413	0.000	0.349	9.294

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	595	595	433	0	536	861	0	1440	840
N.S.	1	1.00	0.73	0.00	0.90	1.45	0.00	2.42	1.41
time (sec)	N/A	0.415	0.205	0.000	0.215	0.387	0.000	0.341	1.935

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	284	284	241	0	381	527	0	714	350
N.S.	1	1.00	0.85	0.00	1.34	1.86	0.00	2.51	1.23
time (sec)	N/A	0.156	0.137	0.000	0.206	0.369	0.000	0.310	1.773

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	333	0	0	0	0	0	0
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.126	0.114	0.000	0.000	0.000	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	263	263	536	0	0	0	0	0	0
N.S.	1	1.00	2.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	0.533	0.000	0.000	0.000	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	573	573	841	0	0	0	0	0	0
N.S.	1	1.00	1.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.876	0.831	0.000	0.000	0.000	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	171	171	155	0	118	179	143	266	140
N.S.	1	1.00	0.91	0.00	0.69	1.05	0.84	1.56	0.82
time (sec)	N/A	0.084	0.093	0.000	0.215	0.338	58.179	0.361	2.214

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	127	0	96	153	116	232	106
N.S.	1	1.00	0.91	0.00	0.69	1.10	0.83	1.67	0.76
time (sec)	N/A	0.069	0.062	0.000	0.192	0.362	18.776	0.360	1.893

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	99	0	74	126	88	81	86
N.S.	1	1.00	0.93	0.00	0.69	1.18	0.82	0.76	0.80
time (sec)	N/A	0.055	0.042	0.000	0.193	0.388	6.590	0.514	2.101

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	63	86	48	90	56	52	44
N.S.	1	1.00	1.19	1.62	0.91	1.70	1.06	0.98	0.83
time (sec)	N/A	0.028	0.025	0.523	0.199	0.340	2.446	0.346	1.621

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	53	0	124	0	0	0	0
N.S.	1	1.00	1.04	0.00	2.43	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	0.004	0.000	0.476	0.000	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	68	63	75	70	391	116	60
N.S.	1	1.00	1.05	0.97	1.15	1.08	6.02	1.78	0.92
time (sec)	N/A	0.036	0.023	0.506	0.200	0.302	137.099	0.311	1.646

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	104	104	0	95	96	0	237	87
N.S.	1	1.00	1.00	0.00	0.91	0.92	0.00	2.28	0.84
time (sec)	N/A	0.050	0.045	0.000	0.201	0.311	0.000	0.321	1.606

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	136	136	132	0	117	123	0	357	113
N.S.	1	1.00	0.97	0.00	0.86	0.90	0.00	2.62	0.83
time (sec)	N/A	0.069	0.058	0.000	0.201	0.310	0.000	0.318	1.670

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	404	404	432	0	0	0	0	0	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.596	0.257	0.000	0.000	0.000	0.000	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	288	288	321	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.380	0.150	0.000	0.000	0.000	0.000	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	152	172	0	0	0	0	0	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.199	0.082	0.000	0.000	0.000	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	386	0	0	0	0	0	0
N.S.	1	1.00	4.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.089	0.226	0.000	0.000	0.000	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	195	298	0	248	235	0	278	193
N.S.	1	1.00	1.53	0.00	1.27	1.21	0.00	1.43	0.99
time (sec)	N/A	0.127	0.193	0.000	0.232	0.295	0.000	0.346	1.747

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	341	341	440	0	321	361	0	578	424
N.S.	1	1.00	1.29	0.00	0.94	1.06	0.00	1.70	1.24
time (sec)	N/A	0.250	0.206	0.000	0.230	0.344	0.000	0.355	1.831

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	480	480	545	0	387	490	0	877	440
N.S.	1	1.00	1.14	0.00	0.81	1.02	0.00	1.83	0.92
time (sec)	N/A	0.307	0.366	0.000	0.227	0.338	0.000	0.407	3.004

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	569	569	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.887	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	260	260	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.338	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.128	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	285	285	558	0	568	541	0	543	357
N.S.	1	1.00	1.96	0.00	1.99	1.90	0.00	1.91	1.25
time (sec)	N/A	0.172	0.411	0.000	0.231	0.339	0.000	0.400	1.824

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	595	595	766	0	732	869	0	1147	846
N.S.	1	1.00	1.29	0.00	1.23	1.46	0.00	1.93	1.42
time (sec)	N/A	0.425	0.650	0.000	0.257	0.373	0.000	0.395	2.029

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	907	907	950	0	864	1203	0	1747	989
N.S.	1	1.00	1.05	0.00	0.95	1.33	0.00	1.93	1.09
time (sec)	N/A	0.659	0.924	0.000	0.266	0.351	0.000	0.411	9.308

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	234	234	218	0	172	201	216	516	189
N.S.	1	1.00	0.93	0.00	0.74	0.86	0.92	2.21	0.81
time (sec)	N/A	0.118	0.149	0.000	0.202	0.358	45.057	0.302	1.763

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	185	185	174	0	140	161	173	390	150
N.S.	1	1.00	0.94	0.00	0.76	0.87	0.94	2.11	0.81
time (sec)	N/A	0.089	0.106	0.000	0.192	0.336	8.682	0.297	1.583

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	136	136	132	0	106	122	131	264	111
N.S.	1	1.00	0.97	0.00	0.78	0.90	0.96	1.94	0.82
time (sec)	N/A	0.061	0.065	0.000	0.192	0.333	1.861	0.315	1.531

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	66	70	77	82	132	65
N.S.	1	1.00	1.00	0.86	0.91	1.00	1.06	1.71	0.84
time (sec)	N/A	0.037	0.034	0.405	0.190	0.317	0.528	0.289	1.438

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	53	0	166	0	0	0	0
N.S.	1	1.00	1.04	0.00	3.25	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	0.004	0.000	0.343	0.000	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	85	0	75	81	0	207	74
N.S.	1	1.00	0.98	0.00	0.86	0.93	0.00	2.38	0.85
time (sec)	N/A	0.044	0.025	0.000	0.222	0.349	0.000	0.305	1.701

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	131	0	106	125	0	343	109
N.S.	1	1.00	0.92	0.00	0.74	0.87	0.00	2.40	0.76
time (sec)	N/A	0.063	0.085	0.000	0.195	0.326	0.000	0.328	1.737

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	192	192	172	0	139	163	0	489	154
N.S.	1	1.00	0.90	0.00	0.72	0.85	0.00	2.55	0.80
time (sec)	N/A	0.081	0.143	0.000	0.207	0.326	0.000	0.314	1.764

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	680	680	427	0	424	674	0	1389	608
N.S.	1	1.00	0.63	0.00	0.62	0.99	0.00	2.04	0.89
time (sec)	N/A	0.456	0.395	0.000	0.206	0.384	0.000	0.323	6.032

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	480	480	317	0	323	484	0	930	431
N.S.	1	1.00	0.66	0.00	0.67	1.01	0.00	1.94	0.90
time (sec)	N/A	0.311	0.239	0.000	0.205	0.365	0.000	0.319	3.007

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	267	267	197	0	217	287	0	465	290
N.S.	1	1.00	0.74	0.00	0.81	1.07	0.00	1.74	1.09
time (sec)	N/A	0.204	0.087	0.000	0.194	0.342	0.000	0.312	1.800

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	195	0	0	0	0	0	0
N.S.	1	1.00	2.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.085	0.086	0.000	0.000	0.000	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	231	231	273	0	0	0	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.274	0.165	0.000	0.000	0.000	0.000	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	405	405	443	0	0	0	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.604	0.308	0.000	0.000	0.000	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1835	1835	1025	0	1064	2183	0	4320	1802
N.S.	1	1.00	0.56	0.00	0.58	1.19	0.00	2.35	0.98
time (sec)	N/A	1.553	0.900	0.000	0.217	0.517	0.000	0.559	9.933

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1357	1357	808	0	867	1688	0	3240	1386
N.S.	1	1.00	0.60	0.00	0.64	1.24	0.00	2.39	1.02
time (sec)	N/A	1.034	0.571	0.000	0.228	0.463	0.000	0.346	9.640

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	907	907	589	0	668	1190	0	2160	979
N.S.	1	1.00	0.65	0.00	0.74	1.31	0.00	2.38	1.08
time (sec)	N/A	0.652	0.322	0.000	0.210	0.393	0.000	0.332	9.193

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	438	438	362	0	455	690	0	1072	558
N.S.	1	1.00	0.83	0.00	1.04	1.58	0.00	2.45	1.27
time (sec)	N/A	0.287	0.151	0.000	0.203	0.338	0.000	0.324	1.879

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	333	0	0	0	0	0	0
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.130	0.126	0.000	0.000	0.000	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	439	439	733	0	0	0	0	0	0
N.S.	1	1.00	1.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.581	0.608	0.000	0.000	0.000	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	765	765	1074	0	0	0	0	0	0
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.818	1.266	0.000	0.000	0.000	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	135	0	108	129	0	254	113
N.S.	1	1.00	0.98	0.00	0.78	0.93	0.00	1.84	0.82
time (sec)	N/A	0.067	0.074	0.000	0.223	0.351	0.000	0.503	2.050

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	130	130	128	0	0	337	0	115	0
N.S.	1	1.00	0.98	0.00	0.00	2.59	0.00	0.88	0.00
time (sec)	N/A	0.053	0.105	0.000	0.000	0.370	0.000	0.351	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	94	0	76	83	95	82	74
N.S.	1	1.00	1.06	0.00	0.85	0.93	1.07	0.92	0.83
time (sec)	N/A	0.047	0.022	0.000	0.199	0.328	103.409	0.368	1.567

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	72	62	0	231	70	72	56
N.S.	1	1.00	1.00	0.86	0.00	3.21	0.97	1.00	0.78
time (sec)	N/A	0.045	0.020	0.391	0.000	0.352	1.900	0.319	1.571

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	55	0	114	0	0	0	0
N.S.	1	1.00	1.00	0.00	2.07	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	0.010	0.000	0.356	0.000	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	59	0	0	208	0	64	0
N.S.	1	1.00	0.87	0.00	0.00	3.06	0.00	0.94	0.00
time (sec)	N/A	0.029	0.015	0.000	0.000	0.346	0.000	0.341	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	93	0	77	85	0	95	74
N.S.	1	1.00	0.99	0.00	0.82	0.90	0.00	1.01	0.79
time (sec)	N/A	0.045	0.027	0.000	0.208	0.357	0.000	0.331	1.826

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	123	65	0	0	313	0	100	0
N.S.	1	1.00	0.53	0.00	0.00	2.54	0.00	0.81	0.00
time (sec)	N/A	0.049	0.012	0.000	0.000	0.372	0.000	0.339	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	482	482	323	0	330	508	0	905	440
N.S.	1	1.00	0.67	0.00	0.68	1.05	0.00	1.88	0.91
time (sec)	N/A	0.314	0.250	0.000	0.201	0.395	0.000	0.612	2.907

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	275	275	239	0	231	304	0	313	299
N.S.	1	1.00	0.87	0.00	0.84	1.11	0.00	1.14	1.09
time (sec)	N/A	0.198	0.104	0.000	0.232	0.371	0.000	0.505	1.726

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	199	0	0	0	0	0	0
N.S.	1	1.00	2.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.087	0.092	0.000	0.000	0.000	0.000	0.000	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	238	238	264	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.285	0.219	0.000	0.000	0.000	0.000	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	412	412	449	0	0	0	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.610	0.328	0.000	0.000	0.000	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	547	547	438	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.497	0.295	0.000	0.000	0.000	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	364	364	319	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	0.174	0.000	0.000	0.000	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	298	298	247	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.260	0.116	0.000	0.000	0.000	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	476	476	473	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.397	0.391	0.000	0.000	0.000	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	640	640	678	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.619	0.762	0.000	0.000	0.000	0.000	0.000	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	913	913	593	0	680	1241	0	2104	992
N.S.	1	1.00	0.65	0.00	0.74	1.36	0.00	2.30	1.09
time (sec)	N/A	0.694	0.566	0.000	0.213	0.489	0.000	0.817	9.190

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	449	449	428	0	484	720	0	755	575
N.S.	1	1.00	0.95	0.00	1.08	1.60	0.00	1.68	1.28
time (sec)	N/A	0.321	0.262	0.000	0.213	0.412	0.000	0.789	1.882

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	339	0	0	0	0	0	0
N.S.	1	1.00	2.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.135	0.140	0.000	0.000	0.000	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	451	451	764	0	0	0	0	0	0
N.S.	1	1.00	1.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.572	0.546	0.000	0.000	0.000	0.000	0.000	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	1552	22	0	75	0	24	24
N.S.	1	1.00	64.67	0.92	0.00	3.12	0.00	1.00	1.00
time (sec)	N/A	1.931	7.796	0.065	0.000	0.354	0.000	0.465	1.438

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	1299	18	0	62	19	20	20
N.S.	1	1.00	64.95	0.90	0.00	3.10	0.95	1.00	1.00
time (sec)	N/A	0.688	5.891	0.053	0.000	0.321	61.662	0.458	1.409

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	1158	22	0	66	0	24	24
N.S.	1	1.00	48.25	0.92	0.00	2.75	0.00	1.00	1.00
time (sec)	N/A	0.323	6.625	0.066	0.000	0.335	0.000	0.440	1.507

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	1385	22	0	66	0	24	24
N.S.	1	1.00	57.71	0.92	0.00	2.75	0.00	1.00	1.00
time (sec)	N/A	1.194	7.583	0.068	0.000	0.325	0.000	0.551	1.456

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	239	239	214	0	162	232	0	169	191
N.S.	1	1.00	0.90	0.00	0.68	0.97	0.00	0.71	0.80
time (sec)	N/A	0.115	0.180	0.000	0.216	0.430	0.000	0.362	1.917

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	190	190	171	0	128	192	162	134	153
N.S.	1	1.00	0.90	0.00	0.67	1.01	0.85	0.71	0.81
time (sec)	N/A	0.086	0.089	0.000	0.189	0.336	57.673	0.355	1.908

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	128	0	96	153	119	103	112
N.S.	1	1.00	0.91	0.00	0.68	1.09	0.84	0.73	0.79
time (sec)	N/A	0.062	0.064	0.000	0.191	0.341	11.428	0.366	1.965

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	79	108	59	107	73	63	59
N.S.	1	1.00	1.13	1.54	0.84	1.53	1.04	0.90	0.84
time (sec)	N/A	0.033	0.033	0.429	0.198	0.332	2.290	0.336	1.707

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	53	0	185	0	0	0	0
N.S.	1	1.00	1.04	0.00	3.63	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	0.004	0.000	0.455	0.000	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	85	0	86	107	0	96	73
N.S.	1	1.00	1.04	0.00	1.05	1.30	0.00	1.17	0.89
time (sec)	N/A	0.043	0.026	0.000	0.195	0.366	0.000	0.370	1.585

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	133	0	117	165	0	127	113
N.S.	1	1.00	0.96	0.00	0.85	1.20	0.00	0.92	0.82
time (sec)	N/A	0.062	0.061	0.000	0.200	0.379	0.000	0.360	1.738

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	187	187	177	0	150	213	0	160	152
N.S.	1	1.00	0.95	0.00	0.80	1.14	0.00	0.86	0.81
time (sec)	N/A	0.086	0.106	0.000	0.198	0.408	0.000	0.494	1.734

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	572	572	603	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.058	0.418	0.000	0.000	0.000	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	400	400	437	0	0	0	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.599	0.228	0.000	0.000	0.000	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	227	227	241	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.312	0.123	0.000	0.000	0.000	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	389	0	0	0	0	0	0
N.S.	1	1.00	4.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.089	0.149	0.000	0.000	0.000	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	269	269	374	0	284	357	0	428	299
N.S.	1	1.00	1.39	0.00	1.06	1.33	0.00	1.59	1.11
time (sec)	N/A	0.218	0.259	0.000	0.225	0.333	0.000	0.338	1.815

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	479	479	551	0	387	597	0	877	439
N.S.	1	1.00	1.15	0.00	0.81	1.25	0.00	1.83	0.92
time (sec)	N/A	0.314	0.367	0.000	0.215	0.393	0.000	0.374	2.997

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	759	759	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.804	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	436	436	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.626	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.136	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	438	438	666	0	640	814	0	846	570
N.S.	1	1.00	1.52	0.00	1.46	1.86	0.00	1.93	1.30
time (sec)	N/A	0.301	0.519	0.000	0.225	0.378	0.000	0.451	1.928

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	907	907	962	0	864	1404	0	1747	992
N.S.	1	1.00	1.06	0.00	0.95	1.55	0.00	1.93	1.09
time (sec)	N/A	0.667	0.874	0.000	0.225	0.449	0.000	0.426	9.280

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	137	0	98	160	0	105	112
N.S.	1	1.00	0.96	0.00	0.69	1.12	0.00	0.73	0.78
time (sec)	N/A	0.066	0.074	0.000	0.194	0.368	0.000	0.379	1.932

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	65	0	0	399	0	103	0
N.S.	1	1.00	0.54	0.00	0.00	3.30	0.00	0.85	0.00
time (sec)	N/A	0.049	0.012	0.000	0.000	0.377	0.000	0.392	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	93	0	63	113	0	69	73
N.S.	1	1.00	0.99	0.00	0.67	1.20	0.00	0.73	0.78
time (sec)	N/A	0.051	0.018	0.000	0.194	0.355	0.000	0.388	1.851

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	53	154	0	279	61	60	51
N.S.	1	1.00	0.82	2.37	0.00	4.29	0.94	0.92	0.78
time (sec)	N/A	0.027	0.011	0.590	0.000	0.350	16.426	0.605	1.651

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	55	0	127	0	0	0	0
N.S.	1	1.00	1.00	0.00	2.31	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	0.010	0.000	0.472	0.000	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	80	0	0	235	0	78	0
N.S.	1	1.00	1.04	0.00	0.00	3.05	0.00	1.01	0.00
time (sec)	N/A	0.033	0.034	0.000	0.000	0.361	0.000	0.361	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	94	0	88	84	0	105	74
N.S.	1	1.00	1.06	0.00	0.99	0.94	0.00	1.18	0.83
time (sec)	N/A	0.045	0.025	0.000	0.200	0.365	0.000	0.408	1.681

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	129	0	0	339	0	111	0
N.S.	1	1.00	0.98	0.00	0.00	2.57	0.00	0.84	0.00
time (sec)	N/A	0.058	0.109	0.000	0.000	0.357	0.000	0.388	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	412	412	830	0	0	0	0	0	0
N.S.	1	1.00	2.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.648	0.433	0.000	0.000	0.000	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	239	239	542	0	0	0	0	0	0
N.S.	1	1.00	2.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.285	0.287	0.000	0.000	0.000	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	1701	0	0	0	0	0	0
N.S.	1	1.00	17.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.089	0.330	0.000	0.000	0.000	0.000	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	276	276	691	0	298	307	0	0	302
N.S.	1	1.00	2.50	0.00	1.08	1.11	0.00	0.00	1.09
time (sec)	N/A	0.205	0.385	0.000	0.200	0.352	0.000	0.000	1.654

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	482	482	988	0	397	513	0	0	440
N.S.	1	1.00	2.05	0.00	0.82	1.06	0.00	0.00	0.91
time (sec)	N/A	0.306	0.542	0.000	0.213	0.370	0.000	0.000	2.809

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	490	490	735	0	0	0	0	0	0
N.S.	1	1.00	1.50	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.541	1.427	0.000	0.000	0.000	0.000	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	309	309	523	0	0	0	0	0	0
N.S.	1	1.00	1.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.294	0.684	0.000	0.000	0.000	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	361	361	609	0	0	0	0	0	0
N.S.	1	1.00	1.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.403	0.989	0.000	0.000	0.000	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	773	773	5557	0	0	0	0	0	0
N.S.	1	1.00	7.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.764	18.529	0.000	0.000	0.000	0.000	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	451	451	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.620	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.131	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	449	449	692	0	684	725	0	0	578
N.S.	1	1.00	1.54	0.00	1.52	1.61	0.00	0.00	1.29
time (sec)	N/A	0.304	0.619	0.000	0.224	0.340	0.000	0.000	1.846

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	5975	22	0	93	0	24	24
N.S.	1	1.00	248.96	0.92	0.00	3.88	0.00	1.00	1.00
time (sec)	N/A	2.035	23.095	0.079	0.000	0.333	0.000	0.762	1.453

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	80	0	20	20
N.S.	1	1.00	1.10	0.90	0.00	4.00	0.00	1.00	1.00
time (sec)	N/A	0.852	4.210	0.072	0.000	0.336	0.000	0.458	1.371

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	5504	22	0	84	0	24	24
N.S.	1	1.00	229.33	0.92	0.00	3.50	0.00	1.00	1.00
time (sec)	N/A	0.891	13.157	0.102	0.000	0.328	0.000	0.915	1.463

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	B	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	6328	22	0	84	0	24	24
N.S.	1	1.00	263.67	0.92	0.00	3.50	0.00	1.00	1.00
time (sec)	N/A	2.352	21.298	0.075	0.000	0.329	0.000	0.782	1.447

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	730	730	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.902	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	551	551	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.618	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	360	360	229	0	0	0	0	0	0
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.372	4.035	0.000	0.000	0.000	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	174	174	130	0	0	0	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.142	0.038	0.000	0.000	0.000	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	23	0	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.05	0.00	1.00	1.00
time (sec)	N/A	0.034	0.103	0.040	0.304	0.351	0.000	0.367	1.597

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	23	0	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.05	0.00	1.00	1.00
time (sec)	N/A	0.037	0.102	0.042	0.304	0.351	0.000	0.406	1.607

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	907	907	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.928	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	677	677	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.669	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	445	445	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.427	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	213	213	182	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.175	0.047	0.000	0.000	0.000	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	33	0	24	24
N.S.	1	1.00	1.08	0.92	1.00	1.38	0.00	1.00	1.00
time (sec)	N/A	0.037	0.109	0.043	0.313	0.338	0.000	1.612	1.570

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	33	0	24	24
N.S.	1	1.00	1.08	0.92	1.00	1.38	0.00	1.00	1.00
time (sec)	N/A	0.034	0.111	0.041	0.317	0.360	0.000	1.619	1.563

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	26	0	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.30	0.00	1.00	1.00
time (sec)	N/A	0.031	0.767	0.064	0.336	0.353	0.000	0.545	1.677

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	24	0	18	18
N.S.	1	1.00	1.11	0.89	1.00	1.33	0.00	1.00	1.00
time (sec)	N/A	0.015	0.031	0.045	0.316	0.354	0.000	0.350	1.494

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	28	0	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.27	0.00	1.00	1.00
time (sec)	N/A	0.036	0.253	0.063	0.327	0.312	0.000	0.421	1.462

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	175	175	131	0	0	0	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.162	0.200	0.000	0.000	0.000	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	552	552	325	0	0	0	0	0	0
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.583	0.836	0.000	0.000	0.000	0.000	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	926	926	525	0	0	0	0	0	0
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.085	3.491	0.000	0.000	0.000	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	35	0	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.59	0.00	1.00	1.00
time (sec)	N/A	0.036	0.236	0.096	0.345	0.341	0.000	2.528	1.651

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	33	0	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.65	0.00	1.00	1.00
time (sec)	N/A	0.019	0.034	0.044	0.334	0.349	0.000	1.396	1.584

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	37	0	24	24
N.S.	1	1.00	1.08	0.92	1.00	1.54	0.00	1.00	1.00
time (sec)	N/A	0.038	0.151	0.046	0.341	0.336	0.000	2.890	1.613

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	216	213	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.201	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	676	676	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.677	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1141	1141	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.169	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1121	1121	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.255	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	831	831	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.905	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	553	553	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.583	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	266	266	174	0	0	0	0	0	0
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.268	0.024	0.000	0.000	0.000	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	23	0	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.05	0.00	1.00	1.00
time (sec)	N/A	0.035	0.121	0.045	0.323	0.337	0.000	0.419	1.698

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	23	0	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.05	0.00	1.00	1.00
time (sec)	N/A	0.033	0.126	0.041	0.322	0.330	0.000	0.403	1.732

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1363	1363	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.410	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1035	1035	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.056	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	673	673	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.640	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	338	338	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.303	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	35	0	24	24
N.S.	1	1.00	1.08	0.92	1.00	1.46	0.00	1.00	1.00
time (sec)	N/A	0.035	0.139	0.057	0.331	0.359	0.000	2.065	1.756

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	35	0	24	24
N.S.	1	1.00	1.08	0.92	1.00	1.46	0.00	1.00	1.00
time (sec)	N/A	0.036	0.115	0.055	0.317	0.345	0.000	2.060	1.725

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	557	557	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.582	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	273	273	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.277	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	23	0	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.05	0.00	1.00	1.00
time (sec)	N/A	0.036	0.125	0.057	0.345	0.346	0.000	0.414	1.703

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	23	0	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.05	0.00	1.00	1.00
time (sec)	N/A	0.036	0.131	0.063	0.362	0.362	0.000	0.404	1.896

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	23	0	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.05	0.00	1.00	1.00
time (sec)	N/A	0.039	0.129	0.045	0.314	0.331	0.000	0.394	1.641

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	19	0	18	18
N.S.	1	1.00	1.11	0.89	1.00	1.06	0.00	1.00	1.00
time (sec)	N/A	0.023	0.028	0.041	0.328	0.326	0.000	0.390	1.512

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	23	0	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.05	0.00	1.00	1.00
time (sec)	N/A	0.047	0.106	0.043	0.327	0.328	0.000	0.390	1.562

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	678	675	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.682	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	350	347	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.318	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	35	0	24	24
N.S.	1	1.00	1.08	0.92	1.00	1.46	0.00	1.00	1.00
time (sec)	N/A	0.038	0.150	0.060	0.334	0.334	0.000	2.619	1.751

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	35	0	24	24
N.S.	1	1.00	1.08	0.92	1.00	1.46	0.00	1.00	1.00
time (sec)	N/A	0.035	0.148	0.051	0.330	0.350	0.000	2.519	1.678

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	35	0	24	24
N.S.	1	1.00	1.08	0.92	1.00	1.46	0.00	1.00	1.00
time (sec)	N/A	0.036	0.149	0.042	0.328	0.353	0.000	2.490	1.597

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	31	0	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.55	0.00	1.00	1.00
time (sec)	N/A	0.021	0.035	0.043	0.316	0.332	0.000	1.490	1.512

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	35	0	24	24
N.S.	1	1.00	1.08	0.92	1.00	1.46	0.00	1.00	1.00
time (sec)	N/A	0.034	0.118	0.043	0.345	0.317	0.000	2.934	1.572

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	26	0	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.30	0.00	1.00	1.00
time (sec)	N/A	0.028	1.012	0.064	0.358	0.316	0.000	0.401	1.722

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	24	0	18	18
N.S.	1	1.00	1.11	0.89	1.00	1.33	0.00	1.00	1.00
time (sec)	N/A	0.021	0.032	0.044	0.354	0.313	0.000	0.391	1.620

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	28	0	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.27	0.00	1.00	1.00
time (sec)	N/A	0.033	0.256	0.065	0.379	0.324	0.000	0.411	1.634

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	267	267	175	0	0	0	0	0	0
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.263	0.277	0.000	0.000	0.000	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	554	554	325	0	0	0	0	0	0
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.561	0.851	0.000	0.000	0.000	0.000	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	832	832	502	0	0	0	0	0	0
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.898	0.918	0.000	0.000	0.000	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	38	0	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.73	0.00	1.00	1.00
time (sec)	N/A	0.031	0.245	0.099	0.366	0.321	0.000	1.507	1.736

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	36	0	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.80	0.00	1.00	1.00
time (sec)	N/A	0.024	0.038	0.047	0.339	0.352	0.000	0.966	1.687

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	40	0	24	24
N.S.	1	1.00	1.08	0.92	1.00	1.67	0.00	1.00	1.00
time (sec)	N/A	0.039	0.152	0.047	0.361	0.324	0.000	1.927	1.636

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	342	339	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.324	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	673	673	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.680	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1036	1036	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.069	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	28	0	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.27	0.00	1.00	1.00
time (sec)	N/A	0.039	0.254	0.066	0.359	0.351	0.000	0.538	1.740

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	28	0	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.27	0.00	1.00	1.00
time (sec)	N/A	0.036	0.138	0.046	0.355	0.319	0.000	0.484	1.457

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	26	0	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.30	0.00	1.00	1.00
time (sec)	N/A	0.029	0.170	0.046	0.357	0.338	0.000	0.533	1.617

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	24	0	18	18
N.S.	1	1.00	1.11	0.89	1.00	1.33	0.00	1.00	1.00
time (sec)	N/A	0.022	0.031	0.046	0.357	0.326	0.000	0.414	1.471

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	28	0	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.27	0.00	1.00	1.00
time (sec)	N/A	0.038	0.137	0.056	0.419	0.333	0.000	0.685	1.621

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	28	0	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.27	0.00	1.00	1.00
time (sec)	N/A	0.039	0.114	0.044	0.374	0.340	0.000	0.465	1.475

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-1)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	0	42	0	24	24
N.S.	1	1.00	1.08	0.92	0.00	1.75	0.00	1.00	1.00
time (sec)	N/A	0.040	0.303	0.112	0.000	0.343	0.000	2.249	1.710

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	42	0	24	24
N.S.	1	1.00	1.08	0.92	1.00	1.75	0.00	1.00	1.00
time (sec)	N/A	0.044	0.174	0.046	0.337	0.345	0.000	2.230	1.454

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	22	40	0	22	22
N.S.	1	1.00	1.09	0.91	1.00	1.82	0.00	1.00	1.00
time (sec)	N/A	0.033	0.202	0.044	0.377	0.326	0.000	2.273	1.561

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	38	0	20	20
N.S.	1	1.00	1.10	0.90	1.00	1.90	0.00	1.00	1.00
time (sec)	N/A	0.025	0.036	0.043	0.343	0.323	0.000	1.784	1.444

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	42	0	24	24
N.S.	1	1.00	1.08	0.92	1.00	1.75	0.00	1.00	1.00
time (sec)	N/A	0.044	0.149	0.046	0.329	0.377	0.000	2.329	1.539

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	22	24	42	0	24	24
N.S.	1	1.00	1.08	0.92	1.00	1.75	0.00	1.00	1.00
time (sec)	N/A	0.041	0.137	0.049	0.352	0.321	0.000	2.297	1.455

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F(-2)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	631	631	373	0	754	1196	0	556	0
N.S.	1	1.00	0.59	0.00	1.19	1.90	0.00	0.88	0.00
time (sec)	N/A	0.650	0.241	0.000	0.287	0.344	0.000	0.389	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F(-2)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	603	603	332	0	731	1162	0	448	0
N.S.	1	1.00	0.55	0.00	1.21	1.93	0.00	0.74	0.00
time (sec)	N/A	0.526	0.319	0.000	0.292	0.368	0.000	0.385	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F(-2)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	588	588	271	0	707	1236	0	471	0
N.S.	1	1.00	0.46	0.00	1.20	2.10	0.00	0.80	0.00
time (sec)	N/A	0.488	0.290	0.000	0.289	0.399	0.000	0.384	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	620	620	235	0	723	1348	0	494	0
N.S.	1	1.00	0.38	0.00	1.17	2.17	0.00	0.80	0.00
time (sec)	N/A	0.529	0.161	0.000	0.297	0.381	0.000	0.398	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	641	641	100	0	739	1369	0	511	0
N.S.	1	1.00	0.16	0.00	1.15	2.14	0.00	0.80	0.00
time (sec)	N/A	0.550	0.052	0.000	0.327	0.412	0.000	0.418	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F(-2)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1002	1002	628	0	1165	2178	0	885	0
N.S.	1	1.00	0.63	0.00	1.16	2.17	0.00	0.88	0.00
time (sec)	N/A	0.863	0.564	0.000	0.319	0.428	0.000	0.389	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F(-2)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	949	949	478	0	1119	2118	0	577	0
N.S.	1	1.00	0.50	0.00	1.18	2.23	0.00	0.61	0.00
time (sec)	N/A	0.830	0.622	0.000	0.315	0.460	0.000	0.431	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F(-2)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	932	932	527	0	1102	2112	0	641	0
N.S.	1	1.00	0.57	0.00	1.18	2.27	0.00	0.69	0.00
time (sec)	N/A	0.809	0.647	0.000	0.303	0.440	0.000	0.440	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	935	935	340	0	1088	2205	0	675	0
N.S.	1	1.00	0.36	0.00	1.16	2.36	0.00	0.72	0.00
time (sec)	N/A	0.790	0.637	0.000	0.313	0.461	0.000	0.426	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	968	968	294	0	1110	2283	0	688	0
N.S.	1	1.00	0.30	0.00	1.15	2.36	0.00	0.71	0.00
time (sec)	N/A	0.800	0.183	0.000	0.312	0.434	0.000	0.438	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1680	1680	1506	0	0	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.279	0.911	0.000	0.000	0.000	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1361	1361	1297	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.333	0.296	0.000	0.000	0.000	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1659	1659	1336	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.759	1.055	0.000	0.000	0.000	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	33	33	33	148	0	35	0	0	0
N.S.	1	1.00	1.00	4.48	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.020	0.012	2.543	0.000	0.352	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	75	75	220	496	0	105	0	0	0
N.S.	1	1.00	2.93	6.61	0.00	1.40	0.00	0.00	0.00
time (sec)	N/A	0.079	0.176	1.418	0.000	0.338	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	161	161	659	0	0	417	0	0	0
N.S.	1	1.00	4.09	0.00	0.00	2.59	0.00	0.00	0.00
time (sec)	N/A	0.144	0.202	0.000	0.000	0.383	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	456	0	0	281	0	0	0
N.S.	1	1.00	3.45	0.00	0.00	2.13	0.00	0.00	0.00
time (sec)	N/A	0.123	0.157	0.000	0.000	0.318	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	102	265	0	0	161	0	0	0
N.S.	1	1.00	2.60	0.00	0.00	1.58	0.00	0.00	0.00
time (sec)	N/A	0.094	0.148	0.000	0.000	0.327	0.000	0.000	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	49	49	49	180	0	69	0	0	0
N.S.	1	1.00	1.00	3.67	0.00	1.41	0.00	0.00	0.00
time (sec)	N/A	0.038	0.009	2.135	0.000	0.309	0.000	0.000	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	44	30	24	30	30
N.S.	1	1.00	1.07	1.00	1.57	1.07	0.86	1.07	1.07
time (sec)	N/A	0.197	0.441	0.079	0.376	0.302	53.375	0.333	1.720

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	91	30	0	30	30
N.S.	1	1.00	1.07	1.00	3.25	1.07	0.00	1.07	1.07
time (sec)	N/A	0.082	2.150	0.085	0.319	0.315	0.000	0.332	1.799

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	238	30	0	30	30
N.S.	1	1.00	1.07	1.00	8.50	1.07	0.00	1.07	1.07
time (sec)	N/A	0.078	8.834	0.063	0.315	0.308	0.000	0.341	2.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	65	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.033	0.020	0.000	0.000	0.000	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	147	132	0	1134	0	261	192
N.S.	1	1.00	0.87	0.78	0.00	6.71	0.00	1.54	1.14
time (sec)	N/A	0.136	0.079	1.463	0.000	1.067	0.000	0.445	1.619

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	105	0	206	235	92	82
N.S.	1	1.00	1.00	1.67	0.00	3.27	3.73	1.46	1.30
time (sec)	N/A	0.035	0.027	0.762	0.000	0.323	95.181	0.338	0.142

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	47	47	54	46	80	64	46
N.S.	1	1.00	1.34	1.34	1.54	1.31	2.29	1.83	1.31
time (sec)	N/A	0.012	0.030	0.518	0.230	0.332	0.287	0.353	1.418

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	56	65	65	65	102	171	67
N.S.	1	1.00	1.24	1.44	1.44	1.44	2.27	3.80	1.49
time (sec)	N/A	0.020	0.040	0.306	0.226	0.350	0.775	0.346	1.440

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	79	111	0	287	0	139	163
N.S.	1	1.00	1.34	1.88	0.00	4.86	0.00	2.36	2.76
time (sec)	N/A	0.027	0.096	0.927	0.000	0.338	0.000	0.533	1.847

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	66	138	0	1169	0	319	499
N.S.	1	1.00	0.40	0.84	0.00	7.08	0.00	1.93	3.02
time (sec)	N/A	0.118	0.349	1.964	0.000	1.060	0.000	0.743	1.689

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	30	19	24	24
N.S.	1	1.00	1.09	1.00	1.09	1.36	0.86	1.09	1.09
time (sec)	N/A	0.005	0.264	0.095	1.330	0.345	18.083	0.433	2.070

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	221	221	743	0	0	0	0	0	0
N.S.	1	1.00	3.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.208	0.921	0.000	0.000	0.000	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	168	168	441	0	0	0	0	0	0
N.S.	1	1.00	2.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.141	0.432	0.000	0.000	0.000	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	115	219	0	0	0	0	0	0
N.S.	1	1.00	1.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.067	0.169	0.000	0.000	0.000	0.000	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	70	77	70	76	107	176	61
N.S.	1	1.00	1.40	1.54	1.40	1.52	2.14	3.52	1.22
time (sec)	N/A	0.035	0.034	0.201	0.211	0.323	0.757	0.321	1.500

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	30	19	24	24
N.S.	1	1.00	1.09	1.00	1.09	1.36	0.86	1.09	1.09
time (sec)	N/A	0.006	0.315	0.078	0.348	0.304	1.148	1.038	1.484

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	148	62	20	24	24
N.S.	1	1.00	1.09	1.00	6.73	2.82	0.91	1.09	1.09
time (sec)	N/A	0.005	0.394	0.076	0.397	0.356	3.460	0.481	1.601

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [134] had the largest ratio of [1.4290000000000005]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	16	0.188
2	A	4	3	1.00	16	0.188
3	A	4	3	1.00	16	0.188
4	A	3	3	1.00	14	0.214
5	A	3	3	1.00	12	0.250
6	A	3	3	1.00	16	0.188
7	A	2	2	1.00	16	0.125
8	A	5	5	1.18	16	0.312
9	A	3	3	1.00	16	0.188
10	A	4	3	1.00	16	0.188
11	A	4	3	1.00	16	0.188
12	A	4	3	1.00	16	0.188
13	A	4	3	1.00	16	0.188
14	A	9	8	1.00	16	0.500
15	A	9	8	1.00	16	0.500
16	A	3	3	1.00	16	0.188
17	A	8	8	1.00	14	0.571
18	A	8	8	1.00	12	0.667
19	A	3	3	1.00	16	0.188
20	A	7	7	1.00	16	0.438
21	A	7	7	1.00	16	0.438
22	A	5	5	1.00	16	0.312
23	A	8	8	1.00	16	0.500
24	A	8	8	1.00	16	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	4	3	1.00	16	0.188
26	A	4	3	1.00	16	0.188
27	A	4	3	1.00	16	0.188
28	A	4	3	1.00	16	0.188
29	A	4	3	1.00	14	0.214
30	A	3	3	1.00	12	0.250
31	A	3	3	1.00	16	0.188
32	A	3	3	1.00	16	0.188
33	A	4	3	1.00	16	0.188
34	A	4	3	1.00	16	0.188
35	A	4	3	1.00	16	0.188
36	A	5	4	1.00	16	0.250
37	A	5	4	1.00	16	0.250
38	A	4	4	1.00	16	0.250
39	A	3	3	1.00	14	0.214
40	A	3	3	1.00	12	0.250
41	A	3	3	1.00	16	0.188
42	A	4	4	1.00	16	0.250
43	A	3	3	1.00	16	0.188
44	A	5	4	1.00	16	0.250
45	A	1	1	1.00	12	0.083
46	A	4	3	1.00	18	0.167
47	A	4	3	1.00	18	0.167
48	A	4	3	1.00	16	0.188
49	A	4	3	1.00	14	0.214
50	A	3	3	1.00	18	0.167
51	A	4	3	1.00	18	0.167
52	A	4	3	1.00	18	0.167
53	A	4	3	1.00	18	0.167
54	A	3	3	1.00	16	0.188
55	A	3	3	1.00	18	0.167
56	A	3	3	1.00	18	0.167
57	A	2	2	1.00	16	0.125
58	A	4	4	1.00	18	0.222
59	A	4	4	1.00	18	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	4	4	1.00	18	0.222
61	A	4	4	1.00	20	0.200
62	A	5	5	1.00	20	0.250
63	A	3	3	1.00	18	0.167
64	A	5	4	1.00	22	0.182
65	A	5	4	1.00	22	0.182
66	A	5	4	1.00	20	0.200
67	A	4	4	1.00	19	0.210
68	A	6	6	1.00	22	0.273
69	A	5	4	1.00	22	0.182
70	A	2	2	1.00	16	0.125
71	A	2	2	1.00	14	0.143
72	A	2	2	1.00	12	0.167
73	A	3	3	1.00	16	0.188
74	A	2	2	1.00	16	0.125
75	A	2	2	1.00	16	0.125
76	A	2	2	1.00	16	0.125
77	A	8	8	1.00	18	0.444
78	A	9	8	1.00	18	0.444
79	A	4	4	1.00	16	0.250
80	A	5	5	1.00	18	0.278
81	A	4	4	1.00	18	0.222
82	A	8	8	1.00	18	0.444
83	A	12	10	1.00	18	0.556
84	A	20	13	1.00	18	0.722
85	A	16	13	1.00	18	0.722
86	A	12	11	1.00	14	0.786
87	A	7	8	1.00	18	0.444
88	A	11	10	1.00	18	0.556
89	A	14	11	1.00	18	0.611
90	A	18	11	1.00	18	0.611
91	A	15	8	1.00	18	0.444
92	A	11	8	1.00	18	0.444
93	A	5	4	1.00	16	0.250
94	A	6	6	1.00	18	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	6	6	1.00	18	0.333
96	A	10	10	1.00	18	0.556
97	A	17	13	1.00	18	0.722
98	N/A	0	0	1.00	18	0.000
99	N/A	0	0	1.00	14	0.000
100	N/A	0	0	1.00	18	0.000
101	N/A	0	0	1.00	18	0.000
102	A	9	7	1.00	18	0.389
103	A	4	4	1.00	16	0.250
104	N/A	0	0	1.00	18	0.000
105	N/A	0	0	1.00	18	0.000
106	N/A	0	0	1.00	18	0.000
107	N/A	0	0	1.00	14	0.000
108	N/A	0	0	1.00	18	0.000
109	A	13	8	1.00	18	0.444
110	A	5	5	1.00	16	0.312
111	N/A	0	0	1.00	18	0.000
112	N/A	0	0	1.00	18	0.000
113	N/A	0	0	1.00	18	0.000
114	N/A	0	0	1.00	14	0.000
115	N/A	0	0	1.00	18	0.000
116	A	18	9	1.00	18	0.500
117	A	6	5	1.00	16	0.312
118	N/A	0	0	1.00	18	0.000
119	N/A	0	0	1.00	18	0.000
120	N/A	0	0	1.00	18	0.000
121	N/A	0	0	1.00	14	0.000
122	N/A	0	0	1.00	18	0.000
123	A	8	7	1.00	16	0.438
124	A	3	3	1.00	14	0.214
125	A	11	8	1.00	16	0.500
126	A	4	4	1.00	14	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
127	A	15	9	1.00	16	0.562
128	A	5	4	1.00	14	0.286
129	A	9	8	1.00	18	0.444
130	A	4	4	1.00	18	0.222
131	A	5	5	1.00	18	0.278
132	A	4	4	1.00	18	0.222
133	A	49	19	1.00	16	1.187
134	A	49	20	1.00	14	1.429
135	A	39	11	1.01	18	0.611
136	A	39	11	1.01	18	0.611
137	A	48	18	1.00	18	1.000
138	A	12	7	1.00	18	0.389
139	A	9	7	1.00	18	0.389
140	A	4	4	1.00	18	0.222
141	N/A	0	0	1.00	18	0.000
142	N/A	0	0	1.00	18	0.000
143	N/A	0	0	1.00	18	0.000
144	N/A	0	0	1.00	16	0.000
145	N/A	0	0	1.00	14	0.000
146	N/A	0	0	1.00	18	0.000
147	N/A	0	0	1.00	18	0.000
148	A	21	8	1.00	18	0.444
149	A	13	8	1.00	18	0.444
150	A	5	5	1.00	18	0.278
151	N/A	0	0	1.00	18	0.000
152	N/A	0	0	1.00	18	0.000
153	N/A	0	0	1.00	18	0.000
154	N/A	0	0	1.00	16	0.000
155	N/A	0	0	1.00	14	0.000
156	N/A	0	0	1.00	18	0.000
157	N/A	0	0	1.00	18	0.000
158	N/A	0	0	1.00	20	0.000
159	N/A	0	0	1.00	20	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
160	A	3	3	1.00	18	0.167
161	N/A	0	0	1.00	20	0.000
162	N/A	0	0	1.00	20	0.000
163	A	9	9	1.00	24	0.375
164	A	10	9	1.00	24	0.375
165	A	5	5	1.00	22	0.227
166	A	6	6	1.00	21	0.286
167	A	5	5	1.00	24	0.208
168	A	9	9	1.00	24	0.375
169	A	1	1	1.00	12	0.083
170	A	3	3	1.00	12	0.250
171	A	3	3	1.00	14	0.214
172	A	3	3	1.00	14	0.214
173	A	3	3	1.00	16	0.188
174	A	5	5	1.00	18	0.278
175	A	6	6	1.00	18	0.333
176	A	3	2	1.00	18	0.111
177	A	3	2	1.00	18	0.111
178	A	3	2	1.00	16	0.125
179	A	2	2	1.00	10	0.200
180	A	3	3	1.00	18	0.167
181	A	4	3	1.00	18	0.167
182	A	3	2	1.00	18	0.111
183	A	3	2	1.00	18	0.111
184	A	6	5	1.00	20	0.250
185	A	6	5	1.00	20	0.250
186	A	6	5	1.00	18	0.278
187	A	3	3	1.00	12	0.250
188	A	9	6	1.00	20	0.300
189	A	6	5	1.00	20	0.250
190	A	6	5	1.00	20	0.250
191	A	13	11	1.00	20	0.550
192	A	12	11	1.00	20	0.550
193	A	11	10	1.00	18	0.556
194	A	8	8	1.00	12	0.667

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
195	A	12	6	1.00	20	0.300
196	A	11	10	1.00	20	0.500
197	A	11	10	1.00	20	0.500
198	A	4	3	1.00	20	0.150
199	A	4	3	1.00	20	0.150
200	A	4	3	1.00	18	0.167
201	A	8	7	1.00	20	0.350
202	A	4	3	1.00	20	0.150
203	A	4	3	1.00	20	0.150
204	A	4	3	1.00	20	0.150
205	A	8	7	1.00	16	0.438
206	A	6	3	1.00	20	0.150
207	A	5	3	1.00	20	0.150
208	A	2	2	1.00	18	0.111
209	A	5	5	1.00	20	0.250
210	A	9	6	1.00	20	0.300
211	N/A	0	0	1.00	20	0.000
212	A	8	4	1.00	20	0.200
213	A	7	4	1.00	20	0.200
214	A	6	4	1.00	18	0.222
215	A	2	2	1.00	12	0.167
216	N/A	0	0	1.00	20	0.000
217	N/A	0	0	1.00	20	0.000
218	N/A	0	0	1.00	20	0.000
219	A	13	8	1.00	21	0.381
220	A	10	8	1.00	21	0.381
221	A	7	7	1.00	19	0.368
222	A	3	3	1.00	18	0.167
223	A	7	8	1.00	21	0.381
224	A	11	10	1.00	21	0.476
225	A	14	10	1.00	21	0.476
226	A	21	15	1.00	23	0.652
227	A	17	13	1.00	23	0.565
228	A	14	10	1.00	21	0.476

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
229	A	9	6	1.00	20	0.300
230	A	14	9	1.00	23	0.391
231	A	16	11	1.00	23	0.478
232	A	21	15	1.00	23	0.652
233	A	33	20	1.00	23	0.870
234	A	30	17	1.00	23	0.739
235	A	22	15	1.00	21	0.714
236	A	12	6	1.00	20	0.300
237	A	17	9	1.00	23	0.391
238	A	24	16	1.00	23	0.696
239	A	31	17	1.00	23	0.739
240	A	21	14	1.00	23	0.609
241	A	17	14	1.00	23	0.609
242	A	13	11	1.00	21	0.524
243	A	8	7	1.00	20	0.350
244	A	13	9	1.00	23	0.391
245	A	16	11	1.00	23	0.478
246	A	20	13	1.00	23	0.565
247	A	25	14	1.00	23	0.609
248	A	21	12	1.00	23	0.522
249	A	18	11	1.00	21	0.524
250	A	13	7	1.00	20	0.350
251	A	18	9	1.00	23	0.391
252	A	22	13	1.00	23	0.565
253	A	25	15	1.00	23	0.652
254	A	37	18	1.00	23	0.783
255	A	34	18	1.00	23	0.783
256	A	26	16	1.00	21	0.762
257	A	16	7	1.00	20	0.350
258	A	21	9	1.00	23	0.391
259	A	30	18	1.00	23	0.783
260	A	39	19	1.00	23	0.826
261	A	16	9	1.00	22	0.409
262	A	12	8	1.00	22	0.364
263	A	8	4	1.00	20	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
264	A	12	12	1.00	22	0.546
265	A	18	12	1.00	22	0.546
266	A	19	8	1.00	24	0.333
267	A	20	10	1.00	24	0.417
268	A	17	6	1.00	22	0.273
269	A	13	6	1.00	22	0.273
270	A	9	6	1.00	20	0.300
271	A	12	8	1.00	22	0.364
272	A	26	13	1.00	22	0.591
273	A	50	15	1.00	24	0.625
274	A	30	15	1.00	22	0.682
275	N/A	0	0	1.00	24	0.000
276	N/A	0	0	1.00	24	0.000
277	N/A	0	0	1.00	22	0.000
278	N/A	0	0	1.00	24	0.000
279	N/A	0	0	1.00	24	0.000
280	N/A	0	0	1.00	24	0.000
281	N/A	0	0	1.00	22	0.000
282	N/A	0	0	1.00	24	0.000
283	N/A	0	0	1.00	24	0.000
284	N/A	0	0	1.00	24	0.000
285	N/A	0	0	1.00	22	0.000
286	N/A	0	0	1.00	24	0.000
287	N/A	0	0	1.00	24	0.000
288	A	17	9	1.00	22	0.409
289	A	13	9	1.00	22	0.409
290	A	9	7	1.00	20	0.350
291	A	29	7	1.00	22	0.318
292	A	47	11	1.00	22	0.500
293	A	55	29	1.00	24	1.208
294	A	47	23	1.00	24	0.958
295	A	23	20	1.00	22	0.909
296	N/A	0	0	1.00	24	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
297	N/A	0	0	1.00	24	0.000
298	N/A	0	0	1.00	24	0.000
299	N/A	0	0	1.00	22	0.000
300	N/A	0	0	1.00	24	0.000
301	N/A	0	0	1.00	24	0.000
302	N/A	0	0	1.00	24	0.000
303	N/A	0	0	1.00	22	0.000
304	N/A	0	0	1.00	24	0.000
305	N/A	0	0	1.00	24	0.000
306	N/A	0	0	1.00	24	0.000
307	N/A	0	0	1.00	22	0.000
308	N/A	0	0	1.00	24	0.000
309	N/A	0	0	1.00	24	0.000
310	A	5	5	1.00	23	0.217
311	A	5	5	1.00	23	0.217
312	A	4	3	1.00	21	0.143
313	A	7	7	1.00	23	0.304
314	A	9	9	1.00	23	0.391
315	A	5	5	1.00	23	0.217
316	A	5	5	1.00	23	0.217
317	A	5	5	1.00	23	0.217
318	A	10	4	1.00	23	0.174
319	A	9	6	1.00	20	0.300
320	A	7	5	1.29	23	0.217
321	A	7	4	1.00	23	0.174
322	A	9	4	1.00	23	0.174
323	A	5	5	1.00	25	0.200
324	A	5	5	1.00	25	0.200
325	A	4	3	1.00	23	0.130
326	A	10	8	1.00	25	0.320
327	A	11	11	1.00	25	0.440
328	A	12	10	1.00	25	0.400
329	A	5	5	1.00	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
330	A	5	5	1.00	25	0.200
331	A	5	5	1.00	25	0.200
332	A	14	4	1.00	25	0.160
333	A	13	6	1.00	22	0.273
334	A	11	6	1.00	25	0.240
335	A	10	6	1.00	25	0.240
336	A	11	4	1.00	25	0.160
337	A	14	4	1.00	25	0.160
338	A	11	9	1.00	25	0.360
339	A	8	8	1.00	25	0.320
340	A	4	4	1.00	23	0.174
341	A	8	9	1.00	25	0.360
342	A	12	11	1.00	25	0.440
343	A	21	13	1.00	25	0.520
344	A	17	11	1.00	25	0.440
345	A	12	8	1.00	22	0.364
346	A	16	10	1.00	25	0.400
347	A	19	11	1.00	25	0.440
348	A	12	11	1.00	25	0.440
349	A	10	9	1.00	25	0.360
350	A	5	4	1.00	23	0.174
351	A	12	10	1.00	25	0.400
352	A	16	11	1.00	25	0.440
353	A	43	16	1.00	25	0.640
354	A	40	14	1.00	25	0.560
355	A	26	13	1.00	22	0.591
356	A	42	15	1.00	25	0.600
357	A	6	7	1.00	22	0.318
358	A	12	8	1.00	18	0.444
359	A	11	7	1.00	18	0.389
360	A	8	7	1.00	25	0.280
361	A	8	7	1.00	25	0.280
362	A	7	7	1.00	23	0.304
363	A	9	9	1.00	25	0.360
364	A	8	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
365	A	11	7	1.00	27	0.259
366	A	11	7	1.00	27	0.259
367	A	10	8	1.00	25	0.320
368	A	12	11	1.00	27	0.407
369	A	11	9	1.00	27	0.333
370	A	13	11	1.00	27	0.407
371	A	8	9	1.00	25	0.360
372	A	5	5	1.00	27	0.185
373	A	9	7	1.00	27	0.259
374	A	19	14	1.00	27	0.518
375	A	12	10	1.00	25	0.400
376	A	10	10	1.00	27	0.370
377	A	17	14	1.00	27	0.518
378	A	4	4	1.00	33	0.121
379	A	3	3	1.00	29	0.103
380	A	4	4	1.00	33	0.121
381	N/A	0	0	1.00	29	0.000
382	N/A	0	0	1.00	27	0.000
383	N/A	0	0	1.00	29	0.000
384	N/A	0	0	1.00	29	0.000
385	N/A	0	0	1.00	29	0.000
386	N/A	0	0	1.00	27	0.000
387	N/A	0	0	1.00	29	0.000
388	N/A	0	0	1.00	29	0.000
389	A	4	4	1.00	14	0.286
390	A	1	1	1.50	12	0.083
391	A	2	2	1.00	14	0.143
392	A	2	2	1.00	16	0.125
393	A	4	4	1.00	14	0.286
394	A	4	4	1.00	16	0.250
395	A	4	4	1.00	18	0.222
396	A	9	8	1.00	18	0.444
397	A	14	8	1.00	20	0.400
398	N/A	0	0	1.00	22	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
399	A	3	3	1.00	22	0.136
400	A	4	3	1.00	22	0.136
401	A	4	3	1.00	22	0.136
402	A	4	3	1.00	20	0.150
403	A	5	3	1.00	18	0.167
404	A	3	3	1.00	22	0.136
405	A	4	3	1.00	22	0.136
406	A	4	3	1.00	22	0.136
407	A	4	3	1.00	22	0.136
408	A	8	8	1.00	24	0.333
409	A	8	8	1.00	22	0.364
410	A	10	8	1.00	20	0.400
411	A	5	5	1.00	24	0.208
412	A	8	8	1.00	24	0.333
413	A	16	10	1.00	24	0.417
414	A	24	10	1.00	24	0.417
415	A	28	8	1.00	24	0.333
416	A	20	8	1.00	22	0.364
417	A	12	8	1.00	20	0.400
418	A	6	6	1.00	24	0.250
419	A	10	10	1.00	24	0.417
420	A	28	14	1.00	24	0.583
421	A	4	3	1.00	22	0.136
422	A	4	3	1.00	22	0.136
423	A	4	3	1.00	20	0.150
424	A	6	4	1.00	18	0.222
425	A	3	3	1.00	22	0.136
426	A	4	3	1.00	22	0.136
427	A	4	3	1.00	22	0.136
428	A	4	3	1.00	22	0.136
429	A	24	10	1.00	24	0.417
430	A	16	10	1.00	22	0.454
431	A	9	9	1.00	20	0.450
432	A	5	5	1.00	24	0.208
433	A	10	8	1.00	24	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
434	A	8	8	1.00	24	0.333
435	A	8	8	1.00	24	0.333
436	A	28	14	1.00	22	0.636
437	A	11	11	1.00	20	0.550
438	A	6	6	1.00	24	0.250
439	A	12	8	1.00	24	0.333
440	A	20	8	1.00	24	0.333
441	A	28	8	1.00	24	0.333
442	A	4	3	1.00	22	0.136
443	A	4	3	1.00	22	0.136
444	A	4	3	1.00	20	0.150
445	A	5	3	1.00	18	0.167
446	A	3	3	1.00	22	0.136
447	A	4	3	1.00	22	0.136
448	A	4	3	1.00	22	0.136
449	A	4	3	1.00	22	0.136
450	A	8	8	1.00	24	0.333
451	A	8	8	1.00	22	0.364
452	A	8	8	1.00	20	0.400
453	A	5	5	1.00	24	0.208
454	A	12	10	1.00	24	0.417
455	A	24	10	1.00	24	0.417
456	A	52	8	1.00	24	0.333
457	A	40	8	1.00	24	0.333
458	A	28	8	1.00	22	0.364
459	A	16	8	1.00	20	0.400
460	A	6	6	1.00	24	0.250
461	A	17	13	1.00	24	0.542
462	A	62	14	1.00	24	0.583
463	A	4	3	1.00	22	0.136
464	A	5	4	1.00	22	0.182
465	A	4	3	1.00	20	0.150
466	A	6	4	1.00	18	0.222
467	A	3	3	1.00	22	0.136
468	A	4	4	1.00	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
469	A	4	3	1.00	22	0.136
470	A	7	4	1.00	22	0.182
471	A	8	8	1.00	24	0.333
472	A	8	8	1.00	22	0.364
473	A	5	5	1.00	24	0.208
474	A	12	10	1.00	24	0.417
475	A	24	10	1.00	24	0.417
476	A	30	14	1.00	24	0.583
477	A	18	14	1.00	20	0.700
478	A	12	11	1.00	24	0.458
479	A	24	12	1.00	24	0.500
480	A	45	12	1.00	24	0.500
481	A	28	8	1.00	24	0.333
482	A	16	8	1.00	22	0.364
483	A	6	6	1.00	24	0.250
484	A	17	13	1.00	24	0.542
485	N/A	0	0	1.00	24	0.000
486	N/A	0	0	1.00	20	0.000
487	N/A	0	0	1.00	24	0.000
488	N/A	0	0	1.00	24	0.000
489	A	4	3	1.00	22	0.136
490	A	4	3	1.00	22	0.136
491	A	4	3	1.00	20	0.150
492	A	6	4	1.00	18	0.222
493	A	3	3	1.00	22	0.136
494	A	4	3	1.00	22	0.136
495	A	4	3	1.00	22	0.136
496	A	4	3	1.00	22	0.136
497	A	36	10	1.00	24	0.417
498	A	24	10	1.00	22	0.454
499	A	13	11	1.00	20	0.550
500	A	5	5	1.00	24	0.208
501	A	8	8	1.00	24	0.333
502	A	8	8	1.00	24	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
503	A	62	14	1.00	22	0.636
504	A	18	14	1.00	20	0.700
505	A	6	6	1.00	24	0.250
506	A	16	8	1.00	24	0.333
507	A	28	8	1.00	24	0.333
508	A	4	3	1.00	22	0.136
509	A	6	5	1.00	22	0.227
510	A	4	3	1.00	20	0.150
511	A	6	5	1.00	18	0.278
512	A	3	3	1.00	22	0.136
513	A	6	5	1.00	22	0.227
514	A	4	3	1.00	22	0.136
515	A	9	5	1.00	22	0.227
516	A	24	10	1.00	24	0.417
517	A	12	10	1.00	22	0.454
518	A	5	5	1.00	24	0.208
519	A	8	8	1.00	24	0.333
520	A	8	8	1.00	24	0.333
521	A	28	17	1.00	24	0.708
522	A	14	13	1.00	20	0.650
523	A	19	14	1.00	24	0.583
524	A	62	14	1.00	24	0.583
525	A	17	13	1.00	22	0.591
526	A	6	6	1.00	24	0.250
527	A	16	8	1.00	24	0.333
528	N/A	0	0	1.00	24	0.000
529	N/A	0	0	1.00	20	0.000
530	N/A	0	0	1.00	24	0.000
531	N/A	0	0	1.00	24	0.000
532	A	27	7	1.00	22	0.318
533	A	21	7	1.00	22	0.318
534	A	15	7	1.00	20	0.350
535	A	9	7	1.00	18	0.389
536	N/A	0	0	1.00	22	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
537	N/A	0	0	1.00	22	0.000
538	A	27	7	1.00	24	0.292
539	A	21	7	1.00	24	0.292
540	A	15	7	1.00	22	0.318
541	A	9	7	1.00	20	0.350
542	N/A	0	0	1.00	24	0.000
543	N/A	0	0	1.00	24	0.000
544	N/A	0	0	1.00	20	0.000
545	N/A	0	0	1.00	18	0.000
546	N/A	0	0	1.00	22	0.000
547	A	9	7	1.00	22	0.318
548	A	21	7	1.00	22	0.318
549	A	33	7	1.00	22	0.318
550	N/A	0	0	1.00	22	0.000
551	N/A	0	0	1.00	20	0.000
552	N/A	0	0	1.00	24	0.000
553	A	9	7	0.99	24	0.292
554	A	21	7	1.00	24	0.292
555	A	33	7	1.00	24	0.292
556	A	39	7	1.00	22	0.318
557	A	30	7	1.00	22	0.318
558	A	21	7	1.00	20	0.350
559	A	12	7	1.00	18	0.389
560	N/A	0	0	1.00	22	0.000
561	N/A	0	0	1.00	22	0.000
562	A	39	7	1.00	24	0.292
563	A	30	7	1.00	24	0.292
564	A	21	7	1.00	22	0.318
565	A	12	7	1.00	20	0.350
566	N/A	0	0	1.00	24	0.000
567	N/A	0	0	1.00	24	0.000
568	A	21	7	1.00	22	0.318
569	A	12	7	1.00	20	0.350

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
570	N/A	0	0	1.00	22	0.000
571	N/A	0	0	1.00	22	0.000
572	N/A	0	0	1.00	22	0.000
573	N/A	0	0	1.00	18	0.000
574	N/A	0	0	1.00	22	0.000
575	A	21	7	1.00	24	0.292
576	A	12	7	0.99	22	0.318
577	N/A	0	0	1.00	24	0.000
578	N/A	0	0	1.00	24	0.000
579	N/A	0	0	1.00	24	0.000
580	N/A	0	0	1.00	20	0.000
581	N/A	0	0	1.00	24	0.000
582	N/A	0	0	1.00	20	0.000
583	N/A	0	0	1.00	18	0.000
584	N/A	0	0	1.00	22	0.000
585	A	12	7	1.00	22	0.318
586	A	21	7	1.00	22	0.318
587	A	30	7	1.00	22	0.318
588	N/A	0	0	1.00	22	0.000
589	N/A	0	0	1.00	20	0.000
590	N/A	0	0	1.00	24	0.000
591	A	12	7	0.99	24	0.292
592	A	21	7	1.00	24	0.292
593	A	30	7	1.00	24	0.292
594	N/A	0	0	1.00	22	0.000
595	N/A	0	0	1.00	22	0.000
596	N/A	0	0	1.00	20	0.000
597	N/A	0	0	1.00	18	0.000
598	N/A	0	0	1.00	22	0.000
599	N/A	0	0	1.00	22	0.000
600	N/A	0	0	1.00	24	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
601	N/A	0	0	1.00	24	0.000
602	N/A	0	0	1.00	22	0.000
603	N/A	0	0	1.00	20	0.000
604	N/A	0	0	1.00	24	0.000
605	N/A	0	0	1.00	24	0.000
606	A	26	12	1.00	29	0.414
607	A	25	12	1.00	29	0.414
608	A	23	10	1.00	29	0.345
609	A	24	11	1.00	29	0.379
610	A	25	11	1.00	29	0.379
611	A	38	13	1.00	31	0.419
612	A	36	12	1.00	31	0.387
613	A	35	12	1.00	31	0.387
614	A	34	11	1.00	31	0.355
615	A	35	11	1.00	31	0.355
616	A	39	20	1.00	31	0.645
617	A	25	11	1.00	31	0.355
618	A	37	19	1.00	31	0.613
619	A	2	2	1.00	18	0.111
620	A	3	3	1.00	23	0.130
621	A	6	5	1.00	28	0.179
622	A	5	5	1.00	28	0.179
623	A	4	4	1.00	26	0.154
624	A	3	3	1.00	20	0.150
625	N/A	0	0	1.00	28	0.000
626	N/A	0	0	1.00	28	0.000
627	N/A	0	0	1.00	28	0.000
628	A	3	3	1.00	16	0.188
629	A	9	9	1.00	16	0.562
630	A	4	4	1.00	16	0.250
631	A	3	3	1.00	14	0.214
632	A	4	4	1.00	16	0.250
633	A	4	4	1.00	16	0.250
634	A	9	9	1.00	16	0.562

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
635	N/A	0	0	1.00	22	0.000
636	A	8	8	1.00	22	0.364
637	A	7	7	1.00	22	0.318
638	A	5	5	1.00	22	0.227
639	A	5	4	1.00	20	0.200
640	N/A	0	0	1.00	22	0.000
641	N/A	0	0	1.00	22	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^4 \log \left(c(a+bx^2)^p\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 80

$$\int x^4 \log \left(c(a+bx^2)^p\right) \, dx = -\frac{2a^2px}{5b^2} + \frac{2apx^3}{15b} - \frac{2px^5}{25} + \frac{2a^{5/2}p \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{5b^{5/2}} + \frac{1}{5}x^5 \log \left(c(a+bx^2)^p\right)$$

```
[Out] -2/5*a^2*p*x/b^2+2/15*a*p*x^3/b-2/25*p*x^5+2/5*a^(5/2)*p*arctan(x*b^(1/2)/a
^(1/2))/b^(5/2)+1/5*x^5*ln(c*(b*x^2+a)^p)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2505, 308, 211}

$$\int x^4 \log \left(c(a+bx^2)^p\right) \, dx = \frac{2a^{5/2}p \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{5b^{5/2}} - \frac{2a^2px}{5b^2} + \frac{1}{5}x^5 \log \left(c(a+bx^2)^p\right) + \frac{2apx^3}{15b} - \frac{2px^5}{25}$$

```
[In] Int[x^4*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (-2*a^2*p*x)/(5*b^2) + (2*a*p*x^3)/(15*b) - (2*p*x^5)/25 + (2*a^(5/2)*p*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(5*b^(5/2)) + (x^5*Log[c*(a + b*x^2)^p])/5
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\log\left(c\left(a+bx^2\right)^p\right)-\frac{1}{5}(2bp)\int\frac{x^6}{a+bx^2}\,dx\\
&= \frac{1}{5}x^5\log\left(c\left(a+bx^2\right)^p\right)-\frac{1}{5}(2bp)\int\left(\frac{a^2}{b^3}-\frac{ax^2}{b^2}+\frac{x^4}{b}-\frac{a^3}{b^3\left(a+bx^2\right)}\right)dx\\
&= -\frac{2a^2px}{5b^2}+\frac{2apx^3}{15b}-\frac{2px^5}{25}+\frac{1}{5}x^5\log\left(c\left(a+bx^2\right)^p\right)+\frac{\left(2a^3p\right)\int\frac{1}{a+bx^2}\,dx}{5b^2}\\
&= -\frac{2a^2px}{5b^2}+\frac{2apx^3}{15b}-\frac{2px^5}{25}+\frac{2a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{5b^{5/2}}+\frac{1}{5}x^5\log\left(c\left(a+bx^2\right)^p\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int x^4\log\left(c\left(a+bx^2\right)^p\right)\,dx=\frac{1}{75}\left(-\frac{30a^2px}{b^2}+\frac{10apx^3}{b}-6px^5+\frac{30a^{5/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+15x^5\log\left(c\left(a+bx^2\right)^p\right)\right)$$

```
[In] Integrate[x^4*Log[c*(a + b*x^2)^p],x]
```

```
[Out] ((-30*a^2*p*x)/b^2 + (10*a*p*x^3)/b - 6*p*x^5 + (30*a^(5/2)*p*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/b^(5/2) + 15*x^5*Log[c*(a + b*x^2)^p])/75
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

method	result
parts	$\frac{x^{5}\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)}{5}-\frac{2pb\left(\frac{\frac{1}{5}x^{5}b^{2}-\frac{1}{3}ab\,x^{3}+a^{2}x}{b^{3}}-\frac{a^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^{3}\sqrt{ab}}\right)}{5}$
risch	$\frac{x^{5}\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{5}-\frac{i\pi\,x^{5}\mathrm{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)^{3}}{10}+\frac{i\pi\,x^{5}\mathrm{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)^{2}\mathrm{csgn}\left(ic\right)}{10}+\frac{i\pi\,x^{5}\,\mathrm{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)^{2}}{10}$

```
[In] int(x^4*ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*ln(c*(b*x^2+a)^p)-2/5*p*b*(1/b^3*(1/5*x^5*b^2-1/3*a*b*x^3+a^2*x)-a^
3/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.35

$$\int x^4\log\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$=\left[\frac{15\,b^2px^5\log\left(bx^2+a\right)-6\,b^2px^5+15\,b^2x^5\log\left(c\right)+10\,abpx^3+15\,a^2p\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-30\,a^2px}{75\,b^2}\right.$$

```
[In] integrate(x^4*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [1/75*(15*b^2*p*x^5*log(b*x^2 + a) - 6*b^2*p*x^5 + 15*b^2*x^5*log(c) + 10*a
*b*p*x^3 + 15*a^2*p*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 +
a)) - 30*a^2*p*x)/b^2, 1/75*(15*b^2*p*x^5*log(b*x^2 + a) - 6*b^2*p*x^5 + 15
*b^2*x^5*log(c) + 10*a*b*p*x^3 + 30*a^2*p*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a)
 - 30*a^2*p*x)/b^2]
```

Sympy [A] (verification not implemented)

Time = 31.80 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.95

$$\int x^4 \log \left(c\left(a+bx^2\right)^p\right)\,dx = \begin{cases} \frac{x^5 \log(0^p c)}{5} & \text{for}\ a=0 \wedge b=0 \\ \frac{x^5 \log(a^p c)}{5} & \text{for}\ b=0 \\ -\frac{2px^5}{25} + \frac{x^5 \log\left(c\left(bx^2\right)^p\right)}{5} & \text{for}\ a=0 \\ \frac{2a^3 p \log\left(x-\sqrt{-\frac{a}{b}}\right)}{5b^3\sqrt{-\frac{a}{b}}} - \frac{a^3 \log\left(c\left(a+bx^2\right)^p\right)}{5b^3\sqrt{-\frac{a}{b}}} - \frac{2a^2px}{5b^2} + \frac{2apx^3}{15b} - \frac{2px^5}{25} + \frac{x^5\log\left(c\left(a+bx^2\right)^p\right)}{5} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**4*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((x**5*log(0**p*c)/5, Eq(a, 0) & Eq(b, 0)), (x**5*log(a**p*c)/5, E
q(b, 0)), (-2*p*x**5/25 + x**5*log(c*(b*x**2)**p)/5, Eq(a, 0)), (2*a**3*p*l
og(x - sqrt(-a/b))/(5*b**3*sqrt(-a/b)) - a**3*log(c*(a + b*x**2)**p)/(5*b**
3*sqrt(-a/b)) - 2*a**2*p*x/(5*b**2) + 2*a*p*x**3/(15*b) - 2*p*x**5/25 + x**
5*log(c*(a + b*x**2)**p)/5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int x^4 \log \left(c\left(a+bx^2\right)^p\right)\,dx = \frac{1}{5}\,x^5 \log\left(\left(bx^2+a\right)^p c\right) + \frac{2}{75}\,bp\left(\frac{15\,a^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} - \frac{3\,b^2x^5 - 5\,abx^3 + 15\,a^2x}{b^3}\right)$$

```
[In] integrate(x^4*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*log((b*x^2 + a)^p*c) + 2/75*b*p*(15*a^3*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*b^3) - (3*b^2*x^5 - 5*a*b*x^3 + 15*a^2*x)/b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int x^4 \log \left(c\left(a+bx^2\right)^p\right) \, dx = \frac{1}{5} px^5 \log \left(bx^2+a\right) - \frac{1}{25} \left(2 \, p - 5 \, \log \left(c\right)\right)x^5 + \frac{2 \, apx^3}{15 \, b} + \frac{2 \, a^3 p \arctan \left(\frac{bx}{\sqrt{ab}}\right)}{5 \, \sqrt{ab} b^2} - \frac{2 \, a^2 px}{5 \, b^2}$$

```
[In] integrate(x^4*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] 1/5*p*x^5*log(b*x^2 + a) - 1/25*(2*p - 5*log(c))*x^5 + 2/15*a*p*x^3/b + 2/5
*a^3*p*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 2/5*a^2*p*x/b^2
```

Mupad [B] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int x^4 \log \left(c\left(a+bx^2\right)^p\right) \, dx = \frac{x^5 \, \ln \left(c \left(b \, x^2 + a\right)^p\right)}{5} - \frac{2 \, p \, x^5}{25} + \frac{2 \, a^{5/2} \, p \, \mathrm{atan}\left(\frac{\sqrt{b} \, x}{\sqrt{a}}\right)}{5 \, b^{5/2}} + \frac{2 \, a \, p \, x^3}{15 \, b} - \frac{2 \, a^2 \, p \, x}{5 \, b^2}$$

```
[In] int(x^4*log(c*(a + b*x^2)^p),x)
```

```
[Out] (x^5*log(c*(a + b*x^2)^p))/5 - (2*p*x^5)/25 + (2*a^(5/2)*p*atan((b^(1/2)*x)
/a^(1/2)))/(5*b^(5/2)) + (2*a*p*x^3)/(15*b) - (2*a^2*p*x)/(5*b^2)
```

3.2 $\int x^3 \log \left(c(a+bx^2)^p\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 59

$$\int x^3 \log \left(c(a+bx^2)^p\right) \, dx = \frac{apx^2}{4b} - \frac{px^4}{8} - \frac{a^2 p \log (a+bx^2)}{4b^2} + \frac{1}{4}x^4 \log \left(c(a+bx^2)^p\right)$$

```
[Out] 1/4*a*p*x^2/b-1/8*p*x^4-1/4*a^2*p*ln(b*x^2+a)/b^2+1/4*x^4*ln(c*(b*x^2+a)^p)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2504, 2442, 45}

$$\int x^3 \log \left(c(a+bx^2)^p\right) \, dx = -\frac{a^2 p \log (a+bx^2)}{4b^2} + \frac{1}{4}x^4 \log \left(c(a+bx^2)^p\right) + \frac{apx^2}{4b} - \frac{px^4}{8}$$

```
[In] Int[x^3*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (a*p*x^2)/(4*b) - (p*x^4)/8 - (a^2*p*Log[a + b*x^2])/(4*b^2) + (x^4*Log[c*(
a + b*x^2)^p])/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
```

```
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x\log\left(c(a+bx)^p\right)\,dx, x, x^2\right)\\
&= \frac{1}{4}x^4\log\left(c\left(a+bx^2\right)^p\right)-\frac{1}{4}(bp)\text{Subst}\left(\int \frac{x^2}{a+bx}\,dx, x, x^2\right)\\
&= \frac{1}{4}x^4\log\left(c\left(a+bx^2\right)^p\right)-\frac{1}{4}(bp)\text{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)\,dx, x, x^2\right)\\
&= \frac{apx^2}{4b}-\frac{px^4}{8}-\frac{a^2p\log\left(a+bx^2\right)}{4b^2}+\frac{1}{4}x^4\log\left(c\left(a+bx^2\right)^p\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int x^3\log\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{apx^2}{4b}-\frac{px^4}{8}-\frac{a^2p\log\left(a+bx^2\right)}{4b^2}+\frac{1}{4}x^4\log\left(c\left(a+bx^2\right)^p\right)$$

```
[In] Integrate[x^3*Log[c*(a + b*x^2)^p],x]
[Out] (a*p*x^2)/(4*b) - (p*x^4)/8 - (a^2*p*Log[a + b*x^2])/(4*b^2) + (x^4*Log[c*(
a + b*x^2)^p])/4
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

method	result	size
parts	$\frac{x^{4} \ln\left(c \left(b \,x^{2}+a \right)^{p}\right)}{4}-\frac{p b \left(-\frac{-\frac{1}{2} b \,x^{4}+x^{2} a}{2 b^{2}}+\frac{a^{2} \ln\left(b \,x^{2}+a \right)}{2 b^{3}}\right)}{2}$	57
parallelrisch	$-\frac{-2 x^{4} \ln\left(c \left(b \,x^{2}+a \right)^{p}\right) b^{2}+b^{2} p \,x^{4}-2 a b p \,x^{2}+2 \ln\left(b \,x^{2}+a \right) a^{2} p +2 a^{2} p}{8 b^{2}}$	63
risch	Expression too large to display	1190

```
[In] int(x^3*ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*ln(c*(b*x^2+a)^p)-1/2*p*b*(-1/2/b^2*(-1/2*b*x^4+x^2*a)+1/2*a^2/b^3*
ln(b*x^2+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int x^3 \log \left(c\left(a+bx^2\right)^p\right) \, dx=-\frac{b^{2} p x^{4} - 2 \, b^{2} x^{4} \log \left(c\right) - 2 \, a b p x^{2} - 2 \, {\left(b^{2} p x^{4} - a^{2} p\right)} \log \left(b x^{2} + a\right)}{8 \, b^{2}}$$

```
[In] integrate(x^3*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] -1/8*(b^2*p*x^4 - 2*b^2*x^4*log(c) - 2*a*b*p*x^2 - 2*(b^2*p*x^4 - a^2*p)*lo
g(b*x^2 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int x^3 \log \left(c\left(a+bx^2\right)^p\right) \, dx=\begin{cases} - \frac{a^{2} \log{\left(c \left(a + b x^{2}\right)^{p} \right)}}{4 b^{2}} + \frac{a p x^{2}}{4 b} - \frac{p x^{4}}{8} + \frac{x^{4} \log{\left(c \left(a + b x^{2}\right)^{p} \right)}}{4} & \text{for}\: b \neq 0 \\\frac{x^{4} \log{\left(a^{p} c \right)}}{4} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((-a**2*log(c*(a + b*x**2)**p)/(4*b**2) + a*p*x**2/(4*b) - p*x**4/
8 + x**4*log(c*(a + b*x**2)**p)/4, Ne(b, 0)), (x**4*log(a**p*c)/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int x^3 \log\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{1}{4}\,x^4 \log\left(\left(bx^2+a\right)^p c\right) - \frac{1}{8}\,bp\left(\frac{2\,a^2 \log\left(bx^2+a\right)}{b^3} + \frac{bx^4 - 2\,ax^2}{b^2}\right)$$

```
[In] integrate(x^3*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((b*x^2 + a)^p*c) - 1/8*b*p*(2*a^2*log(b*x^2 + a)/b^3 + (b*x^4 -
 2*a*x^2)/b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.64

$$\int x^3 \log\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{2\left(bx^2+a\right)^2 p \log\left(bx^2+a\right) - \left(bx^2+a\right)^2 p + 2\left(bx^2+a\right)^2 \log\left(c\right)}{8\,b^2} + \frac{\left(bx^2 - \left(bx^2+a\right)\log\left(bx^2+a\right) + a\right)ap - \left(bx^2+a\right)a \log\left(c\right)}{2\,b^2}$$

```
[In] integrate(x^3*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] 1/8*(2*(b*x^2 + a)^2*p*log(b*x^2 + a) - (b*x^2 + a)^2*p + 2*(b*x^2 + a)^2*l
og(c))/b^2 + 1/2*((b*x^2 - (b*x^2 + a)*log(b*x^2 + a) + a)*a*p - (b*x^2 + a
)*a*log(c))/b^2
```

Mupad [B] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int x^3 \log\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{x^4\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{4} - \frac{p\,x^4}{8} - \frac{a^2\,p\,\ln\left(b\,x^2+a\right)}{4\,b^2} + \frac{a\,p\,x^2}{4\,b}$$

```
[In] int(x^3*log(c*(a + b*x^2)^p),x)
```

```
[Out] (x^4*log(c*(a + b*x^2)^p))/4 - (p*x^4)/8 - (a^2*p*log(a + b*x^2))/(4*b^2) +
 (a*p*x^2)/(4*b)
```

3.3 $\int x^2 \log\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 66

$$\int x^2 \log\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{2apx}{3b} - \frac{2px^3}{9} - \frac{2a^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}} + \frac{1}{3}x^3\log\left(c\left(a+bx^2\right)^p\right)$$

```
[Out] 2/3*a*p*x/b-2/9*p*x^3-2/3*a^(3/2)*p*arctan(x*b^(1/2)/a^(1/2))/b^(3/2)+1/3*x
^3*ln(c*(b*x^2+a)^p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2505, 308, 211}

$$\int x^2 \log\left(c\left(a+bx^2\right)^p\right)\,dx = -\frac{2a^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}} + \frac{1}{3}x^3\log\left(c\left(a+bx^2\right)^p\right) + \frac{2apx}{3b} - \frac{2px^3}{9}$$

```
[In] Int[x^2*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (2*a*p*x)/(3*b) - (2*p*x^3)/9 - (2*a^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
3*b^(3/2)) + (x^3*Log[c*(a + b*x^2)^p])/3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\log\left(c\left(a+bx^2\right)^p\right)-\frac{1}{3}(2bp)\int\frac{x^4}{a+bx^2}\,dx\\
&= \frac{1}{3}x^3\log\left(c\left(a+bx^2\right)^p\right)-\frac{1}{3}(2bp)\int\left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2\left(a+bx^2\right)}\right)dx\\
&= \frac{2apx}{3b}-\frac{2px^3}{9}+\frac{1}{3}x^3\log\left(c\left(a+bx^2\right)^p\right)-\frac{\left(2a^2p\right)\int\frac{1}{a+bx^2}\,dx}{3b}\\
&= \frac{2apx}{3b}-\frac{2px^3}{9}-\frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}}+\frac{1}{3}x^3\log\left(c\left(a+bx^2\right)^p\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

$$\int x^2\log\left(c\left(a+bx^2\right)^p\right)\,dx=\frac{1}{9}\left(\frac{6apx}{b}-2px^3-\frac{6a^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}+3x^3\log\left(c\left(a+bx^2\right)^p\right)\right)$$

```
[In] Integrate[x^2*Log[c*(a + b*x^2)^p],x]
[Out] ((6*a*p*x)/b - 2*p*x^3 - (6*a^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
+ 3*x^3*Log[c*(a + b*x^2)^p])/9
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

method	result
parts	$\frac{x^{3} \ln \left(c \left(b \,x^{2}+a \right)^{p}\right)}{3}-\frac{2 p b \left(-\frac{-\frac{1}{3} b \,x^{3}+a x}{b^{2}}+\frac{a^{2} \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{b^{2} \sqrt{a b}}\right)}{3}$
risch	$\frac{x^{3} \ln \left(\left(b \,x^{2}+a \right)^{p}\right)}{3}+\frac{i \pi \,x^{3} \operatorname{csgn}\left(i \left(b \,x^{2}+a \right)^{p}\right) \operatorname{csgn}\left(i c \left(b \,x^{2}+a \right)^{p}\right)^{2}}{6}-\frac{i \pi \,x^{3} \operatorname{csgn}\left(i \left(b \,x^{2}+a \right)^{p}\right) \operatorname{csgn}\left(i c \left(b \,x^{2}+a \right)^{p}\right) \operatorname{csgn}\left(i c \right)}{6}-\frac{i \pi \,x^{3} \mathrm{cs}}{}$

```
[In] int(x^2*ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*ln(c*(b*x^2+a)^p)-2/3*p*b*(-1/b^2*(-1/3*b*x^3+a*x)+a^2/b^2/(a*b)^(1
/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.30

$$\int x^2 \log \left(c\left(a + bx^2\right)^p\right) \, dx$$
$$= \left[\frac{3\,bpx^3 \log\left(bx^2 + a\right) - 2\,bpx^3 + 3\,bx^3 \log\left(c\right) + 3\,ap\sqrt{-\frac{a}{b}} \log\left(\frac{bx^2 - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) + 6\,apx}{9\,b}, \frac{3\,bpx^3 \log\left(bx^2 + \right.}{}\right.$$

```
[In] integrate(x^2*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [1/9*(3*b*p*x^3*log(b*x^2 + a) - 2*b*p*x^3 + 3*b*x^3*log(c) + 3*a*p*sqrt(-a
/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 6*a*p*x)/b, 1/9*(3*b*
p*x^3*log(b*x^2 + a) - 2*b*p*x^3 + 3*b*x^3*log(c) - 6*a*p*sqrt(a/b)*arctan(
b*x*sqrt(a/b)/a) + 6*a*p*x)/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(63) = 126$.

Time = 7.77 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.14

$$\int x^2 \log \left(c\left(a+bx^2\right)^p\right)\,dx = \begin{cases} \frac{x^3 \log(0^p c)}{3} & \text{for } a=0 \wedge b=0 \\ \frac{x^3 \log(a^p c)}{3} & \text{for } b=0 \\ -\frac{2px^3}{9} + \frac{x^3 \log\left(c(bx^2)^p\right)}{3} & \text{for } a=0 \\ -\frac{2a^2 p \log\left(x-\sqrt{-\frac{a}{b}}\right)}{3b^2\sqrt{-\frac{a}{b}}} + \frac{a^2 \log\left(c(a+bx^2)^p\right)}{3b^2\sqrt{-\frac{a}{b}}} + \frac{2apx}{3b} - \frac{2px^3}{9} + \frac{x^3\log\left(c(a+bx^2)^p\right)}{3} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((x**3*log(0**p*c)/3, Eq(a, 0) & Eq(b, 0)), (x**3*log(a**p*c)/3, E
q(b, 0)), (-2*p*x**3/9 + x**3*log(c*(b*x**2)**p)/3, Eq(a, 0)), (-2*a**2*p*l
og(x - sqrt(-a/b))/(3*b**2*sqrt(-a/b)) + a**2*log(c*(a + b*x**2)**p)/(3*b**
2*sqrt(-a/b)) + 2*a*p*x/(3*b) - 2*p*x**3/9 + x**3*log(c*(a + b*x**2)**p)/3,
 True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int x^2 \log \left(c\left(a+bx^2\right)^p\right)\,dx = \frac{1}{3}\,x^3 \log\left(\left(bx^2+a\right)^p c\right) - \frac{2}{9}\,bp\left(\frac{3\,a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{bx^3 - 3\,ax}{b^2}\right)$$

```
[In] integrate(x^2*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((b*x^2 + a)^p*c) - 2/9*b*p*(3*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*b^2) + (b*x^3 - 3*a*x)/b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int x^2 \log \left(c\left(a+bx^2\right)^p\right)\,dx = \frac{1}{3}\,px^3 \log\left(bx^2+a\right) - \frac{1}{9}\,(2\,p - 3\,\log\left(c\right))x^3 - \frac{2\,a^2 p \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b} + \frac{2\,apx}{3\,b}$$

```
[In] integrate(x^2*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] 1/3*p*x^3*log(b*x^2 + a) - 1/9*(2*p - 3*log(c))*x^3 - 2/3*a^2*p*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*b) + 2/3*a*p*x/b
```

Mupad [B] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int x^2 \log\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{x^3\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{3}-\frac{2\,p\,x^3}{9}-\frac{2\,a^{3/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3\,b^{3/2}}+\frac{2\,a\,p\,x}{3\,b}$$

```
[In] int(x^2*log(c*(a + b*x^2)^p),x)
```

```
[Out] (x^3*log(c*(a + b*x^2)^p))/3 - (2*p*x^3)/9 - (2*a^(3/2)*p*atan((b^(1/2)*x)/
a^(1/2)))/(3*b^(3/2)) + (2*a*p*x)/(3*b)
```

3.4 $\int x \log \left(c\left(a+b x^2\right)^p\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 35

$$\int x \log \left(c\left(a+b x^2\right)^p\right) \, dx=-\frac{p x^2}{2}+\frac{\left(a+b x^2\right) \log \left(c\left(a+b x^2\right)^p\right)}{2 b}$$

```
[Out] -1/2*p*x^2+1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2504, 2436, 2332}

$$\int x \log \left(c\left(a+b x^2\right)^p\right) \, dx=\frac{\left(a+b x^2\right) \log \left(c\left(a+b x^2\right)^p\right)}{2 b}-\frac{p x^2}{2}$$

```
[In] Int[x*Log[c*(a + b*x^2)^p],x]
```

```
[Out] -1/2*(p*x^2) + ((a + b*x^2)*Log[c*(a + b*x^2)^p])/(2*b)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \log\left(c(a+bx)^p\right)\,dx, x, x^2\right) \\
&= \frac{\text{Subst}\left(\int \log\left(cx^p\right)\,dx, x, a+bx^2\right)}{2b} \\
&= -\frac{px^2}{2} + \frac{(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int x\log\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{1}{2}\left(-px^2+\frac{(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b}\right)$$

```
[In] Integrate[x*Log[c*(a + b*x^2)^p],x]
[Out] (-(p*x^2) + ((a + b*x^2)*Log[c*(a + b*x^2)^p])/b)/2
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\ln\left(c\left(b\,x^2+a\right)^p\right)\left(b\,x^2+a\right)-\left(b\,x^2+a\right)p}{2b}$
default	$\frac{\ln\left(c\left(b\,x^2+a\right)^p\right)\left(b\,x^2+a\right)-\left(b\,x^2+a\right)p}{2b}$
norman	$-\frac{p\,x^2}{2}+\frac{x^2\ln\left(c\,\mathrm{e}^{p\ln\left(b\,x^2+a\right)}\right)}{2}+\frac{pa\ln\left(b\,x^2+a\right)}{2b}$
parts	$\frac{x^2\ln\left(c\left(b\,x^2+a\right)^p\right)}{2}-pb\left(\frac{x^2}{2b}-\frac{a\ln\left(b\,x^2+a\right)}{2b^2}\right)$
parallelrisch	$\frac{x^2\ln\left(c\left(b\,x^2+a\right)^p\right)bp-x^2b\,p^2+\ln\left(c\left(b\,x^2+a\right)^p\right)ap+a\,p^2}{2pb}$
risch	$\frac{x^2\ln\left(\left(b\,x^2+a\right)^p\right)}{2}+\frac{i\pi\,x^2\,\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^2}{4}-\frac{i\pi\,x^2\,\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)\mathrm{csgn}(i}{4}$

```
[In] int(x*ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
[Out] 1/2/b*(ln(c*(b*x^2+a)^p)*(b*x^2+a)-(b*x^2+a)*p)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int x\log\left(c\left(a+bx^2\right)^p\right)\,dx=-\frac{bpx^2-bx^2\log\left(c\right)-\left(bpx^2+ap\right)\log\left(bx^2+a\right)}{2\,b}$$

```
[In] integrate(x*log(c*(b*x^2+a)^p),x, algorithm="fricas")
[Out] -1/2*(b*p*x^2 - b*x^2*log(c) - (b*p*x^2 + a*p)*log(b*x^2 + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

$$\int x\log\left(c\left(a+bx^2\right)^p\right)\,dx=\begin{cases}\frac{a\log\left(c\left(a+bx^2\right)^p\right)}{2b}-\frac{px^2}{2}+\frac{x^2\log\left(c\left(a+bx^2\right)^p\right)}{2} & \text{for}\ b\neq 0\\ \frac{x^2\log\left(a^pc\right)}{2} & \text{otherwise}\end{cases}$$

```
[In] integrate(x*ln(c*(b*x**2+a)**p),x)
[Out] Piecewise((a*log(c*(a + b*x**2)**p)/(2*b) - p*x**2/2 + x**2*log(c*(a + b*x*
*2)**p)/2, Ne(b, 0)), (x**2*log(a**p*c)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int x\log\left(c\left(a+bx^2\right)^p\right)\,dx=-\frac{1}{2}\,bp\left(\frac{x^2}{b}-\frac{a\log\left(bx^2+a\right)}{b^2}\right)+\frac{1}{2}\,x^2\log\left(\left(bx^2+a\right)^pc\right)$$

```
[In] integrate(x*log(c*(b*x^2+a)^p),x, algorithm="maxima")
[Out] -1/2*b*p*(x^2/b - a*log(b*x^2 + a)/b^2) + 1/2*x^2*log((b*x^2 + a)^p*c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int x\log\left(c\left(a+bx^2\right)^p\right)\,dx=-\frac{(bx^2-(bx^2+a)\log\left(bx^2+a\right)+a)p-(bx^2+a)\log\left(c\right)}{2\,b}$$

```
[In] integrate(x*log(c*(b*x^2+a)^p),x, algorithm="giac")
[Out] -1/2*((b*x^2 - (b*x^2 + a)*log(b*x^2 + a) + a)*p - (b*x^2 + a)*log(c))/b
```

Mupad [B] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int x\log\left(c\left(a+bx^2\right)^p\right)\,dx=\frac{x^2\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{2}-\frac{p\,x^2}{2}+\frac{a\,p\,\ln\left(b\,x^2+a\right)}{2\,b}$$

```
[In] int(x*log(c*(a + b*x^2)^p),x)
[Out] (x^2*log(c*(a + b*x^2)^p))/2 - (p*x^2)/2 + (a*p*log(a + b*x^2))/(2*b)
```

3.5 $\int \log \left(c(a+bx^2)^p\right) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 45

$$\int \log \left(c(a+bx^2)^p\right) \, dx = -2px + \frac{2\sqrt{a}p \arctan \left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} + x \log \left(c(a+bx^2)^p\right)$$

```
[Out] -2*p*x+x*ln(c*(b*x^2+a)^p)+2*p*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2498, 327, 211}

$$\int \log \left(c(a+bx^2)^p\right) \, dx = \frac{2\sqrt{a}p \arctan \left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} + x \log \left(c(a+bx^2)^p\right) - 2px$$

```
[In] Int[Log[c*(a + b*x^2)^p],x]
```

```
[Out] -2*p*x + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + x*Log[c*(a + b
*x^2)^p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\log\left(c\left(a+bx^2\right)^p\right)-(2bp)\int\frac{x^2}{a+bx^2}\,dx\\ &= -2px+x\log\left(c\left(a+bx^2\right)^p\right)+(2ap)\int\frac{1}{a+bx^2}\,dx\\ &= -2px+\frac{2\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+x\log\left(c\left(a+bx^2\right)^p\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int\log\left(c\left(a+bx^2\right)^p\right)\,dx=-2px+\frac{2\sqrt{a}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+x\log\left(c\left(a+bx^2\right)^p\right)$$

```
[In] Integrate[Log[c*(a + b*x^2)^p],x]
[Out] -2*p*x + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + x*Log[c*(a + b
*x^2)^p]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result
default	$x\ln\left(c\left(b\,x^2+a\right)^p\right)-2pb\left(\frac{x}{b}-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}\right)$
parts	$x\ln\left(c\left(b\,x^2+a\right)^p\right)-2pb\left(\frac{x}{b}-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}\right)$
risch	$x\ln\left(\left(b\,x^2+a\right)^p\right)+\frac{i\operatorname{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^2\operatorname{csgn}\left(i\left(b\,x^2+a\right)^p\right)x\pi}{2}-\frac{i\pi x\operatorname{csgn}\left(i\left(b\,x^2+a\right)^p\right)\operatorname{csgn}\left(ic\left(b\,x^2+a\right)^p\right)\operatorname{csgn}(ic)}{2}-i\pi x$

```
[In] int(ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
[Out] x*ln(c*(b*x^2+a)^p)-2*p*b*(x/b-a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.38

$$\int \log \left(c\left(a+bx^2\right)^p\right)\,dx = \left[px\log \left(bx^2+a\right)+p\sqrt{-\frac{a}{b}}\log \left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-2\,px+x\log \left(c\right),px\log \left(bx^2+a\right)+2\,p\sqrt{\frac{a}{b}}\arctan \left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-2\,px+x\log \left(c\right)\right]$$

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="fricas")
[Out] [p*x*log(b*x^2 + a) + p*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^
2 + a)) - 2*p*x + x*log(c), p*x*log(b*x^2 + a) + 2*p*sqrt(a/b)*arctan(b*x*s
qrt(a/b)/a) - 2*p*x + x*log(c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(44) = 88.

Time = 2.01 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.22

$$\int \log \left(c\left(a+bx^2\right)^p\right)\,dx = \begin{cases} x\log \left(0^{p}c\right) & \text{for}\: a = 0 \wedge b = 0 \\ x\log \left(a^{p}c\right) & \text{for}\: b = 0 \\ -2px + x\log \left(c\left(bx^{2}\right)^{p}\right) & \text{for}\: a = 0 \\ \frac{2ap\log \left(x-\sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}} - \frac{a\log \left(c\left(a+bx^{2}\right)^{p}\right)}{b\sqrt{-\frac{a}{b}}} - 2px + x\log \left(c\left(a+bx^{2}\right)^{p}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(c*(b*x**2+a)**p),x)
[Out] Piecewise((x*log(0**p*c), Eq(a, 0) & Eq(b, 0)), (x*log(a**p*c), Eq(b, 0)),
(-2*p*x + x*log(c*(b*x**2)**p), Eq(a, 0)), (2*a*p*log(x - sqrt(-a/b))/(b*sq
rt(-a/b)) - a*log(c*(a + b*x**2)**p)/(b*sqrt(-a/b)) - 2*p*x + x*log(c*(a +
b*x**2)**p), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \log \left(c\left(a+bx^{2}\right)^{p}\right)\,dx=2\,bp\left(\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}-\frac{x}{b}\right)+x\log\left(\left(bx^{2}+a\right)^{p}c\right)$$

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] 2*b*p*(a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - x/b) + x*log((b*x^2 + a)^p*c
)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \log \left(c\left(a+bx^{2}\right)^{p}\right)\,dx=px\log\left(bx^{2}+a\right)+\frac{2\,ap\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-(2\,p-\log\left(c\right))x$$

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] p*x*log(b*x^2 + a) + 2*a*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) - (2*p - log(c))
*x
```

Mupad [B] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \log \left(c\left(a+bx^{2}\right)^{p}\right)\,dx=x\,\ln\left(c\,\left(b\,x^{2}+a\right)^{p}\right)-2\,p\,x+\frac{2\,\sqrt{a}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

```
[In] int(log(c*(a + b*x^2)^p),x)
```

```
[Out] x*log(c*(a + b*x^2)^p) - 2*p*x + (2*a^(1/2)*p*atan((b^(1/2)*x)/a^(1/2)))/b^
(1/2)
```

3.6 $\int \frac{\log\left(c(a+bx^2)^p\right)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x}\,dx=\frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)+\frac{1}{2}p\,\mathrm{PolyLog}\left(2,1+\frac{bx^2}{a}\right)$$

```
[Out] 1/2*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)+1/2*p*polylog(2,1+b*x^2/a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2441, 2352}

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x}\,dx=\frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)+\frac{1}{2}p\,\mathrm{PolyLog}\left(2,\frac{bx^2}{a}+1\right)$$

```
[In] Int[Log[c*(a + b*x^2)^p]/x,x]
```

```
[Out] (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/2 + (p*PolyLog[2, 1 + (b*x^2)/a])/
2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x}\,dx, x, x^2\right)\\
&= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right) - \frac{1}{2}(bp)\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx, x, x^2\right)\\
&= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right) + \frac{1}{2}p\text{Li}_2\left(1+\frac{bx^2}{a}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$
\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}\,dx = \frac{1}{2}\left(\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right) + p\,\text{PolyLog}\left(2, \frac{a+bx^2}{a}\right)\right)
$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x,x]
[Out] (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + p*PolyLog[2, (a + b*x^2)/a])/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(40) = 80$.

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.50

method	result
parts	$\ln\left(c\left(b\,x^2+a\right)^p\right)\ln(x) - 2pb\left(\frac{\ln(x)\left(\ln\left(\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}\right)+\ln\left(\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}\right)\right)}{2b} + \frac{\text{dilog}\left(\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}\right)+\text{dilog}\left(\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}\right)}{2b}\right)$
risch	$\ln\left(\left(b\,x^2+a\right)^p\right)\ln(x) - p\ln(x)\ln\left(\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}\right) - p\ln(x)\ln\left(\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}\right) - p\,\text{dilog}\left(\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}\right) - p$

```
[In] int(ln(c*(b*x^2+a)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(c*(b*x^2+a)^p)*ln(x)-2*p*b*(1/2*ln(x)*(ln((-b*x+(-a*b)^(1/2))/(-a*b)^(1/
2))+ln((b*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/b+1/2*(dilog((-b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))+dilog((b*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/b)
```

Fricas [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)}{x}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)/x, x)
```

Sympy [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)}{x}\,dx$$

```
[In] integrate(ln(c*(b*x**2+a)**p)/x,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(39) = 78$.

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.82

$$\begin{aligned}\int \frac{\log\left(c(a+bx^2)^p\right)}{x}\,dx &= \frac{1}{2}\,bp\left(\frac{2\,\log\left(bx^2+a\right)\log\left(x\right)}{b} - \frac{2\,\log\left(\frac{bx^2}{a}+1\right)\log\left(x\right) + \mathrm{Li}_2\left(-\frac{bx^2}{a}\right)}{b}\right)\\ &\quad - p\log\left(bx^2+a\right)\log\left(x\right) + \log\left(\left(bx^2+a\right)^p c\right)\log\left(x\right)\end{aligned}$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(2*log(b*x^2 + a)*log(x)/b - (2*log(b*x^2/a + 1)*log(x) + dilog(-b*
x^2/a))/b) - p*log(b*x^2 + a)*log(x) + log((b*x^2 + a)^p*c)*log(x)
```

Giac [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)}{x}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x,x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x}\,dx = \int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)}{x}\,dx$$

```
[In] int(log(c*(a + b*x^2)^p)/x,x)
[Out] int(log(c*(a + b*x^2)^p)/x, x)
```

3.7 $\int \frac{\log\left(c(a+bx^2)^p\right)}{x^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^2}\,dx = \frac{2\sqrt{b}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\log\left(c(a+bx^2)^p\right)}{x}$$

```
[Out] -ln(c*(b*x^2+a)^p)/x+2*p*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2505, 211}

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^2}\,dx = \frac{2\sqrt{b}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\log\left(c(a+bx^2)^p\right)}{x}$$

```
[In] Int[Log[c*(a + b*x^2)^p]/x^2,x]
```

```
[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] - Log[c*(a + b*x^2)^p]/x
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
```

```
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log\left(c(a+bx^2)^p\right)}{x} + (2bp)\int \frac{1}{a+bx^2}\,dx \\
&= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\log\left(c(a+bx^2)^p\right)}{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^2}\,dx = \frac{2\sqrt{b}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\log\left(c(a+bx^2)^p\right)}{x}$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x^2,x]
[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] - Log[c*(a + b*x^2)^p]/x
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)}{x}+\frac{2pb\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$
risch	$-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{x}-\frac{i\pi a\ \mathrm{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)^{2}-i\pi a\ \mathrm{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)\ \mathrm{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)\ \mathrm{csgn}\left(ic\right)-i\pi a\mathrm{csgn}\left(ic\left(b\,x\right.\right.}{2a}$

```
[In] int(ln(c*(b*x^2+a)^p)/x^2,x,method=_RETURNVERBOSE)
[Out] -ln(c*(b*x^2+a)^p)/x+2*p*b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.39

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^2}\,dx$$
$$= \left[\frac{px\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-p\log\left(bx^2+a\right)-\log\left(c\right)}{x}, \frac{2\,px\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-p\log\left(bx^2+a\right)-\log\left(\right.}{x}\right.$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2,x, algorithm="fricas")
```

```
[Out] [(p*x*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - p*log(b*
x^2 + a) - log(c))/x, (2*p*x*sqrt(b/a)*arctan(x*sqrt(b/a)) - p*log(b*x^2 +
a) - log(c))/x]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(39) = 78$.

Time = 7.75 (sec) , antiderivative size = 258, normalized size of antiderivative = 5.86

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^2}\,dx$$
$$= \begin{cases} -\frac{\log\left(0^p c\right)}{x} \\ -\frac{\log\left(a^p c\right)}{x} \\ -\frac{2p}{x}-\frac{\log\left(c\left(bx^2\right)^p\right)}{x} \\ -\frac{\log\left(0^p c\right)}{x} \\ -\frac{a^2\log\left(c\left(a+bx^2\right)^p\right)}{a^2x+abx^3}-\frac{2apx\sqrt{-\frac{a}{b}}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{\frac{a^2x}{b}+ax^3}-\frac{ax^2\log\left(c\left(a+bx^2\right)^p\right)}{\frac{a^2x}{b}+ax^3}+\frac{ax\sqrt{-\frac{a}{b}}\log\left(c\left(a+bx^2\right)^p\right)}{\frac{a^2x}{b}+ax^3}-\frac{2bpx^3\sqrt{-\frac{a}{b}}\log\left(x-\sqrt{-\frac{a}{b}}\right)}{\frac{a^2x}{b}+ax^3}+ \end{cases}$$

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**2,x)
```

```
[Out] Piecewise((-log(0**p*c)/x, Eq(a, 0) & Eq(b, 0)), (-log(a**p*c)/x, Eq(b, 0))
, (-2*p/x - log(c*(b*x**2)**p)/x, Eq(a, 0)), (-log(0**p*c)/x, Eq(a, -b*x**2
)), (-a**2*log(c*(a + b*x**2)**p)/(a**2*x + a*b*x**3) - 2*a*p*x*sqrt(-a/b)*
log(x - sqrt(-a/b))/(a**2*x/b + a*x**3) - a*x**2*log(c*(a + b*x**2)**p)/(a*
*2*x/b + a*x**3) + a*x*sqrt(-a/b)*log(c*(a + b*x**2)**p)/(a**2*x/b + a*x**3
) - 2*b*p*x**3*sqrt(-a/b)*log(x - sqrt(-a/b))/(a**2*x/b + a*x**3) + b*x**3*
sqrt(-a/b)*log(c*(a + b*x**2)**p)/(a**2*x/b + a*x**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^2}\,dx = \frac{2\,bp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} - \frac{\log\left((bx^2+a)^p c\right)}{x}$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2,x, algorithm="maxima")
```

```
[Out] 2*b*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) - log((b*x^2 + a)^p*c)/x
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^2}\,dx = \frac{2\,bp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} - \frac{p\log\left(bx^2+a\right)}{x} - \frac{\log\left(c\right)}{x}$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2,x, algorithm="giac")
```

```
[Out] 2*b*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) - p*log(b*x^2 + a)/x - log(c)/x
```

Mupad [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^2}\,dx = \frac{2\,\sqrt{b}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{x}$$

```
[In] int(log(c*(a + b*x^2)^p)/x^2,x)
```

```
[Out] (2*b^(1/2)*p*atan((b^(1/2)*x)/a^(1/2)))/a^(1/2) - log(c*(a + b*x^2)^p)/x
```

3.8 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 38

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3} dx = \frac{bp\log(x)}{a} - \frac{(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2ax^2}$$

```
[Out] b*p*ln(x)/a-1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/a/x^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2504, 2442, 36, 29, 31}

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3} dx = -\frac{\log\left(c(a+bx^2)^p\right)}{2x^2} - \frac{bp\log(a+bx^2)}{2a} + \frac{bp\log(x)}{a}$$

```
[In] Int[Log[c*(a + b*x^2)^p]/x^3,x]
```

```
[Out] (b*p*Log[x])/a - (b*p*Log[a + b*x^2])/(2*a) - Log[c*(a + b*x^2)^p]/(2*x^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^2}\,dx,x,x^2\right)\\
&= -\frac{\log\left(c(a+bx^2)^p\right)}{2x^2}+\frac{1}{2}(bp)\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx,x,x^2\right)\\
&= -\frac{\log\left(c(a+bx^2)^p\right)}{2x^2}+\frac{(bp)\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2a}-\frac{(b^2p)\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx,x,x^2\right)}{2a}\\
&= \frac{bp\log(x)}{a}-\frac{bp\log\left(a+bx^2\right)}{2a}-\frac{\log\left(c(a+bx^2)^p\right)}{2x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3}\,dx=\frac{bp\log(x)}{a}-\frac{bp\log\left(a+bx^2\right)}{2a}-\frac{\log\left(c(a+bx^2)^p\right)}{2x^2}$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x^3,x]
[Out] (b*p*Log[x])/a - (b*p*Log[a + b*x^2])/(2*a) - Log[c*(a + b*x^2)^p]/(2*x^2)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)}{2x^{2}}+pb\left(\frac{\ln(x)}{a}-\frac{\ln\left(b\,x^{2}+a\right)}{2a}\right)$
parallelrisch	$\frac{2p^{2}b\ln(x)x^{2}-x^{2}\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)bp-\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)ap}{2x^{2}ap}$
risch	$-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{2x^{2}}-\frac{i\pi a\ \mathrm{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)^{2}-i\pi a\ \mathrm{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)\ \mathrm{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)\ \mathrm{csgn}(ic)-i\pi a\mathrm{csgn}(ic}{4x^{2}a}$

```
[In] int(ln(c*(b*x^2+a)^p)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(c*(b*x^2+a)^p)/x^2+p*b*(1/a*ln(x)-1/2/a*ln(b*x^2+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3}\,dx = \frac{2\,bpx^2\log\left(x\right)-\left(bpx^2+ap\right)\log\left(bx^2+a\right)-a\log\left(c\right)}{2\,ax^2}$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*p*x^2*log(x) - (b*p*x^2 + a*p)*log(b*x^2 + a) - a*log(c))/(a*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3}\,dx = \begin{cases} -\frac{\log\left(c(a+bx^2)^p\right)}{2x^2}+\frac{bp\log\left(x\right)}{a}-\frac{b\log\left(c(a+bx^2)^p\right)}{2a} & \text{for}\ a\neq 0 \\ -\frac{p}{2x^2}-\frac{\log\left(c(bx^2)^p\right)}{2x^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**3,x)
```

```
[Out] Piecewise((-log(c*(a + b*x**2)**p)/(2*x**2) + b*p*log(x)/a - b*log(c*(a + b
**x**2)**p)/(2*a), Ne(a, 0)), (-p/(2*x**2) - log(c*(b*x**2)**p)/(2*x**2), Tr
ue))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^3}\,dx = -\frac{1}{2}\,bp\left(\frac{\log \left(bx^2+a\right)}{a} - \frac{\log \left(x^2\right)}{a}\right) - \frac{\log \left(\left(bx^2+a\right)^p c\right)}{2\,x^2}$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x^3,x, algorithm="maxima")
[Out] -1/2*b*p*(log(b*x^2 + a)/a - log(x^2)/a) - 1/2*log((b*x^2 + a)^p*c)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.53

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^3}\,dx = -\frac{\frac{b^2p\log\left(bx^2+a\right)}{a} - \frac{b^2p\log\left(bx^2\right)}{a} + \frac{bp\log\left(bx^2+a\right)}{x^2} + \frac{b\log\left(c\right)}{x^2}}{2\,b}$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x^3,x, algorithm="giac")
[Out] -1/2*(b^2*p*log(b*x^2 + a)/a - b^2*p*log(b*x^2)/a + b*p*log(b*x^2 + a)/x^2
+ b*log(c)/x^2)/b
```

Mupad [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^3}\,dx = \frac{b\,p\,\ln\left(x\right)}{a} - \frac{b\,p\,\ln\left(b\,x^2+a\right)}{2\,a} - \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{2\,x^2}$$

```
[In] int(log(c*(a + b*x^2)^p)/x^3,x)
[Out] (b*p*log(x))/a - (b*p*log(a + b*x^2))/(2*a) - log(c*(a + b*x^2)^p)/(2*x^2)
```

3.9 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^4} dx$

Optimal result

Integrand size = 16, antiderivative size = 60

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^4} dx = -\frac{2bp}{3ax} - \frac{2b^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{\log\left(c(a+bx^2)^p\right)}{3x^3}$$

```
[Out] -2/3*b*p/a/x-2/3*b^(3/2)*p*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)-1/3*ln(c*(b*x^
2+a)^p)/x^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2505, 331, 211}

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^4} dx = -\frac{2b^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{\log\left(c(a+bx^2)^p\right)}{3x^3} - \frac{2bp}{3ax}$$

```
[In] Int[Log[c*(a + b*x^2)^p]/x^4,x]
```

```
[Out] (-2*b*p)/(3*a*x) - (2*b^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*a^(3/2)) -
Log[c*(a + b*x^2)^p]/(3*x^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\log\left(c(a+bx^2)^p\right)}{3x^3}+\frac{1}{3}(2bp)\int\frac{1}{x^2\left(a+bx^2\right)}\,dx\\
&= -\frac{2bp}{3ax}-\frac{\log\left(c(a+bx^2)^p\right)}{3x^3}-\frac{(2b^2p)\int\frac{1}{a+bx^2}\,dx}{3a}\\
&= -\frac{2bp}{3ax}-\frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{\log\left(c(a+bx^2)^p\right)}{3x^3}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

$$
\int\frac{\log\left(c(a+bx^2)^p\right)}{x^4}\,dx=-\frac{2bp\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{bx^2}{a}\right)}{3ax}-\frac{\log\left(c(a+bx^2)^p\right)}{3x^3}
$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x^4,x]
[Out] (-2*b*p*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x^2)/a)])/(3*a*x) - Log[c*(a +
 b*x^2)^p]/(3*x^3)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)}{3x^{3}}+\frac{2pb\left(-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{1}{ax}\right)}{3}$
risch	$-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{3x^{3}}-\frac{i\pi a\ \mathrm{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)^{2}-i\pi a\ \mathrm{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)\mathrm{csgn}\left(ic\right)-i\pi a\mathrm{csgn}\left(ic\left(b\,x^{2}+\right.\right.}{}$

[In] int(ln(c*(b*x^2+a)^p)/x^4,x,method=_RETURNVERBOSE)

[Out] -1/3*ln(c*(b*x^2+a)^p)/x^3+2/3*p*b*(-1/a*b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/a/x)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.25

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^4}\,dx = \left[\frac{bpx^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2\,bpx^2-ap\log\left(bx^2+a\right)-a\log\left(c\right)}{3\,ax^3}, -\frac{2\,bpx^3\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+2\,bpx^2+ap\log\left(bx^2+a\right)+a\log\left(c\right)}{3\,ax^3}\right]$$

[In] integrate(log(c*(b*x^2+a)^p)/x^4,x, algorithm="fricas")

[Out] [1/3*(b*p*x^3*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*b*p*x^2 - a*p*log(b*x^2 + a) - a*log(c))/(a*x^3), -1/3*(2*b*p*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*b*p*x^2 + a*p*log(b*x^2 + a) + a*log(c))/(a*x^3)
]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(56) = 112$.

Time = 38.77 (sec) , antiderivative size = 496, normalized size of antiderivative = 8.27

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^4}\,dx$$
$$= \begin{cases} -\frac{\log(0^pc)}{3x^3} \\ -\frac{\log(a^pc)}{3x^3} \\ -\frac{2p}{9x^3} - \frac{\log\left(c(bx^2)^p\right)}{3x^3} \\ -\frac{\log(0^pc)}{3x^3} \\ -\frac{a^2\sqrt{-\frac{a}{b}}\log\left(c(a+bx^2)^p\right)}{3a^2x^3\sqrt{-\frac{a}{b}}+3abx^5\sqrt{-\frac{a}{b}}} - \frac{2apx^3\log\left(x-\sqrt{-\frac{a}{b}}\right)}{\frac{3a^2x^3\sqrt{-\frac{a}{b}}}{b}+3ax^5\sqrt{-\frac{a}{b}}} - \frac{2apx^2\sqrt{-\frac{a}{b}}}{\frac{3a^2x^3\sqrt{-\frac{a}{b}}}{b}+3ax^5\sqrt{-\frac{a}{b}}} + \frac{ax^3\log\left(c(a+bx^2)^p\right)}{\frac{3a^2x^3\sqrt{-\frac{a}{b}}}{b}+3ax^5\sqrt{-\frac{a}{b}}} - \frac{ax^2\sqrt{-\frac{a}{b}}\log\left(c(a+bx\right.}{\frac{3a^2x^3\sqrt{-\frac{a}{b}}}{b}+3ax^5\sqrt{\right.} \end{cases}$$

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**4,x)
```

```
[Out] Piecewise((-log(0**p*c)/(3*x**3), Eq(a, 0) & Eq(b, 0)), (-log(a**p*c)/(3*x*
*3), Eq(b, 0)), (-2*p/(9*x**3) - log(c*(b*x**2)**p)/(3*x**3), Eq(a, 0)), (-
log(0**p*c)/(3*x**3), Eq(a, -b*x**2)), (-a**2*sqrt(-a/b)*log(c*(a + b*x**2)
**p)/(3*a**2*x**3*sqrt(-a/b) + 3*a*b*x**5*sqrt(-a/b)) - 2*a*p*x**3*log(x -
sqrt(-a/b))/(3*a**2*x**3*sqrt(-a/b)/b + 3*a*x**5*sqrt(-a/b)) - 2*a*p*x**2*s
qrt(-a/b)/(3*a**2*x**3*sqrt(-a/b)/b + 3*a*x**5*sqrt(-a/b)) + a*x**3*log(c*(
a + b*x**2)**p)/(3*a**2*x**3*sqrt(-a/b)/b + 3*a*x**5*sqrt(-a/b)) - a*x**2*s
qrt(-a/b)*log(c*(a + b*x**2)**p)/(3*a**2*x**3*sqrt(-a/b)/b + 3*a*x**5*sqrt(
-a/b)) - 2*b*p*x**5*log(x - sqrt(-a/b))/(3*a**2*x**3*sqrt(-a/b)/b + 3*a*x**
5*sqrt(-a/b)) - 2*b*p*x**4*sqrt(-a/b)/(3*a**2*x**3*sqrt(-a/b)/b + 3*a*x**5*
sqrt(-a/b)) + b*x**5*log(c*(a + b*x**2)**p)/(3*a**2*x**3*sqrt(-a/b)/b + 3*a
*x**5*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^4}\,dx = -\frac{2}{3}\,bp\left(\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} + \frac{1}{ax}\right) - \frac{\log\left((bx^2+a)^pc\right)}{3\,x^3}$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x^4,x, algorithm="maxima")
```

```
[Out] -2/3*b*p*(b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/(a*x)) - 1/3*log((b*x^2
 + a)^p*c)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^4}\,dx = -\frac{2\,b^2p\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{3\,\sqrt{ab}a} - \frac{p\log\left(bx^2+a\right)}{3\,x^3} - \frac{2\,bpx^2+a\log\left(c\right)}{3\,ax^3}$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x^4,x, algorithm="giac")
```

```
[Out] -2/3*b^2*p*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/3*p*log(b*x^2 + a)/x^3 -
 1/3*(2*b*p*x^2 + a*log(c))/(a*x^3)
```

Mupad [B] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^4}\,dx = -\frac{\ln\left(c\,(b\,x^2+a)^p\right)}{3\,x^3} - \frac{2\,b^{3/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3\,a^{3/2}} - \frac{2\,b\,p}{3\,a\,x}$$

```
[In] int(log(c*(a + b*x^2)^p)/x^4,x)
```

```
[Out] - log(c*(a + b*x^2)^p)/(3*x^3) - (2*b^(3/2)*p*atan((b^(1/2)*x)/a^(1/2)))/(3
*a^(3/2)) - (2*b*p)/(3*a*x)
```

3.10 $\int \frac{\log\left(c(a+bx^2)^p\right)}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 64

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^5}\,dx = -\frac{bp}{4ax^2} - \frac{b^2p\log(x)}{2a^2} + \frac{b^2p\log\left(a+bx^2\right)}{4a^2} - \frac{\log\left(c(a+bx^2)^p\right)}{4x^4}$$

```
[Out] -1/4*b*p/a/x^2-1/2*b^2*p*ln(x)/a^2+1/4*b^2*p*ln(b*x^2+a)/a^2-1/4*ln(c*(b*x^
2+a)^p)/x^4
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2442, 46}

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^5}\,dx = \frac{b^2p\log\left(a+bx^2\right)}{4a^2} - \frac{b^2p\log(x)}{2a^2} - \frac{\log\left(c(a+bx^2)^p\right)}{4x^4} - \frac{bp}{4ax^2}$$

```
[In] Int[Log[c*(a + b*x^2)^p]/x^5,x]
```

```
[Out] -1/4*(b*p)/(a*x^2) - (b^2*p*Log[x])/(2*a^2) + (b^2*p*Log[a + b*x^2])/(4*a^2
) - Log[c*(a + b*x^2)^p]/(4*x^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{4x^4} + \frac{1}{4}(bp)\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx, x, x^2\right)\\
&= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{4x^4} + \frac{1}{4}(bp)\text{Subst}\left(\int \left(\frac{1}{ax^2} - \frac{b}{a^2x} + \frac{b^2}{a^2(a+bx)}\right)\,dx, x, x^2\right)\\
&= -\frac{bp}{4ax^2} - \frac{b^2p\log(x)}{2a^2} + \frac{b^2p\log\left(a+bx^2\right)}{4a^2} - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{4x^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^5}\,dx = \frac{1}{4}bp\left(-\frac{1}{ax^2} - \frac{2b\log(x)}{a^2} + \frac{b\log\left(a+bx^2\right)}{a^2}\right) - \frac{\log\left(c\left(a+bx^2\right)^p\right)}{4x^4}$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x^5,x]
[Out] (b*p*(-(1/(a*x^2)) - (2*b*Log[x])/a^2 + (b*Log[a + b*x^2])/a^2))/4 - Log[c*
(a + b*x^2)^p]/(4*x^4)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)}{4x^{4}}+\frac{pb\left(-\frac{1}{2a\,x^{2}}-\frac{b\ln(x)}{a^{2}}+\frac{b\ln\left(b\,x^{2}+a\right)}{2a^{2}}\right)}{2}$
parallelrisch	$-\frac{2b^{2}p^{2}\ln(x)x^{4}-x^{4}\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)b^{2}p-b^{2}p^{2}x^{4}+ab\,p^{2}x^{2}+\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)a^{2}p}{4x^{4}a^{2}p}$
risch	$-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{4x^{4}}-\frac{4b^{2}p\ln(x)x^{4}-2b^{2}p\ln\left(-b\,x^{2}-a\right)x^{4}+i\pi\,a^{2}\operatorname{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)\operatorname{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)^{2}-i\pi\,a^{2}\operatorname{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right.}{}$

```
[In] int(ln(c*(b*x^2+a)^p)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln(c*(b*x^2+a)^p)/x^4+1/2*p*b*(-1/2/a/x^2-1/a^2*b*ln(x)+1/2*b/a^2*ln(b
*x^2+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^5}\,dx = -\frac{2\,b^2px^4\log(x)+abpx^2+a^2\log(c)-\left(b^2px^4-a^2p\right)\log\left(bx^2+a\right)}{4\,a^2x^4}$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*p*x^4*log(x) + a*b*p*x^2 + a^2*log(c) - (b^2*p*x^4 - a^2*p)*log
(b*x^2 + a))/(a^2*x^4)
```

Sympy [A] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.30

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^5}\,dx = \begin{cases} -\frac{\log\left(c(a+bx^2)^p\right)}{4x^4}-\frac{bp}{4ax^2}-\frac{b^2p\log(x)}{2a^2}+\frac{b^2\log\left(c(a+bx^2)^p\right)}{4a^2} & \text{for}\ a\neq 0\\ -\frac{p}{8x^4}-\frac{\log\left(c(bx^2)^p\right)}{4x^4} & \text{otherwise}\end{cases}$$

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**5,x)
```

```
[Out] Piecewise((-log(c*(a + b*x**2)**p)/(4*x**4) - b*p/(4*a*x**2) - b**2*p*log(x
)/(2*a**2) + b**2*log(c*(a + b*x**2)**p)/(4*a**2), Ne(a, 0)), (-p/(8*x**4)
- log(c*(b*x**2)**p)/(4*x**4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^5}\,dx = \frac{1}{4}\,bp\left(\frac{b\log\left(bx^2+a\right)}{a^2} - \frac{b\log\left(x^2\right)}{a^2} - \frac{1}{ax^2}\right) - \frac{\log\left((bx^2+a)^p c\right)}{4\,x^4}$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*b*p*(b*log(b*x^2 + a)/a^2 - b*log(x^2)/a^2 - 1/(a*x^2)) - 1/4*log((b*x^
2 + a)^p*c)/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(56) = 112$.

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.06

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^5}\,dx = -\frac{\frac{b^3p\log(bx^2+a)}{(bx^2+a)^2-2\,(bx^2+a)a+a^2} - \frac{b^3p\log(bx^2+a)}{a^2} + \frac{b^3p\log(bx^2)}{a^2} + \frac{(bx^2+a)b^3p-ab^3p+ab^3\log(c)}{(bx^2+a)^2a-2\,(bx^2+a)a^2+a^3}}{4\,b}$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x^5,x, algorithm="giac")
```

```
[Out] -1/4*(b^3*p*log(b*x^2 + a)/((b*x^2 + a)^2 - 2*(b*x^2 + a)*a + a^2) - b^3*p*
log(b*x^2 + a)/a^2 + b^3*p*log(b*x^2)/a^2 + ((b*x^2 + a)*b^3*p - a*b^3*p +
a*b^3*log(c))/((b*x^2 + a)^2*a - 2*(b*x^2 + a)*a^2 + a^3))/b
```

Mupad [B] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^5}\,dx = \frac{b^2\,p\,\ln\left(b\,x^2+a\right)}{4\,a^2} - \frac{\ln\left(c\,\left(b\,x^2+a\right)^p\right)}{4\,x^4} - \frac{b^2\,p\,\ln\left(x\right)}{2\,a^2} - \frac{b\,p}{4\,a\,x^2}$$

```
[In] int(log(c*(a + b*x^2)^p)/x^5,x)
```

```
[Out] (b^2*p*log(a + b*x^2))/(4*a^2) - log(c*(a + b*x^2)^p)/(4*x^4) - (b^2*p*log(
x))/(2*a^2) - (b*p)/(4*a*x^2)
```

3.11 $\int \frac{\log\left(c(a+bx^2)^p\right)}{x^6} dx$

Optimal result

Integrand size = 16, antiderivative size = 74

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^6} dx = -\frac{2bp}{15ax^3}+\frac{2b^2p}{5a^2x}+\frac{2b^{5/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{5a^{5/2}}-\frac{\log\left(c(a+bx^2)^p\right)}{5x^5}$$

```
[Out] -2/15*b*p/a/x^3+2/5*b^2*p/a^2/x+2/5*b^(5/2)*p*arctan(x*b^(1/2)/a^(1/2))/a^(
5/2)-1/5*ln(c*(b*x^2+a)^p)/x^5
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2505, 331, 211}

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^6} dx = \frac{2b^{5/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{5a^{5/2}}+\frac{2b^2p}{5a^2x}-\frac{\log\left(c(a+bx^2)^p\right)}{5x^5}-\frac{2bp}{15ax^3}$$

```
[In] Int[Log[c*(a + b*x^2)^p]/x^6,x]
```

```
[Out] (-2*b*p)/(15*a*x^3) + (2*b^2*p)/(5*a^2*x) + (2*b^(5/2)*p*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(5*a^(5/2)) - Log[c*(a + b*x^2)^p]/(5*x^5)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log\left(c(a+bx^2)^p\right)}{5x^5}+\frac{1}{5}(2bp)\int\frac{1}{x^4\left(a+bx^2\right)}\,dx\\
&= -\frac{2bp}{15ax^3}-\frac{\log\left(c(a+bx^2)^p\right)}{5x^5}-\frac{(2b^2p)\int\frac{1}{x^2(a+bx^2)}\,dx}{5a}\\
&= -\frac{2bp}{15ax^3}+\frac{2b^2p}{5a^2x}-\frac{\log\left(c(a+bx^2)^p\right)}{5x^5}+\frac{(2b^3p)\int\frac{1}{a+bx^2}\,dx}{5a^2}\\
&= -\frac{2bp}{15ax^3}+\frac{2b^2p}{5a^2x}+\frac{2b^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{5a^{5/2}}-\frac{\log\left(c(a+bx^2)^p\right)}{5x^5}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.66

$$\int\frac{\log\left(c(a+bx^2)^p\right)}{x^6}\,dx=-\frac{2bp\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\frac{bx^2}{a}\right)}{15ax^3}-\frac{\log\left(c(a+bx^2)^p\right)}{5x^5}$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x^6,x]
[Out] (-2*b*p*Hypergeometric2F1[-3/2, 1, -1/2, -((b*x^2)/a)])/(15*a*x^3) - Log[c*
(a + b*x^2)^p]/(5*x^5)
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)}{5x^{5}}+\frac{2pb\left(-\frac{1}{3a\,x^{3}}+\frac{b}{a^{2}x}+\frac{b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^{2}\sqrt{ab}}\right)}{5}$
risch	$-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{5x^{5}}-\frac{-6\left(\sum_{\_R=\operatorname{RootOf}\left(a^{5}\_Z^{2}+b^{5}p^{2}\right)}\_R\ln\left(\left(3\_R^{2}a^{5}+2b^{5}p^{2}\right)x-a^{3}b^{2}p\_R\right)\right)a^{2}x^{5}+3i\pi\,a^{2}\operatorname{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)\operatorname{cs}}{}$

```
[In] int(ln(c*(b*x^2+a)^p)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*ln(c*(b*x^2+a)^p)/x^5+2/5*p*b*(-1/3/a/x^3+b/a^2/x+b^2/a^2/(a*b)^(1/2)*
arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.30

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^6}\,dx = \left[\frac{3\,b^2px^5\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+6\,b^2px^4-2\,abpx^2-3\,a^2p\log\left(bx^2+a\right)-3\,a^2\log\left(c\right)}{15\,a^2x^5},\ \frac{6\,b^2px^5\sqrt{\frac{b}{a}}\,\mathrm{ar}}{}\right.$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x^6,x, algorithm="fricas")
```

```
[Out] [1/15*(3*b^2*p*x^5*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a
)) + 6*b^2*p*x^4 - 2*a*b*p*x^2 - 3*a^2*p*log(b*x^2 + a) - 3*a^2*log(c))/(a^
2*x^5), 1/15*(6*b^2*p*x^5*sqrt(b/a)*arctan(x*sqrt(b/a)) + 6*b^2*p*x^4 - 2*a
*b*p*x^2 - 3*a^2*p*log(b*x^2 + a) - 3*a^2*log(c))/(a^2*x^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(70) = 140$.

Time = 162.33 (sec) , antiderivative size = 583, normalized size of antiderivative = 7.88

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^6}\,dx$$
$$= \begin{cases} -\frac{\log(0^p c)}{5x^5} \\ -\frac{\log(a^p c)}{5x^5} \\ -\frac{2p}{25x^5} - \frac{\log\left(c(bx^2)^p\right)}{5x^5} \\ -\frac{\log(0^p c)}{5x^5} \\ -\frac{3a^3\sqrt{-\frac{a}{b}}\log\left(c(a+bx^2)^p\right)}{15a^3x^5\sqrt{-\frac{a}{b}}+15a^2bx^7\sqrt{-\frac{a}{b}}} - \frac{2a^2px^2\sqrt{-\frac{a}{b}}}{\frac{15a^3x^5\sqrt{-\frac{a}{b}}}{b}+15a^2x^7\sqrt{-\frac{a}{b}}} - \frac{3a^2x^2\sqrt{-\frac{a}{b}}\log\left(c(a+bx^2)^p\right)}{\frac{15a^3x^5\sqrt{-\frac{a}{b}}}{b}+15a^2x^7\sqrt{-\frac{a}{b}}} + \frac{6abpx^5\log\left(x-\sqrt{-\frac{a}{b}}\right)}{\frac{15a^3x^5\sqrt{-\frac{a}{b}}}{b}+15a^2x^7\sqrt{-\frac{a}{b}}} + \frac{4a}{\frac{15a^3x^5\sqrt{}}{b}} \end{cases}$$

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**6,x)
```

```
[Out] Piecewise((-log(0**p*c)/(5*x**5), Eq(a, 0) & Eq(b, 0)), (-log(a**p*c)/(5*x*
*5), Eq(b, 0)), (-2*p/(25*x**5) - log(c*(b*x**2)**p)/(5*x**5), Eq(a, 0)), (
-log(0**p*c)/(5*x**5), Eq(a, -b*x**2)), (-3*a**3*sqrt(-a/b)*log(c*(a + b*x*
*2)**p)/(15*a**3*x**5*sqrt(-a/b) + 15*a**2*b*x**7*sqrt(-a/b)) - 2*a**2*p*x*
*2*sqrt(-a/b)/(15*a**3*x**5*sqrt(-a/b)/b + 15*a**2*x**7*sqrt(-a/b)) - 3*a**
2*x**2*sqrt(-a/b)*log(c*(a + b*x**2)**p)/(15*a**3*x**5*sqrt(-a/b)/b + 15*a*
*2*x**7*sqrt(-a/b)) + 6*a*b*p*x**5*log(x - sqrt(-a/b))/(15*a**3*x**5*sqrt(-
a/b)/b + 15*a**2*x**7*sqrt(-a/b)) + 4*a*b*p*x**4*sqrt(-a/b)/(15*a**3*x**5*s
qrt(-a/b)/b + 15*a**2*x**7*sqrt(-a/b)) - 3*a*b*x**5*log(c*(a + b*x**2)**p)/
(15*a**3*x**5*sqrt(-a/b)/b + 15*a**2*x**7*sqrt(-a/b)) + 6*b**2*p*x**7*log(x
 - sqrt(-a/b))/(15*a**3*x**5*sqrt(-a/b)/b + 15*a**2*x**7*sqrt(-a/b)) + 6*b*
*2*p*x**6*sqrt(-a/b)/(15*a**3*x**5*sqrt(-a/b)/b + 15*a**2*x**7*sqrt(-a/b))
- 3*b**2*x**7*log(c*(a + b*x**2)**p)/(15*a**3*x**5*sqrt(-a/b)/b + 15*a**2*x
**7*sqrt(-a/b)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^6}\,dx = \frac{2}{15}\,bp\left(\frac{3\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{3\,bx^2-a}{a^2x^3}\right) - \frac{\log\left((bx^2+a)^p c\right)}{5\,x^5}$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x^6,x, algorithm="maxima")
```

```
[Out] 2/15*b*p*(3*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + (3*b*x^2 - a)/(a^2*
x^3)) - 1/5*log((b*x^2 + a)^p*c)/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^6}\,dx = \frac{2\,b^3p\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{5\,\sqrt{ab}a^2} - \frac{p\log\left(bx^2+a\right)}{5\,x^5} + \frac{6\,b^2px^4 - 2\,abpx^2 - 3\,a^2\log\left(c\right)}{15\,a^2x^5}$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x^6,x, algorithm="giac")
```

```
[Out] 2/5*b^3*p*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/5*p*log(b*x^2 + a)/x^5
+ 1/15*(6*b^2*p*x^4 - 2*a*b*p*x^2 - 3*a^2*log(c))/(a^2*x^5)
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^6}\,dx = \frac{2\,b^{5/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{5\,a^{5/2}} - \frac{\frac{2\,b\,p}{3\,a} - \frac{2\,b^2\,p\,x^2}{a^2}}{5\,x^3} - \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{5\,x^5}$$

```
[In] int(log(c*(a + b*x^2)^p)/x^6,x)
```

```
[Out] (2*b^(5/2)*p*atan((b^(1/2)*x)/a^(1/2)))/(5*a^(5/2)) - ((2*b*p)/(3*a) - (2*b
^2*p*x^2)/a^2)/(5*x^3) - log(c*(a + b*x^2)^p)/(5*x^5)
```

3.12 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^7}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 78

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^7}\,dx = -\frac{bp}{12ax^4}+\frac{b^2p}{6a^2x^2}+\frac{b^3p\log(x)}{3a^3}-\frac{b^3p\log\left(a+bx^2\right)}{6a^3}-\frac{\log\left(c(a+bx^2)^p\right)}{6x^6}$$

```
[Out] -1/12*b*p/a/x^4+1/6*b^2*p/a^2/x^2+1/3*b^3*p*ln(x)/a^3-1/6*b^3*p*ln(b*x^2+a)
/a^3-1/6*ln(c*(b*x^2+a)^p)/x^6
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2442, 46}

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^7}\,dx = -\frac{b^3p\log\left(a+bx^2\right)}{6a^3}+\frac{b^3p\log(x)}{3a^3}+\frac{b^2p}{6a^2x^2}-\frac{\log\left(c(a+bx^2)^p\right)}{6x^6}-\frac{bp}{12ax^4}$$

```
[In] Int[Log[c*(a + b*x^2)^p]/x^7,x]
```

```
[Out] -1/12*(b*p)/(a*x^4) + (b^2*p)/(6*a^2*x^2) + (b^3*p*Log[x])/(3*a^3) - (b^3*p
*Log[a + b*x^2])/(6*a^3) - Log[c*(a + b*x^2)^p]/(6*x^6)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^4}\,dx,x,x^2\right)\\
&= -\frac{\log\left(c(a+bx^2)^p\right)}{6x^6}+\frac{1}{6}(bp)\text{Subst}\left(\int \frac{1}{x^3(a+bx)}\,dx,x,x^2\right)\\
&= -\frac{\log\left(c(a+bx^2)^p\right)}{6x^6}+\frac{1}{6}(bp)\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)\,dx,x,x^2\right)\\
&= -\frac{bp}{12ax^4}+\frac{b^2p}{6a^2x^2}+\frac{b^3p\log(x)}{3a^3}-\frac{b^3p\log\left(a+bx^2\right)}{6a^3}-\frac{\log\left(c(a+bx^2)^p\right)}{6x^6}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

$$
\int \frac{\log\left(c(a+bx^2)^p\right)}{x^7}\,dx = \frac{1}{3}bp\left(-\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log\left(a+bx^2\right)}{2a^3}\right) - \frac{\log\left(c(a+bx^2)^p\right)}{6x^6}
$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]/x^7,x]
```

```
[Out] (b*p*(-1/4*1/(a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^
2])/(2*a^3)))/3 - Log[c*(a + b*x^2)^p]/(6*x^6)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)}{6x^{6}}+\frac{pb\left(-\frac{1}{4a\,x^{4}}+\frac{b^{2}\ln\left(x\right)}{a^{3}}+\frac{b}{2a^{2}x^{2}}-\frac{b^{2}\ln\left(b\,x^{2}+a\right)}{2a^{3}}\right)}{3}$
parallelrisch	$\frac{4b^{3}p^{2}\ln\left(x\right)x^{6}-2x^{6}\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)b^{3}p-2x^{6}b^{3}p^{2}+2x^{4}a\,b^{2}p^{2}-x^{2}a^{2}b\,p^{2}-2\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)a^{3}p}{12x^{6}p\,a^{3}}$
risch	$-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{6x^{6}}-\frac{2b^{3}p\ln\left(b\,x^{2}+a\right)x^{6}-4b^{3}p\ln\left(x\right)x^{6}+i\pi\,a^{3}\,\mathrm{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{2}+a\right)^{p}\right)^{2}-i\pi\,a^{3}\,\mathrm{csgn}\left(i\left(b\,x^{2}+a\right)^{p}\right)\,\mathrm{csg}$

[In] int(ln(c*(b*x^2+a)^p)/x^7,x,method=_RETURNVERBOSE)

[Out] -1/6*ln(c*(b*x^2+a)^p)/x^6+1/3*p*b*(-1/4/a/x^4+b^2/a^3*ln(x)+1/2*b/a^2/x^2-1/2*b^2/a^3*ln(b*x^2+a))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^7}\,dx = \frac{4\,b^3px^6\log\left(x\right)+2\,ab^2px^4-a^2bpx^2-2\,a^3\log\left(c\right)-2\,\left(b^3px^6+a^3p\right)\log\left(bx^2+a\right)}{12\,a^3x^6}$$

[In] integrate(log(c*(b*x^2+a)^p)/x^7,x, algorithm="fricas")

[Out] 1/12*(4*b^3*p*x^6*log(x) + 2*a*b^2*p*x^4 - a^2*b*p*x^2 - 2*a^3*log(c) - 2*(b^3*p*x^6 + a^3*p)*log(b*x^2 + a))/(a^3*x^6)

Sympy [A] (verification not implemented)

Time = 6.67 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.24

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^7}\,dx = \begin{cases} -\frac{\log\left(c(a+bx^2)^p\right)}{6x^6}-\frac{bp}{12ax^4}+\frac{b^2p}{6a^2x^2}+\frac{b^3p\log\left(x\right)}{3a^3}-\frac{b^3\log\left(c(a+bx^2)^p\right)}{6a^3} & \text{for}\ a\neq 0 \\ -\frac{p}{18x^6}-\frac{\log\left(c(bx^2)^p\right)}{6x^6} & \text{otherwise} \end{cases}$$

[In] integrate(ln(c*(b*x**2+a)**p)/x**7,x)

[Out] Piecewise((-log(c*(a + b*x**2)**p)/(6*x**6) - b*p/(12*a*x**4) + b**2*p/(6*a**2*x**2) + b**3*p*log(x)/(3*a**3) - b**3*log(c*(a + b*x**2)**p)/(6*a**3), Ne(a, 0)), (-p/(18*x**6) - log(c*(b*x**2)**p)/(6*x**6), True))

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^7}\,dx = -\frac{1}{12}\,bp\left(\frac{2\,b^2\log\left(bx^2+a\right)}{a^3} - \frac{2\,b^2\log\left(x^2\right)}{a^3} - \frac{2\,bx^2-a}{a^2x^4}\right) - \frac{\log\left(\left(bx^2+a\right)^pc\right)}{6\,x^6}$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*b*p*(2*b^2*log(b*x^2 + a)/a^3 - 2*b^2*log(x^2)/a^3 - (2*b*x^2 - a)/(a
^2*x^4)) - 1/6*log((b*x^2 + a)^p*c)/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(68) = 136$.

Time = 0.28 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.45

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^7}\,dx = -\frac{\frac{2\,b^4p\log\left(bx^2+a\right)}{\left(bx^2+a\right)^3-3\left(bx^2+a\right)^2a+3\left(bx^2+a\right)a^2-a^3} + \frac{2\,b^4p\log\left(bx^2+a\right)}{a^3} - \frac{2\,b^4p\log\left(bx^2\right)}{a^3} - \frac{2\left(bx^2+a\right)^2b^4p-5\left(bx^2+a\right)ab^4p+3\,a^2b^4p-2\,a^2b^4\log(c}{\left(bx^2+a\right)^3a^2-3\left(bx^2+a\right)^2a^3+3\left(bx^2+a\right)a^4-a^5}}{12\,b}$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x^7,x, algorithm="giac")
```

```
[Out] -1/12*(2*b^4*p*log(b*x^2 + a)/((b*x^2 + a)^3 - 3*(b*x^2 + a)^2*a + 3*(b*x^2
 + a)*a^2 - a^3) + 2*b^4*p*log(b*x^2 + a)/a^3 - 2*b^4*p*log(b*x^2)/a^3 - (2
*(b*x^2 + a)^2*b^4*p - 5*(b*x^2 + a)*a*b^4*p + 3*a^2*b^4*p - 2*a^2*b^4*log(
c))/((b*x^2 + a)^3*a^2 - 3*(b*x^2 + a)^2*a^3 + 3*(b*x^2 + a)*a^4 - a^5))/b
```

Mupad [B] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^7}\,dx = \frac{b^2\,p}{6\,a^2\,x^2} - \frac{b^3\,p\,\ln\left(b\,x^2+a\right)}{6\,a^3} - \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{6\,x^6} + \frac{b^3\,p\,\ln\left(x\right)}{3\,a^3} - \frac{b\,p}{12\,a\,x^4}$$

```
[In] int(log(c*(a + b*x^2)^p)/x^7,x)
```

```
[Out] (b^2*p)/(6*a^2*x^2) - (b^3*p*log(a + b*x^2))/(6*a^3) - log(c*(a + b*x^2)^p)
/(6*x^6) + (b^3*p*log(x))/(3*a^3) - (b*p)/(12*a*x^4)
```

3.13 $\int x^5 \log \left(c \left(a+b x^3\right)^p\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 59

$$\int x^5 \log \left(c \left(a+b x^3\right)^p\right) \, dx=\frac{a p x^3}{6 b}-\frac{p x^6}{12}-\frac{a^2 p \log \left(a+b x^3\right)}{6 b^2}+\frac{1}{6} x^6 \log \left(c \left(a+b x^3\right)^p\right)$$

```
[Out] 1/6*a*p*x^3/b-1/12*p*x^6-1/6*a^2*p*ln(b*x^3+a)/b^2+1/6*x^6*ln(c*(b*x^3+a)^p
)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2442, 45}

$$\int x^5 \log \left(c \left(a+b x^3\right)^p\right) \, dx=-\frac{a^2 p \log \left(a+b x^3\right)}{6 b^2}+\frac{1}{6} x^6 \log \left(c \left(a+b x^3\right)^p\right)+\frac{a p x^3}{6 b}-\frac{p x^6}{12}$$

```
[In] Int[x^5*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (a*p*x^3)/(6*b) - (p*x^6)/12 - (a^2*p*Log[a + b*x^3])/(6*b^2) + (x^6*Log[c*
(a + b*x^3)^p])/6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x\log\left(c(a+bx)^p\right)\,dx,x,x^3\right)\\
&= \frac{1}{6}x^6\log\left(c\left(a+bx^3\right)^p\right)-\frac{1}{6}(bp)\text{Subst}\left(\int\frac{x^2}{a+bx}\,dx,x,x^3\right)\\
&= \frac{1}{6}x^6\log\left(c\left(a+bx^3\right)^p\right)-\frac{1}{6}(bp)\text{Subst}\left(\int\left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)\,dx,x,x^3\right)\\
&= \frac{apx^3}{6b}-\frac{px^6}{12}-\frac{a^2p\log\left(a+bx^3\right)}{6b^2}+\frac{1}{6}x^6\log\left(c\left(a+bx^3\right)^p\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int x^5\log\left(c\left(a+bx^3\right)^p\right)\,dx=\frac{apx^3}{6b}-\frac{px^6}{12}-\frac{a^2p\log\left(a+bx^3\right)}{6b^2}+\frac{1}{6}x^6\log\left(c\left(a+bx^3\right)^p\right)$$

```
[In] Integrate[x^5*Log[c*(a + b*x^3)^p],x]
[Out] (a*p*x^3)/(6*b) - (p*x^6)/12 - (a^2*p*Log[a + b*x^3])/(6*b^2) + (x^6*Log[c*
(a + b*x^3)^p])/6
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

method	result	size
parts	$\frac{x^{6} \ln\left(c \left(b \,x^{3}+a \right)^{p}\right)}{6}-\frac{p b \left(-\frac{-\frac{1}{2} b \,x^{6}+x^{3} a}{3 b^{2}}+\frac{a^{2} \ln\left(b \,x^{3}+a \right)}{3 b^{3}}\right)}{2}$	57
parallelrisch	$-\frac{-2 x^{6} \ln\left(c \left(b \,x^{3}+a \right)^{p}\right) b^{2}+x^{6} b^{2} p -2 a b p \,x^{3}+2 \ln\left(b \,x^{3}+a \right) a^{2} p +2 a^{2} p}{12 b^{2}}$	63
risch	Expression too large to display	1190

```
[In] int(x^5*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x^6*ln(c*(b*x^3+a)^p)-1/2*p*b*(-1/3/b^2*(-1/2*b*x^6+x^3*a)+1/3*a^2/b^3*
ln(b*x^3+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int x^5 \log \left(c\left(a+bx^3\right)^p\right) \, dx = -\frac{b^2 p x^6 - 2\, b^2 x^6 \log \left(c\right) - 2\, a b p x^3 - 2\, \left(b^2 p x^6 - a^2 p\right) \log \left(b x^3 + a\right)}{12\, b^2}$$

```
[In] integrate(x^5*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] -1/12*(b^2*p*x^6 - 2*b^2*x^6*log(c) - 2*a*b*p*x^3 - 2*(b^2*p*x^6 - a^2*p)*l
og(b*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int x^5 \log \left(c\left(a+bx^3\right)^p\right) \, dx = \begin{cases} -\frac{a^2 \log \left(c\left(a+bx^3\right)^p\right)}{6b^2} + \frac{apx^3}{6b} - \frac{px^6}{12} + \frac{x^6 \log \left(c\left(a+bx^3\right)^p\right)}{6} & \text{for}\: b \neq 0 \\ \frac{x^6 \log \left(a^p c\right)}{6} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**5*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Piecewise((-a**2*log(c*(a + b*x**3)**p)/(6*b**2) + a*p*x**3/(6*b) - p*x**6/
12 + x**6*log(c*(a + b*x**3)**p)/6, Ne(b, 0)), (x**6*log(a**p*c)/6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int x^5 \log \left(c\left(a+bx^3\right)^p\right)\,dx=\frac{1}{6}\,x^6\log\left(\left(bx^3+a\right)^pc\right)-\frac{1}{12}\,bp\left(\frac{2\,a^2\log\left(bx^3+a\right)}{b^3}+\frac{bx^6-2\,ax^3}{b^2}\right)$$

```
[In] integrate(x^5*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] 1/6*x^6*log((b*x^3 + a)^p*c) - 1/12*b*p*(2*a^2*log(b*x^3 + a)/b^3 + (b*x^6
- 2*a*x^3)/b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.64

$$\int x^5 \log \left(c\left(a+bx^3\right)^p\right)\,dx=\frac{2\left(bx^3+a\right)^2p\log\left(bx^3+a\right)-\left(bx^3+a\right)^2p+2\left(bx^3+a\right)^2\log\left(c\right)}{12\,b^2}+\frac{\left(bx^3-\left(bx^3+a\right)\log\left(bx^3+a\right)+a\right)ap-\left(bx^3+a\right)a\log\left(c\right)}{3\,b^2}$$

```
[In] integrate(x^5*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] 1/12*(2*(b*x^3 + a)^2*p*log(b*x^3 + a) - (b*x^3 + a)^2*p + 2*(b*x^3 + a)^2*
log(c))/b^2 + 1/3*((b*x^3 - (b*x^3 + a)*log(b*x^3 + a) + a)*a*p - (b*x^3 +
a)*a*log(c))/b^2
```

Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int x^5 \log \left(c\left(a+bx^3\right)^p\right)\,dx=\frac{x^6\,\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{6}-\frac{p\,x^6}{12}-\frac{a^2\,p\,\ln\left(b\,x^3+a\right)}{6\,b^2}+\frac{a\,p\,x^3}{6\,b}$$

```
[In] int(x^5*log(c*(a + b*x^3)^p),x)
```

```
[Out] (x^6*log(c*(a + b*x^3)^p))/6 - (p*x^6)/12 - (a^2*p*log(a + b*x^3))/(6*b^2)
+ (a*p*x^3)/(6*b)
```

3.14 $\int x^4 \log \left(c \left(a+b x^3\right)^p\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 159

$$\int x^4 \log \left(c \left(a+b x^3\right)^p\right) \, dx = \frac{3 a p x^2}{10 b}-\frac{3 p x^5}{25}+\frac{\sqrt{3} a^{5/3} p \arctan \left(\frac{\sqrt[3]{a}-2 \sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{5 b^{5/3}}+\frac{a^{5/3} p \log \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{5 b^{5/3}}-\frac{a^{5/3} p \log \left(a^{2/3}-\sqrt[3]{a} \sqrt[3]{b} x+b^{2/3} x^2\right)}{10 b^{5/3}}+\frac{1}{5} x^5 \log \left(c \left(a+b x^3\right)^p\right)$$

```
[Out] 3/10*a*p*x^2/b-3/25*p*x^5+1/5*a^(5/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(5/3)-1/10*
a^(5/3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(5/3)+1/5*x^5*ln(c*(b
*x^3+a)^p)+1/5*a^(5/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*
3^(1/2)/b^(5/3)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2505, 308, 298, 31, 648, 631, 210, 642}

$$\int x^4 \log \left(c \left(a+b x^3\right)^p\right) \, dx = \frac{\sqrt{3} a^{5/3} p \arctan \left(\frac{\sqrt[3]{a}-2 \sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{5 b^{5/3}}-\frac{a^{5/3} p \log \left(a^{2/3}-\sqrt[3]{a} \sqrt[3]{b} x+b^{2/3} x^2\right)}{10 b^{5/3}}+\frac{a^{5/3} p \log \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{5 b^{5/3}}+\frac{1}{5} x^5 \log \left(c \left(a+b x^3\right)^p\right)+\frac{3 a p x^2}{10 b}-\frac{3 p x^5}{25}$$

```
[In] Int[x^4*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (3*a*p*x^2)/(10*b) - (3*p*x^5)/25 + (Sqrt[3]*a^(5/3)*p*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(5*b^(5/3)) + (a^(5/3)*p*Log[a^(1/3) + b^(1/
3)*x])/(5*b^(5/3)) - (a^(5/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(10*b^(5/3)) + (x^5*Log[c*(a + b*x^3)^p])/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)-\frac{1}{5}(3bp)\int\frac{x^7}{a+bx^3}\,dx\\
&= \frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)-\frac{1}{5}(3bp)\int\left(-\frac{ax}{b^2}+\frac{x^4}{b}+\frac{a^2x}{b^2\left(a+bx^3\right)}\right)dx\\
&= \frac{3apx^2}{10b}-\frac{3px^5}{25}+\frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)-\frac{\left(3a^2p\right)\int\frac{x}{a+bx^3}\,dx}{5b}\\
&= \frac{3apx^2}{10b}-\frac{3px^5}{25}+\frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)\\
&\quad+\frac{\left(a^{5/3}p\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{5b^{4/3}}-\frac{\left(a^{5/3}p\right)\int\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{5b^{4/3}}\\
&= \frac{3apx^2}{10b}-\frac{3px^5}{25}+\frac{a^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5b^{5/3}}+\frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)\\
&\quad-\frac{\left(a^{5/3}p\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{10b^{5/3}}-\frac{\left(3a^2p\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{10b^{4/3}}\\
&= \frac{3apx^2}{10b}-\frac{3px^5}{25}+\frac{a^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5b^{5/3}}-\frac{a^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{10b^{5/3}}\\
&\quad+\frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)-\frac{\left(3a^{5/3}p\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{5b^{5/3}}\\
&= \frac{3apx^2}{10b}-\frac{3px^5}{25}+\frac{\sqrt{3}a^{5/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{5b^{5/3}}+\frac{a^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5b^{5/3}}\\
&\quad-\frac{a^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{10b^{5/3}}+\frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.43

$$\int x^4 \log \left(c\left(a+bx^3\right)^p\right)\,dx = \frac{3apx^2}{10b} - \frac{3px^5}{25} - \frac{3apx^2\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a}\right)}{10b} + \frac{1}{5}x^5\log\left(c\left(a+bx^3\right)^p\right)$$

```
[In] Integrate[x^4*Log[c*(a + b*x^3)^p],x]
[Out] (3*a*p*x^2)/(10*b) - (3*p*x^5)/25 - (3*a*p*x^2*Hypergeometric2F1[2/3, 1, 5/
3, -((b*x^3)/a)])/(10*b) + (x^5*Log[c*(a + b*x^3)^p])/5
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87

method	result
parts	$\frac{x^5\ln\left(c\left(b\,x^3+a\right)^p\right)}{5}-\frac{3pb\left(-\frac{-\frac{1}{5}b\,x^5+\frac{1}{2}x^2a}{b^2}+\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a^2}{b^2}\right)}{5}$
risch	$\frac{x^5\ln\left(\left(b\,x^3+a\right)^p\right)}{5}+\frac{i\pi\,x^5\operatorname{csgn}\left(i\left(b\,x^3+a\right)^p\right)\operatorname{csgn}\left(ic\left(b\,x^3+a\right)^p\right)^2}{10}-\frac{i\pi\,x^5\operatorname{csgn}\left(i\left(b\,x^3+a\right)^p\right)\operatorname{csgn}\left(ic\left(b\,x^3+a\right)^p\right)\operatorname{csgn}\left(ic\right)}{10}-\frac{i\pi\,x^5\mathrm{c}}{}$

```
[In] int(x^4*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*ln(c*(b*x^3+a)^p)-3/5*p*b*(-1/b^2*(-1/5*b*x^5+1/2*x^2*a)+(-1/3/b/(a
/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*a^2
/b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.01

$$\int x^4 \log \left(c\left(a+bx^3\right)^p\right)\,dx = \frac{10\,bpx^5\log\left(bx^3+a\right)-6\,bpx^5+10\,bx^5\log\left(c\right)+15\,apx^2-10\,\sqrt{3}ap\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right)-5\,ap}{50\,b}$$

```
[In] integrate(x^4*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] 1/50*(10*b*p*x^5*log(b*x^3 + a) - 6*b*p*x^5 + 10*b*x^5*log(c) + 15*a*p*x^2
- 10*sqrt(3)*a*p*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3)
- sqrt(3)*a)/a) - 5*a*p*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a
*(a^2/b^2)^(1/3)) + 10*a*p*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/b
```

Sympy [F(-1)]

Timed out.

$$\int x^4 \log \left(c\left(a+bx^3\right)^p\right)\,dx = \text{Timed out}$$

```
[In] integrate(x**4*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.92

$$\int x^4 \log \left(c\left(a+bx^3\right)^p\right)\,dx = \frac{1}{5}\,x^5\log\left(\left(bx^3+a\right)^pc\right) - \frac{1}{50}\,bp\left(\frac{10\,\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\,a^2\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{10\,a^2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{3\,(2\right.$$

```
[In] integrate(x^4*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*log((b*x^3 + a)^p*c) - 1/50*b*p*(10*sqrt(3)*a^2*arctan(1/3*sqrt(3)*
(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(1/3)) + 5*a^2*log(x^2 - x*(a/b
)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(1/3)) - 10*a^2*log(x + (a/b)^(1/3))/(b^3
*(a/b)^(1/3)) + 3*(2*b*x^5 - 5*a*x^2)/b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.02

$$\int x^4 \log \left(c(a+bx^3)^p\right) \, dx$$
$$= \frac{1}{10}\, a^2 b^4 p \left(\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab^5} + \frac{2\sqrt{3}(-ab^2)^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^7} - \frac{(-ab^2)^{\frac{2}{3}} \log\left(x^2 + x(\right.}{ab^7}\right.$$
$$+ \frac{1}{5}\, px^5 \log\left(bx^3 + a\right) - \frac{1}{25}\,(3\,p - 5\,\log\left(c\right))x^5 + \frac{3\,apx^2}{10\,b}$$

```
[In] integrate(x^4*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] 1/10*a^2*b^4*p*(2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5) + 2*sqrt(
3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*
b^7) - (-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^7)) + 1
/5*p*x^5*log(b*x^3 + a) - 1/25*(3*p - 5*log(c))*x^5 + 3/10*a*p*x^2/b
```

Mupad [B] (verification not implemented)

Time = 3.71 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int x^4 \log\left(c(a+bx^3)^p\right) \, dx = \frac{x^5\,\ln\left(c\,(b\,x^3+a)^p\right)}{5} - \frac{3\,p\,x^5}{25} + \frac{a^{5/3}\,p\,\ln\left(b^{1/3}\,x + a^{1/3}\right)}{5\,b^{5/3}} + \frac{3\,a\,p\,x^2}{10\,b}$$
$$+ \frac{a^{5/3}\,p\,\ln\left(\frac{9\,a^4\,p^2\,x}{25\,b} + \frac{9\,a^{13/3}\,p^2\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{25\,b^{4/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{5\,b^{5/3}}$$
$$- \frac{a^{5/3}\,p\,\ln\left(\frac{9\,a^4\,p^2\,x}{25\,b} + \frac{9\,a^{13/3}\,p^2\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{25\,b^{4/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{5\,b^{5/3}}$$

```
[In] int(x^4*log(c*(a + b*x^3)^p),x)
```

```
[Out] (x^5*log(c*(a + b*x^3)^p))/5 - (3*p*x^5)/25 + (a^(5/3)*p*log(b^(1/3)*x + a^
(1/3)))/(5*b^(5/3)) + (3*a*p*x^2)/(10*b) + (a^(5/3)*p*log((9*a^4*p^2*x)/(25
*b) + (9*a^(13/3)*p^2*((3^(1/2)*1i)/2 - 1/2)^2)/(25*b^(4/3)))*((3^(1/2)*1i)
/2 - 1/2))/(5*b^(5/3)) - (a^(5/3)*p*log((9*a^4*p^2*x)/(25*b) + (9*a^(13/3)*
p^2*((3^(1/2)*1i)/2 + 1/2)^2)/(25*b^(4/3)))*((3^(1/2)*1i)/2 + 1/2))/(5*b^(5
/3))
```

3.15 $\int x^3 \log \left(c(a+bx^3)^p\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 157

$$\int x^3 \log \left(c\left(a+bx^3\right)^p\right) \, dx = \frac{3apx}{4b} - \frac{3px^4}{16} + \frac{\sqrt{3}a^{4/3}p \arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{4b^{4/3}} - \frac{a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4b^{4/3}} + \frac{a^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8b^{4/3}} + \frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right)$$

```
[Out] 3/4*a*p*x/b-3/16*p*x^4-1/4*a^(4/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(4/3)+1/8*a^(4
/3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(4/3)+1/4*x^4*ln(c*(b*x^3
+a)^p)+1/4*a^(4/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1
/2)/b^(4/3)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2505, 308, 206, 31, 648, 631, 210, 642}

$$\int x^3 \log \left(c\left(a+bx^3\right)^p\right) \, dx = \frac{\sqrt{3}a^{4/3}p \arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{4b^{4/3}} + \frac{a^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8b^{4/3}} - \frac{a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4b^{4/3}} + \frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right) + \frac{3apx}{4b} - \frac{3px^4}{16}$$

```
[In] Int[x^3*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (3*a*p*x)/(4*b) - (3*p*x^4)/16 + (Sqrt[3]*a^(4/3)*p*ArcTan[(a^(1/3) - 2*b^(
1/3)*x)/(Sqrt[3]*a^(1/3))])/(4*b^(4/3)) - (a^(4/3)*p*Log[a^(1/3) + b^(1/3)*
x])/(4*b^(4/3)) + (a^(4/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(8*b^(4/3)) + (x^4*Log[c*(a + b*x^3)^p])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right)-\frac{1}{4}(3bp)\int\frac{x^6}{a+bx^3}\,dx\\
&= \frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right)-\frac{1}{4}(3bp)\int\left(-\frac{a}{b^2}+\frac{x^3}{b}+\frac{a^2}{b^2\left(a+bx^3\right)}\right)dx\\
&= \frac{3apx}{4b}-\frac{3px^4}{16}+\frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right)-\frac{\left(3a^2p\right)\int\frac{1}{a+bx^3}\,dx}{4b}\\
&= \frac{3apx}{4b}-\frac{3px^4}{16}+\frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right)\\
&\quad-\frac{\left(a^{4/3}p\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{4b}-\frac{\left(a^{4/3}p\right)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{4b}\\
&= \frac{3apx}{4b}-\frac{3px^4}{16}-\frac{a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4b^{4/3}}+\frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right)\\
&\quad+\frac{\left(a^{4/3}p\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{8b^{4/3}}-\frac{\left(3a^{5/3}p\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{8b}\\
&= \frac{3apx}{4b}-\frac{3px^4}{16}-\frac{a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4b^{4/3}}+\frac{a^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8b^{4/3}}\\
&\quad+\frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right)-\frac{\left(3a^{4/3}p\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{4b^{4/3}}\\
&= \frac{3apx}{4b}-\frac{3px^4}{16}+\frac{\sqrt{3}a^{4/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{4b^{4/3}}-\frac{a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4b^{4/3}}\\
&\quad+\frac{a^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8b^{4/3}}+\frac{1}{4}x^4\log\left(c\left(a+bx^3\right)^p\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94

$$\int x^3 \log \left(c(a+bx^3)^p\right) \, dx$$
$$= \frac{12a\sqrt[3]{b}px - 3b^{4/3}px^4 + 4\sqrt{3}a^{4/3}p \arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 4a^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) + 2a^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\right.}{16b^{4/3}}$$

```
[In] Integrate[x^3*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (12*a*b^(1/3)*p*x - 3*b^(4/3)*p*x^4 + 4*Sqrt[3]*a^(4/3)*p*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]] - 4*a^(4/3)*p*Log[a^(1/3) + b^(1/3)*x] + 2*a^(4/
3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 4*b^(4/3)*x^4*Log[c*(
a + b*x^3)^p])/(16*b^(4/3))
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.87

method	result
parts	$\frac{x^4 \ln\left(c\left(b\,x^3+a\right)^p\right)}{4} - \frac{3pb\left(-\frac{-\frac{1}{4}b\,x^4+ax}{b^2} + \frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a^2}{b^2}\right)}{4}$
risch	$\frac{x^4 \ln\left(\left(b\,x^3+a\right)^p\right)}{4} + \frac{i\pi\,x^4\,\mathrm{csgn}\left(i\left(b\,x^3+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^3+a\right)^p\right)^2}{8} - \frac{i\pi\,x^4\,\mathrm{csgn}\left(i\left(b\,x^3+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^3+a\right)^p\right)\mathrm{csgn}\left(ic\right)}{8} - \frac{i\pi\,x^4\mathrm{csg}}{}$

```
[In] int(x^3*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*ln(c*(b*x^3+a)^p)-3/4*p*b*(-1/b^2*(-1/4*b*x^4+a*x)+(1/3/b/(a/b)^(2/
3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/
3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*a^2/b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

$$\int x^3 \log \left(c\left(a+bx^3\right)^p\right) dx$$
$$= \frac{4\,bpx^4 \log \left(bx^3+a\right) - 3\,bpx^4 + 4\,bx^4 \log \left(c\right) + 4\,\sqrt{3}ap\left(-\frac{a}{b}\right)^{\frac{1}{3}} \arctan \left(\frac{2\,\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right) - 2\,ap\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log \left(\right.}{16\,b}$$

```
[In] integrate(x^3*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] 1/16*(4*b*p*x^4*log(b*x^3 + a) - 3*b*p*x^4 + 4*b*x^4*log(c) + 4*sqrt(3)*a*p
*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) - 2*a*
p*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) + 4*a*p*(-a/b)^(1/3
)*log(x - (-a/b)^(1/3)) + 12*a*p*x)/b
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \log \left(c\left(a+bx^3\right)^p\right) dx = \text{Timed out}$$

```
[In] integrate(x**3*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

$$\int x^3 \log \left(c\left(a+bx^3\right)^p\right) dx = \frac{1}{4}\,x^4 \log \left(\left(bx^3+a\right)^p c\right)$$
$$-\frac{1}{16}\,bp\left(\frac{4\,\sqrt{3}a^2 \arctan \left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^3\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{2\,a^2 \log \left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^3\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{4\,a^2 \log \left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^3\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{3\,(bx\right.}{}\right.$$

```
[In] integrate(x^3*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((b*x^3 + a)^p*c) - 1/16*b*p*(4*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(
2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) - 2*a^2*log(x^2 - x*(a/b)
^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 4*a^2*log(x + (a/b)^(1/3))/(b^3*(
a/b)^(2/3)) + 3*(b*x^4 - 4*a*x)/b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.02

$$\int x^3 \log \left(c\left(a+bx^3\right)^p\right) dx$$
$$=\frac{1}{8}a^2b^3p\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab^4}-\frac{2\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^5}-\frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right.\right.}{ab^5}\right.$$
$$+\frac{1}{4}px^4\log\left(bx^3+a\right)-\frac{1}{16}(3p-4\log(c))x^4+\frac{3apx}{4b}$$

```
[In] integrate(x^3*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] 1/8*a^2*b^3*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) - 2*sqrt(3
)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b
^5) - (-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^5)) + 1/
4*p*x^4*log(b*x^3 + a) - 1/16*(3*p - 4*log(c))*x^4 + 3/4*a*p*x/b
```

Mupad [B] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.82

$$\int x^3 \log \left(c\left(a+bx^3\right)^p\right) dx = \frac{x^4\ln\left(c\left(b\,x^3+a\right)^p\right)}{4}-\frac{3\,p\,x^4}{16}+\frac{3\,a\,p\,x}{4\,b}-\frac{a^{4/3}\,p\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{4\,b^{4/3}}$$
$$+\frac{a^{4/3}\,p\,\ln\left(2\,b^{1/3}\,x-a^{1/3}-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{4\,b^{4/3}}$$
$$-\frac{a^{4/3}\,p\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{4\,b^{4/3}}$$

```
[In] int(x^3*log(c*(a + b*x^3)^p),x)
```

```
[Out] (x^4*log(c*(a + b*x^3)^p))/4 - (3*p*x^4)/16 + (3*a*p*x)/(4*b) - (a^(4/3)*p*
log(b^(1/3)*x + a^(1/3)))/(4*b^(4/3)) + (a^(4/3)*p*log(2*b^(1/3)*x - 3^(1/2
```

```
)*a^(1/3)*1i - a^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(4*b^(4/3)) - (a^(4/3)*p*lo
g(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(4*b^
(4/3))
```

3.16 $\int x^2 \log\left(c(a+bx^3)^p\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 35

$$\int x^2 \log\left(c(a+bx^3)^p\right)\,dx = -\frac{px^3}{3} + \frac{(a+bx^3)\log\left(c(a+bx^3)^p\right)}{3b}$$

```
[Out] -1/3*p*x^3+1/3*(b*x^3+a)*ln(c*(b*x^3+a)^p)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2436, 2332}

$$\int x^2 \log\left(c(a+bx^3)^p\right)\,dx = \frac{(a+bx^3)\log\left(c(a+bx^3)^p\right)}{3b} - \frac{px^3}{3}$$

```
[In] Int[x^2*Log[c*(a + b*x^3)^p],x]
```

```
[Out] -1/3*(p*x^3) + ((a + b*x^3)*Log[c*(a + b*x^3)^p])/(3*b)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \log\left(c(a+bx)^p\right)\,dx, x, x^3\right) \\
&= \frac{\text{Subst}\left(\int \log\left(cx^p\right)\,dx, x, a+bx^3\right)}{3b} \\
&= -\frac{px^3}{3} + \frac{(a+bx^3)\log\left(c(a+bx^3)^p\right)}{3b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int x^2\log\left(c\left(a+bx^3\right)^p\right)\,dx = \frac{1}{3}\left(-px^3 + \frac{(a+bx^3)\log\left(c(a+bx^3)^p\right)}{b}\right)$$

```
[In] Integrate[x^2*Log[c*(a + b*x^3)^p],x]
[Out] (-(p*x^3) + ((a + b*x^3)*Log[c*(a + b*x^3)^p])/b)/3
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\ln\left(c\left(b\,x^3+a\right)^p\right)\left(b\,x^3+a\right)-\left(b\,x^3+a\right)p}{3b}$
default	$\frac{\ln\left(c\left(b\,x^3+a\right)^p\right)\left(b\,x^3+a\right)-\left(b\,x^3+a\right)p}{3b}$
parts	$\frac{x^3\ln\left(c\left(b\,x^3+a\right)^p\right)}{3} - pb\left(\frac{x^3}{3b} - \frac{a\ln\left(b\,x^3+a\right)}{3b^2}\right)$
parallelrisch	$\frac{x^3\ln\left(c\left(b\,x^3+a\right)^p\right)bp - x^3 b\,p^2 + \ln\left(c\left(b\,x^3+a\right)^p\right)ap + a\,p^2}{3pb}$
risch	$\frac{x^3\ln\left(\left(b\,x^3+a\right)^p\right)}{3} + \frac{i\pi\,x^3\operatorname{csgn}\left(i\left(b\,x^3+a\right)^p\right)\operatorname{csgn}\left(ic\left(b\,x^3+a\right)^p\right)^2}{6} - \frac{i\pi\,x^3\operatorname{csgn}\left(i\left(b\,x^3+a\right)^p\right)\operatorname{csgn}\left(ic\left(b\,x^3+a\right)^p\right)\operatorname{csgn}(i}{6}$

```
[In] int(x^2*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b*(ln(c*(b*x^3+a)^p)*(b*x^3+a)-(b*x^3+a)*p)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int x^2 \log\left(c\left(a+bx^3\right)^p\right)\,dx = -\frac{bpx^3 - bx^3\log(c) - (bpx^3+ap)\log(bx^3+a)}{3\,b}$$

```
[In] integrate(x^2*log(c*(b*x^3+a)^p),x, algorithm="fricas")
[Out] -1/3*(b*p*x^3 - b*x^3*log(c) - (b*p*x^3 + a*p)*log(b*x^3 + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

$$\int x^2 \log\left(c\left(a+bx^3\right)^p\right)\,dx = \begin{cases} \frac{a\log\left(c(a+bx^3)^p\right)}{3b} - \frac{px^3}{3} + \frac{x^3\log\left(c(a+bx^3)^p\right)}{3} & \text{for } b \neq 0 \\ \frac{x^3\log(a^pc)}{3} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*ln(c*(b*x**3+a)**p),x)
[Out] Piecewise((a*log(c*(a + b*x**3)**p)/(3*b) - p*x**3/3 + x**3*log(c*(a + b*x*
*3)**p)/3, Ne(b, 0)), (x**3*log(a**p*c)/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int x^2 \log\left(c\left(a+bx^3\right)^p\right)\,dx = \frac{1}{3}\,x^3\log\left(\left(bx^3+a\right)^pc\right) - \frac{1}{3}\left(\frac{x^3}{b} - \frac{a\log(bx^3+a)}{b^2}\right)bp$$

```
[In] integrate(x^2*log(c*(b*x^3+a)^p),x, algorithm="maxima")
[Out] 1/3*x^3*log((b*x^3 + a)^p*c) - 1/3*(x^3/b - a*log(b*x^3 + a)/b^2)*b*p
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int x^2 \log\left(c\left(a+bx^3\right)^p\right)\,dx = -\frac{(bx^3-(bx^3+a)\log\left(bx^3+a\right)+a)p-(bx^3+a)\log\left(c\right)}{3\,b}$$

```
[In] integrate(x^2*log(c*(b*x^3+a)^p),x, algorithm="giac")
[Out] -1/3*((b*x^3 - (b*x^3 + a)*log(b*x^3 + a) + a)*p - (b*x^3 + a)*log(c))/b
```

Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int x^2 \log\left(c\left(a+bx^3\right)^p\right)\,dx = \frac{x^3\,\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{3}-\frac{p\,x^3}{3}+\frac{a\,p\,\ln\left(b\,x^3+a\right)}{3\,b}$$

```
[In] int(x^2*log(c*(a + b*x^3)^p),x)
[Out] (x^3*log(c*(a + b*x^3)^p))/3 - (p*x^3)/3 + (a*p*log(a + b*x^3))/(3*b)
```

3.17 $\int x \log \left(c(a+bx^3)^p\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 147

$$\int x \log \left(c(a+bx^3)^p\right) \, dx = -\frac{3px^2}{4} - \frac{\sqrt{3}a^{2/3}p \arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}} - \frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}} + \frac{a^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}} + \frac{1}{2}x^2\log\left(c(a+bx^3)^p\right)$$

```
[Out] -3/4*p*x^2-1/2*a^(2/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)+1/4*a^(2/3)*p*ln(a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(2/3)+1/2*x^2*ln(c*(b*x^3+a)^p)-1/2*a
^(2/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(2/3)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2505, 327, 298, 31, 648, 631, 210, 642}

$$\int x \log \left(c(a+bx^3)^p\right) \, dx = -\frac{\sqrt{3}a^{2/3}p \arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}} + \frac{a^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}} - \frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}} + \frac{1}{2}x^2\log\left(c(a+bx^3)^p\right) - \frac{3px^2}{4}$$

```
[In] Int[x*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (-3*p*x^2)/4 - (Sqrt[3]*a^(2/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a
^(1/3))])/(2*b^(2/3)) - (a^(2/3)*p*Log[a^(1/3) + b^(1/3)*x])/(2*b^(2/3)) +
(a^(2/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*b^(2/3)) + (x
^2*Log[c*(a + b*x^3)^p])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)-\frac{1}{2}(3bp)\int\frac{x^4}{a+bx^3}\,dx\\
&=-\frac{3px^2}{4}+\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)+\frac{1}{2}(3ap)\int\frac{x}{a+bx^3}\,dx\\
&=-\frac{3px^2}{4}+\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)-\frac{\left(a^{2/3}p\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{2\sqrt[3]{b}}+\frac{\left(a^{2/3}p\right)\int\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{b}}\\
&=-\frac{3px^2}{4}-\frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}}+\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)\\
&\quad+\frac{\left(a^{2/3}p\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{4b^{2/3}}+\frac{(3ap)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{4\sqrt[3]{b}}\\
&=-\frac{3px^2}{4}-\frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}}+\frac{a^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}}\\
&\quad+\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)+\frac{\left(3a^{2/3}p\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{2b^{2/3}}\\
&=-\frac{3px^2}{4}-\frac{\sqrt{3}a^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}}-\frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}}\\
&\quad+\frac{a^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}}+\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.36

$$\int x\log\left(c\left(a+bx^3\right)^p\right)\,dx=-\frac{3px^2}{4}+\frac{3}{4}px^2\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a}\right)+\frac{1}{2}x^2\log\left(c\left(a+bx^3\right)^p\right)$$

```
[In] Integrate[x*Log[c*(a + b*x^3)^p],x]
[Out] (-3*p*x^2)/4 + (3*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)])/4 + (
x^2*Log[c*(a + b*x^3)^p])/2
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.87

method	result
parts	$\frac{x^2\ln\left(c\left(b\,x^3+a\right)^p\right)}{2}-\frac{3pb\left(\frac{x^2}{2b}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a}{b}\right)}{2}$
risch	$\frac{x^2\ln\left(\left(b\,x^3+a\right)^p\right)}{2}+\frac{i\operatorname{csgn}\left(ic\left(b\,x^3+a\right)^p\right)^2\operatorname{csgn}\left(i\left(b\,x^3+a\right)^p\right)x^2\pi}{4}-\frac{i\pi\,x^2\operatorname{csgn}\left(i\left(b\,x^3+a\right)^p\right)\operatorname{csgn}\left(ic\left(b\,x^3+a\right)^p\right)\operatorname{csgn}\left(ic\right)}{4}-\frac{i\pi\,x^2\text{cs}}{}$

```
[In] int(x*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*ln(c*(b*x^3+a)^p)-3/2*p*b*(1/2*x^2/b-(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)
^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(
```

```
a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*a/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.02

$$
\begin{aligned}
\int x\log\left(c\left(a+bx^3\right)^p\right)\,dx = {} & \frac{1}{2}\,px^2\log\left(bx^3+a\right)-\frac{3}{4}\,px^2+\frac{1}{2}\,x^2\log\left(c\right) \\
& +\frac{1}{2}\,\sqrt{3}p\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}+\sqrt{3}a}{3\,a}\right) \\
& -\frac{1}{4}\,p\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}-a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right) \\
& +\frac{1}{2}\,p\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax+b\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}\right)
\end{aligned}
$$

```
[In] integrate(x*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] 1/2*p*x^2*log(b*x^3 + a) - 3/4*p*x^2 + 1/2*x^2*log(c) + 1/2*sqrt(3)*p*(-a^2
/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a^2/b^2)^(1/3) + sqrt(3)*a)/a) - 1/
4*p*(-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(2/3) - a*(-a^2/b^2)^(1/3))
 + 1/2*p*(-a^2/b^2)^(1/3)*log(a*x + b*(-a^2/b^2)^(2/3))
```

Sympy [A] (verification not implemented)

Time = 57.15 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.21

$$
\int x\log\left(c\left(a+bx^3\right)^p\right)\,dx
$$

$$
= \begin{cases} \frac{x^2\log\left(0^p c\right)}{2} \\ \frac{x^2\log\left(a^p c\right)}{2} \\ -\frac{3px^2}{4}+\frac{x^2\log\left(c\left(bx^3\right)^p\right)}{2} \\ -\frac{3px^2}{4}+\frac{3p\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{4}-\frac{\sqrt{3}p\left(-\frac{a}{b}\right)^{\frac{2}{3}}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{2}+\frac{x^2\log\left(c\left(a+bx^3\right)^p\right)}{2}-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(c\left(a+\right.\right.}{2} \end{cases}
$$

```
[In] integrate(x*ln(c*(b*x**3+a)**p),x)
```

```
[Out] Piecewise((x**2*log(0**p*c)/2, Eq(a, 0) & Eq(b, 0)), (x**2*log(a**p*c)/2, E
q(b, 0)), (-3*p*x**2/4 + x**2*log(c*(b*x**3)**p)/2, Eq(a, 0)), (-3*p*x**2/4
```

```
 + 3*p*(-a/b)**(2/3)*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/4 -
sqrt(3)*p*(-a/b)**(2/3)*atan(2*sqrt(3)*x/(3*(-a/b)**(1/3)) + sqrt(3)/3)/2 +
 x**2*log(c*(a + b*x**3)**p)/2 - (-a/b)**(2/3)*log(c*(a + b*x**3)**p)/2, Tr
ue))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.89

$$\int x\log\left(c\left(a+bx^3\right)^p\right)\,dx = -\frac{1}{4}\,bp\left(\frac{3\,x^2}{b}-\frac{2\,\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\,a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+\frac{1}{2}\,x^2\log\left(\left(bx^3+a\right)^pc\right)$$

```
[In] integrate(x*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] -1/4*b*p*(3*x^2/b - 2*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(b^2*(a/b)^(1/3)) - a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*
(a/b)^(1/3)) + 2*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(1/3))) + 1/2*x^2*log((b
*x^3 + a)^p*c)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.02

$$\int x\log\left(c\left(a+bx^3\right)^p\right)\,dx = -\frac{1}{4}\,ab^2p\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab^2}+\frac{2\,\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^4}-\frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(\right.\right.}{ab^4}\right.+\frac{1}{2}\,px^2\log\left(bx^3+a\right)-\frac{1}{4}\left(3\,p-2\,\log\left(c\right)\right)x^2$$

```
[In] integrate(x*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] -1/4*a*b^2*p*(2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2) + 2*sqrt(3)
*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^
4) - (-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4)) + 1/2
*p*x^2*log(b*x^3 + a) - 1/4*(3*p - 2*log(c))*x^2
```

Mupad [B] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.82

$$\int x \log \left(c\left(a+bx^3\right)^p\right)\, dx = \frac{x^2\,\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{2}-\frac{3\,p\,x^2}{4}-\frac{a^{2/3}\,p\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{2\,b^{2/3}} - \frac{a^{2/3}\,p\,\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}-\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,b^{2/3}} + \frac{a^{2/3}\,p\,\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}+\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,b^{2/3}}$$

```
[In] int(x*log(c*(a + b*x^3)^p),x)
```

```
[Out] (x^2*log(c*(a + b*x^3)^p))/2 - (3*p*x^2)/4 - (a^(2/3)*p*log(b^(1/3)*x + a^(
1/3)))/(2*b^(2/3)) - (a^(2/3)*p*log(4*b^(1/3)*x - 3^(1/2)*a^(1/3)*2i - 2*a^
(1/3))*((3^(1/2)*1i)/2 - 1/2))/(2*b^(2/3)) + (a^(2/3)*p*log(3^(1/2)*a^(1/3)
*2i + 4*b^(1/3)*x - 2*a^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(2*b^(2/3))
```

3.18 $\int \log\left(c\left(a+bx^3\right)^p\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 133

$$\int \log\left(c\left(a+bx^3\right)^p\right)\,dx=-3px-\frac{\sqrt{3}\sqrt[3]{a}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}}+\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}+x\log\left(c\left(a+bx^3\right)^p\right)$$

```
[Out] -3*p*x+a^(1/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)-1/2*a^(1/3)*p*ln(a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(1/3)+x*ln(c*(b*x^3+a)^p)-a^(1/3)*p*arctan(1/
3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2498, 327, 206, 31, 648, 631, 210, 642}

$$\int \log\left(c\left(a+bx^3\right)^p\right)\,dx=-\frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\sqrt[3]{a}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}}+x\log\left(c\left(a+bx^3\right)^p\right)+\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-3px$$

```
[In] Int[Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*p*x - (Sqrt[3]*a^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/b^(1/3) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (a^(1/3)*p*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)) + x*Log[c*(a + b*x
^3)^p]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log\left(c\left(a+bx^3\right)^p\right)-(3bp)\int\frac{x^3}{a+bx^3}\,dx\\
&= -3px+x\log\left(c\left(a+bx^3\right)^p\right)+(3ap)\int\frac{1}{a+bx^3}\,dx\\
&= -3px+x\log\left(c\left(a+bx^3\right)^p\right)+\left(\sqrt[3]{a}p\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx+\left(\sqrt[3]{a}p\right)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx\\
&= -3px+\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+x\log\left(c\left(a+bx^3\right)^p\right)\\
&\quad+\frac{1}{2}\left(3a^{2/3}p\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx-\frac{\left(\sqrt[3]{a}p\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{b}}\\
&= -3px+\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}\\
&\quad+x\log\left(c\left(a+bx^3\right)^p\right)+\frac{\left(3\sqrt[3]{a}p\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}\\
&= -3px-\frac{\sqrt{3}\sqrt[3]{a}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}}+\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}\\
&\quad-\frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}+x\log\left(c\left(a+bx^3\right)^p\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.97

$$\int \log \left(c\left(a+bx^3\right)^p\right)\, dx = -3px - \frac{\sqrt{3}\sqrt[3]{a}p\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} + \frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} - \frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}} + x\log\left(c\left(a+bx^3\right)^p\right)$$

```
[In] Integrate[Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*p*x - (Sqrt[3]*a^(1/3)*p*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^
(1/3) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (a^(1/3)*p*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)) + x*Log[c*(a + b*x^3)^p]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92

method	result
default	$x\ln\left(c(b\,x^{3}+a)^{p}\right)-3pb\left(\frac{x}{b}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a}{b}\right)$
parts	$x\ln\left(c(b\,x^{3}+a)^{p}\right)-3pb\left(\frac{x}{b}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a}{b}\right)$
risch	$x\ln\left((b\,x^{3}+a)^{p}\right)+\frac{i\operatorname{csgn}\left(ic(b\,x^{3}+a)^{p}\right)^{2}\operatorname{csgn}\left(i(b\,x^{3}+a)^{p}\right)x\pi}{2}-\frac{i\pi x\,\operatorname{csgn}\left(i(b\,x^{3}+a)^{p}\right)\operatorname{csgn}\left(ic(b\,x^{3}+a)^{p}\right)\operatorname{csgn}(ic)}{2}-\frac{i\pi x}{}$

[In] int(ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)

[Out] x*ln(c*(b*x^3+a)^p)-3*p*b*(x/b-(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*a/b)

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$\int \log \left(c\left(a+bx^3\right)^p\right)\,dx = px\log\left(bx^3+a\right)+\sqrt{3}p\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right) - \frac{1}{2}p\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) + p\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-3\,px+x\log\left(c\right)$$

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] p*x*log(b*x^3 + a) + sqrt(3)*p*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^
(2/3) - sqrt(3)*a)/a) - 1/2*p*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(
2/3)) + p*(a/b)^(1/3)*log(x + (a/b)^(1/3)) - 3*p*x + x*log(c)
```

Sympy [A] (verification not implemented)

Time = 24.86 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.24

$$\int \log\left(c\left(a+bx^3\right)^p\right)\,dx = \begin{cases} x\log\left(0^p c\right) \\ -3px+x\log\left(c\left(bx^3\right)^p\right) \\ x\log\left(a^p c\right) \\ -3px+x\log\left(c\left(a+bx^3\right)^p\right)-\frac{3bp\left(-\frac{a}{b}\right)^{\frac{4}{3}}\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a}-\frac{\sqrt{3}bp\left(-\frac{a}{b}\right)^{\frac{4}{3}}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{a}+\frac{b\left(-\frac{a}{b}\right)^{\frac{4}{3}}\log}{} \end{cases}$$

```
[In] integrate(ln(c*(b*x**3+a)**p),x)
```

```
[Out] Piecewise((x*log(0**p*c), Eq(a, 0) & Eq(b, 0)), (-3*p*x + x*log(c*(b*x**3)*
*p), Eq(a, 0)), (x*log(a**p*c), Eq(b, 0)), (-3*p*x + x*log(c*(a + b*x**3)**
p) - 3*b*p*(-a/b)**(4/3)*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/
(2*a) - sqrt(3)*b*p*(-a/b)**(4/3)*atan(2*sqrt(3)*x/(3*(-a/b)**(1/3)) + sqrt
(3)/3)/a + b*(-a/b)**(4/3)*log(c*(a + b*x**3)**p)/a, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.94

$$\int \log \left(c\left(a+bx^3\right)^p\right)\,dx = -\frac{1}{2}\,bp\left(\frac{6\,x}{b}-\frac{2\,\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2\,a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right) + x\log\left(\left(bx^3+a\right)^pc\right)$$

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] -1/2*b*p*(6*x/b - 2*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^
(1/3))/(b^2*(a/b)^(2/3)) + a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a
/b)^(2/3)) - 2*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))) + x*log((b*x^3 + a
)^p*c)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08

$$\int \log \left(c\left(a+bx^3\right)^p\right)\,dx = -\frac{1}{2}\,abp\left(\frac{2\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab}-\frac{2\,\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^2}-\frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x(-\right.}{ab^2}\right. + px\log\left(bx^3+a\right)-(3\,p-\log\left(c\right))x$$

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] -1/2*a*b*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) - 2*sqrt(3)*(-a
*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) -
 (-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)) + p*x*log
(b*x^3 + a) - (3*p - log(c))*x
```

Mupad [B] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.01

$$\int \log \left(c\left(a+bx^3\right)^p\right) \, dx$$
$$= x \ln \left(c \left(b x^3 + a\right)^p\right) - 3 p x - \frac{(-a)^{1/3} p \ln \left(\left(-a\right)^{4/3} + a b^{1/3} x\right)}{b^{1/3}} + \frac{(-a)^{1/3} p \ln \left(2 a b^{1/3} x - (-a)^{4/3} - \sqrt{3} (-a)^{4/3} \, 1\mathrm{i}\right) \left(\frac{1}{2} + \frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)}{b^{1/3}} - \frac{(-a)^{1/3} p \ln \left(2 a b^{1/3} x - (-a)^{4/3} + \sqrt{3} (-a)^{4/3} \, 1\mathrm{i}\right) \left(-\frac{1}{2} + \frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)}{b^{1/3}}$$

```
[In] int(log(c*(a + b*x^3)^p),x)
```

```
[Out] x*log(c*(a + b*x^3)^p) - 3*p*x - ((-a)^(1/3)*p*log((-a)^(4/3) + a*b^(1/3)*x
))/b^(1/3) + ((-a)^(1/3)*p*log(2*a*b^(1/3)*x - 3^(1/2)*(-a)^(4/3)*1i - (-a)
^(4/3))*((3^(1/2)*1i)/2 + 1/2))/b^(1/3) - ((-a)^(1/3)*p*log(3^(1/2)*(-a)^(4
/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/2)*1i)/2 - 1/2))/b^(1/3)
```

3.19 $\int \frac{\log\left(c(a+bx^3)^p\right)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x}\,dx = \frac{1}{3}\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)+\frac{1}{3}p\,\mathrm{PolyLog}\left(2,1+\frac{bx^3}{a}\right)$$

```
[Out] 1/3*ln(-b*x^3/a)*ln(c*(b*x^3+a)^p)+1/3*p*polylog(2,1+b*x^3/a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2441, 2352}

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x}\,dx = \frac{1}{3}\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)+\frac{1}{3}p\,\mathrm{PolyLog}\left(2,\frac{bx^3}{a}+1\right)$$

```
[In] Int[Log[c*(a + b*x^3)^p]/x,x]
```

```
[Out] (Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p])/3 + (p*PolyLog[2, 1 + (b*x^3)/a])/
3
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x}\,dx, x, x^3\right) \\
&= \frac{1}{3}\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right) - \frac{1}{3}(bp)\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx, x, x^3\right) \\
&= \frac{1}{3}\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right) + \frac{1}{3}p\text{Li}_2\left(1+\frac{bx^3}{a}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x}\,dx = \frac{1}{3}\left(\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right) + p\,\text{PolyLog}\left(2, \frac{a+bx^3}{a}\right)\right)$$

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x,x]
[Out] (Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p] + p*PolyLog[2, (a + b*x^3)/a])/3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

method	result
parts	$\ln\left(c(b\,x^3+a)^p\right)\ln\left(x\right)-p\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3+a\right)}\left(\ln\left(x\right)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)$
risch	$\ln\left((b\,x^3+a)^p\right)\ln\left(x\right)-p\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3+a\right)}\left(\ln\left(x\right)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)+\left(\frac{i\pi\ \mathrm{cs}}{}\right.$

```
[In] int(ln(c*(b*x^3+a)^p)/x,x,method=_RETURNVERBOSE)
[Out] ln(c*(b*x^3+a)^p)*ln(x)-p*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=
RootOf(_Z^3*b+a))
```

Fricas [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x}\,dx=\int \frac{\log\left((bx^3+a)^pc\right)}{x}\,dx$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x,x, algorithm="fricas")
[Out] integral(log((b*x^3 + a)^p*c)/x, x)
```

Sympy [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x}\,dx=\int \frac{\log\left(c(a+bx^3)^p\right)}{x}\,dx$$

```
[In] integrate(ln(c*(b*x**3+a)**p)/x,x)
[Out] Integral(log(c*(a + b*x**3)**p)/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(39) = 78$.

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.82

$$\begin{aligned}&\int \frac{\log\left(c(a+bx^3)^p\right)}{x}\,dx\\&=\frac{1}{3}\,bp\left(\frac{3\,\log\left(bx^3+a\right)\log\left(x\right)}{b}-\frac{3\,\log\left(\frac{bx^3}{a}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{bx^3}{a}\right)}{b}\right)\\&\quad-p\log\left(bx^3+a\right)\log\left(x\right)+\log\left(\left(bx^3+a\right)^pc\right)\log\left(x\right)\end{aligned}$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x,x, algorithm="maxima")
```

```
[Out] 1/3*b*p*(3*log(b*x^3 + a)*log(x)/b - (3*log(b*x^3/a + 1)*log(x) + dilog(-b*
x^3/a))/b) - p*log(b*x^3 + a)*log(x) + log((b*x^3 + a)^p*c)*log(x)
```

Giac [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x}\,dx = \int \frac{\log\left((bx^3+a)^p c\right)}{x}\,dx$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^3 + a)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x}\,dx = \int \frac{\ln\left(c\left(b\,x^3+a\right)^p\right)}{x}\,dx$$

```
[In] int(log(c*(a + b*x^3)^p)/x,x)
```

```
[Out] int(log(c*(a + b*x^3)^p)/x, x)
```

3.20 $\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 133

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2}\,dx = -\frac{\sqrt{3}\sqrt[3]{b}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}-\frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}+\frac{\sqrt[3]{b}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}}-\frac{\log\left(c(a+bx^3)^p\right)}{x}$$

```
[Out] -b^(1/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)+1/2*b^(1/3)*p*ln(a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/a^(1/3)-ln(c*(b*x^3+a)^p)/x-b^(1/3)*p*arctan(1/3*(a^(
1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/a^(1/3)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {2505, 298, 31, 648, 631, 210, 642}

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2}\,dx = \frac{\sqrt[3]{b}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[3]{b}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}-\frac{\log\left(c(a+bx^3)^p\right)}{x}-\frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}$$

```
[In] Int[Log[c*(a + b*x^3)^p]/x^2,x]
```

```
[Out] -((Sqrt[3]*b^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(
1/3)) - (b^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + (b^(1/3)*p*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*a^(1/3)) - Log[c*(a + b*x^3)^p]/x
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
```

```
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\log\left(c(a+bx^3)^p\right)}{x}+(3bp)\int \frac{x}{a+bx^3}\,dx \\
&= -\frac{\log\left(c(a+bx^3)^p\right)}{x}-\frac{\left(b^{2/3}p\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{\sqrt[3]{a}}+\frac{\left(b^{2/3}p\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{\sqrt[3]{a}} \\
&= -\frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{\log\left(c(a+bx^3)^p\right)}{x} \\
&\quad +\frac{\left(\sqrt[3]{b}p\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{a}}+\frac{1}{2}\left(3b^{2/3}p\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx \\
&= -\frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}+\frac{\sqrt[3]{b}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}} \\
&\quad -\frac{\log\left(c(a+bx^3)^p\right)}{x}+\frac{\left(3\sqrt[3]{b}p\right)\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}} \\
&= -\frac{\sqrt{3}\sqrt[3]{b}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}-\frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} \\
&\quad +\frac{\sqrt[3]{b}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}}-\frac{\log\left(c(a+bx^3)^p\right)}{x}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.35

$$
\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2}\,dx=\frac{3bpx^2\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a}\right)}{2a}-\frac{\log\left(c(a+bx^3)^p\right)}{x}
$$

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^2,x]
[Out] (3*b*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)])/(2*a) - Log[c*(a +
 b*x^3)^p]/x
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.85

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{x}+3pb\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)$
risch	$-\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)}{x}-\frac{i\pi\,\mathrm{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right)^{2}-i\pi\,\mathrm{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right)\mathrm{csgn}\left(ic\right)-i\pi\mathrm{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right.}{}$

```
[In] int(ln(c*(b*x^3+a)^p)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(c*(b*x^3+a)^p)/x+3*p*b*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b
)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2}\,dx = \frac{2\sqrt{3}px\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-px\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)+2\,px\left(-\frac{b}{a}\right)^{\frac{1}{3}}\mathrm{lo}}{2\,x}$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*p*x*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqr
t(3)) - p*x*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b/a)^(1/3)) + 2
*p*x*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)) - 2*p*log(b*x^3 + a) - 2*log(c)
)/x
```

Sympy [A] (verification not implemented)

Time = 108.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.24

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2}\,dx$$
$$= \begin{cases} -\frac{\log(0^p c)}{x} \\ -\frac{3p}{x}-\frac{\log\left(c(bx^3)^p\right)}{x} \\ -\frac{\log(a^p c)}{x} \\ -\frac{\log\left(c(a+bx^3)^p\right)}{x}+\frac{3bp\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a}-\frac{\sqrt{3}bp\left(-\frac{a}{b}\right)^{\frac{2}{3}}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{a}-\frac{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(c(a+bx^3)^p\right)}{a} \end{cases}$$

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**2,x)
```

```
[Out] Piecewise((-log(0**p*c)/x, Eq(a, 0) & Eq(b, 0)), (-3*p/x - log(c*(b*x**3)**
p)/x, Eq(a, 0)), (-log(a**p*c)/x, Eq(b, 0)), (-log(c*(a + b*x**3)**p)/x + 3
*b*p*(-a/b)**(2/3)*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(2*a)
- sqrt(3)*b*p*(-a/b)**(2/3)*atan(2*sqrt(3)*x/(3*(-a/b)**(1/3)) + sqrt(3)/3)
/a - b*(-a/b)**(2/3)*log(c*(a + b*x**3)**p)/a, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.89

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2}\,dx$$
$$= \frac{1}{2}\,bp\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)$$
$$-\frac{\log\left((bx^3+a)^p c\right)}{x}$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(
a/b)^(1/3)) + log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(1/3)) - 2*lo
g(x + (a/b)^(1/3))/(b*(a/b)^(1/3))) - log((b*x^3 + a)^p*c)/x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.03

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2}\,dx = -\frac{bp\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a} - \frac{\sqrt{3}(-ab^2)^{\frac{2}{3}}\,p\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab} - \frac{p\log\left(bx^3+a\right)}{x} + \frac{(-ab^2)^{\frac{2}{3}}\,p\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,ab} - \frac{\log\left(c\right)}{x}$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2,x, algorithm="giac")
```

```
[Out] -b*p*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a - sqrt(3)*(-a*b^2)^(2/3)*p*a
rctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) - p*log(b*x^3 +
a)/x + 1/2*(-a*b^2)^(2/3)*p*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)
- log(c)/x
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2}\,dx = \frac{(-b)^{1/3}\,p\,\ln\left(a^{1/3}\,(-b)^{8/3}+b^3\,x\right)}{a^{1/3}} - \frac{\ln\left(c\,(b\,x^3+a)^p\right)}{x} + \frac{(-b)^{1/3}\,p\,\ln\left(9\,b^3\,p^2\,x+9\,a^{1/3}\,(-b)^{8/3}\,p^2\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{1/3}} - \frac{(-b)^{1/3}\,p\,\ln\left(9\,b^3\,p^2\,x+9\,a^{1/3}\,(-b)^{8/3}\,p^2\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{1/3}}$$

```
[In] int(log(c*(a + b*x^3)^p)/x^2,x)
```

```
[Out] ((-b)^(1/3)*p*log(a^(1/3)*(-b)^(8/3) + b^3*x))/a^(1/3) - log(c*(a + b*x^3)^
p)/x + ((-b)^(1/3)*p*log(9*b^3*p^2*x + 9*a^(1/3)*(-b)^(8/3)*p^2*((3^(1/2)*1
i)/2 - 1/2)^2)*((3^(1/2)*1i)/2 - 1/2))/a^(1/3) - ((-b)^(1/3)*p*log(9*b^3*p^
2*x + 9*a^(1/3)*(-b)^(8/3)*p^2*((3^(1/2)*1i)/2 + 1/2)^2)*((3^(1/2)*1i)/2 +
1/2))/a^(1/3)
```

3.21 $\int \frac{\log\left(c(a+bx^3)^p\right)}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 139

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^3}\,dx = -\frac{\sqrt{3}b^{2/3}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}} + \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}} - \frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4a^{2/3}} - \frac{\log\left(c(a+bx^3)^p\right)}{2x^2}$$

```
[Out] 1/2*b^(2/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)-1/4*b^(2/3)*p*ln(a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)-1/2*ln(c*(b*x^3+a)^p)/x^2-1/2*b^(2/3)*p*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/a^(2/3)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2505, 206, 31, 648, 631, 210, 642}

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^3}\,dx = -\frac{\sqrt{3}b^{2/3}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}} - \frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4a^{2/3}} + \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}} - \frac{\log\left(c(a+bx^3)^p\right)}{2x^2}$$

```
[In] Int[Log[c*(a + b*x^3)^p]/x^3,x]
```

```
[Out] -1/2*(Sqrt[3]*b^(2/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
a^(2/3) + (b^(2/3)*p*Log[a^(1/3) + b^(1/3)*x])/(2*a^(2/3)) - (b^(2/3)*p*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*a^(2/3)) - Log[c*(a + b*x^3
)^p]/(2*x^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
```

```
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log\left(c(a+bx^3)^p\right)}{2x^2}+\frac{1}{2}(3bp)\int\frac{1}{a+bx^3}\,dx\\
&= -\frac{\log\left(c(a+bx^3)^p\right)}{2x^2}+\frac{(bp)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{2a^{2/3}}+\frac{(bp)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^{2/3}}\\
&= \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}}-\frac{\log\left(c(a+bx^3)^p\right)}{2x^2}\\
&\quad-\frac{\left(b^{2/3}p\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{4a^{2/3}}+\frac{(3bp)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{4\sqrt[3]{a}}\\
&= \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}}-\frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4a^{2/3}}\\
&\quad-\frac{\log\left(c(a+bx^3)^p\right)}{2x^2}+\frac{\left(3b^{2/3}p\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{2a^{2/3}}\\
&= -\frac{\sqrt{3}b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}}+\frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}}\\
&\quad-\frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4a^{2/3}}-\frac{\log\left(c(a+bx^3)^p\right)}{2x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96

$$\int\frac{\log\left(c(a+bx^3)^p\right)}{x^3}\,dx = -\frac{2\sqrt{3}b^{2/3}px^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2b^{2/3}px^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+b^{2/3}px^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+}{4a^{2/3}x^2}$$

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^3,x]
```

[Out] -1/4*(2*Sqrt[3]*b^(2/3)*p*x^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*b^(2/3)*p*x^2*Log[a^(1/3) + b^(1/3)*x] + b^(2/3)*p*x^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a^(2/3)*Log[c*(a + b*x^3)^p])/(a^(2/3)*x^2)

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.81

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{2x^{2}}+\frac{3pb\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{2}$
risch	$-\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)}{2x^{2}}-\frac{i\pi\;\mathrm{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right)^{2}-i\pi\;\mathrm{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)\;\mathrm{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right)\;\mathrm{csgn}\left(ic\right)-i\pi\mathrm{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right.}{}$

[In] int(ln(c*(b*x^3+a)^p)/x^3,x,method=_RETURNVERBOSE)

[Out] -1/2*ln(c*(b*x^3+a)^p)/x^2+3/2*p*b*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.08

$$\int\frac{\log\left(c(a+bx^3)^p\right)}{x^3}\,dx$$
$$=\frac{2\sqrt{3}px^2\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-px^2\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2-abx\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+2px^2\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\mathrm{lo}}{4x^2}$$

[In] integrate(log(c*(b*x^3+a)^p)/x^3,x, algorithm="fricas")

[Out] 1/4*(2*sqrt(3)*p*x^2*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - p*x^2*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(b^2/a^2)^(

```
1/3) + a^2*(b^2/a^2)^(2/3)) + 2*p*x^2*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)
^(1/3)) - 2*p*log(b*x^3 + a) - 2*log(c))/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.86

$$\begin{aligned}\int \frac{\log\left(c(a+bx^3)^p\right)}{x^3}\,dx &= \frac{1}{4}\,bp\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{2\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right) \\ &\quad - \frac{\log\left((bx^3+a)^p c\right)}{2\,x^2}\end{aligned}$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x^3,x, algorithm="maxima")
[Out] 1/4*b*p*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(
a/b)^(2/3)) - log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 2*lo
g(x + (a/b)^(1/3))/(b*(a/b)^(2/3))) - 1/2*log((b*x^3 + a)^p*c)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^3}\,dx = -\frac{1}{4}\,bp\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a} - \frac{2\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab} - \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right.\right.}{ab}\right. - \frac{p\log\left(bx^3+a\right)}{2\,x^2} - \frac{\log\left(c\right)}{2\,x^2}$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*b*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a - 2*sqrt(3)*(-a*b^2)^
(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) - (-a*b^2
)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)) - 1/2*p*log(b*x^3 +
 a)/x^2 - 1/2*log(c)/x^2
```

Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.83

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^3}\,dx = \frac{b^{2/3}\,p\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{2\,a^{2/3}} - \frac{\ln\left(c\,(b\,x^3+a)^p\right)}{2\,x^2} - \frac{b^{2/3}\,p\,\ln\left(2\,b^{1/3}\,x-a^{1/3}-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,a^{2/3}} + \frac{b^{2/3}\,p\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,a^{2/3}}$$

```
[In] int(log(c*(a + b*x^3)^p)/x^3,x)
```

```
[Out] (b^(2/3)*p*log(b^(1/3)*x + a^(1/3)))/(2*a^(2/3)) - log(c*(a + b*x^3)^p)/(2*
x^2) - (b^(2/3)*p*log(2*b^(1/3)*x - 3^(1/2)*a^(1/3)*1i - a^(1/3))*((3^(1/2)
*1i)/2 + 1/2))/(2*a^(2/3)) + (b^(2/3)*p*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*
x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(2*a^(2/3))
```

3.22 $\int \frac{\log\left(c(a+bx^3)^p\right)}{x^4} dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^4} dx = \frac{bp\log(x)}{a} - \frac{bp\log\left(a+bx^3\right)}{3a} - \frac{\log\left(c(a+bx^3)^p\right)}{3x^3}$$

```
[Out] b*p*ln(x)/a-1/3*b*p*ln(b*x^3+a)/a-1/3*ln(c*(b*x^3+a)^p)/x^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2504, 2442, 36, 29, 31}

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^4} dx = -\frac{\log\left(c(a+bx^3)^p\right)}{3x^3} - \frac{bp\log\left(a+bx^3\right)}{3a} + \frac{bp\log(x)}{a}$$

```
[In] Int[Log[c*(a + b*x^3)^p]/x^4,x]
```

```
[Out] (b*p*Log[x])/a - (b*p*Log[a + b*x^3])/(3*a) - Log[c*(a + b*x^3)^p]/(3*x^3)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{\log \left(c(a+bx)^p\right)}{x^2}\,dx, x, x^3\right) \\
&= -\frac{\log \left(c\left(a+bx^3\right)^p\right)}{3x^3} + \frac{1}{3}(bp)\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, x^3\right) \\
&= -\frac{\log \left(c\left(a+bx^3\right)^p\right)}{3x^3} + \frac{(bp)\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^3\right)}{3a} - \frac{\left(b^2p\right)\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^3\right)}{3a} \\
&= \frac{bp\log(x)}{a} - \frac{bp\log\left(a+bx^3\right)}{3a} - \frac{\log \left(c\left(a+bx^3\right)^p\right)}{3x^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$
\int \frac{\log \left(c\left(a+bx^3\right)^p\right)}{x^4}\,dx = \frac{bp\log(x)}{a} - \frac{bp\log\left(a+bx^3\right)}{3a} - \frac{\log \left(c\left(a+bx^3\right)^p\right)}{3x^3}
$$

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^4,x]
[Out] (b*p*Log[x])/a - (b*p*Log[a + b*x^3])/(3*a) - Log[c*(a + b*x^3)^p]/(3*x^3)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{3x^{3}}+pb\left(\frac{\ln\left(x\right)}{a}-\frac{\ln\left(b\,x^{3}+a\right)}{3a}\right)$
parallelrisch	$\frac{3p^{2}b\ln\left(x\right)x^{3}-x^{3}\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)bp-\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)ap}{3x^{3}ap}$
risch	$-\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)}{3x^{3}}-\frac{i\pi a\ \mathrm{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right)^{2}-i\pi a\ \mathrm{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right)\mathrm{csgn}\left(ic\right)-i\pi a\mathrm{csgn}(}{6x^{3}a}$

[In] int(ln(c*(b*x^3+a)^p)/x^4,x,method=_RETURNVERBOSE)

[Out] -1/3*ln(c*(b*x^3+a)^p)/x^3+p*b*(1/a*ln(x)-1/3/a*ln(b*x^3+a))

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^4}\,dx = \frac{3\,bpx^3\log\left(x\right)-\left(bpx^3+ap\right)\log\left(bx^3+a\right)-a\log\left(c\right)}{3\,ax^3}$$

[In] integrate(log(c*(b*x^3+a)^p)/x^4,x, algorithm="fricas")

[Out] 1/3*(3*b*p*x^3*log(x) - (b*p*x^3 + a*p)*log(b*x^3 + a) - a*log(c))/(a*x^3)

Sympy [A] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^4}\,dx = \begin{cases} -\frac{\log\left(c(a+bx^3)^p\right)}{3x^3}+\frac{bp\log\left(x\right)}{a}-\frac{b\log\left(c(a+bx^3)^p\right)}{3a} & \text{for } a\neq 0 \\ -\frac{p}{3x^3}-\frac{\log\left(c(bx^3)^p\right)}{3x^3} & \text{otherwise} \end{cases}$$

[In] integrate(ln(c*(b*x**3+a)**p)/x**4,x)

[Out] Piecewise((-log(c*(a + b*x**3)**p)/(3*x**3) + b*p*log(x)/a - b*log(c*(a + b
*x**3)**p)/(3*a), Ne(a, 0)), (-p/(3*x**3) - log(c*(b*x**3)**p)/(3*x**3), Tr
ue))

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^4}\,dx = -\frac{1}{3}\,bp\left(\frac{\log\left(bx^3+a\right)}{a}-\frac{\log\left(x^3\right)}{a}\right)-\frac{\log\left((bx^3+a)^p c\right)}{3\,x^3}$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x^4,x, algorithm="maxima")
[Out] -1/3*b*p*(log(b*x^3 + a)/a - log(x^3)/a) - 1/3*log((b*x^3 + a)^p*c)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^4}\,dx = -\frac{\frac{b^2p\log\left(bx^3+a\right)}{a}-\frac{b^2p\log\left(bx^3\right)}{a}+\frac{bp\log\left(bx^3+a\right)}{x^3}+\frac{b\log\left(c\right)}{x^3}}{3\,b}$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x^4,x, algorithm="giac")
[Out] -1/3*(b^2*p*log(b*x^3 + a)/a - b^2*p*log(b*x^3)/a + b*p*log(b*x^3 + a)/x^3
+ b*log(c)/x^3)/b
```

Mupad [B] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^4}\,dx = \frac{b\,p\,\ln\left(x\right)}{a}-\frac{b\,p\,\ln\left(b\,x^3+a\right)}{3\,a}-\frac{\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{3\,x^3}$$

```
[In] int(log(c*(a + b*x^3)^p)/x^4,x)
[Out] (b*p*log(x))/a - (b*p*log(a + b*x^3))/(3*a) - log(c*(a + b*x^3)^p)/(3*x^3)
```

3.23 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 151

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^5}\,dx = -\frac{3bp}{4ax}+\frac{\sqrt{3}b^{4/3}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{4a^{4/3}}+\frac{b^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4a^{4/3}} - \frac{b^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8a^{4/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}$$

```
[Out] -3/4*b*p/a/x+1/4*b^(4/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)-1/8*b^(4/3)*p*ln(a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)-1/4*ln(c*(b*x^3+a)^p)/x^4+1/4
*b^(4/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/a^(4/3
)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2505, 331, 298, 31, 648, 631, 210, 642}

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^5}\,dx = \frac{\sqrt{3}b^{4/3}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{4a^{4/3}} - \frac{b^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8a^{4/3}} + \frac{b^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4a^{4/3}}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{4x^4}-\frac{3bp}{4ax}$$

```
[In] Int[Log[c*(a + b*x^3)^p]/x^5,x]
```

```
[Out] (-3*b*p)/(4*a*x) + (Sqrt[3]*b^(4/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(4*a^(4/3)) + (b^(4/3)*p*Log[a^(1/3) + b^(1/3)*x])/(4*a^(4/3)
) - (b^(4/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(8*a^(4/3))
- Log[c*(a + b*x^3)^p]/(4*x^4)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log\left(c(a+bx^3)^p\right)}{4x^4}+\frac{1}{4}(3bp)\int\frac{1}{x^2\left(a+bx^3\right)}\,dx\\
&= -\frac{3bp}{4ax}-\frac{\log\left(c(a+bx^3)^p\right)}{4x^4}-\frac{\left(3b^2p\right)\int\frac{x}{a+bx^3}\,dx}{4a}\\
&= -\frac{3bp}{4ax}-\frac{\log\left(c(a+bx^3)^p\right)}{4x^4}+\frac{\left(b^{5/3}p\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{4a^{4/3}}-\frac{\left(b^{5/3}p\right)\int\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{4a^{4/3}}\\
&= -\frac{3bp}{4ax}+\frac{b^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4a^{4/3}}-\frac{\log\left(c(a+bx^3)^p\right)}{4x^4}\\
&\quad-\frac{\left(b^{4/3}p\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{8a^{4/3}}-\frac{\left(3b^{5/3}p\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{8a}\\
&= -\frac{3bp}{4ax}+\frac{b^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4a^{4/3}}-\frac{b^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8a^{4/3}}\\
&\quad-\frac{\log\left(c(a+bx^3)^p\right)}{4x^4}-\frac{\left(3b^{4/3}p\right)\mathrm{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{4a^{4/3}}\\
&= -\frac{3bp}{4ax}+\frac{\sqrt{3}b^{4/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{4a^{4/3}}+\frac{b^{4/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4a^{4/3}}\\
&\quad-\frac{b^{4/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8a^{4/3}}-\frac{\log\left(c(a+bx^3)^p\right)}{4x^4}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.32

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^5}\,dx = -\frac{3bp\,\text{Hypergeometric2F1}\left(-\frac{1}{3},1,\frac{2}{3},-\frac{bx^3}{a}\right)}{4ax} - \frac{\log\left(c(a+bx^3)^p\right)}{4x^4}$$

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^5,x]
```

```
[Out] (-3*b*p*Hypergeometric2F1[-1/3, 1, 2/3, -((b*x^3)/a)])/(4*a*x) - Log[c*(a + b*x^3)^p]/(4*x^4)
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.85

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{4x^{4}}+\frac{3pb\left(-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b}{a}-\frac{1}{ax}\right)}{4}$
risch	$-\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)}{4x^{4}}-\frac{i\pi a\,\operatorname{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)\operatorname{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right)^{2}-i\pi a\,\operatorname{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)\operatorname{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right)\operatorname{csgn}\left(ic\right)-i\pi a\operatorname{csgn}\left(ic\left(b\,x^{3}+\right.\right.}{}$

```
[In] int(ln(c*(b*x^3+a)^p)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln(c*(b*x^3+a)^p)/x^4+3/4*p*b*(-(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+
1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/
3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))/a*b-1/a/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.91

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^5}\,dx = -\frac{2\sqrt{3}bpx^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+bpx^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-2\,bpx^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\log}{8\,ax^4}$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x^5,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(3)*b*p*x^4*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*
sqrt(3)) + b*p*x^4*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3))
 - 2*b*p*x^4*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)) + 6*b*p*x^3 + 2*a*p*log(b
*x^3 + a) + 2*a*log(c))/(a*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**5,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.84

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^5}\,dx = -\frac{1}{8}\,bp\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{6}{ax}\right) - \frac{\log\left((bx^3+a)^pc\right)}{4\,x^4}$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x^5,x, algorithm="maxima")
```

```
[Out] -1/8*b*p*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*
(a/b)^(1/3)) + log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(1/3)) - 2*l
og(x + (a/b)^(1/3))/(a*(a/b)^(1/3)) + 6/(a*x)) - 1/4*log((b*x^3 + a)^p*c)/x
^4
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^5}\,dx$$

$$= \frac{1}{8}\,b^2p\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2}+\frac{2\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b^2}-\frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right.}{a^2b^2}\right.$$

$$-\frac{p\log\left(bx^3+a\right)}{4\,x^4}-\frac{3\,bpx^3+a\log\left(c\right)}{4\,ax^4}$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*b^2*p*(2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 2*sqrt(3)*(-a*b^
2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2) -
(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^2)) - 1/4*p*
log(b*x^3 + a)/x^4 - 1/4*(3*b*p*x^3 + a*log(c))/(a*x^4)
```

Mupad [B] (verification not implemented)

Time = 3.35 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.83

$$\begin{aligned}\int \frac{\log \left(c(a+bx^3)^p\right)}{x^5}\,dx &= \frac{b^{4/3}\,p\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{4\,a^{4/3}}-\frac{\ln\left(c\left(b\,x^3+a\right)^p\right)}{4\,x^4}-\frac{3\,b\,p}{4\,a\,x}\\&+\frac{b^{4/3}\,p\,\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}-\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{4\,a^{4/3}}\\&-\frac{b^{4/3}\,p\,\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}+\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{4\,a^{4/3}}\end{aligned}$$

```
[In] int(log(c*(a + b*x^3)^p)/x^5,x)
```

```
[Out] (b^(4/3)*p*log(b^(1/3)*x + a^(1/3)))/(4*a^(4/3)) - log(c*(a + b*x^3)^p)/(4*
x^4) - (3*b*p)/(4*a*x) + (b^(4/3)*p*log(4*b^(1/3)*x - 3^(1/2)*a^(1/3)*2i -
2*a^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(4*a^(4/3)) - (b^(4/3)*p*log(3^(1/2)*a^(
1/3)*2i + 4*b^(1/3)*x - 2*a^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(4*a^(4/3))
```

3.24 $\int \frac{\log\left(c(a+bx^3)^p\right)}{x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 151

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^6}\,dx = -\frac{3bp}{10ax^2}+\frac{\sqrt{3}b^{5/3}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{5a^{5/3}}-\frac{b^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5a^{5/3}}+\frac{b^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{10a^{5/3}}-\frac{\log\left(c(a+bx^3)^p\right)}{5x^5}$$

```
[Out] -3/10*b*p/a/x^2-1/5*b^(5/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)+1/10*b^(5/3)*p*
ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)-1/5*ln(c*(b*x^3+a)^p)/x^5
+1/5*b^(5/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/a^
(5/3)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2505, 331, 206, 31, 648, 631, 210, 642}

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^6}\,dx = \frac{\sqrt{3}b^{5/3}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{5a^{5/3}}+\frac{b^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{10a^{5/3}}-\frac{b^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5a^{5/3}}-\frac{\log\left(c(a+bx^3)^p\right)}{5x^5}-\frac{3bp}{10ax^2}$$

```
[In] Int[Log[c*(a + b*x^3)^p]/x^6,x]
```

```
[Out] (-3*b*p)/(10*a*x^2) + (Sqrt[3]*b^(5/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sq
rt[3]*a^(1/3))])/(5*a^(5/3)) - (b^(5/3)*p*Log[a^(1/3) + b^(1/3)*x])/(5*a^(5
/3)) + (b^(5/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(10*a^(5/
3)) - Log[c*(a + b*x^3)^p]/(5*x^5)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log\left(c(a+bx^3)^p\right)}{5x^5}+\frac{1}{5}(3bp)\int\frac{1}{x^3\left(a+bx^3\right)}\,dx\\
&= -\frac{3bp}{10ax^2}-\frac{\log\left(c(a+bx^3)^p\right)}{5x^5}-\frac{(3b^2p)\int\frac{1}{a+bx^3}\,dx}{5a}\\
&= -\frac{3bp}{10ax^2}-\frac{\log\left(c(a+bx^3)^p\right)}{5x^5}-\frac{(b^2p)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{5a^{5/3}}-\frac{(b^2p)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{5a^{5/3}}\\
&= -\frac{3bp}{10ax^2}-\frac{b^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5a^{5/3}}-\frac{\log\left(c(a+bx^3)^p\right)}{5x^5}\\
&\quad+\frac{(b^{5/3}p)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{10a^{5/3}}-\frac{(3b^2p)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{10a^{4/3}}\\
&= -\frac{3bp}{10ax^2}-\frac{b^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5a^{5/3}}+\frac{b^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{10a^{5/3}}\\
&\quad-\frac{\log\left(c(a+bx^3)^p\right)}{5x^5}-\frac{(3b^{5/3}p)\,\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{5a^{5/3}}\\
&= -\frac{3bp}{10ax^2}+\frac{\sqrt{3}b^{5/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{5a^{5/3}}-\frac{b^{5/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{5a^{5/3}}\\
&\quad+\frac{b^{5/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{10a^{5/3}}-\frac{\log\left(c(a+bx^3)^p\right)}{5x^5}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.32

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^6}\,dx = -\frac{3bp\,\text{Hypergeometric2F1}\left(-\frac{2}{3},1,\frac{1}{3},-\frac{bx^3}{a}\right)}{10ax^2} - \frac{\log\left(c(a+bx^3)^p\right)}{5x^5}$$

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^6,x]
```

```
[Out] (-3*b*p*Hypergeometric2F1[-2/3, 1, 1/3, -((b*x^3)/a)])/(10*a*x^2) - Log[c*(
a + b*x^3)^p]/(5*x^5)
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.85

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{5x^{5}}+\frac{3pb\left(-\frac{1}{2a\,x^{2}}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)b}{a}\right)}{5}$
risch	$-\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)}{5x^{5}}-\frac{-2\left(\sum_{\_R=\operatorname{RootOf}\left(a^{5}\_Z^{3}+b^{5}p^{3}\right)}\_R\ln\left(\left(-4\_R^{3}a^{5}-3b^{5}p^{3}\right)x-a^{2}b^{3}p^{2}\_R\right)\right)a\,x^{5}+i\pi a\,\operatorname{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)\operatorname{csgn}}{}$

```
[In] int(ln(c*(b*x^3+a)^p)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*ln(c*(b*x^3+a)^p)/x^5+3/5*p*b*(-1/2/a/x^2-(1/3/b/(a/b)^(2/3)*ln(x+(a/b
)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))/a*b)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.14

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^6}\,dx = \frac{2\sqrt{3}bpx^5\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-bpx^5\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+2}{10\,ax^5}$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x^6,x, algorithm="fricas")
```

```
[Out] 1/10*(2*sqrt(3)*b*p*x^5*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^
2)^(2/3) - sqrt(3)*b)/b) - b*p*x^5*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b
^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 2*b*p*x^5*(-b^2/a^2)^(1/3)*log(b*x
- a*(-b^2/a^2)^(1/3)) - 3*b*p*x^3 - 2*a*p*log(b*x^3 + a) - 2*a*log(c))/(a*x
^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^6}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**6,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.85

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^6}\,dx = -\frac{1}{10}\,bp\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{3}{ax^2}\right)-\frac{\log\left((bx^3+a)^pc\right)}{5\,x^5}$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x^6,x, algorithm="maxima")
```

```
[Out] -1/10*b*p*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a
*(a/b)^(2/3)) - log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/3)) + 2*
log(x + (a/b)^(1/3))/(a*(a/b)^(2/3)) + 3/(a*x^2)) - 1/5*log((b*x^3 + a)^p*c
)/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^6}\,dx = \frac{b^2p\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{5\,a^2} - \frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\,bp\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{5\,a^2} - \frac{(-ab^2)^{\frac{1}{3}}\,bp\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{10\,a^2} - \frac{p\log\left(bx^3+a\right)}{5\,x^5} - \frac{3\,bpx^3+2\,a\log\left(c\right)}{10\,ax^5}$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x^6,x, algorithm="giac")
```

```
[Out] 1/5*b^2*p*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 - 1/5*sqrt(3)*(-a*b^2
)^(1/3)*b*p*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^2 - 1/1
0*(-a*b^2)^(1/3)*b*p*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^2 - 1/5*p*l
og(b*x^3 + a)/x^5 - 1/10*(3*b*p*x^3 + 2*a*log(c))/(a*x^5)
```

Mupad [B] (verification not implemented)

Time = 3.66 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^6}\,dx = \frac{(-b)^{5/3}\,p\,\ln\left(a^{1/3}\,(-b)^{11/3}-b^4\,x\right)}{5\,a^{5/3}} - \frac{\ln\left(c\,(b\,x^3+a)^p\right)}{5\,x^5} - \frac{3\,b\,p}{10\,a\,x^2} + \frac{(-b)^{5/3}\,p\,\ln\left(225\,a^2\,b^4\,p\,x-225\,a^{7/3}\,(-b)^{11/3}\,p\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{5\,a^{5/3}} - \frac{(-b)^{5/3}\,p\,\ln\left(225\,a^2\,b^4\,p\,x+225\,a^{7/3}\,(-b)^{11/3}\,p\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{5\,a^{5/3}}$$

```
[In] int(log(c*(a + b*x^3)^p)/x^6,x)
```

```
[Out] ((-b)^(5/3)*p*log(a^(1/3)*(-b)^(11/3) - b^4*x))/(5*a^(5/3)) - log(c*(a + b*
x^3)^p)/(5*x^5) - (3*b*p)/(10*a*x^2) + ((-b)^(5/3)*p*log(225*a^2*b^4*p*x -
225*a^(7/3)*(-b)^(11/3)*p*((3^(1/2)*1i)/2 - 1/2))*((3^(1/2)*1i)/2 - 1/2))/(
5*a^(5/3)) - ((-b)^(5/3)*p*log(225*a^2*b^4*p*x + 225*a^(7/3)*(-b)^(11/3)*p*
((3^(1/2)*1i)/2 + 1/2))*((3^(1/2)*1i)/2 + 1/2))/(5*a^(5/3))
```

3.25 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^7} dx$

Optimal result

Integrand size = 16, antiderivative size = 64

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^7} dx = -\frac{bp}{6ax^3} - \frac{b^2p\log(x)}{2a^2} + \frac{b^2p\log\left(a+bx^3\right)}{6a^2} - \frac{\log\left(c(a+bx^3)^p\right)}{6x^6}$$

```
[Out] -1/6*b*p/a/x^3-1/2*b^2*p*ln(x)/a^2+1/6*b^2*p*ln(b*x^3+a)/a^2-1/6*ln(c*(b*x^
3+a)^p)/x^6
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2442, 46}

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^7} dx = \frac{b^2p\log\left(a+bx^3\right)}{6a^2} - \frac{b^2p\log(x)}{2a^2} - \frac{\log\left(c(a+bx^3)^p\right)}{6x^6} - \frac{bp}{6ax^3}$$

```
[In] Int[Log[c*(a + b*x^3)^p]/x^7,x]
```

```
[Out] -1/6*(b*p)/(a*x^3) - (b^2*p*Log[x])/(2*a^2) + (b^2*p*Log[a + b*x^3])/(6*a^2
) - Log[c*(a + b*x^3)^p]/(6*x^6)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{\log \left(c(a+bx)^p\right)}{x^3}\,dx,x,x^3\right)\\
&= -\frac{\log \left(c\left(a+bx^3\right)^p\right)}{6x^6}+\frac{1}{6}(bp)\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx,x,x^3\right)\\
&= -\frac{\log \left(c\left(a+bx^3\right)^p\right)}{6x^6}+\frac{1}{6}(bp)\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)\,dx,x,x^3\right)\\
&= -\frac{bp}{6ax^3}-\frac{b^2p\log(x)}{2a^2}+\frac{b^2p\log \left(a+bx^3\right)}{6a^2}-\frac{\log \left(c\left(a+bx^3\right)^p\right)}{6x^6}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{\log \left(c\left(a+bx^3\right)^p\right)}{x^7}\,dx=\frac{1}{6}bp\left(-\frac{1}{ax^3}-\frac{3b\log(x)}{a^2}+\frac{b\log \left(a+bx^3\right)}{a^2}\right)-\frac{\log \left(c\left(a+bx^3\right)^p\right)}{6x^6}$$

```
[In] Integrate[Log[c*(a + b*x^3)^p]/x^7,x]
[Out] (b*p*(-(1/(a*x^3)) - (3*b*Log[x])/a^2 + (b*Log[a + b*x^3])/a^2))/6 - Log[c*
(a + b*x^3)^p]/(6*x^6)
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{6x^{6}}+\frac{pb\left(-\frac{1}{3a\,x^{3}}-\frac{b\ln\left(x\right)}{a^{2}}+\frac{b\ln\left(b\,x^{3}+a\right)}{3a^{2}}\right)}{2}$
parallelrisch	$-\frac{3b^{2}p^{2}\ln\left(x\right)x^{6}-x^{6}\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)b^{2}p-b^{2}p^{2}x^{6}+ab\,p^{2}x^{3}+\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)a^{2}p}{6x^{6}a^{2}p}$
risch	$-\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)}{6x^{6}}-\frac{-2b^{2}p\ln\left(-b\,x^{3}-a\right)x^{6}+6b^{2}p\ln\left(x\right)x^{6}+i\pi\,a^{2}\,\mathrm{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)\mathrm{csgn}\left(ic\left(b\,x^{3}+a\right)^{p}\right)^{2}-i\pi\,a^{2}\,\mathrm{csgn}\left(i\left(b\,x^{3}+a\right)^{p}\right)}{1}$

[In] int(ln(c*(b*x^3+a)^p)/x^7,x,method=_RETURNVERBOSE)

[Out] -1/6*ln(c*(b*x^3+a)^p)/x^6+1/2*p*b*(-1/3/a/x^3-1/a^2*b*ln(x)+1/3*b/a^2*ln(b
*x^3+a))

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^7}\,dx = -\frac{3\,b^2px^6\log\left(x\right)+abpx^3+a^2\log\left(c\right)-\left(b^2px^6-a^2p\right)\log\left(bx^3+a\right)}{6\,a^2x^6}$$

[In] integrate(log(c*(b*x^3+a)^p)/x^7,x, algorithm="fricas")

[Out] -1/6*(3*b^2*p*x^6*log(x) + a*b*p*x^3 + a^2*log(c) - (b^2*p*x^6 - a^2*p)*log
(b*x^3 + a))/(a^2*x^6)

Sympy [A] (verification not implemented)

Time = 7.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.30

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^7}\,dx = \begin{cases} -\frac{\log\left(c(a+bx^3)^p\right)}{6x^6}-\frac{bp}{6ax^3}-\frac{b^2p\log\left(x\right)}{2a^2}+\frac{b^2\log\left(c(a+bx^3)^p\right)}{6a^2} & \text{for}\ a\neq 0 \\ -\frac{p}{12x^6}-\frac{\log\left(c(bx^3)^p\right)}{6x^6} & \text{otherwise} \end{cases}$$

[In] integrate(ln(c*(b*x**3+a)**p)/x**7,x)

[Out] Piecewise((-log(c*(a + b*x**3)**p)/(6*x**6) - b*p/(6*a*x**3) - b**2*p*log(x
)/(2*a**2) + b**2*log(c*(a + b*x**3)**p)/(6*a**2), Ne(a, 0)), (-p/(12*x**6)
 - log(c*(b*x**3)**p)/(6*x**6), True))

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^7}\,dx = \frac{1}{6}\,bp\left(\frac{b\log\left(bx^3+a\right)}{a^2} - \frac{b\log\left(x^3\right)}{a^2} - \frac{1}{ax^3}\right) - \frac{\log\left((bx^3+a)^p c\right)}{6\,x^6}$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*b*p*(b*log(b*x^3 + a)/a^2 - b*log(x^3)/a^2 - 1/(a*x^3)) - 1/6*log((b*x^
3 + a)^p*c)/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(56) = 112.

Time = 0.30 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.06

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^7}\,dx = -\frac{\frac{b^3p\log(bx^3+a)}{(bx^3+a)^2-2\,(bx^3+a)a+a^2} - \frac{b^3p\log(bx^3+a)}{a^2} + \frac{b^3p\log(bx^3)}{a^2} + \frac{(bx^3+a)b^3p-ab^3p+ab^3\log(c)}{(bx^3+a)^2a-2\,(bx^3+a)a^2+a^3}}{6\,b}$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x^7,x, algorithm="giac")
```

```
[Out] -1/6*(b^3*p*log(b*x^3 + a)/((b*x^3 + a)^2 - 2*(b*x^3 + a)*a + a^2) - b^3*p*
log(b*x^3 + a)/a^2 + b^3*p*log(b*x^3)/a^2 + ((b*x^3 + a)*b^3*p - a*b^3*p +
a*b^3*log(c))/((b*x^3 + a)^2*a - 2*(b*x^3 + a)*a^2 + a^3))/b
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^7}\,dx = \frac{b^2\,p\,\ln\left(b\,x^3+a\right)}{6\,a^2} - \frac{\ln\left(c\,\left(b\,x^3+a\right)^p\right)}{6\,x^6} - \frac{b^2\,p\,\ln\left(x\right)}{2\,a^2} - \frac{b\,p}{6\,a\,x^3}$$

```
[In] int(log(c*(a + b*x^3)^p)/x^7,x)
```

```
[Out] (b^2*p*log(a + b*x^3))/(6*a^2) - log(c*(a + b*x^3)^p)/(6*x^6) - (b^2*p*log(
x))/(2*a^2) - (b*p)/(6*a*x^3)
```

3.26 $\int x^4 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx$

Optimal result

Integrand size = 16, antiderivative size = 89

$$\int x^4 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = -\frac{b^4px}{5a^4}+\frac{b^3px^2}{10a^3}-\frac{b^2px^3}{15a^2}+\frac{bpx^4}{20a} + \frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{b^5p\log(b+ax)}{5a^5}$$

```
[Out] -1/5*b^4*p*x/a^4+1/10*b^3*p*x^2/a^3-1/15*b^2*p*x^3/a^2+1/20*b*p*x^4/a+1/5*x
^5*ln(c*(a+b/x)^p)+1/5*b^5*p*ln(a*x+b)/a^5
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2505, 269, 45}

$$\int x^4 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{b^5p\log(ax+b)}{5a^5}-\frac{b^4px}{5a^4}+\frac{b^3px^2}{10a^3}-\frac{b^2px^3}{15a^2} + \frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bpx^4}{20a}$$

```
[In] Int[x^4*Log[c*(a + b/x)^p],x]
```

```
[Out] -1/5*(b^4*p*x)/a^4 + (b^3*p*x^2)/(10*a^3) - (b^2*p*x^3)/(15*a^2) + (b*p*x^4
)/(20*a) + (x^5*Log[c*(a + b/x)^p])/5 + (b^5*p*Log[b + a*x])/(5*a^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{5}(bp)\int\frac{x^3}{a+\frac{b}{x}}\,dx\\
&= \frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{5}(bp)\int\frac{x^4}{b+ax}\,dx\\
&= \frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{5}(bp)\int\left(-\frac{b^3}{a^4}+\frac{b^2x}{a^3}-\frac{bx^2}{a^2}+\frac{x^3}{a}+\frac{b^4}{a^4(b+ax)}\right)dx\\
&= -\frac{b^4px}{5a^4}+\frac{b^3px^2}{10a^3}-\frac{b^2px^3}{15a^2}+\frac{bpx^4}{20a}+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{b^5p\log(b+ax)}{5a^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

$$\int x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$
$$=\frac{abpx(-12b^3+6ab^2x-4a^2bx^2+3a^3x^3)+12b^5p\log\left(a+\frac{b}{x}\right)+12a^5x^5\log\left(c\left(a+\frac{b}{x}\right)^p\right)+12b^5p\log(x)}{60a^5}$$

```
[In] Integrate[x^4*Log[c*(a + b/x)^p],x]
[Out] (a*b*p*x*(-12*b^3 + 6*a*b^2*x - 4*a^2*b*x^2 + 3*a^3*x^3) + 12*b^5*p*Log[a +
 b/x] + 12*a^5*x^5*Log[c*(a + b/x)^p] + 12*b^5*p*Log[x])/(60*a^5)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

method	result	s
parts	$\frac{x^{5} \ln\left(c \left(a+\frac{b}{x}\right)^{p}\right)}{5}+\frac{p b \left(\frac{\frac{1}{4} a^{3} x^{4}-\frac{1}{3} b \,x^{3} a^{2}+\frac{1}{2} a \,b^{2} x^{2}-x \,b^{3}}{a^{4}}+\frac{b^{4} \ln\left(a x +b \right)}{a^{5}}\right)}{5}$	7
parallelrisch	$-\frac{-12 x^{5} \ln\left(c \left(\frac{a x +b}{x}\right)^{p}\right) a^{5} p -3 x^{4} a^{4} b \,p^{2}+4 x^{3} a^{3} b^{2} p^{2}-6 x^{2} a^{2} b^{3} p^{2}-12 \ln\left(x \right) b^{5} p^{2}+12 x a \,b^{4} p^{2}-12 \ln\left(c \left(\frac{a x +b}{x}\right)^{p}\right) b^{5} p -12 b^{5} p^{2}}{60 a^{5} p}$	1

```
[In] int(x^4*ln(c*(a+b/x)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*ln(c*(a+b/x)^p)+1/5*p*b*(1/a^4*(1/4*a^3*x^4-1/3*b*x^3*a^2+1/2*a*b^2
*x^2-x*b^3)+1/a^5*b^4*ln(a*x+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int x^4 \log \left(c \left(a+\frac{b}{x}\right)^p\right) dx = \frac{12 a^5 p x^5 \log \left(\frac{ax+b}{x}\right) + 12 a^5 x^5 \log (c) + 3 a^4 b p x^4 - 4 a^3 b^2 p x^3 + 6 a^2 b^3 p x^2 - 12 a b^4 p x + 12 b^5 p \log (ax+b)}{60 a^5}$$

```
[In] integrate(x^4*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/60*(12*a^5*p*x^5*log((a*x + b)/x) + 12*a^5*x^5*log(c) + 3*a^4*b*p*x^4 - 4
*a^3*b^2*p*x^3 + 6*a^2*b^3*p*x^2 - 12*a*b^4*p*x + 12*b^5*p*log(a*x + b))/a^
5
```

Sympy [A] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.12

$$\int x^4 \log \left(c \left(a+\frac{b}{x}\right)^p\right) dx = \begin{cases} \frac{x^5 \log \left(c \left(a+\frac{b}{x}\right)^p\right)}{5} + \frac{bpx^4}{20a} - \frac{b^2px^3}{15a^2} + \frac{b^3px^2}{10a^3} - \frac{b^4px}{5a^4} + \frac{b^5p \log (ax+b)}{5a^5} & \text{for } a \neq 0 \\ \frac{px^5}{25} + \frac{x^5 \log \left(c \left(\frac{b}{x}\right)^p\right)}{5} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**4*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((x**5*log(c*(a + b/x)**p)/5 + b*p*x**4/(20*a) - b**2*p*x**3/(15*a
**2) + b**3*p*x**2/(10*a**3) - b**4*p*x/(5*a**4) + b**5*p*log(a*x + b)/(5*a
**5), Ne(a, 0)), (p*x**5/25 + x**5*log(c*(b/x)**p)/5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

$$
\begin{aligned}
\int x^4 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx
&= \frac{1}{5} x^5 \log \left(\left(a+\frac{b}{x}\right)^p c\right) \\
&\quad + \frac{1}{60} bp\left(\frac{12 b^4 \log (ax+b)}{a^5} + \frac{3 a^3x^4 - 4 a^2bx^3 + 6 ab^2x^2 - 12 b^3x}{a^4}\right)
\end{aligned}
$$

```
[In] integrate(x^4*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*log((a + b/x)^p*c) + 1/60*b*p*(12*b^4*log(a*x + b)/a^5 + (3*a^3*x^4
 - 4*a^2*b*x^3 + 6*a*b^2*x^2 - 12*b^3*x)/a^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(77) = 154$.

Time = 0.32 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.46

$$
\int x^4 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = -\frac{\frac{12 b^6 p \log\left(\frac{ax+b}{x}\right)}{a^5 - \frac{5(ax+b)a^4}{x} + \frac{10(ax+b)^2a^3}{x^2} - \frac{10(ax+b)^3a^2}{x^3} + \frac{5(ax+b)^4a}{x^4} - \frac{(ax+b)^5}{x^5}} + \frac{12 b^6 p \log\left(-a+\frac{ax+b}{x}\right)}{a^5} - \frac{12 b^6 p \log\left(\frac{ax+b}{x}\right)}{a^5} - \frac{25 a^4b^6p - 12 a^4b^6 \log(c) - \frac{77(ax+b)a^3b^6p}{x} + \frac{94(ax+b)^2a^2b^6p}{x^2} - \frac{54(ax+b)^3ab^6p}{x^3} + \frac{12(ax+b)^4b^6p}{x^4}}{a^9 - \frac{5(ax+b)a^8}{x} + \frac{10(ax+b)^2a^7}{x^2} - \frac{10(ax+b)^3a^6}{x^3} + \frac{5(ax+b)^4a^5}{x^4} - \frac{(ax+b)^5a^4}{x^5}}}{60\, b}
$$

```
[In] integrate(x^4*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] -1/60*(12*b^6*p*log((a*x + b)/x)/(a^5 - 5*(a*x + b)*a^4/x + 10*(a*x + b)^2*
a^3/x^2 - 10*(a*x + b)^3*a^2/x^3 + 5*(a*x + b)^4*a/x^4 - (a*x + b)^5/x^5) +
 12*b^6*p*log(-a + (a*x + b)/x)/a^5 - 12*b^6*p*log((a*x + b)/x)/a^5 - (25*a
^4*b^6*p - 12*a^4*b^6*log(c) - 77*(a*x + b)*a^3*b^6*p/x + 94*(a*x + b)^2*a^
2*b^6*p/x^2 - 54*(a*x + b)^3*a*b^6*p/x^3 + 12*(a*x + b)^4*b^6*p/x^4)/(a^9 -
 5*(a*x + b)*a^8/x + 10*(a*x + b)^2*a^7/x^2 - 10*(a*x + b)^3*a^6/x^3 + 5*(a
*x + b)^4*a^5/x^4 - (a*x + b)^5*a^4/x^5))/b
```

Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.87

$$\int x^4 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{x^5 \ln \left(c \left(a+\frac{b}{x}\right)^p\right)}{5} - \frac{b^2 p x^3}{15 a^2} + \frac{b^3 p x^2}{10 a^3} + \frac{b^5 p \ln (b+a x)}{5 a^5} + \frac{b p x^4}{20 a} - \frac{b^4 p x}{5 a^4}$$

```
[In] int(x^4*log(c*(a + b/x)^p),x)
```

```
[Out] (x^5*log(c*(a + b/x)^p))/5 - (b^2*p*x^3)/(15*a^2) + (b^3*p*x^2)/(10*a^3) +
(b^5*p*log(b + a*x))/(5*a^5) + (b*p*x^4)/(20*a) - (b^4*p*x)/(5*a^4)
```

3.27 $\int x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{b^3px}{4a^3} - \frac{b^2px^2}{8a^2} + \frac{bpx^3}{12a} + \frac{1}{4}x^4 \log \left(c\left(a+\frac{b}{x}\right)^p\right) - \frac{b^4p\log(b+ax)}{4a^4}$$

```
[Out] 1/4*b^3*p*x/a^3-1/8*b^2*p*x^2/a^2+1/12*b*p*x^3/a+1/4*x^4*ln(c*(a+b/x)^p)-1/
4*b^4*p*ln(a*x+b)/a^4
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2505, 269, 45}

$$\int x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = -\frac{b^4p\log(ax+b)}{4a^4} + \frac{b^3px}{4a^3} - \frac{b^2px^2}{8a^2} + \frac{1}{4}x^4 \log \left(c\left(a+\frac{b}{x}\right)^p\right) + \frac{bpx^3}{12a}$$

```
[In] Int[x^3*Log[c*(a + b/x)^p],x]
```

```
[Out] (b^3*p*x)/(4*a^3) - (b^2*p*x^2)/(8*a^2) + (b*p*x^3)/(12*a) + (x^4*Log[c*(a
+ b/x)^p])/4 - (b^4*p*Log[b + a*x])/(4*a^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{4}(bp)\int\frac{x^2}{a+\frac{b}{x}}\,dx\\
&= \frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{4}(bp)\int\frac{x^3}{b+ax}\,dx\\
&= \frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{4}(bp)\int\left(\frac{b^2}{a^3}-\frac{bx}{a^2}+\frac{x^2}{a}-\frac{b^3}{a^3(b+ax)}\right)dx\\
&= \frac{b^3px}{4a^3}-\frac{b^2px^2}{8a^2}+\frac{bpx^3}{12a}+\frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)-\frac{b^4p\log(b+ax)}{4a^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)dx\\
=\frac{abpx(6b^2-3abx+2a^2x^2)-6b^4p\log\left(a+\frac{b}{x}\right)+6a^4x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)-6b^4p\log(x)}{24a^4}$$

```
[In] Integrate[x^3*Log[c*(a + b/x)^p],x]
```

```
[Out] (a*b*p*x*(6*b^2 - 3*a*b*x + 2*a^2*x^2) - 6*b^4*p*Log[a + b/x] + 6*a^4*x^4*L
og[c*(a + b/x)^p] - 6*b^4*p*Log[x])/(24*a^4)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

method	result	size
parts	$\frac{x^{4} \ln\left(c \left(a+\frac{b}{x}\right)^{p}\right)}{4}+\frac{p b \left(\frac{\frac{1}{3} x^{3} a^{2}-\frac{1}{2} a b \,x^{2}+b^{2} x}{a^{3}}-\frac{b^{3} \ln\left(a x +b \right)}{a^{4}}\right)}{4}$	63
parallelrisch	$-\frac{-6 x^{4} \ln\left(c \left(\frac{a x +b}{x}\right)^{p}\right) a^{4} p -2 x^{3} a^{3} b \,p^{2}+3 x^{2} a^{2} b^{2} p^{2}+6 \ln\left(x \right) b^{4} p^{2}-6 x a \,b^{3} p^{2}+6 \ln\left(c \left(\frac{a x +b}{x}\right)^{p}\right) b^{4} p +6 b^{4} p^{2}}{24 a^{4} p}$	107

```
[In] int(x^3*ln(c*(a+b/x)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*ln(c*(a+b/x)^p)+1/4*p*b*(1/a^3*(1/3*x^3*a^2-1/2*a*b*x^2+b^2*x)-1/a^
4*b^3*ln(a*x+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int x^3 \log \left(c \left(a+\frac{b}{x}\right)^p\right) \, dx = \frac{6 \, a^{4} p x^{4} \log \left(\frac{a x + b}{x}\right) + 6 \, a^{4} x^{4} \log \left(c\right) + 2 \, a^{3} b p x^{3} - 3 \, a^{2} b^{2} p x^{2} + 6 \, a b^{3} p x - 6 \, b^{4} p \log \left(a x + b\right)}{24 \, a^{4}}$$

```
[In] integrate(x^3*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a^4*p*x^4*log((a*x + b)/x) + 6*a^4*x^4*log(c) + 2*a^3*b*p*x^3 - 3*a
^2*b^2*p*x^2 + 6*a*b^3*p*x - 6*b^4*p*log(a*x + b))/a^4
```

Sympy [A] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16

$$\int x^3 \log \left(c \left(a+\frac{b}{x}\right)^p\right) \, dx = \begin{cases} \frac{x^{4} \log \left(c \left(a + \frac{b}{x}\right)^{p}\right)}{4} + \frac{b p x^{3}}{12 a} - \frac{b^{2} p x^{2}}{8 a^{2}} + \frac{b^{3} p x}{4 a^{3}} - \frac{b^{4} p \log \left(a x + b\right)}{4 a^{4}} & \text{for}\: a \neq 0 \\ \frac{p x^{4}}{16} + \frac{x^{4} \log \left(c \left(\frac{b}{x}\right)^{p}\right)}{4} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((x**4*log(c*(a + b/x)**p)/4 + b*p*x**3/(12*a) - b**2*p*x**2/(8*a*
*2) + b**3*p*x/(4*a**3) - b**4*p*log(a*x + b)/(4*a**4), Ne(a, 0)), (p*x**4/
16 + x**4*log(c*(b/x)**p)/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85

$$\int x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{1}{4}\,x^4 \log\left(\left(a+\frac{b}{x}\right)^p c\right) - \frac{1}{24}\,bp\left(\frac{6\,b^3 \log\left(ax+b\right)}{a^4} - \frac{2\,a^2x^3 - 3\,abx^2 + 6\,b^2x}{a^3}\right)$$

```
[In] integrate(x^3*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((a + b/x)^p*c) - 1/24*b*p*(6*b^3*log(a*x + b)/a^4 - (2*a^2*x^3
- 3*a*b*x^2 + 6*b^2*x)/a^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(65) = 130$.

Time = 0.33 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.43

$$\int x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{\frac{6\,b^5p\log\left(\frac{ax+b}{x}\right)}{a^4 - \frac{4\,(ax+b)a^3}{x} + \frac{6\,(ax+b)^2a^2}{x^2} - \frac{4\,(ax+b)^3a}{x^3} + \frac{(ax+b)^4}{x^4}} + \frac{6\,b^5p\log\left(-a+\frac{ax+b}{x}\right)}{a^4} - \frac{6\,b^5p\log\left(\frac{ax+b}{x}\right)}{a^4} - \frac{11\,a^3b^5p - 6\,a^3b^5\log\left(c\right) - \frac{26\,(ax+b)a^2b^5p}{x} + 21}{a^7 - \frac{4\,(ax+b)a^6}{x} + \frac{6\,(ax+b)^2a^5}{x^2} - 4\,(ax}}{24\,b}$$

```
[In] integrate(x^3*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] 1/24*(6*b^5*p*log((a*x + b)/x)/(a^4 - 4*(a*x + b)*a^3/x + 6*(a*x + b)^2*a^2
/x^2 - 4*(a*x + b)^3*a/x^3 + (a*x + b)^4/x^4) + 6*b^5*p*log(-a + (a*x + b)/
x)/a^4 - 6*b^5*p*log((a*x + b)/x)/a^4 - (11*a^3*b^5*p - 6*a^3*b^5*log(c) -
26*(a*x + b)*a^2*b^5*p/x + 21*(a*x + b)^2*a*b^5*p/x^2 - 6*(a*x + b)^3*b^5*p
/x^3)/(a^7 - 4*(a*x + b)*a^6/x + 6*(a*x + b)^2*a^5/x^2 - 4*(a*x + b)^3*a^4/
x^3 + (a*x + b)^4*a^3/x^4))/b
```

Mupad [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{x^4\,\ln\left(c\,{\left(a+\frac{b}{x}\right)}^p\right)}{4} - \frac{b^2\,p\,x^2}{8\,a^2} - \frac{b^4\,p\,\ln\left(b+a\,x\right)}{4\,a^4} + \frac{b\,p\,x^3}{12\,a} + \frac{b^3\,p\,x}{4\,a^3}$$

```
[In] int(x^3*log(c*(a + b/x)^p),x)
```

```
[Out] (x^4*log(c*(a + b/x)^p))/4 - (b^2*p*x^2)/(8*a^2) - (b^4*p*log(b + a*x))/(4*
a^4) + (b*p*x^3)/(12*a) + (b^3*p*x)/(4*a^3)
```

3.28 $\int x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 61

$$\int x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx=-\frac{b^2px}{3a^2}+\frac{bpx^2}{6a}+\frac{1}{3}x^3\log \left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{b^3p\log(b+ax)}{3a^3}$$

```
[Out] -1/3*b^2*p*x/a^2+1/6*b*p*x^2/a+1/3*x^3*ln(c*(a+b/x)^p)+1/3*b^3*p*ln(a*x+b)/
a^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2505, 269, 45}

$$\int x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx=\frac{b^3p\log(ax+b)}{3a^3}-\frac{b^2px}{3a^2}+\frac{1}{3}x^3\log \left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bpx^2}{6a}$$

```
[In] Int[x^2*Log[c*(a + b/x)^p],x]
```

```
[Out] -1/3*(b^2*p*x)/a^2 + (b*p*x^2)/(6*a) + (x^3*Log[c*(a + b/x)^p])/3 + (b^3*p*
Log[b + a*x])/(3*a^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{3}(bp)\int\frac{x}{a+\frac{b}{x}}\,dx\\
&= \frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{3}(bp)\int\frac{x^2}{b+ax}\,dx\\
&= \frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{3}(bp)\int\left(-\frac{b}{a^2}+\frac{x}{a}+\frac{b^2}{a^2(b+ax)}\right)dx\\
&= -\frac{b^2px}{3a^2}+\frac{bpx^2}{6a}+\frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{b^3p\log(b+ax)}{3a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02

$$\int x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)dx = \frac{abpx(-2b+ax)+2b^3p\log\left(a+\frac{b}{x}\right)+2a^3x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)+2b^3p\log(x)}{6a^3}$$

```
[In] Integrate[x^2*Log[c*(a + b/x)^p],x]
[Out] (a*b*p*x*(-2*b + a*x) + 2*b^3*p*Log[a + b/x] + 2*a^3*x^3*Log[c*(a + b/x)^p]
 + 2*b^3*p*Log[x])/(6*a^3)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

method	result	size
parts	$\frac{x^{3}\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{3}+\frac{pb\left(\frac{\frac{1}{2}x^{2}a-bx}{a^{2}}+\frac{b^{2}\ln(ax+b)}{a^{3}}\right)}{3}$	52
parallelrisch	$-\frac{-2x^{3}\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)a^{3}p-x^{2}a^{2}b\,p^{2}-2\ln(x)b^{3}p^{2}+2xa\,b^{2}p^{2}-2\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)b^{3}p-2b^{3}p^{2}}{6a^{3}p}$	93

```
[In] int(x^2*ln(c*(a+b/x)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*ln(c*(a+b/x)^p)+1/3*p*b*(1/a^2*(1/2*x^2*a-b*x)+b^2/a^3*ln(a*x+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

$$\int x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{2\,a^3px^3\log\left(\frac{ax+b}{x}\right)+2\,a^3x^3\log\left(c\right)+a^2bpx^2-2\,ab^2px+2\,b^3p\log\left(ax+b\right)}{6\,a^3}$$

```
[In] integrate(x^2*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*p*x^3*log((a*x + b)/x) + 2*a^3*x^3*log(c) + a^2*b*p*x^2 - 2*a*b^
2*p*x + 2*b^3*p*log(a*x + b))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \begin{cases}\frac{x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3}+\frac{bpx^2}{6a}-\frac{b^2px}{3a^2}+\frac{b^3p\log\left(ax+b\right)}{3a^3} & \text{for}\ a\neq 0\\ \frac{px^3}{9}+\frac{x^3\log\left(c\left(\frac{b}{x}\right)^p\right)}{3} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**2*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((x**3*log(c*(a + b/x)**p)/3 + b*p*x**2/(6*a) - b**2*p*x/(3*a**2)
+ b**3*p*log(a*x + b)/(3*a**3), Ne(a, 0)), (p*x**3/9 + x**3*log(c*(b/x)**p)
/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{1}{3}x^3 \log \left(\left(a+\frac{b}{x}\right)^p c\right) + \frac{1}{6}bp\left(\frac{2b^2 \log (ax+b)}{a^3} + \frac{ax^2-2bx}{a^2}\right)$$

```
[In] integrate(x^2*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((a + b/x)^p*c) + 1/6*b*p*(2*b^2*log(a*x + b)/a^3 + (a*x^2 - 2*b
*x)/a^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(53) = 106$.

Time = 0.33 (sec) , antiderivative size = 210, normalized size of antiderivative = 3.44

$$\int x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = -\frac{\frac{2b^4p\log\left(\frac{ax+b}{x}\right)}{a^3-\frac{3(ax+b)a^2}{x}+\frac{3(ax+b)^2a}{x^2}-\frac{(ax+b)^3}{x^3}} + \frac{2b^4p\log\left(-a+\frac{ax+b}{x}\right)}{a^3} - \frac{2b^4p\log\left(\frac{ax+b}{x}\right)}{a^3} - \frac{3a^2b^4p-2a^2b^4\log(c)-\frac{5(ax+b)ab^4p}{x}+\frac{2(ax+b)^2b^4p}{x^2}}{a^5-\frac{3(ax+b)a^4}{x}+\frac{3(ax+b)^2a^3}{x^2}-\frac{(ax+b)^3a^2}{x^3}}}{6b}$$

```
[In] integrate(x^2*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] -1/6*(2*b^4*p*log((a*x + b)/x)/(a^3 - 3*(a*x + b)*a^2/x + 3*(a*x + b)^2*a/x
^2 - (a*x + b)^3/x^3) + 2*b^4*p*log(-a + (a*x + b)/x)/a^3 - 2*b^4*p*log((a*
x + b)/x)/a^3 - (3*a^2*b^4*p - 2*a^2*b^4*log(c) - 5*(a*x + b)*a*b^4*p/x + 2
*(a*x + b)^2*b^4*p/x^2)/(a^5 - 3*(a*x + b)*a^4/x + 3*(a*x + b)^2*a^3/x^2 -
(a*x + b)^3*a^2/x^3))/b
```

Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{x^3 \ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{3} + \frac{b^3\,p\,\ln(b+a\,x)}{3\,a^3} + \frac{b\,p\,x^2}{6\,a} - \frac{b^2\,p\,x}{3\,a^2}$$

```
[In] int(x^2*log(c*(a + b/x)^p),x)
```

```
[Out] (x^3*log(c*(a + b/x)^p))/3 + (b^3*p*log(b + a*x))/(3*a^3) + (b*p*x^2)/(6*a)
 - (b^2*p*x)/(3*a^2)
```

3.29 $\int x \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx$

Optimal result

Integrand size = 14, antiderivative size = 47

$$\int x \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{bpx}{2a}+\frac{1}{2}x^2\log \left(c\left(a+\frac{b}{x}\right)^p\right)-\frac{b^2p\log(b+ax)}{2a^2}$$

```
[Out] 1/2*b*p*x/a+1/2*x^2*ln(c*(a+b/x)^p)-1/2*b^2*p*ln(a*x+b)/a^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2505, 199, 45}

$$\int x \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = -\frac{b^2p\log(ax+b)}{2a^2}+\frac{1}{2}x^2\log \left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bpx}{2a}$$

```
[In] Int[x*Log[c*(a + b/x)^p],x]
[Out] (b*p*x)/(2*a) + (x^2*Log[c*(a + b/x)^p])/2 - (b^2*p*Log[b + a*x])/(2*a^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{2}(bp)\int\frac{1}{a+\frac{b}{x}}\,dx \\
&= \frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{2}(bp)\int\frac{x}{b+ax}\,dx \\
&= \frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{1}{2}(bp)\int\left(\frac{1}{a}-\frac{b}{a(b+ax)}\right)dx \\
&= \frac{bpx}{2a}+\frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)-\frac{b^2p\log(b+ax)}{2a^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$
\int x\log\left(c\left(a+\frac{b}{x}\right)^p\right)dx=\frac{1}{2}\left(x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bp(ax-b\log(b+ax))}{a^2}\right)
$$

```
[In] Integrate[x*Log[c*(a + b/x)^p],x]
[Out] (x^2*Log[c*(a + b/x)^p] + (b*p*(a*x - b*Log[b + a*x]))/a^2)/2
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

method	result	size
parts	$\frac{x^2\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{2}+\frac{pb\left(\frac{x}{a}-\frac{b\ln(ax+b)}{a^2}\right)}{2}$	41
parallelrisch	$-\frac{-x^2\ln\left(c\left(\frac{ax+b}{x}\right)^p\right)a^2p+\ln(x)b^2p^2-xab\,p^2+\ln\left(c\left(\frac{ax+b}{x}\right)^p\right)b^2p+b^2p^2}{2a^2p}$	76

```
[In] int(x*ln(c*(a+b/x)^p),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*ln(c*(a+b/x)^p)+1/2*p*b*(x/a-1/a^2*b*ln(a*x+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int x\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx=\frac{a^2px^2\log\left(\frac{ax+b}{x}\right)+a^2x^2\log\left(c\right)+abpx-b^2p\log\left(ax+b\right)}{2\,a^2}$$

```
[In] integrate(x*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*p*x^2*log((a*x + b)/x) + a^2*x^2*log(c) + a*b*p*x - b^2*p*log(a*x
+ b))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.28

$$\int x\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx=\begin{cases}\frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2}+\frac{bpx}{2a}-\frac{b^2p\log\left(ax+b\right)}{2a^2} & \text{for } a\neq 0\\ \frac{px^2}{4}+\frac{x^2\log\left(c\left(\frac{b}{x}\right)^p\right)}{2} & \text{otherwise}\end{cases}$$

```
[In] integrate(x*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((x**2*log(c*(a + b/x)**p)/2 + b*p*x/(2*a) - b**2*p*log(a*x + b)/(
2*a**2), Ne(a, 0)), (p*x**2/4 + x**2*log(c*(b/x)**p)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int x\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx=\frac{1}{2}\,bp\left(\frac{x}{a}-\frac{b\log\left(ax+b\right)}{a^2}\right)+\frac{1}{2}\,x^2\log\left(\left(a+\frac{b}{x}\right)^pc\right)$$

```
[In] integrate(x*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(x/a - b*log(a*x + b)/a^2) + 1/2*x^2*log((a + b/x)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(41) = 82$.

Time = 0.31 (sec) , antiderivative size = 152, normalized size of antiderivative = 3.23

$$\int x \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{\frac{b^3 p \log \left(\frac{ax+b}{x}\right)}{a^2 - \frac{2\,(ax+b)a}{x} + \frac{(ax+b)^2}{x^2}} + \frac{b^3 p \log \left(-a + \frac{ax+b}{x}\right)}{a^2} - \frac{b^3 p \log \left(\frac{ax+b}{x}\right)}{a^2} - \frac{ab^3 p - ab^3 \log(c) - \frac{(ax+b)b^3 p}{x}}{a^3 - \frac{2\,(ax+b)a^2}{x} + \frac{(ax+b)^2 a}{x^2}}}{2\,b}$$

```
[In] integrate(x*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] 1/2*(b^3*p*log((a*x + b)/x)/(a^2 - 2*(a*x + b)*a/x + (a*x + b)^2/x^2) + b^3
*p*log(-a + (a*x + b)/x)/a^2 - b^3*p*log((a*x + b)/x)/a^2 - (a*b^3*p - a*b^
3*log(c) - (a*x + b)*b^3*p/x)/(a^3 - 2*(a*x + b)*a^2/x + (a*x + b)^2*a/x^2)
)/b
```

Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int x \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{x^2 \ln \left(c\left(a+\frac{b}{x}\right)^p\right)}{2} + \frac{b\,p\,x}{2\,a} - \frac{b^2\,p\,\ln \left(b+a\,x\right)}{2\,a^2}$$

```
[In] int(x*log(c*(a + b/x)^p),x)
```

```
[Out] (x^2*log(c*(a + b/x)^p))/2 + (b*p*x)/(2*a) - (b^2*p*log(b + a*x))/(2*a^2)
```

3.30 $\int \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 27

$$\int \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx = x\log \left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bp\log(b+ax)}{a}$$

```
[Out] x*ln(c*(a+b/x)^p)+b*p*ln(a*x+b)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2498, 269, 31}

$$\int \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx = x\log \left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bp\log(ax+b)}{a}$$

```
[In] Int[Log[c*(a + b/x)^p],x]
```

```
[Out] x*Log[c*(a + b/x)^p] + (b*p*Log[b + a*x])/a
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\log\left(c\left(a+\frac{b}{x}\right)^p\right)+(bp)\int\frac{1}{\left(a+\frac{b}{x}\right)x}\,dx\\ &= x\log\left(c\left(a+\frac{b}{x}\right)^p\right)+(bp)\int\frac{1}{b+ax}\,dx\\ &= x\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bp\log(b+ax)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx=\frac{bp\log\left(a+\frac{b}{x}\right)}{a}+x\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bp\log(x)}{a}$$

```
[In] Integrate[Log[c*(a + b/x)^p],x]
[Out] (b*p*Log[a + b/x])/a + x*Log[c*(a + b/x)^p] + (b*p*Log[x])/a
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
parts	$x\ln\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bp\ln(ax+b)}{a}$	28
default	$x\ln\left(c\left(\frac{ax+b}{x}\right)^p\right)+\frac{bp\ln(ax+b)}{a}$	30
parallelrisch	$-\frac{-\ln(x)b^2p^2-x\ln\left(c\left(\frac{ax+b}{x}\right)^p\right)abp-\ln\left(c\left(\frac{ax+b}{x}\right)^p\right)b^2p}{abp}$	63

```
[In] int(ln(c*(a+b/x)^p),x,method=_RETURNVERBOSE)
[Out] x*ln(c*(a+b/x)^p)+b*p*ln(a*x+b)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = \frac{apx\log\left(\frac{ax+b}{x}\right)+bp\log\left(ax+b\right)+ax\log\left(c\right)}{a}$$

```
[In] integrate(log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] (a*p*x*log((a*x + b)/x) + b*p*log(a*x + b) + a*x*log(c))/a
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = \begin{cases} x\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bp\log\left(ax+b\right)}{a} & \text{for}\ a\neq 0 \\ px+x\log\left(c\left(\frac{b}{x}\right)^p\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((x*log(c*(a + b/x)**p) + b*p*log(a*x + b)/a, Ne(a, 0)), (p*x + x*
log(c*(b/x)**p), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx = x\log\left(\left(a+\frac{b}{x}\right)^p c\right)+\frac{bp\log\left(ax+b\right)}{a}$$

```
[In] integrate(log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] x*log((a + b/x)^p*c) + b*p*log(a*x + b)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(27) = 54$.

Time = 0.34 (sec) , antiderivative size = 96, normalized size of antiderivative = 3.56

$$\int \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = -\frac{\frac{b^2p\log\left(-a+\frac{ax+b}{x}\right)}{a}+\frac{b^2p\log\left(\frac{ax+b}{x}\right)}{a-\frac{ax+b}{x}}-\frac{b^2p\log\left(\frac{ax+b}{x}\right)}{a}+\frac{b^2\log(c)}{a-\frac{ax+b}{x}}}{b}$$

```
[In] integrate(log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] -(b^2*p*log(-a + (a*x + b)/x)/a + b^2*p*log((a*x + b)/x)/(a - (a*x + b)/x)
- b^2*p*log((a*x + b)/x)/a + b^2*log(c)/(a - (a*x + b)/x))/b
```

Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = x\,\ln\left(c\,\left(a+\frac{b}{x}\right)^p\right)+\frac{b\,p\,\ln\left(b+a\,x\right)}{a}$$

```
[In] int(log(c*(a + b/x)^p),x)
```

```
[Out] x*log(c*(a + b/x)^p) + (b*p*log(b + a*x))/a
```

3.31 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx = -\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right) - p\,\mathrm{PolyLog}\left(2,1+\frac{b}{ax}\right)$$

```
[Out] -ln(c*(a+b/x)^p)*ln(-b/a/x)-p*polylog(2,1+b/a/x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2441, 2352}

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx = \log\left(-\frac{b}{ax}\right)\left(-\log\left(c\left(a+\frac{b}{x}\right)^p\right)\right) - p\,\mathrm{PolyLog}\left(2,\frac{b}{ax}+1\right)$$

```
[In] Int[Log[c*(a + b/x)^p]/x,x]
```

```
[Out] -(Log[c*(a + b/x)^p]*Log[-(b/(a*x))]) - p*PolyLog[2, 1 + b/(a*x)]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x, \frac{1}{x}\right)\\
&= -\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)+(bp)\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx, x, \frac{1}{x}\right)\\
&= -\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)-p\text{Li}_2\left(1+\frac{b}{ax}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx = -\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)-p\,\text{PolyLog}\left(2,\frac{a+\frac{b}{x}}{a}\right)$$

```
[In] Integrate[Log[c*(a + b/x)^p]/x,x]
[Out] -(Log[c*(a + b/x)^p]*Log[-(b/(a*x))]) - p*PolyLog[2, (a + b/x)/a]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.68

method	result	size
parts	$\ln\left(c\left(a+\frac{b}{x}\right)^p\right)\ln(x)+pb\left(\frac{\ln(x)^2}{2b}-\frac{a\left(\frac{\operatorname{dilog}\left(\frac{ax+b}{b}\right)}{a}+\frac{\ln(x)\ln\left(\frac{ax+b}{b}\right)}{a}\right)}{b}\right)$	67

```
[In] int(ln(c*(a+b/x)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(c*(a+b/x)^p)*ln(x)+p*b*(1/2/b*ln(x)^2-a/b*(dilog((a*x+b)/b)/a+ln(x)*ln((
a*x+b)/b)/a))
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{x}\,dx$$

```
[In] integrate(log(c*(a+b/x)^p)/x,x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x + b)/x)^p)/x, x)
```

Sympy [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx = \int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx$$

```
[In] integrate(ln(c*(a+b/x)**p)/x,x)
```

```
[Out] Integral(log(c*(a + b/x)**p)/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(39) = 78$.

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.08

$$\begin{aligned}
&\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx \\
&= \frac{1}{2}bp\left(\frac{2\log\left(a+\frac{b}{x}\right)\log(x)}{b} + \frac{\log(x)^2}{b} - \frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x) + \mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)}{b}\right) \\
&\quad - p\log\left(a+\frac{b}{x}\right)\log(x) + \log\left(\left(a+\frac{b}{x}\right)^p c\right)\log(x)
\end{aligned}$$

```
[In] integrate(log(c*(a+b/x)^p)/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(2*log(a + b/x)*log(x)/b + log(x)^2/b - 2*(log(a*x/b + 1)*log(x) +
dilog(-a*x/b))/b) - p*log(a + b/x)*log(x) + log((a + b/x)^p*c)*log(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(39) = 78$.

Time = 0.41 (sec) , antiderivative size = 152, normalized size of antiderivative = 3.80

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx = -\frac{\frac{b^3p\log\left(\frac{ax+b}{x}\right)}{a^2-\frac{2\,(ax+b)a}{x}+\frac{(ax+b)^2}{x^2}}+\frac{b^3p\log\left(-a+\frac{ax+b}{x}\right)}{a^2}-\frac{b^3p\log\left(\frac{ax+b}{x}\right)}{a^2}-\frac{ab^3p-ab^3\log(c)-\frac{(ax+b)b^3p}{x}}{a^3-\frac{2\,(ax+b)a^2}{x}+\frac{(ax+b)^2a}{x^2}}}{2\,b^2}$$

```
[In] integrate(log(c*(a+b/x)^p)/x,x, algorithm="giac")
```

```
[Out] -1/2*(b^3*p*log((a*x + b)/x)/(a^2 - 2*(a*x + b)*a/x + (a*x + b)^2/x^2) + b^
3*p*log(-a + (a*x + b)/x)/a^2 - b^3*p*log((a*x + b)/x)/a^2 - (a*b^3*p - a*b
^3*log(c) - (a*x + b)*b^3*p/x)/(a^3 - 2*(a*x + b)*a^2/x + (a*x + b)^2*a/x^2
))/b^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx = \int \frac{\ln \left(c\,\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx$$

```
[In] int(log(c*(a + b/x)^p)/x,x)
```

```
[Out] int(log(c*(a + b/x)^p)/x, x)
```

3.32 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 30

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx = \frac{p}{x} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b}$$

```
[Out] p/x-(a+b/x)*ln(c*(a+b/x)^p)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2436, 2332}

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx = \frac{p}{x} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b}$$

```
[In] Int[Log[c*(a + b/x)^p]/x^2,x]
```

```
[Out] p/x - ((a + b/x)*Log[c*(a + b/x)^p])/b
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \log\left(c(a+bx)^p\right)\,dx, x, \frac{1}{x}\right)\\ &= -\frac{\text{Subst}\left(\int \log\left(cx^p\right)\,dx, x, a+\frac{b}{x}\right)}{b}\\ &= \frac{p}{x}-\frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx = \frac{p}{x}-\frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b}$$

```
[In] Integrate[Log[c*(a + b/x)^p]/x^2,x]
[Out] p/x - ((a + b/x)*Log[c*(a + b/x)^p])/b
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

method	result	size
derivativedivides	$-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)p}{b}$	37
default	$-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)\left(a+\frac{b}{x}\right)-\left(a+\frac{b}{x}\right)p}{b}$	37
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x}-pb\left(-\frac{1}{bx}-\frac{a\ln(x)}{b^{2}}+\frac{a\ln(ax+b)}{b^{2}}\right)$	51
parallelrisch	$-\frac{x\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)a^{2}p+\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)abp-ab\,p^{2}}{xabp}$	61

```
[In] int(ln(c*(a+b/x)^p)/x^2,x,method=_RETURNVERBOSE)
[Out] -1/b*(ln(c*(a+b/x)^p)*(a+b/x)-(a+b/x)*p)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x^{2}}\,dx=\frac{bp-b\log \left(c\right)-\left(apx+bp\right)\log \left(\frac{ax+b}{x}\right)}{bx}$$

```
[In] integrate(log(c*(a+b/x)^p)/x^2,x, algorithm="fricas")
[Out] (b*p - b*log(c) - (a*p*x + b*p)*log((a*x + b)/x))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x^{2}}\,dx=\begin{cases} -\frac{a\log \left(c\left(a+\frac{b}{x}\right)^{p}\right)}{b}+\frac{p}{x}-\frac{\log \left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x} & \text{for } b\neq 0 \\ -\frac{\log \left(a^{p}c\right)}{x} & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(c*(a+b/x)**p)/x**2,x)
[Out] Piecewise((-a*log(c*(a + b/x)**p)/b + p/x - log(c*(a + b/x)**p)/x, Ne(b, 0)
), (-log(a**p*c)/x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.67

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x^{2}}\,dx=-bp\left(\frac{a\log \left(ax+b\right)}{b^{2}}-\frac{a\log \left(x\right)}{b^{2}}-\frac{1}{bx}\right)-\frac{\log \left(\left(a+\frac{b}{x}\right)^{p}c\right)}{x}$$

```
[In] integrate(log(c*(a+b/x)^p)/x^2,x, algorithm="maxima")
[Out] -b*p*(a*log(a*x + b)/b^2 - a*log(x)/b^2 - 1/(b*x)) - log((a + b/x)^p*c)/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(30) = 60$.

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.10

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx = -\frac{\frac{(ax+b)p\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{x} - \frac{(ax+b)p}{x} + \frac{(ax+b)\log(c)}{x}}{b}$$

```
[In] integrate(log(c*(a+b/x)^p)/x^2,x, algorithm="giac")
```

```
[Out] -((a*x + b)*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/x - (a*x + b)*p/x + (a*x
+ b)*log(c)/x)/b
```

Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.33

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx = \frac{p}{x} - \frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{x} - \frac{2\,a\,p\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b}$$

```
[In] int(log(c*(a + b/x)^p)/x^2,x)
```

```
[Out] p/x - log(c*(a + b/x)^p)/x - (2*a*p*atanh((2*a*x)/b + 1))/b
```

3.33 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 59

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx = \frac{p}{4x^2}-\frac{ap}{2bx}+\frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2}$$

```
[Out] 1/4*p/x^2-1/2*a*p/b/x+1/2*a^2*p*ln(a+b/x)/b^2-1/2*ln(c*(a+b/x)^p)/x^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2442, 45}

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx = \frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2}-\frac{ap}{2bx}+\frac{p}{4x^2}$$

```
[In] Int[Log[c*(a + b/x)^p]/x^3,x]
```

```
[Out] p/(4*x^2) - (a*p)/(2*b*x) + (a^2*p*Log[a + b/x])/(2*b^2) - Log[c*(a + b/x)^
p]/(2*x^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int x \log\left(c(a+bx)^p\right)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2}+\frac{1}{2}(bp)\text{Subst}\left(\int \frac{x^2}{a+bx}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2}+\frac{1}{2}(bp)\text{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)\,dx, x, \frac{1}{x}\right)\\
&= \frac{p}{4x^2}-\frac{ap}{2bx}+\frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx=\frac{p}{4x^2}-\frac{ap}{2bx}+\frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2x^2}$$

```
[In] Integrate[Log[c*(a + b/x)^p]/x^3,x]
[Out] p/(4*x^2) - (a*p)/(2*b*x) + (a^2*p*Log[a + b/x])/(2*b^2) - Log[c*(a + b/x)^
p]/(2*x^2)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07

method	result	size
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{2x^{2}}-\frac{pb\left(-\frac{1}{2b\,x^{2}}+\frac{a^{2}\ln(x)}{b^{3}}+\frac{a}{b^{2}x}-\frac{a^{2}\ln(ax+b)}{b^{3}}\right)}{2}$	63
parallelrisch	$-\frac{2\ln(x)x^{2}a^{2}p-2\ln(ax+b)x^{2}a^{2}p-2x^{2}a^{2}p+2apxb+2\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)b^{2}-b^{2}p}{4x^{2}b^{2}}$	76

```
[In] int(ln(c*(a+b/x)^p)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(c*(a+b/x)^p)/x^2-1/2*p*b*(-1/2/b/x^2+a^2/b^3*ln(x)+a/b^2/x-a^2/b^3*
ln(a*x+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x^{3}}\,dx=-\frac{2\,abpx-b^{2}p+2\,b^{2}\log\left(c\right)-2\,\left(a^{2}px^{2}-b^{2}p\right)\log\left(\frac{ax+b}{x}\right)}{4\,b^{2}x^{2}}$$

```
[In] integrate(log(c*(a+b/x)^p)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*b*p*x - b^2*p + 2*b^2*log(c) - 2*(a^2*p*x^2 - b^2*p)*log((a*x + b
)/x))/(b^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x^{3}}\,dx=\begin{cases}\frac{a^{2}\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{2b^{2}}-\frac{ap}{2bx}+\frac{p}{4x^{2}}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{2x^{2}} & \text{for}\ b\neq 0\\ -\frac{\log\left(a^{p}c\right)}{2x^{2}} & \text{otherwise}\end{cases}$$

```
[In] integrate(ln(c*(a+b/x)**p)/x**3,x)
```

```
[Out] Piecewise((a**2*log(c*(a + b/x)**p)/(2*b**2) - a*p/(2*b*x) + p/(4*x**2) - l
og(c*(a + b/x)**p)/(2*x**2), Ne(b, 0)), (-log(a**p*c)/(2*x**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx = \frac{1}{4}\,bp\left(\frac{2\,a^2\log\left(ax+b\right)}{b^3}-\frac{2\,a^2\log\left(x\right)}{b^3}-\frac{2\,ax-b}{b^2x^2}\right)-\frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{2\,x^2}$$

```
[In] integrate(log(c*(a+b/x)^p)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b*p*(2*a^2*log(a*x + b)/b^3 - 2*a^2*log(x)/b^3 - (2*a*x - b)/(b^2*x^2))
 - 1/2*log((a + b/x)^p*c)/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(51) = 102.

Time = 0.31 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.54

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx$$
$$=\frac{\frac{4\,(ax+b)ap\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{bx}-\frac{4\,(ax+b)ap}{bx}-\frac{2\,(ax+b)^2p\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{bx^2}+\frac{4\,(ax+b)a\log(c)}{bx}+\frac{(ax+b)^2p}{bx^2}-\frac{2\,(ax+b)^2\log(c}{bx^2}}{4\,b}$$

```
[In] integrate(log(c*(a+b/x)^p)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(4*(a*x + b)*a*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b*x) - 4*(a*x + b
)*a*p/(b*x) - 2*(a*x + b)^2*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b*x^2) +
 4*(a*x + b)*a*log(c)/(b*x) + (a*x + b)^2*p/(b*x^2) - 2*(a*x + b)^2*log(c)/
(b*x^2))/b
```

Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx = \frac{\frac{p}{2}-\frac{a\,p\,x}{b}}{2\,x^2}-\frac{\ln\left(c\,\left(a+\frac{b}{x}\right)^p\right)}{2\,x^2}+\frac{a^2\,p\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{b^2}$$

```
[In] int(log(c*(a + b/x)^p)/x^3,x)
```

```
[Out] (p/2 - (a*p*x)/b)/(2*x^2) - log(c*(a + b/x)^p)/(2*x^2) + (a^2*p*atanh((2*a*
x)/b + 1))/b^2
```

3.34 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 73

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4}\,dx=\frac{p}{9x^3}-\frac{ap}{6bx^2}+\frac{a^2p}{3b^2x}-\frac{a^3p\log\left(a+\frac{b}{x}\right)}{3b^3}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3}$$

```
[Out] 1/9*p/x^3-1/6*a*p/b/x^2+1/3*a^2*p/b^2/x-1/3*a^3*p*ln(a+b/x)/b^3-1/3*ln(c*(a
+b/x)^p)/x^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2442, 45}

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4}\,dx=-\frac{a^3p\log\left(a+\frac{b}{x}\right)}{3b^3}+\frac{a^2p}{3b^2x}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3}-\frac{ap}{6bx^2}+\frac{p}{9x^3}$$

```
[In] Int[Log[c*(a + b/x)^p]/x^4,x]
```

```
[Out] p/(9*x^3) - (a*p)/(6*b*x^2) + (a^2*p)/(3*b^2*x) - (a^3*p*Log[a + b/x])/(3*b
^3) - Log[c*(a + b/x)^p]/(3*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int x^2 \log\left(c(a+bx)^p\right)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3} + \frac{1}{3}(bp)\text{Subst}\left(\int \frac{x^3}{a+bx}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3} + \frac{1}{3}(bp)\text{Subst}\left(\int \left(\frac{a^2}{b^3} - \frac{ax}{b^2} + \frac{x^2}{b} - \frac{a^3}{b^3(a+bx)}\right)\,dx, x, \frac{1}{x}\right)\\
&= \frac{p}{9x^3} - \frac{ap}{6bx^2} + \frac{a^2p}{3b^2x} - \frac{a^3p\log\left(a+\frac{b}{x}\right)}{3b^3} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$
\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4}\,dx = \frac{p}{9x^3} - \frac{ap}{6bx^2} + \frac{a^2p}{3b^2x} - \frac{a^3p\log\left(a+\frac{b}{x}\right)}{3b^3} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3x^3}
$$

```
[In] Integrate[Log[c*(a + b/x)^p]/x^4,x]
[Out] p/(9*x^3) - (a*p)/(6*b*x^2) + (a^2*p)/(3*b^2*x) - (a^3*p*Log[a + b/x])/(3*b
^3) - Log[c*(a + b/x)^p]/(3*x^3)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

method	result	size
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{3x^{3}}-\frac{pb\left(-\frac{1}{3b\,x^{3}}-\frac{a^{2}}{b^{3}x}+\frac{a}{2b^{2}x^{2}}-\frac{a^{3}\ln(x)}{b^{4}}+\frac{a^{3}\ln(ax+b)}{b^{4}}\right)}{3}$	75
parallelrisch	$-\frac{6x^{3}\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)a^{3}p+6x^{3}a^{3}p^{2}-6x^{2}a^{2}b\,p^{2}+3xa\,b^{2}p^{2}+6\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)b^{3}p-2b^{3}p^{2}}{18x^{3}b^{3}p}$	97

```
[In] int(ln(c*(a+b/x)^p)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*ln(c*(a+b/x)^p)/x^3-1/3*p*b*(-1/3/b/x^3-a^2/b^3/x+1/2*a/b^2/x^2-a^3/b^
4*ln(x)+a^3/b^4*ln(a*x+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x^{4}}\,dx=\frac{6\,a^{2}bpx^{2}-3\,ab^{2}px+2\,b^{3}p-6\,b^{3}\log\left(c\right)-6\,(a^{3}px^{3}+b^{3}p)\log\left(\frac{ax+b}{x}\right)}{18\,b^{3}x^{3}}$$

```
[In] integrate(log(c*(a+b/x)^p)/x^4,x, algorithm="fricas")
```

```
[Out] 1/18*(6*a^2*b*p*x^2 - 3*a*b^2*p*x + 2*b^3*p - 6*b^3*log(c) - 6*(a^3*p*x^3 +
 b^3*p)*log((a*x + b)/x))/(b^3*x^3)
```

Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x^{4}}\,dx=\begin{cases}-\frac{a^{3}\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{3b^{3}}+\frac{a^{2}p}{3b^{2}x}-\frac{ap}{6bx^{2}}+\frac{p}{9x^{3}}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{3x^{3}} & \text{for}\ b\neq 0\\ -\frac{\log\left(a^{p}c\right)}{3x^{3}} & \text{otherwise}\end{cases}$$

```
[In] integrate(ln(c*(a+b/x)**p)/x**4,x)
```

```
[Out] Piecewise((-a**3*log(c*(a + b/x)**p)/(3*b**3) + a**2*p/(3*b**2*x) - a*p/(6*
b*x**2) + p/(9*x**3) - log(c*(a + b/x)**p)/(3*x**3), Ne(b, 0)), (-log(a**p*
c)/(3*x**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4}\,dx = -\frac{1}{18}\,bp\left(\frac{6\,a^3\log\left(ax+b\right)}{b^4}-\frac{6\,a^3\log\left(x\right)}{b^4}-\frac{6\,a^2x^2-3\,abx+2\,b^2}{b^3x^3}\right) - \frac{\log\left(\left(a+\frac{b}{x}\right)^pc\right)}{3\,x^3}$$

```
[In] integrate(log(c*(a+b/x)^p)/x^4,x, algorithm="maxima")
```

```
[Out] -1/18*b*p*(6*a^3*log(a*x + b)/b^4 - 6*a^3*log(x)/b^4 - (6*a^2*x^2 - 3*a*b*x
 + 2*b^2)/(b^3*x^3)) - 1/3*log((a + b/x)^p*c)/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(63) = 126.

Time = 0.32 (sec) , antiderivative size = 234, normalized size of antiderivative = 3.21

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4}\,dx = -\frac{\frac{18\,(ax+b)a^2p\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{b^2x}-\frac{18\,(ax+b)a^2p}{b^2x}-\frac{18\,(ax+b)^2ap\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{b^2x^2}+\frac{18\,(ax+b)a^2\log\left(c\right)}{b^2x}+\frac{9\,(ax+b)^2ap}{b^2x^2}+}{18\,b}$$

```
[In] integrate(log(c*(a+b/x)^p)/x^4,x, algorithm="giac")
```

```
[Out] -1/18*(18*(a*x + b)*a^2*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b^2*x) - 18*
(a*x + b)*a^2*p/(b^2*x) - 18*(a*x + b)^2*a*p*log(-b*(a/b - (a*x + b)/(b*x))
 + a)/(b^2*x^2) + 18*(a*x + b)*a^2*log(c)/(b^2*x) + 9*(a*x + b)^2*a*p/(b^2*
x^2) + 6*(a*x + b)^3*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b^2*x^3) - 18*(
a*x + b)^2*a*log(c)/(b^2*x^2) - 2*(a*x + b)^3*p/(b^2*x^3) + 6*(a*x + b)^3*l
og(c)/(b^2*x^3))/b
```

Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^4}\,dx = \frac{\frac{p}{3}+\frac{a^2\,p\,x^2}{b^2}-\frac{a\,p\,x}{2\,b}}{3\,x^3}-\frac{\ln\left(c\,\left(a+\frac{b}{x}\right)^p\right)}{3\,x^3}-\frac{2\,a^3\,p\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{3\,b^3}$$

```
[In] int(log(c*(a + b/x)^p)/x^4,x)
```

```
[Out] (p/3 + (a^2*p*x^2)/b^2 - (a*p*x)/(2*b))/(3*x^3) - log(c*(a + b/x)^p)/(3*x^3
) - (2*a^3*p*atanh((2*a*x)/b + 1))/(3*b^3)
```

3.35 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5} dx$

Optimal result

Integrand size = 16, antiderivative size = 87

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5} dx = \frac{p}{16x^4} - \frac{ap}{12bx^3} + \frac{a^2p}{8b^2x^2} - \frac{a^3p}{4b^3x} + \frac{a^4p\log\left(a+\frac{b}{x}\right)}{4b^4} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4}$$

```
[Out] 1/16*p/x^4-1/12*a*p/b/x^3+1/8*a^2*p/b^2/x^2-1/4*a^3*p/b^3/x+1/4*a^4*p*ln(a+
b/x)/b^4-1/4*ln(c*(a+b/x)^p)/x^4
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2442, 45}

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5} dx = \frac{a^4p\log\left(a+\frac{b}{x}\right)}{4b^4} - \frac{a^3p}{4b^3x} + \frac{a^2p}{8b^2x^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4} - \frac{ap}{12bx^3} + \frac{p}{16x^4}$$

```
[In] Int[Log[c*(a + b/x)^p]/x^5,x]
```

```
[Out] p/(16*x^4) - (a*p)/(12*b*x^3) + (a^2*p)/(8*b^2*x^2) - (a^3*p)/(4*b^3*x) + (
a^4*p*Log[a + b/x])/(4*b^4) - Log[c*(a + b/x)^p]/(4*x^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int x^3 \log\left(c(a+bx)^p\right)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4}+\frac{1}{4}(bp)\text{Subst}\left(\int \frac{x^4}{a+bx}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4}+\frac{1}{4}(bp)\text{Subst}\left(\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx, x, \frac{1}{x}\right)\\
&= \frac{p}{16x^4}-\frac{ap}{12bx^3}+\frac{a^2p}{8b^2x^2}-\frac{a^3p}{4b^3x}+\frac{a^4p\log\left(a+\frac{b}{x}\right)}{4b^4}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5}\,dx=\frac{p}{16x^4}-\frac{ap}{12bx^3}+\frac{a^2p}{8b^2x^2}-\frac{a^3p}{4b^3x}+\frac{a^4p\log\left(a+\frac{b}{x}\right)}{4b^4}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4x^4}$$

```
[In] Integrate[Log[c*(a + b/x)^p]/x^5,x]
[Out] p/(16*x^4) - (a*p)/(12*b*x^3) + (a^2*p)/(8*b^2*x^2) - (a^3*p)/(4*b^3*x) + (
a^4*p*Log[a + b/x])/(4*b^4) - Log[c*(a + b/x)^p]/(4*x^4)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

method	result	size
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{4x^{4}}-\frac{pb\left(-\frac{1}{4b\,x^{4}}-\frac{a^{2}}{2b^{3}x^{2}}+\frac{a^{4}\ln(x)}{b^{5}}+\frac{a}{3b^{2}x^{3}}+\frac{a^{3}}{b^{4}x}-\frac{a^{4}\ln(ax+b)}{b^{5}}\right)}{4}$	85
parallelrisch	$-\frac{12\ln(x)x^{4}a^{4}p-12\ln(ax+b)x^{4}a^{4}p-12x^{4}a^{4}p+12x^{3}a^{3}bp-6x^{2}a^{2}b^{2}p+4xa\,b^{3}p+12\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)b^{4}-3b^{4}p}{48x^{4}b^{4}}$	100

```
[In] int(ln(c*(a+b/x)^p)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*ln(c*(a+b/x)^p)/x^4-1/4*p*b*(-1/4/b/x^4-1/2*a^2/b^3/x^2+a^4/b^5*ln(x)+
1/3*a/b^2/x^3+a^3/b^4/x-a^4/b^5*ln(a*x+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x^{5}}\,dx = -\frac{12\,a^{3}bpx^{3}-6\,a^{2}b^{2}px^{2}+4\,ab^{3}px-3\,b^{4}p+12\,b^{4}\log(c)-12\,(a^{4}px^{4}-b^{4}p)\log\left(\frac{ax+b}{x}\right)}{48\,b^{4}x^{4}}$$

```
[In] integrate(log(c*(a+b/x)^p)/x^5,x, algorithm="fricas")
[Out] -1/48*(12*a^3*b*p*x^3 - 6*a^2*b^2*p*x^2 + 4*a*b^3*p*x - 3*b^4*p + 12*b^4*lo
g(c) - 12*(a^4*p*x^4 - b^4*p)*log((a*x + b)/x))/(b^4*x^4)
```

Sympy [A] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x^{5}}\,dx = \begin{cases} \frac{a^{4}\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{4b^{4}}-\frac{a^{3}p}{4b^{3}x}+\frac{a^{2}p}{8b^{2}x^{2}}-\frac{ap}{12bx^{3}}+\frac{p}{16x^{4}}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{4x^{4}} & \text{for}\ b\neq 0\\ -\frac{\log(a^{p}c)}{4x^{4}} & \text{otherwise}\end{cases}$$

```
[In] integrate(ln(c*(a+b/x)**p)/x**5,x)
[Out] Piecewise((a**4*log(c*(a + b/x)**p)/(4*b**4) - a**3*p/(4*b**3*x) + a**2*p/(
8*b**2*x**2) - a*p/(12*b*x**3) + p/(16*x**4) - log(c*(a + b/x)**p)/(4*x**4)
, Ne(b, 0)), (-log(a**p*c)/(4*x**4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5}\,dx = \frac{1}{48}\,bp\left(\frac{12\,a^4\log\left(ax+b\right)}{b^5}-\frac{12\,a^4\log\left(x\right)}{b^5}-\frac{12\,a^3x^3-6\,a^2bx^2+4\,ab^2x-3\,b^3}{b^4x^4}\right) - \frac{\log\left(\left(a+\frac{b}{x}\right)^pc\right)}{4\,x^4}$$

```
[In] integrate(log(c*(a+b/x)^p)/x^5,x, algorithm="maxima")
```

```
[Out] 1/48*b*p*(12*a^4*log(a*x + b)/b^5 - 12*a^4*log(x)/b^5 - (12*a^3*x^3 - 6*a^2
*b*x^2 + 4*a*b^2*x - 3*b^3)/(b^4*x^4)) - 1/4*log((a + b/x)^p*c)/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(75) = 150.

Time = 0.31 (sec) , antiderivative size = 317, normalized size of antiderivative = 3.64

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5}\,dx = \frac{\frac{48\,(ax+b)a^3p\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{b^3x}-\frac{48\,(ax+b)a^3p}{b^3x}-\frac{72\,(ax+b)^2a^2p\log\left(-b\left(\frac{a}{b}-\frac{ax+b}{bx}\right)+a\right)}{b^3x^2}+\frac{48\,(ax+b)a^3\log(c)}{b^3x}+\frac{36\,(ax+b)^2a^2p}{b^3x^2}+}{}$$

```
[In] integrate(log(c*(a+b/x)^p)/x^5,x, algorithm="giac")
```

```
[Out] 1/48*(48*(a*x + b)*a^3*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b^3*x) - 48*(
a*x + b)*a^3*p/(b^3*x) - 72*(a*x + b)^2*a^2*p*log(-b*(a/b - (a*x + b)/(b*x)
) + a)/(b^3*x^2) + 48*(a*x + b)*a^3*log(c)/(b^3*x) + 36*(a*x + b)^2*a^2*p/(
b^3*x^2) + 48*(a*x + b)^3*a*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b^3*x^3)
 - 72*(a*x + b)^2*a^2*log(c)/(b^3*x^2) - 16*(a*x + b)^3*a*p/(b^3*x^3) - 12*
(a*x + b)^4*p*log(-b*(a/b - (a*x + b)/(b*x)) + a)/(b^3*x^4) + 48*(a*x + b)^
3*a*log(c)/(b^3*x^3) + 3*(a*x + b)^4*p/(b^3*x^4) - 12*(a*x + b)^4*log(c)/(b
^3*x^4))/b
```

Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{x^5}\,dx = \frac{\frac{p}{4}+\frac{a^2\,p\,x^2}{2\,b^2}-\frac{a^3\,p\,x^3}{b^3}-\frac{a\,p\,x}{3\,b}}{4\,x^4}-\frac{\ln\left(c\,\left(a+\frac{b}{x}\right)^p\right)}{4\,x^4}+\frac{a^4\,p\,\mathrm{atanh}\left(\frac{2\,a\,x}{b}+1\right)}{2\,b^4}$$

```
[In] int(log(c*(a + b/x)^p)/x^5,x)
```

```
[Out] (p/4 + (a^2*p*x^2)/(2*b^2) - (a^3*p*x^3)/b^3 - (a*p*x)/(3*b))/(4*x^4) - log
(c*(a + b/x)^p)/(4*x^4) + (a^4*p*atanh((2*a*x)/b + 1))/(2*b^4)
```

3.36 $\int x^4 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 72

$$\int x^4 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \, dx = -\frac{2b^2px}{5a^2}+\frac{2bpx^3}{15a}+\frac{2b^{5/2}p\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{5a^{5/2}} + \frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

```
[Out] -2/5*b^2*p*x/a^2+2/15*b*p*x^3/a+2/5*b^(5/2)*p*arctan(x*a^(1/2)/b^(1/2))/a^(
5/2)+1/5*x^5*ln(c*(a+b/x^2)^p)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2505, 269, 308, 211}

$$\int x^4 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \, dx = \frac{2b^{5/2}p\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{5a^{5/2}}-\frac{2b^2px}{5a^2}+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{2bpx^3}{15a}$$

```
[In] Int[x^4*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (-2*b^2*p*x)/(5*a^2) + (2*b*p*x^3)/(15*a) + (2*b^(5/2)*p*ArcTan[(Sqrt[a]*x)
/Sqrt[b]])/(5*a^(5/2)) + (x^5*Log[c*(a + b/x^2)^p])/5
```

<u>Rule 211</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{1}{5}(2bp)\int\frac{x^2}{a+\frac{b}{x^2}}\,dx\\
&= \frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{1}{5}(2bp)\int\frac{x^4}{b+ax^2}\,dx\\
&= \frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{1}{5}(2bp)\int\left(-\frac{b}{a^2}+\frac{x^2}{a}+\frac{b^2}{a^2\left(b+ax^2\right)}\right)dx\\
&= -\frac{2b^2px}{5a^2}+\frac{2bpx^3}{15a}+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{\left(2b^3p\right)\int\frac{1}{b+ax^2}\,dx}{5a^2}\\
&= -\frac{2b^2px}{5a^2}+\frac{2bpx^3}{15a}+\frac{2b^{5/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{5a^{5/2}}+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.68

$$
\begin{aligned}
\int x^4\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx &= \frac{2bpx^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\frac{b}{ax^2}\right)}{15a}\\
&\quad+\frac{1}{5}x^5\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)
\end{aligned}
$$

```
[In] Integrate[x^4*Log[c*(a + b/x^2)^p],x]
[Out] (2*b*p*x^3*Hypergeometric2F1[-3/2, 1, -1/2, -(b/(a*x^2))])/(15*a) + (x^5*Lo
g[c*(a + b/x^2)^p])/5
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

method	result	size
parts	$\frac{x^{5} \ln\left(c\left(a+\frac{b}{x^{2}}\right)^{p}\right)}{5}+\frac{2 p b \left(\frac{\frac{1}{3} x^{3} a-b x}{a^{2}}+\frac{b^{2} \arctan\left(\frac{a x}{\sqrt{a b}}\right)}{a^{2} \sqrt{a b}}\right)}{5}$	60

```
[In] int(x^4*ln(c*(a+b/x^2)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*ln(c*(a+b/x^2)^p)+2/5*p*b*(1/a^2*(1/3*x^3*a-b*x)+b^2/a^2/(a*b)^(1/2
)*arctan(a*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.47

$$\int x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$$
$$= \left[\frac{3\,a^2px^5\log\left(\frac{ax^2+b}{x^2}\right)+3\,a^2x^5\log\left(c\right)+2\,abpx^3+3\,b^2p\sqrt{-\frac{b}{a}}\log\left(\frac{ax^2+2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)-6\,b^2px}{15\,a^2}, \frac{3\,a^2px^5\log\left(\frac{ax}{\ldots}\right.}{\ldots}\right.$$

```
[In] integrate(x^4*log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

```
[Out] [1/15*(3*a^2*p*x^5*log((a*x^2 + b)/x^2) + 3*a^2*x^5*log(c) + 2*a*b*p*x^3 +
3*b^2*p*sqrt(-b/a)*log((a*x^2 + 2*a*x*sqrt(-b/a) - b)/(a*x^2 + b)) - 6*b^2*
p*x)/a^2, 1/15*(3*a^2*p*x^5*log((a*x^2 + b)/x^2) + 3*a^2*x^5*log(c) + 2*a*b
*p*x^3 + 6*b^2*p*sqrt(b/a)*arctan(a*x*sqrt(b/a)/b) - 6*b^2*p*x)/a^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(70) = 140$.

Time = 32.99 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.06

$$\int x^4 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \begin{cases} \frac{x^5 \log(0^p c)}{5} & \text{for } a=0 \wedge b=0 \\ \frac{2px^5}{25} + \frac{x^5 \log\left(c\left(\frac{b}{x^2}\right)^p\right)}{5} & \text{for } a=0 \\ \frac{x^5 \log(a^p c)}{5} & \text{for } b=0 \\ \frac{x^5 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{5} + \frac{2bpx^3}{15a} - \frac{2b^2px}{5a^2} + \frac{b^3p\log\left(x-\sqrt{-\frac{b}{a}}\right)}{5a^3\sqrt{-\frac{b}{a}}} - \frac{b^3p\log\left(x+\sqrt{-\frac{b}{a}}\right)}{5a^3\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**4*ln(c*(a+b/x**2)**p),x)
```

```
[Out] Piecewise((x**5*log(0**p*c)/5, Eq(a, 0) & Eq(b, 0)), (2*p*x**5/25 + x**5*lo
g(c*(b/x**2)**p)/5, Eq(a, 0)), (x**5*log(a**p*c)/5, Eq(b, 0)), (x**5*log(c*
(a + b/x**2)**p)/5 + 2*b*p*x**3/(15*a) - 2*b**2*p*x/(5*a**2) + b**3*p*log(x
 - sqrt(-b/a))/(5*a**3*sqrt(-b/a)) - b**3*p*log(x + sqrt(-b/a))/(5*a**3*sqr
t(-b/a)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int x^4 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{1}{5}\, x^5 \log \left(\left(a+\frac{b}{x^2}\right)^p c\right) + \frac{2}{15}\, bp \left(\frac{3\, b^2 \arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{ax^3 - 3\, bx}{a^2}\right)$$

```
[In] integrate(x^4*log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*log((a + b/x^2)^p*c) + 2/15*b*p*(3*b^2*arctan(a*x/sqrt(a*b))/(sqrt(
a*b)*a^2) + (a*x^3 - 3*b*x)/a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{1}{5}px^5\log\left(ax^2+b\right) - \frac{1}{5}px^5\log\left(x^2\right) + \frac{1}{5}x^5\log\left(c\right) + \frac{2\,bpx^3}{15\,a} + \frac{2\,b^3p\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{5\,\sqrt{ab}a^2} - \frac{2\,b^2px}{5\,a^2}$$

```
[In] integrate(x^4*log(c*(a+b/x^2)^p),x, algorithm="giac")
```

```
[Out] 1/5*p*x^5*log(a*x^2 + b) - 1/5*p*x^5*log(x^2) + 1/5*x^5*log(c) + 2/15*b*p*x
^3/a + 2/5*b^3*p*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 2/5*b^2*p*x/a^2
```

Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int x^4 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{x^5\,\ln\left(c\,{\left(a+\frac{b}{x^2}\right)}^p\right)}{5} + \frac{2\,b^{5/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{5\,a^{5/2}} + \frac{2\,b\,p\,x^3}{15\,a} - \frac{2\,b^2\,p\,x}{5\,a^2}$$

```
[In] int(x^4*log(c*(a + b/x^2)^p),x)
```

```
[Out] (x^5*log(c*(a + b/x^2)^p))/5 + (2*b^(5/2)*p*atan((a^(1/2)*x)/b^(1/2)))/(5*a
^(5/2)) + (2*b*p*x^3)/(15*a) - (2*b^2*p*x)/(5*a^2)
```

3.37 $\int x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 51

$$\int x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \, dx=\frac{bpx^2}{4a}+\frac{1}{4}x^4\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)-\frac{b^2p\log \left(b+ax^2\right)}{4a^2}$$

```
[Out] 1/4*b*p*x^2/a+1/4*x^4*ln(c*(a+b/x^2)^p)-1/4*b^2*p*ln(a*x^2+b)/a^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2505, 269, 272, 45}

$$\int x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \, dx=-\frac{b^2p\log \left(ax^2+b\right)}{4a^2}+\frac{1}{4}x^4\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{bpx^2}{4a}$$

```
[In] Int[x^3*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (b*p*x^2)/(4*a) + (x^4*Log[c*(a + b/x^2)^p])/4 - (b^2*p*Log[b + a*x^2])/(4*
a^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{1}{2}(bp)\int\frac{x}{a+\frac{b}{x^2}}\,dx\\
&= \frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{1}{2}(bp)\int\frac{x^3}{b+ax^2}\,dx\\
&= \frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{1}{4}(bp)\text{Subst}\left(\int\frac{x}{b+ax}\,dx,x,x^2\right)\\
&= \frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{1}{4}(bp)\text{Subst}\left(\int\left(\frac{1}{a}-\frac{b}{a(b+ax)}\right)\,dx,x,x^2\right)\\
&= \frac{bpx^2}{4a}+\frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)-\frac{b^2p\log\left(b+ax^2\right)}{4a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx=\frac{1}{4}x^4\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{1}{4}bp\left(\frac{x^2}{a}-\frac{b\log\left(a+\frac{b}{x^2}\right)}{a^2}-\frac{2b\log(x)}{a^2}\right)$$

```
[In] Integrate[x^3*Log[c*(a + b/x^2)^p],x]
[Out] (x^4*Log[c*(a + b/x^2)^p])/4 + (b*p*(x^2/a - (b*Log[a + b/x^2])/a^2 - (2*b*
Log[x])/a^2))/4
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

method	result	size
parts	$\frac{x^{4} \ln\left(c \left(a+\frac{b}{x^{2}}\right)^{p}\right)}{4}+\frac{p b \left(\frac{x^{2}}{2 a}-\frac{b \ln\left(x^{2} a+b\right)}{2 a^{2}}\right)}{2}$	46
parallelrisch	$-\frac{-x^{4} \ln\left(c \left(\frac{x^{2} a+b}{x^{2}}\right)^{p}\right) a^{2} p-a b\, p^{2} x^{2}+2 \ln \left(x \right) b^{2} p^{2}+\ln\left(c \left(\frac{x^{2} a+b}{x^{2}}\right)^{p}\right) b^{2} p+b^{2} p^{2}}{4 a^{2} p}$	83

```
[In] int(x^3*ln(c*(a+b/x^2)^p),x,method=_RETURNVERBOSE)
[Out] 1/4*x^4*ln(c*(a+b/x^2)^p)+1/2*p*b*(1/2/a*x^2-1/2/a^2*b*ln(a*x^2+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx=\frac{a^2px^4\log\left(\frac{ax^2+b}{x^2}\right)+a^2x^4\log(c)+abpx^2-b^2p\log\left(ax^2+b\right)}{4\,a^2}$$

```
[In] integrate(x^3*log(c*(a+b/x^2)^p),x, algorithm="fricas")
[Out] 1/4*(a^2*p*x^4*log((a*x^2 + b)/x^2) + a^2*x^4*log(c) + a*b*p*x^2 - b^2*p*lo
g(a*x^2 + b))/a^2
```

Sympy [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx=\begin{cases} \frac{x^4\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{4}+\frac{bpx^2}{4a}-\frac{b^2p\log\left(ax^2+b\right)}{4a^2} & \text{for}\: a\neq 0 \\ \frac{px^4}{8}+\frac{x^4\log\left(c\left(\frac{b}{x^2}\right)^p\right)}{4} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*ln(c*(a+b/x**2)**p),x)
[Out] Piecewise((x**4*log(c*(a + b/x**2)**p)/4 + b*p*x**2/(4*a) - b**2*p*log(a*x*
*2 + b)/(4*a**2), Ne(a, 0)), (p*x**4/8 + x**4*log(c*(b/x**2)**p)/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{1}{4}x^4\log\left(\left(a+\frac{b}{x^2}\right)^p c\right) + \frac{1}{4}bp\left(\frac{x^2}{a} - \frac{b\log(ax^2+b)}{a^2}\right)$$

```
[In] integrate(x^3*log(c*(a+b/x^2)^p),x, algorithm="maxima")
[Out] 1/4*x^4*log((a + b/x^2)^p*c) + 1/4*b*p*(x^2/a - b*log(a*x^2 + b)/a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{1}{4}px^4\log\left(ax^2+b\right) - \frac{1}{4}px^4\log\left(x^2\right) + \frac{1}{4}x^4\log(c) + \frac{bpx^2}{4a} - \frac{b^2p\log(ax^2+b)}{4a^2}$$

```
[In] integrate(x^3*log(c*(a+b/x^2)^p),x, algorithm="giac")
[Out] 1/4*p*x^4*log(a*x^2 + b) - 1/4*p*x^4*log(x^2) + 1/4*x^4*log(c) + 1/4*b*p*x^
2/a - 1/4*b^2*p*log(a*x^2 + b)/a^2
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{x^4\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{4} - \frac{b^2\,p\,\ln\left(a\,x^2+b\right)}{4\,a^2} + \frac{b\,p\,x^2}{4\,a}$$

```
[In] int(x^3*log(c*(a + b/x^2)^p),x)
[Out] (x^4*log(c*(a + b/x^2)^p))/4 - (b^2*p*log(b + a*x^2))/(4*a^2) + (b*p*x^2)/(
4*a)
```

3.38 $\int x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 58

$$\int x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx=\frac{2bpx}{3a}-\frac{2b^{3/2}p\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3a^{3/2}}+\frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

```
[Out] 2/3*b*p*x/a-2/3*b^(3/2)*p*arctan(x*a^(1/2)/b^(1/2))/a^(3/2)+1/3*x^3*ln(c*(a
+b/x^2)^p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2505, 199, 327, 211}

$$\int x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx=-\frac{2b^{3/2}p\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3a^{3/2}}+\frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{2bpx}{3a}$$

```
[In] Int[x^2*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (2*b*p*x)/(3*a) - (2*b^(3/2)*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(3*a^(3/2)) + (
x^3*Log[c*(a + b/x^2)^p])/3
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{1}{3}(2bp)\int\frac{1}{a+\frac{b}{x^2}}\,dx\\
&= \frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{1}{3}(2bp)\int\frac{x^2}{b+ax^2}\,dx\\
&= \frac{2bpx}{3a}+\frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)-\frac{(2b^2p)\int\frac{1}{b+ax^2}\,dx}{3a}\\
&= \frac{2bpx}{3a}-\frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3a^{3/2}}+\frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$
\int x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx=\frac{2bpx\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{b}{ax^2}\right)}{3a}+\frac{1}{3}x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)
$$

```
[In] Integrate[x^2*Log[c*(a + b/x^2)^p],x]
[Out] (2*b*p*x*Hypergeometric2F1[-1/2, 1, 1/2, -(b/(a*x^2))])/(3*a) + (x^3*Log[c*
(a + b/x^2)^p])/3
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

method	result	size
parts	$\frac{x^{3} \ln\left(c \left(a+\frac{b}{x^{2}}\right)^{p}\right)}{3}+\frac{2 p b \left(\frac{x}{a}-\frac{b \arctan\left(\frac{a x}{\sqrt{a b}}\right)}{a \sqrt{a b}}\right)}{3}$	49

[In] int(x^2*ln(c*(a+b/x^2)^p),x,method=_RETURNVERBOSE)

[Out] 1/3*x^3*ln(c*(a+b/x^2)^p)+2/3*p*b*(x/a-1/a*b/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.43

$$\int x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx = \left[\frac{apx^3 \log\left(\frac{ax^2+b}{x^2}\right)+ax^3\log(c)+bp\sqrt{-\frac{b}{a}}\log\left(\frac{ax^2-2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)+2\,bpx}{3\,a}, \frac{apx^3\log\left(\frac{ax^2+b}{x^2}\right)+ax^3\log(c)-}{3}\right.$$

[In] integrate(x^2*log(c*(a+b/x^2)^p),x, algorithm="fricas")

[Out] [1/3*(a*p*x^3*log((a*x^2 + b)/x^2) + a*x^3*log(c) + b*p*sqrt(-b/a)*log((a*x^2 - 2*a*x*sqrt(-b/a) - b)/(a*x^2 + b)) + 2*b*p*x)/a, 1/3*(a*p*x^3*log((a*x^2 + b)/x^2) + a*x^3*log(c) - 2*b*p*sqrt(b/a)*arctan(a*x*sqrt(b/a)/b) + 2*b*p*x)/a]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(54) = 108$.

Time = 11.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.29

$$\int x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx = \begin{cases} \frac{x^3 \log(0^p c)}{3} & \text{for } a=0 \wedge b=0 \\ \frac{2px^3}{9} + \frac{x^3 \log\left(c\left(\frac{b}{x^2}\right)^p\right)}{3} & \text{for } a=0 \\ \frac{x^3 \log(a^p c)}{3} & \text{for } b=0 \\ \frac{x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3} + \frac{2bpx}{3a} - \frac{b^2 p \log\left(x-\sqrt{-\frac{b}{a}}\right)}{3a^2\sqrt{-\frac{b}{a}}} + \frac{b^2 p \log\left(x+\sqrt{-\frac{b}{a}}\right)}{3a^2\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*ln(c*(a+b/x**2)**p),x)
```

```
[Out] Piecewise((x**3*log(0**p*c)/3, Eq(a, 0) & Eq(b, 0)), (2*p*x**3/9 + x**3*log
(c*(b/x**2)**p)/3, Eq(a, 0)), (x**3*log(a**p*c)/3, Eq(b, 0)), (x**3*log(c*(
a + b/x**2)**p)/3 + 2*b*p*x/(3*a) - b**2*p*log(x - sqrt(-b/a))/(3*a**2*sqrt
(-b/a)) + b**2*p*log(x + sqrt(-b/a))/(3*a**2*sqrt(-b/a)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx = \frac{1}{3}\,x^3 \log\left(\left(a+\frac{b}{x^2}\right)^p c\right) - \frac{2}{3}\,bp\left(\frac{b\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{x}{a}\right)$$

```
[In] integrate(x^2*log(c*(a+b/x^2)^p),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((a + b/x^2)^p*c) - 2/3*b*p*(b*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*
a) - x/a)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

$$\int x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx = \frac{1}{3}\,px^3 \log\left(ax^2+b\right) - \frac{1}{3}\,px^3 \log\left(x^2\right) + \frac{1}{3}\,x^3 \log\left(c\right) - \frac{2\,b^2 p \arctan\left(\frac{ax}{\sqrt{ab}}\right)}{3\,\sqrt{ab}a} + \frac{2\,bpx}{3\,a}$$

```
[In] integrate(x^2*log(c*(a+b/x^2)^p),x, algorithm="giac")
```

```
[Out] 1/3*p*x^3*log(a*x^2 + b) - 1/3*p*x^3*log(x^2) + 1/3*x^3*log(c) - 2/3*b^2*p*
arctan(a*x/sqrt(a*b))/(sqrt(a*b)*a) + 2/3*b*p*x/a
```

Mupad [B] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \, dx = \frac{x^3 \ln \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{3} - \frac{2 b^{3/2} p \operatorname{atan}\left(\frac{\sqrt{a} x}{\sqrt{b}}\right)}{3 a^{3/2}} + \frac{2 b p x}{3 a}$$

```
[In] int(x^2*log(c*(a + b/x^2)^p),x)
```

```
[Out] (x^3*log(c*(a + b/x^2)^p))/3 - (2*b^(3/2)*p*atan((a^(1/2)*x)/b^(1/2)))/(3*a
^(3/2)) + (2*b*p*x)/(3*a)
```

3.39 $\int x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx$

Optimal result

Integrand size = 14, antiderivative size = 37

$$\int x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{1}{2}x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{bp\log(b+ax^2)}{2a}$$

```
[Out] 1/2*x^2*ln(c*(a+b/x^2)^p)+1/2*b*p*ln(a*x^2+b)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2505, 269, 266}

$$\int x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \frac{1}{2}x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + \frac{bp\log(ax^2+b)}{2a}$$

```
[In] Int[x*Log[c*(a + b/x^2)^p],x]
[Out] (x^2*Log[c*(a + b/x^2)^p])/2 + (b*p*Log[b + a*x^2])/(2*a)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+(bp)\int\frac{1}{\left(a+\frac{b}{x^2}\right)x}\,dx\\&=\frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+(bp)\int\frac{x}{b+ax^2}\,dx\\&=\frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{bp\log\left(b+ax^2\right)}{2a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx=\frac{bp\log\left(a+\frac{b}{x^2}\right)}{2a}+\frac{1}{2}x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{bp\log(x)}{a}$$

```
[In] Integrate[x*Log[c*(a + b/x^2)^p],x]
[Out] (b*p*Log[a + b/x^2])/(2*a) + (x^2*Log[c*(a + b/x^2)^p])/2 + (b*p*Log[x])/a
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
parts	$\frac{x^2\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2}+\frac{bp\ln\left(x^2a+b\right)}{2a}$	34
parallelrisch	$-\frac{-x^2\ln\left(c\left(\frac{x^2a+b}{x^2}\right)^p\right)abp-2\ln(x)b^2p^2-\ln\left(c\left(\frac{x^2a+b}{x^2}\right)^p\right)b^2p}{2abp}$	69

```
[In] int(x*ln(c*(a+b/x^2)^p),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*ln(c*(a+b/x^2)^p)+1/2*b*p*ln(a*x^2+b)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx=\frac{apx^2\log\left(\frac{ax^2+b}{x^2}\right)+ax^2\log(c)+bp\log(ax^2+b)}{2a}$$

[In] integrate(x*log(c*(a+b/x^2)^p),x, algorithm="fricas")

[Out] 1/2*(a*p*x^2*log((a*x^2 + b)/x^2) + a*x^2*log(c) + b*p*log(a*x^2 + b))/a

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

$$\int x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx=\begin{cases}\frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2}+\frac{bp\log(ax^2+b)}{2a} & \text{for } a\neq 0\\ \frac{px^2}{2}+\frac{x^2\log\left(c\left(\frac{b}{x^2}\right)^p\right)}{2} & \text{otherwise}\end{cases}$$

[In] integrate(x*ln(c*(a+b/x**2)**p),x)

[Out] Piecewise((x**2*log(c*(a + b/x**2)**p)/2 + b*p*log(a*x**2 + b)/(2*a), Ne(a, 0)), (p*x**2/2 + x**2*log(c*(b/x**2)**p)/2, True))

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx=\frac{1}{2}x^2\log\left(\left(a+\frac{b}{x^2}\right)^pc\right)+\frac{bp\log(ax^2+b)}{2a}$$

[In] integrate(x*log(c*(a+b/x^2)^p),x, algorithm="maxima")

[Out] 1/2*x^2*log((a + b/x^2)^p*c) + 1/2*b*p*log(a*x^2 + b)/a

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx=\frac{1}{2}\,px^2\log\left(ax^2+b\right)-\frac{1}{2}\,px^2\log\left(x^2\right)+\frac{1}{2}\,x^2\log\left(c\right)+\frac{bp\log\left(ax^2+b\right)}{2\,a}$$

```
[In] integrate(x*log(c*(a+b/x^2)^p),x, algorithm="giac")
```

```
[Out] 1/2*p*x^2*log(a*x^2 + b) - 1/2*p*x^2*log(x^2) + 1/2*x^2*log(c) + 1/2*b*p*lo
g(a*x^2 + b)/a
```

Mupad [B] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx=\frac{x^2\,\ln\left(c\,{\left(a+\frac{b}{x^2}\right)}^p\right)}{2}+\frac{b\,p\,\ln\left(a\,x^2+b\right)}{2\,a}$$

```
[In] int(x*log(c*(a + b/x^2)^p),x)
```

```
[Out] (x^2*log(c*(a + b/x^2)^p))/2 + (b*p*log(b + a*x^2))/(2*a)
```

3.40 $\int \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 41

$$\int \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \, dx = \frac{2\sqrt{b}p \arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}} + x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

```
[Out] x*ln(c*(a+b/x^2)^p)+2*p*arctan(x*a^(1/2)/b^(1/2))*b^(1/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2498, 269, 211}

$$\int \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \, dx = \frac{2\sqrt{b}p \arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}} + x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)$$

```
[In] Int[Log[c*(a + b/x^2)^p],x]
[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/Sqrt[a] + x*Log[c*(a + b/x^2)^p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+(2bp)\int\frac{1}{\left(a+\frac{b}{x^2}\right)x^2}\,dx\\
&= x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+(2bp)\int\frac{1}{b+ax^2}\,dx\\
&= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}}+x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$
\int\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx=-\frac{2\sqrt{b}p\arctan\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{\sqrt{a}}+x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)
$$

```
[In] Integrate[Log[c*(a + b/x^2)^p],x]
[Out] (-2*Sqrt[b]*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[a] + x*Log[c*(a + b/x^2)^p]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

method	result	size
parts	$x\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{2pb\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}}$	34
default	$x\ln\left(c\left(\frac{x^2a+b}{x^2}\right)^p\right)+\frac{2pb\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}}$	38

```
[In] int(ln(c*(a+b/x^2)^p),x,method=_RETURNVERBOSE)
[Out] x*ln(c*(a+b/x^2)^p)+2*p*b/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.61

$$\int \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx = \left[px\log\left(\frac{ax^2+b}{x^2}\right)+p\sqrt{-\frac{b}{a}}\log\left(\frac{ax^2+2\,ax\sqrt{-\frac{b}{a}}-b}{ax^2+b}\right)+x\log(c)\,, px\log\left(\frac{ax^2+b}{x^2}\right)+2\,p\sqrt{\frac{b}{a}}\arctan\left(\frac{ax\sqrt{\frac{b}{a}}}{b}\right)+x\log(c)\right]$$

```
[In] integrate(log(c*(a+b/x^2)^p),x, algorithm="fricas")
```

```
[Out] [p*x*log((a*x^2 + b)/x^2) + p*sqrt(-b/a)*log((a*x^2 + 2*a*x*sqrt(-b/a) - b)
/(a*x^2 + b)) + x*log(c), p*x*log((a*x^2 + b)/x^2) + 2*p*sqrt(b/a)*arctan(a
*x*sqrt(b/a)/b) + x*log(c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(39) = 78.

Time = 3.58 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.32

$$\int \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx = \begin{cases} x\log(0^pc) & \text{for } a=0\wedge b=0 \\ 2px+x\log\left(c\left(\frac{b}{x^2}\right)^p\right) & \text{for } a=0 \\ x\log(a^pc) & \text{for } b=0 \\ x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)+\frac{bp\log\left(x-\sqrt{-\frac{b}{a}}\right)}{a\sqrt{-\frac{b}{a}}}-\frac{bp\log\left(x+\sqrt{-\frac{b}{a}}\right)}{a\sqrt{-\frac{b}{a}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(c*(a+b/x**2)**p),x)
```

```
[Out] Piecewise((x*log(0**p*c), Eq(a, 0) & Eq(b, 0)), (2*p*x + x*log(c*(b/x**2)**
p), Eq(a, 0)), (x*log(a**p*c), Eq(b, 0)), (x*log(c*(a + b/x**2)**p) + b*p*l
og(x - sqrt(-b/a))/(a*sqrt(-b/a)) - b*p*log(x + sqrt(-b/a))/(a*sqrt(-b/a)),
 True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx = \frac{2\,bp\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}} + x\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)$$

```
[In] integrate(log(c*(a+b/x^2)^p),x, algorithm="maxima")
[Out] 2*b*p*arctan(a*x/sqrt(a*b))/sqrt(a*b) + x*log((a + b/x^2)^p*c)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx = px\log\left(ax^2+b\right) - px\log\left(x^2\right) + \frac{2\,bp\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}} + x\log\left(c\right)$$

```
[In] integrate(log(c*(a+b/x^2)^p),x, algorithm="giac")
[Out] p*x*log(a*x^2 + b) - p*x*log(x^2) + 2*b*p*arctan(a*x/sqrt(a*b))/sqrt(a*b) +
 x*log(c)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx = x\,\ln\left(c\,\left(a+\frac{b}{x^2}\right)^p\right) + \frac{2\,\sqrt{b}\,p\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{a}}$$

```
[In] int(log(c*(a + b/x^2)^p),x)
[Out] x*log(c*(a + b/x^2)^p) + (2*b^(1/2)*p*atan((a^(1/2)*x)/b^(1/2)))/a^(1/2)
```

3.41 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx = -\frac{1}{2}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)-\frac{1}{2}p\,\text{PolyLog}\left(2,1+\frac{b}{ax^2}\right)$$

```
[Out] -1/2*ln(c*(a+b/x^2)^p)*ln(-b/a/x^2)-1/2*p*polylog(2,1+b/a/x^2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2441, 2352}

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx = -\frac{1}{2}\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)-\frac{1}{2}p\,\text{PolyLog}\left(2,\frac{b}{ax^2}+1\right)$$

```
[In] Int[Log[c*(a + b/x^2)^p]/x,x]
```

```
[Out] -1/2*(Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))]) - (p*PolyLog[2, 1 + b/(a*x^2)
])/2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x}\,dx, x, \frac{1}{x^2}\right)\right) \\
&= -\frac{1}{2}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)+\frac{1}{2}(bp)\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx, x, \frac{1}{x^2}\right) \\
&= -\frac{1}{2}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)-\frac{1}{2}p\text{Li}_2\left(1+\frac{b}{ax^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx = -\frac{1}{2}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)-\frac{1}{2}p\,\text{PolyLog}\left(2,\frac{a+\frac{b}{x^2}}{a}\right)$$

```
[In] Integrate[Log[c*(a + b/x^2)^p]/x,x]
[Out] -1/2*(Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))]) - (p*PolyLog[2, (a + b/x^2)/a
])/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(40) = 80.

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.86

method	result
parts	$\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)\ln(x)+2pb\left(\frac{\ln(x)^2}{2b}-\frac{a\left(\frac{\ln(x)\left(\ln\left(\frac{-ax+\sqrt{-ab}}{\sqrt{-ab}}\right)+\ln\left(\frac{ax+\sqrt{-ab}}{\sqrt{-ab}}\right)\right)}{2a}+\frac{\operatorname{dilog}\left(\frac{-ax+\sqrt{-ab}}{\sqrt{-ab}}\right)+\operatorname{dilog}\left(\frac{ax+\sqrt{-ab}}{\sqrt{-ab}}\right)}{2a}\right.}{b}\right.$

```
[In] int(ln(c*(a+b/x^2)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(c*(a+b/x^2)^p)*ln(x)+2*p*b*(1/2/b*ln(x)^2-a/b*(1/2*ln(x)*(ln((-a*x+(-a*b
)^(1/2))/(-a*b)^(1/2))+ln((a*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/a+1/2*(dilog((-
a*x+(-a*b)^(1/2))/(-a*b)^(1/2))+dilog((a*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/a))
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{x}\,dx$$

```
[In] integrate(log(c*(a+b/x^2)^p)/x,x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x^2 + b)/x^2)^p)/x, x)
```

Sympy [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx=\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx$$

```
[In] integrate(ln(c*(a+b/x**2)**p)/x,x)
```

```
[Out] Integral(log(c*(a + b/x**2)**p)/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(39) = 78$.

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.02

$$\begin{aligned}&\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx\\&=\frac{1}{2}\,bp\left(\frac{2\,\log\left(a+\frac{b}{x^2}\right)\log(x)}{b}+\frac{2\,\log(x)^2}{b}-\frac{2\,\log\left(\frac{ax^2}{b}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ax^2}{b}\right)}{b}\right)\\&\quad-p\log\left(a+\frac{b}{x^2}\right)\log(x)+\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)\log(x)\end{aligned}$$

```
[In] integrate(log(c*(a+b/x^2)^p)/x,x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(2*log(a + b/x^2)*log(x)/b + 2*log(x)^2/b - (2*log(a*x^2/b + 1)*log
(x) + dilog(-a*x^2/b))/b) - p*log(a + b/x^2)*log(x) + log((a + b/x^2)^p*c)*
log(x)
```

Giac [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{x}\,dx$$

```
[In] integrate(log(c*(a+b/x^2)^p)/x,x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x^2)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx=\int \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx$$

```
[In] int(log(c*(a + b/x^2)^p)/x,x)
```

```
[Out] int(log(c*(a + b/x^2)^p)/x, x)
```

3.42 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 50

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx = \frac{2p}{x} + \frac{2\sqrt{a}p\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}$$

```
[Out] 2*p/x-ln(c*(a+b/x^2)^p)/x+2*p*arctan(x*a^(1/2)/b^(1/2))*a^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2505, 269, 331, 211}

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx = \frac{2\sqrt{a}p\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x} + \frac{2p}{x}$$

```
[In] Int[Log[c*(a + b/x^2)^p]/x^2,x]
```

```
[Out] (2*p)/x + (2*Sqrt[a]*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/Sqrt[b] - Log[c*(a + b/
x^2)^p]/x
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x} - (2bp)\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^4}\,dx \\
&= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x} - (2bp)\int \frac{1}{x^2\left(b+ax^2\right)}\,dx \\
&= \frac{2p}{x} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x} + (2ap)\int \frac{1}{b+ax^2}\,dx \\
&= \frac{2p}{x} + \frac{2\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$
\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx = \frac{2p}{x} - \frac{2\sqrt{a}p\arctan\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{\sqrt{b}} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}
$$

```
[In] Integrate[Log[c*(a + b/x^2)^p]/x^2,x]
[Out] (2*p)/x - (2*Sqrt[a]*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[b] - Log[c*(a + b/
x^2)^p]/x
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

method	result	size
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x^{2}}\right)^{p}\right)}{x}-2pb\left(-\frac{a\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{b\sqrt{ab}}-\frac{1}{bx}\right)$	52

```
[In] int(ln(c*(a+b/x^2)^p)/x^2,x,method=_RETURNVERBOSE)
[Out] -ln(c*(a+b/x^2)^p)/x-2*p*b*(-a/b/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2))-1/b/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.38

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx$$
$$=\left[\frac{px\sqrt{-\frac{a}{b}}\log\left(\frac{ax^2+2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right)-p\log\left(\frac{ax^2+b}{x^2}\right)+2\,p-\log\left(c\right)}{x},\ \frac{2\,px\sqrt{\frac{a}{b}}\arctan\left(x\sqrt{\frac{a}{b}}\right)-p\log\left(\frac{ax^2+b}{x^2}\right)+2\right.}{x}\right.$$

```
[In] integrate(log(c*(a+b/x^2)^p)/x^2,x, algorithm="fricas")
[Out] [(p*x*sqrt(-a/b)*log((a*x^2 + 2*b*x*sqrt(-a/b) - b)/(a*x^2 + b)) - p*log((a
*x^2 + b)/x^2) + 2*p - log(c))/x, (2*p*x*sqrt(a/b)*arctan(x*sqrt(a/b)) - p*
log((a*x^2 + b)/x^2) + 2*p - log(c))/x]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(44) = 88$.

Time = 8.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.94

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx$$
$$=\begin{cases} -\frac{\log\left(0^p c\right)}{x} & \text{for}\: a=0 \wedge b=0 \\ \frac{2p}{x}-\frac{\log\left(c\left(\frac{b}{x^2}\right)^p\right)}{x} & \text{for}\: a=0 \\ -\frac{\log\left(a^p c\right)}{x} & \text{for}\: b=0 \\ \frac{p\log\left(x-\sqrt{-\frac{b}{a}}\right)}{\sqrt{-\frac{b}{a}}}-\frac{p\log\left(x+\sqrt{-\frac{b}{a}}\right)}{\sqrt{-\frac{b}{a}}}+\frac{2p}{x}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x} & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(c*(a+b/x**2)**p)/x**2,x)
```

```
[Out] Piecewise((-log(0**p*c)/x, Eq(a, 0) & Eq(b, 0)), (2*p/x - log(c*(b/x**2)**p
)/x, Eq(a, 0)), (-log(a**p*c)/x, Eq(b, 0)), (p*log(x - sqrt(-b/a))/sqrt(-b/
a) - p*log(x + sqrt(-b/a))/sqrt(-b/a) + 2*p/x - log(c*(a + b/x**2)**p)/x, T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx = 2\,bp\left(\frac{a\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}b}+\frac{1}{bx}\right)-\frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{x}$$

```
[In] integrate(log(c*(a+b/x^2)^p)/x^2,x, algorithm="maxima")
```

```
[Out] 2*b*p*(a*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/(b*x)) - log((a + b/x^2)^p
*c)/x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx = \frac{2\,ap\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}}-\frac{p\log\left(ax^2+b\right)}{x}+\frac{p\log\left(x^2\right)}{x}+\frac{2\,p-\log\left(c\right)}{x}$$

```
[In] integrate(log(c*(a+b/x^2)^p)/x^2,x, algorithm="giac")
```

```
[Out] 2*a*p*arctan(a*x/sqrt(a*b))/sqrt(a*b) - p*log(a*x^2 + b)/x + p*log(x^2)/x +
 (2*p - log(c))/x
```

Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx = \frac{2\,p}{x}-\frac{\ln\left(c\,\left(a+\frac{b}{x^2}\right)^p\right)}{x}+\frac{2\,\sqrt{a}\,p\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{\sqrt{b}}$$

```
[In] int(log(c*(a + b/x^2)^p)/x^2,x)
```

```
[Out] (2*p)/x - log(c*(a + b/x^2)^p)/x + (2*a^(1/2)*p*atan((a^(1/2)*x)/b^(1/2)))/
b^(1/2)
```

3.43 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 35

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}\,dx=\frac{p}{2x^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2b}$$

```
[Out] 1/2*p/x^2-1/2*(a+b/x^2)*ln(c*(a+b/x^2)^p)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2436, 2332}

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}\,dx=\frac{p}{2x^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2b}$$

```
[In] Int[Log[c*(a + b/x^2)^p]/x^3,x]
```

```
[Out] p/(2*x^2) - ((a + b/x^2)*Log[c*(a + b/x^2)^p])/(2*b)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int \log\left(c(a+bx)^p\right)\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\frac{\text{Subst}\left(\int \log\left(cx^p\right)\,dx, x, a+\frac{b}{x^2}\right)}{2b}\\
&= \frac{p}{2x^2} - \frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}\,dx = \frac{1}{2}\left(\frac{p}{x^2} - \frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{b}\right)$$

```
[In] Integrate[Log[c*(a + b/x^2)^p]/x^3,x]
[Out] (p/x^2 - ((a + b/x^2)*Log[c*(a + b/x^2)^p])/b)/2
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$-\frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)\left(a+\frac{b}{x^2}\right)-\left(a+\frac{b}{x^2}\right)p}{2b}$	37
default	$-\frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)\left(a+\frac{b}{x^2}\right)-\left(a+\frac{b}{x^2}\right)p}{2b}$	37
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2x^2}-pb\left(-\frac{1}{2b\,x^2}-\frac{a\ln(x)}{b^2}+\frac{a\ln(x^2a+b)}{2b^2}\right)$	54
parallelrisch	$-\frac{x^2\ln\left(c\left(\frac{x^2a+b}{x^2}\right)^p\right)a^2p+\ln\left(c\left(\frac{x^2a+b}{x^2}\right)^p\right)abp-ab\,p^2}{2x^2apb}$	67

```
[In] int(ln(c*(a+b/x^2)^p)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*(ln(c*(a+b/x^2)^p)*(a+b/x^2)-(a+b/x^2)*p)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3} dx = \frac{bp - b\log(c) - (apx^2 + bp)\log\left(\frac{ax^2+b}{x^2}\right)}{2\,bx^2}$$

```
[In] integrate(log(c*(a+b/x^2)^p)/x^3,x, algorithm="fricas")
[Out] 1/2*(b*p - b*log(c) - (a*p*x^2 + b*p)*log((a*x^2 + b)/x^2))/(b*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3} dx = \begin{cases} -\frac{a\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2b} + \frac{p}{2x^2} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2x^2} & \text{for}\ b \neq 0 \\ -\frac{\log(a^p c)}{2x^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(c*(a+b/x**2)**p)/x**3,x)
[Out] Piecewise((-a*log(c*(a + b/x**2)**p)/(2*b) + p/(2*x**2) - log(c*(a + b/x**2
)**p)/(2*x**2), Ne(b, 0)), (-log(a**p*c)/(2*x**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.54

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3} dx = -\frac{1}{2}\,bp\left(\frac{a\log(ax^2+b)}{b^2} - \frac{a\log(x^2)}{b^2} - \frac{1}{bx^2}\right) - \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{2\,x^2}$$

```
[In] integrate(log(c*(a+b/x^2)^p)/x^3,x, algorithm="maxima")
[Out] -1/2*b*p*(a*log(a*x^2 + b)/b^2 - a*log(x^2)/b^2 - 1/(b*x^2)) - 1/2*log((a +
 b/x^2)^p*c)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.63

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}\,dx=-\frac{p\left(\frac{(ax^2+b)\log\left(\frac{ax^2+b}{x^2}\right)}{x^2}-\frac{ax^2+b}{x^2}\right)+\frac{(ax^2+b)\log(c)}{x^2}}{2\,b}$$

```
[In] integrate(log(c*(a+b/x^2)^p)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(p*((a*x^2 + b)*log((a*x^2 + b)/x^2)/x^2 - (a*x^2 + b)/x^2) + (a*x^2 +
 b)*log(c)/x^2)/b
```

Mupad [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.34

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}\,dx=\frac{p}{2\,x^2}-\frac{\ln\left(c\,\left(a+\frac{b}{x^2}\right)^p\right)}{2\,x^2}-\frac{a\,p\,\ln\left(a\,x^2+b\right)}{2\,b}+\frac{a\,p\,\ln\left(x\right)}{b}$$

```
[In] int(log(c*(a + b/x^2)^p)/x^3,x)
```

```
[Out] p/(2*x^2) - log(c*(a + b/x^2)^p)/(2*x^2) - (a*p*log(b + a*x^2))/(2*b) + (a*
p*log(x))/b
```

3.44 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4} dx$

Optimal result

Integrand size = 16, antiderivative size = 68

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4} dx = \frac{2p}{9x^3} - \frac{2ap}{3bx} - \frac{2a^{3/2}p\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3b^{3/2}} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3}$$

```
[Out] 2/9*p/x^3-2/3*a*p/b/x-2/3*a^(3/2)*p*arctan(x*a^(1/2)/b^(1/2))/b^(3/2)-1/3*l
n(c*(a+b/x^2)^p)/x^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2505, 269, 331, 211}

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4} dx = -\frac{2a^{3/2}p\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3b^{3/2}} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3} - \frac{2ap}{3bx} + \frac{2p}{9x^3}$$

```
[In] Int[Log[c*(a + b/x^2)^p]/x^4,x]
```

```
[Out] (2*p)/(9*x^3) - (2*a*p)/(3*b*x) - (2*a^(3/2)*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])
/(3*b^(3/2)) - Log[c*(a + b/x^2)^p]/(3*x^3)
```

<u>Rule 211</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

<u>Rule 269</u>

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3} - \frac{1}{3}(2bp)\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^6}\,dx \\
&= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3} - \frac{1}{3}(2bp)\int \frac{1}{x^4\left(b+ax^2\right)}\,dx \\
&= \frac{2p}{9x^3} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3} + \frac{1}{3}(2ap)\int \frac{1}{x^2\left(b+ax^2\right)}\,dx \\
&= \frac{2p}{9x^3} - \frac{2ap}{3bx} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3} - \frac{\left(2a^2p\right)\int \frac{1}{b+ax^2}\,dx}{3b} \\
&= \frac{2p}{9x^3} - \frac{2ap}{3bx} - \frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3b^{3/2}} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

$$
\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4}\,dx = \frac{2p}{9x^3} - \frac{2ap}{3bx} + \frac{2a^{3/2}p\arctan\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{3b^{3/2}} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3}
$$

```
[In] Integrate[Log[c*(a + b/x^2)^p]/x^4,x]
[Out] (2*p)/(9*x^3) - (2*a*p)/(3*b*x) + (2*a^(3/2)*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])
/(3*b^(3/2)) - Log[c*(a + b/x^2)^p]/(3*x^3)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

method	result	size
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3}-\frac{2pb\left(-\frac{1}{3b\,x^3}+\frac{a}{b^2x}+\frac{a^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}\right)}{3}$	61

```
[In] int(ln(c*(a+b/x^2)^p)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*ln(c*(a+b/x^2)^p)/x^3-2/3*p*b*(-1/3/b/x^3+a/b^2/x+a^2/b^2/(a*b)^(1/2)*
arctan(a*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.26

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4}\,dx = \left[\frac{3\,apx^3\sqrt{-\frac{a}{b}}\log\left(\frac{ax^2-2\,bx\sqrt{-\frac{a}{b}}-b}{ax^2+b}\right)-6\,apx^2-3\,bp\log\left(\frac{ax^2+b}{x^2}\right)+2\,bp-3\,b\log\left(c\right)}{9\,bx^3},\ -\frac{6\,apx^3\sqrt{\frac{a}{b}}\arctan\left(x\sqrt{\frac{a}{b}}\right)+6\,apx^2+3\,bp\log\left(\frac{ax^2+b}{x^2}\right)-2\,bp+3\,b\log\left(c\right)}{9\,bx^3}\right]$$

```
[In] integrate(log(c*(a+b/x^2)^p)/x^4,x, algorithm="fricas")
```

```
[Out] [1/9*(3*a*p*x^3*sqrt(-a/b)*log((a*x^2 - 2*b*x*sqrt(-a/b) - b)/(a*x^2 + b))
- 6*a*p*x^2 - 3*b*p*log((a*x^2 + b)/x^2) + 2*b*p - 3*b*log(c))/(b*x^3), -1/
9*(6*a*p*x^3*sqrt(a/b)*arctan(x*sqrt(a/b)) + 6*a*p*x^2 + 3*b*p*log((a*x^2 +
 b)/x^2) - 2*b*p + 3*b*log(c))/(b*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(63) = 126$.

Time = 24.50 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.03

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4}\,dx = \begin{cases} -\frac{\log(0^p c)}{3x^3} & \text{for } a=0 \wedge b=0 \\ \frac{2p}{9x^3} - \frac{\log\left(c\left(\frac{b}{x^2}\right)^p\right)}{3x^3} & \text{for } a=0 \\ -\frac{\log(a^p c)}{3x^3} & \text{for } b=0 \\ -\frac{ap\log\left(x-\sqrt{-\frac{b}{a}}\right)}{3b\sqrt{-\frac{b}{a}}} + \frac{ap\log\left(x+\sqrt{-\frac{b}{a}}\right)}{3b\sqrt{-\frac{b}{a}}} - \frac{2ap}{3bx} + \frac{2p}{9x^3} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3x^3} & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(c*(a+b/x**2)**p)/x**4,x)
```

```
[Out] Piecewise((-log(0**p*c)/(3*x**3), Eq(a, 0) & Eq(b, 0)), (2*p/(9*x**3) - log
(c*(b/x**2)**p)/(3*x**3), Eq(a, 0)), (-log(a**p*c)/(3*x**3), Eq(b, 0)), (-a
*p*log(x - sqrt(-b/a))/(3*b*sqrt(-b/a)) + a*p*log(x + sqrt(-b/a))/(3*b*sqrt
(-b/a)) - 2*a*p/(3*b*x) + 2*p/(9*x**3) - log(c*(a + b/x**2)**p)/(3*x**3), T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4}\,dx = -\frac{2}{9}\,bp\left(\frac{3\,a^2\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{3\,ax^2-b}{b^2x^3}\right) - \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{3\,x^3}$$

```
[In] integrate(log(c*(a+b/x^2)^p)/x^4,x, algorithm="maxima")
```

```
[Out] -2/9*b*p*(3*a^2*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b^2) + (3*a*x^2 - b)/(b^2*
x^3)) - 1/3*log((a + b/x^2)^p*c)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4}\,dx = -\frac{2\,a^2p\arctan\left(\frac{ax}{\sqrt{ab}}\right)}{3\,\sqrt{ab}b} - \frac{p\log\left(ax^2+b\right)}{3\,x^3} + \frac{p\log\left(x^2\right)}{3\,x^3} - \frac{6\,apx^2-2\,bp+3\,b\log\left(c\right)}{9\,bx^3}$$

```
[In] integrate(log(c*(a+b/x^2)^p)/x^4,x, algorithm="giac")
```

```
[Out] -2/3*a^2*p*arctan(a*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/3*p*log(a*x^2 + b)/x^3 +
 1/3*p*log(x^2)/x^3 - 1/9*(6*a*p*x^2 - 2*b*p + 3*b*log(c))/(b*x^3)
```

Mupad [B] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^4}\,dx = \frac{\frac{2\,p}{3}-\frac{2\,a\,p\,x^2}{b}}{3\,x^3} - \frac{\ln\left(c\,{\left(a+\frac{b}{x^2}\right)}^p\right)}{3\,x^3} - \frac{2\,a^{3/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{a}\,x}{\sqrt{b}}\right)}{3\,b^{3/2}}$$

```
[In] int(log(c*(a + b/x^2)^p)/x^4,x)
```

```
[Out] ((2*p)/3 - (2*a*p*x^2)/b)/(3*x^3) - log(c*(a + b/x^2)^p)/(3*x^3) - (2*a^(3/
2)*p*atan((a^(1/2)*x)/b^(1/2)))/(3*b^(3/2))
```

3.45 $\int \frac{\log\left(1+\frac{b}{x}\right)}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 8

$$\int \frac{\log\left(1+\frac{b}{x}\right)}{x}\,dx = \text{PolyLog}\left(2,-\frac{b}{x}\right)$$

```
[Out] polylog(2,-b/x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2438}

$$\int \frac{\log\left(1+\frac{b}{x}\right)}{x}\,dx = \text{PolyLog}\left(2,-\frac{b}{x}\right)$$

```
[In] Int[Log[1 + b/x]/x,x]
```

```
[Out] PolyLog[2, -(b/x)]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = \text{Li}_2\left(-\frac{b}{x}\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 34 vs. $2(8) = 16$.

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 4.25

$$\int \frac{\log\left(1+\frac{b}{x}\right)}{x}\,dx = -\log\left(-\frac{b}{x}\right)\log\left(\frac{b+x}{x}\right) - \text{PolyLog}\left(2, -\frac{-b-x}{x}\right)$$

```
[In] Integrate[Log[1 + b/x]/x,x]
[Out] -(Log[-(b/x)]*Log[(b + x)/x]) - PolyLog[2, -((-b - x)/x)]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$\text{dilog}\left(1+\frac{b}{x}\right)$	9
default	$\text{dilog}\left(1+\frac{b}{x}\right)$	9
risch	$\text{dilog}\left(1+\frac{b}{x}\right)$	9
parts	$\ln\left(1+\frac{b}{x}\right)\ln\left(x\right)+b\left(\frac{\ln\left(x\right)^{2}}{2b}-\frac{\text{dilog}\left(\frac{x+b}{b}\right)+\ln\left(x\right)\ln\left(\frac{x+b}{b}\right)}{b}\right)$	50

```
[In] int(ln(1+b/x)/x,x,method=_RETURNVERBOSE)
[Out] dilog(1+b/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.38

$$\int \frac{\log\left(1+\frac{b}{x}\right)}{x}\,dx = \text{Li}_2\left(-\frac{b+x}{x}+1\right)$$

```
[In] integrate(log(1+b/x)/x,x, algorithm="fricas")
[Out] dilog(-(b + x)/x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\log \left(1+\frac{b}{x}\right)}{x}\, dx = \mathrm{Li}_2\left(\frac{be^{i\pi}}{x}\right)$$

```
[In] integrate(ln(1+b/x)/x,x)
[Out] polylog(2, b*exp_polar(I*pi)/x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(7) = 14.

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 4.38

$$\int \frac{\log \left(1+\frac{b}{x}\right)}{x}\, dx = \log \left(b+x\right) \log \left(x\right) - \frac{1}{2} \log \left(x\right)^2 - \log \left(x\right) \log \left(\frac{x}{b}+1\right) - \mathrm{Li}_2\left(-\frac{x}{b}\right)$$

```
[In] integrate(log(1+b/x)/x,x, algorithm="maxima")
[Out] log(b + x)*log(x) - 1/2*log(x)^2 - log(x)*log(x/b + 1) - dilog(-x/b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(7) = 14.

Time = 0.45 (sec) , antiderivative size = 110, normalized size of antiderivative = 13.75

$$\int \frac{\log \left(1+\frac{b}{x}\right)}{x}\, dx = -\frac{b^3\left(\frac{1}{\frac{b+x}{x}-1} - \log \left(\frac{|b+x|}{|x|}\right) + \log \left(\left|\frac{b+x}{x}-1\right|\right)\right) + \frac{b^3 \log \left(-b\left(\frac{\left(b-\frac{1}{\frac{1}{b}-\frac{b+x}{bx}}\right)\left(\frac{1}{b}-\frac{b+x}{bx}\right)}{b}+\frac{1}{b}\right)+1\right)}{\left(\frac{b+x}{x}-1\right)^2}}{2\,b^2}$$

```
[In] integrate(log(1+b/x)/x,x, algorithm="giac")
[Out] -1/2*(b^3*(1/((b + x)/x - 1) - log(abs(b + x)/abs(x)) + log(abs((b + x)/x -
 1))) + b^3*log(-b*((b - 1/(1/b - (b + x)/(b*x)))*(1/b - (b + x)/(b*x))/b +
 1/b) + 1)/((b + x)/x - 1)^2)/b^2
```

Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(1+\frac{b}{x}\right)}{x}\,dx = \operatorname{polylog}\left(2,-\frac{b}{x}\right)$$

```
[In] int(log(b/x + 1)/x,x)
[Out] polylog(2, -b/x)
```

3.46 $\int x^3 \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 153

$$\int x^3 \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx = \frac{a^7 p\sqrt{x}}{4b^7} - \frac{a^6 px}{8b^6} + \frac{a^5 px^{3/2}}{12b^5} - \frac{a^4 px^2}{16b^4} + \frac{a^3 px^{5/2}}{20b^3} - \frac{a^2 px^3}{24b^2} + \frac{apx^{7/2}}{28b} - \frac{px^4}{32} - \frac{a^8 p \log \left(a+b\sqrt{x}\right)}{4b^8} + \frac{1}{4}x^4 \log \left(c\left(a+b\sqrt{x}\right)^p\right)$$

```
[Out] -1/8*a^6*p*x/b^6+1/12*a^5*p*x^(3/2)/b^5-1/16*a^4*p*x^2/b^4+1/20*a^3*p*x^(5/
2)/b^3-1/24*a^2*p*x^3/b^2+1/28*a*p*x^(7/2)/b-1/32*p*x^4-1/4*a^8*p*ln(a+b*x^
(1/2))/b^8+1/4*x^4*ln(c*(a+b*x^(1/2))^p)+1/4*a^7*p*x^(1/2)/b^7
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2504, 2442, 45}

$$\int x^3 \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx = -\frac{a^8 p \log \left(a+b\sqrt{x}\right)}{4b^8} + \frac{a^7 p\sqrt{x}}{4b^7} - \frac{a^6 px}{8b^6} + \frac{a^5 px^{3/2}}{12b^5} - \frac{a^4 px^2}{16b^4} + \frac{a^3 px^{5/2}}{20b^3} - \frac{a^2 px^3}{24b^2} + \frac{1}{4}x^4 \log \left(c\left(a+b\sqrt{x}\right)^p\right) + \frac{apx^{7/2}}{28b} - \frac{px^4}{32}$$

```
[In] Int[x^3*Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (a^7*p*Sqrt[x])/(4*b^7) - (a^6*p*x)/(8*b^6) + (a^5*p*x^(3/2))/(12*b^5) - (a
^4*p*x^2)/(16*b^4) + (a^3*p*x^(5/2))/(20*b^3) - (a^2*p*x^3)/(24*b^2) + (a*p
*x^(7/2))/(28*b) - (p*x^4)/32 - (a^8*p*Log[a + b*Sqrt[x]])/(4*b^8) + (x^4*L
og[c*(a + b*Sqrt[x])^p])/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^7\log\left(c(a+bx)^p\right)\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{4}x^4\log\left(c\left(a+b\sqrt{x}\right)^p\right)-\frac{1}{4}(bp)\text{Subst}\left(\int\frac{x^8}{a+bx}\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{4}x^4\log\left(c\left(a+b\sqrt{x}\right)^p\right)-\frac{1}{4}(bp)\text{Subst}\left(\int\left(-\frac{a^7}{b^8}+\frac{a^6x}{b^7}-\frac{a^5x^2}{b^6}+\frac{a^4x^3}{b^5}-\frac{a^3x^4}{b^4}\right.\right.\\
&\qquad\left.\left.+\frac{a^2x^5}{b^3}-\frac{ax^6}{b^2}+\frac{x^7}{b}+\frac{a^8}{b^8(a+bx)}\right)\,dx,x,\sqrt{x}\right)\\
&= \frac{a^7p\sqrt{x}}{4b^7}-\frac{a^6px}{8b^6}+\frac{a^5px^{3/2}}{12b^5}-\frac{a^4px^2}{16b^4}+\frac{a^3px^{5/2}}{20b^3}-\frac{a^2px^3}{24b^2}\\
&\quad+\frac{apx^{7/2}}{28b}-\frac{px^4}{32}-\frac{a^8p\log\left(a+b\sqrt{x}\right)}{4b^8}+\frac{1}{4}x^4\log\left(c\left(a+b\sqrt{x}\right)^p\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.87

$$\int x^3 \log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx = -\frac{p\left(-840a^7b\sqrt{x}+420a^6b^2x-280a^5b^3x^{3/2}+210a^4b^4x^2-168a^3b^5x^{5/2}+140a^2b^6x^3-120ab^7x^{7/2}+105b\right.}{3360b^8} + \frac{1}{4}x^4\log\left(c\left(a+b\sqrt{x}\right)^p\right)$$

```
[In] Integrate[x^3*Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] -1/3360*(p*(-840*a^7*b*Sqrt[x] + 420*a^6*b^2*x - 280*a^5*b^3*x^(3/2) + 210*
a^4*b^4*x^2 - 168*a^3*b^5*x^(5/2) + 140*a^2*b^6*x^3 - 120*a*b^7*x^(7/2) + 1
05*b^8*x^4 + 840*a^8*Log[a + b*Sqrt[x]]))/b^8 + (x^4*Log[c*(a + b*Sqrt[x])^
p])/4
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

method	result	size
parts	$\frac{x^4\ln\left(c\left(a+b\sqrt{x}\right)^p\right)}{4}-\frac{pb\left(-\frac{2\left(-\frac{x^4b^7}{8}+\frac{a\,x^{\frac{7}{2}}b^6}{7}-\frac{a^2x^3b^5}{6}+\frac{a^3x^{\frac{5}{2}}b^4}{5}-\frac{a^4x^2b^3}{4}+\frac{a^5x^{\frac{3}{2}}b^2}{3}-\frac{b\,a^6x}{2}+a^7\sqrt{x}\right)}{b^8}+\frac{2a^8\ln\left(a+b\sqrt{x}\right)}{b^9}\right)}{8}$	121

```
[In] int(x^3*ln(c*(a+b*x^(1/2))^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*ln(c*(a+b*x^(1/2))^p)-1/8*p*b*(-2/b^8*(-1/8*x^4*b^7+1/7*a*x^(7/2)*b
^6-1/6*a^2*x^3*b^5+1/5*a^3*x^(5/2)*b^4-1/4*a^4*x^2*b^3+1/3*a^5*x^(3/2)*b^2-
1/2*b*a^6*x+a^7*x^(1/2))+2*a^8/b^9*ln(a+b*x^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.84

$$\int x^3 \log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx = -\frac{105\,b^8px^4-840\,b^8x^4\log\left(c\right)+140\,a^2b^6px^3+210\,a^4b^4px^2+420\,a^6b^2px-840\left(b^8px^4-a^8p\right)\log\left(b\sqrt{x}+\right.}{3360\,b^8}$$

```
[In] integrate(x^3*log(c*(a+b*x^(1/2))^p),x, algorithm="fricas")
```

```
[Out] -1/3360*(105*b^8*p*x^4 - 840*b^8*x^4*log(c) + 140*a^2*b^6*p*x^3 + 210*a^4*b
^4*p*x^2 + 420*a^6*b^2*p*x - 840*(b^8*p*x^4 - a^8*p)*log(b*sqrt(x) + a) - 8
*(15*a*b^7*p*x^3 + 21*a^3*b^5*p*x^2 + 35*a^5*b^3*p*x + 105*a^7*b*p)*sqrt(x)
)/b^8
```

Sympy [A] (verification not implemented)

Time = 12.25 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95

$$\int x^3 \log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx = -\frac{bp\left(\frac{2a^8\left(\begin{cases}\frac{\sqrt{x}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{x}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^8} - \frac{2a^7\sqrt{x}}{b^8} + \frac{a^6x}{b^7} - \frac{2a^5x^{\frac{3}{2}}}{3b^6} + \frac{a^4x^2}{2b^5} - \frac{2a^3x^{\frac{5}{2}}}{5b^4} + \frac{a^2x^3}{3b^3} - \frac{2ax^{\frac{7}{2}}}{7b^2} + \frac{x^4}{4b}\right)}{8} + \frac{x^4\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{4}$$

```
[In] integrate(x**3*ln(c*(a+b*x**(1/2))**p),x)
```

```
[Out] -b*p*(2*a**8*Piecewise((sqrt(x)/a, Eq(b, 0)), (log(a + b*sqrt(x))/b, True))
/b**8 - 2*a**7*sqrt(x)/b**8 + a**6*x/b**7 - 2*a**5*x**(3/2)/(3*b**6) + a**4
*x**2/(2*b**5) - 2*a**3*x**(5/2)/(5*b**4) + a**2*x**3/(3*b**3) - 2*a*x**(7/
2)/(7*b**2) + x**4/(4*b))/8 + x**4*log(c*(a + b*sqrt(x))**p)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.78

$$\int x^3 \log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx = \frac{1}{4}\,x^4\log\left(\left(b\sqrt{x}+a\right)^pc\right) - \frac{1}{3360}\,bp\left(\frac{840\,a^8\log\left(b\sqrt{x}+a\right)}{b^9} + \frac{105\,b^7x^4 - 120\,ab^6x^{\frac{7}{2}} + 140\,a^2b^5x^3 - 168\,a^3b^4x^{\frac{5}{2}} + 210\,a^4b^3x^2 - 280\,a^5}{b^8}\right.$$

```
[In] integrate(x^3*log(c*(a+b*x^(1/2))^p),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((b*sqrt(x) + a)^p*c) - 1/3360*b*p*(840*a^8*log(b*sqrt(x) + a)/b
^9 + (105*b^7*x^4 - 120*a*b^6*x^(7/2) + 140*a^2*b^5*x^3 - 168*a^3*b^4*x^(5/
2) + 210*a^4*b^3*x^2 - 280*a^5*b^2*x^(3/2) + 420*a^6*b*x - 840*a^7*sqrt(x))
/b^8)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(121) = 242$.

Time = 0.30 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.22

$$\int x^3 \log \left(c\left(a+b\sqrt{x}\right)^p\right) \,dx = \frac{840\,bx^4\log\left(c\right) + \left(\frac{840\,\left(b\sqrt{x}+a\right)^8\log\left(b\sqrt{x}+a\right)}{b^7} - \frac{6720\,\left(b\sqrt{x}+a\right)^7 a\log\left(b\sqrt{x}+a\right)}{b^7} + \frac{23520\,\left(b\sqrt{x}+a\right)^6 a^2\log\left(b\sqrt{x}+a\right)}{b^7} - \frac{47040\,\left(b\sqrt{x}+a\right)}{}\right.}{}$$

```
[In] integrate(x^3*log(c*(a+b*x^(1/2))^p),x, algorithm="giac")
```

```
[Out] 1/3360*(840*b*x^4*log(c) + (840*(b*sqrt(x) + a)^8*log(b*sqrt(x) + a)/b^7 -
6720*(b*sqrt(x) + a)^7*a*log(b*sqrt(x) + a)/b^7 + 23520*(b*sqrt(x) + a)^6*a
^2*log(b*sqrt(x) + a)/b^7 - 47040*(b*sqrt(x) + a)^5*a^3*log(b*sqrt(x) + a)/
b^7 + 58800*(b*sqrt(x) + a)^4*a^4*log(b*sqrt(x) + a)/b^7 - 47040*(b*sqrt(x)
 + a)^3*a^5*log(b*sqrt(x) + a)/b^7 + 23520*(b*sqrt(x) + a)^2*a^6*log(b*sqrt
(x) + a)/b^7 - 6720*(b*sqrt(x) + a)*a^7*log(b*sqrt(x) + a)/b^7 - 105*(b*sqr
t(x) + a)^8/b^7 + 960*(b*sqrt(x) + a)^7*a/b^7 - 3920*(b*sqrt(x) + a)^6*a^2/
b^7 + 9408*(b*sqrt(x) + a)^5*a^3/b^7 - 14700*(b*sqrt(x) + a)^4*a^4/b^7 + 15
680*(b*sqrt(x) + a)^3*a^5/b^7 - 11760*(b*sqrt(x) + a)^2*a^6/b^7 + 6720*(b*s
qrt(x) + a)*a^7/b^7)*p)/b
```

Mupad [B] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

$$\int x^3 \log \left(c\left(a+b\sqrt{x}\right)^p\right) \,dx = \frac{x^4\,\ln\left(c\,{\left(a+b\,\sqrt{x}\right)}^p\right)}{4} - \frac{p\,x^4}{32} - \frac{a^8\,p\,\ln\left(a+b\,\sqrt{x}\right)}{4\,b^8} - \frac{a^2\,p\,x^3}{24\,b^2} - \frac{a^4\,p\,x^2}{16\,b^4} + \frac{a^3\,p\,x^{5/2}}{20\,b^3} + \frac{a^5\,p\,x^{3/2}}{12\,b^5} + \frac{a^7\,p\,\sqrt{x}}{4\,b^7} + \frac{a\,p\,x^{7/2}}{28\,b} - \frac{a^6\,p\,x}{8\,b^6}$$

```
[In] int(x^3*log(c*(a + b*x^(1/2))^p),x)
```

```
[Out] (x^4*log(c*(a + b*x^(1/2))^p))/4 - (p*x^4)/32 - (a^8*p*log(a + b*x^(1/2)))/
(4*b^8) - (a^2*p*x^3)/(24*b^2) - (a^4*p*x^2)/(16*b^4) + (a^3*p*x^(5/2))/(20
*b^3) + (a^5*p*x^(3/2))/(12*b^5) + (a^7*p*x^(1/2))/(4*b^7) + (a*p*x^(7/2))/
(28*b) - (a^6*p*x)/(8*b^6)
```

3.47 $\int x^2 \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 123

$$\int x^2 \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx = \frac{a^5 p\sqrt{x}}{3b^5} - \frac{a^4 p x}{6b^4} + \frac{a^3 p x^{3/2}}{9b^3} - \frac{a^2 p x^2}{12b^2} + \frac{a p x^{5/2}}{15b} - \frac{p x^3}{18} - \frac{a^6 p \log \left(a+b\sqrt{x}\right)}{3b^6} + \frac{1}{3} x^3 \log \left(c\left(a+b\sqrt{x}\right)^p\right)$$

```
[Out] -1/6*a^4*p*x/b^4+1/9*a^3*p*x^(3/2)/b^3-1/12*a^2*p*x^2/b^2+1/15*a*p*x^(5/2)/
b-1/18*p*x^3-1/3*a^6*p*ln(a+b*x^(1/2))/b^6+1/3*x^3*ln(c*(a+b*x^(1/2))^p)+1/
3*a^5*p*x^(1/2)/b^5
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2504, 2442, 45}

$$\int x^2 \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx = -\frac{a^6 p \log \left(a+b\sqrt{x}\right)}{3b^6} + \frac{a^5 p\sqrt{x}}{3b^5} - \frac{a^4 p x}{6b^4} + \frac{a^3 p x^{3/2}}{9b^3} - \frac{a^2 p x^2}{12b^2} + \frac{1}{3} x^3 \log \left(c\left(a+b\sqrt{x}\right)^p\right) + \frac{a p x^{5/2}}{15b} - \frac{p x^3}{18}$$

```
[In] Int[x^2*Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (a^5*p*Sqrt[x])/(3*b^5) - (a^4*p*x)/(6*b^4) + (a^3*p*x^(3/2))/(9*b^3) - (a^
2*p*x^2)/(12*b^2) + (a*p*x^(5/2))/(15*b) - (p*x^3)/18 - (a^6*p*Log[a + b*Sq
rt[x]])/(3*b^6) + (x^3*Log[c*(a + b*Sqrt[x])^p])/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^5 \log\left(c(a+bx)^p\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{1}{3}x^3\log\left(c\left(a+b\sqrt{x}\right)^p\right) - \frac{1}{3}(bp)\text{Subst}\left(\int \frac{x^6}{a+bx}\,dx, x, \sqrt{x}\right)\\
&= \frac{1}{3}x^3\log\left(c\left(a+b\sqrt{x}\right)^p\right)\\
&\quad - \frac{1}{3}(bp)\text{Subst}\left(\int \left(-\frac{a^5}{b^6}+\frac{a^4x}{b^5}-\frac{a^3x^2}{b^4}+\frac{a^2x^3}{b^3}-\frac{ax^4}{b^2}+\frac{x^5}{b}+\frac{a^6}{b^6(a+bx)}\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{a^5p\sqrt{x}}{3b^5} - \frac{a^4px}{6b^4} + \frac{a^3px^{3/2}}{9b^3} - \frac{a^2px^2}{12b^2} + \frac{apx^{5/2}}{15b} - \frac{px^3}{18}\\
&\quad - \frac{a^6p\log\left(a+b\sqrt{x}\right)}{3b^6} + \frac{1}{3}x^3\log\left(c\left(a+b\sqrt{x}\right)^p\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.91

$$\int x^2 \log \left(c\left(a+b\sqrt{x}\right)^p\right)\,dx$$
$$=\frac{bp\sqrt{x}\left(60a^5-30a^4b\sqrt{x}+20a^3b^2x-15a^2b^3x^{3/2}+12ab^4x^2-10b^5x^{5/2}\right)-60a^6p\log\left(a+b\sqrt{x}\right)+60b^6x^3\log}{180b^6}$$

```
[In] Integrate[x^2*Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (b*p*Sqrt[x]*(60*a^5 - 30*a^4*b*Sqrt[x] + 20*a^3*b^2*x - 15*a^2*b^3*x^(3/2)
 + 12*a*b^4*x^2 - 10*b^5*x^(5/2)) - 60*a^6*p*Log[a + b*Sqrt[x]] + 60*b^6*x^
3*Log[c*(a + b*Sqrt[x])^p])/(180*b^6)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.80

method	result	size
parts	$\frac{x^3 \ln\left(c\left(a+b\sqrt{x}\right)^p\right)}{3}-\frac{pb\left(-\frac{2\left(-\frac{x^3 b^5}{6}+\frac{a\,x^{\frac{5}{2}} b^4}{5}-\frac{a^2 b^3 x^2}{4}+\frac{a^3 x^{\frac{3}{2}} b^2}{3}-\frac{b\,a^4 x}{2}+a^5\sqrt{x}\right)}{b^6}+\frac{2a^6\ln\left(a+b\sqrt{x}\right)}{b^7}\right)}{6}$	99

```
[In] int(x^2*ln(c*(a+b*x^(1/2))^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*ln(c*(a+b*x^(1/2))^p)-1/6*p*b*(-2/b^6*(-1/6*x^3*b^5+1/5*a*x^(5/2)*b
^4-1/4*a^2*b^3*x^2+1/3*a^3*x^(3/2)*b^2-1/2*b*a^4*x+a^5*x^(1/2))+2*a^6/b^7*l
n(a+b*x^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.85

$$\int x^2 \log \left(c\left(a+b\sqrt{x}\right)^p\right)\,dx =$$
$$-\frac{10\,b^6px^3-60\,b^6x^3\log\left(c\right)+15\,a^2b^4px^2+30\,a^4b^2px-60\,\left(b^6px^3-a^6p\right)\log\left(b\sqrt{x}+a\right)-4\,\left(3\,ab^5px^2+5\,a\right.}{180\,b^6}$$

```
[In] integrate(x^2*log(c*(a+b*x^(1/2))^p),x, algorithm="fricas")
```

```
[Out] -1/180*(10*b^6*p*x^3 - 60*b^6*x^3*log(c) + 15*a^2*b^4*p*x^2 + 30*a^4*b^2*p*
x - 60*(b^6*p*x^3 - a^6*p)*log(b*sqrt(x) + a) - 4*(3*a*b^5*p*x^2 + 5*a^3*b^
3*p*x + 15*a^5*b*p)*sqrt(x))/b^6
```

Sympy [A] (verification not implemented)

Time = 3.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.97

$$\int x^2 \log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx = -\frac{bp\left(\frac{2a^6\left(\begin{cases}\frac{\sqrt{x}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{x}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^6}-\frac{2a^5\sqrt{x}}{b^6}+\frac{a^4x}{b^5}-\frac{2a^3x^{\frac{3}{2}}}{3b^4}+\frac{a^2x^2}{2b^3}-\frac{2ax^{\frac{5}{2}}}{5b^2}+\frac{x^3}{3b}\right)}{6} + \frac{x^3\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{3}$$

```
[In] integrate(x**2*ln(c*(a+b*x**(1/2))**p),x)
```

```
[Out] -b*p*(2*a**6*Piecewise((sqrt(x)/a, Eq(b, 0)), (log(a + b*sqrt(x))/b, True))
/b**6 - 2*a**5*sqrt(x)/b**6 + a**4*x/b**5 - 2*a**3*x**(3/2)/(3*b**4) + a**2
*x**2/(2*b**3) - 2*a*x**(5/2)/(5*b**2) + x**3/(3*b))/6 + x**3*log(c*(a + b*
sqrt(x))**p)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int x^2 \log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx = \frac{1}{3}\,x^3\log\left(\left(b\sqrt{x}+a\right)^pc\right) - \frac{1}{180}\,bp\left(\frac{60\,a^6\log\left(b\sqrt{x}+a\right)}{b^7}+\frac{10\,b^5x^3-12\,ab^4x^{\frac{5}{2}}+15\,a^2b^3x^2-20\,a^3b^2x^{\frac{3}{2}}+30\,a^4bx-60\,a^5\sqrt{x}}{b^6}\right)$$

```
[In] integrate(x^2*log(c*(a+b*x^(1/2))^p),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((b*sqrt(x) + a)^p*c) - 1/180*b*p*(60*a^6*log(b*sqrt(x) + a)/b^7
 + (10*b^5*x^3 - 12*a*b^4*x^(5/2) + 15*a^2*b^3*x^2 - 20*a^3*b^2*x^(3/2) + 3
0*a^4*b*x - 60*a^5*sqrt(x))/b^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(97) = 194$.

Time = 0.28 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.07

$$\int x^2 \log \left(c\left(a+b\sqrt{x}\right)^p\right)\,dx$$
$$= \frac{60\,bx^3\log\left(c\right) + \left(\frac{60\,\left(b\sqrt{x}+a\right)^6\log\left(b\sqrt{x}+a\right)}{b^5} - \frac{360\,\left(b\sqrt{x}+a\right)^5 a\log\left(b\sqrt{x}+a\right)}{b^5} + \frac{900\,\left(b\sqrt{x}+a\right)^4 a^2\log\left(b\sqrt{x}+a\right)}{b^5} - \frac{1200\,\left(b\sqrt{x}+a\right)^3 a^3\log\left(b\right.}{b^5}}{}$$

```
[In] integrate(x^2*log(c*(a+b*x^(1/2))^p),x, algorithm="giac")
```

```
[Out] 1/180*(60*b*x^3*log(c) + (60*(b*sqrt(x) + a)^6*log(b*sqrt(x) + a)/b^5 - 360
*(b*sqrt(x) + a)^5*a*log(b*sqrt(x) + a)/b^5 + 900*(b*sqrt(x) + a)^4*a^2*log
(b*sqrt(x) + a)/b^5 - 1200*(b*sqrt(x) + a)^3*a^3*log(b*sqrt(x) + a)/b^5 + 9
00*(b*sqrt(x) + a)^2*a^4*log(b*sqrt(x) + a)/b^5 - 360*(b*sqrt(x) + a)*a^5*l
og(b*sqrt(x) + a)/b^5 - 10*(b*sqrt(x) + a)^6/b^5 + 72*(b*sqrt(x) + a)^5*a/b
^5 - 225*(b*sqrt(x) + a)^4*a^2/b^5 + 400*(b*sqrt(x) + a)^3*a^3/b^5 - 450*(b
*sqrt(x) + a)^2*a^4/b^5 + 360*(b*sqrt(x) + a)*a^5/b^5)*p)/b
```

Mupad [B] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.79

$$\int x^2 \log \left(c\left(a+b\sqrt{x}\right)^p\right)\,dx = \frac{x^3\,\ln\left(c\,\left(a+b\,\sqrt{x}\right)^p\right)}{3} - \frac{p\,x^3}{18} - \frac{a^6\,p\,\ln\left(a+b\,\sqrt{x}\right)}{3\,b^6} - \frac{a^2\,p\,x^2}{12\,b^2} + \frac{a^3\,p\,x^{3/2}}{9\,b^3} + \frac{a^5\,p\,\sqrt{x}}{3\,b^5} + \frac{a\,p\,x^{5/2}}{15\,b} - \frac{a^4\,p\,x}{6\,b^4}$$

```
[In] int(x^2*log(c*(a + b*x^(1/2))^p),x)
```

```
[Out] (x^3*log(c*(a + b*x^(1/2))^p))/3 - (p*x^3)/18 - (a^6*p*log(a + b*x^(1/2)))/
(3*b^6) - (a^2*p*x^2)/(12*b^2) + (a^3*p*x^(3/2))/(9*b^3) + (a^5*p*x^(1/2))/
(3*b^5) + (a*p*x^(5/2))/(15*b) - (a^4*p*x)/(6*b^4)
```

3.48 $\int x \log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 93

$$\int x \log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx = \frac{a^3p\sqrt{x}}{2b^3}-\frac{a^2px}{4b^2}+\frac{apx^{3/2}}{6b}-\frac{px^2}{8}-\frac{a^4p\log\left(a+b\sqrt{x}\right)}{2b^4}+\frac{1}{2}x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)$$

```
[Out] -1/4*a^2*p*x/b^2+1/6*a*p*x^(3/2)/b-1/8*p*x^2-1/2*a^4*p*ln(a+b*x^(1/2))/b^4+
1/2*x^2*ln(c*(a+b*x^(1/2))^p)+1/2*a^3*p*x^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2442, 45}

$$\int x \log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx = -\frac{a^4p\log\left(a+b\sqrt{x}\right)}{2b^4}+\frac{a^3p\sqrt{x}}{2b^3}-\frac{a^2px}{4b^2}+\frac{1}{2}x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)+\frac{apx^{3/2}}{6b}-\frac{px^2}{8}$$

```
[In] Int[x*Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (a^3*p*Sqrt[x])/(2*b^3) - (a^2*p*x)/(4*b^2) + (a*p*x^(3/2))/(6*b) - (p*x^2)
/8 - (a^4*p*Log[a + b*Sqrt[x]])/(2*b^4) + (x^2*Log[c*(a + b*Sqrt[x])^p])/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^3\log\left(c(a+bx)^p\right)\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{2}x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)-\frac{1}{2}(bp)\text{Subst}\left(\int\frac{x^4}{a+bx}\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{2}x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)-\frac{1}{2}(bp)\text{Subst}\left(\int\left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)\,dx,x,\sqrt{x}\right)\\
&= \frac{a^3p\sqrt{x}}{2b^3}-\frac{a^2px}{4b^2}+\frac{apx^{3/2}}{6b}-\frac{px^2}{8}-\frac{a^4p\log\left(a+b\sqrt{x}\right)}{2b^4}+\frac{1}{2}x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$
\int x\log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx
=\frac{bp\sqrt{x}\left(12a^3-6a^2b\sqrt{x}+4ab^2x-3b^3x^{3/2}\right)-12a^4p\log\left(a+b\sqrt{x}\right)+12b^4x^2\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{24b^4}
$$

```
[In] Integrate[x*Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (b*p*Sqrt[x]*(12*a^3 - 6*a^2*b*Sqrt[x] + 4*a*b^2*x - 3*b^3*x^(3/2)) - 12*a^
4*p*Log[a + b*Sqrt[x]] + 12*b^4*x^2*Log[c*(a + b*Sqrt[x])^p])/(24*b^4)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

method	result	size
parts	$\frac{x^{2} \ln\left(c \left(a+b \sqrt{x}\right)^{p}\right)}{2}-\frac{p b \left(-\frac{2 \left(-\frac{b^{3} x^{2}}{4}+\frac{a \,x^{\frac{3}{2}} b^{2}}{3}-\frac{a^{2} b x}{2}+a^{3} \sqrt{x}\right)}{b^{4}}+\frac{2 a^{4} \ln\left(a+b \sqrt{x}\right)}{b^{5}}\right)}{4}$	77

```
[In] int(x*ln(c*(a+b*x^(1/2))^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*ln(c*(a+b*x^(1/2))^p)-1/4*p*b*(-2/b^4*(-1/4*b^3*x^2+1/3*a*x^(3/2)*b
^2-1/2*a^2*b*x+a^3*x^(1/2))+2*a^4/b^5*ln(a+b*x^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int x \log\left(c\left(a+b\sqrt{x}\right)^{p}\right)\,dx = -\frac{3\,b^{4}px^{2}-12\,b^{4}x^{2}\log\left(c\right)+6\,a^{2}b^{2}px-12\,\left(b^{4}px^{2}-a^{4}p\right)\log\left(b\sqrt{x}+a\right)-4\,\left(ab^{3}px+3\,a^{3}bp\right)\sqrt{x}}{24\,b^{4}}$$

```
[In] integrate(x*log(c*(a+b*x^(1/2))^p),x, algorithm="fricas")
```

```
[Out] -1/24*(3*b^4*p*x^2 - 12*b^4*x^2*log(c) + 6*a^2*b^2*p*x - 12*(b^4*p*x^2 - a^
4*p)*log(b*sqrt(x) + a) - 4*(a*b^3*p*x + 3*a^3*b*p)*sqrt(x))/b^4
```

Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int x \log\left(c\left(a+b\sqrt{x}\right)^{p}\right)\,dx = -\frac{bp\left(\frac{2a^{4}\left(\begin{cases}\frac{\sqrt{x}}{a} & \text{for}\ b=0\\ \frac{\log\left(a+b\sqrt{x}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^{4}}-\frac{2a^{3}\sqrt{x}}{b^{4}}+\frac{a^{2}x}{b^{3}}-\frac{2ax^{\frac{3}{2}}}{3b^{2}}+\frac{x^{2}}{2b}\right)}{4}+\frac{x^{2}\log\left(c\left(a+b\sqrt{x}\right)^{p}\right)}{2}$$

```
[In] integrate(x*ln(c*(a+b*x**(1/2))**p),x)
```

```
[Out] -b*p*(2*a**4*Piecewise((sqrt(x)/a, Eq(b, 0)), (log(a + b*sqrt(x))/b, True))
/b**4 - 2*a**3*sqrt(x)/b**4 + a**2*x/b**3 - 2*a*x**(3/2)/(3*b**2) + x**2/(2
*b))/4 + x**2*log(c*(a + b*sqrt(x))**p)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.82

$$\int x\log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx$$
$$=-\frac{1}{24}\,bp\left(\frac{12\,a^4\log\left(b\sqrt{x}+a\right)}{b^5}+\frac{3\,b^3x^2-4\,ab^2x^{\frac{3}{2}}+6\,a^2bx-12\,a^3\sqrt{x}}{b^4}\right)$$
$$+\frac{1}{2}\,x^2\log\left(\left(b\sqrt{x}+a\right)^pc\right)$$

```
[In] integrate(x*log(c*(a+b*x^(1/2))^p),x, algorithm="maxima")
```

```
[Out] -1/24*b*p*(12*a^4*log(b*sqrt(x) + a)/b^5 + (3*b^3*x^2 - 4*a*b^2*x^(3/2) + 6
*a^2*b*x - 12*a^3*sqrt(x))/b^4) + 1/2*x^2*log((b*sqrt(x) + a)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(73) = 146.

Time = 0.35 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.84

$$\int x\log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx$$
$$=\frac{12\,bx^2\log\left(c\right)+\left(\frac{12\,\left(b\sqrt{x}+a\right)^4\log\left(b\sqrt{x}+a\right)}{b^3}-\frac{48\,\left(b\sqrt{x}+a\right)^3a\log\left(b\sqrt{x}+a\right)}{b^3}+\frac{72\,\left(b\sqrt{x}+a\right)^2a^2\log\left(b\sqrt{x}+a\right)}{b^3}-\frac{48\,\left(b\sqrt{x}+a\right)a^3\log\left(b\sqrt{x}+\right.}{b^3}}{24\,b}$$

```
[In] integrate(x*log(c*(a+b*x^(1/2))^p),x, algorithm="giac")
```

```
[Out] 1/24*(12*b*x^2*log(c) + (12*(b*sqrt(x) + a)^4*log(b*sqrt(x) + a)/b^3 - 48*(
b*sqrt(x) + a)^3*a*log(b*sqrt(x) + a)/b^3 + 72*(b*sqrt(x) + a)^2*a^2*log(b*
sqrt(x) + a)/b^3 - 48*(b*sqrt(x) + a)*a^3*log(b*sqrt(x) + a)/b^3 - 3*(b*sqr
t(x) + a)^4/b^3 + 16*(b*sqrt(x) + a)^3*a/b^3 - 36*(b*sqrt(x) + a)^2*a^2/b^3
 + 48*(b*sqrt(x) + a)*a^3/b^3)*p)/b
```

Mupad [B] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int x\log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx=\frac{x^2\,\ln\left(c\,\left(a+b\,\sqrt{x}\right)^p\right)}{2}-\frac{p\,x^2}{8}-\frac{a^4\,p\,\ln\left(a+b\,\sqrt{x}\right)}{2\,b^4}+\frac{a^3\,p\,\sqrt{x}}{2\,b^3}-\frac{a^2\,p\,x}{4\,b^2}+\frac{a\,p\,x^{3/2}}{6\,b}$$

```
[In] int(x*log(c*(a + b*x^(1/2))^p),x)
```

```
[Out] (x^2*log(c*(a + b*x^(1/2))^p))/2 - (p*x^2)/8 - (a^4*p*log(a + b*x^(1/2)))/(
2*b^4) + (a^3*p*x^(1/2))/(2*b^3) - (a^2*p*x)/(4*b^2) + (a*p*x^(3/2))/(6*b)
```

3.49 $\int \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 53

$$\int \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx=\frac{ap\sqrt{x}}{b}-\frac{px}{2}-\frac{a^2p\log \left(a+b\sqrt{x}\right)}{b^2}+x\log \left(c\left(a+b\sqrt{x}\right)^p\right)$$

```
[Out] -1/2*p*x-a^2*p*ln(a+b*x^(1/2))/b^2+x*ln(c*(a+b*x^(1/2))^p)+a*p*x^(1/2)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2498, 272, 45}

$$\int \log \left(c\left(a+b\sqrt{x}\right)^p\right) \, dx=-\frac{a^2p\log \left(a+b\sqrt{x}\right)}{b^2}+x\log \left(c\left(a+b\sqrt{x}\right)^p\right)+\frac{ap\sqrt{x}}{b}-\frac{px}{2}$$

```
[In] Int[Log[c*(a + b*Sqrt[x])^p],x]
```

```
[Out] (a*p*Sqrt[x])/b - (p*x)/2 - (a^2*p*Log[a + b*Sqrt[x]])/b^2 + x*Log[c*(a + b
*Sqrt[x])^p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log\left(c\left(a+b\sqrt{x}\right)^p\right)-\frac{1}{2}(bp)\int\frac{\sqrt{x}}{a+b\sqrt{x}}\,dx\\
&= x\log\left(c\left(a+b\sqrt{x}\right)^p\right)-(bp)\text{Subst}\left(\int\frac{x^2}{a+bx}\,dx,x,\sqrt{x}\right)\\
&= x\log\left(c\left(a+b\sqrt{x}\right)^p\right)-(bp)\text{Subst}\left(\int\left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)\,dx,x,\sqrt{x}\right)\\
&= \frac{ap\sqrt{x}}{b}-\frac{px}{2}-\frac{a^2p\log\left(a+b\sqrt{x}\right)}{b^2}+x\log\left(c\left(a+b\sqrt{x}\right)^p\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int\log\left(c\left(a+b\sqrt{x}\right)^p\right)\,dx=\frac{ap\sqrt{x}}{b}-\frac{px}{2}-\frac{a^2p\log\left(a+b\sqrt{x}\right)}{b^2}+x\log\left(c\left(a+b\sqrt{x}\right)^p\right)$$

```
[In] Integrate[Log[c*(a + b*Sqrt[x])^p],x]
[Out] (a*p*Sqrt[x])/b - (p*x)/2 - (a^2*p*Log[a + b*Sqrt[x]])/b^2 + x*Log[c*(a + b
*Sqrt[x])^p]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

method	result	size
default	$x\ln\left(c\left(a+b\sqrt{x}\right)^{p}\right)-\frac{pb\left(-\frac{2\left(-\frac{bx}{2}+a\sqrt{x}\right)}{b^{2}}+\frac{2a^{2}\ln\left(a+b\sqrt{x}\right)}{b^{3}}\right)}{2}$	52
parts	$x\ln\left(c\left(a+b\sqrt{x}\right)^{p}\right)-\frac{pb\left(-\frac{2\left(-\frac{bx}{2}+a\sqrt{x}\right)}{b^{2}}+\frac{2a^{2}\ln\left(a+b\sqrt{x}\right)}{b^{3}}\right)}{2}$	52

```
[In] int(ln(c*(a+b*x^(1/2))^p),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(c*(a+b*x^(1/2))^p)-1/2*p*b*(-2/b^2*(-1/2*b*x+a*x^(1/2))+2*a^2/b^3*ln(a
+b*x^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int\log\left(c\left(a+b\sqrt{x}\right)^{p}\right)\,dx=-\frac{b^{2}px-2\,b^{2}x\log\left(c\right)-2\,abp\sqrt{x}-2\,\left(b^{2}px-a^{2}p\right)\log\left(b\sqrt{x}+a\right)}{2\,b^{2}}$$

```
[In] integrate(log(c*(a+b*x^(1/2))^p),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*p*x - 2*b^2*x*log(c) - 2*a*b*p*sqrt(x) - 2*(b^2*p*x - a^2*p)*log(
b*sqrt(x) + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int\log\left(c\left(a+b\sqrt{x}\right)^{p}\right)\,dx=-\frac{bp\left(\frac{2a^{2}\left(\begin{cases}\frac{\sqrt{x}}{a} & \text{for}\ b=0\\ \frac{\log\left(a+b\sqrt{x}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^{2}}-\frac{2a\sqrt{x}}{b^{2}}+\frac{x}{b}\right)}{2}+x\log\left(c\left(a+b\sqrt{x}\right)^{p}\right)$$

```
[In] integrate(ln(c*(a+b*x**(1/2))**p),x)
```

```
[Out] -b*p*(2*a**2*Piecewise((sqrt(x)/a, Eq(b, 0)), (log(a + b*sqrt(x))/b, True))
/b**2 - 2*a*sqrt(x)/b**2 + x/b)/2 + x*log(c*(a + b*sqrt(x))**p)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \log \left(c\left(a+b\sqrt{x}\right)^{p}\right)\, dx = -\frac{1}{2}\, bp\left(\frac{2\, a^{2} \log \left(b\sqrt{x}+a\right)}{b^{3}} + \frac{bx - 2\, a\sqrt{x}}{b^{2}}\right) + x \log \left(\left(b\sqrt{x}+a\right)^{p} c\right)$$

```
[In] integrate(log(c*(a+b*x^(1/2))^p),x, algorithm="maxima")
```

```
[Out] -1/2*b*p*(2*a^2*log(b*sqrt(x) + a)/b^3 + (b*x - 2*a*sqrt(x))/b^2) + x*log((
b*sqrt(x) + a)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(45) = 90$.

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.83

$$\int \log \left(c\left(a+b\sqrt{x}\right)^{p}\right)\, dx = \frac{\frac{\left(2\, \left(b\sqrt{x}+a\right)^{2} \log \left(b\sqrt{x}+a\right) - 4\, \left(b\sqrt{x}+a\right) a \log \left(b\sqrt{x}+a\right) - \left(b\sqrt{x}+a\right)^{2} + 4\, \left(b\sqrt{x}+a\right) a\right) p}{b} + \frac{2\, \left(\left(b\sqrt{x}+a\right)^{2} - 2\, \left(b\sqrt{x}+a\right) a\right) \log \left(c\right)}{b}}{2\, b}$$

```
[In] integrate(log(c*(a+b*x^(1/2))^p),x, algorithm="giac")
```

```
[Out] 1/2*((2*(b*sqrt(x) + a)^2*log(b*sqrt(x) + a) - 4*(b*sqrt(x) + a)*a*log(b*sq
rt(x) + a) - (b*sqrt(x) + a)^2 + 4*(b*sqrt(x) + a)*a)*p/b + 2*((b*sqrt(x) +
 a)^2 - 2*(b*sqrt(x) + a)*a)*log(c)/b)/b
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \log \left(c\left(a+b\sqrt{x}\right)^{p}\right)\, dx = x\, \ln \left(c\, \left(a+b\, \sqrt{x}\right)^{p}\right) - \frac{p\, \left(b^{2}\, x + 2\, a^{2}\, \ln \left(a+b\, \sqrt{x}\right) - 2\, a\, b\, \sqrt{x}\right)}{2\, b^{2}}$$

```
[In] int(log(c*(a + b*x^(1/2))^p),x)
```

```
[Out] x*log(c*(a + b*x^(1/2))^p) - (p*(b^2*x + 2*a^2*log(a + b*x^(1/2)) - 2*a*b*x
^(1/2)))/(2*b^2)
```

3.50 $\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 46

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx = 2\log\left(c\left(a+b\sqrt{x}\right)^p\right)\log\left(-\frac{b\sqrt{x}}{a}\right)+2p\,\mathrm{PolyLog}\left(2,1+\frac{b\sqrt{x}}{a}\right)$$

```
[Out] 2*ln(-b*x^(1/2)/a)*ln(c*(a+b*x^(1/2))^p)+2*p*polylog(2,1+b*x^(1/2)/a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2504, 2441, 2352}

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx = 2\log\left(-\frac{b\sqrt{x}}{a}\right)\log\left(c\left(a+b\sqrt{x}\right)^p\right)+2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{x}b}{a}+1\right)$$

```
[In] Int[Log[c*(a + b*Sqrt[x])^p]/x,x]
```

```
[Out] 2*Log[c*(a + b*Sqrt[x])^p]*Log[-((b*Sqrt[x])/a)] + 2*p*PolyLog[2, 1 + (b*Sq
rt[x])/a]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x}\,dx, x, \sqrt{x}\right)\\
&= 2\log\left(c\left(a+b\sqrt{x}\right)^p\right)\log\left(-\frac{b\sqrt{x}}{a}\right) - (2bp)\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx, x, \sqrt{x}\right)\\
&= 2\log\left(c\left(a+b\sqrt{x}\right)^p\right)\log\left(-\frac{b\sqrt{x}}{a}\right) + 2p\text{Li}_2\left(1+\frac{b\sqrt{x}}{a}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx = 2\log\left(c\left(a+b\sqrt{x}\right)^p\right)\log\left(-\frac{b\sqrt{x}}{a}\right) + 2p\,\text{PolyLog}\left(2, \frac{a+b\sqrt{x}}{a}\right)$$

```
[In] Integrate[Log[c*(a + b*Sqrt[x])^p]/x,x]
[Out] 2*Log[c*(a + b*Sqrt[x])^p]*Log[-((b*Sqrt[x])/a)] + 2*p*PolyLog[2, (a + b*Sq
rt[x])/a]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

method	result	size
parts	$\ln\left(c\left(a+b\sqrt{x}\right)^p\right)\ln\left(x\right) - \frac{pb\left(\frac{4\,\text{dilog}\left(\frac{a+b\sqrt{x}}{a}\right)}{b} + \frac{2\ln\left(x\right)\ln\left(\frac{a+b\sqrt{x}}{a}\right)}{b}\right)}{2}$	58

```
[In] int(ln(c*(a+b*x^(1/2))^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(c*(a+b*x^(1/2))^p)*ln(x)-1/2*p*b*(4*dilog((a+b*x^(1/2))/a)/b+2*ln(x)*ln(
(a+b*x^(1/2))/a)/b)
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx = \int \frac{\log\left(\left(b\sqrt{x}+a\right)^p c\right)}{x}\,dx$$

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x,x, algorithm="fricas")
[Out] integral(log((b*sqrt(x) + a)^p*c)/x, x)
```

Sympy [F]

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx = \int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx$$

```
[In] integrate(ln(c*(a+b*x**(1/2))**p)/x,x)
[Out] Integral(log(c*(a + b*sqrt(x))**p)/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(39) = 78$.

Time $= 0.20$ (sec) , antiderivative size $= 79$, normalized size of antiderivative $= 1.72$

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx = bp\left(\frac{\log\left(b\sqrt{x}+a\right)\log\left(x\right)}{b} - \frac{\log\left(x\right)\log\left(\frac{b\sqrt{x}}{a}+1\right)+2\,\mathrm{Li}_2\left(-\frac{b\sqrt{x}}{a}\right)}{b}\right) - p\log\left(b\sqrt{x}+a\right)\log\left(x\right) + \log\left(\left(b\sqrt{x}+a\right)^p c\right)\log\left(x\right)$$

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x,x, algorithm="maxima")
[Out] b*p*(log(b*sqrt(x) + a)*log(x)/b - (log(x)*log(b*sqrt(x)/a + 1) + 2*dilog(-
b*sqrt(x)/a))/b) - p*log(b*sqrt(x) + a)*log(x) + log((b*sqrt(x) + a)^p*c)*l
og(x)
```

Giac [F]

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx = \int \frac{\log\left(\left(b\sqrt{x}+a\right)^p c\right)}{x}\,dx$$

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x,x, algorithm="giac")
[Out] integrate(log((b*sqrt(x) + a)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx = \int \frac{\ln\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}\,dx$$

```
[In] int(log(c*(a + b*x^(1/2))^p)/x,x)
[Out] int(log(c*(a + b*x^(1/2))^p)/x, x)
```

3.51 $\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 63

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2}\,dx=-\frac{bp}{a\sqrt{x}}+\frac{b^2p\log\left(a+b\sqrt{x}\right)}{a^2}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}-\frac{b^2p\log(x)}{2a^2}$$

```
[Out] -1/2*b^2*p*ln(x)/a^2+b^2*p*ln(a+b*x^(1/2))/a^2-ln(c*(a+b*x^(1/2))^p)/x-b*p/
a/x^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2504, 2442, 46}

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2}\,dx=\frac{b^2p\log\left(a+b\sqrt{x}\right)}{a^2}-\frac{b^2p\log(x)}{2a^2}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x}-\frac{bp}{a\sqrt{x}}$$

```
[In] Int[Log[c*(a + b*Sqrt[x])^p]/x^2,x]
```

```
[Out] -((b*p)/(a*Sqrt[x])) + (b^2*p*Log[a + b*Sqrt[x]])/a^2 - Log[c*(a + b*Sqrt[x
])^p]/x - (b^2*p*Log[x])/(2*a^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^3}\,dx, x, \sqrt{x}\right)\\
&= -\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x} + (bp)\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx, x, \sqrt{x}\right)\\
&= -\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x} + (bp)\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx, x, \sqrt{x}\right)\\
&= -\frac{bp}{a\sqrt{x}} + \frac{b^2p\log\left(a+b\sqrt{x}\right)}{a^2} - \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x} - \frac{b^2p\log(x)}{2a^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$
\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2}\,dx = -\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x} - \frac{bp\left(\frac{2a}{\sqrt{x}}-2b\log\left(a+b\sqrt{x}\right)+b\log(x)\right)}{2a^2}
$$

```
[In] Integrate[Log[c*(a + b*Sqrt[x])^p]/x^2,x]
[Out] -(Log[c*(a + b*Sqrt[x])^p]/x) - (b*p*((2*a)/Sqrt[x] - 2*b*Log[a + b*Sqrt[x]
] + b*Log[x]))/(2*a^2)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

method	result	size
parts	$-\frac{\ln\left(c\left(a+b\sqrt{x}\right)^{p}\right)}{x}+\frac{pb\left(-\frac{2}{a\sqrt{x}}-\frac{b\ln(x)}{a^{2}}+\frac{2b\ln\left(a+b\sqrt{x}\right)}{a^{2}}\right)}{2}$	54

```
[In] int(ln(c*(a+b*x^(1/2))^p)/x^2,x,method=_RETURNVERBOSE)
[Out] -ln(c*(a+b*x^(1/2))^p)/x+1/2*p*b*(-2/a/x^(1/2)-1/a^2*b*ln(x)+2/a^2*b*ln(a+b
*x^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^{p}\right)}{x^{2}}\,dx = -\frac{b^{2}px\log\left(\sqrt{x}\right)+abp\sqrt{x}+a^{2}\log\left(c\right)-\left(b^{2}px-a^{2}p\right)\log\left(b\sqrt{x}+a\right)}{a^{2}x}$$

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^2,x, algorithm="fricas")
[Out] -(b^2*p*x*log(sqrt(x)) + a*b*p*sqrt(x) + a^2*log(c) - (b^2*p*x - a^2*p)*log
(b*sqrt(x) + a))/(a^2*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(56) = 112.

Time = 11.00 (sec) , antiderivative size = 352, normalized size of antiderivative = 5.59

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^{p}\right)}{x^{2}}\,dx = \begin{cases} -\frac{\log\left(0^{p}c\right)}{x} \\ -\frac{p}{2x}-\frac{\log\left(c\left(b\sqrt{x}\right)^{p}\right)}{x} \\ -\frac{\log\left(0^{p}c\right)}{x} \\ -\frac{2a^{3}\sqrt{x}\log\left(c\left(a+b\sqrt{x}\right)^{p}\right)}{2a^{3}x^{\frac{3}{2}}+2a^{2}bx^{2}}-\frac{2a^{2}bpx}{2a^{3}x^{\frac{3}{2}}+2a^{2}bx^{2}}-\frac{2a^{2}bx\log\left(c\left(a+b\sqrt{x}\right)^{p}\right)}{2a^{3}x^{\frac{3}{2}}+2a^{2}bx^{2}}-\frac{ab^{2}px^{\frac{3}{2}}\log\left(x\right)}{2a^{3}x^{\frac{3}{2}}+2a^{2}bx^{2}}-\frac{2ab^{2}px^{\frac{3}{2}}}{2a^{3}x^{\frac{3}{2}}+2a^{2}bx^{2}}+\frac{2ab^{2}x^{\frac{3}{2}}\log\left(c\left(a+b\sqrt{x}\right)\right.}{2a^{3}x^{\frac{3}{2}}+2a^{2}bx^{2}} \end{cases}$$

```
[In] integrate(ln(c*(a+b*x**(1/2))**p)/x**2,x)
```

```
[Out] Piecewise((-log(0**p*c)/x, Eq(a, 0) & Eq(b, 0)), (-p/(2*x) - log(c*(b*sqrt(
x))**p)/x, Eq(a, 0)), (-log(0**p*c)/x, Eq(a, -b*sqrt(x))), (-2*a**3*sqrt(x)
*log(c*(a + b*sqrt(x))**p)/(2*a**3*x**(3/2) + 2*a**2*b*x**2) - 2*a**2*b*p*x
/(2*a**3*x**(3/2) + 2*a**2*b*x**2) - 2*a**2*b*x*log(c*(a + b*sqrt(x))**p)/(
2*a**3*x**(3/2) + 2*a**2*b*x**2) - a*b**2*p*x**(3/2)*log(x)/(2*a**3*x**(3/2
) + 2*a**2*b*x**2) - 2*a*b**2*p*x**(3/2)/(2*a**3*x**(3/2) + 2*a**2*b*x**2)
+ 2*a*b**2*x**(3/2)*log(c*(a + b*sqrt(x))**p)/(2*a**3*x**(3/2) + 2*a**2*b*x
**2) - b**3*p*x**2*log(x)/(2*a**3*x**(3/2) + 2*a**2*b*x**2) + 2*b**3*x**2*l
og(c*(a + b*sqrt(x))**p)/(2*a**3*x**(3/2) + 2*a**2*b*x**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2}\,dx = \frac{1}{2}\,bp\left(\frac{2\,b\log\left(b\sqrt{x}+a\right)}{a^2}-\frac{b\log\left(x\right)}{a^2}-\frac{2}{a\sqrt{x}}\right)-\frac{\log\left(\left(b\sqrt{x}+a\right)^p c\right)}{x}$$

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(2*b*log(b*sqrt(x) + a)/a^2 - b*log(x)/a^2 - 2/(a*sqrt(x))) - log((
b*sqrt(x) + a)^p*c)/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(55) = 110$.

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.10

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2}\,dx = -\frac{\frac{b^3p\log\left(b\sqrt{x}+a\right)}{\left(b\sqrt{x}+a\right)^2-2\left(b\sqrt{x}+a\right)a+a^2}-\frac{b^3p\log\left(b\sqrt{x}+a\right)}{a^2}+\frac{b^3p\log\left(b\sqrt{x}\right)}{a^2}+\frac{\left(b\sqrt{x}+a\right)b^3p-ab^3p+ab^3\log\left(c\right)}{\left(b\sqrt{x}+a\right)^2a-2\left(b\sqrt{x}+a\right)a^2+a^3}}{b}$$

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^2,x, algorithm="giac")
```

```
[Out] -(b^3*p*log(b*sqrt(x) + a)/((b*sqrt(x) + a)^2 - 2*(b*sqrt(x) + a)*a + a^2)
- b^3*p*log(b*sqrt(x) + a)/a^2 + b^3*p*log(b*sqrt(x))/a^2 + ((b*sqrt(x) + a
)*b^3*p - a*b^3*p + a*b^3*log(c))/((b*sqrt(x) + a)^2*a - 2*(b*sqrt(x) + a)*
a^2 + a^3))/b
```

Mupad [B] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^2}\,dx = \frac{2\,b^2\,p\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^2} - \frac{\ln\left(c\,\left(a+b\,\sqrt{x}\right)^p\right)}{x} - \frac{b\,p}{a\,\sqrt{x}}$$

```
[In] int(log(c*(a + b*x^(1/2))^p)/x^2,x)
```

```
[Out] (2*b^2*p*atanh((2*b*x^(1/2))/a + 1))/a^2 - log(c*(a + b*x^(1/2))^p)/x - (b*
p)/(a*x^(1/2))
```

3.52 $\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 100

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3}\,dx = -\frac{bp}{6ax^{3/2}}+\frac{b^2p}{4a^2x}-\frac{b^3p}{2a^3\sqrt{x}}+\frac{b^4p\log\left(a+b\sqrt{x}\right)}{2a^4} -\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2x^2}-\frac{b^4p\log(x)}{4a^4}$$

```
[Out] -1/6*b*p/a/x^(3/2)+1/4*b^2*p/a^2/x-1/4*b^4*p*ln(x)/a^4+1/2*b^4*p*ln(a+b*x^(
1/2))/a^4-1/2*ln(c*(a+b*x^(1/2))^p)/x^2-1/2*b^3*p/a^3/x^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2504, 2442, 46}

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3}\,dx = \frac{b^4p\log\left(a+b\sqrt{x}\right)}{2a^4}-\frac{b^4p\log(x)}{4a^4}-\frac{b^3p}{2a^3\sqrt{x}} +\frac{b^2p}{4a^2x}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2x^2}-\frac{bp}{6ax^{3/2}}$$

```
[In] Int[Log[c*(a + b*Sqrt[x])^p]/x^3,x]
```

```
[Out] -1/6*(b*p)/(a*x^(3/2)) + (b^2*p)/(4*a^2*x) - (b^3*p)/(2*a^3*Sqrt[x]) + (b^4
*p*Log[a + b*Sqrt[x]])/(2*a^4) - Log[c*(a + b*Sqrt[x])^p]/(2*x^2) - (b^4*p*
Log[x])/(4*a^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^5}\,dx, x, \sqrt{x}\right)\\
&= -\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2x^2} + \frac{1}{2}(bp)\text{Subst}\left(\int \frac{1}{x^4(a+bx)}\,dx, x, \sqrt{x}\right)\\
&= -\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2x^2}\\
&\quad + \frac{1}{2}(bp)\text{Subst}\left(\int \left(\frac{1}{ax^4} - \frac{b}{a^2x^3} + \frac{b^2}{a^3x^2} - \frac{b^3}{a^4x} + \frac{b^4}{a^4(a+bx)}\right)\,dx, x, \sqrt{x}\right)\\
&= -\frac{bp}{6ax^{3/2}} + \frac{b^2p}{4a^2x} - \frac{b^3p}{2a^3\sqrt{x}} + \frac{b^4p\log\left(a+b\sqrt{x}\right)}{2a^4} - \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{2x^2} - \frac{b^4p\log(x)}{4a^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3}\,dx = -\frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{2x^2} + \frac{1}{4}bp\left(-\frac{2}{3ax^{3/2}}+\frac{b}{a^2x}-\frac{2b^2}{a^3\sqrt{x}}+\frac{2b^3\log\left(a+b\sqrt{x}\right)}{a^4}-\frac{b^3\log(x)}{a^4}\right)$$

```
[In] Integrate[Log[c*(a + b*Sqrt[x])^p]/x^3,x]
```

```
[Out] -1/2*Log[c*(a + b*Sqrt[x])^p]/x^2 + (b*p*(-2/(3*a*x^(3/2)) + b/(a^2*x) - (2
*b^2)/(a^3*Sqrt[x]) + (2*b^3*Log[a + b*Sqrt[x]])/a^4 - (b^3*Log[x])/a^4))/4
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.77

method	result	size
parts	$-\frac{\ln\left(c\left(a+b\sqrt{x}\right)^p\right)}{2x^2}+\frac{pb\left(\frac{2b^3\ln\left(a+b\sqrt{x}\right)}{a^4}-\frac{2}{3a\,x^{\frac{3}{2}}}-\frac{2b^2}{a^3\sqrt{x}}+\frac{b}{a^2x}-\frac{b^3\ln(x)}{a^4}\right)}{4}$	77

```
[In] int(ln(c*(a+b*x^(1/2))^p)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(c*(a+b*x^(1/2))^p)/x^2+1/4*p*b*(2*b^3/a^4*ln(a+b*x^(1/2))-2/3/a/x^(
3/2)-2*b^2/a^3/x^(1/2)+b/a^2/x-b^3/a^4*ln(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.84

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3}\,dx = -\frac{6\,b^4px^2\log\left(\sqrt{x}\right)-3\,a^2b^2px+6\,a^4\log\left(c\right)-6\left(b^4px^2-a^4p\right)\log\left(b\sqrt{x}+a\right)+2\left(3\,ab^3px+a^3bp\right)\sqrt{x}}{12\,a^4x^2}$$

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^3,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^4*p*x^2*log(sqrt(x)) - 3*a^2*b^2*p*x + 6*a^4*log(c) - 6*(b^4*p*x
^2 - a^4*p)*log(b*sqrt(x) + a) + 2*(3*a*b^3*p*x + a^3*b*p)*sqrt(x))/(a^4*x^
2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(90) = 180$.

Time = 90.87 (sec) , antiderivative size = 435, normalized size of antiderivative = 4.35

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3}\,dx = \begin{cases} -\frac{\log(0^p c)}{2x^2} \\ -\frac{p}{8x^2} - \frac{\log\left(c\left(b\sqrt{x}\right)^p\right)}{2x^2} \\ -\frac{\log(0^p c)}{2x^2} \\ -\frac{6a^5\sqrt{x}\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{12a^5x^{\frac{5}{2}}+12a^4bx^3} - \frac{2a^4bpx}{12a^5x^{\frac{5}{2}}+12a^4bx^3} - \frac{6a^4bx\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{12a^5x^{\frac{5}{2}}+12a^4bx^3} + \frac{a^3b^2px^{\frac{3}{2}}}{12a^5x^{\frac{5}{2}}+12a^4bx^3} - \frac{3a^2b^3px^2}{12a^5x^{\frac{5}{2}}+12a^4bx^3} - \frac{3ab^4px^{\frac{5}{2}}\log(x)}{12a^5x^{\frac{5}{2}}+12a^4bx^3} \end{cases}$$

```
[In] integrate(ln(c*(a+b*x**(1/2))**p)/x**3,x)
```

```
[Out] Piecewise((-log(0**p*c)/(2*x**2), Eq(a, 0) & Eq(b, 0)), (-p/(8*x**2) - log(
c*(b*sqrt(x))**p)/(2*x**2), Eq(a, 0)), (-log(0**p*c)/(2*x**2), Eq(a, -b*sqr
t(x))), (-6*a**5*sqrt(x)*log(c*(a + b*sqrt(x))**p)/(12*a**5*x**(5/2) + 12*a
**4*b*x**3) - 2*a**4*b*p*x/(12*a**5*x**(5/2) + 12*a**4*b*x**3) - 6*a**4*b*x
*log(c*(a + b*sqrt(x))**p)/(12*a**5*x**(5/2) + 12*a**4*b*x**3) + a**3*b**2*
p*x**(3/2)/(12*a**5*x**(5/2) + 12*a**4*b*x**3) - 3*a**2*b**3*p*x**2/(12*a**
5*x**(5/2) + 12*a**4*b*x**3) - 3*a*b**4*p*x**(5/2)*log(x)/(12*a**5*x**(5/2)
 + 12*a**4*b*x**3) - 6*a*b**4*p*x**(5/2)/(12*a**5*x**(5/2) + 12*a**4*b*x**3
) + 6*a*b**4*x**(5/2)*log(c*(a + b*sqrt(x))**p)/(12*a**5*x**(5/2) + 12*a**4
*b*x**3) - 3*b**5*p*x**3*log(x)/(12*a**5*x**(5/2) + 12*a**4*b*x**3) + 6*b**
5*x**3*log(c*(a + b*sqrt(x))**p)/(12*a**5*x**(5/2) + 12*a**4*b*x**3), True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.76

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3}\,dx = \frac{1}{12}\,bp\left(\frac{6\,b^3\log\left(b\sqrt{x}+a\right)}{a^4} - \frac{3\,b^3\log(x)}{a^4} - \frac{6\,b^2x - 3\,ab\sqrt{x} + 2\,a^2}{a^3x^{\frac{3}{2}}}\right) - \frac{\log\left(\left(b\sqrt{x}+a\right)^pc\right)}{2\,x^2}$$

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*b*p*(6*b^3*log(b*sqrt(x) + a)/a^4 - 3*b^3*log(x)/a^4 - (6*b^2*x - 3*a*
b*sqrt(x) + 2*a^2)/(a^3*x^(3/2))) - 1/2*log((b*sqrt(x) + a)^p*c)/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(80) = 160$.

Time = 0.30 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.32

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3}\,dx = -\frac{\frac{6\,b^5p\log\left(b\sqrt{x}+a\right)}{\left(b\sqrt{x}+a\right)^4-4\,\left(b\sqrt{x}+a\right)^3a+6\,\left(b\sqrt{x}+a\right)^2a^2-4\,\left(b\sqrt{x}+a\right)a^3+a^4}-\frac{6\,b^5p\log\left(b\sqrt{x}+a\right)}{a^4}+\frac{6\,b^5p\log\left(b\sqrt{x}\right)}{a^4}+\frac{6\,\left(b\sqrt{x}+a\right)^3b^5p-21\,\left(b\sqrt{x}+a\right)^2a}{\left(b\sqrt{x}+a\right)^4a^3-4\,\left(b\sqrt{x}+a\right.}}{12\,b}$$

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^3,x, algorithm="giac")
```

```
[Out] -1/12*(6*b^5*p*log(b*sqrt(x) + a)/((b*sqrt(x) + a)^4 - 4*(b*sqrt(x) + a)^3*
a + 6*(b*sqrt(x) + a)^2*a^2 - 4*(b*sqrt(x) + a)*a^3 + a^4) - 6*b^5*p*log(b*
sqrt(x) + a)/a^4 + 6*b^5*p*log(b*sqrt(x))/a^4 + (6*(b*sqrt(x) + a)^3*b^5*p
- 21*(b*sqrt(x) + a)^2*a*b^5*p + 26*(b*sqrt(x) + a)*a^2*b^5*p - 11*a^3*b^5*
p + 6*a^3*b^5*log(c))/((b*sqrt(x) + a)^4*a^3 - 4*(b*sqrt(x) + a)^3*a^4 + 6*
(b*sqrt(x) + a)^2*a^5 - 4*(b*sqrt(x) + a)*a^6 + a^7))/b
```

Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.72

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^3}\,dx = \frac{b^4\,p\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{a^4}-\frac{\ln\left(c\,{\left(a+b\,\sqrt{x}\right)}^p\right)}{2\,x^2}-\frac{\frac{b\,p}{3\,a}-\frac{b^2\,p\,\sqrt{x}}{2\,a^2}+\frac{b^3\,p\,x}{a^3}}{2\,x^{3/2}}$$

```
[In] int(log(c*(a + b*x^(1/2))^p)/x^3,x)
```

```
[Out] (b^4*p*atanh((2*b*x^(1/2))/a + 1))/a^4 - log(c*(a + b*x^(1/2))^p)/(2*x^2) -
 ((b*p)/(3*a) - (b^2*p*x^(1/2))/(2*a^2) + (b^3*p*x)/a^3)/(2*x^(3/2))
```

3.53 $\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 130

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^4}\,dx = -\frac{bp}{15ax^{5/2}}+\frac{b^2p}{12a^2x^2}-\frac{b^3p}{9a^3x^{3/2}}+\frac{b^4p}{6a^4x}-\frac{b^5p}{3a^5\sqrt{x}}+\frac{b^6p\log\left(a+b\sqrt{x}\right)}{3a^6}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{3x^3}-\frac{b^6p\log(x)}{6a^6}$$

```
[Out] -1/15*b*p/a/x^(5/2)+1/12*b^2*p/a^2/x^2-1/9*b^3*p/a^3/x^(3/2)+1/6*b^4*p/a^4/
x-1/6*b^6*p*ln(x)/a^6+1/3*b^6*p*ln(a+b*x^(1/2))/a^6-1/3*ln(c*(a+b*x^(1/2))^
p)/x^3-1/3*b^5*p/a^5/x^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2504, 2442, 46}

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^4}\,dx = \frac{b^6p\log\left(a+b\sqrt{x}\right)}{3a^6}-\frac{b^6p\log(x)}{6a^6}-\frac{b^5p}{3a^5\sqrt{x}}+\frac{b^4p}{6a^4x}-\frac{b^3p}{9a^3x^{3/2}}+\frac{b^2p}{12a^2x^2}-\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{3x^3}-\frac{bp}{15ax^{5/2}}$$

```
[In] Int[Log[c*(a + b*Sqrt[x])^p]/x^4,x]
```

```
[Out] -1/15*(b*p)/(a*x^(5/2)) + (b^2*p)/(12*a^2*x^2) - (b^3*p)/(9*a^3*x^(3/2)) +
(b^4*p)/(6*a^4*x) - (b^5*p)/(3*a^5*Sqrt[x]) + (b^6*p*Log[a + b*Sqrt[x]])/(3
*a^6) - Log[c*(a + b*Sqrt[x])^p]/(3*x^3) - (b^6*p*Log[x])/(6*a^6)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^7}\,dx, x, \sqrt{x}\right) \\
&= -\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{3x^3} + \frac{1}{3}(bp)\text{Subst}\left(\int \frac{1}{x^6(a+bx)}\,dx, x, \sqrt{x}\right) \\
&= -\frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{3x^3} + \frac{1}{3}(bp)\text{Subst}\left(\int \left(\frac{1}{ax^6} - \frac{b}{a^2x^5} + \frac{b^2}{a^3x^4} - \frac{b^3}{a^4x^3} + \frac{b^4}{a^5x^2}\right.\right. \\
&\qquad \left.\left. - \frac{b^5}{a^6x} + \frac{b^6}{a^6(a+bx)}\right)\,dx, x, \sqrt{x}\right) \\
&= -\frac{bp}{15ax^{5/2}} + \frac{b^2p}{12a^2x^2} - \frac{b^3p}{9a^3x^{3/2}} + \frac{b^4p}{6a^4x} - \frac{b^5p}{3a^5\sqrt{x}} \\
&\quad + \frac{b^6p\log\left(a+b\sqrt{x}\right)}{3a^6} - \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{3x^3} - \frac{b^6p\log(x)}{6a^6}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.88

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^4}\,dx$$
$$= \frac{abp\sqrt{x}\left(-12a^4+15a^3b\sqrt{x}-20a^2b^2x+30ab^3x^{3/2}-60b^4x^2\right)+60b^6px^3\log\left(a+b\sqrt{x}\right)-60a^6\log\left(c\left(a+b\sqrt{\right.\right.}}{180a^6x^3}$$

```
[In] Integrate[Log[c*(a + b*Sqrt[x])^p]/x^4,x]
```

```
[Out] (a*b*p*Sqrt[x]*(-12*a^4 + 15*a^3*b*Sqrt[x] - 20*a^2*b^2*x + 30*a*b^3*x^(3/2
) - 60*b^4*x^2) + 60*b^6*p*x^3*Log[a + b*Sqrt[x]] - 60*a^6*Log[c*(a + b*Sqr
t[x])^p] - 30*b^6*p*x^3*Log[x])/(180*a^6*x^3)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76

method	result	size
parts	$-\frac{\ln\left(c\left(a+b\sqrt{x}\right)^{p}\right)}{3x^{3}}+\frac{pb\left(\frac{2b^{5}\ln\left(a+b\sqrt{x}\right)}{a^{6}}-\frac{2}{5a\,x^{\frac{5}{2}}}-\frac{2b^{4}}{a^{5}\sqrt{x}}-\frac{2b^{2}}{3a^{3}x^{\frac{3}{2}}}-\frac{b^{5}\ln\left(x\right)}{a^{6}}+\frac{b^{3}}{a^{4}x}+\frac{b}{2a^{2}x^{2}}\right)}{6}$	99

```
[In] int(ln(c*(a+b*x^(1/2))^p)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*ln(c*(a+b*x^(1/2))^p)/x^3+1/6*p*b*(2/a^6*b^5*ln(a+b*x^(1/2))-2/5/a/x^(
5/2)-2/a^5*b^4/x^(1/2)-2/3*b^2/a^3/x^(3/2)-1/a^6*b^5*ln(x)+b^3/a^4/x+1/2*b/
a^2/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.84

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^p\right)}{x^4}\,dx =$$
$$-\frac{60\,b^6px^3\log\left(\sqrt{x}\right)-30\,a^2b^4px^2-15\,a^4b^2px+60\,a^6\log\left(c\right)-60\,\left(b^6px^3-a^6p\right)\log\left(b\sqrt{x}+a\right)+4\,(15\,ab^5p}{180\,a^6x^3}$$

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^4,x, algorithm="fricas")
```

```
[Out] -1/180*(60*b^6*p*x^3*log(sqrt(x)) - 30*a^2*b^4*p*x^2 - 15*a^4*b^2*p*x + 60*
a^6*log(c) - 60*(b^6*p*x^3 - a^6*p)*log(b*sqrt(x) + a) + 4*(15*a*b^5*p*x^2
+ 5*a^3*b^3*p*x + 3*a^5*b*p)*sqrt(x))/(a^6*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^{p}\right)}{x^{4}}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(a+b*x**(1/2))**p)/x**4,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.75

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^{p}\right)}{x^{4}}\,dx$$
$$= \frac{1}{180}\,bp\left(\frac{60\,b^{5}\log\left(b\sqrt{x}+a\right)}{a^{6}} - \frac{30\,b^{5}\log\left(x\right)}{a^{6}} - \frac{60\,b^{4}x^{2} - 30\,ab^{3}x^{\frac{3}{2}} + 20\,a^{2}b^{2}x - 15\,a^{3}b\sqrt{x} + 12\,a^{4}}{a^{5}x^{\frac{5}{2}}}\right) - \frac{\log\left(\left(b\sqrt{x}+a\right)^{p}c\right)}{3\,x^{3}}$$

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^4,x, algorithm="maxima")
[Out] 1/180*b*p*(60*b^5*log(b*sqrt(x) + a)/a^6 - 30*b^5*log(x)/a^6 - (60*b^4*x^2
- 30*a*b^3*x^(3/2) + 20*a^2*b^2*x - 15*a^3*b*sqrt(x) + 12*a^4)/(a^5*x^(5/2)
)) - 1/3*log((b*sqrt(x) + a)^p*c)/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(104) = 208.

Time = 0.31 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.49

$$\int \frac{\log \left(c\left(a+b\sqrt{x}\right)^{p}\right)}{x^{4}}\,dx =$$
$$-\frac{\frac{60\,b^{7}p\log\left(b\sqrt{x}+a\right)}{\left(b\sqrt{x}+a\right)^{6}-6\left(b\sqrt{x}+a\right)^{5}a+15\left(b\sqrt{x}+a\right)^{4}a^{2}-20\left(b\sqrt{x}+a\right)^{3}a^{3}+15\left(b\sqrt{x}+a\right)^{2}a^{4}-6\left(b\sqrt{x}+a\right)a^{5}+a^{6}} - \frac{60\,b^{7}p\log\left(b\sqrt{x}+a\right)}{a^{6}} + \frac{60\,b^{7}p\log\left(b\sqrt{\cdots}\right.}{a^{6}}}{180}$$

```
[In] integrate(log(c*(a+b*x^(1/2))^p)/x^4,x, algorithm="giac")
[Out] -1/180*(60*b^7*p*log(b*sqrt(x) + a)/((b*sqrt(x) + a)^6 - 6*(b*sqrt(x) + a)^
5*a + 15*(b*sqrt(x) + a)^4*a^2 - 20*(b*sqrt(x) + a)^3*a^3 + 15*(b*sqrt(x) +
 a)^2*a^4 - 6*(b*sqrt(x) + a)*a^5 + a^6) - 60*b^7*p*log(b*sqrt(x) + a)/a^6
```

```
+ 60*b^7*p*log(b*sqrt(x))/a^6 + (60*(b*sqrt(x) + a)^5*b^7*p - 330*(b*sqrt(x
) + a)^4*a*b^7*p + 740*(b*sqrt(x) + a)^3*a^2*b^7*p - 855*(b*sqrt(x) + a)^2*
a^3*b^7*p + 522*(b*sqrt(x) + a)*a^4*b^7*p - 137*a^5*b^7*p + 60*a^5*b^7*log(
c))/((b*sqrt(x) + a)^6*a^5 - 6*(b*sqrt(x) + a)^5*a^6 + 15*(b*sqrt(x) + a)^4
*a^7 - 20*(b*sqrt(x) + a)^3*a^8 + 15*(b*sqrt(x) + a)^2*a^9 - 6*(b*sqrt(x) +
 a)*a^10 + a^11))/b
```

Mupad [B] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.75

$$\int \frac{\log\left(c\left(a+b\sqrt{x}\right)^p\right)}{x^4}\,dx=\frac{2\,b^6\,p\,\mathrm{atanh}\left(\frac{2\,b\,\sqrt{x}}{a}+1\right)}{3\,a^6}-\frac{\frac{b\,p}{5\,a}-\frac{b^2\,p\,\sqrt{x}}{4\,a^2}+\frac{b^5\,p\,x^2}{a^5}-\frac{b^4\,p\,x^{3/2}}{2\,a^4}+\frac{b^3\,p\,x}{3\,a^3}}{3\,x^{5/2}}-\frac{\ln\left(c\,\left(a+b\,\sqrt{x}\right)^p\right)}{3\,x^3}$$

```
[In] int(log(c*(a + b*x^(1/2))^p)/x^4,x)
```

```
[Out] (2*b^6*p*atanh((2*b*x^(1/2))/a + 1))/(3*a^6) - ((b*p)/(5*a) - (b^2*p*x^(1/2
))/(4*a^2) + (b^5*p*x^2)/a^5 - (b^4*p*x^(3/2))/(2*a^4) + (b^3*p*x)/(3*a^3))
/(3*x^(5/2)) - log(c*(a + b*x^(1/2))^p)/(3*x^3)
```

3.54 $\int \frac{\log(a+b\sqrt{x})}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{\log\left(a+b\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\sqrt{x}+\frac{2\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b}$$

```
[Out] -2*x^(1/2)+2*ln(a+b*x^(1/2))*(a+b*x^(1/2))/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2504, 2436, 2332}

$$\int \frac{\log\left(a+b\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b}-2\sqrt{x}$$

```
[In] Int[Log[a + b*Sqrt[x]]/Sqrt[x],x]
```

```
[Out] -2*Sqrt[x] + (2*(a + b*Sqrt[x])*Log[a + b*Sqrt[x]])/b
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \log(a+bx)\,dx, x, \sqrt{x}\right)\\ &= \frac{2\text{Subst}\left(\int \log(x)\,dx, x, a+b\sqrt{x}\right)}{b}\\ &= -2\sqrt{x}+\frac{2\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{\log\left(a+b\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\left(-\sqrt{x}+\frac{\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b}\right)$$

```
[In] Integrate[Log[a + b*Sqrt[x]]/Sqrt[x],x]
[Out] 2*(-Sqrt[x] + ((a + b*Sqrt[x])*Log[a + b*Sqrt[x]])/b)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{2\left(a+b\sqrt{x}\right)\ln\left(a+b\sqrt{x}\right)-2b\sqrt{x}-2a}{b}$	32
default	$\frac{2\left(a+b\sqrt{x}\right)\ln\left(a+b\sqrt{x}\right)-2b\sqrt{x}-2a}{b}$	32

```
[In] int(ln(a+b*x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/b*((a+b*x^(1/2))*ln(a+b*x^(1/2))-b*x^(1/2)-a)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{\log\left(a+b\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2\left(\left(b\sqrt{x}+a\right)\log\left(b\sqrt{x}+a\right)-b\sqrt{x}\right)}{b}$$

```
[In] integrate(log(a+b*x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] 2*((b*sqrt(x) + a)*log(b*sqrt(x) + a) - b*sqrt(x))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(27) = 54.

Time = 0.24 (sec) , antiderivative size = 156, normalized size of antiderivative = 4.88

$$\int \frac{\log\left(a+b\sqrt{x}\right)}{\sqrt{x}}\,dx = \begin{cases} \tilde{\infty}\sqrt{x} & \text{for } \left(a=0\vee a=-b\sqrt{x}\right)\wedge\left(a= \right. \\ 2\sqrt{x}\log\left(a\right) & \text{for } b=0 \\ \frac{2a^2\log\left(a+b\sqrt{x}\right)}{ab+b^2\sqrt{x}}+\frac{2a^2}{ab+b^2\sqrt{x}}+\frac{4ab\sqrt{x}\log\left(a+b\sqrt{x}\right)}{ab+b^2\sqrt{x}}+\frac{2b^2x\log\left(a+b\sqrt{x}\right)}{ab+b^2\sqrt{x}}-\frac{2b^2x}{ab+b^2\sqrt{x}} & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(a+b*x**(1/2))/x**(1/2),x)
[Out] Piecewise((zoo*sqrt(x), (Eq(a, 0) | Eq(a, -b*sqrt(x))) & (Eq(b, 0) | Eq(a,
-b*sqrt(x)))), (2*sqrt(x)*log(a), Eq(b, 0)), (2*a**2*log(a + b*sqrt(x))/(a*
b + b**2*sqrt(x)) + 2*a**2/(a*b + b**2*sqrt(x)) + 4*a*b*sqrt(x)*log(a + b*s
qrt(x))/(a*b + b**2*sqrt(x)) + 2*b**2*x*log(a + b*sqrt(x))/(a*b + b**2*sqrt
(x)) - 2*b**2*x/(a*b + b**2*sqrt(x)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{\log\left(a+b\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2\left(\left(b\sqrt{x}+a\right)\log\left(b\sqrt{x}+a\right)-b\sqrt{x}-a\right)}{b}$$

```
[In] integrate(log(a+b*x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] 2*((b*sqrt(x) + a)*log(b*sqrt(x) + a) - b*sqrt(x) - a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{\log\left(a+b\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2\left(\left(b\sqrt{x}+a\right)\log\left(b\sqrt{x}+a\right)-b\sqrt{x}-a\right)}{b}$$

```
[In] integrate(log(a+b*x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] 2*((b*sqrt(x) + a)*log(b*sqrt(x) + a) - b*sqrt(x) - a)/b
```

Mupad [B] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{\log\left(a+b\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\,\ln\left(a+b\sqrt{x}\right)-2\sqrt{x}+\frac{2\,a\,\ln\left(a+b\sqrt{x}\right)}{b}$$

```
[In] int(log(a + b*x^(1/2))/x^(1/2),x)
[Out] 2*x^(1/2)*log(a + b*x^(1/2)) - 2*x^(1/2) + (2*a*log(a + b*x^(1/2)))/b
```

3.55 $\int (fx)^m \log \left(c\left(d+ex^3\right)^p\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int (fx)^m \log \left(c\left(d+ex^3\right)^p\right) \, dx = -\frac{3ep(fx)^{4+m} \text{Hypergeometric2F1}\left(1,\frac{4+m}{3},\frac{7+m}{3},-\frac{ex^3}{d}\right)}{df^4(1+m)(4+m)} + \frac{(fx)^{1+m} \log \left(c\left(d+ex^3\right)^p\right)}{f(1+m)}$$

```
[Out] -3*e*p*(f*x)^(4+m)*hypergeom([1, 4/3+1/3*m],[7/3+1/3*m],-e*x^3/d)/d/f^4/(1+
m)/(4+m)+(f*x)^(1+m)*ln(c*(e*x^3+d)^p)/f/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2505, 16, 371}

$$\int (fx)^m \log \left(c\left(d+ex^3\right)^p\right) \, dx = \frac{(fx)^{m+1} \log \left(c\left(d+ex^3\right)^p\right)}{f(m+1)} - \frac{3ep(fx)^{m+4} \text{Hypergeometric2F1}\left(1,\frac{m+4}{3},\frac{m+7}{3},-\frac{ex^3}{d}\right)}{df^4(m+1)(m+4)}$$

```
[In] Int[(f*x)^m*Log[c*(d + e*x^3)^p],x]
```

```
[Out] (-3*e*p*(f*x)^(4 + m)*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, -((e*x^3)/
d)])/(d*f^4*(1 + m)*(4 + m)) + ((f*x)^(1 + m)*Log[c*(d + e*x^3)^p])/(f*(1 +
 m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(fx)^{1+m}\log\left(c(d+ex^3)^p\right)}{f(1+m)} - \frac{(3ep)\int \frac{x^2(fx)^{1+m}}{d+ex^3}\,dx}{f(1+m)} \\
&= \frac{(fx)^{1+m}\log\left(c(d+ex^3)^p\right)}{f(1+m)} - \frac{(3ep)\int \frac{(fx)^{3+m}}{d+ex^3}\,dx}{f^3(1+m)} \\
&= -\frac{3ep(fx)^{4+m}\,{}_2F_1\left(1,\frac{4+m}{3};\frac{7+m}{3};-\frac{ex^3}{d}\right)}{df^4(1+m)(4+m)} + \frac{(fx)^{1+m}\log\left(c(d+ex^3)^p\right)}{f(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int (fx)^m\log\left(c(d+ex^3)^p\right)\,dx$$
$$= \frac{x(fx)^m\left(-3epx^3\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{3},\frac{7+m}{3},-\frac{ex^3}{d}\right)+d(4+m)\log\left(c(d+ex^3)^p\right)\right)}{d(1+m)(4+m)}$$

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^3)^p],x]
```

```
[Out] (x*(f*x)^m*(-3*e*p*x^3*Hypergeometric2F1[1, (4 + m)/3, (7 + m)/3, -((e*x^3)
/d)] + d*(4 + m)*Log[c*(d + e*x^3)^p]))/(d*(1 + m)*(4 + m))
```

Maple [F]

$$\int (fx)^m \ln\left(c\left(e\,x^3+d\right)^p\right)dx$$

```
[In] int((f*x)^m*ln(c*(e*x^3+d)^p),x)
[Out] int((f*x)^m*ln(c*(e*x^3+d)^p),x)
```

Fricas [F]

$$\int (fx)^m \log\left(c\left(d+ex^3\right)^p\right)\,dx = \int (fx)^m \log\left(\left(ex^3+d\right)^p c\right)\,dx$$

```
[In] integrate((f*x)^m*log(c*(e*x^3+d)^p),x, algorithm="fricas")
[Out] integral((f*x)^m*log((e*x^3 + d)^p*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \log\left(c\left(d+ex^3\right)^p\right)\,dx = \text{Timed out}$$

```
[In] integrate((f*x)**m*ln(c*(e*x**3+d)**p),x)
[Out] Timed out
```

Maxima [F]

$$\int (fx)^m \log\left(c\left(d+ex^3\right)^p\right)\,dx = \int (fx)^m \log\left(\left(ex^3+d\right)^p c\right)\,dx$$

```
[In] integrate((f*x)^m*log(c*(e*x^3+d)^p),x, algorithm="maxima")
[Out] f^m*x*x^m*log((e*x^3 + d)^p)/(m + 1) + integrate((((e*f^m*(m + 1)*log(c) - 3
*e*f^m*p)*x^3 + d*f^m*(m + 1)*log(c))*x^m/(e*(m + 1)*x^3 + d*(m + 1)), x)
```

Giac [F]

$$\int (fx)^m \log \left(c(d+ex^3)^p\right)\,dx = \int (fx)^m \log \left((ex^3+d)^p c\right)\,dx$$

```
[In] integrate((f*x)^m*log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] integrate((f*x)^m*log((e*x^3 + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \log \left(c(d+ex^3)^p\right)\,dx = \int \ln \left(c\left(e\,x^3+d\right)^p\right)\,\left(f\,x\right)^m\,dx$$

```
[In] int(log(c*(d + e*x^3)^p)*(f*x)^m,x)
[Out] int(log(c*(d + e*x^3)^p)*(f*x)^m, x)
```

3.56 $\int (fx)^m \log \left(c\left(d+ex^2\right)^p\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int (fx)^m \log \left(c\left(d+ex^2\right)^p\right) \, dx = -\frac{2ep(fx)^{3+m} \operatorname{Hypergeometric2F1}\left(1,\frac{3+m}{2},\frac{5+m}{2},-\frac{ex^2}{d}\right)}{df^3(1+m)(3+m)} + \frac{(fx)^{1+m} \log \left(c\left(d+ex^2\right)^p\right)}{f(1+m)}$$

```
[Out] -2*e*p*(f*x)^(3+m)*hypergeom([1, 3/2+1/2*m],[5/2+1/2*m],-e*x^2/d)/d/f^3/(1+
m)/(3+m)+(f*x)^(1+m)*ln(c*(e*x^2+d)^p)/f/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2505, 16, 371}

$$\int (fx)^m \log \left(c\left(d+ex^2\right)^p\right) \, dx = \frac{(fx)^{m+1} \log \left(c\left(d+ex^2\right)^p\right)}{f(m+1)} - \frac{2ep(fx)^{m+3} \operatorname{Hypergeometric2F1}\left(1,\frac{m+3}{2},\frac{m+5}{2},-\frac{ex^2}{d}\right)}{df^3(m+1)(m+3)}$$

```
[In] Int[(f*x)^m*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (-2*e*p*(f*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)/
d)])/(d*f^3*(1 + m)*(3 + m)) + ((f*x)^(1 + m)*Log[c*(d + e*x^2)^p])/(f*(1 +
 m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(fx)^{1+m}\log\left(c(d+ex^2)^p\right)}{f(1+m)} - \frac{(2ep)\int \frac{x(fx)^{1+m}}{d+ex^2}\,dx}{f(1+m)} \\
&= \frac{(fx)^{1+m}\log\left(c(d+ex^2)^p\right)}{f(1+m)} - \frac{(2ep)\int \frac{(fx)^{2+m}}{d+ex^2}\,dx}{f^2(1+m)} \\
&= -\frac{2ep(fx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};-\frac{ex^2}{d}\right)}{df^3(1+m)(3+m)} + \frac{(fx)^{1+m}\log\left(c(d+ex^2)^p\right)}{f(1+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$
\int (fx)^m \log\left(c(d+ex^2)^p\right)\,dx = \frac{x(fx)^m\left(-2epx^2\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{2},\frac{5+m}{2},-\frac{ex^2}{d}\right)+d(3+m)\log\left(c(d+ex^2)^p\right)\right)}{d(1+m)(3+m)}
$$

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^2)^p],x]
[Out] (x*(f*x)^m*(-2*e*p*x^2*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)
/d)] + d*(3 + m)*Log[c*(d + e*x^2)^p]))/(d*(1 + m)*(3 + m))
```

Maple [F]

$$\int (fx)^m \ln \left(c\left(e\,x^2+d\right)^p\right) dx$$

```
[In] int((f*x)^m*ln(c*(e*x^2+d)^p),x)
[Out] int((f*x)^m*ln(c*(e*x^2+d)^p),x)
```

Fricas [F]

$$\int (fx)^m \log \left(c\left(d+ex^2\right)^p\right)\, dx = \int (fx)^m \log \left(\left(ex^2+d\right)^p c\right)\, dx$$

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="fricas")
[Out] integral((f*x)^m*log((e*x^2 + d)^p*c), x)
```

Sympy [A] (verification not implemented)

Time = 29.45 (sec) , antiderivative size = 377, normalized size of antiderivative = 4.65

$$\int (fx)^m \log \left(c\left(d+ex^2\right)^p\right)\, dx =$$

$$-2ep\left(\begin{cases} \frac{0^m\sqrt{-\frac{d}{e^3}}\log\left(-e\sqrt{-\frac{d}{e^3}}+x\right)}{2} - \frac{0^m\sqrt{-\frac{d}{e^3}}\log\left(e\sqrt{-\frac{d}{e^3}}+x\right)}{2} + \frac{0^m x}{e} \\ \frac{f^{m+1}mx^{m+3}\Phi\left(\frac{ex^2e^{i\pi}}{d},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4dfm\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+4df\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{3f^{m+1}x^{m+3}\Phi\left(\frac{ex^2e^{i\pi}}{d},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4dfm\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+4df\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} \\ -\frac{\begin{cases} -\frac{\operatorname{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } \frac{1}{|x|}<1 \wedge |x|<1 \\ \log(d)\log(x) - \frac{\operatorname{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } |x|<1 \\ -\log(d)\log\left(\frac{1}{x}\right) - \frac{\operatorname{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } \frac{1}{|x|}<1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(d) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(d) - \frac{\operatorname{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{otherwise} \end{cases}}{2ef} \end{cases}\right)$$

$$+\left(\begin{cases} 0^m x & \text{for } f=0 \\ \frac{\begin{cases} \frac{(fx)^{m+1}}{m+1} & \text{for } m\neq -1 \\ \log(fx) & \text{otherwise} \end{cases}}{f} & \text{otherwise} \end{cases}\right)\log\left(c\left(d+ex^2\right)^p\right)$$

```
[In] integrate((f*x)**m*ln(c*(e*x**2+d)**p),x)
```

```
[Out] -2*e*p*Piecewise((0**m*sqrt(-d/e**3)*log(-e*sqrt(-d/e**3) + x)/2 - 0**m*sqr
t(-d/e**3)*log(e*sqrt(-d/e**3) + x)/2 + 0**m*x/e, Eq(f, 0) | (Eq(f, 0) & Ne
(m, -1))), (f**(m + 1)*m*x**(m + 3)*lerchphi(e*x**2*exp_polar(I*pi)/d, 1, m
/2 + 3/2)*gamma(m/2 + 3/2)/(4*d*f*m*gamma(m/2 + 5/2) + 4*d*f*gamma(m/2 + 5/
2)) + 3*f**(m + 1)*x**(m + 3)*lerchphi(e*x**2*exp_polar(I*pi)/d, 1, m/2 + 3
/2)*gamma(m/2 + 3/2)/(4*d*f*m*gamma(m/2 + 5/2) + 4*d*f*gamma(m/2 + 5/2)), (
m > -oo) & (m < oo) & Ne(m, -1)), (-Piecewise((-polylog(2, e*x**2*exp_polar
(I*pi)/d)/2, (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - polylog(2, e*
x**2*exp_polar(I*pi)/d)/2, Abs(x) < 1), (-log(d)*log(1/x) - polylog(2, e*x*
*2*exp_polar(I*pi)/d)/2, 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()
), x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(2, e
*x**2*exp_polar(I*pi)/d)/2, True))/(2*e*f) + log(f*x)*log(d + e*x**2)/(2*e*
f), True)) + Piecewise((0**m*x, Eq(f, 0)), (Piecewise(((f*x)**(m + 1)/(m +
1), Ne(m, -1)), (log(f*x), True))/f, True))*log(c*(d + e*x**2)**p)
```

Maxima [F]

$$\int (fx)^m \log\left(c(d+ex^2)^p\right)\,dx = \int (fx)^m \log\left((ex^2+d)^p c\right)\,dx$$

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] f^m*x*x^m*log((e*x^2 + d)^p)/(m + 1) + integrate((d*f^m*(m + 1)*log(c) + (e
*f^m*(m + 1)*log(c) - 2*e*f^m*p)*x^2)*x^m/(e*(m + 1)*x^2 + d*(m + 1)), x)
```

Giac [F]

$$\int (fx)^m \log\left(c(d+ex^2)^p\right)\,dx = \int (fx)^m \log\left((ex^2+d)^p c\right)\,dx$$

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((e*x^2 + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \log\left(c(d+ex^2)^p\right) \, dx = \int \ln\left(c\left(e\,x^2+d\right)^p\right)\,(f\,x)^m \, dx$$

```
[In] int(log(c*(d + e*x^2)^p)*(f*x)^m,x)
[Out] int(log(c*(d + e*x^2)^p)*(f*x)^m, x)
```

3.57 $\int (fx)^m \log (c(d+ex)^p) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 69

$$\int (fx)^m \log (c(d+ex)^p) \, dx = -\frac{ep(fx)^{2+m} \text{Hypergeometric2F1}\left(1,2+m,3+m,-\frac{ex}{d}\right)}{df^2(1+m)(2+m)} + \frac{(fx)^{1+m} \log (c(d+ex)^p)}{f(1+m)}$$

```
[Out] -e*p*(f*x)^(2+m)*hypergeom([1, 2+m],[3+m],-e*x/d)/d/f^2/(1+m)/(2+m)+(f*x)^(
1+m)*ln(c*(e*x+d)^p)/f/(1+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2442, 66}

$$\int (fx)^m \log (c(d+ex)^p) \, dx = \frac{(fx)^{m+1} \log (c(d+ex)^p)}{f(m+1)} - \frac{ep(fx)^{m+2} \text{Hypergeometric2F1}\left(1,m+2,m+3,-\frac{ex}{d}\right)}{df^2(m+1)(m+2)}$$

```
[In] Int[(f*x)^m*Log[c*(d + e*x)^p],x]
```

```
[Out] -((e*p*(f*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, -((e*x)/d)])/(d*f^2
*(1 + m)*(2 + m))) + ((f*x)^(1 + m)*Log[c*(d + e*x)^p])/(f*(1 + m))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
```

```
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(fx)^{1+m}\log\left(c(d+ex)^p\right)}{f(1+m)} - \frac{(ep)\int\frac{(fx)^{1+m}}{d+ex}\,dx}{f(1+m)}\\ &= -\frac{ep(fx)^{2+m}\,{}_2F_1\left(1,2+m;3+m;-\frac{ex}{d}\right)}{df^2(1+m)(2+m)} + \frac{(fx)^{1+m}\log\left(c(d+ex)^p\right)}{f(1+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int (fx)^m\log\left(c(d+ex)^p\right)\,dx = \frac{x(fx)^m\left(-epx\,\text{Hypergeometric2F1}\left(1,2+m,3+m,-\frac{ex}{d}\right)+d(2+m)\log\left(c(d+ex)^p\right)\right)}{d(1+m)(2+m)}$$

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x)^p],x]
[Out] (x*(f*x)^m*(-(e*p*x*Hypergeometric2F1[1, 2 + m, 3 + m, -((e*x)/d)]) + d*(2
+ m)*Log[c*(d + e*x)^p]))/(d*(1 + m)*(2 + m))
```

Maple [F]

$$\int (fx)^m \ln\left(c(ex+d)^p\right)dx$$

```
[In] int((f*x)^m*ln(c*(e*x+d)^p),x)
[Out] int((f*x)^m*ln(c*(e*x+d)^p),x)
```

Fricas [F]

$$\int (fx)^m \log \left(c(d+ex)^p\right) \, dx = \int (fx)^m \log \left((ex+d)^p c\right) \, dx$$

[In] integrate((f*x)^m*log(c*(e*x+d)^p),x, algorithm="fricas")

[Out] integral((f*x)^m*log((e*x + d)^p*c), x)

Sympy [F]

$$\int (fx)^m \log \left(c(d+ex)^p\right) \, dx = \int (fx)^m \log \left(c(d+ex)^p\right) dx$$

[In] integrate((f*x)**m*ln(c*(e*x+d)**p),x)

[Out] Integral((f*x)**m*log(c*(d + e*x)**p), x)

Maxima [F]

$$\int (fx)^m \log \left(c(d+ex)^p\right) \, dx = \int (fx)^m \log \left((ex+d)^p c\right) \, dx$$

[In] integrate((f*x)^m*log(c*(e*x+d)^p),x, algorithm="maxima")

[Out] f^m*x*x^m*log((e*x + d)^p)/(m + 1) + integrate((d*f^m*(m + 1)*log(c) + (e*f
^m*(m + 1)*log(c) - e*f^m*p)*x)*x^m/(e*(m + 1)*x + d*(m + 1)), x)

Giac [F]

$$\int (fx)^m \log \left(c(d+ex)^p\right) \, dx = \int (fx)^m \log \left((ex+d)^p c\right) \, dx$$

[In] integrate((f*x)^m*log(c*(e*x+d)^p),x, algorithm="giac")

[Out] integrate((f*x)^m*log((e*x + d)^p*c), x)

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \log \left(c(d+ex)^p\right) \, dx = \int \ln \left(c\,(d+e\,x)^p\right) \, (f\,x)^m \, dx$$

```
[In] int(log(c*(d + e*x)^p)*(f*x)^m,x)
[Out] int(log(c*(d + e*x)^p)*(f*x)^m, x)
```

3.58 $\int (fx)^m \log \left(c\left(d+\frac{e}{x}\right)^p\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 67

$$\int (fx)^m \log \left(c\left(d+\frac{e}{x}\right)^p\right) \, dx = \frac{ep(fx)^m \text{Hypergeometric2F1}\left(1,-m,1-m,-\frac{e}{dx}\right)}{dm(1+m)} + \frac{(fx)^{1+m} \log \left(c\left(d+\frac{e}{x}\right)^p\right)}{f(1+m)}$$

```
[Out] e*p*(f*x)^m*hypergeom([1, -m],[1-m],-e/d/x)/d/m/(1+m)+(f*x)^(1+m)*ln(c*(d+e
/x)^p)/f/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2505, 16, 346, 66}

$$\int (fx)^m \log \left(c\left(d+\frac{e}{x}\right)^p\right) \, dx = \frac{(fx)^{m+1} \log \left(c\left(d+\frac{e}{x}\right)^p\right)}{f(m+1)} + \frac{ep(fx)^m \text{Hypergeometric2F1}\left(1,-m,1-m,-\frac{e}{dx}\right)}{dm(m+1)}$$

```
[In] Int[(f*x)^m*Log[c*(d + e/x)^p],x]
```

```
[Out] (e*p*(f*x)^m*Hypergeometric2F1[1, -m, 1 - m, -(e/(d*x))])/(d*m*(1 + m)) + (
(f*x)^(1 + m)*Log[c*(d + e/x)^p])/(f*(1 + m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 346

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1), Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x
, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(1+m)}+\frac{(ep)\int\frac{(fx)^{1+m}}{\left(d+\frac{e}{x}\right)x^2}\,dx}{f(1+m)}\\
&= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(1+m)}+\frac{(efp)\int\frac{(fx)^{-1+m}}{d+\frac{e}{x}}\,dx}{1+m}\\
&= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(1+m)}-\frac{\left(ep\left(\frac{1}{x}\right)^m(fx)^m\right)\text{Subst}\left(\int\frac{x^{-1-m}}{d+ex}\,dx,x,\frac{1}{x}\right)}{1+m}\\
&= \frac{ep(fx)^m\,{}_2F_1\left(1,-m;1-m;-\frac{e}{dx}\right)}{dm(1+m)}+\frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int(fx)^m\log\left(c\left(d+\frac{e}{x}\right)^p\right)\,dx$$
$$=\frac{(fx)^m\left(ep\,\text{Hypergeometric2F1}\left(1,-m,1-m,-\frac{e}{dx}\right)+dmx\log\left(c\left(d+\frac{e}{x}\right)^p\right)\right)}{dm(1+m)}$$

```
[In] Integrate[(f*x)^m*Log[c*(d + e/x)^p],x]
```

```
[Out] ((f*x)^m*(e*p*Hypergeometric2F1[1, -m, 1 - m, -(e/(d*x))] + d*m*x*Log[c*(d
+ e/x)^p]))/(d*m*(1 + m))
```

Maple [F]

$$\int (fx)^m \ln \left(c\left(d+\frac{e}{x}\right)^p\right) dx$$

[In] int((f*x)^m*ln(c*(d+e/x)^p),x)

[Out] int((f*x)^m*ln(c*(d+e/x)^p),x)

Fricas [F]

$$\int (fx)^m \log \left(c\left(d+\frac{e}{x}\right)^p\right) dx = \int (fx)^m \log \left(c\left(d+\frac{e}{x}\right)^p\right) dx$$

[In] integrate((f*x)^m*log(c*(d+e/x)^p),x, algorithm="fricas")

[Out] integral((f*x)^m*log(c*((d*x + e)/x)^p), x)

Sympy [A] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.45

$$\int (fx)^m \log \left(c\left(d+\frac{e}{x}\right)^p\right) dx$$
$$= ep\left(\begin{cases} \frac{0^m \log(dx+e)}{d} \\ \frac{d^{m-1}f^{m+1}mx^m\Phi\left(\frac{ee^{i\pi}}{dx},1,me^{i\pi}\right)\Gamma(-m)}{d^m fm\Gamma(1-m)+d^m f\Gamma(1-m)} \\ \frac{\begin{cases} -\frac{1}{dx} & \text{for} \\ \begin{cases} \operatorname{Li}_2\left(\frac{ee^{i\pi}}{dx}\right) & \text{for } \frac{1}{|x|}<1 \wedge |x|<1 \\ \log(d)\log(x)+\operatorname{Li}_2\left(\frac{ee^{i\pi}}{dx}\right) & \text{for } |x|<1 \\ -\log(d)\log\left(\frac{1}{x}\right)+\operatorname{Li}_2\left(\frac{ee^{i\pi}}{dx}\right) & \text{for } \frac{1}{|x|}<1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(d)+G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(d)+\operatorname{Li}_2\left(\frac{ee^{i\pi}}{dx}\right) & \text{otherwise} \end{cases} \\ \hline e & \text{ot} \end{cases}}{f} \end{cases}\right)$$
$$+\left(\begin{cases} 0^m x & \text{for } f=0 \\ \frac{\begin{cases} \frac{(fx)^{m+1}}{m+1} & \text{for } m\neq -1 \\ \log(fx) & \text{otherwise} \end{cases}}{f} & \text{otherwise} \end{cases}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)$$

```
[In] integrate((f*x)**m*ln(c*(d+e/x)**p),x)
```

```
[Out] e*p*Piecewise((0**m*log(d*x + e)/d, Eq(f, 0) | (Eq(f, 0) & Ne(m, -1))), (d*
*(m - 1)*f**(m + 1)*m*x**m*lerchphi(e*exp_polar(I*pi)/(d*x), 1, m*exp_polar
(I*pi))*gamma(-m)/(d**m*f*m*gamma(1 - m) + d**m*f*gamma(1 - m)), (m > -oo)
& (m < oo) & Ne(m, -1)), (Piecewise((-1/(d*x), Eq(e, 0)), (Piecewise((polyl
og(2, e*exp_polar(I*pi)/(d*x)), (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log
(x) + polylog(2, e*exp_polar(I*pi)/(d*x)), Abs(x) < 1), (-log(d)*log(1/x) +
 polylog(2, e*exp_polar(I*pi)/(d*x)), 1/Abs(x) < 1), (-meijerg(((), (1, 1))
, ((0, 0), ()), x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) +
 polylog(2, e*exp_polar(I*pi)/(d*x)), True))/e, True))/f - Piecewise((1/(d*
x), Eq(e, 0)), (log(d + e/x)/e, True))*log(f*x)/f, True)) + Piecewise((0**m
*x, Eq(f, 0)), (Piecewise(((f*x)**(m + 1)/(m + 1), Ne(m, -1)), (log(f*x), T
rue))/f, True))*log(c*(d + e/x)**p)
```

Maxima [F]

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right)\,dx = \int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right)\,dx$$

```
[In] integrate((f*x)^m*log(c*(d+e/x)^p),x, algorithm="maxima")
```

```
[Out] (f^m*x*x^m*log((d*x + e)^p) - f^m*x*x^m*log(x^p))/(m + 1) + integrate((d*f^
m*(m + 1)*x*log(c) + e*f^m*(m + 1)*log(c) + e*f^m*p)*x^m/(d*(m + 1)*x + e*(
m + 1)), x)
```

Giac [F]

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right)\,dx = \int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right)\,dx$$

```
[In] integrate((f*x)^m*log(c*(d+e/x)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log(c*(d + e/x)^p), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x}\right)^p\right)\,dx = \int \ln\left(c\left(d+\frac{e}{x}\right)^p\right)\,(f\,x)^m\,dx$$

```
[In] int(log(c*(d + e/x)^p)*(f*x)^m,x)
[Out] int(log(c*(d + e/x)^p)*(f*x)^m, x)
```

3.59 $\int (fx)^m \log\left(c\left(d+\frac{e}{x^2}\right)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 82

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^2}\right)^p\right)\,dx = -\frac{2efp(fx)^{-1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1-m}{2},\frac{3-m}{2},-\frac{e}{dx^2}\right)}{d\left(1-m^2\right)} + \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(1+m)}$$

```
[Out] -2*e*f*p*(f*x)^(-1+m)*hypergeom([1, 1/2-1/2*m],[3/2-1/2*m],-e/d/x^2)/d/(-m^
2+1)+(f*x)^(1+m)*ln(c*(d+e/x^2)^p)/f/(1+m)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2505, 16, 346, 371}

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^2}\right)^p\right)\,dx = \frac{(fx)^{m+1}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(m+1)} - \frac{2efp(fx)^{m-1}\,\text{Hypergeometric2F1}\left(1,\frac{1-m}{2},\frac{3-m}{2},-\frac{e}{dx^2}\right)}{d\left(1-m^2\right)}$$

```
[In] Int[(f*x)^m*Log[c*(d + e/x^2)^p],x]
```

```
[Out] (-2*e*f*p*(f*x)^(-1 + m)*Hypergeometric2F1[1, (1 - m)/2, (3 - m)/2, -(e/(d*
x^2))])/(d*(1 - m^2)) + ((f*x)^(1 + m)*Log[c*(d + e/x^2)^p])/(f*(1 + m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 346

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1), Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x
, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(1+m)}+\frac{(2ep)\int\frac{(fx)^{1+m}}{\left(d+\frac{e}{x^2}\right)x^3}\,dx}{f(1+m)}\\
&= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(1+m)}+\frac{(2ef^2p)\int\frac{(fx)^{-2+m}}{d+\frac{e}{x^2}}\,dx}{1+m}\\
&= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(1+m)}-\frac{\left(2efp\left(\frac{1}{x}\right)^{-1+m}(fx)^{-1+m}\right)\text{Subst}\left(\int\frac{x^{-m}}{d+ex^2}\,dx,x,\frac{1}{x}\right)}{1+m}\\
&= -\frac{2efp(fx)^{-1+m}\,{}_2F_1\left(1,\frac{1-m}{2};\frac{3-m}{2};-\frac{e}{dx^2}\right)}{d\left(1-m^2\right)}+\frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int(fx)^m\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)\,dx = \frac{(fx)^m\left(2ep\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{m}{2},\frac{3}{2}-\frac{m}{2},-\frac{e}{dx^2}\right)+d(-1+m)x^2\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)\right)}{d(-1+m)(1+m)x}$$

```
[In] Integrate[(f*x)^m*Log[c*(d + e/x^2)^p],x]
[Out] ((f*x)^m*(2*e*p*Hypergeometric2F1[1, 1/2 - m/2, 3/2 - m/2, -(e/(d*x^2))] +
d*(-1 + m)*x^2*Log[c*(d + e/x^2)^p]))/(d*(-1 + m)*(1 + m)*x)
```

Maple [F]

$$\int (fx)^m \ln\left(c\left(d+\frac{e}{x^2}\right)^p\right)dx$$

```
[In] int((f*x)^m*ln(c*(d+e/x^2)^p),x)
[Out] int((f*x)^m*ln(c*(d+e/x^2)^p),x)
```

Fricas [F]

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^2}\right)^p\right)\,dx = \int (fx)^m \log\left(c\left(d+\frac{e}{x^2}\right)^p\right)\,dx$$

```
[In] integrate((f*x)^m*log(c*(d+e/x^2)^p),x, algorithm="fricas")
[Out] integral((f*x)^m*log(c*((d*x^2 + e)/x^2)^p), x)
```

Sympy [A] (verification not implemented)

Time = 28.76 (sec) , antiderivative size = 366, normalized size of antiderivative = 4.46

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^2}\right)^p\right)\,dx$$
$$= 2ep\left(\begin{cases} -\frac{0^m\sqrt{-\frac{1}{de}}\log\left(-e\sqrt{-\frac{1}{de}}+x\right)}{2}+\frac{0^m\sqrt{-\frac{1}{de}}\log\left(e\sqrt{-\frac{1}{de}}+x\right)}{2} \\ \frac{f^{m+1}mx^{m-1}\Phi\left(\frac{ee^{i\pi}}{dx^2},1,\frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4dfm\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)+4df\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)}-\frac{f^{m+1}x^{m-1}\Phi\left(\frac{ee^{i\pi}}{dx^2},1,\frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{4dfm\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)+4df\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)} \\ \frac{\begin{cases} \frac{\mathrm{Li}_2\left(\frac{ee^{i\pi}}{dx^2}\right)}{2} & \text{for } \frac{1}{|x|}<1\wedge|x|<1 \\ \log(d)\log(x)+\frac{\mathrm{Li}_2\left(\frac{ee^{i\pi}}{dx^2}\right)}{2} & \text{for } |x|<1 \\ -\log(d)\log\left(\frac{1}{x}\right)+\frac{\mathrm{Li}_2\left(\frac{ee^{i\pi}}{dx^2}\right)}{2} & \text{for } \frac{1}{|x|}<1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(d)+G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(d)+\frac{\mathrm{Li}_2\left(\frac{ee^{i\pi}}{dx^2}\right)}{2} & \text{otherwise} \end{cases}}{2ef} \end{cases}\right. - \frac{\log}{}$$
$$+\left(\begin{cases} 0^m x & \text{for } f=0 \\ \frac{\begin{cases} \frac{(fx)^{m+1}}{m+1} & \text{for } m\neq -1 \\ \log(fx) & \text{otherwise} \end{cases}}{f} & \text{otherwise} \end{cases}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)$$

```
[In] integrate((f*x)**m*ln(c*(d+e/x**2)**p),x)
```

```
[Out] 2*e*p*Piecewise((-0**m*sqrt(-1/(d*e))*log(-e*sqrt(-1/(d*e)) + x)/2 + 0**m*s
qrt(-1/(d*e))*log(e*sqrt(-1/(d*e)) + x)/2, Eq(f, 0) | (Eq(f, 0) & Ne(m, -1)
)), (f**(m + 1)*m*x**(m - 1)*lerchphi(e*exp_polar(I*pi)/(d*x**2), 1, 1/2 -
m/2)*gamma(1/2 - m/2)/(4*d*f*m*gamma(3/2 - m/2) + 4*d*f*gamma(3/2 - m/2)) -
 f**(m + 1)*x**(m - 1)*lerchphi(e*exp_polar(I*pi)/(d*x**2), 1, 1/2 - m/2)*g
amma(1/2 - m/2)/(4*d*f*m*gamma(3/2 - m/2) + 4*d*f*gamma(3/2 - m/2)), (m > -
oo) & (m < oo) & Ne(m, -1)), (Piecewise((polylog(2, e*exp_polar(I*pi)/(d*x*
*2))/2, (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) + polylog(2, e*exp_p
olar(I*pi)/(d*x**2))/2, Abs(x) < 1), (-log(d)*log(1/x) + polylog(2, e*exp_p
olar(I*pi)/(d*x**2))/2, 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ())
, x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) + polylog(2, e*
exp_polar(I*pi)/(d*x**2))/2, True))/(2*e*f) - log(f*x)*log(d + e/x**2)/(2*e
*f), True)) + Piecewise((0**m*x, Eq(f, 0)), (Piecewise(((f*x)**(m + 1)/(m +
 1), Ne(m, -1)), (log(f*x), True))/f, True))*log(c*(d + e/x**2)**p)
```

Maxima [F]

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^2}\right)^p\right)\,dx = \int (fx)^m \log\left(c\left(d+\frac{e}{x^2}\right)^p\right)\,dx$$

```
[In] integrate((f*x)^m*log(c*(d+e/x^2)^p),x, algorithm="maxima")
```

```
[Out] (f^m*x*x^m*log((d*x^2 + e)^p) - 2*f^m*x*x^m*log(x^p))/(m + 1) + integrate((
d*f^m*(m + 1)*x^2*log(c) + e*f^m*(m + 1)*log(c) + 2*e*f^m*p)*x^m/(d*(m + 1)
*x^2 + e*(m + 1)), x)
```

Giac [F]

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^2}\right)^p\right)\,dx = \int (fx)^m \log\left(c\left(d+\frac{e}{x^2}\right)^p\right)\,dx$$

```
[In] integrate((f*x)^m*log(c*(d+e/x^2)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log(c*(d + e/x^2)^p), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^2}\right)^p\right)\,dx = \int \ln\left(c\left(d+\frac{e}{x^2}\right)^p\right)(f\,x)^m\,dx$$

```
[In] int(log(c*(d + e/x^2)^p)*(f*x)^m,x)
[Out] int(log(c*(d + e/x^2)^p)*(f*x)^m, x)
```

3.60 $\int (fx)^m \log \left(c\left(d+\frac{e}{x^3}\right)^p\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 85

$$\int (fx)^m \log \left(c\left(d+\frac{e}{x^3}\right)^p\right) \, dx = -\frac{3ef^2p(fx)^{-2+m} \text{Hypergeometric2F1}\left(1,\frac{2-m}{3},\frac{5-m}{3},-\frac{e}{dx^3}\right)}{d\left(2+m-m^2\right)} + \frac{(fx)^{1+m} \log \left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(1+m)}$$

```
[Out] -3*e*f^2*p*(f*x)^(-2+m)*hypergeom([1, 2/3-1/3*m],[5/3-1/3*m],-e/d/x^3)/d/(-
m^2+m+2)+(f*x)^(1+m)*ln(c*(d+e/x^3)^p)/f/(1+m)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2505, 16, 346, 371}

$$\int (fx)^m \log \left(c\left(d+\frac{e}{x^3}\right)^p\right) \, dx = \frac{(fx)^{m+1} \log \left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(m+1)} - \frac{3ef^2p(fx)^{m-2} \text{Hypergeometric2F1}\left(1,\frac{2-m}{3},\frac{5-m}{3},-\frac{e}{dx^3}\right)}{d\left(-m^2+m+2\right)}$$

```
[In] Int[(f*x)^m*Log[c*(d + e/x^3)^p],x]
```

```
[Out] (-3*e*f^2*p*(f*x)^(-2 + m)*Hypergeometric2F1[1, (2 - m)/3, (5 - m)/3, -(e/(
d*x^3))])/(d*(2 + m - m^2)) + ((f*x)^(1 + m)*Log[c*(d + e/x^3)^p])/(f*(1 +
m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 346

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1), Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x
, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(1+m)}+\frac{(3ep)\int\frac{(fx)^{1+m}}{\left(d+\frac{e}{x^3}\right)x^4}\,dx}{f(1+m)}\\
&= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(1+m)}+\frac{(3ef^3p)\int\frac{(fx)^{-3+m}}{d+\frac{e}{x^3}}\,dx}{1+m}\\
&= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(1+m)}-\frac{\left(3ef^2p\left(\frac{1}{x}\right)^{-2+m}(fx)^{-2+m}\right)\text{Subst}\left(\int\frac{x^{1-m}}{d+ex^3}\,dx,x,\frac{1}{x}\right)}{1+m}\\
&= -\frac{3ef^2p(fx)^{-2+m}\,{}_2F_1\left(1,\frac{2-m}{3};\frac{5-m}{3};-\frac{e}{dx^3}\right)}{d\left(2+m-m^2\right)}+\frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)}{f(1+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^3}\right)^p\right)\,dx$$
$$=\frac{(fx)^m\left(3ep\,\text{Hypergeometric2F1}\left(1,\frac{2}{3}-\frac{m}{3},\frac{5}{3}-\frac{m}{3},-\frac{e}{dx^3}\right)+d(-2+m)x^3\log\left(c\left(d+\frac{e}{x^3}\right)^p\right)\right)}{d(-2+m)(1+m)x^2}$$

```
[In] Integrate[(f*x)^m*Log[c*(d + e/x^3)^p],x]
[Out] ((f*x)^m*(3*e*p*Hypergeometric2F1[1, 2/3 - m/3, 5/3 - m/3, -(e/(d*x^3))] +
d*(-2 + m)*x^3*Log[c*(d + e/x^3)^p]))/(d*(-2 + m)*(1 + m)*x^2)
```

Maple [F]

$$\int (fx)^m \ln\left(c\left(d+\frac{e}{x^3}\right)^p\right)dx$$

```
[In] int((f*x)^m*ln(c*(d+e/x^3)^p),x)
[Out] int((f*x)^m*ln(c*(d+e/x^3)^p),x)
```

Fricas [F]

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^3}\right)^p\right)\,dx=\int (fx)^m \log\left(c\left(d+\frac{e}{x^3}\right)^p\right)\,dx$$

```
[In] integrate((f*x)^m*log(c*(d+e/x^3)^p),x, algorithm="fricas")
[Out] integral((f*x)^m*log(c*((d*x^3 + e)/x^3)^p), x)
```

Sympy [F(-1)]

Timed out.

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^3}\right)^p\right)\,dx=\text{Timed out}$$

```
[In] integrate((f*x)**m*ln(c*(d+e/x**3)**p),x)
[Out] Timed out
```

Maxima [F]

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^3}\right)^p\right)\,dx = \int (fx)^m \log\left(c\left(d+\frac{e}{x^3}\right)^p\right)\,dx$$

```
[In] integrate((f*x)^m*log(c*(d+e/x^3)^p),x, algorithm="maxima")
```

```
[Out] (f^m*x*x^m*log((d*x^3 + e)^p) - 3*f^m*x*x^m*log(x^p))/(m + 1) + integrate((
d*f^m*(m + 1)*x^3*log(c) + e*f^m*(m + 1)*log(c) + 3*e*f^m*p)*x^m/(d*(m + 1)
*x^3 + e*(m + 1)), x)
```

Giac [F]

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^3}\right)^p\right)\,dx = \int (fx)^m \log\left(c\left(d+\frac{e}{x^3}\right)^p\right)\,dx$$

```
[In] integrate((f*x)^m*log(c*(d+e/x^3)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log(c*(d + e/x^3)^p), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \log\left(c\left(d+\frac{e}{x^3}\right)^p\right)\,dx = \int \ln\left(c\left(d+\frac{e}{x^3}\right)^p\right)(f\,x)^m\,dx$$

```
[In] int(log(c*(d + e/x^3)^p)*(f*x)^m,x)
```

```
[Out] int(log(c*(d + e/x^3)^p)*(f*x)^m, x)
```

3.61 $\int (fx)^m \log \left(c\left(d+e\sqrt{x}\right)^p\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 83

$$\int (fx)^m \log \left(c\left(d+e\sqrt{x}\right)^p\right) \, dx = -\frac{epx^{3/2}(fx)^m \text{Hypergeometric2F1}\left(1,3+2m,2(2+m),-\frac{e\sqrt{x}}{d}\right)}{d\left(3+5m+2m^2\right)} + \frac{(fx)^{1+m} \log \left(c\left(d+e\sqrt{x}\right)^p\right)}{f(1+m)}$$

```
[Out] -e*p*x^(3/2)*(f*x)^m*hypergeom([1, 3+2*m],[4+2*m],-e*x^(1/2)/d)/d/(1+m)/(3+
2*m)+(f*x)^(1+m)*ln(c*(d+e*x^(1/2))^p)/f/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2505, 20, 348, 66}

$$\int (fx)^m \log \left(c\left(d+e\sqrt{x}\right)^p\right) \, dx = \frac{(fx)^{m+1} \log \left(c\left(d+e\sqrt{x}\right)^p\right)}{f(m+1)} - \frac{epx^{3/2}(fx)^m \text{Hypergeometric2F1}\left(1,2m+3,2(m+2),-\frac{e\sqrt{x}}{d}\right)}{d\left(2m^2+5m+3\right)}$$

```
[In] Int[(f*x)^m*Log[c*(d + e*Sqrt[x])^p],x]
```

```
[Out] -((e*p*x^(3/2)*(f*x)^m*Hypergeometric2F1[1, 3 + 2*m, 2*(2 + m), -((e*Sqrt[x
])/d)])/(d*(3 + 5*m + 2*m^2))) + ((f*x)^(1 + m)*Log[c*(d + e*Sqrt[x])^p])/(
f*(1 + m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(fx)^{1+m}\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f(1+m)} - \frac{(ep)\int \frac{(fx)^{1+m}}{\left(d+e\sqrt{x}\right)\sqrt{x}}\,dx}{2f(1+m)} \\
&= \frac{(fx)^{1+m}\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f(1+m)} - \frac{\left(epx^{-m}(fx)^m\right)\int \frac{x^{\frac{1}{2}+m}}{d+e\sqrt{x}}\,dx}{2(1+m)} \\
&= \frac{(fx)^{1+m}\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f(1+m)} - \frac{\left(epx^{-m}(fx)^m\right)\text{Subst}\left(\int \frac{x^{-1+2\left(\frac{3}{2}+m\right)}}{d+ex}\,dx, x, \sqrt{x}\right)}{1+m} \\
&= -\frac{epx^{3/2}(fx)^m\,{}_2F_1\left(1,3+2m;2(2+m);-\frac{e\sqrt{x}}{d}\right)}{d\left(3+5m+2m^2\right)} + \frac{(fx)^{1+m}\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92

$$\int (fx)^m \log \left(c\left(d+e\sqrt{x}\right)^p\right) \, dx = \frac{x(fx)^m \left(-ep\sqrt{x}\,\text{Hypergeometric2F1}\left(1,3+2m,4+2m,-\frac{e\sqrt{x}}{d}\right)+d(3+2m)\log\left(c\left(d+e\sqrt{x}\right)^p\right)\right)}{d(1+m)(3+2m)}$$

```
[In] Integrate[(f*x)^m*Log[c*(d + e*Sqrt[x])^p],x]
[Out] (x*(f*x)^m*(-(e*p*Sqrt[x]*Hypergeometric2F1[1, 3 + 2*m, 4 + 2*m, -((e*Sqrt[
x])/d)]) + d*(3 + 2*m)*Log[c*(d + e*Sqrt[x])^p]))/(d*(1 + m)*(3 + 2*m))
```

Maple [F]

$$\int (fx)^m \ln \left(c\left(d+e\sqrt{x}\right)^p\right) dx$$

```
[In] int((f*x)^m*ln(c*(d+e*x^(1/2))^p),x)
[Out] int((f*x)^m*ln(c*(d+e*x^(1/2))^p),x)
```

Fricas [F]

$$\int (fx)^m \log \left(c\left(d+e\sqrt{x}\right)^p\right) \, dx = \int (fx)^m \log \left(\left(e\sqrt{x}+d\right)^p c\right) \, dx$$

```
[In] integrate((f*x)^m*log(c*(d+e*x^(1/2))^p),x, algorithm="fricas")
[Out] integral((f*x)^m*log((e*sqrt(x) + d)^p*c), x)
```

Sympy [F]

$$\int (fx)^m \log \left(c\left(d+e\sqrt{x}\right)^p\right) \, dx = \int (fx)^m \log \left(c\left(d+e\sqrt{x}\right)^p\right) dx$$

```
[In] integrate((f*x)**m*ln(c*(d+e*x**(1/2))**p),x)
[Out] Integral((f*x)**m*log(c*(d + e*sqrt(x))**p), x)
```

Maxima [F]

$$\int (fx)^m \log \left(c(d+e\sqrt{x})^p\right) \, dx = \int (fx)^m \log \left((e\sqrt{x}+d)^p c\right) \, dx$$

```
[In] integrate((f*x)^m*log(c*(d+e*x^(1/2))^p),x, algorithm="maxima")
```

```
[Out] e^2*f^m*p*integrate(1/2*x*x^m/(d*e*(m + 1)*sqrt(x) + d^2*(m + 1)), x) + (d*
f^m*(2*m + 3)*x*x^m*log((e*sqrt(x) + d)^p) + d*f^m*(2*m + 3)*x*x^m*log(c) -
 e*f^m*p*x^(3/2)*x^m)/((2*m^2 + 5*m + 3)*d)
```

Giac [F]

$$\int (fx)^m \log \left(c(d+e\sqrt{x})^p\right) \, dx = \int (fx)^m \log \left((e\sqrt{x}+d)^p c\right) \, dx$$

```
[In] integrate((f*x)^m*log(c*(d+e*x^(1/2))^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((e*sqrt(x) + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \log \left(c(d+e\sqrt{x})^p\right) \, dx = \int \ln \left(c \left(d+e\sqrt{x}\right)^p\right) (f\,x)^m \, dx$$

```
[In] int(log(c*(d + e*x^(1/2))^p)*(f*x)^m,x)
```

```
[Out] int(log(c*(d + e*x^(1/2))^p)*(f*x)^m, x)
```

3.62 $\int (fx)^m \log \left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 70

$$\int (fx)^m \log \left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right) dx = \frac{px(fx)^m \operatorname{Hypergeometric2F1}\left(1,2(1+m),3+2m,-\frac{d\sqrt{x}}{e}\right)}{2(1+m)^2} + \frac{(fx)^{1+m} \log \left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(1+m)}$$

```
[Out] 1/2*p*x*(f*x)^m*hypergeom([1, 2+2*m],[3+2*m],-d*x^(1/2)/e)/(1+m)^2+(f*x)^(1
+m)*ln(c*(d+e/x^(1/2))^p)/f/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2505, 20, 269, 348, 66}

$$\int (fx)^m \log \left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right) dx = \frac{(fx)^{m+1} \log \left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(m+1)} + \frac{px(fx)^m \operatorname{Hypergeometric2F1}\left(1,2(m+1),2m+3,-\frac{d\sqrt{x}}{e}\right)}{2(m+1)^2}$$

```
[In] Int[(f*x)^m*Log[c*(d + e/Sqrt[x])^p],x]
```

```
[Out] (p*x*(f*x)^m*Hypergeometric2F1[1, 2*(1 + m), 3 + 2*m, -((d*Sqrt[x])/e)])/(2
*(1 + m)^2) + ((f*x)^(1 + m)*Log[c*(d + e/Sqrt[x])^p])/(f*(1 + m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(1+m)}+\frac{(ep)\int\frac{(fx)^{1+m}}{\left(d+\frac{e}{\sqrt{x}}\right)x^{3/2}}\,dx}{2f(1+m)}\\
&= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(1+m)}+\frac{\left(epx^{-m}(fx)^m\right)\int\frac{x^{-\frac{1}{2}+m}}{d+\frac{e}{\sqrt{x}}}\,dx}{2(1+m)}\\
&= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(1+m)}+\frac{\left(epx^{-m}(fx)^m\right)\int\frac{x^m}{e+d\sqrt{x}}\,dx}{2(1+m)}
\end{aligned}$$

$$= \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(1+m)} + \frac{(epx^{-m}(fx)^m)\,\mathrm{Subst}\left(\int\frac{x^{-1+2(1+m)}}{e+dx}\,dx,x,\sqrt{x}\right)}{1+m}$$
$$= \frac{px(fx)^m\,{}_2F_1\left(1,2(1+m);3+2m;-\frac{d\sqrt{x}}{e}\right)}{2(1+m)^2} + \frac{(fx)^{1+m}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f(1+m)}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10

$$\int (fx)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\,dx$$
$$= \frac{\sqrt{x}(fx)^m\left(ep\,\mathrm{Hypergeometric2F1}\left(1,-1-2m,-2m,-\frac{e}{d\sqrt{x}}\right)+d(1+2m)\sqrt{x}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\right)}{d(1+m)(1+2m)}$$

```
[In] Integrate[(f*x)^m*Log[c*(d + e/Sqrt[x])^p],x]
[Out] (Sqrt[x]*(f*x)^m*(e*p*Hypergeometric2F1[1, -1 - 2*m, -2*m, -(e/(d*Sqrt[x]))
] + d*(1 + 2*m)*Sqrt[x]*Log[c*(d + e/Sqrt[x])^p]))/(d*(1 + m)*(1 + 2*m))
```

Maple [F]

$$\int (fx)^m \ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)dx$$

```
[In] int((f*x)^m*ln(c*(d+e/x^(1/2))^p),x)
[Out] int((f*x)^m*ln(c*(d+e/x^(1/2))^p),x)
```

Fricas [F]

$$\int (fx)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\,dx = \int (fx)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\,dx$$

```
[In] integrate((f*x)^m*log(c*(d+e/x^(1/2))^p),x, algorithm="fricas")
[Out] integral((f*x)^m*log(c*((d*x + e*sqrt(x))/x)^p), x)
```

Sympy [F]

$$\int (fx)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right) dx = \int (fx)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right) dx$$

```
[In] integrate((f*x)**m*ln(c*(d+e/x**(1/2))**p),x)
[Out] Integral((f*x)**m*log(c*(d + e/sqrt(x))**p), x)
```

Maxima [F]

$$\int (fx)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right) dx = \int (fx)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right) dx$$

```
[In] integrate((f*x)^m*log(c*(d+e/x^(1/2))^p),x, algorithm="maxima")
[Out] d^2*f^m*p*integrate(1/2*x*x^m/(d*e*(m + 1)*sqrt(x) + e^2*(m + 1)), x) + 1/2
*(2*(2*m^2 + 5*m + 3)*e*f^m*x*x^m*log((d*sqrt(x) + e)^p) - 2*(2*m^2 + 5*m +
 3)*e*f^m*x*x^m*log(x^(1/2*p)) - 2*(m*p + p)*d*f^m*x^(3/2)*x^m + (2*(2*m^2
+ 5*m + 3)*e*f^m*log(c) + (2*m*p + 3*p)*e*f^m)*x*x^m)/((2*m^3 + 7*m^2 + 8*m
 + 3)*e)
```

Giac [F]

$$\int (fx)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right) dx = \int (fx)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right) dx$$

```
[In] integrate((f*x)^m*log(c*(d+e/x^(1/2))^p),x, algorithm="giac")
[Out] integrate((f*x)^m*log(c*(d + e/sqrt(x))^p), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right) dx = \int \ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right) (f\,x)^m\,dx$$

```
[In] int(log(c*(d + e/x^(1/2))^p)*(f*x)^m,x)
[Out] int(log(c*(d + e/x^(1/2))^p)*(f*x)^m, x)
```

3.63 $\int (fx)^m \log\left(c(d+ex^n)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 87

$$\int (fx)^m \log\left(c(d+ex^n)^p\right)\,dx = -\frac{enpx^{1+n}(fx)^m \,\text{Hypergeometric2F1}\left(1,\frac{1+m+n}{n},\frac{1+m+2n}{n},-\frac{ex^n}{d}\right)}{d(1+m)(1+m+n)} + \frac{(fx)^{1+m}\log\left(c(d+ex^n)^p\right)}{f(1+m)}$$

```
[Out] -e*n*p*x^(1+n)*(f*x)^m*hypergeom([1, (1+m+n)/n],[(1+m+2*n)/n],-e*x^n/d)/d/(
1+m)/(1+m+n)+(f*x)^(1+m)*ln(c*(d+e*x^n)^p)/f/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2505, 20, 371}

$$\int (fx)^m \log\left(c(d+ex^n)^p\right)\,dx = \frac{(fx)^{m+1}\log\left(c(d+ex^n)^p\right)}{f(m+1)} - \frac{enpx^{n+1}(fx)^m \,\text{Hypergeometric2F1}\left(1,\frac{m+n+1}{n},\frac{m+2n+1}{n},-\frac{ex^n}{d}\right)}{d(m+1)(m+n+1)}$$

```
[In] Int[(f*x)^m*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*n*p*x^(1 + n)*(f*x)^m*Hypergeometric2F1[1, (1 + m + n)/n, (1 + m + 2*n
)/n, -((e*x^n)/d)])/(d*(1 + m)*(1 + m + n))) + ((f*x)^(1 + m)*Log[c*(d + e*
x^n)^p])/(f*(1 + m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(fx)^{1+m}\log\left(c(d+ex^n)^p\right)}{f(1+m)} - \frac{(enp)\int\frac{x^{-1+n}(fx)^{1+m}}{d+ex^n}\,dx}{f(1+m)}\\ &= \frac{(fx)^{1+m}\log\left(c(d+ex^n)^p\right)}{f(1+m)} - \frac{\left(enpx^{-m}(fx)^m\right)\int\frac{x^{m+n}}{d+ex^n}\,dx}{1+m}\\ &= -\frac{enpx^{1+n}(fx)^m\,{}_2F_1\left(1,\frac{1+m+n}{n};\frac{1+m+2n}{n};-\frac{ex^n}{d}\right)}{d(1+m)(1+m+n)} + \frac{(fx)^{1+m}\log\left(c(d+ex^n)^p\right)}{f(1+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

$$\int(fx)^m\log\left(c(d+ex^n)^p\right)\,dx$$
$$=\frac{x(fx)^m\left(-enpx^n\,\text{Hypergeometric2F1}\left(1,\frac{1+m+n}{n},\frac{1+m+2n}{n},-\frac{ex^n}{d}\right)+d(1+m+n)\log\left(c(d+ex^n)^p\right)\right)}{d(1+m)(1+m+n)}$$

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^n)^p],x]
[Out] (x*(f*x)^m*(-(e*n*p*x^n*Hypergeometric2F1[1, (1 + m + n)/n, (1 + m + 2*n)/n
, -((e*x^n)/d)]) + d*(1 + m + n)*Log[c*(d + e*x^n)^p]))/(d*(1 + m)*(1 + m +
 n))
```

Maple [F]

$$\int (fx)^m \ln (c(d+e\,x^n)^p)\,dx$$

```
[In] int((f*x)^m*ln(c*(d+e*x^n)^p),x)
[Out] int((f*x)^m*ln(c*(d+e*x^n)^p),x)
```

Fricas [F]

$$\int (fx)^m \log (c(d+ex^n)^p)\ dx = \int (fx)^m \log ((ex^n+d)^p c)\ dx$$

```
[In] integrate((f*x)^m*log(c*(d+e*x^n)^p),x, algorithm="fricas")
[Out] integral((f*x)^m*log((e*x^n + d)^p*c), x)
```

Sympy [F]

$$\int (fx)^m \log (c(d+ex^n)^p)\ dx = \int (fx)^m \log (c(d+ex^n)^p)\,dx$$

```
[In] integrate((f*x)**m*ln(c*(d+e*x**n)**p),x)
[Out] Integral((f*x)**m*log(c*(d + e*x**n)**p), x)
```

Maxima [F]

$$\int (fx)^m \log (c(d+ex^n)^p)\ dx = \int (fx)^m \log ((ex^n+d)^p c)\ dx$$

```
[In] integrate((f*x)^m*log(c*(d+e*x^n)^p),x, algorithm="maxima")
[Out] d*f^m*n*p*integrate(x^m/(e*(m + 1)*x^n + d*(m + 1)), x) + (f^m*(m + 1)*x*x^
m*log((e*x^n + d)^p) - (f^m*n*p - f^m*(m + 1)*log(c))*x*x^m)/(m^2 + 2*m + 1
)
```

Giac [F]

$$\int (fx)^m \log(c(d+ex^n)^p)\,dx = \int (fx)^m \log((ex^n+d)^p c)\,dx$$

```
[In] integrate((f*x)^m*log(c*(d+e*x^n)^p),x, algorithm="giac")
[Out] integrate((f*x)^m*log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \log(c(d+ex^n)^p)\,dx = \int \ln\left(c\,(d+e\,x^n)^p\right)\,(f\,x)^m\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)*(f*x)^m,x)
[Out] int(log(c*(d + e*x^n)^p)*(f*x)^m, x)
```

3.64 $\int (fx)^{-1+3n} \log (c(d+ex^n)^p) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 141

$$\int (fx)^{-1+3n} \log (c(d+ex^n)^p) \, dx = -\frac{p(fx)^{3n}}{9fn} - \frac{d^2px^{-2n}(fx)^{3n}}{3e^2fn} + \frac{dpx^{-n}(fx)^{3n}}{6efn} + \frac{d^3px^{-3n}(fx)^{3n}\log (d+ex^n)}{3e^3fn} + \frac{(fx)^{3n}\log (c(d+ex^n)^p)}{3fn}$$

```
[Out] -1/9*p*(f*x)^(3*n)/f/n-1/3*d^2*p*(f*x)^(3*n)/e^2/f/n/(x^(2*n))+1/6*d*p*(f*x
)^(3*n)/e/f/n/(x^n)+1/3*d^3*p*(f*x)^(3*n)*ln(d+e*x^n)/e^3/f/n/(x^(3*n))+1/3
*(f*x)^(3*n)*ln(c*(d+e*x^n)^p)/f/n
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2505, 20, 272, 45}

$$\int (fx)^{-1+3n} \log (c(d+ex^n)^p) \, dx = \frac{(fx)^{3n}\log (c(d+ex^n)^p)}{3fn} + \frac{d^3px^{-3n}(fx)^{3n}\log (d+ex^n)}{3e^3fn} - \frac{d^2px^{-2n}(fx)^{3n}}{3e^2fn} + \frac{dpx^{-n}(fx)^{3n}}{6efn} - \frac{p(fx)^{3n}}{9fn}$$

```
[In] Int[(f*x)^(-1 + 3*n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -1/9*(p*(f*x)^(3*n))/(f*n) - (d^2*p*(f*x)^(3*n))/(3*e^2*f*n*x^(2*n)) + (d*p
*(f*x)^(3*n))/(6*e*f*n*x^n) + (d^3*p*(f*x)^(3*n)*Log[d + e*x^n])/(3*e^3*f*n
*x^(3*n)) + ((f*x)^(3*n)*Log[c*(d + e*x^n)^p])/(3*f*n)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(fx)^{3n}\log\left(c(d+ex^n)^p\right)}{3fn} - \frac{(ep)\int \frac{x^{-1+n}(fx)^{3n}}{d+ex^n}\,dx}{3f} \\
&= \frac{(fx)^{3n}\log\left(c(d+ex^n)^p\right)}{3fn} - \frac{\left(epx^{-3n}(fx)^{3n}\right)\int \frac{x^{-1+4n}}{d+ex^n}\,dx}{3f} \\
&= \frac{(fx)^{3n}\log\left(c(d+ex^n)^p\right)}{3fn} - \frac{\left(epx^{-3n}(fx)^{3n}\right)\operatorname{Subst}\left(\int \frac{x^3}{d+ex}\,dx, x, x^n\right)}{3fn} \\
&= \frac{(fx)^{3n}\log\left(c(d+ex^n)^p\right)}{3fn} - \frac{\left(epx^{-3n}(fx)^{3n}\right)\operatorname{Subst}\left(\int \left(\frac{d^2}{e^3}-\frac{dx}{e^2}+\frac{x^2}{e}-\frac{d^3}{e^3(d+ex)}\right)dx, x, x^n\right)}{3fn} \\
&= -\frac{p(fx)^{3n}}{9fn} - \frac{d^2px^{-2n}(fx)^{3n}}{3e^2fn} + \frac{dpx^{-n}(fx)^{3n}}{6efn} \\
&\quad + \frac{d^3px^{-3n}(fx)^{3n}\log(d+ex^n)}{3e^3fn} + \frac{(fx)^{3n}\log\left(c(d+ex^n)^p\right)}{3fn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.65

$$\int (fx)^{-1+3n}\log \left(c(d+ex^n)^p\right)\,dx$$
$$=\frac{x^{-3n}(fx)^{3n}\left(-epx^n(6d^2-3dex^n+2e^2x^{2n})+6d^3p\log\left(d+ex^n\right)+6e^3x^{3n}\log\left(c(d+ex^n)^p\right)\right)}{18e^3fn}$$

```
[In] Integrate[(f*x)^(-1 + 3*n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] ((f*x)^(3*n)*(-(e*p*x^n*(6*d^2 - 3*d*e*x^n + 2*e^2*x^(2*n))) + 6*d^3*p*Log[
d + e*x^n] + 6*e^3*x^(3*n)*Log[c*(d + e*x^n)^p]))/(18*e^3*f*n*x^(3*n))
```

Maple [F]

$$\int (fx)^{-1+3n}\ln\left(c(d+e\,x^n)^p\right)dx$$

```
[In] int((f*x)^(-1+3*n)*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int((f*x)^(-1+3*n)*ln(c*(d+e*x^n)^p),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.79

$$\int (fx)^{-1+3n}\log \left(c(d+ex^n)^p\right)\,dx$$
$$=\frac{3\,de^2f^{3\,n-1}px^{2\,n}-6\,d^2ef^{3\,n-1}px^n-2\,(e^3p-3\,e^3\log\left(c\right))f^{3\,n-1}x^{3\,n}+6\,(e^3f^{3\,n-1}px^{3\,n}+d^3f^{3\,n-1}p)\log\left(ex^n+\right.}{18\,e^3n}$$

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] 1/18*(3*d*e^2*f^(3*n - 1)*p*x^(2*n) - 6*d^2*e*f^(3*n - 1)*p*x^n - 2*(e^3*p
- 3*e^3*log(c))*f^(3*n - 1)*x^(3*n) + 6*(e^3*f^(3*n - 1)*p*x^(3*n) + d^3*f^
(3*n - 1)*p)*log(e*x^n + d))/(e^3*n)
```

Sympy [F]

$$\int (fx)^{-1+3n} \log \left(c(d+ex^n)^p\right) \, dx = \int (fx)^{3n-1} \log \left(c(d+ex^n)^p\right) dx$$

```
[In] integrate((f*x)**(-1+3*n)*ln(c*(d+e*x**n)**p),x)
[Out] Integral((f*x)**(3*n - 1)*log(c*(d + e*x**n)**p), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.82

$$\int (fx)^{-1+3n} \log \left(c(d+ex^n)^p\right) \, dx = \frac{ep\left(\frac{6\,d^3 f^{3\,n} \log\left(\frac{ex^n+d}{e}\right)}{e^4 n} - \frac{2\,e^2 f^{3\,n} x^{3\,n} - 3\,def^{3\,n} x^{2\,n} + 6\,d^2 f^{3\,n} x^n}{e^3 n}\right)}{18\,f} + \frac{(fx)^{3\,n} \log\left((ex^n+d)^p c\right)}{3\,fn}$$

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
[Out] 1/18*e*p*(6*d^3*f^(3*n)*log((e*x^n + d)/e)/(e^4*n) - (2*e^2*f^(3*n)*x^(3*n)
 - 3*d*e*f^(3*n)*x^(2*n) + 6*d^2*f^(3*n)*x^n)/(e^3*n))/f + 1/3*(f*x)^(3*n)*
log((e*x^n + d)^p*c)/(f*n)
```

Giac [F]

$$\int (fx)^{-1+3n} \log \left(c(d+ex^n)^p\right) \, dx = \int (fx)^{3\,n-1} \log \left((ex^n+d)^p c\right) \, dx$$

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
[Out] integrate((f*x)^(3*n - 1)*log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1+3n} \log \left(c(d+ex^n)^p\right)\,dx = \int \ln \left(c\left(d+e\,x^n\right)^p\right)\,\left(f\,x\right)^{3\,n-1}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)*(f*x)^(3*n - 1),x)
[Out] int(log(c*(d + e*x^n)^p)*(f*x)^(3*n - 1), x)
```

3.65 $\int (fx)^{-1+2n} \log \left(c(d+ex^n)^p\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 112

$$\int (fx)^{-1+2n} \log \left(c(d+ex^n)^p\right) \, dx = -\frac{p(fx)^{2n}}{4fn} + \frac{dpx^{-n}(fx)^{2n}}{2efn} - \frac{d^2px^{-2n}(fx)^{2n} \log (d+ex^n)}{2e^2fn} + \frac{(fx)^{2n} \log \left(c(d+ex^n)^p\right)}{2fn}$$

```
[Out] -1/4*p*(f*x)^(2*n)/f/n+1/2*d*p*(f*x)^(2*n)/e/f/n/(x^n)-1/2*d^2*p*(f*x)^(2*n
)*ln(d+e*x^n)/e^2/f/n/(x^(2*n))+1/2*(f*x)^(2*n)*ln(c*(d+e*x^n)^p)/f/n
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2505, 20, 272, 45}

$$\int (fx)^{-1+2n} \log \left(c(d+ex^n)^p\right) \, dx = \frac{(fx)^{2n} \log \left(c(d+ex^n)^p\right)}{2fn} - \frac{d^2px^{-2n}(fx)^{2n} \log (d+ex^n)}{2e^2fn} + \frac{dpx^{-n}(fx)^{2n}}{2efn} - \frac{p(fx)^{2n}}{4fn}$$

```
[In] Int[(f*x)^(-1 + 2*n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -1/4*(p*(f*x)^(2*n))/(f*n) + (d*p*(f*x)^(2*n))/(2*e*f*n*x^n) - (d^2*p*(f*x)
^(2*n)*Log[d + e*x^n])/(2*e^2*f*n*x^(2*n)) + ((f*x)^(2*n)*Log[c*(d + e*x^n)
^p])/(2*f*n)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
```

```
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(fx)^{2n}\log\left(c(d+ex^n)^p\right)}{2fn} - \frac{(ep)\int\frac{x^{-1+n}(fx)^{2n}}{d+ex^n}\,dx}{2f} \\
&= \frac{(fx)^{2n}\log\left(c(d+ex^n)^p\right)}{2fn} - \frac{\left(epx^{-2n}(fx)^{2n}\right)\int\frac{x^{-1+3n}}{d+ex^n}\,dx}{2f} \\
&= \frac{(fx)^{2n}\log\left(c(d+ex^n)^p\right)}{2fn} - \frac{\left(epx^{-2n}(fx)^{2n}\right)\mathrm{Subst}\left(\int\frac{x^2}{d+ex}\,dx,x,x^n\right)}{2fn} \\
&= \frac{(fx)^{2n}\log\left(c(d+ex^n)^p\right)}{2fn} - \frac{\left(epx^{-2n}(fx)^{2n}\right)\mathrm{Subst}\left(\int\left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)\,dx,x,x^n\right)}{2fn} \\
&= -\frac{p(fx)^{2n}}{4fn} + \frac{dpx^{-n}(fx)^{2n}}{2efn} - \frac{d^2px^{-2n}(fx)^{2n}\log\left(d+ex^n\right)}{2e^2fn} + \frac{(fx)^{2n}\log\left(c(d+ex^n)^p\right)}{2fn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.66

$$\int (fx)^{-1+2n}\log\left(c(d+ex^n)^p\right)\,dx$$
$$=-\frac{x^{-2n}(fx)^{2n}\left(2d^2p\log\left(d+ex^n\right)+ex^n(-2dp+epx^n-2ex^n\log\left(c(d+ex^n)^p\right))\right)}{4e^2fn}$$

```
[In] Integrate[(f*x)^(-1 + 2*n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -1/4*((f*x)^(2*n)*(2*d^2*p*Log[d + e*x^n] + e*x^n*(-2*d*p + e*p*x^n - 2*e*x
^n*Log[c*(d + e*x^n)^p])))/(e^2*f*n*x^(2*n))
```

Maple [F]

$$\int (fx)^{-1+2n}\ln\left(c(d+e\,x^n)^p\right)dx$$

```
[In] int((f*x)^(-1+2*n)*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int((f*x)^(-1+2*n)*ln(c*(d+e*x^n)^p),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.82

$$\int (fx)^{-1+2n}\log\left(c(d+ex^n)^p\right)\,dx$$
$$=\frac{2\,def^{2\,n-1}px^n-\left(e^2p-2\,e^2\log\left(c\right)\right)f^{2\,n-1}x^{2\,n}+2\left(e^2f^{2\,n-1}px^{2\,n}-d^2f^{2\,n-1}p\right)\log\left(ex^n+d\right)}{4\,e^2n}$$

```
[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d*e*f^(2*n - 1)*p*x^n - (e^2*p - 2*e^2*log(c))*f^(2*n - 1)*x^(2*n) +
 2*(e^2*f^(2*n - 1)*p*x^(2*n) - d^2*f^(2*n - 1)*p)*log(e*x^n + d))/(e^2*n)
```

Sympy [F]

$$\int (fx)^{-1+2n} \log \left(c(d+ex^n)^p\right) \, dx = \int (fx)^{2n-1} \log \left(c(d+ex^n)^p\right) dx$$

```
[In] integrate((f*x)**(-1+2*n)*ln(c*(d+e*x**n)**p),x)
[Out] Integral((f*x)**(2*n - 1)*log(c*(d + e*x**n)**p), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.85

$$\int (fx)^{-1+2n} \log \left(c(d+ex^n)^p\right) \, dx = -\frac{ep\left(\frac{2\,d^2 f^{2\,n} \log\left(\frac{ex^n+d}{e}\right)}{e^3 n} + \frac{ef^{2\,n}x^{2\,n} - 2\,df^{2\,n}x^n}{e^2 n}\right)}{4\,f} + \frac{(fx)^{2\,n} \log\left((ex^n+d)^p c\right)}{2\,fn}$$

```
[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
[Out] -1/4*e*p*(2*d^2*f^(2*n)*log((e*x^n + d)/e)/(e^3*n) + (e*f^(2*n)*x^(2*n) - 2
*d*f^(2*n)*x^n)/(e^2*n))/f + 1/2*(f*x)^(2*n)*log((e*x^n + d)^p*c)/(f*n)
```

Giac [F]

$$\int (fx)^{-1+2n} \log \left(c(d+ex^n)^p\right) \, dx = \int (fx)^{2\,n-1} \log \left((ex^n+d)^p c\right) \, dx$$

```
[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
[Out] integrate((f*x)^(2*n - 1)*log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1+2n} \log \left(c(d+ex^n)^p\right) \, dx = \int \ln \left(c\,(d+e\,x^n)^p\right) \, (f\,x)^{2\,n-1} \, dx$$

```
[In] int(log(c*(d + e*x^n)^p)*(f*x)^(2*n - 1),x)
[Out] int(log(c*(d + e*x^n)^p)*(f*x)^(2*n - 1), x)
```

3.66 $\int (fx)^{-1+n}\log \left(c(d+ex^n)^p\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int (fx)^{-1+n}\log \left(c(d+ex^n)^p\right)\,dx=-\frac{p(fx)^n}{fn}+\frac{dpx^{-n}(fx)^n\log(d+ex^n)}{efn}+\frac{(fx)^n\log\left(c(d+ex^n)^p\right)}{fn}$$

```
[Out] -p*(f*x)^n/f/n+d*p*(f*x)^n*ln(d+e*x^n)/e/f/n/(x^n)+(f*x)^n*ln(c*(d+e*x^n)^p
)/f/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2505, 20, 272, 45}

$$\int (fx)^{-1+n}\log \left(c(d+ex^n)^p\right)\,dx=\frac{(fx)^n\log\left(c(d+ex^n)^p\right)}{fn}+\frac{dpx^{-n}(fx)^n\log(d+ex^n)}{efn}-\frac{p(fx)^n}{fn}$$

```
[In] Int[(f*x)^(-1 + n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((p*(f*x)^n)/(f*n)) + (d*p*(f*x)^n*Log[d + e*x^n])/(e*f*n*x^n) + ((f*x)^n*
Log[c*(d + e*x^n)^p])/(f*n)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
```

```
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(fx)^n \log\left(c(d+ex^n)^p\right)}{fn} - \frac{(ep)\int \frac{x^{-1+n}(fx)^n}{d+ex^n}\,dx}{f} \\
&= \frac{(fx)^n \log\left(c(d+ex^n)^p\right)}{fn} - \frac{\left(epx^{-n}(fx)^n\right)\int \frac{x^{-1+2n}}{d+ex^n}\,dx}{f} \\
&= \frac{(fx)^n \log\left(c(d+ex^n)^p\right)}{fn} - \frac{\left(epx^{-n}(fx)^n\right)\operatorname{Subst}\left(\int \frac{x}{d+ex}\,dx, x, x^n\right)}{fn} \\
&= \frac{(fx)^n \log\left(c(d+ex^n)^p\right)}{fn} - \frac{\left(epx^{-n}(fx)^n\right)\operatorname{Subst}\left(\int \left(\frac{1}{e} - \frac{d}{e(d+ex)}\right)dx, x, x^n\right)}{fn} \\
&= -\frac{p(fx)^n}{fn} + \frac{dpx^{-n}(fx)^n \log(d+ex^n)}{efn} + \frac{(fx)^n \log\left(c(d+ex^n)^p\right)}{fn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.70

$$\int (fx)^{-1+n}\log\left(c(d+ex^n)^p\right)\,dx=\frac{x^{1-n}(fx)^{-1+n}\left(-px^n+\frac{(d+ex^n)\log(c(d+ex^n)^p)}{e}\right)}{n}$$

```
[In] Integrate[(f*x)^(-1 + n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] (x^(1 - n)*(f*x)^(-1 + n)*(-(p*x^n) + ((d + e*x^n)*Log[c*(d + e*x^n)^p])/e)
)/n
```

Maple [F]

$$\int (fx)^{n-1}\ln\left(c(d+e\,x^n)^p\right)dx$$

```
[In] int((f*x)^(n-1)*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int((f*x)^(n-1)*ln(c*(d+e*x^n)^p),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int (fx)^{-1+n}\log\left(c(d+ex^n)^p\right)\,dx$$
$$=-\frac{(ep-e\log(c))f^{n-1}x^n-(ef^{n-1}px^n+df^{n-1}p)\log(ex^n+d)}{en}$$

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] -((e*p - e*log(c))*f^(n - 1)*x^n - (e*f^(n - 1)*p*x^n + d*f^(n - 1)*p)*log(
e*x^n + d))/(e*n)
```

Sympy [F]

$$\int (fx)^{-1+n}\log\left(c(d+ex^n)^p\right)\,dx=\int (fx)^{n-1}\log\left(c(d+ex^n)^p\right)dx$$

```
[In] integrate((f*x)**(-1+n)*ln(c*(d+e*x**n)**p),x)
```

```
[Out] Integral((f*x)**(n - 1)*log(c*(d + e*x**n)**p), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int (fx)^{-1+n} \log \left(c(d+ex^n)^p\right) \, dx = -\frac{ep\left(\frac{f^n x^n}{en} - \frac{df^n \log\left(\frac{ex^n+d}{e}\right)}{e^2 n}\right)}{f} + \frac{(fx)^n \log\left((ex^n+d)^p c\right)}{fn}$$

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] -e*p*(f^n*x^n/(e*n) - d*f^n*log((e*x^n + d)/e)/(e^2*n))/f + (f*x)^n*log((e*
x^n + d)^p*c)/(f*n)
```

Giac [F]

$$\int (fx)^{-1+n} \log \left(c(d+ex^n)^p\right) \, dx = \int (fx)^{n-1} \log \left((ex^n+d)^p c\right) \, dx$$

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^(n - 1)*log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1+n} \log \left(c(d+ex^n)^p\right) \, dx = \int \ln \left(c\,{\left(d+e\,x^n\right)}^p\right)\,{\left(f\,x\right)}^{n-1} \,d x$$

```
[In] int(log(c*(d + e*x^n)^p)*(f*x)^(n - 1),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)*(f*x)^(n - 1), x)
```

3.67 $\int \frac{\log(c(d+ex^n)^p)}{fx}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{fx}\,dx=\frac{\log \left(-\frac{ex^n}{d}\right)\log \left(c(d+ex^n)^p\right)}{fn}+\frac{p\,\mathrm{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{fn}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/f/n+p*polylog(2,1+e*x^n/d)/f/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {12, 2504, 2441, 2352}

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{fx}\,dx=\frac{\log \left(-\frac{ex^n}{d}\right)\log \left(c(d+ex^n)^p\right)}{fn}+\frac{p\,\mathrm{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{fn}$$

```
[In] Int[Log[c*(d + e*x^n)^p]/(f*x),x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f*n) + (p*PolyLog[2, 1 + (e*x^n)/
d])/(f*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{\log(c(d+ex^n)^p)}{x}\,dx}{f}\\
&= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{fn}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{fn} - \frac{(ep)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^n\right)}{fn}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{fn} + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{fx}\,dx = \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right) + p\,\text{PolyLog}\left(2, \frac{d+ex^n}{d}\right)}{fn}$$

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(f*x),x]
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*PolyLog[2, (d + e*x^n)/d])/(f*n
)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.64

method	result
risch	$\frac{\ln(x)\ln((d+e\,x^{n})^{p})}{f}+\frac{\left(\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^{n})^{p})\mathrm{csgn}(ic(d+e\,x^{n})^{p})^{2}}{2}-\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^{n})^{p})\ \mathrm{csgn}(ic(d+e\,x^{n})^{p})\ \mathrm{csgn}(ic)}{2}-\frac{i\pi\mathrm{csgn}(ic(d+e\,x^{n})^{p})^{3}}{2}+\right.}{f}$

```
[In] int(ln(c*(d+e*x^n)^p)/f/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*ln(x)*ln((d+e*x^n)^p)+1/f*(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x
^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I
*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c
))*ln(x)-1/f*p/n*dilog((d+e*x^n)/d)-1/f*p*ln(x)*ln((d+e*x^n)/d)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{fx}\,dx = \frac{np\log\left(ex^n+d\right)\log\left(x\right)-np\log\left(x\right)\log\left(\frac{ex^n+d}{d}\right)+n\log\left(c\right)\log\left(x\right)-p\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)}{fn}$$

```
[In] integrate(log(c*(d+e*x^n)^p)/f/x,x, algorithm="fricas")
```

```
[Out] (n*p*log(e*x^n + d)*log(x) - n*p*log(x)*log((e*x^n + d)/d) + n*log(c)*log(x
) - p*dilog(-(e*x^n + d)/d + 1))/(f*n)
```

Sympy [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{fx}\,dx = \frac{\int \frac{\log\left(c(d+ex^n)^p\right)}{x}\,dx}{f}$$

```
[In] integrate(ln(c*(d+e*x**n)**p)/f/x,x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)/x, x)/f
```

Maxima [F]

$$\int \frac{\log(c(d+ex^n)^p)}{fx}\,dx = \int \frac{\log((ex^n+d)^p c)}{fx}\,dx$$

[In] integrate(log(c*(d+e*x^n)^p)/f/x,x, algorithm="maxima")

[Out] 1/2*(2*d*n*p*integrate(log(x)/(e*x*x^n + d*x), x) - n*p*log(x)^2 + 2*log((e*x^n + d)^p)*log(x) + 2*log(c)*log(x))/f

Giac [F]

$$\int \frac{\log(c(d+ex^n)^p)}{fx}\,dx = \int \frac{\log((ex^n+d)^p c)}{fx}\,dx$$

[In] integrate(log(c*(d+e*x^n)^p)/f/x,x, algorithm="giac")

[Out] integrate(log((e*x^n + d)^p*c)/(f*x), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c(d+ex^n)^p)}{fx}\,dx = \int \frac{\ln(c\,(d+e\,x^n)^p)}{f\,x}\,dx$$

[In] int(log(c*(d + e*x^n)^p)/(f*x),x)

[Out] int(log(c*(d + e*x^n)^p)/(f*x), x)

3.68 $\int (fx)^{-1-n}\log\left(c(d+ex^n)^p\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 80

$$\int (fx)^{-1-n}\log\left(c(d+ex^n)^p\right)\,dx=\frac{epx^n(fx)^{-n}\log(x)}{df}-\frac{epx^n(fx)^{-n}\log\left(d+ex^n\right)}{dfn}-\frac{(fx)^{-n}\log\left(c(d+ex^n)^p\right)}{fn}$$

```
[Out] e*p*x^n*ln(x)/d/f/((f*x)^n)-e*p*x^n*ln(d+e*x^n)/d/f/n/((f*x)^n)-ln(c*(d+e*x
^n)^p)/f/n/((f*x)^n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2505, 19, 272, 36, 29, 31}

$$\int (fx)^{-1-n}\log\left(c(d+ex^n)^p\right)\,dx=-\frac{(fx)^{-n}\log\left(c(d+ex^n)^p\right)}{fn}+\frac{epx^n\log(x)(fx)^{-n}}{df}-\frac{epx^n(fx)^{-n}\log\left(d+ex^n\right)}{dfn}$$

```
[In] Int[(f*x)^(-1 - n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] (e*p*x^n*Log[x])/(d*f*(f*x)^n) - (e*p*x^n*Log[d + e*x^n])/(d*f*n*(f*x)^n) -
 Log[c*(d + e*x^n)^p]/(f*n*(f*x)^n)
```

Rule 19

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + n)*
((b*v)^n/(a*v)^n), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !I
```

```
ntegerQ[m] &&  !IntegerQ[n] && IntegerQ[m + n]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(fx)^{-n}\log\left(c(d+ex^n)^p\right)}{fn}+\frac{(ep)\int \frac{x^{-1+n}(fx)^{-n}}{d+ex^n}\,dx}{f}\\
&= -\frac{(fx)^{-n}\log\left(c(d+ex^n)^p\right)}{fn}+\frac{\left(epx^n(fx)^{-n}\right)\int \frac{1}{x(d+ex^n)}\,dx}{f}\\
&= -\frac{(fx)^{-n}\log\left(c(d+ex^n)^p\right)}{fn}+\frac{\left(epx^n(fx)^{-n}\right)\text{Subst}\left(\int \frac{1}{x(d+ex)}\,dx,x,x^n\right)}{fn}\\
&= -\frac{(fx)^{-n}\log\left(c(d+ex^n)^p\right)}{fn}+\frac{\left(epx^n(fx)^{-n}\right)\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^n\right)}{dfn}\\
&\quad -\frac{\left(e^2px^n(fx)^{-n}\right)\text{Subst}\left(\int \frac{1}{d+ex}\,dx,x,x^n\right)}{dfn}\\
&= \frac{epx^n(fx)^{-n}\log(x)}{df}-\frac{epx^n(fx)^{-n}\log\left(d+ex^n\right)}{dfn}-\frac{(fx)^{-n}\log\left(c(d+ex^n)^p\right)}{fn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int (fx)^{-1-n}\log\left(c(d+ex^n)^p\right)\,dx$$
$$=-\frac{(fx)^{-n}\left(-enpx^n\log(x)+epx^n\log\left(d+ex^n\right)+d\log\left(c(d+ex^n)^p\right)\right)}{dfn}$$

```
[In] Integrate[(f*x)^(-1 - n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((-(e*n*p*x^n*Log[x]) + e*p*x^n*Log[d + e*x^n] + d*Log[c*(d + e*x^n)^p])/(
d*f*n*(f*x)^n))
```

Maple [F]

$$\int (fx)^{-1-n}\ln\left(c(d+e\,x^n)^p\right)dx$$

```
[In] int((f*x)^(-1-n)*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int((f*x)^(-1-n)*ln(c*(d+e*x^n)^p),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int (fx)^{-1-n}\log\left(c(d+ex^n)^p\right)\,dx$$
$$=\frac{ef^{-n-1}npx^n\log\left(x\right)-df^{-n-1}\log\left(c\right)-\left(ef^{-n-1}px^n+df^{-n-1}p\right)\log\left(ex^n+d\right)}{dnx^n}$$

```
[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] (e*f^(-n - 1)*n*p*x^n*log(x) - d*f^(-n - 1)*log(c) - (e*f^(-n - 1)*p*x^n +
d*f^(-n - 1)*p)*log(e*x^n + d))/(d*n*x^n)
```

Sympy [F(-2)]

Exception generated.

$$\int (fx)^{-1-n} \log (c(d+ex^n)^p) \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f*x)**(-1-n)*ln(c*(d+e*x**n)**p),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int (fx)^{-1-n} \log (c(d+ex^n)^p) \, dx = \frac{ep\left(\frac{\log(x)}{df^n} - \frac{\log\left(\frac{ex^n+d}{e}\right)}{df^n n}\right)}{f} - \frac{\log((ex^n+d)^p c)}{(fx)^n fn}$$

```
[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
[Out] e*p*(log(x)/(d*f^n) - log((e*x^n + d)/e)/(d*f^n*n))/f - log((e*x^n + d)^p*c
)/((f*x)^n*f*n)
```

Giac [F]

$$\int (fx)^{-1-n} \log (c(d+ex^n)^p) \, dx = \int (fx)^{-n-1} \log ((ex^n+d)^p c) \, dx$$

```
[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
[Out] integrate((f*x)^(-n - 1)*log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1-n} \log (c(d+ex^n)^p) \, dx = \int \frac{\ln (c\,(d+e\,x^n)^p)}{(f\,x)^{n+1}} \,dx$$

```
[In] int(log(c*(d + e*x^n)^p)/(f*x)^(n + 1),x)
[Out] int(log(c*(d + e*x^n)^p)/(f*x)^(n + 1), x)
```

3.69 $\int (fx)^{-1-2n} \log (c(d+ex^n)^p) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 120

$$\int (fx)^{-1-2n} \log (c(d+ex^n)^p) \, dx = -\frac{epx^n(fx)^{-2n}}{2dfn} - \frac{e^2px^{2n}(fx)^{-2n}\log(x)}{2d^2f} + \frac{e^2px^{2n}(fx)^{-2n}\log(d+ex^n)}{2d^2fn} - \frac{(fx)^{-2n}\log(c(d+ex^n)^p)}{2fn}$$

```
[Out] -1/2*e*p*x^n/d/f/n/((f*x)^(2*n))-1/2*e^2*p*x^(2*n)*ln(x)/d^2/f/((f*x)^(2*n)
)+1/2*e^2*p*x^(2*n)*ln(d+e*x^n)/d^2/f/n/((f*x)^(2*n))-1/2*ln(c*(d+e*x^n)^p)
/f/n/((f*x)^(2*n))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2505, 20, 272, 46}

$$\int (fx)^{-1-2n} \log (c(d+ex^n)^p) \, dx = -\frac{(fx)^{-2n}\log(c(d+ex^n)^p)}{2fn} - \frac{e^2px^{2n}\log(x)(fx)^{-2n}}{2d^2f} + \frac{e^2px^{2n}(fx)^{-2n}\log(d+ex^n)}{2d^2fn} - \frac{epx^n(fx)^{-2n}}{2dfn}$$

```
[In] Int[(f*x)^(-1 - 2*n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -1/2*(e*p*x^n)/(d*f*n*(f*x)^(2*n)) - (e^2*p*x^(2*n)*Log[x])/(2*d^2*f*(f*x)^
(2*n)) + (e^2*p*x^(2*n)*Log[d + e*x^n])/(2*d^2*f*n*(f*x)^(2*n)) - Log[c*(d
+ e*x^n)^p]/(2*f*n*(f*x)^(2*n))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(fx)^{-2n}\log\left(c(d+ex^n)^p\right)}{2fn}+\frac{(ep)\int\frac{x^{-1+n}(fx)^{-2n}}{d+ex^n}\,dx}{2f}\\
&= -\frac{(fx)^{-2n}\log\left(c(d+ex^n)^p\right)}{2fn}+\frac{\left(epx^{2n}(fx)^{-2n}\right)\int\frac{x^{-1-n}}{d+ex^n}\,dx}{2f}\\
&= -\frac{(fx)^{-2n}\log\left(c(d+ex^n)^p\right)}{2fn}+\frac{\left(epx^{2n}(fx)^{-2n}\right)\text{Subst}\left(\int\frac{1}{x^2(d+ex)}\,dx,x,x^n\right)}{2fn}\\
&= -\frac{(fx)^{-2n}\log\left(c(d+ex^n)^p\right)}{2fn}+\frac{\left(epx^{2n}(fx)^{-2n}\right)\text{Subst}\left(\int\left(\frac{1}{dx^2}-\frac{e}{d^2x}+\frac{e^2}{d^2(d+ex)}\right)dx,x,x^n\right)}{2fn}\\
&= -\frac{epx^n(fx)^{-2n}}{2dfn}-\frac{e^2px^{2n}(fx)^{-2n}\log(x)}{2d^2f}\\
&\quad+\frac{e^2px^{2n}(fx)^{-2n}\log\left(d+ex^n\right)}{2d^2fn}-\frac{(fx)^{-2n}\log\left(c(d+ex^n)^p\right)}{2fn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.63

$$\int (fx)^{-1-2n}\log\left(c(d+ex^n)^p\right)\,dx = -\frac{(fx)^{-2n}\left(e^2npx^{2n}\log(x)-e^2px^{2n}\log\left(d+ex^n\right)+d(epx^n+d\log\left(c(d+ex^n)^p\right))\right)}{2d^2fn}$$

```
[In] Integrate[(f*x)^(-1 - 2*n)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -1/2*(e^2*n*p*x^(2*n)*Log[x] - e^2*p*x^(2*n)*Log[d + e*x^n] + d*(e*p*x^n +
d*Log[c*(d + e*x^n)^p]))/(d^2*f*n*(f*x)^(2*n))
```

Maple [F]

$$\int (fx)^{-1-2n}\ln\left(c(d+e\,x^n)^p\right)dx$$

```
[In] int((f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int((f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int (fx)^{-1-2n}\log\left(c(d+ex^n)^p\right)\,dx = -\frac{e^2f^{-2\,n-1}npx^{2\,n}\log\left(x\right)+def^{-2\,n-1}px^n+d^2f^{-2\,n-1}\log\left(c\right)-\left(e^2f^{-2\,n-1}px^{2\,n}-d^2f^{-2\,n-1}p\right)\log\left(ex^n+d\right)}{2\,d^2nx^{2\,n}}$$

```
[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] -1/2*(e^2*f^(-2*n - 1)*n*p*x^(2*n)*log(x) + d*e*f^(-2*n - 1)*p*x^n + d^2*f^
(-2*n - 1)*log(c) - (e^2*f^(-2*n - 1)*p*x^(2*n) - d^2*f^(-2*n - 1)*p)*log(e
*x^n + d))/(d^2*n*x^(2*n))
```

Sympy [F(-2)]

Exception generated.

$$\int (fx)^{-1-2n}\log\left(c(d+ex^n)^p\right)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((f*x)**(-1-2*n)*ln(c*(d+e*x**n)**p),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.82

$$\int (fx)^{-1-2n}\log\left(c(d+ex^n)^p\right)\,dx = -\frac{ep\left(\frac{e\log(x)}{d^2f^{2n}} - \frac{e\log\left(\frac{ex^n+d}{e}\right)}{d^2f^{2n}n} + \frac{1}{df^{2n}nx^n}\right)}{2f} - \frac{\log\left((ex^n+d)^pc\right)}{2\,(fx)^{2n}\,fn}$$

```
[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
[Out] -1/2*e*p*(e*log(x)/(d^2*f^(2*n)) - e*log((e*x^n + d)/e)/(d^2*f^(2*n)*n) + 1
/(d*f^(2*n)*n*x^n))/f - 1/2*log((e*x^n + d)^p*c)/((f*x)^(2*n)*f*n)
```

Giac [F]

$$\int (fx)^{-1-2n}\log\left(c(d+ex^n)^p\right)\,dx = \int (fx)^{-2n-1}\log\left((ex^n+d)^pc\right)\,dx$$

```
[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p),x, algorithm="giac")
[Out] integrate((f*x)^(-2*n - 1)*log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1-2n}\log\left(c(d+ex^n)^p\right)\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)}{(f\,x)^{2\,n+1}}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)/(f*x)^(2*n + 1),x)
[Out] int(log(c*(d + e*x^n)^p)/(f*x)^(2*n + 1), x)
```

3.70 $\int x^2 \log (c(d+ex^n)^p)\, dx$

Optimal result

Integrand size = 16, antiderivative size = 65

$$\int x^2 \log \left(c(d+ex^n)^p\right)\, dx = -\frac{enpx^{3+n}\,\text{Hypergeometric2F1}\left(1,\frac{3+n}{n},2+\frac{3}{n},-\frac{ex^n}{d}\right)}{3d(3+n)} + \frac{1}{3}x^3 \log \left(c(d+ex^n)^p\right)$$

```
[Out] -1/3*e*n*p*x^(3+n)*hypergeom([1, (3+n)/n],[2+3/n],-e*x^n/d)/d/(3+n)+1/3*x^3
*ln(c*(d+e*x^n)^p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2505, 371}

$$\int x^2 \log \left(c(d+ex^n)^p\right)\, dx = \frac{1}{3}x^3 \log \left(c(d+ex^n)^p\right) - \frac{enpx^{n+3}\,\text{Hypergeometric2F1}\left(1,\frac{n+3}{n},2+\frac{3}{n},-\frac{ex^n}{d}\right)}{3d(n+3)}$$

```
[In] Int[x^2*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -1/3*(e*n*p*x^(3 + n)*Hypergeometric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)
])/(d*(3 + n)) + (x^3*Log[c*(d + e*x^n)^p])/3
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3 \log \left(c(d+ex^n)^p\right) - \frac{1}{3}(enp) \int \frac{x^{2+n}}{d+ex^n}\,dx \\
&= -\frac{enpx^{3+n}\,{}_2F_1\left(1,\frac{3+n}{n};2+\frac{3}{n};-\frac{ex^n}{d}\right)}{3d(3+n)} + \frac{1}{3}x^3 \log \left(c(d+ex^n)^p\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$
\int x^2 \log \left(c(d+ex^n)^p\right)\,dx = \frac{1}{3}x^3 \left(-\frac{enpx^n \operatorname{Hypergeometric2F1}\left(1,\frac{3+n}{n},2+\frac{3}{n},-\frac{ex^n}{d}\right)}{d(3+n)} + \log \left(c(d+ex^n)^p\right)\right)
$$

```
[In] Integrate[x^2*Log[c*(d + e*x^n)^p],x]
[Out] (x^3*(-((e*n*p*x^n*Hypergeometric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/
(d*(3 + n))) + Log[c*(d + e*x^n)^p]))/3
```

Maple [F]

$$
\int x^2 \ln \left(c(d+e\,x^n)^p\right) dx
$$

```
[In] int(x^2*ln(c*(d+e*x^n)^p),x)
[Out] int(x^2*ln(c*(d+e*x^n)^p),x)
```

Fricas [F]

$$\int x^2 \log \left(c(d+ex^n)^p\right) \, dx = \int x^2 \log \left((ex^n+d)^p c\right) \, dx$$

[In] integrate(x^2*log(c*(d+e*x^n)^p),x, algorithm="fricas")

[Out] integral(x^2*log((e*x^n + d)^p*c), x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.88 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.97

$$\int x^2 \log \left(c(d+ex^n)^p\right) \, dx = -\frac{d^{-2-\frac{3}{n}} d^{1+\frac{3}{n}} epx^{n+3} \Phi\left(\frac{ex^n e^{i\pi}}{d}, 1, 1+\frac{3}{n}\right) \Gamma\left(1+\frac{3}{n}\right)}{3\Gamma\left(2+\frac{3}{n}\right)} - \frac{d^{-2-\frac{3}{n}} d^{1+\frac{3}{n}} epx^{n+3} \Phi\left(\frac{ex^n e^{i\pi}}{d}, 1, 1+\frac{3}{n}\right) \Gamma\left(1+\frac{3}{n}\right)}{n\Gamma\left(2+\frac{3}{n}\right)} + \frac{x^3 \log \left(c(d+ex^n)^p\right)}{3}$$

[In] integrate(x**2*ln(c*(d+e*x**n)**p),x)

[Out] -d**(-2 - 3/n)*d**(1 + 3/n)*e*p*x**(n + 3)*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 3/n)*gamma(1 + 3/n)/(3*gamma(2 + 3/n)) - d**(-2 - 3/n)*d**(1 + 3/n)*e*p*x**(n + 3)*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 3/n)*gamma(1 + 3/n)/(n*gamma(2 + 3/n)) + x**3*log(c*(d + e*x**n)**p)/3

Maxima [F]

$$\int x^2 \log \left(c(d+ex^n)^p\right) \, dx = \int x^2 \log \left((ex^n+d)^p c\right) \, dx$$

[In] integrate(x^2*log(c*(d+e*x^n)^p),x, algorithm="maxima")

[Out] -1/9*(n*p - 3*log(c))*x^3 + d*n*p*integrate(1/3*x^2/(e*x^n + d), x) + 1/3*x^3*log((e*x^n + d)^p)

Giac [F]

$$\int x^2 \log\left(c(d+ex^n)^p\right)\,dx = \int x^2 \log\left((ex^n+d)^p c\right)\,dx$$

```
[In] integrate(x^2*log(c*(d+e*x^n)^p),x, algorithm="giac")
[Out] integrate(x^2*log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \log\left(c(d+ex^n)^p\right)\,dx = \int x^2\,\ln\left(c\,(d+e\,x^n)^p\right)\,dx$$

```
[In] int(x^2*log(c*(d + e*x^n)^p),x)
[Out] int(x^2*log(c*(d + e*x^n)^p), x)
```

3.71 $\int x \log \left(c(d+e x^n)^p\right) dx$

Optimal result

Integrand size = 14, antiderivative size = 65

$$\int x \log \left(c(d+e x^n)^p\right) dx = -\frac{enpx^{2+n}\,\text{Hypergeometric2F1}\left(1, \frac{2+n}{n}, 2\left(1+\frac{1}{n}\right), -\frac{ex^n}{d}\right)}{2d(2+n)} + \frac{1}{2}x^2 \log \left(c(d+e x^n)^p\right)$$

```
[Out] -1/2*e*n*p*x^(2+n)*hypergeom([1, (2+n)/n],[2+2/n],-e*x^n/d)/d/(2+n)+1/2*x^2
*ln(c*(d+e*x^n)^p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2505, 371}

$$\int x \log \left(c(d+e x^n)^p\right) dx = \frac{1}{2}x^2 \log \left(c(d+e x^n)^p\right) - \frac{enpx^{n+2}\,\text{Hypergeometric2F1}\left(1, \frac{n+2}{n}, 2\left(1+\frac{1}{n}\right), -\frac{ex^n}{d}\right)}{2d(n+2)}$$

```
[In] Int[x*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -1/2*(e*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/n, 2*(1 + n^(-1)), -((e*
x^n)/d)])/(d*(2 + n)) + (x^2*Log[c*(d + e*x^n)^p])/2
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2\log\left(c(d+ex^n)^p\right)-\frac{1}{2}(enp)\int\frac{x^{1+n}}{d+ex^n}\,dx\\ &= -\frac{enpx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(2+n)}+\frac{1}{2}x^2\log\left(c(d+ex^n)^p\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int x\log\left(c(d+ex^n)^p\right)\,dx=\frac{1}{2}x^2\left(-\frac{enpx^n\,\text{Hypergeometric2F1}\left(1,\frac{2+n}{n},2+\frac{2}{n},-\frac{ex^n}{d}\right)}{d(2+n)}+\log\left(c(d+ex^n)^p\right)\right)$$

```
[In] Integrate[x*Log[c*(d + e*x^n)^p],x]
[Out] (x^2*(-((e*n*p*x^n*Hypergeometric2F1[1, (2 + n)/n, 2 + 2/n, -((e*x^n)/d)])/
(d*(2 + n))) + Log[c*(d + e*x^n)^p]))/2
```

Maple [F]

$$\int x\ln\left(c\left(d+e\,x^n\right)^p\right)dx$$

```
[In] int(x*ln(c*(d+e*x^n)^p),x)
[Out] int(x*ln(c*(d+e*x^n)^p),x)
```

Fricas [F]

$$\int x \log \left(c(d+ex^n)^p\right) \, dx = \int x \log \left((ex^n+d)^p c\right) \, dx$$

[In] integrate(x*log(c*(d+e*x^n)^p),x, algorithm="fricas")

[Out] integral(x*log((e*x^n + d)^p*c), x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.88 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.97

$$\int x \log \left(c(d+ex^n)^p\right) \, dx = -\frac{d^{-2-\frac{2}{n}} d^{1+\frac{2}{n}} e p x^{n+2} \Phi\left(\frac{e x^n e^{i\pi}}{d}, 1, 1+\frac{2}{n}\right) \Gamma\left(1+\frac{2}{n}\right)}{2\Gamma\left(2+\frac{2}{n}\right)} - \frac{d^{-2-\frac{2}{n}} d^{1+\frac{2}{n}} e p x^{n+2} \Phi\left(\frac{e x^n e^{i\pi}}{d}, 1, 1+\frac{2}{n}\right) \Gamma\left(1+\frac{2}{n}\right)}{n\Gamma\left(2+\frac{2}{n}\right)} + \frac{x^2 \log \left(c(d+ex^n)^p\right)}{2}$$

[In] integrate(x*ln(c*(d+e*x**n)**p),x)

[Out] -d**(-2 - 2/n)*d**(1 + 2/n)*e*p*x**(n + 2)*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(2*gamma(2 + 2/n)) - d**(-2 - 2/n)*d**(1 + 2/n)*e*p*x**(n + 2)*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(n*gamma(2 + 2/n)) + x**2*log(c*(d + e*x**n)**p)/2

Maxima [F]

$$\int x \log \left(c(d+ex^n)^p\right) \, dx = \int x \log \left((ex^n+d)^p c\right) \, dx$$

[In] integrate(x*log(c*(d+e*x^n)^p),x, algorithm="maxima")

[Out] d*n*p*integrate(1/2*x/(e*x^n + d), x) - 1/4*(n*p - 2*log(c))*x^2 + 1/2*x^2*log((e*x^n + d)^p)

Giac [F]

$$\int x \log (c(d+ex^n)^p) \, dx = \int x \log ((ex^n+d)^p c) \, dx$$

```
[In] integrate(x*log(c*(d+e*x^n)^p),x, algorithm="giac")
[Out] integrate(x*log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \log (c(d+ex^n)^p) \, dx = \int x \ln (c (d+e x^n)^p) \, dx$$

```
[In] int(x*log(c*(d + e*x^n)^p),x)
[Out] int(x*log(c*(d + e*x^n)^p), x)
```

3.72 $\int \log (c(d+ex^n)^p)\, dx$

Optimal result

Integrand size = 12, antiderivative size = 54

$$\int \log (c(d+ex^n)^p)\, dx = -\frac{enpx^{1+n}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(1+n)} + x\log (c(d+ex^n)^p)$$

```
[Out] -e*n*p*x^(1+n)*hypergeom([1, 1+1/n],[2+1/n],-e*x^n/d)/d/(1+n)+x*ln(c*(d+e*x
^n)^p)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2498, 371}

$$\int \log (c(d+ex^n)^p)\, dx = x\log (c(d+ex^n)^p) - \frac{enpx^{n+1}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(n+1)}$$

```
[In] Int[Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)/d
)])/(d*(1 + n))) + x*Log[c*(d + e*x^n)^p]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\log\left(c(d+ex^n)^p\right)-(enp)\int\frac{x^n}{d+ex^n}\,dx\\ &= -\frac{enpx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)}+x\log\left(c(d+ex^n)^p\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int\log\left(c(d+ex^n)^p\right)\,dx = x\left(-\frac{enpx^n\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(1+n)}+\log\left(c(d+ex^n)^p\right)\right)$$

```
[In] Integrate[Log[c*(d + e*x^n)^p],x]
[Out] x*(-((e*n*p*x^n*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)/d)])
/(d*(1 + n))) + Log[c*(d + e*x^n)^p])
```

Maple [F]

$$\int\ln\left(c(d+e\,x^n)^p\right)dx$$

```
[In] int(ln(c*(d+e*x^n)^p),x)
[Out] int(ln(c*(d+e*x^n)^p),x)
```

Fricas [F]

$$\int \log \left(c(d+ex^n)^p\right) \, dx = \int \log \left((ex^n+d)^p c\right) \, dx$$

```
[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="fricas")
[Out] integral(log((e*x^n + d)^p*c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.41

$$\int \log \left(c(d+ex^n)^p\right) \, dx = x\log \left(c(d+ex^n)^p\right) + \frac{d^{-\frac{1}{n}} d^{1+\frac{1}{n}} e e^{\frac{1}{n}} e^{-1-\frac{1}{n}} px\Phi\left(\frac{dx^{-n}e^{i\pi}}{e}, 1, \frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{dn\Gamma\left(1+\frac{1}{n}\right)}$$

```
[In] integrate(ln(c*(d+e*x**n)**p),x)
[Out] x*log(c*(d + e*x**n)**p) + d**(1 + 1/n)*e*e**(1/n)*e**(-1 - 1/n)*p*x*lerchp
hi(d*exp_polar(I*pi)/(e*x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/(d*d**(1/n)
*n*gamma(1 + 1/n))
```

Maxima [F]

$$\int \log \left(c(d+ex^n)^p\right) \, dx = \int \log \left((ex^n+d)^p c\right) \, dx$$

```
[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="maxima")
[Out] d*n*p*integrate(1/(e*x^n + d), x) - (n*p - log(c))*x + x*log((e*x^n + d)^p)
```

Giac [F]

$$\int \log \left(c(d+ex^n)^p\right) \, dx = \int \log \left((ex^n+d)^p c\right) \, dx$$

```
[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="giac")
[Out] integrate(log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(c(d+ex^n)^p\right) \, dx = \int \ln \left(c\,(d+e\,x^n)^p\right) \, dx$$

```
[In] int(log(c*(d + e*x^n)^p),x)
[Out] int(log(c*(d + e*x^n)^p), x)
```

3.73 $\int \frac{\log(c(d+ex^n)^p)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{\log(c(d+ex^n)^p)}{x}\,dx = \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{p\,\mathrm{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/n+p*polylog(2,1+e*x^n/d)/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2441, 2352}

$$\int \frac{\log(c(d+ex^n)^p)}{x}\,dx = \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}$$

```
[In] Int[Log[c*(d + e*x^n)^p]/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (p*PolyLog[2, 1 + (e*x^n)/d])/
n
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}\\ &= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n} - \frac{(ep)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^n\right)}{n}\\ &= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n} + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x}\,dx = \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right) + p\,\text{PolyLog}\left(2, \frac{d+ex^n}{d}\right)}{n}$$

```
[In] Integrate[Log[c*(d + e*x^n)^p]/x,x]
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*PolyLog[2, (d + e*x^n)/d])/n
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.86

method	result
risch	$\ln(x)\ln\left((d+e\,x^n)^p\right)+\left(\frac{i\pi\,\text{csgn}(i(d+e\,x^n)^p)\text{csgn}(ic(d+e\,x^n)^p)^2}{2}-\frac{i\pi\,\text{csgn}(i(d+e\,x^n)^p)\,\text{csgn}(ic(d+e\,x^n)^p)\,\text{csgn}(ic)}{2}-\right.$

```
[In] int(ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*ln((d+e*x^n)^p)+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2
-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn
(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))*ln(x)
-p/n*dilog((d+e*x^n)/d)-p*ln(x)*ln((d+e*x^n)/d)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x}\,dx = \frac{np\log \left(ex^n+d\right)\log \left(x\right)-np\log \left(x\right)\log \left(\frac{ex^n+d}{d}\right)+n\log \left(c\right)\log \left(x\right)-p\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)}{n}$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] (n*p*log(e*x^n + d)*log(x) - n*p*log(x)*log((e*x^n + d)/d) + n*log(c)*log(x
) - p*dilog(-(e*x^n + d)/d + 1))/n
```

Sympy [F]

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{\log \left(c(d+ex^n)^p\right)}{x}\,dx$$

```
[In] integrate(ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)/x, x)
```

Maxima [F]

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{\log \left((ex^n+d)^p c\right)}{x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] d*n*p*integrate(log(x)/(e*x*x^n + d*x), x) - 1/2*n*p*log(x)^2 + log((e*x^n
+ d)^p)*log(x) + log(c)*log(x)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
[Out] integrate(log((e*x^n + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{\ln\left(c\left(d+e\,x^n\right)^p\right)}{x}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)/x,x)
[Out] int(log(c*(d + e*x^n)^p)/x, x)
```

3.74 $\int \frac{\log(c(d+ex^n)^p)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 66

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x^2}\,dx = -\frac{enpx^{-1+n}\,\text{Hypergeometric2F1}\left(1,-\frac{1-n}{n},2-\frac{1}{n},-\frac{ex^n}{d}\right)}{d(1-n)} - \frac{\log \left(c(d+ex^n)^p\right)}{x}$$

```
[Out] -e*n*p*x^(-1+n)*hypergeom([1, (-1+n)/n],[2-1/n],-e*x^n/d)/d/(1-n)-ln(c*(d+e
*x^n)^p)/x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2505, 371}

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x^2}\,dx = -\frac{\log \left(c(d+ex^n)^p\right)}{x} - \frac{enpx^{n-1}\,\text{Hypergeometric2F1}\left(1,-\frac{1-n}{n},2-\frac{1}{n},-\frac{ex^n}{d}\right)}{d(1-n)}$$

```
[In] Int[Log[c*(d + e*x^n)^p]/x^2,x]
```

```
[Out] -((e*n*p*x^(-1 + n)*Hypergeometric2F1[1, -((1 - n)/n), 2 - n^(-1), -((e*x^n
)/d)])/(d*(1 - n))) - Log[c*(d + e*x^n)^p]/x
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\log\left(c(d+ex^n)^p\right)}{x}+(enp)\int\frac{x^{-2+n}}{d+ex^n}\,dx\\ &= -\frac{enpx^{-1+n}\,{}_2F_1\left(1,-\frac{1-n}{n};2-\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1-n)}-\frac{\log\left(c(d+ex^n)^p\right)}{x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int\frac{\log\left(c(d+ex^n)^p\right)}{x^2}\,dx=\frac{\frac{enpx^n\,\text{Hypergeometric2F1}\left(1,\frac{-1+n}{n},2-\frac{1}{n},-\frac{ex^n}{d}\right)}{d(-1+n)}-\log\left(c(d+ex^n)^p\right)}{x}$$

```
[In] Integrate[Log[c*(d + e*x^n)^p]/x^2,x]
[Out] ((e*n*p*x^n*Hypergeometric2F1[1, (-1 + n)/n, 2 - n^(-1), -((e*x^n)/d)])/(d*
(-1 + n)) - Log[c*(d + e*x^n)^p])/x
```

Maple [F]

$$\int\frac{\ln\left(c(d+e\,x^n)^p\right)}{x^2}dx$$

```
[In] int(ln(c*(d+e*x^n)^p)/x^2,x)
[Out] int(ln(c*(d+e*x^n)^p)/x^2,x)
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x^2}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{x^2}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x^2,x, algorithm="fricas")
[Out] integral(log((e*x^n + d)^p*c)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.46 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x^2}\,dx = -\frac{\log\left(c(d+ex^n)^p\right)}{x} + \frac{d^{\frac{1}{n}} d^{1-\frac{1}{n}} e e^{-\frac{1}{n}} e^{-1+\frac{1}{n}} p \Phi\left(\frac{dx^{-n}e^{i\pi}}{e}, 1, \frac{1}{n}\right)\Gamma\left(-\frac{1}{n}\right)}{dnx\Gamma\left(1-\frac{1}{n}\right)}$$

```
[In] integrate(ln(c*(d+e*x**n)**p)/x**2,x)
[Out] -log(c*(d + e*x**n)**p)/x + d**(1/n)*d**(1 - 1/n)*e*e**(-1 + 1/n)*p*lerchph
i(d*exp_polar(I*pi)/(e*x**n), 1, 1/n)*gamma(-1/n)/(d*e**(1/n)*n*x*gamma(1 -
 1/n))
```

Maxima [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x^2}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{x^2}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x^2,x, algorithm="maxima")
[Out] -d*n*p*integrate(1/(e*x^2*x^n + d*x^2), x) - (n*p + log((e*x^n + d)^p) + lo
g(c))/x
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x^2}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{x^2}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x^2,x, algorithm="giac")
[Out] integrate(log((e*x^n + d)^p*c)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log (c(d+ex^n)^p)}{x^2}\,dx = \int \frac{\ln \left(c\,{\left(d+e\,x^n\right)}^p\right)}{x^2}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)/x^2,x)
[Out] int(log(c*(d + e*x^n)^p)/x^2, x)
```

3.75 $\int \frac{\log(c(d+ex^n)^p)}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 72

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x^3}\,dx = -\frac{enpx^{-2+n}\,\text{Hypergeometric2F1}\left(1,-\frac{2-n}{n},2\left(1-\frac{1}{n}\right),-\frac{ex^n}{d}\right)}{2d(2-n)} - \frac{\log\left(c(d+ex^n)^p\right)}{2x^2}$$

```
[Out] -1/2*e*n*p*x^(-2+n)*hypergeom([1, (-2+n)/n],[2-2/n],-e*x^n/d)/d/(2-n)-1/2*l
n(c*(d+e*x^n)^p)/x^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2505, 371}

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x^3}\,dx = -\frac{\log\left(c(d+ex^n)^p\right)}{2x^2} - \frac{enpx^{n-2}\,\text{Hypergeometric2F1}\left(1,-\frac{2-n}{n},2\left(1-\frac{1}{n}\right),-\frac{ex^n}{d}\right)}{2d(2-n)}$$

```
[In] Int[Log[c*(d + e*x^n)^p]/x^3,x]
```

```
[Out] -1/2*(e*n*p*x^(-2 + n)*Hypergeometric2F1[1, -((2 - n)/n), 2*(1 - n^(-1)), -
((e*x^n)/d)])/(d*(2 - n)) - Log[c*(d + e*x^n)^p]/(2*x^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\log\left(c(d+ex^n)^p\right)}{2x^2}+\frac{1}{2}(enp)\int\frac{x^{-3+n}}{d+ex^n}\,dx\\ &= -\frac{enpx^{-2+n}\,{}_2F_1\left(1,-\frac{2-n}{n};2\left(1-\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(2-n)}-\frac{\log\left(c(d+ex^n)^p\right)}{2x^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int\frac{\log\left(c(d+ex^n)^p\right)}{x^3}\,dx=\frac{\frac{enpx^n\,\text{Hypergeometric2F1}\left(1,\frac{-2+n}{n},2-\frac{2}{n},-\frac{ex^n}{d}\right)}{d(-2+n)}-\log\left(c(d+ex^n)^p\right)}{2x^2}$$

```
[In] Integrate[Log[c*(d + e*x^n)^p]/x^3,x]
[Out] ((e*n*p*x^n*Hypergeometric2F1[1, (-2 + n)/n, 2 - 2/n, -((e*x^n)/d)])/(d*(-2
 + n)) - Log[c*(d + e*x^n)^p])/(2*x^2)
```

Maple [F]

$$\int\frac{\ln\left(c(d+e\,x^n)^p\right)}{x^3}dx$$

```
[In] int(ln(c*(d+e*x^n)^p)/x^3,x)
[Out] int(ln(c*(d+e*x^n)^p)/x^3,x)
```

Fricas [F]

$$\int \frac{\log(c(d+ex^n)^p)}{x^3}\,dx = \int \frac{\log((ex^n+d)^p c)}{x^3}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x^3,x, algorithm="fricas")
[Out] integral(log((e*x^n + d)^p*c)/x^3, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.77 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08

$$\int \frac{\log(c(d+ex^n)^p)}{x^3}\,dx = -\frac{\log(c(d+ex^n)^p)}{2x^2} + \frac{d^{\frac{2}{n}} d^{1-\frac{2}{n}} e e^{-\frac{2}{n}} e^{-1+\frac{2}{n}} p \Phi\left(\frac{dx^{-n}e^{i\pi}}{e}, 1, \frac{2}{n}\right)\Gamma\left(-\frac{2}{n}\right)}{dnx^2\Gamma\left(1-\frac{2}{n}\right)}$$

```
[In] integrate(ln(c*(d+e*x**n)**p)/x**3,x)
[Out] -log(c*(d + e*x**n)**p)/(2*x**2) + d**(2/n)*d**(1 - 2/n)*e*e**(-1 + 2/n)*p*
lerchphi(d*exp_polar(I*pi)/(e*x**n), 1, 2/n)*gamma(-2/n)/(d*e**(2/n)*n*x**2
*gamma(1 - 2/n))
```

Maxima [F]

$$\int \frac{\log(c(d+ex^n)^p)}{x^3}\,dx = \int \frac{\log((ex^n+d)^p c)}{x^3}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x^3,x, algorithm="maxima")
[Out] -d*n*p*integrate(1/2/(e*x^3*x^n + d*x^3), x) - 1/4*(n*p + 2*log((e*x^n + d)
^p) + 2*log(c))/x^2
```

Giac [F]

$$\int \frac{\log(c(d+ex^n)^p)}{x^3}\,dx = \int \frac{\log((ex^n+d)^p c)}{x^3}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x^3,x, algorithm="giac")
[Out] integrate(log((e*x^n + d)^p*c)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x^3}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)}{x^3}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)/x^3,x)
[Out] int(log(c*(d + e*x^n)^p)/x^3, x)
```

3.76 $\int \frac{\log(c(d+ex^n)^p)}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 70

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x^4}\,dx = -\frac{enpx^{-3+n}\,\text{Hypergeometric2F1}\left(1,-\frac{3-n}{n},2-\frac{3}{n},-\frac{ex^n}{d}\right)}{3d(3-n)} - \frac{\log\left(c(d+ex^n)^p\right)}{3x^3}$$

```
[Out] -1/3*e*n*p*x^(-3+n)*hypergeom([1, (-3+n)/n],[2-3/n],-e*x^n/d)/d/(3-n)-1/3*l
n(c*(d+e*x^n)^p)/x^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2505, 371}

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x^4}\,dx = -\frac{\log\left(c(d+ex^n)^p\right)}{3x^3} - \frac{enpx^{n-3}\,\text{Hypergeometric2F1}\left(1,-\frac{3-n}{n},2-\frac{3}{n},-\frac{ex^n}{d}\right)}{3d(3-n)}$$

```
[In] Int[Log[c*(d + e*x^n)^p]/x^4,x]
```

```
[Out] -1/3*(e*n*p*x^(-3 + n)*Hypergeometric2F1[1, -((3 - n)/n), 2 - 3/n, -((e*x^n
)/d)])/(d*(3 - n)) - Log[c*(d + e*x^n)^p]/(3*x^3)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\log\left(c(d+ex^n)^p\right)}{3x^3}+\frac{1}{3}(enp)\int\frac{x^{-4+n}}{d+ex^n}\,dx\\ &= -\frac{enpx^{-3+n}\,{}_2F_1\left(1,-\frac{3-n}{n};2-\frac{3}{n};-\frac{ex^n}{d}\right)}{3d(3-n)}-\frac{\log\left(c(d+ex^n)^p\right)}{3x^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int\frac{\log\left(c(d+ex^n)^p\right)}{x^4}\,dx=\frac{\frac{enpx^n\,\text{Hypergeometric2F1}\left(1,\frac{-3+n}{n},2-\frac{3}{n},-\frac{ex^n}{d}\right)}{d(-3+n)}-\log\left(c(d+ex^n)^p\right)}{3x^3}$$

```
[In] Integrate[Log[c*(d + e*x^n)^p]/x^4,x]
[Out] ((e*n*p*x^n*Hypergeometric2F1[1, (-3 + n)/n, 2 - 3/n, -((e*x^n)/d)])/(d*(-3
 + n)) - Log[c*(d + e*x^n)^p])/(3*x^3)
```

Maple [F]

$$\int\frac{\ln\left(c(d+e\,x^n)^p\right)}{x^4}dx$$

```
[In] int(ln(c*(d+e*x^n)^p)/x^4,x)
[Out] int(ln(c*(d+e*x^n)^p)/x^4,x)
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x^4}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{x^4}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x^4,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x^4}\,dx = -\frac{\log\left(c(d+ex^n)^p\right)}{3x^3} + \frac{d^{\frac{3}{n}}d^{1-\frac{3}{n}}ee^{-\frac{3}{n}}e^{-1+\frac{3}{n}}p\Phi\left(\frac{dx^{-n}e^{i\pi}}{e},1,\frac{3}{n}\right)\Gamma\left(-\frac{3}{n}\right)}{dnx^3\Gamma\left(1-\frac{3}{n}\right)}$$

```
[In] integrate(ln(c*(d+e*x**n)**p)/x**4,x)
```

```
[Out] -log(c*(d + e*x**n)**p)/(3*x**3) + d**(3/n)*d**(1 - 3/n)*e*e**(-1 + 3/n)*p*
lerchphi(d*exp_polar(I*pi)/(e*x**n), 1, 3/n)*gamma(-3/n)/(d*e**(3/n)*n*x**3
*gamma(1 - 3/n))
```

Maxima [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x^4}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{x^4}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x^4,x, algorithm="maxima")
```

```
[Out] -d*n*p*integrate(1/3/(e*x^4*x^n + d*x^4), x) - 1/9*(n*p + 3*log((e*x^n + d)
^p) + 3*log(c))/x^3
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x^4}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{x^4}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x^4,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log (c(d+ex^n)^p)}{x^4}\,dx = \int \frac{\ln \left(c\,{\left(d+e\,x^n\right)}^p\right)}{x^4}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)/x^4,x)
[Out] int(log(c*(d + e*x^n)^p)/x^4, x)
```

3.77 $\int x^5 \log^2\left(c(a+bx^2)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 215

$$\begin{aligned}\int x^5 \log^2\left(c(a+bx^2)^p\right)\,dx &= \frac{a^2p^2x^2}{b^2}-\frac{ap^2(a+bx^2)^2}{4b^3}+\frac{p^2(a+bx^2)^3}{27b^3}\\&\quad-\frac{a^3p^2\log^2(a+bx^2)}{6b^3}-\frac{a^2p(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^3}\\&\quad+\frac{ap(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{2b^3}\\&\quad-\frac{p(a+bx^2)^3\log\left(c(a+bx^2)^p\right)}{9b^3}\\&\quad+\frac{a^3p\log(a+bx^2)\log\left(c(a+bx^2)^p\right)}{3b^3}+\frac{1}{6}x^6\log^2\left(c(a+bx^2)^p\right)\end{aligned}$$

```
[Out] a^2*p^2*x^2/b^2-1/4*a*p^2*(b*x^2+a)^2/b^3+1/27*p^2*(b*x^2+a)^3/b^3-1/6*a^3*
p^2*ln(b*x^2+a)^2/b^3-a^2*p*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b^3+1/2*a*p*(b*x^2+
a)^2*ln(c*(b*x^2+a)^p)/b^3-1/9*p*(b*x^2+a)^3*ln(c*(b*x^2+a)^p)/b^3+1/3*a^3*
p*ln(b*x^2+a)*ln(c*(b*x^2+a)^p)/b^3+1/6*x^6*ln(c*(b*x^2+a)^p)^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used

= {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$\int x^5 \log^2 \left(c\left(a+bx^2\right)^p\right) \, dx = \frac{a^3 p \log\left(a+bx^2\right) \log\left(c\left(a+bx^2\right)^p\right)}{3b^3} - \frac{a^3 p^2 \log^2\left(a+bx^2\right)}{6b^3} - \frac{a^2 p \left(a+bx^2\right) \log\left(c\left(a+bx^2\right)^p\right)}{b^3} + \frac{a^2 p^2 x^2}{b^2} - \frac{p\left(a+bx^2\right)^3 \log\left(c\left(a+bx^2\right)^p\right)}{9b^3} + \frac{ap\left(a+bx^2\right)^2 \log\left(c\left(a+bx^2\right)^p\right)}{2b^3} + \frac{p^2\left(a+bx^2\right)^3}{27b^3} - \frac{ap^2\left(a+bx^2\right)^2}{4b^3} + \frac{1}{6}x^6 \log^2\left(c\left(a+bx^2\right)^p\right)$$

```
[In] Int[x^5*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] (a^2*p^2*x^2)/b^2 - (a*p^2*(a + b*x^2)^2)/(4*b^3) + (p^2*(a + b*x^2)^3)/(27
*b^3) - (a^3*p^2*Log[a + b*x^2]^2)/(6*b^3) - (a^2*p*(a + b*x^2)*Log[c*(a +
b*x^2)^p])/b^3 + (a*p*(a + b*x^2)^2*Log[c*(a + b*x^2)^p])/(2*b^3) - (p*(a +
 b*x^2)^3*Log[c*(a + b*x^2)^p])/(9*b^3) + (a^3*p*Log[a + b*x^2]*Log[c*(a +
b*x^2)^p])/(3*b^3) + (x^6*Log[c*(a + b*x^2)^p]^2)/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x^2\log^2\left(c(a+bx)^p\right)\,dx,x,x^2\right)\\
&= \frac{1}{6}x^6\log^2\left(c\left(a+bx^2\right)^p\right)-\frac{1}{3}(bp)\text{Subst}\left(\int\frac{x^3\log\left(c(a+bx)^p\right)}{a+bx}\,dx,x,x^2\right)\\
&= \frac{1}{6}x^6\log^2\left(c\left(a+bx^2\right)^p\right)-\frac{1}{3}p\text{Subst}\left(\int\frac{\left(-\frac{a}{b}+\frac{x}{b}\right)^3\log\left(cx^p\right)}{x}\,dx,x,a+bx^2\right)
\end{aligned}
$$

$$= -\frac{a^2p(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^3} + \frac{ap(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{2b^3} - \frac{p(a+bx^2)^3\log\left(c(a+bx^2)^p\right)}{9b^3} + \frac{a^3p\log(a+bx^2)\log\left(c(a+bx^2)^p\right)}{3b^3} + \frac{1}{6}x^6\log^2\left(c\left(a+bx^2\right)^p\right) + \frac{1}{3}p^2\text{Subst}\left(\int\frac{18a^2x-9ax^2+2x^3-6a^3\log(x)}{6b^3x}\,dx,x,a+bx^2\right)$$

$$= -\frac{a^2p(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^3} + \frac{ap(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{2b^3} - \frac{p(a+bx^2)^3\log\left(c(a+bx^2)^p\right)}{9b^3} + \frac{a^3p\log(a+bx^2)\log\left(c(a+bx^2)^p\right)}{3b^3} + \frac{1}{6}x^6\log^2\left(c\left(a+bx^2\right)^p\right) + \frac{p^2\text{Subst}\left(\int\frac{18a^2x-9ax^2+2x^3-6a^3\log(x)}{x}\,dx,x,a+bx^2\right)}{18b^3}$$

$$= -\frac{a^2p(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^3} + \frac{ap(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{2b^3} - \frac{p(a+bx^2)^3\log\left(c(a+bx^2)^p\right)}{9b^3} + \frac{a^3p\log(a+bx^2)\log\left(c(a+bx^2)^p\right)}{3b^3} + \frac{1}{6}x^6\log^2\left(c\left(a+bx^2\right)^p\right) + \frac{p^2\text{Subst}\left(\int\left(18a^2-9ax+2x^2-\frac{6a^3\log(x)}{x}\right)dx,x,a+bx^2\right)}{18b^3}$$

$$= \frac{a^2p^2x^2}{b^2} - \frac{ap^2(a+bx^2)^2}{4b^3} + \frac{p^2(a+bx^2)^3}{27b^3} - \frac{a^2p(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^3} + \frac{ap(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{2b^3} - \frac{p(a+bx^2)^3\log\left(c(a+bx^2)^p\right)}{9b^3} + \frac{a^3p\log(a+bx^2)\log\left(c(a+bx^2)^p\right)}{3b^3} + \frac{1}{6}x^6\log^2\left(c\left(a+bx^2\right)^p\right) - \frac{(a^3p^2)\,\text{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,a+bx^2\right)}{3b^3}$$

$$= \frac{a^2p^2x^2}{b^2} - \frac{ap^2(a+bx^2)^2}{4b^3} + \frac{p^2(a+bx^2)^3}{27b^3} - \frac{a^3p^2\log^2(a+bx^2)}{6b^3} - \frac{a^2p(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^3} + \frac{ap(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{2b^3} - \frac{p(a+bx^2)^3\log\left(c(a+bx^2)^p\right)}{9b^3} + \frac{a^3p\log(a+bx^2)\log\left(c(a+bx^2)^p\right)}{3b^3} + \frac{1}{6}x^6\log^2\left(c\left(a+bx^2\right)^p\right)$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.59

$$\int x^5 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$=\frac{bp^2x^2(66a^2-15abx^2+4b^2x^4)-30a^3p^2\log\left(a+bx^2\right)-6p(6a^3+6a^2bx^2-3ab^2x^4+2b^3x^6)\log\left(c\left(a+bx^2\right)^{p}\right.}{108b^3}$$

```
[In] Integrate[x^5*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] (b*p^2*x^2*(66*a^2 - 15*a*b*x^2 + 4*b^2*x^4) - 30*a^3*p^2*Log[a + b*x^2] -
6*p*(6*a^3 + 6*a^2*b*x^2 - 3*a*b^2*x^4 + 2*b^3*x^6)*Log[c*(a + b*x^2)^p] +
18*(a^3 + b^3*x^6)*Log[c*(a + b*x^2)^p]^2)/(108*b^3)
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.88

method	result
parallelrisch	$-\frac{-18x^6\ln\left(c\left(b\,x^2+a\right)^p\right)^2b^3+12x^6\ln\left(c\left(b\,x^2+a\right)^p\right)b^3p-4x^6b^3p^2-18x^4\ln\left(c\left(b\,x^2+a\right)^p\right)a\,b^2p+15x^4a\,b^2p^2+36x^2\ln\left(c\left(b\,x^2+a\right)^p\right)a}{108b^3}$
risch	Expression too large to display

```
[In] int(x^5*ln(c*(b*x^2+a)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/108*(-18*x^6*ln(c*(b*x^2+a)^p)^2*b^3+12*x^6*ln(c*(b*x^2+a)^p)*b^3*p-4*x^
6*b^3*p^2-18*x^4*ln(c*(b*x^2+a)^p)*a*b^2*p+15*x^4*a*b^2*p^2+36*x^2*ln(c*(b*
x^2+a)^p)*a^2*b*p-66*x^2*a^2*b*p^2+102*ln(b*x^2+a)*a^3*p^2-18*ln(c*(b*x^2+a
)^p)^2*a^3-36*ln(c*(b*x^2+a)^p)*a^3*p+66*a^3*p^2)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.88

$$\int x^5 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$=\frac{4\,b^3p^2x^6+18\,b^3x^6\log\left(c\right)^2-15\,ab^2p^2x^4+66\,a^2bp^2x^2+18\,(b^3p^2x^6+a^3p^2)\log\left(bx^2+a\right)^2-6\,(2\,b^3p^2x^6-3\,a}{1}$$

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/108*(4*b^3*p^2*x^6 + 18*b^3*x^6*log(c)^2 - 15*a*b^2*p^2*x^4 + 66*a^2*b*p^
2*x^2 + 18*(b^3*p^2*x^6 + a^3*p^2)*log(b*x^2 + a)^2 - 6*(2*b^3*p^2*x^6 - 3*
```

```
a*b^2*p^2*x^4 + 6*a^2*b*p^2*x^2 + 11*a^3*p^2 - 6*(b^3*p*x^6 + a^3*p)*log(c)
)*log(b*x^2 + a) - 6*(2*b^3*p*x^6 - 3*a*b^2*p*x^4 + 6*a^2*b*p*x^2)*log(c))/
b^3
```

Sympy [A] (verification not implemented)

Time = 3.22 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.85

$$\int x^5 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx = \begin{cases} -\frac{11a^3p\log\left(c(a+bx^2)^p\right)}{18b^3} + \frac{a^3\log\left(c(a+bx^2)^p\right)^2}{6b^3} + \frac{11a^2p^2x^2}{18b^2} - \frac{a^2px^2\log\left(c(a+bx^2)^p\right)}{3b^2} - \frac{5ap^2x^4}{36b} + \frac{apx^4\log\left(c(a+bx^2)^p\right)}{6b} + \frac{p^2x^6}{27} \\ \frac{x^6\log\left(a^pc\right)^2}{6} \end{cases}$$

```
[In] integrate(x**5*ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Piecewise((-11*a**3*p*log(c*(a + b*x**2)**p)/(18*b**3) + a**3*log(c*(a + b*
x**2)**p)**2/(6*b**3) + 11*a**2*p**2*x**2/(18*b**2) - a**2*p*x**2*log(c*(a
+ b*x**2)**p)/(3*b**2) - 5*a*p**2*x**4/(36*b) + a*p*x**4*log(c*(a + b*x**2)
**p)/(6*b) + p**2*x**6/27 - p*x**6*log(c*(a + b*x**2)**p)/9 + x**6*log(c*(a
 + b*x**2)**p)**2/6, Ne(b, 0)), (x**6*log(a**p*c)**2/6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.67

$$\begin{aligned}\int x^5 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx &= \frac{1}{6}\,x^6\log\left(\left(bx^2+a\right)^pc\right)^2 \\ &\quad + \frac{1}{18}\,bp\left(\frac{6\,a^3\log\left(bx^2+a\right)}{b^4} - \frac{2\,b^2x^6-3\,abx^4+6\,a^2x^2}{b^3}\right)\log\left(\left(bx^2+a\right)^pc\right) \\ &\quad + \frac{\left(4\,b^3x^6-15\,ab^2x^4+66\,a^2bx^2-18\,a^3\log\left(bx^2+a\right)^2-66\,a^3\log\left(bx^2+a\right)\right)p^2}{108\,b^3}\end{aligned}$$

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*log((b*x^2 + a)^p*c)^2 + 1/18*b*p*(6*a^3*log(b*x^2 + a)/b^4 - (2*b^
2*x^6 - 3*a*b*x^4 + 6*a^2*x^2)/b^3)*log((b*x^2 + a)^p*c) + 1/108*(4*b^3*x^6
 - 15*a*b^2*x^4 + 66*a^2*b*x^2 - 18*a^3*log(b*x^2 + a)^2 - 66*a^3*log(b*x^2
 + a))*p^2/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.72

$$\int x^5 \log^2\left(c(a+bx^2)^p\right)\,dx = \frac{(bx^2+a)^3p^2\log(bx^2+a)^2}{6\,b^3} - \frac{(bx^2+a)^2ap^2\log(bx^2+a)^2}{2\,b^3} - \frac{(bx^2+a)^3p^2\log(bx^2+a)}{9\,b^3} + \frac{(bx^2+a)^2ap^2\log(bx^2+a)}{2\,b^3} + \frac{(bx^2+a)^3p\log(bx^2+a)\log(c)}{3\,b^3} - \frac{(bx^2+a)^2ap\log(bx^2+a)\log(c)}{b^3} + \frac{(bx^2+a)^3p^2}{27\,b^3} - \frac{(bx^2+a)^2ap^2}{4\,b^3} - \frac{(bx^2+a)^3p\log(c)}{9\,b^3} + \frac{(bx^2+a)^2ap\log(c)}{2\,b^3} + \frac{(bx^2+a)^3\log(c)^2}{6\,b^3} - \frac{(bx^2+a)^2a\log(c)^2}{2\,b^3} + \frac{\left(2\,bx^2+(bx^2+a)\log(bx^2+a)^2-2\,(bx^2+a)\log(bx^2+a)+2\,a\right)a^2p^2-2\,(bx^2-(bx^2+a)\log(bx^2+a}{2\,b^3}$$

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] 1/6*(b*x^2 + a)^3*p^2*log(b*x^2 + a)^2/b^3 - 1/2*(b*x^2 + a)^2*a*p^2*log(b*
x^2 + a)^2/b^3 - 1/9*(b*x^2 + a)^3*p^2*log(b*x^2 + a)/b^3 + 1/2*(b*x^2 + a)
^2*a*p^2*log(b*x^2 + a)/b^3 + 1/3*(b*x^2 + a)^3*p*log(b*x^2 + a)*log(c)/b^3
 - (b*x^2 + a)^2*a*p*log(b*x^2 + a)*log(c)/b^3 + 1/27*(b*x^2 + a)^3*p^2/b^3
 - 1/4*(b*x^2 + a)^2*a*p^2/b^3 - 1/9*(b*x^2 + a)^3*p*log(c)/b^3 + 1/2*(b*x^
2 + a)^2*a*p*log(c)/b^3 + 1/6*(b*x^2 + a)^3*log(c)^2/b^3 - 1/2*(b*x^2 + a)^
2*a*log(c)^2/b^3 + 1/2*((2*b*x^2 + (b*x^2 + a)*log(b*x^2 + a)^2 - 2*(b*x^2
+ a)*log(b*x^2 + a) + 2*a)*a^2*p^2 - 2*(b*x^2 - (b*x^2 + a)*log(b*x^2 + a)
+ a)*a^2*p*log(c) + (b*x^2 + a)*a^2*log(c)^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.59

$$\int x^5 \log^2\left(c(a+bx^2)^p\right)\,dx = \frac{p^2\,x^6}{27} + \ln\left(c\,\left(b\,x^2+a\right)^p\right)^2\,\left(\frac{x^6}{6}+\frac{a^3}{6\,b^3}\right) - \ln\left(c\,\left(b\,x^2+a\right)^p\right)\,\left(\frac{p\,x^6}{9}+\frac{a^2\,p\,x^2}{3\,b^2}-\frac{a\,p\,x^4}{6\,b}\right) - \frac{5\,a\,p^2\,x^4}{36\,b} - \frac{11\,a^3\,p^2\,\ln\left(b\,x^2+a\right)}{18\,b^3} + \frac{11\,a^2\,p^2\,x^2}{18\,b^2}$$

```
[In] int(x^5*log(c*(a + b*x^2)^p)^2,x)
```

```
[Out] (p^2*x^6)/27 + log(c*(a + b*x^2)^p)^2*(x^6/6 + a^3/(6*b^3)) - log(c*(a + b*
x^2)^p)*((p*x^6)/9 + (a^2*p*x^2)/(3*b^2) - (a*p*x^4)/(6*b)) - (5*a*p^2*x^4)
/(36*b) - (11*a^3*p^2*log(a + b*x^2))/(18*b^3) + (11*a^2*p^2*x^2)/(18*b^2)
```

3.78 $\int x^3 \log^2\left(c(a+bx^2)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 145

$$\int x^3 \log^2\left(c(a+bx^2)^p\right)\,dx = -\frac{ap^2x^2}{b} + \frac{p^2(a+bx^2)^2}{8b^2} + \frac{ap(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^2} - \frac{p(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{4b^2} - \frac{a(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b^2} + \frac{(a+bx^2)^2\log^2\left(c(a+bx^2)^p\right)}{4b^2}$$

```
[Out] -a*p^2*x^2/b+1/8*p^2*(b*x^2+a)^2/b^2+a*p*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b^2-1/
4*p*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)/b^2-1/2*a*(b*x^2+a)*ln(c*(b*x^2+a)^p)^2/b
^2+1/4*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)^2/b^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\int x^3 \log^2\left(c(a+bx^2)^p\right)\,dx = \frac{(a+bx^2)^2\log^2\left(c(a+bx^2)^p\right)}{4b^2} - \frac{a(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b^2} - \frac{p(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{4b^2} + \frac{ap(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^2} + \frac{p^2(a+bx^2)^2}{8b^2} - \frac{ap^2x^2}{b}$$

```
[In] Int[x^3*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] -((a*p^2*x^2)/b) + (p^2*(a + b*x^2)^2)/(8*b^2) + (a*p*(a + b*x^2)*Log[c*(a
+ b*x^2)^p])/b^2 - (p*(a + b*x^2)^2*Log[c*(a + b*x^2)^p])/(4*b^2) - (a*(a +
 b*x^2)*Log[c*(a + b*x^2)^p]^2)/(2*b^2) + ((a + b*x^2)^2*Log[c*(a + b*x^2)^
p]^2)/(4*b^2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
```

```
d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x\log^2\left(c(a+bx)^p\right)\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a\log^2\left(c(a+bx)^p\right)}{b}+\frac{(a+bx)\log^2\left(c(a+bx)^p\right)}{b}\right)\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int(a+bx)\log^2\left(c(a+bx)^p\right)\,dx,x,x^2\right)}{2b}-\frac{a\text{Subst}\left(\int\log^2\left(c(a+bx)^p\right)\,dx,x,x^2\right)}{2b}\\
&= \frac{\text{Subst}\left(\int x\log^2\left(cx^p\right)\,dx,x,a+bx^2\right)}{2b^2}-\frac{a\text{Subst}\left(\int\log^2\left(cx^p\right)\,dx,x,a+bx^2\right)}{2b^2}\\
&= -\frac{a(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b^2}+\frac{(a+bx^2)^2\log^2\left(c(a+bx^2)^p\right)}{4b^2}\\
&\quad-\frac{p\text{Subst}\left(\int x\log\left(cx^p\right)\,dx,x,a+bx^2\right)}{2b^2}+\frac{(ap)\text{Subst}\left(\int\log\left(cx^p\right)\,dx,x,a+bx^2\right)}{b^2}\\
&= -\frac{ap^2x^2}{b}+\frac{p^2(a+bx^2)^2}{8b^2}+\frac{ap(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^2}\\
&\quad-\frac{p(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{4b^2}\\
&\quad-\frac{a(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b^2}+\frac{(a+bx^2)^2\log^2\left(c(a+bx^2)^p\right)}{4b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.72

$$\int x^3\log^2\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$=\frac{bp^2x^2(-6a+bx^2)+2a^2p^2\log\left(a+bx^2\right)+2p(2a^2+2abx^2-b^2x^4)\log\left(c(a+bx^2)^p\right)-2(a^2-b^2x^4)\log^2\left(c\right.}{8b^2}$$

```
[In] Integrate[x^3*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] (b*p^2*x^2*(-6*a + b*x^2) + 2*a^2*p^2*Log[a + b*x^2] + 2*p*(2*a^2 + 2*a*b*x
^2 - b^2*x^4)*Log[c*(a + b*x^2)^p] - 2*(a^2 - b^2*x^4)*Log[c*(a + b*x^2)^p]
^2)/(8*b^2)
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{2x^4\ln\left(c\left(b\,x^2+a\right)^p\right)^2b^2-2x^4\ln\left(c\left(b\,x^2+a\right)^p\right)b^2p+b^2p^2x^4+4x^2\ln\left(c\left(b\,x^2+a\right)^p\right)abp-6ab\,p^2x^2+10\ln\left(b\,x^2+a\right)a^2p^2-2\ln\left(c\left(b\,x^2+a\right)\right.}{8b^2}$
risch	Expression too large to display

```
[In] int(x^3*ln(c*(b*x^2+a)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(2*x^4*ln(c*(b*x^2+a)^p)^2*b^2-2*x^4*ln(c*(b*x^2+a)^p)*b^2*p+b^2*p^2*x^
4+4*x^2*ln(c*(b*x^2+a)^p)*a*b*p-6*a*b*p^2*x^2+10*ln(b*x^2+a)*a^2*p^2-2*ln(c
*(b*x^2+a)^p)^2*a^2-4*ln(c*(b*x^2+a)^p)*a^2*p+6*a^2*p^2)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.02

$$\int x^3 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$= \frac{b^2p^2x^4+2\,b^2x^4\log\left(c\right)^2-6\,abp^2x^2+2\left(b^2p^2x^4-a^2p^2\right)\log\left(bx^2+a\right)^2-2\left(b^2p^2x^4-2\,abp^2x^2-3\,a^2p^2-2\left(b\right.\right.}{8\,b^2}$$

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*p^2*x^4 + 2*b^2*x^4*log(c)^2 - 6*a*b*p^2*x^2 + 2*(b^2*p^2*x^4 - a^
2*p^2)*log(b*x^2 + a)^2 - 2*(b^2*p^2*x^4 - 2*a*b*p^2*x^2 - 3*a^2*p^2 - 2*(b
^2*p*x^4 - a^2*p)*log(c))*log(b*x^2 + a) - 2*(b^2*p*x^4 - 2*a*b*p*x^2)*log(
c))/b^2
```

Sympy [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.96

$$\int x^3 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$= \begin{cases} \frac{3a^2p\log\left(c\left(a+bx^2\right)^p\right)}{4b^2} - \frac{a^2\log\left(c\left(a+bx^2\right)^p\right)^2}{4b^2} - \frac{3ap^2x^2}{4b} + \frac{apx^2\log\left(c\left(a+bx^2\right)^p\right)}{2b} + \frac{p^2x^4}{8} - \frac{px^4\log\left(c\left(a+bx^2\right)^p\right)}{4} + \frac{x^4\log\left(c\left(a+bx^2\right)\right.}{4} \\ \frac{x^4\log\left(a^pc\right)^2}{4} \end{cases}$$

```
[In] integrate(x**3*ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Piecewise((3*a**2*p*log(c*(a + b*x**2)**p)/(4*b**2) - a**2*log(c*(a + b*x**
2)**p)**2/(4*b**2) - 3*a*p**2*x**2/(4*b) + a*p*x**2*log(c*(a + b*x**2)**p)/
(2*b) + p**2*x**4/8 - p*x**4*log(c*(a + b*x**2)**p)/4 + x**4*log(c*(a + b*x
**2)**p)**2/4, Ne(b, 0)), (x**4*log(a**p*c)**2/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.83

$$\int x^3 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{1}{4}\,x^4\log\left(\left(bx^2+a\right)^pc\right)^2 - \frac{1}{4}\,bp\left(\frac{2\,a^2\log\left(bx^2+a\right)}{b^3} + \frac{bx^4-2\,ax^2}{b^2}\right)\log\left(\left(bx^2+a\right)^pc\right) + \frac{\left(b^2x^4-6\,abx^2+2\,a^2\log\left(bx^2+a\right)^2+6\,a^2\log\left(bx^2+a\right)\right)p^2}{8\,b^2}$$

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log((b*x^2 + a)^p*c)^2 - 1/4*b*p*(2*a^2*log(b*x^2 + a)/b^3 + (b*x^4
 - 2*a*x^2)/b^2)*log((b*x^2 + a)^p*c) + 1/8*(b^2*x^4 - 6*a*b*x^2 + 2*a^2*lo
g(b*x^2 + a)^2 + 6*a^2*log(b*x^2 + a))*p^2/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.49

$$\int x^3 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$= \frac{2\left(bx^2+a\right)^2 p^2 \log\left(bx^2+a\right)^2 - 2\left(bx^2+a\right)^2 p^2 \log\left(bx^2+a\right) + 4\left(bx^2+a\right)^2 p \log\left(bx^2+a\right)\log\left(c\right) + \left(bx^2+a\right)^2}{8\,b^2}$$
$$- \frac{\left(2\,bx^2 + \left(bx^2+a\right)\log\left(bx^2+a\right)^2 - 2\left(bx^2+a\right)\log\left(bx^2+a\right) + 2\,a\right)ap^2 - 2\left(bx^2 - \left(bx^2+a\right)\log\left(bx^2+a\right)\right)}{2\,b^2}$$

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] 1/8*(2*(b*x^2 + a)^2*p^2*log(b*x^2 + a)^2 - 2*(b*x^2 + a)^2*p^2*log(b*x^2 +
 a) + 4*(b*x^2 + a)^2*p*log(b*x^2 + a)*log(c) + (b*x^2 + a)^2*p^2 - 2*(b*x^
2 + a)^2*p*log(c) + 2*(b*x^2 + a)^2*log(c)^2)/b^2 - 1/2*((2*b*x^2 + (b*x^2
+ a)*log(b*x^2 + a)^2 - 2*(b*x^2 + a)*log(b*x^2 + a) + 2*a)*a*p^2 - 2*(b*x^
2 - (b*x^2 + a)*log(b*x^2 + a) + a)*a*p*log(c) + (b*x^2 + a)*a*log(c)^2)/b^
2
```

Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.69

$$\int x^3 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{p^2\,x^4}{8} - \ln\left(c\left(b\,x^2+a\right)^p\right)\left(\frac{p\,x^4}{4} - \frac{a\,p\,x^2}{2\,b}\right) + \ln\left(c\left(b\,x^2+a\right)^p\right)^2\left(\frac{x^4}{4} - \frac{a^2}{4\,b^2}\right) - \frac{3\,a\,p^2\,x^2}{4\,b} + \frac{3\,a^2\,p^2\,\ln\left(b\,x^2+a\right)}{4\,b^2}$$

```
[In] int(x^3*log(c*(a + b*x^2)^p)^2,x)
```

```
[Out] (p^2*x^4)/8 - log(c*(a + b*x^2)^p)*((p*x^4)/4 - (a*p*x^2)/(2*b)) + log(c*(a
 + b*x^2)^p)^2*(x^4/4 - a^2/(4*b^2)) - (3*a*p^2*x^2)/(4*b) + (3*a^2*p^2*log
(a + b*x^2))/(4*b^2)
```

3.79 $\int x \log^2\left(c(a+bx^2)^p\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 61

$$\int x\log^2\left(c\left(a+bx^2\right)^p\right)\,dx=p^2x^2-\frac{p(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b}+\frac{(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b}$$

```
[Out] p^2*x^2-p*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b+1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)^2/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2504, 2436, 2333, 2332}

$$\int x\log^2\left(c\left(a+bx^2\right)^p\right)\,dx=\frac{(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b}-\frac{p(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b}+p^2x^2$$

```
[In] Int[x*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] p^2*x^2 - (p*(a + b*x^2)*Log[c*(a + b*x^2)^p])/b + ((a + b*x^2)*Log[c*(a + 
b*x^2)^p]^2)/(2*b)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
```

```
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \log^2\left(c(a+bx)^p\right)\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \log^2\left(cx^p\right)\,dx, x, a+bx^2\right)}{2b}\\
&= \frac{(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b} - \frac{p\text{Subst}\left(\int \log\left(cx^p\right)\,dx, x, a+bx^2\right)}{b}\\
&= p^2x^2 - \frac{p(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b} + \frac{(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$
\int x\log^2\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{1}{2}\left(\frac{(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{b} - 2p\left(-px^2 + \frac{(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b}\right)\right)
$$

```
[In] Integrate[x*Log[c*(a + b*x^2)^p]^2,x]
[Out] (((a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/b - 2*p*(-(p*x^2) + ((a + b*x^2)*Log[
c*(a + b*x^2)^p])/b))/2
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.72

method	result	size
parallelrisch	$\frac{x^{2} \ln\left(c \left(b \,x^{2}+a \right)^{p}\right)^{2} a b p -2 x^{2} \ln\left(c \left(b \,x^{2}+a \right)^{p}\right) a b \,p^{2}+2 x^{2} a b \,p^{3}+\ln\left(c \left(b \,x^{2}+a \right)^{p}\right)^{2} a^{2} p -2 \ln\left(c \left(b \,x^{2}+a \right)^{p}\right) a^{2} p^{2}}{2 a b p}$	105
risch	Expression too large to display	1034

```
[In] int(x*ln(c*(b*x^2+a)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(x^2*ln(c*(b*x^2+a)^p)^2*a*b*p-2*x^2*ln(c*(b*x^2+a)^p)*a*b*p^2+2*x^2*a*
b*p^3+ln(c*(b*x^2+a)^p)^2*a^2*p-2*ln(c*(b*x^2+a)^p)*a^2*p^2)/a/b/p
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.57

$$\int x \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$=\frac{2\,bp^2x^2-2\,bpx^2\log\left(c\right)+bx^2\log\left(c\right)^2+\left(bp^2x^2+ap^2\right)\log\left(bx^2+a\right)^2-2\left(bp^2x^2+ap^2-\left(bpx^2+ap\right)\log\left(c\right.\right.}{2\,b}$$

```
[In] integrate(x*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*p^2*x^2 - 2*b*p*x^2*log(c) + b*x^2*log(c)^2 + (b*p^2*x^2 + a*p^2)*
log(b*x^2 + a)^2 - 2*(b*p^2*x^2 + a*p^2 - (b*p*x^2 + a*p)*log(c))*log(b*x^2
 + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.48

$$\int x \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$=\begin{cases} -\frac{ap\log\left(c\left(a+bx^2\right)^p\right)}{b}+\frac{a\log\left(c\left(a+bx^2\right)^p\right)^2}{2b}+p^2x^2-px^2\log\left(c\left(a+bx^2\right)^p\right)+\frac{x^2\log\left(c\left(a+bx^2\right)^p\right)^2}{2} & \text{for}\ b\neq 0 \\ \frac{x^2\log\left(a^pc\right)^2}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Piecewise((-a*p*log(c*(a + b*x**2)**p)/b + a*log(c*(a + b*x**2)**p)**2/(2*b
) + p**2*x**2 - p*x**2*log(c*(a + b*x**2)**p) + x**2*log(c*(a + b*x**2)**p)
**2/2, Ne(b, 0)), (x**2*log(a**p*c)**2/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.59

$$\int x\log^2\left(c\left(a+bx^2\right)^p\right)\,dx = -bp\left(\frac{x^2}{b}-\frac{a\log\left(bx^2+a\right)}{b^2}\right)\log\left(\left(bx^2+a\right)^pc\right) + \frac{1}{2}x^2\log\left(\left(bx^2+a\right)^pc\right)^2 + \frac{\left(2\,bx^2-a\log\left(bx^2+a\right)^2-2\,a\log\left(bx^2+a\right)\right)p^2}{2\,b}$$

[In] integrate(x*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")

[Out] -b*p*(x^2/b - a*log(b*x^2 + a)/b^2)*log((b*x^2 + a)^p*c) + 1/2*x^2*log((b*x
^2 + a)^p*c)^2 + 1/2*(2*b*x^2 - a*log(b*x^2 + a)^2 - 2*a*log(b*x^2 + a))*p^
2/b

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.57

$$\int x\log^2\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{\left(2\,bx^2+\left(bx^2+a\right)\log\left(bx^2+a\right)^2-2\left(bx^2+a\right)\log\left(bx^2+a\right)+2\,a\right)p^2-2\left(bx^2-\left(bx^2+a\right)\log\left(bx^2+a\right)+a\right.}{2\,b}$$

[In] integrate(x*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")

[Out] 1/2*((2*b*x^2 + (b*x^2 + a)*log(b*x^2 + a)^2 - 2*(b*x^2 + a)*log(b*x^2 + a)
 + 2*a)*p^2 - 2*(b*x^2 - (b*x^2 + a)*log(b*x^2 + a) + a)*p*log(c) + (b*x^2
+ a)*log(c)^2)/b

Mupad [B] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

$$\int x\log^2\left(c\left(a+bx^2\right)^p\right)\,dx = p^2\,x^2+\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2\,\left(\frac{a}{2\,b}+\frac{x^2}{2}\right) - p\,x^2\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)-\frac{a\,p^2\,\ln\left(b\,x^2+a\right)}{b}$$

[In] int(x*log(c*(a + b*x^2)^p)^2,x)

[Out] p^2*x^2 + log(c*(a + b*x^2)^p)^2*(a/(2*b) + x^2/2) - p*x^2*log(c*(a + b*x^2
)^p) - (a*p^2*log(a + b*x^2))/b

3.80 $\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 72

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x}\,dx = \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c(a+bx^2)^p\right) + p\log\left(c(a+bx^2)^p\right)\text{PolyLog}\left(2,1+\frac{bx^2}{a}\right) - p^2\,\text{PolyLog}\left(3,1+\frac{bx^2}{a}\right)$$

```
[Out] 1/2*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)^2+p*ln(c*(b*x^2+a)^p)*polylog(2,1+b*x^2/
a)-p^2*polylog(3,1+b*x^2/a)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2504, 2443, 2481, 2421, 6724}

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x}\,dx = p\,\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)\log\left(c(a+bx^2)^p\right) + \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c(a+bx^2)^p\right) + p^2\left(-\text{PolyLog}\left(3,\frac{bx^2}{a}+1\right)\right)$$

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x,x]
```

```
[Out] (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p]^2)/2 + p*Log[c*(a + b*x^2)^p]*PolyL
og[2, 1 + (b*x^2)/a] - p^2*PolyLog[3, 1 + (b*x^2)/a]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{\log^2\left(c(a+bx)^p\right)}{x}\,dx, x, x^2\right)$$

$$=\frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)-(bp)\text{Subst}\left(\int\frac{\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{a+bx}\,dx,x,x^2\right)$$

$$=\frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)-p\text{Subst}\left(\int\frac{\log\left(cx^p\right)\log\left(-\frac{b\left(-\frac{a}{b}+\frac{x}{b}\right)}{a}\right)}{x}\,dx,x,a+bx^2\right)$$

$$=\frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)+p\log\left(c\left(a+bx^2\right)^p\right)\text{Li}_2\left(1+\frac{bx^2}{a}\right)$$
$$-p^2\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{x}{a}\right)}{x}\,dx,x,a+bx^2\right)$$

$$=\frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^2\left(c\left(a+bx^2\right)^p\right)+p\log\left(c\left(a+bx^2\right)^p\right)\text{Li}_2\left(1+\frac{bx^2}{a}\right)-p^2\text{Li}_3\left(1+\frac{bx^2}{a}\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 163 vs. $2(72) = 144$.

Time = 0.08 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.26

$$\int\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x}\,dx=\log(x)\left(-p\log\left(a+bx^2\right)+\log\left(c\left(a+bx^2\right)^p\right)\right)^2+2p\left(-p\log\left(a+bx^2\right)\right.$$
$$\left.+\log\left(c\left(a+bx^2\right)^p\right)\right)\left(\log(x)\left(\log\left(a+bx^2\right)-\log\left(1+\frac{bx^2}{a}\right)\right)\right.$$
$$\left.-\frac{1}{2}\text{PolyLog}\left(2,-\frac{bx^2}{a}\right)\right)+\frac{1}{2}p^2\left(\log\left(-\frac{bx^2}{a}\right)\log^2\left(a+bx^2\right)\right.$$
$$\left.+2\log\left(a+bx^2\right)\text{PolyLog}\left(2,1+\frac{bx^2}{a}\right)-2\,\text{PolyLog}\left(3,1+\frac{bx^2}{a}\right)\right)$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x,x]
```

```
[Out] Log[x]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2 + 2*p*(-(p*Log[a + b*
x^2]) + Log[c*(a + b*x^2)^p])*(Log[x]*(Log[a + b*x^2] - Log[1 + (b*x^2)/a])
 - PolyLog[2, -((b*x^2)/a)]/2) + (p^2*(Log[-((b*x^2)/a)]*Log[a + b*x^2]^2 +
 2*Log[a + b*x^2]*PolyLog[2, 1 + (b*x^2)/a] - 2*PolyLog[3, 1 + (b*x^2)/a]))
/2
```

Maple [F]

$$\int \frac{\ln \left(c(b\,x^2+a)^p\right)^2}{x}dx$$

```
[In] int(ln(c*(b*x^2+a)^p)^2/x,x)
[Out] int(ln(c*(b*x^2+a)^p)^2/x,x)
```

Fricas [F]

$$\int \frac{\log^2 \left(c(a+bx^2)^p\right)}{x}\,dx = \int \frac{\log \left((bx^2+a)^p c\right)^2}{x}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x,x, algorithm="fricas")
[Out] integral(log((b*x^2 + a)^p*c)^2/x, x)
```

Sympy [F]

$$\int \frac{\log^2 \left(c(a+bx^2)^p\right)}{x}\,dx = \int \frac{\log \left(c(a+bx^2)^p\right)^2}{x}\,dx$$

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x,x)
[Out] Integral(log(c*(a + b*x**2)**p)**2/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.64

$$\begin{aligned}&\int \frac{\log^2 \left(c(a+bx^2)^p\right)}{x}\,dx \\ &= \frac{1}{2}\left(\log \left(bx^2+a\right)^2 \log \left(-\frac{bx^2+a}{a}+1\right)+2\,\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\log \left(bx^2+a\right)-2\,\mathrm{Li}_3(\frac{bx^2+a}{a})\right)p^2 \\ &\quad+\left(\log \left(bx^2+a\right)\log \left(-\frac{bx^2+a}{a}+1\right)+\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\right)p\log \left(c\right)+\log \left(c\right)^2\log \left(x\right)\end{aligned}$$

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x,x, algorithm="maxima")
[Out] 1/2*(log(b*x^2 + a)^2*log(-(b*x^2 + a)/a + 1) + 2*dilog((b*x^2 + a)/a)*log(
b*x^2 + a) - 2*polylog(3, (b*x^2 + a)/a))*p^2 + (log(b*x^2 + a)*log(-(b*x^2
 + a)/a + 1) + dilog((b*x^2 + a)/a))*p*log(c) + log(c)^2*log(x)
```

Giac [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x,x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x}\,dx = \int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2}{x}\,dx$$

```
[In] int(log(c*(a + b*x^2)^p)^2/x,x)
[Out] int(log(c*(a + b*x^2)^p)^2/x, x)
```

3.81 $\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^3}\,dx = \frac{bp\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{a} - \frac{(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2ax^2} + \frac{bp^2\operatorname{PolyLog}\left(2,1+\frac{bx^2}{a}\right)}{a}$$

```
[Out] b*p*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)/a-1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)^2/a/x^
2+b*p^2*polylog(2,1+b*x^2/a)/a
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2504, 2444, 2441, 2352}

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^3}\,dx = -\frac{(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2ax^2} + \frac{bp\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{a} + \frac{bp^2\operatorname{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{a}$$

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^3,x]
```

```
[Out] (b*p*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/a - ((a + b*x^2)*Log[c*(a + b*
x^2)^p]^2)/(2*a*x^2) + (b*p^2*PolyLog[2, 1 + (b*x^2)/a])/a
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\log^2\left(c(a+bx)^p\right)}{x^2}\,dx, x, x^2\right)\\
&= -\frac{(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2ax^2} + \frac{(bp)\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x, x^2\right)}{a}\\
&= \frac{bp\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{a} - \frac{(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2ax^2}\\
&\quad - \frac{(b^2p^2)\,\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx, x, x^2\right)}{a}\\
&= \frac{bp\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{a} - \frac{(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2ax^2} + \frac{bp^2\text{Li}_2\left(1+\frac{bx^2}{a}\right)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.16

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^3}\,dx = \frac{bp\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{a} - \frac{b\log^2\left(c(a+bx^2)^p\right)}{2a} - \frac{\log^2\left(c(a+bx^2)^p\right)}{2x^2} + \frac{bp^2\,\mathrm{PolyLog}\left(2,\frac{a+bx^2}{a}\right)}{a}$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^3,x]
```

```
[Out] (b*p*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/a - (b*Log[c*(a + b*x^2)^p]^2)
/(2*a) - Log[c*(a + b*x^2)^p]^2/(2*x^2) + (b*p^2*PolyLog[2, (a + b*x^2)/a])
/a
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 481, normalized size of antiderivative = 6.01

method	result
risch	$-\frac{\ln\left(\left(b\,x^2+a\right)^p\right)^2}{2x^2}+\frac{2pb\ln\left(\left(b\,x^2+a\right)^p\right)\ln(x)}{a}-\frac{pb\ln\left(\left(b\,x^2+a\right)^p\right)\ln\left(b\,x^2+a\right)}{a}-\frac{2p^2b\ln(x)\ln\left(\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}\right)}{a}-\frac{2p^2b\ln(x)\ln\left(\frac{bx+}{\sqrt{-}}\right.}{a}$

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln((b*x^2+a)^p)^2/x^2+2*p*b*ln((b*x^2+a)^p)/a*ln(x)-p*b*ln((b*x^2+a)^p
)/a*ln(b*x^2+a)-2*p^2*b/a*ln(x)*ln((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-2*p^2*
b/a*ln(x)*ln((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-2*p^2*b/a*dilog((-b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))-2*p^2*b/a*dilog((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+1/2*p
^2*b/a*ln(b*x^2+a)^2+(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi
*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a
)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c))*(-1/2/x^2*ln((b*x^2+
a)^p)+p*b*(1/a*ln(x)-1/2/a*ln(b*x^2+a)))-1/8*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn
(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c
)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(
c))^2/x^2
```

Fricas [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^3}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x^3}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^3,x, algorithm="fricas")
[Out] integral(log((b*x^2 + a)^p*c)^2/x^3, x)
```

Sympy [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^3}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)^2}{x^3}\,dx$$

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**3,x)
[Out] Integral(log(c*(a + b*x**2)**p)**2/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.48

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^3}\,dx = \frac{1}{2}\,b^2p^2\left(\frac{\log\left(bx^2+a\right)^2}{ab} - \frac{2\left(2\,\log\left(\frac{bx^2}{a}+1\right)\log\left(x\right) + \mathrm{Li}_2\left(-\frac{bx^2}{a}\right)\right)}{ab}\right) - bp\left(\frac{\log\left(bx^2+a\right)}{a} - \frac{\log\left(x^2\right)}{a}\right)\log\left(\left(bx^2+a\right)^p c\right) - \frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{2\,x^2}$$

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^3,x, algorithm="maxima")
[Out] 1/2*b^2*p^2*(log(b*x^2 + a)^2/(a*b) - 2*(2*log(b*x^2/a + 1)*log(x) + dilog(
-b*x^2/a))/(a*b)) - b*p*(log(b*x^2 + a)/a - log(x^2)/a)*log((b*x^2 + a)^p*c
) - 1/2*log((b*x^2 + a)^p*c)^2/x^2
```

Giac [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^3}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x^3}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^3,x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^3}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)^2}{x^3}\,dx$$

```
[In] int(log(c*(a + b*x^2)^p)^2/x^3,x)
[Out] int(log(c*(a + b*x^2)^p)^2/x^3, x)
```

3.82 $\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 129

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^5}\,dx = \frac{b^2p^2\log(x)}{a^2} - \frac{bp(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2a^2x^2} - \frac{\log^2\left(c(a+bx^2)^p\right)}{4x^4} - \frac{b^2p\log\left(c(a+bx^2)^p\right)\log\left(1-\frac{a}{a+bx^2}\right)}{2a^2} + \frac{b^2p^2\operatorname{PolyLog}\left(2,\frac{a}{a+bx^2}\right)}{2a^2}$$

```
[Out] b^2*p^2*ln(x)/a^2-1/2*b*p*(b*x^2+a)*ln(c*(b*x^2+a)^p)/a^2/x^2-1/4*ln(c*(b*x
^2+a)^p)^2/x^4-1/2*b^2*p*ln(c*(b*x^2+a)^p)*ln(1-a/(b*x^2+a))/a^2+1/2*b^2*p^
2*polylog(2,a/(b*x^2+a))/a^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31}

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^5}\,dx = -\frac{b^2p\log\left(1-\frac{a}{a+bx^2}\right)\log\left(c(a+bx^2)^p\right)}{2a^2} + \frac{b^2p^2\operatorname{PolyLog}\left(2,\frac{a}{bx^2+a}\right)}{2a^2} + \frac{b^2p^2\log(x)}{a^2} - \frac{bp(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2a^2x^2} - \frac{\log^2\left(c(a+bx^2)^p\right)}{4x^4}$$

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^5,x]
```

```
[Out] (b^2*p^2*Log[x])/a^2 - (b*p*(a + b*x^2)*Log[c*(a + b*x^2)^p])/(2*a^2*x^2) -
 Log[c*(a + b*x^2)^p]^2/(4*x^4) - (b^2*p*Log[c*(a + b*x^2)^p]*Log[1 - a/(a
+ b*x^2)])/(2*a^2) + (b^2*p^2*PolyLog[2, a/(a + b*x^2)])/(2*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\log^2\left(c(a+bx)^p\right)}{x^3}\,dx, x, x^2\right) \\
&= -\frac{\log^2\left(c(a+bx^2)^p\right)}{4x^4} + \frac{1}{2}(bp)\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^2(a+bx)}\,dx, x, x^2\right) \\
&= -\frac{\log^2\left(c(a+bx^2)^p\right)}{4x^4} + \frac{1}{2}p\text{Subst}\left(\int \frac{\log\left(cx^p\right)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx^2\right) \\
&= -\frac{\log^2\left(c(a+bx^2)^p\right)}{4x^4} + \frac{p\text{Subst}\left(\int \frac{\log(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx^2\right)}{2a} \\
&\quad - \frac{(bp)\text{Subst}\left(\int \frac{\log(cx^p)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)}\,dx, x, a+bx^2\right)}{2a} \\
&= -\frac{bp(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2a^2x^2} - \frac{\log^2\left(c(a+bx^2)^p\right)}{4x^4} - \frac{b^2p\log\left(c(a+bx^2)^p\right)\log\left(1-\frac{a}{a+bx^2}\right)}{2a^2} \\
&\quad + \frac{(bp^2)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx^2\right)}{2a^2} + \frac{(b^2p^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{a}{x}\right)}{x}\,dx, x, a+bx^2\right)}{2a^2} \\
&= \frac{b^2p^2\log(x)}{a^2} - \frac{bp(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2a^2x^2} - \frac{\log^2\left(c(a+bx^2)^p\right)}{4x^4} \\
&\quad - \frac{b^2p\log\left(c(a+bx^2)^p\right)\log\left(1-\frac{a}{a+bx^2}\right)}{2a^2} + \frac{b^2p^2\text{Li}_2\left(\frac{a}{a+bx^2}\right)}{2a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.09

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^5}\,dx$$

$$= \frac{-\log^2\left(c(a+bx^2)^p\right) + \frac{bx^2\left(4bp^2x^2\log(x) - 2bp^2x^2\log(a+bx^2) - 2ap\log\left(c(a+bx^2)^p\right) + bx^2\log^2\left(c(a+bx^2)^p\right) - 2bpx^2\left(\log\left(-\frac{bx^2}{a}\right)\log(c(a\right.\right.}{a^2}}{4x^4}$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^5,x]
```

```
[Out] (-Log[c*(a + b*x^2)^p]^2 + (b*x^2*(4*b*p^2*x^2*Log[x] - 2*b*p^2*x^2*Log[a +
 b*x^2] - 2*a*p*Log[c*(a + b*x^2)^p] + b*x^2*Log[c*(a + b*x^2)^p]^2 - 2*b*p
*x^2*(Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + p*PolyLog[2, 1 + (b*x^2)/a])
))/a^2)/(4*x^4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.64 (sec) , antiderivative size = 554, normalized size of antiderivative = 4.29

method	result
risch	$-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)^{2}}{4x^{4}}-\frac{pb\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{2a\,x^{2}}-\frac{p\,b^{2}\ln\left(\left(b\,x^{2}+a\right)^{p}\right)\ln\left(x\right)}{a^{2}}+\frac{p\,b^{2}\ln\left(\left(b\,x^{2}+a\right)^{p}\right)\ln\left(b\,x^{2}+a\right)}{2a^{2}}-\frac{p^{2}b^{2}\ln\left(b\,x^{2}+a\right)^{2}}{4a^{2}}+\frac{b^{2}p^{2}}{}$

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln((b*x^2+a)^p)^2/x^4-1/2*p*b*ln((b*x^2+a)^p)/a/x^2-p*b^2*ln((b*x^2+a)
^p)/a^2*ln(x)+1/2*p*b^2*ln((b*x^2+a)^p)/a^2*ln(b*x^2+a)-1/4*p^2*b^2/a^2*ln(
b*x^2+a)^2+b^2*p^2*ln(x)/a^2-1/2*p^2*b^2/a^2*ln(b*x^2+a)+p^2*b^2/a^2*ln(x)*
ln((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+p^2*b^2/a^2*ln(x)*ln((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))+p^2*b^2/a^2*dilog((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+p^2*b^2
/a^2*dilog((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(
I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)
-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c
))*(-1/4/x^4*ln((b*x^2+a)^p)+1/2*p*b*(-1/2/a/x^2-1/a^2*b*ln(x)+1/2*b/a^2*ln
(b*x^2+a)))-1/16*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csg
n(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)
^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c))^2/x^4
```

Fricas [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^5}\,dx=\int \frac{\log\left(\left(bx^2+a\right)^pc\right)^2}{x^5}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^5,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^5, x)
```

Sympy [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^5}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)^2}{x^5}\,dx$$

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**5,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**2/x**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.10

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^5}\,dx =$$
$$-\frac{1}{4}\,b^2p^2\left(\frac{\log\left(bx^2+a\right)^2}{a^2}-\frac{2\left(2\,\log\left(\frac{bx^2}{a}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{bx^2}{a}\right)\right)}{a^2}+\frac{2\,\log\left(bx^2+a\right)}{a^2}-\frac{4\,\log\left(x\right)}{a^2}\right)$$
$$+\frac{1}{2}\,bp\left(\frac{b\log\left(bx^2+a\right)}{a^2}-\frac{b\log\left(x^2\right)}{a^2}-\frac{1}{ax^2}\right)\log\left(\left(bx^2+a\right)^pc\right)-\frac{\log\left(\left(bx^2+a\right)^pc\right)^2}{4\,x^4}$$

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*p^2*(log(b*x^2 + a)^2/a^2 - 2*(2*log(b*x^2/a + 1)*log(x) + dilog(-
b*x^2/a))/a^2 + 2*log(b*x^2 + a)/a^2 - 4*log(x)/a^2) + 1/2*b*p*(b*log(b*x^2
 + a)/a^2 - b*log(x^2)/a^2 - 1/(a*x^2))*log((b*x^2 + a)^p*c) - 1/4*log((b*x
^2 + a)^p*c)^2/x^4
```

Giac [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^5}\,dx = \int \frac{\log\left(\left(bx^2+a\right)^pc\right)^2}{x^5}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^5}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)^2}{x^5}\,dx$$

```
[In] int(log(c*(a + b*x^2)^p)^2/x^5,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^2/x^5, x)
```

3.83 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^7}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 193

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^7}\,dx = -\frac{b^2p^2}{6a^2x^2} - \frac{b^3p^2\log(x)}{a^3} + \frac{b^3p^2\log\left(a+bx^2\right)}{6a^3} - \frac{bp\log\left(c(a+bx^2)^p\right)}{6ax^4} + \frac{b^2p(a+bx^2)\log\left(c(a+bx^2)^p\right)}{3a^3x^2} - \frac{\log^2\left(c(a+bx^2)^p\right)}{6x^6} + \frac{b^3p\log\left(c(a+bx^2)^p\right)\log\left(1-\frac{a}{a+bx^2}\right)}{3a^3} - \frac{b^3p^2\operatorname{PolyLog}\left(2,\frac{a}{a+bx^2}\right)}{3a^3}$$

```
[Out] -1/6*b^2*p^2/a^2/x^2-b^3*p^2*ln(x)/a^3+1/6*b^3*p^2*ln(b*x^2+a)/a^3-1/6*b*p*
ln(c*(b*x^2+a)^p)/a/x^4+1/3*b^2*p*(b*x^2+a)*ln(c*(b*x^2+a)^p)/a^3/x^2-1/6*l
n(c*(b*x^2+a)^p)^2/x^6+1/3*b^3*p*ln(c*(b*x^2+a)^p)*ln(1-a/(b*x^2+a))/a^3-1/
3*b^3*p^2*polylog(2,a/(b*x^2+a))/a^3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^7}\,dx = \frac{b^3p\log\left(1-\frac{a}{a+bx^2}\right)\log\left(c(a+bx^2)^p\right)}{3a^3} - \frac{b^3p^2\operatorname{PolyLog}\left(2,\frac{a}{bx^2+a}\right)}{3a^3} + \frac{b^3p^2\log\left(a+bx^2\right)}{6a^3} - \frac{b^3p^2\log(x)}{a^3} + \frac{b^2p(a+bx^2)\log\left(c(a+bx^2)^p\right)}{3a^3x^2} - \frac{b^2p^2}{6a^2x^2} - \frac{\log^2\left(c(a+bx^2)^p\right)}{6x^6} - \frac{bp\log\left(c(a+bx^2)^p\right)}{6ax^4}$$

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^7,x]
```

```
[Out] -1/6*(b^2*p^2)/(a^2*x^2) - (b^3*p^2*Log[x])/a^3 + (b^3*p^2*Log[a + b*x^2])/
(6*a^3) - (b*p*Log[c*(a + b*x^2)^p])/(6*a*x^4) + (b^2*p*(a + b*x^2)*Log[c*(
a + b*x^2)^p])/(3*a^3*x^2) - Log[c*(a + b*x^2)^p]^2/(6*x^6) + (b^3*p*Log[c*
(a + b*x^2)^p]*Log[1 - a/(a + b*x^2)])/(3*a^3) - (b^3*p^2*PolyLog[2, a/(a +
 b*x^2)])/(3*a^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\log^2\left(c(a+bx)^p\right)}{x^4}\,dx,x,x^2\right)\\
&= -\frac{\log^2\left(c(a+bx^2)^p\right)}{6x^6}+\frac{1}{3}(bp)\text{Subst}\left(\int \frac{\log\left(c(a+bx)^p\right)}{x^3(a+bx)}\,dx,x,x^2\right)\\
&= -\frac{\log^2\left(c(a+bx^2)^p\right)}{6x^6}+\frac{1}{3}p\text{Subst}\left(\int \frac{\log\left(cx^p\right)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx,x,a+bx^2\right)\\
&= -\frac{\log^2\left(c(a+bx^2)^p\right)}{6x^6}+\frac{p\text{Subst}\left(\int \frac{\log(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx,x,a+bx^2\right)}{3a}\\
&\quad-\frac{(bp)\text{Subst}\left(\int \frac{\log(cx^p)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx,x,a+bx^2\right)}{3a}
\end{aligned}
$$

$$= -\frac{bp\log\left(c(a+bx^2)^p\right)}{6ax^4} - \frac{\log^2\left(c(a+bx^2)^p\right)}{6x^6} - \frac{(bp)\text{Subst}\left(\int \frac{\log(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx^2\right)}{3a^2} + \frac{(b^2p)\,\text{Subst}\left(\int \frac{\log(cx^p)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)}\,dx, x, a+bx^2\right)}{3a^2} + \frac{(bp^2)\,\text{Subst}\left(\int \frac{1}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx^2\right)}{6a}$$

$$= -\frac{bp\log\left(c(a+bx^2)^p\right)}{6ax^4} + \frac{b^2p(a+bx^2)\log\left(c(a+bx^2)^p\right)}{3a^3x^2} - \frac{\log^2\left(c(a+bx^2)^p\right)}{6x^6} + \frac{b^3p\log\left(c(a+bx^2)^p\right)\log\left(1-\frac{a}{a+bx^2}\right)}{3a^3} + \frac{(bp^2)\,\text{Subst}\left(\int \left(\frac{b^2}{a(a-x)^2}+\frac{b^2}{a^2(a-x)}+\frac{b^2}{a^2x}\right)dx, x, a+bx^2\right)}{6a} - \frac{(b^2p^2)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx^2\right)}{3a^3} - \frac{(b^3p^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{a}{x}\right)}{x}\,dx, x, a+bx^2\right)}{3a^3}$$

$$= -\frac{b^2p^2}{6a^2x^2} - \frac{b^3p^2\log(x)}{a^3} + \frac{b^3p^2\log\left(a+bx^2\right)}{6a^3} - \frac{bp\log\left(c(a+bx^2)^p\right)}{6ax^4} + \frac{b^2p(a+bx^2)\log\left(c(a+bx^2)^p\right)}{3a^3x^2} - \frac{\log^2\left(c(a+bx^2)^p\right)}{6x^6} + \frac{b^3p\log\left(c(a+bx^2)^p\right)\log\left(1-\frac{a}{a+bx^2}\right)}{3a^3} - \frac{b^3p^2\text{Li}_2\left(\frac{a}{a+bx^2}\right)}{3a^3}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.98

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^7}\,dx = -\frac{ab^2p^2x^4 + 6b^3p^2x^6\log(x) - 3b^3p^2x^6\log\left(a+bx^2\right) + a^2bpx^2\log\left(c(a+bx^2)^p\right) - 2ab^2px^4\log\left(c(a+bx^2)^p\right)}{}$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^7,x]
```

```
[Out] -1/6*(a*b^2*p^2*x^4 + 6*b^3*p^2*x^6*Log[x] - 3*b^3*p^2*x^6*Log[a + b*x^2] +
 a^2*b*p*x^2*Log[c*(a + b*x^2)^p] - 2*a*b^2*p*x^4*Log[c*(a + b*x^2)^p] - 2*
b^3*p*x^6*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + a^3*Log[c*(a + b*x^2)^p]
^2 + b^3*x^6*Log[c*(a + b*x^2)^p]^2 - 2*b^3*p^2*x^6*PolyLog[2, 1 + (b*x^2)/
a])/(a^3*x^6)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.30 (sec) , antiderivative size = 607, normalized size of antiderivative = 3.15

method	result
risch	$-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)^{2}}{6x^{6}}-\frac{pb\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{6a\,x^{4}}+\frac{2p\,b^{3}\ln\left(\left(b\,x^{2}+a\right)^{p}\right)\ln\left(x\right)}{3a^{3}}+\frac{p\,b^{2}\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{3a^{2}x^{2}}-\frac{p\,b^{3}\ln\left(\left(b\,x^{2}+a\right)^{p}\right)\ln\left(b\,x^{2}+a\right)}{3a^{3}}-$

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*ln((b*x^2+a)^p)^2/x^6-1/6*p*b*ln((b*x^2+a)^p)/a/x^4+2/3*p*b^3*ln((b*x^
2+a)^p)/a^3*ln(x)+1/3*p*b^2*ln((b*x^2+a)^p)/a^2/x^2-1/3*p*b^3*ln((b*x^2+a)^
p)/a^3*ln(b*x^2+a)-1/6*b^2*p^2/a^2/x^2-b^3*p^2*ln(x)/a^3+1/2*b^3*p^2*ln(b*x
^2+a)/a^3-2/3*p^2*b^3/a^3*ln(x)*ln((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-2/3*p^
2*b^3/a^3*ln(x)*ln((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-2/3*p^2*b^3/a^3*dilog((
-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))-2/3*p^2*b^3/a^3*dilog((b*x+(-a*b)^(1/2))/(
-a*b)^(1/2))+1/6*p^2*b^3/a^3*ln(b*x^2+a)^2+(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I
*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-
I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)
)*(-1/6/x^6*ln((b*x^2+a)^p)+1/3*p*b*(-1/4/a/x^4+b^2/a^3*ln(x)+1/2*b/a^2/x^2
-1/2*b^2/a^3*ln(b*x^2+a)))-1/24*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a
)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I
*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c))^2/x^6
```

Fricas [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^7}\,dx = \int \frac{\log\left((bx^2+a)^pc\right)^2}{x^7}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^7,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^7, x)
```

Sympy [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^7}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)^2}{x^7}\,dx$$

[In] integrate(ln(c*(b*x**2+a)**p)**2/x**7,x)

[Out] Integral(log(c*(a + b*x**2)**p)**2/x**7, x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.90

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^7}\,dx = -\frac{1}{6}\,b^2p^2\left(\frac{2\left(2\,\log\left(\frac{bx^2}{a}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{bx^2}{a}\right)\right)b}{a^3} - \frac{3\,b\log\left(bx^2+a\right)}{a^3} - \frac{bx^2\log\left(bx^2+a\right)^2 - 6\,bx^2\log\left(x\right)\right. \\ - \frac{1}{6}\,bp\left(\frac{2\,b^2\log\left(bx^2+a\right)}{a^3} - \frac{2\,b^2\log\left(x^2\right)}{a^3} - \frac{2\,bx^2-a}{a^2x^4}\right)\log\left(\left(bx^2+a\right)^pc\right) \\ - \frac{\log\left(\left(bx^2+a\right)^pc\right)^2}{6\,x^6}$$

[In] integrate(log(c*(b*x^2+a)^p)^2/x^7,x, algorithm="maxima")

[Out] -1/6*b^2*p^2*(2*(2*log(b*x^2/a + 1)*log(x) + dilog(-b*x^2/a))*b/a^3 - 3*b*l
og(b*x^2 + a)/a^3 - (b*x^2*log(b*x^2 + a)^2 - 6*b*x^2*log(x) - a)/(a^3*x^2)
) - 1/6*b*p*(2*b^2*log(b*x^2 + a)/a^3 - 2*b^2*log(x^2)/a^3 - (2*b*x^2 - a)/
(a^2*x^4))*log((b*x^2 + a)^p*c) - 1/6*log((b*x^2 + a)^p*c)^2/x^6

Giac [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^7}\,dx = \int \frac{\log\left(\left(bx^2+a\right)^pc\right)^2}{x^7}\,dx$$

[In] integrate(log(c*(b*x^2+a)^p)^2/x^7,x, algorithm="giac")

[Out] integrate(log((b*x^2 + a)^p*c)^2/x^7, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^7}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)^2}{x^7}\,dx$$

```
[In] int(log(c*(a + b*x^2)^p)^2/x^7,x)
[Out] int(log(c*(a + b*x^2)^p)^2/x^7, x)
```

3.84 $\int x^4 \log^2\left(c\left(a+bx^2\right)^p\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 336

$$\int x^4 \log^2\left(c\left(a+bx^2\right)^p\right) \, dx = \frac{184a^2p^2x}{75b^2} - \frac{64ap^2x^3}{225b} + \frac{8p^2x^5}{125} - \frac{184a^{5/2}p^2 \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{75b^{5/2}} + \frac{4ia^{5/2}p^2 \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5b^{5/2}} + \frac{8a^{5/2}p^2 \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) \log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{5b^{5/2}} - \frac{4a^2px \log\left(c\left(a+bx^2\right)^p\right)}{5b^2} + \frac{4apx^3 \log\left(c\left(a+bx^2\right)^p\right)}{15b} - \frac{4}{25}px^5 \log\left(c\left(a+bx^2\right)^p\right) + \frac{4a^{5/2}p \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) \log\left(c\left(a+bx^2\right)^p\right)}{5b^{5/2}} + \frac{1}{5}x^5 \log^2\left(c\left(a+bx^2\right)^p\right) + \frac{4ia^{5/2}p^2 \operatorname{PolyL}\ldots}{\ldots}$$

```
[Out] 184/75*a^2*p^2*x/b^2-64/225*a*p^2*x^3/b+8/125*p^2*x^5-184/75*a^(5/2)*p^2*ar
ctan(x*b^(1/2)/a^(1/2))/b^(5/2)+4/5*I*a^(5/2)*p^2*arctan(x*b^(1/2)/a^(1/2))
^2/b^(5/2)-4/5*a^2*p*x*ln(c*(b*x^2+a)^p)/b^2+4/15*a*p*x^3*ln(c*(b*x^2+a)^p)
/b-4/25*p*x^5*ln(c*(b*x^2+a)^p)+4/5*a^(5/2)*p*arctan(x*b^(1/2)/a^(1/2))*ln(
c*(b*x^2+a)^p)/b^(5/2)+1/5*x^5*ln(c*(b*x^2+a)^p)^2+8/5*a^(5/2)*p^2*arctan(x
*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/b^(5/2)+4/5*I*a^(5/2)
*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/b^(5/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {2507, 2526, 2498, 327, 211, 2505, 308, 2520, 12, 5040, 4964, 2449, 2352}

$$\int x^4 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx=\frac{4a^{5/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{5b^{5/2}}+\frac{4ia^{5/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5b^{5/2}}-\frac{184a^{5/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{75b^{5/2}}+\frac{8a^{5/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{5b^{5/2}}+\frac{4ia^{5/2}p^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}\right)}{5b^{5/2}}-\frac{4a^2px\log\left(c(a+bx^2)^p\right)}{5b^2}+\frac{184a^2p^2x}{75b^2}+\frac{1}{5}x^5\log^2\left(c\left(a+bx^2\right)^p\right)-\frac{4}{25}px^5\log\left(c\left(a+bx^2\right)^p\right)+\frac{4apx^3\log\left(c(a+bx^2)^p\right)}{15b}-\frac{64ap^2x^3}{225b}+\frac{8p^2x^5}{125}$$

```
[In] Int[x^4*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] (184*a^2*p^2*x)/(75*b^2) - (64*a*p^2*x^3)/(225*b) + (8*p^2*x^5)/125 - (184*
a^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(75*b^(5/2)) + (((4*I)/5)*a^(5/2)*
p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/b^(5/2) + (8*a^(5/2)*p^2*ArcTan[(Sqrt[b]
*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(5*b^(5/2)) - (4*a^2
*p*x*Log[c*(a + b*x^2)^p])/(5*b^2) + (4*a*p*x^3*Log[c*(a + b*x^2)^p])/(15*b
) - (4*p*x^5*Log[c*(a + b*x^2)^p])/25 + (4*a^(5/2)*p*ArcTan[(Sqrt[b]*x)/Sqr
t[a]]*Log[c*(a + b*x^2)^p])/(5*b^(5/2)) + (x^5*Log[c*(a + b*x^2)^p]^2)/5 +
(((4*I)/5)*a^(5/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])
/b^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\log^2\left(c\left(a+bx^2\right)^p\right)-\frac{1}{5}(4bp)\int\frac{x^6\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx\\
&= \frac{1}{5}x^5\log^2\left(c\left(a+bx^2\right)^p\right)-\frac{1}{5}(4bp)\int\left(\frac{a^2\log\left(c\left(a+bx^2\right)^p\right)}{b^3}-\frac{ax^2\log\left(c\left(a+bx^2\right)^p\right)}{b^2}\right.\\
&\qquad\left.+\frac{x^4\log\left(c\left(a+bx^2\right)^p\right)}{b}-\frac{a^3\log\left(c\left(a+bx^2\right)^p\right)}{b^3\left(a+bx^2\right)}\right)dx\\
&= \frac{1}{5}x^5\log^2\left(c\left(a+bx^2\right)^p\right)-\frac{1}{5}(4p)\int x^4\log\left(c\left(a+bx^2\right)^p\right)dx\\
&\quad-\frac{\left(4a^2p\right)\int\log\left(c\left(a+bx^2\right)^p\right)dx}{5b^2}+\frac{\left(4a^3p\right)\int\frac{\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx}{5b^2}\\
&\quad+\frac{(4ap)\int x^2\log\left(c\left(a+bx^2\right)^p\right)dx}{5b}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4a^2px\log\left(c(a+bx^2)^p\right)}{5b^2}+\frac{4apx^3\log\left(c(a+bx^2)^p\right)}{15b} \\
&\quad -\frac{4}{25}px^5\log\left(c(a+bx^2)^p\right)+\frac{4a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{5b^{5/2}} \\
&\quad +\frac{1}{5}x^5\log^2\left(c(a+bx^2)^p\right)-\frac{1}{15}(8ap^2)\int\frac{x^4}{a+bx^2}\,dx+\frac{(8a^2p^2)\int\frac{x^2}{a+bx^2}\,dx}{5b} \\
&\quad -\frac{(8a^3p^2)\int\frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(a+bx^2)}\,dx}{5b}+\frac{1}{25}(8bp^2)\int\frac{x^6}{a+bx^2}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{8a^2p^2x}{5b^2}-\frac{4a^2px\log\left(c(a+bx^2)^p\right)}{5b^2}+\frac{4apx^3\log\left(c(a+bx^2)^p\right)}{15b} \\
&\quad -\frac{4}{25}px^5\log\left(c(a+bx^2)^p\right)+\frac{4a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{5b^{5/2}} \\
&\quad +\frac{1}{5}x^5\log^2\left(c(a+bx^2)^p\right)-\frac{1}{15}(8ap^2)\int\left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2(a+bx^2)}\right)dx \\
&\quad -\frac{(8a^3p^2)\int\frac{1}{a+bx^2}\,dx}{5b^2}-\frac{(8a^{5/2}p^2)\int\frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a+bx^2}\,dx}{5b^{3/2}} \\
&\quad +\frac{1}{25}(8bp^2)\int\left(\frac{a^2}{b^3}-\frac{ax^2}{b^2}+\frac{x^4}{b}-\frac{a^3}{b^3(a+bx^2)}\right)dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{184a^2p^2x}{75b^2}-\frac{64ap^2x^3}{225b}+\frac{8p^2x^5}{125}-\frac{8a^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{5b^{5/2}} \\
&\quad +\frac{4ia^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5b^{5/2}}-\frac{4a^2px\log\left(c(a+bx^2)^p\right)}{5b^2}+\frac{4apx^3\log\left(c(a+bx^2)^p\right)}{15b} \\
&\quad -\frac{4}{25}px^5\log\left(c(a+bx^2)^p\right)+\frac{4a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{5b^{5/2}}+\frac{1}{5}x^5\log^2\left(c(a+bx^2)^p\right)+\frac{(8a^2p^2)\ \cdots}{}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{184a^2p^2x}{75b^2}-\frac{64ap^2x^3}{225b}+\frac{8p^2x^5}{125}-\frac{184a^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{75b^{5/2}}+\frac{4ia^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5b^{5/2}} \\
&\quad +\frac{8a^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{5b^{5/2}}-\frac{4a^2px\log\left(c(a+bx^2)^p\right)}{5b^2}+\frac{4apx^3\log\left(c(a+bx^2)^p\right)}{15b} \\
&\quad -\frac{4}{25}px^5\log\left(c(a+bx^2)^p\right)+\frac{4a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{5b^{5/2}}+\frac{1}{5}x^5\log^2\left(c(a+bx^2)^p\right)-\frac{(8a^2p^2)\ \cdots}{}
\end{aligned}
$$

$$= \frac{184a^2p^2x}{75b^2} - \frac{64ap^2x^3}{225b} + \frac{8p^2x^5}{125} - \frac{184a^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{75b^{5/2}} + \frac{4ia^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5b^{5/2}} + \frac{8a^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{5b^{5/2}} - \frac{4a^2px\log\left(c(a+bx^2)^p\right)}{5b^2} + \frac{4apx^3\log\left(c(a+bx^2)^p\right)}{15b} - \frac{4}{25}px^5\log\left(c(a+bx^2)^p\right) + \frac{4a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{5b^{5/2}} + \frac{1}{5}x^5\log^2\left(c(a+bx^2)^p\right) + \frac{(8ia^{5/2}}{}$$

$$= \frac{184a^2p^2x}{75b^2} - \frac{64ap^2x^3}{225b} + \frac{8p^2x^5}{125} - \frac{184a^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{75b^{5/2}} + \frac{4ia^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5b^{5/2}} + \frac{8a^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{5b^{5/2}} - \frac{4a^2px\log\left(c(a+bx^2)^p\right)}{5b^2} + \frac{4apx^3\log\left(c(a+bx^2)^p\right)}{15b} - \frac{4}{25}px^5\log\left(c(a+bx^2)^p\right) + \frac{4a^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{5b^{5/2}} + \frac{1}{5}x^5\log^2\left(c(a+bx^2)^p\right) + \frac{4ia^{5/2}p}{}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.74

$$\int x^4\log^2\left(c(a+bx^2)^p\right)\,dx = \frac{900ia^{5/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2 + 60a^{5/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-46p+30p\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)+15\log\left(c(a+bx^2)^p\right)\right)+}{}$$

```
[In] Integrate[x^4*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] ((900*I)*a^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + 60*a^(5/2)*p*ArcTan[(S
qrt[b]*x)/Sqrt[a]]*(-46*p + 30*p*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)] +
 15*Log[c*(a + b*x^2)^p]) + Sqrt[b]*x*(8*p^2*(345*a^2 - 40*a*b*x^2 + 9*b^2*
x^4) - 60*p*(15*a^2 - 5*a*b*x^2 + 3*b^2*x^4)*Log[c*(a + b*x^2)^p] + 225*b^2
*x^4*Log[c*(a + b*x^2)^p]^2) + (900*I)*a^(5/2)*p^2*PolyLog[2, (I*Sqrt[a] +
Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(1125*b^(5/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.86 (sec) , antiderivative size = 612, normalized size of antiderivative = 1.82

method	result
risch	$\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)^{2} x^{5}}{5}-\frac{4 p\,x^{5} \ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{25}+\frac{4 p a\,x^{3} \ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{15 b}-\frac{4 p\,a^{2} x \ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{5 b^{2}}-\frac{4 p^{2} a^{3} \arctan\left(\frac{b x}{\sqrt{a b}}\right) \ln\left(b\,x^{2}+a\right)}{5 b^{2} \sqrt{a b}}+$

```
[In] int(x^4*ln(c*(b*x^2+a)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*ln((b*x^2+a)^p)^2*x^5-4/25*p*x^5*ln((b*x^2+a)^p)+4/15*p/b*a*x^3*ln((b*x
^2+a)^p)-4/5*p/b^2*a^2*x*ln((b*x^2+a)^p)-4/5*p^2/b^2*a^3/(a*b)^(1/2)*arctan
(b*x/(a*b)^(1/2))*ln(b*x^2+a)+4/5*p/b^2*a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1
/2))*ln((b*x^2+a)^p)+8/125*p^2*x^5-64/225*a*p^2*x^3/b+184/75*a^2*p^2*x/b^2-
184/75*p^2/b^2*a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-4/5*p^2*b*Sum(-1/2*(
ln(x-_alpha)*ln(b*x^2+a)-2*b*(1/4/_alpha/b*ln(x-_alpha)^2+1/2*_alpha/a*ln(x
-_alpha)*ln(1/2*(x+_alpha)/_alpha)+1/2*_alpha/a*dilog(1/2*(x+_alpha)/_alpha
)))*a^3/b^4/_alpha,_alpha=RootOf(_Z^2*b+a))+(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(
I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)
-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c
))*(1/5*x^5*ln((b*x^2+a)^p)-2/5*p*b*(1/b^3*(1/5*x^5*b^2-1/3*a*b*x^3+a^2*x)-
a^3/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))+1/20*(I*Pi*csgn(I*(b*x^2+a)^p
)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*cs
gn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)
+2*ln(c))^2*x^5
```

Fricas [F]

$$\int x^4 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx=\int x^4 \log\left(\left(bx^2+a\right)^p c\right)^2\,dx$$

```
[In] integrate(x^4*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] integral(x^4*log((b*x^2 + a)^p*c)^2, x)
```

Sympy [F]

$$\int x^4 \log^2\left(c(a+bx^2)^p\right)\, dx = \int x^4 \log\left(c(a+bx^2)^p\right)^2 dx$$

[In] integrate(x**4*ln(c*(b*x**2+a)**p)**2,x)

[Out] Integral(x**4*log(c*(a + b*x**2)**p)**2, x)

Maxima [F]

$$\int x^4 \log^2\left(c(a+bx^2)^p\right)\, dx = \int x^4 \log\left((bx^2+a)^p c\right)^2\, dx$$

[In] integrate(x^4*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")

[Out] 1/5*p^2*x^5*log(b*x^2 + a)^2 + integrate(1/5*(5*b*x^6*log(c)^2 + 5*a*x^4*log(c)^2 - 2*((2*p^2 - 5*p*log(c))*b*x^6 - 5*a*p*x^4*log(c))*log(b*x^2 + a))/(b*x^2 + a), x)

Giac [F]

$$\int x^4 \log^2\left(c(a+bx^2)^p\right)\, dx = \int x^4 \log\left((bx^2+a)^p c\right)^2\, dx$$

[In] integrate(x^4*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")

[Out] integrate(x^4*log((b*x^2 + a)^p*c)^2, x)

Mupad [F(-1)]

Timed out.

$$\int x^4 \log^2\left(c(a+bx^2)^p\right)\, dx = \int x^4 \ln\left(c\left(b\,x^2+a\right)^p\right)^2 dx$$

[In] int(x^4*log(c*(a + b*x^2)^p)^2,x)

[Out] int(x^4*log(c*(a + b*x^2)^p)^2, x)

3.85 $\int x^2 \log^2 \left(c(a+bx^2)^p\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 294

$$\int x^2 \log^2 \left(c\left(a+bx^2\right)^p\right) \, dx = -\frac{32ap^2x}{9b} + \frac{8p^2x^3}{27} + \frac{32a^{3/2}p^2 \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}} - \frac{4ia^{3/2}p^2 \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}} - \frac{8a^{3/2}p^2 \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) \log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3b^{3/2}} + \frac{4apx \log\left(c\left(a+bx^2\right)^p\right)}{3b} - \frac{4}{9}px^3 \log\left(c\left(a+bx^2\right)^p\right) - \frac{4a^{3/2}p \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) \log\left(c\left(a+bx^2\right)^p\right)}{3b^{3/2}} + \frac{1}{3}x^3 \log^2\left(c\left(a+bx^2\right)^p\right) - \frac{4ia^{3/2}p^2 \operatorname{PolyLog}}{3}$$

```
[Out] -32/9*a*p^2*x/b+8/27*p^2*x^3+32/9*a^(3/2)*p^2*arctan(x*b^(1/2)/a^(1/2))/b^(
3/2)-4/3*I*a^(3/2)*p^2*arctan(x*b^(1/2)/a^(1/2))^2/b^(3/2)+4/3*a*p*x*ln(c*(
b*x^2+a)^p)/b-4/9*p*x^3*ln(c*(b*x^2+a)^p)-4/3*a^(3/2)*p*arctan(x*b^(1/2)/a^
(1/2))*ln(c*(b*x^2+a)^p)/b^(3/2)+1/3*x^3*ln(c*(b*x^2+a)^p)^2-8/3*a^(3/2)*p^
2*arctan(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/b^(3/2)-4/3
*I*a^(3/2)*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/b^(3/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules

used = {2507, 2526, 2498, 327, 211, 2505, 308, 2520, 12, 5040, 4964, 2449, 2352}

$$\begin{aligned}
&\int x^2 \log^2\left(c\left(a+bx^2\right)^p\right)\,dx \\
&= -\frac{4a^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3b^{3/2}} - \frac{4ia^{3/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}} \\
&\quad + \frac{32a^{3/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}} - \frac{8a^{3/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3b^{3/2}} \\
&\quad - \frac{4ia^{3/2}p^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}\right)}{3b^{3/2}} + \frac{4apx\log\left(c\left(a+bx^2\right)^p\right)}{3b} \\
&\quad + \frac{1}{3}x^3\log^2\left(c\left(a+bx^2\right)^p\right) - \frac{4}{9}px^3\log\left(c\left(a+bx^2\right)^p\right) - \frac{32ap^2x}{9b} + \frac{8p^2x^3}{27}
\end{aligned}$$

```
[In] Int[x^2*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] (-32*a*p^2*x)/(9*b) + (8*p^2*x^3)/27 + (32*a^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(9*b^(3/2)) - (((4*I)/3)*a^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2
)/b^(3/2) - (8*a^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqr
t[a] + I*Sqrt[b]*x)])/(3*b^(3/2)) + (4*a*p*x*Log[c*(a + b*x^2)^p])/(3*b) -
(4*p*x^3*Log[c*(a + b*x^2)^p])/9 - (4*a^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
*Log[c*(a + b*x^2)^p])/(3*b^(3/2)) + (x^3*Log[c*(a + b*x^2)^p]^2)/3 - (((4*
I)/3)*a^(3/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/b^(3
/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
```

```
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\log^2\left(c\left(a+bx^2\right)^p\right)-\frac{1}{3}(4bp)\int\frac{x^4\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx\\
&= \frac{1}{3}x^3\log^2\left(c\left(a+bx^2\right)^p\right)-\frac{1}{3}(4bp)\int\left(-\frac{a\log\left(c\left(a+bx^2\right)^p\right)}{b^2}+\frac{x^2\log\left(c\left(a+bx^2\right)^p\right)}{b}\right.\\
&\qquad\left.+\frac{a^2\log\left(c\left(a+bx^2\right)^p\right)}{b^2\left(a+bx^2\right)}\right)dx\\
&= \frac{1}{3}x^3\log^2\left(c\left(a+bx^2\right)^p\right)-\frac{1}{3}(4p)\int x^2\log\left(c\left(a+bx^2\right)^p\right)\,dx\\
&\quad+\frac{(4ap)\int\log\left(c\left(a+bx^2\right)^p\right)\,dx}{3b}-\frac{\left(4a^2p\right)\int\frac{\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx}{3b}\\
&= \frac{4apx\log\left(c\left(a+bx^2\right)^p\right)}{3b}-\frac{4}{9}px^3\log\left(c\left(a+bx^2\right)^p\right)\\
&\quad-\frac{4a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3b^{3/2}}\\
&\quad+\frac{1}{3}x^3\log^2\left(c\left(a+bx^2\right)^p\right)-\frac{1}{3}\left(8ap^2\right)\int\frac{x^2}{a+bx^2}\,dx\\
&\quad+\frac{1}{3}\left(8a^2p^2\right)\int\frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\left(a+bx^2\right)}\,dx+\frac{1}{9}\left(8bp^2\right)\int\frac{x^4}{a+bx^2}\,dx
\end{aligned}
$$

$$= -\frac{8ap^2x}{3b} + \frac{4apx\log\left(c\left(a+bx^2\right)^p\right)}{3b} - \frac{4}{9}px^3\log\left(c\left(a+bx^2\right)^p\right) - \frac{4a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3b^{3/2}} + \frac{1}{3}x^3\log^2\left(c\left(a+bx^2\right)^p\right) + \frac{(8a^2p^2)\int\frac{1}{a+bx^2}\,dx}{3b} + \frac{(8a^{3/2}p^2)\int\frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a+bx^2}\,dx}{3\sqrt{b}} + \frac{1}{9}(8bp^2)\int\left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2\left(a+bx^2\right)}\right)dx$$

$$= -\frac{32ap^2x}{9b} + \frac{8p^2x^3}{27} + \frac{8a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}} - \frac{4ia^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}} + \frac{4apx\log\left(c\left(a+bx^2\right)^p\right)}{3b} - \frac{4}{9}px^3\log\left(c\left(a+bx^2\right)^p\right) - \frac{4a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3b^{3/2}} + \frac{1}{3}x^3\log^2\left(c\left(a+bx^2\right)^p\right) - \frac{(8ap^2)\int}{}$$

$$= -\frac{32ap^2x}{9b} + \frac{8p^2x^3}{27} + \frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}} - \frac{4ia^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}} - \frac{8a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3b^{3/2}} + \frac{4apx\log\left(c\left(a+bx^2\right)^p\right)}{3b} - \frac{4}{9}px^3\log\left(c\left(a+bx^2\right)^p\right) - \frac{4a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3b^{3/2}} + \frac{1}{3}x^3\log^2\left(c\left(a+bx^2\right)^p\right) + \frac{(8ap^2)\int}{}$$

$$= -\frac{32ap^2x}{9b} + \frac{8p^2x^3}{27} + \frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}} - \frac{4ia^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}} - \frac{8a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3b^{3/2}} + \frac{4apx\log\left(c\left(a+bx^2\right)^p\right)}{3b} - \frac{4}{9}px^3\log\left(c\left(a+bx^2\right)^p\right) - \frac{4a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3b^{3/2}} + \frac{1}{3}x^3\log^2\left(c\left(a+bx^2\right)^p\right) - \frac{(8ia^{3/2}p^2}{}$$

$$= -\frac{32ap^2x}{9b} + \frac{8p^2x^3}{27} + \frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}} - \frac{4ia^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}} - \frac{8a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3b^{3/2}} + \frac{4apx\log\left(c\left(a+bx^2\right)^p\right)}{3b} - \frac{4}{9}px^3\log\left(c\left(a+bx^2\right)^p\right) - \frac{4a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3b^{3/2}} + \frac{1}{3}x^3\log^2\left(c\left(a+bx^2\right)^p\right) - \frac{4ia^{3/2}p^2\mathrm{L}}{}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.76

$$\int x^2 \log^2\left(c(a+bx^2)^p\right)\,dx$$
$$= \frac{-36ia^{3/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2 - 12a^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-8p+6p\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)+3\log\left(c(a+bx^2)^p\right)\right)+\sqrt{b}\cdots}{\cdots}$$

```
[In] Integrate[x^2*Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] ((-36*I)*a^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 - 12*a^(3/2)*p*ArcTan[(S
qrt[b]*x)/Sqrt[a]]*(-8*p + 6*p*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)] + 3
*Log[c*(a + b*x^2)^p]) + Sqrt[b]*x*(8*p^2*(-12*a + b*x^2) + 12*p*(3*a - b*x
^2)*Log[c*(a + b*x^2)^p] + 9*b*x^2*Log[c*(a + b*x^2)^p]^2) - (36*I)*a^(3/2)
*p^2*PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(27*b^
(3/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.59 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.92

method	result
risch	$\frac{\ln\left(\left(b\,x^2+a\right)^p\right)^2 x^3}{3}-\frac{4p\,x^3\ln\left(\left(b\,x^2+a\right)^p\right)}{9}+\frac{4pax\ln\left(\left(b\,x^2+a\right)^p\right)}{3b}+\frac{4p^2a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)\ln\left(b\,x^2+a\right)}{3b\sqrt{ab}}-\frac{4p\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)\ln(\cdots}{3b\sqrt{ab}}$

```
[In] int(x^2*ln(c*(b*x^2+a)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln((b*x^2+a)^p)^2*x^3-4/9*p*x^3*ln((b*x^2+a)^p)+4/3*p/b*a*x*ln((b*x^2+a
)^p)+4/3*p^2/b*a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*ln(b*x^2+a)-4/3*p/b*
a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*ln((b*x^2+a)^p)+8/27*p^2*x^3-32/9*a
*p^2*x/b+32/9*p^2/b*a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-4/3*p^2*b*Sum(1
/2*(ln(x-_alpha)*ln(b*x^2+a)-2*b*(1/4/_alpha/b*ln(x-_alpha)^2+1/2*_alpha/a*
ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+1/2*_alpha/a*dilog(1/2*(x+_alpha)/_a
lpha)))*a^2/b^3/_alpha,_alpha=RootOf(_Z^2*b+a))+(I*Pi*csgn(I*(b*x^2+a)^p)*c
sgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(
I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*
ln(c))*(1/3*x^3*ln((b*x^2+a)^p)-2/3*p*b*(1/b^2*(1/3*b*x^3-a*x)+a^2/b^2/(a*b
)^(1/2)*arctan(b*x/(a*b)^(1/2))))+1/12*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(
b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi
```

```
*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c))^2*
x^3
```

Fricas [F]

$$\int x^2 \log^2\left(c(a+bx^2)^p\right)\,dx = \int x^2 \log\left((bx^2+a)^p c\right)^2\,dx$$

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
[Out] integral(x^2*log((b*x^2 + a)^p*c)^2, x)
```

Sympy [F]

$$\int x^2 \log^2\left(c(a+bx^2)^p\right)\,dx = \int x^2 \log\left(c(a+bx^2)^p\right)^2\,dx$$

```
[In] integrate(x**2*ln(c*(b*x**2+a)**p)**2,x)
[Out] Integral(x**2*log(c*(a + b*x**2)**p)**2, x)
```

Maxima [F]

$$\int x^2 \log^2\left(c(a+bx^2)^p\right)\,dx = \int x^2 \log\left((bx^2+a)^p c\right)^2\,dx$$

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
[Out] 1/3*p^2*x^3*log(b*x^2 + a)^2 + integrate(1/3*(3*b*x^4*log(c)^2 + 3*a*x^2*lo
g(c)^2 - 2*((2*p^2 - 3*p*log(c))*b*x^4 - 3*a*p*x^2*log(c))*log(b*x^2 + a))/
(b*x^2 + a), x)
```

Giac [F]

$$\int x^2 \log^2\left(c(a+bx^2)^p\right)\,dx = \int x^2 \log\left((bx^2+a)^p c\right)^2\,dx$$

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
[Out] integrate(x^2*log((b*x^2 + a)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \log^2\left(c(a+bx^2)^p\right)\,dx=\int x^2\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2\,dx$$

```
[In] int(x^2*log(c*(a + b*x^2)^p)^2,x)
[Out] int(x^2*log(c*(a + b*x^2)^p)^2, x)
```

3.86 $\int \log^2\left(c(a+bx^2)^p\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 237

$$\begin{aligned}\int \log^2\left(c\left(a+bx^2\right)^p\right)\,dx &= 8p^2x-\frac{8\sqrt{a}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{4i\sqrt{a}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{b}}\\&\quad+\frac{8\sqrt{a}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}-4px\log\left(c\left(a+bx^2\right)^p\right)\\&\quad+\frac{4\sqrt{a}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}}\\&\quad+x\log^2\left(c\left(a+bx^2\right)^p\right)+\frac{4i\sqrt{a}p^2\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}\end{aligned}$$

```
[Out] 8*p^2*x-4*p*x*ln(c*(b*x^2+a)^p)+x*ln(c*(b*x^2+a)^p)^2-8*p^2*arctan(x*b^(1/2
)/a^(1/2))*a^(1/2)/b^(1/2)+4*I*p^2*arctan(x*b^(1/2)/a^(1/2))^2*a^(1/2)/b^(1
/2)+4*p*arctan(x*b^(1/2)/a^(1/2))*ln(c*(b*x^2+a)^p)*a^(1/2)/b^(1/2)+8*p^2*a
rctan(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))*a^(1/2)/b^(1/2
)+4*I*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))*a^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {2500, 2526, 2498, 327, 211, 2520, 12, 5040, 4964, 2449, 2352}

$$\begin{aligned}\int \log^2\left(c\left(a+bx^2\right)^p\right)\,dx = &\frac{4\sqrt{a}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}}+\frac{4i\sqrt{a}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{b}}\\&-\frac{8\sqrt{a}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{8\sqrt{a}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}\\&+x\log^2\left(c\left(a+bx^2\right)^p\right)-4px\log\left(c\left(a+bx^2\right)^p\right)\\&+\frac{4i\sqrt{a}p^2\,\text{PolyLog}\left(2,1-\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}\right)}{\sqrt{b}}+8p^2x\end{aligned}$$

```
[In] Int[Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] 8*p^2*x - (8*Sqrt[a]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + ((4*I)*Sqrt
[a]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/Sqrt[b] + (8*Sqrt[a]*p^2*ArcTan[(Sqr
t[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[b] - 4*p*x*
Log[c*(a + b*x^2)^p] + (4*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a +
b*x^2)^p])/Sqrt[b] + x*Log[c*(a + b*x^2)^p]^2 + ((4*I)*Sqrt[a]*p^2*PolyLog[
2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2500

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[x^n*(
(a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log^2\left(c\left(a+bx^2\right)^p\right)-(4bp)\int\frac{x^2\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx\\
&= x\log^2\left(c\left(a+bx^2\right)^p\right)-(4bp)\int\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{b}-\frac{a\log\left(c\left(a+bx^2\right)^p\right)}{b\left(a+bx^2\right)}\right)dx\\
&= x\log^2\left(c\left(a+bx^2\right)^p\right)-(4p)\int\log\left(c\left(a+bx^2\right)^p\right)\,dx+(4ap)\int\frac{\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx\\
&= -4px\log\left(c\left(a+bx^2\right)^p\right)+\frac{4\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}}\\
&\quad+x\log^2\left(c\left(a+bx^2\right)^p\right)+\left(8bp^2\right)\int\frac{x^2}{a+bx^2}\,dx-\left(8abp^2\right)\int\frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\left(a+bx^2\right)}\,dx\\
&= 8p^2x-4px\log\left(c\left(a+bx^2\right)^p\right)+\frac{4\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}}\\
&\quad+x\log^2\left(c\left(a+bx^2\right)^p\right)-\left(8ap^2\right)\int\frac{1}{a+bx^2}\,dx-\left(8\sqrt{a}\sqrt{b}p^2\right)\int\frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a+bx^2}\,dx\\
&= 8p^2x-\frac{8\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{4i\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{b}}-4px\log\left(c\left(a\right.\right.\\
&\qquad\left.\left.+bx^2\right)^p\right)+\frac{4\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}}+x\log^2\left(c\left(a\right.\right.\\
&\qquad\left.\left.+bx^2\right)^p\right)+\left(8p^2\right)\int\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{i-\frac{\sqrt{b}x}{\sqrt{a}}}\,dx\\
&= 8p^2x-\frac{8\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{4i\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{b}}\\
&\quad+\frac{8\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}-4px\log\left(c\left(a+bx^2\right)^p\right)\\
&\quad+\frac{4\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}}\\
&\quad+x\log^2\left(c\left(a+bx^2\right)^p\right)-\left(8p^2\right)\int\frac{\log\left(\frac{2}{1+\frac{i\sqrt{b}x}{\sqrt{a}}}\right)}{1+\frac{bx^2}{a}}\,dx
\end{aligned}
$$

$$= 8p^2x - \frac{8\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{4i\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{b}} + \frac{8\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}} - 4px\log\left(c\left(a+bx^2\right)^p\right) + \frac{4\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{b}} + x\log^2\left(c\left(a+bx^2\right)^p\right) + \frac{\left(8i\sqrt{a}p^2\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+\frac{i\sqrt{b}x}{\sqrt{a}}}\right)}{\sqrt{b}}$$

$$= 8p^2x - \frac{8\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{4i\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{b}} + \frac{8\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}} - 4px\log\left(c\left(a+bx^2\right)^p\right) + \frac{4\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{b}} + x\log^2\left(c\left(a+bx^2\right)^p\right) + \frac{4i\sqrt{a}p^2\text{Li}_2\left(1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.81

$$\int \log^2\left(c\left(a+bx^2\right)^p\right)\,dx$$

$$= \frac{4i\sqrt{a}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2 + 4\sqrt{a}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-2p+2p\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)+\log\left(c\left(a+bx^2\right)^p\right)\right)+\sqrt{b}x\left(8p^2-\right.}{\sqrt{b}}$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] ((4*I)*Sqrt[a]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + 4*Sqrt[a]*p*ArcTan[(Sqrt
[b]*x)/Sqrt[a]]*(-2*p + 2*p*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)] + Log[
c*(a + b*x^2)^p]) + Sqrt[b]*x*(8*p^2 - 4*p*Log[c*(a + b*x^2)^p] + Log[c*(a
+ b*x^2)^p]^2) + (4*I)*Sqrt[a]*p^2*PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)/((-I)
*Sqrt[a] + Sqrt[b]*x)])/Sqrt[b]
```

Maple [F]

$$\int \ln \left(c\left(b\,x^{2}+a\right)^{p}\right)^{2}dx$$

```
[In] int(ln(c*(b*x^2+a)^p)^2,x)
[Out] int(ln(c*(b*x^2+a)^p)^2,x)
```

Fricas [F]

$$\int \log^2 \left(c\left(a+bx^2\right)^p\right)\,dx = \int \log \left(\left(bx^2+a\right)^p c\right)^2\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
[Out] integral(log((b*x^2 + a)^p*c)^2, x)
```

Sympy [F]

$$\int \log^2 \left(c\left(a+bx^2\right)^p\right)\,dx = \int \log \left(c\left(a+bx^2\right)^p\right)^2 dx$$

```
[In] integrate(ln(c*(b*x**2+a)**p)**2,x)
[Out] Integral(log(c*(a + b*x**2)**p)**2, x)
```

Maxima [F]

$$\int \log^2 \left(c\left(a+bx^2\right)^p\right)\,dx = \int \log \left(\left(bx^2+a\right)^p c\right)^2\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
[Out] p^2*x*log(b*x^2 + a)^2 + integrate((b*x^2*log(c)^2 + a*log(c)^2 - 2*((2*p^2
 - p*log(c))*b*x^2 - a*p*log(c))*log(b*x^2 + a))/(b*x^2 + a), x)
```

Giac [F]

$$\int \log^2\left(c\left(a+bx^2\right)^p\right)\,dx=\int \log\left(\left(bx^2+a\right)^pc\right)^2\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \log^2\left(c\left(a+bx^2\right)^p\right)\,dx=\int \ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2\,dx$$

```
[In] int(log(c*(a + b*x^2)^p)^2,x)
[Out] int(log(c*(a + b*x^2)^p)^2, x)
```

3.87 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 190

$$\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx = \frac{4i\sqrt{b}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}} + \frac{8\sqrt{b}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}} + \frac{4\sqrt{b}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{a}} - \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x} + \frac{4i\sqrt{b}p^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}}$$

```
[Out] -ln(c*(b*x^2+a)^p)^2/x+4*I*p^2*arctan(x*b^(1/2)/a^(1/2))^2*b^(1/2)/a^(1/2)+
4*p*arctan(x*b^(1/2)/a^(1/2))*ln(c*(b*x^2+a)^p)*b^(1/2)/a^(1/2)+8*p^2*arcta
n(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))*b^(1/2)/a^(1/2)+4*
I*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))*b^(1/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used

$= \{2507, 211, 2520, 12, 5040, 4964, 2449, 2352\}$

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^2}\,dx = \frac{4\sqrt{b}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{a}} + \frac{4i\sqrt{b}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}} + \frac{8\sqrt{b}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}} - \frac{\log^2\left(c(a+bx^2)^p\right)}{x} + \frac{4i\sqrt{b}p^2\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^2,x]
```

```
[Out] ((4*I)*Sqrt[b]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/Sqrt[a] + (8*Sqrt[b]*p^2*
ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[
a] + (4*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^p])/Sqrt[a]
 - Log[c*(a + b*x^2)^p]^2/x + ((4*I)*Sqrt[b]*p^2*PolyLog[2, 1 - (2*Sqrt[a])
/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[a]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
```

```
e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\log^2\left(c(a+bx^2)^p\right)}{x} + (4bp)\int \frac{\log\left(c(a+bx^2)^p\right)}{a+bx^2}\,dx \\
&= \frac{4\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{a}} - \frac{\log^2\left(c(a+bx^2)^p\right)}{x} - \left(8b^2p^2\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,(a+bx^2)}\,dx \\
&= \frac{4\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{a}} - \frac{\log^2\left(c(a+bx^2)^p\right)}{x} - \frac{\left(8b^{3/2}p^2\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a+bx^2}\,dx}{\sqrt{a}} \\
&= \frac{4i\sqrt{b}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}} + \frac{4\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{a}} \\
&\quad - \frac{\log^2\left(c(a+bx^2)^p\right)}{x} + \frac{\left(8bp^2\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{i-\frac{\sqrt{b}x}{\sqrt{a}}}\,dx}{a}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4i\sqrt{b}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}} + \frac{8\sqrt{b}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}} \\
&\quad + \frac{4\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{a}} \\
&\quad - \frac{\log^2\left(c(a+bx^2)^p\right)}{x} - \frac{(8bp^2)\int\frac{\log\left(\frac{2}{1+\frac{i\sqrt{b}x}{\sqrt{a}}}\right)}{1+\frac{bx^2}{a}}\,dx}{a} \\
&= \frac{4i\sqrt{b}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}} + \frac{8\sqrt{b}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}} \\
&\quad + \frac{4\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{a}} - \frac{\log^2\left(c(a+bx^2)^p\right)}{x} \\
&\quad + \frac{\left(8i\sqrt{b}p^2\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+\frac{i\sqrt{b}x}{\sqrt{a}}}\right)}{\sqrt{a}} \\
&= \frac{4i\sqrt{b}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}} + \frac{8\sqrt{b}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}} \\
&\quad + \frac{4\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{a}} \\
&\quad - \frac{\log^2\left(c(a+bx^2)^p\right)}{x} + \frac{4i\sqrt{b}p^2\text{Li}_2\left(1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.91

$$
\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^2}\,dx
= \frac{4i\sqrt{b}p^2x\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2 - \sqrt{a}\log^2\left(c(a+bx^2)^p\right) + 4\sqrt{b}px\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(2p\log\left(\frac{2i}{i-\frac{\sqrt{b}x}{\sqrt{a}}}\right) + \log(c(a+bx^2)\right.}{\sqrt{a}x}
$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^2,x]
```

```
[Out] ((4*I)*Sqrt[b]*p^2*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 - Sqrt[a]*Log[c*(a + b*x
^2)^p]^2 + 4*Sqrt[b]*p*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(2*p*Log[(2*I)/(I - (S
qrt[b]*x)/Sqrt[a])] + Log[c*(a + b*x^2)^p]) + (4*I)*Sqrt[b]*p^2*x*PolyLog[2
, (I*Sqrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(Sqrt[a]*x)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.49 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.35

method	result
risch	$-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)^{2}}{x}-\frac{4p^{2}b\arctan\left(\frac{bx}{\sqrt{ab}}\right)\ln\left(b\,x^{2}+a\right)}{\sqrt{ab}}+\frac{4pb\arctan\left(\frac{bx}{\sqrt{ab}}\right)\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{\sqrt{ab}}+p^{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\_Z^{2}+a\right)}\frac{2\ln(x-}{}\right.$

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x*ln((b*x^2+a)^p)^2-4*p^2*b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*ln(b*x^2
+a)+4*p*b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*ln((b*x^2+a)^p)+p^2*sum(1/_al
pha*(2*ln(x-_alpha)*ln(b*x^2+a)-b*(1/_alpha/b*ln(x-_alpha)^2+2*_alpha/a*ln(
x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+2*_alpha/a*dilog(1/2*(x+_alpha)/_alpha)
)),_alpha=RootOf(_Z^2*b+a))+(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)
^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(
b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c))*(-1/x*ln((b*x
^2+a)^p)+2*p*b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/4*(I*Pi*csgn(I*(b*x^2
+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^
p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn
(I*c)+2*ln(c))^2/x
```

Fricas [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^2}\,dx = \int \frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^2}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^2, x)
```

Sympy [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^2}\,dx = \int \frac{\log\left(c\left(a+bx^2\right)^p\right)^2}{x^2}\,dx$$

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**2,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**2/x**2, x)
```

Maxima [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^2}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x^2}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^2,x, algorithm="maxima")
[Out] -p^2*log(b*x^2 + a)^2/x + integrate((b*x^2*log(c)^2 + a*log(c)^2 + 2*((2*p^
2 + p*log(c))*b*x^2 + a*p*log(c))*log(b*x^2 + a))/(b*x^4 + a*x^2), x)
```

Giac [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^2}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x^2}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^2,x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^2}\,dx = \int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2}{x^2}\,dx$$

```
[In] int(log(c*(a + b*x^2)^p)^2/x^2,x)
[Out] int(log(c*(a + b*x^2)^p)^2/x^2, x)
```

3.88 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 254

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^4}\,dx = \frac{8b^{3/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{4ib^{3/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3a^{3/2}} - \frac{8b^{3/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3a^{3/2}} - \frac{4bp\log\left(c(a+bx^2)^p\right)}{3ax} - \frac{4b^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{3a^{3/2}} - \frac{\log^2\left(c(a+bx^2)^p\right)}{3x^3} - \frac{4ib^{3/2}p^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3a^{3/2}}$$

```
[Out] 8/3*b^(3/2)*p^2*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)-4/3*I*b^(3/2)*p^2*arctan(
x*b^(1/2)/a^(1/2))^2/a^(3/2)-4/3*b*p*ln(c*(b*x^2+a)^p)/a/x-4/3*b^(3/2)*p*ar
ctan(x*b^(1/2)/a^(1/2))*ln(c*(b*x^2+a)^p)/a^(3/2)-1/3*ln(c*(b*x^2+a)^p)^2/x
^3-8/3*b^(3/2)*p^2*arctan(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1
/2)))/a^(3/2)-4/3*I*b^(3/2)*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2))
)/a^(3/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {2507, 2526, 2505, 211, 2520, 12, 5040, 4964, 2449, 2352}

$$\begin{aligned}\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^4}dx = & -\frac{4b^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^{3/2}}-\frac{4ib^{3/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3a^{3/2}}\\ &+\frac{8b^{3/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{8b^{3/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3a^{3/2}}\\ &-\frac{4ib^{3/2}p^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}\right)}{3a^{3/2}}\\ &-\frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{3ax}-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{3x^3}\end{aligned}$$

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^4,x]
```

```
[Out] (8*b^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*a^(3/2)) - (((4*I)/3)*b^(3/2
)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/a^(3/2) - (8*b^(3/2)*p^2*ArcTan[(Sqrt[
b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(3*a^(3/2)) - (4*b
*p*Log[c*(a + b*x^2)^p])/(3*a*x) - (4*b^(3/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
*Log[c*(a + b*x^2)^p])/(3*a^(3/2)) - Log[c*(a + b*x^2)^p]^2/(3*x^3) - (((4*
I)/3)*b^(3/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/a^(3
/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{3x^3} + \frac{1}{3}(4bp)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2\left(a+bx^2\right)}\,dx$$

$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{3x^3} + \frac{1}{3}(4bp)\int \left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{ax^2} - \frac{b\log\left(c\left(a+bx^2\right)^p\right)}{a\left(a+bx^2\right)}\right)dx$$

$$= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{3x^3} + \frac{(4bp)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx}{3a} - \frac{\left(4b^2p\right)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx}{3a}$$

$$= -\frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{3ax} - \frac{4b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^{3/2}} - \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{3x^3} + \frac{\left(8b^2p^2\right)\int \frac{1}{a+bx^2}\,dx}{3a} + \frac{\left(8b^3p^2\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\left(a+bx^2\right)}\,dx}{3a}$$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{3ax} - \frac{4b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^{3/2}} - \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{3x^3} + \frac{\left(8b^{5/2}p^2\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a+bx^2}\,dx}{3a^{3/2}}$$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{4ib^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3a^{3/2}} - \frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{3ax} - \frac{4b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^{3/2}} - \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{3x^3} - \frac{\left(8b^2p^2\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{i-\frac{\sqrt{b}x}{\sqrt{a}}}\,dx}{3a^2}$$

$$= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{4ib^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3a^{3/2}} - \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3a^{3/2}} - \frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{3ax} - \frac{4b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3a^{3/2}} - \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{3x^3} + \frac{\left(8b^2p^2\right)\int \frac{\log\left(\frac{2}{1+\frac{i\sqrt{b}x}{\sqrt{a}}}\right)}{1+\frac{bx^2}{a}}\,dx}{3a^2}$$

$$
\begin{aligned}
&= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{4ib^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3a^{3/2}} - \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3a^{3/2}} \\
&\quad - \frac{4bp\log\left(c(a+bx^2)^p\right)}{3ax} - \frac{4b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{3a^{3/2}} \\
&\quad - \frac{\log^2\left(c(a+bx^2)^p\right)}{3x^3} - \frac{\left(8ib^{3/2}p^2\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+\frac{i\sqrt{b}x}{\sqrt{a}}}\right)}{3a^{3/2}} \\
&= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{3/2}} - \frac{4ib^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3a^{3/2}} - \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3a^{3/2}} \\
&\quad - \frac{4bp\log\left(c(a+bx^2)^p\right)}{3ax} - \frac{4b^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{3a^{3/2}} \\
&\quad - \frac{\log^2\left(c(a+bx^2)^p\right)}{3x^3} - \frac{4ib^{3/2}p^2\text{Li}_2\left(1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3a^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.81

$$
\int\frac{\log^2\left(c(a+bx^2)^p\right)}{x^4}\,dx = \frac{-4ib^{3/2}p^2x^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2 - 4b^{3/2}px^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-2p+2p\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)+\log\left(c(a+bx^2)^p\right)\right) - \sqrt{a}}{3a^{3/2}x^3}
$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^4,x]
```

```
[Out] ((-4*I)*b^(3/2)*p^2*x^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 - 4*b^(3/2)*p*x^3*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]]*(-2*p + 2*p*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x
)] + Log[c*(a + b*x^2)^p]) - Sqrt[a]*Log[c*(a + b*x^2)^p]*(4*b*p*x^2 + a*Lo
g[c*(a + b*x^2)^p]) - (4*I)*b^(3/2)*p^2*x^3*PolyLog[2, (I*Sqrt[a] + Sqrt[b]
*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(3*a^(3/2)*x^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.56 (sec) , antiderivative size = 522, normalized size of antiderivative = 2.06

method	result
risch	$-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)^{2}}{3x^{3}}+\frac{4p^{2}b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)\ln\left(b\,x^{2}+a\right)}{3a\sqrt{ab}}-\frac{4p\,b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{3a\sqrt{ab}}-\frac{4pb\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{3ax}+\frac{8p^{2}b^{2}\arctan(}{3a\sqrt{ab}}$

[In] int(ln(c*(b*x^2+a)^p)^2/x^4,x,method=_RETURNVERBOSE)

[Out] -1/3*ln((b*x^2+a)^p)^2/x^3+4/3*p^2*b^2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*ln(b*x^2+a)-4/3*p*b^2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*ln((b*x^2+a)^p)-4/3*p*b*ln((b*x^2+a)^p)/a/x+8/3*p^2*b^2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+4/3*p^2*b*Sum(-1/2*(ln(x-_alpha)*ln(b*x^2+a)-2*b*(1/4/_alpha/b*ln(x-_alpha)^2+1/2*_alpha/a*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+1/2*_alpha/a*dilog(1/2*(x+_alpha)/_alpha)))/a/_alpha,_alpha=RootOf(_Z^2*b+a))+(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c))*(-1/3/x^3*ln((b*x^2+a)^p)+2/3*p*b*(-1/a*b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/a/x))-1/12*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c))^2/x^3

Fricas [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^4}\,dx=\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^4}\,dx$$

[In] integrate(log(c*(b*x^2+a)^p)^2/x^4,x, algorithm="fricas")

[Out] integral(log((b*x^2 + a)^p*c)^2/x^4, x)

Sympy [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^4}\,dx=\int \frac{\log\left(c\left(a+bx^2\right)^p\right)^2}{x^4}\,dx$$

[In] integrate(ln(c*(b*x**2+a)**p)**2/x**4,x)

[Out] Integral(log(c*(a + b*x**2)**p)**2/x**4, x)

Maxima [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^4}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x^4}\,dx$$

[In] integrate(log(c*(b*x^2+a)^p)^2/x^4,x, algorithm="maxima")

[Out] -1/3*p^2*log(b*x^2 + a)^2/x^3 + integrate(1/3*(3*b*x^2*log(c)^2 + 3*a*log(c)^2 + 2*((2*p^2 + 3*p*log(c))*b*x^2 + 3*a*p*log(c))*log(b*x^2 + a))/(b*x^6 + a*x^4), x)

Giac [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^4}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x^4}\,dx$$

[In] integrate(log(c*(b*x^2+a)^p)^2/x^4,x, algorithm="giac")

[Out] integrate(log((b*x^2 + a)^p*c)^2/x^4, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^4}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)^2}{x^4}\,dx$$

[In] int(log(c*(a + b*x^2)^p)^2/x^4,x)

[Out] int(log(c*(a + b*x^2)^p)^2/x^4, x)

3.89 $\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^6}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 296

$$\begin{aligned}\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^6}\,dx = &-\frac{8b^2p^2}{15a^2x}-\frac{32b^{5/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{15a^{5/2}}+\frac{4ib^{5/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5a^{5/2}}\\&+\frac{8b^{5/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{5a^{5/2}}-\frac{4bp\log\left(c(a+bx^2)^p\right)}{15ax^3}\\&+\frac{4b^2p\log\left(c(a+bx^2)^p\right)}{5a^2x}+\frac{4b^{5/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{5a^{5/2}}\\&-\frac{\log^2\left(c(a+bx^2)^p\right)}{5x^5}+\frac{4ib^{5/2}p^2\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{5a^{5/2}}\end{aligned}$$

```
[Out] -8/15*b^2*p^2/a^2/x-32/15*b^(5/2)*p^2*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)+4/5
*I*b^(5/2)*p^2*arctan(x*b^(1/2)/a^(1/2))^2/a^(5/2)-4/15*b*p*ln(c*(b*x^2+a)^
p)/a/x^3+4/5*b^2*p*ln(c*(b*x^2+a)^p)/a^2/x+4/5*b^(5/2)*p*arctan(x*b^(1/2)/a
^(1/2))*ln(c*(b*x^2+a)^p)/a^(5/2)-1/5*ln(c*(b*x^2+a)^p)^2/x^5+8/5*b^(5/2)*p
^2*arctan(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/a^(5/2)+4/
5*I*b^(5/2)*p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/a^(5/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {2507, 2526, 2505, 331, 211, 2520, 12, 5040, 4964, 2449, 2352}

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^6}\,dx=\frac{4b^{5/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{5a^{5/2}}+\frac{4ib^{5/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5a^{5/2}}-\frac{32b^{5/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{15a^{5/2}}+\frac{8b^{5/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{5a^{5/2}}+\frac{4ib^{5/2}p^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}\right)}{5a^{5/2}}+\frac{4b^2p\log\left(c(a+bx^2)^p\right)}{5a^2x}-\frac{8b^2p^2}{15a^2x}-\frac{\log^2\left(c(a+bx^2)^p\right)}{5x^5}-\frac{4bp\log\left(c(a+bx^2)^p\right)}{15ax^3}$$

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^6,x]
```

```
[Out] (-8*b^2*p^2)/(15*a^2*x) - (32*b^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(15*
a^(5/2)) + (((4*I)/5)*b^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/a^(5/2) +
(8*b^(5/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqr
t[b]*x)])/(5*a^(5/2)) - (4*b*p*Log[c*(a + b*x^2)^p])/(15*a*x^3) + (4*b^2*p*
Log[c*(a + b*x^2)^p])/(5*a^2*x) + (4*b^(5/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*
Log[c*(a + b*x^2)^p])/(5*a^(5/2)) - Log[c*(a + b*x^2)^p]^2/(5*x^5) + (((4*I
)/5)*b^(5/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/a^(5/
2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{5x^5}+\frac{1}{5}(4bp)\int\frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^4\left(a+bx^2\right)}\,dx\\
&= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{5x^5}\\
&\quad+\frac{1}{5}(4bp)\int\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{ax^4}-\frac{b\log\left(c\left(a+bx^2\right)^p\right)}{a^2x^2}+\frac{b^2\log\left(c\left(a+bx^2\right)^p\right)}{a^2\left(a+bx^2\right)}\right)dx\\
&= -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{5x^5}+\frac{(4bp)\int\frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^4}\,dx}{5a}\\
&\quad-\frac{\left(4b^2p\right)\int\frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx}{5a^2}+\frac{\left(4b^3p\right)\int\frac{\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx}{5a^2}\\
&= -\frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{15ax^3}+\frac{4b^2p\log\left(c\left(a+bx^2\right)^p\right)}{5a^2x}+\frac{4b^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{5a^{5/2}}\\
&\quad-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{5x^5}+\frac{\left(8b^2p^2\right)\int\frac{1}{x^2\left(a+bx^2\right)}\,dx}{15a}-\frac{\left(8b^3p^2\right)\int\frac{1}{a+bx^2}\,dx}{5a^2}-\frac{\left(8b^4p^2\right)\int\frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\left(a+bx^2\right)}\,dx}{5a^2}\\
&= -\frac{8b^2p^2}{15a^2x}-\frac{8b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{5a^{5/2}}-\frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{15ax^3}\\
&\quad+\frac{4b^2p\log\left(c\left(a+bx^2\right)^p\right)}{5a^2x}+\frac{4b^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{5a^{5/2}}\\
&\quad-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{5x^5}-\frac{\left(8b^3p^2\right)\int\frac{1}{a+bx^2}\,dx}{15a^2}-\frac{\left(8b^{7/2}p^2\right)\int\frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a+bx^2}\,dx}{5a^{5/2}}\\
&= -\frac{8b^2p^2}{15a^2x}-\frac{32b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{15a^{5/2}}+\frac{4ib^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5a^{5/2}}-\frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{15ax^3}\\
&\quad+\frac{4b^2p\log\left(c\left(a+bx^2\right)^p\right)}{5a^2x}+\frac{4b^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{5a^{5/2}}\\
&\quad-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{5x^5}+\frac{\left(8b^3p^2\right)\int\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{i-\frac{\sqrt{b}x}{\sqrt{a}}}\,dx}{5a^3}
\end{aligned}$$

$$= -\frac{8b^2p^2}{15a^2x} - \frac{32b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{15a^{5/2}} + \frac{4ib^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5a^{5/2}} + \frac{8b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{5a^{5/2}} - \frac{4bp\log\left(c(a+bx^2)^p\right)}{15ax^3} + \frac{4b^2p\log\left(c(a+bx^2)^p\right)}{5a^2x} + \frac{4b^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{5a^{5/2}} - \frac{\log^2\left(c(a+bx^2)^p\right)}{5x^5} - \frac{(8b^3p^2)\int\frac{\log\left(\frac{2}{1+\frac{i\sqrt{b}x}{\sqrt{a}}}\right)}{1+\frac{bx^2}{a}}dx}{5a^3}$$

$$= -\frac{8b^2p^2}{15a^2x} - \frac{32b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{15a^{5/2}} + \frac{4ib^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5a^{5/2}} + \frac{8b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{5a^{5/2}} - \frac{4bp\log\left(c(a+bx^2)^p\right)}{15ax^3} + \frac{4b^2p\log\left(c(a+bx^2)^p\right)}{5a^2x} + \frac{4b^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{5a^{5/2}} - \frac{\log^2\left(c(a+bx^2)^p\right)}{5x^5} + \frac{(8ib^{5/2}p^2)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}dx, x, \frac{1}{1+\frac{i\sqrt{b}x}{\sqrt{a}}}\right)}{5a^{5/2}}$$

$$= -\frac{8b^2p^2}{15a^2x} - \frac{32b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{15a^{5/2}} + \frac{4ib^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{5a^{5/2}} + \frac{8b^{5/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{5a^{5/2}} - \frac{4bp\log\left(c(a+bx^2)^p\right)}{15ax^3} + \frac{4b^2p\log\left(c(a+bx^2)^p\right)}{5a^2x} + \frac{4b^{5/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{5a^{5/2}} - \frac{\log^2\left(c(a+bx^2)^p\right)}{5x^5} + \frac{4ib^{5/2}p^2\text{Li}_2\left(1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{5a^{5/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.12 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.99

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^6}\,dx = -\frac{\log^2\left(c(a+bx^2)^p\right)}{5x^5} + \frac{4}{5}bp\left(-\frac{2b^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}} - \frac{2bp\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{bx^2}{a}\right)}{3a^2x} - \frac{\log\left(c(a+bx^2)^p\right)}{3ax^3} + \frac{b\log\left(c(a+bx^2)^p\right)}{a^2x} + \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{a^{5/2}} + \frac{p\left(ib^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2 + 2b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2i\sqrt{a}}{i\sqrt{a}-\sqrt{b}x}\right) + ib^{3/2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{a}+\sqrt{b}x}{i\sqrt{a}-\sqrt{b}x}\right)\right)}{a^{5/2}}\right)$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^6,x]
```

```
[Out] -1/5*Log[c*(a + b*x^2)^p]^2/x^5 + (4*b*p*((-2*b^(3/2)*p*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/a^(5/2) - (2*b*p*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x^2)/a)])/(
3*a^2*x) - Log[c*(a + b*x^2)^p]/(3*a*x^3) + (b*Log[c*(a + b*x^2)^p])/(a^2*x
) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^p])/a^(5/2) + (p
*(I*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + 2*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]]*Log[((2*I)*Sqrt[a])/(I*Sqrt[a] - Sqrt[b]*x)] + I*b^(3/2)*PolyLog[2,
-((I*Sqrt[a] + Sqrt[b]*x)/(I*Sqrt[a] - Sqrt[b]*x))]))/a^(5/2)))/5
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.13 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.92

method	result
risch	$-\frac{\ln\left(\left(b\,x^2+a\right)^p\right)^2}{5x^5} - \frac{4pb\ln\left(\left(b\,x^2+a\right)^p\right)}{15a\,x^3} + \frac{4p\,b^2\ln\left(\left(b\,x^2+a\right)^p\right)}{5a^2x} - \frac{4p^2b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)\ln\left(b\,x^2+a\right)}{5a^2\sqrt{ab}} + \frac{4p\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)\ln\left((b\,x\right.}{5a^2\sqrt{ab}}$

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*ln((b*x^2+a)^p)^2/x^5-4/15*p*b*ln((b*x^2+a)^p)/a/x^3+4/5*p*b^2*ln((b*x
^2+a)^p)/a^2/x-4/5*p^2*b^3/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*ln(b*x^2
+a)+4/5*p*b^3/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))*ln((b*x^2+a)^p)-8/15*
b^2*p^2/a^2/x-32/15*p^2*b^3/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+4/5*p^2
```

```
*b*Sum(1/2*(ln(x-_alpha)*ln(b*x^2+a)-2*b*(1/4/_alpha/b*ln(x-_alpha)^2+1/2*_
alpha/a*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+1/2*_alpha/a*dilog(1/2*(x+_a
lpha)/_alpha)))/a^2*b/_alpha,_alpha=RootOf(_Z^2*b+a))+(I*Pi*csgn(I*(b*x^2+a
)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)
*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I
*c)+2*ln(c))*(-1/5/x^5*ln((b*x^2+a)^p)+2/5*p*b*(-1/3/a/x^3+b/a^2/x+b^2/a^2/
(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))-1/20*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I
*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-
I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)
)^2/x^5
```

Fricas [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^6}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x^6}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^6,x, algorithm="fricas")
[Out] integral(log((b*x^2 + a)^p*c)^2/x^6, x)
```

Sympy [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^6}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)^2}{x^6}\,dx$$

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**6,x)
[Out] Integral(log(c*(a + b*x**2)**p)**2/x**6, x)
```

Maxima [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^6}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x^6}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^6,x, algorithm="maxima")
[Out] -1/5*p^2*log(b*x^2 + a)^2/x^5 + integrate(1/5*(5*b*x^2*log(c)^2 + 5*a*log(c
)^2 + 2*((2*p^2 + 5*p*log(c))*b*x^2 + 5*a*p*log(c))*log(b*x^2 + a))/(b*x^8
+ a*x^6), x)
```

Giac [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^6}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^2}{x^6}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^6,x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^6}\,dx = \int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2}{x^6}\,dx$$

```
[In] int(log(c*(a + b*x^2)^p)^2/x^6,x)
[Out] int(log(c*(a + b*x^2)^p)^2/x^6, x)
```

3.90 $\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^8}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 338

$$\begin{aligned}\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{x^8}\,dx &= -\frac{8b^2p^2}{105a^2x^3}+\frac{64b^3p^2}{105a^3x}+\frac{184b^{7/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{105a^{7/2}} \\ &\quad -\frac{4ib^{7/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{7a^{7/2}}-\frac{8b^{7/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{7a^{7/2}} \\ &\quad -\frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{35ax^5}+\frac{4b^2p\log\left(c\left(a+bx^2\right)^p\right)}{21a^2x^3} \\ &\quad -\frac{4b^3p\log\left(c\left(a+bx^2\right)^p\right)}{7a^3x}-\frac{4b^{7/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{7a^{7/2}} \\ &\quad -\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{7x^7}-\frac{4ib^{7/2}p^2\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{7a^{7/2}}\end{aligned}$$

```
[Out] -8/105*b^2*p^2/a^2/x^3+64/105*b^3*p^2/a^3/x+184/105*b^(7/2)*p^2*arctan(x*b^
(1/2)/a^(1/2))/a^(7/2)-4/7*I*b^(7/2)*p^2*arctan(x*b^(1/2)/a^(1/2))^2/a^(7/2
)-4/35*b*p*ln(c*(b*x^2+a)^p)/a/x^5+4/21*b^2*p*ln(c*(b*x^2+a)^p)/a^2/x^3-4/7
*b^3*p*ln(c*(b*x^2+a)^p)/a^3/x-4/7*b^(7/2)*p*arctan(x*b^(1/2)/a^(1/2))*ln(c
*(b*x^2+a)^p)/a^(7/2)-1/7*ln(c*(b*x^2+a)^p)^2/x^7-8/7*b^(7/2)*p^2*arctan(x*
b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/a^(7/2)-4/7*I*b^(7/2)*
p^2*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/a^(7/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {2507, 2526, 2505, 331, 211, 2520, 12, 5040, 4964, 2449, 2352}

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^8}\,dx = -\frac{4b^{7/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{7a^{7/2}} - \frac{4ib^{7/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{7a^{7/2}} + \frac{184b^{7/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{105a^{7/2}} - \frac{8b^{7/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{7a^{7/2}} - \frac{4ib^{7/2}p^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}\right)}{7a^{7/2}} - \frac{4b^3p\log\left(c(a+bx^2)^p\right)}{7a^3x} + \frac{64b^3p^2}{105a^3x} + \frac{4b^2p\log\left(c(a+bx^2)^p\right)}{21a^2x^3} - \frac{8b^2p^2}{105a^2x^3} - \frac{\log^2\left(c(a+bx^2)^p\right)}{7x^7} - \frac{4bp\log\left(c(a+bx^2)^p\right)}{35ax^5}$$

```
[In] Int[Log[c*(a + b*x^2)^p]^2/x^8,x]
```

```
[Out] (-8*b^2*p^2)/(105*a^2*x^3) + (64*b^3*p^2)/(105*a^3*x) + (184*b^(7/2)*p^2*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(105*a^(7/2)) - (((4*I)/7)*b^(7/2)*p^2*ArcTan[(S
qrt[b]*x)/Sqrt[a]]^2)/a^(7/2) - (8*b^(7/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*
Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(7*a^(7/2)) - (4*b*p*Log[c*(a + b
*x^2)^p])/(35*a*x^5) + (4*b^2*p*Log[c*(a + b*x^2)^p])/(21*a^2*x^3) - (4*b^3
*p*Log[c*(a + b*x^2)^p])/(7*a^3*x) - (4*b^(7/2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a
]]*Log[c*(a + b*x^2)^p])/(7*a^(7/2)) - Log[c*(a + b*x^2)^p]^2/(7*x^7) - (((
4*I)/7)*b^(7/2)*p^2*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/a^
(7/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
```

```
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log^2\left(c(a+bx^2)^p\right)}{7x^7}+\frac{1}{7}(4bp)\int\frac{\log\left(c(a+bx^2)^p\right)}{x^6\left(a+bx^2\right)}\,dx\\
&= -\frac{\log^2\left(c(a+bx^2)^p\right)}{7x^7}+\frac{1}{7}(4bp)\int\left(\frac{\log\left(c(a+bx^2)^p\right)}{ax^6}-\frac{b\log\left(c(a+bx^2)^p\right)}{a^2x^4}\right.\\
&\qquad\left.+\frac{b^2\log\left(c(a+bx^2)^p\right)}{a^3x^2}-\frac{b^3\log\left(c(a+bx^2)^p\right)}{a^3\left(a+bx^2\right)}\right)dx\\
&= -\frac{\log^2\left(c(a+bx^2)^p\right)}{7x^7}+\frac{(4bp)\int\frac{\log\left(c(a+bx^2)^p\right)}{x^6}\,dx}{7a}-\frac{(4b^2p)\int\frac{\log\left(c(a+bx^2)^p\right)}{x^4}\,dx}{7a^2}\\
&\quad+\frac{(4b^3p)\int\frac{\log\left(c(a+bx^2)^p\right)}{x^2}\,dx}{7a^3}-\frac{(4b^4p)\int\frac{\log\left(c(a+bx^2)^p\right)}{a+bx^2}\,dx}{7a^3}\\
&= -\frac{4bp\log\left(c(a+bx^2)^p\right)}{35ax^5}+\frac{4b^2p\log\left(c(a+bx^2)^p\right)}{21a^2x^3}-\frac{4b^3p\log\left(c(a+bx^2)^p\right)}{7a^3x}\\
&\quad-\frac{4b^{7/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{7a^{7/2}}-\frac{\log^2\left(c(a+bx^2)^p\right)}{7x^7}+\frac{(8b^2p^2)\int\frac{1}{x^4(a+bx^2)}\,dx}{35a}\\
&\quad-\frac{(8b^3p^2)\int\frac{1}{x^2(a+bx^2)}\,dx}{21a^2}+\frac{(8b^4p^2)\int\frac{1}{a+bx^2}\,dx}{7a^3}+\frac{(8b^5p^2)\int\frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(a+bx^2)}\,dx}{7a^3}\\
&= -\frac{8b^2p^2}{105a^2x^3}+\frac{8b^3p^2}{21a^3x}+\frac{8b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{7a^{7/2}}-\frac{4bp\log\left(c(a+bx^2)^p\right)}{35ax^5}\\
&\quad+\frac{4b^2p\log\left(c(a+bx^2)^p\right)}{21a^2x^3}-\frac{4b^3p\log\left(c(a+bx^2)^p\right)}{7a^3x}\\
&\quad-\frac{4b^{7/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{7a^{7/2}}-\frac{\log^2\left(c(a+bx^2)^p\right)}{7x^7}\\
&\quad-\frac{(8b^3p^2)\int\frac{1}{x^2(a+bx^2)}\,dx}{35a^2}+\frac{(8b^4p^2)\int\frac{1}{a+bx^2}\,dx}{21a^3}+\frac{(8b^{9/2}p^2)\int\frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a+bx^2}\,dx}{7a^{7/2}}
\end{aligned}$$

$$=-\frac{8b^2p^2}{105a^2x^3}+\frac{64b^3p^2}{105a^3x}+\frac{32b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{21a^{7/2}}-\frac{4ib^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{7a^{7/2}}-\frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{35ax^5}+\frac{4b^2p\log\left(c\left(a+bx^2\right)^p\right)}{21a^2x^3}-\frac{4b^3p\log\left(c\left(a+bx^2\right)^p\right)}{7a^3x}-\frac{4b^{7/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{7a^{7/2}}-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{7x^7}-\frac{\left(8b^4p^2\right)\int\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{i-\frac{\sqrt{b}x}{\sqrt{a}}}\,dx}{7a^4}+\frac{\left(8b^4p^2\right)\int\frac{1}{a+bx^2}\,dx}{35a^3}$$

$$=-\frac{8b^2p^2}{105a^2x^3}+\frac{64b^3p^2}{105a^3x}+\frac{184b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{105a^{7/2}}-\frac{4ib^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{7a^{7/2}}-\frac{8b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{7a^{7/2}}-\frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{35ax^5}+\frac{4b^2p\log\left(c\left(a+bx^2\right)^p\right)}{21a^2x^3}-\frac{4b^3p\log\left(c\left(a+bx^2\right)^p\right)}{7a^3x}-\frac{4b^{7/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{7a^{7/2}}-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{7x^7}+\frac{\left(8b^4p^2\right)\int\frac{\log\left(\frac{2}{1+\frac{i\sqrt{b}x}{\sqrt{a}}}\right)}{1+\frac{bx^2}{a}}\,dx}{7a^4}$$

$$=-\frac{8b^2p^2}{105a^2x^3}+\frac{64b^3p^2}{105a^3x}+\frac{184b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{105a^{7/2}}-\frac{4ib^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{7a^{7/2}}-\frac{8b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{7a^{7/2}}-\frac{4bp\log\left(c\left(a+bx^2\right)^p\right)}{35ax^5}+\frac{4b^2p\log\left(c\left(a+bx^2\right)^p\right)}{21a^2x^3}-\frac{4b^3p\log\left(c\left(a+bx^2\right)^p\right)}{7a^3x}-\frac{4b^{7/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{7a^{7/2}}-\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{7x^7}-\frac{\left(8ib^{7/2}p^2\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+\frac{i\sqrt{b}x}{\sqrt{a}}}\right)}{7a^{7/2}}$$

$$= -\frac{8b^2p^2}{105a^2x^3} + \frac{64b^3p^2}{105a^3x} + \frac{184b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{105a^{7/2}} - \frac{4ib^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{7a^{7/2}} - \frac{8b^{7/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{7a^{7/2}} - \frac{4bp\log\left(c(a+bx^2)^p\right)}{35ax^5} + \frac{4b^2p\log\left(c(a+bx^2)^p\right)}{21a^2x^3} - \frac{4b^3p\log\left(c(a+bx^2)^p\right)}{7a^3x} - \frac{4b^{7/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{7a^{7/2}} - \frac{\log^2\left(c(a+bx^2)^p\right)}{7x^7} - \frac{4ib^{7/2}p^2\text{Li}_2\left(1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{7a^{7/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.04

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^8}\,dx = -\frac{\log^2\left(c(a+bx^2)^p\right)}{7x^7} + \frac{4}{7}bp\left(\frac{2b^{5/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}} - \frac{2bp\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\frac{bx^2}{a}\right)}{15a^2x^3} + \frac{2b^2p\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{bx^2}{a}\right)}{3a^3x} - \frac{\log\left(c(a+bx^2)^p\right)}{5ax^5} + \frac{b\log\left(c(a+bx^2)^p\right)}{3a^2x^3} - \frac{b^2\log\left(c(a+bx^2)^p\right)}{a^3x} - \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{a^{7/2}} - \frac{p\left(ib^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2 + 2b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2i\sqrt{a}}{i\sqrt{a}-\sqrt{b}x}\right) + ib^{5/2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{a}+\sqrt{b}x}{i\sqrt{a}-\sqrt{b}x}\right)\right)}{a^{7/2}}\right)$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]^2/x^8,x]
```

```
[Out] -1/7*Log[c*(a + b*x^2)^p]^2/x^7 + (4*b*p*((2*b^(5/2)*p*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/a^(7/2) - (2*b*p*Hypergeometric2F1[-3/2, 1, -1/2, -((b*x^2)/a)])/(
15*a^2*x^3) + (2*b^2*p*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x^2)/a)])/(3*a^
3*x) - Log[c*(a + b*x^2)^p]/(5*a*x^5) + (b*Log[c*(a + b*x^2)^p])/(3*a^2*x^3
) - (b^2*Log[c*(a + b*x^2)^p])/(a^3*x) - (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]]*Log[c*(a + b*x^2)^p])/a^(7/2) - (p*(I*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]
]^2 + 2*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[((2*I)*Sqrt[a])/(I*Sqrt[a]
- Sqrt[b]*x)] + I*b^(5/2)*PolyLog[2, -((I*Sqrt[a] + Sqrt[b]*x)/(I*Sqrt[a] -
 Sqrt[b]*x))]))/a^(7/2)))/7
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.92 (sec) , antiderivative size = 619, normalized size of antiderivative = 1.83

method	result
risch	$-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)^{2}}{7x^{7}}-\frac{4pb\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{35a\,x^{5}}-\frac{4p\,b^{3}\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{7a^{3}x}+\frac{4p\,b^{2}\ln\left(\left(b\,x^{2}+a\right)^{p}\right)}{21a^{2}x^{3}}+\frac{4p^{2}b^{4}\arctan\left(\frac{bx}{\sqrt{ab}}\right)\ln\left(b\,x^{2}+a\right)}{7a^{3}\sqrt{ab}}-$

```
[In] int(ln(c*(b*x^2+a)^p)^2/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*ln((b*x^2+a)^p)^2/x^7-4/35*p*b*ln((b*x^2+a)^p)/a/x^5-4/7*p*b^3*ln((b*x
^2+a)^p)/a^3/x+4/21*p*b^2*ln((b*x^2+a)^p)/a^2/x^3+4/7*p^2*b^4/a^3/(a*b)^(1/
2)*arctan(b*x/(a*b)^(1/2))*ln(b*x^2+a)-4/7*p*b^4/a^3/(a*b)^(1/2)*arctan(b*x
/(a*b)^(1/2))*ln((b*x^2+a)^p)+64/105*b^3*p^2/a^3/x+184/105*p^2*b^4/a^3/(a*b
)^(1/2)*arctan(b*x/(a*b)^(1/2))-8/105*b^2*p^2/a^2/x^3+4/7*p^2*b*Sum(-1/2*(l
n(x-_alpha)*ln(b*x^2+a)-2*b*(1/4/_alpha/b*ln(x-_alpha)^2+1/2*_alpha/a*ln(x-
_alpha)*ln(1/2*(x+_alpha)/_alpha)+1/2*_alpha/a*dilog(1/2*(x+_alpha)/_alpha)
))/a^3*b^2/_alpha,_alpha=RootOf(_Z^2*b+a))+(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I
*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-
I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)
)*(-1/7/x^7*ln((b*x^2+a)^p)+2/7*p*b*(-1/5/a/x^5-b^2/a^3/x+1/3*b/a^2/x^3-b^3
/a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))-1/28*(I*Pi*csgn(I*(b*x^2+a)^p)*c
sgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(
I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*
ln(c))^2/x^7
```

Fricas [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^8}\,dx=\int \frac{\log\left(\left(bx^2+a\right)^p c\right)^2}{x^8}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^8,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^2/x^8, x)
```

Sympy [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^8}\,dx=\int \frac{\log\left(c(a+bx^2)^p\right)^2}{x^8}\,dx$$

```
[In] integrate(ln(c*(b*x**2+a)**p)**2/x**8,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**2/x**8, x)
```

Maxima [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^8}\,dx=\int \frac{\log\left((bx^2+a)^pc\right)^2}{x^8}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^8,x, algorithm="maxima")
```

```
[Out] -1/7*p^2*log(b*x^2 + a)^2/x^7 + integrate(1/7*(7*b*x^2*log(c)^2 + 7*a*log(c
)^2 + 2*((2*p^2 + 7*p*log(c))*b*x^2 + 7*a*p*log(c))*log(b*x^2 + a))/(b*x^10
 + a*x^8), x)
```

Giac [F]

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^8}\,dx=\int \frac{\log\left((bx^2+a)^pc\right)^2}{x^8}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^2/x^8,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^2/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^8}\,dx=\int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^2}{x^8}\,dx$$

```
[In] int(log(c*(a + b*x^2)^p)^2/x^8,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^2/x^8, x)
```

3.91 $\int x^5 \log^3\left(c(a+bx^2)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 334

$$
\begin{aligned}
\int x^5 \log^3\left(c(a+bx^2)^p\right)\,dx = & -\frac{3a^2p^3x^2}{b^2} + \frac{3ap^3(a+bx^2)^2}{8b^3} - \frac{p^3(a+bx^2)^3}{27b^3} \\
& + \frac{3a^2p^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^3} \\
& - \frac{3ap^2(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{4b^3} \\
& + \frac{p^2(a+bx^2)^3\log\left(c(a+bx^2)^p\right)}{9b^3} \\
& - \frac{3a^2p(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b^3} \\
& + \frac{3ap(a+bx^2)^2\log^2\left(c(a+bx^2)^p\right)}{4b^3} \\
& - \frac{p(a+bx^2)^3\log^2\left(c(a+bx^2)^p\right)}{6b^3} \\
& + \frac{a^2(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2b^3} \\
& - \frac{a(a+bx^2)^2\log^3\left(c(a+bx^2)^p\right)}{2b^3} \\
& + \frac{(a+bx^2)^3\log^3\left(c(a+bx^2)^p\right)}{6b^3}
\end{aligned}
$$

```
[Out] -3*a^2*p^3*x^2/b^2+3/8*a*p^3*(b*x^2+a)^2/b^3-1/27*p^3*(b*x^2+a)^3/b^3+3*a^2
*p^2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b^3-3/4*a*p^2*(b*x^2+a)^2*ln(c*(b*x^2+a)^p
)/b^3+1/9*p^2*(b*x^2+a)^3*ln(c*(b*x^2+a)^p)/b^3-3/2*a^2*p*(b*x^2+a)*ln(c*(b
*x^2+a)^p)^2/b^3+3/4*a*p*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)^2/b^3-1/6*p*(b*x^2+a
)^3*ln(c*(b*x^2+a)^p)^2/b^3+1/2*a^2*(b*x^2+a)*ln(c*(b*x^2+a)^p)^3/b^3-1/2*a
```

```
*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)^3/b^3+1/6*(b*x^2+a)^3*ln(c*(b*x^2+a)^p)^3/b^
3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\begin{aligned}\int x^5 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx = & \frac{3a^2p^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^3} \\ & + \frac{a^2(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2b^3} \\ & - \frac{3a^2p(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b^3} \\ & - \frac{3a^2p^3x^2}{b^2} + \frac{p^2(a+bx^2)^3\log\left(c(a+bx^2)^p\right)}{9b^3} \\ & - \frac{3ap^2(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{4b^3} \\ & + \frac{(a+bx^2)^3\log^3\left(c(a+bx^2)^p\right)}{6b^3} \\ & - \frac{a(a+bx^2)^2\log^3\left(c(a+bx^2)^p\right)}{2b^3} \\ & - \frac{p(a+bx^2)^3\log^2\left(c(a+bx^2)^p\right)}{6b^3} \\ & + \frac{3ap(a+bx^2)^2\log^2\left(c(a+bx^2)^p\right)}{4b^3} \\ & - \frac{p^3(a+bx^2)^3}{27b^3} + \frac{3ap^3(a+bx^2)^2}{8b^3}\end{aligned}$$

```
[In] Int[x^5*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (-3*a^2*p^3*x^2)/b^2 + (3*a*p^3*(a + b*x^2)^2)/(8*b^3) - (p^3*(a + b*x^2)^3
)/(27*b^3) + (3*a^2*p^2*(a + b*x^2)*Log[c*(a + b*x^2)^p])/b^3 - (3*a*p^2*(a
 + b*x^2)^2*Log[c*(a + b*x^2)^p])/(4*b^3) + (p^2*(a + b*x^2)^3*Log[c*(a + b
*x^2)^p])/(9*b^3) - (3*a^2*p*(a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/(2*b^3) +
(3*a*p*(a + b*x^2)^2*Log[c*(a + b*x^2)^p]^2)/(4*b^3) - (p*(a + b*x^2)^3*Log
[c*(a + b*x^2)^p]^2)/(6*b^3) + (a^2*(a + b*x^2)*Log[c*(a + b*x^2)^p]^3)/(2*
b^3) - (a*(a + b*x^2)^2*Log[c*(a + b*x^2)^p]^3)/(2*b^3) + ((a + b*x^2)^3*Lo
g[c*(a + b*x^2)^p]^3)/(6*b^3)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x^2\log^3\left(c(a+bx)^p\right)\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int\left(\frac{a^2\log^3\left(c(a+bx)^p\right)}{b^2}-\frac{2a(a+bx)\log^3\left(c(a+bx)^p\right)}{b^2}\right.\right.\\
&\qquad\left.\left.+\frac{(a+bx)^2\log^3\left(c(a+bx)^p\right)}{b^2}\right)\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int(a+bx)^2\log^3\left(c(a+bx)^p\right)\,dx,x,x^2\right)}{2b^2}\\
&\quad-\frac{a\text{Subst}\left(\int(a+bx)\log^3\left(c(a+bx)^p\right)\,dx,x,x^2\right)}{b^2}\\
&\quad+\frac{a^2\text{Subst}\left(\int\log^3\left(c(a+bx)^p\right)\,dx,x,x^2\right)}{2b^2}\\
&= \frac{\text{Subst}\left(\int x^2\log^3\left(cx^p\right)\,dx,x,a+bx^2\right)}{2b^3}-\frac{a\text{Subst}\left(\int x\log^3\left(cx^p\right)\,dx,x,a+bx^2\right)}{b^3}\\
&\quad+\frac{a^2\text{Subst}\left(\int\log^3\left(cx^p\right)\,dx,x,a+bx^2\right)}{2b^3}\\
&= \frac{a^2(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2b^3}-\frac{a(a+bx^2)^2\log^3\left(c(a+bx^2)^p\right)}{2b^3}\\
&\quad+\frac{(a+bx^2)^3\log^3\left(c(a+bx^2)^p\right)}{6b^3}-\frac{p\text{Subst}\left(\int x^2\log^2\left(cx^p\right)\,dx,x,a+bx^2\right)}{2b^3}\\
&\quad+\frac{(3ap)\text{Subst}\left(\int x\log^2\left(cx^p\right)\,dx,x,a+bx^2\right)}{2b^3}\\
&\quad-\frac{(3a^2p)\,\text{Subst}\left(\int\log^2\left(cx^p\right)\,dx,x,a+bx^2\right)}{2b^3}\\
&= -\frac{3a^2p(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b^3}+\frac{3ap(a+bx^2)^2\log^2\left(c(a+bx^2)^p\right)}{4b^3}\\
&\quad-\frac{p(a+bx^2)^3\log^2\left(c(a+bx^2)^p\right)}{6b^3}+\frac{a^2(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2b^3}\\
&\quad-\frac{a(a+bx^2)^2\log^3\left(c(a+bx^2)^p\right)}{2b^3}+\frac{(a+bx^2)^3\log^3\left(c(a+bx^2)^p\right)}{6b^3}\\
&\quad+\frac{p^2\text{Subst}\left(\int x^2\log\left(cx^p\right)\,dx,x,a+bx^2\right)}{3b^3}\\
&\quad-\frac{(3ap^2)\,\text{Subst}\left(\int x\log\left(cx^p\right)\,dx,x,a+bx^2\right)}{2b^3}\\
&\quad+\frac{(3a^2p^2)\,\text{Subst}\left(\int\log\left(cx^p\right)\,dx,x,a+bx^2\right)}{b^3}
\end{aligned}
$$

$$= -\frac{3a^2p^3x^2}{b^2} + \frac{3ap^3(a+bx^2)^2}{8b^3} - \frac{p^3(a+bx^2)^3}{27b^3} + \frac{3a^2p^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^3} - \frac{3ap^2(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{4b^3} + \frac{p^2(a+bx^2)^3\log\left(c(a+bx^2)^p\right)}{9b^3} - \frac{3a^2p(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b^3} + \frac{3ap(a+bx^2)^2\log^2\left(c(a+bx^2)^p\right)}{4b^3} - \frac{p(a+bx^2)^3\log^2\left(c(a+bx^2)^p\right)}{6b^3} + \frac{a^2(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2b^3} - \frac{a(a+bx^2)^2\log^3\left(c(a+bx^2)^p\right)}{2b^3} + \frac{(a+bx^2)^3\log^3\left(c(a+bx^2)^p\right)}{6b^3}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.53

$$\int x^5\log^3\left(c(a+bx^2)^p\right)\,dx = \frac{bp^3x^2(-510a^2+57abx^2-8b^2x^4)+114a^3p^3\log(a+bx^2)+6p^2(66a^3+66a^2bx^2-15ab^2x^4+4b^3x^6)\log\left(c(\right.}{216b}$$

```
[In] Integrate[x^5*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (b*p^3*x^2*(-510*a^2 + 57*a*b*x^2 - 8*b^2*x^4) + 114*a^3*p^3*Log[a + b*x^2]
 + 6*p^2*(66*a^3 + 66*a^2*b*x^2 - 15*a*b^2*x^4 + 4*b^3*x^6)*Log[c*(a + b*x^
2)^p] - 18*p*(11*a^3 + 6*a^2*b*x^2 - 3*a*b^2*x^4 + 2*b^3*x^6)*Log[c*(a + b*
x^2)^p]^2 + 36*(a^3 + b^3*x^6)*Log[c*(a + b*x^2)^p]^3)/(216*b^3)
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.87

method	result
parallelrisch	$\frac{36x^6\ln(c(bx^2+a)^p)^3b^3-36x^6\ln(c(bx^2+a)^p)^2b^3p+24x^6\ln(c(bx^2+a)^p)b^3p^2-8b^3p^3x^6+54x^4\ln(c(bx^2+a)^p)^2ab^2p-90x^4\ln(c(\right.}{}$
risch	Expression too large to display

```
[In] int(x^5*ln(c*(b*x^2+a)^p)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/216*(36*x^6*ln(c*(b*x^2+a)^p)^3*b^3-36*x^6*ln(c*(b*x^2+a)^p)^2*b^3*p+24*x
^6*ln(c*(b*x^2+a)^p)*b^3*p^2-8*b^3*p^3*x^6+54*x^4*ln(c*(b*x^2+a)^p)^2*a*b^2
*p-90*x^4*ln(c*(b*x^2+a)^p)*a*b^2*p^2+57*a*b^2*p^3*x^4-108*x^2*ln(c*(b*x^2+
a)^p)^2*a^2*b*p+396*x^2*ln(c*(b*x^2+a)^p)*a^2*b*p^2-510*a^2*b*p^3*x^2+906*l
n(b*x^2+a)*a^3*p^3+36*ln(c*(b*x^2+a)^p)^3*a^3-198*ln(c*(b*x^2+a)^p)^2*a^3*p
-396*ln(c*(b*x^2+a)^p)*a^3*p^2+510*a^3*p^3)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.07

$$\int x^5 \log^3\left(c(a+bx^2)^p\right)\,dx = -\frac{8\,b^3p^3x^6 - 36\,b^3x^6\log\left(c\right)^3 - 57\,ab^2p^3x^4 + 510\,a^2bp^3x^2 - 36\,(b^3p^3x^6 + a^3p^3)\log\left(bx^2+a\right)^3 + 18\,(2\,b^3p^3x^6}{}$$

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] -1/216*(8*b^3*p^3*x^6 - 36*b^3*x^6*log(c)^3 - 57*a*b^2*p^3*x^4 + 510*a^2*b*
p^3*x^2 - 36*(b^3*p^3*x^6 + a^3*p^3)*log(b*x^2 + a)^3 + 18*(2*b^3*p^3*x^6 -
 3*a*b^2*p^3*x^4 + 6*a^2*b*p^3*x^2 + 11*a^3*p^3 - 6*(b^3*p^2*x^6 + a^3*p^2)
*log(c))*log(b*x^2 + a)^2 + 18*(2*b^3*p*x^6 - 3*a*b^2*p*x^4 + 6*a^2*b*p*x^2
)*log(c)^2 - 6*(4*b^3*p^3*x^6 - 15*a*b^2*p^3*x^4 + 66*a^2*b*p^3*x^2 + 85*a^
3*p^3 + 18*(b^3*p*x^6 + a^3*p)*log(c)^2 - 6*(2*b^3*p^2*x^6 - 3*a*b^2*p^2*x^
4 + 6*a^2*b*p^2*x^2 + 11*a^3*p^2)*log(c))*log(b*x^2 + a) - 6*(4*b^3*p^2*x^6
 - 15*a*b^2*p^2*x^4 + 66*a^2*b*p^2*x^2)*log(c))/b^3
```

Sympy [A] (verification not implemented)

Time = 5.00 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.87

$$\int x^5 \log^3\left(c(a+bx^2)^p\right)\,dx = \begin{cases} \frac{85a^3p^2\log\left(c(a+bx^2)^p\right)}{36b^3} - \frac{11a^3p\log\left(c(a+bx^2)^p\right)^2}{12b^3} + \frac{a^3\log\left(c(a+bx^2)^p\right)^3}{6b^3} - \frac{85a^2p^3x^2}{36b^2} + \frac{11a^2p^2x^2\log\left(c(a+bx^2)^p\right)}{6b^2} - \frac{a^2px^2\log\left(c(a+bx^2)^p\right)^2}{2b^2} \\ \frac{x^6\log\left(a^pc\right)^3}{6} \end{cases}$$

```
[In] integrate(x**5*ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Piecewise((85*a**3*p**2*log(c*(a + b*x**2)**p)/(36*b**3) - 11*a**3*p*log(c*
(a + b*x**2)**p)**2/(12*b**3) + a**3*log(c*(a + b*x**2)**p)**3/(6*b**3) - 8
5*a**2*p**3*x**2/(36*b**2) + 11*a**2*p**2*x**2*log(c*(a + b*x**2)**p)/(6*b*
*2) - a**2*p*x**2*log(c*(a + b*x**2)**p)**2/(2*b**2) + 19*a*p**3*x**4/(72*b
) - 5*a*p**2*x**4*log(c*(a + b*x**2)**p)/(12*b) + a*p*x**4*log(c*(a + b*x**
2)**p)**2/(4*b) - p**3*x**6/27 + p**2*x**6*log(c*(a + b*x**2)**p)/9 - p*x**
6*log(c*(a + b*x**2)**p)**2/6 + x**6*log(c*(a + b*x**2)**p)**3/6, Ne(b, 0))
, (x**6*log(a**p*c)**3/6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.72

$$\int x^5 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{1}{6}\,x^6\log\left(\left(bx^2+a\right)^p c\right)^3 + \frac{1}{12}\,bp\left(\frac{6\,a^3\log\left(bx^2+a\right)}{b^4} - \frac{2\,b^2x^6 - 3\,abx^4 + 6\,a^2x^2}{b^3}\right)\log\left(\left(bx^2+a\right)^p c\right)^2 - \frac{1}{216}\,bp\left(\frac{\left(8\,b^3x^6 - 57\,ab^2x^4 - 36\,a^3\log\left(bx^2+a\right)^3 + 510\,a^2bx^2 - 198\,a^3\log\left(bx^2+a\right)^2 - 510\,a^3\log\left(bx^2+\right.\right.}{b^4}\right.$$

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*log((b*x^2 + a)^p*c)^3 + 1/12*b*p*(6*a^3*log(b*x^2 + a)/b^4 - (2*b^
2*x^6 - 3*a*b*x^4 + 6*a^2*x^2)/b^3)*log((b*x^2 + a)^p*c)^2 - 1/216*b*p*((8*
b^3*x^6 - 57*a*b^2*x^4 - 36*a^3*log(b*x^2 + a)^3 + 510*a^2*b*x^2 - 198*a^3*
log(b*x^2 + a)^2 - 510*a^3*log(b*x^2 + a))*p^2/b^4 - 6*(4*b^3*x^6 - 15*a*b^
2*x^4 + 66*a^2*b*x^2 - 18*a^3*log(b*x^2 + a)^2 - 66*a^3*log(b*x^2 + a))*p*l
og((b*x^2 + a)^p*c)/b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. 2(314) = 628.

Time = 0.32 (sec) , antiderivative size = 662, normalized size of antiderivative = 1.98

$$\int x^5 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{\left(bx^2+a\right)^3 p^3 \log\left(bx^2+a\right)^3}{6\,b^3} - \frac{\left(bx^2+a\right)^2 a p^3 \log\left(bx^2+a\right)^3}{2\,b^3}$$
$$- \frac{\left(bx^2+a\right)^3 p^3 \log\left(bx^2+a\right)^2}{6\,b^3} + \frac{3\left(bx^2+a\right)^2 a p^3 \log\left(bx^2+a\right)^2}{4\,b^3}$$
$$+ \frac{\left(bx^2+a\right)^3 p^2 \log\left(bx^2+a\right)^2 \log(c)}{2\,b^3} - \frac{3\left(bx^2+a\right)^2 a p^2 \log\left(bx^2+a\right)^2 \log(c)}{2\,b^3}$$
$$+ \frac{\left(bx^2+a\right)^3 p^3 \log\left(bx^2+a\right)}{9\,b^3} - \frac{3\left(bx^2+a\right)^2 a p^3 \log\left(bx^2+a\right)}{4\,b^3}$$
$$- \frac{\left(bx^2+a\right)^3 p^2 \log\left(bx^2+a\right)\log(c)}{3\,b^3} + \frac{3\left(bx^2+a\right)^2 a p^2 \log\left(bx^2+a\right)\log(c)}{2\,b^3}$$
$$+ \frac{\left(bx^2+a\right)^3 p \log\left(bx^2+a\right)\log(c)^2}{2\,b^3} - \frac{3\left(bx^2+a\right)^2 a p \log\left(bx^2+a\right)\log(c)^2}{2\,b^3}$$
$$- \frac{\left(bx^2+a\right)^3 p^3}{27\,b^3} + \frac{3\left(bx^2+a\right)^2 a p^3}{8\,b^3} + \frac{\left(bx^2+a\right)^3 p^2 \log(c)}{9\,b^3} - \frac{3\left(bx^2+a\right)^2 a p^2 \log(c)}{4\,b^3}$$
$$- \frac{\left(bx^2+a\right)^3 p \log(c)^2}{6\,b^3} + \frac{3\left(bx^2+a\right)^2 a p \log(c)^2}{4\,b^3} + \frac{\left(bx^2+a\right)^3 \log(c)^3}{6\,b^3} - \frac{\left(bx^2+a\right)^2 a \log(c)^3}{2\,b^3}$$
$$+ \frac{\left(\left(bx^2+a\right)\log\left(bx^2+a\right)^3 - 6\,bx^2 - 3\left(bx^2+a\right)\log\left(bx^2+a\right)^2 + 6\left(bx^2+a\right)\log\left(bx^2+a\right) - 6\,a\right)a^2p^3 + 3}{}$$

```
[In] integrate(x^5*log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] 1/6*(b*x^2 + a)^3*p^3*log(b*x^2 + a)^3/b^3 - 1/2*(b*x^2 + a)^2*a*p^3*log(b*
x^2 + a)^3/b^3 - 1/6*(b*x^2 + a)^3*p^3*log(b*x^2 + a)^2/b^3 + 3/4*(b*x^2 +
a)^2*a*p^3*log(b*x^2 + a)^2/b^3 + 1/2*(b*x^2 + a)^3*p^2*log(b*x^2 + a)^2*lo
g(c)/b^3 - 3/2*(b*x^2 + a)^2*a*p^2*log(b*x^2 + a)^2*log(c)/b^3 + 1/9*(b*x^2
 + a)^3*p^3*log(b*x^2 + a)/b^3 - 3/4*(b*x^2 + a)^2*a*p^3*log(b*x^2 + a)/b^3
 - 1/3*(b*x^2 + a)^3*p^2*log(b*x^2 + a)*log(c)/b^3 + 3/2*(b*x^2 + a)^2*a*p^
2*log(b*x^2 + a)*log(c)/b^3 + 1/2*(b*x^2 + a)^3*p*log(b*x^2 + a)*log(c)^2/b
^3 - 3/2*(b*x^2 + a)^2*a*p*log(b*x^2 + a)*log(c)^2/b^3 - 1/27*(b*x^2 + a)^3
*p^3/b^3 + 3/8*(b*x^2 + a)^2*a*p^3/b^3 + 1/9*(b*x^2 + a)^3*p^2*log(c)/b^3 -
 3/4*(b*x^2 + a)^2*a*p^2*log(c)/b^3 - 1/6*(b*x^2 + a)^3*p*log(c)^2/b^3 + 3/
4*(b*x^2 + a)^2*a*p*log(c)^2/b^3 + 1/6*(b*x^2 + a)^3*log(c)^3/b^3 - 1/2*(b*
x^2 + a)^2*a*log(c)^3/b^3 + 1/2*(((b*x^2 + a)*log(b*x^2 + a)^3 - 6*b*x^2 -
3*(b*x^2 + a)*log(b*x^2 + a)^2 + 6*(b*x^2 + a)*log(b*x^2 + a) - 6*a)*a^2*p^
3 + 3*(2*b*x^2 + (b*x^2 + a)*log(b*x^2 + a)^2 - 2*(b*x^2 + a)*log(b*x^2 + a
) + 2*a)*a^2*p^2*log(c) - 3*(b*x^2 - (b*x^2 + a)*log(b*x^2 + a) + a)*a^2*p*
log(c)^2 + (b*x^2 + a)*a^2*log(c)^3)/b^3
```

Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.56

$$\int x^5 \log^3\left(c(a+bx^2)^p\right)\,dx = \ln\left(c\left(b\,x^2+a\right)^p\right)^3\left(\frac{x^6}{6}+\frac{a^3}{6\,b^3}\right) - \ln\left(c\left(b\,x^2+a\right)^p\right)^2\left(\frac{p\,x^6}{6}+\frac{11\,a^3\,p}{12\,b^3}+\frac{a^2\,p\,x^2}{2\,b^2}-\frac{a\,p\,x^4}{4\,b}\right) - \frac{p^3\,x^6}{27}+\frac{\ln\left(c\left(b\,x^2+a\right)^p\right)\left(\frac{b\,p^2\,x^6}{3}-\frac{5\,a\,p^2\,x^4}{4}+\frac{11\,a^2\,p^2\,x^2}{2\,b}\right)}{3\,b} + \frac{19\,a\,p^3\,x^4}{72\,b}+\frac{85\,a^3\,p^3\,\ln\left(b\,x^2+a\right)}{36\,b^3}-\frac{85\,a^2\,p^3\,x^2}{36\,b^2}$$

```
[In] int(x^5*log(c*(a + b*x^2)^p)^3,x)
```

```
[Out] log(c*(a + b*x^2)^p)^3*(x^6/6 + a^3/(6*b^3)) - log(c*(a + b*x^2)^p)^2*((p*x
^6)/6 + (11*a^3*p)/(12*b^3) + (a^2*p*x^2)/(2*b^2) - (a*p*x^4)/(4*b)) - (p^3
*x^6)/27 + (log(c*(a + b*x^2)^p)*((b*p^2*x^6)/3 - (5*a*p^2*x^4)/4 + (11*a^2
*p^2*x^2)/(2*b)))/(3*b) + (19*a*p^3*x^4)/(72*b) + (85*a^3*p^3*log(a + b*x^2
))/(36*b^3) - (85*a^2*p^3*x^2)/(36*b^2)
```

3.92 $\int x^3 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 211

$$\begin{aligned}\int x^3 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx = {} & \frac{3ap^3x^2}{b} - \frac{3p^3\left(a+bx^2\right)^2}{16b^2} - \frac{3ap^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^2} \\ & + \frac{3p^2\left(a+bx^2\right)^2\log\left(c\left(a+bx^2\right)^p\right)}{8b^2} \\ & + \frac{3ap\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^2} \\ & - \frac{3p\left(a+bx^2\right)^2\log^2\left(c\left(a+bx^2\right)^p\right)}{8b^2} \\ & - \frac{a\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2b^2} + \frac{\left(a+bx^2\right)^2\log^3\left(c\left(a+bx^2\right)^p\right)}{4b^2}\end{aligned}$$

```
[Out] 3*a*p^3*x^2/b-3/16*p^3*(b*x^2+a)^2/b^2-3*a*p^2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/
b^2+3/8*p^2*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)/b^2+3/2*a*p*(b*x^2+a)*ln(c*(b*x^2
+a)^p)^2/b^2-3/8*p*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)^2/b^2-1/2*a*(b*x^2+a)*ln(c
*(b*x^2+a)^p)^3/b^2+1/4*(b*x^2+a)^2*ln(c*(b*x^2+a)^p)^3/b^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used

$= \{2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341\}$

$$\int x^3 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{3p^2\left(a+bx^2\right)^2 \log\left(c\left(a+bx^2\right)^p\right)}{8b^2} - \frac{3ap^2\left(a+bx^2\right)\log\left(c\left(a+bx^2\right)^p\right)}{b^2} + \frac{\left(a+bx^2\right)^2 \log^3\left(c\left(a+bx^2\right)^p\right)}{4b^2} - \frac{a\left(a+bx^2\right)\log^3\left(c\left(a+bx^2\right)^p\right)}{2b^2} - \frac{3p\left(a+bx^2\right)^2 \log^2\left(c\left(a+bx^2\right)^p\right)}{8b^2} + \frac{3ap\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{2b^2} - \frac{3p^3\left(a+bx^2\right)^2}{16b^2} + \frac{3ap^3x^2}{b}$$

```
[In] Int[x^3*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (3*a*p^3*x^2)/b - (3*p^3*(a + b*x^2)^2)/(16*b^2) - (3*a*p^2*(a + b*x^2)*Log
[c*(a + b*x^2)^p])/b^2 + (3*p^2*(a + b*x^2)^2*Log[c*(a + b*x^2)^p])/(8*b^2)
 + (3*a*p*(a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/(2*b^2) - (3*p*(a + b*x^2)^2*
Log[c*(a + b*x^2)^p]^2)/(8*b^2) - (a*(a + b*x^2)*Log[c*(a + b*x^2)^p]^3)/(2
*b^2) + ((a + b*x^2)^2*Log[c*(a + b*x^2)^p]^3)/(4*b^2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x\log^3\left(c(a+bx)^p\right)\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int\left(-\frac{a\log^3\left(c(a+bx)^p\right)}{b}+\frac{(a+bx)\log^3\left(c(a+bx)^p\right)}{b}\right)\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int(a+bx)\log^3\left(c(a+bx)^p\right)\,dx,x,x^2\right)}{2b}-\frac{a\text{Subst}\left(\int\log^3\left(c(a+bx)^p\right)\,dx,x,x^2\right)}{2b}\\
&= \frac{\text{Subst}\left(\int x\log^3\left(cx^p\right)\,dx,x,a+bx^2\right)}{2b^2}-\frac{a\text{Subst}\left(\int\log^3\left(cx^p\right)\,dx,x,a+bx^2\right)}{2b^2}\\
&= -\frac{a(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2b^2}+\frac{(a+bx^2)^2\log^3\left(c(a+bx^2)^p\right)}{4b^2}\\
&\quad-\frac{(3p)\text{Subst}\left(\int x\log^2\left(cx^p\right)\,dx,x,a+bx^2\right)}{4b^2}\\
&\quad+\frac{(3ap)\text{Subst}\left(\int\log^2\left(cx^p\right)\,dx,x,a+bx^2\right)}{2b^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3ap(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b^2} - \frac{3p(a+bx^2)^2\log^2\left(c(a+bx^2)^p\right)}{8b^2} \\
&\quad - \frac{a(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2b^2} + \frac{(a+bx^2)^2\log^3\left(c(a+bx^2)^p\right)}{4b^2} \\
&\quad + \frac{(3p^2)\,\text{Subst}\left(\int x\log\left(cx^p\right)\,dx, x, a+bx^2\right)}{4b^2} \\
&\quad - \frac{(3ap^2)\,\text{Subst}\left(\int \log\left(cx^p\right)\,dx, x, a+bx^2\right)}{b^2} \\
&= \frac{3ap^3x^2}{b} - \frac{3p^3(a+bx^2)^2}{16b^2} - \frac{3ap^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b^2} \\
&\quad + \frac{3p^2(a+bx^2)^2\log\left(c(a+bx^2)^p\right)}{8b^2} \\
&\quad + \frac{3ap(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b^2} - \frac{3p(a+bx^2)^2\log^2\left(c(a+bx^2)^p\right)}{8b^2} \\
&\quad - \frac{a(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2b^2} + \frac{(a+bx^2)^2\log^3\left(c(a+bx^2)^p\right)}{4b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.69

$$
\int x^3\log^3\left(c(a+bx^2)^p\right)\,dx = -\frac{3bp^3x^2(-14a+bx^2)+6a^2p^3\log(a+bx^2)+6p^2(6a^2+6abx^2-b^2x^4)\log\left(c(a+bx^2)^p\right)-6p(3a^2+2abx^2}{16b^2}
$$

```
[In] Integrate[x^3*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] -1/16*(3*b*p^3*x^2*(-14*a + b*x^2) + 6*a^2*p^3*Log[a + b*x^2] + 6*p^2*(6*a^
2 + 6*a*b*x^2 - b^2*x^4)*Log[c*(a + b*x^2)^p] - 6*p*(3*a^2 + 2*a*b*x^2 - b^
2*x^4)*Log[c*(a + b*x^2)^p]^2 + 4*(a^2 - b^2*x^4)*Log[c*(a + b*x^2)^p]^3)/b
^2
```

Maple [A] (verified)

Time = 11.03 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.06

method	result
parallelrisch	$-\frac{-4x^4\ln\left(c\left(bx^2+a\right)^p\right)^3b^2+6x^4\ln\left(c\left(bx^2+a\right)^p\right)^2b^2p-6x^4\ln\left(c\left(bx^2+a\right)^p\right)b^2p^2+3b^2p^3x^4-12x^2\ln\left(c\left(bx^2+a\right)^p\right)^2abp+36x^2\ln(c(}{1}$
risch	Expression too large to display

```
[In] int(x^3*ln(c*(b*x^2+a)^p)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(-4*x^4*ln(c*(b*x^2+a)^p)^3*b^2+6*x^4*ln(c*(b*x^2+a)^p)^2*b^2*p-6*x^4
*ln(c*(b*x^2+a)^p)*b^2*p^2+3*b^2*p^3*x^4-12*x^2*ln(c*(b*x^2+a)^p)^2*a*b*p+3
6*x^2*ln(c*(b*x^2+a)^p)*a*b*p^2-42*x^2*a*b*p^3+78*ln(b*x^2+a)*a^2*p^3+4*ln(
c*(b*x^2+a)^p)^3*a^2-18*ln(c*(b*x^2+a)^p)^2*a^2*p-36*ln(c*(b*x^2+a)^p)*a^2*
p^2+42*a^2*p^3)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.30

$$\int x^3 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx =$$
$$-\frac{3\,b^2p^3x^4 - 4\,b^2x^4\log\left(c\right)^3 - 42\,abp^3x^2 - 4\left(b^2p^3x^4 - a^2p^3\right)\log\left(bx^2+a\right)^3 + 6\left(b^2p^3x^4 - 2\,abp^3x^2 - 3\,a^2p^3\right.}{}$$

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(3*b^2*p^3*x^4 - 4*b^2*x^4*log(c)^3 - 42*a*b*p^3*x^2 - 4*(b^2*p^3*x^4
 - a^2*p^3)*log(b*x^2 + a)^3 + 6*(b^2*p^3*x^4 - 2*a*b*p^3*x^2 - 3*a^2*p^3 -
 2*(b^2*p^2*x^4 - a^2*p^2)*log(c))*log(b*x^2 + a)^2 + 6*(b^2*p*x^4 - 2*a*b*
p*x^2)*log(c)^2 - 6*(b^2*p^3*x^4 - 6*a*b*p^3*x^2 - 7*a^2*p^3 + 2*(b^2*p*x^4
 - a^2*p)*log(c)^2 - 2*(b^2*p^2*x^4 - 2*a*b*p^2*x^2 - 3*a^2*p^2)*log(c))*lo
g(b*x^2 + a) - 6*(b^2*p^2*x^4 - 6*a*b*p^2*x^2)*log(c))/b^2
```

Sympy [A] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.06

$$\int x^3 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$= \begin{cases} -\frac{21a^2p^2\log\left(c\left(a+bx^2\right)^p\right)}{8b^2} + \frac{9a^2p\log\left(c\left(a+bx^2\right)^p\right)^2}{8b^2} - \frac{a^2\log\left(c\left(a+bx^2\right)^p\right)^3}{4b^2} + \frac{21ap^3x^2}{8b} - \frac{9ap^2x^2\log\left(c\left(a+bx^2\right)^p\right)}{4b} + \frac{3apx^2\log\left(c\left(a+\right.\right.}{4b} \\ \frac{x^4\log\left(a^pc\right)^3}{4} \end{cases}$$

```
[In] integrate(x**3*ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Piecewise((-21*a**2*p**2*log(c*(a + b*x**2)**p)/(8*b**2) + 9*a**2*p*log(c*(
a + b*x**2)**p)**2/(8*b**2) - a**2*log(c*(a + b*x**2)**p)**3/(4*b**2) + 21*
a*p**3*x**2/(8*b) - 9*a*p**2*x**2*log(c*(a + b*x**2)**p)/(4*b) + 3*a*p*x**2
*log(c*(a + b*x**2)**p)**2/(4*b) - 3*p**3*x**4/16 + 3*p**2*x**4*log(c*(a +
b*x**2)**p)/8 - 3*p*x**4*log(c*(a + b*x**2)**p)**2/8 + x**4*log(c*(a + b*x*
*2)**p)**3/4, Ne(b, 0)), (x**4*log(a**p*c)**3/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.96

$$\int x^3 \log^3 \left(c\left(a+bx^2\right)^p\right) \, dx$$
$$= \frac{1}{4} x^4 \log \left(\left(bx^2+a\right)^p c\right)^3 - \frac{3}{8} bp \left(\frac{2 a^2 \log \left(bx^2+a\right)}{b^3} + \frac{bx^4 - 2 ax^2}{b^2}\right) \log \left(\left(bx^2+a\right)^p c\right)^2$$
$$- \frac{1}{16} bp \left(\frac{\left(3 b^2 x^4 + 4 a^2 \log \left(bx^2+a\right)^3 - 42 abx^2 + 18 a^2 \log \left(bx^2+a\right)^2 + 42 a^2 \log \left(bx^2+a\right)\right) p^2}{b^3} - \frac{6 \left(b^2 x^4 \right.}{}\right.$$

[In] integrate(x^3*log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")

[Out] 1/4*x^4*log((b*x^2 + a)^p*c)^3 - 3/8*b*p*(2*a^2*log(b*x^2 + a)/b^3 + (b*x^4 - 2*a*x^2)/b^2)*log((b*x^2 + a)^p*c)^2 - 1/16*b*p*((3*b^2*x^4 + 4*a^2*log(b*x^2 + a)^3 - 42*a*b*x^2 + 18*a^2*log(b*x^2 + a)^2 + 42*a^2*log(b*x^2 + a))*p^2/b^3 - 6*(b^2*x^4 - 6*a*b*x^2 + 2*a^2*log(b*x^2 + a)^2 + 6*a^2*log(b*x^2 + a))*p*log((b*x^2 + a)^p*c)/b^3)

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.82

$$\int x^3 \log^3 \left(c\left(a+bx^2\right)^p\right) \, dx$$
$$= \frac{4 \left(bx^2+a\right)^2 p^3 \log \left(bx^2+a\right)^3 - 6 \left(bx^2+a\right)^2 p^3 \log \left(bx^2+a\right)^2 + 12 \left(bx^2+a\right)^2 p^2 \log \left(bx^2+a\right)^2 \log \left(c\right) + 6 \left(bx^2\right.}{}$$
$$- \frac{\left(\left(bx^2+a\right) \log \left(bx^2+a\right)^3 - 6 bx^2 - 3 \left(bx^2+a\right) \log \left(bx^2+a\right)^2 + 6 \left(bx^2+a\right) \log \left(bx^2+a\right) - 6 a\right) ap^3 + 3 \left(\right.}{}$$

[In] integrate(x^3*log(c*(b*x^2+a)^p)^3,x, algorithm="giac")

[Out] 1/16*(4*(b*x^2 + a)^2*p^3*log(b*x^2 + a)^3 - 6*(b*x^2 + a)^2*p^3*log(b*x^2 + a)^2 + 12*(b*x^2 + a)^2*p^2*log(b*x^2 + a)^2*log(c) + 6*(b*x^2 + a)^2*p^3*log(b*x^2 + a) - 12*(b*x^2 + a)^2*p^2*log(b*x^2 + a)*log(c) + 12*(b*x^2 + a)^2*p*log(b*x^2 + a)*log(c)^2 - 3*(b*x^2 + a)^2*p^3 + 6*(b*x^2 + a)^2*p^2*log(c) - 6*(b*x^2 + a)^2*p*log(c)^2 + 4*(b*x^2 + a)^2*log(c)^3)/b^2 - 1/2*(((b*x^2 + a)*log(b*x^2 + a)^3 - 6*b*x^2 - 3*(b*x^2 + a)*log(b*x^2 + a)^2 + 6*(b*x^2 + a)*log(b*x^2 + a) - 6*a)*a*p^3 + 3*(2*b*x^2 + (b*x^2 + a)*log(b*x^2 + a)^2 - 2*(b*x^2 + a)*log(b*x^2 + a) + 2*a)*a*p^2*log(c) - 3*(b*x^2 - (b*x^2 + a)*log(b*x^2 + a) + a)*a*p*log(c)^2 + (b*x^2 + a)*a*log(c)^3)/b^2

Mupad [B] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.68

$$\int x^3 \log^3 \left(c\left(a+bx^2\right)^p\right) \, dx = \ln\left(c\left(b\,x^2+a\right)^p\right)^2 \left(\frac{9\,a^2\,p}{8\,b^2}-\frac{3\,p\,x^4}{8}+\frac{3\,a\,p\,x^2}{4\,b}\right) - \frac{3\,p^3\,x^4}{16} + \ln\left(c\left(b\,x^2+a\right)^p\right)\left(\frac{3\,p^2\,x^4}{8}-\frac{9\,a\,p^2\,x^2}{4\,b}\right) + \ln\left(c\left(b\,x^2+a\right)^p\right)^3\left(\frac{x^4}{4}-\frac{a^2}{4\,b^2}\right) + \frac{21\,a\,p^3\,x^2}{8\,b} - \frac{21\,a^2\,p^3\,\ln\left(b\,x^2+a\right)}{8\,b^2}$$

```
[In] int(x^3*log(c*(a + b*x^2)^p)^3,x)
```

```
[Out] log(c*(a + b*x^2)^p)^2*((9*a^2*p)/(8*b^2) - (3*p*x^4)/8 + (3*a*p*x^2)/(4*b)
) - (3*p^3*x^4)/16 + log(c*(a + b*x^2)^p)*((3*p^2*x^4)/8 - (9*a*p^2*x^2)/(4
*b)) + log(c*(a + b*x^2)^p)^3*(x^4/4 - a^2/(4*b^2)) + (21*a*p^3*x^2)/(8*b)
- (21*a^2*p^3*log(a + b*x^2))/(8*b^2)
```

3.93 $\int x \log^3\left(c(a+bx^2)^p\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 93

$$\int x \log^3\left(c(a+bx^2)^p\right)\,dx = -3p^3x^2 + \frac{3p^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b} - \frac{3p(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b} + \frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2b}$$

```
[Out] -3*p^3*x^2+3*p^2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b-3/2*p*(b*x^2+a)*ln(c*(b*x^2+
a)^p)^2/b+1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)^3/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2504, 2436, 2333, 2332}

$$\int x \log^3\left(c(a+bx^2)^p\right)\,dx = \frac{3p^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b} + \frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2b} - \frac{3p(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b} - 3p^3x^2$$

```
[In] Int[x*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] -3*p^3*x^2 + (3*p^2*(a + b*x^2)*Log[c*(a + b*x^2)^p])/b - (3*p*(a + b*x^2)*
Log[c*(a + b*x^2)^p]^2)/(2*b) + ((a + b*x^2)*Log[c*(a + b*x^2)^p]^3)/(2*b)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \log^3\left(c(a+bx)^p\right)\, dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \log^3\left(cx^p\right)\, dx, x, a+bx^2\right)}{2b}\\
&= \frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2b} - \frac{(3p)\text{Subst}\left(\int \log^2\left(cx^p\right)\, dx, x, a+bx^2\right)}{2b}\\
&= -\frac{3p(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b} + \frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2b}\\
&\quad + \frac{(3p^2)\,\text{Subst}\left(\int \log\left(cx^p\right)\, dx, x, a+bx^2\right)}{b}\\
&= -3p^3x^2 + \frac{3p^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b}\\
&\quad - \frac{3p(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2b} + \frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int x\log^3\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$=\frac{-6bp^3x^2+6p^2(a+bx^2)\log\left(c(a+bx^2)^p\right)-3p(a+bx^2)\log^2\left(c(a+bx^2)^p\right)+(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2b}$$

```
[In] Integrate[x*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (-6*b*p^3*x^2 + 6*p^2*(a + b*x^2)*Log[c*(a + b*x^2)^p] - 3*p*(a + b*x^2)*Lo
g[c*(a + b*x^2)^p]^2 + (a + b*x^2)*Log[c*(a + b*x^2)^p]^3)/(2*b)
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.62

method	result
parallelrisch	$\frac{x^{2}\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)^{3}abp-3x^{2}\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)^{2}ab\,p^{2}+6x^{2}\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)ab\,p^{3}-6x^{2}ab\,p^{4}+\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)^{3}a^{2}p-3\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)^{2}}{2abp}$
risch	Expression too large to display

```
[In] int(x*ln(c*(b*x^2+a)^p)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(x^2*ln(c*(b*x^2+a)^p)^3*a*b*p-3*x^2*ln(c*(b*x^2+a)^p)^2*a*b*p^2+6*x^2*
ln(c*(b*x^2+a)^p)*a*b*p^3-6*x^2*a*b*p^4+ln(c*(b*x^2+a)^p)^3*a^2*p-3*ln(c*(b
*x^2+a)^p)^2*a^2*p^2+6*ln(c*(b*x^2+a)^p)*a^2*p^3)/a/b/p
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.89

$$\int x\log^3\left(c\left(a+bx^2\right)^p\right)\,dx =$$
$$-\frac{6\,bp^3x^2-6\,bp^2x^2\log\left(c\right)+3\,bpx^2\log\left(c\right)^2-bx^2\log\left(c\right)^3-\left(bp^3x^2+ap^3\right)\log\left(bx^2+a\right)^3+3\left(bp^3x^2+ap^3-\right.}{}$$

```
[In] integrate(x*log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*b*p^3*x^2 - 6*b*p^2*x^2*log(c) + 3*b*p*x^2*log(c)^2 - b*x^2*log(c)^
3 - (b*p^3*x^2 + a*p^3)*log(b*x^2 + a)^3 + 3*(b*p^3*x^2 + a*p^3 - (b*p^2*x^
2 + a*p^2)*log(c))*log(b*x^2 + a)^2 - 3*(2*b*p^3*x^2 + 2*a*p^3 + (b*p*x^2 +
 a*p)*log(c)^2 - 2*(b*p^2*x^2 + a*p^2)*log(c))*log(b*x^2 + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.54

$$\int x\log^3\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$=\begin{cases}\frac{3ap^2\log\left(c\left(a+bx^2\right)^p\right)}{b}-\frac{3ap\log\left(c\left(a+bx^2\right)^p\right)^2}{2b}+\frac{a\log\left(c\left(a+bx^2\right)^p\right)^3}{2b}-3p^3x^2+3p^2x^2\log\left(c\left(a+bx^2\right)^p\right)-\frac{3px^2\log\left(c\left(a+bx^2\right)^p\right)^2}{2} \\ \frac{x^2\log\left(a^pc\right)^3}{2}\end{cases}$$

```
[In] integrate(x*ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Piecewise((3*a*p**2*log(c*(a + b*x**2)**p)/b - 3*a*p*log(c*(a + b*x**2)**p)
**2/(2*b) + a*log(c*(a + b*x**2)**p)**3/(2*b) - 3*p**3*x**2 + 3*p**2*x**2*l
og(c*(a + b*x**2)**p) - 3*p*x**2*log(c*(a + b*x**2)**p)**2/2 + x**2*log(c*(
a + b*x**2)**p)**3/2, Ne(b, 0)), (x**2*log(a**p*c)**3/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.76

$$\int x\log^3\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$=-\frac{3}{2}\,bp\left(\frac{x^2}{b}-\frac{a\log\left(bx^2+a\right)}{b^2}\right)\log\left(\left(bx^2+a\right)^pc\right)^2+\frac{1}{2}\,x^2\log\left(\left(bx^2+a\right)^pc\right)^3$$
$$+\frac{1}{2}\,bp\left(\frac{\left(a\log\left(bx^2+a\right)^3-6\,bx^2+3\,a\log\left(bx^2+a\right)^2+6\,a\log\left(bx^2+a\right)\right)p^2}{b^2}+\frac{3\left(2\,bx^2-a\log\left(bx^2+a\right)^2\right.}{}\right.$$

```
[In] integrate(x*log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -3/2*b*p*(x^2/b - a*log(b*x^2 + a)/b^2)*log((b*x^2 + a)^p*c)^2 + 1/2*x^2*lo
g((b*x^2 + a)^p*c)^3 + 1/2*b*p*((a*log(b*x^2 + a)^3 - 6*b*x^2 + 3*a*log(b*x
^2 + a)^2 + 6*a*log(b*x^2 + a))*p^2/b^2 + 3*(2*b*x^2 - a*log(b*x^2 + a)^2 -
 2*a*log(b*x^2 + a))*p*log((b*x^2 + a)^p*c)/b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.82

$$\int x \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$=\frac{\left(\left(bx^2+a\right)\log\left(bx^2+a\right)^3-6\,bx^2-3\left(bx^2+a\right)\log\left(bx^2+a\right)^2+6\left(bx^2+a\right)\log\left(bx^2+a\right)-6\,a\right)p^3+3\left(2\,bx\right.}{}$$

```
[In] integrate(x*log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] 1/2*(((b*x^2 + a)*log(b*x^2 + a)^3 - 6*b*x^2 - 3*(b*x^2 + a)*log(b*x^2 + a)
^2 + 6*(b*x^2 + a)*log(b*x^2 + a) - 6*a)*p^3 + 3*(2*b*x^2 + (b*x^2 + a)*log
(b*x^2 + a)^2 - 2*(b*x^2 + a)*log(b*x^2 + a) + 2*a)*p^2*log(c) - 3*(b*x^2 -
 (b*x^2 + a)*log(b*x^2 + a) + a)*p*log(c)^2 + (b*x^2 + a)*log(c)^3)/b
```

Mupad [B] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.11

$$\int x\log^3\left(c\left(a+bx^2\right)^p\right)\,dx=\ln\left(c\left(b\,x^2+a\right)^p\right)^3\left(\frac{a}{2\,b}+\frac{x^2}{2}\right)-\ln\left(c\left(b\,x^2+a\right)^p\right)^2\left(\frac{3\,p\,x^2}{2}+\frac{3\,a\,p}{2\,b}\right)-3\,p^3\,x^2+3\,p^2\,x^2\,\ln\left(c\left(b\,x^2+a\right)^p\right)+\frac{3\,a\,p^3\,\ln\left(b\,x^2+a\right)}{b}$$

```
[In] int(x*log(c*(a + b*x^2)^p)^3,x)
```

```
[Out] log(c*(a + b*x^2)^p)^3*(a/(2*b) + x^2/2) - log(c*(a + b*x^2)^p)^2*((3*p*x^2
)/2 + (3*a*p)/(2*b)) - 3*p^3*x^2 + 3*p^2*x^2*log(c*(a + b*x^2)^p) + (3*a*p^
3*log(a + b*x^2))/b
```

3.94 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 106

$$\begin{aligned}\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x} \, dx = {} & \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right) \\ & + \frac{3}{2}p\log^2\left(c\left(a+bx^2\right)^p\right)\operatorname{PolyLog}\left(2,1+\frac{bx^2}{a}\right) \\ & - 3p^2\log\left(c\left(a+bx^2\right)^p\right)\operatorname{PolyLog}\left(3,1+\frac{bx^2}{a}\right) \\ & + 3p^3\operatorname{PolyLog}\left(4,1+\frac{bx^2}{a}\right)\end{aligned}$$

```
[Out] 1/2*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)^3+3/2*p*ln(c*(b*x^2+a)^p)^2*polylog(2,1+
b*x^2/a)-3*p^2*ln(c*(b*x^2+a)^p)*polylog(3,1+b*x^2/a)+3*p^3*polylog(4,1+b*x
^2/a)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2504, 2443, 2481, 2421, 2430, 6724}

$$\begin{aligned}\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x} \, dx = {} & -3p^2\operatorname{PolyLog}\left(3,\frac{bx^2}{a}+1\right)\log\left(c\left(a+bx^2\right)^p\right) \\ & + \frac{3}{2}p\operatorname{PolyLog}\left(2,\frac{bx^2}{a}+1\right)\log^2\left(c\left(a+bx^2\right)^p\right) \\ & + \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right) + 3p^3\operatorname{PolyLog}\left(4,\frac{bx^2}{a}+1\right)\end{aligned}$$

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x,x]
```

```
[Out] (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p]^3)/2 + (3*p*Log[c*(a + b*x^2)^p]^2*
PolyLog[2, 1 + (b*x^2)/a])/2 - 3*p^2*Log[c*(a + b*x^2)^p]*PolyLog[3, 1 + (b
*x^2)/a] + 3*p^3*PolyLog[4, 1 + (b*x^2)/a]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\log^3\left(c(a+bx)^p\right)}{x}\,dx, x, x^2\right)\\
&= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right) - \frac{1}{2}(3bp)\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)\log^2\left(c(a+bx)^p\right)}{a+bx}\,dx, x, x^2\right)\\
&= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right)\\
&\quad - \frac{1}{2}(3p)\text{Subst}\left(\int \frac{\log^2\left(cx^p\right)\log\left(-\frac{b\left(-\frac{a}{b}+\frac{x}{b}\right)}{a}\right)}{x}\,dx, x, a+bx^2\right)\\
&= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right) + \frac{3}{2}p\log^2\left(c\left(a+bx^2\right)^p\right)\text{Li}_2\left(1+\frac{bx^2}{a}\right)\\
&\quad - \left(3p^2\right)\text{Subst}\left(\int \frac{\log\left(cx^p\right)\text{Li}_2\left(\frac{x}{a}\right)}{x}\,dx, x, a+bx^2\right)\\
&= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right) + \frac{3}{2}p\log^2\left(c\left(a+bx^2\right)^p\right)\text{Li}_2\left(1+\frac{bx^2}{a}\right)\\
&\quad - 3p^2\log\left(c\left(a+bx^2\right)^p\right)\text{Li}_3\left(1+\frac{bx^2}{a}\right) + \left(3p^3\right)\text{Subst}\left(\int \frac{\text{Li}_3\left(\frac{x}{a}\right)}{x}\,dx, x, a+bx^2\right)\\
&= \frac{1}{2}\log\left(-\frac{bx^2}{a}\right)\log^3\left(c\left(a+bx^2\right)^p\right) + \frac{3}{2}p\log^2\left(c\left(a+bx^2\right)^p\right)\text{Li}_2\left(1+\frac{bx^2}{a}\right)\\
&\quad - 3p^2\log\left(c\left(a+bx^2\right)^p\right)\text{Li}_3\left(1+\frac{bx^2}{a}\right) + 3p^3\text{Li}_4\left(1+\frac{bx^2}{a}\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 279 vs. $2(106) = 212$.

Time = 0.11 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.63

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x}\,dx = \log(x)\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^3+3p\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^2\left(\log(x)\left(\log\left(a+bx^2\right)-\log\left(1+\frac{bx^2}{a}\right)\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,-\frac{bx^2}{a}\right)\right)-\frac{3}{2}p^2\left(p\log\left(a+bx^2\right)-\log\left(c(a+bx^2)^p\right)\right)\left(\log\left(-\frac{bx^2}{a}\right)\log^2\left(a+bx^2\right)+2\log\left(a+bx^2\right)\operatorname{PolyLog}\left(2,1+\frac{bx^2}{a}\right)-2\operatorname{PolyLog}\left(3,1+\frac{bx^2}{a}\right)\right)+\frac{1}{2}p^3\left(\log\left(-\frac{bx^2}{a}\right)\log^3\left(a+bx^2\right)+3\log^2\left(a+bx^2\right)\operatorname{PolyLog}\left(2,1+\frac{bx^2}{a}\right)-6\log\left(a+bx^2\right)\operatorname{PolyLog}\left(3,1+\frac{bx^2}{a}\right)+6\operatorname{PolyLog}\left(4,1+\frac{bx^2}{a}\right)\right)$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x,x]
```

```
[Out] Log[x]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^3 + 3*p*(-(p*Log[a + b*
x^2]) + Log[c*(a + b*x^2)^p])^2*(Log[x]*(Log[a + b*x^2] - Log[1 + (b*x^2)/a
]) - PolyLog[2, -((b*x^2)/a)]/2) - (3*p^2*(p*Log[a + b*x^2] - Log[c*(a + b*
x^2)^p])*(Log[-((b*x^2)/a)]*Log[a + b*x^2]^2 + 2*Log[a + b*x^2]*PolyLog[2,
1 + (b*x^2)/a] - 2*PolyLog[3, 1 + (b*x^2)/a]))/2 + (p^3*(Log[-((b*x^2)/a)]*
Log[a + b*x^2]^3 + 3*Log[a + b*x^2]^2*PolyLog[2, 1 + (b*x^2)/a] - 6*Log[a +
 b*x^2]*PolyLog[3, 1 + (b*x^2)/a] + 6*PolyLog[4, 1 + (b*x^2)/a]))/2
```

Maple [F]

$$\int \frac{\ln\left(c(b\,x^2+a)^p\right)^3}{x}dx$$

```
[In] int(ln(c*(b*x^2+a)^p)^3/x,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3/x,x)
```

Fricas [F]

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x}\,dx = \int \frac{\log\left((bx^2+a)^pc\right)^3}{x}\,dx$$

[In] integrate(log(c*(b*x^2+a)^p)^3/x,x, algorithm="fricas")

[Out] integral(log((b*x^2 + a)^p*c)^3/x, x)

Sympy [F]

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)^3}{x}\,dx$$

[In] integrate(ln(c*(b*x**2+a)**p)**3/x,x)

[Out] Integral(log(c*(a + b*x**2)**p)**3/x, x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(101) = 202.

Time = 0.21 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.05

$$\begin{aligned}&\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x}\,dx\\&=\frac{1}{2}\left(\log\left(bx^2+a\right)^3\log\left(-\frac{bx^2+a}{a}+1\right)+3\,\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\log\left(bx^2+a\right)^2-6\,\log\left(bx^2+a\right)\mathrm{Li}_3\left(\frac{bx^2+a}{a}\right)\right.\\&\quad+\frac{3}{2}\left(\log\left(bx^2+a\right)^2\log\left(-\frac{bx^2+a}{a}+1\right)+2\,\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\log\left(bx^2+a\right)-2\,\mathrm{Li}_3\left(\frac{bx^2+a}{a}\right)\right)p^2\log\left(c\right)\\&\quad+\frac{3}{2}\left(\log\left(bx^2+a\right)\log\left(-\frac{bx^2+a}{a}+1\right)+\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\right)p\log\left(c\right)^2+\log\left(c\right)^3\log\left(x\right)\end{aligned}$$

[In] integrate(log(c*(b*x^2+a)^p)^3/x,x, algorithm="maxima")

[Out] 1/2*(log(b*x^2 + a)^3*log(-(b*x^2 + a)/a + 1) + 3*dilog((b*x^2 + a)/a)*log(b*x^2 + a)^2 - 6*log(b*x^2 + a)*polylog(3, (b*x^2 + a)/a) + 6*polylog(4, (b*x^2 + a)/a))*p^3 + 3/2*(log(b*x^2 + a)^2*log(-(b*x^2 + a)/a + 1) + 2*dilog((b*x^2 + a)/a)*log(b*x^2 + a) - 2*polylog(3, (b*x^2 + a)/a))*p^2*log(c) + 3/2*(log(b*x^2 + a)*log(-(b*x^2 + a)/a + 1) + dilog((b*x^2 + a)/a))*p*log(c)^2 + log(c)^3*log(x)

Giac [F]

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^3}{x}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x,x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x}\,dx = \int \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}{x}\,dx$$

```
[In] int(log(c*(a + b*x^2)^p)^3/x,x)
[Out] int(log(c*(a + b*x^2)^p)^3/x, x)
```

3.95 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 119

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^3}\,dx=\frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c(a+bx^2)^p\right)}{2a}-\frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2ax^2}+\frac{3bp^2\log\left(c(a+bx^2)^p\right)\operatorname{PolyLog}\left(2,1+\frac{bx^2}{a}\right)}{a}-\frac{3bp^3\operatorname{PolyLog}\left(3,1+\frac{bx^2}{a}\right)}{a}$$

```
[Out] 3/2*b*p*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)^2/a-1/2*(b*x^2+a)*ln(c*(b*x^2+a)^p)^
3/a/x^2+3*b*p^2*ln(c*(b*x^2+a)^p)*polylog(2,1+b*x^2/a)/a-3*b*p^3*polylog(3,
1+b*x^2/a)/a
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2504, 2444, 2443, 2481, 2421, 6724}

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^3}\,dx=\frac{3bp^2\operatorname{PolyLog}\left(2,\frac{bx^2}{a}+1\right)\log\left(c(a+bx^2)^p\right)}{a}-\frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2ax^2}+\frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c(a+bx^2)^p\right)}{2a}-\frac{3bp^3\operatorname{PolyLog}\left(3,\frac{bx^2}{a}+1\right)}{a}$$

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x^3,x]
```

```
[Out] (3*b*p*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p]^2)/(2*a) - ((a + b*x^2)*Log[c
*(a + b*x^2)^p]^3)/(2*a*x^2) + (3*b*p^2*Log[c*(a + b*x^2)^p]*PolyLog[2, 1 +
 (b*x^2)/a])/a - (3*b*p^3*PolyLog[3, 1 + (b*x^2)/a])/a
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\log^3\left(c(a+bx)^p\right)}{x^2}\,dx,x,x^2\right)\\
&= -\frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2ax^2}+\frac{(3bp)\text{Subst}\left(\int \frac{\log^2(c(a+bx)^p)}{x}\,dx,x,x^2\right)}{2a}\\
&= \frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c(a+bx^2)^p\right)}{2a}-\frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2ax^2}\\
&\quad-\frac{(3b^2p^2)\,\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{a+bx}\,dx,x,x^2\right)}{a}\\
&= \frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c(a+bx^2)^p\right)}{2a}-\frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2ax^2}\\
&\quad-\frac{(3bp^2)\,\text{Subst}\left(\int \frac{\log(cx^p)\log\left(-\frac{b\left(-\frac{a}{b}+\frac{x}{b}\right)}{a}\right)}{x}\,dx,x,a+bx^2\right)}{a}\\
&= \frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c(a+bx^2)^p\right)}{2a}-\frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2ax^2}\\
&\quad+\frac{3bp^2\log\left(c(a+bx^2)^p\right)\text{Li}_2\left(1+\frac{bx^2}{a}\right)}{a}-\frac{(3bp^3)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{x}{a}\right)}{x}\,dx,x,a+bx^2\right)}{a}\\
&= \frac{3bp\log\left(-\frac{bx^2}{a}\right)\log^2\left(c(a+bx^2)^p\right)}{2a}-\frac{(a+bx^2)\log^3\left(c(a+bx^2)^p\right)}{2ax^2}\\
&\quad+\frac{3bp^2\log\left(c(a+bx^2)^p\right)\text{Li}_2\left(1+\frac{bx^2}{a}\right)}{a}-\frac{3bp^3\text{Li}_3\left(1+\frac{bx^2}{a}\right)}{a}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 302 vs. $2(119) = 238$.

Time = 0.24 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.54

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^3}\,dx =$$
$$-\frac{-6bp^3x^2\log(x)\log^2\left(a+bx^2\right)+3bp^3x^2\log\left(-\frac{bx^2}{a}\right)\log^2\left(a+bx^2\right)+bp^3x^2\log^3\left(a+bx^2\right)+12bp^2x^2\log(x)}{}$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x^3,x]
```

```
[Out] -1/2*(-6*b*p^3*x^2*Log[x]*Log[a + b*x^2]^2 + 3*b*p^3*x^2*Log[-((b*x^2)/a)]*
Log[a + b*x^2]^2 + b*p^3*x^2*Log[a + b*x^2]^3 + 12*b*p^2*x^2*Log[x]*Log[a +
 b*x^2]*Log[c*(a + b*x^2)^p] - 6*b*p^2*x^2*Log[-((b*x^2)/a)]*Log[a + b*x^2]
*Log[c*(a + b*x^2)^p] - 3*b*p^2*x^2*Log[a + b*x^2]^2*Log[c*(a + b*x^2)^p] -
 6*b*p*x^2*Log[x]*Log[c*(a + b*x^2)^p]^2 + 3*b*p*x^2*Log[a + b*x^2]*Log[c*(
a + b*x^2)^p]^2 + a*Log[c*(a + b*x^2)^p]^3 - 6*b*p^2*x^2*Log[c*(a + b*x^2)^
p]*PolyLog[2, 1 + (b*x^2)/a] + 6*b*p^3*x^2*PolyLog[3, 1 + (b*x^2)/a])/(a*x^
2)
```

Maple [F]

$$\int \frac{\ln\left(c(b\,x^2+a)^p\right)^3}{x^3}dx$$

```
[In] int(ln(c*(b*x^2+a)^p)^3/x^3,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3/x^3,x)
```

Fricas [F]

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^3}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^3}{x^3}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^3/x^3, x)
```

Sympy [F]

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^3}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)^3}{x^3}\,dx$$

[In] integrate(ln(c*(b*x**2+a)**p)**3/x**3,x)

[Out] Integral(log(c*(a + b*x**2)**p)**3/x**3, x)

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.70

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^3}\,dx$$
$$= \frac{1}{2}\left(\frac{3\left(\log\left(bx^2+a\right)^2\log\left(-\frac{bx^2+a}{a}+1\right)+2\operatorname{Li}_2\left(\frac{bx^2+a}{a}\right)\log\left(bx^2+a\right)-2\operatorname{Li}_3\left(\frac{bx^2+a}{a}\right)\right)p^2}{a}+\frac{6\left(\log\left(bx^2+a\right)\right.}{}\right.$$
$$-\frac{\log\left(\left(bx^2+a\right)^pc\right)^3}{2\,x^2}$$

[In] integrate(log(c*(b*x^2+a)^p)^3/x^3,x, algorithm="maxima")

[Out] 1/2*(3*(log(b*x^2 + a)^2*log(-(b*x^2 + a)/a + 1) + 2*dilog((b*x^2 + a)/a)*l
og(b*x^2 + a) - 2*polylog(3, (b*x^2 + a)/a))*p^2/a + 6*(log(b*x^2 + a)*log(
-(b*x^2 + a)/a + 1) + dilog((b*x^2 + a)/a))*p*log(c)/a + 6*log(c)^2*log(x)/
a - (p^2*log(b*x^2 + a)^3 + 3*p*log(b*x^2 + a)^2*log(c) + 3*log(b*x^2 + a)*
log(c)^2)/a)*b*p - 1/2*log((b*x^2 + a)^p*c)^3/x^2

Giac [F]

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^3}\,dx = \int \frac{\log\left(\left(bx^2+a\right)^pc\right)^3}{x^3}\,dx$$

[In] integrate(log(c*(b*x^2+a)^p)^3/x^3,x, algorithm="giac")

[Out] integrate(log((b*x^2 + a)^p*c)^3/x^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^3}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)^3}{x^3}\,dx$$

```
[In] int(log(c*(a + b*x^2)^p)^3/x^3,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^3/x^3, x)
```

3.96 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^5} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 219

$$\begin{aligned}\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^5} \, dx = {} & \frac{3b^2p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2} \\ & - \frac{3bp\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{4a^2x^2} - \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{4x^4} \\ & - \frac{3b^2p\log^2\left(c\left(a+bx^2\right)^p\right)\log\left(1-\frac{a}{a+bx^2}\right)}{4a^2} \\ & + \frac{3b^2p^2\log\left(c\left(a+bx^2\right)^p\right)\operatorname{PolyLog}\left(2,\frac{a}{a+bx^2}\right)}{2a^2} \\ & + \frac{3b^2p^3\operatorname{PolyLog}\left(2,1+\frac{bx^2}{a}\right)}{2a^2} + \frac{3b^2p^3\operatorname{PolyLog}\left(3,\frac{a}{a+bx^2}\right)}{2a^2}\end{aligned}$$

```
[Out] 3/2*b^2*p^2*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)/a^2-3/4*b*p*(b*x^2+a)*ln(c*(b*x^
2+a)^p)^2/a^2/x^2-1/4*ln(c*(b*x^2+a)^p)^3/x^4-3/4*b^2*p*ln(c*(b*x^2+a)^p)^2
*ln(1-a/(b*x^2+a))/a^2+3/2*b^2*p^2*ln(c*(b*x^2+a)^p)*polylog(2,a/(b*x^2+a))
/a^2+3/2*b^2*p^3*polylog(2,1+b*x^2/a)/a^2+3/2*b^2*p^3*polylog(3,a/(b*x^2+a)
)/a^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438}

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^5}\,dx = \frac{3b^2p^2\operatorname{PolyLog}\left(2,\frac{a}{bx^2+a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2} + \frac{3b^2p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2a^2} - \frac{3b^2p\log\left(1-\frac{a}{a+bx^2}\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{4a^2} + \frac{3b^2p^3\operatorname{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2a^2} + \frac{3b^2p^3\operatorname{PolyLog}\left(3,\frac{a}{bx^2+a}\right)}{2a^2} - \frac{3bp\left(a+bx^2\right)\log^2\left(c\left(a+bx^2\right)^p\right)}{4a^2x^2} - \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{4x^4}$$

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x^5,x]
```

```
[Out] (3*b^2*p^2*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(2*a^2) - (3*b*p*(a + b*
x^2)*Log[c*(a + b*x^2)^p]^2)/(4*a^2*x^2) - Log[c*(a + b*x^2)^p]^3/(4*x^4) -
 (3*b^2*p*Log[c*(a + b*x^2)^p]^2*Log[1 - a/(a + b*x^2)])/(4*a^2) + (3*b^2*p
^2*Log[c*(a + b*x^2)^p]*PolyLog[2, a/(a + b*x^2)])/(2*a^2) + (3*b^2*p^3*Pol
yLog[2, 1 + (b*x^2)/a])/(2*a^2) + (3*b^2*p^3*PolyLog[3, a/(a + b*x^2)])/(2*
a^2)
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
```

```
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\log^3\left(c(a+bx)^p\right)}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\log^3\left(c(a+bx^2)^p\right)}{4x^4} + \frac{1}{4}(3bp)\text{Subst}\left(\int \frac{\log^2\left(c(a+bx)^p\right)}{x^2(a+bx)}\,dx, x, x^2\right)\\
&= -\frac{\log^3\left(c(a+bx^2)^p\right)}{4x^4} + \frac{1}{4}(3p)\text{Subst}\left(\int \frac{\log^2\left(cx^p\right)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx^2\right)\\
&= -\frac{\log^3\left(c(a+bx^2)^p\right)}{4x^4} + \frac{(3p)\text{Subst}\left(\int \frac{\log^2(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx^2\right)}{4a}\\
&\quad - \frac{(3bp)\text{Subst}\left(\int \frac{\log^2(cx^p)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)}\,dx, x, a+bx^2\right)}{4a}\\
&= -\frac{3bp(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{4a^2x^2} - \frac{\log^3\left(c(a+bx^2)^p\right)}{4x^4}\\
&\quad - \frac{3b^2p\log^2\left(c(a+bx^2)^p\right)\log\left(1-\frac{a}{a+bx^2}\right)}{4a^2} + \frac{(3bp^2)\,\text{Subst}\left(\int \frac{\log(cx^p)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx^2\right)}{2a^2}\\
&\quad + \frac{(3b^2p^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{a}{x}\right)\log(cx^p)}{x}\,dx, x, a+bx^2\right)}{2a^2}\\
&= \frac{3b^2p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2a^2} - \frac{3bp(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{4a^2x^2}\\
&\quad - \frac{\log^3\left(c(a+bx^2)^p\right)}{4x^4} - \frac{3b^2p\log^2\left(c(a+bx^2)^p\right)\log\left(1-\frac{a}{a+bx^2}\right)}{4a^2} + \frac{3b^2p^2\log\left(c(a+bx^2)^p\right)\text{Li}_2\left(\frac{a}{a+bx^2}\right)}{2a^2}\\
&\quad - \frac{(3b^2p^3)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{x}{a}\right)}{x}\,dx, x, a+bx^2\right)}{2a^2} - \frac{(3b^2p^3)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{a}{x}\right)}{x}\,dx, x, a+bx^2\right)}{2a^2}\\
&= \frac{3b^2p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2a^2} - \frac{3bp(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{4a^2x^2}\\
&\quad - \frac{\log^3\left(c(a+bx^2)^p\right)}{4x^4} - \frac{3b^2p\log^2\left(c(a+bx^2)^p\right)\log\left(1-\frac{a}{a+bx^2}\right)}{4a^2}\\
&\quad + \frac{3b^2p^2\log\left(c(a+bx^2)^p\right)\text{Li}_2\left(\frac{a}{a+bx^2}\right)}{2a^2} + \frac{3b^2p^3\text{Li}_2\left(1+\frac{bx^2}{a}\right)}{2a^2} + \frac{3b^2p^3\text{Li}_3\left(\frac{a}{a+bx^2}\right)}{2a^2}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 477 vs. $2(219) = 438$.

Time = 0.28 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.18

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^5}\,dx = -\frac{3bp\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^2}{4ax^2} - \frac{3b^2p\log(x)\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^2}{2a^2} + \frac{3b^2p\log\left(a+bx^2\right)\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^2}{4a^2} - \frac{3p\log\left(a+bx^2\right)\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^2}{4x^4} - \frac{\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^3}{4x^4} + 3p^2\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)\left(-\frac{\log^2\left(a+bx^2\right)}{4x^4} + \frac{b\left(4bx^2\log(x)+\log\left(a+bx^2\right)\left(-2(a+bx^2)-2bx^2\log\left(-\frac{bx^2}{a}\right)+bx^2\log\left(a+bx^2\right)\right)-2bx^2\,\mathrm{PolyLog}\left(2\right.\right.}{4a^2x^2}\right. + \frac{b^2p^3\left(\frac{(a+bx^2)\left(a\left(3-2\log(a+bx^2)\right)+(a+bx^2)\left(-3+\log(a+bx^2)\right)\right)\log^2(a+bx^2)}{b^2x^4} - 3\left(-2+\log\left(a+bx^2\right)\right)\log\left(a+bx^2\right)\log\left(1\right.\right.}{4a^2}$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x^5,x]
```

```
[Out] (-3*b*p*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2)/(4*a*x^2) - (3*b^2*
p*Log[x]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2)/(2*a^2) + (3*b^2*p
*Log[a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2)/(4*a^2) - (
3*p*Log[a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2)/(4*x^4)
- (-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^3/(4*x^4) + 3*p^2*(-(p*Log[a
 + b*x^2]) + Log[c*(a + b*x^2)^p])*(-1/4*Log[a + b*x^2]^2/x^4 + (b*(4*b*x^2
*Log[x] + Log[a + b*x^2]*(-2*(a + b*x^2) - 2*b*x^2*Log[-((b*x^2)/a)] + b*x^
2*Log[a + b*x^2]) - 2*b*x^2*PolyLog[2, 1 + (b*x^2)/a]))/(4*a^2*x^2)) + (b^2
*p^3*(((a + b*x^2)*(a*(3 - 2*Log[a + b*x^2]) + (a + b*x^2)*(-3 + Log[a + b*
x^2]))*Log[a + b*x^2]^2)/(b^2*x^4) - 3*(-2 + Log[a + b*x^2])*Log[a + b*x^2]
*Log[1 - (a + b*x^2)/a] - 6*(-1 + Log[a + b*x^2])*PolyLog[2, (a + b*x^2)/a]
 + 6*PolyLog[3, (a + b*x^2)/a]))/(4*a^2)
```

Maple [F]

$$\int \frac{\ln \left(c(b\,x^2+a)^p\right)^3}{x^5}dx$$

[In] int(ln(c*(b*x^2+a)^p)^3/x^5,x)

[Out] int(ln(c*(b*x^2+a)^p)^3/x^5,x)

Fricas [F]

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^5}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^3}{x^5}\,dx$$

[In] integrate(log(c*(b*x^2+a)^p)^3/x^5,x, algorithm="fricas")

[Out] integral(log((b*x^2 + a)^p*c)^3/x^5, x)

Sympy [F]

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^5}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)^3}{x^5}\,dx$$

[In] integrate(ln(c*(b*x**2+a)**p)**3/x**5,x)

[Out] Integral(log(c*(a + b*x**2)**p)**3/x**5, x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.23

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^5}\,dx = -\frac{1}{4}\left(\frac{3\left(\log\left(bx^2+a\right)^2\log\left(-\frac{bx^2+a}{a}+1\right)+2\,\mathrm{Li}_2\left(\frac{bx^2+a}{a}\right)\log\left(bx^2+a\right)-2\,\mathrm{Li}_3\left(\frac{bx^2+a}{a}\right)\right)bp^2}{a^2} - \frac{6\left(p^2-p\log\left(\right.\right.}{}\right. - \frac{\log\left((bx^2+a)^p c\right)^3}{4\,x^4}$$

[In] integrate(log(c*(b*x^2+a)^p)^3/x^5,x, algorithm="maxima")

```
[Out] -1/4*(3*(log(b*x^2 + a)^2*log(-(b*x^2 + a)/a + 1) + 2*dilog((b*x^2 + a)/a)*
log(b*x^2 + a) - 2*polylog(3, (b*x^2 + a)/a))*b*p^2/a^2 - 6*(p^2 - p*log(c)
)*(log(b*x^2 + a)*log(-(b*x^2 + a)/a + 1) + dilog((b*x^2 + a)/a))*b/a^2 - 6
*(2*p*log(c) - log(c)^2)*b*log(x)/a^2 - (b*p^2*x^2*log(b*x^2 + a)^3 - 3*((p
^2 - p*log(c))*b*x^2 + a*p^2)*log(b*x^2 + a)^2 - 3*a*log(c)^2 - 3*((2*p*log
(c) - log(c)^2)*b*x^2 + 2*a*p*log(c))*log(b*x^2 + a))/(a^2*x^2))*b*p - 1/4*
log((b*x^2 + a)^p*c)^3/x^4
```

Giac [F]

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^5}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^3}{x^5}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^5,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^3/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^5}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)^3}{x^5}\,dx$$

```
[In] int(log(c*(a + b*x^2)^p)^3/x^5,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^3/x^5, x)
```

3.97 $\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^7}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 352

$$\begin{aligned}\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^7}\,dx &= \frac{b^3p^3\log(x)}{a^3} - \frac{b^2p^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2a^3x^2} \\ &\quad - \frac{b^3p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{a^3} - \frac{bp\log^2\left(c(a+bx^2)^p\right)}{4ax^4} \\ &\quad + \frac{b^2p(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2a^3x^2} - \frac{\log^3\left(c(a+bx^2)^p\right)}{6x^6} \\ &\quad - \frac{b^3p^2\log\left(c(a+bx^2)^p\right)\log\left(1-\frac{a}{a+bx^2}\right)}{2a^3} \\ &\quad + \frac{b^3p\log^2\left(c(a+bx^2)^p\right)\log\left(1-\frac{a}{a+bx^2}\right)}{2a^3} + \frac{b^3p^3\,\text{PolyLog}\left(2,\frac{a}{a+bx^2}\right)}{2a^3} \\ &\quad - \frac{b^3p^2\log\left(c(a+bx^2)^p\right)\text{PolyLog}\left(2,\frac{a}{a+bx^2}\right)}{a^3} \\ &\quad - \frac{b^3p^3\,\text{PolyLog}\left(2,1+\frac{bx^2}{a}\right)}{a^3} - \frac{b^3p^3\,\text{PolyLog}\left(3,\frac{a}{a+bx^2}\right)}{a^3}\end{aligned}$$

```
[Out] b^3*p^3*ln(x)/a^3-1/2*b^2*p^2*(b*x^2+a)*ln(c*(b*x^2+a)^p)/a^3/x^2-b^3*p^2*l
n(-b*x^2/a)*ln(c*(b*x^2+a)^p)/a^3-1/4*b*p*ln(c*(b*x^2+a)^p)^2/a/x^4+1/2*b^2
*p*(b*x^2+a)*ln(c*(b*x^2+a)^p)^2/a^3/x^2-1/6*ln(c*(b*x^2+a)^p)^3/x^6-1/2*b^
3*p^2*ln(c*(b*x^2+a)^p)*ln(1-a/(b*x^2+a))/a^3+1/2*b^3*p*ln(c*(b*x^2+a)^p)^2
*ln(1-a/(b*x^2+a))/a^3+1/2*b^3*p^3*polylog(2,a/(b*x^2+a))/a^3-b^3*p^2*ln(c*
(b*x^2+a)^p)*polylog(2,a/(b*x^2+a))/a^3-b^3*p^3*polylog(2,1+b*x^2/a)/a^3-b^
3*p^3*polylog(3,a/(b*x^2+a))/a^3
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31}

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^7}\,dx = -\frac{b^3p^2\operatorname{PolyLog}\left(2,\frac{a}{bx^2+a}\right)\log\left(c(a+bx^2)^p\right)}{a^3} - \frac{b^3p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{a^3} - \frac{b^3p^2\log\left(1-\frac{a}{a+bx^2}\right)\log\left(c(a+bx^2)^p\right)}{2a^3} + \frac{b^3p\log\left(1-\frac{a}{a+bx^2}\right)\log^2\left(c(a+bx^2)^p\right)}{2a^3} + \frac{b^3p^3\operatorname{PolyLog}\left(2,\frac{a}{bx^2+a}\right)}{2a^3} - \frac{b^3p^3\operatorname{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{a^3} - \frac{b^3p^3\operatorname{PolyLog}\left(3,\frac{a}{bx^2+a}\right)}{a^3} + \frac{b^3p^3\log(x)}{a^3} - \frac{b^2p^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2a^3x^2} + \frac{b^2p(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2a^3x^2} - \frac{\log^3\left(c(a+bx^2)^p\right)}{6x^6} - \frac{bp\log^2\left(c(a+bx^2)^p\right)}{4ax^4}$$

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x^7,x]
```

```
[Out] (b^3*p^3*Log[x])/a^3 - (b^2*p^2*(a + b*x^2)*Log[c*(a + b*x^2)^p])/(2*a^3*x^
2) - (b^3*p^2*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/a^3 - (b*p*Log[c*(a +
 b*x^2)^p]^2)/(4*a*x^4) + (b^2*p*(a + b*x^2)*Log[c*(a + b*x^2)^p]^2)/(2*a^3
*x^2) - Log[c*(a + b*x^2)^p]^3/(6*x^6) - (b^3*p^2*Log[c*(a + b*x^2)^p]*Log[
1 - a/(a + b*x^2)])/(2*a^3) + (b^3*p*Log[c*(a + b*x^2)^p]^2*Log[1 - a/(a +
b*x^2)])/(2*a^3) + (b^3*p^3*PolyLog[2, a/(a + b*x^2)])/(2*a^3) - (b^3*p^2*L
og[c*(a + b*x^2)^p]*PolyLog[2, a/(a + b*x^2)])/a^3 - (b^3*p^3*PolyLog[2, 1
+ (b*x^2)/a])/a^3 - (b^3*p^3*PolyLog[3, a/(a + b*x^2)])/a^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
```

```
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\log^3\left(c(a+bx)^p\right)}{x^4}\,dx, x, x^2\right)\\
&= -\frac{\log^3\left(c(a+bx^2)^p\right)}{6x^6} + \frac{1}{2}(bp)\text{Subst}\left(\int \frac{\log^2\left(c(a+bx)^p\right)}{x^3(a+bx)}\,dx, x, x^2\right)\\
&= -\frac{\log^3\left(c(a+bx^2)^p\right)}{6x^6} + \frac{1}{2}p\text{Subst}\left(\int \frac{\log^2\left(cx^p\right)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx^2\right)
\end{aligned}
$$

$$= -\frac{\log^3\left(c(a+bx^2)^p\right)}{6x^6} + \frac{p\text{Subst}\left(\int \frac{\log^2(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^3}\,dx, x, a+bx^2\right)}{2a} - \frac{(bp)\text{Subst}\left(\int \frac{\log^2(cx^p)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx^2\right)}{2a}$$

$$= -\frac{bp\log^2\left(c(a+bx^2)^p\right)}{4ax^4} - \frac{\log^3\left(c(a+bx^2)^p\right)}{6x^6} - \frac{(bp)\text{Subst}\left(\int \frac{\log^2(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx^2\right)}{2a^2} + \frac{(b^2p)\,\text{Subst}\left(\int \frac{\log^2(cx^p)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)}\,dx, x, a+bx^2\right)}{2a^2} + \frac{(bp^2)\,\text{Subst}\left(\int \frac{\log(cx^p)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx^2\right)}{2a}$$

$$= -\frac{bp\log^2\left(c(a+bx^2)^p\right)}{4ax^4} + \frac{b^2p(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2a^3x^2} - \frac{\log^3\left(c(a+bx^2)^p\right)}{6x^6} + \frac{b^3p\log^2\left(c(a+bx^2)^p\right)\log\left(1-\frac{a}{a+bx^2}\right)}{2a^3} + \frac{(bp^2)\,\text{Subst}\left(\int \frac{\log(cx^p)}{\left(-\frac{a}{b}+\frac{x}{b}\right)^2}\,dx, x, a+bx^2\right)}{2a^2} - \frac{(b^2p^2)\,\text{Subst}\left(\int \frac{\log(cx^p)}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx^2\right)}{a^3} - \frac{(b^2p^2)\,\text{Subst}\left(\int \frac{\log(cx^p)}{x\left(-\frac{a}{b}+\frac{x}{b}\right)}\,dx, x, a+bx^2\right)}{2a^2} - \frac{(b^3p^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{a}{x}\right)\log(cx^p)}{x}\,dx, x, a+bx^2\right)}{a^3}$$

$$= -\frac{b^2p^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2a^3x^2} - \frac{b^3p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{a^3} - \frac{bp\log^2\left(c(a+bx^2)^p\right)}{4ax^4} + \frac{b^2p(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2a^3x^2} - \frac{\log^3\left(c(a+bx^2)^p\right)}{6x^6} - \frac{b^3p^2\log\left(c(a+bx^2)^p\right)\log\left(1-\frac{a}{a+bx^2}\right)}{2a^3} + \frac{b^3p\log^2\left(c(a+bx^2)^p\right)\log\left(1-\frac{a}{a+bx^2}\right)}{2a^3} - \frac{b^3p^2\log\left(c(a+bx^2)^p\right)\text{Li}_2\left(\frac{a}{a+bx^2}\right)}{a^3} + \frac{(b^2p^3)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x}{b}}\,dx, x, a+bx^2\right)}{2a^3} + \frac{(b^3p^3)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{a}{x}\right)}{x}\,dx, x, a+bx^2\right)}{2a^3} + \frac{(b^3p^3)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{x}{a}\right)}{x}\,dx, x, a+bx^2\right)}{a^3} + \frac{(b^3p^3)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{a}{x}\right)}{x}\,dx, x, a+bx^2\right)}{a^3}$$

$$
\begin{aligned}
&= \frac{b^3p^3\log(x)}{a^3} - \frac{b^2p^2(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2a^3x^2} - \frac{b^3p^2\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{a^3} \\
&\quad - \frac{bp\log^2\left(c(a+bx^2)^p\right)}{4ax^4} + \frac{b^2p(a+bx^2)\log^2\left(c(a+bx^2)^p\right)}{2a^3x^2} - \frac{\log^3\left(c(a+bx^2)^p\right)}{6x^6} \\
&\quad - \frac{b^3p^2\log\left(c(a+bx^2)^p\right)\log\left(1-\frac{a}{a+bx^2}\right)}{2a^3} + \frac{b^3p\log^2\left(c(a+bx^2)^p\right)\log\left(1-\frac{a}{a+bx^2}\right)}{2a^3} \\
&\quad + \frac{b^3p^3\mathrm{Li}_2\left(\frac{a}{a+bx^2}\right)}{2a^3} - \frac{b^3p^2\log\left(c(a+bx^2)^p\right)\mathrm{Li}_2\left(\frac{a}{a+bx^2}\right)}{a^3} - \frac{b^3p^3\mathrm{Li}_2\left(1+\frac{bx^2}{a}\right)}{a^3} \\
&\quad - \frac{b^3p^3\mathrm{Li}_3\left(\frac{a}{a+bx^2}\right)}{a^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.62

$$
\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^7}\,dx =
$$

$$
-\frac{-6b^3p^3x^6\log\left(-\frac{bx^2}{a}\right)+6b^3p^3x^6\log\left(a+bx^2\right)-36b^3p^3x^6\log(x)\log\left(a+bx^2\right)+18b^3p^3x^6\log\left(-\frac{bx^2}{a}\right)\log}{}
$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x^7,x]
```

```
[Out] -1/12*(-6*b^3*p^3*x^6*Log[-((b*x^2)/a)] + 6*b^3*p^3*x^6*Log[a + b*x^2] - 36
*b^3*p^3*x^6*Log[x]*Log[a + b*x^2] + 18*b^3*p^3*x^6*Log[-((b*x^2)/a)]*Log[a
 + b*x^2] + 9*b^3*p^3*x^6*Log[a + b*x^2]^2 - 12*b^3*p^3*x^6*Log[x]*Log[a +
b*x^2]^2 + 6*b^3*p^3*x^6*Log[-((b*x^2)/a)]*Log[a + b*x^2]^2 + 2*b^3*p^3*x^6
*Log[a + b*x^2]^3 + 6*a*b^2*p^2*x^4*Log[c*(a + b*x^2)^p] + 36*b^3*p^2*x^6*L
og[x]*Log[c*(a + b*x^2)^p] - 18*b^3*p^2*x^6*Log[a + b*x^2]*Log[c*(a + b*x^2
)^p] + 24*b^3*p^2*x^6*Log[x]*Log[a + b*x^2]*Log[c*(a + b*x^2)^p] - 12*b^3*p
^2*x^6*Log[-((b*x^2)/a)]*Log[a + b*x^2]*Log[c*(a + b*x^2)^p] - 6*b^3*p^2*x^
6*Log[a + b*x^2]^2*Log[c*(a + b*x^2)^p] + 3*a^2*b*p*x^2*Log[c*(a + b*x^2)^p
]^2 - 6*a*b^2*p*x^4*Log[c*(a + b*x^2)^p]^2 - 12*b^3*p*x^6*Log[x]*Log[c*(a +
 b*x^2)^p]^2 + 6*b^3*p*x^6*Log[a + b*x^2]*Log[c*(a + b*x^2)^p]^2 + 2*a^3*Lo
g[c*(a + b*x^2)^p]^3 + 6*b^3*p^2*x^6*(3*p - 2*Log[c*(a + b*x^2)^p])*PolyLog
[2, 1 + (b*x^2)/a] + 12*b^3*p^3*x^6*PolyLog[3, 1 + (b*x^2)/a])/(a^3*x^6)
```

Maple [F]

$$\int \frac{\ln \left(c(b\,x^{2}+a)^{p}\right)^{3}}{x^{7}}dx$$

```
[In] int(ln(c*(b*x^2+a)^p)^3/x^7,x)
[Out] int(ln(c*(b*x^2+a)^p)^3/x^7,x)
```

Fricas [F]

$$\int \frac{\log^{3}\left(c(a+bx^{2})^{p}\right)}{x^{7}}\,dx = \int \frac{\log\left((bx^{2}+a)^{p}c\right)^{3}}{x^{7}}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^7,x, algorithm="fricas")
[Out] integral(log((b*x^2 + a)^p*c)^3/x^7, x)
```

Sympy [F]

$$\int \frac{\log^{3}\left(c(a+bx^{2})^{p}\right)}{x^{7}}\,dx = \int \frac{\log\left(c(a+bx^{2})^{p}\right)^{3}}{x^{7}}\,dx$$

```
[In] integrate(ln(c*(b*x**2+a)**p)**3/x**7,x)
[Out] Integral(log(c*(a + b*x**2)**p)**3/x**7, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.96

$$\int \frac{\log^{3}\left(c(a+bx^{2})^{p}\right)}{x^{7}}\,dx = \frac{1}{12}\left(\frac{6\left(\log\left(bx^{2}+a\right)^{2}\log\left(-\frac{bx^{2}+a}{a}+1\right)+2\,\mathrm{Li}_{2}\left(\frac{bx^{2}+a}{a}\right)\log\left(bx^{2}+a\right)-2\,\mathrm{Li}_{3}\left(\frac{bx^{2}+a}{a}\right)\right)b^{2}p^{2}}{a^{3}} - \frac{6\left(3\,p^{2}-2\,p\,\mathrm{lo}\right.}{}\right. - \frac{\log\left((bx^{2}+a)^{p}c\right)^{3}}{6\,x^{6}}$$

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^7,x, algorithm="maxima")
```

```
[Out] 1/12*(6*(log(b*x^2 + a)^2*log(-(b*x^2 + a)/a + 1) + 2*dilog((b*x^2 + a)/a)*
log(b*x^2 + a) - 2*polylog(3, (b*x^2 + a)/a))*b^2*p^2/a^3 - 6*(3*p^2 - 2*p*
log(c))*(log(b*x^2 + a)*log(-(b*x^2 + a)/a + 1) + dilog((b*x^2 + a)/a))*b^2
/a^3 + 12*(p^2 - 3*p*log(c) + log(c)^2)*b^2*log(x)/a^3 - (2*b^2*p^2*x^4*log
(b*x^2 + a)^3 + 6*(p*log(c) - log(c)^2)*a*b*x^2 + 3*a^2*log(c)^2 - 3*((3*p^
2 - 2*p*log(c))*b^2*x^4 + 2*a*b*p^2*x^2 - a^2*p^2)*log(b*x^2 + a)^2 + 6*((p
^2 - 3*p*log(c) + log(c)^2)*b^2*x^4 + (p^2 - 2*p*log(c))*a*b*x^2 + a^2*p*lo
g(c))*log(b*x^2 + a))/(a^3*x^4))*b*p - 1/6*log((b*x^2 + a)^p*c)^3/x^6
```

Giac [F]

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^7}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^3}{x^7}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^7,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^3/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^7}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)^3}{x^7}\,dx$$

```
[In] int(log(c*(a + b*x^2)^p)^3/x^7,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)^3/x^7, x)
```

3.98 $\int x^2 \log^3\left(c(a+bx^2)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^2 \log^3\left(c(a+bx^2)^p\right)\,dx$$
$$= \frac{208ap^3x}{9b} - \frac{16p^3x^3}{27} - \frac{208a^{3/2}p^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}} + \frac{32ia^{3/2}p^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}}$$
$$+ \frac{64a^{3/2}p^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3b^{3/2}} - \frac{32ap^2x\log\left(c(a+bx^2)^p\right)}{3b}$$
$$+ \frac{8}{9}p^2x^3\log\left(c(a+bx^2)^p\right) + \frac{32a^{3/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{3b^{3/2}} + \frac{2apx\log^2\left(c(a+bx^2)^p\right)}{b} - \frac{2}{3}px^3\log^2\left(c\right.$$

```
[Out] 208/9*a*p^3*x/b-16/27*p^3*x^3-208/9*a^(3/2)*p^3*arctan(x*b^(1/2)/a^(1/2))/b
^(3/2)+32/3*I*a^(3/2)*p^3*arctan(x*b^(1/2)/a^(1/2))^2/b^(3/2)-32/3*a*p^2*x*
ln(c*(b*x^2+a)^p)/b+8/9*p^2*x^3*ln(c*(b*x^2+a)^p)+32/3*a^(3/2)*p^2*arctan(x
*b^(1/2)/a^(1/2))*ln(c*(b*x^2+a)^p)/b^(3/2)+2*a*p*x*ln(c*(b*x^2+a)^p)^2/b-2
/3*p*x^3*ln(c*(b*x^2+a)^p)^2+1/3*x^3*ln(c*(b*x^2+a)^p)^3+64/3*a^(3/2)*p^3*a
rctan(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/b^(3/2)+32/3*I
*a^(3/2)*p^3*polylog(2,1-2*a^(1/2)/(a^(1/2)+I*x*b^(1/2)))/b^(3/2)-2*a^2*p*U
nintegrable(ln(c*(b*x^2+a)^p)^2/(b*x^2+a),x)/b
```

Rubi [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx = \int x^2 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$$

```
[In] Int[x^2*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (208*a*p^3*x)/(9*b) - (16*p^3*x^3)/27 - (208*a^(3/2)*p^3*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(9*b^(3/2)) + (((32*I)/3)*a^(3/2)*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]
]^2)/b^(3/2) + (64*a^(3/2)*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/
(Sqrt[a] + I*Sqrt[b]*x)])/(3*b^(3/2)) - (32*a*p^2*x*Log[c*(a + b*x^2)^p])/(
3*b) + (8*p^2*x^3*Log[c*(a + b*x^2)^p])/9 + (32*a^(3/2)*p^2*ArcTan[(Sqrt[b]
*x)/Sqrt[a]]*Log[c*(a + b*x^2)^p])/(3*b^(3/2)) + (2*a*p*x*Log[c*(a + b*x^2)
^p]^2)/b - (2*p*x^3*Log[c*(a + b*x^2)^p]^2)/3 + (x^3*Log[c*(a + b*x^2)^p]^3
)/3 + (((32*I)/3)*a^(3/2)*p^3*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[
b]*x)])/b^(3/2) - (2*a^2*p*Defer[Int][Log[c*(a + b*x^2)^p]^2/(a + b*x^2), x
])/b
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\log^3\left(c\left(a+bx^2\right)^p\right) - (2bp)\int \frac{x^4\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx \\
&= \frac{1}{3}x^3\log^3\left(c\left(a+bx^2\right)^p\right) - (2bp)\int \left(-\frac{a\log^2\left(c\left(a+bx^2\right)^p\right)}{b^2} + \frac{x^2\log^2\left(c\left(a+bx^2\right)^p\right)}{b} \right.\\
&\qquad \left. + \frac{a^2\log^2\left(c\left(a+bx^2\right)^p\right)}{b^2\left(a+bx^2\right)}\right)dx \\
&= \frac{1}{3}x^3\log^3\left(c\left(a+bx^2\right)^p\right) - (2p)\int x^2\log^2\left(c\left(a+bx^2\right)^p\right)\,dx \\
&\quad + \frac{(2ap)\int \log^2\left(c\left(a+bx^2\right)^p\right)\,dx}{b} - \frac{\left(2a^2p\right)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx}{b} \\
&= \frac{2apx\log^2\left(c\left(a+bx^2\right)^p\right)}{b} - \frac{2}{3}px^3\log^2\left(c\left(a+bx^2\right)^p\right) \\
&\quad + \frac{1}{3}x^3\log^3\left(c\left(a+bx^2\right)^p\right) - \frac{\left(2a^2p\right)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx}{b} \\
&\quad - \left(8ap^2\right)\int \frac{x^2\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx + \frac{1}{3}\left(8bp^2\right)\int \frac{x^4\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx
\end{aligned}$$

$$= \frac{2apx\log^2\left(c\left(a+bx^2\right)^p\right)}{b} - \frac{2}{3}px^3\log^2\left(c\left(a+bx^2\right)^p\right) + \frac{1}{3}x^3\log^3\left(c\left(a+bx^2\right)^p\right)$$
$$- \frac{\left(2a^2p\right)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx}{b} - \left(8ap^2\right)\int\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{b}\right.$$
$$\left.- \frac{a\log\left(c\left(a+bx^2\right)^p\right)}{b\left(a+bx^2\right)}\right)dx + \frac{1}{3}\left(8bp^2\right)\int\left(-\frac{a\log\left(c\left(a+bx^2\right)^p\right)}{b^2}\right.$$
$$\left.+ \frac{x^2\log\left(c\left(a+bx^2\right)^p\right)}{b} + \frac{a^2\log\left(c\left(a+bx^2\right)^p\right)}{b^2\left(a+bx^2\right)}\right)dx$$

$$= \frac{2apx\log^2\left(c\left(a+bx^2\right)^p\right)}{b} - \frac{2}{3}px^3\log^2\left(c\left(a+bx^2\right)^p\right) + \frac{1}{3}x^3\log^3\left(c\left(a+bx^2\right)^p\right)$$
$$- \frac{\left(2a^2p\right)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx}{b} + \frac{1}{3}\left(8p^2\right)\int x^2\log\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$- \frac{\left(8ap^2\right)\int\log\left(c\left(a+bx^2\right)^p\right)\,dx}{3b} - \frac{\left(8ap^2\right)\int\log\left(c\left(a+bx^2\right)^p\right)\,dx}{b}$$
$$+ \frac{\left(8a^2p^2\right)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx}{3b} + \frac{\left(8a^2p^2\right)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx}{b}$$

$$= -\frac{32ap^2x\log\left(c\left(a+bx^2\right)^p\right)}{3b} + \frac{8}{9}p^2x^3\log\left(c\left(a+bx^2\right)^p\right)$$
$$+ \frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{3b^{3/2}} + \frac{2apx\log^2\left(c\left(a+bx^2\right)^p\right)}{b}$$
$$- \frac{2}{3}px^3\log^2\left(c\left(a+bx^2\right)^p\right) + \frac{1}{3}x^3\log^3\left(c\left(a+bx^2\right)^p\right)$$
$$- \frac{\left(2a^2p\right)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx}{b} + \frac{1}{3}\left(16ap^3\right)\int\frac{x^2}{a+bx^2}\,dx$$
$$+ \left(16ap^3\right)\int\frac{x^2}{a+bx^2}\,dx - \frac{1}{3}\left(16a^2p^3\right)\int\frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\left(a+bx^2\right)}\,dx$$
$$- \left(16a^2p^3\right)\int\frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\left(a+bx^2\right)}\,dx - \frac{1}{9}\left(16bp^3\right)\int\frac{x^4}{a+bx^2}\,dx$$

$$= \frac{64ap^3x}{3b} - \frac{32ap^2x\log\left(c(a+bx^2)^p\right)}{3b} + \frac{8}{9}p^2x^3\log\left(c(a+bx^2)^p\right) + \frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{3b^{3/2}} + \frac{2apx\log^2\left(c(a+bx^2)^p\right)}{b} - \frac{2}{3}px^3\log^2\left(c(a+bx^2)^p\right) + \frac{1}{3}x^3\log^3\left(c(a+bx^2)^p\right) - \frac{(2a^2p)\int\frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2}\,dx}{b} - \frac{(16a^2p^3)\int\frac{1}{a+bx^2}\,dx}{3b} - \frac{(16a^2p^3)\int\frac{1}{a+bx^2}\,dx}{b} - \frac{(16a^{3/2}p^3)\int\frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a+bx^2}\,dx}{3\sqrt{b}} - \frac{(16a^{3/2}p^3)\int\frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a+bx^2}\,dx}{\sqrt{b}} - \frac{1}{9}(16bp^3)\int\left(-\frac{a}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2(a+bx^2)}\right)dx$$

$$= \frac{208ap^3x}{9b} - \frac{16p^3x^3}{27} - \frac{64a^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}} + \frac{32ia^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}} - \frac{32ap^2x\log\left(c(a+bx^2)^p\right)}{3b} + \frac{8}{9}p^2x^3\log\left(c(a+bx^2)^p\right) + \frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{3b^{3/2}} + \frac{2apx\log^2\left(c(a+bx^2)^p\right)}{b} - \frac{2}{3}p$$

$$= \frac{208ap^3x}{9b} - \frac{16p^3x^3}{27} - \frac{208a^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}} + \frac{32ia^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}} + \frac{64a^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3b^{3/2}} - \frac{32ap^2x\log\left(c(a+bx^2)^p\right)}{3b}$$

$$+\frac{8}{9}p^2x^3\log\left(c(a+bx^2)^p\right) + \frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{3b^{3/2}} + \frac{2apx\log^2\left(c(a+bx^2)^p\right)}{b} - \frac{2}{3}p$$

$$= \frac{208ap^3x}{9b} - \frac{16p^3x^3}{27} - \frac{208a^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}} + \frac{32ia^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}} + \frac{64a^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3b^{3/2}} - \frac{32ap^2x\log\left(c(a+bx^2)^p\right)}{3b}$$

$$+\frac{8}{9}p^2x^3\log\left(c(a+bx^2)^p\right) + \frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{3b^{3/2}} + \frac{2apx\log^2\left(c(a+bx^2)^p\right)}{b} - \frac{2}{3}p$$

$$= \frac{208ap^3x}{9b} - \frac{16p^3x^3}{27} - \frac{208a^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{9b^{3/2}} + \frac{32ia^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{3b^{3/2}}$$
$$+ \frac{64a^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{3b^{3/2}} - \frac{32ap^2x\log\left(c(a+bx^2)^p\right)}{3b}$$
$$+ \frac{8}{9}p^2x^3\log\left(c(a+bx^2)^p\right) + \frac{32a^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{3b^{3/2}} + \frac{2apx\log^2\left(c(a+bx^2)^p\right)}{b} - \frac{2}{3}px^3$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 909 vs. 2(380) = 760.

Time = 3.13 (sec) , antiderivative size = 909, normalized size of antiderivative = 50.50

$$\int x^2\log^3\left(c(a+bx^2)^p\right)\,dx = \frac{2apx\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^2}{b}$$
$$- \frac{2a^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^2}{b^{3/2}}$$
$$+ px^3\log\left(a+bx^2\right)\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^2$$
$$+ \frac{1}{3}x^3\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^2\left(-2p-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)$$
$$+3p^2\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)\left(\frac{1}{3}x^3\log^2\left(a+bx^2\right) - \frac{4\left(9ia^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2 + 3a^{3/2}\arctan\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{}\right.$$

```
[In] Integrate[x^2*Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (2*a*p*x*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2)/b - (2*a^(3/2)*p*A
rcTan[(Sqrt[b]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2)/
b^(3/2) + p*x^3*Log[a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])
^2 + (x^3*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2*(-2*p - p*Log[a +
b*x^2] + Log[c*(a + b*x^2)^p]))/3 + 3*p^2*(-(p*Log[a + b*x^2]) + Log[c*(a +
 b*x^2)^p])*((x^3*Log[a + b*x^2]^2)/3 - (4*((9*I)*a^(3/2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]]^2 + 3*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-8 + 6*Log[(2*Sqrt[a]
)/(Sqrt[a] + I*Sqrt[b]*x)] + 3*Log[a + b*x^2]) + Sqrt[b]*x*(24*a - 2*b*x^2
+ (-9*a + 3*b*x^2)*Log[a + b*x^2]) + (9*I)*a^(3/2)*PolyLog[2, (I*Sqrt[a] +
Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)]))/(27*b^(3/2))) + (p^3*(416*Sqrt[-a]
*a^(3/2)*Sqrt[(b*x^2)/(a + b*x^2)]*Sqrt[a + b*x^2]*ArcSin[Sqrt[a]/Sqrt[a +
b*x^2]] + (2*Sqrt[-a]*b*x^2*(624*a - 16*b*x^2 + (-288*a + 24*b*x^2)*Log[a +
 b*x^2] + 18*(3*a - b*x^2)*Log[a + b*x^2]^2 + 9*b*x^2*Log[a + b*x^2]^3))/3
+ 36*Sqrt[-a]*a^(3/2)*Sqrt[(b*x^2)/(a + b*x^2)]*(8*Sqrt[a]*HypergeometricPF
Q[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, a/(a + b*x^2)] + Log[a + b*x^2]*(4
```

```
*Sqrt[a]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, a/(a + b*x^2)] + Sq
rt[a + b*x^2]*ArcSin[Sqrt[a]/Sqrt[a + b*x^2]]*Log[a + b*x^2])) - 48*a^2*(4*
Sqrt[b*x^2]*ArcTanh[Sqrt[b*x^2]/Sqrt[-a]]*(Log[a + b*x^2] - Log[1 + (b*x^2)
/a]) - Sqrt[-a]*Sqrt[-((b*x^2)/a)]*(Log[1 + (b*x^2)/a]^2 - 4*Log[1 + (b*x^2
)/a]*Log[(1 + Sqrt[-((b*x^2)/a)])/2] + 2*Log[(1 + Sqrt[-((b*x^2)/a)])/2]^2
- 4*PolyLog[2, 1/2 - Sqrt[-((b*x^2)/a)]/2]))))/(18*Sqrt[-a]*b^2*x)
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^2 \ln\left(c\left(b\,x^2+a\right)^p\right)^3 dx$$

```
[In] int(x^2*ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] int(x^2*ln(c*(b*x^2+a)^p)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^2 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx = \int x^2 \log\left(\left(bx^2+a\right)^p c\right)^3\,dx$$

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2*log((b*x^2 + a)^p*c)^3, x)
```

Sympy [N/A]

Not integrable

Time = 3.88 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int x^2 \log^3\left(c\left(a+bx^2\right)^p\right)\,dx = \int x^2 \log\left(c\left(a+bx^2\right)^p\right)^3 dx$$

```
[In] integrate(x**2*ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(x**2*log(c*(a + b*x**2)**p)**3, x)
```

Maxima [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 6.83

$$\int x^2 \log^3 \left(c(a+bx^2)^p\right) \, dx = \int x^2 \log \left((bx^2+a)^p c\right)^3 \, dx$$

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] 1/3*p^3*x^3*log(b*x^2 + a)^3 + integrate((b*x^4*log(c)^3 + a*x^2*log(c)^3 -
 ((2*p^3 - 3*p^2*log(c))*b*x^4 - 3*a*p^2*x^2*log(c))*log(b*x^2 + a)^2 + 3*(
b*p*x^4*log(c)^2 + a*p*x^2*log(c)^2)*log(b*x^2 + a))/(b*x^2 + a), x)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^2 \log^3 \left(c(a+bx^2)^p\right) \, dx = \int x^2 \log \left((bx^2+a)^p c\right)^3 \, dx$$

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*log((b*x^2 + a)^p*c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^2 \log^3 \left(c(a+bx^2)^p\right) \, dx = \int x^2 \ln \left(c\left(b\,x^2+a\right)^p\right)^3 \, dx$$

```
[In] int(x^2*log(c*(a + b*x^2)^p)^3,x)
```

```
[Out] int(x^2*log(c*(a + b*x^2)^p)^3, x)
```

3.99 $\int \log^3\left(c(a+bx^2)^p\right) dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\begin{aligned}\int \log^3\left(c(a+bx^2)^p\right) dx &= -48p^3x+\frac{48\sqrt{a}p^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}-\frac{24i\sqrt{a}p^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{b}}\\&\quad-\frac{48\sqrt{a}p^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}+24p^2x\log\left(c(a+bx^2)^p\right)\\&\quad-\frac{24\sqrt{a}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{b}}-6px\log^2\left(c(a+bx^2)^p\right)\\&\quad+x\log^3\left(c(a+bx^2)^p\right)-\frac{24i\sqrt{a}p^3\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}}\\&\quad+6ap\operatorname{Int}\left(\frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2},x\right)\end{aligned}$$

```
[Out] -48*p^3*x+24*p^2*x*ln(c*(b*x^2+a)^p)-6*p*x*ln(c*(b*x^2+a)^p)^2+x*ln(c*(b*x^
2+a)^p)^3+48*p^3*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(1/2)-24*I*p^3*arctan(
x*b^(1/2)/a^(1/2))^2*a^(1/2)/b^(1/2)-24*p^2*arctan(x*b^(1/2)/a^(1/2))*ln(c*
(b*x^2+a)^p)*a^(1/2)/b^(1/2)-48*p^3*arctan(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)/
(a^(1/2)+I*x*b^(1/2)))*a^(1/2)/b^(1/2)-24*I*p^3*polylog(2,1-2*a^(1/2)/(a^(1
/2)+I*x*b^(1/2)))*a^(1/2)/b^(1/2)+6*a*p*Unintegrable(ln(c*(b*x^2+a)^p)^2/(b
*x^2+a),x)
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \log^3\left(c\left(a+bx^2\right)^p\right)\,dx = \int \log^3\left(c\left(a+bx^2\right)^p\right)\,dx$$

```
[In] Int[Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] -48*p^3*x + (48*Sqrt[a]*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] - ((24*I)*
Sqrt[a]*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/Sqrt[b] - (48*Sqrt[a]*p^3*ArcTan
[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/Sqrt[b] + 2
4*p^2*x*Log[c*(a + b*x^2)^p] - (24*Sqrt[a]*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*
Log[c*(a + b*x^2)^p])/Sqrt[b] - 6*p*x*Log[c*(a + b*x^2)^p]^2 + x*Log[c*(a +
 b*x^2)^p]^3 - ((24*I)*Sqrt[a]*p^3*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*
Sqrt[b]*x)])/Sqrt[b] + 6*a*p*Defer[Int][Log[c*(a + b*x^2)^p]^2/(a + b*x^2),
 x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log^3\left(c\left(a+bx^2\right)^p\right)-(6bp)\int \frac{x^2\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx\\
&= x\log^3\left(c\left(a+bx^2\right)^p\right)-(6bp)\int \left(\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{b}-\frac{a\log^2\left(c\left(a+bx^2\right)^p\right)}{b\left(a+bx^2\right)}\right)dx\\
&= x\log^3\left(c\left(a+bx^2\right)^p\right)-(6p)\int \log^2\left(c\left(a+bx^2\right)^p\right)\,dx+(6ap)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx\\
&= -6px\log^2\left(c\left(a+bx^2\right)^p\right)+x\log^3\left(c\left(a+bx^2\right)^p\right)\\
&\quad +(6ap)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx+\left(24bp^2\right)\int \frac{x^2\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx\\
&= -6px\log^2\left(c\left(a+bx^2\right)^p\right)+x\log^3\left(c\left(a+bx^2\right)^p\right)+(6ap)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx\\
&\quad +\left(24bp^2\right)\int \left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{b}-\frac{a\log\left(c\left(a+bx^2\right)^p\right)}{b\left(a+bx^2\right)}\right)dx\\
&= -6px\log^2\left(c\left(a+bx^2\right)^p\right)+x\log^3\left(c\left(a+bx^2\right)^p\right)+(6ap)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx\\
&\quad +\left(24p^2\right)\int \log\left(c\left(a+bx^2\right)^p\right)\,dx-\left(24ap^2\right)\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx
\end{aligned}$$

$$= 24p^2 x \log\left(c\left(a+bx^2\right)^p\right) - \frac{24\sqrt{a}p^2 \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{b}}$$
$$- 6px\log^2\left(c\left(a+bx^2\right)^p\right) + x\log^3\left(c\left(a+bx^2\right)^p\right) + (6ap)\int \frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2}\,dx$$
$$- \left(48bp^3\right)\int \frac{x^2}{a+bx^2}\,dx + \left(48abp^3\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,(a+bx^2)}\,dx$$

$$= -48p^3 x + 24p^2 x\log\left(c\left(a+bx^2\right)^p\right) - \frac{24\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{b}}$$
$$- 6px\log^2\left(c\left(a+bx^2\right)^p\right) + x\log^3\left(c\left(a+bx^2\right)^p\right) + (6ap)\int \frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2}\,dx$$
$$+ \left(48ap^3\right)\int \frac{1}{a+bx^2}\,dx + \left(48\sqrt{a}\sqrt{b}p^3\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a+bx^2}\,dx$$

$$= -48p^3 x + \frac{48\sqrt{a}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} - \frac{24i\sqrt{a}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{b}} + 24p^2 x\log\left(c\left(a+bx^2\right)^p\right)$$
$$- \frac{24\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{b}} - 6px\log^2\left(c\left(a+bx^2\right)^p\right)$$
$$+ x\log^3\left(c\left(a+bx^2\right)^p\right) + (6ap)\int \frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2}\,dx - \left(48p^3\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{i-\frac{\sqrt{b}x}{\sqrt{a}}}\,dx$$

$$= -48p^3 x + \frac{48\sqrt{a}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} - \frac{24i\sqrt{a}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{b}}$$
$$- \frac{48\sqrt{a}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}} + 24p^2 x\log\left(c\left(a+bx^2\right)^p\right)$$
$$- \frac{24\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{\sqrt{b}} - 6px\log^2\left(c\left(a+bx^2\right)^p\right)$$
$$+ x\log^3\left(c\left(a+bx^2\right)^p\right) + (6ap)\int \frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2}\,dx + \left(48p^3\right)\int \frac{\log\left(\frac{2}{1+\frac{i\sqrt{b}x}{\sqrt{a}}}\right)}{1+\frac{bx^2}{a}}\,dx$$

$$
\begin{aligned}
&= -48p^3x + \frac{48\sqrt{a}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} - \frac{24i\sqrt{a}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{b}} \\
&\quad - \frac{48\sqrt{a}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}} + 24p^2x\log\left(c\left(a+bx^2\right)^p\right) \\
&\quad - \frac{24\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}} - 6px\log^2\left(c\left(a+bx^2\right)^p\right) \\
&\quad + x\log^3\left(c\left(a+bx^2\right)^p\right) + (6ap)\int\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx \\
&\quad - \frac{\left(48i\sqrt{a}p^3\right)\mathrm{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+\frac{i\sqrt{b}x}{\sqrt{a}}}\right)}{\sqrt{b}}
\end{aligned}
$$

$$
\begin{aligned}
&= -48p^3x + \frac{48\sqrt{a}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} - \frac{24i\sqrt{a}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{b}} \\
&\quad - \frac{48\sqrt{a}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}} + 24p^2x\log\left(c\left(a+bx^2\right)^p\right) \\
&\quad - \frac{24\sqrt{a}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^p\right)}{\sqrt{b}} - 6px\log^2\left(c\left(a+bx^2\right)^p\right) \\
&\quad + x\log^3\left(c\left(a+bx^2\right)^p\right) - \frac{24i\sqrt{a}p^3\mathrm{Li}_2\left(1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{b}} + (6ap)\int\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 789 vs. $2(290) = 580$.

Time = 2.67 (sec) , antiderivative size = 789, normalized size of antiderivative = 56.36

$$
\begin{aligned}
&\int\log^3\left(c\left(a+bx^2\right)^p\right)\,dx = \frac{6\sqrt{a}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-p\log\left(a+bx^2\right)+\log\left(c\left(a+bx^2\right)^p\right)\right)^2}{\sqrt{b}} \\
&\quad + 3px\log\left(a+bx^2\right)\left(-p\log\left(a+bx^2\right)+\log\left(c\left(a+bx^2\right)^p\right)\right)^2 \\
&\quad + x\left(-p\log\left(a+bx^2\right)+\log\left(c\left(a+bx^2\right)^p\right)\right)^2\left(-6p-p\log\left(a+bx^2\right)+\log\left(c\left(a+bx^2\right)^p\right)\right) \\
&\quad - \frac{3p^2\left(p\log\left(a+bx^2\right)-\log\left(c\left(a+bx^2\right)^p\right)\right)\left(4i\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2+4\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-2+2\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}\right.\right.\right.}{\sqrt{b}} \\
&\quad + \frac{p^3\left(-48\sqrt{-a^2}\sqrt{\frac{bx^2}{a+bx^2}}\sqrt{a+bx^2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+bx^2}}\right)+\sqrt{-a}bx^2\left(-48+24\log\left(a+bx^2\right)-6\log^2\left(a+bx^2\right)+\right.\right.}{}
\end{aligned}
$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] (6*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a +
b*x^2)^p])^2)/Sqrt[b] + 3*p*x*Log[a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(
a + b*x^2)^p])^2 + x*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2*(-6*p -
 p*Log[a + b*x^2] + Log[c*(a + b*x^2)^p]) - (3*p^2*(p*Log[a + b*x^2] - Log[
c*(a + b*x^2)^p])*((4*I)*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + 4*Sqrt[a]*
ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-2 + 2*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)
] + Log[a + b*x^2]) + Sqrt[b]*x*(8 - 4*Log[a + b*x^2] + Log[a + b*x^2]^2) +
 (4*I)*Sqrt[a]*PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x
)]))/Sqrt[b] + (p^3*(-48*Sqrt[-a^2]*Sqrt[(b*x^2)/(a + b*x^2)]*Sqrt[a + b*x^
2]*ArcSin[Sqrt[a]/Sqrt[a + b*x^2]] + Sqrt[-a]*b*x^2*(-48 + 24*Log[a + b*x^2
] - 6*Log[a + b*x^2]^2 + Log[a + b*x^2]^3) - 6*Sqrt[-a^2]*Sqrt[(b*x^2)/(a +
 b*x^2)]*(8*Sqrt[a]*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}
, a/(a + b*x^2)] + Log[a + b*x^2]*(4*Sqrt[a]*HypergeometricPFQ[{1/2, 1/2, 1
/2}, {3/2, 3/2}, a/(a + b*x^2)] + Sqrt[a + b*x^2]*ArcSin[Sqrt[a]/Sqrt[a + b
*x^2]]*Log[a + b*x^2])) + 24*a*Sqrt[b*x^2]*ArcTanh[Sqrt[b*x^2]/Sqrt[-a]]*(L
og[a + b*x^2] - Log[1 + (b*x^2)/a]) + 6*(-a)^(3/2)*Sqrt[-((b*x^2)/a)]*(Log[
1 + (b*x^2)/a]^2 - 4*Log[1 + (b*x^2)/a]*Log[(1 + Sqrt[-((b*x^2)/a)])/2] + 2
*Log[(1 + Sqrt[-((b*x^2)/a)])/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[-((b*x^2)/a)]/
2])))/(Sqrt[-a]*b*x)
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \ln \left(c\left(b\,x^{2}+a\right)^{p}\right)^{3}dx$$

```
[In] int(ln(c*(b*x^2+a)^p)^3,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \log^{3}\left(c\left(a+bx^{2}\right)^{p}\right)\,dx=\int \log\left(\left(bx^{2}+a\right)^{p}c\right)^{3}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^3, x)
```

Sympy [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \log^3\left(c(a+bx^2)^p\right)\,dx = \int \log\left(c(a+bx^2)^p\right)^3\,dx$$

```
[In] integrate(ln(c*(b*x**2+a)**p)**3,x)
[Out] Integral(log(c*(a + b*x**2)**p)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 111, normalized size of antiderivative = 7.93

$$\int \log^3\left(c(a+bx^2)^p\right)\,dx = \int \log\left((bx^2+a)^p c\right)^3\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
[Out] p^3*x*log(b*x^2 + a)^3 + integrate((b*x^2*log(c)^3 + a*log(c)^3 - 3*((2*p^3
 - p^2*log(c))*b*x^2 - a*p^2*log(c))*log(b*x^2 + a)^2 + 3*(b*p*x^2*log(c)^2
 + a*p*log(c)^2)*log(b*x^2 + a))/(b*x^2 + a), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \log^3\left(c(a+bx^2)^p\right)\,dx = \int \log\left((bx^2+a)^p c\right)^3\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \log^3\left(c\left(a+bx^2\right)^p\right)\,dx=\int \ln\left(c\,\left(b\,x^2+a\right)^p\right)^3\,dx$$

```
[In] int(log(c*(a + b*x^2)^p)^3,x)
[Out] int(log(c*(a + b*x^2)^p)^3, x)
```

3.100 $\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx=-\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x}+6bp\text{Int}\left(\frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2},x\right)$$

```
[Out] -ln(c*(b*x^2+a)^p)^3/x+6*b*p*Unintegrable(ln(c*(b*x^2+a)^p)^2/(b*x^2+a),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx=\int \frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx$$

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x^2,x]
```

```
[Out] -(Log[c*(a + b*x^2)^p]^3/x) + 6*b*p*Defer[Int][Log[c*(a + b*x^2)^p]^2/(a + b*x^2), x]
```

Rubi steps

$$\text{integral}=-\frac{\log^3\left(c\left(a+bx^2\right)^p\right)}{x}+(6bp)\int \frac{\log^2\left(c\left(a+bx^2\right)^p\right)}{a+bx^2}\,dx$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 505, normalized size of antiderivative = 28.06

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^2}\,dx$$
$$= \frac{p^3\left(-96\sqrt{a}\sqrt{1-\frac{a}{a+bx^2}}\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2},\frac{3}{2};\frac{a}{a+bx^2}\right)-48\sqrt{a}\sqrt{1-\frac{a}{a+bx^2}}\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};\frac{a}{a+bx^2}\right)\log\left(a+b\right.}{2\sqrt{a}x}$$
$$+\frac{6\sqrt{b}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^2}{\sqrt{a}}$$
$$-\frac{3p\log\left(a+bx^2\right)\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^2}{x}$$
$$-\frac{\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^3}{x}$$
$$+3p^2\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)\left(-\frac{\log^2\left(a+bx^2\right)}{x}\right.$$
$$\left.+\frac{4\sqrt{b}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(i\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+2\log\left(\frac{2i}{i-\frac{\sqrt{b}x}{\sqrt{a}}}\right)+\log\left(a+bx^2\right)\right)+i\,\text{PolyLog}\left(2,\frac{i\sqrt{a}+\sqrt{b}x}{-i\sqrt{a}+\sqrt{b}x}\right)\right)}{\sqrt{a}}\right)$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x^2,x]
```

```
[Out] (p^3*(-96*Sqrt[a]*Sqrt[1 - a/(a + b*x^2)]*HypergeometricPFQ[{1/2, 1/2, 1/2,
 1/2}, {3/2, 3/2, 3/2}, a/(a + b*x^2)] - 48*Sqrt[a]*Sqrt[1 - a/(a + b*x^2)]
*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, a/(a + b*x^2)]*Log[a + b*x^
2] - 2*Log[a + b*x^2]^2*(6*Sqrt[a + b*x^2]*Sqrt[1 - a/(a + b*x^2)]*ArcSin[S
qrt[a]/Sqrt[a + b*x^2]] + Sqrt[a]*Log[a + b*x^2])))/(2*Sqrt[a]*x) + (6*Sqrt
[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^
p])^2)/Sqrt[a] - (3*p*Log[a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^
2)^p])^2)/x - (-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^3/x + 3*p^2*(-(p
*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])*(-(Log[a + b*x^2]^2/x) + (4*Sqrt[b
]*(ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(I*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 2*Log[(2*I)
/(I - (Sqrt[b]*x)/Sqrt[a])] + Log[a + b*x^2]) + I*PolyLog[2, (I*Sqrt[a] + S
qrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)]))/Sqrt[a])
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\ln \left(c(b\,x^2+a)^p\right)^3}{x^2}dx$$

```
[In] int(ln(c*(b*x^2+a)^p)^3/x^2,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^2}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^3}{x^2}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^p*c)^3/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 2.95 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^2}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)^3}{x^2}\,dx$$

```
[In] integrate(ln(c*(b*x**2+a)**p)**3/x**2,x)
```

```
[Out] Integral(log(c*(a + b*x**2)**p)**3/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^2}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^3}{x^2}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^2,x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)^3/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^2}\,dx = \int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^3}{x^2}\,dx$$

```
[In] int(log(c*(a + b*x^2)^p)^3/x^2,x)
[Out] int(log(c*(a + b*x^2)^p)^3/x^2, x)
```

3.101 $\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\begin{aligned}\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^4}\,dx =& \frac{8ib^{3/2}p^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{a^{3/2}}+\frac{16b^{3/2}p^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{a^{3/2}}\\&+\frac{8b^{3/2}p^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{a^{3/2}}\\&-\frac{2bp\log^2\left(c(a+bx^2)^p\right)}{ax}-\frac{\log^3\left(c(a+bx^2)^p\right)}{3x^3}\\&+\frac{8ib^{3/2}p^3\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{a^{3/2}}-\frac{2b^2p\,\mathrm{Int}\left(\frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2},x\right)}{a}\end{aligned}$$

```
[Out] 8*I*b^(3/2)*p^3*arctan(x*b^(1/2)/a^(1/2))^2/a^(3/2)+8*b^(3/2)*p^2*arctan(x*
b^(1/2)/a^(1/2))*ln(c*(b*x^2+a)^p)/a^(3/2)-2*b*p*ln(c*(b*x^2+a)^p)^2/a/x-1/
3*ln(c*(b*x^2+a)^p)^3/x^3+16*b^(3/2)*p^3*arctan(x*b^(1/2)/a^(1/2))*ln(2*a^(
1/2)/(a^(1/2)+I*x*b^(1/2)))/a^(3/2)+8*I*b^(3/2)*p^3*polylog(2,1-2*a^(1/2)/(
a^(1/2)+I*x*b^(1/2)))/a^(3/2)-2*b^2*p*Unintegrable(ln(c*(b*x^2+a)^p)^2/(b*x
^2+a),x)/a
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^4}\,dx = \int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^4}\,dx$$

```
[In] Int[Log[c*(a + b*x^2)^p]^3/x^4,x]
```

```
[Out] ((8*I)*b^(3/2)*p^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/a^(3/2) + (16*b^(3/2)*p^3
*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/a^(3
/2) + (8*b^(3/2)*p^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^p])/a^(3
/2) - (2*b*p*Log[c*(a + b*x^2)^p]^2)/(a*x) - Log[c*(a + b*x^2)^p]^3/(3*x^3)
 + ((8*I)*b^(3/2)*p^3*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/
a^(3/2) - (2*b^2*p*Defer[Int][Log[c*(a + b*x^2)^p]^2/(a + b*x^2), x])/a
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\log^3\left(c(a+bx^2)^p\right)}{3x^3} + (2bp)\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^2\left(a+bx^2\right)}\,dx \\
&= -\frac{\log^3\left(c(a+bx^2)^p\right)}{3x^3} + (2bp)\int \left(\frac{\log^2\left(c(a+bx^2)^p\right)}{ax^2} - \frac{b\log^2\left(c(a+bx^2)^p\right)}{a\left(a+bx^2\right)}\right)dx \\
&= -\frac{\log^3\left(c(a+bx^2)^p\right)}{3x^3} + \frac{(2bp)\int \frac{\log^2\left(c(a+bx^2)^p\right)}{x^2}\,dx}{a} - \frac{(2b^2p)\int \frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2}\,dx}{a} \\
&= -\frac{2bp\log^2\left(c(a+bx^2)^p\right)}{ax} - \frac{\log^3\left(c(a+bx^2)^p\right)}{3x^3} \\
&\quad - \frac{(2b^2p)\int \frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2}\,dx}{a} + \frac{(8b^2p^2)\int \frac{\log\left(c(a+bx^2)^p\right)}{a+bx^2}\,dx}{a} \\
&= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{a^{3/2}} - \frac{2bp\log^2\left(c(a+bx^2)^p\right)}{ax} \\
&\quad - \frac{\log^3\left(c(a+bx^2)^p\right)}{3x^3} - \frac{(2b^2p)\int \frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2}\,dx}{a} - \frac{(16b^3p^3)\int \frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(a+bx^2)}\,dx}{a} \\
&= \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{a^{3/2}} - \frac{2bp\log^2\left(c(a+bx^2)^p\right)}{ax} \\
&\quad - \frac{\log^3\left(c(a+bx^2)^p\right)}{3x^3} - \frac{(2b^2p)\int \frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2}\,dx}{a} - \frac{(16b^{5/2}p^3)\int \frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a+bx^2}\,dx}{a^{3/2}}
\end{aligned}
$$

$$= \frac{8ib^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{a^{3/2}} + \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{a^{3/2}} - \frac{2bp\log^2\left(c(a+bx^2)^p\right)}{ax} - \frac{\log^3\left(c(a+bx^2)^p\right)}{3x^3} - \frac{(2b^2p)\int\frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2}dx}{a} + \frac{(16b^2p^3)\int\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{i-\frac{\sqrt{b}x}{\sqrt{a}}}dx}{a^2}$$

$$= \frac{8ib^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{a^{3/2}} + \frac{16b^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{a^{3/2}} + \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{a^{3/2}} - \frac{2bp\log^2\left(c(a+bx^2)^p\right)}{ax} - \frac{\log^3\left(c(a+bx^2)^p\right)}{3x^3} - \frac{(2b^2p)\int\frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2}dx}{a} - \frac{(16b^2p^3)\int\frac{\log\left(\frac{2}{1+\frac{i\sqrt{b}x}{\sqrt{a}}}\right)}{1+\frac{bx^2}{a}}dx}{a^2}$$

$$= \frac{8ib^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{a^{3/2}} + \frac{16b^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{a^{3/2}} + \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{a^{3/2}} - \frac{2bp\log^2\left(c(a+bx^2)^p\right)}{ax} - \frac{\log^3\left(c(a+bx^2)^p\right)}{3x^3} - \frac{(2b^2p)\int\frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2}dx}{a} + \frac{(16ib^{3/2}p^3)\,\mathrm{Subst}\left(\int\frac{\log(2x)}{1-2x}dx, x, \frac{1}{1+\frac{i\sqrt{b}x}{\sqrt{a}}}\right)}{a^{3/2}}$$

$$= \frac{8ib^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{a^{3/2}} + \frac{16b^{3/2}p^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{a^{3/2}} + \frac{8b^{3/2}p^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^p\right)}{a^{3/2}} - \frac{2bp\log^2\left(c(a+bx^2)^p\right)}{ax} - \frac{\log^3\left(c(a+bx^2)^p\right)}{3x^3} + \frac{8ib^{3/2}p^3\mathrm{Li}_2\left(1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{a^{3/2}} - \frac{(2b^2p)\int\frac{\log^2\left(c(a+bx^2)^p\right)}{a+bx^2}dx}{a}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 851 vs. $2(254) = 508$.

Time = 2.19 (sec) , antiderivative size = 851, normalized size of antiderivative = 47.28

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^4}\,dx$$

$$= \frac{a^2\left(p\log\left(a+bx^2\right)-\log\left(c(a+bx^2)^p\right)\right)^3-6abpx^2\left(-p\log\left(a+bx^2\right)+\log\left(c(a+bx^2)^p\right)\right)^2-6\sqrt{a}b^{3/2}px^3\,\mathrm{ar}}{}$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]^3/x^4,x]
```

```
[Out] (a^2*(p*Log[a + b*x^2] - Log[c*(a + b*x^2)^p])^3 - 6*a*b*p*x^2*(-(p*Log[a +
 b*x^2]) + Log[c*(a + b*x^2)^p])^2 - 6*Sqrt[a]*b^(3/2)*p*x^3*ArcTan[(Sqrt[b
]*x)/Sqrt[a]]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2 - 3*a^2*p*Log[
a + b*x^2]*(-(p*Log[a + b*x^2]) + Log[c*(a + b*x^2)^p])^2 + 3*Sqrt[a]*p^2*(
p*Log[a + b*x^2] - Log[c*(a + b*x^2)^p])*(a^(3/2)*Log[a + b*x^2]^2 + 4*b*x^
2*(I*Sqrt[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2 + Sqrt[a]*Log[a + b*x^2] + Sqr
t[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(-2 + 2*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqr
t[b]*x)] + Log[a + b*x^2]) + I*Sqrt[b]*x*PolyLog[2, (I*Sqrt[a] + Sqrt[b]*x)
/((-I)*Sqrt[a] + Sqrt[b]*x)])) + p^3*(48*a*b*x^2*Sqrt[(b*x^2)/(a + b*x^2)]*
HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, a/(a + b*x^2)] + 2
4*Sqrt[-a]*(b*x^2)^(3/2)*ArcTanh[Sqrt[b*x^2]/Sqrt[-a]]*Log[a + b*x^2] + 24*
a*b*x^2*Sqrt[(b*x^2)/(a + b*x^2)]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2,
3/2}, a/(a + b*x^2)]*Log[a + b*x^2] - 6*a*b*x^2*Log[a + b*x^2]^2 + 6*Sqrt[a
]*((b*x^2)/(a + b*x^2))^(3/2)*(a + b*x^2)^(3/2)*ArcSin[Sqrt[a]/Sqrt[a + b*x
^2]]*Log[a + b*x^2]^2 - a^2*Log[a + b*x^2]^3 - 24*Sqrt[-a]*(b*x^2)^(3/2)*Ar
cTanh[Sqrt[b*x^2]/Sqrt[-a]]*Log[1 + (b*x^2)/a] - 6*a^2*(-((b*x^2)/a))^(3/2)
*Log[1 + (b*x^2)/a]^2 + 24*a^2*(-((b*x^2)/a))^(3/2)*Log[1 + (b*x^2)/a]*Log[
(1 + Sqrt[-((b*x^2)/a)])/2] - 12*a^2*(-((b*x^2)/a))^(3/2)*Log[(1 + Sqrt[-((
b*x^2)/a)])/2]^2 + 24*a^2*(-((b*x^2)/a))^(3/2)*PolyLog[2, 1/2 - Sqrt[-((b*x
^2)/a)]/2]))/(3*a^2*x^3)
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c(b\,x^2+a)^p\right)^3}{x^4}dx$$

```
[In] int(ln(c*(b*x^2+a)^p)^3/x^4,x)
```

```
[Out] int(ln(c*(b*x^2+a)^p)^3/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^4}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^3}{x^4}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^4,x, algorithm="fricas")
[Out] integral(log((b*x^2 + a)^p*c)^3/x^4, x)
```

Sympy [N/A]

Not integrable

Time = 4.76 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^4}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)^3}{x^4}\,dx$$

```
[In] integrate(ln(c*(b*x**2+a)**p)**3/x**4,x)
[Out] Integral(log(c*(a + b*x**2)**p)**3/x**4, x)
```

Maxima [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 117, normalized size of antiderivative = 6.50

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^4}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^3}{x^4}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^4,x, algorithm="maxima")
[Out] -1/3*p^3*log(b*x^2 + a)^3/x^3 + integrate((b*x^2*log(c)^3 + a*log(c)^3 + ((
2*p^3 + 3*p^2*log(c))*b*x^2 + 3*a*p^2*log(c))*log(b*x^2 + a)^2 + 3*(b*p*x^2
*log(c)^2 + a*p*log(c)^2)*log(b*x^2 + a))/(b*x^6 + a*x^4), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^4}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)^3}{x^4}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)^3/x^4,x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)^3/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\log^3\left(c(a+bx^2)^p\right)}{x^4}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)^3}{x^4}\,dx$$

```
[In] int(log(c*(a + b*x^2)^p)^3/x^4,x)
[Out] int(log(c*(a + b*x^2)^p)^3/x^4, x)
```

3.102 $\int \frac{x^3}{\log(c(a+bx^2)^p)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int \frac{x^3}{\log(c(a+bx^2)^p)}\,dx = -\frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)}{2b^2p} + \frac{(a+bx^2)^2\left(c(a+bx^2)^p\right)^{-2/p}\text{ExpIntegralEi}\left(\frac{2\log(c(a+bx^2)^p)}{p}\right)}{2b^2p}$$

```
[Out] -1/2*a*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b^2/p/((c*(b*x^2+a)^p)^(1/p))+1/2*
(b*x^2+a)^2*Ei(2*ln(c*(b*x^2+a)^p)/p)/b^2/p/((c*(b*x^2+a)^p)^(2/p))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2504, 2446, 2436, 2337, 2209, 2437, 2347}

$$\int \frac{x^3}{\log(c(a+bx^2)^p)}\,dx = \frac{(a+bx^2)^2\left(c(a+bx^2)^p\right)^{-2/p}\text{ExpIntegralEi}\left(\frac{2\log(c(bx^2+a)^p)}{p}\right)}{2b^2p} - \frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log(c(bx^2+a)^p)}{p}\right)}{2b^2p}$$

```
[In] Int[x^3/Log[c*(a + b*x^2)^p],x]
```

```
[Out] -1/2*(a*(a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(b^2*p*(c*(a + b
*x^2)^p)^p^(-1)) + ((a + b*x^2)^2*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/p]
)/(2*b^2*p*(c*(a + b*x^2)^p)^(2/p))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\log\left(c(a+bx)^p\right)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b\log\left(c(a+bx)^p\right)}+\frac{a+bx}{b\log\left(c(a+bx)^p\right)}\right)\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{a+bx}{\log(c(a+bx)^p)}\,dx, x, x^2\right)}{2b}-\frac{a\text{Subst}\left(\int \frac{1}{\log(c(a+bx)^p)}\,dx, x, x^2\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \frac{x}{\log(cx^p)}\,dx, x, a+bx^2\right)}{2b^2}-\frac{a\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, a+bx^2\right)}{2b^2}\\
&= \frac{\left((a+bx^2)^2\left(c(a+bx^2)^p\right)^{-2/p}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{p}}}{x}\,dx, x, \log\left(c(a+bx^2)^p\right)\right)}{2b^2p}\\
&\quad-\frac{\left(a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c(a+bx^2)^p\right)\right)}{2b^2p}\\
&= -\frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c(a+bx^2)^p\right)}{p}\right)}{2b^2p}\\
&\quad+\frac{(a+bx^2)^2\left(c(a+bx^2)^p\right)^{-2/p}\text{Ei}\left(\frac{2\log\left(c(a+bx^2)^p\right)}{p}\right)}{2b^2p}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90

$$
\int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = -\frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-2/p}\left(a\left(c(a+bx^2)^p\right)^{\frac{1}{p}}\text{ExpIntegralEi}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)-(a+bx^2)\text{ExpIntegralEi}\left(\right.\right.}{2b^2p}
$$

```
[In] Integrate[x^3/Log[c*(a + b*x^2)^p],x]
```

```
[Out] -1/2*((a + b*x^2)*(a*(c*(a + b*x^2)^p)^p^(-1)*ExpIntegralEi[Log[c*(a + b*x^
2)^p]/p] - (a + b*x^2)*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/p]))/(b^2*p*(
c*(a + b*x^2)^p)^(2/p))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.88 (sec) , antiderivative size = 547, normalized size of antiderivative = 5.11

method	result
risch	$-\frac{(bx^2+a)^2 c^{-\frac{2}{p}}\left((bx^2+a)^p\right)^{-\frac{2}{p}} e^{\frac{i\pi \operatorname{csgn}\left(ic\left(bx^2+a\right)^p\right)\left(-\operatorname{csgn}\left(ic\left(bx^2+a\right)^p\right)+\operatorname{csgn}(ic)\right)\left(-\operatorname{csgn}\left(ic\left(bx^2+a\right)^p\right)+\operatorname{csgn}\left(i\left(bx^2+a\right)^p\right)\right)}{p}} \operatorname{Ei}_1\left(\right.}{}$

```
[In] int(x^3/ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b^2/p*(b*x^2+a)^2*c^(-2/p)*((b*x^2+a)^p)^(-2/p)*exp(I*Pi*csgn(I*c*(b*x
^2+a)^p)*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*c))*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*
(b*x^2+a)^p))/p)*Ei(1,-2*ln(b*x^2+a)-(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*
x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*c
sgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln(
(b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)+1/2/b^2*a/p*(b*x^2+a)*c^(-1/p)*((b*x^2+a)^
p)^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)*(-csgn(I*c*(b*x^2+a)^p)+csgn(I
*c))*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*(b*x^2+a)^p))/p)*Ei(1,-ln(b*x^2+a)-1/2*
(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*
csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*
(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = -\frac{ac^{\left(\frac{1}{p}\right)}\,\mathrm{log\_integral}\left((bx^2+a)c^{\left(\frac{1}{p}\right)}\right) - \mathrm{log\_integral}\left((b^2x^4+2\,abx^2+a^2)c^{\frac{2}{p}}\right)}{2\,b^2c^{\frac{2}{p}}p}$$

```
[In] integrate(x^3/log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] -1/2*(a*c^(1/p)*log_integral((b*x^2 + a)*c^(1/p)) - log_integral((b^2*x^4 +
 2*a*b*x^2 + a^2)*c^(2/p)))/(b^2*c^(2/p)*p)
```

Sympy [F]

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

```
[In] integrate(x**3/ln(c*(b*x**2+a)**p),x)
[Out] Integral(x**3/log(c*(a + b*x**2)**p), x)
```

Maxima [F]

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^3}{\log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

```
[In] integrate(x^3/log(c*(b*x^2+a)^p),x, algorithm="maxima")
[Out] integrate(x^3/log((b*x^2 + a)^p*c), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = -\frac{a\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)}{2\,b^2c^{\left(\frac{1}{p}\right)}p} + \frac{\mathrm{Ei}\left(\frac{2\log(c)}{p}+2\,\log\left(bx^2+a\right)\right)}{2\,b^2c^{\frac{2}{p}}p}$$

```
[In] integrate(x^3/log(c*(b*x^2+a)^p),x, algorithm="giac")
[Out] -1/2*a*Ei(log(c)/p + log(b*x^2 + a))/(b^2*c^(1/p)*p) + 1/2*Ei(2*log(c)/p +
2*log(b*x^2 + a))/(b^2*c^(2/p)*p)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^3}{\ln\left(c\left(b\,x^2+a\right)^p\right)}\,dx$$

```
[In] int(x^3/log(c*(a + b*x^2)^p),x)
[Out] int(x^3/log(c*(a + b*x^2)^p), x)
```

3.103 $\int \frac{x}{\log(c(a+bx^2)^p)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 51

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{p}\right)}{2bp}$$

```
[Out] 1/2*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b/p/((c*(b*x^2+a)^p)^(1/p))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2504, 2436, 2337, 2209}

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c\left(bx^2+a\right)^p\right)}{p}\right)}{2bp}$$

```
[In] Int[x/Log[c*(a + b*x^2)^p],x]
```

```
[Out] ((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(2*b*p*(c*(a + b*x^2)^p
)^p^(-1))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\log\left(c(a+bx)^p\right)}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, a+bx^2\right)}{2b}\\
&= \frac{\left((a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c(a+bx^2)^p\right)\right)}{2bp}\\
&= \frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c(a+bx^2)^p\right)}{p}\right)}{2bp}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c(a+bx^2)^p\right)}{p}\right)}{2bp}$$

```
[In] Integrate[x/Log[c*(a + b*x^2)^p],x]
```

```
[Out] ((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(2*b*p*(c*(a + b*x^2)^p
)^p^(-1))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.95 (sec) , antiderivative size = 272, normalized size of antiderivative = 5.33

method	result
risch	$-\frac{(b\,x^2+a)\,c^{-\frac{1}{p}}\left((b\,x^2+a)^p\right)^{-\frac{1}{p}}\,\mathrm{e}^{\frac{i\pi\,\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)\left(-\,\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)+\mathrm{csgn}(ic)\right)\left(-\,\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)+\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\right)}{2p}}\,\mathrm{Ei}_1\left(-\right.}{}$

[In] int(x/ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)

[Out] -1/2/b/p*(b*x^2+a)*c^(-1/p)*((b*x^2+a)^p)^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*c))*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*(b*x^2+a)^p))/p)*Ei(1,-ln(b*x^2+a)-1/2*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \frac{\text{log\_integral}\left((bx^2+a)c^{\left(\frac{1}{p}\right)}\right)}{2\,bc^{\left(\frac{1}{p}\right)}p}$$

[In] integrate(x/log(c*(b*x^2+a)^p),x, algorithm="fricas")

[Out] 1/2*log_integral((b*x^2 + a)*c^(1/p))/(b*c^(1/p)*p)

Sympy [F]

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

[In] integrate(x/ln(c*(b*x**2+a)**p),x)

[Out] Integral(x/log(c*(a + b*x**2)**p), x)

Maxima [F]

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x}{\log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

```
[In] integrate(x/log(c*(b*x^2+a)^p),x, algorithm="maxima")
[Out] integrate(x/log((b*x^2 + a)^p*c), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \frac{\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)}{2\,bc^{\left(\frac{1}{p}\right)}p}$$

```
[In] integrate(x/log(c*(b*x^2+a)^p),x, algorithm="giac")
[Out] 1/2*Ei(log(c)/p + log(b*x^2 + a))/(b*c^(1/p)*p)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x}{\ln\left(c\left(b\,x^2+a\right)^p\right)}\,dx$$

```
[In] int(x/log(c*(a + b*x^2)^p),x)
[Out] int(x/log(c*(a + b*x^2)^p), x)
```

3.104 $\int \frac{1}{x \log(c(a+bx^2)^p)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x \log\left(c\left(a+bx^2\right)^p\right)} \, dx = \text{Int}\left(\frac{1}{x \log\left(c\left(a+bx^2\right)^p\right)}, x\right)$$

```
[Out] Unintegrable(1/x/ln(c*(b*x^2+a)^p),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x \log\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x \log\left(c\left(a+bx^2\right)^p\right)} \, dx$$

```
[In] Int[1/(x*Log[c*(a + b*x^2)^p]),x]
[Out] Defer[Int][1/(x*Log[c*(a + b*x^2)^p]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x \log\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \log \left(c \left(a+b x^{2}\right)^{p}\right)} d x=\int \frac{1}{x \log \left(c \left(a+b x^{2}\right)^{p}\right)} d x$$

```
[In] Integrate[1/(x*Log[c*(a + b*x^2)^p]),x]
[Out] Integrate[1/(x*Log[c*(a + b*x^2)^p]), x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \ln \left(c \left(b \,x^{2}+a \right)^{p}\right)}d x$$

```
[In] int(1/x/ln(c*(b*x^2+a)^p),x)
[Out] int(1/x/ln(c*(b*x^2+a)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \log \left(c \left(a+b x^{2}\right)^{p}\right)} d x=\int \frac{1}{x \log \left(\left(b x^{2}+a\right)^{p} c\right)} d x$$

```
[In] integrate(1/x/log(c*(b*x^2+a)^p),x, algorithm="fricas")
[Out] integral(1/(x*log((b*x^2 + a)^p*c)), x)
```

Sympy [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{x \log \left(c \left(a+b x^{2}\right)^{p}\right)} d x=\int \frac{1}{x \log \left(c \left(a+b x^{2}\right)^{p}\right)} d x$$

```
[In] integrate(1/x/ln(c*(b*x**2+a)**p),x)
[Out] Integral(1/(x*log(c*(a + b*x**2)**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x \log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

```
[In] integrate(1/x/log(c*(b*x^2+a)^p),x, algorithm="maxima")
[Out] integrate(1/(x*log((b*x^2 + a)^p*c)), x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x \log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

```
[In] integrate(1/x/log(c*(b*x^2+a)^p),x, algorithm="giac")
[Out] integrate(1/(x*log((b*x^2 + a)^p*c)), x)
```

Mupad [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x\,\ln\left(c\,\left(b\,x^2+a\right)^p\right)}\,dx$$

```
[In] int(1/(x*log(c*(a + b*x^2)^p)),x)
[Out] int(1/(x*log(c*(a + b*x^2)^p)), x)
```

3.105 $\int \frac{1}{x^3 \log\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3 \log\left(c\left(a+bx^2\right)^p\right)}\,dx = \operatorname{Int}\left(\frac{1}{x^3 \log\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^3/ln(c*(b*x^2+a)^p),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^3 \log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^3 \log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

```
[In] Int[1/(x^3*Log[c*(a + b*x^2)^p]),x]
[Out] Defer[Int][1/(x^3*Log[c*(a + b*x^2)^p]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^3 \log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log \left(c \left(a+b x^2\right)^p\right)} \, dx=\int \frac{1}{x^3 \log \left(c \left(a+b x^2\right)^p\right)} \, dx$$

```
[In] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]),x]
[Out] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]), x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3} \ln \left(c \left(b \,x^{2}+a \right)^{p}\right)}d x$$

```
[In] int(1/x^3/ln(c*(b*x^2+a)^p),x)
[Out] int(1/x^3/ln(c*(b*x^2+a)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log \left(c \left(a+b x^2\right)^p\right)} \, dx=\int \frac{1}{x^{3} \log \left({\left(b x^{2} + a\right)}^{p} c\right)} \,d x$$

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p),x, algorithm="fricas")
[Out] integral(1/(x^3*log((b*x^2 + a)^p*c)), x)
```

Sympy [N/A]

Not integrable

Time = 4.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3 \log \left(c \left(a+b x^2\right)^p\right)} \, dx=\int \frac{1}{x^{3} \log \left(c \left(a + b x^{2}\right)^{p}\right)}\, dx$$

```
[In] integrate(1/x**3/ln(c*(b*x**2+a)**p),x)
[Out] Integral(1/(x**3*log(c*(a + b*x**2)**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log \left(c \left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^3 \log \left(\left(bx^2+a\right)^p c\right)} \, dx$$

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p),x, algorithm="maxima")
[Out] integrate(1/(x^3*log((b*x^2 + a)^p*c)), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log \left(c \left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^3 \log \left(\left(bx^2+a\right)^p c\right)} \, dx$$

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p),x, algorithm="giac")
[Out] integrate(1/(x^3*log((b*x^2 + a)^p*c)), x)
```

Mupad [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log \left(c \left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^3 \, \ln \left(c \left(b \, x^2+a\right)^p\right)} \, dx$$

```
[In] int(1/(x^3*log(c*(a + b*x^2)^p)),x)
[Out] int(1/(x^3*log(c*(a + b*x^2)^p)), x)
```

3.106 $\int \frac{x^2}{\log(c(a+bx^2)^p)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \mathrm{Int}\left(\frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(x^2/ln(c*(b*x^2+a)^p),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

```
[In] Int[x^2/Log[c*(a + b*x^2)^p],x]
[Out] Defer[Int][x^2/Log[c*(a + b*x^2)^p], x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

```
[In] Integrate[x^2/Log[c*(a + b*x^2)^p],x]
[Out] Integrate[x^2/Log[c*(a + b*x^2)^p], x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\ln\left(c\left(b\,x^2+a\right)^p\right)}dx$$

```
[In] int(x^2/ln(c*(b*x^2+a)^p),x)
[Out] int(x^2/ln(c*(b*x^2+a)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^2}{\log\left(\left(bx^2+a\right)^pc\right)}\,dx$$

```
[In] integrate(x^2/log(c*(b*x^2+a)^p),x, algorithm="fricas")
[Out] integral(x^2/log((b*x^2 + a)^p*c), x)
```

Sympy [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx$$

```
[In] integrate(x**2/ln(c*(b*x**2+a)**p),x)
[Out] Integral(x**2/log(c*(a + b*x**2)**p), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^2}{\log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

```
[In] integrate(x^2/log(c*(b*x^2+a)^p),x, algorithm="maxima")
[Out] integrate(x^2/log((b*x^2 + a)^p*c), x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^2}{\log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

```
[In] integrate(x^2/log(c*(b*x^2+a)^p),x, algorithm="giac")
[Out] integrate(x^2/log((b*x^2 + a)^p*c), x)
```

Mupad [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^2}{\ln\left(c\left(b\,x^2+a\right)^p\right)}\,dx$$

```
[In] int(x^2/log(c*(a + b*x^2)^p),x)
[Out] int(x^2/log(c*(a + b*x^2)^p), x)
```

3.107 $\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)} \, dx = \text{Int}\left(\frac{1}{\log\left(c\left(a+bx^2\right)^p\right)}, x\right)$$

```
[Out] Unintegrable(1/ln(c*(b*x^2+a)^p),x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)} \, dx$$

```
[In] Int[Log[c*(a + b*x^2)^p]^(-1),x]
```

```
[Out] Defer[Int][Log[c*(a + b*x^2)^p]^(-1), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log \left(c \left(a+b x^2\right)^p\right)} \, dx=\int \frac{1}{\log \left(c \left(a+b x^2\right)^p\right)} \, dx$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]^(-1),x]
[Out] Integrate[Log[c*(a + b*x^2)^p]^(-1), x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\ln \left(c \left(b \,x^{2}+a \right)^{p}\right)}d x$$

```
[In] int(1/ln(c*(b*x^2+a)^p),x)
[Out] int(1/ln(c*(b*x^2+a)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log \left(c \left(a+b x^2\right)^p\right)} \, dx=\int \frac{1}{\log \left({\left(b x^{2} + a\right)}^{p} c\right)} \, dx$$

```
[In] integrate(1/log(c*(b*x^2+a)^p),x, algorithm="fricas")
[Out] integral(1/log((b*x^2 + a)^p*c), x)
```

Sympy [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log \left(c \left(a+b x^2\right)^p\right)} \, dx=\int \frac{1}{\log \left(c \left(a + b x^{2}\right)^{p} \right)}\, dx$$

```
[In] integrate(1/ln(c*(b*x**2+a)**p),x)
[Out] Integral(1/log(c*(a + b*x**2)**p), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log \left(c \left(a+b x^2\right)^p\right)}\, dx = \int \frac{1}{\log \left(\left(b x^2+a\right)^p c\right)}\, dx$$

```
[In] integrate(1/log(c*(b*x^2+a)^p),x, algorithm="maxima")
[Out] integrate(1/log((b*x^2 + a)^p*c), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log \left(c \left(a+b x^2\right)^p\right)}\, dx = \int \frac{1}{\log \left(\left(b x^2+a\right)^p c\right)}\, dx$$

```
[In] integrate(1/log(c*(b*x^2+a)^p),x, algorithm="giac")
[Out] integrate(1/log((b*x^2 + a)^p*c), x)
```

Mupad [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log \left(c \left(a+b x^2\right)^p\right)}\, dx = \int \frac{1}{\ln \left(c\, \left(b\, x^2+a\right)^p\right)}\, dx$$

```
[In] int(1/log(c*(a + b*x^2)^p),x)
[Out] int(1/log(c*(a + b*x^2)^p), x)
```

3.108 $\int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)} \, dx = \operatorname{Int}\left(\frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)}, x\right)$$

```
[Out] Unintegrable(1/x^2/ln(c*(b*x^2+a)^p),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)} \, dx$$

```
[In] Int[1/(x^2*Log[c*(a + b*x^2)^p]),x]
[Out] Defer[Int][1/(x^2*Log[c*(a + b*x^2)^p]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log \left(c \left(a+b x^2\right)^p\right)} \, dx=\int \frac{1}{x^2 \log \left(c \left(a+b x^2\right)^p\right)} \, dx$$

```
[In] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]),x]
[Out] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]), x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{2} \ln \left(c \left(b \,x^{2}+a \right)^{p}\right)}d x$$

```
[In] int(1/x^2/ln(c*(b*x^2+a)^p),x)
[Out] int(1/x^2/ln(c*(b*x^2+a)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log \left(c \left(a+b x^2\right)^p\right)} \, dx=\int \frac{1}{x^{2} \log \left({\left(b x^{2} + a\right)}^{p} c\right)} \, dx$$

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p),x, algorithm="fricas")
[Out] integral(1/(x^2*log((b*x^2 + a)^p*c)), x)
```

Sympy [N/A]

Not integrable

Time = 3.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2 \log \left(c \left(a+b x^2\right)^p\right)} \, dx=\int \frac{1}{x^{2} \log \left(c \left(a + b x^{2}\right)^{p}\right)}\, dx$$

```
[In] integrate(1/x**2/ln(c*(b*x**2+a)**p),x)
[Out] Integral(1/(x**2*log(c*(a + b*x**2)**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^2 \log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p),x, algorithm="maxima")
[Out] integrate(1/(x^2*log((b*x^2 + a)^p*c)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^2 \log\left(\left(bx^2+a\right)^p c\right)}\,dx$$

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p),x, algorithm="giac")
[Out] integrate(1/(x^2*log((b*x^2 + a)^p*c)), x)
```

Mupad [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^2\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}\,dx$$

```
[In] int(1/(x^2*log(c*(a + b*x^2)^p)),x)
[Out] int(1/(x^2*log(c*(a + b*x^2)^p)), x)
```

3.109 $\int \frac{x^3}{\log^2(c(a+bx^2)^p)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 138

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = -\frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\operatorname{ExpIntegralEi}\left(\frac{\log\left(c(a+bx^2)^p\right)}{p}\right)}{2b^2p^2} + \frac{(a+bx^2)^2\left(c(a+bx^2)^p\right)^{-2/p}\operatorname{ExpIntegralEi}\left(\frac{2\log\left(c(a+bx^2)^p\right)}{p}\right)}{b^2p^2} - \frac{x^2(a+bx^2)}{2bp\log\left(c\left(a+bx^2\right)^p\right)}$$

```
[Out] -1/2*a*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b^2/p^2/((c*(b*x^2+a)^p)^(1/p))+(b
*x^2+a)^2*Ei(2*ln(c*(b*x^2+a)^p)/p)/b^2/p^2/((c*(b*x^2+a)^p)^(2/p))-1/2*x^2
*(b*x^2+a)/b/p/ln(c*(b*x^2+a)^p)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2504, 2447, 2446, 2436, 2337, 2209, 2437, 2347}

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \frac{(a+bx^2)^2\left(c(a+bx^2)^p\right)^{-2/p}\operatorname{ExpIntegralEi}\left(\frac{2\log\left(c(bx^2+a)^p\right)}{p}\right)}{b^2p^2} - \frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\operatorname{ExpIntegralEi}\left(\frac{\log\left(c(bx^2+a)^p\right)}{p}\right)}{2b^2p^2} - \frac{x^2(a+bx^2)}{2bp\log\left(c\left(a+bx^2\right)^p\right)}$$

```
[In] Int[x^3/Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] -1/2*(a*(a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(b^2*p^2*(c*(a +
 b*x^2)^p)^p^(-1)) + ((a + b*x^2)^2*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/
p])/(b^2*p^2*(c*(a + b*x^2)^p)^(2/p)) - (x^2*(a + b*x^2))/(2*b*p*Log[c*(a +
 b*x^2)^p])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
```

```
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x}{\log^2\left(c(a+bx)^p\right)}\,dx, x, x^2\right)$$

$$= -\frac{x^2(a+bx^2)}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{x}{\log(c(a+bx)^p)}\,dx, x, x^2\right)}{p} + \frac{a\text{Subst}\left(\int \frac{1}{\log(c(a+bx)^p)}\,dx, x, x^2\right)}{2bp}$$

$$= -\frac{x^2(a+bx^2)}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \left(-\frac{a}{b\log(c(a+bx)^p)} + \frac{a+bx}{b\log(c(a+bx)^p)}\right)\,dx, x, x^2\right)}{p} + \frac{a\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, a+bx^2\right)}{2b^2p}$$

$$= -\frac{x^2(a+bx^2)}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{a+bx}{\log(c(a+bx)^p)}\,dx, x, x^2\right)}{bp} - \frac{a\text{Subst}\left(\int \frac{1}{\log(c(a+bx)^p)}\,dx, x, x^2\right)}{bp} + \frac{\left(a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c(a+b\right.\right.}{2b^2p^2}$$

$$= \frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c(a+bx^2)^p\right)}{p}\right)}{2b^2p^2} - \frac{x^2(a+bx^2)}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{x}{\log(cx^p)}\,dx, x, a+bx^2\right)}{b^2p} - \frac{a\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, a+bx^2\right)}{b^2p}$$

$$= \frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)}{2b^2p^2} - \frac{x^2(a+bx^2)}{2bp\log\left(c\left(a+bx^2\right)^p\right)}$$
$$+ \frac{\left((a+bx^2)^2\left(c(a+bx^2)^p\right)^{-2/p}\right)\operatorname{Subst}\left(\int\frac{e^{\frac{2x}{p}}}{x}\,dx, x, \log\left(c(a+bx^2)^p\right)\right)}{b^2p^2}$$
$$- \frac{\left(a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\right)\operatorname{Subst}\left(\int\frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c(a+bx^2)^p\right)\right)}{b^2p^2}$$
$$= -\frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)}{2b^2p^2}$$
$$+ \frac{(a+bx^2)^2\left(c(a+bx^2)^p\right)^{-2/p}\operatorname{Ei}\left(\frac{2\log(c(a+bx^2)^p)}{p}\right)}{b^2p^2} - \frac{x^2(a+bx^2)}{2bp\log\left(c\left(a+bx^2\right)^p\right)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.14

$$\int\frac{x^3}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx =$$
$$-\frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-2/p}\left(bpx^2\left(c(a+bx^2)^p\right)^{2/p} + a\left(c(a+bx^2)^p\right)^{\frac{1}{p}}\operatorname{ExpIntegralEi}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)\log\right.}{2b^2p^2\log\left(c\left(a+bx^2\right)^p\right)}$$

```
[In] Integrate[x^3/Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] -1/2*((a + b*x^2)*(b*p*x^2*(c*(a + b*x^2)^p)^(2/p) + a*(c*(a + b*x^2)^p)^p^
(-1)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p]*Log[c*(a + b*x^2)^p] - 2*(a + b*
x^2)*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/p]*Log[c*(a + b*x^2)^p]))/(b^2*
p^2*(c*(a + b*x^2)^p)^(2/p)*Log[c*(a + b*x^2)^p])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.85 (sec) , antiderivative size = 1487, normalized size of antiderivative = 10.78

method	result	size
risch	Expression too large to display	1487

```
[In] int(x^3/ln(c*(b*x^2+a)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/p/b*x^2*(b*x^2+a)/(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi
*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a
```

```
)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p))-1/
p^2*((b*x^2+a)^p)^(-2/p)*c^(-2/p)*exp(I*Pi*csgn(I*c*(b*x^2+a)^p)*(-csgn(I*c
*(b*x^2+a)^p)+csgn(I*c))*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*(b*x^2+a)^p))/p)*Ei
(1,-2*ln(b*x^2+a)-(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*cs
gn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p
)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p)-2*p*ln
(b*x^2+a))/p)*x^4-2/p^2/b*((b*x^2+a)^p)^(-2/p)*c^(-2/p)*exp(I*Pi*csgn(I*c*(
b*x^2+a)^p)*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*c))*(-csgn(I*c*(b*x^2+a)^p)+csgn
(I*(b*x^2+a)^p))/p)*Ei(1,-2*ln(b*x^2+a)-(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*P
i*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*
ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)*a*x^2-1/p^2/b^2*((b*x^2+a)^p)^(-2/p)*c^
(-2/p)*exp(I*Pi*csgn(I*c*(b*x^2+a)^p)*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*c))*(-
csgn(I*c*(b*x^2+a)^p)+csgn(I*(b*x^2+a)^p))/p)*Ei(1,-2*ln(b*x^2+a)-(I*Pi*csg
n(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*
(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)
^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)*a^2+1/2/p^2/b
*a*((b*x^2+a)^p)^(-1/p)*c^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)*(-csgn(
I*c*(b*x^2+a)^p)+csgn(I*c))*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*(b*x^2+a)^p))/p)
*Ei(1,-ln(b*x^2+a)-1/2*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*
Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2
+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p)-2
*p*ln(b*x^2+a))/p)*x^2+1/2/p^2/b^2*a^2*((b*x^2+a)^p)^(-1/p)*c^(-1/p)*exp(1/
2*I*Pi*csgn(I*c*(b*x^2+a)^p)*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*c))*(-csgn(I*c*
(b*x^2+a)^p)+csgn(I*(b*x^2+a)^p))/p)*Ei(1,-ln(b*x^2+a)-1/2*(I*Pi*csgn(I*(b*
x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+
a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*c
sgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = -\frac{(ap\log\left(bx^2+a\right)+a\log\left(c\right))c^{\left(\frac{1}{p}\right)}\,\mathrm{log\_integral}\left((bx^2+a)c^{\left(\frac{1}{p}\right)}\right)+(b^2px^4+abpx^2)c^{\frac{2}{p}}-2\left(p\log\left(bx^2+a\right)\right.}{2\left(b^2p^3\log\left(bx^2+a\right)+b^2p^2\log\left(c\right)\right)c^{\frac{2}{p}}}$$

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a*p*log(b*x^2 + a) + a*log(c))*c^(1/p)*log_integral((b*x^2 + a)*c^(1
/p)) + (b^2*p*x^4 + a*b*p*x^2)*c^(2/p) - 2*(p*log(b*x^2 + a) + log(c))*log_
```

```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)*c^(2/p)))/((b^2*p^3*log(b*x^2 + a) + b
^2*p^2*log(c))*c^(2/p))
```

Sympy [F]

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)^2}\,dx$$

```
[In] integrate(x**3/ln(c*(b*x**2+a)**p)**2,x)
[Out] Integral(x**3/log(c*(a + b*x**2)**p)**2, x)
```

Maxima [F]

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^3}{\log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
[Out] -1/2*(b*x^4 + a*x^2)/(b*p^2*log(b*x^2 + a) + b*p*log(c)) + integrate((2*b*x
^3 + a*x)/(b*p^2*log(b*x^2 + a) + b*p*log(c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(136) = 272$.

Time = 0.30 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.27

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$
$$=\frac{1}{2}a\left(\frac{(bx^2+a)p}{b^2p^3\log\left(bx^2+a\right)+b^2p^2\log\left(c\right)}-\frac{p\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\log\left(bx^2+a\right)}{\left(b^2p^3\log\left(bx^2+a\right)+b^2p^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}-\frac{\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2\right.\right.}{\left(b^2p^3\log\left(bx^2+a\right)+b\right.}\right.$$
$$-\frac{\frac{(bx^2+a)^2p}{bp^3\log(bx^2+a)+bp^2\log(c)}-\frac{2\,p\mathrm{Ei}\left(\frac{2\,\log(c)}{p}+2\,\log(bx^2+a)\right)\log(bx^2+a)}{(bp^3\log(bx^2+a)+bp^2\log(c))c^{\frac{2}{p}}}-\frac{2\,\mathrm{Ei}\left(\frac{2\,\log(c)}{p}+2\,\log(bx^2+a)\right)\log(c)}{(bp^3\log(bx^2+a)+bp^2\log(c))c^{\frac{2}{p}}}}{2\,b}$$

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
[Out] 1/2*a*((b*x^2 + a)*p/(b^2*p^3*log(b*x^2 + a) + b^2*p^2*log(c)) - p*Ei(log(c
)/p + log(b*x^2 + a))*log(b*x^2 + a)/((b^2*p^3*log(b*x^2 + a) + b^2*p^2*log
(c))*c^(1/p)) - Ei(log(c)/p + log(b*x^2 + a))*log(c)/((b^2*p^3*log(b*x^2 +
a) + b^2*p^2*log(c))*c^(1/p))) - 1/2*((b*x^2 + a)^2*p/(b*p^3*log(b*x^2 + a)
 + b*p^2*log(c)) - 2*p*Ei(2*log(c)/p + 2*log(b*x^2 + a))*log(b*x^2 + a)/((b
*p^3*log(b*x^2 + a) + b*p^2*log(c))*c^(2/p)) - 2*Ei(2*log(c)/p + 2*log(b*x^
2 + a))*log(c)/((b*p^3*log(b*x^2 + a) + b*p^2*log(c))*c^(2/p)))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^3}{\ln\left(c\left(b\,x^2+a\right)^p\right)^2}\,dx$$

```
[In] int(x^3/log(c*(a + b*x^2)^p)^2,x)
[Out] int(x^3/log(c*(a + b*x^2)^p)^2, x)
```

3.110 $\int \frac{x}{\log^2(c(a+bx^2)^p)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 83

$$\int \frac{x}{\log^2(c(a+bx^2)^p)}\,dx = \frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)}{2bp^2} - \frac{a+bx^2}{2bp\log(c(a+bx^2)^p)}$$

```
[Out] 1/2*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b/p^2/((c*(b*x^2+a)^p)^(1/p))+1/2*(-b
*x^2-a)/b/p/ln(c*(b*x^2+a)^p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2504, 2436, 2334, 2337, 2209}

$$\int \frac{x}{\log^2(c(a+bx^2)^p)}\,dx = \frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log(c(bx^2+a)^p)}{p}\right)}{2bp^2} - \frac{a+bx^2}{2bp\log(c(a+bx^2)^p)}$$

```
[In] Int[x/Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] ((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(2*b*p^2*(c*(a + b*x^2)
^p)^p^(-1)) - (a + b*x^2)/(2*b*p*Log[c*(a + b*x^2)^p])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\log^2\left(c(a+bx)^p\right)}\,dx, x, x^2\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{\log^2(cx^p)}\,dx, x, a+bx^2\right)}{2b} \\
&= -\frac{a+bx^2}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, a+bx^2\right)}{2bp} \\
&= -\frac{a+bx^2}{2bp\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\left(\left(a+bx^2\right)\left(c(a+bx^2)^p\right)^{-1/p}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c(a+bx^2)^p\right)\right)}{2bp^2}
\end{aligned}$$

$$= \frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)}{2bp^2} - \frac{a+bx^2}{2bp\log\left(c\left(a+bx^2\right)^p\right)}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = -\frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\left(p\left(c(a+bx^2)^p\right)^{\frac{1}{p}} - \text{ExpIntegralEi}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)\log\left(c(a+bx^2)^p\right)\right)}{2bp^2\log\left(c\left(a+bx^2\right)^p\right)}$$

```
[In] Integrate[x/Log[c*(a + b*x^2)^p]^2,x]
```

```
[Out] -1/2*((a + b*x^2)*(p*(c*(a + b*x^2)^p)^p^(-1) - ExpIntegralEi[Log[c*(a + b*
x^2)^p]/p]*Log[c*(a + b*x^2)^p]))/(b*p^2*(c*(a + b*x^2)^p)^p^(-1)*Log[c*(a
+ b*x^2)^p])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.77 (sec) , antiderivative size = 421, normalized size of antiderivative = 5.07

method	result
risch	$-\frac{b\,x^2+a}{\left(i\pi\ \mathrm{csgn}(i(b\,x^2+a)^p)\mathrm{csgn}(ic(b\,x^2+a)^p)^2-i\pi\ \mathrm{csgn}(i(b\,x^2+a)^p)\ \mathrm{csgn}(ic(b\,x^2+a)^p)\ \mathrm{csgn}(ic)-i\pi\mathrm{csgn}(ic(b\,x^2+a)^p)^3+i\pi\mathrm{csgn}(ic(b\,x^2\right.}$

```
[In] int(x/ln(c*(b*x^2+a)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^
p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I
*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p))/p/b*(b*x^2+a)-1/2/p^
2/b*(b*x^2+a)*c^(-1/p)*((b*x^2+a)^p)^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(b*x^2+a)
^p)*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*c))*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*(b*x^
2+a)^p))/p)*Ei(1,-ln(b*x^2+a)-1/2*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2
+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn
(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*
x^2+a)^p)-2*p*ln(b*x^2+a))/p)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = -\frac{(bpx^2+ap)c^{\left(\frac{1}{p}\right)} - (p\log\left(bx^2+a\right)+\log\left(c\right))\,\mathrm{log\_integral}\left((bx^2+a)c^{\left(\frac{1}{p}\right)}\right)}{2\,(bp^3\log\left(bx^2+a\right)+bp^2\log\left(c\right))c^{\left(\frac{1}{p}\right)}}$$

```
[In] integrate(x/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((b*p*x^2 + a*p)*c^(1/p) - (p*log(b*x^2 + a) + log(c))*log_integral((b
*x^2 + a)*c^(1/p)))/((b*p^3*log(b*x^2 + a) + b*p^2*log(c))*c^(1/p))
```

Sympy [F]

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)^2}\,dx$$

```
[In] integrate(x/ln(c*(b*x**2+a)**p)**2,x)
```

```
[Out] Integral(x/log(c*(a + b*x**2)**p)**2, x)
```

Maxima [F]

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x}{\log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

```
[In] integrate(x/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 + a)/(b*p^2*log(b*x^2 + a) + b*p*log(c)) + integrate(x/(p^2*log
(b*x^2 + a) + p*log(c)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.70

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = -\frac{(bx^2+a)p}{2\left(bp^3\log\left(bx^2+a\right)+bp^2\log\left(c\right)\right)} + \frac{p\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\log\left(bx^2+a\right)}{2\left(bp^3\log\left(bx^2+a\right)+bp^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}} + \frac{\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\log\left(c\right)}{2\left(bp^3\log\left(bx^2+a\right)+bp^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}$$

```
[In] integrate(x/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b*x^2 + a)*p/(b*p^3*log(b*x^2 + a) + b*p^2*log(c)) + 1/2*p*Ei(log(c)/
p + log(b*x^2 + a))*log(b*x^2 + a)/((b*p^3*log(b*x^2 + a) + b*p^2*log(c))*c
^(1/p)) + 1/2*Ei(log(c)/p + log(b*x^2 + a))*log(c)/((b*p^3*log(b*x^2 + a) +
 b*p^2*log(c))*c^(1/p))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x}{\ln\left(c\left(b\,x^2+a\right)^p\right)^2}\,dx$$

```
[In] int(x/log(c*(a + b*x^2)^p)^2,x)
```

```
[Out] int(x/log(c*(a + b*x^2)^p)^2, x)
```

3.111 $\int \frac{1}{x\log^2(c(a+bx^2)^p)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\operatorname{Int}\left(\frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x/ln(c*(b*x^2+a)^p)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

```
[In] Int[1/(x*Log[c*(a + b*x^2)^p]^2),x]
[Out] Defer[Int][1/(x*Log[c*(a + b*x^2)^p]^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

```
[In] Integrate[1/(x*Log[c*(a + b*x^2)^p]^2),x]
[Out] Integrate[1/(x*Log[c*(a + b*x^2)^p]^2), x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\ln\left(c\left(b\,x^2+a\right)^p\right)^2}dx$$

```
[In] int(1/x/ln(c*(b*x^2+a)^p)^2,x)
[Out] int(1/x/ln(c*(b*x^2+a)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

```
[In] integrate(1/x/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
[Out] integral(1/(x*log((b*x^2 + a)^p*c)^2), x)
```

Sympy [N/A]

Not integrable

Time = 3.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)^2}\,dx$$

```
[In] integrate(1/x/ln(c*(b*x**2+a)**p)**2,x)
[Out] Integral(1/(x*log(c*(a + b*x**2)**p)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.83

$$\int \frac{1}{x \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x \log\left(\left(bx^2+a\right)^p c\right)^2} \, dx$$

```
[In] integrate(1/x/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -a*integrate(1/(b*p^2*x^3*log(b*x^2 + a) + b*p*x^3*log(c)), x) - 1/2*(b*x^2
 + a)/(b*p^2*x^2*log(b*x^2 + a) + b*p*x^2*log(c))
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x \log\left(\left(bx^2+a\right)^p c\right)^2} \, dx$$

```
[In] integrate(1/x/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x*log((b*x^2 + a)^p*c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x \ln\left(c\left(b\,x^2+a\right)^p\right)^2} \, dx$$

```
[In] int(1/(x*log(c*(a + b*x^2)^p)^2),x)
```

```
[Out] int(1/(x*log(c*(a + b*x^2)^p)^2), x)
```

3.112 $\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx = \operatorname{Int}\left(\frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)}, x\right)$$

`[Out] Unintegrable(1/x^3/ln(c*(b*x^2+a)^p)^2,x)`

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$$

`[In] Int[1/(x^3*Log[c*(a + b*x^2)^p]^2),x]`

`[Out] Defer[Int][1/(x^3*Log[c*(a + b*x^2)^p]^2), x]`

Rubi steps

$$\text{integral} = \int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

```
[In] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]^2),x]
[Out] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]^2), x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 \ln\left(c\left(b\,x^2+a\right)^p\right)^2}dx$$

```
[In] int(1/x^3/ln(c*(b*x^2+a)^p)^2,x)
[Out] int(1/x^3/ln(c*(b*x^2+a)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^3 \log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
[Out] integral(1/(x^3*log((b*x^2 + a)^p*c)^2), x)
```

Sympy [N/A]

Not integrable

Time = 5.81 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^3 \log\left(c\left(a+bx^2\right)^p\right)^2}\,dx$$

```
[In] integrate(1/x**3/ln(c*(b*x**2+a)**p)**2,x)
[Out] Integral(1/(x**3*log(c*(a + b*x**2)**p)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 4.33

$$\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^3 \log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
[Out] -1/2*(b*x^2 + a)/(b*p^2*x^4*log(b*x^2 + a) + b*p*x^4*log(c)) - integrate((b
*x^2 + 2*a)/(b*p^2*x^5*log(b*x^2 + a) + b*p*x^5*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^3 \log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
[Out] integrate(1/(x^3*log((b*x^2 + a)^p*c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^3 \ln\left(c\left(b\,x^2+a\right)^p\right)^2}\,dx$$

```
[In] int(1/(x^3*log(c*(a + b*x^2)^p)^2),x)
[Out] int(1/(x^3*log(c*(a + b*x^2)^p)^2), x)
```

3.113 $\int \frac{x^2}{\log^2(c(a+bx^2)^p)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \operatorname{Int}\left(\frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)},x\right)$$

`[Out] Unintegrable(x^2/ln(c*(b*x^2+a)^p)^2,x)`

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

`[In] Int[x^2/Log[c*(a + b*x^2)^p]^2,x]`

`[Out] Defer[Int][x^2/Log[c*(a + b*x^2)^p]^2, x]`

Rubi steps

$$\text{integral} = \int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log^2(c(a+bx^2)^p)}\,dx = \int \frac{x^2}{\log^2(c(a+bx^2)^p)}\,dx$$

```
[In] Integrate[x^2/Log[c*(a + b*x^2)^p]^2,x]
[Out] Integrate[x^2/Log[c*(a + b*x^2)^p]^2, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\ln(c(b\,x^2+a)^p)^2}dx$$

```
[In] int(x^2/ln(c*(b*x^2+a)^p)^2,x)
[Out] int(x^2/ln(c*(b*x^2+a)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log^2(c(a+bx^2)^p)}\,dx = \int \frac{x^2}{\log((bx^2+a)^p c)^2}\,dx$$

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
[Out] integral(x^2/log((b*x^2 + a)^p*c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 2.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{\log^2(c(a+bx^2)^p)}\,dx = \int \frac{x^2}{\log(c(a+bx^2)^p)^2}\,dx$$

```
[In] integrate(x**2/ln(c*(b*x**2+a)**p)**2,x)
[Out] Integral(x**2/log(c*(a + b*x**2)**p)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.67

$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^2}{\log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^3 + a*x)/(b*p^2*log(b*x^2 + a) + b*p*log(c)) + integrate(1/2*(3*b
*x^2 + a)/(b*p^2*log(b*x^2 + a) + b*p*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^2}{\log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/log((b*x^2 + a)^p*c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^2}{\ln\left(c\left(b\,x^2+a\right)^p\right)^2}\,dx$$

```
[In] int(x^2/log(c*(a + b*x^2)^p)^2,x)
```

```
[Out] int(x^2/log(c*(a + b*x^2)^p)^2, x)
```

3.114 $\int \frac{1}{\log^2(c(a+bx^2)^p)} dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)} dx = \mathrm{Int}\left(\frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}, x\right)$$

`[Out] Unintegrable(1/ln(c*(b*x^2+a)^p)^2,x)`

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)} dx = \int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)} dx$$

`[In] Int[Log[c*(a + b*x^2)^p]^(-2),x]`

`[Out] Defer[Int][Log[c*(a + b*x^2)^p]^(-2), x]`

Rubi steps

$$\text{integral} = \int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)} dx$$

Mathematica [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]^(-2),x]
[Out] Integrate[Log[c*(a + b*x^2)^p]^(-2), x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\ln\left(c\left(b\,x^2+a\right)^p\right)^2}dx$$

```
[In] int(1/ln(c*(b*x^2+a)^p)^2,x)
[Out] int(1/ln(c*(b*x^2+a)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{\log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

```
[In] integrate(1/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
[Out] integral(log((b*x^2 + a)^p*c)^(-2), x)
```

Sympy [N/A]

Not integrable

Time = 1.89 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)^2}\,dx$$

```
[In] integrate(1/ln(c*(b*x**2+a)**p)**2,x)
[Out] Integral(log(c*(a + b*x**2)**p)**(-2), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 5.21

$$\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{\log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

```
[In] integrate(1/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 + a)/(b*p^2*x*log(b*x^2 + a) + b*p*x*log(c)) + integrate(1/2*(b
*x^2 - a)/(b*p^2*x^2*log(b*x^2 + a) + b*p*x^2*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{\log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

```
[In] integrate(1/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)^p*c)^(-2), x)
```

Mupad [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{\ln\left(c\left(b\,x^2+a\right)^p\right)^2}\,dx$$

```
[In] int(1/log(c*(a + b*x^2)^p)^2,x)
```

```
[Out] int(1/log(c*(a + b*x^2)^p)^2, x)
```

3.115 $\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx = \operatorname{Int}\left(\frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)}, x\right)$$

```
[Out] Unintegrable(1/x^2/ln(c*(b*x^2+a)^p)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$$

```
[In] Int[1/(x^2*Log[c*(a + b*x^2)^p]^2),x]
```

```
[Out] Defer[Int][1/(x^2*Log[c*(a + b*x^2)^p]^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx$$

```
[In] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]^2),x]
[Out] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]^2), x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \ln\left(c\left(b\,x^2+a\right)^p\right)^2}dx$$

```
[In] int(1/x^2/ln(c*(b*x^2+a)^p)^2,x)
[Out] int(1/x^2/ln(c*(b*x^2+a)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^2 \log\left(\left(bx^2+a\right)^p c\right)^2} \, dx$$

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="fricas")
[Out] integral(1/(x^2*log((b*x^2 + a)^p*c)^2), x)
```

Sympy [N/A]

Not integrable

Time = 4.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)^2} \, dx$$

```
[In] integrate(1/x**2/ln(c*(b*x**2+a)**p)**2,x)
[Out] Integral(1/(x**2*log(c*(a + b*x**2)**p)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 4.39

$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^2 \log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 + a)/(b*p^2*x^3*log(b*x^2 + a) + b*p*x^3*log(c)) - integrate(1/
2*(b*x^2 + 3*a)/(b*p^2*x^4*log(b*x^2 + a) + b*p*x^4*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^2 \log\left(\left(bx^2+a\right)^p c\right)^2}\,dx$$

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*log((b*x^2 + a)^p*c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^2\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^2 \ln\left(c\left(b\,x^2+a\right)^p\right)^2}\,dx$$

```
[In] int(1/(x^2*log(c*(a + b*x^2)^p)^2),x)
```

```
[Out] int(1/(x^2*log(c*(a + b*x^2)^p)^2), x)
```

3.116 $\int \frac{x^3}{\log^3(c(a+bx^2)^p)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 204

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = -\frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\operatorname{ExpIntegralEi}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)}{4b^2p^3} + \frac{(a+bx^2)^2\left(c(a+bx^2)^p\right)^{-2/p}\operatorname{ExpIntegralEi}\left(\frac{2\log(c(a+bx^2)^p)}{p}\right)}{b^2p^3} - \frac{x^2(a+bx^2)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a(a+bx^2)}{4b^2p^2\log\left(c\left(a+bx^2\right)^p\right)} - \frac{x^2(a+bx^2)}{2bp^2\log\left(c\left(a+bx^2\right)^p\right)}$$

```
[Out] -1/4*a*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b^2/p^3/((c*(b*x^2+a)^p)^(1/p))+(b
*x^2+a)^2*Ei(2*ln(c*(b*x^2+a)^p)/p)/b^2/p^3/((c*(b*x^2+a)^p)^(2/p))-1/4*x^2
*(b*x^2+a)/b/p/ln(c*(b*x^2+a)^p)^2-1/4*a*(b*x^2+a)/b^2/p^2/ln(c*(b*x^2+a)^p
)-1/2*x^2*(b*x^2+a)/b/p^2/ln(c*(b*x^2+a)^p)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used

$= \{2504, 2447, 2446, 2436, 2337, 2209, 2437, 2347, 2334\}$

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \frac{\left(a+bx^2\right)^2\left(c(a+bx^2)^p\right)^{-2/p}\text{ExpIntegralEi}\left(\frac{2\log\left(c(bx^2+a)^p\right)}{p}\right)}{b^2p^3} - \frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c(bx^2+a)^p\right)}{p}\right)}{4b^2p^3} - \frac{a(a+bx^2)}{4b^2p^2\log\left(c\left(a+bx^2\right)^p\right)} - \frac{x^2(a+bx^2)}{2bp^2\log\left(c\left(a+bx^2\right)^p\right)} - \frac{x^2(a+bx^2)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)}$$

```
[In] Int[x^3/Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] -1/4*(a*(a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(b^2*p^3*(c*(a + 
 b*x^2)^p)^p^(-1)) + ((a + b*x^2)^2*ExpIntegralEi[(2*Log[c*(a + b*x^2)^p])/
p])/(b^2*p^3*(c*(a + b*x^2)^p)^(2/p)) - (x^2*(a + b*x^2))/(4*b*p*Log[c*(a + 
 b*x^2)^p]^2) - (a*(a + b*x^2))/(4*b^2*p^2*Log[c*(a + b*x^2)^p]) - (x^2*(a 
+ b*x^2))/(2*b*p^2*Log[c*(a + b*x^2)^p])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\log^3\left(c(a+bx)^p\right)}\,dx,x,x^2\right)\\ &= -\frac{x^2(a+bx^2)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)}+\frac{\text{Subst}\left(\int \frac{x}{\log^2(c(a+bx)^p)}\,dx,x,x^2\right)}{2p}+\frac{a\text{Subst}\left(\int \frac{1}{\log^2(c(a+bx)^p)}\,dx,x,x^2\right)}{4bp}\end{aligned}$$

$$= -\frac{x^2(a+bx^2)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{x^2(a+bx^2)}{2bp^2\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{x}{\log(c(a+bx)^p)}\,dx, x, x^2\right)}{p^2} + \frac{a\text{Subst}\left(\int \frac{1}{\log(c(a+bx)^p)}\,dx, x, x^2\right)}{2bp^2} + \frac{a\text{Subst}\left(\int \frac{1}{\log^2(cx^p)}\,dx, x, a+bx^2\right)}{4b^2p}$$

$$= -\frac{x^2(a+bx^2)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a(a+bx^2)}{4b^2p^2\log\left(c\left(a+bx^2\right)^p\right)} - \frac{x^2(a+bx^2)}{2bp^2\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \left(-\frac{a}{b\log(c(a+bx)^p)} + \frac{a+bx}{b\log(c(a+bx)^p)}\right)\,dx, x, x^2\right)}{p^2} + \frac{a\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, a+bx^2\right)}{4b^2p^2} + \frac{a\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, a+bx^2\right)}{2b^2p^2}$$

$$= -\frac{x^2(a+bx^2)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a(a+bx^2)}{4b^2p^2\log\left(c\left(a+bx^2\right)^p\right)} - \frac{x^2(a+bx^2)}{2bp^2\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{a+bx}{\log(c(a+bx)^p)}\,dx, x, x^2\right)}{bp^2} - \frac{a\text{Subst}\left(\int \frac{1}{\log(c(a+bx)^p)}\,dx, x, x^2\right)}{bp^2} + \frac{\left(a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c(a+bx^2)^p\right)\right)}{4b^2p^3} + \frac{\left(a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c(a+bx^2)^p\right)\right)}{2b^2p^3}$$

$$= \frac{3a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{Ei}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)}{4b^2p^3} - \frac{x^2(a+bx^2)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a(a+bx^2)}{4b^2p^2\log\left(c\left(a+bx^2\right)^p\right)} - \frac{x^2(a+bx^2)}{2bp^2\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{x}{\log(cx^p)}\,dx, x, a+bx^2\right)}{b^2p^2} - \frac{a\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, a+bx^2\right)}{b^2p^2}$$

$$= \frac{3a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\text{Ei}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)}{4b^2p^3} - \frac{x^2(a+bx^2)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a(a+bx^2)}{4b^2p^2\log\left(c\left(a+bx^2\right)^p\right)} - \frac{x^2(a+bx^2)}{2bp^2\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\left((a+bx^2)^2\left(c(a+bx^2)^p\right)^{-2/p}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{p}}}{x}\,dx, x, \log\left(c(a+bx^2)^p\right)\right)}{b^2p^3} - \frac{\left(a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c(a+bx^2)^p\right)\right)}{b^2p^3}$$

$$= -\frac{a(a+bx^2)\left(c(a+bx^2)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)}{4b^2p^3} + \frac{(a+bx^2)^2\left(c(a+bx^2)^p\right)^{-2/p}\operatorname{Ei}\left(\frac{2\log(c(a+bx^2)^p)}{p}\right)}{b^2p^3} - \frac{x^2(a+bx^2)}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a(a+bx^2)}{4b^2p^2\log\left(c\left(a+bx^2\right)^p\right)} - \frac{x^2(a+bx^2)}{2bp^2\log\left(c\left(a+bx^2\right)^p\right)}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = -\frac{(a+bx^2)\left(c(a+bx^2)^p\right)^{-2/p}\left(a\left(c(a+bx^2)^p\right)^{\frac{1}{p}}\operatorname{ExpIntegralEi}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)\log^2\left(c(a+bx^2)^p\right) - 4(a+\right.}{4b^2p^3\,\mathrm{l}}$$

```
[In] Integrate[x^3/Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] -1/4*((a + b*x^2)*(a*(c*(a + b*x^2)^p)^p^(-1)*ExpIntegralEi[Log[c*(a + b*x^
2)^p]/p]*Log[c*(a + b*x^2)^p]^2 - 4*(a + b*x^2)*ExpIntegralEi[(2*Log[c*(a +
 b*x^2)^p])/p]*Log[c*(a + b*x^2)^p]^2 + p*(c*(a + b*x^2)^p)^(2/p)*(b*p*x^2
+ (a + 2*b*x^2)*Log[c*(a + b*x^2)^p])))/(b^2*p^3*(c*(a + b*x^2)^p)^(2/p)*Lo
g[c*(a + b*x^2)^p]^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.90 (sec) , antiderivative size = 1969, normalized size of antiderivative = 9.65

method	result	size
risch	Expression too large to display	1969

```
[In] int(x^3/ln(c*(b*x^2+a)^p)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(2*b^2*p*x^4+2*a*b*p*x^2+I*Pi*a^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+3*
I*Pi*a*b*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-2*I*Pi*b^2*x^4*csgn(I*c*(b*x
^2+a)^p)^3+2*I*Pi*b^2*x^4*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)-I*Pi*a^2*csgn(I
*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)+I*Pi*a^2*csgn(I*(b*x^2+a)^p)*
csgn(I*c*(b*x^2+a)^p)^2-3*I*Pi*a*b*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+
a)^p)*csgn(I*c)-I*Pi*a^2*csgn(I*c*(b*x^2+a)^p)^3-2*I*Pi*b^2*x^4*csgn(I*(b*x
^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-3*I*Pi*a*b*x^2*csgn(I*c*(b*x^2+a)^
p)^3+3*I*Pi*a*b*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+2*I*Pi*b^2*
```

```
x^4*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2+4*ln(c)*b^2*x^4+4*b^2*x^4*l
n((b*x^2+a)^p)+6*ln(c)*a*b*x^2+6*a*b*x^2*ln((b*x^2+a)^p)+2*ln(c)*a^2+2*a^2*
ln((b*x^2+a)^p))/(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csg
n(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)
^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p))^2/b^2/
p^2-1/p^3*c^(-2/p)*((b*x^2+a)^p)^(-2/p)*exp(I*Pi*csgn(I*c*(b*x^2+a)^p)*(-cs
gn(I*c*(b*x^2+a)^p)+csgn(I*c))*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*(b*x^2+a)^p))
/p)*Ei(1,-2*ln(b*x^2+a)-(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I
*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^
2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p)-
2*p*ln(b*x^2+a))/p)*x^4-2/b/p^3*c^(-2/p)*((b*x^2+a)^p)^(-2/p)*exp(I*Pi*csgn
(I*c*(b*x^2+a)^p)*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*c))*(-csgn(I*c*(b*x^2+a)^p
)+csgn(I*(b*x^2+a)^p))/p)*Ei(1,-2*ln(b*x^2+a)-(I*Pi*csgn(I*(b*x^2+a)^p)*csg
n(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*
c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln
(c)+2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)*a*x^2-1/b^2/p^3*c^(-2/p)*((b*x^2+
a)^p)^(-2/p)*exp(I*Pi*csgn(I*c*(b*x^2+a)^p)*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*
c))*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*(b*x^2+a)^p))/p)*Ei(1,-2*ln(b*x^2+a)-(I*
Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csg
n(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*
x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)*a^2+1/4
/b/p^3*a*c^(-1/p)*((b*x^2+a)^p)^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)*(
-csgn(I*c*(b*x^2+a)^p)+csgn(I*c))*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*(b*x^2+a)^
p))/p)*Ei(1,-ln(b*x^2+a)-1/2*(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p
)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*
(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a
)^p)-2*p*ln(b*x^2+a))/p)*x^2+1/4/b^2/p^3*a^2*c^(-1/p)*((b*x^2+a)^p)^(-1/p)*
exp(1/2*I*Pi*csgn(I*c*(b*x^2+a)^p)*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*c))*(-csg
n(I*c*(b*x^2+a)^p)+csgn(I*(b*x^2+a)^p))/p)*Ei(1,-ln(b*x^2+a)-1/2*(I*Pi*csgn
(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(
b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^
p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.32

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx =$$

$$-\frac{\left(ap^2\log\left(bx^2+a\right)^2+2\,ap\log\left(bx^2+a\right)\log\left(c\right)+a\log\left(c\right)^2\right)c^{\left(\frac{1}{p}\right)}\operatorname{log\_integral}\left(\left(bx^2+a\right)c^{\left(\frac{1}{p}\right)}\right)+\left(b^2p^2x^4\right.}{}$$

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((a*p^2*log(b*x^2 + a)^2 + 2*a*p*log(b*x^2 + a)*log(c) + a*log(c)^2)*c
^(1/p)*log_integral((b*x^2 + a)*c^(1/p)) + (b^2*p^2*x^4 + a*b*p^2*x^2 + (2*
b^2*p^2*x^4 + 3*a*b*p^2*x^2 + a^2*p^2)*log(b*x^2 + a) + (2*b^2*p*x^4 + 3*a*
b*p*x^2 + a^2*p)*log(c))*c^(2/p) - 4*(p^2*log(b*x^2 + a)^2 + 2*p*log(b*x^2
+ a)*log(c) + log(c)^2)*log_integral((b^2*x^4 + 2*a*b*x^2 + a^2)*c^(2/p)))/
((b^2*p^5*log(b*x^2 + a)^2 + 2*b^2*p^4*log(b*x^2 + a)*log(c) + b^2*p^3*log(
c)^2)*c^(2/p))
```

Sympy [F]

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^3}{\log\left(c\left(a+bx^2\right)^p\right)^3}\,dx$$

```
[In] integrate(x**3/ln(c*(b*x**2+a)**p)**3,x)
```

```
[Out] Integral(x**3/log(c*(a + b*x**2)**p)**3, x)
```

Maxima [F]

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^3}{\log\left(\left(bx^2+a\right)^p c\right)^3}\,dx$$

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(p + 2*log(c))*x^4 + a*b*(p + 3*log(c))*x^2 + a^2*log(c) + (2*b^2
*p*x^4 + 3*a*b*p*x^2 + a^2*p)*log(b*x^2 + a))/(b^2*p^4*log(b*x^2 + a)^2 + 2
*b^2*p^3*log(b*x^2 + a)*log(c) + b^2*p^2*log(c)^2) + integrate(1/2*(4*b*x^3
 + 3*a*x)/(b*p^3*log(b*x^2 + a) + b*p^2*log(c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 874 vs. $2(198) = 396$.

Time = 0.38 (sec) , antiderivative size = 874, normalized size of antiderivative = 4.28

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] 1/4*((b*x^2 + a)*p^2*log(b*x^2 + a)/(b^2*p^5*log(b*x^2 + a)^2 + 2*b^2*p^4*l
og(b*x^2 + a)*log(c) + b^2*p^3*log(c)^2) - p^2*Ei(log(c)/p + log(b*x^2 + a)
```

```
)*log(b*x^2 + a)^2/((b^2*p^5*log(b*x^2 + a)^2 + 2*b^2*p^4*log(b*x^2 + a)*lo
g(c) + b^2*p^3*log(c)^2)*c^(1/p)) + (b*x^2 + a)*p^2/(b^2*p^5*log(b*x^2 + a)
^2 + 2*b^2*p^4*log(b*x^2 + a)*log(c) + b^2*p^3*log(c)^2) + (b*x^2 + a)*p*lo
g(c)/(b^2*p^5*log(b*x^2 + a)^2 + 2*b^2*p^4*log(b*x^2 + a)*log(c) + b^2*p^3*
log(c)^2) - 2*p*Ei(log(c)/p + log(b*x^2 + a))*log(b*x^2 + a)*log(c)/((b^2*p
^5*log(b*x^2 + a)^2 + 2*b^2*p^4*log(b*x^2 + a)*log(c) + b^2*p^3*log(c)^2)*c
^(1/p)) - Ei(log(c)/p + log(b*x^2 + a))*log(c)^2/((b^2*p^5*log(b*x^2 + a)^2
 + 2*b^2*p^4*log(b*x^2 + a)*log(c) + b^2*p^3*log(c)^2)*c^(1/p)))*a - 1/4*(2
*(b*x^2 + a)^2*p^2*log(b*x^2 + a)/(b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x
^2 + a)*log(c) + b*p^3*log(c)^2) + (b*x^2 + a)^2*p^2/(b*p^5*log(b*x^2 + a)^
2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2) - 4*p^2*Ei(2*log(c)/p +
 2*log(b*x^2 + a))*log(b*x^2 + a)^2/((b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(
b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(2/p)) + 2*(b*x^2 + a)^2*p*log(c)/(b*
p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2) - 8*
p*Ei(2*log(c)/p + 2*log(b*x^2 + a))*log(b*x^2 + a)*log(c)/((b*p^5*log(b*x^2
 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(2/p)) - 4*Ei(2
*log(c)/p + 2*log(b*x^2 + a))*log(c)^2/((b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*l
og(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(2/p)))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^3}{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}\,dx$$

```
[In] int(x^3/log(c*(a + b*x^2)^p)^3,x)
[Out] int(x^3/log(c*(a + b*x^2)^p)^3, x)
```

3.117 $\int \frac{x}{\log^3(c(a+bx^2)^p)} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 114

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\frac{\left(a+bx^2\right)\left(c(a+bx^2)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c(a+bx^2)^p\right)}{p}\right)}{4bp^3}-\frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)}-\frac{a+bx^2}{4bp^2\log\left(c\left(a+bx^2\right)^p\right)}$$

```
[Out] 1/4*(b*x^2+a)*Ei(ln(c*(b*x^2+a)^p)/p)/b/p^3/((c*(b*x^2+a)^p)^(1/p))+1/4*(-b
*x^2-a)/b/p/ln(c*(b*x^2+a)^p)^2+1/4*(-b*x^2-a)/b/p^2/ln(c*(b*x^2+a)^p)
```

Rubi [A] (verified)

Time = 0.06 (sec), antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2504, 2436, 2334, 2337, 2209}

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\frac{\left(a+bx^2\right)\left(c(a+bx^2)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c(bx^2+a)^p\right)}{p}\right)}{4bp^3}-\frac{a+bx^2}{4bp^2\log\left(c\left(a+bx^2\right)^p\right)}-\frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)}$$

```
[In] Int[x/Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] ((a + b*x^2)*ExpIntegralEi[Log[c*(a + b*x^2)^p]/p])/(4*b*p^3*(c*(a + b*x^2)
^p)^p^(-1)) - (a + b*x^2)/(4*b*p*Log[c*(a + b*x^2)^p]^2) - (a + b*x^2)/(4*b
*p^2*Log[c*(a + b*x^2)^p])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\log^3\left(c(a+bx)^p\right)}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\log^3(cx^p)}\,dx, x, a+bx^2\right)}{2b}$$

$$= -\frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{1}{\log^2(cx^p)}\,dx, x, a+bx^2\right)}{4bp}$$

$$= -\frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a+bx^2}{4bp^2\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, a+bx^2\right)}{4bp^2}$$

$$= -\frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a+bx^2}{4bp^2\log\left(c\left(a+bx^2\right)^p\right)} + \frac{\left(\left(a+bx^2\right)\left(c(a+bx^2)^p\right)^{-1/p}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c(a+bx^2)^p\right)\right)}{4bp^3}$$

$$= \frac{\left(a+bx^2\right)\left(c(a+bx^2)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c(a+bx^2)^p\right)}{p}\right)}{4bp^3} - \frac{a+bx^2}{4bp\log^2\left(c\left(a+bx^2\right)^p\right)} - \frac{a+bx^2}{4bp^2\log\left(c\left(a+bx^2\right)^p\right)}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.99

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = -\frac{\left(a+bx^2\right)\left(c(a+bx^2)^p\right)^{-1/p}\left(-\text{ExpIntegralEi}\left(\frac{\log(c(a+bx^2)^p)}{p}\right)\log^2\left(c(a+bx^2)^p\right) + p\left(c(a+bx^2)^p\right)^{\frac{1}{p}}\left(p+\right.\right.}{4bp^3\log^2\left(c\left(a+bx^2\right)^p\right)}$$

```
[In] Integrate[x/Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] -1/4*((a + b*x^2)*(-(ExpIntegralEi[Log[c*(a + b*x^2)^p]/p]*Log[c*(a + b*x^2
)^p]^2) + p*(c*(a + b*x^2)^p)^p^(-1)*(p + Log[c*(a + b*x^2)^p])))/(b*p^3*(c
*(a + b*x^2)^p)^p^(-1)*Log[c*(a + b*x^2)^p]^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.95 (sec) , antiderivative size = 716, normalized size of antiderivative = 6.28

method	result
risch	$-\frac{i\pi b\,x^2\,\text{csgn}\left(i\left(b\,x^2+a\right)^p\right)\text{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^2 - i\pi b\,x^2\,\text{csgn}\left(i\left(b\,x^2+a\right)^p\right)\text{csgn}\left(ic\left(b\,x^2+a\right)^p\right)\text{csgn}(ic) - i\pi b\,x^2\text{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^3 + i\pi}{2p^2\left(i\pi\,\text{csgn}\left(i\left(b\,x^2+a\right.\right.\right.}$

```
[In] int(x/ln(c*(b*x^2+a)^p)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(I*Pi*b*x^2*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*b*x^2*csg
n(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*b*x^2*csgn(I*c*(b*x^2
+a)^p)^3+I*Pi*b*x^2*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+I*Pi*a*csgn(I*(b*x^2+
a)^p)*csgn(I*c*(b*x^2+a)^p)^2-I*Pi*a*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)
^p)*csgn(I*c)-I*Pi*a*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*a*csgn(I*c*(b*x^2+a)^p)^2
```

*csgn(I*c)+2*ln(c)*b*x^2+2*b*x^2*ln((b*x^2+a)^p)+2*ln(c)*a+2*a*ln((b*x^2+a)
^p)+2*x^2*p*b+2*a*p)/p^2/(I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)^2-
I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*c)-I*Pi*csgn(I*c*(b*x
^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln(c)+2*ln((b*x^2+a)^p)
)^2/b-1/4/p^3/b*(b*x^2+a)*c^(-1/p)*((b*x^2+a)^p)^(-1/p)*exp(1/2*I*Pi*csgn(I
c(b*x^2+a)^p)*(-csgn(I*c*(b*x^2+a)^p)+csgn(I*c))*(-csgn(I*c*(b*x^2+a)^p)+
csgn(I*(b*x^2+a)^p))/p)*Ei(1,-ln(b*x^2+a)-1/2*(I*Pi*csgn(I*(b*x^2+a)^p)*csg
n(I*c*(b*x^2+a)^p)^2-I*Pi*csgn(I*(b*x^2+a)^p)*csgn(I*c*(b*x^2+a)^p)*csgn(I*
c)-I*Pi*csgn(I*c*(b*x^2+a)^p)^3+I*Pi*csgn(I*c*(b*x^2+a)^p)^2*csgn(I*c)+2*ln
(c)+2*ln((b*x^2+a)^p)-2*p*ln(b*x^2+a))/p)

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.38

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = -\frac{\left(bp^2x^2+ap^2+\left(bp^2x^2+ap^2\right)\log\left(bx^2+a\right)+\left(bpx^2+ap\right)\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}-\Big(p^2\log\left(bx^2+a\right)^2+2\,p\log\left(bx^2 \cdots\right.}{4\,\Big(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c \cdots\right.}$$

[In] integrate(x/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")

[Out] -1/4*((b*p^2*x^2 + a*p^2 + (b*p^2*x^2 + a*p^2)*log(b*x^2 + a) + (b*p*x^2 + a*p)*log(c))*c^(1/p) - (p^2*log(b*x^2 + a)^2 + 2*p*log(b*x^2 + a)*log(c) + log(c)^2)*log_integral((b*x^2 + a)*c^(1/p)))/((b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(1/p))

Sympy [F]

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x}{\log\left(c\left(a+bx^2\right)^p\right)^3}\,dx$$

[In] integrate(x/ln(c*(b*x**2+a)**p)**3,x)

[Out] Integral(x/log(c*(a + b*x**2)**p)**3, x)

Maxima [F]

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x}{\log\left(\left(bx^2+a\right)^p c\right)^3}\,dx$$

```
[In] integrate(x/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b*(p + log(c))*x^2 + a*(p + log(c)) + (b*p*x^2 + a*p)*log(b*x^2 + a))
/(b*p^4*log(b*x^2 + a)^2 + 2*b*p^3*log(b*x^2 + a)*log(c) + b*p^2*log(c)^2)
+ integrate(1/2*x/(p^3*log(b*x^2 + a) + p^2*log(c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(108) = 216$.

Time = 0.34 (sec) , antiderivative size = 406, normalized size of antiderivative = 3.56

$$\begin{aligned}
&\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx \\
&= -\frac{(bx^2+a)p^2\log\left(bx^2+a\right)}{4\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2\right)} \\
&\quad+\frac{p^2\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\log\left(bx^2+a\right)^2}{4\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2\right)c^{\left(\frac{1}{p}\right)}} \\
&\quad-\frac{(bx^2+a)p^2}{4\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2\right)} \\
&\quad-\frac{(bx^2+a)p\log\left(c\right)}{4\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2\right)} \\
&\quad+\frac{p\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\log\left(bx^2+a\right)\log\left(c\right)}{2\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2\right)c^{\left(\frac{1}{p}\right)}} \\
&\quad+\frac{\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(bx^2+a\right)\right)\log\left(c\right)^2}{4\left(bp^5\log\left(bx^2+a\right)^2+2\,bp^4\log\left(bx^2+a\right)\log\left(c\right)+bp^3\log\left(c\right)^2\right)c^{\left(\frac{1}{p}\right)}}
\end{aligned}$$

```
[In] integrate(x/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] -1/4*(b*x^2 + a)*p^2*log(b*x^2 + a)/(b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b
*x^2 + a)*log(c) + b*p^3*log(c)^2) + 1/4*p^2*Ei(log(c)/p + log(b*x^2 + a))*
log(b*x^2 + a)^2/((b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) +
 b*p^3*log(c)^2)*c^(1/p)) - 1/4*(b*x^2 + a)*p^2/(b*p^5*log(b*x^2 + a)^2 + 2
*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2) - 1/4*(b*x^2 + a)*p*log(c)/(
```

```
b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2) +
1/2*p*Ei(log(c)/p + log(b*x^2 + a))*log(b*x^2 + a)*log(c)/((b*p^5*log(b*x^2
 + a)^2 + 2*b*p^4*log(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(1/p)) + 1/4*Ei
(log(c)/p + log(b*x^2 + a))*log(c)^2/((b*p^5*log(b*x^2 + a)^2 + 2*b*p^4*log
(b*x^2 + a)*log(c) + b*p^3*log(c)^2)*c^(1/p))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x}{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)^3}\,dx$$

```
[In] int(x/log(c*(a + b*x^2)^p)^3,x)
[Out] int(x/log(c*(a + b*x^2)^p)^3, x)
```

3.118 $\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \text{Int}\left(\frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x/ln(c*(b*x^2+a)^p)^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

```
[In] Int[1/(x*Log[c*(a + b*x^2)^p]^3),x]
[Out] Defer[Int][1/(x*Log[c*(a + b*x^2)^p]^3), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

```
[In] Integrate[1/(x*Log[c*(a + b*x^2)^p]^3),x]
[Out] Integrate[1/(x*Log[c*(a + b*x^2)^p]^3), x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\ln\left(c\left(b\,x^2+a\right)^p\right)^3}dx$$

```
[In] int(1/x/ln(c*(b*x^2+a)^p)^3,x)
[Out] int(1/x/ln(c*(b*x^2+a)^p)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\log\left(\left(bx^2+a\right)^pc\right)^3}\,dx$$

```
[In] integrate(1/x/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
[Out] integral(1/(x*log((b*x^2 + a)^p*c)^3), x)
```

Sympy [N/A]

Not integrable

Time = 4.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\log\left(c\left(a+bx^2\right)^p\right)^3}\,dx$$

```
[In] integrate(1/x/ln(c*(b*x**2+a)**p)**3,x)
[Out] Integral(1/(x*log(c*(a + b*x**2)**p)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 161, normalized size of antiderivative = 8.94

$$\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\log\left(\left(bx^2+a\right)^p c\right)^3}\,dx$$

```
[In] integrate(1/x/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*p*x^4 + a*b*(p - log(c))*x^2 - a^2*log(c) - (a*b*p*x^2 + a^2*p)*l
og(b*x^2 + a))/(b^2*p^4*x^4*log(b*x^2 + a)^2 + 2*b^2*p^3*x^4*log(b*x^2 + a)
*log(c) + b^2*p^2*x^4*log(c)^2) + integrate(1/2*(a*b*x^2 + 2*a^2)/(b^2*p^3*
x^5*log(b*x^2 + a) + b^2*p^2*x^5*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\log\left(\left(bx^2+a\right)^p c\right)^3}\,dx$$

```
[In] integrate(1/x/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x*log((b*x^2 + a)^p*c)^3), x)
```

Mupad [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{x\ln\left(c\left(b\,x^2+a\right)^p\right)^3}\,dx$$

```
[In] int(1/(x*log(c*(a + b*x^2)^p)^3),x)
```

```
[Out] int(1/(x*log(c*(a + b*x^2)^p)^3), x)
```

3.119 $\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)} \, dx = \text{Int}\left(\frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)}, x\right)$$

```
[Out] Unintegrable(1/x^3/ln(c*(b*x^2+a)^p)^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)} \, dx$$

```
[In] Int[1/(x^3*Log[c*(a + b*x^2)^p]^3),x]
[Out] Defer[Int][1/(x^3*Log[c*(a + b*x^2)^p]^3), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^3 \log^3\left(c\left(a+bx^2\right)^p\right)} \, dx$$

Mathematica [N/A]

Not integrable

Time = 1.92 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^3\left(c \left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^3 \log^3\left(c \left(a+bx^2\right)^p\right)} \, dx$$

```
[In] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]^3),x]
[Out] Integrate[1/(x^3*Log[c*(a + b*x^2)^p]^3), x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 \ln\left(c \left(b\,x^2+a\right)^p\right)^3}dx$$

```
[In] int(1/x^3/ln(c*(b*x^2+a)^p)^3,x)
[Out] int(1/x^3/ln(c*(b*x^2+a)^p)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^3\left(c \left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^3 \log\left(\left(bx^2+a\right)^p c\right)^3} \, dx$$

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
[Out] integral(1/(x^3*log((b*x^2 + a)^p*c)^3), x)
```

Sympy [N/A]

Not integrable

Time = 7.81 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3 \log^3\left(c \left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^3 \log\left(c \left(a+bx^2\right)^p\right)^3} \, dx$$

```
[In] integrate(1/x**3/ln(c*(b*x**2+a)**p)**3,x)
[Out] Integral(1/(x**3*log(c*(a + b*x**2)**p)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 184, normalized size of antiderivative = 10.22

$$\int \frac{1}{x^3 \log^3 \left(c \left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^3 \log \left(\left(bx^2+a\right)^p c\right)^3} \, dx$$

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(p - log(c))*x^4 + a*b*(p - 3*log(c))*x^2 - 2*a^2*log(c) - (b^2*p
*x^4 + 3*a*b*p*x^2 + 2*a^2*p)*log(b*x^2 + a))/(b^2*p^4*x^6*log(b*x^2 + a)^2
 + 2*b^2*p^3*x^6*log(b*x^2 + a)*log(c) + b^2*p^2*x^6*log(c)^2) + integrate(
1/2*(b^2*x^4 + 6*a*b*x^2 + 6*a^2)/(b^2*p^3*x^7*log(b*x^2 + a) + b^2*p^2*x^7
*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^3 \left(c \left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^3 \log \left(\left(bx^2+a\right)^p c\right)^3} \, dx$$

```
[In] integrate(1/x^3/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*log((b*x^2 + a)^p*c)^3), x)
```

Mupad [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^3 \left(c \left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^3 \ln \left(c \left(b\,x^2+a\right)^p\right)^3} \, dx$$

```
[In] int(1/(x^3*log(c*(a + b*x^2)^p)^3),x)
```

```
[Out] int(1/(x^3*log(c*(a + b*x^2)^p)^3), x)
```

3.120 $\int \frac{x^2}{\log^3(c(a+bx^2)^p)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \operatorname{Int}\left(\frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(x^2/ln(c*(b*x^2+a)^p)^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

```
[In] Int[x^2/Log[c*(a + b*x^2)^p]^3,x]
```

```
[Out] Defer[Int][x^2/Log[c*(a + b*x^2)^p]^3, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

```
[In] Integrate[x^2/Log[c*(a + b*x^2)^p]^3,x]
[Out] Integrate[x^2/Log[c*(a + b*x^2)^p]^3, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^{2}}{\ln \left(c \left(b\,x^{2}+a \right)^{p}\right)^{3}}d x$$

```
[In] int(x^2/ln(c*(b*x^2+a)^p)^3,x)
[Out] int(x^2/ln(c*(b*x^2+a)^p)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^{2}}{\log \left({\left(b x^{2}+a\right)}^{p} c\right)^{3}}\,dx$$

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
[Out] integral(x^2/log((b*x^2 + a)^p*c)^3, x)
```

Sympy [N/A]

Not integrable

Time = 3.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{x^{2}}{\log \left(c \left(a+bx^{2}\right)^{p}\right)^{3}}\,dx$$

```
[In] integrate(x**2/ln(c*(b*x**2+a)**p)**3,x)
[Out] Integral(x**2/log(c*(a + b*x**2)**p)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 180, normalized size of antiderivative = 10.00

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^2}{\log\left(\left(bx^2+a\right)^p c\right)^3}\,dx$$

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(b^2*(2*p + 3*log(c))*x^4 + 2*a*b*(p + 2*log(c))*x^2 + a^2*log(c) + (3
*b^2*p*x^4 + 4*a*b*p*x^2 + a^2*p)*log(b*x^2 + a))/(b^2*p^4*x*log(b*x^2 + a)
^2 + 2*b^2*p^3*x*log(b*x^2 + a)*log(c) + b^2*p^2*x*log(c)^2) + integrate(1/
8*(9*b^2*x^4 + 4*a*b*x^2 - a^2)/(b^2*p^3*x^2*log(b*x^2 + a) + b^2*p^2*x^2*l
og(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^2}{\log\left(\left(bx^2+a\right)^p c\right)^3}\,dx$$

```
[In] integrate(x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2/log((b*x^2 + a)^p*c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{x^2}{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}\,dx$$

```
[In] int(x^2/log(c*(a + b*x^2)^p)^3,x)
```

```
[Out] int(x^2/log(c*(a + b*x^2)^p)^3, x)
```

3.121 $\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx = \text{Int}\left(\frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)}, x\right)$$

```
[Out] Unintegrable(1/ln(c*(b*x^2+a)^p)^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx = \int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

```
[In] Int[Log[c*(a + b*x^2)^p]^(-3),x]
```

```
[Out] Defer[Int][Log[c*(a + b*x^2)^p]^(-3), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)} dx$$

Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]^(-3),x]
[Out] Integrate[Log[c*(a + b*x^2)^p]^(-3), x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}dx$$

```
[In] int(1/ln(c*(b*x^2+a)^p)^3,x)
[Out] int(1/ln(c*(b*x^2+a)^p)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{\log\left(\left(bx^2+a\right)^pc\right)^3}\,dx$$

```
[In] integrate(1/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
[Out] integral(log((b*x^2 + a)^p*c)^(-3), x)
```

Sympy [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx=\int \frac{1}{\log\left(c\left(a+bx^2\right)^p\right)^3}\,dx$$

```
[In] integrate(1/ln(c*(b*x**2+a)**p)**3,x)
[Out] Integral(log(c*(a + b*x**2)**p)**(-3), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 164, normalized size of antiderivative = 11.71

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{\log\left(\left(bx^2+a\right)^p c\right)^3}\,dx$$

[In] integrate(1/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")

[Out] -1/8*(b^2*(2*p + log(c))*x^4 + 2*a*b*p*x^2 - a^2*log(c) + (b^2*p*x^4 - a^2*p)*log(b*x^2 + a))/(b^2*p^4*x^3*log(b*x^2 + a)^2 + 2*b^2*p^3*x^3*log(b*x^2 + a)*log(c) + b^2*p^2*x^3*log(c)^2) + integrate(1/8*(b^2*x^4 + 3*a^2)/(b^2*p^3*x^4*log(b*x^2 + a) + b^2*p^2*x^4*log(c)), x)

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{\log\left(\left(bx^2+a\right)^p c\right)^3}\,dx$$

[In] integrate(1/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")

[Out] integrate(log((b*x^2 + a)^p*c)^(-3), x)

Mupad [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{\ln\left(c\left(b\,x^2+a\right)^p\right)^3}\,dx$$

[In] int(1/log(c*(a + b*x^2)^p)^3,x)

[Out] int(1/log(c*(a + b*x^2)^p)^3, x)

3.122 $\int \frac{1}{x^2 \log^3(c(a+bx^2)^p)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \text{Int}\left(\frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^2/ln(c*(b*x^2+a)^p)^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

```
[In] Int[1/(x^2*Log[c*(a + b*x^2)^p]^3),x]
```

```
[Out] Defer[Int][1/(x^2*Log[c*(a + b*x^2)^p]^3), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx$$

```
[In] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]^3),x]
[Out] Integrate[1/(x^2*Log[c*(a + b*x^2)^p]^3), x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \ln\left(c\left(b\,x^2+a\right)^p\right)^3}dx$$

```
[In] int(1/x^2/ln(c*(b*x^2+a)^p)^3,x)
[Out] int(1/x^2/ln(c*(b*x^2+a)^p)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^2 \log\left(\left(bx^2+a\right)^p c\right)^3}\,dx$$

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="fricas")
[Out] integral(1/(x^2*log((b*x^2 + a)^p*c)^3), x)
```

Sympy [N/A]

Not integrable

Time = 5.81 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)}\,dx = \int \frac{1}{x^2 \log\left(c\left(a+bx^2\right)^p\right)^3}\,dx$$

```
[In] integrate(1/x**2/ln(c*(b*x**2+a)**p)**3,x)
[Out] Integral(1/(x**2*log(c*(a + b*x**2)**p)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 187, normalized size of antiderivative = 10.39

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^2 \log\left(\left(bx^2+a\right)^p c\right)^3} \, dx$$

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(b^2*(2*p - log(c))*x^4 + 2*a*b*(p - 2*log(c))*x^2 - 3*a^2*log(c) - (b
^2*p*x^4 + 4*a*b*p*x^2 + 3*a^2*p)*log(b*x^2 + a))/(b^2*p^4*x^5*log(b*x^2 +
a)^2 + 2*b^2*p^3*x^5*log(b*x^2 + a)*log(c) + b^2*p^2*x^5*log(c)^2) + integr
ate(1/8*(b^2*x^4 + 12*a*b*x^2 + 15*a^2)/(b^2*p^3*x^6*log(b*x^2 + a) + b^2*p
^2*x^6*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^2 \log\left(\left(bx^2+a\right)^p c\right)^3} \, dx$$

```
[In] integrate(1/x^2/log(c*(b*x^2+a)^p)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*log((b*x^2 + a)^p*c)^3), x)
```

Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^3\left(c\left(a+bx^2\right)^p\right)} \, dx = \int \frac{1}{x^2 \ln\left(c\left(b\,x^2+a\right)^p\right)^3} \, dx$$

```
[In] int(1/(x^2*log(c*(a + b*x^2)^p)^3),x)
```

```
[Out] int(1/(x^2*log(c*(a + b*x^2)^p)^3), x)
```

3.123 $\int \frac{x^3}{\log(c(a+bx^2))}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)\right)}\,dx = \frac{\text{ExpIntegralEi}\left(2\log\left(c(a+bx^2)\right)\right)}{2b^2c^2} - \frac{a\,\text{LogIntegral}\left(c(a+bx^2)\right)}{2b^2c}$$

```
[Out] 1/2*Ei(2*ln(c*(b*x^2+a)))/b^2/c^2-1/2*a*Li(c*(b*x^2+a))/b^2/c
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2504, 2446, 2436, 2335, 2437, 2346, 2209}

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)\right)}\,dx = \frac{\text{ExpIntegralEi}\left(2\log\left(c(bx^2+a)\right)\right)}{2b^2c^2} - \frac{a\,\text{LogIntegral}\left(c(bx^2+a)\right)}{2b^2c}$$

```
[In] Int[x^3/Log[c*(a + b*x^2)],x]
```

```
[Out] ExpIntegralEi[2*Log[c*(a + b*x^2)]]/(2*b^2*c^2) - (a*LogIntegral[c*(a + b*x
^2)])/(2*b^2*c)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\log(c(a+bx))}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a}{b\log(c(a+bx))}+\frac{a+bx}{b\log(c(a+bx))}\right)\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{a+bx}{\log(c(a+bx))}\,dx, x, x^2\right)}{2b}-\frac{a\text{Subst}\left(\int \frac{1}{\log(c(a+bx))}\,dx, x, x^2\right)}{2b}
\end{aligned}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\log(cx)}\,dx, x, a+bx^2\right)}{2b^2} - \frac{a\text{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, a+bx^2\right)}{2b^2}$$

$$= -\frac{a\text{li}(c(a+bx^2))}{2b^2c} + \frac{\text{Subst}\left(\int \frac{e^{2x}}{x}\,dx, x, \log\left(c(a+bx^2)\right)\right)}{2b^2c^2}$$

$$= \frac{\text{Ei}(2\log\left(c(a+bx^2)\right))}{2b^2c^2} - \frac{a\text{li}(c(a+bx^2))}{2b^2c}$$

Mathematica [F]

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)\right)}\,dx = \int \frac{x^3}{\log\left(c\left(a+bx^2\right)\right)}\,dx$$

```
[In] Integrate[x^3/Log[c*(a + b*x^2)],x]
[Out] Integrate[x^3/Log[c*(a + b*x^2)], x]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{-\operatorname{Ei}_1\left(-2\ln\left(c\left(b\,x^2+a\right)\right)\right)+ca\,\operatorname{Ei}_1\left(-\ln\left(c\left(b\,x^2+a\right)\right)\right)}{2c^2b^2}$	43
risch	$\frac{a\,\operatorname{Ei}_1\left(-\ln\left(c\left(b\,x^2+a\right)\right)\right)}{2c\,b^2} - \frac{\operatorname{Ei}_1\left(-2\ln\left(c\left(b\,x^2+a\right)\right)\right)}{2c^2b^2}$	47

```
[In] int(x^3/ln(c*(b*x^2+a)),x,method=_RETURNVERBOSE)
[Out] 1/2/c^2/b^2*(-Ei(1,-2*ln(c*(b*x^2+a)))+c*a*Ei(1,-ln(c*(b*x^2+a))))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)\right)}\,dx = -\frac{ac\,\text{log\_integral}\left(bcx^2+ac\right) - \text{log\_integral}\left(b^2c^2x^4+2\,abc^2x^2+a^2c^2\right)}{2\,b^2c^2}$$

```
[In] integrate(x^3/log(c*(b*x^2+a)),x, algorithm="fricas")
[Out] -1/2*(a*c*log_integral(b*c*x^2 + a*c) - log_integral(b^2*c^2*x^4 + 2*a*b*c^
2*x^2 + a^2*c^2))/(b^2*c^2)
```

Sympy [F]

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)\right)}\,dx = \int \frac{x^3}{\log\left(ac+bcx^2\right)}\,dx$$

```
[In] integrate(x**3/ln(c*(b*x**2+a)),x)
[Out] Integral(x**3/log(a*c + b*c*x**2), x)
```

Maxima [F]

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)\right)}\,dx = \int \frac{x^3}{\log\left(\left(bx^2+a\right)c\right)}\,dx$$

```
[In] integrate(x^3/log(c*(b*x^2+a)),x, algorithm="maxima")
[Out] integrate(x^3/log((b*x^2 + a)*c), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)\right)}\,dx = -\frac{a\mathrm{Ei}\left(\log\left(bcx^2+ac\right)\right)}{2\,b^2c} + \frac{\mathrm{Ei}\left(2\,\log\left(bcx^2+ac\right)\right)}{2\,b^2c^2}$$

```
[In] integrate(x^3/log(c*(b*x^2+a)),x, algorithm="giac")
[Out] -1/2*a*Ei(log(b*c*x^2 + a*c))/(b^2*c) + 1/2*Ei(2*log(b*c*x^2 + a*c))/(b^2*c
^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\log\left(c\left(a+bx^2\right)\right)}\,dx = \int \frac{x^3}{\ln\left(c\,\left(b\,x^2+a\right)\right)}\,dx$$

```
[In] int(x^3/log(c*(a + b*x^2)),x)
[Out] int(x^3/log(c*(a + b*x^2)), x)
```

3.124 $\int \frac{x}{\log(c(a+bx^2))}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 20

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)\right)}\,dx = \frac{\text{LogIntegral}\left(c(a+bx^2)\right)}{2bc}$$

```
[Out] 1/2*Li(c*(b*x^2+a))/b/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2504, 2436, 2335}

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)\right)}\,dx = \frac{\text{LogIntegral}\left(c(bx^2+a)\right)}{2bc}$$

```
[In] Int[x/Log[c*(a + b*x^2)],x]
```

```
[Out] LogIntegral[c*(a + b*x^2)]/(2*b*c)
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\log(c(a+bx))}\,dx, x, x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, a+bx^2\right)}{2b}\\ &= \frac{\text{li}(c(a+bx^2))}{2bc}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)\right)}\,dx = \frac{\text{LogIntegral}\left(c(a+bx^2)\right)}{2bc}$$

```
[In] Integrate[x/Log[c*(a + b*x^2)],x]
[Out] LogIntegral[c*(a + b*x^2)]/(2*b*c)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

method	result	size
derivativedivides	$-\frac{\text{Ei}_1\left(-\ln\left(c\left(b\,x^2+a\right)\right)\right)}{2bc}$	23
default	$-\frac{\text{Ei}_1\left(-\ln\left(c\left(b\,x^2+a\right)\right)\right)}{2bc}$	23
risch	$-\frac{\text{Ei}_1\left(-\ln\left(c\left(b\,x^2+a\right)\right)\right)}{2bc}$	23

```
[In] int(x/ln(c*(b*x^2+a)),x,method=_RETURNVERBOSE)
[Out] -1/2/b/c*Ei(1,-ln(c*(b*x^2+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{x}{\log \left(c\left(a+bx^2\right)\right)}\,dx = \frac{\text{log\_integral}\left(bcx^2+ac\right)}{2\,bc}$$

```
[In] integrate(x/log(c*(b*x^2+a)),x, algorithm="fricas")
[Out] 1/2*log_integral(b*c*x^2 + a*c)/(b*c)
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{x}{\log \left(c\left(a+bx^2\right)\right)}\,dx = \begin{cases} \frac{x^2}{2\log(ac)} & \text{for } b=0 \\ 0 & \text{for } c=0 \\ \frac{\text{Ei}\left(\log\left(ac+bcx^2\right)\right)}{2bc} & \text{otherwise} \end{cases}$$

```
[In] integrate(x/ln(c*(b*x**2+a)),x)
[Out] Piecewise((x**2/(2*log(a*c)), Eq(b, 0)), (0, Eq(c, 0)), (Ei(log(a*c + b*c*x
**2))/(2*b*c), True))
```

Maxima [F]

$$\int \frac{x}{\log \left(c\left(a+bx^2\right)\right)}\,dx = \int \frac{x}{\log \left((bx^2+a)c\right)}\,dx$$

```
[In] integrate(x/log(c*(b*x^2+a)),x, algorithm="maxima")
[Out] integrate(x/log((b*x^2 + a)*c), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)\right)}\,dx = \frac{\mathrm{Ei}\left(\log\left(bcx^2+ac\right)\right)}{2\,bc}$$

```
[In] integrate(x/log(c*(b*x^2+a)),x, algorithm="giac")
```

```
[Out] 1/2*Ei(log(b*c*x^2 + a*c))/(b*c)
```

Mupad [B] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x}{\log\left(c\left(a+bx^2\right)\right)}\,dx = \frac{\mathrm{logint}\left(c\left(b\,x^2+a\right)\right)}{2\,b\,c}$$

```
[In] int(x/log(c*(a + b*x^2)),x)
```

```
[Out] logint(c*(a + b*x^2))/(2*b*c)
```

3.125 $\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 71

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx = \frac{\operatorname{ExpIntegralEi}\left(2\log\left(c(a+bx^2)\right)\right)}{b^2c^2} - \frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} - \frac{a\operatorname{LogIntegral}\left(c(a+bx^2)\right)}{2b^2c}$$

```
[Out] Ei(2*ln(c*(b*x^2+a)))/b^2/c^2-1/2*a*Li(c*(b*x^2+a))/b^2/c-1/2*x^2*(b*x^2+a)
/b/ln(c*(b*x^2+a))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2504, 2447, 2446, 2436, 2335, 2437, 2346, 2209}

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx = \frac{\operatorname{ExpIntegralEi}\left(2\log\left(c(bx^2+a)\right)\right)}{b^2c^2} - \frac{a\operatorname{LogIntegral}\left(c(bx^2+a)\right)}{2b^2c} - \frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)}$$

```
[In] Int[x^3/Log[c*(a + b*x^2)]^2,x]
```

```
[Out] ExpIntegralEi[2*Log[c*(a + b*x^2)]]/(b^2*c^2) - (x^2*(a + b*x^2))/(2*b*Log[
c*(a + b*x^2)]) - (a*LogIntegral[c*(a + b*x^2)])/(2*b^2*c)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x}{\log^2(c(a+bx))}\,dx, x, x^2\right)$$

$$= -\frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{a\text{Subst}\left(\int \frac{1}{\log(c(a+bx))}\,dx, x, x^2\right)}{2b} + \text{Subst}\left(\int \frac{x}{\log(c(a+bx))}\,dx, x, x^2\right)$$

$$= -\frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{a\text{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, a+bx^2\right)}{2b^2} + \text{Subst}\left(\int \left(-\frac{a}{b\log(c(a+bx))} + \frac{a+bx}{b\log(c(a+bx))}\right)\,dx, x, x^2\right)$$

$$= -\frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{a\,\text{li}(c(a+bx^2))}{2b^2c} + \frac{\text{Subst}\left(\int \frac{a+bx}{\log(c(a+bx))}\,dx, x, x^2\right)}{b} - \frac{a\text{Subst}\left(\int \frac{1}{\log(c(a+bx))}\,dx, x, x^2\right)}{b}$$

$$= -\frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{a\,\text{li}(c(a+bx^2))}{2b^2c} + \frac{\text{Subst}\left(\int \frac{x}{\log(cx)}\,dx, x, a+bx^2\right)}{b^2} - \frac{a\text{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, a+bx^2\right)}{b^2}$$

$$= -\frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} - \frac{a\,\text{li}(c(a+bx^2))}{2b^2c} + \frac{\text{Subst}\left(\int \frac{e^{2x}}{x}\,dx, x, \log\left(c(a+bx^2)\right)\right)}{b^2c^2}$$

$$= \frac{\text{Ei}\left(2\log\left(c(a+bx^2)\right)\right)}{b^2c^2} - \frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} - \frac{a\,\text{li}(c(a+bx^2))}{2b^2c}$$

Mathematica [F]

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx = \int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx$$

```
[In] Integrate[x^3/Log[c*(a + b*x^2)]^2,x]
[Out] Integrate[x^3/Log[c*(a + b*x^2)]^2, x]
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

method	result	size
risch	$-\frac{x^{2}\left(b\,x^{2}+a\right)}{2 b \ln\left(c\left(b\,x^{2}+a\right)\right)}+\frac{a\,\mathrm{Ei}_{1}\left(-\ln\left(c\left(b\,x^{2}+a\right)\right)\right)}{2 c\,b^{2}}-\frac{\mathrm{Ei}_{1}\left(-2\ln\left(c\left(b\,x^{2}+a\right)\right)\right)}{c^{2} b^{2}}$	74
default	$\frac{-\frac{c^{2}\left(b\,x^{2}+a\right)^{2}}{\ln\left(c\left(b\,x^{2}+a\right)\right)}-2\,\mathrm{Ei}_{1}\left(-2\ln\left(c\left(b\,x^{2}+a\right)\right)\right)-ca\left(-\frac{c\left(b\,x^{2}+a\right)}{\ln\left(c\left(b\,x^{2}+a\right)\right)}-\mathrm{Ei}_{1}\left(-\ln\left(c\left(b\,x^{2}+a\right)\right)\right)\right)}{2 c^{2} b^{2}}$	95

```
[In] int(x^3/ln(c*(b*x^2+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^2*(b*x^2+a)/b/ln(c*(b*x^2+a))+1/2/c/b^2*a*Ei(1,-ln(c*(b*x^2+a)))-1/c
^2/b^2*Ei(1,-2*ln(c*(b*x^2+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.39

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx = -\frac{b^2c^2x^4+abc^2x^2+\left(ac\log\_integral\left(bcx^2+ac\right)-2\,\log\_integral\left(b^2c^2x^4+2\,abc^2x^2+a^2c^2\right)\right)\log\left(bcx^2+\right.}{2\,b^2c^2\log\left(bcx^2+ac\right)}$$

```
[In] integrate(x^3/log(c*(b*x^2+a))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*c^2*x^4 + a*b*c^2*x^2 + (a*c*log_integral(b*c*x^2 + a*c) - 2*log_
integral(b^2*c^2*x^4 + 2*a*b*c^2*x^2 + a^2*c^2))*log(b*c*x^2 + a*c))/(b^2*c
^2*log(b*c*x^2 + a*c))
```

Sympy [F]

$$\int \frac{x^3}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx = \frac{-ax^2-bx^4}{2b\log\left(c\left(a+bx^2\right)\right)}+\frac{\int \frac{ax}{\log\left(ac+bcx^2\right)}\,dx+\int \frac{2bx^3}{\log\left(ac+bcx^2\right)}\,dx}{b}$$

```
[In] integrate(x**3/ln(c*(b*x**2+a))**2,x)
```

```
[Out] (-a*x**2 - b*x**4)/(2*b*log(c*(a + b*x**2))) + (Integral(a*x/log(a*c + b*c*
x**2), x) + Integral(2*b*x**3/log(a*c + b*c*x**2), x))/b
```

Maxima [F]

$$\int \frac{x^3}{\log^2(c(a+bx^2))}\,dx = \int \frac{x^3}{\log\left((bx^2+a)c\right)^2}\,dx$$

```
[In] integrate(x^3/log(c*(b*x^2+a))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^4 + a*x^2)/(b*log(b*x^2 + a) + b*log(c)) + integrate((2*b*x^3 + a
*x)/(b*log(b*x^2 + a) + b*log(c)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.41

$$\int \frac{x^3}{\log^2(c(a+bx^2))}\,dx = \frac{a\left(\frac{bcx^2+ac}{\log(bcx^2+ac)} - \operatorname{Ei}(\log(bcx^2+ac))\right)}{2\,b^2c} - \frac{\frac{(bcx^2+ac)^2}{\log(bcx^2+ac)} - 2\operatorname{Ei}(2\log(bcx^2+ac))}{2\,b^2c^2}$$

```
[In] integrate(x^3/log(c*(b*x^2+a))^2,x, algorithm="giac")
```

```
[Out] 1/2*a*((b*c*x^2 + a*c)/log(b*c*x^2 + a*c) - Ei(log(b*c*x^2 + a*c)))/(b^2*c)
 - 1/2*((b*c*x^2 + a*c)^2/log(b*c*x^2 + a*c) - 2*Ei(2*log(b*c*x^2 + a*c)))/
(b^2*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\log^2(c(a+bx^2))}\,dx = \int \frac{x^3}{\ln\left(c\,\left(b\,x^2+a\right)\right)^2}\,dx$$

```
[In] int(x^3/log(c*(a + b*x^2))^2,x)
```

```
[Out] int(x^3/log(c*(a + b*x^2))^2, x)
```

3.126 $\int \frac{x}{\log^2(c(a+bx^2))} dx$

Optimal result

Integrand size = 14, antiderivative size = 47

$$\int \frac{x}{\log^2(c(a+bx^2))} dx = -\frac{a+bx^2}{2b\log(c(a+bx^2))} + \frac{\text{LogIntegral}(c(a+bx^2))}{2bc}$$

```
[Out] 1/2*Li(c*(b*x^2+a))/b/c+1/2*(-b*x^2-a)/b/ln(c*(b*x^2+a))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2504, 2436, 2334, 2335}

$$\int \frac{x}{\log^2(c(a+bx^2))} dx = \frac{\text{LogIntegral}(c(bx^2+a))}{2bc} - \frac{a+bx^2}{2b\log(c(a+bx^2))}$$

```
[In] Int[x/Log[c*(a + b*x^2)]^2,x]
```

```
[Out] -1/2*(a + b*x^2)/(b*Log[c*(a + b*x^2)]) + LogIntegral[c*(a + b*x^2)]/(2*b*c
)
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\log^2(c(a+bx))}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\log^2(cx)}\,dx, x, a+bx^2\right)}{2b}\\
&= -\frac{a+bx^2}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{\text{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, a+bx^2\right)}{2b}\\
&= -\frac{a+bx^2}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{\text{li}(c(a+bx^2))}{2bc}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx = \frac{-\frac{a+bx^2}{\log(c(a+bx^2))} + \frac{\text{LogIntegral}(c(a+bx^2))}{c}}{2b}$$

```
[In] Integrate[x/Log[c*(a + b*x^2)]^2,x]
[Out] (-((a + b*x^2)/Log[c*(a + b*x^2)]) + LogIntegral[c*(a + b*x^2)]/c)/(2*b)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{-\frac{c\left(b\,x^{2}+a\right)}{\ln\left(c\left(b\,x^{2}+a\right)\right)}-\mathrm{Ei}_{1}\left(-\ln\left(c\left(b\,x^{2}+a\right)\right)\right)}{2bc}$	48
default	$\frac{-\frac{c\left(b\,x^{2}+a\right)}{\ln\left(c\left(b\,x^{2}+a\right)\right)}-\mathrm{Ei}_{1}\left(-\ln\left(c\left(b\,x^{2}+a\right)\right)\right)}{2bc}$	48
risch	$-\frac{b\,x^{2}+a}{2\ln\left(c\left(b\,x^{2}+a\right)\right)b}-\frac{\mathrm{Ei}_{1}\left(-\ln\left(c\left(b\,x^{2}+a\right)\right)\right)}{2bc}$	48

```
[In] int(x/ln(c*(b*x^2+a))^2,x,method=_RETURNVERBOSE)
[Out] 1/2/b/c*(-1/ln(c*(b*x^2+a))*c*(b*x^2+a)-Ei(1,-ln(c*(b*x^2+a))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.17

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx = -\frac{bcx^2+ac-\log\left(bcx^2+ac\right)\operatorname{log\_integral}\left(bcx^2+ac\right)}{2\,bc\log\left(bcx^2+ac\right)}$$

```
[In] integrate(x/log(c*(b*x^2+a))^2,x, algorithm="fricas")
[Out] -1/2*(b*c*x^2 + a*c - log(b*c*x^2 + a*c)*log_integral(b*c*x^2 + a*c))/(b*c*
log(b*c*x^2 + a*c))
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx = \begin{cases} \frac{x^2}{2\log\left(ac\right)} & \text{for}\ b=0 \\ 0 & \text{for}\ c=0 \\ \frac{\mathrm{Ei}\left(\log\left(ac+bcx^2\right)\right)}{2bc} & \text{otherwise} \end{cases} + \frac{-a-bx^2}{2b\log\left(c\left(a+bx^2\right)\right)}$$

```
[In] integrate(x/ln(c*(b*x**2+a))**2,x)
[Out] Piecewise((x**2/(2*log(a*c)), Eq(b, 0)), (0, Eq(c, 0)), (Ei(log(a*c + b*c*x
**2))/(2*b*c), True)) + (-a - b*x**2)/(2*b*log(c*(a + b*x**2)))
```

Maxima [F]

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx = \int \frac{x}{\log\left((bx^2+a)c\right)^2}\,dx$$

```
[In] integrate(x/log(c*(b*x^2+a))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x^2 + a)/(b*log(b*x^2 + a) + b*log(c)) + integrate(x/(log(b*x^2 + a
) + log(c)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx = -\frac{\frac{bcx^2+ac}{\log(bcx^2+ac)} - \mathrm{Ei}\left(\log\left(bcx^2+ac\right)\right)}{2\,bc}$$

```
[In] integrate(x/log(c*(b*x^2+a))^2,x, algorithm="giac")
```

```
[Out] -1/2*((b*c*x^2 + a*c)/log(b*c*x^2 + a*c) - Ei(log(b*c*x^2 + a*c)))/(b*c)
```

Mupad [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{x}{\log^2\left(c\left(a+bx^2\right)\right)}\,dx = \frac{\mathrm{logint}\left(c\left(b\,x^2+a\right)\right)}{2\,b\,c} - \frac{\frac{b\,x^2}{2}+\frac{a}{2}}{b\,\ln\left(c\,\left(b\,x^2+a\right)\right)}$$

```
[In] int(x/log(c*(a + b*x^2))^2,x)
```

```
[Out] logint(c*(a + b*x^2))/(2*b*c) - (a/2 + (b*x^2)/2)/(b*log(c*(a + b*x^2)))
```

3.127 $\int \frac{x^3}{\log^3(c(a+bx^2))} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 127

$$\int \frac{x^3}{\log^3(c(a+bx^2))} \, dx = \frac{\text{ExpIntegralEi}(2\log(c(a+bx^2)))}{b^2c^2} - \frac{x^2(a+bx^2)}{4b\log^2(c(a+bx^2))} - \frac{a(a+bx^2)}{4b^2\log(c(a+bx^2))} - \frac{x^2(a+bx^2)}{2b\log(c(a+bx^2))} - \frac{a\,\text{LogIntegral}(c(a+bx^2))}{4b^2c}$$

```
[Out] Ei(2*ln(c*(b*x^2+a)))/b^2/c^2-1/4*a*Li(c*(b*x^2+a))/b^2/c-1/4*x^2*(b*x^2+a)
/b/ln(c*(b*x^2+a))^2-1/4*a*(b*x^2+a)/b^2/ln(c*(b*x^2+a))-1/2*x^2*(b*x^2+a)/
b/ln(c*(b*x^2+a))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {2504, 2447, 2446, 2436, 2335, 2437, 2346, 2209, 2334}

$$\int \frac{x^3}{\log^3(c(a+bx^2))} \, dx = \frac{\text{ExpIntegralEi}(2\log(c(bx^2+a)))}{b^2c^2} - \frac{a\,\text{LogIntegral}(c(bx^2+a))}{4b^2c} - \frac{a(a+bx^2)}{4b^2\log(c(a+bx^2))} - \frac{x^2(a+bx^2)}{4b\log^2(c(a+bx^2))} - \frac{x^2(a+bx^2)}{2b\log(c(a+bx^2))}$$

```
[In] Int[x^3/Log[c*(a + b*x^2)]^3,x]
```

```
[Out] ExpIntegralEi[2*Log[c*(a + b*x^2)]]/(b^2*c^2) - (x^2*(a + b*x^2))/(4*b*Log[
c*(a + b*x^2)]^2) - (a*(a + b*x^2))/(4*b^2*Log[c*(a + b*x^2)]) - (x^2*(a +
b*x^2))/(2*b*Log[c*(a + b*x^2)]) - (a*LogIntegral[c*(a + b*x^2)])/(4*b^2*c)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
```

```
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\log^3(c(a+bx))}\,dx, x, x^2\right) \\
&= -\frac{x^2(a+bx^2)}{4b\log^2\left(c\left(a+bx^2\right)\right)} + \frac{1}{2}\text{Subst}\left(\int \frac{x}{\log^2(c(a+bx))}\,dx, x, x^2\right) \\
&\quad + \frac{a\text{Subst}\left(\int \frac{1}{\log^2(c(a+bx))}\,dx, x, x^2\right)}{4b} \\
&= -\frac{x^2(a+bx^2)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{a\text{Subst}\left(\int \frac{1}{\log^2(cx)}\,dx, x, a+bx^2\right)}{4b^2} \\
&\quad + \frac{a\text{Subst}\left(\int \frac{1}{\log(c(a+bx))}\,dx, x, x^2\right)}{2b} + \text{Subst}\left(\int \frac{x}{\log(c(a+bx))}\,dx, x, x^2\right) \\
&= -\frac{x^2(a+bx^2)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a(a+bx^2)}{4b^2\log\left(c\left(a+bx^2\right)\right)} - \frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} \\
&\quad + \frac{a\text{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, a+bx^2\right)}{4b^2} + \frac{a\text{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, a+bx^2\right)}{2b^2} \\
&\quad + \text{Subst}\left(\int \left(-\frac{a}{b\log(c(a+bx))} + \frac{a+bx}{b\log(c(a+bx))}\right)dx, x, x^2\right) \\
&= -\frac{x^2(a+bx^2)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a(a+bx^2)}{4b^2\log\left(c\left(a+bx^2\right)\right)} - \frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{3a\text{li}(c(a+bx^2))}{4b^2c} \\
&\quad + \frac{\text{Subst}\left(\int \frac{a+bx}{\log(c(a+bx))}\,dx, x, x^2\right)}{b} - \frac{a\text{Subst}\left(\int \frac{1}{\log(c(a+bx))}\,dx, x, x^2\right)}{b}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{x^2(a+bx^2)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a(a+bx^2)}{4b^2\log\left(c\left(a+bx^2\right)\right)} - \frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} + \frac{3a\operatorname{li}(c(a+bx^2))}{4b^2c} \\
&\quad + \frac{\operatorname{Subst}\left(\int \frac{x}{\log(cx)}\,dx, x, a+bx^2\right)}{b^2} - \frac{a\operatorname{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, a+bx^2\right)}{b^2} \\
&= -\frac{x^2(a+bx^2)}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a(a+bx^2)}{4b^2\log\left(c\left(a+bx^2\right)\right)} - \frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} \\
&\quad - \frac{a\operatorname{li}(c(a+bx^2))}{4b^2c} + \frac{\operatorname{Subst}\left(\int \frac{e^{2x}}{x}\,dx, x, \log\left(c(a+bx^2)\right)\right)}{b^2c^2} \\
&= \frac{\operatorname{Ei}\left(2\log\left(c(a+bx^2)\right)\right)}{b^2c^2} - \frac{x^2(a+bx^2)}{4b\log^2\left(c\left(a+bx^2\right)\right)} \\
&\quad - \frac{a(a+bx^2)}{4b^2\log\left(c\left(a+bx^2\right)\right)} - \frac{x^2(a+bx^2)}{2b\log\left(c\left(a+bx^2\right)\right)} - \frac{a\operatorname{li}(c(a+bx^2))}{4b^2c}
\end{aligned}
$$

Mathematica [F]

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = \int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx$$

```
[In] Integrate[x^3/Log[c*(a + b*x^2)]^3,x]
[Out] Integrate[x^3/Log[c*(a + b*x^2)]^3, x]
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.83

method	result	si
risch	$-\frac{\left(b\,x^2+a\right)\left(2\ln\left(c\left(b\,x^2+a\right)\right)b\,x^2+b\,x^2+\ln\left(c\left(b\,x^2+a\right)\right)a\right)}{4b^2\ln\left(c\left(b\,x^2+a\right)\right)^2}+\frac{a\ \mathrm{Ei}_1\left(-\ln\left(c\left(b\,x^2+a\right)\right)\right)}{4c\,b^2}-\frac{\mathrm{Ei}_1\left(-2\ln\left(c\left(b\,x^2+a\right)\right)\right)}{c^2b^2}$	10
default	$\frac{-\frac{c^2\left(b\,x^2+a\right)^2}{2\ln\left(c\left(b\,x^2+a\right)\right)^2}-\frac{c^2\left(b\,x^2+a\right)^2}{\ln\left(c\left(b\,x^2+a\right)\right)}-2\ \mathrm{Ei}_1\left(-2\ln\left(c\left(b\,x^2+a\right)\right)\right)-ca\left(-\frac{c\left(b\,x^2+a\right)}{2\ln\left(c\left(b\,x^2+a\right)\right)^2}-\frac{c\left(b\,x^2+a\right)}{2\ln\left(c\left(b\,x^2+a\right)\right)}-\frac{\mathrm{Ei}_1\left(-\ln\left(c\left(b\,x^2+a\right)\right)\right)}{2}\right)}{2c^2b^2}$	14

```
[In] int(x^3/ln(c*(b*x^2+a))^3,x,method=_RETURNVERBOSE)
[Out] -1/4*(b*x^2+a)*(2*ln(c*(b*x^2+a))*b*x^2+b*x^2+ln(c*(b*x^2+a))*a)/b^2/ln(c*(
b*x^2+a))^2+1/4/c/b^2*a*Ei(1,-ln(c*(b*x^2+a)))-1/c^2/b^2*Ei(1,-2*ln(c*(b*x^
2+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.12

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = -\frac{b^2c^2x^4+abc^2x^2+\left(ac\log\_integral\left(bcx^2+ac\right)-4\log\_integral\left(b^2c^2x^4+2abc^2x^2+a^2c^2\right)\right)\log\left(bcx^2+\right.}{4b^2c^2\log\left(bcx^2+ac\right)^2}$$

[In] integrate(x^3/log(c*(b*x^2+a))^3,x, algorithm="fricas")

[Out] -1/4*(b^2*c^2*x^4 + a*b*c^2*x^2 + (a*c*log_integral(b*c*x^2 + a*c) - 4*log_integral(b^2*c^2*x^4 + 2*a*b*c^2*x^2 + a^2*c^2))*log(b*c*x^2 + a*c)^2 + (2*b^2*c^2*x^4 + 3*a*b*c^2*x^2 + a^2*c^2)*log(b*c*x^2 + a*c))/(b^2*c^2*log(b*c*x^2 + a*c)^2)

Sympy [F]

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = \frac{\int \frac{3ax}{\log\left(ac+bcx^2\right)}\,dx+\int \frac{4bx^3}{\log\left(ac+bcx^2\right)}\,dx}{2b} + \frac{-abx^2-b^2x^4+\left(-a^2-3abx^2-2b^2x^4\right)\log\left(c(a+bx^2)\right)}{4b^2\log\left(c\left(a+bx^2\right)\right)^2}$$

[In] integrate(x**3/ln(c*(b*x**2+a))**3,x)

[Out] (Integral(3*a*x/log(a*c + b*c*x**2), x) + Integral(4*b*x**3/log(a*c + b*c*x**2), x))/(2*b) + (-a*b*x**2 - b**2*x**4 + (-a**2 - 3*a*b*x**2 - 2*b**2*x**4)*log(c*(a + b*x**2)))/(4*b**2*log(c*(a + b*x**2))**2)

Maxima [F]

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = \int \frac{x^3}{\log\left(\left(bx^2+a\right)c\right)^3}\,dx$$

[In] integrate(x^3/log(c*(b*x^2+a))^3,x, algorithm="maxima")

[Out] -1/4*(b^2*x^4*(2*log(c) + 1) + a*b*x^2*(3*log(c) + 1) + a^2*log(c) + (2*b^2*x^4 + 3*a*b*x^2 + a^2)*log(b*x^2 + a))/(b^2*log(b*x^2 + a)^2 + 2*b^2*log(b*x^2 + a)*log(c) + b^2*log(c)^2) + integrate(1/2*(4*b*x^3 + 3*a*x)/(b*log(b*x^2 + a) + b*log(c)), x)

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.19

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = \frac{a\left(\frac{bcx^2+ac}{\log(bcx^2+ac)} + \frac{bcx^2+ac}{\log(bcx^2+ac)^2} - \mathrm{Ei}(\log\left(bcx^2+ac\right))\right)}{4\,b^2c} - \frac{\frac{2\left(bcx^2+ac\right)^2}{\log(bcx^2+ac)} + \frac{\left(bcx^2+ac\right)^2}{\log(bcx^2+ac)^2} - 4\,\mathrm{Ei}(2\,\log\left(bcx^2+ac\right))}{4\,b^2c^2}$$

```
[In] integrate(x^3/log(c*(b*x^2+a))^3,x, algorithm="giac")
```

```
[Out] 1/4*a*((b*c*x^2 + a*c)/log(b*c*x^2 + a*c) + (b*c*x^2 + a*c)/log(b*c*x^2 + a
*c)^2 - Ei(log(b*c*x^2 + a*c)))/(b^2*c) - 1/4*(2*(b*c*x^2 + a*c)^2/log(b*c*
x^2 + a*c) + (b*c*x^2 + a*c)^2/log(b*c*x^2 + a*c)^2 - 4*Ei(2*log(b*c*x^2 +
a*c)))/(b^2*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = \int \frac{x^3}{\ln\left(c\,\left(b\,x^2+a\right)\right)^3}\,dx$$

```
[In] int(x^3/log(c*(a + b*x^2))^3,x)
```

```
[Out] int(x^3/log(c*(a + b*x^2))^3, x)
```

3.128 $\int \frac{x}{\log^3(c(a+bx^2))}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 73

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = -\frac{a+bx^2}{4b\log^2\left(c\left(a+bx^2\right)\right)}-\frac{a+bx^2}{4b\log\left(c\left(a+bx^2\right)\right)}+\frac{\text{LogIntegral}\left(c(a+bx^2)\right)}{4bc}$$

```
[Out] 1/4*Li(c*(b*x^2+a))/b/c+1/4*(-b*x^2-a)/b/ln(c*(b*x^2+a))^2+1/4*(-b*x^2-a)/b
/ln(c*(b*x^2+a))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2504, 2436, 2334, 2335}

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = \frac{\text{LogIntegral}\left(c(bx^2+a)\right)}{4bc}-\frac{a+bx^2}{4b\log^2\left(c\left(a+bx^2\right)\right)}-\frac{a+bx^2}{4b\log\left(c\left(a+bx^2\right)\right)}$$

```
[In] Int[x/Log[c*(a + b*x^2)]^3,x]
```

```
[Out] -1/4*(a + b*x^2)/(b*Log[c*(a + b*x^2)]^2) - (a + b*x^2)/(4*b*Log[c*(a + b*x
^2)]) + LogIntegral[c*(a + b*x^2)]/(4*b*c)
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
```

```
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\log^3(c(a+bx))}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\log^3(cx)}\,dx, x, a+bx^2\right)}{2b}\\
&= -\frac{a+bx^2}{4b\log^2\left(c\left(a+bx^2\right)\right)} + \frac{\text{Subst}\left(\int \frac{1}{\log^2(cx)}\,dx, x, a+bx^2\right)}{4b}\\
&= -\frac{a+bx^2}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a+bx^2}{4b\log\left(c\left(a+bx^2\right)\right)} + \frac{\text{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, a+bx^2\right)}{4b}\\
&= -\frac{a+bx^2}{4b\log^2\left(c\left(a+bx^2\right)\right)} - \frac{a+bx^2}{4b\log\left(c\left(a+bx^2\right)\right)} + \frac{\text{li}(c(a+bx^2))}{4bc}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx=\frac{-\frac{\left(a+bx^2\right)\left(1+\log\left(c\left(a+bx^2\right)\right)\right)}{\log^2\left(c\left(a+bx^2\right)\right)}+\frac{\text{LogIntegral}\left(c\left(a+bx^2\right)\right)}{c}}{4b}$$

```
[In] Integrate[x/Log[c*(a + b*x^2)]^3,x]
```

```
[Out] (-(((a + b*x^2)*(1 + Log[c*(a + b*x^2)]))/Log[c*(a + b*x^2)]^2) + LogIntegr
al[c*(a + b*x^2)]/c)/(4*b)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{-\frac{c\left(b\,x^2+a\right)}{2\ln\left(c\left(b\,x^2+a\right)\right)^2}-\frac{c\left(b\,x^2+a\right)}{2\ln\left(c\left(b\,x^2+a\right)\right)}-\frac{\mathrm{Ei}_1\left(-\ln\left(c\left(b\,x^2+a\right)\right)\right)}{2}}{2bc}$	70
default	$\frac{-\frac{c\left(b\,x^2+a\right)}{2\ln\left(c\left(b\,x^2+a\right)\right)^2}-\frac{c\left(b\,x^2+a\right)}{2\ln\left(c\left(b\,x^2+a\right)\right)}-\frac{\mathrm{Ei}_1\left(-\ln\left(c\left(b\,x^2+a\right)\right)\right)}{2}}{2bc}$	70
risch	$-\frac{\ln(c(b\,x^2+a))b\,x^2+b\,x^2+\ln(c(b\,x^2+a))a+a}{4b\ln(c(b\,x^2+a))^2}-\frac{\mathrm{Ei}_1\left(-\ln\left(c\left(b\,x^2+a\right)\right)\right)}{4bc}$	75

```
[In] int(x/ln(c*(b*x^2+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b/c*(-1/2/ln(c*(b*x^2+a))^2*c*(b*x^2+a)-1/2/ln(c*(b*x^2+a))*c*(b*x^2+a)
-1/2*Ei(1,-ln(c*(b*x^2+a))))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = -\frac{bcx^2-\log\left(bcx^2+ac\right)^2\text{log\_integral}\left(bcx^2+ac\right)+ac+\left(bcx^2+ac\right)\log\left(bcx^2+ac\right)}{4\,bc\log\left(bcx^2+ac\right)^2}$$

```
[In] integrate(x/log(c*(b*x^2+a))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*c*x^2 - log(b*c*x^2 + a*c)^2*log_integral(b*c*x^2 + a*c) + a*c + (b
*c*x^2 + a*c)*log(b*c*x^2 + a*c))/(b*c*log(b*c*x^2 + a*c)^2)
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = \frac{\begin{cases} \frac{x^2}{2\log\left(ac\right)} & \text{for } b=0 \\ 0 & \text{for } c=0 \\ \frac{\operatorname{Ei}\left(\log\left(ac+bcx^2\right)\right)}{2bc} & \text{otherwise} \end{cases}}{2} + \frac{-a-bx^2+\left(-a-bx^2\right)\log\left(c(a+bx^2)\right)}{4b\log\left(c\left(a+bx^2\right)\right)^2}$$

```
[In] integrate(x/ln(c*(b*x**2+a))**3,x)
```

```
[Out] Piecewise((x**2/(2*log(a*c)), Eq(b, 0)), (0, Eq(c, 0)), (Ei(log(a*c + b*c*x
**2))/(2*b*c), True))/2 + (-a - b*x**2 + (-a - b*x**2)*log(c*(a + b*x**2)))
/(4*b*log(c*(a + b*x**2))**2)
```

Maxima [F]

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = \int \frac{x}{\log\left(\left(bx^2+a\right)c\right)^3}\,dx$$

```
[In] integrate(x/log(c*(b*x^2+a))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b*x^2*(log(c) + 1) + a*(log(c) + 1) + (b*x^2 + a)*log(b*x^2 + a))/(b*
log(b*x^2 + a)^2 + 2*b*log(b*x^2 + a)*log(c) + b*log(c)^2) + integrate(1/2*
x/(log(b*x^2 + a) + log(c)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = -\frac{\frac{bcx^2+ac}{\log(bcx^2+ac)} + \frac{bcx^2+ac}{\log(bcx^2+ac)^2} - \operatorname{Ei}\left(\log\left(bcx^2+ac\right)\right)}{4\,bc}$$

```
[In] integrate(x/log(c*(b*x^2+a))^3,x, algorithm="giac")
```

```
[Out] -1/4*((b*c*x^2 + a*c)/log(b*c*x^2 + a*c) + (b*c*x^2 + a*c)/log(b*c*x^2 + a*
c)^2 - Ei(log(b*c*x^2 + a*c)))/(b*c)
```

Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int \frac{x}{\log^3\left(c\left(a+bx^2\right)\right)}\,dx = \frac{\mathrm{logint}(c\,(b\,x^2+a))}{4\,b\,c} - \frac{\frac{a\,c}{4}+\ln\left(c\,(b\,x^2+a)\right)\,\left(\frac{b\,c\,x^2}{4}+\frac{a\,c}{4}\right)+\frac{b\,c\,x^2}{4}}{b\,c\ln\left(c\,\left(b\,x^2+a\right)\right)^2}$$

```
[In] int(x/log(c*(a + b*x^2))^3,x)
```

```
[Out] logint(c*(a + b*x^2))/(4*b*c) - ((a*c)/4 + log(c*(a + b*x^2))*((a*c)/4 + (b
*c*x^2)/4) + (b*c*x^2)/4)/(b*c*log(c*(a + b*x^2))^2)
```

3.129 $\int x^5 \log^2\left(c(d+ex^3)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 150

$$\int x^5 \log^2\left(c(d+ex^3)^p\right)\,dx = -\frac{2dp^2x^3}{3e}+\frac{p^2(d+ex^3)^2}{12e^2}+\frac{2dp(d+ex^3)\log\left(c(d+ex^3)^p\right)}{3e^2} - \frac{p(d+ex^3)^2\log\left(c(d+ex^3)^p\right)}{6e^2} - \frac{d(d+ex^3)\log^2\left(c(d+ex^3)^p\right)}{3e^2} + \frac{(d+ex^3)^2\log^2\left(c(d+ex^3)^p\right)}{6e^2}$$

```
[Out] -2/3*d*p^2*x^3/e+1/12*p^2*(e*x^3+d)^2/e^2+2/3*d*p*(e*x^3+d)*ln(c*(e*x^3+d)^
p)/e^2-1/6*p*(e*x^3+d)^2*ln(c*(e*x^3+d)^p)/e^2-1/3*d*(e*x^3+d)*ln(c*(e*x^3+
d)^p)^2/e^2+1/6*(e*x^3+d)^2*ln(c*(e*x^3+d)^p)^2/e^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\int x^5 \log^2\left(c(d+ex^3)^p\right)\,dx = \frac{(d+ex^3)^2\log^2\left(c(d+ex^3)^p\right)}{6e^2} - \frac{d(d+ex^3)\log^2\left(c(d+ex^3)^p\right)}{3e^2} - \frac{p(d+ex^3)^2\log\left(c(d+ex^3)^p\right)}{6e^2} + \frac{2dp(d+ex^3)\log\left(c(d+ex^3)^p\right)}{3e^2}+\frac{p^2(d+ex^3)^2}{12e^2}-\frac{2dp^2x^3}{3e}$$

```
[In] Int[x^5*Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (-2*d*p^2*x^3)/(3*e) + (p^2*(d + e*x^3)^2)/(12*e^2) + (2*d*p*(d + e*x^3)*Lo
g[c*(d + e*x^3)^p])/(3*e^2) - (p*(d + e*x^3)^2*Log[c*(d + e*x^3)^p])/(6*e^2
) - (d*(d + e*x^3)*Log[c*(d + e*x^3)^p]^2)/(3*e^2) + ((d + e*x^3)^2*Log[c*(
d + e*x^3)^p]^2)/(6*e^2)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
```

```
+ e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x\log^2\left(c(d+ex)^p\right)\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int\left(-\frac{d\log^2\left(c(d+ex)^p\right)}{e}+\frac{(d+ex)\log^2\left(c(d+ex)^p\right)}{e}\right)\,dx, x, x^3\right)\\
&= \frac{\text{Subst}\left(\int(d+ex)\log^2\left(c(d+ex)^p\right)\,dx, x, x^3\right)}{3e}-\frac{d\text{Subst}\left(\int\log^2\left(c(d+ex)^p\right)\,dx, x, x^3\right)}{3e}\\
&= \frac{\text{Subst}\left(\int x\log^2\left(cx^p\right)\,dx, x, d+ex^3\right)}{3e^2}-\frac{d\text{Subst}\left(\int\log^2\left(cx^p\right)\,dx, x, d+ex^3\right)}{3e^2}\\
&= -\frac{d(d+ex^3)\log^2\left(c(d+ex^3)^p\right)}{3e^2}+\frac{(d+ex^3)^2\log^2\left(c(d+ex^3)^p\right)}{6e^2}\\
&\quad-\frac{p\text{Subst}\left(\int x\log\left(cx^p\right)\,dx, x, d+ex^3\right)}{3e^2}+\frac{(2dp)\text{Subst}\left(\int\log\left(cx^p\right)\,dx, x, d+ex^3\right)}{3e^2}\\
&= -\frac{2dp^2x^3}{3e}+\frac{p^2(d+ex^3)^2}{12e^2}+\frac{2dp(d+ex^3)\log\left(c(d+ex^3)^p\right)}{3e^2}\\
&\quad-\frac{p(d+ex^3)^2\log\left(c(d+ex^3)^p\right)}{6e^2}\\
&\quad-\frac{d(d+ex^3)\log^2\left(c(d+ex^3)^p\right)}{3e^2}+\frac{(d+ex^3)^2\log^2\left(c(d+ex^3)^p\right)}{6e^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.70

$$\int x^5 \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = \frac{ep^2x^3(-6d+ex^3)+2d^2p^2\log\left(d+ex^3\right)+2p(2d^2+2dex^3-e^2x^6)\log\left(c\left(d+ex^3\right)^p\right)-2(d^2-e^2x^6)\log^2\left(\right.}{12e^2}$$

```
[In] Integrate[x^5*Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (e*p^2*x^3*(-6*d + e*x^3) + 2*d^2*p^2*Log[d + e*x^3] + 2*p*(2*d^2 + 2*d*e*x
^3 - e^2*x^6)*Log[c*(d + e*x^3)^p] - 2*(d^2 - e^2*x^6)*Log[c*(d + e*x^3)^p]
^2)/(12*e^2)
```

Maple [A] (verified)

Time = 4.79 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01

method	result
parallelrisch	$\frac{2x^6\ln\left(c\left(e\,x^3+d\right)^p\right)^2e^2-2x^6\ln\left(c\left(e\,x^3+d\right)^p\right)e^2p+e^2p^2x^6+4x^3\ln\left(c\left(e\,x^3+d\right)^p\right)dep-6de\,p^2x^3+10d^2p^2\ln\left(e\,x^3+d\right)-2\ln\left(c\left(e\,x^3+\right.\right.}{12e^2}$
risch	Expression too large to display

```
[In] int(x^5*ln(c*(e*x^3+d)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(2*x^6*ln(c*(e*x^3+d)^p)^2*e^2-2*x^6*ln(c*(e*x^3+d)^p)*e^2*p+e^2*p^2*x
^6+4*x^3*ln(c*(e*x^3+d)^p)*d*e*p-6*d*e*p^2*x^3+10*d^2*p^2*ln(e*x^3+d)-2*ln(
c*(e*x^3+d)^p)^2*d^2-4*ln(c*(e*x^3+d)^p)*d^2*p+6*d^2*p^2)/e^2
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99

$$\int x^5 \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = \frac{e^2p^2x^6+2\,e^2x^6\log\left(c\right)^2-6\,dep^2x^3+2\left(e^2p^2x^6-d^2p^2\right)\log\left(ex^3+d\right)^2-2\left(e^2p^2x^6-2\,dep^2x^3-3\,d^2p^2-2\right.}{12\,e^2}$$

```
[In] integrate(x^5*log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(e^2*p^2*x^6 + 2*e^2*x^6*log(c)^2 - 6*d*e*p^2*x^3 + 2*(e^2*p^2*x^6 - d
^2*p^2)*log(e*x^3 + d)^2 - 2*(e^2*p^2*x^6 - 2*d*e*p^2*x^3 - 3*d^2*p^2 - 2*(
e^2*p*x^6 - d^2*p)*log(c))*log(e*x^3 + d) - 2*(e^2*p*x^6 - 2*d*e*p*x^3)*log
(c))/e^2
```

Sympy [A] (verification not implemented)

Time = 3.67 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.91

$$\int x^5 \log^2\left(c\left(d+ex^3\right)^p\right)\,dx$$
$$=\begin{cases} \frac{d^2p\log\left(c\left(d+ex^3\right)^p\right)}{2e^2}-\frac{d^2\log\left(c\left(d+ex^3\right)^p\right)^2}{6e^2}-\frac{dp^2x^3}{2e}+\frac{dpx^3\log\left(c\left(d+ex^3\right)^p\right)}{3e}+\frac{p^2x^6}{12}-\frac{px^6\log\left(c\left(d+ex^3\right)^p\right)}{6}+\frac{x^6\log\left(c\left(d+ex^3\right)^p\right)^2}{6} \\ \frac{x^6\log\left(cd^p\right)^2}{6} \end{cases}$$

```
[In] integrate(x**5*ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Piecewise((d**2*p*log(c*(d + e*x**3)**p)/(2*e**2) - d**2*log(c*(d + e*x**3)
**p)**2/(6*e**2) - d*p**2*x**3/(2*e) + d*p*x**3*log(c*(d + e*x**3)**p)/(3*e
) + p**2*x**6/12 - p*x**6*log(c*(d + e*x**3)**p)/6 + x**6*log(c*(d + e*x**3
)**p)**2/6, Ne(e, 0)), (x**6*log(c*d**p)**2/6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.80

$$\int x^5\log^2\left(c\left(d+ex^3\right)^p\right)\,dx=\frac{1}{6}\,x^6\log\left(\left(ex^3+d\right)^pc\right)^2-\frac{1}{6}\,ep\left(\frac{2\,d^2\log\left(ex^3+d\right)}{e^3}+\frac{ex^6-2\,dx^3}{e^2}\right)\log\left(\left(ex^3+d\right)^pc\right)+\frac{\left(e^2x^6-6\,dex^3+2\,d^2\log\left(ex^3+d\right)^2+6\,d^2\log\left(ex^3+d\right)\right)p^2}{12\,e^2}$$

```
[In] integrate(x^5*log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*log((e*x^3 + d)^p*c)^2 - 1/6*e*p*(2*d^2*log(e*x^3 + d)/e^3 + (e*x^6
 - 2*d*x^3)/e^2)*log((e*x^3 + d)^p*c) + 1/12*(e^2*x^6 - 6*d*e*x^3 + 2*d^2*l
og(e*x^3 + d)^2 + 6*d^2*log(e*x^3 + d))*p^2/e^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.44

$$\int x^5 \log^2\left(c(d+ex^3)^p\right)\,dx$$
$$= \frac{2\left(ex^3+d\right)^2 p^2 \log\left(ex^3+d\right)^2 - 2\left(ex^3+d\right)^2 p^2 \log\left(ex^3+d\right) + 4\left(ex^3+d\right)^2 p \log\left(ex^3+d\right)\log\left(c\right) + \left(ex^3+d\right.}{12\,e^2}$$
$$- \frac{\left(2\,ex^3 + \left(ex^3+d\right)\log\left(ex^3+d\right)^2 - 2\left(ex^3+d\right)\log\left(ex^3+d\right) + 2\,d\right)dp^2 - 2\left(ex^3 - \left(ex^3+d\right)\log\left(ex^3+\right.\right.}{3\,e^2}$$

```
[In] integrate(x^5*log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] 1/12*(2*(e*x^3 + d)^2*p^2*log(e*x^3 + d)^2 - 2*(e*x^3 + d)^2*p^2*log(e*x^3
+ d) + 4*(e*x^3 + d)^2*p*log(e*x^3 + d)*log(c) + (e*x^3 + d)^2*p^2 - 2*(e*x
^3 + d)^2*p*log(c) + 2*(e*x^3 + d)^2*log(c)^2)/e^2 - 1/3*((2*e*x^3 + (e*x^3
 + d)*log(e*x^3 + d)^2 - 2*(e*x^3 + d)*log(e*x^3 + d) + 2*d)*d*p^2 - 2*(e*x
^3 - (e*x^3 + d)*log(e*x^3 + d) + d)*d*p*log(c) + (e*x^3 + d)*d*log(c)^2)/e
^2
```

Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.67

$$\int x^5 \log^2\left(c(d+ex^3)^p\right)\,dx = \frac{p^2\,x^6}{12} - \ln\left(c\left(e\,x^3+d\right)^p\right)\left(\frac{p\,x^6}{6} - \frac{d\,p\,x^3}{3\,e}\right) + \ln\left(c\left(e\,x^3+d\right)^p\right)^2\left(\frac{x^6}{6} - \frac{d^2}{6\,e^2}\right) - \frac{d\,p^2\,x^3}{2\,e} + \frac{d^2\,p^2\,\ln\left(e\,x^3+d\right)}{2\,e^2}$$

```
[In] int(x^5*log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] (p^2*x^6)/12 - log(c*(d + e*x^3)^p)*((p*x^6)/6 - (d*p*x^3)/(3*e)) + log(c*(
d + e*x^3)^p)^2*(x^6/6 - d^2/(6*e^2)) - (d*p^2*x^3)/(2*e) + (d^2*p^2*log(d
+ e*x^3))/(2*e^2)
```

3.130 $\int x^2 \log^2\left(c(d+ex^3)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 66

$$\int x^2 \log^2\left(c(d+ex^3)^p\right)\,dx = \frac{2p^2x^3}{3} - \frac{2p(d+ex^3)\log\left(c(d+ex^3)^p\right)}{3e} + \frac{(d+ex^3)\log^2\left(c(d+ex^3)^p\right)}{3e}$$

```
[Out] 2/3*p^2*x^3-2/3*p*(e*x^3+d)*ln(c*(e*x^3+d)^p)/e+1/3*(e*x^3+d)*ln(c*(e*x^3+d
)^p)^2/e
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2504, 2436, 2333, 2332}

$$\int x^2 \log^2\left(c(d+ex^3)^p\right)\,dx = \frac{(d+ex^3)\log^2\left(c(d+ex^3)^p\right)}{3e} - \frac{2p(d+ex^3)\log\left(c(d+ex^3)^p\right)}{3e} + \frac{2p^2x^3}{3}$$

```
[In] Int[x^2*Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (2*p^2*x^3)/3 - (2*p*(d + e*x^3)*Log[c*(d + e*x^3)^p])/(3*e) + ((d + e*x^3)
*Log[c*(d + e*x^3)^p]^2)/(3*e)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \log^2\left(c(d+ex)^p\right)\,dx, x, x^3\right)\\
&= \frac{\text{Subst}\left(\int \log^2\left(cx^p\right)\,dx, x, d+ex^3\right)}{3e}\\
&= \frac{(d+ex^3)\log^2\left(c\left(d+ex^3\right)^p\right)}{3e} - \frac{(2p)\text{Subst}\left(\int \log\left(cx^p\right)\,dx, x, d+ex^3\right)}{3e}\\
&= \frac{2p^2x^3}{3} - \frac{2p(d+ex^3)\log\left(c\left(d+ex^3\right)^p\right)}{3e} + \frac{(d+ex^3)\log^2\left(c\left(d+ex^3\right)^p\right)}{3e}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$
\int x^2\log^2\left(c\left(d+ex^3\right)^p\right)\,dx = \frac{1}{3}\left(\frac{(d+ex^3)\log^2\left(c\left(d+ex^3\right)^p\right)}{e} - 2p\left(-px^3 + \frac{(d+ex^3)\log\left(c\left(d+ex^3\right)^p\right)}{e}\right)\right)
$$

```
[In] Integrate[x^2*Log[c*(d + e*x^3)^p]^2,x]
[Out] (((d + e*x^3)*Log[c*(d + e*x^3)^p]^2)/e - 2*p*(-(p*x^3) + ((d + e*x^3)*Log[
c*(d + e*x^3)^p])/e))/3
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.53

method	result	size
parallelrisch	$\frac{x^{3} \ln\left(c \left(e \,x^{3}+d \right)^{p}\right)^{2} e p -2 x^{3} \ln\left(c \left(e \,x^{3}+d \right)^{p}\right) e \,p^{2}+2 x^{3} e \,p^{3}+\ln\left(c \left(e \,x^{3}+d \right)^{p}\right)^{2} d p -2 \ln\left(c \left(e \,x^{3}+d \right)^{p}\right) d \,p^{2}-2 d \,p^{3}}{3 p e}$	101
risch	Expression too large to display	1036

```
[In] int(x^2*ln(c*(e*x^3+d)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(x^3*ln(c*(e*x^3+d)^p)^2*e*p-2*x^3*ln(c*(e*x^3+d)^p)*e*p^2+2*x^3*e*p^3+
ln(c*(e*x^3+d)^p)^2*d*p-2*ln(c*(e*x^3+d)^p)*d*p^2-2*d*p^3)/p/e
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.45

$$\int x^2 \log^2\left(c\left(d+ex^3\right)^p\right)\,dx$$
$$=\frac{2\,ep^2x^3-2\,epx^3\log(c)+ex^3\log(c)^2+(ep^2x^3+dp^2)\log(ex^3+d)^2-2\,(ep^2x^3+dp^2-(epx^3+dp)\log(c)}{3\,e}$$

```
[In] integrate(x^2*log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*e*p^2*x^3 - 2*e*p*x^3*log(c) + e*x^3*log(c)^2 + (e*p^2*x^3 + d*p^2)*
log(e*x^3 + d)^2 - 2*(e*p^2*x^3 + d*p^2 - (e*p*x^3 + d*p)*log(c))*log(e*x^3
 + d))/e
```

Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.52

$$\int x^2 \log^2\left(c\left(d+ex^3\right)^p\right)\,dx$$
$$=\begin{cases} -\frac{2dp\log\left(c(d+ex^3)^p\right)}{3e}+\frac{d\log\left(c(d+ex^3)^p\right)^2}{3e}+\frac{2p^2x^3}{3}-\frac{2px^3\log\left(c(d+ex^3)^p\right)}{3}+\frac{x^3\log\left(c(d+ex^3)^p\right)^2}{3} & \text{for } e\neq 0 \\ \frac{x^3\log\left(cd^p\right)^2}{3} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Piecewise((-2*d*p*log(c*(d + e*x**3)**p)/(3*e) + d*log(c*(d + e*x**3)**p)**
2/(3*e) + 2*p**2*x**3/3 - 2*p*x**3*log(c*(d + e*x**3)**p)/3 + x**3*log(c*(d
 + e*x**3)**p)**2/3, Ne(e, 0)), (x**3*log(c*d**p)**2/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.47

$$\int x^2 \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = \frac{1}{3}\,x^3\log\left(\left(ex^3+d\right)^p c\right)^2 - \frac{2}{3}\left(\frac{x^3}{e} - \frac{d\log\left(ex^3+d\right)}{e^2}\right)ep\log\left(\left(ex^3+d\right)^p c\right) + \frac{\left(2\,ex^3 - d\log\left(ex^3+d\right)^2 - 2\,d\log\left(ex^3+d\right)\right)p^2}{3\,e}$$

```
[In] integrate(x^2*log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((e*x^3 + d)^p*c)^2 - 2/3*(x^3/e - d*log(e*x^3 + d)/e^2)*e*p*log
((e*x^3 + d)^p*c) + 1/3*(2*e*x^3 - d*log(e*x^3 + d)^2 - 2*d*log(e*x^3 + d))
*p^2/e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.45

$$\int x^2 \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = \frac{\left(2\,ex^3 + \left(ex^3+d\right)\log\left(ex^3+d\right)^2 - 2\left(ex^3+d\right)\log\left(ex^3+d\right) + 2\,d\right)p^2 - 2\left(ex^3 - \left(ex^3+d\right)\log\left(ex^3+d\right) -}{3\,e}$$

```
[In] integrate(x^2*log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] 1/3*((2*e*x^3 + (e*x^3 + d)*log(e*x^3 + d)^2 - 2*(e*x^3 + d)*log(e*x^3 + d)
 + 2*d)*p^2 - 2*(e*x^3 - (e*x^3 + d)*log(e*x^3 + d) + d)*p*log(c) + (e*x^3
+ d)*log(c)^2)/e
```

Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

$$\int x^2 \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = \frac{2\,p^2\,x^3}{3} + \ln\left(c\left(e\,x^3+d\right)^p\right)^2\left(\frac{d}{3\,e} + \frac{x^3}{3}\right) - \frac{2\,p\,x^3\,\ln\left(c\left(e\,x^3+d\right)^p\right)}{3} - \frac{2\,d\,p^2\,\ln\left(e\,x^3+d\right)}{3\,e}$$

```
[In] int(x^2*log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] (2*p^2*x^3)/3 + log(c*(d + e*x^3)^p)^2*(d/(3*e) + x^3/3) - (2*p*x^3*log(c*(
d + e*x^3)^p))/3 - (2*d*p^2*log(d + e*x^3))/(3*e)
```

3.131 $\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 77

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x}\,dx = \frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right) + \frac{2}{3}p\log\left(c\left(d+ex^3\right)^p\right)\operatorname{PolyLog}\left(2,1+\frac{ex^3}{d}\right) - \frac{2}{3}p^2\operatorname{PolyLog}\left(3,1+\frac{ex^3}{d}\right)$$

```
[Out] 1/3*ln(-e*x^3/d)*ln(c*(e*x^3+d)^p)^2+2/3*p*ln(c*(e*x^3+d)^p)*polylog(2,1+e*
x^3/d)-2/3*p^2*polylog(3,1+e*x^3/d)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2504, 2443, 2481, 2421, 6724}

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x}\,dx = \frac{2}{3}p\operatorname{PolyLog}\left(2,\frac{ex^3}{d}+1\right)\log\left(c\left(d+ex^3\right)^p\right) + \frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right) - \frac{2}{3}p^2\operatorname{PolyLog}\left(3,\frac{ex^3}{d}+1\right)$$

```
[In] Int[Log[c*(d + e*x^3)^p]^2/x,x]
```

```
[Out] (Log[-((e*x^3)/d)]*Log[c*(d + e*x^3)^p]^2)/3 + (2*p*Log[c*(d + e*x^3)^p]*Po
lyLog[2, 1 + (e*x^3)/d])/3 - (2*p^2*PolyLog[3, 1 + (e*x^3)/d])/3
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{\log^2\left(c(d+ex)^p\right)}{x}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right) - \frac{1}{3}(2ep)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)\log\left(c(d+ex)^p\right)}{d+ex}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right) - \frac{1}{3}(2p)\text{Subst}\left(\int \frac{\log\left(cx^p\right)\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}dx, x, d+ex^3\right)$$

$$= \frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right) + \frac{2}{3}p\log\left(c\left(d+ex^3\right)^p\right)\text{Li}_2\left(1+\frac{ex^3}{d}\right) - \frac{1}{3}\left(2p^2\right)\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{x}{d}\right)}{x}dx, x, d+ex^3\right)$$

$$= \frac{1}{3}\log\left(-\frac{ex^3}{d}\right)\log^2\left(c\left(d+ex^3\right)^p\right) + \frac{2}{3}p\log\left(c\left(d+ex^3\right)^p\right)\text{Li}_2\left(1+\frac{ex^3}{d}\right) - \frac{2}{3}p^2\text{Li}_3\left(1+\frac{ex^3}{d}\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 2965, normalized size of antiderivative = 38.51

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x}dx = \text{Result too large to show}$$

```
[In] Integrate[Log[c*(d + e*x^3)^p]^2/x,x]
```

```
[Out] Log[x]*(-(p*Log[d + e*x^3]) + Log[c*(d + e*x^3)^p])^2 + 2*p*(-(p*Log[d + e*
x^3]) + Log[c*(d + e*x^3)^p])*(Log[x]*(Log[d + e*x^3] - Log[1 + (e*x^3)/d])
 - PolyLog[2, -((e*x^3)/d)]/3) + p^2*(Log[-((e^(1/3)*x)/d^(1/3))]*Log[d^(1/
3)/e^(1/3) + x]^2 + 2*Log[-((e^(1/3)*x)/d^(1/3))]*Log[d^(1/3)/e^(1/3) + x]*
Log[-(((-1)^(1/3)*d^(1/3))/e^(1/3)) + x] + Log[-(((-1)^(2/3)*e^(1/3)*x)/d^(
1/3))]*Log[-(((-1)^(1/3)*d^(1/3))/e^(1/3)) + x]^2 + 2*Log[-((e^(1/3)*x)/d^(
1/3))]*Log[d^(1/3)/e^(1/3) + x]*Log[((-1)^(2/3)*d^(1/3))/e^(1/3) + x] + 2*L
og[-(((-1)^(2/3)*e^(1/3)*x)/d^(1/3))]*Log[-(((-1)^(1/3)*d^(1/3))/e^(1/3)) +
 x]*Log[((-1)^(2/3)*d^(1/3))/e^(1/3) + x] + Log[((-1)^(1/3)*e^(1/3)*x)/d^(1
/3)]*Log[((-1)^(2/3)*d^(1/3))/e^(1/3) + x]^2 + Log[((-1)^(2/3)*(((-1)^(2/3)
*d^(1/3))/e^(1/3) + x))/(-(((-1)^(1/3)*d^(1/3))/e^(1/3)) + x)]^2*(Log[-(((-
1)^(2/3)*e^(1/3)*x)/d^(1/3))] + Log[(I*Sqrt[3]*d^(1/3))/((-1)^(1/3)*d^(1/3)
 - e^(1/3)*x)] - Log[((-1)^(2/3)*(1 + (-1)^(1/3))*e^(1/3)*x)/((-1)^(1/3)*d^
(1/3) - e^(1/3)*x)]) + (Log[-((e^(1/3)*x)/d^(1/3))] + Log[-(((-1 + (-1)^(2/
3))*d^(1/3))/(d^(1/3) + e^(1/3)*x))] - Log[((1 + (-1)^(1/3))*e^(1/3)*x)/(d^
(1/3) + e^(1/3)*x)])*Log[(d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/(d^(1/3) + e^(1/3
)*x)]^2 + (Log[2] + Log[-((e^(1/3)*x)/d^(1/3))] + Log[((1 + (-1)^(1/3))*d^(
```

```
1/3))/(d^(1/3) + e^(1/3)*x)] - Log[((3 - I*Sqrt[3])*e^(1/3)*x)/(d^(1/3) + e
^(1/3)*x)])*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/(d^(1/3) + e^(1/3)*x)]^2 +
 2*(Log[((-1)^(1/3)*e^(1/3)*x)/d^(1/3)] - Log[-(((-1)^(2/3)*e^(1/3)*x)/d^(1
/3))])*Log[((-1)^(2/3)*(((-1)^(2/3)*d^(1/3))/e^(1/3) + x))/(-(((-1)^(1/3)*d
^(1/3))/e^(1/3)) + x)]*Log[1 - ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)] + 2*(-Log[-(
(e^(1/3)*x)/d^(1/3))] + Log[((-1)^(1/3)*e^(1/3)*x)/d^(1/3)])*Log[(d^(1/3) -
 (-1)^(1/3)*e^(1/3)*x)/(d^(1/3) + e^(1/3)*x)]*Log[1 - ((-1)^(1/3)*e^(1/3)*x
)/d^(1/3)] + (Log[-((e^(1/3)*x)/d^(1/3))] - Log[((-1)^(1/3)*e^(1/3)*x)/d^(1
/3)])*Log[1 - ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)]*(-2*Log[d^(1/3)/e^(1/3) + x]
+ Log[1 - ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)]) + (-Log[((-1)^(1/3)*e^(1/3)*x)/d
^(1/3)] + Log[-(((-1)^(2/3)*e^(1/3)*x)/d^(1/3))])*Log[1 - ((-1)^(1/3)*e^(1/
3)*x)/d^(1/3)]*(-2*Log[-(((-1)^(1/3)*d^(1/3))/e^(1/3)) + x] + Log[1 - ((-1)
^(1/3)*e^(1/3)*x)/d^(1/3)]) + 2*(-Log[-((e^(1/3)*x)/d^(1/3))] + Log[-(((-1)
^(2/3)*e^(1/3)*x)/d^(1/3))])*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/(d^(1/3)
+ e^(1/3)*x)]*Log[1 + ((-1)^(2/3)*e^(1/3)*x)/d^(1/3)] + (Log[-((e^(1/3)*x)/
d^(1/3))] - Log[-(((-1)^(2/3)*e^(1/3)*x)/d^(1/3))])*Log[1 + ((-1)^(2/3)*e^(
1/3)*x)/d^(1/3)]*(-2*Log[d^(1/3)/e^(1/3) + x] + Log[1 + ((-1)^(2/3)*e^(1/3)
*x)/d^(1/3)]) + Log[x]*(Log[d^(1/3)/e^(1/3) + x] + Log[-(((-1)^(1/3)*d^(1/3
))/e^(1/3)) + x] + Log[((-1)^(2/3)*d^(1/3))/e^(1/3) + x] - Log[d + e*x^3])^
2 - 2*(Log[d^(1/3)/e^(1/3) + x] + Log[-(((-1)^(1/3)*d^(1/3))/e^(1/3)) + x]
+ Log[((-1)^(2/3)*d^(1/3))/e^(1/3) + x] - Log[d + e*x^3])*(Log[x]*Log[d^(1/
3)/e^(1/3) + x] + Log[x]*Log[-(((-1)^(1/3)*d^(1/3))/e^(1/3)) + x] + Log[x]*
Log[((-1)^(2/3)*d^(1/3))/e^(1/3) + x] - Log[x]*Log[1 + (e^(1/3)*x)/d^(1/3)]
 - Log[x]*Log[1 - ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)] - Log[x]*Log[1 + ((-1)^(2
/3)*e^(1/3)*x)/d^(1/3)] - PolyLog[2, -((e^(1/3)*x)/d^(1/3))] - PolyLog[2, (
(-1)^(1/3)*e^(1/3)*x)/d^(1/3)] - PolyLog[2, -(((-1)^(2/3)*e^(1/3)*x)/d^(1/3
))]) + 2*Log[((-1)^(2/3)*(((-1)^(2/3)*d^(1/3))/e^(1/3) + x))/(-(((-1)^(1/3)
*d^(1/3))/e^(1/3)) + x)]*(-PolyLog[2, ((-1)^(2/3)*(((-1)^(2/3)*d^(1/3))/e^(
1/3) + x))/(-(((-1)^(1/3)*d^(1/3))/e^(1/3)) + x)] + PolyLog[2, ((-1)^(2/3)*
d^(1/3) + e^(1/3)*x)/(-((-1)^(1/3)*d^(1/3)) + e^(1/3)*x)]) + 2*Log[(d^(1/3)
 - (-1)^(1/3)*e^(1/3)*x)/(d^(1/3) + e^(1/3)*x)]*(PolyLog[2, ((-1)^(2/3)*d^(
1/3) + e^(1/3)*x)/(d^(1/3) + e^(1/3)*x)] - PolyLog[2, (d^(1/3) - (-1)^(1/3)
*e^(1/3)*x)/(d^(1/3) + e^(1/3)*x)]) + 2*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x
)/(d^(1/3) + e^(1/3)*x)]*(PolyLog[2, (-((-1)^(1/3)*d^(1/3)) + e^(1/3)*x)/(d
^(1/3) + e^(1/3)*x)] - PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/(d^(1/3)
 + e^(1/3)*x)]) + 2*Log[d^(1/3)/e^(1/3) + x]*PolyLog[2, 1 + (e^(1/3)*x)/d^(
1/3)] + 2*(Log[((-1)^(2/3)*d^(1/3))/e^(1/3) + x] - Log[(d^(1/3) - (-1)^(1/3
)*e^(1/3)*x)/(d^(1/3) + e^(1/3)*x)])*PolyLog[2, 1 + (e^(1/3)*x)/d^(1/3)] +
2*(Log[-(((-1)^(1/3)*d^(1/3))/e^(1/3)) + x] - Log[(d^(1/3) + (-1)^(2/3)*e^(
1/3)*x)/(d^(1/3) + e^(1/3)*x)])*PolyLog[2, 1 + (e^(1/3)*x)/d^(1/3)] + 2*Log
[((-1)^(2/3)*d^(1/3))/e^(1/3) + x]*PolyLog[2, 1 - ((-1)^(1/3)*e^(1/3)*x)/d^
(1/3)] + 2*(Log[-(((-1)^(1/3)*d^(1/3))/e^(1/3)) + x] + Log[((-1)^(2/3)*(((-
1)^(2/3)*d^(1/3))/e^(1/3) + x))/(-(((-1)^(1/3)*d^(1/3))/e^(1/3)) + x)])*Pol
yLog[2, 1 - ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)] + 2*(Log[d^(1/3)/e^(1/3) + x] +
 Log[(d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/(d^(1/3) + e^(1/3)*x)])*PolyLog[2, 1
```

```
- ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)] + 2*Log[-(((-1)^(1/3)*d^(1/3))/e^(1/3)) +
 x]*PolyLog[2, 1 + ((-1)^(2/3)*e^(1/3)*x)/d^(1/3)] + 2*(Log[((-1)^(2/3)*d^(
1/3))/e^(1/3) + x] - Log[((-1)^(2/3)*(((-1)^(2/3)*d^(1/3))/e^(1/3) + x))/(-
(((-1)^(1/3)*d^(1/3))/e^(1/3)) + x)])*PolyLog[2, 1 + ((-1)^(2/3)*e^(1/3)*x)
/d^(1/3)] + 2*(Log[d^(1/3)/e^(1/3) + x] + Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)
*x)/(d^(1/3) + e^(1/3)*x)])*PolyLog[2, 1 + ((-1)^(2/3)*e^(1/3)*x)/d^(1/3)]
+ 2*PolyLog[3, ((-1)^(2/3)*(((-1)^(2/3)*d^(1/3))/e^(1/3) + x))/(-(((-1)^(1/
3)*d^(1/3))/e^(1/3)) + x)] - 2*PolyLog[3, (-((-1)^(1/3)*d^(1/3)) + e^(1/3)*
x)/(d^(1/3) + e^(1/3)*x)] - 2*PolyLog[3, ((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/(
d^(1/3) + e^(1/3)*x)] - 2*PolyLog[3, ((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/(-((-
1)^(1/3)*d^(1/3)) + e^(1/3)*x)] + 2*PolyLog[3, (d^(1/3) - (-1)^(1/3)*e^(1/3
)*x)/(d^(1/3) + e^(1/3)*x)] + 2*PolyLog[3, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)
/(d^(1/3) + e^(1/3)*x)] - 6*PolyLog[3, 1 + (e^(1/3)*x)/d^(1/3)] - 6*PolyLog
[3, 1 - ((-1)^(1/3)*e^(1/3)*x)/d^(1/3)] - 6*PolyLog[3, 1 + ((-1)^(2/3)*e^(1
/3)*x)/d^(1/3)])
```

Maple [F]

$$\int \frac{\ln \left(c(e\,x^{3}+d)^{p}\right)^{2}}{x}dx$$

[In] `int(ln(c*(e*x^3+d)^p)^2/x,x)`

[Out] `int(ln(c*(e*x^3+d)^p)^2/x,x)`

Fricas [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x}\,dx=\int \frac{\log\left((ex^3+d)^p c\right)^2}{x}\,dx$$

[In] `integrate(log(c*(e*x^3+d)^p)^2/x,x, algorithm="fricas")`

[Out] `integral(log((e*x^3 + d)^p*c)^2/x, x)`

Sympy [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x}\,dx=\int \frac{\log\left(c(d+ex^3)^p\right)^2}{x}\,dx$$

[In] `integrate(ln(c*(e*x**3+d)**p)**2/x,x)`

[Out] `Integral(log(c*(d + e*x**3)**p)**2/x, x)`

Maxima [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x}\,dx = \int \frac{\log\left((ex^3+d)^p c\right)^2}{x}\,dx$$

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x,x, algorithm="maxima")
[Out] integrate(log((e*x^3 + d)^p*c)^2/x, x)
```

Giac [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x}\,dx = \int \frac{\log\left((ex^3+d)^p c\right)^2}{x}\,dx$$

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x,x, algorithm="giac")
[Out] integrate(log((e*x^3 + d)^p*c)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x}\,dx = \int \frac{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}{x}\,dx$$

```
[In] int(log(c*(d + e*x^3)^p)^2/x,x)
[Out] int(log(c*(d + e*x^3)^p)^2/x, x)
```

3.132 $\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 86

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^4}\,dx = \frac{2ep\log\left(-\frac{ex^3}{d}\right)\log\left(c(d+ex^3)^p\right)}{3d} - \frac{(d+ex^3)\log^2\left(c(d+ex^3)^p\right)}{3dx^3} + \frac{2ep^2\operatorname{PolyLog}\left(2,1+\frac{ex^3}{d}\right)}{3d}$$

```
[Out] 2/3*e*p*ln(-e*x^3/d)*ln(c*(e*x^3+d)^p)/d-1/3*(e*x^3+d)*ln(c*(e*x^3+d)^p)^2/
d/x^3+2/3*e*p^2*polylog(2,1+e*x^3/d)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2504, 2444, 2441, 2352}

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^4}\,dx = -\frac{(d+ex^3)\log^2\left(c(d+ex^3)^p\right)}{3dx^3} + \frac{2ep\log\left(-\frac{ex^3}{d}\right)\log\left(c(d+ex^3)^p\right)}{3d} + \frac{2ep^2\operatorname{PolyLog}\left(2,\frac{ex^3}{d}+1\right)}{3d}$$

```
[In] Int[Log[c*(d + e*x^3)^p]^2/x^4,x]
```

```
[Out] (2*e*p*Log[-((e*x^3)/d)]*Log[c*(d + e*x^3)^p])/(3*d) - ((d + e*x^3)*Log[c*(
d + e*x^3)^p]^2)/(3*d*x^3) + (2*e*p^2*PolyLog[2, 1 + (e*x^3)/d])/(3*d)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{\log^2\left(c(d+ex)^p\right)}{x^2}\,dx, x, x^3\right)\\
&= -\frac{(d+ex^3)\log^2\left(c(d+ex^3)^p\right)}{3dx^3}+\frac{(2ep)\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^3\right)}{3d}\\
&= \frac{2ep\log\left(-\frac{ex^3}{d}\right)\log\left(c(d+ex^3)^p\right)}{3d}-\frac{(d+ex^3)\log^2\left(c(d+ex^3)^p\right)}{3dx^3}\\
&\quad -\frac{(2e^2p^2)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^3\right)}{3d}\\
&= \frac{2ep\log\left(-\frac{ex^3}{d}\right)\log\left(c(d+ex^3)^p\right)}{3d}-\frac{(d+ex^3)\log^2\left(c(d+ex^3)^p\right)}{3dx^3}+\frac{2ep^2\text{Li}_2\left(1+\frac{ex^3}{d}\right)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^4}\,dx = \frac{2ep\log\left(-\frac{ex^3}{d}\right)\log\left(c(d+ex^3)^p\right)}{3d} - \frac{e\log^2\left(c(d+ex^3)^p\right)}{3d} - \frac{\log^2\left(c(d+ex^3)^p\right)}{3x^3} + \frac{2ep^2\,\mathrm{PolyLog}\left(2,\frac{d+ex^3}{d}\right)}{3d}$$

[In] Integrate[Log[c*(d + e*x^3)^p]^2/x^4,x]

[Out] (2*e*p*Log[-((e*x^3)/d)]*Log[c*(d + e*x^3)^p])/(3*d) - (e*Log[c*(d + e*x^3)^p]^2)/(3*d) - Log[c*(d + e*x^3)^p]^2/(3*x^3) + (2*e*p^2*PolyLog[2, (d + e*x^3)/d])/(3*d)

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.82 (sec) , antiderivative size = 411, normalized size of antiderivative = 4.78

method	result
risch	$-\frac{\ln\left(\left(e\,x^3+d\right)^p\right)^2}{3x^3}+\frac{2pe\ln\left(\left(e\,x^3+d\right)^p\right)\ln(x)}{d}-\frac{2pe\ln\left(\left(e\,x^3+d\right)^p\right)\ln\left(e\,x^3+d\right)}{3d}-\frac{2p^2e\left(\sum_{\_R1=\mathrm{RootOf}\left(\_Z^3e+d\right)}\left(\ln(x)\ln\left(\frac{\_R1}{\_R}\right.\right.\right.}{d}$

[In] int(ln(c*(e*x^3+d)^p)^2/x^4,x,method=_RETURNVERBOSE)

[Out] -1/3*ln((e*x^3+d)^p)^2/x^3+2*p*e*ln((e*x^3+d)^p)/d*ln(x)-2/3*p*e*ln((e*x^3+d)^p)/d*ln(e*x^3+d)-2*p^2*e/d*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=RootOf(_Z^3*e+d))+1/3*p^2*e/d*ln(e*x^3+d)^2+(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c))*(-1/3*ln((e*x^3+d)^p)/x^3+p*e*(1/d*ln(x)-1/3/d*ln(e*x^3+d)))-1/12*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c))^2/x^3

Fricas [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^4}\,dx = \int \frac{\log\left((ex^3+d)^p c\right)^2}{x^4}\,dx$$

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^3 + d)^p*c)^2/x^4, x)
```

Sympy [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^4}\,dx = \int \frac{\log\left(c(d+ex^3)^p\right)^2}{x^4}\,dx$$

```
[In] integrate(ln(c*(e*x**3+d)**p)**2/x**4,x)
```

```
[Out] Integral(log(c*(d + e*x**3)**p)**2/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.37

$$\begin{aligned}
&\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^4}\,dx \\
&= \frac{1}{3}\,e^2p^2\left(\frac{\log\left(ex^3+d\right)^2}{de} - \frac{2\left(3\,\log\left(\frac{ex^3}{d}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{ex^3}{d}\right)\right)}{de}\right) \\
&\quad - \frac{2}{3}\,ep\left(\frac{\log\left(ex^3+d\right)}{d} - \frac{\log\left(x^3\right)}{d}\right)\log\left(\left(ex^3+d\right)^p c\right) - \frac{\log\left(\left(ex^3+d\right)^p c\right)^2}{3\,x^3}
\end{aligned}$$

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*e^2*p^2*(log(e*x^3 + d)^2/(d*e) - 2*(3*log(e*x^3/d + 1)*log(x) + dilog(
-e*x^3/d))/(d*e)) - 2/3*e*p*(log(e*x^3 + d)/d - log(x^3)/d)*log((e*x^3 + d)
^p*c) - 1/3*log((e*x^3 + d)^p*c)^2/x^3
```

Giac [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^4}\,dx = \int \frac{\log\left((ex^3+d)^p c\right)^2}{x^4}\,dx$$

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^4,x, algorithm="giac")
[Out] integrate(log((e*x^3 + d)^p*c)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^4}\,dx = \int \frac{\ln\left(c\,(e\,x^3+d)^p\right)^2}{x^4}\,dx$$

```
[In] int(log(c*(d + e*x^3)^p)^2/x^4,x)
[Out] int(log(c*(d + e*x^3)^p)^2/x^4, x)
```

3.133 $\int x \log^2\left(c(d+ex^3)^p\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 1294

$$\int x\log^2\left(c\left(d+ex^3\right)^p\right)\,dx$$

$$= \frac{9p^2x^2}{4} + \frac{3\sqrt{3}d^{2/3}p^2\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2e^{2/3}} + \frac{3d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2e^{2/3}}$$
$$+ \frac{d^{2/3}p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2e^{2/3}} + \frac{d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{e^{2/3}}$$
$$- \frac{\sqrt[3]{-1}d^{2/3}p^2\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{e^{2/3}}$$
$$- \frac{\sqrt[3]{-1}d^{2/3}p^2\log^2\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{2e^{2/3}}$$
$$+ \frac{(-1)^{2/3}d^{2/3}p^2\log\left(-\frac{(-1)^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{e^{2/3}}$$
$$+ \frac{(-1)^{2/3}d^{2/3}p^2\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{e^{2/3}}$$
$$+ \frac{(-1)^{2/3}d^{2/3}p^2\log^2\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{2e^{2/3}}$$
$$+ \frac{d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)}{e^{2/3}}$$
$$- \frac{(-1)^{2/3}d^{2/3}p^2\log\left(-\frac{(-1)^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)\log\left(\frac{\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{e^{2/3}}$$
$$- \frac{\sqrt[3]{-1}d^{2/3}p^2\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(-\frac{(-1)^{2/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{e^{2/3}}$$
$$- \frac{3d^{2/3}p^2\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{4e^{2/3}}$$

$$-\frac{3}{2}px^2\log\left(c\left(d+ex^3\right)^p\right) - \frac{d^{2/3}p\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)^p\right)}{e^{2/3}} + \frac{\sqrt[3]{-1}d^{2/3}p\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(\right.}{e^{2/3}}$$

```
[Out] d^(2/3)*p^2*ln(d^(1/3)+e^(1/3)*x)*ln((-(-1)^(2/3)*d^(1/3)-e^(1/3)*x)/(1-(-1
)^(2/3))/d^(1/3))/e^(2/3)-1/2*(-1)^(1/3)*d^(2/3)*p^2*ln(d^(1/3)-(-1)^(1/3)*
e^(1/3)*x)^2/e^(2/3)+1/2*(-1)^(2/3)*d^(2/3)*p^2*ln(d^(1/3)+(-1)^(2/3)*e^(1/
3)*x)^2/e^(2/3)+3/2*d^(2/3)*p^2*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)/d^(1/3)*3^
(1/2))*3^(1/2)/e^(2/3)+1/2*x^2*ln(c*(e*x^3+d)^p)^2-3/2*p*x^2*ln(c*(e*x^3+d)
^p)+d^(2/3)*p^2*polylog(2,2*(d^(1/3)+e^(1/3)*x)/d^(1/3)/(3-I*3^(1/2)))/e^(2
/3)+d^(2/3)*p^2*polylog(2,(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/e^(2/
3)+3/2*d^(2/3)*p^2*ln(d^(1/3)+e^(1/3)*x)/e^(2/3)+1/2*d^(2/3)*p^2*ln(d^(1/3)
+e^(1/3)*x)^2/e^(2/3)-3/4*d^(2/3)*p^2*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*
x^2)/e^(2/3)+9/4*p^2*x^2+d^(2/3)*p^2*ln(d^(1/3)+e^(1/3)*x)*ln((-1)^(1/3)*(d
^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/e^(2/3)-d^(2/3)*p*ln(d
^(1/3)+e^(1/3)*x)*ln(c*(e*x^3+d)^p)/e^(2/3)-(-1)^(1/3)*d^(2/3)*p^2*ln((-1)^
(1/3)*(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(d^(1/3)-(-1)^(1/3)*e^(
1/3)*x)/e^(2/3)+(-1)^(2/3)*d^(2/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^
(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/e^(2/3)+(
-1)^(2/3)*d^(2/3)*p^2*ln(-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(
1/3))*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/e^(2/3)-(-1)^(2/3)*d^(2/3)*p^2*ln((-
1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln((d^(1/3)
+(-1)^(2/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/e^(2/3)-(-1)^(2/3)*d^(2/3)*p
^2*ln(-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln((d^(1/3)+(
-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/e^(2/3)+(-1)^(1/3)*d^(2/3)*p^2
*ln(-(-1)^(1/3)*((-1)^(2/3)*d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(-
(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/e^(2/3)-(
-1)^(1/3)*d^(2/3)*p^2*ln(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)*ln(-(-1)^(2/3)*(d^(1
/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/e^(2/3)+(-1)^(1/3)*d^(2/3
)*p*ln(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)*ln(c*(e*x^3+d)^p)/e^(2/3)-(-1)^(2/3)*d
^(2/3)*p*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)*ln(c*(e*x^3+d)^p)/e^(2/3)-(-1)^(2
/3)*d^(2/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^
(1/3))/e^(2/3)-(-1)^(1/3)*d^(2/3)*p^2*polylog(2,(d^(1/3)-(-1)^(1/3)*e^(1/3)
*x)/(1+(-1)^(1/3))/d^(1/3))/e^(2/3)-(-1)^(2/3)*d^(2/3)*p^2*polylog(2,(-1)^(
1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/e^(2/3)+(-1)^(1
/3)*d^(2/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1
)^(2/3))/d^(1/3))/e^(2/3)
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 1300, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 1.188$, Rules used = {2507, 2526, 2505, 327, 298, 31, 648, 631, 210, 642, 2512, 266, 2463, 2437, 2338,

2441, 2440, 2438, 12}

$$\int x\log^2\left(c\left(d+ex^3\right)^p\right)\,dx$$

$$= \frac{9x^2p^2}{4}+\frac{d^{2/3}\log^2\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)p^2}{2e^{2/3}}-\frac{\sqrt[3]{-1}d^{2/3}\log^2\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)p^2}{2e^{2/3}}$$

$$+\frac{(-1)^{2/3}d^{2/3}\log^2\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)p^2}{2e^{2/3}}+\frac{3\sqrt{3}d^{2/3}\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)p^2}{2e^{2/3}}$$

$$+\frac{3d^{2/3}\log\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)p^2}{2e^{2/3}}+\frac{d^{2/3}\log\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)\log\left(-\frac{\sqrt[3]{e}x+(-1)^{2/3}\sqrt[3]{d}}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)p^2}{e^{2/3}}$$

$$-\frac{\sqrt[3]{-1}d^{2/3}\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)p^2}{e^{2/3}}$$

$$+\frac{(-1)^{2/3}d^{2/3}\log\left(-\frac{(-1)^{2/3}\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)\log\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)p^2}{e^{2/3}}$$

$$+\frac{(-1)^{2/3}d^{2/3}\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)p^2}{e^{2/3}}$$

$$-\frac{(-1)^{2/3}d^{2/3}\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(\frac{(-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)p^2}{e^{2/3}}$$

$$+\frac{d^{2/3}\log\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)\log\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)p^2}{e^{2/3}}$$

$$-\frac{\sqrt[3]{-1}d^{2/3}\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(-\frac{(-1)^{2/3}\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)p^2}{e^{2/3}}$$

$$-\frac{3d^{2/3}\log\left(e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}\right)p^2}{4e^{2/3}}$$

$$+\frac{d^{2/3}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{e}x+\sqrt[3]{d}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)p^2}{e^{2/3}}+\frac{d^{2/3}\,\mathrm{PolyLog}\left(2,\frac{2\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(3-i\sqrt{3}\right)\sqrt[3]{d}}\right)p^2}{e^{2/3}}$$

$$-\frac{\sqrt[3]{-1}d^{2/3}\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[3]{-1}\left(\sqrt[3]{e}x+(-1)^{2/3}\sqrt[3]{d}\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)p^2}{e^{2/3}}$$

$$\sqrt[3]{-1}d^{2/3}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)p^2$$

```
[In] Int[x*Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (9*p^2*x^2)/4 + (3*Sqrt[3]*d^(2/3)*p^2*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt
[3]*d^(1/3))])/(2*e^(2/3)) + (3*d^(2/3)*p^2*Log[d^(1/3) + e^(1/3)*x])/(2*e^
(2/3)) + (d^(2/3)*p^2*Log[d^(1/3) + e^(1/3)*x]^2)/(2*e^(2/3)) + (d^(2/3)*p^
2*Log[d^(1/3) + e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1
)^(2/3))*d^(1/3)))])/e^(2/3) - ((-1)^(1/3)*d^(2/3)*p^2*Log[((-1)^(1/3)*(d^(
1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) - (-1)^(1/3)*e^(
1/3)*x])/e^(2/3) - ((-1)^(1/3)*d^(2/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)
*x]^2)/(2*e^(2/3)) + ((-1)^(2/3)*d^(2/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e
^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]
)/e^(2/3) + ((-1)^(2/3)*d^(2/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e
^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])
/e^(2/3) + ((-1)^(2/3)*d^(2/3)*p^2*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]^2)/(
2*e^(2/3)) - ((-1)^(2/3)*d^(2/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*
e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x
)/((1 + (-1)^(1/3))*d^(1/3))])/e^(2/3) + (d^(2/3)*p^2*Log[d^(1/3) + e^(1/3)
*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(
1/3))])/e^(2/3) - ((-1)^(1/3)*d^(2/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*
x]*Log[-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^
(1/3)))])/e^(2/3) - (3*d^(2/3)*p^2*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3
)*x^2])/(4*e^(2/3)) - (3*p*x^2*Log[c*(d + e*x^3)^p])/2 - (d^(2/3)*p*Log[d^(
1/3) + e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(2/3) + ((-1)^(1/3)*d^(2/3)*p*Log
[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(2/3) - ((-1)^(2/3
)*d^(2/3)*p*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(2/
3) + (x^2*Log[c*(d + e*x^3)^p]^2)/2 + (d^(2/3)*p^2*PolyLog[2, (d^(1/3) + e^
(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/e^(2/3) + (d^(2/3)*p^2*PolyLog[2, (2*
(d^(1/3) + e^(1/3)*x))/((3 - I*Sqrt[3])*d^(1/3))])/e^(2/3) - ((-1)^(1/3)*d^
(2/3)*p^2*PolyLog[2, -(((-1)^(1/3)*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((1 -
(-1)^(2/3))*d^(1/3)))])/e^(2/3) - ((-1)^(1/3)*d^(2/3)*p^2*PolyLog[2, (d^(1/
3) - (-1)^(1/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/e^(2/3) - ((-1)^(2/
3)*d^(2/3)*p^2*PolyLog[2, ((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1
 + (-1)^(1/3))*d^(1/3))])/e^(2/3) + ((-1)^(2/3)*d^(2/3)*p^2*PolyLog[2, (d^(
1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))])/e^(2/3)
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 210</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
```

```
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\log^2\left(c\left(d+ex^3\right)^p\right)-(3ep)\int \frac{x^4\log\left(c\left(d+ex^3\right)^p\right)}{d+ex^3}\,dx\\
&= \frac{1}{2}x^2\log^2\left(c\left(d+ex^3\right)^p\right)-(3ep)\int \left(\frac{x\log\left(c\left(d+ex^3\right)^p\right)}{e}-\frac{dx\log\left(c\left(d+ex^3\right)^p\right)}{e\left(d+ex^3\right)}\right)dx\\
&= \frac{1}{2}x^2\log^2\left(c\left(d+ex^3\right)^p\right)-(3p)\int x\log\left(c\left(d+ex^3\right)^p\right)\,dx+(3dp)\int \frac{x\log\left(c\left(d+ex^3\right)^p\right)}{d+ex^3}\,dx\\
&= -\frac{3}{2}px^2\log\left(c\left(d+ex^3\right)^p\right)+\frac{1}{2}x^2\log^2\left(c\left(d+ex^3\right)^p\right)\\
&\quad+(3dp)\int \left(-\frac{\log\left(c\left(d+ex^3\right)^p\right)}{3\sqrt[3]{d}\sqrt[3]{e}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}-\frac{(-1)^{2/3}\log\left(c\left(d+ex^3\right)^p\right)}{3\sqrt[3]{d}\sqrt[3]{e}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}\right.\\
&\qquad\left.+\frac{\sqrt[3]{-1}\log\left(c\left(d+ex^3\right)^p\right)}{3\sqrt[3]{d}\sqrt[3]{e}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}\right)dx+\frac{1}{2}\left(9ep^2\right)\int \frac{x^4}{d+ex^3}\,dx\\
&= \frac{9p^2x^2}{4}-\frac{3}{2}px^2\log\left(c\left(d+ex^3\right)^p\right)+\frac{1}{2}x^2\log^2\left(c\left(d+ex^3\right)^p\right)\\
&\quad-\frac{\left(d^{2/3}p\right)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{\sqrt[3]{e}}+\frac{\left(\sqrt[3]{-1}d^{2/3}p\right)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x}\,dx}{\sqrt[3]{e}}\\
&\quad-\frac{\left((-1)^{2/3}d^{2/3}p\right)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x}\,dx}{\sqrt[3]{e}}-\frac{1}{2}\left(9dp^2\right)\int \frac{x}{d+ex^3}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{9p^2x^2}{4} - \frac{3}{2}px^2\log\left(c(d+ex^3)^p\right) - \frac{d^{2/3}p\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{e^{2/3}} \\
&\quad + \frac{\sqrt[3]{-1}d^{2/3}p\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{e^{2/3}} \\
&\quad - \frac{(-1)^{2/3}d^{2/3}p\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{e^{2/3}} \\
&\quad + \frac{1}{2}x^2\log^2\left(c(d+ex^3)^p\right) + \frac{(3d^{2/3}p^2)\int\frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{2\sqrt[3]{e}} - \frac{(3d^{2/3}p^2)\int\frac{\sqrt[3]{d}+\sqrt[3]{e}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{2\sqrt[3]{e}} + (3d^{2/3}\sqrt[3]{}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{9p^2x^2}{4} + \frac{3d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2e^{2/3}} \\
&\quad - \frac{3}{2}px^2\log\left(c(d+ex^3)^p\right) - \frac{d^{2/3}p\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{e^{2/3}} \\
&\quad + \frac{\sqrt[3]{-1}d^{2/3}p\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{e^{2/3}} \\
&\quad - \frac{(-1)^{2/3}d^{2/3}p\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{e^{2/3}} \\
&\quad + \frac{1}{2}x^2\log^2\left(c(d+ex^3)^p\right) - \frac{(3d^{2/3}p^2)\int\frac{-\sqrt[3]{d}\sqrt[3]{e}+2e^{2/3}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{4e^{2/3}} - \frac{(9dp^2)\int\frac{1}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{4\sqrt[3]{e}} + (
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{9p^2x^2}{4} + \frac{3d^{2/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2e^{2/3}} - \frac{3d^{2/3}p^2\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{4e^{2/3}} \\
&\quad - \frac{3}{2}px^2\log\left(c(d+ex^3)^p\right) - \frac{d^{2/3}p\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{e^{2/3}} + \frac{\sqrt[3]{-1}d^{2/3}p\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{}\right.}{e^{2/3}} \\
&= \text{Too large to display}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.52 (sec) , antiderivative size = 1041, normalized size of antiderivative = 0.80

$$\int x\log^2\left(c\left(d+ex^3\right)^p\right)\,dx$$

$$= \frac{1}{2}x^2\log^2\left(c\left(d+ex^3\right)^p\right) - 3ep\left(-\frac{3px^2}{4e} + \frac{3px^2\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{ex^3}{d}\right)}{4e}\right.$$

$$-\frac{d^{2/3}p\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)}{6e^{5/3}} - \frac{d^{2/3}p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{3e^{5/3}}$$

$$-\frac{d^{2/3}p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)}{3e^{5/3}} + \frac{x^2\log\left(c\left(d+ex^3\right)^p\right)}{2e}$$

$$+\frac{d^{2/3}\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)^p\right)}{3e^{5/3}}$$

$$-\frac{\sqrt[3]{-1}d^{2/3}\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)^p\right)}{3e^{5/3}}$$

$$+\frac{(-1)^{2/3}d^{2/3}\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)^p\right)}{3e^{5/3}}$$

$$-\frac{d^{2/3}p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)}{3e^{5/3}} - \frac{d^{2/3}p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{3e^{5/3}}$$

$$+\frac{\sqrt[3]{-1}d^{2/3}p\left(\frac{2\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)}{e^{2/3}} + \frac{\log^2\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)}{e^{2/3}} + \frac{2\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}\right.}{}\right.}{6e}$$

$$-\frac{(-1)^{2/3}d^{2/3}p\left(\frac{2\log\left(-\frac{(-1)^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)}{e^{2/3}} + \frac{2\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(-\sqrt[3]{d}-(\right.}{e^{2/3}}\right.}{6e}$$

```
[In] Integrate[x*Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (x^2*Log[c*(d + e*x^3)^p]^2)/2 - 3*e*p*((-3*p*x^2)/(4*e) + (3*p*x^2*Hyperge
ometric2F1[2/3, 1, 5/3, -((e*x^3)/d)])/(4*e) - (d^(2/3)*p*Log[-d^(1/3) - e^
(1/3)*x]^2)/(6*e^(5/3)) - (d^(2/3)*p*Log[-d^(1/3) - e^(1/3)*x]*Log[-(((-1)^
(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3)))])/(3*e^(5/3)) - (d^(
2/3)*p*Log[-d^(1/3) - e^(1/3)*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^(1
/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/(3*e^(5/3)) + (x^2*Log[c*(d + e*x^3)^p
])/(2*e) + (d^(2/3)*Log[-d^(1/3) - e^(1/3)*x]*Log[c*(d + e*x^3)^p])/(3*e^(5
/3)) - ((-1)^(1/3)*d^(2/3)*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[c*(d +
e*x^3)^p])/(3*e^(5/3)) + ((-1)^(2/3)*d^(2/3)*Log[-d^(1/3) - (-1)^(2/3)*e^(1
/3)*x]*Log[c*(d + e*x^3)^p])/(3*e^(5/3)) - (d^(2/3)*p*PolyLog[2, (d^(1/3) +
 e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/(3*e^(5/3)) - (d^(2/3)*p*PolyLog[2
, (d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))])/(3*e^(5/3)) + ((-1)^(1
/3)*d^(2/3)*p*((2*Log[((-1)^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*
d^(1/3))]*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x])/e^(2/3) + Log[-d^(1/3) + (-
1)^(1/3)*e^(1/3)*x]^2/e^(2/3) + (2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log
[-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3))
)])/e^(2/3) + (2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 + (-1)^(1/
3))*d^(1/3))])/e^(2/3) + (2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1
 - (-1)^(2/3))*d^(1/3))])/e^(2/3)))/(6*e) - ((-1)^(2/3)*d^(2/3)*p*((2*Log[-
(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))]*Log[-d^(1/
3) - (-1)^(2/3)*e^(1/3)*x])/e^(2/3) + (2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1
/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/
3)*x])/e^(2/3) + Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]^2/e^(2/3) + (2*PolyLo
g[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/e^(2/3)
+ (2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))
])/e^(2/3)))/(6*e))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.83 (sec) , antiderivative size = 1957, normalized size of antiderivative = 1.51

method	result	size
risch	Expression too large to display	1957

```
[In] int(x*ln(c*(e*x^3+d)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln((e*x^3+d)^p)^2*x^2-3/2*p*x^2*ln((e*x^3+d)^p)+p^2/e*d/(d/e)^(1/3)*ln(
x+(d/e)^(1/3))*ln(e*x^3+d)-p/e*d/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*ln((e*x^3+d)
^p)-1/2*p^2/e*d/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*ln(e*x^3+d)+1
/2*p/e*d/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*ln((e*x^3+d)^p)-p^2/
e*d*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*ln(e*x^3+d)
+p/e*d*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*ln((e*x^
```

```
3+d)^p)+9/4*p^2*x^2+3/2*p^2/e*d/(d/e)^(1/3)*ln(x+(d/e)^(1/3))-3/4*p^2/e*d/(
d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))-3/2*p^2/e*d*3^(1/2)/(d/e)^(1/3
)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))-3*p^2*e*Sum(-1/3*(ln(x-_alpha)*ln
(e*x^3+d)-3*e*(1/6/_alpha^2/e*ln(x-_alpha)^2+1/3*_alpha*ln(x-_alpha)*(2*Roo
tOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2
,index=2)*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(
_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))+2*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,
index=1)*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*ln((RootOf(_Z^2+3*_Z*_
alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,inde
x=2))+3*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*ln((RootOf(_Z^2+3*_Z*_a
lpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index
=1))*_alpha+6*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*ln((RootOf(_Z^2+3
*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2
,index=2))*_alpha+6*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*ln((RootOf(
_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_a
lpha^2,index=1))*_alpha+3*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*ln((R
ootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alph
a+3*_alpha^2,index=2))*_alpha+9*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,inde
x=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))*_alpha^2+9*ln((
RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alp
ha+3*_alpha^2,index=2))*_alpha^2)/(3*_alpha+2*RootOf(_Z^2+3*_Z*_alpha+3*_al
pha^2,index=1))/d/(3*_alpha+2*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2))+
1/3*_alpha*(2*dilog((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/
RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2))*RootOf(_Z^2+3*_Z*_alpha+3*_alp
ha^2,index=1)*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)+6*dilog((RootOf(_
Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_al
pha^2,index=2))*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*_alpha+3*dilog(
(RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_al
pha+3*_alpha^2,index=2))*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*_alpha
+9*dilog((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2
+3*_Z*_alpha+3*_alpha^2,index=2))*_alpha^2+2*RootOf(_Z^2+3*_Z*_alpha+3*_alp
ha^2,index=1)*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*dilog((RootOf(_Z^
2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alph
a^2,index=1))+3*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*dilog((RootOf(_
Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_al
pha^2,index=1))*_alpha+6*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*dilog(
(RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_al
pha+3*_alpha^2,index=1))*_alpha+9*dilog((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2
,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))*_alpha^2)/
(3*_alpha+2*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))/d/(3*_alpha+2*Root
Of(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2))))*d/_alpha/e^2,_alpha=RootOf(_Z^3*
e+d))+(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+
d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csg
n(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c))*(1/2*ln((e*x^3+d)^p)*x^2-3/2*p*e*(1
/2*x^2/e-(-1/3/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))+1/6/e/(d/e)^(1/3)*ln(x^2-(d/
```

```
e)^(1/3)*x+(d/e)^(2/3))+1/3*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/
e)^(1/3)*x-1)))*d/e))+1/8*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2
-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*
x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c))^2*x^2
```

Fricas [F]

$$\int x \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = \int x \log\left(\left(ex^3+d\right)^p c\right)^2\,dx$$

```
[In] integrate(x*log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
[Out] integral(x*log((e*x^3 + d)^p*c)^2, x)
```

Sympy [F]

$$\int x \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = \int x \log\left(c\left(d+ex^3\right)^p\right)^2\,dx$$

```
[In] integrate(x*ln(c*(e*x**3+d)**p)**2,x)
[Out] Integral(x*log(c*(d + e*x**3)**p)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x \log^2\left(c\left(d+ex^3\right)^p\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int x \log^2 \left(c(d+ex^3)^p\right) \, dx = \int x \log \left((ex^3+d)^p c\right)^2 \, dx$$

```
[In] integrate(x*log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
[Out] integrate(x*log((e*x^3 + d)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x \log^2 \left(c(d+ex^3)^p\right) \, dx = \int x \ln \left(c \left(e\,x^3+d\right)^p\right)^2 dx$$

```
[In] int(x*log(c*(d + e*x^3)^p)^2,x)
[Out] int(x*log(c*(d + e*x^3)^p)^2, x)
```

3.134 $\int \log^2\left(c\left(d+ex^3\right)^p\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 1304

$$\int \log^2\left(c\left(d+ex^3\right)^p\right)\,dx$$

$$= 18p^2x + \frac{6\sqrt{3}\sqrt[3]{d}p^2\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt[3]{e}} - \frac{\sqrt[3]{d}p^2\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)}{\sqrt[3]{e}}$$

$$- \frac{6\sqrt[3]{d}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\sqrt[3]{e}} - \frac{2\sqrt[3]{d}p^2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{\sqrt[3]{e}}$$

$$- \frac{2(-1)^{2/3}\sqrt[3]{d}p^2\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\sqrt[3]{e}}$$

$$- \frac{(-1)^{2/3}\sqrt[3]{d}p^2\log^2\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\sqrt[3]{e}}$$

$$+ \frac{2\sqrt[3]{-1}\sqrt[3]{d}p^2\log\left(-\frac{(-1)^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)}{\sqrt[3]{e}}$$

$$+ \frac{2\sqrt[3]{-1}\sqrt[3]{d}p^2\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)}{\sqrt[3]{e}}$$

$$+ \frac{\sqrt[3]{-1}\sqrt[3]{d}p^2\log^2\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)}{\sqrt[3]{e}}$$

$$- \frac{2\sqrt[3]{d}p^2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)}{\sqrt[3]{e}}$$

$$- \frac{2\sqrt[3]{-1}\sqrt[3]{d}p^2\log\left(-\frac{(-1)^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)\log\left(\frac{\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{\sqrt[3]{e}}$$

$$- \frac{2(-1)^{2/3}\sqrt[3]{d}p^2\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(-\frac{(-1)^{2/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{\sqrt[3]{e}}$$

$$+ \frac{3\sqrt[3]{d}p^2\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{\sqrt[3]{e}}$$

$$-6px\log\left(c\left(d+ex^3\right)^p\right) + \frac{2\sqrt[3]{d}p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{e}} + \frac{2(-1)^{2/3}\sqrt[3]{d}p\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right.}{\sqrt[3]{e}}$$

```
[Out] 2*(-1)^(2/3)*d^(1/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*
x)/(1-(-1)^(2/3))/d^(1/3))/e^(1/3)-2*(-1)^(1/3)*d^(1/3)*p^2*polylog(2,(-1)^
(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/e^(1/3)-2*(-1)
^(2/3)*d^(1/3)*p^2*polylog(2,(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/
d^(1/3))/e^(1/3)-2*d^(1/3)*p^2*ln(-d^(1/3)-e^(1/3)*x)*ln((-1)^(1/3)*(d^(1/3
)+(-1)^(2/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/e^(1/3)+2*d^(1/3)*p*ln(-d^(
1/3)-e^(1/3)*x)*ln(c*(e*x^3+d)^p)/e^(1/3)+6*d^(1/3)*p^2*arctan(1/3*(d^(1/3)
-2*e^(1/3)*x)/d^(1/3)*3^(1/2))*3^(1/2)/e^(1/3)-2*(-1)^(1/3)*d^(1/3)*p^2*pol
ylog(2,-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/e^(1/3)-2*d^
(1/3)*p^2*ln(-d^(1/3)-e^(1/3)*x)*ln((-(-1)^(2/3)*d^(1/3)-e^(1/3)*x)/(1-(-1)
^(2/3))/d^(1/3))/e^(1/3)+18*p^2*x-6*p*x*ln(c*(e*x^3+d)^p)+3*d^(1/3)*p^2*ln(
d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/e^(1/3)-2*d^(1/3)*p^2*polylog(2,2*(d
^(1/3)+e^(1/3)*x)/d^(1/3)/(3-I*3^(1/2)))/e^(1/3)-6*d^(1/3)*p^2*ln(d^(1/3)+e
^(1/3)*x)/e^(1/3)-2*d^(1/3)*p^2*polylog(2,(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3)
)/d^(1/3))/e^(1/3)-d^(1/3)*p^2*ln(-d^(1/3)-e^(1/3)*x)^2/e^(1/3)-2*(-1)^(2/3
)*d^(1/3)*p^2*ln((-1)^(1/3)*(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(
-d^(1/3)+(-1)^(1/3)*e^(1/3)*x)/e^(1/3)+2*(-1)^(1/3)*d^(1/3)*p^2*ln(-(-1)^(2
/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(-d^(1/3)-(-1)^(2/3)*e^(1
/3)*x)/e^(1/3)+2*(-1)^(1/3)*d^(1/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e
^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(-d^(1/3)-(-1)^(2/3)*e^(1/3)*x)/e^(1/3)
-2*(-1)^(1/3)*d^(1/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(
-1)^(1/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3
))/e^(1/3)-2*(-1)^(1/3)*d^(1/3)*p^2*ln(-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(
-1)^(2/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3
))/e^(1/3)+2*(-1)^(2/3)*d^(1/3)*p^2*ln(-(-1)^(1/3)*((-1)^(2/3)*d^(1/3)+e^(1
/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x
)/(1-(-1)^(2/3))/d^(1/3))/e^(1/3)-2*(-1)^(2/3)*d^(1/3)*p^2*ln(-d^(1/3)+(-1)
^(1/3)*e^(1/3)*x)*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/
3))/d^(1/3))/e^(1/3)+2*(-1)^(2/3)*d^(1/3)*p*ln(-d^(1/3)+(-1)^(1/3)*e^(1/3)*
x)*ln(c*(e*x^3+d)^p)/e^(1/3)-2*(-1)^(1/3)*d^(1/3)*p*ln(-d^(1/3)-(-1)^(2/3)*
e^(1/3)*x)*ln(c*(e*x^3+d)^p)/e^(1/3)+(-1)^(1/3)*d^(1/3)*p^2*ln(-d^(1/3)-(-1
)^(2/3)*e^(1/3)*x)^2/e^(1/3)-(-1)^(2/3)*d^(1/3)*p^2*ln(-d^(1/3)+(-1)^(1/3)*
e^(1/3)*x)^2/e^(1/3)+x*ln(c*(e*x^3+d)^p)^2
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 1310, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 1.429$, Rules used = {2500, 2526, 2498, 327, 206, 31, 648, 631, 210, 642, 2521, 2512, 266, 2463, 2437,

2338, 2441, 2440, 2438, 12}

$$\int \log^2\left(c\left(d+ex^3\right)^p\right)\,dx$$

$$= -\frac{\sqrt[3]{d}\log^2\left(-\sqrt[3]{e}x-\sqrt[3]{d}\right)p^2}{\sqrt[3]{e}} - \frac{(-1)^{2/3}\sqrt[3]{d}\log^2\left(\sqrt[3]{-1}\sqrt[3]{e}x-\sqrt[3]{d}\right)p^2}{\sqrt[3]{e}}$$

$$+ \frac{\sqrt[3]{-1}\sqrt[3]{d}\log^2\left(-(-1)^{2/3}\sqrt[3]{e}x-\sqrt[3]{d}\right)p^2}{\sqrt[3]{e}} + 18xp^2 + \frac{6\sqrt{3}\sqrt[3]{d}\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)p^2}{\sqrt[3]{e}}$$

$$- \frac{6\sqrt[3]{d}\log\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)p^2}{\sqrt[3]{e}} - \frac{2\sqrt[3]{d}\log\left(-\sqrt[3]{e}x-\sqrt[3]{d}\right)\log\left(-\frac{\sqrt[3]{e}x+(-1)^{2/3}\sqrt[3]{d}}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)p^2}{\sqrt[3]{e}}$$

$$- \frac{2(-1)^{2/3}\sqrt[3]{d}\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(\sqrt[3]{-1}\sqrt[3]{e}x-\sqrt[3]{d}\right)p^2}{\sqrt[3]{e}}$$

$$+ \frac{2\sqrt[3]{-1}\sqrt[3]{d}\log\left(-\frac{(-1)^{2/3}\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)\log\left(-(-1)^{2/3}\sqrt[3]{e}x-\sqrt[3]{d}\right)p^2}{\sqrt[3]{e}}$$

$$+ \frac{2\sqrt[3]{-1}\sqrt[3]{d}\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(-(-1)^{2/3}\sqrt[3]{e}x-\sqrt[3]{d}\right)p^2}{\sqrt[3]{e}}$$

$$- \frac{2\sqrt[3]{-1}\sqrt[3]{d}\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(\frac{(-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)p^2}{\sqrt[3]{e}}$$

$$- \frac{2\sqrt[3]{d}\log\left(-\sqrt[3]{e}x-\sqrt[3]{d}\right)\log\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)p^2}{\sqrt[3]{e}}$$

$$- \frac{2(-1)^{2/3}\sqrt[3]{d}\log\left(\sqrt[3]{-1}\sqrt[3]{e}x-\sqrt[3]{d}\right)\log\left(-\frac{(-1)^{2/3}\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)p^2}{\sqrt[3]{e}}$$

$$+ \frac{3\sqrt[3]{d}\log\left(e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}\right)p^2}{\sqrt[3]{e}}$$

$$- \frac{2\sqrt[3]{d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{e}x+\sqrt[3]{d}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)p^2}{\sqrt[3]{e}} - \frac{2\sqrt[3]{d}\,\mathrm{PolyLog}\left(2,\frac{2\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(3-i\sqrt{3}\right)\sqrt[3]{d}}\right)p^2}{\sqrt[3]{e}}$$

$$- \frac{2(-1)^{2/3}\sqrt[3]{d}\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[3]{-1}\left(\sqrt[3]{e}x+(-1)^{2/3}\sqrt[3]{d}\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)p^2}{\sqrt[3]{e}}$$

```
[In] Int[Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] 18*p^2*x + (6*Sqrt[3]*d^(1/3)*p^2*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d
^(1/3))])/e^(1/3) - (d^(1/3)*p^2*Log[-d^(1/3) - e^(1/3)*x]^2)/e^(1/3) - (6*
d^(1/3)*p^2*Log[d^(1/3) + e^(1/3)*x])/e^(1/3) - (2*d^(1/3)*p^2*Log[-d^(1/3)
 - e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1
/3)))])/e^(1/3) - (2*(-1)^(2/3)*d^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) + e^(1
/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x])/e
^(1/3) - ((-1)^(2/3)*d^(1/3)*p^2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]^2)/e^
(1/3) + (2*(-1)^(1/3)*d^(1/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/
((1 - (-1)^(2/3))*d^(1/3)))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/e^(1/3)
+ (2*(-1)^(1/3)*d^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x
))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/e^(1/3
) + ((-1)^(1/3)*d^(1/3)*p^2*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]^2)/e^(1/3)
 - (2*(-1)^(1/3)*d^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*
x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 +
(-1)^(1/3))*d^(1/3))])/e^(1/3) - (2*d^(1/3)*p^2*Log[-d^(1/3) - e^(1/3)*x]*L
og[((-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))
])/e^(1/3) - (2*(-1)^(2/3)*d^(1/3)*p^2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]
*Log[-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1
/3)))])/e^(1/3) + (3*d^(1/3)*p^2*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*
x^2])/e^(1/3) - 6*p*x*Log[c*(d + e*x^3)^p] + (2*d^(1/3)*p*Log[-d^(1/3) - e^
(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(1/3) + (2*(-1)^(2/3)*d^(1/3)*p*Log[-d^(1/
3) + (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(1/3) - (2*(-1)^(1/3)*d^
(1/3)*p*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/e^(1/3)
+ x*Log[c*(d + e*x^3)^p]^2 - (2*d^(1/3)*p^2*PolyLog[2, (d^(1/3) + e^(1/3)*x
)/((1 + (-1)^(1/3))*d^(1/3))])/e^(1/3) - (2*d^(1/3)*p^2*PolyLog[2, (2*(d^(1
/3) + e^(1/3)*x))/((3 - I*Sqrt[3])*d^(1/3))])/e^(1/3) - (2*(-1)^(2/3)*d^(1/
3)*p^2*PolyLog[2, -(((-1)^(1/3)*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((1 - (-1
)^(2/3))*d^(1/3)))])/e^(1/3) - (2*(-1)^(2/3)*d^(1/3)*p^2*PolyLog[2, (d^(1/3
) - (-1)^(1/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/e^(1/3) - (2*(-1)^(1
/3)*d^(1/3)*p^2*PolyLog[2, ((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((
1 + (-1)^(1/3))*d^(1/3))])/e^(1/3) + (2*(-1)^(1/3)*d^(1/3)*p^2*PolyLog[2, (
d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))])/e^(1/3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2500

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[x^n*(
```

```
(a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log^2\left(c\left(d+ex^3\right)^p\right)-(6ep)\int \frac{x^3\log\left(c\left(d+ex^3\right)^p\right)}{d+ex^3}\,dx\\
&= x\log^2\left(c\left(d+ex^3\right)^p\right)-(6ep)\int \left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{e}-\frac{d\log\left(c\left(d+ex^3\right)^p\right)}{e\left(d+ex^3\right)}\right)dx\\
&= x\log^2\left(c\left(d+ex^3\right)^p\right)-(6p)\int \log\left(c\left(d+ex^3\right)^p\right)dx+(6dp)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{d+ex^3}\,dx\\
&= -6px\log\left(c\left(d+ex^3\right)^p\right)+x\log^2\left(c\left(d+ex^3\right)^p\right)+(6dp)\int \left(-\frac{\log\left(c\left(d+ex^3\right)^p\right)}{3d^{2/3}\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)}\right.\\
&\qquad -\frac{\log\left(c\left(d+ex^3\right)^p\right)}{3d^{2/3}\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)}\\
&\qquad \left.-\frac{\log\left(c\left(d+ex^3\right)^p\right)}{3d^{2/3}\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)}\right)dx+\left(18ep^2\right)\int \frac{x^3}{d+ex^3}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= 18p^2x - 6px\log\left(c\left(d+ex^3\right)^p\right) + x\log^2\left(c\left(d+ex^3\right)^p\right) \\
&\quad - \left(2\sqrt[3]{d}p\right)\int \frac{\log\left(c(d+ex^3)^p\right)}{-\sqrt[3]{d}-\sqrt[3]{e}x}\,dx - \left(2\sqrt[3]{d}p\right)\int \frac{\log\left(c(d+ex^3)^p\right)}{-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x}\,dx \\
&\quad - \left(2\sqrt[3]{d}p\right)\int \frac{\log\left(c(d+ex^3)^p\right)}{-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x}\,dx - \left(18dp^2\right)\int \frac{1}{d+ex^3}\,dx \\
&= 18p^2x - 6px\log\left(c\left(d+ex^3\right)^p\right) + \frac{2\sqrt[3]{d}p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{e}} \\
&\quad + \frac{2(-1)^{2/3}\sqrt[3]{d}p\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{e}} \\
&\quad - \frac{2\sqrt[3]{-1}\sqrt[3]{d}p\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{e}} + x\log^2\left(c\left(d+ex^3\right)^p\right) \\
&\quad - \left(6\sqrt[3]{d}p^2\right)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx - \left(6\sqrt[3]{d}p^2\right)\int \frac{2\sqrt[3]{d}-\sqrt[3]{e}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx \\
&\quad - \left(6\sqrt[3]{d}e^{2/3}p^2\right)\int \frac{x^2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)}{d+ex^3}\,dx + \left(6\sqrt[3]{-1}\sqrt[3]{d}e^{2/3}p^2\right)\int \frac{x^2\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)}{d+ex^3} \\
&= 18p^2x - \frac{6\sqrt[3]{d}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\sqrt[3]{e}} - 6px\log\left(c\left(d+ex^3\right)^p\right) \\
&\quad + \frac{2\sqrt[3]{d}p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{e}} \\
&\quad + \frac{2(-1)^{2/3}\sqrt[3]{d}p\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{e}} \\
&\quad - \frac{2\sqrt[3]{-1}\sqrt[3]{d}p\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{e}} + x\log^2\left(c\left(d+ex^3\right)^p\right) \\
&\quad - \left(9d^{2/3}p^2\right)\int \frac{1}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx + \frac{\left(3\sqrt[3]{d}p^2\right)\int \frac{-\sqrt[3]{d}\sqrt[3]{e}+2e^{2/3}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{\sqrt[3]{e}} - \left(6\sqrt[3]{d}e^{2/3}p^2\right)\int
\end{aligned}
$$

$$= 18p^2x - \frac{6\sqrt[3]{d}p^2 \log\left(\sqrt[3]{d} + \sqrt[3]{e}x\right)}{\sqrt[3]{e}} + \frac{3\sqrt[3]{d}p^2 \log\left(d^{2/3} - \sqrt[3]{d}\sqrt[3]{e}x + e^{2/3}x^2\right)}{\sqrt[3]{e}}$$
$$- 6px \log\left(c\left(d + ex^3\right)^p\right) + \frac{2\sqrt[3]{d}p \log\left(-\sqrt[3]{d} - \sqrt[3]{e}x\right) \log\left(c(d + ex^3)^p\right)}{\sqrt[3]{e}}$$
$$+ \frac{2(-1)^{2/3}\sqrt[3]{d}p \log\left(-\sqrt[3]{d} + \sqrt[3]{-1}\sqrt[3]{e}x\right) \log\left(c(d + ex^3)^p\right)}{\sqrt[3]{e}}$$
$$- \frac{2\sqrt[3]{-1}\sqrt[3]{d}p \log\left(-\sqrt[3]{d} - (-1)^{2/3}\sqrt[3]{e}x\right) \log\left(c(d + ex^3)^p\right)}{\sqrt[3]{e}}$$
$$+ x \log^2\left(c\left(d+ex^3\right)^p\right) - \left(2\sqrt[3]{d}p^2\right) \int \frac{\log\left(-\sqrt[3]{d} - \sqrt[3]{e}x\right)}{\sqrt[3]{d} + \sqrt[3]{e}x}\,dx - \left(2\sqrt[3]{d}p^2\right) \int \frac{\log\left(-\sqrt[3]{d} - \sqrt[3]{e}x\right)}{-\sqrt[3]{-1}\sqrt[3]{d} + \sqrt[3]{e}x}\,dx - \left(\right.$$
$$= \text{Too large to display}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 1101, normalized size of antiderivative = 0.84

$$\int \log^2\left(c\left(d + ex^3\right)^p\right)\,dx$$
$$= \frac{18\sqrt[3]{e}p^2x + 6\sqrt{3}\sqrt[3]{d}p^2 \arctan\left(\frac{1 - \frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right) - \sqrt[3]{d}p^2 \log^2\left(-\sqrt[3]{d} - \sqrt[3]{e}x\right) - 2\sqrt[3]{d}p^2 \log\left(-\sqrt[3]{d} - \sqrt[3]{e}x\right) \log\left(\right.}{}$$

```
[In] Integrate[Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (18*e^(1/3)*p^2*x + 6*Sqrt[3]*d^(1/3)*p^2*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3)
)/Sqrt[3]] - d^(1/3)*p^2*Log[-d^(1/3) - e^(1/3)*x]^2 - 2*d^(1/3)*p^2*Log[-d
^(1/3) - e^(1/3)*x]*Log[((-1)^(1/3)*d^(1/3) - e^(1/3)*x)/((1 + (-1)^(1/3))*
d^(1/3))] - 6*d^(1/3)*p^2*Log[d^(1/3) + e^(1/3)*x] - 2*(-1)^(2/3)*d^(1/3)*p
^2*Log[((-1)^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-
d^(1/3) + (-1)^(1/3)*e^(1/3)*x] - (-1)^(2/3)*d^(1/3)*p^2*Log[-d^(1/3) + (-1
)^(1/3)*e^(1/3)*x]^2 + 2*(-1)^(1/3)*d^(1/3)*p^2*Log[((-1)^(2/3)*(d^(1/3) +
e^(1/3)*x))/((-1 + (-1)^(2/3))*d^(1/3))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*
x] + 2*(-1)^(1/3)*d^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)
*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x] + (-1
)^(1/3)*d^(1/3)*p^2*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]^2 - 2*(-1)^(2/3)*d
^(1/3)*p^2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[((-1)^(2/3)*(d^(1/3) +
(-1)^(2/3)*e^(1/3)*x))/((-1 + (-1)^(2/3))*d^(1/3))] - 2*d^(1/3)*p^2*Log[-d^
(1/3) - e^(1/3)*x]*Log[(I + Sqrt[3] - ((2*I)*e^(1/3)*x)/d^(1/3))/(3*I + Sqr
```

```
t[3])] + 3*d^(1/3)*p^2*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2] - 6*e
^(1/3)*p*x*Log[c*(d + e*x^3)^p] + 2*d^(1/3)*p*Log[-d^(1/3) - e^(1/3)*x]*Log
[c*(d + e*x^3)^p] + 2*(-1)^(2/3)*d^(1/3)*p*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3
)*x]*Log[c*(d + e*x^3)^p] - 2*(-1)^(1/3)*d^(1/3)*p*Log[-d^(1/3) - (-1)^(2/3
)*e^(1/3)*x]*Log[c*(d + e*x^3)^p] + e^(1/3)*x*Log[c*(d + e*x^3)^p]^2 - 2*d^
(1/3)*p^2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] - 2*
(-1)^(2/3)*d^(1/3)*p^2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 + (-
1)^(1/3))*d^(1/3))] - 2*(-1)^(2/3)*d^(1/3)*p^2*PolyLog[2, (-d^(1/3) + (-1)^
(1/3)*e^(1/3)*x)/((-1 + (-1)^(2/3))*d^(1/3))] + 2*(-1)^(1/3)*d^(1/3)*p^2*Po
lyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*(
-1)^(1/3)*d^(1/3)*p^2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1
)^(2/3))*d^(1/3))] - 2*d^(1/3)*p^2*PolyLog[2, ((2*I)*(1 + (e^(1/3)*x)/d^(1/
3)))/(3*I + Sqrt[3])])/e^(1/3)
```

Maple [F]

$$\int \ln \left(c(e\,x^{3}+d)^{p}\right)^{2}dx$$

```
[In] int(ln(c*(e*x^3+d)^p)^2,x)
[Out] int(ln(c*(e*x^3+d)^p)^2,x)
```

Fricas [F]

$$\int \log^2\left(c\left(d+ex^3\right)^p\right)\,dx=\int \log\left(\left(ex^3+d\right)^pc\right)^2\,dx$$

```
[In] integrate(log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
[Out] integral(log((e*x^3 + d)^p*c)^2, x)
```

Sympy [F]

$$\int \log^2\left(c\left(d+ex^3\right)^p\right)\,dx=\int \log\left(c\left(d+ex^3\right)^p\right)^2dx$$

```
[In] integrate(ln(c*(e*x**3+d)**p)**2,x)
[Out] Integral(log(c*(d + e*x**3)**p)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \log^2\left(c(d+ex^3)^p\right) dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \log^2\left(c(d+ex^3)^p\right) dx = \int \log\left((ex^3+d)^p c\right)^2 dx$$

```
[In] integrate(log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^3 + d)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \log^2\left(c(d+ex^3)^p\right) dx = \int \ln\left(c\left(e\,x^3+d\right)^p\right)^2 dx$$

```
[In] int(log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] int(log(c*(d + e*x^3)^p)^2, x)
```

3.135 $\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 1137

$$
\begin{aligned}
&\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^2}\,dx \\
&= \frac{\sqrt[3]{e}p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\sqrt[3]{d}} + \frac{2\sqrt[3]{e}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{\sqrt[3]{d}} \\
&- \frac{2\sqrt[3]{-1}\sqrt[3]{e}p^2\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\sqrt[3]{d}} \\
&- \frac{\sqrt[3]{-1}\sqrt[3]{e}p^2\log^2\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\sqrt[3]{d}} \\
&+ \frac{2(-1)^{2/3}\sqrt[3]{e}p^2\log\left(-\frac{(-1)^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\sqrt[3]{d}} \\
&+ \frac{2(-1)^{2/3}\sqrt[3]{e}p^2\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\sqrt[3]{d}} \\
&+ \frac{(-1)^{2/3}\sqrt[3]{e}p^2\log^2\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\sqrt[3]{d}} \\
&+ \frac{2\sqrt[3]{e}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)}{\sqrt[3]{d}} \\
&- \frac{2(-1)^{2/3}\sqrt[3]{e}p^2\log\left(-\frac{(-1)^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)\log\left(\frac{\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{\sqrt[3]{d}} \\
&- \frac{2\sqrt[3]{-1}\sqrt[3]{e}p^2\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(-\frac{(-1)^{2/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{\sqrt[3]{d}} \\
&- \frac{2\sqrt[3]{e}p\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{d}} \\
&+ \frac{2\sqrt[3]{-1}\sqrt[3]{e}p\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{d}} \\
&- \frac{2(-1)^{2/3}\sqrt[3]{e}p\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{d}}
\end{aligned}
$$

```
[Out] e^(1/3)*p^2*ln(d^(1/3)+e^(1/3)*x)^2/d^(1/3)+2*e^(1/3)*p^2*ln(d^(1/3)+e^(1/3
)*x)*ln((-(-1)^(2/3)*d^(1/3)-e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(1/3)-2*(
-1)^(1/3)*e^(1/3)*p^2*ln((-1)^(1/3)*(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1
/3))*ln(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/d^(1/3)-(-1)^(1/3)*e^(1/3)*p^2*ln(d^(
1/3)-(-1)^(1/3)*e^(1/3)*x)^2/d^(1/3)+2*(-1)^(2/3)*e^(1/3)*p^2*ln(-(-1)^(2/3
)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)
*x)/d^(1/3)+2*(-1)^(2/3)*e^(1/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1
/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/d^(1/3)+(-1
)^(2/3)*e^(1/3)*p^2*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)^2/d^(1/3)-2*(-1)^(2/3)
*e^(1/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^
(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(1/3)+2*
e^(1/3)*p^2*ln(d^(1/3)+e^(1/3)*x)*ln((-1)^(1/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)
*x)/(1+(-1)^(1/3))/d^(1/3))/d^(1/3)-2*(-1)^(2/3)*e^(1/3)*p^2*ln(-(-1)^(2/3)
*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/3)
*x)/(1-(-1)^(2/3))/d^(1/3))/d^(1/3)+2*(-1)^(1/3)*e^(1/3)*p^2*ln(-(-1)^(1/3)
*((-1)^(2/3)*d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(-(-1)^(2/3)*(d^(
1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(1/3)-2*(-1)^(1/3)*e^(
1/3)*p^2*ln(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3
)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(1/3)-2*e^(1/3)*p*ln(d^(1/3)+e^(1/3)
*x)*ln(c*(e*x^3+d)^p)/d^(1/3)+2*(-1)^(1/3)*e^(1/3)*p*ln(d^(1/3)-(-1)^(1/3)*
e^(1/3)*x)*ln(c*(e*x^3+d)^p)/d^(1/3)-2*(-1)^(2/3)*e^(1/3)*p*ln(d^(1/3)+(-1)
^(2/3)*e^(1/3)*x)*ln(c*(e*x^3+d)^p)/d^(1/3)-ln(c*(e*x^3+d)^p)^2/x+2*e^(1/3)
*p^2*polylog(2,(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(1/3)-2*(-1)^(
2/3)*e^(1/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d
^(1/3))/d^(1/3)-2*(-1)^(1/3)*e^(1/3)*p^2*polylog(2,(d^(1/3)-(-1)^(1/3)*e^(1
/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(1/3)-2*(-1)^(2/3)*e^(1/3)*p^2*polylog(2,(
-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(1/3)+2*
(-1)^(1/3)*e^(1/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)
/(1-(-1)^(2/3))/d^(1/3))/d^(1/3)+2*e^(1/3)*p^2*polylog(2,2*(d^(1/3)+e^(1/3)
*x)/d^(1/3)/(3-I*3^(1/2)))/d^(1/3)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 1143, normalized size of antiderivative = 1.01, number of steps used = 39, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules

used = {2507, 2526, 2512, 266, 2463, 2437, 2338, 2441, 2440, 2438, 12}

$$
\begin{aligned}
&\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^2}\,dx \\
&= \frac{\sqrt[3]{e}\log^2\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)p^2}{\sqrt[3]{d}} - \frac{\sqrt[3]{-1}\sqrt[3]{e}\log^2\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)p^2}{\sqrt[3]{d}} \\
&\quad + \frac{(-1)^{2/3}\sqrt[3]{e}\log^2\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)p^2}{\sqrt[3]{d}} \\
&\quad + \frac{2\sqrt[3]{e}\log\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)\log\left(-\frac{\sqrt[3]{e}x+(-1)^{2/3}\sqrt[3]{d}}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)p^2}{\sqrt[3]{d}} \\
&\quad - \frac{2\sqrt[3]{-1}\sqrt[3]{e}\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)p^2}{\sqrt[3]{d}} \\
&\quad + \frac{2(-1)^{2/3}\sqrt[3]{e}\log\left(-\frac{(-1)^{2/3}\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)\log\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)p^2}{\sqrt[3]{d}} \\
&\quad + \frac{2(-1)^{2/3}\sqrt[3]{e}\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)p^2}{\sqrt[3]{d}} \\
&\quad - \frac{2(-1)^{2/3}\sqrt[3]{e}\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(\frac{(-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)p^2}{\sqrt[3]{d}} \\
&\quad + \frac{2\sqrt[3]{e}\log\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)\log\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)p^2}{\sqrt[3]{d}} \\
&\quad - \frac{2\sqrt[3]{-1}\sqrt[3]{e}\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(-\frac{(-1)^{2/3}\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)p^2}{\sqrt[3]{d}} \\
&\quad + \frac{2\sqrt[3]{e}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{e}x+\sqrt[3]{d}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)p^2}{\sqrt[3]{d}} + \frac{2\sqrt[3]{e}\,\mathrm{PolyLog}\left(2,\frac{2\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(3-i\sqrt{3}\right)\sqrt[3]{d}}\right)p^2}{\sqrt[3]{d}} \\
&\quad - \frac{2\sqrt[3]{-1}\sqrt[3]{e}\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[3]{-1}\left(\sqrt[3]{e}x+(-1)^{2/3}\sqrt[3]{d}\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)p^2}{\sqrt[3]{d}} \\
&\quad 2\sqrt[3]{-1}\sqrt[3]{e}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)p^2
\end{aligned}
$$

```
[In] Int[Log[c*(d + e*x^3)^p]^2/x^2,x]
```

```
[Out] (e^(1/3)*p^2*Log[d^(1/3) + e^(1/3)*x]^2)/d^(1/3) + (2*e^(1/3)*p^2*Log[d^(1/
3) + e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^
(1/3)))])/d^(1/3) - (2*(-1)^(1/3)*e^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) + e^
(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x])/
d^(1/3) - ((-1)^(1/3)*e^(1/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]^2)/d^
(1/3) + (2*(-1)^(2/3)*e^(1/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/
((1 - (-1)^(2/3))*d^(1/3)))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/d^(1/3) +
 (2*(-1)^(2/3)*e^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x)
)/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/d^(1/3)
+ ((-1)^(2/3)*e^(1/3)*p^2*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]^2)/d^(1/3) -
(2*(-1)^(2/3)*e^(1/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))
/((1 + (-1)^(1/3))*d^(1/3))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1
)^(1/3))*d^(1/3))])/d^(1/3) + (2*e^(1/3)*p^2*Log[d^(1/3) + e^(1/3)*x]*Log[(
(-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/d
^(1/3) - (2*(-1)^(1/3)*e^(1/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[
-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))
])/d^(1/3) - (2*e^(1/3)*p*Log[d^(1/3) + e^(1/3)*x]*Log[c*(d + e*x^3)^p])/d^
(1/3) + (2*(-1)^(1/3)*e^(1/3)*p*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[c*(
d + e*x^3)^p])/d^(1/3) - (2*(-1)^(2/3)*e^(1/3)*p*Log[d^(1/3) + (-1)^(2/3)*e
^(1/3)*x]*Log[c*(d + e*x^3)^p])/d^(1/3) - Log[c*(d + e*x^3)^p]^2/x + (2*e^(
1/3)*p^2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/d^(1
/3) + (2*e^(1/3)*p^2*PolyLog[2, (2*(d^(1/3) + e^(1/3)*x))/((3 - I*Sqrt[3])*
d^(1/3))])/d^(1/3) - (2*(-1)^(1/3)*e^(1/3)*p^2*PolyLog[2, -(((-1)^(1/3)*((-
1)^(2/3)*d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/d^(1/3) - (2*(
-1)^(1/3)*e^(1/3)*p^2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 + (-1
)^(1/3))*d^(1/3))])/d^(1/3) - (2*(-1)^(2/3)*e^(1/3)*p^2*PolyLog[2, ((-1)^(1
/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/d^(1/3)
+ (2*(-1)^(2/3)*e^(1/3)*p^2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1
 - (-1)^(2/3))*d^(1/3))])/d^(1/3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log^2\left(c(d+ex^3)^p\right)}{x} + (6ep)\int \frac{x\log\left(c(d+ex^3)^p\right)}{d+ex^3}\,dx \\
&= -\frac{\log^2\left(c(d+ex^3)^p\right)}{x} + (6ep)\int \left(-\frac{\log\left(c(d+ex^3)^p\right)}{3\sqrt[3]{d}\sqrt[3]{e}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}\right. \\
&\qquad \left. -\frac{(-1)^{2/3}\log\left(c(d+ex^3)^p\right)}{3\sqrt[3]{d}\sqrt[3]{e}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)} + \frac{\sqrt[3]{-1}\log\left(c(d+ex^3)^p\right)}{3\sqrt[3]{d}\sqrt[3]{e}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}\right)dx \\
&= -\frac{\log^2\left(c(d+ex^3)^p\right)}{x} - \frac{\left(2e^{2/3}p\right)\int \frac{\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{\sqrt[3]{d}} \\
&\quad + \frac{\left(2\sqrt[3]{-1}e^{2/3}p\right)\int \frac{\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x}\,dx}{\sqrt[3]{d}} - \frac{\left(2(-1)^{2/3}e^{2/3}p\right)\int \frac{\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x}\,dx}{\sqrt[3]{d}} \\
&= -\frac{2\sqrt[3]{e}p\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{d}} \\
&\quad + \frac{2\sqrt[3]{-1}\sqrt[3]{e}p\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{d}} \\
&\quad - \frac{2(-1)^{2/3}\sqrt[3]{e}p\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{d}} - \frac{\log^2\left(c(d+ex^3)^p\right)}{x} \\
&\quad + \frac{\left(6e^{4/3}p^2\right)\int \frac{x^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{d+ex^3}\,dx}{\sqrt[3]{d}} - \frac{\left(6\sqrt[3]{-1}e^{4/3}p^2\right)\int \frac{x^2\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{d+ex^3}\,dx}{\sqrt[3]{d}} \\
&\quad + \frac{\left(6(-1)^{2/3}e^{4/3}p^2\right)\int \frac{x^2\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{d+ex^3}\,dx}{\sqrt[3]{d}}
\end{aligned}$$

$$= -\frac{2\sqrt[3]{e}p\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{d}}$$
$$+\frac{2\sqrt[3]{-1}\sqrt[3]{e}p\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{d}}$$
$$-\frac{2(-1)^{2/3}\sqrt[3]{e}p\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{d}}-\frac{\log^2\left(c(d+ex^3)^p\right)}{x}$$
$$+\frac{\left(6e^{4/3}p^2\right)\int\left(\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3e^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3e^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x\right)}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3e^{2/3}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}\right)dx}{\sqrt[3]{d}}$$
$$-\frac{\left(6\sqrt[3]{-1}e^{4/3}p^2\right)\int\left(\frac{\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{3e^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}+\frac{\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{3e^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x\right)}+\frac{\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{3e^{2/3}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}\right)dx}{\sqrt[3]{d}}$$
$$+\frac{\left(6(-1)^{2/3}e^{4/3}p^2\right)\int\left(\frac{\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{3e^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}+\frac{\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{3e^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x\right)}+\frac{\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{3e^{2/3}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}\right)d}{\sqrt[3]{d}}$$

$$= -\frac{2\sqrt[3]{e}p\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{d}} + \frac{2\sqrt[3]{-1}\sqrt[3]{e}p\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{d}} - \frac{2(-1)^{2/3}\sqrt[3]{e}p\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{\sqrt[3]{d}} - \frac{\log^2\left(c(d+ex^3)^p\right)}{x} + \frac{\left(2e^{2/3}p^2\right)\int\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\sqrt[3]{d}+\sqrt[3]{e}x}dx}{\sqrt[3]{d}} + \frac{\left(2e^{2/3}p^2\right)\int\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x}dx}{\sqrt[3]{d}} + \frac{\left(2e^{2/3}p^2\right)\int\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}dx}{\sqrt[3]{d}} - \frac{\left(2\sqrt[3]{-1}e^{2/3}p^2\right)\int\frac{\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\sqrt[3]{d}+\sqrt[3]{e}x}dx}{\sqrt[3]{d}} - \frac{\left(2\sqrt[3]{-1}e^{2/3}p^2\right)\int\frac{\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x}dx}{\sqrt[3]{d}} - \frac{\left(2\sqrt[3]{-1}e^{2/3}p^2\right)\int\frac{\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}dx}{\sqrt[3]{d}} + \frac{\left(2(-1)^{2/3}e^{2/3}p^2\right)\int\frac{\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\sqrt[3]{d}+\sqrt[3]{e}x}dx}{\sqrt[3]{d}} + \frac{\left(2(-1)^{2/3}e^{2/3}p^2\right)\int\frac{\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x}dx}{\sqrt[3]{d}} + \frac{\left(2(-1)^{2/3}e^{2/3}p^2\right)\int\frac{\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}dx}{\sqrt[3]{d}}$$

$=$ Too large to display

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 972, normalized size of antiderivative = 0.85

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^2}\,dx = -\frac{\log^2\left(c(d+ex^3)^p\right)}{x} + 6ep\left(\frac{p\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)}{6\sqrt[3]{d}e^{2/3}} + \frac{p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{3\sqrt[3]{d}e^{2/3}} + \frac{p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)}{3\sqrt[3]{d}e^{2/3}} - \frac{\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{3\sqrt[3]{d}e^{2/3}} + \frac{\sqrt[3]{-1}\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{3\sqrt[3]{d}e^{2/3}} - \frac{(-1)^{2/3}\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{3\sqrt[3]{d}e^{2/3}} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)}{3\sqrt[3]{d}e^{2/3}} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{3\sqrt[3]{d}e^{2/3}} - \frac{\sqrt[3]{-1}p\left(\frac{2\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)}{e^{2/3}} + \frac{\log^2\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)}{e^{2/3}} + \frac{2\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\cdots}{\cdots}\right.}{6\sqrt[3]{d}\cdots} + \frac{(-1)^{2/3}p\left(\frac{2\log\left(-\frac{(-1)^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)}{e^{2/3}} + \frac{2\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(-\sqrt[3]{d}-(-1)^{2\cdots}\right.}{e^{2/3}}\right.}{6\sqrt[3]{d}\cdots}$$

```
[In] Integrate[Log[c*(d + e*x^3)^p]^2/x^2,x]
```

```
[Out] -(Log[c*(d + e*x^3)^p]^2/x) + 6*e*p*((p*Log[-d^(1/3) - e^(1/3)*x]^2)/(6*d^(
1/3)*e^(2/3)) + (p*Log[-d^(1/3) - e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^
(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3)))])/(3*d^(1/3)*e^(2/3)) + (p*Log[-d^(1/3
) - e^(1/3)*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)
^(1/3))*d^(1/3))])/(3*d^(1/3)*e^(2/3)) - (Log[-d^(1/3) - e^(1/3)*x]*Log[c*(
d + e*x^3)^p])/(3*d^(1/3)*e^(2/3)) + ((-1)^(1/3)*Log[-d^(1/3) + (-1)^(1/3)*
e^(1/3)*x]*Log[c*(d + e*x^3)^p])/(3*d^(1/3)*e^(2/3)) - ((-1)^(2/3)*Log[-d^(
1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/(3*d^(1/3)*e^(2/3)) + (p
*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/(3*d^(1/3)*e
^(2/3)) + (p*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))])/
(3*d^(1/3)*e^(2/3)) - ((-1)^(1/3)*p*((2*Log[((-1)^(1/3)*(d^(1/3) + e^(1/3)*
x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x])/e^(2/
3) + Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]^2/e^(2/3) + (2*Log[-d^(1/3) + (-1
)^(1/3)*e^(1/3)*x]*Log[-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1
- (-1)^(2/3))*d^(1/3)))])/e^(2/3) + (2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(
1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/e^(2/3) + (2*PolyLog[2, (d^(1/3) - (-1
)^(1/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))])/e^(2/3)))/(6*d^(1/3)) + ((-
1)^(2/3)*p*((2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d
^(1/3)))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/e^(2/3) + (2*Log[((-1)^(1/3
)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3
) - (-1)^(2/3)*e^(1/3)*x])/e^(2/3) + Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]^2
/e^(2/3) + (2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))
*d^(1/3))])/e^(2/3) + (2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 -
(-1)^(2/3))*d^(1/3))])/e^(2/3)))/(6*d^(1/3)))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.64 (sec) , antiderivative size = 1787, normalized size of antiderivative = 1.57

method	result	size
risch	Expression too large to display	1787

```
[In] int(ln(c*(e*x^3+d)^p)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x*ln((e*x^3+d)^p)^2+2*p^2/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*ln(e*x^3+d)-2*p/
(d/e)^(1/3)*ln(x+(d/e)^(1/3))*ln((e*x^3+d)^p)-p^2/(d/e)^(1/3)*ln(x^2-(d/e)^
(1/3)*x+(d/e)^(2/3))*ln(e*x^3+d)+p/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(
2/3))*ln((e*x^3+d)^p)-2*p^2*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)
^(1/3)*x-1))*ln(e*x^3+d)+2*p*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e
)^(1/3)*x-1))*ln((e*x^3+d)^p)+p^2*sum(1/_alpha*(2*ln(x-_alpha)*ln(e*x^3+d)-
e*(1/_alpha^2/e*ln(x-_alpha)^2+2*_alpha*ln(x-_alpha)*(2*RootOf(_Z^2+3*_Z*_a
lpha+3*_alpha^2,index=1)*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*ln((Ro
```

```
otOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha
+3*_alpha^2,index=1))+2*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*RootOf(
_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2
,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2))+3*RootOf(_
Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,
index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))*_alpha+6*Ro
otOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*ln((RootOf(_Z^2+3*_Z*_alpha+3*_al
pha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2))*_alph
a+6*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*ln((RootOf(_Z^2+3*_Z*_alpha
+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))
*_alpha+3*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*ln((RootOf(_Z^2+3*_Z*
_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,ind
ex=2))*_alpha+9*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/R
ootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))*_alpha^2+9*ln((RootOf(_Z^2+3*_Z
*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,in
dex=2))*_alpha^2)/(3*_alpha+2*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))/
d/(3*_alpha+2*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2))+2*_alpha*(2*dilo
g((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_
alpha+3*_alpha^2,index=2))*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*Root
Of(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)+6*dilog((RootOf(_Z^2+3*_Z*_alpha+3*
_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2))*Ro
otOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*_alpha+3*dilog((RootOf(_Z^2+3*_Z*
_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,ind
ex=2))*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*_alpha+9*dilog((RootOf(_
Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_al
pha^2,index=2))*_alpha^2+2*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*Root
Of(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*dilog((RootOf(_Z^2+3*_Z*_alpha+3*_a
lpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))+3*Ro
otOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*dilog((RootOf(_Z^2+3*_Z*_alpha+3*
_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))*_a
lpha+6*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*dilog((RootOf(_Z^2+3*_Z*
_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,ind
ex=1))*_alpha+9*dilog((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha
)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))*_alpha^2)/(3*_alpha+2*RootOf
(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))/d/(3*_alpha+2*RootOf(_Z^2+3*_Z*_alph
a+3*_alpha^2,index=2)))),_alpha=RootOf(_Z^3*e+d))+(I*Pi*csgn(I*(e*x^3+d)^p)
*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csg
n(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+
2*ln(c))*(-ln((e*x^3+d)^p)/x+3*p*e*(-1/3/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))+1/
6/e/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))+1/3*3^(1/2)/e/(d/e)^(1/3)
*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))))-1/4*(I*Pi*csgn(I*(e*x^3+d)^p)*cs
gn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I
*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*l
n(c))^2/x
```

Fricas [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^2}\,dx = \int \frac{\log\left((ex^3+d)^p c\right)^2}{x^2}\,dx$$

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^2,x, algorithm="fricas")
[Out] integral(log((e*x^3 + d)^p*c)^2/x^2, x)
```

Sympy [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^2}\,dx = \int \frac{\log\left(c(d+ex^3)^p\right)^2}{x^2}\,dx$$

```
[In] integrate(ln(c*(e*x**3+d)**p)**2/x**2,x)
[Out] Integral(log(c*(d + e*x**3)**p)**2/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^2}\,dx = \int \frac{\log\left((ex^3+d)^p c\right)^2}{x^2}\,dx$$

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^2,x, algorithm="giac")
[Out] integrate(log((e*x^3 + d)^p*c)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^2}\,dx = \int \frac{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}{x^2}\,dx$$

```
[In] int(log(c*(d + e*x^3)^p)^2/x^2,x)
[Out] int(log(c*(d + e*x^3)^p)^2/x^2, x)
```

3.136 $\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^3} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 1170

$$\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^3}\,dx$$

$$= -\frac{e^{2/3}p^2\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)}{2d^{2/3}} - \frac{e^{2/3}p^2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{d^{2/3}}$$

$$- \frac{(-1)^{2/3}e^{2/3}p^2\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)}{d^{2/3}}$$

$$- \frac{(-1)^{2/3}e^{2/3}p^2\log^2\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)}{2d^{2/3}}$$

$$+ \frac{\sqrt[3]{-1}e^{2/3}p^2\log\left(-\frac{(-1)^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)}{d^{2/3}}$$

$$+ \frac{\sqrt[3]{-1}e^{2/3}p^2\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)}{d^{2/3}}$$

$$+ \frac{\sqrt[3]{-1}e^{2/3}p^2\log^2\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)}{2d^{2/3}}$$

$$- \frac{e^{2/3}p^2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)}{d^{2/3}}$$

$$- \frac{\sqrt[3]{-1}e^{2/3}p^2\log\left(-\frac{(-1)^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)\log\left(\frac{\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{d^{2/3}}$$

$$- \frac{(-1)^{2/3}e^{2/3}p^2\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(-\frac{(-1)^{2/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{d^{2/3}}$$

$$+ \frac{e^{2/3}p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)^p\right)}{d^{2/3}}$$

$$+ \frac{(-1)^{2/3}e^{2/3}p\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)^p\right)}{d^{2/3}}$$

$$- \frac{\sqrt[3]{-1}e^{2/3}p\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)\log\left(c\left(d+ex^3\right)^p\right)}{d^{2/3}}$$

$$- \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{2x^2} - \frac{e^{2/3}p^2\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)}{d^{2/3}}$$

```
[Out] -1/2*e^(2/3)*p^2*ln(-d^(1/3)-e^(1/3)*x)^2/d^(2/3)-e^(2/3)*p^2*ln(-d^(1/3)-e
^(1/3)*x)*ln((-(-1)^(2/3)*d^(1/3)-e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(2/3
)-(-1)^(2/3)*e^(2/3)*p^2*ln((-1)^(1/3)*(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d
^(1/3))*ln(-d^(1/3)+(-1)^(1/3)*e^(1/3)*x)/d^(2/3)-1/2*(-1)^(2/3)*e^(2/3)*p^
2*ln(-d^(1/3)+(-1)^(1/3)*e^(1/3)*x)^2/d^(2/3)+(-1)^(1/3)*e^(2/3)*p^2*ln(-(-
1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(-d^(1/3)-(-1)^(2/3)
*e^(1/3)*x)/d^(2/3)+(-1)^(1/3)*e^(2/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3
)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(-d^(1/3)-(-1)^(2/3)*e^(1/3)*x)/d^(2
/3)+1/2*(-1)^(1/3)*e^(2/3)*p^2*ln(-d^(1/3)-(-1)^(2/3)*e^(1/3)*x)^2/d^(2/3)-
(-1)^(1/3)*e^(2/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)
^(1/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/
d^(2/3)-e^(2/3)*p^2*ln(-d^(1/3)-e^(1/3)*x)*ln((-1)^(1/3)*(d^(1/3)+(-1)^(2/3
)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(2/3)-(-1)^(1/3)*e^(2/3)*p^2*ln(-(-1
)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln((d^(1/3)+(-1)^(2/3)*
e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(2/3)+(-1)^(2/3)*e^(2/3)*p^2*ln(-(-1)^
(1/3)*((-1)^(2/3)*d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln(-(-1)^(2/3)
*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(2/3)-(-1)^(2/3)*
e^(2/3)*p^2*ln(-d^(1/3)+(-1)^(1/3)*e^(1/3)*x)*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^
(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(2/3)+e^(2/3)*p*ln(-d^(1/3)-e^(1
/3)*x)*ln(c*(e*x^3+d)^p)/d^(2/3)+(-1)^(2/3)*e^(2/3)*p*ln(-d^(1/3)+(-1)^(1/3
)*e^(1/3)*x)*ln(c*(e*x^3+d)^p)/d^(2/3)-(-1)^(1/3)*e^(2/3)*p*ln(-d^(1/3)-(-1
)^(2/3)*e^(1/3)*x)*ln(c*(e*x^3+d)^p)/d^(2/3)-1/2*ln(c*(e*x^3+d)^p)^2/x^2-e^
(2/3)*p^2*polylog(2,(d^(1/3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(2/3)-(-1
)^(1/3)*e^(2/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3)
)/d^(1/3))/d^(2/3)-(-1)^(2/3)*e^(2/3)*p^2*polylog(2,(d^(1/3)-(-1)^(1/3)*e^(
1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(2/3)-(-1)^(1/3)*e^(2/3)*p^2*polylog(2,(-
1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(2/3)+(-1
)^(2/3)*e^(2/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1
-(-1)^(2/3))/d^(1/3))/d^(2/3)-e^(2/3)*p^2*polylog(2,2*(d^(1/3)+e^(1/3)*x)/d
^(1/3)/(3-I*3^(1/2)))/d^(2/3)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 1176, normalized size of antiderivative = 1.01, number of steps used = 39, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules

used = {2507, 2521, 2512, 266, 2463, 2437, 2338, 2441, 2440, 2438, 12}

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^3}\,dx$$

$$= -\frac{e^{2/3}\log^2\left(-\sqrt[3]{e}x-\sqrt[3]{d}\right)p^2}{2d^{2/3}} - \frac{(-1)^{2/3}e^{2/3}\log^2\left(\sqrt[3]{-1}\sqrt[3]{e}x-\sqrt[3]{d}\right)p^2}{2d^{2/3}}$$

$$+ \frac{\sqrt[3]{-1}e^{2/3}\log^2\left(-(-1)^{2/3}\sqrt[3]{e}x-\sqrt[3]{d}\right)p^2}{2d^{2/3}}$$

$$- \frac{e^{2/3}\log\left(-\sqrt[3]{e}x-\sqrt[3]{d}\right)\log\left(-\frac{\sqrt[3]{e}x+(-1)^{2/3}\sqrt[3]{d}}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)p^2}{d^{2/3}}$$

$$- \frac{(-1)^{2/3}e^{2/3}\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(\sqrt[3]{-1}\sqrt[3]{e}x-\sqrt[3]{d}\right)p^2}{d^{2/3}}$$

$$+ \frac{\sqrt[3]{-1}e^{2/3}\log\left(-\frac{(-1)^{2/3}\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)\log\left(-(-1)^{2/3}\sqrt[3]{e}x-\sqrt[3]{d}\right)p^2}{d^{2/3}}$$

$$+ \frac{\sqrt[3]{-1}e^{2/3}\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(-(-1)^{2/3}\sqrt[3]{e}x-\sqrt[3]{d}\right)p^2}{d^{2/3}}$$

$$- \frac{\sqrt[3]{-1}e^{2/3}\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(\frac{(-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)p^2}{d^{2/3}}$$

$$- \frac{e^{2/3}\log\left(-\sqrt[3]{e}x-\sqrt[3]{d}\right)\log\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)p^2}{d^{2/3}}$$

$$- \frac{(-1)^{2/3}e^{2/3}\log\left(\sqrt[3]{-1}\sqrt[3]{e}x-\sqrt[3]{d}\right)\log\left(-\frac{(-1)^{2/3}\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)p^2}{d^{2/3}}$$

$$- \frac{e^{2/3}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{e}x+\sqrt[3]{d}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)p^2}{d^{2/3}} - \frac{e^{2/3}\,\mathrm{PolyLog}\left(2,\frac{2\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(3-i\sqrt{3}\right)\sqrt[3]{d}}\right)p^2}{d^{2/3}}$$

$$- \frac{(-1)^{2/3}e^{2/3}\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[3]{-1}\left(\sqrt[3]{e}x+(-1)^{2/3}\sqrt[3]{d}\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)p^2}{d^{2/3}}$$

$$- \frac{(-1)^{2/3}e^{2/3}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)p^2}{d^{2/3}}$$

```
[In] Int[Log[c*(d + e*x^3)^p]^2/x^3,x]
```

```
[Out] -1/2*(e^(2/3)*p^2*Log[-d^(1/3) - e^(1/3)*x]^2)/d^(2/3) - (e^(2/3)*p^2*Log[-
d^(1/3) - e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3
))*d^(1/3)))])/d^(2/3) - ((-1)^(2/3)*e^(2/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) +
 e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*
x])/d^(2/3) - ((-1)^(2/3)*e^(2/3)*p^2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]^
2)/(2*d^(2/3)) + ((-1)^(1/3)*e^(2/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/
3)*x))/((1 - (-1)^(2/3))*d^(1/3)))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/d
^(2/3) + ((-1)^(1/3)*e^(2/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1
/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x])/d
^(2/3) + ((-1)^(1/3)*e^(2/3)*p^2*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]^2)/(2
*d^(2/3)) - ((-1)^(1/3)*e^(2/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e
^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*x)
/((1 + (-1)^(1/3))*d^(1/3))])/d^(2/3) - (e^(2/3)*p^2*Log[-d^(1/3) - e^(1/3)
*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(
1/3))])/d^(2/3) - ((-1)^(2/3)*e^(2/3)*p^2*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)
*x]*Log[-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^(2/3))*d
^(1/3)))])/d^(2/3) + (e^(2/3)*p*Log[-d^(1/3) - e^(1/3)*x]*Log[c*(d + e*x^3)
^p])/d^(2/3) + ((-1)^(2/3)*e^(2/3)*p*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*L
og[c*(d + e*x^3)^p])/d^(2/3) - ((-1)^(1/3)*e^(2/3)*p*Log[-d^(1/3) - (-1)^(2
/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])/d^(2/3) - Log[c*(d + e*x^3)^p]^2/(2*x^
2) - (e^(2/3)*p^2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3
))])/d^(2/3) - (e^(2/3)*p^2*PolyLog[2, (2*(d^(1/3) + e^(1/3)*x))/((3 - I*Sq
rt[3])*d^(1/3))])/d^(2/3) - ((-1)^(2/3)*e^(2/3)*p^2*PolyLog[2, -(((-1)^(1/3
)*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/d^(2/3) -
 ((-1)^(2/3)*e^(2/3)*p^2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 +
(-1)^(1/3))*d^(1/3))])/d^(2/3) - ((-1)^(1/3)*e^(2/3)*p^2*PolyLog[2, ((-1)^(
1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/d^(2/3)
 + ((-1)^(1/3)*e^(2/3)*p^2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1
- (-1)^(2/3))*d^(1/3))])/d^(2/3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log^2\left(c(d+ex^3)^p\right)}{2x^2} + (3ep)\int \frac{\log\left(c(d+ex^3)^p\right)}{d+ex^3}\,dx \\
&= -\frac{\log^2\left(c(d+ex^3)^p\right)}{2x^2} + (3ep)\int \left(-\frac{\log\left(c(d+ex^3)^p\right)}{3d^{2/3}\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)}\right. \\
&\qquad \left. -\frac{\log\left(c(d+ex^3)^p\right)}{3d^{2/3}\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)} - \frac{\log\left(c(d+ex^3)^p\right)}{3d^{2/3}\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)}\right) dx \\
&= -\frac{\log^2\left(c(d+ex^3)^p\right)}{2x^2} - \frac{(ep)\int \frac{\log\left(c(d+ex^3)^p\right)}{-\sqrt[3]{d}-\sqrt[3]{e}x}\,dx}{d^{2/3}} \\
&\quad - \frac{(ep)\int \frac{\log\left(c(d+ex^3)^p\right)}{-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x}\,dx}{d^{2/3}} - \frac{(ep)\int \frac{\log\left(c(d+ex^3)^p\right)}{-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x}\,dx}{d^{2/3}} \\
&= \frac{e^{2/3}p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{d^{2/3}} \\
&\quad + \frac{(-1)^{2/3}e^{2/3}p\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{d^{2/3}} \\
&\quad - \frac{\sqrt[3]{-1}e^{2/3}p\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{d^{2/3}} - \frac{\log^2\left(c(d+ex^3)^p\right)}{2x^2} \\
&\quad - \frac{\left(3e^{5/3}p^2\right)\int \frac{x^2\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)}{d+ex^3}\,dx}{d^{2/3}} + \frac{\left(3\sqrt[3]{-1}e^{5/3}p^2\right)\int \frac{x^2\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)}{d+ex^3}\,dx}{d^{2/3}} \\
&\quad - \frac{\left(3(-1)^{2/3}e^{5/3}p^2\right)\int \frac{x^2\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)}{d+ex^3}\,dx}{d^{2/3}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{e^{2/3} p \log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right) \log\left(c(d+ex^3)^p\right)}{d^{2/3}} \\
&+ \frac{(-1)^{2/3} e^{2/3} p \log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right) \log\left(c(d+ex^3)^p\right)}{d^{2/3}} \\
&- \frac{\sqrt[3]{-1} e^{2/3} p \log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right) \log\left(c(d+ex^3)^p\right)}{d^{2/3}} - \frac{\log^2\left(c(d+ex^3)^p\right)}{2x^2} \\
&- \frac{\left(3e^{5/3}p^2\right) \int \left(\frac{\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3e^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3e^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3e^{2/3}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}\right) dx}{d^{2/3}} \\
&+ \frac{\left(3\sqrt[3]{-1}e^{5/3}p^2\right) \int \left(\frac{\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)}{3e^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)}{3e^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)}{3e^{2/3}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}\right) dx}{d^{2/3}} \\
&- \frac{\left(3(-1)^{2/3}e^{5/3}p^2\right) \int \left(\frac{\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)}{3e^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)}{3e^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)}{3e^{2/3}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}\right) d}{d^{2/3}}
\end{aligned}
$$

$$= \frac{e^{2/3}p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{d^{2/3}} + \frac{(-1)^{2/3}e^{2/3}p\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{d^{2/3}} - \frac{\sqrt[3]{-1}e^{2/3}p\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{d^{2/3}} - \frac{\log^2\left(c(d+ex^3)^p\right)}{2x^2} - \frac{(ep^2)\int\frac{\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{d^{2/3}} - \frac{(ep^2)\int\frac{\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)}{-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{d^{2/3}} - \frac{(ep^2)\int\frac{\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)}{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{d^{2/3}} + \frac{\left(\sqrt[3]{-1}ep^2\right)\int\frac{\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{d^{2/3}} + \frac{\left(\sqrt[3]{-1}ep^2\right)\int\frac{\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)}{-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{d^{2/3}} + \frac{\left(\sqrt[3]{-1}ep^2\right)\int\frac{\log\left(-\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{e}x\right)}{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{d^{2/3}} - \frac{\left((-1)^{2/3}ep^2\right)\int\frac{\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{d^{2/3}} - \frac{\left((-1)^{2/3}ep^2\right)\int\frac{\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)}{-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{d^{2/3}} - \frac{\left((-1)^{2/3}ep^2\right)\int\frac{\log\left(-\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{e}x\right)}{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{d^{2/3}}$$

$$= \text{Too large to display}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 766, normalized size of antiderivative = 0.65

$$\int\frac{\log^2\left(c(d+ex^3)^p\right)}{x^3}\,dx = -\frac{\log^2\left(c(d+ex^3)^p\right)}{2x^2} - \frac{e^{2/3}p\left(p\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)+2p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\log\left(\frac{\sqrt[3]{-1}\sqrt[3]{d}-\sqrt[3]{e}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)+2p\log\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right)\mathrm{lo}\right.}{}$$

```
[In] Integrate[Log[c*(d + e*x^3)^p]^2/x^3,x]
```

```
[Out] -1/2*Log[c*(d + e*x^3)^p]^2/x^2 - (e^(2/3)*p*(p*Log[-d^(1/3) - e^(1/3)*x]^2
 + 2*p*Log[-d^(1/3) - e^(1/3)*x]*Log[((-1)^(1/3)*d^(1/3) - e^(1/3)*x)/((1 +
 (-1)^(1/3))*d^(1/3))] + 2*p*Log[-d^(1/3) - e^(1/3)*x]*Log[(I + Sqrt[3] - (
(2*I)*e^(1/3)*x)/d^(1/3))/(3*I + Sqrt[3])] - 2*Log[-d^(1/3) - e^(1/3)*x]*Lo
g[c*(d + e*x^3)^p] - 2*(-1)^(2/3)*Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*Log[
c*(d + e*x^3)^p] + 2*(-1)^(1/3)*Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*
(d + e*x^3)^p] + 2*p*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(
1/3))] + (-1)^(2/3)*p*(Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*(2*Log[((-1)^(1
/3)*(d^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))] + Log[-d^(1/3) + (-1
)^(1/3)*e^(1/3)*x] + 2*Log[((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((
-1 + (-1)^(2/3))*d^(1/3))]) + 2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)
/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, (-d^(1/3) + (-1)^(1/3)*e^(1/3)*
x)/((-1 + (-1)^(2/3))*d^(1/3))]) - (-1)^(1/3)*p*(Log[-d^(1/3) - (-1)^(2/3)*
e^(1/3)*x]*(2*Log[((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((-1 + (-1)^(2/3))*d^(
1/3))] + 2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/
3))*d^(1/3))] + Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]) + 2*PolyLog[2, (d^(1/
3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, (d^(1
/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))]) + 2*p*PolyLog[2, (
(2*I)*(1 + (e^(1/3)*x)/d^(1/3)))/(3*I + Sqrt[3])]))/(2*d^(2/3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.77 (sec) , antiderivative size = 1787, normalized size of antiderivative = 1.53

method	result	size
risch	Expression too large to display	1787

```
[In] int(ln(c*(e*x^3+d)^p)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2*ln((e*x^3+d)^p)^2-p^2/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*ln(e*x^3+d)+p/
(d/e)^(2/3)*ln(x+(d/e)^(1/3))*ln((e*x^3+d)^p)+1/2*p^2/(d/e)^(2/3)*ln(x^2-(d
/e)^(1/3)*x+(d/e)^(2/3))*ln(e*x^3+d)-1/2*p/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x
+(d/e)^(2/3))*ln((e*x^3+d)^p)-p^2/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(d/e)^(1/3)*x-1))*ln(e*x^3+d)+p/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/
(d/e)^(1/3)*x-1))*ln((e*x^3+d)^p)+1/2*p^2*sum(1/_alpha^2*(2*ln(x-_alpha)*ln
(e*x^3+d)-e*(1/_alpha^2/e*ln(x-_alpha)^2+2*_alpha*ln(x-_alpha)*(2*RootOf(_Z
^2+3*_Z*_alpha+3*_alpha^2,index=1)*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index
=2)*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3
*_Z*_alpha+3*_alpha^2,index=1))+2*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=
1)*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*ln((RootOf(_Z^2+3*_Z*_alpha+
3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2))+
3*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*ln((RootOf(_Z^2+3*_Z*_alpha+3
*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))*_
alpha+6*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*ln((RootOf(_Z^2+3*_Z*_a
```

```
lpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index
=2))*_alpha+6*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*ln((RootOf(_Z^2+3
*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2
,index=1))*_alpha+3*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*ln((RootOf(
_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_a
lpha^2,index=2))*_alpha+9*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x
+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))*_alpha^2+9*ln((RootOf
(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_
alpha^2,index=2))*_alpha^2)/(3*_alpha+2*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,
index=1))/d/(3*_alpha+2*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2))+2*_alp
ha*(2*dilog((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_
Z^2+3*_Z*_alpha+3*_alpha^2,index=2))*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,ind
ex=1)*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)+6*dilog((RootOf(_Z^2+3*_Z
*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,in
dex=2))*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*_alpha+3*dilog((RootOf(
_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_a
lpha^2,index=2))*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*_alpha+9*dilog
((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_a
lpha+3*_alpha^2,index=2))*_alpha^2+2*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,ind
ex=1)*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*dilog((RootOf(_Z^2+3*_Z*_
alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,inde
x=1))+3*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*dilog((RootOf(_Z^2+3*_Z
*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,in
dex=1))*_alpha+6*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*dilog((RootOf(
_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_a
lpha^2,index=1))*_alpha+9*dilog((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1
)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))*_alpha^2)/(3*_alph
a+2*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))/d/(3*_alpha+2*RootOf(_Z^2+
3*_Z*_alpha+3*_alpha^2,index=2)))),_alpha=RootOf(_Z^3*e+d))+(I*Pi*csgn(I*(e
*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3
+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*
csgn(I*c)+2*ln(c))*(-1/2*ln((e*x^3+d)^p)/x^2+3/2*p*e*(1/3/e/(d/e)^(2/3)*ln(
x+(d/e)^(1/3))-1/6/e/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))+1/3/e/(d
/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))))-1/8*(I*Pi*csgn(
I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e
*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p
)^2*csgn(I*c)+2*ln(c))^2/x^2
```

Fricas [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^3}\,dx = \int \frac{\log\left((ex^3+d)^p c\right)^2}{x^3}\,dx$$

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^3,x, algorithm="fricas")
[Out] integral(log((e*x^3 + d)^p*c)^2/x^3, x)
```

Sympy [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^3}\,dx = \int \frac{\log\left(c(d+ex^3)^p\right)^2}{x^3}\,dx$$

```
[In] integrate(ln(c*(e*x**3+d)**p)**2/x**3,x)
[Out] Integral(log(c*(d + e*x**3)**p)**2/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^3}\,dx = \int \frac{\log\left((ex^3+d)^p c\right)^2}{x^3}\,dx$$

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^3,x, algorithm="giac")
[Out] integrate(log((e*x^3 + d)^p*c)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^3}\,dx = \int \frac{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}{x^3}\,dx$$

[In] int(log(c*(d + e*x^3)^p)^2/x^3,x)

[Out] int(log(c*(d + e*x^3)^p)^2/x^3, x)

3.137 $\int \frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{x^5}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 1328

$$
\begin{aligned}
&\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^5}\,dx \\
&= -\frac{3\sqrt{3}e^{4/3}p^2\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2d^{4/3}}-\frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} \\
&\quad-\frac{e^{4/3}p^2\log^2\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{4d^{4/3}}-\frac{e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(-\frac{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{2d^{4/3}} \\
&\quad+\frac{\sqrt[3]{-1}e^{4/3}p^2\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{2d^{4/3}} \\
&\quad+\frac{\sqrt[3]{-1}e^{4/3}p^2\log^2\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{4d^{4/3}} \\
&\quad-\frac{(-1)^{2/3}e^{4/3}p^2\log\left(-\frac{(-1)^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{2d^{4/3}} \\
&\quad-\frac{(-1)^{2/3}e^{4/3}p^2\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{2d^{4/3}} \\
&\quad-\frac{(-1)^{2/3}e^{4/3}p^2\log^2\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{4d^{4/3}} \\
&\quad-\frac{e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)}{2d^{4/3}} \\
&\quad+\frac{(-1)^{2/3}e^{4/3}p^2\log\left(-\frac{(-1)^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)\log\left(\frac{\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{2d^{4/3}} \\
&\quad+\frac{\sqrt[3]{-1}e^{4/3}p^2\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(-\frac{(-1)^{2/3}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\left(1-(-1)^{2/3}\right)\sqrt[3]{d}}\right)}{2d^{4/3}} \\
&\quad+\frac{3e^{4/3}p^2\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{4d^{4/3}}-\frac{3ep\log\left(c(d+ex^3)^p\right)}{2dx} \\
&\quad+\frac{e^{4/3}p\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{2d^{4/3}} \\
&\quad-\frac{\sqrt[3]{-1}e^{4/3}p\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{2d^{4/3}} \\
&\quad (-1)^{2/3}e^{4/3}p\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)
\end{aligned}
$$

```
[Out] 1/2*(-1)^(2/3)*e^(4/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1
)^(2/3))/d^(1/3))/d^(4/3)+1/2*(-1)^(1/3)*e^(4/3)*p^2*polylog(2,(d^(1/3)-(-1
)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(4/3)+1/2*(-1)^(2/3)*e^(4/3)*p
^2*polylog(2,(-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/
3))/d^(4/3)-1/2*(-1)^(1/3)*e^(4/3)*p^2*polylog(2,-(-1)^(2/3)*(d^(1/3)+(-1)^
(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(4/3)-3/2*e^(4/3)*p^2*arctan(1/3
*(d^(1/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2))*3^(1/2)/d^(4/3)+1/2*e^(4/3)*p*ln(d^
(1/3)+e^(1/3)*x)*ln(c*(e*x^3+d)^p)/d^(4/3)-1/2*e^(4/3)*p^2*ln(d^(1/3)+e^(1/
3)*x)*ln((-1)^(1/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/
d^(4/3)-3/2*e*p*ln(c*(e*x^3+d)^p)/d/x-1/4*(-1)^(2/3)*e^(4/3)*p^2*ln(d^(1/3)
+(-1)^(2/3)*e^(1/3)*x)^2/d^(4/3)-1/2*e^(4/3)*p^2*ln(d^(1/3)+e^(1/3)*x)*ln((
-(-1)^(2/3)*d^(1/3)-e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(4/3)+1/4*(-1)^(1/
3)*e^(4/3)*p^2*ln(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)^2/d^(4/3)-1/4*ln(c*(e*x^3+d
)^p)^2/x^4-1/2*e^(4/3)*p^2*polylog(2,2*(d^(1/3)+e^(1/3)*x)/d^(1/3)/(3-I*3^(
1/2)))/d^(4/3)-3/2*e^(4/3)*p^2*ln(d^(1/3)+e^(1/3)*x)/d^(4/3)-1/4*e^(4/3)*p^
2*ln(d^(1/3)+e^(1/3)*x)^2/d^(4/3)+3/4*e^(4/3)*p^2*ln(d^(2/3)-d^(1/3)*e^(1/3
)*x+e^(2/3)*x^2)/d^(4/3)-1/2*e^(4/3)*p^2*polylog(2,(d^(1/3)+e^(1/3)*x)/(1+(
-1)^(1/3))/d^(1/3))/d^(4/3)+1/2*(-1)^(1/3)*e^(4/3)*p^2*ln((-1)^(1/3)*(d^(1/
3)+e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/d^(4
/3)-1/2*(-1)^(2/3)*e^(4/3)*p^2*ln(-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(
2/3))/d^(1/3))*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/d^(4/3)-1/2*(-1)^(2/3)*e^(4
/3)*p^2*ln((-1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3)
)*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/d^(4/3)+1/2*(-1)^(2/3)*e^(4/3)*p^2*ln((-
1)^(1/3)*(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))*ln((d^(1/3)
+(-1)^(2/3)*e^(1/3)*x)/(1+(-1)^(1/3))/d^(1/3))/d^(4/3)+1/2*(-1)^(2/3)*e^(4/
3)*p^2*ln(-(-1)^(2/3)*(d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))*ln((d^(1/
3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(4/3)-1/2*(-1)^(1/3)*e^(
4/3)*p^2*ln(-(-1)^(1/3)*((-1)^(2/3)*d^(1/3)+e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/
3))*ln(-(-1)^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d
^(4/3)+1/2*(-1)^(1/3)*e^(4/3)*p^2*ln(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)*ln(-(-1)
^(2/3)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)/(1-(-1)^(2/3))/d^(1/3))/d^(4/3)-1/2*(
-1)^(1/3)*e^(4/3)*p*ln(d^(1/3)-(-1)^(1/3)*e^(1/3)*x)*ln(c*(e*x^3+d)^p)/d^(4
/3)+1/2*(-1)^(2/3)*e^(4/3)*p*ln(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)*ln(c*(e*x^3+d
)^p)/d^(4/3)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 1334, normalized size of antiderivative = 1.00, number of steps used = 48, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {2507, 2526, 2505, 298, 31, 648, 631, 210, 642, 2512, 266, 2463, 2437, 2338, 2441,

2440, 2438, 12}

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^5}\,dx$$

$$= -\frac{e^{4/3}\log^2\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)p^2}{4d^{4/3}} + \frac{\sqrt[3]{-1}e^{4/3}\log^2\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)p^2}{4d^{4/3}}$$

$$- \frac{(-1)^{2/3}e^{4/3}\log^2\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)p^2}{4d^{4/3}} - \frac{3\sqrt{3}e^{4/3}\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)p^2}{2d^{4/3}}$$

$$- \frac{3e^{4/3}\log\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)p^2}{2d^{4/3}} - \frac{e^{4/3}\log\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)\log\left(-\frac{\sqrt[3]{e}x+(-1)^{2/3}\sqrt[3]{d}}{(1-(-1)^{2/3})\sqrt[3]{d}}\right)p^2}{2d^{4/3}}$$

$$+ \frac{\sqrt[3]{-1}e^{4/3}\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)p^2}{2d^{4/3}}$$

$$- \frac{(-1)^{2/3}e^{4/3}\log\left(-\frac{(-1)^{2/3}\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{(1-(-1)^{2/3})\sqrt[3]{d}}\right)\log\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)p^2}{2d^{4/3}}$$

$$- \frac{(-1)^{2/3}e^{4/3}\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)p^2}{2d^{4/3}}$$

$$+ \frac{(-1)^{2/3}e^{4/3}\log\left(\frac{\sqrt[3]{-1}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)\log\left(\frac{(-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)p^2}{2d^{4/3}}$$

$$- \frac{e^{4/3}\log\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)\log\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)p^2}{2d^{4/3}}$$

$$+ \frac{\sqrt[3]{-1}e^{4/3}\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(-\frac{(-1)^{2/3}\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)}{(1-(-1)^{2/3})\sqrt[3]{d}}\right)p^2}{2d^{4/3}}$$

$$+ \frac{3e^{4/3}\log\left(e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}\right)p^2}{4d^{4/3}}$$

$$- \frac{e^{4/3}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{e}x+\sqrt[3]{d}}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{d}}\right)p^2}{2d^{4/3}} - \frac{e^{4/3}\,\mathrm{PolyLog}\left(2,\frac{2\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(3-i\sqrt{3}\right)\sqrt[3]{d}}\right)p^2}{2d^{4/3}}$$

$$+ \frac{\sqrt[3]{-1}e^{4/3}\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[3]{-1}\left(\sqrt[3]{e}x+(-1)^{2/3}\sqrt[3]{d}\right)}{(1-(-1)^{2/3})\sqrt[3]{d}}\right)p^2}{2d^{4/3}}$$

$$\sqrt[3]{-1}e^{4/3}\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x}{\dots}\right)p^2$$

```
[In] Int[Log[c*(d + e*x^3)^p]^2/x^5,x]
```

```
[Out] (-3*Sqrt[3]*e^(4/3)*p^2*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/
(2*d^(4/3)) - (3*e^(4/3)*p^2*Log[d^(1/3) + e^(1/3)*x])/(2*d^(4/3)) - (e^(4/
3)*p^2*Log[d^(1/3) + e^(1/3)*x]^2)/(4*d^(4/3)) - (e^(4/3)*p^2*Log[d^(1/3) +
 e^(1/3)*x]*Log[-(((-1)^(2/3)*d^(1/3) + e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3
)))])/(2*d^(4/3)) + ((-1)^(1/3)*e^(4/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) + e^(1
/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x])/(2
*d^(4/3)) + ((-1)^(1/3)*e^(4/3)*p^2*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]^2)/
(4*d^(4/3)) - ((-1)^(2/3)*e^(4/3)*p^2*Log[-(((-1)^(2/3)*(d^(1/3) + e^(1/3)*
x))/((1 - (-1)^(2/3))*d^(1/3)))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/(2*d^
(4/3)) - ((-1)^(2/3)*e^(4/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1
/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x])/(2
*d^(4/3)) - ((-1)^(2/3)*e^(4/3)*p^2*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]^2)/
(4*d^(4/3)) + ((-1)^(2/3)*e^(4/3)*p^2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)
*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))]*Log[(d^(1/3) + (-1)^(2/3)*e^(1/3)*
x)/((1 + (-1)^(1/3))*d^(1/3))])/(2*d^(4/3)) - (e^(4/3)*p^2*Log[d^(1/3) + e^
(1/3)*x]*Log[((-1)^(1/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 + (-1)^(1/3)
)*d^(1/3))])/(2*d^(4/3)) + ((-1)^(1/3)*e^(4/3)*p^2*Log[d^(1/3) - (-1)^(1/3)
*e^(1/3)*x]*Log[-(((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((1 - (-1)^
(2/3))*d^(1/3)))])/(2*d^(4/3)) + (3*e^(4/3)*p^2*Log[d^(2/3) - d^(1/3)*e^(1/
3)*x + e^(2/3)*x^2])/(4*d^(4/3)) - (3*e*p*Log[c*(d + e*x^3)^p])/(2*d*x) + (
e^(4/3)*p*Log[d^(1/3) + e^(1/3)*x]*Log[c*(d + e*x^3)^p])/(2*d^(4/3)) - ((-1
)^(1/3)*e^(4/3)*p*Log[d^(1/3) - (-1)^(1/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p])
/(2*d^(4/3)) + ((-1)^(2/3)*e^(4/3)*p*Log[d^(1/3) + (-1)^(2/3)*e^(1/3)*x]*Lo
g[c*(d + e*x^3)^p])/(2*d^(4/3)) - Log[c*(d + e*x^3)^p]^2/(4*x^4) - (e^(4/3)
*p^2*PolyLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))])/(2*d^(4/
3)) - (e^(4/3)*p^2*PolyLog[2, (2*(d^(1/3) + e^(1/3)*x))/((3 - I*Sqrt[3])*d^
(1/3))])/(2*d^(4/3)) + ((-1)^(1/3)*e^(4/3)*p^2*PolyLog[2, -(((-1)^(1/3)*((-
1)^(2/3)*d^(1/3) + e^(1/3)*x))/((1 - (-1)^(2/3))*d^(1/3)))])/(2*d^(4/3)) +
((-1)^(1/3)*e^(4/3)*p^2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 + (
-1)^(1/3))*d^(1/3))])/(2*d^(4/3)) + ((-1)^(2/3)*e^(4/3)*p^2*PolyLog[2, ((-1
)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))])/(2*d
^(4/3)) - ((-1)^(2/3)*e^(4/3)*p^2*PolyLog[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*
x)/((1 - (-1)^(2/3))*d^(1/3))])/(2*d^(4/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
```

```
qQ[e*f - d*g, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
```

```
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{4x^4}+\frac{1}{2}(3ep)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{x^2\left(d+ex^3\right)}\,dx\\
&= -\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{4x^4}+\frac{1}{2}(3ep)\int \left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{dx^2}-\frac{ex\log\left(c\left(d+ex^3\right)^p\right)}{d\left(d+ex^3\right)}\right)dx\\
&= -\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{4x^4}+\frac{(3ep)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{x^2}\,dx}{2d}-\frac{\left(3e^2p\right)\int \frac{x\log\left(c\left(d+ex^3\right)^p\right)}{d+ex^3}\,dx}{2d}\\
&= -\frac{3ep\log\left(c\left(d+ex^3\right)^p\right)}{2dx}-\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{4x^4}\\
&\quad-\frac{\left(3e^2p\right)\int \left(-\frac{\log\left(c\left(d+ex^3\right)^p\right)}{3\sqrt[3]{d}\sqrt[3]{e}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}-\frac{(-1)^{2/3}\log\left(c\left(d+ex^3\right)^p\right)}{3\sqrt[3]{d}\sqrt[3]{e}\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}+\frac{\sqrt[3]{-1}\log\left(c\left(d+ex^3\right)^p\right)}{3\sqrt[3]{d}\sqrt[3]{e}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}\right)dx}{2d}\\
&\quad+\frac{\left(9e^2p^2\right)\int \frac{x}{d+ex^3}\,dx}{2d}\\
&= -\frac{3ep\log\left(c\left(d+ex^3\right)^p\right)}{2dx}-\frac{\log^2\left(c\left(d+ex^3\right)^p\right)}{4x^4}+\frac{\left(e^{5/3}p\right)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{2d^{4/3}}\\
&\quad-\frac{\left(\sqrt[3]{-1}e^{5/3}p\right)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x}\,dx}{2d^{4/3}}+\frac{\left((-1)^{2/3}e^{5/3}p\right)\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x}\,dx}{2d^{4/3}}\\
&\quad-\frac{\left(3e^{5/3}p^2\right)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{2d^{4/3}}+\frac{\left(3e^{5/3}p^2\right)\int \frac{\sqrt[3]{d}+\sqrt[3]{e}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{2d^{4/3}}
\end{aligned}$$

$$= -\frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} - \frac{3ep\log\left(c(d+ex^3)^p\right)}{2dx}$$
$$+ \frac{e^{4/3}p\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{2d^{4/3}}$$
$$- \frac{\sqrt[3]{-1}e^{4/3}p\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{2d^{4/3}}$$
$$+ \frac{(-1)^{2/3}e^{4/3}p\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{2d^{4/3}} - \frac{\log^2\left(c(d+ex^3)^p\right)}{4x^4}$$
$$+ \frac{\left(3e^{4/3}p^2\right)\int\frac{-\sqrt[3]{d}\sqrt[3]{e}+2e^{2/3}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{4d^{4/3}} + \frac{\left(9e^{5/3}p^2\right)\int\frac{1}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{4d}$$
$$- \frac{\left(3e^{7/3}p^2\right)\int\frac{x^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{d+ex^3}\,dx}{2d^{4/3}} + \frac{\left(3\sqrt[3]{-1}e^{7/3}p^2\right)\int\frac{x^2\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{d+ex^3}\,dx}{2d^{4/3}}$$
$$- \frac{\left(3(-1)^{2/3}e^{7/3}p^2\right)\int\frac{x^2\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{d+ex^3}\,dx}{2d^{4/3}}$$

$$= -\frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} + \frac{3e^{4/3}p^2\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{4d^{4/3}}$$
$$-\frac{3ep\log\left(c(d+ex^3)^p\right)}{2dx} + \frac{e^{4/3}p\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{2d^{4/3}}$$
$$-\frac{\sqrt[3]{-1}e^{4/3}p\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{2d^{4/3}}$$
$$+\frac{(-1)^{2/3}e^{4/3}p\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{2d^{4/3}}$$
$$-\frac{\log^2\left(c(d+ex^3)^p\right)}{4x^4} + \frac{(9e^{4/3}p^2)\,\mathrm{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{2d^{4/3}}$$
$$-\frac{(3e^{7/3}p^2)\int\left(\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3e^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3e^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x\right)}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3e^{2/3}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}\right)dx}{2d^{4/3}}$$
$$+\frac{(3\sqrt[3]{-1}e^{7/3}p^2)\int\left(\frac{\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{3e^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}+\frac{\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{3e^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x\right)}+\frac{\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{3e^{2/3}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}\right)dx}{2d^{4/3}}$$
$$-\frac{(3(-1)^{2/3}e^{7/3}p^2)\int\left(\frac{\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{3e^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}+\frac{\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{3e^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x\right)}+\frac{\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{3e^{2/3}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}\right)}{2d^{4/3}}$$

$$
\begin{aligned}
&= -\frac{3\sqrt{3}e^{4/3}p^2\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{2d^{4/3}} - \frac{3e^{4/3}p^2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{2d^{4/3}} \\
&\quad + \frac{3e^{4/3}p^2\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{4d^{4/3}} - \frac{3ep\log\left(c(d+ex^3)^p\right)}{2dx} \\
&\quad + \frac{e^{4/3}p\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{2d^{4/3}} \\
&\quad - \frac{\sqrt[3]{-1}e^{4/3}p\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{2d^{4/3}} \\
&\quad + \frac{(-1)^{2/3}e^{4/3}p\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)\log\left(c(d+ex^3)^p\right)}{2d^{4/3}} - \frac{\log^2\left(c(d+ex^3)^p\right)}{4x^4} \\
&\quad - \frac{\left(e^{5/3}p^2\right)\int\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{2d^{4/3}} - \frac{\left(e^{5/3}p^2\right)\int\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{2d^{4/3}} \\
&\quad - \frac{\left(e^{5/3}p^2\right)\int\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{2d^{4/3}} + \frac{\left(\sqrt[3]{-1}e^{5/3}p^2\right)\int\frac{\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{2d^{4/3}} \\
&\quad + \frac{\left(\sqrt[3]{-1}e^{5/3}p^2\right)\int\frac{\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{2d^{4/3}} + \frac{\left(\sqrt[3]{-1}e^{5/3}p^2\right)\int\frac{\log\left(\sqrt[3]{d}-\sqrt[3]{-1}\sqrt[3]{e}x\right)}{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{2d^{4/3}} \\
&\quad - \frac{\left((-1)^{2/3}e^{5/3}p^2\right)\int\frac{\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{2d^{4/3}} \\
&\quad - \frac{\left((-1)^{2/3}e^{5/3}p^2\right)\int\frac{\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{2d^{4/3}} \\
&\quad - \frac{\left((-1)^{2/3}e^{5/3}p^2\right)\int\frac{\log\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{2d^{4/3}} \\
&= \text{Too large to display}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.01 (sec) , antiderivative size = 912, normalized size of antiderivative = 0.69

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^5}\,dx$$

$$= \frac{-\log^2\left(c(d+ex^3)^p\right) + \frac{epx^3\left(9epx^3\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{ex^3}{d}\right) - d^{2/3}\sqrt[3]{e}px\log^2\left(-\sqrt[3]{d}-\sqrt[3]{e}x\right) - 2d^{2/3}\sqrt[3]{e}px\log\left(-\sqrt[3]{d}-\right.\right.}{}}{}$$

```
[In] Integrate[Log[c*(d + e*x^3)^p]^2/x^5,x]
```

```
[Out] (-Log[c*(d + e*x^3)^p]^2 + (e*p*x^3*(9*e*p*x^3*Hypergeometric2F1[2/3, 1, 5/
3, -((e*x^3)/d)] - d^(2/3)*e^(1/3)*p*x*Log[-d^(1/3) - e^(1/3)*x]^2 - 2*d^(2
/3)*e^(1/3)*p*x*Log[-d^(1/3) - e^(1/3)*x]*Log[((-1)^(1/3)*d^(1/3) - e^(1/3)
*x)/((1 + (-1)^(1/3))*d^(1/3))] - 2*d^(2/3)*e^(1/3)*p*x*Log[-d^(1/3) - e^(1
/3)*x]*Log[(I + Sqrt[3] - ((2*I)*e^(1/3)*x)/d^(1/3))/(3*I + Sqrt[3])] - 6*d
*Log[c*(d + e*x^3)^p] + 2*d^(2/3)*e^(1/3)*x*Log[-d^(1/3) - e^(1/3)*x]*Log[c
*(d + e*x^3)^p] - 2*(-1)^(1/3)*d^(2/3)*e^(1/3)*x*Log[-d^(1/3) + (-1)^(1/3)*
e^(1/3)*x]*Log[c*(d + e*x^3)^p] + 2*(-1)^(2/3)*d^(2/3)*e^(1/3)*x*Log[-d^(1/
3) - (-1)^(2/3)*e^(1/3)*x]*Log[c*(d + e*x^3)^p] - 2*d^(2/3)*e^(1/3)*p*x*Pol
yLog[2, (d^(1/3) + e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + (-1)^(1/3)*d^(2
/3)*e^(1/3)*p*x*(Log[-d^(1/3) + (-1)^(1/3)*e^(1/3)*x]*(2*Log[((-1)^(1/3)*(d
^(1/3) + e^(1/3)*x))/((1 + (-1)^(1/3))*d^(1/3))] + Log[-d^(1/3) + (-1)^(1/3
)*e^(1/3)*x] + 2*Log[((-1)^(2/3)*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/((-1 + (
-1)^(2/3))*d^(1/3))]) + 2*PolyLog[2, (d^(1/3) - (-1)^(1/3)*e^(1/3)*x)/((1 +
 (-1)^(1/3))*d^(1/3))] + 2*PolyLog[2, (-d^(1/3) + (-1)^(1/3)*e^(1/3)*x)/((-
1 + (-1)^(2/3))*d^(1/3))]) - (-1)^(2/3)*d^(2/3)*e^(1/3)*p*x*(Log[-d^(1/3) -
 (-1)^(2/3)*e^(1/3)*x]*(2*Log[((-1)^(2/3)*(d^(1/3) + e^(1/3)*x))/((-1 + (-1
)^(2/3))*d^(1/3))] + 2*Log[((-1)^(1/3)*(d^(1/3) - (-1)^(1/3)*e^(1/3)*x))/((
1 + (-1)^(1/3))*d^(1/3))] + Log[-d^(1/3) - (-1)^(2/3)*e^(1/3)*x]) + 2*PolyL
og[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 + (-1)^(1/3))*d^(1/3))] + 2*Poly
Log[2, (d^(1/3) + (-1)^(2/3)*e^(1/3)*x)/((1 - (-1)^(2/3))*d^(1/3))]) - 2*d^
(2/3)*e^(1/3)*p*x*PolyLog[2, ((2*I)*(1 + (e^(1/3)*x)/d^(1/3)))/(3*I + Sqrt[
3])]))/d^2)/(4*x^4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.17 (sec) , antiderivative size = 1954, normalized size of antiderivative = 1.47

method	result	size
risch	Expression too large to display	1954

```
[In] int(ln(c*(e*x^3+d)^p)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln((e*x^3+d)^p)^2/x^4-3/2*p*e*ln((e*x^3+d)^p)/d/x-1/2*p^2*e/d/(d/e)^(1
/3)*ln(x+(d/e)^(1/3))*ln(e*x^3+d)+1/2*p*e/d/(d/e)^(1/3)*ln(x+(d/e)^(1/3))*l
n((e*x^3+d)^p)+1/4*p^2*e/d/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*ln
(e*x^3+d)-1/4*p*e/d/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*ln((e*x^3
+d)^p)+1/2*p^2*e/d*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-
1))*ln(e*x^3+d)-1/2*p*e/d*3^(1/2)/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(
1/3)*x-1))*ln((e*x^3+d)^p)-3/2*p^2*e/d/(d/e)^(1/3)*ln(x+(d/e)^(1/3))+3/4*p^
2*e/d/(d/e)^(1/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))+3/2*p^2*e/d*3^(1/2)/(d/
e)^(1/3)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))+3/2*p^2*e*Sum(-1/3*(ln(x-_
alpha)*ln(e*x^3+d)-3*e*(1/6/_alpha^2/e*ln(x-_alpha)^2+1/3*_alpha*ln(x-_alph
a)*(2*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*RootOf(_Z^2+3*_Z*_alpha+3
*_alpha^2,index=2)*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha
)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))+2*RootOf(_Z^2+3*_Z*_alpha+3*
_alpha^2,index=1)*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*ln((RootOf(_Z
^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alp
ha^2,index=2))+3*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*ln((RootOf(_Z^
2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alph
a^2,index=1))*_alpha+6*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*ln((Root
Of(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3
*_alpha^2,index=2))*_alpha+6*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*ln
((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_a
lpha+3*_alpha^2,index=1))*_alpha+3*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index
=2)*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3
*_Z*_alpha+3*_alpha^2,index=2))*_alpha+9*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alp
ha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))*_alpha
^2+9*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+
3*_Z*_alpha+3*_alpha^2,index=2))*_alpha^2)/(3*_alpha+2*RootOf(_Z^2+3*_Z*_al
pha+3*_alpha^2,index=1))/d/(3*_alpha+2*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,i
ndex=2))+1/3*_alpha*(2*dilog((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x
+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2))*RootOf(_Z^2+3*_Z*_alp
ha+3*_alpha^2,index=1)*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)+6*dilog(
(RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_al
pha+3*_alpha^2,index=2))*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*_alpha
+3*dilog((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2
+3*_Z*_alpha+3*_alpha^2,index=2))*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=
```

```
2)*_alpha+9*dilog((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/Ro
otOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2))*_alpha^2+2*RootOf(_Z^2+3*_Z*_alp
ha+3*_alpha^2,index=1)*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*dilog((R
ootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alph
a+3*_alpha^2,index=1))+3*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*dilog(
(RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_al
pha+3*_alpha^2,index=1))*_alpha+6*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=
2)*dilog((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2
+3*_Z*_alpha+3*_alpha^2,index=1))*_alpha+9*dilog((RootOf(_Z^2+3*_Z*_alpha+3
*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))*_
alpha^2)/(3*_alpha+2*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))/d/(3*_alp
ha+2*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)))))/d/_alpha,_alpha=RootOf(
_Z^3*e+d))+(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e
*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*P
i*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c))*(-1/4*ln((e*x^3+d)^p)/x^4+3/4*
p*e*(-1/d/x-(-1/3/e/(d/e)^(1/3)*ln(x+(d/e)^(1/3))+1/6/e/(d/e)^(1/3)*ln(x^2-
(d/e)^(1/3)*x+(d/e)^(2/3))+1/3*3^(1/2)/e/(d/e)^(1/3)*arctan(1/3*3^(1/2)*(2/
(d/e)^(1/3)*x-1)))*e/d))-1/16*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^
p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c
*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c))^2/x^4
```

Fricas [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^5}\,dx = \int \frac{\log\left((ex^3+d)^p c\right)^2}{x^5}\,dx$$

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^5,x, algorithm="fricas")
[Out] integral(log((e*x^3 + d)^p*c)^2/x^5, x)
```

Sympy [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^5}\,dx = \int \frac{\log\left(c(d+ex^3)^p\right)^2}{x^5}\,dx$$

```
[In] integrate(ln(c*(e*x**3+d)**p)**2/x**5,x)
[Out] Integral(log(c*(d + e*x**3)**p)**2/x**5, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^5}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^5}\,dx = \int \frac{\log\left((ex^3+d)^p c\right)^2}{x^5}\,dx$$

```
[In] integrate(log(c*(e*x^3+d)^p)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^3 + d)^p*c)^2/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(d+ex^3)^p\right)}{x^5}\,dx = \int \frac{\ln\left(c\,(e\,x^3+d)^p\right)^2}{x^5}\,dx$$

```
[In] int(log(c*(d + e*x^3)^p)^2/x^5,x)
```

```
[Out] int(log(c*(d + e*x^3)^p)^2/x^5, x)
```

3.138 $\int \frac{x^8}{\log(c(d+ex^3)^p)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 164

$$\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\frac{d^2(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c(d+ex^3)^p\right)}{p}\right)}{3e^3p}-\frac{2d(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\text{ExpIntegralEi}\left(\frac{2\log\left(c(d+ex^3)^p\right)}{p}\right)}{3e^3p}+\frac{(d+ex^3)^3\left(c(d+ex^3)^p\right)^{-3/p}\text{ExpIntegralEi}\left(\frac{3\log\left(c(d+ex^3)^p\right)}{p}\right)}{3e^3p}$$

```
[Out] 1/3*d^2*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e^3/p/((c*(e*x^3+d)^p)^(1/p))-2/3
*d*(e*x^3+d)^2*Ei(2*ln(c*(e*x^3+d)^p)/p)/e^3/p/((c*(e*x^3+d)^p)^(2/p))+1/3*
(e*x^3+d)^3*Ei(3*ln(c*(e*x^3+d)^p)/p)/e^3/p/((c*(e*x^3+d)^p)^(3/p))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2504, 2446, 2436, 2337, 2209, 2437, 2347}

$$\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\frac{d^2(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c(ex^3+d)^p\right)}{p}\right)}{3e^3p}+\frac{(d+ex^3)^3\left(c(d+ex^3)^p\right)^{-3/p}\text{ExpIntegralEi}\left(\frac{3\log\left(c(ex^3+d)^p\right)}{p}\right)}{3e^3p}-\frac{2d(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\text{ExpIntegralEi}\left(\frac{2\log\left(c(ex^3+d)^p\right)}{p}\right)}{3e^3p}$$

```
[In] Int[x^8/Log[c*(d + e*x^3)^p],x]
```

```
[Out] (d^2*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e^3*p*(c*(d + e*
x^3)^p)^p^(-1)) - (2*d*(d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])
/p])/(3*e^3*p*(c*(d + e*x^3)^p)^(2/p)) + ((d + e*x^3)^3*ExpIntegralEi[(3*Lo
g[c*(d + e*x^3)^p])/p])/(3*e^3*p*(c*(d + e*x^3)^p)^(3/p))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\log\left(c(d+ex)^p\right)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{d^2}{e^2\log\left(c(d+ex)^p\right)} - \frac{2d(d+ex)}{e^2\log\left(c(d+ex)^p\right)} + \frac{(d+ex)^2}{e^2\log\left(c(d+ex)^p\right)}\right)\,dx, x, x^3\right)\\
&= \frac{\text{Subst}\left(\int \frac{(d+ex)^2}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3e^2} - \frac{(2d)\text{Subst}\left(\int \frac{d+ex}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3e^2}\\
&\quad + \frac{d^2\text{Subst}\left(\int \frac{1}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3e^2}\\
&= \frac{\text{Subst}\left(\int \frac{x^2}{\log(cx^p)}\,dx, x, d+ex^3\right)}{3e^3} - \frac{(2d)\text{Subst}\left(\int \frac{x}{\log(cx^p)}\,dx, x, d+ex^3\right)}{3e^3}\\
&\quad + \frac{d^2\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, d+ex^3\right)}{3e^3}\\
&= \frac{\left(\left(d+ex^3\right)^3\left(c\left(d+ex^3\right)^p\right)^{-3/p}\right)\text{Subst}\left(\int \frac{e^{\frac{3x}{p}}}{x}\,dx, x, \log\left(c\left(d+ex^3\right)^p\right)\right)}{3e^3p}\\
&\quad - \frac{\left(2d\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{p}}}{x}\,dx, x, \log\left(c\left(d+ex^3\right)^p\right)\right)}{3e^3p}\\
&\quad + \frac{\left(d^2\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c\left(d+ex^3\right)^p\right)\right)}{3e^3p}\\
&= \frac{d^2\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3e^3p}\\
&\quad - \frac{2d\left(d+ex^3\right)^2\left(c\left(d+ex^3\right)^p\right)^{-2/p}\text{Ei}\left(\frac{2\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3e^3p}\\
&\quad + \frac{\left(d+ex^3\right)^3\left(c\left(d+ex^3\right)^p\right)^{-3/p}\text{Ei}\left(\frac{3\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3e^3p}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.89

$$\int \frac{x^8}{\log \left(c \left(d+e x^3\right)^p\right)} \, dx = \frac{\left(d+ex^3\right)\left(c(d+ex^3)^p\right)^{-3/p}\left(d^2\left(c(d+ex^3)^p\right)^{2/p}\text{ExpIntegralEi}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)-\left(d+ex^3\right)\left(2d\left(c(d+ex^3)^p\right)^{1/p}\right.\right.}{3e^3p}$$

[In] Integrate[x^8/Log[c*(d + e*x^3)^p],x]

[Out] ((d + e*x^3)*(d^2*(c*(d + e*x^3)^p)^(2/p)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p] - (d + e*x^3)*(2*d*(c*(d + e*x^3)^p)^p^(-1)*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/p] - (d + e*x^3)*ExpIntegralEi[(3*Log[c*(d + e*x^3)^p])/p])))/(3*e^3*p*(c*(d + e*x^3)^p)^(3/p))

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.03 (sec) , antiderivative size = 823, normalized size of antiderivative = 5.02

method	result	size
risch	Expression too large to display	823

[In] int(x^8/ln(c*(e*x^3+d)^p),x,method=_RETURNVERBOSE)

[Out] -1/3/e^3/p*(e*x^3+d)^3*c^(-3/p)*((e*x^3+d)^p)^(-3/p)*exp(3/2*I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-3*ln(e*x^3+d)-3/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)-1/3/e^3*d^2/p*(e*x^3+d)*c^(-1/p)*((e*x^3+d)^p)^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-ln(e*x^3+d)-1/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)+2/3/e^3*d/p*(e*x^3+d)^2*c^(-2/p)*((e*x^3+d)^p)^(-2/p)*exp(I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-2*ln(e*x^3+d)-(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.71

$$\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$
$$= \frac{c^{\frac{2}{p}}d^2\,\mathrm{log\_integral}\left((ex^3+d)c^{\left(\frac{1}{p}\right)}\right) - 2\,c^{\left(\frac{1}{p}\right)}d\,\mathrm{log\_integral}\left((e^2x^6+2\,dex^3+d^2)c^{\frac{2}{p}}\right) + \mathrm{log\_integral}\left((e^3\right.}{3\,c^{\frac{3}{p}}e^3p}$$

```
[In] integrate(x^8/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

```
[Out] 1/3*(c^(2/p)*d^2*log_integral((e*x^3 + d)*c^(1/p)) - 2*c^(1/p)*d*log_integr
al((e^2*x^6 + 2*d*e*x^3 + d^2)*c^(2/p)) + log_integral((e^3*x^9 + 3*d*e^2*x
^6 + 3*d^2*e*x^3 + d^3)*c^(3/p)))/(c^(3/p)*e^3*p)
```

Sympy [F]

$$\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

```
[In] integrate(x**8/ln(c*(e*x**3+d)**p),x)
```

```
[Out] Integral(x**8/log(c*(d + e*x**3)**p), x)
```

Maxima [F]

$$\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^8}{\log\left(\left(ex^3+d\right)^pc\right)}\,dx$$

```
[In] integrate(x^8/log(c*(e*x^3+d)^p),x, algorithm="maxima")
```

```
[Out] integrate(x^8/log((e*x^3 + d)^p*c), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.66

$$\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{d^2\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(ex^3+d\right)\right)}{3\,c^{\left(\frac{1}{p}\right)}e^3p} - \frac{2\,d\mathrm{Ei}\left(\frac{2\,\log(c)}{p}+2\,\log\left(ex^3+d\right)\right)}{3\,c^{\frac{2}{p}}e^3p} + \frac{\mathrm{Ei}\left(\frac{3\,\log(c)}{p}+3\,\log\left(ex^3+d\right)\right)}{3\,c^{\frac{3}{p}}e^3p}$$

```
[In] integrate(x^8/log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] 1/3*d^2*Ei(log(c)/p + log(e*x^3 + d))/(c^(1/p)*e^3*p) - 2/3*d*Ei(2*log(c)/p
 + 2*log(e*x^3 + d))/(c^(2/p)*e^3*p) + 1/3*Ei(3*log(c)/p + 3*log(e*x^3 + d)
)/(c^(3/p)*e^3*p)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^8}{\ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

```
[In] int(x^8/log(c*(d + e*x^3)^p),x)
[Out] int(x^8/log(c*(d + e*x^3)^p), x)
```

3.139 $\int \frac{x^5}{\log(c(d+ex^3)^p)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = -\frac{d(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\operatorname{ExpIntegralEi}\left(\frac{\log\left(c(d+ex^3)^p\right)}{p}\right)}{3e^2p} + \frac{(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\operatorname{ExpIntegralEi}\left(\frac{2\log\left(c(d+ex^3)^p\right)}{p}\right)}{3e^2p}$$

```
[Out] -1/3*d*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e^2/p/((c*(e*x^3+d)^p)^(1/p))+1/3*
(e*x^3+d)^2*Ei(2*ln(c*(e*x^3+d)^p)/p)/e^2/p/((c*(e*x^3+d)^p)^(2/p))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2504, 2446, 2436, 2337, 2209, 2437, 2347}

$$\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\operatorname{ExpIntegralEi}\left(\frac{2\log\left(c(ex^3+d)^p\right)}{p}\right)}{3e^2p} - \frac{d(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\operatorname{ExpIntegralEi}\left(\frac{\log\left(c(ex^3+d)^p\right)}{p}\right)}{3e^2p}$$

```
[In] Int[x^5/Log[c*(d + e*x^3)^p],x]
```

```
[Out] -1/3*(d*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(e^2*p*(c*(d + e
*x^3)^p)^p^(-1)) + ((d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/p]
)/(3*e^2*p*(c*(d + e*x^3)^p)^(2/p))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{\log\left(c(d+ex)^p\right)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{d}{e\log\left(c(d+ex)^p\right)}+\frac{d+ex}{e\log\left(c(d+ex)^p\right)}\right)\,dx, x, x^3\right)\\
&= \frac{\text{Subst}\left(\int \frac{d+ex}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3e}-\frac{d\text{Subst}\left(\int \frac{1}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3e}\\
&= \frac{\text{Subst}\left(\int \frac{x}{\log(cx^p)}\,dx, x, d+ex^3\right)}{3e^2}-\frac{d\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, d+ex^3\right)}{3e^2}\\
&= \frac{\left((d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{p}}}{x}\,dx, x, \log\left(c(d+ex^3)^p\right)\right)}{3e^2p}\\
&\quad-\frac{\left(d(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c(d+ex^3)^p\right)\right)}{3e^2p}\\
&= -\frac{d(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\text{Ei}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)}{3e^2p}\\
&\quad+\frac{(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\text{Ei}\left(\frac{2\log(c(d+ex^3)^p)}{p}\right)}{3e^2p}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90

$$\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = -\frac{(d+ex^3)\left(c(d+ex^3)^p\right)^{-2/p}\left(d\left(c(d+ex^3)^p\right)^{\frac{1}{p}}\text{ExpIntegralEi}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)-(d+ex^3)\,\text{ExpIntegralEi}}{3e^2p}$$

```
[In] Integrate[x^5/Log[c*(d + e*x^3)^p],x]
```

```
[Out] -1/3*((d + e*x^3)*(d*(c*(d + e*x^3)^p)^p^(-1)*ExpIntegralEi[Log[c*(d + e*x^
3)^p]/p] - (d + e*x^3)*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/p]))/(e^2*p*(
c*(d + e*x^3)^p)^(2/p))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.89 (sec) , antiderivative size = 547, normalized size of antiderivative = 5.11

method	result
risch	$-\frac{\left(e\,x^{3}+d\right)^{2} c^{-\frac{2}{p}} \left(\left(e\,x^{3}+d\right)^{p}\right)^{-\frac{2}{p}} \mathrm{e}^{\frac{i\pi\,\mathrm{csgn}\left(ic\left(e\,x^{3}+d\right)^{p}\right)\left(-\mathrm{csgn}\left(ic\left(e\,x^{3}+d\right)^{p}\right)+\mathrm{csgn}(ic)\right)\left(-\mathrm{csgn}\left(ic\left(e\,x^{3}+d\right)^{p}\right)+\mathrm{csgn}\left(i\left(e\,x^{3}+d\right)^{p}\right)\right)}{p}} \mathrm{Ei}_{1}\left(-\right.}{}$

```
[In] int(x^5/ln(c*(e*x^3+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/e^2/p*(e*x^3+d)^2*c^(-2/p)*((e*x^3+d)^p)^(-2/p)*exp(I*Pi*csgn(I*c*(e*x
^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*
(e*x^3+d)^p))/p)*Ei(1,-2*ln(e*x^3+d)-(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*
x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*c
sgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln(
(e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)+1/3/e^2*d/p*(e*x^3+d)*c^(-1/p)*((e*x^3+d)^
p)^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I
*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-ln(e*x^3+d)-1/2*
(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*
csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*
(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.64

$$\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = -\frac{c^{\left(\frac{1}{p}\right)} d\,\mathrm{log\_integral}\left((ex^3+d)c^{\left(\frac{1}{p}\right)}\right) - \mathrm{log\_integral}\left((e^2x^6+2\,dex^3+d^2)c^{\frac{2}{p}}\right)}{3\,c^{\frac{2}{p}}e^2p}$$

```
[In] integrate(x^5/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

```
[Out] -1/3*(c^(1/p)*d*log_integral((e*x^3 + d)*c^(1/p)) - log_integral((e^2*x^6 +
 2*d*e*x^3 + d^2)*c^(2/p)))/(c^(2/p)*e^2*p)
```

Sympy [F]

$$\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

```
[In] integrate(x**5/ln(c*(e*x**3+d)**p),x)
[Out] Integral(x**5/log(c*(d + e*x**3)**p), x)
```

Maxima [F]

$$\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^5}{\log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

```
[In] integrate(x^5/log(c*(e*x^3+d)^p),x, algorithm="maxima")
[Out] integrate(x^5/log((e*x^3 + d)^p*c), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.64

$$\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = -\frac{d\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(ex^3+d\right)\right)}{3\,c^{\left(\frac{1}{p}\right)}e^2p} + \frac{\mathrm{Ei}\left(\frac{2\,\log(c)}{p}+2\,\log\left(ex^3+d\right)\right)}{3\,c^{\frac{2}{p}}e^2p}$$

```
[In] integrate(x^5/log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] -1/3*d*Ei(log(c)/p + log(e*x^3 + d))/(c^(1/p)*e^2*p) + 1/3*Ei(2*log(c)/p +
2*log(e*x^3 + d))/(c^(2/p)*e^2*p)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^5}{\ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

```
[In] int(x^5/log(c*(d + e*x^3)^p),x)
[Out] int(x^5/log(c*(d + e*x^3)^p), x)
```

3.140 $\int \frac{x^2}{\log(c(d+ex^3)^p)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 51

$$\int \frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3ep}$$

```
[Out] 1/3*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e/p/((c*(e*x^3+d)^p)^(1/p))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2504, 2436, 2337, 2209}

$$\int \frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3ep}$$

```
[In] Int[x^2/Log[c*(d + e*x^3)^p],x]
```

```
[Out] ((d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e*p*(c*(d + e*x^3)^p
)^p^(-1))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\log\left(c(d+ex)^p\right)}\,dx, x, x^3\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, d+ex^3\right)}{3e} \\
&= \frac{\left(\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c\left(d+ex^3\right)^p\right)\right)}{3ep} \\
&= \frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3ep}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3ep}$$

```
[In] Integrate[x^2/Log[c*(d + e*x^3)^p],x]
[Out] ((d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e*p*(c*(d + e*x^3)^p
)^p^(-1))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.79 (sec) , antiderivative size = 272, normalized size of antiderivative = 5.33

method	result
risch	$-\frac{(e\,x^3+d)\left((e\,x^3+d)^p\right)^{-\frac{1}{p}} c^{-\frac{1}{p}} \mathrm{e}^{\frac{i\pi\ \mathrm{csgn}\left(ic\left(e\,x^3+d\right)^p\right)\left(-\mathrm{csgn}\left(ic\left(e\,x^3+d\right)^p\right)+\mathrm{csgn}(ic)\right)\left(-\mathrm{csgn}\left(ic\left(e\,x^3+d\right)^p\right)+\mathrm{csgn}\left(i\left(e\,x^3+d\right)^p\right)\right)}{2p}}\ \mathrm{Ei}_1\left(-\mathrm{l}\right.}{}$

```
[In] int(x^2/ln(c*(e*x^3+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/e/p*(e*x^3+d)*((e*x^3+d)^p)^(-1/p)*c^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(e*x
^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*
(e*x^3+d)^p))/p)*Ei(1,-ln(e*x^3+d)-1/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(
e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi
*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*l
n((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

$$\int \frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{\mathrm{log\_integral}\left((ex^3+d)c^{\left(\frac{1}{p}\right)}\right)}{3\,c^{\left(\frac{1}{p}\right)}ep}$$

```
[In] integrate(x^2/log(c*(e*x^3+d)^p),x, algorithm="fricas")
```

```
[Out] 1/3*log_integral((e*x^3 + d)*c^(1/p))/(c^(1/p)*e*p)
```

Sympy [F]

$$\int \frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

```
[In] integrate(x**2/ln(c*(e*x**3+d)**p),x)
```

```
[Out] Integral(x**2/log(c*(d + e*x**3)**p), x)
```

Maxima [F]

$$\int \frac{x^2}{\log \left(c \left(d+e x^3\right)^p\right)} \, dx = \int \frac{x^2}{\log \left(\left(e x^3+d\right)^p c\right)} \, dx$$

```
[In] integrate(x^2/log(c*(e*x^3+d)^p),x, algorithm="maxima")
[Out] integrate(x^2/log((e*x^3 + d)^p*c), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{\log \left(c \left(d+e x^3\right)^p\right)} \, dx = \frac{\mathrm{Ei}\left(\frac{\log(c)}{p} + \log \left(e x^3+d\right)\right)}{3 \, c^{\left(\frac{1}{p}\right)} e p}$$

```
[In] integrate(x^2/log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] 1/3*Ei(log(c)/p + log(e*x^3 + d))/(c^(1/p)*e*p)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\log \left(c \left(d+e x^3\right)^p\right)} \, dx = \int \frac{x^2}{\ln \left(c \, {\left(e \, x^3+d\right)}^p\right)} \,d x$$

```
[In] int(x^2/log(c*(d + e*x^3)^p),x)
[Out] int(x^2/log(c*(d + e*x^3)^p), x)
```

3.141 $\int \frac{1}{x\log\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\operatorname{Int}\left(\frac{1}{x\log\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x/ln(c*(e*x^3+d)^p),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{x\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

```
[In] Int[1/(x*Log[c*(d + e*x^3)^p]),x]
```

```
[Out] Defer[Int][1/(x*Log[c*(d + e*x^3)^p]), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{1}{x \log \left(c \left(d+e x^3\right)^p\right)} \, dx$$

```
[In] Integrate[1/(x*Log[c*(d + e*x^3)^p]),x]
[Out] Integrate[1/(x*Log[c*(d + e*x^3)^p]), x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \ln \left(c \left(e\, x^{3}+d\right)^{p}\right)}d x$$

```
[In] int(1/x/ln(c*(e*x^3+d)^p),x)
[Out] int(1/x/ln(c*(e*x^3+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{1}{x \log \left({\left(e x^{3} + d\right)}^{p} c\right)} \, dx$$

```
[In] integrate(1/x/log(c*(e*x^3+d)^p),x, algorithm="fricas")
[Out] integral(1/(x*log((e*x^3 + d)^p*c)), x)
```

Sympy [N/A]

Not integrable

Time = 9.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{x \log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{1}{x \log \left(c \left(d + e x^{3}\right)^{p}\right)}\, dx$$

```
[In] integrate(1/x/ln(c*(e*x**3+d)**p),x)
[Out] Integral(1/(x*log(c*(d + e*x**3)**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \log \left(c \left(d+e x^3\right)^p\right)} \, dx = \int \frac{1}{x \log \left(\left(e x^3+d\right)^p c\right)} \, dx$$

```
[In] integrate(1/x/log(c*(e*x^3+d)^p),x, algorithm="maxima")
[Out] integrate(1/(x*log((e*x^3 + d)^p*c)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \log \left(c \left(d+e x^3\right)^p\right)} \, dx = \int \frac{1}{x \log \left(\left(e x^3+d\right)^p c\right)} \, dx$$

```
[In] integrate(1/x/log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] integrate(1/(x*log((e*x^3 + d)^p*c)), x)
```

Mupad [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x \log \left(c \left(d+e x^3\right)^p\right)} \, dx = \int \frac{1}{x \ln \left(c \left(e\, x^3+d\right)^p\right)} \, dx$$

```
[In] int(1/(x*log(c*(d + e*x^3)^p)),x)
[Out] int(1/(x*log(c*(d + e*x^3)^p)), x)
```

3.142 $\int \frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)}\,dx = \text{Int}\left(\frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^4/ln(c*(e*x^3+d)^p),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

```
[In] Int[1/(x^4*Log[c*(d + e*x^3)^p]),x]
[Out] Defer[Int][1/(x^4*Log[c*(d + e*x^3)^p]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 \log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{1}{x^4 \log \left(c \left(d+e x^3\right)^p\right)} \, dx$$

```
[In] Integrate[1/(x^4*Log[c*(d + e*x^3)^p]),x]
[Out] Integrate[1/(x^4*Log[c*(d + e*x^3)^p]), x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{4} \ln \left(c \left(e \,x^{3}+d \right)^{p}\right)}d x$$

```
[In] int(1/x^4/ln(c*(e*x^3+d)^p),x)
[Out] int(1/x^4/ln(c*(e*x^3+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 \log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int { \frac{1}{x^{4} \log \left({\left(e x^{3} + d\right)}^{p} c\right)} \,d x }$$

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p),x, algorithm="fricas")
[Out] integral(1/(x^4*log((e*x^3 + d)^p*c)), x)
```

Sympy [N/A]

Not integrable

Time = 32.74 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4 \log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{1}{x^{4} \log {\left(c \left(d + e x^{3}\right)^{p} \right)}}\, dx$$

```
[In] integrate(1/x**4/ln(c*(e*x**3+d)**p),x)
[Out] Integral(1/(x**4*log(c*(d + e*x**3)**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^4 \log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p),x, algorithm="maxima")
[Out] integrate(1/(x^4*log((e*x^3 + d)^p*c)), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^4 \log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] integrate(1/(x^4*log((e*x^3 + d)^p*c)), x)
```

Mupad [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^4\, \ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

```
[In] int(1/(x^4*log(c*(d + e*x^3)^p)),x)
[Out] int(1/(x^4*log(c*(d + e*x^3)^p)), x)
```

3.143 $\int \frac{x^3}{\log(c(d+ex^3)^p)} dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\operatorname{Int}\left(\frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(x^3/ln(c*(e*x^3+d)^p),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

```
[In] Int[x^3/Log[c*(d + e*x^3)^p],x]
```

```
[Out] Defer[Int][x^3/Log[c*(d + e*x^3)^p], x]
```

Rubi steps

$$\text{integral}=\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\log(c(d+ex^3)^p)}\,dx = \int \frac{x^3}{\log(c(d+ex^3)^p)}\,dx$$

```
[In] Integrate[x^3/Log[c*(d + e*x^3)^p],x]
[Out] Integrate[x^3/Log[c*(d + e*x^3)^p], x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\ln(c(e\,x^3+d)^p)}dx$$

```
[In] int(x^3/ln(c*(e*x^3+d)^p),x)
[Out] int(x^3/ln(c*(e*x^3+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\log(c(d+ex^3)^p)}\,dx = \int \frac{x^3}{\log((ex^3+d)^p c)}\,dx$$

```
[In] integrate(x^3/log(c*(e*x^3+d)^p),x, algorithm="fricas")
[Out] integral(x^3/log((e*x^3 + d)^p*c), x)
```

Sympy [N/A]

Not integrable

Time = 10.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{\log(c(d+ex^3)^p)}\,dx = \int \frac{x^3}{\log(c(d+ex^3)^p)}\,dx$$

```
[In] integrate(x**3/ln(c*(e*x**3+d)**p),x)
[Out] Integral(x**3/log(c*(d + e*x**3)**p), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^3}{\log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

```
[In] integrate(x^3/log(c*(e*x^3+d)^p),x, algorithm="maxima")
[Out] integrate(x^3/log((e*x^3 + d)^p*c), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^3}{\log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

```
[In] integrate(x^3/log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] integrate(x^3/log((e*x^3 + d)^p*c), x)
```

Mupad [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^3}{\ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

```
[In] int(x^3/log(c*(d + e*x^3)^p),x)
[Out] int(x^3/log(c*(d + e*x^3)^p), x)
```

3.144 $\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)} \, dx = \text{Int}\left(\frac{x}{\log\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(x/ln(c*(e*x^3+d)^p),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)} \, dx = \int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)} \, dx$$

```
[In] Int[x/Log[c*(d + e*x^3)^p],x]
[Out] Defer[Int][x/Log[c*(d + e*x^3)^p], x]
```

Rubi steps

$$\text{integral} = \int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{\log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{x}{\log \left(c \left(d+e x^3\right)^p\right)} \, dx$$

```
[In] Integrate[x/Log[c*(d + e*x^3)^p],x]
[Out] Integrate[x/Log[c*(d + e*x^3)^p], x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{\ln \left(c \left(e\,x^{3}+d\right)^{p}\right)}d x$$

```
[In] int(x/ln(c*(e*x^3+d)^p),x)
[Out] int(x/ln(c*(e*x^3+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{\log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{x}{\log \left({\left(e x^{3} + d\right)}^{p} c\right)} \, dx$$

```
[In] integrate(x/log(c*(e*x^3+d)^p),x, algorithm="fricas")
[Out] integral(x/log((e*x^3 + d)^p*c), x)
```

Sympy [N/A]

Not integrable

Time = 4.80 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{\log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{x}{\log \left(c \left(d + e x^{3}\right)^{p}\right)}\, dx$$

```
[In] integrate(x/ln(c*(e*x**3+d)**p),x)
[Out] Integral(x/log(c*(d + e*x**3)**p), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x}{\log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

```
[In] integrate(x/log(c*(e*x^3+d)^p),x, algorithm="maxima")
[Out] integrate(x/log((e*x^3 + d)^p*c), x)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x}{\log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

```
[In] integrate(x/log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] integrate(x/log((e*x^3 + d)^p*c), x)
```

Mupad [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x}{\ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

```
[In] int(x/log(c*(d + e*x^3)^p),x)
[Out] int(x/log(c*(d + e*x^3)^p), x)
```

3.145 $\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)} dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)} dx = \text{Int}\left(\frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}, x\right)$$

```
[Out] Unintegrable(1/ln(c*(e*x^3+d)^p),x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)} dx = \int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)} dx$$

```
[In] Int[Log[c*(d + e*x^3)^p]^(-1),x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^3)^p]^(-1), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)} dx$$

Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

```
[In] Integrate[Log[c*(d + e*x^3)^p]^(-1),x]
[Out] Integrate[Log[c*(d + e*x^3)^p]^(-1), x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\ln\left(c\left(e\,x^3+d\right)^p\right)}dx$$

```
[In] int(1/ln(c*(e*x^3+d)^p),x)
[Out] int(1/ln(c*(e*x^3+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{\log\left(\left(ex^3+d\right)^pc\right)}\,dx$$

```
[In] integrate(1/log(c*(e*x^3+d)^p),x, algorithm="fricas")
[Out] integral(1/log((e*x^3 + d)^p*c), x)
```

Sympy [N/A]

Not integrable

Time = 4.79 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx$$

```
[In] integrate(1/ln(c*(e*x**3+d)**p),x)
[Out] Integral(1/log(c*(d + e*x**3)**p), x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{\log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

```
[In] integrate(1/log(c*(e*x^3+d)^p),x, algorithm="maxima")
[Out] integrate(1/log((e*x^3 + d)^p*c), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{\log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

```
[In] integrate(1/log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] integrate(1/log((e*x^3 + d)^p*c), x)
```

Mupad [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{\ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

```
[In] int(1/log(c*(d + e*x^3)^p),x)
[Out] int(1/log(c*(d + e*x^3)^p), x)
```

3.146 $\int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)} \, dx = \operatorname{Int}\left(\frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)}, x\right)$$

```
[Out] Unintegrable(1/x^2/ln(c*(e*x^3+d)^p),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)} \, dx = \int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)} \, dx$$

```
[In] Int[1/(x^2*Log[c*(d + e*x^3)^p]),x]
[Out] Defer[Int][1/(x^2*Log[c*(d + e*x^3)^p]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{1}{x^2 \log \left(c \left(d+e x^3\right)^p\right)} \, dx$$

```
[In] Integrate[1/(x^2*Log[c*(d + e*x^3)^p]),x]
[Out] Integrate[1/(x^2*Log[c*(d + e*x^3)^p]), x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{2} \ln \left(c \left(e \,x^{3}+d \right)^{p}\right)}d x$$

```
[In] int(1/x^2/ln(c*(e*x^3+d)^p),x)
[Out] int(1/x^2/ln(c*(e*x^3+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{1}{x^2 \log \left((e x^3+d)^p c\right)} \, dx$$

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p),x, algorithm="fricas")
[Out] integral(1/(x^2*log((e*x^3 + d)^p*c)), x)
```

Sympy [N/A]

Not integrable

Time = 18.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2 \log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{1}{x^2 \log \left(c \left(d+e x^3\right)^p\right)} \, dx$$

```
[In] integrate(1/x**2/ln(c*(e*x**3+d)**p),x)
[Out] Integral(1/(x**2*log(c*(d + e*x**3)**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^2 \log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p),x, algorithm="maxima")
[Out] integrate(1/(x^2*log((e*x^3 + d)^p*c)), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^2 \log\left(\left(ex^3+d\right)^p c\right)}\,dx$$

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] integrate(1/(x^2*log((e*x^3 + d)^p*c)), x)
```

Mupad [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^2\, \ln\left(c\left(e\,x^3+d\right)^p\right)}\,dx$$

```
[In] int(1/(x^2*log(c*(d + e*x^3)^p)),x)
[Out] int(1/(x^2*log(c*(d + e*x^3)^p)), x)
```

3.147 $\int \frac{1}{x^3 \log \left(c \left(d+e x^3\right)^p\right)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3 \log \left(c \left(d+e x^3\right)^p\right)} \, dx=\text{Int}\left(\frac{1}{x^3 \log \left(c \left(d+e x^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^3/ln(c*(e*x^3+d)^p),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^3 \log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{1}{x^3 \log \left(c \left(d+e x^3\right)^p\right)} \, dx$$

```
[In] Int[1/(x^3*Log[c*(d + e*x^3)^p]),x]
```

```
[Out] Defer[Int][1/(x^3*Log[c*(d + e*x^3)^p]), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^3 \log \left(c \left(d+e x^3\right)^p\right)} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{1}{x^3 \log \left(c \left(d+e x^3\right)^p\right)} \, dx$$

```
[In] Integrate[1/(x^3*Log[c*(d + e*x^3)^p]),x]
[Out] Integrate[1/(x^3*Log[c*(d + e*x^3)^p]), x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3} \ln \left(c \left(e \,x^{3}+d \right)^{p}\right)}dx$$

```
[In] int(1/x^3/ln(c*(e*x^3+d)^p),x)
[Out] int(1/x^3/ln(c*(e*x^3+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{1}{x^{3} \log \left({\left(e x^{3}+d\right)}^{p} c\right)} \, dx$$

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p),x, algorithm="fricas")
[Out] integral(1/(x^3*log((e*x^3 + d)^p*c)), x)
```

Sympy [N/A]

Not integrable

Time = 24.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3 \log \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{1}{x^{3} \log \left(c \left(d+e x^{3}\right)^{p}\right)} \, dx$$

```
[In] integrate(1/x**3/ln(c*(e*x**3+d)**p),x)
[Out] Integral(1/(x**3*log(c*(d + e*x**3)**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log \left(c \left(d+e x^3\right)^p\right)} \, dx = \int \frac{1}{x^3 \log \left(\left(e x^3+d\right)^p c\right)} \, dx$$

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p),x, algorithm="maxima")
[Out] integrate(1/(x^3*log((e*x^3 + d)^p*c)), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log \left(c \left(d+e x^3\right)^p\right)} \, dx = \int \frac{1}{x^3 \log \left(\left(e x^3+d\right)^p c\right)} \, dx$$

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p),x, algorithm="giac")
[Out] integrate(1/(x^3*log((e*x^3 + d)^p*c)), x)
```

Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log \left(c \left(d+e x^3\right)^p\right)} \, dx = \int \frac{1}{x^3 \ln \left(c \left(e x^3+d\right)^p\right)} \, dx$$

```
[In] int(1/(x^3*log(c*(d + e*x^3)^p)),x)
[Out] int(1/(x^3*log(c*(d + e*x^3)^p)), x)
```

3.148 $\int \frac{x^8}{\log^2(c(d+ex^3)^p)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 195

$$\int \frac{x^8}{\log^2(c(d+ex^3)^p)}\,dx = \frac{d^2(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\operatorname{ExpIntegralEi}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)}{3e^3p^2} - \frac{4d(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\operatorname{ExpIntegralEi}\left(\frac{2\log(c(d+ex^3)^p)}{p}\right)}{3e^3p^2} + \frac{(d+ex^3)^3\left(c(d+ex^3)^p\right)^{-3/p}\operatorname{ExpIntegralEi}\left(\frac{3\log(c(d+ex^3)^p)}{p}\right)}{e^3p^2} - \frac{x^6(d+ex^3)}{3ep\log(c(d+ex^3)^p)}$$

```
[Out] 1/3*d^2*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e^3/p^2/((c*(e*x^3+d)^p)^(1/p))-4
/3*d*(e*x^3+d)^2*Ei(2*ln(c*(e*x^3+d)^p)/p)/e^3/p^2/((c*(e*x^3+d)^p)^(2/p))+
(e*x^3+d)^3*Ei(3*ln(c*(e*x^3+d)^p)/p)/e^3/p^2/((c*(e*x^3+d)^p)^(3/p))-1/3*x
^6*(e*x^3+d)/e/p/ln(c*(e*x^3+d)^p)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used

$= \{2504, 2447, 2446, 2436, 2337, 2209, 2437, 2347\}$

$$\int \frac{x^8}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{d^2(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log\left(c(ex^3+d)^p\right)}{p}\right)}{3e^3p^2} + \frac{(d+ex^3)^3\left(c(d+ex^3)^p\right)^{-3/p}\text{ExpIntegralEi}\left(\frac{3\log\left(c(ex^3+d)^p\right)}{p}\right)}{e^3p^2} - \frac{4d(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\text{ExpIntegralEi}\left(\frac{2\log\left(c(ex^3+d)^p\right)}{p}\right)}{3e^3p^2} - \frac{x^6(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)}$$

```
[In] Int[x^8/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] (d^2*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e^3*p^2*(c*(d +
e*x^3)^p)^p^(-1)) - (4*d*(d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p
])/p])/(3*e^3*p^2*(c*(d + e*x^3)^p)^(2/p)) + ((d + e*x^3)^3*ExpIntegralEi[(
3*Log[c*(d + e*x^3)^p])/p])/(e^3*p^2*(c*(d + e*x^3)^p)^(3/p)) - (x^6*(d + e
*x^3))/(3*e*p*Log[c*(d + e*x^3)^p])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\log^2\left(c(d+ex)^p\right)}\,dx, x, x^3\right)\\
&= -\frac{x^6(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)}+\frac{\text{Subst}\left(\int \frac{x^2}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{p}+\frac{(2d)\text{Subst}\left(\int \frac{x}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3ep}\\
&= -\frac{x^6(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)}\\
&\quad+\frac{\text{Subst}\left(\int \left(\frac{d^2}{e^2\log(c(d+ex)^p)}-\frac{2d(d+ex)}{e^2\log(c(d+ex)^p)}+\frac{(d+ex)^2}{e^2\log(c(d+ex)^p)}\right)\,dx, x, x^3\right)}{p}\\
&\quad+\frac{(2d)\text{Subst}\left(\int \left(-\frac{d}{e\log(c(d+ex)^p)}+\frac{d+ex}{e\log(c(d+ex)^p)}\right)\,dx, x, x^3\right)}{3ep}
\end{aligned}$$

$$=-\frac{x^6(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)}+\frac{\text{Subst}\left(\int\frac{(d+ex)^2}{\log(c(d+ex)^p)}\,dx,x,x^3\right)}{e^2p}+\frac{(2d)\text{Subst}\left(\int\frac{d+ex}{\log(c(d+ex)^p)}\,dx,x,x^3\right)}{3e^2p}-\frac{(2d)\text{Subst}\left(\int\frac{d+ex}{\log(c(d+ex)^p)}\,dx,x,x^3\right)}{e^2p}-\frac{(2d^2)\,\text{Subst}\left(\int\frac{1}{\log(c(d+ex)^p)}\,dx,x,x^3\right)}{3e^2p}+\frac{d^2\text{Subst}\left(\int\frac{1}{\log(c(d+ex)^p)}\,dx,x,x^3\right)}{e^2p}$$

$$=-\frac{x^6(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)}+\frac{\text{Subst}\left(\int\frac{x^2}{\log(cx^p)}\,dx,x,d+ex^3\right)}{e^3p}+\frac{(2d)\text{Subst}\left(\int\frac{x}{\log(cx^p)}\,dx,x,d+ex^3\right)}{3e^3p}-\frac{(2d)\text{Subst}\left(\int\frac{x}{\log(cx^p)}\,dx,x,d+ex^3\right)}{e^3p}-\frac{(2d^2)\,\text{Subst}\left(\int\frac{1}{\log(cx^p)}\,dx,x,d+ex^3\right)}{3e^3p}+\frac{d^2\text{Subst}\left(\int\frac{1}{\log(cx^p)}\,dx,x,d+ex^3\right)}{e^3p}$$

$$=-\frac{x^6(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)}+\frac{\left((d+ex^3)^3\left(c(d+ex^3)^p\right)^{-3/p}\right)\text{Subst}\left(\int\frac{e^{\frac{3x}{p}}}{x}\,dx,x,\log\left(c(d+ex^3)^p\right)\right)}{e^3p^2}+\frac{\left(2d(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{p}}}{x}\,dx,x,\log\left(c(d+ex^3)^p\right)\right)}{3e^3p^2}-\frac{\left(2d(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{p}}}{x}\,dx,x,\log\left(c(d+ex^3)^p\right)\right)}{e^3p^2}-\frac{\left(2d^2(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{p}}}{x}\,dx,x,\log\left(c(d+ex^3)^p\right)\right)}{3e^3p^2}+\frac{\left(d^2(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{p}}}{x}\,dx,x,\log\left(c(d+ex^3)^p\right)\right)}{e^3p^2}$$

$$=\frac{d^2(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c(d+ex^3)^p\right)}{p}\right)}{3e^3p^2}-\frac{4d(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\text{Ei}\left(\frac{2\log\left(c(d+ex^3)^p\right)}{p}\right)}{3e^3p^2}+\frac{(d+ex^3)^3\left(c(d+ex^3)^p\right)^{-3/p}\text{Ei}\left(\frac{3\log\left(c(d+ex^3)^p\right)}{p}\right)}{e^3p^2}-\frac{x^6(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.49

$$\int \frac{x^8}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-3/p}\left(-e^2px^6\left(c\left(d+ex^3\right)^p\right)^{3/p}+d^2\left(c\left(d+ex^3\right)^p\right)^{2/p}\text{ExpIntegralEi}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)\right.}{}$$

[In] Integrate[x^8/Log[c*(d + e*x^3)^p]^2,x]

[Out] ((d + e*x^3)*(-(e^2*p*x^6*(c*(d + e*x^3)^p)^(3/p)) + d^2*(c*(d + e*x^3)^p)^(2/p)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p]*Log[c*(d + e*x^3)^p] - 4*d*(d + e*x^3)*(c*(d + e*x^3)^p)^p^(-1)*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/p]*Log[c*(d + e*x^3)^p] + 3*d^2*ExpIntegralEi[(3*Log[c*(d + e*x^3)^p])/p]*Log[c*(d + e*x^3)^p] + 6*d*e*x^3*ExpIntegralEi[(3*Log[c*(d + e*x^3)^p])/p]*Log[c*(d + e*x^3)^p] + 3*e^2*x^6*ExpIntegralEi[(3*Log[c*(d + e*x^3)^p])/p]*Log[c*(d + e*x^3)^p]))/(3*e^3*p^2*(c*(d + e*x^3)^p)^(3/p)*Log[c*(d + e*x^3)^p])

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.14 (sec) , antiderivative size = 2564, normalized size of antiderivative = 13.15

method	result	size
risch	Expression too large to display	2564

[In] int(x^8/ln(c*(e*x^3+d)^p)^2,x,method=_RETURNVERBOSE)

[Out] -2/3/p/e*x^6*(e*x^3+d)/(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p))-1/3/p^2/e^2*d^2*((e*x^3+d)^p)^(-1/p)*c^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-ln(e*x^3+d)-1/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)*x^3-1/3/p^2/e^3*d^3*((e*x^3+d)^p)^(-1/p)*c^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-ln(e*x^3+d)-1/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)-1/p^2*((e*x^3+d)^p)^(-3/p)*c^(-3/p)*exp(3/2*I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*

```
(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(
1,-3*ln(e*x^3+d)-3/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi
*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d
)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p
*ln(e*x^3+d))/p)*x^9-3/p^2/e*((e*x^3+d)^p)^(-3/p)*c^(-3/p)*exp(3/2*I*Pi*csg
n(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^
p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-3*ln(e*x^3+d)-3/2*(I*Pi*csgn(I*(e*x^3+d)^p
)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*cs
gn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)
+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)*d*x^6-3/p^2/e^2*((e*x^3+d)^p
)^(-3/p)*c^(-3/p)*exp(3/2*I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p
)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-3*ln(e*x
^3+d)-3/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*
x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi
*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d
))/p)*d^2*x^3-1/p^2/e^3*((e*x^3+d)^p)^(-3/p)*c^(-3/p)*exp(3/2*I*Pi*csgn(I*c
*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+cs
gn(I*(e*x^3+d)^p))/p)*Ei(1,-3*ln(e*x^3+d)-3/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csg
n(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*
c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln
(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)*d^3+4/3/p^2/e*d*((e*x^3+d)^p)^(-2
/p)*c^(-2/p)*exp(I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*
c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-2*ln(e*x^3+d)-(I*
Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csg
n(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*
x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)*x^6+8/3
/p^2/e^2*d^2*((e*x^3+d)^p)^(-2/p)*c^(-2/p)*exp(I*Pi*csgn(I*c*(e*x^3+d)^p)*(
-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^
p))/p)*Ei(1,-2*ln(e*x^3+d)-(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^
2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e
*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^
p)-2*p*ln(e*x^3+d))/p)*x^3+4/3/p^2/e^3*d^3*((e*x^3+d)^p)^(-2/p)*c^(-2/p)*ex
p(I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*
(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-2*ln(e*x^3+d)-(I*Pi*csgn(I*(e*x^
3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)
^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csg
n(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.08

$$\int \frac{x^8}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = -\frac{4\,(dp\log\left(ex^3+d\right)+d\log\left(c\right))c^{\left(\frac{1}{p}\right)}\,\mathrm{log\_integral}\left(\left(e^2x^6+2\,dex^3+d^2\right)c^{\frac{2}{p}}\right)-\left(d^2p\log\left(ex^3+d\right)+d^2\log\right.}{}$$

```
[In] integrate(x^8/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(4*(d*p*log(e*x^3 + d) + d*log(c))*c^(1/p)*log_integral((e^2*x^6 + 2*d
*e*x^3 + d^2)*c^(2/p)) - (d^2*p*log(e*x^3 + d) + d^2*log(c))*c^(2/p)*log_in
tegral((e*x^3 + d)*c^(1/p)) + (e^3*p*x^9 + d*e^2*p*x^6)*c^(3/p) - 3*(p*log(
e*x^3 + d) + log(c))*log_integral((e^3*x^9 + 3*d*e^2*x^6 + 3*d^2*e*x^3 + d^
3)*c^(3/p)))/((e^3*p^3*log(e*x^3 + d) + e^3*p^2*log(c))*c^(3/p))
```

Sympy [F]

$$\int \frac{x^8}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^8}{\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

```
[In] integrate(x**8/ln(c*(e*x**3+d)**p)**2,x)
```

```
[Out] Integral(x**8/log(c*(d + e*x**3)**p)**2, x)
```

Maxima [F]

$$\int \frac{x^8}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^8}{\log\left(\left(ex^3+d\right)^pc\right)^2}\,dx$$

```
[In] integrate(x^8/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^9 + d*x^6)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)) + integrate((3*e
*x^8 + 2*d*x^5)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(193) = 386$.

Time = 0.33 (sec) , antiderivative size = 487, normalized size of antiderivative = 2.50

$$\int \frac{x^8}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = -\frac{1}{3}\,d^2\left(\frac{(ex^3+d)p}{e^3p^3\log\left(ex^3+d\right)+e^3p^2\log\left(c\right)} - \frac{p\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(ex^3+d\right)\right)\log\left(ex^3+d\right)}{\left(e^3p^3\log\left(ex^3+d\right)+e^3p^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}} - \frac{\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(ex^3+d\right)\right)\log\left(c\right)}{\left(e^3p^3\log\left(ex^3+d\right)+e^3p^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}\right) - \frac{\left(ex^3+d\right)^3p}{3\left(e^3p^3\log\left(ex^3+d\right)+e^3p^2\log\left(c\right)\right)} + \frac{2\left(ex^3+d\right)^2dp}{3\left(e^3p^3\log\left(ex^3+d\right)+e^3p^2\log\left(c\right)\right)} - \frac{4\,dp\mathrm{Ei}\left(\frac{2\,\log(c)}{p}+2\,\log\left(ex^3+d\right)\right)\log\left(ex^3+d\right)}{3\left(e^3p^3\log\left(ex^3+d\right)+e^3p^2\log\left(c\right)\right)c^{\frac{2}{p}}} + \frac{p\mathrm{Ei}\left(\frac{3\,\log(c)}{p}+3\,\log\left(ex^3+d\right)\right)\log\left(ex^3+d\right)}{\left(e^3p^3\log\left(ex^3+d\right)+e^3p^2\log\left(c\right)\right)c^{\frac{3}{p}}} - \frac{4\,d\mathrm{Ei}\left(\frac{2\,\log(c)}{p}+2\,\log\left(ex^3+d\right)\right)\log\left(c\right)}{3\left(e^3p^3\log\left(ex^3+d\right)+e^3p^2\log\left(c\right)\right)c^{\frac{2}{p}}} + \frac{\mathrm{Ei}\left(\frac{3\,\log(c)}{p}+3\,\log\left(ex^3+d\right)\right)\log\left(c\right)}{\left(e^3p^3\log\left(ex^3+d\right)+e^3p^2\log\left(c\right)\right)c^{\frac{3}{p}}}$$

```
[In] integrate(x^8/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] -1/3*d^2*((e*x^3 + d)*p/(e^3*p^3*log(e*x^3 + d) + e^3*p^2*log(c)) - p*Ei(lo
g(c)/p + log(e*x^3 + d))*log(e*x^3 + d)/((e^3*p^3*log(e*x^3 + d) + e^3*p^2*
log(c))*c^(1/p)) - Ei(log(c)/p + log(e*x^3 + d))*log(c)/((e^3*p^3*log(e*x^3
 + d) + e^3*p^2*log(c))*c^(1/p))) - 1/3*(e*x^3 + d)^3*p/(e^3*p^3*log(e*x^3
+ d) + e^3*p^2*log(c)) + 2/3*(e*x^3 + d)^2*d*p/(e^3*p^3*log(e*x^3 + d) + e^
3*p^2*log(c)) - 4/3*d*p*Ei(2*log(c)/p + 2*log(e*x^3 + d))*log(e*x^3 + d)/((
e^3*p^3*log(e*x^3 + d) + e^3*p^2*log(c))*c^(2/p)) + p*Ei(3*log(c)/p + 3*log
(e*x^3 + d))*log(e*x^3 + d)/((e^3*p^3*log(e*x^3 + d) + e^3*p^2*log(c))*c^(3
/p)) - 4/3*d*Ei(2*log(c)/p + 2*log(e*x^3 + d))*log(c)/((e^3*p^3*log(e*x^3 +
 d) + e^3*p^2*log(c))*c^(2/p)) + Ei(3*log(c)/p + 3*log(e*x^3 + d))*log(c)/(
(e^3*p^3*log(e*x^3 + d) + e^3*p^2*log(c))*c^(3/p))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^8}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

```
[In] int(x^8/log(c*(d + e*x^3)^p)^2,x)
[Out] int(x^8/log(c*(d + e*x^3)^p)^2, x)
```

3.149 $\int \frac{x^5}{\log^2(c(d+ex^3)^p)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 141

$$\int \frac{x^5}{\log^2(c(d+ex^3)^p)}\,dx = -\frac{d(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)}{3e^2p^2} + \frac{2(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\text{ExpIntegralEi}\left(\frac{2\log(c(d+ex^3)^p)}{p}\right)}{3e^2p^2} - \frac{x^3(d+ex^3)}{3ep\log(c(d+ex^3)^p)}$$

```
[Out] -1/3*d*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e^2/p^2/((c*(e*x^3+d)^p)^(1/p))+2/
3*(e*x^3+d)^2*Ei(2*ln(c*(e*x^3+d)^p)/p)/e^2/p^2/((c*(e*x^3+d)^p)^(2/p))-1/3
*x^3*(e*x^3+d)/e/p/ln(c*(e*x^3+d)^p)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2504, 2447, 2446, 2436, 2337, 2209, 2437, 2347}

$$\int \frac{x^5}{\log^2(c(d+ex^3)^p)}\,dx = \frac{2(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\text{ExpIntegralEi}\left(\frac{2\log(c(ex^3+d)^p)}{p}\right)}{3e^2p^2} - \frac{d(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\text{ExpIntegralEi}\left(\frac{\log(c(ex^3+d)^p)}{p}\right)}{3e^2p^2} - \frac{x^3(d+ex^3)}{3ep\log(c(d+ex^3)^p)}$$

```
[In] Int[x^5/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] -1/3*(d*(d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(e^2*p^2*(c*(d +
 e*x^3)^p)^p^(-1)) + (2*(d + e*x^3)^2*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p]
)/p])/(3*e^2*p^2*(c*(d + e*x^3)^p)^(2/p)) - (x^3*(d + e*x^3))/(3*e*p*Log[c*
(d + e*x^3)^p])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2446

```
Int[((f_.) + (g_.)*(x_))^(q_.)/((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)
]*(b_.)), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q/(a + b*Log[c*(d + e*
x)^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] &
& IGtQ[q, 0]
```

Rule 2447

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(d + e*x)*(f + g*x)^q*((a + b*Log[c*(d + e
```

```
*x)^n])^(p + 1)/(b*e*n*(p + 1))), x] + (-Dist[(q + 1)/(b*n*(p + 1)), Int[(f
 + g*x)^q*(a + b*Log[c*(d + e*x)^n])^(p + 1), x], x] + Dist[q*((e*f - d*g)/
(b*e*n*(p + 1))), Int[(f + g*x)^(q - 1)*(a + b*Log[c*(d + e*x)^n])^(p + 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && LtQ[
p, -1] && GtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{x}{\log^2\left(c(d+ex)^p\right)}\,dx, x, x^3\right)$$

$$= -\frac{x^3(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{2\text{Subst}\left(\int \frac{x}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3p} + \frac{d\text{Subst}\left(\int \frac{1}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3ep}$$

$$= -\frac{x^3(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{2\text{Subst}\left(\int \left(-\frac{d}{e\log(c(d+ex)^p)} + \frac{d+ex}{e\log(c(d+ex)^p)}\right)\,dx, x, x^3\right)}{3p} + \frac{d\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, d+ex^3\right)}{3e^2p}$$

$$= -\frac{x^3(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{2\text{Subst}\left(\int \frac{d+ex}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3ep} - \frac{(2d)\text{Subst}\left(\int \frac{1}{\log(c(d+ex)^p)}\,dx, x, x^3\right)}{3ep} + \frac{\left(d(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c(d+ex^3)^p\right)\right)}{3e^2p^2}$$

$$= \frac{d(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\text{Ei}\left(\frac{\log\left(c(d+ex^3)^p\right)}{p}\right)}{3e^2p^2} - \frac{x^3(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)} + \frac{2\text{Subst}\left(\int \frac{x}{\log(cx^p)}\,dx, x, d+ex^3\right)}{3e^2p} - \frac{(2d)\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, d+ex^3\right)}{3e^2p}$$

$$= \frac{d(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\text{Ei}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)}{3e^2p^2} - \frac{x^3(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)}$$
$$+ \frac{\left(2(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{p}}}{x}\,dx, x, \log\left(c(d+ex^3)^p\right)\right)}{3e^2p^2}$$
$$- \frac{\left(2d(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c(d+ex^3)^p\right)\right)}{3e^2p^2}$$
$$= -\frac{d(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\text{Ei}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)}{3e^2p^2}$$
$$+ \frac{2(d+ex^3)^2\left(c(d+ex^3)^p\right)^{-2/p}\text{Ei}\left(\frac{2\log(c(d+ex^3)^p)}{p}\right)}{3e^2p^2} - \frac{x^3(d+ex^3)}{3ep\log\left(c\left(d+ex^3\right)^p\right)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.11

$$\int \frac{x^5}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = -\frac{(d+ex^3)\left(c(d+ex^3)^p\right)^{-2/p}\left(epx^3\left(c(d+ex^3)^p\right)^{2/p} + d\left(c(d+ex^3)^p\right)^{\frac{1}{p}}\text{ExpIntegralEi}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)\log\right.}{3e^2p^2\log\left(c\left(d+ex^3\right)^p\right)}$$

```
[In] Integrate[x^5/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] -1/3*((d + e*x^3)*(e*p*x^3*(c*(d + e*x^3)^p)^(2/p) + d*(c*(d + e*x^3)^p)^p^
(-1)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p]*Log[c*(d + e*x^3)^p] - 2*(d + e*
x^3)*ExpIntegralEi[(2*Log[c*(d + e*x^3)^p])/p]*Log[c*(d + e*x^3)^p]))/(e^2*
p^2*(c*(d + e*x^3)^p)^(2/p)*Log[c*(d + e*x^3)^p])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.96 (sec) , antiderivative size = 1487, normalized size of antiderivative = 10.55

method	result	size
risch	Expression too large to display	1487

```
[In] int(x^5/ln(c*(e*x^3+d)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/p/e*x^3*(e*x^3+d)/(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*
Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3
```

```
+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p))-
2/3/p^2*c^(-2/p)*((e*x^3+d)^p)^(-2/p)*exp(I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn
(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p
)*Ei(1,-2*ln(e*x^3+d)-(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*P
i*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+
d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*
p*ln(e*x^3+d))/p)*x^6-4/3/p^2/e*c^(-2/p)*((e*x^3+d)^p)^(-2/p)*exp(I*Pi*csgn
(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p
)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-2*ln(e*x^3+d)-(I*Pi*csgn(I*(e*x^3+d)^p)*csg
n(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*
c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln
(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)*d*x^3-2/3/p^2/e^2*c^(-2/p)*((e*x^
3+d)^p)^(-2/p)*exp(I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(
I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-2*ln(e*x^3+d)-(
I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*c
sgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(
e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)*d^2+1
/3/p^2/e*d*c^(-1/p)*((e*x^3+d)^p)^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(e*x^3+d)^p)
*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d
)^p))/p)*Ei(1,-ln(e*x^3+d)-1/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)
^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*
c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3
+d)^p)-2*p*ln(e*x^3+d))/p)*x^3+1/3/p^2/e^2*d^2*c^(-1/p)*((e*x^3+d)^p)^(-1/p
)*exp(1/2*I*Pi*csgn(I*c*(e*x^3+d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-c
sgn(I*c*(e*x^3+d)^p)+csgn(I*(e*x^3+d)^p))/p)*Ei(1,-ln(e*x^3+d)-1/2*(I*Pi*cs
gn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c
*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d
)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = -\frac{(dp\log\left(ex^3+d\right)+d\log\left(c\right))c^{\left(\frac{1}{p}\right)}\operatorname{log\_integral}\left(\left(ex^3+d\right)c^{\left(\frac{1}{p}\right)}\right)+\left(e^2px^6+depx^3\right)c^{\frac{2}{p}}-2\left(p\log\left(ex^3+d\right)\right.}{3\left(e^2p^3\log\left(ex^3+d\right)+e^2p^2\log\left(c\right)\right)c^{\frac{2}{p}}}$$

```
[In] integrate(x^5/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
```

```
[Out] -1/3*((d*p*log(e*x^3 + d) + d*log(c))*c^(1/p)*log_integral((e*x^3 + d)*c^(1
/p)) + (e^2*p*x^6 + d*e*p*x^3)*c^(2/p) - 2*(p*log(e*x^3 + d) + log(c))*log_
```

```
integral((e^2*x^6 + 2*d*e*x^3 + d^2)*c^(2/p)))/((e^2*p^3*log(e*x^3 + d) + e
^2*p^2*log(c))*c^(2/p))
```

Sympy [F]

$$\int \frac{x^5}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^5}{\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

```
[In] integrate(x**5/ln(c*(e*x**3+d)**p)**2,x)
[Out] Integral(x**5/log(c*(d + e*x**3)**p)**2, x)
```

Maxima [F]

$$\int \frac{x^5}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^5}{\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

```
[In] integrate(x^5/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
[Out] -1/3*(e*x^6 + d*x^3)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)) + integrate((2*e
*x^5 + d*x^2)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(137) = 274$.

Time = 0.31 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.22

$$\int \frac{x^5}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$
$$=\frac{1}{3}d\left(\frac{\left(ex^3+d\right)p}{e^2p^3\log\left(ex^3+d\right)+e^2p^2\log\left(c\right)}-\frac{p\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(ex^3+d\right)\right)\log\left(ex^3+d\right)}{\left(e^2p^3\log\left(ex^3+d\right)+e^2p^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}-\frac{\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(ex^3\right.\right.}{\left(e^2p^3\log\left(ex^3+d\right)+\right.}\right.$$
$$-\frac{\frac{\left(ex^3+d\right)^2p}{ep^3\log(ex^3+d)+ep^2\log(c)}-\frac{2\,p\mathrm{Ei}\left(\frac{2\,\log(c)}{p}+2\,\log(ex^3+d)\right)\log(ex^3+d)}{\left(ep^3\log(ex^3+d)+ep^2\log(c)\right)c^{\frac{2}{p}}}-\frac{2\,\mathrm{Ei}\left(\frac{2\,\log(c)}{p}+2\,\log(ex^3+d)\right)\log(c)}{\left(ep^3\log(ex^3+d)+ep^2\log(c)\right)c^{\frac{2}{p}}}}{3\,e}$$

```
[In] integrate(x^5/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
[Out] 1/3*d*((e*x^3 + d)*p/(e^2*p^3*log(e*x^3 + d) + e^2*p^2*log(c)) - p*Ei(log(c
)/p + log(e*x^3 + d))*log(e*x^3 + d)/((e^2*p^3*log(e*x^3 + d) + e^2*p^2*log
(c))*c^(1/p)) - Ei(log(c)/p + log(e*x^3 + d))*log(c)/((e^2*p^3*log(e*x^3 +
d) + e^2*p^2*log(c))*c^(1/p))) - 1/3*((e*x^3 + d)^2*p/(e*p^3*log(e*x^3 + d)
 + e*p^2*log(c)) - 2*p*Ei(2*log(c)/p + 2*log(e*x^3 + d))*log(e*x^3 + d)/((e
*p^3*log(e*x^3 + d) + e*p^2*log(c))*c^(2/p)) - 2*Ei(2*log(c)/p + 2*log(e*x^
3 + d))*log(c)/((e*p^3*log(e*x^3 + d) + e*p^2*log(c))*c^(2/p)))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x^5}{\ln\left(c\,{\left(e\,x^3+d\right)}^p\right)^2}\,dx$$

```
[In] int(x^5/log(c*(d + e*x^3)^p)^2,x)
[Out] int(x^5/log(c*(d + e*x^3)^p)^2, x)
```

3.150 $\int \frac{x^2}{\log^2(c(d+ex^3)^p)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 83

$$\int \frac{x^2}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\operatorname{ExpIntegralEi}\left(\frac{\log\left(c\left(d+ex^3\right)^p\right)}{p}\right)}{3ep^2}-\frac{d+ex^3}{3ep\log\left(c\left(d+ex^3\right)^p\right)}$$

```
[Out] 1/3*(e*x^3+d)*Ei(ln(c*(e*x^3+d)^p)/p)/e/p^2/((c*(e*x^3+d)^p)^(1/p))+1/3*(-e
*x^3-d)/e/p/ln(c*(e*x^3+d)^p)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2504, 2436, 2334, 2337, 2209}

$$\int \frac{x^2}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\frac{\left(d+ex^3\right)\left(c\left(d+ex^3\right)^p\right)^{-1/p}\operatorname{ExpIntegralEi}\left(\frac{\log\left(c\left(ex^3+d\right)^p\right)}{p}\right)}{3ep^2}-\frac{d+ex^3}{3ep\log\left(c\left(d+ex^3\right)^p\right)}$$

```
[In] Int[x^2/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] ((d + e*x^3)*ExpIntegralEi[Log[c*(d + e*x^3)^p]/p])/(3*e*p^2*(c*(d + e*x^3)
^p)^p^(-1)) - (d + e*x^3)/(3*e*p*Log[c*(d + e*x^3)^p])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\log^2\left(c(d+ex)^p\right)}\,dx, x, x^3\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\log^2(cx^p)}\,dx, x, d+ex^3\right)}{3e}\\
&= -\frac{d+ex^3}{3ep\log\left(c\left(d+ex^3\right)^p\right)}+\frac{\text{Subst}\left(\int \frac{1}{\log(cx^p)}\,dx, x, d+ex^3\right)}{3ep}\\
&= -\frac{d+ex^3}{3ep\log\left(c\left(d+ex^3\right)^p\right)}+\frac{\left(\left(d+ex^3\right)\left(c(d+ex^3)^p\right)^{-1/p}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{p}}}{x}\,dx, x, \log\left(c(d+ex^3)^p\right)\right)}{3ep^2}
\end{aligned}$$

$$= \frac{(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\operatorname{Ei}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)}{3ep^2} - \frac{d+ex^3}{3ep\log\left(c\left(d+ex^3\right)^p\right)}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17

$$\int \frac{x^2}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = -\frac{(d+ex^3)\left(c(d+ex^3)^p\right)^{-1/p}\left(p\left(c(d+ex^3)^p\right)^{\frac{1}{p}} - \text{ExpIntegralEi}\left(\frac{\log(c(d+ex^3)^p)}{p}\right)\log\left(c(d+ex^3)^p\right)\right)}{3ep^2\log\left(c\left(d+ex^3\right)^p\right)}$$

```
[In] Integrate[x^2/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] -1/3*((d + e*x^3)*(p*(c*(d + e*x^3)^p)^p^(-1) - ExpIntegralEi[Log[c*(d + e*
x^3)^p]/p]*Log[c*(d + e*x^3)^p]))/(e*p^2*(c*(d + e*x^3)^p)^p^(-1)*Log[c*(d
+ e*x^3)^p])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.84 (sec) , antiderivative size = 421, normalized size of antiderivative = 5.07

method	result
risch	$-\frac{2(e x^3+d)}{3(i\pi \operatorname{csgn}(i(e x^3+d)^p)\operatorname{csgn}(ic(e x^3+d)^p)^2-i\pi \operatorname{csgn}(i(e x^3+d)^p)\operatorname{csgn}(ic(e x^3+d)^p)\operatorname{csgn}(ic)-i\pi\operatorname{csgn}(ic(e x^3+d)^p)^3+i\pi\operatorname{csgn}(ic(e x}$

```
[In] int(x^2/ln(c*(e*x^3+d)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d
)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn
(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((e*x^3+d)^p))/p/e*(e*x^3+d)-1/3/
p^2/e*(e*x^3+d)*c^(-1/p)*((e*x^3+d)^p)^(-1/p)*exp(1/2*I*Pi*csgn(I*c*(e*x^3+
d)^p)*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*c))*(-csgn(I*c*(e*x^3+d)^p)+csgn(I*(e*
x^3+d)^p))/p)*Ei(1,-ln(e*x^3+d)-1/2*(I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x
^3+d)^p)^2-I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)*csgn(I*c)-I*Pi*cs
gn(I*c*(e*x^3+d)^p)^3+I*Pi*csgn(I*c*(e*x^3+d)^p)^2*csgn(I*c)+2*ln(c)+2*ln((
e*x^3+d)^p)-2*p*ln(e*x^3+d))/p)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = -\frac{(epx^3+dp)c^{\left(\frac{1}{p}\right)} - (p\log\left(ex^3+d\right)+\log\left(c\right))\,\mathrm{log\_integral}\left((ex^3+d)c^{\left(\frac{1}{p}\right)}\right)}{3\,(ep^3\log\left(ex^3+d\right)+ep^2\log\left(c\right))c^{\left(\frac{1}{p}\right)}}$$

```
[In] integrate(x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
[Out] -1/3*((e*p*x^3 + d*p)*c^(1/p) - (p*log(e*x^3 + d) + log(c))*log_integral((e
*x^3 + d)*c^(1/p)))/((e*p^3*log(e*x^3 + d) + e*p^2*log(c))*c^(1/p))
```

Sympy [F]

$$\int \frac{x^2}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^2}{\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

```
[In] integrate(x**2/ln(c*(e*x**3+d)**p)**2,x)
[Out] Integral(x**2/log(c*(d + e*x**3)**p)**2, x)
```

Maxima [F]

$$\int \frac{x^2}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^2}{\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

```
[In] integrate(x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
[Out] -1/3*(e*x^3 + d)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)) + integrate(x^2/(p*l
og((e*x^3 + d)^p) + p*log(c)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.70

$$\int \frac{x^2}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = -\frac{(ex^3+d)p}{3\left(ep^3\log\left(ex^3+d\right)+ep^2\log\left(c\right)\right)} + \frac{p\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(ex^3+d\right)\right)\log\left(ex^3+d\right)}{3\left(ep^3\log\left(ex^3+d\right)+ep^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}} + \frac{\mathrm{Ei}\left(\frac{\log(c)}{p}+\log\left(ex^3+d\right)\right)\log\left(c\right)}{3\left(ep^3\log\left(ex^3+d\right)+ep^2\log\left(c\right)\right)c^{\left(\frac{1}{p}\right)}}$$

```
[In] integrate(x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] -1/3*(e*x^3 + d)*p/(e*p^3*log(e*x^3 + d) + e*p^2*log(c)) + 1/3*p*Ei(log(c)/
p + log(e*x^3 + d))*log(e*x^3 + d)/((e*p^3*log(e*x^3 + d) + e*p^2*log(c))*c
^(1/p)) + 1/3*Ei(log(c)/p + log(e*x^3 + d))*log(c)/((e*p^3*log(e*x^3 + d) +
 e*p^2*log(c))*c^(1/p))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^2}{{\ln\left(c\,{\left(e\,x^3+d\right)}^p\right)}^2}\,dx$$

```
[In] int(x^2/log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] int(x^2/log(c*(d + e*x^3)^p)^2, x)
```

3.151 $\int \frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\operatorname{Int}\left(\frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x/ln(c*(e*x^3+d)^p)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

```
[In] Int[1/(x*Log[c*(d + e*x^3)^p]^2),x]
```

```
[Out] Defer[Int][1/(x*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

```
[In] Integrate[1/(x*Log[c*(d + e*x^3)^p]^2),x]
[Out] Integrate[1/(x*Log[c*(d + e*x^3)^p]^2), x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\ln\left(c\left(e\,x^3+d\right)^p\right)^2}dx$$

```
[In] int(1/x/ln(c*(e*x^3+d)^p)^2,x)
[Out] int(1/x/ln(c*(e*x^3+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{x\log\left(\left(ex^3+d\right)^pc\right)^2}\,dx$$

```
[In] integrate(1/x/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
[Out] integral(1/(x*log((e*x^3 + d)^p*c)^2), x)
```

Sympy [N/A]

Not integrable

Time = 16.37 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{x\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

```
[In] integrate(1/x/ln(c*(e*x**3+d)**p)**2,x)
[Out] Integral(1/(x*log(c*(d + e*x**3)**p)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.83

$$\int \frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

```
[In] integrate(1/x/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -d*integrate(1/(e*p*x^4*log((e*x^3 + d)^p) + e*p*x^4*log(c)), x) - 1/3*(e*x
^3 + d)/(e*p*x^3*log((e*x^3 + d)^p) + e*p*x^3*log(c))
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

```
[In] integrate(1/x/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x*log((e*x^3 + d)^p*c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x\ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

```
[In] int(1/(x*log(c*(d + e*x^3)^p)^2),x)
```

```
[Out] int(1/(x*log(c*(d + e*x^3)^p)^2), x)
```

3.152 $\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx = \operatorname{Int}\left(\frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)}, x\right)$$

```
[Out] Unintegrable(1/x^4/ln(c*(e*x^3+d)^p)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx = \int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

```
[In] Int[1/(x^4*Log[c*(d + e*x^3)^p]^2),x]
[Out] Defer[Int][1/(x^4*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

```
[In] Integrate[1/(x^4*Log[c*(d + e*x^3)^p]^2),x]
[Out] Integrate[1/(x^4*Log[c*(d + e*x^3)^p]^2), x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 \ln\left(c\left(e\,x^3+d\right)^p\right)^2}dx$$

```
[In] int(1/x^4/ln(c*(e*x^3+d)^p)^2,x)
[Out] int(1/x^4/ln(c*(e*x^3+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^4 \log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
[Out] integral(1/(x^4*log((e*x^3 + d)^p*c)^2), x)
```

Sympy [N/A]

Not integrable

Time = 39.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^4 \log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

```
[In] integrate(1/x**4/ln(c*(e*x**3+d)**p)**2,x)
[Out] Integral(1/(x**4*log(c*(d + e*x**3)**p)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 4.33

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^4 \log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^3 + d)/(e*p*x^6*log((e*x^3 + d)^p) + e*p*x^6*log(c)) - integrate(
(e*x^3 + 2*d)/(e*p*x^7*log((e*x^3 + d)^p) + e*p*x^7*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^4 \log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

```
[In] integrate(1/x^4/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^4*log((e*x^3 + d)^p*c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^4 \ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

```
[In] int(1/(x^4*log(c*(d + e*x^3)^p)^2),x)
```

```
[Out] int(1/(x^4*log(c*(d + e*x^3)^p)^2), x)
```

3.153 $\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)} \, dx = \text{Int}\left(\frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(x^3/ln(c*(e*x^3+d)^p)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)} \, dx = \int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

```
[In] Int[x^3/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] Defer[Int][x^3/Log[c*(d + e*x^3)^p]^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

```
[In] Integrate[x^3/Log[c*(d + e*x^3)^p]^2,x]
[Out] Integrate[x^3/Log[c*(d + e*x^3)^p]^2, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}dx$$

```
[In] int(x^3/ln(c*(e*x^3+d)^p)^2,x)
[Out] int(x^3/ln(c*(e*x^3+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^3}{\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

```
[In] integrate(x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
[Out] integral(x^3/log((e*x^3 + d)^p*c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 15.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^3}{\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

```
[In] integrate(x**3/ln(c*(e*x**3+d)**p)**2,x)
[Out] Integral(x**3/log(c*(d + e*x**3)**p)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.67

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^3}{\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

```
[In] integrate(x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^4 + d*x)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)) + integrate(1/3*(4
*e*x^3 + d)/(e*p*log((e*x^3 + d)^p) + e*p*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^3}{\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

```
[In] integrate(x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/log((e*x^3 + d)^p*c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{x^3}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

```
[In] int(x^3/log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] int(x^3/log(c*(d + e*x^3)^p)^2, x)
```

3.154 $\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)} \, dx = \operatorname{Int}\left(\frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(x/ln(c*(e*x^3+d)^p)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)} \, dx = \int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

```
[In] Int[x/Log[c*(d + e*x^3)^p]^2,x]
```

```
[Out] Defer[Int][x/Log[c*(d + e*x^3)^p]^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

```
[In] Integrate[x/Log[c*(d + e*x^3)^p]^2,x]
[Out] Integrate[x/Log[c*(d + e*x^3)^p]^2, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}dx$$

```
[In] int(x/ln(c*(e*x^3+d)^p)^2,x)
[Out] int(x/ln(c*(e*x^3+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x}{\log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

```
[In] integrate(x/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
[Out] integral(x/log((e*x^3 + d)^p*c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 9.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x}{\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

```
[In] integrate(x/ln(c*(e*x**3+d)**p)**2,x)
[Out] Integral(x/log(c*(d + e*x**3)**p)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 4.62

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x}{\log\left(\left(ex^3+d\right)^pc\right)^2}\,dx$$

```
[In] integrate(x/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^3 + d)/(e*p*x*log((e*x^3 + d)^p) + e*p*x*log(c)) + integrate(1/3*
(2*e*x^3 - d)/(e*p*x^2*log((e*x^3 + d)^p) + e*p*x^2*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x}{\log\left(\left(ex^3+d\right)^pc\right)^2}\,dx$$

```
[In] integrate(x/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(x/log((e*x^3 + d)^p*c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{x}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{x}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

```
[In] int(x/log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] int(x/log(c*(d + e*x^3)^p)^2, x)
```

3.155 $\int \frac{1}{\log^2(c(d+ex^3)^p)} dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)} dx = \text{Int}\left(\frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}, x\right)$$

```
[Out] Unintegrable(1/ln(c*(e*x^3+d)^p)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)} dx = \int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)} dx$$

```
[In] Int[Log[c*(d + e*x^3)^p]^(-2),x]
[Out] Defer[Int][Log[c*(d + e*x^3)^p]^(-2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)} dx$$

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

```
[In] Integrate[Log[c*(d + e*x^3)^p]^(-2),x]
[Out] Integrate[Log[c*(d + e*x^3)^p]^(-2), x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}dx$$

```
[In] int(1/ln(c*(e*x^3+d)^p)^2,x)
[Out] int(1/ln(c*(e*x^3+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{\log\left(\left(ex^3+d\right)^pc\right)^2}\,dx$$

```
[In] integrate(1/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
[Out] integral(log((e*x^3 + d)^p*c)^(-2), x)
```

Sympy [N/A]

Not integrable

Time = 9.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{\log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

```
[In] integrate(1/ln(c*(e*x**3+d)**p)**2,x)
[Out] Integral(log(c*(d + e*x**3)**p)**(-2), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 5.50

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{\log\left(\left(ex^3+d\right)^pc\right)^2}\,dx$$

```
[In] integrate(1/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^3 + d)/(e*p*x^2*log((e*x^3 + d)^p) + e*p*x^2*log(c)) + integrate(
1/3*(e*x^3 - 2*d)/(e*p*x^3*log((e*x^3 + d)^p) + e*p*x^3*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{\log\left(\left(ex^3+d\right)^pc\right)^2}\,dx$$

```
[In] integrate(1/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^3 + d)^p*c)^(-2), x)
```

Mupad [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{\log^2\left(c\left(d+ex^3\right)^p\right)}\,dx=\int \frac{1}{\ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

```
[In] int(1/log(c*(d + e*x^3)^p)^2,x)
```

```
[Out] int(1/log(c*(d + e*x^3)^p)^2, x)
```

3.156 $\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx = \text{Int}\left(\frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)}, x\right)$$

```
[Out] Unintegrable(1/x^2/ln(c*(e*x^3+d)^p)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx = \int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

```
[In] Int[1/(x^2*Log[c*(d + e*x^3)^p]^2),x]
[Out] Defer[Int][1/(x^2*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx$$

```
[In] Integrate[1/(x^2*Log[c*(d + e*x^3)^p]^2),x]
[Out] Integrate[1/(x^2*Log[c*(d + e*x^3)^p]^2), x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \ln\left(c\left(e\,x^3+d\right)^p\right)^2}dx$$

```
[In] int(1/x^2/ln(c*(e*x^3+d)^p)^2,x)
[Out] int(1/x^2/ln(c*(e*x^3+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^2 \log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
[Out] integral(1/(x^2*log((e*x^3 + d)^p*c)^2), x)
```

Sympy [N/A]

Not integrable

Time = 22.47 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^2 \log\left(c\left(d+ex^3\right)^p\right)^2}\,dx$$

```
[In] integrate(1/x**2/ln(c*(e*x**3+d)**p)**2,x)
[Out] Integral(1/(x**2*log(c*(d + e*x**3)**p)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 4.39

$$\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^2 \log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^3 + d)/(e*p*x^4*log((e*x^3 + d)^p) + e*p*x^4*log(c)) - integrate(
1/3*(e*x^3 + 4*d)/(e*p*x^5*log((e*x^3 + d)^p) + e*p*x^5*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^2 \log\left(\left(ex^3+d\right)^p c\right)^2}\,dx$$

```
[In] integrate(1/x^2/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*log((e*x^3 + d)^p*c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \log^2\left(c\left(d+ex^3\right)^p\right)}\,dx = \int \frac{1}{x^2 \ln\left(c\left(e\,x^3+d\right)^p\right)^2}\,dx$$

```
[In] int(1/(x^2*log(c*(d + e*x^3)^p)^2),x)
```

```
[Out] int(1/(x^2*log(c*(d + e*x^3)^p)^2), x)
```

3.157 $\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3 \log^2 \left(c \left(d+e x^3\right)^p\right)} \, dx=\text{Int}\left(\frac{1}{x^3 \log^2 \left(c \left(d+e x^3\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/x^3/ln(c*(e*x^3+d)^p)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^3 \log^2 \left(c \left(d+e x^3\right)^p\right)} \, dx=\int \frac{1}{x^3 \log^2 \left(c \left(d+e x^3\right)^p\right)} \, dx$$

```
[In] Int[1/(x^3*Log[c*(d + e*x^3)^p]^2),x]
```

```
[Out] Defer[Int][1/(x^3*Log[c*(d + e*x^3)^p]^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^3 \log^2 \left(c \left(d+e x^3\right)^p\right)} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)}\, dx = \int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)}\, dx$$

```
[In] Integrate[1/(x^3*Log[c*(d + e*x^3)^p]^2),x]
[Out] Integrate[1/(x^3*Log[c*(d + e*x^3)^p]^2), x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 \ln\left(c\left(e\,x^3+d\right)^p\right)^2}dx$$

```
[In] int(1/x^3/ln(c*(e*x^3+d)^p)^2,x)
[Out] int(1/x^3/ln(c*(e*x^3+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)}\, dx = \int \frac{1}{x^3 \log\left(\left(ex^3+d\right)^p c\right)^2}\, dx$$

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="fricas")
[Out] integral(1/(x^3*log((e*x^3 + d)^p*c)^2), x)
```

Sympy [N/A]

Not integrable

Time = 30.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)}\, dx = \int \frac{1}{x^3 \log\left(c\left(d+ex^3\right)^p\right)^2}\, dx$$

```
[In] integrate(1/x**3/ln(c*(e*x**3+d)**p)**2,x)
[Out] Integral(1/(x**3*log(c*(d + e*x**3)**p)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 4.44

$$\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx = \int \frac{1}{x^3 \log\left(\left(ex^3+d\right)^p c\right)^2} \, dx$$

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(e*x^3 + d)/(e*p*x^5*log((e*x^3 + d)^p) + e*p*x^5*log(c)) - integrate(
1/3*(2*e*x^3 + 5*d)/(e*p*x^6*log((e*x^3 + d)^p) + e*p*x^6*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx = \int \frac{1}{x^3 \log\left(\left(ex^3+d\right)^p c\right)^2} \, dx$$

```
[In] integrate(1/x^3/log(c*(e*x^3+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*log((e*x^3 + d)^p*c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \log^2\left(c\left(d+ex^3\right)^p\right)} \, dx = \int \frac{1}{x^3 \ln\left(c\left(e\,x^3+d\right)^p\right)^2} \, dx$$

```
[In] int(1/(x^3*log(c*(d + e*x^3)^p)^2),x)
```

```
[Out] int(1/(x^3*log(c*(d + e*x^3)^p)^2), x)
```

3.158 $\int (fx)^m \log^3\left(c\left(d+ex^2\right)^p\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (fx)^m \log^3\left(c\left(d+ex^2\right)^p\right)\,dx = \frac{(fx)^{1+m}\log^3\left(c\left(d+ex^2\right)^p\right)}{f(1+m)} - \frac{6ep\text{Int}\left(\frac{(fx)^{2+m}\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2},x\right)}{f^2(1+m)}$$

```
[Out] (f*x)^(1+m)*ln(c*(e*x^2+d)^p)^3/f/(1+m)-6*e*p*Unintegrable((f*x)^(2+m)*ln(c
*(e*x^2+d)^p)^2/(e*x^2+d),x)/f^2/(1+m)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (fx)^m \log^3\left(c\left(d+ex^2\right)^p\right)\,dx = \int (fx)^m \log^3\left(c\left(d+ex^2\right)^p\right)\,dx$$

```
[In] Int[(f*x)^m*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] ((f*x)^(1 + m)*Log[c*(d + e*x^2)^p]^3)/(f*(1 + m)) - (6*e*p*Defer[Int][((f*
x)^(2 + m)*Log[c*(d + e*x^2)^p]^2)/(d + e*x^2), x])/(f^2*(1 + m))
```

Rubi steps

$$\text{integral} = \frac{(fx)^{1+m}\log^3\left(c\left(d+ex^2\right)^p\right)}{f(1+m)} - \frac{(6ep)\int \frac{(fx)^{2+m}\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx}{f^2(1+m)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 994 vs. $2(77) = 154$.

Time = 1.82 (sec) , antiderivative size = 994, normalized size of antiderivative = 49.70

$$\int (fx)^m \log^3\left(c(d+ex^2)^p\right)\,dx$$

$$= \frac{(fx)^m \left((1+m)p^3x^2\log^3(d+ex^2) + \frac{6p^3\left(-\frac{ex^2}{d}\right)^{\frac{1-m}{2}}\left(-\left((1+m)(d+ex^2)\,{}_4F_3\left(1,1,1,\frac{1}{2}-\frac{m}{2};2,2,2;1+\frac{ex^2}{d}\right)\right)+(1+m)(d+ex^2)\,{}_3F_2\left(\right.\right.}{e}\right.}{}$$

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] ((f*x)^m*((1 + m)*p^3*x^2*Log[d + e*x^2]^3 + (6*p^3*(-((e*x^2)/d))^((1 - m)
/2)*(-((1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1, 1, 1/2 - m/2}, {2, 2, 2
}, 1 + (e*x^2)/d]) + (1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1, 1/2 - m/2
}, {2, 2}, 1 + (e*x^2)/d]*Log[d + e*x^2] + d*(-1 + (-((e*x^2)/d))^((1 + m)/
2))*Log[d + e*x^2]^2))/e + (6*d*(1 + m)*p^3*((e*x^2)/(d + e*x^2))^(1/2 - m/
2)*(8*HypergeometricPFQ[{1/2 - m/2, 1/2 - m/2, 1/2 - m/2, 1/2 - m/2}, {3/2
- m/2, 3/2 - m/2, 3/2 - m/2}, d/(d + e*x^2)] + (-1 + m)*Log[d + e*x^2]*(-4*
HypergeometricPFQ[{1/2 - m/2, 1/2 - m/2, 1/2 - m/2}, {3/2 - m/2, 3/2 - m/2}
, d/(d + e*x^2)] + (-1 + m)*Hypergeometric2F1[1/2 - m/2, 1/2 - m/2, 3/2 - m
/2, d/(d + e*x^2)]*Log[d + e*x^2])))/(e*(-1 + m)^3) - (3*p^2*(-((e*x^2)/d))
^((1 - m)/2)*(-((1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1, 1, 1/2 - m/2},
 {2, 2, 2}, 1 + (e*x^2)/d]) + (1 + m)*(d + e*x^2)*HypergeometricPFQ[{1, 1,
1/2 - m/2}, {2, 2}, 1 + (e*x^2)/d]*Log[d + e*x^2] + d*(-1 + (-((e*x^2)/d))^
((1 + m)/2))*Log[d + e*x^2]^2)*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])
)/e - (3*m*p^2*(-((e*x^2)/d))^((1 - m)/2)*(-((1 + m)*(d + e*x^2)*Hypergeome
tricPFQ[{1, 1, 1, 1/2 - m/2}, {2, 2, 2}, 1 + (e*x^2)/d]) + (1 + m)*(d + e*x
^2)*HypergeometricPFQ[{1, 1, 1/2 - m/2}, {2, 2}, 1 + (e*x^2)/d]*Log[d + e*x
^2] + d*(-1 + (-((e*x^2)/d))^((1 + m)/2))*Log[d + e*x^2]^2)*(-(p*Log[d + e*
x^2]) + Log[c*(d + e*x^2)^p]))/e + (3*p*x^2*(-2*e*x^2*Hypergeometric2F1[1,
(3 + m)/2, (5 + m)/2, -((e*x^2)/d)] + d*(3 + m)*Log[d + e*x^2])*(-(p*Log[d
+ e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(d*(3 + m)) + (3*m*p*x^2*(-2*e*x^2*Hyp
ergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)/d)] + d*(3 + m)*Log[d + e
*x^2])*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(d*(3 + m)) + x^2*(-
(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^3 + m*x^2*(-(p*Log[d + e*x^2]) +
 Log[c*(d + e*x^2)^p])^3))/((1 + m)^2*x)
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (fx)^m \ln \left(c\left(e\,x^2+d\right)^p\right)^3 dx$$

`[In] int((f*x)^m*ln(c*(e*x^2+d)^p)^3,x)`

`[Out] int((f*x)^m*ln(c*(e*x^2+d)^p)^3,x)`

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (fx)^m \log^3\left(c\left(d+ex^2\right)^p\right) \, dx = \int (fx)^m \log\left(\left(ex^2+d\right)^p c\right)^3 \, dx$$

`[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^3,x, algorithm="fricas")`

`[Out] integral((f*x)^m*log((e*x^2 + d)^p*c)^3, x)`

Sympy [N/A]

Not integrable

Time = 82.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (fx)^m \log^3\left(c\left(d+ex^2\right)^p\right) \, dx = \int (fx)^m \log\left(c\left(d+ex^2\right)^p\right)^3 dx$$

`[In] integrate((f*x)**m*ln(c*(e*x**2+d)**p)**3,x)`

`[Out] Integral((f*x)**m*log(c*(d + e*x**2)**p)**3, x)`

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 171, normalized size of antiderivative = 8.55

$$\int (fx)^m \log^3\left(c\left(d+ex^2\right)^p\right) \, dx = \int (fx)^m \log\left(\left(ex^2+d\right)^p c\right)^3 \, dx$$

`[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^3,x, algorithm="maxima")`

```
[Out] f^m*x*x^m*log((e*x^2 + d)^p)^3/(m + 1) + integrate((3*(d*f^m*(m + 1)*log(c)
 + (e*f^m*(m + 1)*log(c) - 2*e*f^m*p)*x^2)*x^m*log((e*x^2 + d)^p)^2 + 3*(e*
f^m*(m + 1)*x^2*log(c)^2 + d*f^m*(m + 1)*log(c)^2)*x^m*log((e*x^2 + d)^p) +
 (e*f^m*(m + 1)*x^2*log(c)^3 + d*f^m*(m + 1)*log(c)^3)*x^m)/(e*(m + 1)*x^2
+ d*(m + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (fx)^m \log^3\left(c\left(d+ex^2\right)^p\right)\,dx = \int (fx)^m \log\left(\left(ex^2+d\right)^p c\right)^3\,dx$$

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^3,x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((e*x^2 + d)^p*c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (fx)^m \log^3\left(c\left(d+ex^2\right)^p\right)\,dx = \int \ln\left(c\left(e\,x^2+d\right)^p\right)^3\,{\left(f\,x\right)}^m\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)^3*(f*x)^m,x)
```

```
[Out] int(log(c*(d + e*x^2)^p)^3*(f*x)^m, x)
```

3.159 $\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right)\, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right)\, dx = \frac{(fx)^{1+m}\log^2\left(c\left(d+ex^2\right)^p\right)}{f(1+m)} - \frac{4ep\text{Int}\left(\frac{(fx)^{2+m}\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2},x\right)}{f^2(1+m)}$$

```
[Out] (f*x)^(1+m)*ln(c*(e*x^2+d)^p)^2/f/(1+m)-4*e*p*Unintegrable((f*x)^(2+m)*ln(c
*(e*x^2+d)^p)/(e*x^2+d),x)/f^2/(1+m)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right)\, dx = \int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right)\, dx$$

```
[In] Int[(f*x)^m*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] ((f*x)^(1 + m)*Log[c*(d + e*x^2)^p]^2)/(f*(1 + m)) - (4*e*p*Defer[Int][((f*
x)^(2 + m)*Log[c*(d + e*x^2)^p])/(d + e*x^2), x])/(f^2*(1 + m))
```

Rubi steps

$$\text{integral} = \frac{(fx)^{1+m}\log^2\left(c\left(d+ex^2\right)^p\right)}{f(1+m)} - \frac{(4ep)\int \frac{(fx)^{2+m}\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\, dx}{f^2(1+m)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 466 vs. $2(75) = 150$.

Time = 0.53 (sec) , antiderivative size = 466, normalized size of antiderivative = 23.30

$$\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$=\frac{(fx)^m\left(4p^2x\left(\frac{2ex^2\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{2},\frac{5+m}{2},-\frac{ex^2}{d}\right)}{d(3+m)}-\log\left(d+ex^2\right)\right)+(1+m)p^2x\log^2\left(d+ex^2\right)+\frac{4d(1+m)p}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] ((f*x)^m*(4*p^2*x*((2*e*x^2*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e
*x^2)/d)])/(d*(3 + m)) - Log[d + e*x^2]) + (1 + m)*p^2*x*Log[d + e*x^2]^2 +
 (4*d*(1 + m)*p^2*((e*x^2)/(d + e*x^2))^(1/2 - m/2)*(-2*HypergeometricPFQ[{
1/2 - m/2, 1/2 - m/2, 1/2 - m/2}, {3/2 - m/2, 3/2 - m/2}, d/(d + e*x^2)] +
(-1 + m)*Hypergeometric2F1[1/2 - m/2, 1/2 - m/2, 3/2 - m/2, d/(d + e*x^2)]*
Log[d + e*x^2]))/(e*(-1 + m)^2*x) + (2*p*(2*e*x^3*Hypergeometric2F1[1, (3 +
 m)/2, (5 + m)/2, -((e*x^2)/d)] - d*(3 + m)*x*Log[d + e*x^2])*(p*Log[d + e*
x^2] - Log[c*(d + e*x^2)^p]))/(d*(3 + m)) - (2*m*p*(-2*e*x^3*Hypergeometric
2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)/d)] + d*(3 + m)*x*Log[d + e*x^2])*(p
*Log[d + e*x^2] - Log[c*(d + e*x^2)^p]))/(d*(3 + m)) + x*(-(p*Log[d + e*x^2
]) + Log[c*(d + e*x^2)^p])^2 + m*x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)
^p])^2))/(1 + m)^2
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (fx)^m \ln\left(c\left(e\,x^2+d\right)^p\right)^2 dx$$

```
[In] int((f*x)^m*ln(c*(e*x^2+d)^p)^2,x)
```

```
[Out] int((f*x)^m*ln(c*(e*x^2+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (fx)^m \log^2\left(c(d+ex^2)^p\right)\,dx = \int (fx)^m \log\left(\left(ex^2+d\right)^p c\right)^2\,dx$$

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
[Out] integral((f*x)^m*log((e*x^2 + d)^p*c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 46.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (fx)^m \log^2\left(c(d+ex^2)^p\right)\,dx = \int (fx)^m \log\left(c\left(d+ex^2\right)^p\right)^2\,dx$$

```
[In] integrate((f*x)**m*ln(c*(e*x**2+d)**p)**2,x)
[Out] Integral((f*x)**m*log(c*(d + e*x**2)**p)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 126, normalized size of antiderivative = 6.30

$$\int (fx)^m \log^2\left(c(d+ex^2)^p\right)\,dx = \int (fx)^m \log\left(\left(ex^2+d\right)^p c\right)^2\,dx$$

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
[Out] f^m*x*x^m*log((e*x^2 + d)^p)^2/(m + 1) + integrate((2*(d*f^m*(m + 1)*log(c)
 + (e*f^m*(m + 1)*log(c) - 2*e*f^m*p)*x^2)*x^m*log((e*x^2 + d)^p) + (e*f^m*
(m + 1)*x^2*log(c)^2 + d*f^m*(m + 1)*log(c)^2)*x^m)/(e*(m + 1)*x^2 + d*(m +
 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right)\,dx = \int (fx)^m \log\left(\left(ex^2+d\right)^p c\right)^2\,dx$$

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
[Out] integrate((f*x)^m*log((e*x^2 + d)^p*c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (fx)^m \log^2\left(c\left(d+ex^2\right)^p\right)\,dx = \int \ln\left(c\left(e\,x^2+d\right)^p\right)^2\,(f\,x)^m\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)^2*(f*x)^m,x)
[Out] int(log(c*(d + e*x^2)^p)^2*(f*x)^m, x)
```

3.160 $\int (fx)^m \log\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int (fx)^m \log\left(c(d+ex^2)^p\right)\,dx = -\frac{2ep(fx)^{3+m}\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{2},\frac{5+m}{2},-\frac{ex^2}{d}\right)}{df^3(1+m)(3+m)} + \frac{(fx)^{1+m}\log\left(c(d+ex^2)^p\right)}{f(1+m)}$$

```
[Out] -2*e*p*(f*x)^(3+m)*hypergeom([1, 3/2+1/2*m],[5/2+1/2*m],-e*x^2/d)/d/f^3/(1+
m)/(3+m)+(f*x)^(1+m)*ln(c*(e*x^2+d)^p)/f/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2505, 16, 371}

$$\int (fx)^m \log\left(c(d+ex^2)^p\right)\,dx = \frac{(fx)^{m+1}\log\left(c(d+ex^2)^p\right)}{f(m+1)} - \frac{2ep(fx)^{m+3}\,\text{Hypergeometric2F1}\left(1,\frac{m+3}{2},\frac{m+5}{2},-\frac{ex^2}{d}\right)}{df^3(m+1)(m+3)}$$

```
[In] Int[(f*x)^m*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (-2*e*p*(f*x)^(3 + m)*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)/
d)])/(d*f^3*(1 + m)*(3 + m)) + ((f*x)^(1 + m)*Log[c*(d + e*x^2)^p])/(f*(1 +
 m))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(fx)^{1+m}\log\left(c\left(d+ex^2\right)^p\right)}{f(1+m)}-\frac{(2ep)\int\frac{x(fx)^{1+m}}{d+ex^2}\,dx}{f(1+m)}\\ &= \frac{(fx)^{1+m}\log\left(c\left(d+ex^2\right)^p\right)}{f(1+m)}-\frac{(2ep)\int\frac{(fx)^{2+m}}{d+ex^2}\,dx}{f^2(1+m)}\\ &= -\frac{2ep(fx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{2};\frac{5+m}{2};-\frac{ex^2}{d}\right)}{df^3(1+m)(3+m)}+\frac{(fx)^{1+m}\log\left(c\left(d+ex^2\right)^p\right)}{f(1+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int(fx)^m\log\left(c\left(d+ex^2\right)^p\right)\,dx\\ =\frac{x(fx)^m\left(-2epx^2\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{2},\frac{5+m}{2},-\frac{ex^2}{d}\right)+d(3+m)\log\left(c\left(d+ex^2\right)^p\right)\right)}{d(1+m)(3+m)}$$

```
[In] Integrate[(f*x)^m*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (x*(f*x)^m*(-2*e*p*x^2*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((e*x^2)
/d)] + d*(3 + m)*Log[c*(d + e*x^2)^p]))/(d*(1 + m)*(3 + m))
```

Maple [F]

$$\int (fx)^m \ln \left(c\left(e\,x^2+d\right)^p\right)dx$$

```
[In] int((f*x)^m*ln(c*(e*x^2+d)^p),x)
[Out] int((f*x)^m*ln(c*(e*x^2+d)^p),x)
```

Fricas [F]

$$\int (fx)^m \log \left(c\left(d+ex^2\right)^p\right)\,dx = \int (fx)^m \log \left(\left(ex^2+d\right)^p c\right)\,dx$$

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="fricas")
[Out] integral((f*x)^m*log((e*x^2 + d)^p*c), x)
```

Sympy [A] (verification not implemented)

Time = 28.95 (sec) , antiderivative size = 377, normalized size of antiderivative = 4.65

$$\int (fx)^m \log \left(c\left(d+ex^2\right)^p\right)\,dx =$$

$$-2ep\left(\begin{cases} \frac{0^m\sqrt{-\frac{d}{e^3}}\log\left(-e\sqrt{-\frac{d}{e^3}}+x\right)}{2} - \frac{0^m\sqrt{-\frac{d}{e^3}}\log\left(e\sqrt{-\frac{d}{e^3}}+x\right)}{2} + \frac{0^m x}{e} \\ \frac{f^{m+1}mx^{m+3}\Phi\left(\frac{ex^2e^{i\pi}}{d},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4dfm\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+4df\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{3f^{m+1}x^{m+3}\Phi\left(\frac{ex^2e^{i\pi}}{d},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4dfm\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)+4df\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} \\ -\frac{\begin{cases} -\frac{\operatorname{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } \frac{1}{|x|}<1 \wedge |x|<1 \\ \log(d)\log(x) - \frac{\operatorname{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } |x|<1 \\ -\log(d)\log\left(\frac{1}{x}\right) - \frac{\operatorname{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{for } \frac{1}{|x|}<1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle|x\right)\log(d) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle|x\right)\log(d) - \frac{\operatorname{Li}_2\left(\frac{ex^2e^{i\pi}}{d}\right)}{2} & \text{otherwise} \end{cases}}{2ef} \end{cases}\right)$$

$$+\left(\begin{cases} 0^m x & \text{for } f=0 \\ \frac{\begin{cases} \frac{(fx)^{m+1}}{m+1} & \text{for } m\neq -1 \\ \log(fx) & \text{otherwise} \end{cases}}{f} & \text{otherwise} \end{cases}\right)\log\left(c\left(d+ex^2\right)^p\right)$$

```
[In] integrate((f*x)**m*ln(c*(e*x**2+d)**p),x)
```

```
[Out] -2*e*p*Piecewise((0**m*sqrt(-d/e**3)*log(-e*sqrt(-d/e**3) + x)/2 - 0**m*sqr
t(-d/e**3)*log(e*sqrt(-d/e**3) + x)/2 + 0**m*x/e, Eq(f, 0) | (Eq(f, 0) & Ne
(m, -1))), (f**(m + 1)*m*x**(m + 3)*lerchphi(e*x**2*exp_polar(I*pi)/d, 1, m
/2 + 3/2)*gamma(m/2 + 3/2)/(4*d*f*m*gamma(m/2 + 5/2) + 4*d*f*gamma(m/2 + 5/
2)) + 3*f**(m + 1)*x**(m + 3)*lerchphi(e*x**2*exp_polar(I*pi)/d, 1, m/2 + 3
/2)*gamma(m/2 + 3/2)/(4*d*f*m*gamma(m/2 + 5/2) + 4*d*f*gamma(m/2 + 5/2)), (
m > -oo) & (m < oo) & Ne(m, -1)), (-Piecewise((-polylog(2, e*x**2*exp_polar
(I*pi)/d)/2, (Abs(x) < 1) & (1/Abs(x) < 1)), (log(d)*log(x) - polylog(2, e*
x**2*exp_polar(I*pi)/d)/2, Abs(x) < 1), (-log(d)*log(1/x) - polylog(2, e*x*
*2*exp_polar(I*pi)/d)/2, 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()
), x)*log(d) + meijerg(((1, 1), ()), ((), (0, 0)), x)*log(d) - polylog(2, e
*x**2*exp_polar(I*pi)/d)/2, True))/(2*e*f) + log(f*x)*log(d + e*x**2)/(2*e*
f), True)) + Piecewise((0**m*x, Eq(f, 0)), (Piecewise(((f*x)**(m + 1)/(m +
1), Ne(m, -1)), (log(f*x), True))/f, True))*log(c*(d + e*x**2)**p)
```

Maxima [F]

$$\int (fx)^m \log \left(c(d+ex^2)^p\right) \, dx = \int (fx)^m \log \left((ex^2+d)^p c\right) \, dx$$

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] f^m*x*x^m*log((e*x^2 + d)^p)/(m + 1) + integrate((d*f^m*(m + 1)*log(c) + (e
*f^m*(m + 1)*log(c) - 2*e*f^m*p)*x^2)*x^m/(e*(m + 1)*x^2 + d*(m + 1)), x)
```

Giac [F]

$$\int (fx)^m \log \left(c(d+ex^2)^p\right) \, dx = \int (fx)^m \log \left((ex^2+d)^p c\right) \, dx$$

```
[In] integrate((f*x)^m*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*log((e*x^2 + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^m \log \left(c\left(d+ex^2\right)^p\right) \, dx = \int \ln \left(c\left(e\,x^2+d\right)^p\right) \,(f\,x)^m \, dx$$

```
[In] int(log(c*(d + e*x^2)^p)*(f*x)^m,x)
[Out] int(log(c*(d + e*x^2)^p)*(f*x)^m, x)
```

3.161 $\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx = \operatorname{Int}\left(\frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable((f*x)^m/ln(c*(e*x^2+d)^p),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] Int[(f*x)^m/Log[c*(d + e*x^2)^p],x]
```

```
[Out] Defer[Int][(f*x)^m/Log[c*(d + e*x^2)^p], x]
```

Rubi steps

$$\text{integral} = \int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(fx)^m}{\log \left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(fx)^m}{\log \left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] Integrate[(f*x)^m/Log[c*(d + e*x^2)^p],x]
[Out] Integrate[(f*x)^m/Log[c*(d + e*x^2)^p], x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m}{\ln \left(c\left(e\,x^2+d\right)^p\right)}dx$$

```
[In] int((f*x)^m/ln(c*(e*x^2+d)^p),x)
[Out] int((f*x)^m/ln(c*(e*x^2+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(fx)^m}{\log \left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(fx)^m}{\log \left(\left(ex^2+d\right)^p c\right)}\,dx$$

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p),x, algorithm="fricas")
[Out] integral((f*x)^m/log((e*x^2 + d)^p*c), x)
```

Sympy [N/A]

Not integrable

Time = 12.99 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(fx)^m}{\log \left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(fx)^m}{\log \left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] integrate((f*x)**m/ln(c*(e*x**2+d)**p),x)
[Out] Integral((f*x)**m/log(c*(d + e*x**2)**p), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{(fx)^m}{\log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] integrate((f*x)^m/log((e*x^2 + d)^p*c), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{(fx)^m}{\log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p),x, algorithm="giac")
[Out] integrate((f*x)^m/log((e*x^2 + d)^p*c), x)
```

Mupad [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{(f\,x)^m}{\ln\left(c\left(e\,x^2+d\right)^p\right)}\,dx$$

```
[In] int((f*x)^m/log(c*(d + e*x^2)^p),x)
[Out] int((f*x)^m/log(c*(d + e*x^2)^p), x)
```

3.162 $\int \frac{(fx)^m}{\log^2(c(d+ex^2)^p)} dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(fx)^m}{\log^2(c(d+ex^2)^p)} dx = \text{Int}\left(\frac{(fx)^m}{\log^2(c(d+ex^2)^p)}, x\right)$$

```
[Out] Unintegrable((f*x)^m/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m}{\log^2(c(d+ex^2)^p)} dx = \int \frac{(fx)^m}{\log^2(c(d+ex^2)^p)} dx$$

```
[In] Int[(f*x)^m/Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] Defer[Int][(f*x)^m/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(fx)^m}{\log^2(c(d+ex^2)^p)} dx$$

Mathematica [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] Integrate[(f*x)^m/Log[c*(d + e*x^2)^p]^2,x]
[Out] Integrate[(f*x)^m/Log[c*(d + e*x^2)^p]^2, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m}{\ln\left(c\left(e\,x^2+d\right)^p\right)^2}dx$$

```
[In] int((f*x)^m/ln(c*(e*x^2+d)^p)^2,x)
[Out] int((f*x)^m/ln(c*(e*x^2+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(fx)^m}{\log\left(\left(ex^2+d\right)^pc\right)^2}\,dx$$

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
[Out] integral((f*x)^m/log((e*x^2 + d)^p*c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 29.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(fx)^m}{\log\left(c\left(d+ex^2\right)^p\right)^2}\,dx$$

```
[In] integrate((f*x)**m/ln(c*(e*x**2+d)**p)**2,x)
[Out] Integral((f*x)**m/log(c*(d + e*x**2)**p)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 4.85

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{(fx)^m}{\log\left(\left(ex^2+d\right)^p c\right)^2}\,dx$$

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*f^m*x^2 + d*f^m)*x^m/(e*p*x*log((e*x^2 + d)^p) + e*p*x*log(c)) + in
tegrate(1/2*(e*f^m*(m + 1)*x^2 + d*f^m*(m - 1))*x^m/(e*p*x^2*log((e*x^2 + d
)^p) + e*p*x^2*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{(fx)^m}{\log\left(\left(ex^2+d\right)^p c\right)^2}\,dx$$

```
[In] integrate((f*x)^m/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^m/log((e*x^2 + d)^p*c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(fx)^m}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{(f\,x)^m}{\ln\left(c\left(e\,x^2+d\right)^p\right)^2}\,dx$$

```
[In] int((f*x)^m/log(c*(d + e*x^2)^p)^2,x)
```

```
[Out] int((f*x)^m/log(c*(d + e*x^2)^p)^2, x)
```

3.163 $\int (fx)^{-1+3n} \log^2 (c(d+ex^n)^p)\, dx$

Optimal result

Integrand size = 24, antiderivative size = 372

$$\begin{aligned}\int (fx)^{-1+3n} \log^2 (c(d+ex^n)^p)\, dx = {} & \frac{2d^2p^2x^{1-2n}(fx)^{-1+3n}}{e^2n} - \frac{dp^2x^{1-3n}(fx)^{-1+3n}(d+ex^n)^2}{2e^3n} \\ & + \frac{2p^2x^{1-3n}(fx)^{-1+3n}(d+ex^n)^3}{27e^3n} \\ & - \frac{d^3p^2x^{1-3n}(fx)^{-1+3n}\log^2(d+ex^n)}{3e^3n} \\ & - \frac{2d^2px^{1-3n}(fx)^{-1+3n}(d+ex^n)\log(c(d+ex^n)^p)}{e^3n} \\ & + \frac{dpx^{1-3n}(fx)^{-1+3n}(d+ex^n)^2\log(c(d+ex^n)^p)}{e^3n} \\ & - \frac{2px^{1-3n}(fx)^{-1+3n}(d+ex^n)^3\log(c(d+ex^n)^p)}{9e^3n} \\ & + \frac{2d^3px^{1-3n}(fx)^{-1+3n}\log(d+ex^n)\log(c(d+ex^n)^p)}{3e^3n} \\ & + \frac{x(fx)^{-1+3n}\log^2(c(d+ex^n)^p)}{3n}\end{aligned}$$

```
[Out] 2*d^2*p^2*x^(1-2*n)*(f*x)^(-1+3*n)/e^2/n-1/2*d*p^2*x^(1-3*n)*(f*x)^(-1+3*n)
*(d+e*x^n)^2/e^3/n+2/27*p^2*x^(1-3*n)*(f*x)^(-1+3*n)*(d+e*x^n)^3/e^3/n-1/3*
d^3*p^2*x^(1-3*n)*(f*x)^(-1+3*n)*ln(d+e*x^n)^2/e^3/n-2*d^2*p*x^(1-3*n)*(f*x
)^(-1+3*n)*(d+e*x^n)*ln(c*(d+e*x^n)^p)/e^3/n+d*p*x^(1-3*n)*(f*x)^(-1+3*n)*(
d+e*x^n)^2*ln(c*(d+e*x^n)^p)/e^3/n-2/9*p*x^(1-3*n)*(f*x)^(-1+3*n)*(d+e*x^n)
^3*ln(c*(d+e*x^n)^p)/e^3/n+2/3*d^3*p*x^(1-3*n)*(f*x)^(-1+3*n)*ln(d+e*x^n)*l
n(c*(d+e*x^n)^p)/e^3/n+1/3*x*(f*x)^(-1+3*n)*ln(c*(d+e*x^n)^p)^2/n
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2506, 2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$\begin{aligned}\int (fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)\,dx = &\frac{2d^3px^{1-3n}(fx)^{3n-1}\log(d+ex^n)\log\left(c(d+ex^n)^p\right)}{3e^3n}\\ &-\frac{2d^2px^{1-3n}(fx)^{3n-1}(d+ex^n)\log\left(c(d+ex^n)^p\right)}{e^3n}\\ &-\frac{2px^{1-3n}(fx)^{3n-1}(d+ex^n)^3\log\left(c(d+ex^n)^p\right)}{9e^3n}\\ &+\frac{dpx^{1-3n}(fx)^{3n-1}(d+ex^n)^2\log\left(c(d+ex^n)^p\right)}{e^3n}\\ &+\frac{x(fx)^{3n-1}\log^2\left(c(d+ex^n)^p\right)}{3n}\\ &-\frac{d^3p^2x^{1-3n}(fx)^{3n-1}\log^2(d+ex^n)}{3e^3n}\\ &+\frac{2d^2p^2x^{1-2n}(fx)^{3n-1}}{e^2n}+\frac{2p^2x^{1-3n}(fx)^{3n-1}(d+ex^n)^3}{27e^3n}\\ &-\frac{dp^2x^{1-3n}(fx)^{3n-1}(d+ex^n)^2}{2e^3n}\end{aligned}$$

```
[In] Int[(f*x)^(-1 + 3*n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] (2*d^2*p^2*x^(1 - 2*n)*(f*x)^(-1 + 3*n))/(e^2*n) - (d*p^2*x^(1 - 3*n)*(f*x)
^(-1 + 3*n)*(d + e*x^n)^2)/(2*e^3*n) + (2*p^2*x^(1 - 3*n)*(f*x)^(-1 + 3*n)*
(d + e*x^n)^3)/(27*e^3*n) - (d^3*p^2*x^(1 - 3*n)*(f*x)^(-1 + 3*n)*Log[d + e
*x^n]^2)/(3*e^3*n) - (2*d^2*p*x^(1 - 3*n)*(f*x)^(-1 + 3*n)*(d + e*x^n)*Log[
c*(d + e*x^n)^p])/(e^3*n) + (d*p*x^(1 - 3*n)*(f*x)^(-1 + 3*n)*(d + e*x^n)^2
*Log[c*(d + e*x^n)^p])/(e^3*n) - (2*p*x^(1 - 3*n)*(f*x)^(-1 + 3*n)*(d + e*x
^n)^3*Log[c*(d + e*x^n)^p])/(9*e^3*n) + (2*d^3*p*x^(1 - 3*n)*(f*x)^(-1 + 3*
n)*Log[d + e*x^n]*Log[c*(d + e*x^n)^p])/(3*e^3*n) + (x*(f*x)^(-1 + 3*n)*Log
[c*(d + e*x^n)^p]^2)/(3*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2506

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(a + b*Log[c*(d + e*x^n)^
p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simpl
ify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(x^{1-3n}(fx)^{-1+3n}\right)\int x^{-1+3n}\log^2\left(c(d+ex^n)^p\right)\,dx \\
&= \frac{\left(x^{1-3n}(fx)^{-1+3n}\right)\text{Subst}\left(\int x^2\log^2\left(c(d+ex)^p\right)\,dx,x,x^n\right)}{n} \\
&= \frac{x(fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)}{3n}-\frac{\left(2epx^{1-3n}(fx)^{-1+3n}\right)\text{Subst}\left(\int \frac{x^3\log(c(d+ex)^p)}{d+ex}\,dx,x,x^n\right)}{3n} \\
&= \frac{x(fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)}{3n}-\frac{\left(2px^{1-3n}(fx)^{-1+3n}\right)\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^3\log(cx^p)}{x}\,dx,x,d+ex^n\right)}{3n} \\
&= -\frac{2d^2px^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)\log\left(c(d+ex^n)^p\right)}{e^3n} \\
&\quad+\frac{dpx^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)^2\log\left(c(d+ex^n)^p\right)}{e^3n} \\
&\quad-\frac{2px^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)^3\log\left(c(d+ex^n)^p\right)}{9e^3n} \\
&\quad+\frac{2d^3px^{1-3n}(fx)^{-1+3n}\log\left(d+ex^n\right)\log\left(c(d+ex^n)^p\right)}{3e^3n} \\
&\quad+\frac{x(fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)}{3n} \\
&\quad+\frac{\left(2p^2x^{1-3n}(fx)^{-1+3n}\right)\text{Subst}\left(\int \frac{18d^2x-9dx^2+2x^3-6d^3\log(x)}{6e^3x}\,dx,x,d+ex^n\right)}{3n} \\
&= -\frac{2d^2px^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)\log\left(c(d+ex^n)^p\right)}{e^3n} \\
&\quad+\frac{dpx^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)^2\log\left(c(d+ex^n)^p\right)}{e^3n} \\
&\quad-\frac{2px^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)^3\log\left(c(d+ex^n)^p\right)}{9e^3n} \\
&\quad+\frac{2d^3px^{1-3n}(fx)^{-1+3n}\log\left(d+ex^n\right)\log\left(c(d+ex^n)^p\right)}{3e^3n} \\
&\quad+\frac{x(fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)}{3n} \\
&\quad+\frac{\left(p^2x^{1-3n}(fx)^{-1+3n}\right)\text{Subst}\left(\int \frac{18d^2x-9dx^2+2x^3-6d^3\log(x)}{x}\,dx,x,d+ex^n\right)}{9e^3n}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2d^2px^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)\log\left(c(d+ex^n)^p\right)}{e^3n} \\
&\quad+\frac{dpx^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)^2\log\left(c(d+ex^n)^p\right)}{e^3n} \\
&\quad-\frac{2px^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)^3\log\left(c(d+ex^n)^p\right)}{9e^3n} \\
&\quad+\frac{2d^3px^{1-3n}(fx)^{-1+3n}\log\left(d+ex^n\right)\log\left(c(d+ex^n)^p\right)}{3e^3n} \\
&\quad+\frac{x(fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)}{3n} \\
&\quad+\frac{\left(p^2x^{1-3n}(fx)^{-1+3n}\right)\text{Subst}\left(\int\left(18d^2-9dx+2x^2-\frac{6d^3\log(x)}{x}\right)\,dx,x,d+ex^n\right)}{9e^3n} \\
&= \frac{2d^2p^2x^{1-2n}(fx)^{-1+3n}}{e^2n}-\frac{dp^2x^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)^2}{2e^3n} \\
&\quad+\frac{2p^2x^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)^3}{27e^3n}-\frac{2d^2px^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)\log\left(c(d+ex^n)^p\right)}{e^3n} \\
&\quad+\frac{dpx^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)^2\log\left(c(d+ex^n)^p\right)}{e^3n} \\
&\quad-\frac{2px^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)^3\log\left(c(d+ex^n)^p\right)}{9e^3n} \\
&\quad+\frac{2d^3px^{1-3n}(fx)^{-1+3n}\log\left(d+ex^n\right)\log\left(c(d+ex^n)^p\right)}{3e^3n} \\
&\quad+\frac{x(fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)}{3n} \\
&\quad-\frac{\left(2d^3p^2x^{1-3n}(fx)^{-1+3n}\right)\text{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,d+ex^n\right)}{3e^3n} \\
&= \frac{2d^2p^2x^{1-2n}(fx)^{-1+3n}}{e^2n}-\frac{dp^2x^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)^2}{2e^3n} \\
&\quad+\frac{2p^2x^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)^3}{27e^3n}-\frac{d^3p^2x^{1-3n}(fx)^{-1+3n}\log^2\left(d+ex^n\right)}{3e^3n} \\
&\quad-\frac{2d^2px^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)\log\left(c(d+ex^n)^p\right)}{e^3n} \\
&\quad+\frac{dpx^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)^2\log\left(c(d+ex^n)^p\right)}{e^3n} \\
&\quad-\frac{2px^{1-3n}(fx)^{-1+3n}\left(d+ex^n\right)^3\log\left(c(d+ex^n)^p\right)}{9e^3n} \\
&\quad+\frac{2d^3px^{1-3n}(fx)^{-1+3n}\log\left(d+ex^n\right)\log\left(c(d+ex^n)^p\right)}{3e^3n} \\
&\quad+\frac{x(fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)}{3n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.46

$$\int (fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)\,dx$$
$$=\frac{x^{-3n}(fx)^{3n}\left(-18d^3p^2\log^2(d+ex^n)+6d^3p\log(d+ex^n)\left(-11p+6\log\left(c(d+ex^n)^p\right)\right)+ex^n\left(p^2(66d^2-1\right.\right.}{54e^3fn}$$

```
[In] Integrate[(f*x)^(-1 + 3*n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] ((f*x)^(3*n)*(-18*d^3*p^2*Log[d + e*x^n]^2 + 6*d^3*p*Log[d + e*x^n]*(-11*p
+ 6*Log[c*(d + e*x^n)^p]) + e*x^n*(p^2*(66*d^2 - 15*d*e*x^n + 4*e^2*x^(2*n)
) - 6*p*(6*d^2 - 3*d*e*x^n + 2*e^2*x^(2*n))*Log[c*(d + e*x^n)^p] + 18*e^2*x
^(2*n)*Log[c*(d + e*x^n)^p]^2)))/(54*e^3*f*n*x^(3*n))
```

Maple [F]

$$\int (fx)^{-1+3n}\ln\left(c(d+e\,x^n)^p\right)^2dx$$

```
[In] int((f*x)^(-1+3*n)*ln(c*(d+e*x^n)^p)^2,x)
```

```
[Out] int((f*x)^(-1+3*n)*ln(c*(d+e*x^n)^p)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.72

$$\int (fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)\,dx$$
$$=\frac{2\left(2\,e^3p^2-6\,e^3p\log(c)+9\,e^3\log(c)^2\right)f^{3\,n-1}x^{3\,n}-3\left(5\,de^2p^2-6\,de^2p\log(c)\right)f^{3\,n-1}x^{2\,n}+6\left(11\,d^2ep^2-6\,d\right.}{}$$

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/54*(2*(2*e^3*p^2 - 6*e^3*p*log(c) + 9*e^3*log(c)^2)*f^(3*n - 1)*x^(3*n) -
 3*(5*d*e^2*p^2 - 6*d*e^2*p*log(c))*f^(3*n - 1)*x^(2*n) + 6*(11*d^2*e*p^2 -
 6*d^2*e*p*log(c))*f^(3*n - 1)*x^n + 18*(e^3*f^(3*n - 1)*p^2*x^(3*n) + d^3*
f^(3*n - 1)*p^2)*log(e*x^n + d)^2 + 6*(3*d*e^2*f^(3*n - 1)*p^2*x^(2*n) - 6*
d^2*e*f^(3*n - 1)*p^2*x^n - 2*(e^3*p^2 - 3*e^3*p*log(c))*f^(3*n - 1)*x^(3*n
) - (11*d^3*p^2 - 6*d^3*p*log(c))*f^(3*n - 1))*log(e*x^n + d))/(e^3*n)
```

Sympy [F]

$$\int (fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)\,dx=\int (fx)^{3n-1}\log\left(c(d+ex^n)^p\right)^2dx$$

```
[In] integrate((f*x)**(-1+3*n)*ln(c*(d+e*x**n)**p)**2,x)
[Out] Integral((f*x)**(3*n - 1)*log(c*(d + e*x**n)**p)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.64

$$\int (fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)\,dx$$
$$=\frac{ep\left(\frac{6\,d^3f^{3\,n}\log\left(\frac{ex^n+d}{e}\right)}{e^4n}-\frac{2\,e^2f^{3\,n}x^{3\,n}-3\,def^{3\,n}x^{2\,n}+6\,d^2f^{3\,n}x^n}{e^3n}\right)\log\left((ex^n+d)^pc\right)}{9\,f}$$
$$+\frac{(fx)^{3\,n}\log\left((ex^n+d)^pc\right)^2}{3\,fn}$$
$$-\frac{\left(18\,d^3f^{3\,n}\log\left(ex^n+d\right)^2-4\,e^3f^{3\,n}x^{3\,n}+15\,de^2f^{3\,n}x^{2\,n}-66\,d^2ef^{3\,n}x^n-6\,(6\,f^{3\,n}\log\left(e\right)-11\,f^{3\,n})d^3\log\left(\right.\right.}{54\,e^3fn}$$

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
[Out] 1/9*e*p*(6*d^3*f^(3*n)*log((e*x^n + d)/e)/(e^4*n) - (2*e^2*f^(3*n)*x^(3*n)
- 3*d*e*f^(3*n)*x^(2*n) + 6*d^2*f^(3*n)*x^n)/(e^3*n))*log((e*x^n + d)^p*c)/
f + 1/3*(f*x)^(3*n)*log((e*x^n + d)^p*c)^2/(f*n) - 1/54*(18*d^3*f^(3*n)*log
(e*x^n + d)^2 - 4*e^3*f^(3*n)*x^(3*n) + 15*d*e^2*f^(3*n)*x^(2*n) - 66*d^2*e
*f^(3*n)*x^n - 6*(6*f^(3*n)*log(e) - 11*f^(3*n))*d^3*log(e*x^n + d))*p^2/(e
^3*f*n)
```

Giac [F]

$$\int (fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)\,dx=\int (fx)^{3\,n-1}\log\left((ex^n+d)^pc\right)^2\,dx$$

```
[In] integrate((f*x)^(-1+3*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")
[Out] integrate((f*x)^(3*n - 1)*log((e*x^n + d)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1+3n}\log^2\left(c(d+ex^n)^p\right)\,dx=\int \ln\left(c\,{\left(d+e\,x^n\right)}^p\right)^2\,{\left(f\,x\right)}^{3\,n-1}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)^2*(f*x)^(3*n - 1),x)
[Out] int(log(c*(d + e*x^n)^p)^2*(f*x)^(3*n - 1), x)
```

3.164 $\int (fx)^{-1+2n} \log^2\left(c(d+ex^n)^p\right) dx$

Optimal result

Integrand size = 24, antiderivative size = 255

$$\begin{aligned}\int (fx)^{-1+2n} \log^2\left(c(d+ex^n)^p\right) dx = &-\frac{2dp^2x^{1-n}(fx)^{-1+2n}}{en} + \frac{p^2x^{1-2n}(fx)^{-1+2n}(d+ex^n)^2}{4e^2n} \\ &+ \frac{2dpx^{1-2n}(fx)^{-1+2n}(d+ex^n)\log\left(c(d+ex^n)^p\right)}{e^2n} \\ &- \frac{px^{1-2n}(fx)^{-1+2n}(d+ex^n)^2\log\left(c(d+ex^n)^p\right)}{2e^2n} \\ &- \frac{dx^{1-2n}(fx)^{-1+2n}(d+ex^n)\log^2\left(c(d+ex^n)^p\right)}{e^2n} \\ &+ \frac{x^{1-2n}(fx)^{-1+2n}(d+ex^n)^2\log^2\left(c(d+ex^n)^p\right)}{2e^2n}\end{aligned}$$

```
[Out] -2*d*p^2*x^(1-n)*(f*x)^(-1+2*n)/e/n+1/4*p^2*x^(1-2*n)*(f*x)^(-1+2*n)*(d+e*x
^n)^2/e^2/n+2*d*p*x^(1-2*n)*(f*x)^(-1+2*n)*(d+e*x^n)*ln(c*(d+e*x^n)^p)/e^2/
n-1/2*p*x^(1-2*n)*(f*x)^(-1+2*n)*(d+e*x^n)^2*ln(c*(d+e*x^n)^p)/e^2/n-d*x^(1
-2*n)*(f*x)^(-1+2*n)*(d+e*x^n)*ln(c*(d+e*x^n)^p)^2/e^2/n+1/2*x^(1-2*n)*(f*x
)^(-1+2*n)*(d+e*x^n)^2*ln(c*(d+e*x^n)^p)^2/e^2/n
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used

$= \{2506, 2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341\}$

$$\int (fx)^{-1+2n} \log^2 \left(c(d+ex^n)^p\right) dx = \frac{x^{1-2n}(fx)^{2n-1}(d+ex^n)^2 \log^2\left(c(d+ex^n)^p\right)}{2e^2n} - \frac{dx^{1-2n}(fx)^{2n-1}(d+ex^n)\log^2\left(c(d+ex^n)^p\right)}{e^2n} - \frac{px^{1-2n}(fx)^{2n-1}(d+ex^n)^2\log\left(c(d+ex^n)^p\right)}{2e^2n} + \frac{2dpx^{1-2n}(fx)^{2n-1}(d+ex^n)\log\left(c(d+ex^n)^p\right)}{e^2n} + \frac{p^2x^{1-2n}(fx)^{2n-1}(d+ex^n)^2}{4e^2n} - \frac{2dp^2x^{1-n}(fx)^{2n-1}}{en}$$

```
[In] Int[(f*x)^(-1 + 2*n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] (-2*d*p^2*x^(1 - n)*(f*x)^(-1 + 2*n))/(e*n) + (p^2*x^(1 - 2*n)*(f*x)^(-1 +
2*n)*(d + e*x^n)^2)/(4*e^2*n) + (2*d*p*x^(1 - 2*n)*(f*x)^(-1 + 2*n)*(d + e*
x^n)*Log[c*(d + e*x^n)^p])/(e^2*n) - (p*x^(1 - 2*n)*(f*x)^(-1 + 2*n)*(d + e
*x^n)^2*Log[c*(d + e*x^n)^p])/(2*e^2*n) - (d*x^(1 - 2*n)*(f*x)^(-1 + 2*n)*(
d + e*x^n)*Log[c*(d + e*x^n)^p]^2)/(e^2*n) + (x^(1 - 2*n)*(f*x)^(-1 + 2*n)*
(d + e*x^n)^2*Log[c*(d + e*x^n)^p]^2)/(2*e^2*n)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2506

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(a + b*Log[c*(d + e*x^n)^
p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simpl
ify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(x^{1-2n}(fx)^{-1+2n}\right)\int x^{-1+2n}\log^2\left(c(d+ex^n)^p\right)\,dx\\
&= \frac{\left(x^{1-2n}(fx)^{-1+2n}\right)\operatorname{Subst}\left(\int x\log^2\left(c(d+ex)^p\right)\,dx,x,x^n\right)}{n}\\
&= \frac{\left(x^{1-2n}(fx)^{-1+2n}\right)\operatorname{Subst}\left(\int\left(-\frac{d\log^2(c(d+ex)^p)}{e}+\frac{(d+ex)\log^2(c(d+ex)^p)}{e}\right)\,dx,x,x^n\right)}{n}\\
&= \frac{\left(x^{1-2n}(fx)^{-1+2n}\right)\operatorname{Subst}\left(\int(d+ex)\log^2\left(c(d+ex)^p\right)\,dx,x,x^n\right)}{en}\\
&\quad-\frac{\left(dx^{1-2n}(fx)^{-1+2n}\right)\operatorname{Subst}\left(\int\log^2\left(c(d+ex)^p\right)\,dx,x,x^n\right)}{en}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\left(x^{1-2n}(fx)^{-1+2n}\right)\mathrm{Subst}\left(\int x\log^2\left(cx^p\right)\,dx,x,d+ex^n\right)}{e^2n} \\
&\quad - \frac{\left(dx^{1-2n}(fx)^{-1+2n}\right)\mathrm{Subst}\left(\int \log^2\left(cx^p\right)\,dx,x,d+ex^n\right)}{e^2n} \\
&= -\frac{dx^{1-2n}(fx)^{-1+2n}\left(d+ex^n\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{e^2n} \\
&\quad + \frac{x^{1-2n}(fx)^{-1+2n}\left(d+ex^n\right)^2\log^2\left(c\left(d+ex^n\right)^p\right)}{2e^2n} \\
&\quad - \frac{\left(px^{1-2n}(fx)^{-1+2n}\right)\mathrm{Subst}\left(\int x\log\left(cx^p\right)\,dx,x,d+ex^n\right)}{e^2n} \\
&\quad + \frac{\left(2dpx^{1-2n}(fx)^{-1+2n}\right)\mathrm{Subst}\left(\int \log\left(cx^p\right)\,dx,x,d+ex^n\right)}{e^2n} \\
&= -\frac{2dp^2x^{1-n}(fx)^{-1+2n}}{en} + \frac{p^2x^{1-2n}(fx)^{-1+2n}\left(d+ex^n\right)^2}{4e^2n} \\
&\quad + \frac{2dpx^{1-2n}(fx)^{-1+2n}\left(d+ex^n\right)\log\left(c\left(d+ex^n\right)^p\right)}{e^2n} \\
&\quad - \frac{px^{1-2n}(fx)^{-1+2n}\left(d+ex^n\right)^2\log\left(c\left(d+ex^n\right)^p\right)}{2e^2n} \\
&\quad - \frac{dx^{1-2n}(fx)^{-1+2n}\left(d+ex^n\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{e^2n} \\
&\quad + \frac{x^{1-2n}(fx)^{-1+2n}\left(d+ex^n\right)^2\log^2\left(c\left(d+ex^n\right)^p\right)}{2e^2n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.55

$$
\int (fx)^{-1+2n}\log^2\left(c\left(d+ex^n\right)^p\right)\,dx
$$

$$
= \frac{x^{-2n}(fx)^{2n}\left(2d^2p^2\log^2\left(d+ex^n\right)+2d^2p\log\left(d+ex^n\right)\left(3p-2\log\left(c\left(d+ex^n\right)^p\right)\right)+ex^n\left(p^2\left(-6d+ex^n\right)+2\right.\right.}{4e^2fn}
$$

```
[In] Integrate[(f*x)^(-1 + 2*n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] ((f*x)^(2*n)*(2*d^2*p^2*Log[d + e*x^n]^2 + 2*d^2*p*Log[d + e*x^n]*(3*p - 2*
Log[c*(d + e*x^n)^p]) + e*x^n*(p^2*(-6*d + e*x^n) + 2*p*(2*d - e*x^n)*Log[c
*(d + e*x^n)^p] + 2*e*x^n*Log[c*(d + e*x^n)^p]^2)))/(4*e^2*f*n*x^(2*n))
```

Maple [F]

$$\int (fx)^{-1+2n} \ln \left(c(d+e\,x^{n})^{p}\right)^{2} dx$$

[In] int((f*x)^(-1+2*n)*ln(c*(d+e*x^n)^p)^2,x)

[Out] int((f*x)^(-1+2*n)*ln(c*(d+e*x^n)^p)^2,x)

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.80

$$\int (fx)^{-1+2n} \log^2\left(c(d+ex^n)^p\right)\,dx$$
$$= \frac{\left(e^2p^2 - 2\,e^2p\log\left(c\right) + 2\,e^2\log\left(c\right)^2\right)f^{2\,n-1}x^{2\,n} - 2\left(3\,dep^2 - 2\,dep\log\left(c\right)\right)f^{2\,n-1}x^n + 2\left(e^2f^{2\,n-1}p^2x^{2\,n} - d^2f^2\right.}{}$$

[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")

[Out] 1/4*((e^2*p^2 - 2*e^2*p*log(c) + 2*e^2*log(c)^2)*f^(2*n - 1)*x^(2*n) - 2*(3*d*e*p^2 - 2*d*e*p*log(c))*f^(2*n - 1)*x^n + 2*(e^2*f^(2*n - 1)*p^2*x^(2*n) - d^2*f^(2*n - 1)*p^2)*log(e*x^n + d)^2 + 2*(2*d*e*f^(2*n - 1)*p^2*x^n - (e^2*p^2 - 2*e^2*p*log(c))*f^(2*n - 1)*x^(2*n) + (3*d^2*p^2 - 2*d^2*p*log(c))*f^(2*n - 1))*log(e*x^n + d))/(e^2*n)

Sympy [F]

$$\int (fx)^{-1+2n} \log^2\left(c(d+ex^n)^p\right)\,dx = \int (fx)^{2n-1} \log\left(c(d+ex^n)^p\right)^2\,dx$$

[In] integrate((f*x)**(-1+2*n)*ln(c*(d+e*x**n)**p)**2,x)

[Out] Integral((f*x)**(2*n - 1)*log(c*(d + e*x**n)**p)**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.78

$$\int (fx)^{-1+2n}\log^2\left(c(d+ex^n)^p\right)\,dx = -\frac{ep\left(\frac{2\,d^2f^{2\,n}\log\left(\frac{ex^n+d}{e}\right)}{e^3n}+\frac{ef^{2\,n}x^{2\,n}-2\,df^{2\,n}x^n}{e^2n}\right)\log\left((ex^n+d)^pc\right)}{2\,f}+\frac{(fx)^{2\,n}\log\left((ex^n+d)^pc\right)^2}{2\,fn}+\frac{\left(2\,d^2f^{2\,n}\log\left(ex^n+d\right)^2+e^2f^{2\,n}x^{2\,n}-6\,def^{2\,n}x^n-2\,\left(2\,f^{2\,n}\log\left(e\right)-3\,f^{2\,n}\right)d^2\log\left(ex^n+d\right)\right)p^2}{4\,e^2fn}$$

```
[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*e*p*(2*d^2*f^(2*n)*log((e*x^n + d)/e)/(e^3*n) + (e*f^(2*n)*x^(2*n) - 2
*d*f^(2*n)*x^n)/(e^2*n))*log((e*x^n + d)^p*c)/f + 1/2*(f*x)^(2*n)*log((e*x^
n + d)^p*c)^2/(f*n) + 1/4*(2*d^2*f^(2*n)*log(e*x^n + d)^2 + e^2*f^(2*n)*x^(
2*n) - 6*d*e*f^(2*n)*x^n - 2*(2*f^(2*n)*log(e) - 3*f^(2*n))*d^2*log(e*x^n +
 d))*p^2/(e^2*f*n)
```

Giac [F]

$$\int (fx)^{-1+2n}\log^2\left(c(d+ex^n)^p\right)\,dx = \int (fx)^{2\,n-1}\log\left((ex^n+d)^pc\right)^2\,dx$$

```
[In] integrate((f*x)^(-1+2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((f*x)^(2*n - 1)*log((e*x^n + d)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1+2n}\log^2\left(c(d+ex^n)^p\right)\,dx = \int \ln\left(c\,{\left(d+e\,x^n\right)}^p\right)^2\,{\left(f\,x\right)}^{2\,n-1}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)^2*(f*x)^(2*n - 1),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)^2*(f*x)^(2*n - 1), x)
```

3.165 $\int (fx)^{-1+n} \log^2(c(d+ex^n)^p)\, dx$

Optimal result

Integrand size = 22, antiderivative size = 101

$$\int (fx)^{-1+n} \log^2(c(d+ex^n)^p)\, dx = \frac{2p^2x(fx)^{-1+n}}{n} - \frac{2px^{1-n}(fx)^{-1+n}(d+ex^n)\log(c(d+ex^n)^p)}{en} + \frac{x^{1-n}(fx)^{-1+n}(d+ex^n)\log^2(c(d+ex^n)^p)}{en}$$

```
[Out] 2*p^2*x*(f*x)^(-1+n)/n-2*p*x^(1-n)*(f*x)^(-1+n)*(d+e*x^n)*ln(c*(d+e*x^n)^p)
/e/n+x^(1-n)*(f*x)^(-1+n)*(d+e*x^n)*ln(c*(d+e*x^n)^p)^2/e/n
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2506, 2504, 2436, 2333, 2332}

$$\int (fx)^{-1+n} \log^2(c(d+ex^n)^p)\, dx = \frac{x^{1-n}(fx)^{n-1}(d+ex^n)\log^2(c(d+ex^n)^p)}{en} - \frac{2px^{1-n}(fx)^{n-1}(d+ex^n)\log(c(d+ex^n)^p)}{en} + \frac{2p^2x(fx)^{n-1}}{n}$$

```
[In] Int[(f*x)^(-1 + n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] (2*p^2*x*(f*x)^(-1 + n))/n - (2*p*x^(1 - n)*(f*x)^(-1 + n)*(d + e*x^n)*Log[
c*(d + e*x^n)^p])/(e*n) + (x^(1 - n)*(f*x)^(-1 + n)*(d + e*x^n)*Log[c*(d +
e*x^n)^p]^2)/(e*n)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2506

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(a + b*Log[c*(d + e*x^n)^
p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simpl
ify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(x^{1-n}(fx)^{-1+n}\right)\int x^{-1+n}\log^2\left(c(d+ex^n)^p\right)\,dx\\
&= \frac{\left(x^{1-n}(fx)^{-1+n}\right)\text{Subst}\left(\int \log^2\left(c(d+ex)^p\right)\,dx,x,x^n\right)}{n}\\
&= \frac{\left(x^{1-n}(fx)^{-1+n}\right)\text{Subst}\left(\int \log^2\left(cx^p\right)\,dx,x,d+ex^n\right)}{en}\\
&= \frac{x^{1-n}(fx)^{-1+n}\left(d+ex^n\right)\log^2\left(c(d+ex^n)^p\right)}{en}\\
&\quad-\frac{\left(2px^{1-n}(fx)^{-1+n}\right)\text{Subst}\left(\int \log\left(cx^p\right)\,dx,x,d+ex^n\right)}{en}
\end{aligned}
$$

$$= \frac{2p^2x(fx)^{-1+n}}{n} - \frac{2px^{1-n}(fx)^{-1+n}(d+ex^n)\log(c(d+ex^n)^p)}{en} + \frac{x^{1-n}(fx)^{-1+n}(d+ex^n)\log^2(c(d+ex^n)^p)}{en}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.73

$$\int (fx)^{-1+n}\log^2(c(d+ex^n)^p)\,dx = \frac{x^{-n}(fx)^n\left(2ep^2x^n-2p(d+ex^n)\log(c(d+ex^n)^p)+(d+ex^n)\log^2(c(d+ex^n)^p)\right)}{efn}$$

```
[In] Integrate[(f*x)^(-1 + n)*Log[c*(d + e*x^n)^p]^2,x]
[Out] ((f*x)^n*(2*e*p^2*x^n - 2*p*(d + e*x^n)*Log[c*(d + e*x^n)^p] + (d + e*x^n)*
Log[c*(d + e*x^n)^p]^2))/(e*f*n*x^n)
```

Maple [F]

$$\int (fx)^{n-1}\ln\left(c(d+e\,x^n)^p\right)^2dx$$

```
[In] int((f*x)^(n-1)*ln(c*(d+e*x^n)^p)^2,x)
[Out] int((f*x)^(n-1)*ln(c*(d+e*x^n)^p)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.20

$$\int (fx)^{-1+n}\log^2(c(d+ex^n)^p)\,dx = \frac{\left(2\,ep^2-2\,ep\log(c)+e\log(c)^2\right)f^{n-1}x^n+\left(ef^{n-1}p^2x^n+df^{n-1}p^2\right)\log\left(ex^n+d\right)^2-2\left((ep^2-ep\log(c))f^{n-1}\right.}{en}$$

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
[Out] ((2*e*p^2 - 2*e*p*log(c) + e*log(c)^2)*f^(n - 1)*x^n + (e*f^(n - 1)*p^2*x^n
 + d*f^(n - 1)*p^2)*log(e*x^n + d)^2 - 2*((e*p^2 - e*p*log(c))*f^(n - 1)*x^
n + (d*p^2 - d*p*log(c))*f^(n - 1))*log(e*x^n + d))/(e*n)
```

Sympy [F]

$$\int (fx)^{-1+n}\log^2\left(c(d+ex^n)^p\right)\,dx = \int (fx)^{n-1}\log\left(c(d+ex^n)^p\right)^2 dx$$

```
[In] integrate((f*x)**(-1+n)*ln(c*(d+e*x**n)**p)**2,x)
[Out] Integral((f*x)**(n - 1)*log(c*(d + e*x**n)**p)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.45

$$\begin{aligned}&\int (fx)^{-1+n}\log^2\left(c(d+ex^n)^p\right)\,dx\\&= -\frac{2\,ep\left(\frac{f^nx^n}{en}-\frac{df^n\log\left(\frac{ex^n+d}{e}\right)}{e^2n}\right)\log\left((ex^n+d)^pc\right)}{f}+\frac{(fx)^n\log\left((ex^n+d)^pc\right)^2}{fn}\\&\quad-\frac{\left(df^n\log\left(ex^n+d\right)^2-2\,ef^nx^n-2\left(f^n\log\left(e\right)-f^n\right)d\log\left(ex^n+d\right)\right)p^2}{efn}\end{aligned}$$

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
[Out] -2*e*p*(f^n*x^n/(e*n) - d*f^n*log((e*x^n + d)/e)/(e^2*n))*log((e*x^n + d)^p
*c)/f + (f*x)^n*log((e*x^n + d)^p*c)^2/(f*n) - (d*f^n*log(e*x^n + d)^2 - 2*
e*f^n*x^n - 2*(f^n*log(e) - f^n)*d*log(e*x^n + d))*p^2/(e*f*n)
```

Giac [F]

$$\int (fx)^{-1+n}\log^2\left(c(d+ex^n)^p\right)\,dx = \int (fx)^{n-1}\log\left((ex^n+d)^pc\right)^2 dx$$

```
[In] integrate((f*x)^(-1+n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")
[Out] integrate((f*x)^(n - 1)*log((e*x^n + d)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1+n}\log^2\left(c(d+ex^n)^p\right)\,dx=\int \ln\left(c\,(d+e\,x^n)^p\right)^2\,(f\,x)^{n-1}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)^2*(f*x)^(n - 1),x)
[Out] int(log(c*(d + e*x^n)^p)^2*(f*x)^(n - 1), x)
```

3.166 $\int \frac{\log^2(c(d+ex^n)^p)}{fx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 88

$$\int \frac{\log^2\left(c(d+ex^n)^p\right)}{fx}\,dx = \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)}{fn} + \frac{2p\log\left(c(d+ex^n)^p\right)\operatorname{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{fn} - \frac{2p^2\operatorname{PolyLog}\left(3,1+\frac{ex^n}{d}\right)}{fn}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)^2/f/n+2*p*ln(c*(d+e*x^n)^p)*polylog(2,1+e*x^
n/d)/f/n-2*p^2*polylog(3,1+e*x^n/d)/f/n
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {12, 2504, 2443, 2481, 2421, 6724}

$$\int \frac{\log^2\left(c(d+ex^n)^p\right)}{fx}\,dx = \frac{2p\operatorname{PolyLog}\left(2,\frac{ex^n}{d}+1\right)\log\left(c(d+ex^n)^p\right)}{fn} + \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)}{fn} - \frac{2p^2\operatorname{PolyLog}\left(3,\frac{ex^n}{d}+1\right)}{fn}$$

```
[In] Int[Log[c*(d + e*x^n)^p]^2/(f*x),x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p]^2)/(f*n) + (2*p*Log[c*(d + e*x^n)^p
]*PolyLog[2, 1 + (e*x^n)/d])/(f*n) - (2*p^2*PolyLog[3, 1 + (e*x^n)/d])/(f*n
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\log^2(c(d+ex^n)^p)}{x}\,dx}{f}$$

$$=\frac{\text{Subst}\left(\int \frac{\log^2(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{fn}$$

$$=\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)}{fn}-\frac{(2ep)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)\log(c(d+ex)^p)}{d+ex}\,dx,x,x^n\right)}{fn}$$

$$=\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)}{fn}-\frac{(2p)\text{Subst}\left(\int \frac{\log(cx^p)\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx,x,d+ex^n\right)}{fn}$$

$$=\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)}{fn}+\frac{2p\log\left(c(d+ex^n)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn}-\frac{(2p^2)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{x}{d}\right)}{x}\,dx,x,d+ex^n\right)}{fn}$$

$$=\frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)}{fn}+\frac{2p\log\left(c(d+ex^n)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn}-\frac{2p^2\text{Li}_3\left(1+\frac{ex^n}{d}\right)}{fn}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.91

$$\int \frac{\log^2\left(c(d+ex^n)^p\right)}{fx}\,dx$$
$$=\frac{\log(x)\left(-p\log\left(d+ex^n\right)+\log\left(c(d+ex^n)^p\right)\right)^2+2p(-p\log\left(d+ex^n\right)+\log\left(c(d+ex^n)^p\right))\left(\log(x)\left(\log(d\right.\right.}{}$$

```
[In] Integrate[Log[c*(d + e*x^n)^p]^2/(f*x),x]
```

```
[Out] (Log[x]*(-(p*Log[d + e*x^n]) + Log[c*(d + e*x^n)^p])^2 + 2*p*(-(p*Log[d + e
*x^n]) + Log[c*(d + e*x^n)^p])*(Log[x]*(Log[d + e*x^n] - Log[1 + (e*x^n)/d]
) - PolyLog[2, -((e*x^n)/d)]/n) + (p^2*(Log[-((e*x^n)/d)]*Log[d + e*x^n]^2
+ 2*Log[d + e*x^n]*PolyLog[2, 1 + (e*x^n)/d] - 2*PolyLog[3, 1 + (e*x^n)/d])
)/n)/f
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.30 (sec) , antiderivative size = 614, normalized size of antiderivative = 6.98

method	result
risch	$-\frac{2\ln\left(-\frac{e\,x^{n}}{d}\right)\ln(d+e\,x^{n})^{2}p^{2}}{nf}+\frac{\ln(e\,x^{n})\ln(d+e\,x^{n})^{2}p^{2}}{nf}+\frac{\ln\left(1-\frac{d+e\,x^{n}}{d}\right)\ln(d+e\,x^{n})^{2}p^{2}}{nf}-\frac{2\,\mathrm{dilog}\left(-\frac{e\,x^{n}}{d}\right)\ln(d+e\,x^{n})p^{2}}{nf}+$

[In] int(ln(c*(d+e*x^n)^p)^2/f/x,x,method=_RETURNVERBOSE)

[Out] -2/n/f*ln(-e*x^n/d)*ln(d+e*x^n)^2*p^2+1/n/f*ln(e*x^n)*ln(d+e*x^n)^2*p^2+1/n /f*ln(1-(d+e*x^n)/d)*ln(d+e*x^n)^2*p^2-2/n/f*dilog(-e*x^n/d)*ln(d+e*x^n)*p^ 2+2/n/f*ln(-e*x^n/d)*ln((d+e*x^n)^p)*ln(d+e*x^n)*p-2/n/f*ln(e*x^n)*ln((d+e* x^n)^p)*ln(d+e*x^n)*p+2/n/f*polylog(2,(d+e*x^n)/d)*ln(d+e*x^n)*p^2+2/n/f*di log(-e*x^n/d)*ln((d+e*x^n)^p)*p+1/n/f*ln(e*x^n)*ln((d+e*x^n)^p)^2-2/n/f*pol ylog(3,(d+e*x^n)/d)*p^2+1/f*(I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p) ^2-I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-I*Pi*csgn(I*c*(d+e*x^n)^p)^3+I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+2*ln(c))/n*(ln(x^n)*ln ((d+e*x^n)^p)-p*e*(dilog((d+e*x^n)/d)/e+ln(x^n)*ln((d+e*x^n)/d)/e))+1/4/f*(I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-I*Pi*csgn(I*(d+e*x^n)^p)*c sgn(I*c*(d+e*x^n)^p)*csgn(I*c)-I*Pi*csgn(I*c*(d+e*x^n)^p)^3+I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+2*ln(c))^2*ln(x)

Fricas [F]

$$\int \frac{\log^2\left(c(d+ex^n)^p\right)}{fx}\,dx=\int \frac{\log\left((ex^n+d)^pc\right)^2}{fx}\,dx$$

[In] integrate(log(c*(d+e*x^n)^p)^2/f/x,x, algorithm="fricas")

[Out] integral(log((e*x^n + d)^p*c)^2/(f*x), x)

Sympy [F]

$$\int \frac{\log^2\left(c(d+ex^n)^p\right)}{fx}\,dx=\frac{\int \frac{\log\left(c(d+ex^n)^p\right)^2}{x}\,dx}{f}$$

[In] integrate(ln(c*(d+e*x**n)**p)**2/f/x,x)

[Out] Integral(log(c*(d + e*x**n)**p)**2/x, x)/f

Maxima [F]

$$\int \frac{\log^2\left(c(d+ex^n)^p\right)}{fx}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)^2}{fx}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)^2/f/x,x, algorithm="maxima")
```

```
[Out] (log((e*x^n + d)^p)^2*log(x) - integrate(-(e*x^n*log(c)^2 + d*log(c)^2 - 2*
((e*n*p*log(x) - e*log(c))*x^n - d*log(c))*log((e*x^n + d)^p))/(e*x*x^n + d
*x), x))/f
```

Giac [F]

$$\int \frac{\log^2\left(c(d+ex^n)^p\right)}{fx}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)^2}{fx}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)^2/f/x,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)^2/(f*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(d+ex^n)^p\right)}{fx}\,dx = \int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)^2}{f\,x}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)^2/(f*x),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)^2/(f*x), x)
```

3.167 $\int (fx)^{-1-n} \log^2 (c(d+ex^n)^p) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 124

$$\int (fx)^{-1-n} \log^2 (c(d+ex^n)^p) \, dx = \frac{2epx^{1+n}(fx)^{-1-n} \log \left(-\frac{ex^n}{d}\right) \log (c(d+ex^n)^p)}{dn} - \frac{x(fx)^{-1-n} (d+ex^n) \log^2 (c(d+ex^n)^p)}{dn} + \frac{2ep^2x^{1+n}(fx)^{-1-n} \operatorname{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{dn}$$

```
[Out] 2*e*p*x^(1+n)*(f*x)^(-1-n)*ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/d/n-x*(f*x)^(-1-n
)*(d+e*x^n)*ln(c*(d+e*x^n)^p)^2/d/n+2*e*p^2*x^(1+n)*(f*x)^(-1-n)*polylog(2,
1+e*x^n/d)/d/n
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2506, 2504, 2444, 2441, 2352}

$$\int (fx)^{-1-n} \log^2 (c(d+ex^n)^p) \, dx = -\frac{x(fx)^{-n-1} (d+ex^n) \log^2 (c(d+ex^n)^p)}{dn} + \frac{2epx^{n+1}(fx)^{-n-1} \log \left(-\frac{ex^n}{d}\right) \log (c(d+ex^n)^p)}{dn} + \frac{2ep^2x^{n+1}(fx)^{-n-1} \operatorname{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{dn}$$

```
[In] Int[(f*x)^(-1 - n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] (2*e*p*x^(1 + n)*(f*x)^(-1 - n)*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(d*
n) - (x*(f*x)^(-1 - n)*(d + e*x^n)*Log[c*(d + e*x^n)^p]^2)/(d*n) + (2*e*p^2
*x^(1 + n)*(f*x)^(-1 - n)*PolyLog[2, 1 + (e*x^n)/d])/(d*n)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2444

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] :> Simp[(d + e*x)*((a + b*Log[c*(d + e*x)^n])^p/((e*f
- d*g)*(f + g*x))), x] - Dist[b*e*n*(p/(e*f - d*g)), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
 NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2506

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(a + b*Log[c*(d + e*x^n)^
p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simpl
ify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Rubi steps

$$\text{integral} = \left(x^{1+n}(fx)^{-1-n}\right)\int x^{-1-n}\log^2\left(c(d+ex^n)^p\right)\,dx$$

$$= \frac{\left(x^{1+n}(fx)^{-1-n}\right)\text{Subst}\left(\int\frac{\log^2(c(d+ex)^p)}{x^2}\,dx,x,x^n\right)}{n}$$

$$=-\frac{x(fx)^{-1-n}\left(d+ex^n\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{dn}+\frac{\left(2epx^{1+n}(fx)^{-1-n}\right)\mathrm{Subst}\left(\int\frac{\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{dn}$$

$$=\frac{2epx^{1+n}(fx)^{-1-n}\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{dn}-\frac{x(fx)^{-1-n}\left(d+ex^n\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{dn}-\frac{\left(2e^2p^2x^{1+n}(fx)^{-1-n}\right)\mathrm{Subst}\left(\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx,x,x^n\right)}{dn}$$

$$=\frac{2epx^{1+n}(fx)^{-1-n}\log\left(-\frac{ex^n}{d}\right)\log\left(c\left(d+ex^n\right)^p\right)}{dn}-\frac{x(fx)^{-1-n}\left(d+ex^n\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{dn}+\frac{2ep^2x^{1+n}(fx)^{-1-n}\mathrm{Li}_2\left(1+\frac{ex^n}{d}\right)}{dn}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.21

$$\int(fx)^{-1-n}\log^2\left(c\left(d+ex^n\right)^p\right)\,dx=$$

$$-\frac{x^{1+n}(fx)^{-1-n}\left(x^{-n}\log^2\left(c\left(d+ex^n\right)^p\right)-2ep\left(-\frac{p\log\left(-\frac{dx^{-n}}{e}\right)\log\left(-e-dx^{-n}\right)}{d}+\frac{p\log^2\left(-e-dx^{-n}\right)}{2d}-\frac{\log\left(-e-dx^{-n}\right)\log\right.}{d}\right.\right.}{n}$$

```
[In] Integrate[(f*x)^(-1 - n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] -((x^(1 + n)*(f*x)^(-1 - n)*(Log[c*(d + e*x^n)^p]^2/x^n - 2*e*p*(-((p*Log[-
(d/(e*x^n))]*Log[-e - d/x^n])/d) + (p*Log[-e - d/x^n]^2)/(2*d) - (Log[-e -
d/x^n]*Log[c*(d + e*x^n)^p])/d - (p*PolyLog[2, (e + d/x^n)/e])/d)))/n)
```

Maple [F]

$$\int(fx)^{-1-n}\ln\left(c\left(d+e\,x^n\right)^p\right)^2dx$$

```
[In] int((f*x)^(-1-n)*ln(c*(d+e*x^n)^p)^2,x)
```

```
[Out] int((f*x)^(-1-n)*ln(c*(d+e*x^n)^p)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.59

$$\int (fx)^{-1-n}\log^2\left(c(d+ex^n)^p\right)\,dx = -\frac{2\,ef^{-n-1}np^2x^n\log(x)\log\left(\frac{ex^n+d}{d}\right)-2\,ef^{-n-1}npx^n\log(c)\log(x)+2\,ef^{-n-1}p^2x^n\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)+df^{-}$$

```
[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
```

```
[Out] -(2*e*f^(-n - 1)*n*p^2*x^n*log(x)*log((e*x^n + d)/d) - 2*e*f^(-n - 1)*n*p*x
^n*log(c)*log(x) + 2*e*f^(-n - 1)*p^2*x^n*dilog(-(e*x^n + d)/d + 1) + d*f^(
-n - 1)*log(c)^2 + (e*f^(-n - 1)*p^2*x^n + d*f^(-n - 1)*p^2)*log(e*x^n + d)
^2 + 2*(d*f^(-n - 1)*p*log(c) - (e*n*p^2*log(x) - e*p*log(c))*f^(-n - 1)*x^
n)*log(e*x^n + d))/(d*n*x^n)
```

Sympy [F]

$$\int (fx)^{-1-n}\log^2\left(c(d+ex^n)^p\right)\,dx = \int (fx)^{-n-1}\log\left(c(d+ex^n)^p\right)^2\,dx$$

```
[In] integrate((f*x)**(-1-n)*ln(c*(d+e*x**n)**p)**2,x)
```

```
[Out] Integral((f*x)**(-n - 1)*log(c*(d + e*x**n)**p)**2, x)
```

Maxima [F]

$$\int (fx)^{-1-n}\log^2\left(c(d+ex^n)^p\right)\,dx = \int (fx)^{-n-1}\log\left((ex^n+d)^pc\right)^2\,dx$$

```
[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")
```

```
[Out] -(e*n^2*p^2*x^n*log(x)^2 - e*p^2*x^n*log(e*x^n + d)^2 + d*log((e*x^n + d)^p
)^2 + d*log(c)^2 - 2*(e*n*p*x^n*log(x) - e*p*x^n*log(e*x^n + d) - d*log(c))
*log((e*x^n + d)^p))*f^(-n - 1)/(d*n*x^n) + integrate(2*(e*n*p^2*log(x) + e
*p*log(c))/(e*f^(n + 1)*x*x^n + d*f^(n + 1)*x), x)
```

Giac [F]

$$\int (fx)^{-1-n}\log^2\left(c(d+ex^n)^p\right)\,dx=\int (fx)^{-n-1}\log\left((ex^n+d)^pc\right)^2\,dx$$

[In] integrate((f*x)^(-1-n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")

[Out] integrate((f*x)^(-n - 1)*log((e*x^n + d)^p*c)^2, x)

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1-n}\log^2\left(c(d+ex^n)^p\right)\,dx=\int \frac{\ln\left(c\left(d+e\,x^n\right)^p\right)^2}{\left(f\,x\right)^{n+1}}\,dx$$

[In] int(log(c*(d + e*x^n)^p)^2/(f*x)^(n + 1),x)

[Out] int(log(c*(d + e*x^n)^p)^2/(f*x)^(n + 1), x)

3.168 $\int (fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 200

$$\begin{aligned}\int (fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)\,dx &= \frac{e^2p^2x^{1+2n}(fx)^{-1-2n}\log(x)}{d^2}\\&-\frac{epx^{1+n}(fx)^{-1-2n}(d+ex^n)\log\left(c(d+ex^n)^p\right)}{d^2n}\\&-\frac{x(fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)}{2n}\\&-\frac{e^2px^{1+2n}(fx)^{-1-2n}\log\left(c(d+ex^n)^p\right)\log\left(1-\frac{d}{d+ex^n}\right)}{d^2n}\\&+\frac{e^2p^2x^{1+2n}(fx)^{-1-2n}\operatorname{PolyLog}\left(2,\frac{d}{d+ex^n}\right)}{d^2n}\end{aligned}$$

```
[Out] e^2*p^2*x^(1+2*n)*(f*x)^(-1-2*n)*ln(x)/d^2-e*p*x^(1+n)*(f*x)^(-1-2*n)*(d+e*
x^n)*ln(c*(d+e*x^n)^p)/d^2/n-1/2*x*(f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p)^2/n-e^2
*p*x^(1+2*n)*(f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p)*ln(1-d/(d+e*x^n))/d^2/n+e^2*p
^2*x^(1+2*n)*(f*x)^(-1-2*n)*polylog(2,d/(d+e*x^n))/d^2/n
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used

$= \{2506, 2504, 2445, 2458, 2389, 2379, 2438, 2351, 31\}$

$$\int (fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)\,dx = -\frac{e^2px^{2n+1}(fx)^{-2n-1}\log\left(1-\frac{d}{d+ex^n}\right)\log\left(c(d+ex^n)^p\right)}{d^2n} - \frac{epx^{n+1}(fx)^{-2n-1}\left(d+ex^n\right)\log\left(c(d+ex^n)^p\right)}{d^2n} - \frac{x(fx)^{-2n-1}\log^2\left(c(d+ex^n)^p\right)}{2n} + \frac{e^2p^2x^{2n+1}(fx)^{-2n-1}\operatorname{PolyLog}\left(2,\frac{d}{ex^n+d}\right)}{d^2n} + \frac{e^2p^2x^{2n+1}\log(x)(fx)^{-2n-1}}{d^2}$$

```
[In] Int[(f*x)^(-1 - 2*n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] (e^2*p^2*x^(1 + 2*n)*(f*x)^(-1 - 2*n)*Log[x])/d^2 - (e*p*x^(1 + n)*(f*x)^(-
1 - 2*n)*(d + e*x^n)*Log[c*(d + e*x^n)^p])/(d^2*n) - (x*(f*x)^(-1 - 2*n)*Lo
g[c*(d + e*x^n)^p]^2)/(2*n) - (e^2*p*x^(1 + 2*n)*(f*x)^(-1 - 2*n)*Log[c*(d
+ e*x^n)^p]*Log[1 - d/(d + e*x^n)])/(d^2*n) + (e^2*p^2*x^(1 + 2*n)*(f*x)^(-
1 - 2*n)*PolyLog[2, d/(d + e*x^n)])/(d^2*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2506

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_)*(
x_))^(m_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[x^m*(a + b*Log[c*(d + e*x^n)^
p])^q, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && IntegerQ[Simpl
ify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(x^{1+2n}(fx)^{-1-2n}\right)\int x^{-1-2n}\log^2\left(c(d+ex^n)^p\right)\,dx\\
&= \frac{\left(x^{1+2n}(fx)^{-1-2n}\right)\operatorname{Subst}\left(\int \frac{\log^2(c(d+ex)^p)}{x^3}\,dx,x,x^n\right)}{n}\\
&= -\frac{x(fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)}{2n}+\frac{\left(epx^{1+2n}(fx)^{-1-2n}\right)\operatorname{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^2(d+ex)}\,dx,x,x^n\right)}{n}
\end{aligned}$$

$$=-\frac{x(fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)}{2n}+\frac{(px^{1+2n}(fx)^{-1-2n})\operatorname{Subst}\left(\int\frac{\log(cx^p)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+ex^n\right)}{n}$$

$$=-\frac{x(fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)}{2n}+\frac{(px^{1+2n}(fx)^{-1-2n})\operatorname{Subst}\left(\int\frac{\log(cx^p)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+ex^n\right)}{dn}-\frac{(epx^{1+2n}(fx)^{-1-2n})\operatorname{Subst}\left(\int\frac{\log(cx^p)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx,x,d+ex^n\right)}{dn}$$

$$=-\frac{epx^{1+n}(fx)^{-1-2n}\left(d+ex^n\right)\log\left(c(d+ex^n)^p\right)}{d^2n}-\frac{x(fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)}{2n}-\frac{e^2px^{1+2n}(fx)^{-1-2n}\log\left(c(d+ex^n)^p\right)\log\left(1-\frac{d}{d+ex^n}\right)}{d^2n}+\frac{(ep^2x^{1+2n}(fx)^{-1-2n})\operatorname{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x}{e}}\,dx,x,d+ex^n\right)}{d^2n}+\frac{(e^2p^2x^{1+2n}(fx)^{-1-2n})\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{d}{x}\right)}{x}\,dx,x,d+ex^n\right)}{d^2n}$$

$$=\frac{e^2p^2x^{1+2n}(fx)^{-1-2n}\log(x)}{d^2}-\frac{epx^{1+n}(fx)^{-1-2n}\left(d+ex^n\right)\log\left(c(d+ex^n)^p\right)}{d^2n}-\frac{x(fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)}{2n}-\frac{e^2px^{1+2n}(fx)^{-1-2n}\log\left(c(d+ex^n)^p\right)\log\left(1-\frac{d}{d+ex^n}\right)}{d^2n}+\frac{e^2p^2x^{1+2n}(fx)^{-1-2n}\operatorname{Li}_2\left(\frac{d}{d+ex^n}\right)}{d^2n}$$

Mathematica [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.44

$$\int(fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)\,dx$$
$$=\frac{(fx)^{-2n}\left(e^2n^2p^2x^{2n}\log^2(x)+e^2p^2x^{2n}\log^2\left(e+dx^{-n}\right)-2e^2p^2x^{2n}\log\left(e-ex^{-n}\right)-2e^2p^2x^{2n}\log\left(e+dx^{-n}\right)\mathrm{l}\right.}{}$$

```
[In] Integrate[(f*x)^(-1 - 2*n)*Log[c*(d + e*x^n)^p]^2,x]
```

```
[Out] (e^2*n^2*p^2*x^(2*n)*Log[x]^2 + e^2*p^2*x^(2*n)*Log[e + d/x^n]^2 - 2*e^2*p^
2*x^(2*n)*Log[e - e/x^n] - 2*e^2*p^2*x^(2*n)*Log[e + d/x^n]*Log[e - e/x^n]
```

```
- 2*d*e*p*x^n*Log[c*(d + e*x^n)^p] + 2*e^2*p*x^(2*n)*Log[e - e/x^n]*Log[c*(
d + e*x^n)^p] - d^2*Log[c*(d + e*x^n)^p]^2 + 2*e^2*n*p*x^(2*n)*Log[x]*(p +
p*Log[e + d/x^n] - p*Log[e - e/x^n] - Log[c*(d + e*x^n)^p] + p*Log[1 + (e*x
^n)/d]) + 2*e^2*p^2*x^(2*n)*PolyLog[2, -((e*x^n)/d)])/(2*d^2*f*n*(f*x)^(2*n
))
```

Maple [F]

$$\int (fx)^{-1-2n} \ln\left(c(d+e\,x^{n})^{p}\right)^{2} dx$$

```
[In] int((f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p)^2,x)
```

```
[Out] int((f*x)^(-1-2*n)*ln(c*(d+e*x^n)^p)^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.40

$$\int (fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)\,dx = \frac{2\,e^2f^{-2\,n-1}np^2x^{2\,n}\log\left(x\right)\log\left(\frac{ex^n+d}{d}\right)+2\,e^2f^{-2\,n-1}p^2x^{2\,n}\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)-2\,def^{-2\,n-1}px^n\log\left(c\right)-d^2f^{-}\cdots}{\cdots}$$

```
[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*e^2*f^(-2*n - 1)*n*p^2*x^(2*n)*log(x)*log((e*x^n + d)/d) + 2*e^2*f^(
-2*n - 1)*p^2*x^(2*n)*dilog(-(e*x^n + d)/d + 1) - 2*d*e*f^(-2*n - 1)*p*x^n*
log(c) - d^2*f^(-2*n - 1)*log(c)^2 + 2*(e^2*n*p^2 - e^2*n*p*log(c))*f^(-2*n
 - 1)*x^(2*n)*log(x) + (e^2*f^(-2*n - 1)*p^2*x^(2*n) - d^2*f^(-2*n - 1)*p^2
)*log(e*x^n + d)^2 - 2*(d*e*f^(-2*n - 1)*p^2*x^n + d^2*f^(-2*n - 1)*p*log(c
) + (e^2*n*p^2*log(x) + e^2*p^2 - e^2*p*log(c))*f^(-2*n - 1)*x^(2*n))*log(e
*x^n + d))/(d^2*n*x^(2*n))
```

Sympy [F]

$$\int (fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)\,dx = \int (fx)^{-2n-1}\log\left(c(d+ex^n)^p\right)^2\,dx$$

```
[In] integrate((f*x)**(-1-2*n)*ln(c*(d+e*x**n)**p)**2,x)
```

```
[Out] Integral((f*x)**(-2*n - 1)*log(c*(d + e*x**n)**p)**2, x)
```

Maxima [F]

$$\int (fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)\,dx=\int (fx)^{-2n-1}\log\left((ex^n+d)^pc\right)^2\,dx$$

[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="maxima")

[Out] 1/2*(e^2*n^2*p^2*x^(2*n)*log(x)^2 - e^2*p^2*x^(2*n)*log(e*x^n + d)^2 - 2*d*e*p*x^n*log(c) - d^2*log((e*x^n + d)^p)^2 - d^2*log(c)^2 - 2*(e^2*n*p*x^(2*n)*log(x) - e^2*p*x^(2*n)*log(e*x^n + d) + d*e*p*x^n + d^2*log(c))*log((e*x^n + d)^p))*f^(-2*n - 1)/(d^2*n*x^(2*n)) - integrate((e^2*n*p^2*log(x) - e^2*p^2 + e^2*p*log(c))/(d*e*f^(2*n + 1)*x*x^n + d^2*f^(2*n + 1)*x), x)

Giac [F]

$$\int (fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)\,dx=\int (fx)^{-2n-1}\log\left((ex^n+d)^pc\right)^2\,dx$$

[In] integrate((f*x)^(-1-2*n)*log(c*(d+e*x^n)^p)^2,x, algorithm="giac")

[Out] integrate((f*x)^(-2*n - 1)*log((e*x^n + d)^p*c)^2, x)

Mupad [F(-1)]

Timed out.

$$\int (fx)^{-1-2n}\log^2\left(c(d+ex^n)^p\right)\,dx=\int \frac{\ln\left(c\,(d+e\,x^n)^p\right)^2}{(f\,x)^{2\,n+1}}\,dx$$

[In] int(log(c*(d + e*x^n)^p)^2/(f*x)^(2*n + 1),x)

[Out] int(log(c*(d + e*x^n)^p)^2/(f*x)^(2*n + 1), x)

3.169 $\int \frac{\log(1+ex^n)}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 13

$$\int \frac{\log(1+ex^n)}{x}\,dx = -\frac{\text{PolyLog}(2,-ex^n)}{n}$$

```
[Out] -polylog(2,-e*x^n)/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2438}

$$\int \frac{\log(1+ex^n)}{x}\,dx = -\frac{\text{PolyLog}(2,-ex^n)}{n}$$

```
[In] Int[Log[1 + e*x^n]/x,x]
```

```
[Out] -(PolyLog[2, -(e*x^n)]/n)
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\text{integral} = -\frac{\text{Li}_2(-ex^n)}{n}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\log(1+ex^n)}{x}\,dx = -\frac{\text{PolyLog}(2,-ex^n)}{n}$$

```
[In] Integrate[Log[1 + e*x^n]/x,x]
[Out] -(PolyLog[2, -(e*x^n)]/n)
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{\operatorname{dilog}(1+e\,x^{n})}{n}$	14
default	$-\frac{\operatorname{dilog}(1+e\,x^{n})}{n}$	14
meijerg	$-\frac{\operatorname{Li}_2(-e\,x^{n})}{n}$	14
risch	$-\frac{\operatorname{dilog}(1+e\,x^{n})}{n}$	14

```
[In] int(ln(1+e*x^n)/x,x,method=_RETURNVERBOSE)
[Out] -1/n*dilog(1+e*x^n)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{\log(1+ex^n)}{x}\,dx = -\frac{\operatorname{Li}_2(-ex^n)}{n}$$

```
[In] integrate(log(1+e*x^n)/x,x, algorithm="fricas")
[Out] -dilog(-e*x^n)/n
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\log(1+ex^n)}{x}\,dx = -\frac{\mathrm{Li}_2\left(ex^n e^{i\pi}\right)}{n}$$

```
[In] integrate(ln(1+e*x**n)/x,x)
[Out] -polylog(2, e*x**n*exp_polar(I*pi))/n
```

Maxima [F]

$$\int \frac{\log(1+ex^n)}{x}\,dx = \int \frac{\log(ex^n+1)}{x}\,dx$$

```
[In] integrate(log(1+e*x^n)/x,x, algorithm="maxima")
[Out] -1/2*n*log(x)^2 + n*integrate(log(x)/(e*x*x^n + x), x) + log(e*x^n + 1)*log
(x)
```

Giac [F]

$$\int \frac{\log(1+ex^n)}{x}\,dx = \int \frac{\log(ex^n+1)}{x}\,dx$$

```
[In] integrate(log(1+e*x^n)/x,x, algorithm="giac")
[Out] integrate(log(e*x^n + 1)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(1+ex^n)}{x}\,dx = \int \frac{\ln(e\,x^n+1)}{x}\,dx$$

```
[In] int(log(e*x^n + 1)/x,x)
[Out] int(log(e*x^n + 1)/x, x)
```

3.170 $\int \frac{\log(2+ex^n)}{x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 21

$$\int \frac{\log(2+ex^n)}{x} \, dx = \log(2)\log(x) - \frac{\text{PolyLog}\left(2, -\frac{ex^n}{2}\right)}{n}$$

```
[Out] ln(2)*ln(x)-polylog(2,-1/2*e*x^n)/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2504, 2439, 2438}

$$\int \frac{\log(2+ex^n)}{x} \, dx = \log(2)\log(x) - \frac{\text{PolyLog}\left(2, -\frac{ex^n}{2}\right)}{n}$$

```
[In] Int[Log[2 + e*x^n]/x,x]
```

```
[Out] Log[2]*Log[x] - PolyLog[2, -1/2*(e*x^n)]/n
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + e*(x/d)]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\log(2+ex)}{x}\,dx, x, x^n\right)}{n}$$

$$= \log(2)\log(x) + \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{2}\right)}{x}\,dx, x, x^n\right)}{n}$$

$$= \log(2)\log(x) - \frac{\text{Li}_2\left(-\frac{ex^n}{2}\right)}{n}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\log(2+ex^n)}{x}\,dx = \log(2)\log(x) - \frac{\text{PolyLog}\left(2, -\frac{ex^n}{2}\right)}{n}$$

```
[In] Integrate[Log[2 + e*x^n]/x,x]
[Out] Log[2]*Log[x] - PolyLog[2, -1/2*(e*x^n)]/n
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(19) = 38.

Time = 0.84 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

method	result	size
risch	$\ln(x)\ln(2+e\,x^n) - \frac{\operatorname{dilog}\left(\frac{e\,x^n}{2}+1\right)}{n} - \ln(x)\ln\left(\frac{e\,x^n}{2}+1\right)$	40
derivativedivides	$\frac{\left(\ln(2+e\,x^n)-\ln\left(\frac{e\,x^n}{2}+1\right)\right)\ln\left(-\frac{e\,x^n}{2}\right)-\operatorname{dilog}\left(\frac{e\,x^n}{2}+1\right)}{n}$	45
default	$\frac{\left(\ln(2+e\,x^n)-\ln\left(\frac{e\,x^n}{2}+1\right)\right)\ln\left(-\frac{e\,x^n}{2}\right)-\operatorname{dilog}\left(\frac{e\,x^n}{2}+1\right)}{n}$	45

```
[In] int(ln(2+e*x^n)/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*ln(2+e*x^n)-1/n*dilog(1/2*e*x^n+1)-ln(x)*ln(1/2*e*x^n+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(18) = 36$.

Time = 0.33 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

$$\int \frac{\log(2+ex^n)}{x}\,dx = \frac{n\log(ex^n+2)\log(x) - n\log\left(\frac{1}{2}ex^n+1\right)\log(x) - \mathrm{Li}_2\left(-\frac{1}{2}ex^n\right)}{n}$$

```
[In] integrate(log(2+e*x^n)/x,x, algorithm="fricas")
```

```
[Out] (n*log(e*x^n + 2)*log(x) - n*log(1/2*e*x^n + 1)*log(x) - dilog(-1/2*e*x^n))
/n
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.65 (sec) , antiderivative size = 100, normalized size of antiderivative = 4.76

$$\int \frac{\log(2+ex^n)}{x}\,dx$$
$$= \begin{cases} -\frac{\mathrm{Li}_2\left(\frac{ex^ne^{i\pi}}{2}\right)}{n} & \text{for } \frac{1}{|x|} < 1 \wedge |x| < 1 \\ \log(2)\log(x) - \frac{\mathrm{Li}_2\left(\frac{ex^ne^{i\pi}}{2}\right)}{n} & \text{for } |x| < 1 \\ -\log(2)\log\left(\frac{1}{x}\right) - \frac{\mathrm{Li}_2\left(\frac{ex^ne^{i\pi}}{2}\right)}{n} & \text{for } \frac{1}{|x|} < 1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x\right)\log(2) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x\right)\log(2) - \frac{\mathrm{Li}_2\left(\frac{ex^ne^{i\pi}}{2}\right)}{n} & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(2+e*x**n)/x,x)
```

```
[Out] Piecewise((-polylog(2, e*x**n*exp_polar(I*pi)/2)/n, (Abs(x) < 1) & (1/Abs(x
) < 1)), (log(2)*log(x) - polylog(2, e*x**n*exp_polar(I*pi)/2)/n, Abs(x) <
1), (-log(2)*log(1/x) - polylog(2, e*x**n*exp_polar(I*pi)/2)/n, 1/Abs(x) <
1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(2) + meijerg(((1, 1), ()),
 ((), (0, 0)), x)*log(2) - polylog(2, e*x**n*exp_polar(I*pi)/2)/n, True))
```

Maxima [F]

$$\int \frac{\log(2+ex^n)}{x}\,dx = \int \frac{\log(ex^n+2)}{x}\,dx$$

```
[In] integrate(log(2+e*x^n)/x,x, algorithm="maxima")
[Out] -1/2*n*log(x)^2 + 2*n*integrate(log(x)/(e*x*x^n + 2*x), x) + log(e*x^n + 2)
*log(x)
```

Giac [F]

$$\int \frac{\log(2+ex^n)}{x}\,dx = \int \frac{\log(ex^n+2)}{x}\,dx$$

```
[In] integrate(log(2+e*x^n)/x,x, algorithm="giac")
[Out] integrate(log(e*x^n + 2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(2+ex^n)}{x}\,dx = \int \frac{\ln(e\,x^n+2)}{x}\,dx$$

```
[In] int(log(e*x^n + 2)/x,x)
[Out] int(log(e*x^n + 2)/x, x)
```

3.171 $\int \frac{\log(2(3+ex^n))}{x} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 21

$$\int \frac{\log (2(3+ex^n))}{x} \, dx = \log(6)\log(x) - \frac{\operatorname{PolyLog}\left(2, -\frac{ex^n}{3}\right)}{n}$$

```
[Out] ln(6)*ln(x)-polylog(2,-1/3*e*x^n)/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2504, 2439, 2438}

$$\int \frac{\log (2(3+ex^n))}{x} \, dx = \log(6)\log(x) - \frac{\operatorname{PolyLog}\left(2, -\frac{ex^n}{3}\right)}{n}$$

```
[In] Int[Log[2*(3 + e*x^n)]/x,x]
```

```
[Out] Log[6]*Log[x] - PolyLog[2, -1/3*(e*x^n)]/n
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] :> Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + e*(x/d)]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\log(2(3+ex))}{x}\,dx, x, x^n\right)}{n} \\
&= \log(6)\log(x) + \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{3}\right)}{x}\,dx, x, x^n\right)}{n} \\
&= \log(6)\log(x) - \frac{\text{Li}_2\left(-\frac{ex^n}{3}\right)}{n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(2(3+ex^n)\right)}{x}\,dx = \log(6)\log(x) - \frac{\text{PolyLog}\left(2, -\frac{ex^n}{3}\right)}{n}$$

```
[In] Integrate[Log[2*(3 + e*x^n)]/x,x]
[Out] Log[6]*Log[x] - PolyLog[2, -1/3*(e*x^n)]/n
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(19) = 38.

Time = 0.88 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.95

method	result	size
risch	$\ln(x)\ln(6+2e\,x^n) - \frac{\text{dilog}\left(\frac{e\,x^n}{3}+1\right)}{n} - \ln(x)\ln\left(\frac{e\,x^n}{3}+1\right)$	41
derivativedivides	$\frac{\left(\ln(6+2e\,x^n)-\ln\left(\frac{e\,x^n}{3}+1\right)\right)\ln\left(-\frac{e\,x^n}{3}\right)-\text{dilog}\left(\frac{e\,x^n}{3}+1\right)}{n}$	46
default	$\frac{\left(\ln(6+2e\,x^n)-\ln\left(\frac{e\,x^n}{3}+1\right)\right)\ln\left(-\frac{e\,x^n}{3}\right)-\text{dilog}\left(\frac{e\,x^n}{3}+1\right)}{n}$	46

```
[In] int(ln(6+2*e*x^n)/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*ln(6+2*e*x^n)-1/n*dilog(1/3*e*x^n+1)-ln(x)*ln(1/3*e*x^n+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(18) = 36$.

Time = 0.32 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.95

$$\int \frac{\log(2(3+ex^n))}{x}\,dx = \frac{n\log(2\,ex^n+6)\log(x) - n\log\left(\frac{1}{3}\,ex^n+1\right)\log(x) - \mathrm{Li}_2\left(-\frac{1}{3}\,ex^n\right)}{n}$$

```
[In] integrate(log(6+2*e*x^n)/x,x, algorithm="fricas")
```

```
[Out] (n*log(2*e*x^n + 6)*log(x) - n*log(1/3*e*x^n + 1)*log(x) - dilog(-1/3*e*x^n
))/n
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.63 (sec) , antiderivative size = 100, normalized size of antiderivative = 4.76

$$\int \frac{\log(2(3+ex^n))}{x}\,dx = \begin{cases} -\frac{\mathrm{Li}_2\left(\frac{ex^n e^{i\pi}}{3}\right)}{n} & \text{for } \frac{1}{|x|} < 1 \wedge |x| < 1 \\ \log(6)\log(x) - \frac{\mathrm{Li}_2\left(\frac{ex^n e^{i\pi}}{3}\right)}{n} & \text{for } |x| < 1 \\ -\log(6)\log\left(\frac{1}{x}\right) - \frac{\mathrm{Li}_2\left(\frac{ex^n e^{i\pi}}{3}\right)}{n} & \text{for } \frac{1}{|x|} < 1 \\ -G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle|\, x\right)\log(6) + G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle|\, x\right)\log(6) - \frac{\mathrm{Li}_2\left(\frac{ex^n e^{i\pi}}{3}\right)}{n} & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(6+2*e*x**n)/x,x)
```

```
[Out] Piecewise((-polylog(2, e*x**n*exp_polar(I*pi)/3)/n, (Abs(x) < 1) & (1/Abs(x
) < 1)), (log(6)*log(x) - polylog(2, e*x**n*exp_polar(I*pi)/3)/n, Abs(x) <
1), (-log(6)*log(1/x) - polylog(2, e*x**n*exp_polar(I*pi)/3)/n, 1/Abs(x) <
1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)*log(6) + meijerg(((1, 1), ()),
 ((), (0, 0)), x)*log(6) - polylog(2, e*x**n*exp_polar(I*pi)/3)/n, True))
```

Maxima [F]

$$\int \frac{\log(2(3+ex^n))}{x}\,dx = \int \frac{\log(2\,ex^n+6)}{x}\,dx$$

```
[In] integrate(log(6+2*e*x^n)/x,x, algorithm="maxima")
```

```
[Out] -1/2*n*log(x)^2 + 3*n*integrate(log(x)/(e*x*x^n + 3*x), x) + log(2)*log(x)
+ log(e*x^n + 3)*log(x)
```

Giac [F]

$$\int \frac{\log(2(3+ex^n))}{x}\,dx = \int \frac{\log(2\,ex^n+6)}{x}\,dx$$

```
[In] integrate(log(6+2*e*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(log(2*e*x^n + 6)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(2(3+ex^n))}{x}\,dx = \int \frac{\ln(2\,e\,x^n+6)}{x}\,dx$$

```
[In] int(log(2*e*x^n + 6)/x,x)
```

```
[Out] int(log(2*e*x^n + 6)/x, x)
```

3.172 $\int \frac{\log(c(d+ex^n))}{x} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 41

$$\int \frac{\log(c(d+ex^n))}{x} \, dx = \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n))}{n} + \frac{\operatorname{PolyLog}\left(2, 1+\frac{ex^n}{d}\right)}{n}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n))/n+polylog(2,1+e*x^n/d)/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2504, 2441, 2352}

$$\int \frac{\log(c(d+ex^n))}{x} \, dx = \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n))}{n} + \frac{\operatorname{PolyLog}\left(2, \frac{ex^n}{d}+1\right)}{n}$$

```
[In] Int[Log[c*(d + e*x^n)]/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)])/n + PolyLog[2, 1 + (e*x^n)/d]/n
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{x}\,dx,x,x^n\right)}{n}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)\right)}{n}-\frac{e\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx,x,x^n\right)}{n}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)\right)}{n}+\frac{\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{\log\left(c(d+ex^n)\right)}{x}\,dx=\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)\right)+\text{PolyLog}\left(2,\frac{d+ex^n}{d}\right)}{n}$$

```
[In] Integrate[Log[c*(d + e*x^n)]/x,x]
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)] + PolyLog[2, (d + e*x^n)/d])/n
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\operatorname{dilog}\left(-\frac{e\,x^{n}}{d}\right)+\ln\left(ce\,x^{n}+cd\right)\ln\left(-\frac{e\,x^{n}}{d}\right)}{n}$
default	$\frac{\operatorname{dilog}\left(-\frac{e\,x^{n}}{d}\right)+\ln\left(ce\,x^{n}+cd\right)\ln\left(-\frac{e\,x^{n}}{d}\right)}{n}$
risch	$\ln\left(x\right)\ln\left(d+e\,x^{n}\right)+\left(\frac{i\pi\ \operatorname{csgn}\left(i(d+e\,x^{n})\right)\operatorname{csgn}\left(ic(d+e\,x^{n})\right)^{2}}{2}-\frac{i\pi\ \operatorname{csgn}\left(i(d+e\,x^{n})\right)\operatorname{csgn}\left(ic(d+e\,x^{n})\right)\operatorname{csgn}\left(ic\right)}{2}\right.$

```
[In] int(ln(c*(d+e*x^n))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(dilog(-e*x^n/d)+ln(c*e*x^n+c*d)*ln(-e*x^n/d))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int \frac{\log\left(c(d+ex^n)\right)}{x}\,dx = \frac{n\log\left(cex^n+cd\right)\log\left(x\right) - n\log\left(x\right)\log\left(\frac{ex^n+d}{d}\right) - \mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)}{n}$$

```
[In] integrate(log(c*(d+e*x^n))/x,x, algorithm="fricas")
```

```
[Out] (n*log(c*e*x^n + c*d)*log(x) - n*log(x)*log((e*x^n + d)/d) - dilog(-(e*x^n
+ d)/d + 1))/n
```

Sympy [F]

$$\int \frac{\log\left(c(d+ex^n)\right)}{x}\,dx = \int \frac{\log\left(cd+cex^n\right)}{x}\,dx$$

```
[In] integrate(ln(c*(d+e*x**n))/x,x)
```

```
[Out] Integral(log(c*d + c*e*x**n)/x, x)
```

Maxima [F]

$$\int \frac{\log\left(c(d+ex^n)\right)}{x}\,dx = \int \frac{\log\left((ex^n+d)c\right)}{x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n))/x,x, algorithm="maxima")
```

```
[Out] d*n*integrate(log(x)/(e*x*x^n + d*x), x) - 1/2*n*log(x)^2 + log(e*x^n + d)*
log(x) + log(c)*log(x)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^n)\right)}{x}\,dx = \int \frac{\log\left((ex^n+d)c\right)}{x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n))/x,x, algorithm="giac")
[Out] integrate(log((e*x^n + d)*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)\right)}{x}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)\right)}{x}\,dx$$

```
[In] int(log(c*(d + e*x^n))/x,x)
[Out] int(log(c*(d + e*x^n))/x, x)
```

3.173 $\int \frac{\log(c(d+ex^n)^p)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{\log(c(d+ex^n)^p)}{x}\,dx = \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{p\,\text{PolyLog}\left(2, 1+\frac{ex^n}{d}\right)}{n}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/n+p*polylog(2,1+e*x^n/d)/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2504, 2441, 2352}

$$\int \frac{\log(c(d+ex^n)^p)}{x}\,dx = \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{p\,\text{PolyLog}\left(2, \frac{ex^n}{d}+1\right)}{n}$$

```
[In] Int[Log[c*(d + e*x^n)^p]/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (p*PolyLog[2, 1 + (e*x^n)/d])/
n
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n} - \frac{(ep)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^n\right)}{n}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n} + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$
\int \frac{\log\left(c(d+ex^n)^p\right)}{x}\,dx = \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right) + p\,\text{PolyLog}\left(2, \frac{d+ex^n}{d}\right)}{n}
$$

```
[In] Integrate[Log[c*(d + e*x^n)^p]/x,x]
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*PolyLog[2, (d + e*x^n)/d])/n
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.86

method	result
risch	$\ln(x)\ln\left((d+e\,x^n)^p\right) + \left(\frac{i\pi\ \text{csgn}(i(d+e\,x^n)^p)\text{csgn}(ic(d+e\,x^n)^p)^2}{2} - \frac{i\pi\ \text{csgn}(i(d+e\,x^n)^p)\,\text{csgn}(ic(d+e\,x^n)^p)\,\text{csgn}(ic)}{2} -\right.$

```
[In] int(ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*ln((d+e*x^n)^p)+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2
-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn
(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))*ln(x)
-p/n*dilog((d+e*x^n)/d)-p*ln(x)*ln((d+e*x^n)/d)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x}\,dx = \frac{np\log \left(ex^n+d\right)\log \left(x\right)-np\log \left(x\right)\log \left(\frac{ex^n+d}{d}\right)+n\log \left(c\right)\log \left(x\right)-p\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)}{n}$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] (n*p*log(e*x^n + d)*log(x) - n*p*log(x)*log((e*x^n + d)/d) + n*log(c)*log(x
) - p*dilog(-(e*x^n + d)/d + 1))/n
```

Sympy [F]

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{\log \left(c(d+ex^n)^p\right)}{x}\,dx$$

```
[In] integrate(ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)/x, x)
```

Maxima [F]

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{\log \left((ex^n+d)^p c\right)}{x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] d*n*p*integrate(log(x)/(e*x*x^n + d*x), x) - 1/2*n*p*log(x)^2 + log((e*x^n
+ d)^p)*log(x) + log(c)*log(x)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
[Out] integrate(log((e*x^n + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)}{x}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)/x,x)
[Out] int(log(c*(d + e*x^n)^p)/x, x)
```

3.174 $\int \frac{\log^2(c(d+ex^n)^p)}{x} dx$

Optimal result

Integrand size = 18, antiderivative size = 79

$$\int \frac{\log^2(c(d+ex^n)^p)}{x} dx = \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)}{n} + \frac{2p\log\left(c(d+ex^n)^p\right)\operatorname{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n} - \frac{2p^2\operatorname{PolyLog}\left(3,1+\frac{ex^n}{d}\right)}{n}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)^2/n+2*p*ln(c*(d+e*x^n)^p)*polylog(2,1+e*x^n/
d)/n-2*p^2*polylog(3,1+e*x^n/d)/n
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2504, 2443, 2481, 2421, 6724}

$$\int \frac{\log^2(c(d+ex^n)^p)}{x} dx = \frac{2p\operatorname{PolyLog}\left(2,\frac{ex^n}{d}+1\right)\log\left(c(d+ex^n)^p\right)}{n} + \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)}{n} - \frac{2p^2\operatorname{PolyLog}\left(3,\frac{ex^n}{d}+1\right)}{n}$$

```
[In] Int[Log[c*(d + e*x^n)^p]^2/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p]^2)/n + (2*p*Log[c*(d + e*x^n)^p]*Po
lyLog[2, 1 + (e*x^n)/d])/n - (2*p^2*PolyLog[3, 1 + (e*x^n)/d])/n
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\log^2\left(c(d+ex)^p\right)}{x}\,dx, x, x^n\right)}{n}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c\left(d+ex^n\right)^p\right)}{n}-\frac{(2ep)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)\log\left(c(d+ex)^p\right)}{d+ex}\,dx, x, x^n\right)}{n}
\end{aligned}
$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)}{n} - \frac{(2p)\text{Subst}\left(\int \frac{\log(cx^p)\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}dx, x, d+ex^n\right)}{n}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)}{n} + \frac{2p\log\left(c(d+ex^n)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n} - \frac{(2p^2)\,\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{x}{d}\right)}{x}dx, x, d+ex^n\right)}{n}$$

$$= \frac{\log\left(-\frac{ex^n}{d}\right)\log^2\left(c(d+ex^n)^p\right)}{n} + \frac{2p\log\left(c(d+ex^n)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n} - \frac{2p^2\text{Li}_3\left(1+\frac{ex^n}{d}\right)}{n}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 164 vs. $2(79) = 158$.

Time = 0.04 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.08

$$\int \frac{\log^2\left(c(d+ex^n)^p\right)}{x}dx = \log(x)\left(-p\log\left(d+ex^n\right)+\log\left(c(d+ex^n)^p\right)\right)^2 + 2p(-p\log\left(d+ex^n\right) + \log\left(c(d+ex^n)^p\right))\left(\log(x)\left(\log\left(d+ex^n\right)-\log\left(1+\frac{ex^n}{d}\right)\right) - \frac{\text{PolyLog}\left(2,-\frac{ex^n}{d}\right)}{n}\right) + \frac{p^2\left(\log\left(-\frac{ex^n}{d}\right)\log^2\left(d+ex^n\right)+2\log\left(d+ex^n\right)\text{PolyLog}\left(2,1+\frac{ex^n}{d}\right)-2\,\text{PolyLog}\left(3,1+\frac{ex^n}{d}\right)\right)}{n}$$

```
[In] Integrate[Log[c*(d + e*x^n)^p]^2/x,x]
```

```
[Out] Log[x]*(-(p*Log[d + e*x^n]) + Log[c*(d + e*x^n)^p])^2 + 2*p*(-(p*Log[d + e*
x^n]) + Log[c*(d + e*x^n)^p])*(Log[x]*(Log[d + e*x^n] - Log[1 + (e*x^n)/d])
 - PolyLog[2, -((e*x^n)/d)]/n) + (p^2*(Log[-((e*x^n)/d)]*Log[d + e*x^n]^2 +
 2*Log[d + e*x^n]*PolyLog[2, 1 + (e*x^n)/d] - 2*PolyLog[3, 1 + (e*x^n)/d]))
/n
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.29 (sec) , antiderivative size = 578, normalized size of antiderivative = 7.32

method	result
risch	$\frac{\ln\left(1-\frac{d+e\,x^n}{d}\right)\ln(d+e\,x^n)^2p^2}{n} - \frac{2\ln(d+e\,x^n)^2\ln\left(-\frac{e\,x^n}{d}\right)p^2}{n} + \frac{\ln(d+e\,x^n)^2\ln(e\,x^n)p^2}{n} + \frac{2\,\text{Li}_2\left(\frac{d+e\,x^n}{d}\right)\ln(d+e\,x^n)p^2}{n} + \frac{2\ln((\cdots}{}$

```
[In] int(ln(c*(d+e*x^n)^p)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*ln(1-(d+e*x^n)/d)*ln(d+e*x^n)^2*p^2-2/n*ln(d+e*x^n)^2*ln(-e*x^n/d)*p^2+
1/n*ln(d+e*x^n)^2*ln(e*x^n)*p^2+2/n*polylog(2,(d+e*x^n)/d)*ln(d+e*x^n)*p^2+
2/n*ln((d+e*x^n)^p)*ln(d+e*x^n)*ln(-e*x^n/d)*p-2/n*ln((d+e*x^n)^p)*ln(d+e*x
^n)*ln(e*x^n)*p-2/n*ln(d+e*x^n)*dilog(-e*x^n/d)*p^2-2/n*polylog(3,(d+e*x^n)
/d)*p^2+1/n*ln((d+e*x^n)^p)^2*ln(e*x^n)+2/n*ln((d+e*x^n)^p)*dilog(-e*x^n/d)
*p+(I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-I*Pi*csgn(I*(d+e*x^n)^
p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-I*Pi*csgn(I*c*(d+e*x^n)^p)^3+I*Pi*csgn(I
*c*(d+e*x^n)^p)^2*csgn(I*c)+2*ln(c))/n*(ln(x^n)*ln((d+e*x^n)^p)-p*e*(dilog(
(d+e*x^n)/d)/e+ln(x^n)*ln((d+e*x^n)/d)/e))+1/4*(I*Pi*csgn(I*(d+e*x^n)^p)*cs
gn(I*c*(d+e*x^n)^p)^2-I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I
*c)-I*Pi*csgn(I*c*(d+e*x^n)^p)^3+I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+2*l
n(c))^2*ln(x)
```

Fricas [F]

$$\int \frac{\log^2(c(d+ex^n)^p)}{x}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)^2}{x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)^2/x,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)^2/x, x)
```

Sympy [F]

$$\int \frac{\log^2(c(d+ex^n)^p)}{x}\,dx = \int \frac{\log\left(c(d+ex^n)^p\right)^2}{x}\,dx$$

```
[In] integrate(ln(c*(d+e*x**n)**p)**2/x,x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)**2/x, x)
```

Maxima [F]

$$\int \frac{\log^2(c(d+ex^n)^p)}{x}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)^2}{x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)^2/x,x, algorithm="maxima")
```

```
[Out] log((e*x^n + d)^p)^2*log(x) - integrate(-(e*x^n*log(c)^2 + d*log(c)^2 - 2*(
(e*n*p*log(x) - e*log(c))*x^n - d*log(c))*log((e*x^n + d)^p))/(e*x*x^n + d*
x), x)
```

Giac [F]

$$\int \frac{\log^2\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)^2}{x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)^2/x,x, algorithm="giac")
[Out] integrate(log((e*x^n + d)^p*c)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)^2}{x}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)^2/x,x)
[Out] int(log(c*(d + e*x^n)^p)^2/x, x)
```

3.175 $\int \frac{\log^3(c(d+ex^n)^p)}{x} dx$

Optimal result

Integrand size = 18, antiderivative size = 113

$$\int \frac{\log^3(c(d+ex^n)^p)}{x} dx = \frac{\log\left(-\frac{ex^n}{d}\right)\log^3(c(d+ex^n)^p)}{n} + \frac{3p\log^2(c(d+ex^n)^p)\operatorname{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n} - \frac{6p^2\log(c(d+ex^n)^p)\operatorname{PolyLog}\left(3,1+\frac{ex^n}{d}\right)}{n} + \frac{6p^3\operatorname{PolyLog}\left(4,1+\frac{ex^n}{d}\right)}{n}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)^3/n+3*p*ln(c*(d+e*x^n)^p)^2*polylog(2,1+e*x^
n/d)/n-6*p^2*ln(c*(d+e*x^n)^p)*polylog(3,1+e*x^n/d)/n+6*p^3*polylog(4,1+e*x
^n/d)/n
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2443, 2481, 2421, 2430, 6724}

$$\int \frac{\log^3(c(d+ex^n)^p)}{x} dx = -\frac{6p^2\operatorname{PolyLog}\left(3,\frac{ex^n}{d}+1\right)\log(c(d+ex^n)^p)}{n} + \frac{3p\operatorname{PolyLog}\left(2,\frac{ex^n}{d}+1\right)\log^2(c(d+ex^n)^p)}{n} + \frac{\log\left(-\frac{ex^n}{d}\right)\log^3(c(d+ex^n)^p)}{n} + \frac{6p^3\operatorname{PolyLog}\left(4,\frac{ex^n}{d}+1\right)}{n}$$

```
[In] Int[Log[c*(d + e*x^n)^p]^3/x,x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p]^3)/n + (3*p*Log[c*(d + e*x^n)^p]^2*
PolyLog[2, 1 + (e*x^n)/d])/n - (6*p^2*Log[c*(d + e*x^n)^p]*PolyLog[3, 1 + (
e*x^n)/d])/n + (6*p^3*PolyLog[4, 1 + (e*x^n)/d])/n
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\log^3(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c(d+ex^n)^p\right)}{n} - \frac{(3ep)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)\log^2(c(d+ex)^p)}{d+ex}\,dx, x, x^n\right)}{n}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c(d+ex^n)^p\right)}{n} - \frac{(3p)\text{Subst}\left(\int \frac{\log^2(cx^p)\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx, x, d+ex^n\right)}{n}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c(d+ex^n)^p\right)}{n} + \frac{3p\log^2\left(c(d+ex^n)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}\\
&\quad - \frac{(6p^2)\,\text{Subst}\left(\int \frac{\log(cx^p)\text{Li}_2\left(\frac{x}{d}\right)}{x}\,dx, x, d+ex^n\right)}{n}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c(d+ex^n)^p\right)}{n} + \frac{3p\log^2\left(c(d+ex^n)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}\\
&\quad - \frac{6p^2\log\left(c(d+ex^n)^p\right)\text{Li}_3\left(1+\frac{ex^n}{d}\right)}{n} + \frac{(6p^3)\,\text{Subst}\left(\int \frac{\text{Li}_3\left(\frac{x}{d}\right)}{x}\,dx, x, d+ex^n\right)}{n}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log^3\left(c(d+ex^n)^p\right)}{n} + \frac{3p\log^2\left(c(d+ex^n)^p\right)\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}\\
&\quad - \frac{6p^2\log\left(c(d+ex^n)^p\right)\text{Li}_3\left(1+\frac{ex^n}{d}\right)}{n} + \frac{6p^3\text{Li}_4\left(1+\frac{ex^n}{d}\right)}{n}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 270 vs. 2(113) = 226.

Time = 0.11 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.39

$$\int \frac{\log^3\left(c(d+ex^n)^p\right)}{x}\,dx = \frac{-np^3\log(x)\log^3(d+ex^n) + p^3\log\left(-\frac{ex^n}{d}\right)\log^3(d+ex^n) + 3np^2\log(x)\log^2(d+ex^n)\log\left(c(d+ex^n)^p\right) -}{}$$

```
[In] Integrate[Log[c*(d + e*x^n)^p]^3/x,x]
```

```
[Out] (-(n*p^3*Log[x]*Log[d + e*x^n]^3) + p^3*Log[-((e*x^n)/d)]*Log[d + e*x^n]^3
+ 3*n*p^2*Log[x]*Log[d + e*x^n]^2*Log[c*(d + e*x^n)^p] - 3*p^2*Log[-((e*x^n
)/d)]*Log[d + e*x^n]^2*Log[c*(d + e*x^n)^p] - 3*n*p*Log[x]*Log[d + e*x^n]*L
og[c*(d + e*x^n)^p]^2 + 3*p*Log[-((e*x^n)/d)]*Log[d + e*x^n]*Log[c*(d + e*x
^n)^p]^2 + n*Log[x]*Log[c*(d + e*x^n)^p]^3 + 3*p*Log[c*(d + e*x^n)^p]^2*Pol
yLog[2, 1 + (e*x^n)/d] - 6*p^2*Log[c*(d + e*x^n)^p]*PolyLog[3, 1 + (e*x^n)/
d] + 6*p^3*PolyLog[4, 1 + (e*x^n)/d])/n
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.99 (sec) , antiderivative size = 1409, normalized size of antiderivative = 12.47

method	result	size
risch	Expression too large to display	1409

```
[In] int(ln(c*(d+e*x^n)^p)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/n*ln(e*x^n)*ln(d+e*x^n)^3*p^3+3/n*ln(-e*x^n/d)*ln(d+e*x^n)^3*p^3-2/n*ln(
1-(d+e*x^n)/d)*ln(d+e*x^n)^3*p^3+3/n*ln(e*x^n)*ln((d+e*x^n)^p)*ln(d+e*x^n)^
2*p^2+3/n*dilog(-e*x^n/d)*ln(d+e*x^n)^2*p^3-6/n*ln(-e*x^n/d)*ln((d+e*x^n)^p
)*ln(d+e*x^n)^2*p^2+3/n*ln(1-(d+e*x^n)/d)*ln((d+e*x^n)^p)*ln(d+e*x^n)^2*p^2
-3/n*polylog(2,(d+e*x^n)/d)*ln(d+e*x^n)^2*p^3-3/n*ln(e*x^n)*ln((d+e*x^n)^p)
^2*ln(d+e*x^n)*p-6/n*dilog(-e*x^n/d)*ln((d+e*x^n)^p)*ln(d+e*x^n)*p^2+3/n*ln
(-e*x^n/d)*ln((d+e*x^n)^p)^2*ln(d+e*x^n)*p+6/n*polylog(2,(d+e*x^n)/d)*ln((d
+e*x^n)^p)*ln(d+e*x^n)*p^2+1/n*ln(e*x^n)*ln((d+e*x^n)^p)^3+3/n*dilog(-e*x^n
/d)*ln((d+e*x^n)^p)^2*p-6/n*polylog(3,(d+e*x^n)/d)*ln((d+e*x^n)^p)*p^2+6/n*
polylog(4,(d+e*x^n)/d)*p^3+1/8*(I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)
^p)^2-I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-I*Pi*csgn(I*
c*(d+e*x^n)^p)^3+I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+2*ln(c))^3*ln(x)+(3
/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-3/2*I*Pi*csgn(I*(d+e*x^
n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-3/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+3/2*
I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+3*ln(c))/n*((ln((d+e*x^n)^p)-p*ln(d+
e*x^n))^2*ln(e*x^n)+p^2*(ln(d+e*x^n)^2*ln(1-(d+e*x^n)/d)+2*ln(d+e*x^n)*poly
log(2,(d+e*x^n)/d)-2*polylog(3,(d+e*x^n)/d))+2*p*(ln((d+e*x^n)^p)-p*ln(d+e*
x^n))*(dilog(-e*x^n/d)+ln(d+e*x^n)*ln(-e*x^n/d)))+(-3/4*Pi^2*csgn(I*(d+e*x^
n)^p)^2*csgn(I*c*(d+e*x^n)^p)^4+3/2*Pi^2*csgn(I*(d+e*x^n)^p)^2*csgn(I*c*(d+
e*x^n)^p)^3*csgn(I*c)-3/4*Pi^2*csgn(I*(d+e*x^n)^p)^2*csgn(I*c*(d+e*x^n)^p)^
2*csgn(I*c)^2+3/2*Pi^2*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^5-3*Pi^2*c
sgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^4*csgn(I*c)+3/2*Pi^2*csgn(I*(d+e*x
^n)^p)*csgn(I*c*(d+e*x^n)^p)^3*csgn(I*c)^2-3/4*Pi^2*csgn(I*c*(d+e*x^n)^p)^6
+3/2*Pi^2*csgn(I*c*(d+e*x^n)^p)^5*csgn(I*c)-3/4*Pi^2*csgn(I*c*(d+e*x^n)^p)^
4*csgn(I*c)^2+3*I*Pi*ln(c)*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-3*I*
Pi*ln(c)*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-3*I*Pi*ln(c)*c
sgn(I*c*(d+e*x^n)^p)^3+3*I*Pi*ln(c)*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+3*ln(
```

```
c)^2)/n*(ln(x^n)*ln((d+e*x^n)^p)-p*e*(dilog((d+e*x^n)/d)/e+ln(x^n)*ln((d+e*
x^n)/d)/e))
```

Fricas [F]

$$\int \frac{\log^3(c(d+ex^n)^p)}{x}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)^3}{x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)^3/x,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)^3/x, x)
```

Sympy [F]

$$\int \frac{\log^3(c(d+ex^n)^p)}{x}\,dx = \int \frac{\log\left(c(d+ex^n)^p\right)^3}{x}\,dx$$

```
[In] integrate(ln(c*(d+e*x**n)**p)**3/x,x)
```

```
[Out] Integral(log(c*(d + e*x**n)**p)**3/x, x)
```

Maxima [F]

$$\int \frac{\log^3(c(d+ex^n)^p)}{x}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)^3}{x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)^3/x,x, algorithm="maxima")
```

```
[Out] log((e*x^n + d)^p)^3*log(x) - integrate(-(e*x^n*log(c)^3 + d*log(c)^3 - 3*(
(e*n*p*log(x) - e*log(c))*x^n - d*log(c))*log((e*x^n + d)^p)^2 + 3*(e*x^n*l
og(c)^2 + d*log(c)^2)*log((e*x^n + d)^p))/(e*x*x^n + d*x), x)
```

Giac [F]

$$\int \frac{\log^3(c(d+ex^n)^p)}{x}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)^3}{x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)^3/x,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^3\left(c(d+ex^n)^p\right)}{x}\,dx=\int \frac{\ln\left(c\,{\left(d+e\,x^n\right)}^p\right)^3}{x}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)^3/x,x)
[Out] int(log(c*(d + e*x^n)^p)^3/x, x)
```

3.176 $\int (d+ex)^3 \log (c(a+bx)^p) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 140

$$\begin{aligned}\int (d+ex)^3 \log (c(a+bx)^p) \, dx = & -\frac{(bd-ae)^3 px}{4b^3} - \frac{(bd-ae)^2 p(d+ex)^2}{8b^2 e} \\ & - \frac{(bd-ae)p(d+ex)^3}{12be} - \frac{p(d+ex)^4}{16e} \\ & - \frac{(bd-ae)^4 p\log(a+bx)}{4b^4 e} + \frac{(d+ex)^4 \log (c(a+bx)^p)}{4e}\end{aligned}$$

```
[Out] -1/4*(-a*e+b*d)^3*p*x/b^3-1/8*(-a*e+b*d)^2*p*(e*x+d)^2/b^2/e-1/12*(-a*e+b*d
)*p*(e*x+d)^3/b/e-1/16*p*(e*x+d)^4/e-1/4*(-a*e+b*d)^4*p*ln(b*x+a)/b^4/e+1/4
*(e*x+d)^4*ln(c*(b*x+a)^p)/e
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2442, 45}

$$\begin{aligned}\int (d+ex)^3 \log (c(a+bx)^p) \, dx = & -\frac{p(bd-ae)^4 \log(a+bx)}{4b^4 e} - \frac{px(bd-ae)^3}{4b^3} \\ & - \frac{p(d+ex)^2(bd-ae)^2}{8b^2 e} + \frac{(d+ex)^4 \log (c(a+bx)^p)}{4e} \\ & - \frac{p(d+ex)^3(bd-ae)}{12be} - \frac{p(d+ex)^4}{16e}\end{aligned}$$

```
[In] Int[(d + e*x)^3*Log[c*(a + b*x)^p],x]
```

```
[Out] -1/4*((b*d - a*e)^3*p*x)/b^3 - ((b*d - a*e)^2*p*(d + e*x)^2)/(8*b^2*e) - ((
b*d - a*e)*p*(d + e*x)^3)/(12*b*e) - (p*(d + e*x)^4)/(16*e) - ((b*d - a*e)^
4*p*Log[a + b*x])/(4*b^4*e) + ((d + e*x)^4*Log[c*(a + b*x)^p])/(4*e)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^4\log\left(c(a+bx)^p\right)}{4e} - \frac{(bp)\int\frac{(d+ex)^4}{a+bx}\,dx}{4e} \\
&= \frac{(d+ex)^4\log\left(c(a+bx)^p\right)}{4e} \\
&\quad - \frac{(bp)\int\left(\frac{e(bd-ae)^3}{b^4}+\frac{(bd-ae)^4}{b^4(a+bx)}+\frac{e(bd-ae)^2(d+ex)}{b^3}+\frac{e(bd-ae)(d+ex)^2}{b^2}+\frac{e(d+ex)^3}{b}\right)dx}{4e} \\
&= -\frac{(bd-ae)^3px}{4b^3} - \frac{(bd-ae)^2p(d+ex)^2}{8b^2e} - \frac{(bd-ae)p(d+ex)^3}{12be} \\
&\quad - \frac{p(d+ex)^4}{16e} - \frac{(bd-ae)^4p\log(a+bx)}{4b^4e} + \frac{(d+ex)^4\log\left(c(a+bx)^p\right)}{4e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.32

$$\int(d+ex)^3\log\left(c(a+bx)^p\right)\,dx =$$
$$-\frac{bpx(-12a^3e^3+6a^2be^2(8d+ex)-4ab^2e(18d^2+6dex+e^2x^2)+b^3(48d^3+36d^2ex+16de^2x^2+3e^3x^3))}{}$$

```
[In] Integrate[(d + e*x)^3*Log[c*(a + b*x)^p],x]
```

```
[Out] -1/48*(b*p*x*(-12*a^3*e^3 + 6*a^2*b*e^2*(8*d + e*x) - 4*a*b^2*e*(18*d^2 + 6
*d*e*x + e^2*x^2) + b^3*(48*d^3 + 36*d^2*e*x + 16*d*e^2*x^2 + 3*e^3*x^3)) +
 12*a^2*e*(6*b^2*d^2 - 4*a*b*d*e + a^2*e^2)*p*Log[a + b*x] - 12*b^3*(4*a*d^
3 + b*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3))*Log[c*(a + b*x)^p])/b^
4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(128) = 256$.

Time = 1.36 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.03

method	result
parts	$\frac{\ln(c(bx+a)^p)e^3x^4}{4} + \ln\left(c(bx+a)^p\right)e^2d\,x^3 + \frac{3\ln(c(bx+a)^p)e\,d^2x^2}{2} + d^3\ln\left(c(bx+a)^p\right)x + \frac{\ln(c(bx+a)^p)d}{4e}$
parallelrisch	$-\frac{-12x^4\ln(c(bx+a)^p)a\,b^4e^3+3x^4a\,b^4e^3p-48x^3\ln(c(bx+a)^p)a\,b^4d\,e^2-4x^3a^2b^3e^3p+16x^3a\,b^4d\,e^2p-72x^2\ln(c(bx+a)^p)a\,b^4d^2}{}$
risch	$\frac{ie^2\pi d\,x^3\,\mathrm{csgn}(ic(bx+a)^p)^2\,\mathrm{csgn}(ic)}{2} + \frac{ie^2\pi d\,x^3\,\mathrm{csgn}(i(bx+a)^p)\,\mathrm{csgn}(ic(bx+a)^p)^2}{2} - \frac{i\pi\,d^3x\,\mathrm{csgn}(i(bx+a)^p)\,\mathrm{csgn}(ic(bx+a)^p}{2}$

```
[In] int((e*x+d)^3*ln(c*(b*x+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(c*(b*x+a)^p)*e^3*x^4+ln(c*(b*x+a)^p)*e^2*d*x^3+3/2*ln(c*(b*x+a)^p)*e
*d^2*x^2+d^3*ln(c*(b*x+a)^p)*x+1/4*ln(c*(b*x+a)^p)/e*d^4-1/4/e*p*b*(-e/b^4*
(-1/4*b^3*e^3*x^4+1/3*((a*e-2*b*d)*e^2*b^2-2*b^3*d*e^2)*x^3+1/2*(2*(a*e-2*b
*d)*d*e*b^2-b*e*(a^2*e^2-2*a*b*d*e+2*b^2*d^2))*x^2+x*(a*e-2*b*d)*(a^2*e^2-2
*a*b*d*e+2*b^2*d^2))+(a^4*e^4-4*a^3*b*d*e^3+6*a^2*b^2*d^2*e^2-4*a*b^3*d^3*e
+b^4*d^4)/b^5*ln(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(128) = 256$.

Time = 0.34 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.92

$$\int (d+ex)^3\log\left(c(a+bx)^p\right)\,dx =$$
$$-\frac{3\,b^4e^3px^4 + 4\,(4\,b^4de^2 - ab^3e^3)px^3 + 6\,(6\,b^4d^2e - 4\,ab^3de^2 + a^2b^2e^3)px^2 + 12\,(4\,b^4d^3 - 6\,ab^3d^2e + 4\,a^2b^2}{}$$

```
[In] integrate((e*x+d)^3*log(c*(b*x+a)^p),x, algorithm="fricas")
```

```
[Out] -1/48*(3*b^4*e^3*p*x^4 + 4*(4*b^4*d*e^2 - a*b^3*e^3)*p*x^3 + 6*(6*b^4*d^2*e
 - 4*a*b^3*d*e^2 + a^2*b^2*e^3)*p*x^2 + 12*(4*b^4*d^3 - 6*a*b^3*d^2*e + 4*a
^2*b^2*d*e^2 - a^3*b*e^3)*p*x - 12*(b^4*e^3*p*x^4 + 4*b^4*d*e^2*p*x^3 + 6*b
^4*d^2*e*p*x^2 + 4*b^4*d^3*p*x + (4*a*b^3*d^3 - 6*a^2*b^2*d^2*e + 4*a^3*b*d
*e^2 - a^4*e^3)*p)*log(b*x + a) - 12*(b^4*e^3*x^4 + 4*b^4*d*e^2*x^3 + 6*b^4
*d^2*e*x^2 + 4*b^4*d^3*x)*log(c))/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(117) = 234.

Time = 0.93 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.39

$$\int (d+ex)^3 \log \left(c(a+bx)^p\right) \, dx$$
$$= \begin{cases} -\frac{a^4e^3\log\left(c(a+bx)^p\right)}{4b^4} + \frac{a^3de^2\log\left(c(a+bx)^p\right)}{b^3} + \frac{a^3e^3px}{4b^3} - \frac{3a^2d^2e\log\left(c(a+bx)^p\right)}{2b^2} - \frac{a^2de^2px}{b^2} - \frac{a^2e^3px^2}{8b^2} + \frac{ad^3\log\left(c(a+bx)^p\right)}{b} + 3ac \\ \left(d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4}\right)\log\left(a^pc\right) \end{cases}$$

```
[In] integrate((e*x+d)**3*ln(c*(b*x+a)**p),x)
```

```
[Out] Piecewise((-a**4*e**3*log(c*(a + b*x)**p)/(4*b**4) + a**3*d*e**2*log(c*(a +
 b*x)**p)/b**3 + a**3*e**3*p*x/(4*b**3) - 3*a**2*d**2*e*log(c*(a + b*x)**p)
/(2*b**2) - a**2*d*e**2*p*x/b**2 - a**2*e**3*p*x**2/(8*b**2) + a*d**3*log(c
*(a + b*x)**p)/b + 3*a*d**2*e*p*x/(2*b) + a*d*e**2*p*x**2/(2*b) + a*e**3*p*
x**3/(12*b) - d**3*p*x + d**3*x*log(c*(a + b*x)**p) - 3*d**2*e*p*x**2/4 + 3
*d**2*e*x**2*log(c*(a + b*x)**p)/2 - d*e**2*p*x**3/3 + d*e**2*x**3*log(c*(a
 + b*x)**p) - e**3*p*x**4/16 + e**3*x**4*log(c*(a + b*x)**p)/4, Ne(b, 0)),
((d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4)*log(a**p*c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.53

$$\int (d+ex)^3 \log \left(c(a+bx)^p\right) \, dx =$$
$$-\frac{1}{48}\,bp\left(\frac{3\,b^3e^3x^4 + 4\,(4\,b^3de^2 - ab^2e^3)x^3 + 6\,(6\,b^3d^2e - 4\,ab^2de^2 + a^2be^3)x^2 + 12\,(4\,b^3d^3 - 6\,ab^2d^2e + 4\,a^2b}{b^4}\right.$$
$$+\frac{1}{4}\left(e^3x^4 + 4\,de^2x^3 + 6\,d^2ex^2 + 4\,d^3x\right)\log\left((bx+a)^pc\right)$$

```
[In] integrate((e*x+d)^3*log(c*(b*x+a)^p),x, algorithm="maxima")
```

```
[Out] -1/48*b*p*((3*b^3*e^3*x^4 + 4*(4*b^3*d*e^2 - a*b^2*e^3)*x^3 + 6*(6*b^3*d^2*
e - 4*a*b^2*d*e^2 + a^2*b*e^3)*x^2 + 12*(4*b^3*d^3 - 6*a*b^2*d^2*e + 4*a^2*
b*d*e^2 - a^3*e^3)*x)/b^4 - 12*(4*a*b^3*d^3 - 6*a^2*b^2*d^2*e + 4*a^3*b*d*e
^2 - a^4*e^3)*log(b*x + a)/b^5) + 1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2
+ 4*d^3*x)*log((b*x + a)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(128) = 256$.

Time = 0.31 (sec) , antiderivative size = 573, normalized size of antiderivative = 4.09

$$\int (d+ex)^3\log \left(c(a+bx)^p\right)\,dx = \frac{(bx+a)d^3p\log(bx+a)}{b} + \frac{3\,(bx+a)^2d^2ep\log(bx+a)}{2\,b^2} - \frac{3\,(bx+a)ad^2ep\log(bx+a)}{b^2} + \frac{(bx+a)^3de^2p\log(bx+a)}{b^3} - \frac{3\,(bx+a)^2ade^2p\log(bx+a)}{b^3} + \frac{3\,(bx+a)a^2de^2p\log(bx+a)}{b^3} + \frac{(bx+a)^4e^3p\log(bx+a)}{4\,b^4} - \frac{(bx+a)^3ae^3p\log(bx+a)}{b^4} + \frac{3\,(bx+a)^2a^2e^3p\log(bx+a)}{2\,b^4} - \frac{(bx+a)a^3e^3p\log(bx+a)}{b^4} - \frac{(bx+a)d^3p}{b} - \frac{3\,(bx+a)^2d^2ep}{4\,b^2} + \frac{3\,(bx+a)ad^2ep}{b^2} - \frac{(bx+a)^3de^2p}{3\,b^3} + \frac{3\,(bx+a)^2ade^2p}{2\,b^3} - \frac{3\,(bx+a)a^2de^2p}{b^3} - \frac{(bx+a)^4e^3p}{16\,b^4} + \frac{(bx+a)^3ae^3p}{3\,b^4} - \frac{3\,(bx+a)^2a^2e^3p}{4\,b^4} + \frac{(bx+a)a^3e^3p}{b^4} + \frac{(bx+a)d^3\log(c)}{b} + \frac{3\,(bx+a)^2d^2e\log(c)}{2\,b^2} - \frac{3\,(bx+a)ad^2e\log(c)}{b^2} + \frac{(bx+a)^3de^2\log(c)}{b^3} - \frac{3\,(bx+a)^2ade^2\log(c)}{b^3} + \frac{3\,(bx+a)a^2de^2\log(c)}{b^3} + \frac{(bx+a)^4e^3\log(c)}{4\,b^4} - \frac{(bx+a)^3ae^3\log(c)}{b^4} + \frac{3\,(bx+a)^2a^2e^3\log(c)}{2\,b^4} - \frac{(bx+a)a^3e^3\log(c)}{b^4}$$

```
[In] integrate((e*x+d)^3*log(c*(b*x+a)^p),x, algorithm="giac")
```

```
[Out] (b*x + a)*d^3*p*log(b*x + a)/b + 3/2*(b*x + a)^2*d^2*e*p*log(b*x + a)/b^2 -
 3*(b*x + a)*a*d^2*e*p*log(b*x + a)/b^2 + (b*x + a)^3*d*e^2*p*log(b*x + a)/
b^3 - 3*(b*x + a)^2*a*d*e^2*p*log(b*x + a)/b^3 + 3*(b*x + a)*a^2*d*e^2*p*lo
g(b*x + a)/b^3 + 1/4*(b*x + a)^4*e^3*p*log(b*x + a)/b^4 - (b*x + a)^3*a*e^3
```

```
*p*log(b*x + a)/b^4 + 3/2*(b*x + a)^2*a^2*e^3*p*log(b*x + a)/b^4 - (b*x + a
)*a^3*e^3*p*log(b*x + a)/b^4 - (b*x + a)*d^3*p/b - 3/4*(b*x + a)^2*d^2*e*p/
b^2 + 3*(b*x + a)*a*d^2*e*p/b^2 - 1/3*(b*x + a)^3*d*e^2*p/b^3 + 3/2*(b*x +
a)^2*a*d*e^2*p/b^3 - 3*(b*x + a)*a^2*d*e^2*p/b^3 - 1/16*(b*x + a)^4*e^3*p/b
^4 + 1/3*(b*x + a)^3*a*e^3*p/b^4 - 3/4*(b*x + a)^2*a^2*e^3*p/b^4 + (b*x + a
)*a^3*e^3*p/b^4 + (b*x + a)*d^3*log(c)/b + 3/2*(b*x + a)^2*d^2*e*log(c)/b^2
 - 3*(b*x + a)*a*d^2*e*log(c)/b^2 + (b*x + a)^3*d*e^2*log(c)/b^3 - 3*(b*x +
 a)^2*a*d*e^2*log(c)/b^3 + 3*(b*x + a)*a^2*d*e^2*log(c)/b^3 + 1/4*(b*x + a)
^4*e^3*log(c)/b^4 - (b*x + a)^3*a*e^3*log(c)/b^4 + 3/2*(b*x + a)^2*a^2*e^3*
log(c)/b^4 - (b*x + a)*a^3*e^3*log(c)/b^4
```

Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.49

$$\int (d+ex)^3 \log \left(c(a+bx)^p\right) \, dx = \ln \left(c\,{\left(a+b\,x\right)}^p\right)\,\left(d^3\,x+\frac{3\,d^2\,e\,x^2}{2}+d\,e^2\,x^3+\frac{e^3\,x^4}{4}\right)+x^2\,\left(\frac{a\,\left(d\,e^2\,p-\frac{a\,e^3\,p}{4\,b}\right)}{2\,b}-\frac{3\,d^2\,e\,p}{4}\right) - x\,\left(d^3\,p+\frac{a\,\left(\frac{a\,\left(d\,e^2\,p-\frac{a\,e^3\,p}{4\,b}\right)}{b}-\frac{3\,d^2\,e\,p}{2}\right)}{b}\right)-x^3\,\left(\frac{d\,e^2\,p}{3}-\frac{a\,e^3\,p}{12\,b}\right) - \frac{e^3\,p\,x^4}{16}-\frac{\ln \left(a+b\,x\right)\,\left(p\,a^4\,e^3-4\,p\,a^3\,b\,d\,e^2+6\,p\,a^2\,b^2\,d^2\,e-4\,p\,a\,b^3\,d^3\right)}{4\,b^4}$$

```
[In] int(log(c*(a + b*x)^p)*(d + e*x)^3,x)
```

```
[Out] log(c*(a + b*x)^p)*(d^3*x + (e^3*x^4)/4 + (3*d^2*e*x^2)/2 + d*e^2*x^3) + x^
2*((a*(d*e^2*p - (a*e^3*p)/(4*b)))/(2*b) - (3*d^2*e*p)/4) - x*(d^3*p + (a*(
(a*(d*e^2*p - (a*e^3*p)/(4*b)))/b - (3*d^2*e*p)/2))/b) - x^3*((d*e^2*p)/3 -
 (a*e^3*p)/(12*b)) - (e^3*p*x^4)/16 - (log(a + b*x)*(a^4*e^3*p - 4*a*b^3*d^
3*p - 4*a^3*b*d*e^2*p + 6*a^2*b^2*d^2*e*p))/(4*b^4)
```

3.177 $\int (d+ex)^2 \log \left(c(a+bx)^p\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 112

$$\int (d+ex)^2 \log \left(c(a+bx)^p\right) \, dx = -\frac{(bd-ae)^2px}{3b^2} - \frac{(bd-ae)p(d+ex)^2}{6be} - \frac{p(d+ex)^3}{9e} - \frac{(bd-ae)^3p\log(a+bx)}{3b^3e} + \frac{(d+ex)^3 \log \left(c(a+bx)^p\right)}{3e}$$

```
[Out] -1/3*(-a*e+b*d)^2*p*x/b^2-1/6*(-a*e+b*d)*p*(e*x+d)^2/b/e-1/9*p*(e*x+d)^3/e-
1/3*(-a*e+b*d)^3*p*ln(b*x+a)/b^3/e+1/3*(e*x+d)^3*ln(c*(b*x+a)^p)/e
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2442, 45}

$$\int (d+ex)^2 \log \left(c(a+bx)^p\right) \, dx = -\frac{p(bd-ae)^3 \log(a+bx)}{3b^3e} - \frac{px(bd-ae)^2}{3b^2} + \frac{(d+ex)^3 \log \left(c(a+bx)^p\right)}{3e} - \frac{p(d+ex)^2(bd-ae)}{6be} - \frac{p(d+ex)^3}{9e}$$

```
[In] Int[(d + e*x)^2*Log[c*(a + b*x)^p],x]
```

```
[Out] -1/3*((b*d - a*e)^2*p*x)/b^2 - ((b*d - a*e)*p*(d + e*x)^2)/(6*b*e) - (p*(d
+ e*x)^3)/(9*e) - ((b*d - a*e)^3*p*Log[a + b*x])/(3*b^3*e) + ((d + e*x)^3*L
og[c*(a + b*x)^p])/(3*e)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^3\log\left(c(a+bx)^p\right)}{3e} - \frac{(bp)\int\frac{(d+ex)^3}{a+bx}\,dx}{3e} \\
&= \frac{(d+ex)^3\log\left(c(a+bx)^p\right)}{3e} - \frac{(bp)\int\left(\frac{e(bd-ae)^2}{b^3} + \frac{(bd-ae)^3}{b^3(a+bx)} + \frac{e(bd-ae)(d+ex)}{b^2} + \frac{e(d+ex)^2}{b}\right)dx}{3e} \\
&= -\frac{(bd-ae)^2px}{3b^2} - \frac{(bd-ae)p(d+ex)^2}{6be} - \frac{p(d+ex)^3}{9e} \\
&\quad - \frac{(bd-ae)^3p\log(a+bx)}{3b^3e} + \frac{(d+ex)^3\log\left(c(a+bx)^p\right)}{3e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08

$$\int (d+ex)^2\log\left(c(a+bx)^p\right)\,dx$$
$$= \frac{6a^2e(-3bd+ae)p\log(a+bx) + b(-px(6a^2e^2 - 3abe(6d+ex) + b^2(18d^2+9dex+2e^2x^2)) + 6b(3ad^2 + bx}{18b^3}$$

```
[In] Integrate[(d + e*x)^2*Log[c*(a + b*x)^p],x]
```

```
[Out] (6*a^2*e*(-3*b*d + a*e)*p*Log[a + b*x] + b*(-(p*x*(6*a^2*e^2 - 3*a*b*e*(6*d
 + e*x) + b^2*(18*d^2 + 9*d*e*x + 2*e^2*x^2))) + 6*b*(3*a*d^2 + b*x*(3*d^2
+ 3*d*e*x + e^2*x^2))*Log[c*(a + b*x)^p]))/(18*b^3)
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.64

method	result
parts	$\frac{\ln(c(bx+a)^p)e^2x^3}{3}+\ln\left(c(bx+a)^p\right)ed\,x^2+d^2\ln\left(c(bx+a)^p\right)x+\frac{\ln(c(bx+a)^p)d^3}{3e}-\frac{pb\left(\frac{e\left(\frac{1}{3}x^3b^2e^2-\frac{1}{2}abe\right.}{}\right.}{}$
parallelrisch	$\frac{6x^3\ln(c(bx+a)^p)b^3e^2-2x^3b^3e^2p+18x^2\ln(c(bx+a)^p)b^3de+3x^2ab^2e^2p-9x^2b^3dep+6\ln(bx+a)a^3e^2p-18\ln(bx+a)a^2bdep+36}{}$
risch	$-\frac{i\pi d^2x\,\mathrm{csgn}(i(bx+a)^p)\,\mathrm{csgn}(ic(bx+a)^p)\,\mathrm{csgn}(ic)}{2}+\frac{(ex+d)^3\ln((bx+a)^p)}{3e}+\frac{ie^2\pi x^3\,\mathrm{csgn}(ic(bx+a)^p)^2\,\mathrm{csgn}(ic)}{6}+\frac{ie^2\pi x}{}$

```
[In] int((e*x+d)^2*ln(c*(b*x+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(c*(b*x+a)^p)*e^2*x^3+ln(c*(b*x+a)^p)*e*d*x^2+d^2*ln(c*(b*x+a)^p)*x+1
/3*ln(c*(b*x+a)^p)/e*d^3-1/3/e*p*b*(e/b^3*(1/3*x^3*b^2*e^2-1/2*a*b*e^2*x^2+
3/2*d*e*b^2*x^2+x*a^2*e^2-3*a*b*d*e*x+3*x*b^2*d^2)+(-a^3*e^3+3*a^2*b*d*e^2-
3*a*b^2*d^2*e+b^3*d^3)/b^4*ln(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.54

$$\int (d+ex)^2\log\left(c(a+bx)^p\right)\,dx =$$
$$-\frac{2\,b^3e^2px^3+3\,(3\,b^3de-ab^2e^2)px^2+6\,(3\,b^3d^2-3\,ab^2de+a^2be^2)px-6\,(b^3e^2px^3+3\,b^3depx^2+3\,b^3d^2px}{18\,b^3}$$

```
[In] integrate((e*x+d)^2*log(c*(b*x+a)^p),x, algorithm="fricas")
```

```
[Out] -1/18*(2*b^3*e^2*p*x^3 + 3*(3*b^3*d*e - a*b^2*e^2)*p*x^2 + 6*(3*b^3*d^2 - 3
*a*b^2*d*e + a^2*b*e^2)*p*x - 6*(b^3*e^2*p*x^3 + 3*b^3*d*e*p*x^2 + 3*b^3*d^
2*p*x + (3*a*b^2*d^2 - 3*a^2*b*d*e + a^3*e^2)*p)*log(b*x + a) - 6*(b^3*e^2*
x^3 + 3*b^3*d*e*x^2 + 3*b^3*d^2*x)*log(c))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(94) = 188$.

Time = 0.57 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.80

$$\int (d+ex)^2\log\left(c(a+bx)^p\right)\,dx$$
$$=\begin{cases}\frac{a^3e^2\log\left(c(a+bx)^p\right)}{3b^3}-\frac{a^2de\log\left(c(a+bx)^p\right)}{b^2}-\frac{a^2e^2px}{3b^2}+\frac{ad^2\log\left(c(a+bx)^p\right)}{b}+\frac{adepx}{b}+\frac{ae^2px^2}{6b}-d^2px+d^2x\log\left(c(a+bx)^p\right. \\ \left(d^2x+dex^2+\frac{e^2x^3}{3}\right)\log\left(a^pc\right)\end{cases}$$

```
[In] integrate((e*x+d)**2*ln(c*(b*x+a)**p),x)
```

```
[Out] Piecewise((a**3*e**2*log(c*(a + b*x)**p)/(3*b**3) - a**2*d*e*log(c*(a + b*x
)**p)/b**2 - a**2*e**2*p*x/(3*b**2) + a*d**2*log(c*(a + b*x)**p)/b + a*d*e*
p*x/b + a*e**2*p*x**2/(6*b) - d**2*p*x + d**2*x*log(c*(a + b*x)**p) - d*e*p
*x**2/2 + d*e*x**2*log(c*(a + b*x)**p) - e**2*p*x**3/9 + e**2*x**3*log(c*(a
 + b*x)**p)/3, Ne(b, 0)), ((d**2*x + d*e*x**2 + e**2*x**3/3)*log(a**p*c), T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.21

$$\int (d+ex)^2 \log \left(c(a+bx)^p\right) \, dx = -\frac{1}{18}\, bp\Bigg(\frac{2\, b^2e^2x^3 + 3\,(3\, b^2de - abe^2)x^2 + 6\,(3\, b^2d^2 - 3\, abde + a^2e^2)x}{b^3} - \frac{6\,(3\, ab^2d^2 - 3\, a^2bde + a^3e^2)\log (bx}{b^4} + \frac{1}{3}\left(e^2x^3 + 3\, dex^2 + 3\, d^2x\right)\log \left((bx+a)^pc\right)$$

```
[In] integrate((e*x+d)^2*log(c*(b*x+a)^p),x, algorithm="maxima")
```

```
[Out] -1/18*b*p*((2*b^2*e^2*x^3 + 3*(3*b^2*d*e - a*b*e^2)*x^2 + 6*(3*b^2*d^2 - 3*
a*b*d*e + a^2*e^2)*x)/b^3 - 6*(3*a*b^2*d^2 - 3*a^2*b*d*e + a^3*e^2)*log(b*x
 + a)/b^4) + 1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)*log((b*x + a)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(102) = 204$.

Time = 0.30 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.82

$$\int (d+ex)^2 \log \left(c(a+bx)^p\right) \, dx = \frac{(bx+a)d^2p\log(bx+a)}{b} + \frac{(bx+a)^2dep\log(bx+a)}{b^2} - \frac{2(bx+a)adep\log(bx+a)}{b^2} + \frac{(bx+a)^3e^2p\log(bx+a)}{3b^3} - \frac{(bx+a)^2ae^2p\log(bx+a)}{b^3} + \frac{(bx+a)a^2e^2p\log(bx+a)}{b^3} - \frac{(bx+a)d^2p}{b} - \frac{(bx+a)^2dep}{2b^2} + \frac{2(bx+a)adep}{b^2} - \frac{(bx+a)^3e^2p}{9b^3} + \frac{(bx+a)^2ae^2p}{2b^3} - \frac{(bx+a)a^2e^2p}{b^3} + \frac{(bx+a)d^2\log(c)}{b} + \frac{(bx+a)^2de\log(c)}{b^2} - \frac{2(bx+a)ade\log(c)}{b^2} + \frac{(bx+a)^3e^2\log(c)}{3b^3} - \frac{(bx+a)^2ae^2\log(c)}{b^3} + \frac{(bx+a)a^2e^2\log(c)}{b^3}$$

```
[In] integrate((e*x+d)^2*log(c*(b*x+a)^p),x, algorithm="giac")
```

```
[Out] (b*x + a)*d^2*p*log(b*x + a)/b + (b*x + a)^2*d*e*p*log(b*x + a)/b^2 - 2*(b*
x + a)*a*d*e*p*log(b*x + a)/b^2 + 1/3*(b*x + a)^3*e^2*p*log(b*x + a)/b^3 -
(b*x + a)^2*a*e^2*p*log(b*x + a)/b^3 + (b*x + a)*a^2*e^2*p*log(b*x + a)/b^3
 - (b*x + a)*d^2*p/b - 1/2*(b*x + a)^2*d*e*p/b^2 + 2*(b*x + a)*a*d*e*p/b^2
- 1/9*(b*x + a)^3*e^2*p/b^3 + 1/2*(b*x + a)^2*a*e^2*p/b^3 - (b*x + a)*a^2*e
^2*p/b^3 + (b*x + a)*d^2*log(c)/b + (b*x + a)^2*d*e*log(c)/b^2 - 2*(b*x + a
)*a*d*e*log(c)/b^2 + 1/3*(b*x + a)^3*e^2*log(c)/b^3 - (b*x + a)^2*a*e^2*log
(c)/b^3 + (b*x + a)*a^2*e^2*log(c)/b^3
```

Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.17

$$\int (d+ex)^2 \log \left(c(a+bx)^p\right) \, dx = \ln\left(c\,(a+b\,x)^p\right)\,\left(d^2\,x+d\,e\,x^2+\frac{e^2\,x^3}{3}\right) - x^2\,\left(\frac{d\,e\,p}{2}-\frac{a\,e^2\,p}{6\,b}\right) - x\,\left(d^2\,p-\frac{a\,\left(d\,e\,p-\frac{a\,e^2\,p}{3\,b}\right)}{b}\right) - \frac{e^2\,p\,x^3}{9} + \frac{\ln\left(a+b\,x\right)\,\left(p\,a^3\,e^2-3\,p\,a^2\,b\,d\,e+3\,p\,a\,b^2\,d^2\right)}{3\,b^3}$$

```
[In] int(log(c*(a + b*x)^p)*(d + e*x)^2,x)
```

```
[Out] log(c*(a + b*x)^p)*(d^2*x + (e^2*x^3)/3 + d*e*x^2) - x^2*((d*e*p)/2 - (a*e^
2*p)/(6*b)) - x*(d^2*p - (a*(d*e*p - (a*e^2*p)/(3*b)))/b) - (e^2*p*x^3)/9 +
 (log(a + b*x)*(a^3*e^2*p + 3*a*b^2*d^2*p - 3*a^2*b*d*e*p))/(3*b^3)
```

3.178 $\int (d+ex)\log\left(c(a+bx)^p\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 84

$$\int (d+ex)\log\left(c(a+bx)^p\right)\,dx = -\frac{(bd-ae)px}{2b}-\frac{p(d+ex)^2}{4e}-\frac{(bd-ae)^2p\log(a+bx)}{2b^2e}+\frac{(d+ex)^2\log\left(c(a+bx)^p\right)}{2e}$$

```
[Out] -1/2*(-a*e+b*d)*p*x/b-1/4*p*(e*x+d)^2/e-1/2*(-a*e+b*d)^2*p*ln(b*x+a)/b^2/e+
1/2*(e*x+d)^2*ln(c*(b*x+a)^p)/e
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2442, 45}

$$\int (d+ex)\log\left(c(a+bx)^p\right)\,dx = -\frac{p(bd-ae)^2\log(a+bx)}{2b^2e}+\frac{(d+ex)^2\log\left(c(a+bx)^p\right)}{2e}-\frac{px(bd-ae)}{2b}-\frac{p(d+ex)^2}{4e}$$

```
[In] Int[(d + e*x)*Log[c*(a + b*x)^p],x]
```

```
[Out] -1/2*((b*d - a*e)*p*x)/b - (p*(d + e*x)^2)/(4*e) - ((b*d - a*e)^2*p*Log[a +
 b*x])/(2*b^2*e) + ((d + e*x)^2*Log[c*(a + b*x)^p])/(2*e)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^2\log\left(c(a+bx)^p\right)}{2e} - \frac{(bp)\int\frac{(d+ex)^2}{a+bx}\,dx}{2e}\\
&= \frac{(d+ex)^2\log\left(c(a+bx)^p\right)}{2e} - \frac{(bp)\int\left(\frac{e(bd-ae)}{b^2}+\frac{(bd-ae)^2}{b^2(a+bx)}+\frac{e(d+ex)}{b}\right)dx}{2e}\\
&= -\frac{(bd-ae)px}{2b} - \frac{p(d+ex)^2}{4e} - \frac{(bd-ae)^2p\log(a+bx)}{2b^2e} + \frac{(d+ex)^2\log\left(c(a+bx)^p\right)}{2e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93

$$\int(d+ex)\log\left(c(a+bx)^p\right)\,dx = -dpx - \frac{1}{2}ep\left(-\frac{ax}{b}+\frac{x^2}{2}+\frac{a^2\log(a+bx)}{b^2}\right) + \frac{1}{2}ex^2\log\left(c(a+bx)^p\right) + \frac{d(a+bx)\log\left(c(a+bx)^p\right)}{b}$$

```
[In] Integrate[(d + e*x)*Log[c*(a + b*x)^p],x]
[Out] -(d*p*x) - (e*p*(-((a*x)/b) + x^2/2 + (a^2*Log[a + b*x])/b^2))/2 + (e*x^2*L
og[c*(a + b*x)^p])/2 + (d*(a + b*x)*Log[c*(a + b*x)^p])/b
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

method	result
parts	$\frac{\ln(c(bx+a)^p)e\,x^2}{2}+d\ln\left(c(bx+a)^p\right)x-\frac{pb\left(-\frac{-\frac{1}{2}be\,x^2+aex-2bdx}{b^2}+\frac{a(ae-2bd)\ln(bx+a)}{b^3}\right)}{2}$
norman	$dx\ln\left(c\,\mathrm{e}^{p\ln(bx+a)}\right)-\frac{ep\,x^2}{4}+\frac{e\,x^2\ln\left(c\,\mathrm{e}^{p\ln(bx+a)}\right)}{2}+\frac{p(ae-2bd)x}{2b}-\frac{p(a^2e-2abd)\ln(bx+a)}{2b^2}$
default	$d\ln\left(c(bx+a)^p\right)x-dpx+\frac{dpa\ln(bx+a)}{b}+\frac{e\,x^2\ln\left(c\,\mathrm{e}^{p\ln(bx+a)}\right)}{2}-\frac{ep\,x^2}{4}-\frac{p\,a^2e\ln(bx+a)}{2b^2}+\frac{aepx}{2b}$
parallelrisch	$-\frac{-2x^2\ln(c(bx+a)^p)b^2e+b^2ep\,x^2+2\ln(bx+a)a^2ep-8\ln(bx+a)abdp-4x\ln(c(bx+a)^p)b^2d-2abepx+4b^2dpx+4\ln(c(bx+a)^p)a}{4b^2}$
risch	$\left(\frac{1}{2}e\,x^2+dx\right)\ln\left((bx+a)^p\right)+\frac{i\pi dx\,\mathrm{csgn}(i(bx+a)^p)\,\mathrm{csgn}(ic(bx+a)^p)^2}{2}-\frac{i\pi e\,x^2\,\mathrm{csgn}(i(bx+a)^p)\,\mathrm{csgn}(ic(bx+a)^p)\,\mathrm{c}}{4}$

```
[In] int((e*x+d)*ln(c*(b*x+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(c*(b*x+a)^p)*e*x^2+d*ln(c*(b*x+a)^p)*x-1/2*p*b*(-1/b^2*(-1/2*b*e*x^2
+a*e*x-2*b*d*x)+a*(a*e-2*b*d)/b^3*ln(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.08

$$\int (d+ex)\log\left(c(a+bx)^p\right)\,dx =$$
$$-\frac{b^2epx^2+2\left(2\,b^2d-abe\right)px-2\left(b^2epx^2+2\,b^2dpx+\left(2\,abd-a^2e\right)p\right)\log\left(bx+a\right)-2\left(b^2ex^2+2\,b^2dx\right)\log}{4\,b^2}$$

```
[In] integrate((e*x+d)*log(c*(b*x+a)^p),x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*e*p*x^2 + 2*(2*b^2*d - a*b*e)*p*x - 2*(b^2*e*p*x^2 + 2*b^2*d*p*x
+ (2*a*b*d - a^2*e)*p)*log(b*x + a) - 2*(b^2*e*x^2 + 2*b^2*d*x)*log(c))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.25

$$\int (d+ex)\log\left(c(a+bx)^p\right)\,dx$$
$$=\begin{cases}-\frac{a^2e\log\left(c(a+bx)^p\right)}{2b^2}+\frac{ad\log\left(c(a+bx)^p\right)}{b}+\frac{aepx}{2b}-dpx+dx\log\left(c(a+bx)^p\right)-\frac{epx^2}{4}+\frac{ex^2\log\left(c(a+bx)^p\right)}{2} & \text{for } b\neq 0\\ \left(dx+\frac{ex^2}{2}\right)\log\left(a^pc\right) & \text{otherwise}\end{cases}$$

```
[In] integrate((e*x+d)*ln(c*(b*x+a)**p),x)
```

```
[Out] Piecewise((-a**2*e*log(c*(a + b*x)**p)/(2*b**2) + a*d*log(c*(a + b*x)**p)/b
 + a*e*p*x/(2*b) - d*p*x + d*x*log(c*(a + b*x)**p) - e*p*x**2/4 + e*x**2*lo
g(c*(a + b*x)**p)/2, Ne(b, 0)), ((d*x + e*x**2/2)*log(a**p*c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88

$$\int (d+ex)\log \left(c(a+bx)^p\right) \, dx = -\frac{1}{4}\, bp\left(\frac{bex^2+2\,(2\,bd-ae)x}{b^2}-\frac{2\,(2\,abd-a^2e)\log\left(bx+a\right)}{b^3}\right) + \frac{1}{2}\,\left(ex^2+2\,dx\right)\log\left((bx+a)^pc\right)$$

```
[In] integrate((e*x+d)*log(c*(b*x+a)^p),x, algorithm="maxima")
```

```
[Out] -1/4*b*p*((b*e*x^2 + 2*(2*b*d - a*e)*x)/b^2 - 2*(2*a*b*d - a^2*e)*log(b*x +
 a)/b^3) + 1/2*(e*x^2 + 2*d*x)*log((b*x + a)^p*c)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.62

$$\int (d+ex)\log \left(c(a+bx)^p\right) \, dx = \frac{(bx+a)dp\log\left(bx+a\right)}{b}+\frac{(bx+a)^2ep\log\left(bx+a\right)}{2\,b^2}-\frac{(bx+a)aep\log\left(bx+a\right)}{b^2}-\frac{(bx+a)dp}{b}-\frac{(bx+a)^2ep}{4\,b^2}+\frac{(bx+a)aep}{b^2}+\frac{(bx+a)d\log\left(c\right)}{b}+\frac{(bx+a)^2e\log\left(c\right)}{2\,b^2}-\frac{(bx+a)ae\log\left(c\right)}{b^2}$$

```
[In] integrate((e*x+d)*log(c*(b*x+a)^p),x, algorithm="giac")
```

```
[Out] (b*x + a)*d*p*log(b*x + a)/b + 1/2*(b*x + a)^2*e*p*log(b*x + a)/b^2 - (b*x
+ a)*a*e*p*log(b*x + a)/b^2 - (b*x + a)*d*p/b - 1/4*(b*x + a)^2*e*p/b^2 + (
b*x + a)*a*e*p/b^2 + (b*x + a)*d*log(c)/b + 1/2*(b*x + a)^2*e*log(c)/b^2 -
(b*x + a)*a*e*log(c)/b^2
```

Mupad [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

$$\int (d+ex)\log\left(c(a+bx)^p\right)\,dx = \ln\left(c\,(a+b\,x)^p\right)\,\left(\frac{e\,x^2}{2}+d\,x\right)-x\,\left(d\,p-\frac{a\,e\,p}{2\,b}\right)-\frac{e\,p\,x^2}{4}-\frac{\ln\left(a+b\,x\right)\,\left(a^2\,e\,p-2\,a\,b\,d\,p\right)}{2\,b^2}$$

```
[In] int(log(c*(a + b*x)^p)*(d + e*x),x)
```

```
[Out] log(c*(a + b*x)^p)*(d*x + (e*x^2)/2) - x*(d*p - (a*e*p)/(2*b)) - (e*p*x^2)/
4 - (log(a + b*x)*(a^2*e*p - 2*a*b*d*p))/(2*b^2)
```

3.179 $\int \log(c(a+bx)^p)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 24

$$\int \log(c(a+bx)^p)\,dx = -px + \frac{(a+bx)\log(c(a+bx)^p)}{b}$$

```
[Out] -p*x+(b*x+a)*ln(c*(b*x+a)^p)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2436, 2332}

$$\int \log(c(a+bx)^p)\,dx = \frac{(a+bx)\log(c(a+bx)^p)}{b} - px$$

```
[In] Int[Log[c*(a + b*x)^p],x]
```

```
[Out] -(p*x) + ((a + b*x)*Log[c*(a + b*x)^p])/b
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \log\left(cx^p\right)\,dx, x, a+bx\right)}{b}$$
$$= -px + \frac{(a+bx)\log\left(c(a+bx)^p\right)}{b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \log\left(c(a+bx)^p\right)\,dx = -px + \frac{(a+bx)\log\left(c(a+bx)^p\right)}{b}$$

```
[In] Integrate[Log[c*(a + b*x)^p],x]
[Out] -(p*x) + ((a + b*x)*Log[c*(a + b*x)^p])/b
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

method	result
norman	$x\ln\left(c\,\mathrm{e}^{p\ln(bx+a)}\right)+\frac{pa\ln(bx+a)}{b}-px$
default	$\ln\left(c(bx+a)^p\right)x-pb\left(\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}\right)$
parts	$\ln\left(c(bx+a)^p\right)x-pb\left(\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}\right)$
parallelrisch	$\frac{x\ln(c(bx+a)^p)abp-ab\,p^2x+\ln(c(bx+a)^p)a^2p}{abp}$
risch	$x\ln\left((bx+a)^p\right)+\frac{i\pi x\ \mathrm{csgn}(i(bx+a)^p)\,\mathrm{csgn}(ic(bx+a)^p)^2}{2}-\frac{i\pi x\ \mathrm{csgn}(i(bx+a)^p)\,\mathrm{csgn}(ic(bx+a)^p)\,\mathrm{csgn}(ic)}{2}-$

```
[In] int(ln(c*(b*x+a)^p),x,method=_RETURNVERBOSE)
[Out] x*ln(c*exp(p*ln(b*x+a)))+p*a/b*ln(b*x+a)-p*x
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \log \left(c(a+bx)^p\right) \, dx = -\frac{bpx - bx\log\left(c\right) - \left(bpx + ap\right)\log\left(bx + a\right)}{b}$$

```
[In] integrate(log(c*(b*x+a)^p),x, algorithm="fricas")
```

```
[Out] -(b*p*x - b*x*log(c) - (b*p*x + a*p)*log(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \log \left(c(a+bx)^p\right) \, dx = \begin{cases} \frac{a\log\left(c(a+bx)^p\right)}{b} - px + x\log\left(c(a+bx)^p\right) & \text{for}\ b \neq 0 \\ x\log\left(a^pc\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(c*(b*x+a)**p),x)
```

```
[Out] Piecewise((a*log(c*(a + b*x)**p)/b - p*x + x*log(c*(a + b*x)**p), Ne(b, 0))
, (x*log(a**p*c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \log \left(c(a+bx)^p\right) \, dx = -bp\left(\frac{x}{b} - \frac{a\log\left(bx + a\right)}{b^2}\right) + x\log\left(\left(bx + a\right)^pc\right)$$

```
[In] integrate(log(c*(b*x+a)^p),x, algorithm="maxima")
```

```
[Out] -b*p*(x/b - a*log(b*x + a)/b^2) + x*log((b*x + a)^p*c)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \log \left(c(a+bx)^p\right) \, dx = \frac{(bx+a)p\log \left(bx+a\right)}{b} - \frac{(bx+a)p}{b} + \frac{(bx+a)\log \left(c\right)}{b}$$

```
[In] integrate(log(c*(b*x+a)^p),x, algorithm="giac")
```

```
[Out] (b*x + a)*p*log(b*x + a)/b - (b*x + a)*p/b + (b*x + a)*log(c)/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \log \left(c(a+bx)^p\right) \, dx = x\,\ln\left(c\,{\left(a+b\,x\right)}^p\right) - p\,x + \frac{a\,p\,\ln\left(a+b\,x\right)}{b}$$

```
[In] int(log(c*(a + b*x)^p),x)
```

```
[Out] x*log(c*(a + b*x)^p) - p*x + (a*p*log(a + b*x))/b
```

3.180 $\int \frac{\log(c(a+bx)^p)}{d+ex} dx$

Optimal result

Integrand size = 18, antiderivative size = 58

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx=\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}+\frac{p\,\mathrm{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e}$$

```
[Out] ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d))/e+p*polylog(2,-e*(b*x+a)/(-a*e+b*d
))/e
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2441, 2440, 2438}

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx=\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e}+\frac{p\,\mathrm{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e}$$

```
[In] Int[Log[c*(a + b*x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e + (p*PolyLog[2, -((e*
(a + b*x))/(b*d - a*e))])/e
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e} - \frac{(bp)\int\frac{\log\left(\frac{b(d+ex)}{bd-ae}\right)}{a+bx}\,dx}{e}\\ &= \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e} - \frac{p\text{Subst}\left(\int\frac{\log\left(1+\frac{ex}{bd-ae}\right)}{x}\,dx, x, a+bx\right)}{e}\\ &= \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e} + \frac{p\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{e}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int\frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx = \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e} + \frac{p\,\text{PolyLog}\left(2,\frac{e(a+bx)}{-bd+ae}\right)}{e}$$

```
[In] Integrate[Log[c*(a + b*x)^p]/(d + e*x),x]
[Out] (Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e + (p*PolyLog[2, (e*(a
 + b*x))/(-(b*d) + a*e)])/e
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.69

method	result
parts	$\frac{\ln(c(bx+a)^p)\ln(ex+d)}{e}-\frac{pb\left(\frac{\operatorname{dilog}\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{b}+\frac{\ln(ex+d)\ln\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{b}\right)}{e}$
risch	$\frac{\ln((bx+a)^p)\ln(ex+d)}{e}-\frac{p\operatorname{dilog}\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{e}-\frac{p\ln(ex+d)\ln\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{e}+\frac{\left(\frac{i\pi\ \operatorname{csgn}(i(bx+a)^p)\operatorname{csgn}(ic(bx+a)^p)^2}{2}-\frac{i\pi\ \mathrm{cs}}{}\right.}{}$

[In] int(ln(c*(b*x+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)

[Out] ln(c*(b*x+a)^p)/e*ln(e*x+d)-1/e*p*b*(dilog(((e*x+d)*b+a*e-b*d)/(a*e-b*d))/b +ln(e*x+d)*ln(((e*x+d)*b+a*e-b*d)/(a*e-b*d))/b)

Fricas [F]

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx=\int \frac{\log\left((bx+a)^pc\right)}{ex+d}\,dx$$

[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")

[Out] integral(log((b*x + a)^p*c)/(e*x + d), x)

Sympy [F]

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx=\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx$$

[In] integrate(ln(c*(b*x+a)**p)/(e*x+d),x)

[Out] Integral(log(c*(a + b*x)**p)/(d + e*x), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(57) = 114.

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.03

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx=\frac{bp\left(\frac{\log(bx+a)\log(ex+d)}{b}-\frac{\log(ex+d)\log\left(-\frac{bex+bd}{bd-ae}+1\right)+\mathrm{Li}_2\left(\frac{bex+bd}{bd-ae}\right)}{b}\right)}{e}-\frac{p\log\left(bx+a\right)\log\left(ex+d\right)}{e}+\frac{\log\left((bx+a)^pc\right)\log\left(ex+d\right)}{e}$$

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*p*(log(b*x + a)*log(e*x + d)/b - (log(e*x + d)*log(-(b*e*x + b*d)/(b*d -
a*e) + 1) + dilog((b*e*x + b*d)/(b*d - a*e)))/b)/e - p*log(b*x + a)*log(e*x
 + d)/e + log((b*x + a)^p*c)*log(e*x + d)/e
```

Giac [F]

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{\log\left((bx+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{\ln\left(c\,(a+b\,x)^p\right)}{d+e\,x}\,dx$$

```
[In] int(log(c*(a + b*x)^p)/(d + e*x),x)
```

```
[Out] int(log(c*(a + b*x)^p)/(d + e*x), x)
```

3.181 $\int \frac{\log(c(a+bx)^p)}{(d+ex)^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 68

$$\int \frac{\log\left(c(a+bx)^p\right)}{(d+ex)^2}\,dx = \frac{bp\log(a+bx)}{e(bd-ae)} - \frac{\log\left(c(a+bx)^p\right)}{e(d+ex)} - \frac{bp\log(d+ex)}{e(bd-ae)}$$

```
[Out] b*p*ln(b*x+a)/e/(-a*e+b*d)-ln(c*(b*x+a)^p)/e/(e*x+d)-b*p*ln(e*x+d)/e/(-a*e+
b*d)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2442, 36, 31}

$$\int \frac{\log\left(c(a+bx)^p\right)}{(d+ex)^2}\,dx = -\frac{\log\left(c(a+bx)^p\right)}{e(d+ex)} + \frac{bp\log(a+bx)}{e(bd-ae)} - \frac{bp\log(d+ex)}{e(bd-ae)}$$

```
[In] Int[Log[c*(a + b*x)^p]/(d + e*x)^2,x]
```

```
[Out] (b*p*Log[a + b*x])/(e*(b*d - a*e)) - Log[c*(a + b*x)^p]/(e*(d + e*x)) - (b*
p*Log[d + e*x])/(e*(b*d - a*e))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log\left(c(a+bx)^p\right)}{e(d+ex)}+\frac{(bp)\int\frac{1}{(a+bx)(d+ex)}\,dx}{e}\\
&= -\frac{\log\left(c(a+bx)^p\right)}{e(d+ex)}-\frac{(bp)\int\frac{1}{d+ex}\,dx}{bd-ae}+\frac{\left(b^2p\right)\int\frac{1}{a+bx}\,dx}{e(bd-ae)}\\
&= \frac{bp\log(a+bx)}{e(bd-ae)}-\frac{\log\left(c(a+bx)^p\right)}{e(d+ex)}-\frac{bp\log(d+ex)}{e(bd-ae)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int\frac{\log\left(c(a+bx)^p\right)}{(d+ex)^2}\,dx=\frac{\frac{bp\log(a+bx)}{bd-ae}-\frac{\log(c(a+bx)^p)}{d+ex}+\frac{bp\log(d+ex)}{-bd+ae}}{e}$$

```
[In] Integrate[Log[c*(a + b*x)^p]/(d + e*x)^2,x]
[Out] ((b*p*Log[a + b*x])/(b*d - a*e) - Log[c*(a + b*x)^p]/(d + e*x) + (b*p*Log[d
 + e*x])/(-(b*d) + a*e))/e
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

method	result
parts	$-\frac{\ln(c(bx+a)^p)}{e(ex+d)}+\frac{pb\left(\frac{\ln(ex+d)}{ae-bd}-\frac{\ln(bx+a)}{ae-bd}\right)}{e}$
parallelrisch	$-\frac{\ln(bx+a)x\,b^2ep-\ln(ex+d)x\,b^2ep+\ln(bx+a)b^2dp-\ln(ex+d)b^2dp+\ln(c(bx+a)^p)abe-\ln(c(bx+a)^p)b^2d}{(ae-bd)(ex+d)be}$
risch	$-\frac{\ln((bx+a)^p)}{e(ex+d)}-\frac{i\pi ae\ \mathrm{csgn}(i(bx+a)^p)\,\mathrm{csgn}(ic(bx+a)^p)^2-i\pi ae\ \mathrm{csgn}(i(bx+a)^p)\,\mathrm{csgn}(ic(bx+a)^p)\,\mathrm{csgn}(ic)-i\pi ae\,\mathrm{csgn}(ic(bx+}{}$

```
[In] int(ln(c*(b*x+a)^p)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(c*(b*x+a)^p)/e/(e*x+d)+p*b/e*(1/(a*e-b*d)*ln(e*x+d)-1/(a*e-b*d)*ln(b*x+
a))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

$$\int \frac{\log\left(c(a+bx)^p\right)}{(d+ex)^2}\,dx = \frac{(bepx+aep)\log\left(bx+a\right)-(bepx+bdp)\log\left(ex+d\right)-(bd-ae)\log\left(c\right)}{bd^2e-ade^2+(bde^2-ae^3)x}$$

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] ((b*e*p*x + a*e*p)*log(b*x + a) - (b*e*p*x + b*d*p)*log(e*x + d) - (b*d - a
*e)*log(c))/(b*d^2*e - a*d*e^2 + (b*d*e^2 - a*e^3)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(53) = 106$.

Time = 1.47 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.47

$$\int \frac{\log\left(c(a+bx)^p\right)}{(d+ex)^2}\,dx = \begin{cases} \frac{\frac{a\log\left(c(a+bx)^p\right)}{b}-px+x\log\left(c(a+bx)^p\right)}{d^2} & \text{for } e=0 \\ -\frac{p}{de+e^2x}-\frac{\log\left(c\left(\frac{bd}{e}+bx\right)^p\right)}{de+e^2x} & \text{for } a=\frac{bd}{e} \\ -\frac{ae\log\left(c(a+bx)^p\right)}{ade^2+ae^3x-bd^2e-bde^2x}+\frac{bdp\log\left(\frac{d}{e}+x\right)}{ade^2+ae^3x-bd^2e-bde^2x}+\frac{bepx\log\left(\frac{d}{e}+x\right)}{ade^2+ae^3x-bd^2e-bde^2x}-\frac{bex\log\left(c(a+bx)^p\right)}{ade^2+ae^3x-bd^2e-bde^2x} & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(c*(b*x+a)**p)/(e*x+d)**2,x)
```

```
[Out] Piecewise(((a*log(c*(a + b*x)**p)/b - p*x + x*log(c*(a + b*x)**p))/d**2, Eq
(e, 0)), (-p/(d*e + e**2*x) - log(c*(b*d/e + b*x)**p)/(d*e + e**2*x), Eq(a,
 b*d/e)), (-a*e*log(c*(a + b*x)**p)/(a*d*e**2 + a*e**3*x - b*d**2*e - b*d*e
**2*x) + b*d*p*log(d/e + x)/(a*d*e**2 + a*e**3*x - b*d**2*e - b*d*e**2*x) +
 b*e*p*x*log(d/e + x)/(a*d*e**2 + a*e**3*x - b*d**2*e - b*d*e**2*x) - b*e*x
*log(c*(a + b*x)**p)/(a*d*e**2 + a*e**3*x - b*d**2*e - b*d*e**2*x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \frac{\log\left(c(a+bx)^p\right)}{(d+ex)^2}\,dx = \frac{bp\left(\frac{\log(bx+a)}{bd-ae} - \frac{\log(ex+d)}{bd-ae}\right)}{e} - \frac{\log\left((bx+a)^p c\right)}{(ex+d)e}$$

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] b*p*(log(b*x + a)/(b*d - a*e) - log(e*x + d)/(b*d - a*e))/e - log((b*x + a)
^p*c)/((e*x + d)*e)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

$$\int \frac{\log\left(c(a+bx)^p\right)}{(d+ex)^2}\,dx = \frac{bp\log(bx+a)}{bde-ae^2} - \frac{bp\log(ex+d)}{bde-ae^2} - \frac{p\log(bx+a)}{e^2x+de} - \frac{\log(c)}{e^2x+de}$$

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] b*p*log(b*x + a)/(b*d*e - a*e^2) - b*p*log(e*x + d)/(b*d*e - a*e^2) - p*log
(b*x + a)/(e^2*x + d*e) - log(c)/(e^2*x + d*e)
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

$$\int \frac{\log\left(c(a+bx)^p\right)}{(d+ex)^2}\,dx = -\frac{\ln\left(c\,(a+b\,x)^p\right)}{e\,(d+e\,x)} + \frac{b\,p\,\mathrm{atan}\left(\frac{a\,e\,1\mathrm{i}+b\,d\,1\mathrm{i}+b\,e\,x\,2\mathrm{i}}{a\,e-b\,d}\right)\,2\mathrm{i}}{a\,e^2-b\,d\,e}$$

```
[In] int(log(c*(a + b*x)^p)/(d + e*x)^2,x)
```

```
[Out] (b*p*atan((a*e*1i + b*d*1i + b*e*x*2i)/(a*e - b*d))*2i)/(a*e^2 - b*d*e) - l
og(c*(a + b*x)^p)/(e*(d + e*x))
```

3.182 $\int \frac{\log(c(a+bx)^p)}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 105

$$\int \frac{\log(c(a+bx)^p)}{(d+ex)^3}\,dx = \frac{bp}{2e(bd-ae)(d+ex)} + \frac{b^2p\log(a+bx)}{2e(bd-ae)^2} - \frac{\log(c(a+bx)^p)}{2e(d+ex)^2} - \frac{b^2p\log(d+ex)}{2e(bd-ae)^2}$$

```
[Out] 1/2*b*p/e/(-a*e+b*d)/(e*x+d)+1/2*b^2*p*ln(b*x+a)/e/(-a*e+b*d)^2-1/2*ln(c*(b
*x+a)^p)/e/(e*x+d)^2-1/2*b^2*p*ln(e*x+d)/e/(-a*e+b*d)^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2442, 46}

$$\int \frac{\log(c(a+bx)^p)}{(d+ex)^3}\,dx = \frac{b^2p\log(a+bx)}{2e(bd-ae)^2} - \frac{b^2p\log(d+ex)}{2e(bd-ae)^2} - \frac{\log(c(a+bx)^p)}{2e(d+ex)^2} + \frac{bp}{2e(d+ex)(bd-ae)}$$

```
[In] Int[Log[c*(a + b*x)^p]/(d + e*x)^3,x]
```

```
[Out] (b*p)/(2*e*(b*d - a*e)*(d + e*x)) + (b^2*p*Log[a + b*x])/(2*e*(b*d - a*e)^2
) - Log[c*(a + b*x)^p]/(2*e*(d + e*x)^2) - (b^2*p*Log[d + e*x])/(2*e*(b*d -
 a*e)^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log\left(c(a+bx)^p\right)}{2e(d+ex)^2}+\frac{(bp)\int\frac{1}{(a+bx)(d+ex)^2}\,dx}{2e}\\
&= -\frac{\log\left(c(a+bx)^p\right)}{2e(d+ex)^2}+\frac{(bp)\int\left(\frac{b^2}{(bd-ae)^2(a+bx)}-\frac{e}{(bd-ae)(d+ex)^2}-\frac{be}{(bd-ae)^2(d+ex)}\right)dx}{2e}\\
&= \frac{bp}{2e(bd-ae)(d+ex)}+\frac{b^2p\log(a+bx)}{2e(bd-ae)^2}-\frac{\log\left(c(a+bx)^p\right)}{2e(d+ex)^2}-\frac{b^2p\log(d+ex)}{2e(bd-ae)^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.76

$$\int\frac{\log\left(c(a+bx)^p\right)}{(d+ex)^3}\,dx=\frac{-\log\left(c(a+bx)^p\right)+\frac{bp(d+ex)(bd-ae+b(d+ex)\log(a+bx)-b(d+ex)\log(d+ex))}{(bd-ae)^2}}{2e(d+ex)^2}$$

```
[In] Integrate[Log[c*(a + b*x)^p]/(d + e*x)^3,x]
[Out] (-Log[c*(a + b*x)^p] + (b*p*(d + e*x)*(b*d - a*e + b*(d + e*x)*Log[a + b*x]
 - b*(d + e*x)*Log[d + e*x]))/(b*d - a*e)^2)/(2*e*(d + e*x)^2)
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.84

method	result
parts	$-\frac{\ln(c(bx+a)^{p})}{2e(ex+d)^{2}}+\frac{pb\left(-\frac{1}{(ae-bd)(ex+d)}-\frac{b\ln(ex+d)}{(ae-bd)^{2}}+\frac{b\ln(bx+a)}{(ae-bd)^{2}}\right)}{2e}$
parallelrisch	$-\frac{2\ln(ex+d)x\,b^{3}d\,e^{2}p-2\ln(bx+a)x\,b^{3}d\,e^{2}p+a\,b^{2}d\,e^{2}p+xa\,b^{2}e^{3}p-x\,b^{3}d\,e^{2}p-2\ln(c(bx+a)^{p})a\,b^{2}d\,e^{2}-\ln(bx+a)x^{2}b^{3}e^{3}p+\ln(ex+}{2(a^{2}e^{2}-2adeb+b^{2}d^{2})(ex+d)^{2}e}$
risch	$-\frac{\ln((bx+a)^{p})}{2e(ex+d)^{2}}-\frac{2i\pi abde\ \mathrm{csgn}(i(bx+a)^{p})\,\mathrm{csgn}(ic(bx+a)^{p})\,\mathrm{csgn}(ic)-4\ln(-bx-a)b^{2}depx+4\ln(ex+d)b^{2}depx-i\pi\,b^{2}d^{2}\,\mathrm{csgn}(ic}{}$

```
[In] int(ln(c*(b*x+a)^p)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(c*(b*x+a)^p)/e/(e*x+d)^2+1/2*p*b/e*(-1/(a*e-b*d)/(e*x+d)-b/(a*e-b*d
)^2*ln(e*x+d)+b/(a*e-b*d)^2*ln(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(97) = 194.

Time = 0.32 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.25

$$\int \frac{\log\left(c(a+bx)^{p}\right)}{(d+ex)^{3}}\,dx$$
$$=\frac{(b^{2}de-abe^{2})px+(b^{2}d^{2}-abde)p+(b^{2}e^{2}px^{2}+2\,b^{2}depx+(2\,abde-a^{2}e^{2})p)\log(bx+a)-(b^{2}e^{2}px^{2}+2\,b^{2}}{2\,(b^{2}d^{4}e-2\,abd^{3}e^{2}+a^{2}d^{2}e^{3}+(b^{2}d^{2}e^{3}-2\,abde^{4}+a^{2}e^{5})x^{2}+2\,(b^{2}d^{3}e^{2}}$$

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*((b^2*d*e - a*b*e^2)*p*x + (b^2*d^2 - a*b*d*e)*p + (b^2*e^2*p*x^2 + 2*b
^2*d*e*p*x + (2*a*b*d*e - a^2*e^2)*p)*log(b*x + a) - (b^2*e^2*p*x^2 + 2*b^2
*d*e*p*x + b^2*d^2*p)*log(e*x + d) - (b^2*d^2 - 2*a*b*d*e + a^2*e^2)*log(c)
)/(b^2*d^4*e - 2*a*b*d^3*e^2 + a^2*d^2*e^3 + (b^2*d^2*e^3 - 2*a*b*d*e^4 + a
^2*e^5)*x^2 + 2*(b^2*d^3*e^2 - 2*a*b*d^2*e^3 + a^2*d*e^4)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1518 vs. 2(85) = 170.

Time = 5.25 (sec) , antiderivative size = 1518, normalized size of antiderivative = 14.46

$$\int \frac{\log\left(c(a+bx)^{p}\right)}{(d+ex)^{3}}\,dx = \text{Too large to display}$$

```
[In] integrate(ln(c*(b*x+a)**p)/(e*x+d)**3,x)
```

```
[Out] Piecewise(((a*log(c*(a + b*x)**p)/b - p*x + x*log(c*(a + b*x)**p))/d**3, Eq
(e, 0)), (-p/(4*d**2*e + 8*d*e**2*x + 4*e**3*x**2) - 2*log(c*(b*d/e + b*x)*
*p)/(4*d**2*e + 8*d*e**2*x + 4*e**3*x**2), Eq(a, b*d/e)), (-a**2*e**2*log(c
*(a + b*x)**p)/(2*a**2*d**2*e**3 + 4*a**2*d*e**4*x + 2*a**2*e**5*x**2 - 4*a
*b*d**3*e**2 - 8*a*b*d**2*e**3*x - 4*a*b*d*e**4*x**2 + 2*b**2*d**4*e + 4*b*
*2*d**3*e**2*x + 2*b**2*d**2*e**3*x**2) - a*b*d*e*p/(2*a**2*d**2*e**3 + 4*a
**2*d*e**4*x + 2*a**2*e**5*x**2 - 4*a*b*d**3*e**2 - 8*a*b*d**2*e**3*x - 4*a
*b*d*e**4*x**2 + 2*b**2*d**4*e + 4*b**2*d**3*e**2*x + 2*b**2*d**2*e**3*x**2
) + 2*a*b*d*e*log(c*(a + b*x)**p)/(2*a**2*d**2*e**3 + 4*a**2*d*e**4*x + 2*a
**2*e**5*x**2 - 4*a*b*d**3*e**2 - 8*a*b*d**2*e**3*x - 4*a*b*d*e**4*x**2 + 2
*b**2*d**4*e + 4*b**2*d**3*e**2*x + 2*b**2*d**2*e**3*x**2) - a*b*e**2*p*x/(
2*a**2*d**2*e**3 + 4*a**2*d*e**4*x + 2*a**2*e**5*x**2 - 4*a*b*d**3*e**2 - 8
*a*b*d**2*e**3*x - 4*a*b*d*e**4*x**2 + 2*b**2*d**4*e + 4*b**2*d**3*e**2*x +
 2*b**2*d**2*e**3*x**2) - b**2*d**2*p*log(d/e + x)/(2*a**2*d**2*e**3 + 4*a*
*2*d*e**4*x + 2*a**2*e**5*x**2 - 4*a*b*d**3*e**2 - 8*a*b*d**2*e**3*x - 4*a*
b*d*e**4*x**2 + 2*b**2*d**4*e + 4*b**2*d**3*e**2*x + 2*b**2*d**2*e**3*x**2)
 + b**2*d**2*p/(2*a**2*d**2*e**3 + 4*a**2*d*e**4*x + 2*a**2*e**5*x**2 - 4*a
*b*d**3*e**2 - 8*a*b*d**2*e**3*x - 4*a*b*d*e**4*x**2 + 2*b**2*d**4*e + 4*b*
*2*d**3*e**2*x + 2*b**2*d**2*e**3*x**2) - 2*b**2*d*e*p*x*log(d/e + x)/(2*a*
*2*d**2*e**3 + 4*a**2*d*e**4*x + 2*a**2*e**5*x**2 - 4*a*b*d**3*e**2 - 8*a*b
*d**2*e**3*x - 4*a*b*d*e**4*x**2 + 2*b**2*d**4*e + 4*b**2*d**3*e**2*x + 2*b
**2*d**2*e**3*x**2) + b**2*d*e*p*x/(2*a**2*d**2*e**3 + 4*a**2*d*e**4*x + 2*
a**2*e**5*x**2 - 4*a*b*d**3*e**2 - 8*a*b*d**2*e**3*x - 4*a*b*d*e**4*x**2 +
2*b**2*d**4*e + 4*b**2*d**3*e**2*x + 2*b**2*d**2*e**3*x**2) + 2*b**2*d*e*x*
log(c*(a + b*x)**p)/(2*a**2*d**2*e**3 + 4*a**2*d*e**4*x + 2*a**2*e**5*x**2
- 4*a*b*d**3*e**2 - 8*a*b*d**2*e**3*x - 4*a*b*d*e**4*x**2 + 2*b**2*d**4*e +
 4*b**2*d**3*e**2*x + 2*b**2*d**2*e**3*x**2) - b**2*e**2*p*x**2*log(d/e + x
)/(2*a**2*d**2*e**3 + 4*a**2*d*e**4*x + 2*a**2*e**5*x**2 - 4*a*b*d**3*e**2
- 8*a*b*d**2*e**3*x - 4*a*b*d*e**4*x**2 + 2*b**2*d**4*e + 4*b**2*d**3*e**2*
x + 2*b**2*d**2*e**3*x**2) + b**2*e**2*x**2*log(c*(a + b*x)**p)/(2*a**2*d**
2*e**3 + 4*a**2*d*e**4*x + 2*a**2*e**5*x**2 - 4*a*b*d**3*e**2 - 8*a*b*d**2*
e**3*x - 4*a*b*d*e**4*x**2 + 2*b**2*d**4*e + 4*b**2*d**3*e**2*x + 2*b**2*d*
*2*e**3*x**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14

$$\int \frac{\log \left(c(a+bx)^p\right)}{(d+ex)^3}\,dx = \frac{bp\left(\frac{b\log(bx+a)}{b^2d^2-2\,abde+a^2e^2} - \frac{b\log(ex+d)}{b^2d^2-2\,abde+a^2e^2} + \frac{1}{bd^2-ade+(bde-ae^2)x}\right)}{2\,e} - \frac{\log\left((bx+a)^pc\right)}{2\,(ex+d)^2e}$$

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(b*log(b*x + a)/(b^2*d^2 - 2*a*b*d*e + a^2*e^2) - b*log(e*x + d)/(b
^2*d^2 - 2*a*b*d*e + a^2*e^2) + 1/(b*d^2 - a*d*e + (b*d*e - a*e^2)*x))/e -
1/2*log((b*x + a)^p*c)/((e*x + d)^2*e)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.76

$$\int \frac{\log(c(a+bx)^p)}{(d+ex)^3}\,dx = \frac{b^2p\log(bx+a)}{2\,(b^2d^2e-2\,abde^2+a^2e^3)} - \frac{b^2p\log(ex+d)}{2\,(b^2d^2e-2\,abde^2+a^2e^3)} - \frac{p\log(bx+a)}{2\,(e^3x^2+2\,de^2x+d^2e)} + \frac{bepx+bdp-bd\log(c)+ae\log(c)}{2\,(bde^3x^2-ae^4x^2+2\,bd^2e^2x-2\,ade^3x+bd^3e-ad^2e^2)}$$

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^2*p*log(b*x + a)/(b^2*d^2*e - 2*a*b*d*e^2 + a^2*e^3) - 1/2*b^2*p*log(
e*x + d)/(b^2*d^2*e - 2*a*b*d*e^2 + a^2*e^3) - 1/2*p*log(b*x + a)/(e^3*x^2
+ 2*d*e^2*x + d^2*e) + 1/2*(b*e*p*x + b*d*p - b*d*log(c) + a*e*log(c))/(b*d
*e^3*x^2 - a*e^4*x^2 + 2*b*d^2*e^2*x - 2*a*d*e^3*x + b*d^3*e - a*d^2*e^2)
```

Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \frac{\log(c(a+bx)^p)}{(d+ex)^3}\,dx = -\frac{\ln\left(c\,(a+b\,x)^p\right)}{2\,e\,(d+e\,x)^2} - \frac{b\,p}{2\,e\,(a\,e-b\,d)\,(d+e\,x)} - \frac{b^2\,p\,\mathrm{atan}\left(\frac{a\,e\,1\mathrm{i}+b\,d\,1\mathrm{i}+b\,e\,x\,2\mathrm{i}}{a\,e-b\,d}\right)\,1\mathrm{i}}{e\,(a\,e-b\,d)^2}$$

```
[In] int(log(c*(a + b*x)^p)/(d + e*x)^3,x)
```

```
[Out] - log(c*(a + b*x)^p)/(2*e*(d + e*x)^2) - (b*p)/(2*e*(a*e - b*d)*(d + e*x))
- (b^2*p*atan((a*e*1i + b*d*1i + b*e*x*2i)/(a*e - b*d))*1i)/(e*(a*e - b*d)^
2)
```

3.183 $\int \frac{\log(c(a+bx)^p)}{(d+ex)^4} dx$

Optimal result

Integrand size = 18, antiderivative size = 133

$$\int \frac{\log(c(a+bx)^p)}{(d+ex)^4} dx = \frac{bp}{6e(bd-ae)(d+ex)^2} + \frac{b^2p}{3e(bd-ae)^2(d+ex)} + \frac{b^3p\log(a+bx)}{3e(bd-ae)^3} - \frac{\log(c(a+bx)^p)}{3e(d+ex)^3} - \frac{b^3p\log(d+ex)}{3e(bd-ae)^3}$$

```
[Out] 1/6*b*p/e/(-a*e+b*d)/(e*x+d)^2+1/3*b^2*p/e/(-a*e+b*d)^2/(e*x+d)+1/3*b^3*p*l
n(b*x+a)/e/(-a*e+b*d)^3-1/3*ln(c*(b*x+a)^p)/e/(e*x+d)^3-1/3*b^3*p*ln(e*x+d)
/e/(-a*e+b*d)^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2442, 46}

$$\int \frac{\log(c(a+bx)^p)}{(d+ex)^4} dx = \frac{b^3p\log(a+bx)}{3e(bd-ae)^3} - \frac{b^3p\log(d+ex)}{3e(bd-ae)^3} + \frac{b^2p}{3e(d+ex)(bd-ae)^2} - \frac{\log(c(a+bx)^p)}{3e(d+ex)^3} + \frac{bp}{6e(d+ex)^2(bd-ae)}$$

```
[In] Int[Log[c*(a + b*x)^p]/(d + e*x)^4,x]
```

```
[Out] (b*p)/(6*e*(b*d - a*e)*(d + e*x)^2) + (b^2*p)/(3*e*(b*d - a*e)^2*(d + e*x))
 + (b^3*p*Log[a + b*x])/(3*e*(b*d - a*e)^3) - Log[c*(a + b*x)^p]/(3*e*(d +
e*x)^3) - (b^3*p*Log[d + e*x])/(3*e*(b*d - a*e)^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log\left(c(a+bx)^p\right)}{3e(d+ex)^3}+\frac{(bp)\int\frac{1}{(a+bx)(d+ex)^3}\,dx}{3e}\\
&= -\frac{\log\left(c(a+bx)^p\right)}{3e(d+ex)^3}+\frac{(bp)\int\left(\frac{b^3}{(bd-ae)^3(a+bx)}-\frac{e}{(bd-ae)(d+ex)^3}-\frac{be}{(bd-ae)^2(d+ex)^2}-\frac{b^2e}{(bd-ae)^3(d+ex)}\right)dx}{3e}\\
&= \frac{bp}{6e(bd-ae)(d+ex)^2}+\frac{b^2p}{3e(bd-ae)^2(d+ex)}\\
&\quad+\frac{b^3p\log(a+bx)}{3e(bd-ae)^3}-\frac{\log\left(c(a+bx)^p\right)}{3e(d+ex)^3}-\frac{b^3p\log(d+ex)}{3e(bd-ae)^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.79

$$\int\frac{\log\left(c(a+bx)^p\right)}{(d+ex)^4}\,dx = \frac{-2\log\left(c(a+bx)^p\right)+\frac{bp(d+ex)\left((bd-ae)(3bd-ae+2bex)+2b^2(d+ex)^2\log(a+bx)-2b^2(d+ex)^2\log(d+ex)\right)}{(bd-ae)^3}}{6e(d+ex)^3}$$

```
[In] Integrate[Log[c*(a + b*x)^p]/(d + e*x)^4,x]
```

```
[Out] (-2*Log[c*(a + b*x)^p] + (b*p*(d + e*x)*((b*d - a*e)*(3*b*d - a*e + 2*b*e*x
) + 2*b^2*(d + e*x)^2*Log[a + b*x] - 2*b^2*(d + e*x)^2*Log[d + e*x]))/(b*d
- a*e)^3)/(6*e*(d + e*x)^3)
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

method	result
parts	$-\frac{\ln(c(bx+a)^p)}{3e(ex+d)^3}+\frac{pb\left(-\frac{1}{2(ae-bd)(ex+d)^2}+\frac{b^2\ln(ex+d)}{(ae-bd)^3}+\frac{b}{(ae-bd)^2(ex+d)}-\frac{b^2\ln(bx+a)}{(ae-bd)^3}\right)}{3e}$
parallelrisch	$-\frac{-6xa\,b^3d\,e^4p+6\ln(bx+a)x^2b^4d\,e^4p-6\ln(ex+d)x^2b^4d\,e^4p+6\ln(bx+a)x\,b^4d^2e^3p-6\ln(ex+d)x\,b^4d^2e^3p+2\ln(bx+a)x^3b^4e^5p}{}$
risch	$-\frac{\ln((bx+a)^p)}{3e(ex+d)^3}+\frac{-6\ln(bx+a)b^3d\,e^2p\,x^2+6\ln(-ex-d)b^3d\,e^2p\,x^2-6\ln(bx+a)b^3d^2epx+6\ln(-ex-d)b^3d^2epx-3b^3d^3p-2\ln(}{}$

```
[In] int(ln(c*(b*x+a)^p)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*ln(c*(b*x+a)^p)/e/(e*x+d)^3+1/3*p*b/e*(-1/2/(a*e-b*d)/(e*x+d)^2+b^2/(a
*e-b*d)^3*ln(e*x+d)+b/(a*e-b*d)^2/(e*x+d)-b^2/(a*e-b*d)^3*ln(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(123) = 246$.

Time = 0.39 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.33

$$\int \frac{\log\left(c(a+bx)^p\right)}{(d+ex)^4}\,dx$$
$$=\frac{2\left(b^3de^2-ab^2e^3\right)px^2+\left(5\,b^3d^2e-6\,ab^2de^2+a^2be^3\right)px+\left(3\,b^3d^3-4\,ab^2d^2e+a^2bde^2\right)p+2\left(b^3e^3px^3+3\,b^3\right.}{6\left(b^3d^6e-3\,ab^2d^5e^2+3\,a^2bd^4e^3-a^3d^3e^4+\left(b^3d^3e^4-3\,a\right.\right.}$$

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(b^3*d*e^2 - a*b^2*e^3)*p*x^2 + (5*b^3*d^2*e - 6*a*b^2*d*e^2 + a^2*b
*e^3)*p*x + (3*b^3*d^3 - 4*a*b^2*d^2*e + a^2*b*d*e^2)*p + 2*(b^3*e^3*p*x^3
+ 3*b^3*d*e^2*p*x^2 + 3*b^3*d^2*e*p*x + (3*a*b^2*d^2*e - 3*a^2*b*d*e^2 + a^
3*e^3)*p)*log(b*x + a) - 2*(b^3*e^3*p*x^3 + 3*b^3*d*e^2*p*x^2 + 3*b^3*d^2*e
*p*x + b^3*d^3*p)*log(e*x + d) - 2*(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2
 - a^3*e^3)*log(c))/(b^3*d^6*e - 3*a*b^2*d^5*e^2 + 3*a^2*b*d^4*e^3 - a^3*d^
3*e^4 + (b^3*d^3*e^4 - 3*a*b^2*d^2*e^5 + 3*a^2*b*d*e^6 - a^3*e^7)*x^3 + 3*(
b^3*d^4*e^3 - 3*a*b^2*d^3*e^4 + 3*a^2*b*d^2*e^5 - a^3*d*e^6)*x^2 + 3*(b^3*d
^5*e^2 - 3*a*b^2*d^4*e^3 + 3*a^2*b*d^3*e^4 - a^3*d^2*e^5)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4571 vs. $2(109) = 218$.

Time = 17.52 (sec) , antiderivative size = 4571, normalized size of antiderivative = 34.37

$$\int \frac{\log \left(c(a+bx)^p\right)}{(d+ex)^4} \, dx = \text{Too large to display}$$

```
[In] integrate(ln(c*(b*x+a)**p)/(e*x+d)**4,x)
```

```
[Out] Piecewise(((a*log(c*(a + b*x)**p)/b - p*x + x*log(c*(a + b*x)**p))/d**4, Eq
(e, 0)), (-p/(9*d**3*e + 27*d**2*e**2*x + 27*d*e**3*x**2 + 9*e**4*x**3) - 3
*log(c*(b*d/e + b*x)**p)/(9*d**3*e + 27*d**2*e**2*x + 27*d*e**3*x**2 + 9*e*
*4*x**3), Eq(a, b*d/e)), (-2*a**3*e**3*log(c*(a + b*x)**p)/(6*a**3*d**3*e**
4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*
b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*
d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*
e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x
- 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) - a**2*b*d*e**2*p/(6*a**3
*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 -
 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 1
8*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b
**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**
5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) + 6*a**2*b*d*e**
2*log(c*(a + b*x)**p)/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e
**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x -
 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 5
4*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3
- 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**
3*e**4*x**3) - a**2*b*e**3*p*x/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18
*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3
*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5
*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e
**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6
*b**3*d**3*e**4*x**3) + 4*a*b**2*d**2*e*p/(6*a**3*d**3*e**4 + 18*a**3*d**2*
e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*
a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*
a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*
b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e*
*3*x**2 - 6*b**3*d**3*e**4*x**3) - 6*a*b**2*d**2*e*log(c*(a + b*x)**p)/(6*a
**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**
3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2
- 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*
a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*
d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) + 6*a*b**2*d*
```

```
e**2*p*x/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*
a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d*
*2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4
*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6
*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3)
+ 2*a*b**2*e**3*p*x**2/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*
e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x
- 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 +
54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3
 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d*
*3*e**4*x**3) + 2*b**3*d**3*p*log(d/e + x)/(6*a**3*d**3*e**4 + 18*a**3*d**2
*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54
*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18
*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a
*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e
**3*x**2 - 6*b**3*d**3*e**4*x**3) - 3*b**3*d**3*p/(6*a**3*d**3*e**4 + 18*a*
*3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e*
*3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x*
*3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2
 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3
*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) + 6*b**3*d**2*e*p*x*log(d/e + x)/(
6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*
x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x*
*2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x +
54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b*
*3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) - 5*b**3*d
**2*e*p*x/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6
*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d
**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**
4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**
6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3)
 - 6*b**3*d**2*e*x*log(c*(a + b*x)**p)/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**
5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**
2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b
**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**
2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*
x**2 - 6*b**3*d**3*e**4*x**3) + 6*b**3*d*e**2*p*x**2*log(d/e + x)/(6*a**3*d
**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 1
8*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*
a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**
2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*
e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) - 2*b**3*d*e**2*p*
x**2/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3
*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e
**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**
```

```
3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e -
 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) - 6*
b**3*d*e**2*x**2*log(c*(a + b*x)**p)/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*
x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*
b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**
2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*
d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*x*
*2 - 6*b**3*d**3*e**4*x**3) + 2*b**3*e**3*p*x**3*log(d/e + x)/(6*a**3*d**3*
e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x**2 + 6*a**3*e**7*x**3 - 18*a*
*2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a**2*b*d**2*e**5*x**2 - 18*a**2
*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b**2*d**4*e**3*x + 54*a*b**2*d*
*3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b**3*d**6*e - 18*b**3*d**5*e**2
*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**4*x**3) - 2*b**3*e**3*x**3*log
(c*(a + b*x)**p)/(6*a**3*d**3*e**4 + 18*a**3*d**2*e**5*x + 18*a**3*d*e**6*x
**2 + 6*a**3*e**7*x**3 - 18*a**2*b*d**4*e**3 - 54*a**2*b*d**3*e**4*x - 54*a
**2*b*d**2*e**5*x**2 - 18*a**2*b*d*e**6*x**3 + 18*a*b**2*d**5*e**2 + 54*a*b
**2*d**4*e**3*x + 54*a*b**2*d**3*e**4*x**2 + 18*a*b**2*d**2*e**5*x**3 - 6*b
**3*d**6*e - 18*b**3*d**5*e**2*x - 18*b**3*d**4*e**3*x**2 - 6*b**3*d**3*e**
4*x**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.74

$$\int \frac{\log \left(c(a+bx)^p\right)}{(d+ex)^4}\,dx$$
$$= \frac{\left(\frac{2\,b^2\log(bx+a)}{b^3d^3-3\,ab^2d^2e+3\,a^2bde^2-a^3e^3} - \frac{2\,b^2\log(ex+d)}{b^3d^3-3\,ab^2d^2e+3\,a^2bde^2-a^3e^3} + \frac{2\,bex+3\,bd-ae}{b^2d^4-2\,abd^3e+a^2d^2e^2+(b^2d^2e^2-2\,abde^3+a^2e^4)x^2+2\,(b^2d^3e-2\,abd^2e^2-}\right.}{6\,e}$$
$$-\frac{\log\left((bx+a)^pc\right)}{3\,(ex+d)^3e}$$

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^2*log(b*x + a)/(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3)
 - 2*b^2*log(e*x + d)/(b^3*d^3 - 3*a*b^2*d^2*e + 3*a^2*b*d*e^2 - a^3*e^3) +
 (2*b*e*x + 3*b*d - a*e)/(b^2*d^4 - 2*a*b*d^3*e + a^2*d^2*e^2 + (b^2*d^2*e^
2 - 2*a*b*d*e^3 + a^2*e^4)*x^2 + 2*(b^2*d^3*e - 2*a*b*d^2*e^2 + a^2*d*e^3)*
x))*b*p/e - 1/3*log((b*x + a)^p*c)/((e*x + d)^3*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(123) = 246$.

Time = 0.32 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.74

$$\int \frac{\log(c(a+bx)^p)}{(d+ex)^4}\,dx = \frac{b^3p\log(bx+a)}{3(b^3d^3e-3ab^2d^2e^2+3a^2bde^3-a^3e^4)} - \frac{b^3p\log(ex+d)}{3(b^3d^3e-3ab^2d^2e^2+3a^2bde^3-a^3e^4)} - \frac{p\log(bx+a)}{3(e^4x^3+3de^3x^2+3d^2e^2x+d^3e)} + \frac{2b^2e^2px^2+5b^2depx-abe^2px+3b^2d^2p-abdep-2b^2d^2\log(c)+4abde\log(c)-}{6(b^2d^2e^4x^3-2abde^5x^3+a^2e^6x^3+3b^2d^3e^3x^2-6abd^2e^4x^2+3a^2de^5x^2+3b^2d^4e^2x-6abd^3e^3x+3a^2}$$

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] 1/3*b^3*p*log(b*x + a)/(b^3*d^3*e - 3*a*b^2*d^2*e^2 + 3*a^2*b*d*e^3 - a^3*e
^4) - 1/3*b^3*p*log(e*x + d)/(b^3*d^3*e - 3*a*b^2*d^2*e^2 + 3*a^2*b*d*e^3 -
 a^3*e^4) - 1/3*p*log(b*x + a)/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e
) + 1/6*(2*b^2*e^2*p*x^2 + 5*b^2*d*e*p*x - a*b*e^2*p*x + 3*b^2*d^2*p - a*b*
d*e*p - 2*b^2*d^2*log(c) + 4*a*b*d*e*log(c) - 2*a^2*e^2*log(c))/(b^2*d^2*e^
4*x^3 - 2*a*b*d*e^5*x^3 + a^2*e^6*x^3 + 3*b^2*d^3*e^3*x^2 - 6*a*b*d^2*e^4*x
^2 + 3*a^2*d*e^5*x^2 + 3*b^2*d^4*e^2*x - 6*a*b*d^3*e^3*x + 3*a^2*d^2*e^4*x
+ b^2*d^5*e - 2*a*b*d^4*e^2 + a^2*d^3*e^3)
```

Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.09

$$\int \frac{\log(c(a+bx)^p)}{(d+ex)^4}\,dx = \frac{b^2\,p\,x}{3\,(a\,e-b\,d)^2\,(d+e\,x)^2} - \frac{\ln\left(c\,(a+b\,x)^p\right)}{3\,e\,(d+e\,x)^3} - \frac{a\,b\,p}{6\,(a\,e-b\,d)^2\,(d+e\,x)^2} + \frac{b^2\,d\,p}{2\,e\,(a\,e-b\,d)^2\,(d+e\,x)^2} + \frac{b^3\,p\,\mathrm{atan}\left(\frac{a\,e\,1\mathrm{i}+b\,d\,1\mathrm{i}+b\,e\,x\,2\mathrm{i}}{a\,e-b\,d}\right)\,2\mathrm{i}}{3\,e\,(a\,e-b\,d)^3}$$

```
[In] int(log(c*(a + b*x)^p)/(d + e*x)^4,x)
```

```
[Out] (b^2*p*x)/(3*(a*e - b*d)^2*(d + e*x)^2) - log(c*(a + b*x)^p)/(3*e*(d + e*x)
^3) + (b^3*p*atan((a*e*1i + b*d*1i + b*e*x*2i)/(a*e - b*d))*2i)/(3*e*(a*e -
 b*d)^3) - (a*b*p)/(6*(a*e - b*d)^2*(d + e*x)^2) + (b^2*d*p)/(2*e*(a*e - b*
d)^2*(d + e*x)^2)
```

3.184 $\int (d+ex)^3 \log \left(c\left(a+bx^2\right)^p\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 178

$$\int (d+ex)^3 \log \left(c\left(a+bx^2\right)^p\right) \, dx = -\frac{2d\left(bd^2-ae^2\right)px}{b} - \frac{e\left(6bd^2-ae^2\right)px^2}{4b} - \frac{2}{3}de^2px^3 - \frac{1}{8}e^3px^4 + \frac{2\sqrt{a}d\left(bd^2-ae^2\right)p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}} - \frac{\left(b^2d^4-6abd^2e^2+a^2e^4\right)p\log\left(a+bx^2\right)}{4b^2e} + \frac{(d+ex)^4\log\left(c\left(a+bx^2\right)^p\right)}{4e}$$

```
[Out] -2*d*(-a*e^2+b*d^2)*p*x/b-1/4*e*(-a*e^2+6*b*d^2)*p*x^2/b-2/3*d*e^2*p*x^3-1/
8*e^3*p*x^4-1/4*(a^2*e^4-6*a*b*d^2*e^2+b^2*d^4)*p*ln(b*x^2+a)/b^2/e+1/4*(e*
x+d)^4*ln(c*(b*x^2+a)^p)/e+2*d*(-a*e^2+b*d^2)*p*arctan(x*b^(1/2)/a^(1/2))*a
^(1/2)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{2513, 815, 649, 211, 266\}$

$$\int (d+ex)^3 \log\left(c\left(a+bx^2\right)^p\right)\,dx = -\frac{p(a^2e^4-6abd^2e^2+b^2d^4)\log\left(a+bx^2\right)}{4b^2e} + \frac{2\sqrt{a}dp\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bd^2-ae^2)}{b^{3/2}} + \frac{(d+ex)^4\log\left(c(a+bx^2)^p\right)}{4e} - \frac{epx^2(6bd^2-ae^2)}{4b} - \frac{2dpx(bd^2-ae^2)}{b} - \frac{2}{3}de^2px^3 - \frac{1}{8}e^3px^4$$

```
[In] Int[(d + e*x)^3*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (-2*d*(b*d^2 - a*e^2)*p*x)/b - (e*(6*b*d^2 - a*e^2)*p*x^2)/(4*b) - (2*d*e^2
*p*x^3)/3 - (e^3*p*x^4)/8 + (2*Sqrt[a]*d*(b*d^2 - a*e^2)*p*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/b^(3/2) - ((b^2*d^4 - 6*a*b*d^2*e^2 + a^2*e^4)*p*Log[a + b*x^2
])/(4*b^2*e) + ((d + e*x)^4*Log[c*(a + b*x^2)^p])/(4*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
```

```
&& (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d+ex)^4\log\left(c(a+bx^2)^p\right)}{4e}-\frac{(bp)\int\frac{x(d+ex)^4}{a+bx^2}\,dx}{2e}\\
&= \frac{(d+ex)^4\log\left(c(a+bx^2)^p\right)}{4e}\\
&\quad-\frac{(bp)\int\left(\frac{4de(bd^2-ae^2)}{b^2}+\frac{e^2(6bd^2-ae^2)x}{b^2}+\frac{4de^3x^2}{b}+\frac{e^4x^3}{b}-\frac{4ade(bd^2-ae^2)-(b^2d^4-6abd^2e^2+a^2e^4)x}{b^2(a+bx^2)}\right)dx}{2e}\\
&= -\frac{2d(bd^2-ae^2)\,px}{b}-\frac{e(6bd^2-ae^2)\,px^2}{4b}-\frac{2}{3}de^2px^3-\frac{1}{8}e^3px^4\\
&\quad+\frac{(d+ex)^4\log\left(c(a+bx^2)^p\right)}{4e}+\frac{p\int\frac{4ade(bd^2-ae^2)-(b^2d^4-6abd^2e^2+a^2e^4)x}{a+bx^2}\,dx}{2be}\\
&= -\frac{2d(bd^2-ae^2)\,px}{b}-\frac{e(6bd^2-ae^2)\,px^2}{4b}-\frac{2}{3}de^2px^3-\frac{1}{8}e^3px^4\\
&\quad+\frac{(d+ex)^4\log\left(c(a+bx^2)^p\right)}{4e}+\frac{(2ad(bd^2-ae^2)\,p)\int\frac{1}{a+bx^2}\,dx}{b}\\
&\quad+\frac{((-b^2d^4+6abd^2e^2-a^2e^4)\,p)\int\frac{x}{a+bx^2}\,dx}{2be}\\
&= -\frac{2d(bd^2-ae^2)\,px}{b}-\frac{e(6bd^2-ae^2)\,px^2}{4b}-\frac{2}{3}de^2px^3\\
&\quad-\frac{1}{8}e^3px^4+\frac{2\sqrt{a}d(bd^2-ae^2)\,p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\\
&\quad-\frac{(b^2d^4-6abd^2e^2+a^2e^4)\,p\log(a+bx^2)}{4b^2e}+\frac{(d+ex)^4\log\left(c(a+bx^2)^p\right)}{4e}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.40

$$
\int(d+ex)^3\log\left(c(a+bx^2)^p\right)\,dx
$$
$$
=\frac{-6\left(b^2d^4+4\sqrt{-a}b^{3/2}d^3e-6abd^2e^2+4(-a)^{3/2}\sqrt{b}de^3+a^2e^4\right)p\log\left(\sqrt{-a}-\sqrt{b}x\right)-6\left(b^2d^4-4\sqrt{-a}b^{3/2}d\right.}{}
$$

```
[In] Integrate[(d + e*x)^3*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (-6*(b^2*d^4 + 4*Sqrt[-a]*b^(3/2)*d^3*e - 6*a*b*d^2*e^2 + 4*(-a)^(3/2)*Sqrt
[b]*d*e^3 + a^2*e^4)*p*Log[Sqrt[-a] - Sqrt[b]*x] - 6*(b^2*d^4 - 4*Sqrt[-a]*
```

```
b^(3/2)*d^3*e - 6*a*b*d^2*e^2 + 4*Sqrt[-a]*a*Sqrt[b]*d*e^3 + a^2*e^4)*p*Log
[Sqrt[-a] + Sqrt[b]*x] + b*(6*a*e^3*p*x*(8*d + e*x) - b*e*p*x*(48*d^3 + 36*
d^2*e*x + 16*d*e^2*x^2 + 3*e^3*x^3) + 6*b*(d + e*x)^4*Log[c*(a + b*x^2)^p])
)/(24*b^2*e)
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.37

method	result
parts	$\frac{\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)e^{3}x^{4}}{4}+\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)e^{2}d\,x^{3}+\frac{3\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)e\,d^{2}x^{2}}{2}+d^{3}\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)x+\frac{\ln\left(c\left(b\,x^{2}+\right.\right.}{4e}$
risch	Expression too large to display

```
[In] int((e*x+d)^3*ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(c*(b*x^2+a)^p)*e^3*x^4+ln(c*(b*x^2+a)^p)*e^2*d*x^3+3/2*ln(c*(b*x^2+a
)^p)*e*d^2*x^2+d^3*ln(c*(b*x^2+a)^p)*x+1/4*ln(c*(b*x^2+a)^p)/e*d^4-1/2*p*b/
e*(-e/b^2*(-1/4*x^4*b*e^3-4/3*x^3*b*d*e^2+1/2*x^2*a*e^3-3*e*d^2*b*x^2+4*x*a
*d*e^2-4*b*d^3*x)+1/b^2*(1/2*(a^2*e^4-6*a*b*d^2*e^2+b^2*d^4)/b*ln(b*x^2+a)+
(4*a^2*d*e^3-4*a*b*d^3*e)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 498, normalized size of antiderivative = 2.80

$$\int (d+ex)^3 \log \left(c\left(a+bx^2\right)^p\right)\, dx$$

$$= \left[-\frac{3\,b^2e^3px^4+16\,b^2de^2px^3+6\,(6\,b^2d^2e-abe^3)px^2-24\,(b^2d^3-abde^2)p\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+48\,(\cdots}{\cdots}\right.$$

$$-\frac{3\,b^2e^3px^4+16\,b^2de^2px^3+6\,(6\,b^2d^2e-abe^3)px^2-48\,(b^2d^3-abde^2)p\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+48\,(b^2d^3-\cdots}{\cdots}$$

```
[In] integrate((e*x+d)^3*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*b^2*e^3*p*x^4 + 16*b^2*d*e^2*p*x^3 + 6*(6*b^2*d^2*e - a*b*e^3)*p*
x^2 - 24*(b^2*d^3 - a*b*d*e^2)*p*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) -
```

```
a)/(b*x^2 + a)) + 48*(b^2*d^3 - a*b*d*e^2)*p*x - 6*(b^2*e^3*p*x^4 + 4*b^2*
d*e^2*p*x^3 + 6*b^2*d^2*e*p*x^2 + 4*b^2*d^3*p*x + (6*a*b*d^2*e - a^2*e^3)*p
)*log(b*x^2 + a) - 6*(b^2*e^3*x^4 + 4*b^2*d*e^2*x^3 + 6*b^2*d^2*e*x^2 + 4*b
^2*d^3*x)*log(c))/b^2, -1/24*(3*b^2*e^3*p*x^4 + 16*b^2*d*e^2*p*x^3 + 6*(6*b
^2*d^2*e - a*b*e^3)*p*x^2 - 48*(b^2*d^3 - a*b*d*e^2)*p*sqrt(a/b)*arctan(b*x
*sqrt(a/b)/a) + 48*(b^2*d^3 - a*b*d*e^2)*p*x - 6*(b^2*e^3*p*x^4 + 4*b^2*d*e
^2*p*x^3 + 6*b^2*d^2*e*p*x^2 + 4*b^2*d^3*p*x + (6*a*b*d^2*e - a^2*e^3)*p)*l
og(b*x^2 + a) - 6*(b^2*e^3*x^4 + 4*b^2*d*e^2*x^3 + 6*b^2*d^2*e*x^2 + 4*b^2*
d^3*x)*log(c))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(170) = 340$.

Time = 17.80 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.96

$$\int (d+ex)^3 \log \left(c\left(a+bx^2\right)^p\right)\,dx$$
$$= \begin{cases} \left(d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4}\right)\log\left(0^pc\right) \\ \left(d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4}\right)\log\left(a^pc\right) \\ -2d^3px + d^3x\log\left(c\left(bx^2\right)^p\right) - \frac{3d^2epx^2}{2} + \frac{3d^2ex^2\log\left(c\left(bx^2\right)^p\right)}{2} - \frac{2de^2px^3}{3} + de^2x^3\log\left(c\left(bx^2\right)^p\right) - \frac{e^3px^4}{8} + \frac{e^3x^4\log\left(\cdots\right.}{4} \\ -\frac{2a^2de^2p\log\left(x-\sqrt{-\frac{a}{b}}\right)}{b^2\sqrt{-\frac{a}{b}}} + \frac{a^2de^2\log\left(c\left(a+bx^2\right)^p\right)}{b^2\sqrt{-\frac{a}{b}}} - \frac{a^2e^3\log\left(c\left(a+bx^2\right)^p\right)}{4b^2} + \frac{2ad^3p\log\left(x-\sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}} - \frac{ad^3\log\left(c\left(a+bx^2\right)^p\right)}{b\sqrt{-\frac{a}{b}}} + \frac{3ad^2e\log\cdots}{} \end{cases}$$

```
[In] integrate((e*x+d)**3*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise(((d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4)*log(0**p*
c), Eq(a, 0) & Eq(b, 0)), ((d**3*x + 3*d**2*e*x**2/2 + d*e**2*x**3 + e**3*x
**4/4)*log(a**p*c), Eq(b, 0)), (-2*d**3*p*x + d**3*x*log(c*(b*x**2)**p) - 3
*d**2*e*p*x**2/2 + 3*d**2*e*x**2*log(c*(b*x**2)**p)/2 - 2*d*e**2*p*x**3/3 +
 d*e**2*x**3*log(c*(b*x**2)**p) - e**3*p*x**4/8 + e**3*x**4*log(c*(b*x**2)*
*p)/4, Eq(a, 0)), (-2*a**2*d*e**2*p*log(x - sqrt(-a/b))/(b**2*sqrt(-a/b)) +
 a**2*d*e**2*log(c*(a + b*x**2)**p)/(b**2*sqrt(-a/b)) - a**2*e**3*log(c*(a
+ b*x**2)**p)/(4*b**2) + 2*a*d**3*p*log(x - sqrt(-a/b))/(b*sqrt(-a/b)) - a*
d**3*log(c*(a + b*x**2)**p)/(b*sqrt(-a/b)) + 3*a*d**2*e*log(c*(a + b*x**2)*
*p)/(2*b) + 2*a*d*e**2*p*x/b + a*e**3*p*x**2/(4*b) - 2*d**3*p*x + d**3*x*lo
g(c*(a + b*x**2)**p) - 3*d**2*e*p*x**2/2 + 3*d**2*e*x**2*log(c*(a + b*x**2)
**p)/2 - 2*d*e**2*p*x**3/3 + d*e**2*x**3*log(c*(a + b*x**2)**p) - e**3*p*x*
*4/8 + e**3*x**4*log(c*(a + b*x**2)**p)/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.99

$$\int (d+ex)^3 \log \left(c(a+bx^2)^p\right) \, dx$$
$$= \frac{1}{24}\,bp\left(\frac{48\,(abd^3 - a^2de^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} - \frac{3\,be^3x^4 + 16\,bde^2x^3 + 6\,(6\,bd^2e - ae^3)x^2 + 48\,(bd^3 - ade^2)x}{b^2} + \right.$$
$$+ \frac{1}{4}\left(e^3x^4 + 4\,de^2x^3 + 6\,d^2ex^2 + 4\,d^3x\right)\log\left((bx^2+a)^pc\right)$$

```
[In] integrate((e*x+d)^3*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] 1/24*b*p*(48*(a*b*d^3 - a^2*d*e^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) -
(3*b*e^3*x^4 + 16*b*d*e^2*x^3 + 6*(6*b*d^2*e - a*e^3)*x^2 + 48*(b*d^3 - a*d
*e^2)*x)/b^2 + 6*(6*a*b*d^2*e - a^2*e^3)*log(b*x^2 + a)/b^3) + 1/4*(e^3*x^4
 + 4*d*e^2*x^3 + 6*d^2*e*x^2 + 4*d^3*x)*log((b*x^2 + a)^p*c)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.22

$$\int (d+ex)^3 \log \left(c(a+bx^2)^p\right) \, dx = -\frac{1}{8}\left(e^3p - 2\,e^3\log(c)\right)x^4 - \frac{1}{3}\left(2\,de^2p - 3\,de^2\log(c)\right)x^3$$
$$- \frac{(6\,bd^2ep - ae^3p - 6\,bd^2e\log(c))x^2}{4\,b}$$
$$+ \frac{1}{4}\left(e^3px^4 + 4\,de^2px^3 + 6\,d^2epx^2 + 4\,d^3px\right)\log\left(bx^2+a\right)$$
$$- \frac{(2\,bd^3p - 2\,ade^2p - bd^3\log(c))x}{b}$$
$$+ \frac{2\,(abd^3p - a^2de^2p)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$
$$+ \frac{(6\,abd^2ep - a^2e^3p)\log(bx^2+a)}{4\,b^2}$$

```
[In] integrate((e*x+d)^3*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] -1/8*(e^3*p - 2*e^3*log(c))*x^4 - 1/3*(2*d*e^2*p - 3*d*e^2*log(c))*x^3 - 1/
4*(6*b*d^2*e*p - a*e^3*p - 6*b*d^2*e*log(c))*x^2/b + 1/4*(e^3*p*x^4 + 4*d*e
^2*p*x^3 + 6*d^2*e*p*x^2 + 4*d^3*p*x)*log(b*x^2 + a) - (2*b*d^3*p - 2*a*d*e
^2*p - b*d^3*log(c))*x/b + 2*(a*b*d^3*p - a^2*d*e^2*p)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b) + 1/4*(6*a*b*d^2*e*p - a^2*e^3*p)*log(b*x^2 + a)/b^2
```

Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.25

$$\int (d+ex)^3\log\left(c\left(a+bx^2\right)^p\right)\,dx=\frac{e^3\,x^4\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{4}-2\,d^3\,p\,x-\frac{e^3\,p\,x^4}{8}+d^3\,x\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)+\frac{3\,d^2\,e\,x^2\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{2}+d\,e^2\,x^3\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)-\frac{3\,d^2\,e\,p\,x^2}{2}-\frac{2\,d\,e^2\,p\,x^3}{3}+\frac{a\,e^3\,p\,x^2}{4\,b}+\frac{2\,\sqrt{a}\,d^3\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}-\frac{a^2\,e^3\,p\,\ln\left(b\,x^2+a\right)}{4\,b^2}-\frac{2\,a^{3/2}\,d\,e^2\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}+\frac{2\,a\,d\,e^2\,p\,x}{b}+\frac{3\,a\,d^2\,e\,p\,\ln\left(b\,x^2+a\right)}{2\,b}$$

```
[In] int(log(c*(a + b*x^2)^p)*(d + e*x)^3,x)
```

```
[Out] (e^3*x^4*log(c*(a + b*x^2)^p))/4 - 2*d^3*p*x - (e^3*p*x^4)/8 + d^3*x*log(c*
(a + b*x^2)^p) + (3*d^2*e*x^2*log(c*(a + b*x^2)^p))/2 + d*e^2*x^3*log(c*(a
+ b*x^2)^p) - (3*d^2*e*p*x^2)/2 - (2*d*e^2*p*x^3)/3 + (a*e^3*p*x^2)/(4*b) +
 (2*a^(1/2)*d^3*p*atan((b^(1/2)*x)/a^(1/2)))/b^(1/2) - (a^2*e^3*p*log(a + b
*x^2))/(4*b^2) - (2*a^(3/2)*d*e^2*p*atan((b^(1/2)*x)/a^(1/2)))/b^(3/2) + (2
*a*d*e^2*p*x)/b + (3*a*d^2*e*p*log(a + b*x^2))/(2*b)
```

3.185 $\int (d+ex)^2 \log \left(c\left(a+bx^2\right)^p\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 141

$$\int (d+ex)^2 \log \left(c\left(a+bx^2\right)^p\right) \, dx = -\frac{2(3bd^2-ae^2)\,px}{3b} - depx^2 - \frac{2}{9}e^2px^3 + \frac{2\sqrt{a}(3bd^2-ae^2)\,p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}} - \frac{d(bd^2-3ae^2)\,p\log\left(a+bx^2\right)}{3be} + \frac{(d+ex)^3\log\left(c\left(a+bx^2\right)^p\right)}{3e}$$

```
[Out] -2/3*(-a*e^2+3*b*d^2)*p*x/b-d*e*p*x^2-2/9*e^2*p*x^3-1/3*d*(-3*a*e^2+b*d^2)*
p*ln(b*x^2+a)/b/e+1/3*(e*x+d)^3*ln(c*(b*x^2+a)^p)/e+2/3*(-a*e^2+3*b*d^2)*p*
arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2513, 815, 649, 211, 266}

$$\int (d+ex)^2 \log \left(c\left(a+bx^2\right)^p\right) \, dx = \frac{2\sqrt{a}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(3bd^2-ae^2)}{3b^{3/2}} + \frac{(d+ex)^3\log\left(c\left(a+bx^2\right)^p\right)}{3e} - \frac{dp(bd^2-3ae^2)\log\left(a+bx^2\right)}{3be} - \frac{2px(3bd^2-ae^2)}{3b} - depx^2 - \frac{2}{9}e^2px^3$$

```
[In] Int[(d + e*x)^2*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (-2*(3*b*d^2 - a*e^2)*p*x)/(3*b) - d*e*p*x^2 - (2*e^2*p*x^3)/9 + (2*Sqrt[a]
*(3*b*d^2 - a*e^2)*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*b^(3/2)) - (d*(b*d^2 -
 3*a*e^2)*p*Log[a + b*x^2])/(3*b*e) + ((d + e*x)^3*Log[c*(a + b*x^2)^p])/(3
*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^3\log\left(c\left(a+bx^2\right)^p\right)}{3e}-\frac{(2bp)\int\frac{x(d+ex)^3}{a+bx^2}\,dx}{3e}\\
&= \frac{(d+ex)^3\log\left(c\left(a+bx^2\right)^p\right)}{3e}\\
&\quad-\frac{(2bp)\int\left(\frac{e\left(3bd^2-ae^2\right)}{b^2}+\frac{3de^2x}{b}+\frac{e^3x^2}{b}-\frac{ae\left(3bd^2-ae^2\right)-bd\left(bd^2-3ae^2\right)x}{b^2\left(a+bx^2\right)}\right)dx}{3e}
\end{aligned}$$

$$= -\frac{2(3bd^2 - ae^2)\,px}{3b} - depx^2 - \frac{2}{9}e^2px^3 + \frac{(d+ex)^3\log\left(c(a+bx^2)^p\right)}{3e} + \frac{(2p)\int \frac{ae(3bd^2-ae^2)-bd(bd^2-3ae^2)x}{a+bx^2}\,dx}{3be}$$

$$= -\frac{2(3bd^2 - ae^2)\,px}{3b} - depx^2 - \frac{2}{9}e^2px^3 + \frac{(d+ex)^3\log\left(c(a+bx^2)^p\right)}{3e} - \frac{(2d(bd^2-3ae^2)\,p)\int \frac{x}{a+bx^2}\,dx}{3e} + \frac{(2a(3bd^2-ae^2)\,p)\int \frac{1}{a+bx^2}\,dx}{3b}$$

$$= -\frac{2(3bd^2 - ae^2)\,px}{3b} - depx^2 - \frac{2}{9}e^2px^3 + \frac{2\sqrt{a}(3bd^2-ae^2)\,p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}} - \frac{d(bd^2-3ae^2)\,p\log\left(a+bx^2\right)}{3be} + \frac{(d+ex)^3\log\left(c(a+bx^2)^p\right)}{3e}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.50

$$\int (d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$= \frac{3\left(-b^{3/2}d^3 - 3\sqrt{-a}bd^2e + 3a\sqrt{b}de^2 + \sqrt{-a}ae^3\right)p\log\left(\sqrt{-a}-\sqrt{b}x\right) - 3\left(b^{3/2}d^3 - 3\sqrt{-a}bd^2e - 3a\sqrt{b}de^2\right.}{}$$

```
[In] Integrate[(d + e*x)^2*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (3*(-(b^(3/2)*d^3) - 3*Sqrt[-a]*b*d^2*e + 3*a*Sqrt[b]*d*e^2 + Sqrt[-a]*a*e^
3)*p*Log[Sqrt[-a] - Sqrt[b]*x] - 3*(b^(3/2)*d^3 - 3*Sqrt[-a]*b*d^2*e - 3*a*
Sqrt[b]*d*e^2 + Sqrt[-a]*a*e^3)*p*Log[Sqrt[-a] + Sqrt[b]*x] + Sqrt[b]*(6*a*
e^3*p*x - b*e*p*x*(18*d^2 + 9*d*e*x + 2*e^2*x^2) + 3*b*(d + e*x)^3*Log[c*(a
 + b*x^2)^p]))/(9*b^(3/2)*e)
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.33

method	result
parts	$\frac{\ln\left(c\left(b\,x^2+a\right)^p\right)e^2x^3}{3}+\ln\left(c\left(b\,x^2+a\right)^p\right)e\,d\,x^2+d^2\ln\left(c\left(b\,x^2+a\right)^p\right)x+\frac{\ln\left(c\left(b\,x^2+a\right)^p\right)d^3}{3e}-\frac{2pb\left(-\frac{e\left(-\frac{1}{3}x^3b\,e^2\right.}{}\right.}{}$
risch	$-\frac{ie^2\pi\,x^3\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^3}{6}-\frac{ix\pi\,d^2\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^3}{2}-\frac{ie\pi d\,x^2\,\mathrm{csgn}\left(i\left(b\,x^2+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^2+a\right)^p\right)\mathrm{csgn}(ic)}{2}+\frac{2xap\,e^2}{3b}$

```
[In] int((e*x+d)^2*ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(c*(b*x^2+a)^p)*e^2*x^3+ln(c*(b*x^2+a)^p)*e*d*x^2+d^2*ln(c*(b*x^2+a)^
p)*x+1/3*ln(c*(b*x^2+a)^p)/e*d^3-2/3*p*b/e*(-e/b^2*(-1/3*x^3*b*e^2-3/2*b*d*
e*x^2+x*a*e^2-3*b*d^2*x)+1/b^2*(1/2*(-3*a*b*d*e^2+b^2*d^3)/b*ln(b*x^2+a)+(a
^2*e^3-3*a*b*d^2*e)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.27

$$\int (d+ex)^2 \log \left(c\left(a+bx^2\right)^p\right) \, dx$$

$$= \left[-\frac{2\,be^2px^3 + 9\,bdepx^2 - 3\,(3\,bd^2 - ae^2)p\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right) + 6\,(3\,bd^2 - ae^2)px - 3\,(be^2px^3 + 3\,bd}{9\,b} \right.$$

$$-\frac{2\,be^2px^3 + 9\,bdepx^2 - 6\,(3\,bd^2 - ae^2)p\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) + 6\,(3\,bd^2 - ae^2)px - 3\,(be^2px^3 + 3\,bdepx^2 +}{9\,b}$$

```
[In] integrate((e*x+d)^2*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [-1/9*(2*b*e^2*p*x^3 + 9*b*d*e*p*x^2 - 3*(3*b*d^2 - a*e^2)*p*sqrt(-a/b)*log
((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 6*(3*b*d^2 - a*e^2)*p*x - 3*
(b*e^2*p*x^3 + 3*b*d*e*p*x^2 + 3*b*d^2*p*x + 3*a*d*e*p)*log(b*x^2 + a) - 3*
(b*e^2*x^3 + 3*b*d*e*x^2 + 3*b*d^2*x)*log(c))/b, -1/9*(2*b*e^2*p*x^3 + 9*b*
d*e*p*x^2 - 6*(3*b*d^2 - a*e^2)*p*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 6*(3*
b*d^2 - a*e^2)*p*x - 3*(b*e^2*p*x^3 + 3*b*d*e*p*x^2 + 3*b*d^2*p*x + 3*a*d*e
*p)*log(b*x^2 + a) - 3*(b*e^2*x^3 + 3*b*d*e*x^2 + 3*b*d^2*x)*log(c))/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(131) = 262$.

Time = 8.66 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.65

$$\int (d+ex)^2 \log \left(c\left(a+bx^2\right)^p\right) \, dx$$
$$= \begin{cases} \left(d^2x+dex^2+\frac{e^2x^3}{3}\right)\log\left(0^pc\right) \\ \left(d^2x+dex^2+\frac{e^2x^3}{3}\right)\log\left(a^pc\right) \\ -2d^2px+d^2x\log\left(c(bx^2)^p\right)-depx^2+dex^2\log\left(c(bx^2)^p\right)-\frac{2e^2px^3}{9}+\frac{e^2x^3\log\left(c(bx^2)^p\right)}{3} \\ -\frac{2a^2e^2p\log\left(x-\sqrt{-\frac{a}{b}}\right)}{3b^2\sqrt{-\frac{a}{b}}}+\frac{a^2e^2\log\left(c(a+bx^2)^p\right)}{3b^2\sqrt{-\frac{a}{b}}}+\frac{2ad^2p\log\left(x-\sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}}-\frac{ad^2\log\left(c(a+bx^2)^p\right)}{b\sqrt{-\frac{a}{b}}}+\frac{ade\log\left(c(a+bx^2)^p\right)}{b}+\frac{2ae^2px}{3b} \end{cases}$$

```
[In] integrate((e*x+d)**2*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise(((d**2*x + d*e*x**2 + e**2*x**3/3)*log(0**p*c), Eq(a, 0) & Eq(b,
0)), ((d**2*x + d*e*x**2 + e**2*x**3/3)*log(a**p*c), Eq(b, 0)), (-2*d**2*p*
x + d**2*x*log(c*(b*x**2)**p) - d*e*p*x**2 + d*e*x**2*log(c*(b*x**2)**p) -
2*e**2*p*x**3/9 + e**2*x**3*log(c*(b*x**2)**p)/3, Eq(a, 0)), (-2*a**2*e**2*
p*log(x - sqrt(-a/b))/(3*b**2*sqrt(-a/b)) + a**2*e**2*log(c*(a + b*x**2)**p
)/(3*b**2*sqrt(-a/b)) + 2*a*d**2*p*log(x - sqrt(-a/b))/(b*sqrt(-a/b)) - a*d
**2*log(c*(a + b*x**2)**p)/(b*sqrt(-a/b)) + a*d*e*log(c*(a + b*x**2)**p)/b
+ 2*a*e**2*p*x/(3*b) - 2*d**2*p*x + d**2*x*log(c*(a + b*x**2)**p) - d*e*p*x
**2 + d*e*x**2*log(c*(a + b*x**2)**p) - 2*e**2*p*x**3/9 + e**2*x**3*log(c*(
a + b*x**2)**p)/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93

$$\int (d+ex)^2 \log \left(c\left(a+bx^2\right)^p\right) \, dx$$
$$= \frac{1}{9}\left(\frac{9\,ade\log\left(bx^2+a\right)}{b^2}+\frac{6\left(3\,abd^2-a^2e^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2}-\frac{2\,be^2x^3+9\,bdex^2+6\left(3\,bd^2-ae^2\right)x}{b^2}\right)bp$$
$$+\frac{1}{3}\left(e^2x^3+3\,dex^2+3\,d^2x\right)\log\left(\left(bx^2+a\right)^pc\right)$$

```
[In] integrate((e*x+d)^2*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] 1/9*(9*a*d*e*log(b*x^2 + a)/b^2 + 6*(3*a*b*d^2 - a^2*e^2)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*b^2) - (2*b*e^2*x^3 + 9*b*d*e*x^2 + 6*(3*b*d^2 - a*e^2)*x)/
b^2)*b*p + 1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)*log((b*x^2 + a)^p*c)
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.08

$$\int (d+ex)^2 \log \left(c\left(a+bx^2\right)^p\right) \, dx = -\frac{1}{9} \left(2 e^2 p - 3 e^2 \log \left(c\right)\right) x^3 + \frac{adep \log \left(bx^2+a\right)}{b} - \left(dep - de \log \left(c\right)\right) x^2 + \frac{1}{3} \left(e^2 p x^3 + 3 depx^2 + 3 d^2 px\right) \log \left(bx^2+a\right) - \frac{\left(6 bd^2 p - 2 ae^2 p - 3 bd^2 \log \left(c\right)\right) x}{3 b} + \frac{2 \left(3 abd^2 p - a^2 e^2 p\right) \arctan \left(\frac{bx}{\sqrt{ab}}\right)}{3 \sqrt{ab} b}$$

```
[In] integrate((e*x+d)^2*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] -1/9*(2*e^2*p - 3*e^2*log(c))*x^3 + a*d*e*p*log(b*x^2 + a)/b - (d*e*p - d*e
*log(c))*x^2 + 1/3*(e^2*p*x^3 + 3*d*e*p*x^2 + 3*d^2*p*x)*log(b*x^2 + a) - 1
/3*(6*b*d^2*p - 2*a*e^2*p - 3*b*d^2*log(c))*x/b + 2/3*(3*a*b*d^2*p - a^2*e^
2*p)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

Mupad [B] (verification not implemented)

Time = 4.45 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.87

$$\int (d+ex)^2 \log \left(c\left(a+bx^2\right)^p\right) \, dx = \frac{e^2 \, x^3 \, \ln \left(c \left(b \, x^2+a\right)^p\right)}{3} - 2 \, d^2 \, p \, x - \frac{2 \, e^2 \, p \, x^3}{9} + d^2 \, x \, \ln \left(c \left(b \, x^2+a\right)^p\right) + d \, e \, x^2 \, \ln \left(c \left(b \, x^2+a\right)^p\right) - d \, e \, p \, x^2 + \frac{2 \, a \, e^2 \, p \, x}{3 \, b} - \frac{2 \sqrt{a} \, d^2 \, p \, \mathrm{atan}\left(\frac{3 \sqrt{a} \, b^{3/2} \, d^2 \, p \, x}{a^2 \, e^2 \, p - 3 \, a \, b \, d^2 \, p} - \frac{a^{3/2} \sqrt{b} \, e^2 \, p \, x}{a^2 \, e^2 \, p - 3 \, a \, b \, d^2 \, p}\right)}{\sqrt{b}} + \frac{2 \, a^{3/2} \, e^2 \, p \, \mathrm{atan}\left(\frac{3 \sqrt{a} \, b^{3/2} \, d^2 \, p \, x}{a^2 \, e^2 \, p - 3 \, a \, b \, d^2 \, p} - \frac{a^{3/2} \sqrt{b} \, e^2 \, p \, x}{a^2 \, e^2 \, p - 3 \, a \, b \, d^2 \, p}\right)}{3 \, b^{3/2}} + \frac{a \, d \, e \, p \, \ln \left(b \, x^2+a\right)}{b}$$

```
[In] int(log(c*(a + b*x^2)^p)*(d + e*x)^2,x)
```

```
[Out] (e^2*x^3*log(c*(a + b*x^2)^p))/3 - 2*d^2*p*x - (2*e^2*p*x^3)/9 + d^2*x*log(
c*(a + b*x^2)^p) + d*e*x^2*log(c*(a + b*x^2)^p) - d*e*p*x^2 + (2*a*e^2*p*x)
/(3*b) - (2*a^(1/2)*d^2*p*atan((3*a^(1/2)*b^(3/2)*d^2*p*x)/(a^2*e^2*p - 3*a
*b*d^2*p) - (a^(3/2)*b^(1/2)*e^2*p*x)/(a^2*e^2*p - 3*a*b*d^2*p)))/b^(1/2) +
 (2*a^(3/2)*e^2*p*atan((3*a^(1/2)*b^(3/2)*d^2*p*x)/(a^2*e^2*p - 3*a*b*d^2*p
) - (a^(3/2)*b^(1/2)*e^2*p*x)/(a^2*e^2*p - 3*a*b*d^2*p)))/(3*b^(3/2)) + (a*
d*e*p*log(a + b*x^2))/b
```

3.186 $\int (d+ex)\log\left(c(a+bx^2)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 99

$$\int (d+ex)\log\left(c(a+bx^2)^p\right)\,dx = -2dpx-\frac{1}{2}epx^2+\frac{2\sqrt{a}dp\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} -\frac{(bd^2-ae^2)\,p\log(a+bx^2)}{2be}+\frac{(d+ex)^2\log\left(c(a+bx^2)^p\right)}{2e}$$

```
[Out] -2*d*p*x-1/2*e*p*x^2-1/2*(-a*e^2+b*d^2)*p*ln(b*x^2+a)/b/e+1/2*(e*x+d)^2*ln(
c*(b*x^2+a)^p)/e+2*d*p*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2513, 815, 649, 211, 266}

$$\int (d+ex)\log\left(c(a+bx^2)^p\right)\,dx = \frac{2\sqrt{a}dp\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{(d+ex)^2\log\left(c(a+bx^2)^p\right)}{2e} -\frac{p(bd^2-ae^2)\log(a+bx^2)}{2be}-2dpx-\frac{1}{2}epx^2$$

```
[In] Int[(d + e*x)*Log[c*(a + b*x^2)^p],x]
```

```
[Out] -2*d*p*x - (e*p*x^2)/2 + (2*Sqrt[a]*d*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b
] - ((b*d^2 - a*e^2)*p*Log[a + b*x^2])/(2*b*e) + ((d + e*x)^2*Log[c*(a + b*
x^2)^p])/(2*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e}-\frac{(bp)\int\frac{x(d+ex)^2}{a+bx^2}\,dx}{e}\\
&= \frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e}-\frac{(bp)\int\left(\frac{2de}{b}+\frac{e^2x}{b}-\frac{2ade-\left(bd^2-ae^2\right)x}{b\left(a+bx^2\right)}\right)dx}{e}\\
&= -2dpx-\frac{1}{2}epx^2+\frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e}+\frac{p\int\frac{2ade-\left(bd^2-ae^2\right)x}{a+bx^2}\,dx}{e}\\
&= -2dpx-\frac{1}{2}epx^2+\frac{(d+ex)^2\log\left(c\left(a+bx^2\right)^p\right)}{2e}\\
&\quad+(2adp)\int\frac{1}{a+bx^2}\,dx+\frac{\left(\left(-bd^2+ae^2\right)p\right)\int\frac{x}{a+bx^2}\,dx}{e}
\end{aligned}
$$

$$= -2dpx - \frac{1}{2}epx^2 + \frac{2\sqrt{a}dp\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} - \frac{(bd^2 - ae^2)\,p\log(a + bx^2)}{2be} + \frac{(d + ex)^2\log(c(a + bx^2)^p)}{2e}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

$$\int (d + ex)\log\left(c(a + bx^2)^p\right)\,dx = -2dpx + \frac{2\sqrt{a}dp\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} + dx\log\left(c(a + bx^2)^p\right) + \frac{1}{2}e\left(-px^2 + \frac{(a + bx^2)\log(c(a + bx^2)^p)}{b}\right)$$

[In] Integrate[(d + e*x)*Log[c*(a + b*x^2)^p],x]

[Out] -2*d*p*x + (2*Sqrt[a]*d*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + d*x*Log[c*(a + b*x^2)^p] + (e*(-(p*x^2) + ((a + b*x^2)*Log[c*(a + b*x^2)^p])/b))/2

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.79

method	result
default	$d\ln\left(c(b\,x^2 + a)^p\right)x - 2dpx + \frac{2dpa\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{e\left(\ln(c(b\,x^2+a)^p)(b\,x^2+a) - (b\,x^2+a)p\right)}{2b}$
parts	$\frac{\ln(c(b\,x^2+a)^p)e\,x^2}{2} + d\ln\left(c(b\,x^2 + a)^p\right)x - pb\left(\frac{\frac{1}{2}e\,x^2 + 2dx}{b} - \frac{a\left(\frac{e\ln(b\,x^2+a)}{2b} + \frac{2d\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{b}\right)$
risch	$\left(\frac{1}{2}e\,x^2 + dx\right)\ln\left((b\,x^2 + a)^p\right) - \frac{ix\pi d\,\mathrm{csgn}\left(i(b\,x^2+a)^p\right)\mathrm{csgn}\left(ic(b\,x^2+a)^p\right)\mathrm{csgn}(ic)}{2} + \frac{i\,\mathrm{csgn}(ic)\mathrm{csgn}\left(ic(b\,x^2+a)^p\right)^2x^2e\pi}{4}$

[In] int((e*x+d)*ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)

[Out] d*ln(c*(b*x^2+a)^p)*x-2*d*p*x+2*d*p*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+1/2*e/b*(ln(c*(b*x^2+a)^p)*(b*x^2+a)-(b*x^2+a)*p)

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.00

$$\int (d+ex)\log\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$=\left[-\frac{bepx^2-2\,bdp\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+4\,bdpx-(bepx^2+2\,bdpx+aep)\log\left(bx^2+a\right)-(bex^2+2}{2\,b}\right.$$
$$-\frac{bepx^2-4\,bdp\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+4\,bdpx-(bepx^2+2\,bdpx+aep)\log\left(bx^2+a\right)-(bex^2+2\,bdx)\log}{2\,b}$$

```
[In] integrate((e*x+d)*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [-1/2*(b*e*p*x^2 - 2*b*d*p*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b
*x^2 + a)) + 4*b*d*p*x - (b*e*p*x^2 + 2*b*d*p*x + a*e*p)*log(b*x^2 + a) - (
b*e*x^2 + 2*b*d*x)*log(c))/b, -1/2*(b*e*p*x^2 - 4*b*d*p*sqrt(a/b)*arctan(b*
x*sqrt(a/b)/a) + 4*b*d*p*x - (b*e*p*x^2 + 2*b*d*p*x + a*e*p)*log(b*x^2 + a)
 - (b*e*x^2 + 2*b*d*x)*log(c))/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(92) = 184.

Time = 4.32 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.01

$$\int (d+ex)\log\left(c\left(a+bx^2\right)^p\right)\,dx$$
$$=\begin{cases}\left(dx+\frac{ex^2}{2}\right)\log\left(0^pc\right)\\ \left(dx+\frac{ex^2}{2}\right)\log\left(a^pc\right)\\ -2dpx+dx\log\left(c\left(bx^2\right)^p\right)-\frac{epx^2}{2}+\frac{ex^2\log\left(c\left(bx^2\right)^p\right)}{2}\\ \frac{2adp\log\left(x-\sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}}-\frac{ad\log\left(c\left(a+bx^2\right)^p\right)}{b\sqrt{-\frac{a}{b}}}+\frac{ae\log\left(c\left(a+bx^2\right)^p\right)}{2b}-2dpx+dx\log\left(c\left(a+bx^2\right)^p\right)-\frac{epx^2}{2}+\frac{ex^2\log\left(c\left(a+bx^2\right.\right.}{2}\end{cases}$$

```
[In] integrate((e*x+d)*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise(((d*x + e*x**2/2)*log(0**p*c), Eq(a, 0) & Eq(b, 0)), ((d*x + e*x*
*2/2)*log(a**p*c), Eq(b, 0)), (-2*d*p*x + d*x*log(c*(b*x**2)**p) - e*p*x**2
```

```
/2 + e*x**2*log(c*(b*x**2)**p)/2, Eq(a, 0)), (2*a*d*p*log(x - sqrt(-a/b))/(
b*sqrt(-a/b)) - a*d*log(c*(a + b*x**2)**p)/(b*sqrt(-a/b)) + a*e*log(c*(a +
b*x**2)**p)/(2*b) - 2*d*p*x + d*x*log(c*(a + b*x**2)**p) - e*p*x**2/2 + e*x
**2*log(c*(a + b*x**2)**p)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.81

$$\int (d+ex)\log \left(c\left(a+bx^2\right)^p\right)\,dx = \frac{1}{2}\left(\frac{4\,ad\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}+\frac{ae\log\left(bx^2+a\right)}{b^2}-\frac{ex^2+4\,dx}{b}\right)bp + \frac{1}{2}\left(ex^2+2\,dx\right)\log\left(\left(bx^2+a\right)^pc\right)$$

```
[In] integrate((e*x+d)*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*d*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + a*e*log(b*x^2 + a)/b^2 - (
e*x^2 + 4*d*x)/b)*b*p + 1/2*(e*x^2 + 2*d*x)*log((b*x^2 + a)^p*c)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int (d+ex)\log \left(c\left(a+bx^2\right)^p\right)\,dx = \frac{2\,adp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} - \frac{1}{2}\left(ep-e\log\left(c\right)\right)x^2 + \frac{aep\log\left(bx^2+a\right)}{2\,b} - \left(2\,dp-d\log\left(c\right)\right)x + \frac{1}{2}\left(epx^2+2\,dpx\right)\log\left(bx^2+a\right)$$

```
[In] integrate((e*x+d)*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] 2*a*d*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) - 1/2*(e*p - e*log(c))*x^2 + 1/2*a*
e*p*log(b*x^2 + a)/b - (2*d*p - d*log(c))*x + 1/2*(e*p*x^2 + 2*d*p*x)*log(b
*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.82

$$\int (d+ex)\log \left(c\big(a+bx^2\big)^p\right)\,dx = d\,x\,\ln\left(c\,\left(b\,x^2+a\right)^p\right)-\frac{e\,p\,x^2}{2}-2\,d\,p\,x+\frac{e\,x^2\,\ln\left(c\,\left(b\,x^2+a\right)^p\right)}{2}+\frac{a\,e\,p\,\ln\left(b\,x^2+a\right)}{2\,b}+\frac{2\,\sqrt{a}\,d\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

```
[In] int(log(c*(a + b*x^2)^p)*(d + e*x),x)
[Out] d*x*log(c*(a + b*x^2)^p) - (e*p*x^2)/2 - 2*d*p*x + (e*x^2*log(c*(a + b*x^2)
^p))/2 + (a*e*p*log(a + b*x^2))/(2*b) + (2*a^(1/2)*d*p*atan((b^(1/2)*x)/a^(
1/2)))/b^(1/2)
```

3.187 $\int \log \left(c(a+bx^2)^p\right) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 45

$$\int \log \left(c(a+bx^2)^p\right) \, dx = -2px + \frac{2\sqrt{a}p \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} + x \log \left(c(a+bx^2)^p\right)$$

```
[Out] -2*p*x+x*ln(c*(b*x^2+a)^p)+2*p*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2498, 327, 211}

$$\int \log \left(c(a+bx^2)^p\right) \, dx = \frac{2\sqrt{a}p \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} + x \log \left(c(a+bx^2)^p\right) - 2px$$

```
[In] Int[Log[c*(a + b*x^2)^p],x]
```

```
[Out] -2*p*x + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + x*Log[c*(a + b
*x^2)^p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\log\left(c\left(a+bx^2\right)^p\right)-(2bp)\int\frac{x^2}{a+bx^2}\,dx\\ &= -2px+x\log\left(c\left(a+bx^2\right)^p\right)+(2ap)\int\frac{1}{a+bx^2}\,dx\\ &= -2px+\frac{2\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+x\log\left(c\left(a+bx^2\right)^p\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int\log\left(c\left(a+bx^2\right)^p\right)\,dx=-2px+\frac{2\sqrt{a}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+x\log\left(c\left(a+bx^2\right)^p\right)$$

```
[In] Integrate[Log[c*(a + b*x^2)^p],x]
[Out] -2*p*x + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + x*Log[c*(a + b
*x^2)^p]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result
default	$x\ln\left(c\left(b\,x^2+a\right)^p\right)-2pb\left(\frac{x}{b}-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}\right)$
parts	$x\ln\left(c\left(b\,x^2+a\right)^p\right)-2pb\left(\frac{x}{b}-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}\right)$
risch	$x\ln\left(\left(b\,x^2+a\right)^p\right)+\frac{i\operatorname{csgn}\left(ic\left(b\,x^2+a\right)^p\right)^2\operatorname{csgn}\left(i\left(b\,x^2+a\right)^p\right)x\pi}{2}-\frac{i\pi x\operatorname{csgn}\left(i\left(b\,x^2+a\right)^p\right)\operatorname{csgn}\left(ic\left(b\,x^2+a\right)^p\right)\operatorname{csgn}\left(ic\right)}{2}-\frac{i\pi x}{}$

```
[In] int(ln(c*(b*x^2+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(c*(b*x^2+a)^p)-2*p*b*(x/b-a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.38

$$\int \log\left(c(a+bx^2)^p\right)\,dx = \left[px\log\left(bx^2+a\right)+p\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-2\,px+x\log\left(c\right),px\log\left(bx^2+a\right)+2\,p\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-2\,px+x\log\left(c\right)\right]$$

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] [p*x*log(b*x^2 + a) + p*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^
2 + a)) - 2*p*x + x*log(c), p*x*log(b*x^2 + a) + 2*p*sqrt(a/b)*arctan(b*x*s
qrt(a/b)/a) - 2*p*x + x*log(c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(44) = 88.

Time = 2.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.22

$$\int \log\left(c(a+bx^2)^p\right)\,dx = \begin{cases} x\log\left(0^p c\right) & \text{for}\: a=0 \wedge b=0 \\ x\log\left(a^p c\right) & \text{for}\: b=0 \\ -2px+x\log\left(c(bx^2)^p\right) & \text{for}\: a=0 \\ \frac{2ap\log\left(x-\sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}}-\frac{a\log\left(c(a+bx^2)^p\right)}{b\sqrt{-\frac{a}{b}}}-2px+x\log\left(c(a+bx^2)^p\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(c*(b*x**2+a)**p),x)
```

```
[Out] Piecewise((x*log(0**p*c), Eq(a, 0) & Eq(b, 0)), (x*log(a**p*c), Eq(b, 0)),
(-2*p*x + x*log(c*(b*x**2)**p), Eq(a, 0)), (2*a*p*log(x - sqrt(-a/b))/(b*sq
rt(-a/b)) - a*log(c*(a + b*x**2)**p)/(b*sqrt(-a/b)) - 2*p*x + x*log(c*(a +
b*x**2)**p), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \log \left(c\left(a+bx^2\right)^p\right) \,dx = 2\,bp\left(\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}-\frac{x}{b}\right)+x\log\left(\left(bx^2+a\right)^pc\right)$$

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] 2*b*p*(a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - x/b) + x*log((b*x^2 + a)^p*c
)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \log \left(c\left(a+bx^2\right)^p\right) \,dx = px\log\left(bx^2+a\right)+\frac{2\,ap\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}-(2\,p-\log\left(c\right))x$$

```
[In] integrate(log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] p*x*log(b*x^2 + a) + 2*a*p*arctan(b*x/sqrt(a*b))/sqrt(a*b) - (2*p - log(c))
*x
```

Mupad [B] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \log \left(c\left(a+bx^2\right)^p\right) \,dx = x\,\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)-2\,p\,x+\frac{2\,\sqrt{a}\,p\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}$$

```
[In] int(log(c*(a + b*x^2)^p),x)
```

```
[Out] x*log(c*(a + b*x^2)^p) - 2*p*x + (2*a^(1/2)*p*atan((b^(1/2)*x)/a^(1/2)))/b^
(1/2)
```

3.188 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 201

$$\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx = -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e}$$

```
[Out] ln(e*x+d)*ln(c*(b*x^2+a)^p)/e-p*ln(e*x+d)*ln(e*((-a)^(1/2)-x*b^(1/2))/(e*(-
a)^(1/2)+d*b^(1/2)))/e-p*ln(e*x+d)*ln(-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(1
/2)+d*b^(1/2)))/e-p*polylog(2,(e*x+d)*b^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/e-
p*polylog(2,(e*x+d)*b^(1/2)/(e*(-a)^(1/2)+d*b^(1/2)))/e
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used

= {2512, 266, 2463, 2441, 2440, 2438}

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e} - \frac{p\,\text{PolyLog}\left(2, \frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e} - \frac{p\,\text{PolyLog}\left(2, \frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e} - \frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e} - \frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e}$$

```
[In] Int[Log[c*(a + b*x^2)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])
/e) - (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d
+ e*x])/e + (Log[d + e*x]*Log[c*(a + b*x^2)^p])/e - (p*PolyLog[2, (Sqrt[b]*
(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e - (p*PolyLog[2, (Sqrt[b]*(d + e*x))
/(Sqrt[b]*d + Sqrt[-a]*e)])/e
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{(2bp)\int \frac{x\log(d+ex)}{a+bx^2}\,dx}{e} \\
&= \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{(2bp)\int \left(-\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{e} \\
&= \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e} + \frac{\left(\sqrt{b}p\right)\int \frac{\log(d+ex)}{\sqrt{-a}-\sqrt{b}x}\,dx}{e} - \frac{\left(\sqrt{b}p\right)\int \frac{\log(d+ex)}{\sqrt{-a}+\sqrt{b}x}\,dx}{e} \\
&= -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e} \\
&\quad - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e} \\
&\quad + p\int \frac{\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)}{d+ex}\,dx + p\int \frac{\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{-\sqrt{b}d+\sqrt{-a}e}\right)}{d+ex}\,dx \\
&= -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e} \\
&\quad + \frac{\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e} + \frac{p\,\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{b}x}{-\sqrt{b}d+\sqrt{-a}e}\right)}{x}\,dx, x, d+ex\right)}{e} \\
&\quad + \frac{p\,\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{b}x}{\sqrt{b}d+\sqrt{-a}e}\right)}{x}\,dx, x, d+ex\right)}{e}
\end{aligned}
$$

$$= -\frac{p\log\left(\frac{e(\sqrt{-a}-\sqrt{b}x)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(\sqrt{-a}+\sqrt{b}x)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e} - \frac{p\operatorname{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e} - \frac{p\operatorname{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = -\frac{p\log\left(\frac{e(\sqrt{-a}-\sqrt{b}x)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(\sqrt{-a}+\sqrt{b}x)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e} - \frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e} - \frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e}$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])
/e) - (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d
+ e*x])/e + (Log[d + e*x]*Log[c*(a + b*x^2)^p])/e - (p*PolyLog[2, (Sqrt[b]*
(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e - (p*PolyLog[2, (Sqrt[b]*(d + e*x))
/(Sqrt[b]*d + Sqrt[-a]*e)])/e
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.98

method	result
parts	$\frac{\ln(ex+d)\ln\left(c(bx^2+a)^p\right)}{e} - \frac{2pb\left(\frac{\ln(ex+d)\left(\ln\left(\frac{e\sqrt{-ab}-(ex+d)b+bd}{e\sqrt{-ab}+bd}\right)+\ln\left(\frac{e\sqrt{-ab}+(ex+d)b-bd}{e\sqrt{-ab}-bd}\right)\right)}{2b}+\frac{\operatorname{dilog}\left(\frac{e\sqrt{-ab}-(ex+d)b+bd}{e\sqrt{-ab}+bd}\right)+\operatorname{dilog}\left(\cdots\right.}{2b}\cdots\right.}{e}\cdots$
risch	$\frac{\ln\left((bx^2+a)^p\right)\ln(ex+d)}{e} - \frac{p\ln(ex+d)\ln\left(\frac{e\sqrt{-ab}-(ex+d)b+bd}{e\sqrt{-ab}+bd}\right)}{e} - \frac{p\ln(ex+d)\ln\left(\frac{e\sqrt{-ab}+(ex+d)b-bd}{e\sqrt{-ab}-bd}\right)}{e} - \frac{p\operatorname{dilog}\left(\frac{e\sqrt{-ab}-(ex+d)\cdots}{e\sqrt{-ab}+bd}\right.\cdots}{e}\cdots$

```
[In] int(ln(c*(b*x^2+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] ln(e*x+d)*ln(c*(b*x^2+a)^p)/e-2*p*b/e*(1/2*ln(e*x+d)*(ln((e*(-a*b)^(1/2)-(e
*x+d)*b+b*d)/(e*(-a*b)^(1/2)+b*d))+ln((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a
*b)^(1/2)-b*d)))/b+1/2*(dilog((e*(-a*b)^(1/2)-(e*x+d)*b+b*d)/(e*(-a*b)^(1/2
)+b*d))+dilog((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1/2)-b*d)))/b)
```

Fricas [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx$$

```
[In] integrate(ln(c*(b*x**2+a)**p)/(e*x+d),x)
[Out] Integral(log(c*(a + b*x**2)**p)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)}{d+e\,x}\,dx$$

```
[In] int(log(c*(a + b*x^2)^p)/(d + e*x),x)
[Out] int(log(c*(a + b*x^2)^p)/(d + e*x), x)
```

3.189 $\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 119

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^2}\,dx = \frac{2\sqrt{a}\sqrt{b}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{bd^2+ae^2} - \frac{2bdp\log(d+ex)}{e\left(bd^2+ae^2\right)} + \frac{bdp\log\left(a+bx^2\right)}{e\left(bd^2+ae^2\right)} - \frac{\log\left(c(a+bx^2)^p\right)}{e(d+ex)}$$

```
[Out] -2*b*d*p*ln(e*x+d)/e/(a*e^2+b*d^2)+b*d*p*ln(b*x^2+a)/e/(a*e^2+b*d^2)-ln(c*(
b*x^2+a)^p)/e/(e*x+d)+2*p*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)*b^(1/2)/(a*e^2+
b*d^2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2513, 815, 649, 211, 266}

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^2}\,dx = \frac{2\sqrt{a}\sqrt{b}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{ae^2+bd^2} - \frac{\log\left(c(a+bx^2)^p\right)}{e(d+ex)} + \frac{bdp\log\left(a+bx^2\right)}{e\left(ae^2+bd^2\right)} - \frac{2bdp\log(d+ex)}{e\left(ae^2+bd^2\right)}$$

```
[In] Int[Log[c*(a + b*x^2)^p]/(d + e*x)^2,x]
```

```
[Out] (2*Sqrt[a]*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b*d^2 + a*e^2) - (2*b*d*
p*Log[d + e*x])/(e*(b*d^2 + a*e^2)) + (b*d*p*Log[a + b*x^2])/(e*(b*d^2 + a*
e^2)) - Log[c*(a + b*x^2)^p]/(e*(d + e*x))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\log\left(c(a+bx^2)^p\right)}{e(d+ex)}+\frac{(2bp)\int\frac{x}{(d+ex)(a+bx^2)}\,dx}{e}\\
&= -\frac{\log\left(c(a+bx^2)^p\right)}{e(d+ex)}+\frac{(2bp)\int\left(-\frac{de}{(bd^2+ae^2)(d+ex)}+\frac{ae+bdx}{(bd^2+ae^2)(a+bx^2)}\right)dx}{e}\\
&= -\frac{2bdp\log(d+ex)}{e\left(bd^2+ae^2\right)}-\frac{\log\left(c(a+bx^2)^p\right)}{e(d+ex)}+\frac{(2bp)\int\frac{ae+bdx}{a+bx^2}\,dx}{e\left(bd^2+ae^2\right)}\\
&= -\frac{2bdp\log(d+ex)}{e\left(bd^2+ae^2\right)}-\frac{\log\left(c(a+bx^2)^p\right)}{e(d+ex)}+\frac{(2abp)\int\frac{1}{a+bx^2}\,dx}{bd^2+ae^2}+\frac{(2b^2dp)\int\frac{x}{a+bx^2}\,dx}{e\left(bd^2+ae^2\right)}\\
&= \frac{2\sqrt{a}\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{bd^2+ae^2}-\frac{2bdp\log(d+ex)}{e\left(bd^2+ae^2\right)}+\frac{bdp\log\left(a+bx^2\right)}{e\left(bd^2+ae^2\right)}-\frac{\log\left(c(a+bx^2)^p\right)}{e(d+ex)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.15

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^2}\,dx$$
$$= \frac{2\sqrt{a}\sqrt{b}ep(d+ex)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-2bdp(d+ex)\log(d+ex)+bd^2p\log\left(a+bx^2\right)+bdepx\log\left(a+bx^2\right)-b}{e\left(bd^2+ae^2\right)(d+ex)}$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(d + e*x)^2,x]
```

```
[Out] (2*Sqrt[a]*Sqrt[b]*e*p*(d + e*x)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] - 2*b*d*p*(d +
 e*x)*Log[d + e*x] + b*d^2*p*Log[a + b*x^2] + b*d*e*p*x*Log[a + b*x^2] - b*
d^2*Log[c*(a + b*x^2)^p] - a*e^2*Log[c*(a + b*x^2)^p])/(e*(b*d^2 + a*e^2)*(
d + e*x))
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.83

method	result	size
parts	$-\frac{\ln\left(c\left(b\,x^2+a\right)^p\right)}{e(ex+d)}+\frac{2pb\left(-\frac{d\ln(ex+d)}{a\,e^2+b\,d^2}+\frac{\frac{d\ln\left(b\,x^2+a\right)}{2}+\frac{ae\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{a\,e^2+b\,d^2}\right)}{e}$	99
risch	Expression too large to display	1233

```
[In] int(ln(c*(b*x^2+a)^p)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(c*(b*x^2+a)^p)/e/(e*x+d)+2*p*b/e*(-d/(a*e^2+b*d^2)*ln(e*x+d)+1/(a*e^2+b
*d^2)*(1/2*d*ln(b*x^2+a)+a*e/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.19

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^2}\,dx$$
$$= \left[\frac{(e^2px+dep)\sqrt{-ab}\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)+(bdepx-ae^2p)\log\left(bx^2+a\right)-2\left(bdepx+bd^2p\right)\log\left(ex+d\right)-}{bd^3e+ade^3+(bd^2e^2+ae^4)x}\right.$$

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [((e^2*p*x + d*e*p)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) + (b*d*e*p*x - a*e^2*p)*log(b*x^2 + a) - 2*(b*d*e*p*x + b*d^2*p)*log(e*x
+ d) - (b*d^2 + a*e^2)*log(c))/(b*d^3*e + a*d*e^3 + (b*d^2*e^2 + a*e^4)*x),
 (2*(e^2*p*x + d*e*p)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (b*d*e*p*x - a*e^2*
p)*log(b*x^2 + a) - 2*(b*d*e*p*x + b*d^2*p)*log(e*x + d) - (b*d^2 + a*e^2)*
log(c))/(b*d^3*e + a*d*e^3 + (b*d^2*e^2 + a*e^4)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(b*x**2+a)**p)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.91

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^2}\,dx = \frac{\left(\frac{2\,ae\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(bd^2+ae^2)\sqrt{ab}} + \frac{d\log(bx^2+a)}{bd^2+ae^2} - \frac{2\,d\log(ex+d)}{bd^2+ae^2}\right)bp}{e} - \frac{\log\left((bx^2+a)^p c\right)}{(ex+d)e}$$

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] (2*a*e*arctan(b*x/sqrt(a*b))/((b*d^2 + a*e^2)*sqrt(a*b)) + d*log(b*x^2 + a)
/(b*d^2 + a*e^2) - 2*d*log(e*x + d)/(b*d^2 + a*e^2))*b*p/e - log((b*x^2 + a
)^p*c)/((e*x + d)*e)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^2}\,dx = \frac{bdp\log\left(bx^2+a\right)}{bd^2e+ae^3} - \frac{2\,bdp\log\left(ex+d\right)}{bd^2e+ae^3} + \frac{2\,abp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(bd^2+ae^2)\sqrt{ab}} - \frac{p\log\left(bx^2+a\right)}{e^2x+de} - \frac{\log\left(c\right)}{e^2x+de}$$

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] b*d*p*log(b*x^2 + a)/(b*d^2*e + a*e^3) - 2*b*d*p*log(e*x + d)/(b*d^2*e + a*
e^3) + 2*a*b*p*arctan(b*x/sqrt(a*b))/((b*d^2 + a*e^2)*sqrt(a*b)) - p*log(b*
x^2 + a)/(e^2*x + d*e) - log(c)/(e^2*x + d*e)
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.83

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^2}\,dx$$
$$= \frac{\ln\left(\frac{4\,b^3\,p^2\,x}{e}-\frac{p\,\left(b\,d+e\,\sqrt{-a\,b}\right)\,\left(2\,a\,b^2\,e\,p+2\,b^3\,d\,p\,x-\frac{2\,b^2\,e\,p\,\left(b\,d+e\,\sqrt{-a\,b}\right)\,\left(-b\,x\,d^2+4\,a\,d\,e+3\,a\,x\,e^2\right)}{b\,d^2\,e+a\,e^3}\right)}{b\,d^2\,e+a\,e^3}\right)\,\left(b\,d\,p+e\,p\,\sqrt{-a\,b}\right)}{b\,d^2\,e+a\,e^3}$$
$$-\frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{e\,\left(d+e\,x\right)}$$
$$+\frac{\ln\left(\frac{4\,b^3\,p^2\,x}{e}-\frac{p\,\left(b\,d-e\,\sqrt{-a\,b}\right)\,\left(2\,a\,b^2\,e\,p+2\,b^3\,d\,p\,x-\frac{2\,b^2\,e\,p\,\left(b\,d-e\,\sqrt{-a\,b}\right)\,\left(-b\,x\,d^2+4\,a\,d\,e+3\,a\,x\,e^2\right)}{b\,d^2\,e+a\,e^3}\right)}{b\,d^2\,e+a\,e^3}\right)\,\left(b\,d\,p-e\,p\,\sqrt{-a\,b}\right)}{b\,d^2\,e+a\,e^3}$$
$$-\frac{2\,b\,d\,p\,\ln\left(d+e\,x\right)}{b\,d^2\,e+a\,e^3}$$

```
[In] int(log(c*(a + b*x^2)^p)/(d + e*x)^2,x)
```

```
[Out] (log((4*b^3*p^2*x)/e - (p*(b*d + e*(-a*b)^(1/2))*(2*a*b^2*e*p + 2*b^3*d*p*x
 - (2*b^2*e*p*(b*d + e*(-a*b)^(1/2))*(4*a*d*e + 3*a*e^2*x - b*d^2*x))/(a*e^
3 + b*d^2*e)))/(a*e^3 + b*d^2*e))*(b*d*p + e*p*(-a*b)^(1/2)))/(a*e^3 + b*d^
2*e) - log(c*(a + b*x^2)^p)/(e*(d + e*x)) + (log((4*b^3*p^2*x)/e - (p*(b*d
- e*(-a*b)^(1/2))*(2*a*b^2*e*p + 2*b^3*d*p*x - (2*b^2*e*p*(b*d - e*(-a*b)^(
1/2))*(4*a*d*e + 3*a*e^2*x - b*d^2*x))/(a*e^3 + b*d^2*e)))/(a*e^3 + b*d^2*e
))*(b*d*p - e*p*(-a*b)^(1/2)))/(a*e^3 + b*d^2*e) - (2*b*d*p*log(d + e*x))/(
a*e^3 + b*d^2*e)
```

3.190 $\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 174

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^3}\,dx = \frac{bdp}{e\left(bd^2+ae^2\right)(d+ex)} + \frac{2\sqrt{a}b^{3/2}dp\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\left(bd^2+ae^2\right)^2} - \frac{b(bd^2-ae^2)\,p\log(d+ex)}{e\left(bd^2+ae^2\right)^2} + \frac{b(bd^2-ae^2)\,p\log\left(a+bx^2\right)}{2e\left(bd^2+ae^2\right)^2} - \frac{\log\left(c(a+bx^2)^p\right)}{2e(d+ex)^2}$$

```
[Out] b*d*p/e/(a*e^2+b*d^2)/(e*x+d)-b*(-a*e^2+b*d^2)*p*ln(e*x+d)/e/(a*e^2+b*d^2)^
2+1/2*b*(-a*e^2+b*d^2)*p*ln(b*x^2+a)/e/(a*e^2+b*d^2)^2-1/2*ln(c*(b*x^2+a)^p
)/e/(e*x+d)^2+2*b^(3/2)*d*p*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/(a*e^2+b*d^2)
^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2513, 815, 649, 211, 266}

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^3}\,dx = \frac{2\sqrt{a}b^{3/2}dp\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\left(ae^2+bd^2\right)^2} - \frac{\log\left(c(a+bx^2)^p\right)}{2e(d+ex)^2} + \frac{bp(bd^2-ae^2)\log\left(a+bx^2\right)}{2e\left(ae^2+bd^2\right)^2} + \frac{bdp}{e(d+ex)\left(ae^2+bd^2\right)} - \frac{bp(bd^2-ae^2)\log(d+ex)}{e\left(ae^2+bd^2\right)^2}$$

```
[In] Int[Log[c*(a + b*x^2)^p]/(d + e*x)^3,x]
```

```
[Out] (b*d*p)/(e*(b*d^2 + a*e^2)*(d + e*x)) + (2*Sqrt[a]*b^(3/2)*d*p*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(b*d^2 + a*e^2)^2 - (b*(b*d^2 - a*e^2)*p*Log[d + e*x])/(e*
(b*d^2 + a*e^2)^2) + (b*(b*d^2 - a*e^2)*p*Log[a + b*x^2])/(2*e*(b*d^2 + a*e
^2)^2) - Log[c*(a + b*x^2)^p]/(2*e*(d + e*x)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2e(d+ex)^2}+\frac{(bp)\int\frac{x}{(d+ex)^2\left(a+bx^2\right)}\,dx}{e}\\
&= -\frac{\log\left(c\left(a+bx^2\right)^p\right)}{2e(d+ex)^2}+\frac{(bp)\int\left(-\frac{de}{\left(bd^2+ae^2\right)(d+ex)^2}+\frac{e\left(-bd^2+ae^2\right)}{\left(bd^2+ae^2\right)^2(d+ex)}+\frac{b\left(2ade+\left(bd^2-ae^2\right)x\right)}{\left(bd^2+ae^2\right)^2\left(a+bx^2\right)}\right)dx}{e}
\end{aligned}$$

$$= \frac{bdp}{e\left(bd^2+ae^2\right)(d+ex)} - \frac{b(bd^2-ae^2)\,p\log(d+ex)}{e\left(bd^2+ae^2\right)^2} - \frac{\log\left(c(a+bx^2)^p\right)}{2e(d+ex)^2} + \frac{(b^2p)\int\frac{2ade+(bd^2-ae^2)x}{a+bx^2}\,dx}{e\left(bd^2+ae^2\right)^2}$$

$$= \frac{bdp}{e\left(bd^2+ae^2\right)(d+ex)} - \frac{b(bd^2-ae^2)\,p\log(d+ex)}{e\left(bd^2+ae^2\right)^2} - \frac{\log\left(c(a+bx^2)^p\right)}{2e(d+ex)^2} + \frac{(2ab^2dp)\int\frac{1}{a+bx^2}\,dx}{\left(bd^2+ae^2\right)^2} + \frac{\left(b^2(bd^2-ae^2)\,p\right)\int\frac{x}{a+bx^2}\,dx}{e\left(bd^2+ae^2\right)^2}$$

$$= \frac{bdp}{e\left(bd^2+ae^2\right)(d+ex)} + \frac{2\sqrt{a}b^{3/2}dp\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\left(bd^2+ae^2\right)^2} - \frac{b(bd^2-ae^2)\,p\log(d+ex)}{e\left(bd^2+ae^2\right)^2} + \frac{b(bd^2-ae^2)\,p\log\left(a+bx^2\right)}{2e\left(bd^2+ae^2\right)^2} - \frac{\log\left(c(a+bx^2)^p\right)}{2e(d+ex)^2}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.25

$$\int\frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^3}\,dx = \frac{\frac{bp(d+ex)\left(\left(\sqrt{-a}bd^2+2a\sqrt{b}de+(-a)^{3/2}e^2\right)(d+ex)\log\left(\sqrt{-a}-\sqrt{b}x\right)+\left(\sqrt{-a}bd^2-2a\sqrt{b}de+(-a)^{3/2}e^2\right)(d+ex)\log\left(\sqrt{-a}+\sqrt{b}x\right)+2\sqrt{-a}(bd^3+ac}{\sqrt{-a}(bd^2+ae^2)^2}}{2e(d+ex)^2}$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(d + e*x)^3,x]
```

```
[Out] ((b*p*(d + e*x)*((Sqrt[-a]*b*d^2 + 2*a*Sqrt[b]*d*e + (-a)^(3/2)*e^2)*(d + e
*x)*Log[Sqrt[-a] - Sqrt[b]*x] + (Sqrt[-a]*b*d^2 - 2*a*Sqrt[b]*d*e + (-a)^(3
/2)*e^2)*(d + e*x)*Log[Sqrt[-a] + Sqrt[b]*x] + 2*Sqrt[-a]*(b*d^3 + a*d*e^2
- (b*d^2 - a*e^2)*(d + e*x)*Log[d + e*x])))/(Sqrt[-a]*(b*d^2 + a*e^2)^2) -
Log[c*(a + b*x^2)^p])/(2*e*(d + e*x)^2)
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.84

method	result	size
parts	$-\frac{\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)}{2e\left(ex+d\right)^{2}}+\frac{pb\left(\frac{\left(a\,e^{2}-b\,d^{2}\right)\ln\left(ex+d\right)}{\left(a\,e^{2}+b\,d^{2}\right)^{2}}+\frac{d}{\left(a\,e^{2}+b\,d^{2}\right)\left(ex+d\right)}+\frac{b\left(\frac{\left(-a\,e^{2}+b\,d^{2}\right)\ln\left(b\,x^{2}+a\right)}{2b}+\frac{2ade\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{\left(a\,e^{2}+b\,d^{2}\right)^{2}}\right)}{e}$	147
risch	Expression too large to display	2684

```
[In] int(ln(c*(b*x^2+a)^p)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(c*(b*x^2+a)^p)/e/(e*x+d)^2+p*b/e*((a*e^2-b*d^2)/(a*e^2+b*d^2)^2*ln(
e*x+d)+d/(a*e^2+b*d^2)/(e*x+d)+b/(a*e^2+b*d^2)^2*(1/2*(-a*e^2+b*d^2)/b*ln(b
*x^2+a)+2*a*d*e/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(162) = 324.

Time = 0.36 (sec) , antiderivative size = 744, normalized size of antiderivative = 4.28

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^3}\,dx$$
$$= \left[\frac{2\left(b^2d^3e+abde^3\right)px+2\left(bde^3px^2+2\,bd^2e^2px+bd^3ep\right)\sqrt{-ab}\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)+2\left(b^2d^4+abd^2e^2\right)p+(\ldots}{2\,(b^2\ldots}\right.$$

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2*d^3*e + a*b*d*e^3)*p*x + 2*(b*d*e^3*p*x^2 + 2*b*d^2*e^2*p*x +
b*d^3*e*p)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(b^
2*d^4 + a*b*d^2*e^2)*p + ((b^2*d^2*e^2 - a*b*e^4)*p*x^2 + 2*(b^2*d^3*e - a*
b*d*e^3)*p*x - (3*a*b*d^2*e^2 + a^2*e^4)*p)*log(b*x^2 + a) - 2*((b^2*d^2*e^
2 - a*b*e^4)*p*x^2 + 2*(b^2*d^3*e - a*b*d*e^3)*p*x + (b^2*d^4 - a*b*d^2*e^2
)*p)*log(e*x + d) - (b^2*d^4 + 2*a*b*d^2*e^2 + a^2*e^4)*log(c))/(b^2*d^6*e
+ 2*a*b*d^4*e^3 + a^2*d^2*e^5 + (b^2*d^4*e^3 + 2*a*b*d^2*e^5 + a^2*e^7)*x^2
 + 2*(b^2*d^5*e^2 + 2*a*b*d^3*e^4 + a^2*d*e^6)*x), 1/2*(2*(b^2*d^3*e + a*b*
d*e^3)*p*x + 4*(b*d*e^3*p*x^2 + 2*b*d^2*e^2*p*x + b*d^3*e*p)*sqrt(a*b)*arct
an(sqrt(a*b)*x/a) + 2*(b^2*d^4 + a*b*d^2*e^2)*p + ((b^2*d^2*e^2 - a*b*e^4)*
p*x^2 + 2*(b^2*d^3*e - a*b*d*e^3)*p*x - (3*a*b*d^2*e^2 + a^2*e^4)*p)*log(b*
x^2 + a) - 2*((b^2*d^2*e^2 - a*b*e^4)*p*x^2 + 2*(b^2*d^3*e - a*b*d*e^3)*p*x
 + (b^2*d^4 - a*b*d^2*e^2)*p)*log(e*x + d) - (b^2*d^4 + 2*a*b*d^2*e^2 + a^2
*e^4)*log(c))/(b^2*d^6*e + 2*a*b*d^4*e^3 + a^2*d^2*e^5 + (b^2*d^4*e^3 + 2*a
*b*d^2*e^5 + a^2*e^7)*x^2 + 2*(b^2*d^5*e^2 + 2*a*b*d^3*e^4 + a^2*d*e^6)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^3}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(b*x**2+a)**p)/(e*x+d)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.18

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^3}\,dx = \frac{\left(\frac{4\,abde\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2d^4+2\,abd^2e^2+a^2e^4)\sqrt{ab}} + \frac{(bd^2-ae^2)\log(bx^2+a)}{b^2d^4+2\,abd^2e^2+a^2e^4} - \frac{2\,(bd^2-ae^2)\log(ex+d)}{b^2d^4+2\,abd^2e^2+a^2e^4} + \frac{2\,d}{bd^3+ade^2+(bd^2e+ae^3)x}\right)bp}{2\,e} - \frac{\log\left((bx^2+a)^p c\right)}{2\,(ex+d)^2 e}$$

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^3,x, algorithm="maxima")
[Out] 1/2*(4*a*b*d*e*arctan(b*x/sqrt(a*b))/((b^2*d^4 + 2*a*b*d^2*e^2 + a^2*e^4)*s
qrt(a*b)) + (b*d^2 - a*e^2)*log(b*x^2 + a)/(b^2*d^4 + 2*a*b*d^2*e^2 + a^2*e
^4) - 2*(b*d^2 - a*e^2)*log(e*x + d)/(b^2*d^4 + 2*a*b*d^2*e^2 + a^2*e^4) +
2*d/(b*d^3 + a*d*e^2 + (b*d^2*e + a*e^3)*x))*b*p/e - 1/2*log((b*x^2 + a)^p*
c)/((e*x + d)^2*e)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.60

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^3}\,dx = \frac{2\,ab^2dp\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2d^4+2\,abd^2e^2+a^2e^4)\sqrt{ab}} + \frac{(b^2d^2p-abe^2p)\log(bx^2+a)}{2\,(b^2d^4e+2\,abd^2e^3+a^2e^5)} - \frac{p\log(bx^2+a)}{2\,(e^3x^2+2\,de^2x+d^2e)} - \frac{(b^2d^2p-abe^2p)\log(ex+d)}{b^2d^4e+2\,abd^2e^3+a^2e^5} + \frac{2\,bdepx+2\,bd^2p-bd^2\log(c)-ae^2\log(c)}{2\,(bd^2e^3x^2+ae^5x^2+2\,bd^3e^2x+2\,ade^4x+bd^4e+ad^2e^3)}$$

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 2*a*b^2*d*p*arctan(b*x/sqrt(a*b))/((b^2*d^4 + 2*a*b*d^2*e^2 + a^2*e^4)*sqrt
(a*b)) + 1/2*(b^2*d^2*p - a*b*e^2*p)*log(b*x^2 + a)/(b^2*d^4*e + 2*a*b*d^2*
e^3 + a^2*e^5) - 1/2*p*log(b*x^2 + a)/(e^3*x^2 + 2*d*e^2*x + d^2*e) - (b^2*
d^2*p - a*b*e^2*p)*log(e*x + d)/(b^2*d^4*e + 2*a*b*d^2*e^3 + a^2*e^5) + 1/2
*(2*b*d*e*p*x + 2*b*d^2*p - b*d^2*log(c) - a*e^2*log(c))/(b*d^2*e^3*x^2 + a
*e^5*x^2 + 2*b*d^3*e^2*x + 2*a*d*e^4*x + b*d^4*e + a*d^2*e^3)
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.56

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{(d+ex)^3}\,dx = \frac{\ln\left(b^2\,x+\sqrt{-a\,b^3}\right)\,\left(b^2\,d^2\,p-a\,b\,e^2\,p+2\,d\,e\,p\,\sqrt{-a\,b^3}\right)}{2\,\left(a^2\,e^5+2\,a\,b\,d^2\,e^3+b^2\,d^4\,e\right)} - \frac{\ln\left(d+e\,x\right)\,\left(b^2\,d^2\,p-a\,b\,e^2\,p\right)}{a^2\,e^5+2\,a\,b\,d^2\,e^3+b^2\,d^4\,e} - \frac{\ln\left(c\,\left(b\,x^2+a\right)^p\right)}{2\,e\,\left(d^2+2\,d\,e\,x+e^2\,x^2\right)} - \frac{\ln\left(b^2\,x-\sqrt{-a\,b^3}\right)\,\left(a\,b\,e^2\,p-b^2\,d^2\,p+2\,d\,e\,p\,\sqrt{-a\,b^3}\right)}{2\,\left(a^2\,e^5+2\,a\,b\,d^2\,e^3+b^2\,d^4\,e\right)} + \frac{b\,d\,p}{\left(x\,e^2+d\,e\right)\,\left(b\,d^2+a\,e^2\right)}$$

```
[In] int(log(c*(a + b*x^2)^p)/(d + e*x)^3,x)
```

```
[Out] (log(b^2*x + (-a*b^3)^(1/2))*(b^2*d^2*p - a*b*e^2*p + 2*d*e*p*(-a*b^3)^(1/2
)))/(2*(a^2*e^5 + b^2*d^4*e + 2*a*b*d^2*e^3)) - (log(d + e*x)*(b^2*d^2*p -
a*b*e^2*p))/(a^2*e^5 + b^2*d^4*e + 2*a*b*d^2*e^3) - log(c*(a + b*x^2)^p)/(2
*e*(d^2 + e^2*x^2 + 2*d*e*x)) - (log(b^2*x - (-a*b^3)^(1/2))*(a*b*e^2*p - b
^2*d^2*p + 2*d*e*p*(-a*b^3)^(1/2)))/(2*(a^2*e^5 + b^2*d^4*e + 2*a*b*d^2*e^3
)) + (b*d*p)/((d*e + e^2*x)*(a*e^2 + b*d^2))
```

3.191 $\int (d+ex)^3 \log \left(c(a+bx^3)^p\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 320

$$\begin{aligned}
&\int (d+ex)^3 \log \left(c(a+bx^3)^p\right) \, dx \\
&= -\frac{3(4bd^3-ae^3)\,px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 \\
&\quad - \frac{\sqrt{3}\sqrt[3]{a}\left(4bd^3+6\sqrt[3]{a}b^{2/3}d^2e-ae^3\right)p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{4b^{4/3}} \\
&\quad + \frac{\sqrt[3]{a}\left(4bd^3-6\sqrt[3]{a}b^{2/3}d^2e-ae^3\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4b^{4/3}} \\
&\quad - \frac{\sqrt[3]{a}\left(4bd^3-6\sqrt[3]{a}b^{2/3}d^2e-ae^3\right)p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8b^{4/3}} \\
&\quad - \frac{d(bd^3-4ae^3)\,p\log(a+bx^3)}{4be} + \frac{(d+ex)^4\log\left(c(a+bx^3)^p\right)}{4e}
\end{aligned}$$

```
[Out] -3/4*(-a*e^3+4*b*d^3)*p*x/b-9/4*d^2*e*p*x^2-d*e^2*p*x^3-3/16*e^3*p*x^4+1/4*
a^(1/3)*(4*b*d^3-6*a^(1/3)*b^(2/3)*d^2*e-a*e^3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(
4/3)-1/8*a^(1/3)*(4*b*d^3-6*a^(1/3)*b^(2/3)*d^2*e-a*e^3)*p*ln(a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/b^(4/3)-1/4*d*(-4*a*e^3+b*d^3)*p*ln(b*x^3+a)/b/e+
1/4*(e*x+d)^4*ln(c*(b*x^3+a)^p)/e-1/4*a^(1/3)*(4*b*d^3+6*a^(1/3)*b^(2/3)*d^
2*e-a*e^3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(4
/3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {2513, 1850, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int (d+ex)^3 \log \left(c\left(a+bx^3\right)^p\right) \, dx$$
$$= -\frac{\sqrt[3]{a}p\left(-6\sqrt[3]{a}b^{2/3}d^2e-ae^3+4bd^3\right)\log \left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8b^{4/3}}$$
$$-\frac{\sqrt{3}\sqrt[3]{a}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(6\sqrt[3]{a}b^{2/3}d^2e-ae^3+4bd^3\right)}{4b^{4/3}}$$
$$+\frac{\sqrt[3]{a}p\left(-6\sqrt[3]{a}b^{2/3}d^2e-ae^3+4bd^3\right)\log \left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4b^{4/3}}+\frac{(d+ex)^4\log \left(c\left(a+bx^3\right)^p\right)}{4e}$$
$$-\frac{dp\left(bd^3-4ae^3\right)\log \left(a+bx^3\right)}{4be}-\frac{3px\left(4bd^3-ae^3\right)}{4b}-\frac{9}{4}d^2epx^2-de^2px^3-\frac{3}{16}e^3px^4$$

```
[In] Int[(d + e*x)^3*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (-3*(4*b*d^3 - a*e^3)*p*x)/(4*b) - (9*d^2*e*p*x^2)/4 - d*e^2*p*x^3 - (3*e^3
*p*x^4)/16 - (Sqrt[3]*a^(1/3)*(4*b*d^3 + 6*a^(1/3)*b^(2/3)*d^2*e - a*e^3)*p
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(4*b^(4/3)) + (a^(1/3)*
(4*b*d^3 - 6*a^(1/3)*b^(2/3)*d^2*e - a*e^3)*p*Log[a^(1/3) + b^(1/3)*x])/(4*
b^(4/3)) - (a^(1/3)*(4*b*d^3 - 6*a^(1/3)*b^(2/3)*d^2*e - a*e^3)*p*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(8*b^(4/3)) - (d*(b*d^3 - 4*a*e^3)*p
*Log[a + b*x^3])/(4*b*e) + ((d + e*x)^4*Log[c*(a + b*x^3)^p])/(4*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d+ex)^4\log\left(c\left(a+bx^3\right)^p\right)}{4e}-\frac{(3bp)\int\frac{x^2(d+ex)^4}{a+bx^3}\,dx}{4e}\\
&=-\frac{3}{16}e^3px^4+\frac{(d+ex)^4\log\left(c\left(a+bx^3\right)^p\right)}{4e}-\frac{(3p)\int\frac{x^2\left(4bd^4+4e\left(4bd^3-ae^3\right)x+24bd^2e^2x^2+16bde^3x^3\right)}{a+bx^3}\,dx}{16e}\\
&=-de^2px^3-\frac{3}{16}e^3px^4+\frac{(d+ex)^4\log\left(c\left(a+bx^3\right)^p\right)}{4e}\\
&\quad-\frac{p\int\frac{x^2\left(12bd\left(bd^3-4ae^3\right)+12be\left(4bd^3-ae^3\right)x+72b^2d^2e^2x^2\right)}{a+bx^3}\,dx}{16be}\\
&=-de^2px^3-\frac{3}{16}e^3px^4+\frac{(d+ex)^4\log\left(c\left(a+bx^3\right)^p\right)}{4e}\\
&\quad-\frac{p\int\left(12e\left(4bd^3-ae^3\right)+72bd^2e^2x-\frac{12\left(ae\left(4bd^3-ae^3\right)+6abd^2e^2x-bd\left(bd^3-4ae^3\right)x^2\right)}{a+bx^3}\right)dx}{16be}\\
&=-\frac{3\left(4bd^3-ae^3\right)px}{4b}-\frac{9}{4}d^2epx^2-de^2px^3-\frac{3}{16}e^3px^4\\
&\quad+\frac{(d+ex)^4\log\left(c\left(a+bx^3\right)^p\right)}{4e}+\frac{(3p)\int\frac{ae\left(4bd^3-ae^3\right)+6abd^2e^2x-bd\left(bd^3-4ae^3\right)x^2}{a+bx^3}\,dx}{4be}\\
&=-\frac{3\left(4bd^3-ae^3\right)px}{4b}-\frac{9}{4}d^2epx^2-de^2px^3-\frac{3}{16}e^3px^4+\frac{(d+ex)^4\log\left(c\left(a+bx^3\right)^p\right)}{4e}\\
&\quad+\frac{(3p)\int\frac{ae\left(4bd^3-ae^3\right)+6abd^2e^2x}{a+bx^3}\,dx}{4be}-\frac{\left(3d\left(bd^3-4ae^3\right)p\right)\int\frac{x^2}{a+bx^3}\,dx}{4e}
\end{aligned}
$$

$$= -\frac{3(4bd^3 - ae^3)\,px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 - \frac{d(bd^3 - 4ae^3)\,p\log(a + bx^3)}{4be} + \frac{(d + ex)^4\log\left(c(a + bx^3)^p\right)}{4e} + \frac{p\int \frac{\sqrt[3]{a}\left(6a^{4/3}bd^2e^2 + 2a\sqrt[3]{b}e(4bd^3 - ae^3)\right) + \sqrt[3]{b}\left(6a^{4/3}bd^2e^2 - a\sqrt[3]{b}e(4bd^3 - ae^3)\right)x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx}{4a^{2/3}b^{4/3}e} + \frac{\left(\sqrt[3]{a}\left(4bd^3 - 6\sqrt[3]{a}b^{2/3}d^2e - ae^3\right)p\right)\int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x}\,dx}{4b}$$

$$= -\frac{3(4bd^3 - ae^3)\,px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 + \frac{\sqrt[3]{a}\left(4bd^3 - 6\sqrt[3]{a}b^{2/3}d^2e - ae^3\right)p\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{4b^{4/3}} - \frac{d(bd^3 - 4ae^3)\,p\log(a + bx^3)}{4be} + \frac{(d + ex)^4\log\left(c(a + bx^3)^p\right)}{4e} - \frac{\left(\sqrt[3]{a}\left(4bd^3 - 6\sqrt[3]{a}b^{2/3}d^2e - ae^3\right)p\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx}{8b^{4/3}} + \frac{\left(3a^{2/3}\left(4bd^3 + 6\sqrt[3]{a}b^{2/3}d^2e - ae^3\right)p\right)\int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx}{8b}$$

$$= -\frac{3(4bd^3 - ae^3)\,px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 + \frac{\sqrt[3]{a}\left(4bd^3 - 6\sqrt[3]{a}b^{2/3}d^2e - ae^3\right)p\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{4b^{4/3}} - \frac{\sqrt[3]{a}\left(4bd^3 - 6\sqrt[3]{a}b^{2/3}d^2e - ae^3\right)p\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{8b^{4/3}} - \frac{d(bd^3 - 4ae^3)\,p\log(a + bx^3)}{4be} + \frac{(d + ex)^4\log\left(c(a + bx^3)^p\right)}{4e} + \frac{\left(3\sqrt[3]{a}\left(4bd^3 + 6\sqrt[3]{a}b^{2/3}d^2e - ae^3\right)p\right)\text{Subst}\left(\int \frac{1}{-3 - x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{4b^{4/3}}$$

$$= -\frac{3(4bd^3-ae^3)\,px}{4b} - \frac{9}{4}d^2epx^2 - de^2px^3 - \frac{3}{16}e^3px^4 - \frac{\sqrt{3}\sqrt[3]{a}\left(4bd^3+6\sqrt[3]{a}b^{2/3}d^2e-ae^3\right)p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{4b^{4/3}} + \frac{\sqrt[3]{a}\left(4bd^3-6\sqrt[3]{a}b^{2/3}d^2e-ae^3\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{4b^{4/3}} - \frac{\sqrt[3]{a}\left(4bd^3-6\sqrt[3]{a}b^{2/3}d^2e-ae^3\right)p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{8b^{4/3}} - \frac{d(bd^3-4ae^3)\,p\log\left(a+bx^3\right)}{4be} + \frac{(d+ex)^4\log\left(c(a+bx^3)^p\right)}{4e}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.32 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.89

$$\int (d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)\,dx$$

$$= \frac{\frac{3e(-4bd^3+ae^3)px}{b} - 9d^2e^2px^2 - 4de^3px^3 - \frac{3}{4}e^4px^4 + \frac{\sqrt{3}\sqrt[3]{a}e(-4bd^3+ae^3)p\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{4/3}} + 9d^2e^2px^2\,\text{Hypergeom}}{}$$

```
[In] Integrate[(d + e*x)^3*Log[c*(a + b*x^3)^p],x]
```

```
[Out] ((3*e*(-4*b*d^3 + a*e^3)*p*x)/b - 9*d^2*e^2*p*x^2 - 4*d*e^3*p*x^3 - (3*e^4*
p*x^4)/4 + (Sqrt[3]*a^(1/3)*e*(-4*b*d^3 + a*e^3)*p*ArcTan[(1 - (2*b^(1/3)*x
)/a^(1/3))/Sqrt[3]])/b^(4/3) + 9*d^2*e^2*p*x^2*Hypergeometric2F1[2/3, 1, 5/
3, -((b*x^3)/a)] + (a^(1/3)*e*(4*b*d^3 - a*e^3)*p*Log[a^(1/3) + b^(1/3)*x])
/b^(4/3) + (a^(1/3)*e*(-4*b*d^3 + a*e^3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(2*b^(4/3)) - (d*(b*d^3 - 4*a*e^3)*p*Log[a + b*x^3])/b + (d
 + e*x)^4*Log[c*(a + b*x^3)^p])/(4*e)
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.24

method	result
parts	$\frac{\ln(c(b\,x^3+a)^p)e^3x^4}{4}+\ln\left(c(b\,x^3+a)^p\right)e^2d\,x^3+\frac{3\ln(c(b\,x^3+a)^p)e\,d^2x^2}{2}+d^3\ln\left(c(b\,x^3+a)^p\right)x+\frac{\ln(c(b\,x^3+a}{4e}$
risch	$-\frac{ie^3\pi\,x^4\text{csgn}(ic(b\,x^3+a)^p)^3}{8}-\frac{i\pi\,d^3x\text{csgn}(ic(b\,x^3+a)^p)^3}{2}+\frac{(ex+d)^4\ln((b\,x^3+a)^p)}{4e}-3d^3px+\frac{3e^3apx}{4b}+\frac{e^3\ln(c)x^4}{4}-$

```
[In] int((e*x+d)^3*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)

[Out] 1/4*ln(c*(b*x^3+a)^p)*e^3*x^4+ln(c*(b*x^3+a)^p)*e^2*d*x^3+3/2*ln(c*(b*x^3+a
)^p)*e*d^2*x^2+d^3*ln(c*(b*x^3+a)^p)*x+1/4*ln(c*(b*x^3+a)^p)/e*d^4-3/4*p*b/
e*(-e/b^2*(-1/4*x^4*b*e^3-4/3*x^3*b*d*e^2-3*e*d^2*b*x^2+x*a*e^3-4*b*d^3*x)+
((a^2*e^4-4*a*b*d^3*e)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/
3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1)))-6*a*b*d^2*e^2*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(
1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/
b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*(-4*a*b*d*e^3+b^2*d^4
)/b*ln(b*x^3+a))/b^2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.18 (sec) , antiderivative size = 8840, normalized size of antiderivative = 27.62

$$\int (d+ex)^3 \log\left(c\left(a+bx^3\right)^p\right)\,dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^3*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 23.69 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.83

$$\begin{aligned}\int (d+ex)^3 \log\left(c\left(a+bx^3\right)^p\right)\,dx = & -\frac{3a^2e^3p\operatorname{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)}{4b} \\ & + 3ad^3p\operatorname{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right) \\ & + \frac{9ad^2ep\operatorname{RootSum}\left(27t^3ab^2+1,\left(t\mapsto t\log\left(9t^2ab+x\right)\right)\right)}{2} \\ & + ade^2p\left(\begin{cases}\frac{x^3}{a} & \text{for } b=0 \\ \frac{\log\left(a+bx^3\right)}{b} & \text{otherwise}\end{cases}\right) \\ & + \frac{3ae^3px}{4b} - 3d^3px + d^3x\log\left(c\left(a+bx^3\right)^p\right) \\ & - \frac{9d^2epx^2}{4} + \frac{3d^2ex^2\log\left(c\left(a+bx^3\right)^p\right)}{2} \\ & - de^2px^3 + de^2x^3\log\left(c\left(a+bx^3\right)^p\right) \\ & - \frac{3e^3px^4}{16} + \frac{e^3x^4\log\left(c\left(a+bx^3\right)^p\right)}{4}\end{aligned}$$

```
[In] integrate((e*x+d)**3*ln(c*(b*x**3+a)**p),x)
```

```
[Out] -3*a**2*e**3*p*RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x)))
/(4*b) + 3*a*d**3*p*RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a +
 x))) + 9*a*d**2*e*p*RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2
*a*b + x)))/2 + a*d*e**2*p*Piecewise((x**3/a, Eq(b, 0)), (log(a + b*x**3)/b
, True)) + 3*a*e**3*p*x/(4*b) - 3*d**3*p*x + d**3*x*log(c*(a + b*x**3)**p)
- 9*d**2*e*p*x**2/4 + 3*d**2*e*x**2*log(c*(a + b*x**3)**p)/2 - d*e**2*p*x**
3 + d*e**2*x**3*log(c*(a + b*x**3)**p) - 3*e**3*p*x**4/16 + e**3*x**4*log(c
*(a + b*x**3)**p)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.04

$$\int (d+ex)^3 \log \left(c\left(a+bx^3\right)^p\right)\, dx$$

$$= \frac{1}{16}\, bp \left(\frac{4\,\sqrt{3}\left(6\,abd^2e\left(\frac{a}{b}\right)^{\frac{2}{3}} + 4\,abd^3\left(\frac{a}{b}\right)^{\frac{1}{3}} - a^2e^3\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) \arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^2} - \frac{3\,be^3x^4 + 16\,bde^2x^3 + 36\right.$$

$$+ \frac{1}{4}\left(e^3x^4 + 4\,de^2x^3 + 6\,d^2ex^2 + 4\,d^3x\right) \log\left(\left(bx^3+a\right)^p c\right)$$

```
[In] integrate((e*x+d)^3*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] 1/16*b*p*(4*sqrt(3)*(6*a*b*d^2*e*(a/b)^(2/3) + 4*a*b*d^3*(a/b)^(1/3) - a^2*
e^3*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2
) - (3*b*e^3*x^4 + 16*b*d*e^2*x^3 + 36*b*d^2*e*x^2 + 12*(4*b*d^3 - a*e^3)*x
)/b^2 + 2*(8*a*b*d*e^2*(a/b)^(2/3) + 6*a*b*d^2*e*(a/b)^(1/3) - 4*a*b*d^3 +
a^2*e^3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 4*(4*a*
b*d*e^2*(a/b)^(2/3) - 6*a*b*d^2*e*(a/b)^(1/3) + 4*a*b*d^3 - a^2*e^3)*log(x
+ (a/b)^(1/3))/(b^3*(a/b)^(2/3))) + 1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^
2 + 4*d^3*x)*log((b*x^3 + a)^p*c)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.19

$$\int (d+ex)^3 \log\left(c\left(a+bx^3\right)^p\right)\,dx$$
$$= -\frac{1}{16}\left(3\,e^3p-4\,e^3\log\left(c\right)\right)x^4+\frac{ade^2p\log\left(\left|bx^3+a\right|\right)}{b}-\left(de^2p-de^2\log\left(c\right)\right)x^3$$
$$-\frac{3}{4}\left(3\,d^2ep-2\,d^2e\log\left(c\right)\right)x^2+\frac{1}{4}\left(e^3px^4+4\,de^2px^3+6\,d^2epx^2+4\,d^3px\right)\log\left(bx^3+a\right)$$
$$-\frac{\left(12\,bd^3p-3\,ae^3p-4\,bd^3\log\left(c\right)\right)x}{4\,b}$$
$$+\frac{\sqrt{3}\left(4\left(-ab^2\right)^{\frac{1}{3}}bd^3p-\left(-ab^2\right)^{\frac{1}{3}}ae^3p-6\left(-ab^2\right)^{\frac{2}{3}}d^2ep\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{4\,b^2}$$
$$+\frac{\left(4\left(-ab^2\right)^{\frac{1}{3}}bd^3p-\left(-ab^2\right)^{\frac{1}{3}}ae^3p+6\left(-ab^2\right)^{\frac{2}{3}}d^2ep\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{8\,b^2}$$
$$-\frac{\left(6\,ab^3d^2ep\left(-\frac{a}{b}\right)^{\frac{1}{3}}+4\,ab^3d^3p-a^2b^2e^3p\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{4\,ab^3}$$

```
[In] integrate((e*x+d)^3*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] -1/16*(3*e^3*p - 4*e^3*log(c))*x^4 + a*d*e^2*p*log(abs(b*x^3 + a))/b - (d*e
^2*p - d*e^2*log(c))*x^3 - 3/4*(3*d^2*e*p - 2*d^2*e*log(c))*x^2 + 1/4*(e^3*
p*x^4 + 4*d*e^2*p*x^3 + 6*d^2*e*p*x^2 + 4*d^3*p*x)*log(b*x^3 + a) - 1/4*(12
*b*d^3*p - 3*a*e^3*p - 4*b*d^3*log(c))*x/b + 1/4*sqrt(3)*(4*(-a*b^2)^(1/3)*
b*d^3*p - (-a*b^2)^(1/3)*a*e^3*p - 6*(-a*b^2)^(2/3)*d^2*e*p)*arctan(1/3*sqr
t(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^2 + 1/8*(4*(-a*b^2)^(1/3)*b*d^3*p
 - (-a*b^2)^(1/3)*a*e^3*p + 6*(-a*b^2)^(2/3)*d^2*e*p)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/b^2 - 1/4*(6*a*b^3*d^2*e*p*(-a/b)^(1/3) + 4*a*b^3*d^3*p
 - a^2*b^2*e^3*p)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.68

$$\int (d+ex)^3 \log \left(c\left(a+bx^3\right)^p\right) \, dx = \ln \left(c\, \left(b\, x^3+a\right)^p\right)\, \left(d^3\, x+\frac{3\, d^2\, e\, x^2}{2}+d\, e^2\, x^3+\frac{e^3\, x^4}{4}\right) - x\, \left(3\, d^3\, p-\frac{3\, a\, e^3\, p}{4\, b}\right)+\left(\sum_{k=1}^{3}\ln\left(x\, \left(\frac{9\, a^3\, d\, e^5\, p^2}{4}+\frac{45\, b\, a^2\, d^4\, e^2\, p^2}{4}\right) +\mathrm{root}\left(64\, b^4\, c^3-192\, a\, b^3\, c^2\, d\, e^2\, p+288\, a\, b^3\, c\, d^5\, e\, p^2+120\, a^2\, b^2\, c\, d^2\, e^4\, p^2-4\, a^3\, b\, d^3\, e^6\, p^3-24\, a^2\, b^2\, d^6\, e^3\, p^3-6 + \frac{45\, a^3\, d^2\, e^4\, p^2}{8}+\frac{27\, a^2\, b\, d^5\, e\, p^2}{2}\right)\, \mathrm{root}\left(64\, b^4\, c^3-192\, a\, b^3\, c^2\, d\, e^2\, p+288\, a\, b^3\, c\, d^5\, e\, p^2 + 120\, a^2\, b^2\, c\, d^2\, e^4\, p^2-4\, a^3\, b\, d^3\, e^6\, p^3-24\, a^2\, b^2\, d^6\, e^3\, p^3-64\, a\, b^3\, d^9\, p^3+a^4\, e^9\, p^3,c,k\right)\right) - \frac{3\, e^3\, p\, x^4}{16}-\frac{9\, d^2\, e\, p\, x^2}{4}-d\, e^2\, p\, x^3$$

```
[In] int(log(c*(a + b*x^3)^p)*(d + e*x)^3,x)
```

```
[Out] log(c*(a + b*x^3)^p)*(d^3*x + (e^3*x^4)/4 + (3*d^2*e*x^2)/2 + d*e^2*x^3) -
x*(3*d^3*p - (3*a*e^3*p)/(4*b)) + symsum(log(x*((9*a^3*d*e^5*p^2)/4 + (45*a
^2*b*d^4*e^2*p^2)/4) + root(64*b^4*c^3 - 192*a*b^3*c^2*d*e^2*p + 288*a*b^3*
c*d^5*e*p^2 + 120*a^2*b^2*c*d^2*e^4*p^2 - 4*a^3*b*d^3*e^6*p^3 - 24*a^2*b^2*
d^6*e^3*p^3 - 64*a*b^3*d^9*p^3 + a^4*e^9*p^3, c, k)*(x*(9*a*b^2*d^3*p - (9*
a^2*b*e^3*p)/4) + 9*root(64*b^4*c^3 - 192*a*b^3*c^2*d*e^2*p + 288*a*b^3*c*d
^5*e*p^2 + 120*a^2*b^2*c*d^2*e^4*p^2 - 4*a^3*b*d^3*e^6*p^3 - 24*a^2*b^2*d^6
*e^3*p^3 - 64*a*b^3*d^9*p^3 + a^4*e^9*p^3, c, k)*a*b^2 - 18*a^2*b*d*e^2*p)
+ (45*a^3*d^2*e^4*p^2)/8 + (27*a^2*b*d^5*e*p^2)/2)*root(64*b^4*c^3 - 192*a*
b^3*c^2*d*e^2*p + 288*a*b^3*c*d^5*e*p^2 + 120*a^2*b^2*c*d^2*e^4*p^2 - 4*a^3
*b*d^3*e^6*p^3 - 24*a^2*b^2*d^6*e^3*p^3 - 64*a*b^3*d^9*p^3 + a^4*e^9*p^3, c
, k), k, 1, 3) - (3*e^3*p*x^4)/16 - (9*d^2*e*p*x^2)/4 - d*e^2*p*x^3
```

3.192 $\int (d+ex)^2 \log \left(c(a+bx^3)^p\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 250

$$\begin{aligned}\int (d+ex)^2 \log \left(c(a+bx^3)^p\right) \, dx = & -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 \\ & - \frac{\sqrt{3}\sqrt[3]{a}d\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}} \\ & + \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} \\ & - \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2b^{2/3}} \\ & - \frac{(bd^3-ae^3)\,p\log(a+bx^3)}{3be} \\ & + \frac{(d+ex)^3\log\left(c(a+bx^3)^p\right)}{3e}\end{aligned}$$

```
[Out] -3*d^2*p*x-3/2*d*e*p*x^2-1/3*e^2*p*x^3+a^(1/3)*d*(b^(1/3)*d-a^(1/3)*e)*p*ln
(a^(1/3)+b^(1/3)*x)/b^(2/3)-1/2*a^(1/3)*d*(b^(1/3)*d-a^(1/3)*e)*p*ln(a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(2/3)-1/3*(-a*e^3+b*d^3)*p*ln(b*x^3+a)/b
/e+1/3*(e*x+d)^3*ln(c*(b*x^3+a)^p)/e-a^(1/3)*d*(b^(1/3)*d+a^(1/3)*e)*p*arct
an(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(2/3)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {2513, 1850, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int (d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)\,dx=-\frac{\sqrt[3]{a}dp\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2b^{2/3}}-\frac{\sqrt{3}\sqrt[3]{a}dp\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)}{b^{2/3}}+\frac{\sqrt[3]{a}dp\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+\frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e}-\frac{p\left(bd^3-ae^3\right)\log\left(a+bx^3\right)}{3be}-3d^2px-\frac{3}{2}depx^2-\frac{1}{3}e^2px^3$$

```
[In] Int[(d + e*x)^2*Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*d^2*p*x - (3*d*e*p*x^2)/2 - (e^2*p*x^3)/3 - (Sqrt[3]*a^(1/3)*d*(b^(1/3)*
d + a^(1/3)*e)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/b^(2/3)
 + (a^(1/3)*d*(b^(1/3)*d - a^(1/3)*e)*p*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) -
 (a^(1/3)*d*(b^(1/3)*d - a^(1/3)*e)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(2*b^(2/3)) - ((b*d^3 - a*e^3)*p*Log[a + b*x^3])/(3*b*e) + ((d +
 e*x)^3*Log[c*(a + b*x^3)^p])/(3*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e} - \frac{(bp)\int\frac{x^2(d+ex)^3}{a+bx^3}\,dx}{e} \\
&= -\frac{1}{3}e^2px^3 + \frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e} - \frac{p\int\frac{x^2\left(3\left(bd^3-ae^3\right)+9bd^2ex+9bde^2x^2\right)}{a+bx^3}\,dx}{3e} \\
&= -\frac{1}{3}e^2px^3 + \frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e} - \frac{p\int\left(9d^2e+9de^2x-\frac{3\left(3ad^2e+3ade^2x-\left(bd^3-ae^3\right)x^2\right)}{a+bx^3}\right)dx}{3e} \\
&= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 + \frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e} + \frac{p\int\frac{3ad^2e+3ade^2x-\left(bd^3-ae^3\right)x^2}{a+bx^3}\,dx}{e} \\
&= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 + \frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e} \\
&\quad + \frac{p\int\frac{3ad^2e+3ade^2x}{a+bx^3}\,dx}{e} - \frac{\left(\left(bd^3-ae^3\right)p\right)\int\frac{x^2}{a+bx^3}\,dx}{e} \\
&= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 - \frac{\left(bd^3-ae^3\right)p\log\left(a+bx^3\right)}{3be} \\
&\quad + \frac{(d+ex)^3\log\left(c\left(a+bx^3\right)^p\right)}{3e} \\
&\quad + \frac{p\int\frac{\sqrt[3]{a}\left(6a\sqrt[3]{b}d^2e+3a^{4/3}de^2\right)+\sqrt[3]{b}\left(-3a\sqrt[3]{b}d^2e+3a^{4/3}de^2\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}\sqrt[3]{b}e} \\
&\quad - \frac{\left(\left(-3a\sqrt[3]{b}d^2e+3a^{4/3}de^2\right)p\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}\sqrt[3]{b}e}
\end{aligned}
$$

$$= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 + \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}d - \sqrt[3]{a}e\right)p\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{b^{2/3}} - \frac{(bd^3 - ae^3)\,p\log(a + bx^3)}{3be} + \frac{(d + ex)^3\log\left(c(a + bx^3)^p\right)}{3e} - \frac{\left(\sqrt[3]{a}d\left(\sqrt[3]{b}d - \sqrt[3]{a}e\right)p\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2b^{2/3}} + \frac{1}{2}\left(3a^{2/3}d\left(d + \frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)p\right)\int\frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}\,dx$$

$$= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 + \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}d - \sqrt[3]{a}e\right)p\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{b^{2/3}} - \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}d - \sqrt[3]{a}e\right)p\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{2b^{2/3}} - \frac{(bd^3 - ae^3)\,p\log(a + bx^3)}{3be} + \frac{(d + ex)^3\log\left(c(a + bx^3)^p\right)}{3e} + \frac{\left(3\sqrt[3]{a}d\left(\sqrt[3]{b}d + \sqrt[3]{a}e\right)p\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{b^{2/3}}$$

$$= -3d^2px - \frac{3}{2}depx^2 - \frac{1}{3}e^2px^3 - \frac{\sqrt{3}\sqrt[3]{a}d\left(\sqrt[3]{b}d + \sqrt[3]{a}e\right)p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}} + \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}d - \sqrt[3]{a}e\right)p\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{b^{2/3}} - \frac{\sqrt[3]{a}d\left(\sqrt[3]{b}d - \sqrt[3]{a}e\right)p\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{2b^{2/3}} - \frac{(bd^3 - ae^3)\,p\log(a + bx^3)}{3be} + \frac{(d + ex)^3\log\left(c(a + bx^3)^p\right)}{3e}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.92

$$\int (d + ex)^2\log\left(c\left(a + bx^3\right)^p\right)\,dx$$

$$= \frac{-\frac{p\left(18bd^2ex+9bde^2x^2+2be^3x^3+6\sqrt{3}\sqrt[3]{a}b^{2/3}d^2e\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-9bde^2x^2\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a}\right)-6\sqrt[3]{a}b^{2/3}d^2e\log\left(\sqrt[3]{a}+\right.\right.}{2b}}{3e}$$

```
[In] Integrate[(d + e*x)^2*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (-1/2*(p*(18*b*d^2*e*x + 9*b*d*e^2*x^2 + 2*b*e^3*x^3 + 6*Sqrt[3]*a^(1/3)*b^
(2/3)*d^2*e*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 9*b*d*e^2*x^2*Hyp
ergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)] - 6*a^(1/3)*b^(2/3)*d^2*e*Log[a^(
1/3) + b^(1/3)*x] + 3*a^(1/3)*b^(2/3)*d^2*e*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2] + 2*(b*d^3 - a*e^3)*Log[a + b*x^3]))/b + (d + e*x)^3*Log[c*
(a + b*x^3)^p])/(3*e)
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.34

method	result
parts	$\frac{\ln\left(c\left(b\,x^3+a\right)^p\right)e^2x^3}{3}+\ln\left(c\left(b\,x^3+a\right)^p\right)ed\,x^2+d^2\ln\left(c\left(b\,x^3+a\right)^p\right)x+\frac{\ln\left(c\left(b\,x^3+a\right)^p\right)d^3}{3e}-\frac{pb\left(\frac{e\left(\frac{1}{3}e^2x^3+\frac{3}{2}de\right.}{b}\right.}{}$
risch	$\frac{(ex+d)^3\ln\left(\left(b\,x^3+a\right)^p\right)}{3e}-\frac{ix\pi\,d^2\,\mathrm{csgn}\left(i\left(b\,x^3+a\right)^p\right)\mathrm{csgn}\left(ic\left(b\,x^3+a\right)^p\right)\mathrm{csgn}(ic)}{2}+\frac{ix\pi\,d^2\mathrm{csgn}\left(ic\left(b\,x^3+a\right)^p\right)^2\mathrm{csgn}(ic)}{2}-\frac{ie\pi d\,x^2}{}$

```
[In] int((e*x+d)^2*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(c*(b*x^3+a)^p)*e^2*x^3+ln(c*(b*x^3+a)^p)*e*d*x^2+d^2*ln(c*(b*x^3+a)^
p)*x+1/3*ln(c*(b*x^3+a)^p)/e*d^3-p*b/e*(e/b*(1/3*e^2*x^3+3/2*d*e*x^2+3*d^2*
x)+(-3*e*d^2*a*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^
2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(a/b)^(1/3)*x-1)))-3*e^2*d*a*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/
(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+1/3*(-a*e^3+b*d^3)/b*ln(b*x^3+a))/b)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.32 (sec) , antiderivative size = 5799, normalized size of antiderivative = 23.20

$$\int (d+ex)^2 \log \left(c\left(a+bx^3\right)^p\right) \, dx = \text{Too large to display}$$

```
[In] integrate((e*x+d)^2*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [A] (verification not implemented)

Time = 15.87 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.69

$$\begin{aligned}\int (d+ex)^2 \log \left(c\left(a+bx^3\right)^p\right) \, dx &= 3ad^2p \operatorname{RootSum}\left(27t^3a^2b-1, \left(t \mapsto t\log\left(3ta+x\right)\right)\right) \\ &\quad + 3adep \operatorname{RootSum}\left(27t^3ab^2+1, \left(t \mapsto t\log\left(9t^2ab+x\right)\right)\right) \\ &\quad + \frac{ae^2p\left(\begin{cases}\frac{x^3}{a} & \text{for}\ b=0 \\ \frac{\log\left(a+bx^3\right)}{b} & \text{otherwise}\end{cases}\right)}{3} \\ &\quad - 3d^2px + d^2x\log\left(c\left(a+bx^3\right)^p\right) - \frac{3depx^2}{2} \\ &\quad + dex^2\log\left(c\left(a+bx^3\right)^p\right) - \frac{e^2px^3}{3} + \frac{e^2x^3\log\left(c\left(a+bx^3\right)^p\right)}{3}\end{aligned}$$

```
[In] integrate((e*x+d)**2*ln(c*(b*x**3+a)**p),x)
```

```
[Out] 3*a*d**2*p*RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x))) + 3
*a*d*e*p*RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)))
+ a*e**2*p*Piecewise((x**3/a, Eq(b, 0)), (log(a + b*x**3)/b, True))/3 - 3*d
**2*p*x + d**2*x*log(c*(a + b*x**3)**p) - 3*d*e*p*x**2/2 + d*e*x**2*log(c*(
a + b*x**3)**p) - e**2*p*x**3/3 + e**2*x**3*log(c*(a + b*x**3)**p)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00

$$\int (d+ex)^2 \log \left(c\left(a+bx^3\right)^p\right)\, dx =$$

$$-\frac{1}{6}\,bp\left(\frac{2\,e^2x^3+9\,dex^2+18\,d^2x}{b}-\frac{6\,\sqrt{3}\left(abde\left(\frac{a}{b}\right)^{\frac{2}{3}}+abd^2\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^2}-\frac{\left(2\,ae^2\left(\frac{a}{b}\right)^{\frac{2}{3}}\right.}{}\right.$$

$$+\frac{1}{3}\left(e^2x^3+3\,dex^2+3\,d^2x\right)\log\left(\left(bx^3+a\right)^pc\right)$$

```
[In] integrate((e*x+d)^2*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] -1/6*b*p*((2*e^2*x^3 + 9*d*e*x^2 + 18*d^2*x)/b - 6*sqrt(3)*(a*b*d*e*(a/b)^(
2/3) + a*b*d^2*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1
/3))/(a*b^2) - (2*a*e^2*(a/b)^(2/3) + 3*a*d*e*(a/b)^(1/3) - 3*a*d^2)*log(x^
2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) - 2*(a*e^2*(a/b)^(2/3) -
 3*a*d*e*(a/b)^(1/3) + 3*a*d^2)*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))) + 1
/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)*log((b*x^3 + a)^p*c)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.10

$$\begin{aligned}
&\int (d+ex)^2 \log \left(c\left(a+bx^3\right)^p\right)\, dx \\
&= -\frac{1}{3}\left(e^2p-e^2\log\left(c\right)\right)x^3+\frac{ae^2p\log\left(\left|bx^3+a\right|\right)}{3\,b}-\frac{1}{2}\left(3\,dep-2\,de\log\left(c\right)\right)x^2 \\
&\quad -\left(3\,d^2p-d^2\log\left(c\right)\right)x+\frac{1}{3}\left(e^2px^3+3\,depx^2+3\,d^2px\right)\log\left(bx^3+a\right) \\
&\quad +\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bd^2p-\left(-ab^2\right)^{\frac{2}{3}}dep\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2} \\
&\quad -\frac{\left(abdep\left(-\frac{a}{b}\right)^{\frac{1}{3}}+abd^2p\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab} \\
&\quad +\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}bd^2p+\left(-ab^2\right)^{\frac{2}{3}}dep\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^2}
\end{aligned}$$

```
[In] integrate((e*x+d)^2*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] -1/3*(e^2*p - e^2*log(c))*x^3 + 1/3*a*e^2*p*log(abs(b*x^3 + a))/b - 1/2*(3*
d*e*p - 2*d*e*log(c))*x^2 - (3*d^2*p - d^2*log(c))*x + 1/3*(e^2*p*x^3 + 3*d
*e*p*x^2 + 3*d^2*p*x)*log(b*x^3 + a) + sqrt(3)*((-a*b^2)^(1/3)*b*d^2*p - (-
a*b^2)^(2/3)*d*e*p)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b
^2 - (a*b*d*e*p*(-a/b)^(1/3) + a*b*d^2*p)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(
1/3)))/(a*b) + 1/2*((-a*b^2)^(1/3)*b*d^2*p + (-a*b^2)^(2/3)*d*e*p)*log(x^2
+ x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^2
```

Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.43

$$\int (d+ex)^2 \log \left(c\left(a+bx^3\right)^p\right) \, dx$$
$$= \left(\sum_{k=1}^{3} \ln \left(\mathrm{root}\left(27\,b^3\,c^3-27\,a\,b^2\,c^2\,e^2\,p+81\,a\,b^2\,c\,d^3\,e\,p^2+9\,a^2\,b\,c\,e^4\,p^2-27\,a\,b^2\,d^6\,p^3-a^3\,e^6\,p^3,c,k\right)\,\left(\mathrm{root}\left(27\,b^3 \cdots\right.\right.\right.\right.$$
$$+\,a^3\,e^4\,p^2+9\,a^2\,b\,d^3\,e\,p^2+6\,a^2\,b\,d^2\,e^2\,p^2\,x\Big)\,\mathrm{root}\big(27\,b^3\,c^3-27\,a\,b^2\,c^2\,e^2\,p+81\,a\,b^2\,c\,d^3\,e\,p^2$$
$$+\,9\,a^2\,b\,c\,e^4\,p^2-27\,a\,b^2\,d^6\,p^3-a^3\,e^6\,p^3,c,k\big)\Bigg)$$
$$+\ln\left(c\,\left(b\,x^3+a\right)^p\right)\,\left(d^2\,x+d\,e\,x^2+\frac{e^2\,x^3}{3}\right)-3\,d^2\,p\,x-\frac{e^2\,p\,x^3}{3}-\frac{3\,d\,e\,p\,x^2}{2}$$

```
[In] int(log(c*(a + b*x^3)^p)*(d + e*x)^2,x)
```

```
[Out] symsum(log(root(27*b^3*c^3 - 27*a*b^2*c^2*e^2*p + 81*a*b^2*c*d^3*e*p^2 + 9*
a^2*b*c*e^4*p^2 - 27*a*b^2*d^6*p^3 - a^3*e^6*p^3, c, k)*(9*root(27*b^3*c^3
- 27*a*b^2*c^2*e^2*p + 81*a*b^2*c*d^3*e*p^2 + 9*a^2*b*c*e^4*p^2 - 27*a*b^2*
d^6*p^3 - a^3*e^6*p^3, c, k)*a*b^2 - 6*a^2*b*e^2*p + 9*a*b^2*d^2*p*x) + a^3
*e^4*p^2 + 9*a^2*b*d^3*e*p^2 + 6*a^2*b*d^2*e^2*p^2*x)*root(27*b^3*c^3 - 27*
a*b^2*c^2*e^2*p + 81*a*b^2*c*d^3*e*p^2 + 9*a^2*b*c*e^4*p^2 - 27*a*b^2*d^6*p
^3 - a^3*e^6*p^3, c, k), k, 1, 3) + log(c*(a + b*x^3)^p)*(d^2*x + (e^2*x^3)
/3 + d*e*x^2) - 3*d^2*p*x - (e^2*p*x^3)/3 - (3*d*e*p*x^2)/2
```

3.193 $\int (d+ex)\log\left(c\left(a+bx^3\right)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 229

$$\begin{aligned}\int (d+ex)\log\left(c\left(a+bx^3\right)^p\right)\,dx &= -3dpx-\frac{3}{4}epx^2\\ &\quad-\frac{\sqrt{3}\sqrt[3]{a}\left(2\sqrt[3]{b}d+\sqrt[3]{a}e\right)p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}}\\ &\quad+\frac{\sqrt[3]{a}\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}}\\ &\quad-\frac{\sqrt[3]{a}\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}}\\ &\quad-\frac{d^2p\log\left(a+bx^3\right)}{2e}+\frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e}\end{aligned}$$

```
[Out] -3*d*p*x-3/4*e*p*x^2+1/2*a^(1/3)*(2*b^(1/3)*d-a^(1/3)*e)*p*ln(a^(1/3)+b^(1/
3)*x)/b^(2/3)-1/4*a^(1/3)*(2*b^(1/3)*d-a^(1/3)*e)*p*ln(a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/b^(2/3)-1/2*d^2*p*ln(b*x^3+a)/e+1/2*(e*x+d)^2*ln(c*(b*x^
3+a)^p)/e-1/2*a^(1/3)*(2*b^(1/3)*d+a^(1/3)*e)*p*arctan(1/3*(a^(1/3)-2*b^(1/
3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(2/3)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {2513, 1901, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int (d+ex)\log\left(c\left(a+bx^3\right)^p\right)\,dx=-\frac{\sqrt[3]{a}p\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}}-\frac{\sqrt{3}\sqrt[3]{a}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(\sqrt[3]{a}e+2\sqrt[3]{b}d\right)}{2b^{2/3}}+\frac{\sqrt[3]{a}p\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}}+\frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e}-\frac{d^2p\log\left(a+bx^3\right)}{2e}-3dpx-\frac{3}{4}epx^2$$

```
[In] Int[(d + e*x)*Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*d*p*x - (3*e*p*x^2)/4 - (Sqrt[3]*a^(1/3)*(2*b^(1/3)*d + a^(1/3)*e)*p*Arc
Tan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2*b^(2/3)) + (a^(1/3)*(2*b
^(1/3)*d - a^(1/3)*e)*p*Log[a^(1/3) + b^(1/3)*x])/(2*b^(2/3)) - (a^(1/3)*(2
*b^(1/3)*d - a^(1/3)*e)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
4*b^(2/3)) - (d^2*p*Log[a + b*x^3])/(2*e) + ((d + e*x)^2*Log[c*(a + b*x^3)^
p])/(2*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e} - \frac{(3bp)\int\frac{x^2(d+ex)^2}{a+bx^3}\,dx}{2e}$$

$$= \frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e} - \frac{(3bp)\int\left(\frac{2de}{b}+\frac{e^2x}{b}-\frac{2ade+ae^2x-bd^2x^2}{b(a+bx^3)}\right)dx}{2e}$$

$$= -3dpx - \frac{3}{4}epx^2 + \frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e} + \frac{(3p)\int\frac{2ade+ae^2x-bd^2x^2}{a+bx^3}\,dx}{2e}$$

$$= -3dpx - \frac{3}{4}epx^2 + \frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e} + \frac{(3p)\int\frac{2ade+ae^2x}{a+bx^3}\,dx}{2e} - \frac{(3bd^2p)\int\frac{x^2}{a+bx^3}\,dx}{2e}$$

$$= -3dpx - \frac{3}{4}epx^2 - \frac{d^2p\log\left(a+bx^3\right)}{2e} + \frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e} + \frac{p\int\frac{\sqrt[3]{a}\left(4a\sqrt[3]{b}de+a^{4/3}e^2\right)+\sqrt[3]{b}\left(-2a\sqrt[3]{b}de+a^{4/3}e^2\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^{2/3}\sqrt[3]{b}e} + \frac{1}{2}\left(\sqrt[3]{a}\left(2d-\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)p\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx$$

$$= -3dpx - \frac{3}{4}epx^2 + \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}} - \frac{d^2p\log\left(a+bx^3\right)}{2e} + \frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e} - \frac{\left(\sqrt[3]{a}\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)p\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{4b^{2/3}} + \frac{1}{4}\left(3a^{2/3}\left(2d+\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)p\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx$$

$$= -3dpx - \frac{3}{4}epx^2 + \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}} - \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}d-\sqrt[3]{a}e\right)p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}} - \frac{d^2p\log\left(a+bx^3\right)}{2e} + \frac{(d+ex)^2\log\left(c\left(a+bx^3\right)^p\right)}{2e} + \frac{\left(3\sqrt[3]{a}\left(2\sqrt[3]{b}d+\sqrt[3]{a}e\right)p\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{2b^{2/3}}$$

$$= -3dpx - \frac{3}{4}epx^2 - \frac{\sqrt{3}\sqrt[3]{a}\left(2\sqrt[3]{b}d + \sqrt[3]{a}e\right)p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}} + \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}d - \sqrt[3]{a}e\right)p\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{2b^{2/3}} - \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}d - \sqrt[3]{a}e\right)p\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{4b^{2/3}} - \frac{d^2p\log(a+bx^3)}{2e} + \frac{(d+ex)^2\log\left(c(a+bx^3)^p\right)}{2e}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.89

$$\int (d+ex)\log\left(c(a+bx^3)^p\right)\,dx = -3dpx - \frac{3}{4}epx^2 + \frac{\sqrt{3}\sqrt[3]{a}dp\arctan\left(\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}\right)}{\sqrt[3]{b}} + \frac{3}{4}epx^2\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a}\right) + \frac{\sqrt[3]{a}dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} - \frac{\sqrt[3]{a}dp\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}} + dx\log\left(c(a+bx^3)^p\right) + \frac{1}{2}ex^2\log\left(c(a+bx^3)^p\right)$$

```
[In] Integrate[(d + e*x)*Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*d*p*x - (3*e*p*x^2)/4 + (Sqrt[3]*a^(1/3)*d*p*ArcTan[(-(a^(1/3)*b^(1/3))
+ 2*b^(2/3)*x)/(Sqrt[3]*a^(1/3)*b^(1/3))]])/b^(1/3) + (3*e*p*x^2*Hypergeomet
ric2F1[2/3, 1, 5/3, -((b*x^3)/a)])/4 + (a^(1/3)*d*p*Log[a^(1/3) + b^(1/3)*x
])/b^(1/3) - (a^(1/3)*d*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
2*b^(1/3)) + d*x*Log[c*(a + b*x^3)^p] + (e*x^2*Log[c*(a + b*x^3)^p])/2
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.08

method	result
parts	$\frac{\ln(c(b\,x^3+a)^p)e\,x^2}{2}+d\ln\left(c(b\,x^3+a)^p\right)x-\frac{3pb\left(\frac{\frac{1}{2}e\,x^2+2dx}{b}-\frac{\left(\left(2d\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}}{}\right.}{3b(}\right.\right.\right.}{}\right.}{}$
risch	$\left(\frac{1}{2}e\,x^2+dx\right)\ln\left((b\,x^3+a)^p\right)+\frac{i\operatorname{csgn}\left(ic(b\,x^3+a)^p\right)^2\operatorname{csgn}\left(i(b\,x^3+a)^p\right)x^2e\pi}{4}-\frac{i\pi e\,x^2\operatorname{csgn}\left(i(b\,x^3+a)^p\right)\operatorname{csgn}\left(ic(b\,x^3+a)^p\right.}{4}$

```
[In] int((e*x+d)*ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(c*(b*x^3+a)^p)*e*x^2+d*ln(c*(b*x^3+a)^p)*x-3/2*p*b*(1/b*(1/2*e*x^2+2
*d*x)-(2*d*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a
/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1)))+e*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*
ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1
/2)*(2/(a/b)^(1/3)*x-1))))*a/b)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 2284, normalized size of antiderivative = 9.97

$$\int (d+ex)\log\left(c\left(a+bx^3\right)^p\right)\,dx=\text{Too large to display}$$

```
[In] integrate((e*x+d)*log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] -3/4*e*p*x^2 - 3*d*p*x + 1/4*(4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8
*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (
1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2
)^(1/3)*(I*sqrt(3) + 1))*log(4*a*d*e^2*p^2 + 2*(4*(1/2)^(2/3)*a*d*e*p^2*(-I
*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)
/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3
- a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))*b*d^2*p + 1/4*(4*(1/2)^(2/3)*a*d
*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^
2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*
b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))^2*b*e + (8*b*d^3 + a*e
^3)*p^2*x) - 1/8*(4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e
^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*(
(8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*s
qrt(3) + 1) - sqrt(3)*sqrt(-(32*a*d*e*p^2 + (4*(1/2)^(2/3)*a*d*e*p^2*(-I*sq
rt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^
2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a
^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))^2*b)/b))*log(-2*a*d*e^2*p^2 - (4*(1
/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b
*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^
3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))*b*d^2*p -
 1/8*(4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^
2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a
*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))
^2*b*e + (8*b*d^3 + a*e^3)*p^2*x + 1/8*sqrt(3)*(8*b*d^2*p - (4*(1/2)^(2/3)*
a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 -
 a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8
*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))*b*e)*sqrt(-(32*a*d*
e*p^2 + (4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3
/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3
+ a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) +
1))^2*b)/b)) - 1/8*(4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a
*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)
*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I
*sqrt(3) + 1) + sqrt(3)*sqrt(-(32*a*d*e*p^2 + (4*(1/2)^(2/3)*a*d*e*p^2*(-I*
sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/
b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 -
 a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))^2*b)/b))*log(-2*a*d*e^2*p^2 - (4*
(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a
*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*
p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))*b*d^2*p
 - 1/8*(4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/
b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 +
 a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1
))^2*b*e + (8*b*d^3 + a*e^3)*p^2*x - 1/8*sqrt(3)*(8*b*d^2*p - (4*(1/2)^(2/3
)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3
```

```
- a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^3 + a*e^3)*a*p^3/b^2 +
(8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3) + 1))*b*e)*sqrt(-(32*a*
d*e*p^2 + (4*(1/2)^(2/3)*a*d*e*p^2*(-I*sqrt(3) + 1)/(((8*b*d^3 + a*e^3)*a*p
^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*b) - (1/2)^(1/3)*((8*b*d^
3 + a*e^3)*a*p^3/b^2 + (8*a*b*d^3*p^3 - a^2*e^3*p^3)/b^2)^(1/3)*(I*sqrt(3)
+ 1))^2*b)/b)) + 1/2*(e*p*x^2 + 2*d*p*x)*log(b*x^3 + a) + 1/2*(e*x^2 + 2*d*
x)*log(c)
```

Sympy [A] (verification not implemented)

Time = 10.94 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.49

$$\int (d+ex)\log\left(c\left(a+bx^3\right)^p\right)\,dx = 3adp\,\mathrm{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right) + \frac{3aep\,\mathrm{RootSum}\left(27t^3ab^2+1,\left(t\mapsto t\log\left(9t^2ab+x\right)\right)\right)}{2} - 3dpx + dx\log\left(c\left(a+bx^3\right)^p\right) - \frac{3epx^2}{4} + \frac{ex^2\log\left(c\left(a+bx^3\right)^p\right)}{2}$$

```
[In] integrate((e*x+d)*ln(c*(b*x**3+a)**p),x)
```

```
[Out] 3*a*d*p*RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x))) + 3*a*
e*p*RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)))/2 - 3
*d*p*x + d*x*log(c*(a + b*x**3)**p) - 3*e*p*x**2/4 + e*x**2*log(c*(a + b*x*
*3)**p)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.82

$$\int (d+ex)\log\left(c\left(a+bx^3\right)^p\right)\,dx = -\frac{1}{4}\,bp\left(\frac{3\left(ex^2+4\,dx\right)}{b} - \frac{2\sqrt{3}\left(ae\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(ae\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,ad\right)\log\left(x^2-x\left(\frac{a}{b}\right.\right.}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right. + \frac{1}{2}\left(ex^2+2\,dx\right)\log\left(\left(bx^3+a\right)^pc\right)$$

```
[In] integrate((e*x+d)*log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] -1/4*b*p*(3*(e*x^2 + 4*d*x)/b - 2*sqrt(3)*(a*e*(a/b)^(1/3) + 2*a*d)*arctan(
1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^2*(a/b)^(2/3)) - (a*e*(a/b)
^(1/3) - 2*a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) +
2*(a*e*(a/b)^(1/3) - 2*a*d)*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))) + 1/2*(
e*x^2 + 2*d*x)*log((b*x^3 + a)^p*c)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.93

$$\begin{aligned}\int (d+ex)\log\left(c\left(a+bx^3\right)^p\right)\,dx &= -\frac{1}{4}\,(3\,ep-2\,e\log\left(c\right))x^2-(3\,dp-d\log\left(c\right))x+\frac{1}{2}\,\left(epx^2+2\,dpx\right)\log\left(bx^3+a\right)\\ &\quad-\frac{\left(aep\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,adp\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{2\,a}\\ &\quad+\frac{\left(2\sqrt{3}(-ab^2)^{\frac{1}{3}}\,bdp-\sqrt{3}(-ab^2)^{\frac{2}{3}}\,ep\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{2\,b^2}\\ &\quad+\frac{\left(2\,(-ab^2)^{\frac{1}{3}}\,bdp+(-ab^2)^{\frac{2}{3}}\,ep\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{4\,b^2}\end{aligned}$$

```
[In] integrate((e*x+d)*log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] -1/4*(3*e*p - 2*e*log(c))*x^2 - (3*d*p - d*log(c))*x + 1/2*(e*p*x^2 + 2*d*p
*x)*log(b*x^3 + a) - 1/2*(a*e*p*(-a/b)^(1/3) + 2*a*d*p)*(-a/b)^(1/3)*log(ab
s(x - (-a/b)^(1/3)))/a + 1/2*(2*sqrt(3)*(-a*b^2)^(1/3)*b*d*p - sqrt(3)*(-a*
b^2)^(2/3)*e*p)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^2 +
 1/4*(2*(-a*b^2)^(1/3)*b*d*p + (-a*b^2)^(2/3)*e*p)*log(x^2 + x*(-a/b)^(1/3)
 + (-a/b)^(2/3))/b^2
```

Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.92

$$\int (d+ex)\log \left(c\left(a+bx^3\right)^p\right)\,dx$$
$$= \left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(8\,b^2\,c^3+12\,a\,b\,c\,d\,e\,p^2-8\,a\,b\,d^3\,p^3+a^2\,e^3\,p^3,c,k\right)\,\left(\mathrm{root}\left(8\,b^2\,c^3+12\,a\,b\,c\,d\,e\,p^2-8\,a\,b\,d^3\,p^3+a^2\,e\right.\right.\right.\right.$$
$$\left.\left.+\frac{9\,a^2\,b\,d\,e\,p^2}{2}+\frac{9\,a^2\,b\,e^2\,p^2\,x}{4}\right)\mathrm{root}\left(8\,b^2\,c^3+12\,a\,b\,c\,d\,e\,p^2-8\,a\,b\,d^3\,p^3+a^2\,e^3\,p^3,c,k\right)\right)$$
$$+\ln\left(c\,\left(b\,x^3+a\right)^p\right)\,\left(\frac{e\,x^2}{2}+d\,x\right)-\frac{3\,e\,p\,x^2}{4}-3\,d\,p\,x$$

```
[In] int(log(c*(a + b*x^3)^p)*(d + e*x),x)
```

```
[Out] symsum(log(root(8*b^2*c^3 + 12*a*b*c*d*e*p^2 - 8*a*b*d^3*p^3 + a^2*e^3*p^3,
 c, k)*(9*root(8*b^2*c^3 + 12*a*b*c*d*e*p^2 - 8*a*b*d^3*p^3 + a^2*e^3*p^3,
c, k)*a*b^2 + 9*a*b^2*d*p*x) + (9*a^2*b*d*e*p^2)/2 + (9*a^2*b*e^2*p^2*x)/4)
*root(8*b^2*c^3 + 12*a*b*c*d*e*p^2 - 8*a*b*d^3*p^3 + a^2*e^3*p^3, c, k), k,
 1, 3) + log(c*(a + b*x^3)^p)*(d*x + (e*x^2)/2) - (3*e*p*x^2)/4 - 3*d*p*x
```

3.194 $\int \log \left(c(a+bx^3)^p\right) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 133

$$\int \log \left(c(a+bx^3)^p\right) \, dx = -3px - \frac{\sqrt{3}\sqrt[3]{a}p \arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}} + \frac{\sqrt[3]{a}p \log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} - \frac{\sqrt[3]{a}p \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}} + x \log \left(c(a+bx^3)^p\right)$$

```
[Out] -3*p*x+a^(1/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)-1/2*a^(1/3)*p*ln(a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(1/3)+x*ln(c*(b*x^3+a)^p)-a^(1/3)*p*arctan(1/
3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2498, 327, 206, 31, 648, 631, 210, 642}

$$\int \log \left(c(a+bx^3)^p\right) \, dx = -\frac{\sqrt[3]{a}p \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}} - \frac{\sqrt{3}\sqrt[3]{a}p \arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}} + x \log \left(c(a+bx^3)^p\right) + \frac{\sqrt[3]{a}p \log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} - 3px$$

```
[In] Int[Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*p*x - (Sqrt[3]*a^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/b^(1/3) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (a^(1/3)*p*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)) + x*Log[c*(a + b*x
^3)^p]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\log\left(c\left(a+bx^3\right)^p\right)-(3bp)\int\frac{x^3}{a+bx^3}\,dx\\
&= -3px+x\log\left(c\left(a+bx^3\right)^p\right)+(3ap)\int\frac{1}{a+bx^3}\,dx\\
&= -3px+x\log\left(c\left(a+bx^3\right)^p\right)+\left(\sqrt[3]{a}p\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx+\left(\sqrt[3]{a}p\right)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx\\
&= -3px+\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+x\log\left(c\left(a+bx^3\right)^p\right)\\
&\quad+\frac{1}{2}\left(3a^{2/3}p\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx-\frac{\left(\sqrt[3]{a}p\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{b}}\\
&= -3px+\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}\\
&\quad+x\log\left(c\left(a+bx^3\right)^p\right)+\frac{\left(3\sqrt[3]{a}p\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}\\
&= -3px-\frac{\sqrt{3}\sqrt[3]{a}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}}+\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}\\
&\quad-\frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}+x\log\left(c\left(a+bx^3\right)^p\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.97

$$\int \log \left(c\left(a+bx^3\right)^p\right) \, dx = -3px - \frac{\sqrt{3}\sqrt[3]{a}p \arctan \left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} + \frac{\sqrt[3]{a}p \log \left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} - \frac{\sqrt[3]{a}p \log \left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}} + x \log \left(c\left(a+bx^3\right)^p\right)$$

```
[In] Integrate[Log[c*(a + b*x^3)^p],x]
```

```
[Out] -3*p*x - (Sqrt[3]*a^(1/3)*p*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^
(1/3) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (a^(1/3)*p*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)) + x*Log[c*(a + b*x^3)^p]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92

method	result
default	$x\ln\left(c(b\,x^{3}+a)^{p}\right)-3pb\left(\frac{x}{b}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a}{b}\right)$
parts	$x\ln\left(c(b\,x^{3}+a)^{p}\right)-3pb\left(\frac{x}{b}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a}{b}\right)$
risch	$x\ln\left((b\,x^{3}+a)^{p}\right)+\frac{i\operatorname{csgn}\left(ic(b\,x^{3}+a)^{p}\right)^{2}\operatorname{csgn}\left(i(b\,x^{3}+a)^{p}\right)x\pi}{2}-\frac{i\pi x\,\operatorname{csgn}\left(i(b\,x^{3}+a)^{p}\right)\operatorname{csgn}\left(ic(b\,x^{3}+a)^{p}\right)\operatorname{csgn}(ic)}{2}-\frac{i\pi x}{}$

```
[In] int(ln(c*(b*x^3+a)^p),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(c*(b*x^3+a)^p)-3*p*b*(x/b-(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(
a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))*a/b)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$\int \log \left(c\left(a+bx^3\right)^p\right) \, dx = px\log \left(bx^3+a\right)+\sqrt{3}p\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan \left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right) - \frac{1}{2}p\left(\frac{a}{b}\right)^{\frac{1}{3}}\log \left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) + p\left(\frac{a}{b}\right)^{\frac{1}{3}}\log \left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-3\,px+x\log \left(c\right)$$

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="fricas")
```

```
[Out] p*x*log(b*x^3 + a) + sqrt(3)*p*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^
(2/3) - sqrt(3)*a)/a) - 1/2*p*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(
2/3)) + p*(a/b)^(1/3)*log(x + (a/b)^(1/3)) - 3*p*x + x*log(c)
```

Sympy [A] (verification not implemented)

Time = 24.71 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.24

$$\int \log \left(c\left(a+bx^3\right)^p\right) \, dx = \begin{cases} x\log \left(0^pc\right) \\ -3px+x\log \left(c\left(bx^3\right)^p\right) \\ x\log \left(a^pc\right) \\ -3px+x\log \left(c\left(a+bx^3\right)^p\right)-\frac{3bp\left(-\frac{a}{b}\right)^{\frac{4}{3}}\log \left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a}-\frac{\sqrt{3}bp\left(-\frac{a}{b}\right)^{\frac{4}{3}}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{a}+\frac{b\left(-\frac{a}{b}\right)^{\frac{4}{3}}\log}{} \end{cases}$$

```
[In] integrate(ln(c*(b*x**3+a)**p),x)
```

```
[Out] Piecewise((x*log(0**p*c), Eq(a, 0) & Eq(b, 0)), (-3*p*x + x*log(c*(b*x**3)*
*p), Eq(a, 0)), (x*log(a**p*c), Eq(b, 0)), (-3*p*x + x*log(c*(a + b*x**3)**
p) - 3*b*p*(-a/b)**(4/3)*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/
(2*a) - sqrt(3)*b*p*(-a/b)**(4/3)*atan(2*sqrt(3)*x/(3*(-a/b)**(1/3)) + sqrt
(3)/3)/a + b*(-a/b)**(4/3)*log(c*(a + b*x**3)**p)/a, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.94

$$\int \log\left(c\left(a+bx^3\right)^p\right)\,dx = -\frac{1}{2}\,bp\left(\frac{6\,x}{b}-\frac{2\,\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2\,a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^2\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right) + x\log\left(\left(bx^3+a\right)^pc\right)$$

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="maxima")
```

```
[Out] -1/2*b*p*(6*x/b - 2*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^
(1/3))/(b^2*(a/b)^(2/3)) + a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a
/b)^(2/3)) - 2*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))) + x*log((b*x^3 + a
)^p*c)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08

$$\int \log\left(c\left(a+bx^3\right)^p\right)\,dx = -\frac{1}{2}\,abp\left(\frac{2\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab}-\frac{2\,\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^2}-\frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x(-\right.}{ab^2}\right. + px\log\left(bx^3+a\right)-(3\,p-\log\left(c\right))x$$

```
[In] integrate(log(c*(b*x^3+a)^p),x, algorithm="giac")
```

```
[Out] -1/2*a*b*p*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) - 2*sqrt(3)*(-a
*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) -
 (-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)) + p*x*log
(b*x^3 + a) - (3*p - log(c))*x
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.01

$$\int \log \left(c\left(a+bx^3\right)^p\right) \, dx$$
$$= x \ln \left(c \left(b x^3+a\right)^p\right)-3 p x-\frac{(-a)^{1/3} p \ln \left(\left(-a\right)^{4/3}+a b^{1/3} x\right)}{b^{1/3}}$$
$$+\frac{(-a)^{1/3} p \ln \left(2 a b^{1/3} x-(-a)^{4/3}-\sqrt{3} (-a)^{4/3} 1\mathrm{i}\right) \left(\frac{1}{2}+\frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)}{b^{1/3}}$$
$$-\frac{(-a)^{1/3} p \ln \left(2 a b^{1/3} x-(-a)^{4/3}+\sqrt{3} (-a)^{4/3} 1\mathrm{i}\right) \left(-\frac{1}{2}+\frac{\sqrt{3}\, 1\mathrm{i}}{2}\right)}{b^{1/3}}$$

```
[In] int(log(c*(a + b*x^3)^p),x)
```

```
[Out] x*log(c*(a + b*x^3)^p) - 3*p*x - ((-a)^(1/3)*p*log((-a)^(4/3) + a*b^(1/3)*x
))/b^(1/3) + ((-a)^(1/3)*p*log(2*a*b^(1/3)*x - 3^(1/2)*(-a)^(4/3)*1i - (-a)
^(4/3))*((3^(1/2)*1i)/2 + 1/2))/b^(1/3) - ((-a)^(1/3)*p*log(3^(1/2)*(-a)^(4
/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/2)*1i)/2 - 1/2))/b^(1/3)
```

3.195 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 308

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx = -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e}$$

```
[Out] -p*ln(-e*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/e-p*ln(-e*((-
1)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/e-p
*ln((-1)^(1/3)*e*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/
3)*e))*ln(e*x+d)/e+ln(e*x+d)*ln(c*(b*x^3+a)^p)/e-p*polylog(2,b^(1/3)*(e*x+d
```

```
)/(b^(1/3)*d-a^(1/3)*e))/e-p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3
)*a^(1/3)*e))/e-p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e
))/e
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2512, 266, 2463, 2441, 2440, 2438}

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{d+ex}\,dx=\frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e}-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e}-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e}-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e}-\frac{p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e}$$

```
[In] Int[Log[c*(a + b*x^3)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x]
)/e) - (p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3
)*a^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)
*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e + (Log[d +
 e*x]*Log[c*(a + b*x^3)^p])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d - a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1
/3)*a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2
/3)*a^(1/3)*e)])/e
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e}-\frac{(3bp)\int\frac{x^2\log(d+ex)}{a+bx^3}\,dx}{e}\\
&= \frac{\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e}\\
&\quad-\frac{(3bp)\int\left(\frac{\log(d+ex)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\log(d+ex)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\log(d+ex)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx}{e}
\end{aligned}$$

$$= \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e} - \frac{\left(\sqrt[3]{b}p\right)\int \frac{\log(d+ex)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e} - \frac{\left(\sqrt[3]{b}p\right)\int \frac{\log(d+ex)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e} - \frac{\left(\sqrt[3]{b}p\right)\int \frac{\log(d+ex)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e}$$

$$= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e} + p\int \frac{\log\left(\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d+\sqrt[3]{a}e}\right)}{d+ex}\,dx + p\int \frac{\log\left(\frac{e\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d-\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d+ex}\,dx + p\int \frac{\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d+(-1)^{2/3}\sqrt[3]{a}e}\right)}{d+ex}\,dx$$

$$
= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e}
- \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e}
+ \frac{p\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d+\sqrt[3]{a}e}\right)}{x}dx, x, d+ex\right)}{e}
+ \frac{p\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d-\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{x}dx, x, d+ex\right)}{e}
+ \frac{p\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d+(-1)^{2/3}\sqrt[3]{a}e}\right)}{x}dx, x, d+ex\right)}{e}
$$

$$
= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e}
- \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e}
- \frac{p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e} - \frac{p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e} - \frac{p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.02

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e}$$

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x]
)/e) - (p*Log[-(((-1)^(2/3)*e*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(b^(1/3)*d
- (-1)^(2/3)*a^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(a^(1/3) +
 (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e
 + (Log[d + e*x]*Log[c*(a + b*x^3)^p])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x)
)/(b^(1/3)*d - a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d + (-1)^(1/3)*a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d - (-1)^(2/3)*a^(1/3)*e)])/e
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.33

method	result
parts	$\frac{\ln(ex+d)\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{e}-\frac{p\left(\sum_{\_R1=\mathrm{RootOf}\left(\_Z^{3}b-3bd\_Z^{2}+3b\,d^{2}\_Z+a\,e^{3}-b\,d^{3}\right)}\left(\ln(ex+d)\ln\left(\frac{-ex+\_R1-d}{\_R1}\right)+\mathrm{dilog}\left(\frac{-ex+\_}{\_R}\right.\right.\right.}{e}$
risch	$\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)\ln(ex+d)}{e}-\frac{p\left(\sum_{\_R1=\mathrm{RootOf}\left(\_Z^{3}b-3bd\_Z^{2}+3b\,d^{2}\_Z+a\,e^{3}-b\,d^{3}\right)}\left(\ln(ex+d)\ln\left(\frac{-ex+\_R1-d}{\_R1}\right)+\mathrm{dilog}\left(\frac{-ex+\_R}{\_R}\right.\right.\right.}{e}$

[In] int(ln(c*(b*x^3+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)

[Out] ln(e*x+d)*ln(c*(b*x^3+a)^p)/e-p/e*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))

Fricas [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx=\int \frac{\log\left((bx^3+a)^p c\right)}{ex+d}\,dx$$

[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")

[Out] integral(log((b*x^3 + a)^p*c)/(e*x + d), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx=\text{Timed out}$$

[In] integrate(ln(c*(b*x**3+a)**p)/(e*x+d),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{\log\left((bx^3+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{\log\left((bx^3+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{\ln\left(c\,(b\,x^3+a)^p\right)}{d+e\,x}\,dx$$

```
[In] int(log(c*(a + b*x^3)^p)/(d + e*x),x)
[Out] int(log(c*(a + b*x^3)^p)/(d + e*x), x)
```

3.196 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 292

$$\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{(d+ex)^2}\,dx = -\frac{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}d^2+\sqrt[3]{a}\sqrt[3]{b}de+a^{2/3}e^2} + \frac{\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{bd^3-ae^3} - \frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)} - \frac{\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}d+\sqrt[3]{a}e\right)p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\left(bd^3-ae^3\right)} + \frac{bd^2p\log\left(a+bx^3\right)}{e\left(bd^3-ae^3\right)} - \frac{\log\left(c\left(a+bx^3\right)^p\right)}{e(d+ex)}$$

```
[Out] a^(1/3)*b^(1/3)*(b^(1/3)*d+a^(1/3)*e)*p*ln(a^(1/3)+b^(1/3)*x)/(-a*e^3+b*d^3
)-3*b*d^2*p*ln(e*x+d)/e/(-a*e^3+b*d^3)-1/2*a^(1/3)*b^(1/3)*(b^(1/3)*d+a^(1/
3)*e)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-a*e^3+b*d^3)+b*d^2*p*ln
(b*x^3+a)/e/(-a*e^3+b*d^3)-ln(c*(b*x^3+a)^p)/e/(e*x+d)-a^(1/3)*b^(1/3)*p*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/(b^(2/3)*d^2+a^(1/3
)*b^(1/3)*d*e+a^(2/3)*e^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2513, 6857, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{(d+ex)^2}\,dx = -\frac{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}e^2+\sqrt[3]{a}\sqrt[3]{b}de+b^{2/3}d^2} - \frac{\sqrt[3]{a}\sqrt[3]{b}p\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\left(bd^3-ae^3\right)} - \frac{\log\left(c(a+bx^3)^p\right)}{e(d+ex)} + \frac{\sqrt[3]{a}\sqrt[3]{b}p\left(\sqrt[3]{a}e+\sqrt[3]{b}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{bd^3-ae^3} + \frac{bd^2p\log\left(a+bx^3\right)}{e\left(bd^3-ae^3\right)} - \frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)}$$

```
[In] Int[Log[c*(a + b*x^3)^p]/(d + e*x)^2,x]
```

```
[Out] -((Sqrt[3]*a^(1/3)*b^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(b^(2/3)*d^2 + a^(1/3)*b^(1/3)*d*e + a^(2/3)*e^2)) + (a^(1/3)*b^(1/3)*
(b^(1/3)*d + a^(1/3)*e)*p*Log[a^(1/3) + b^(1/3)*x])/(b*d^3 - a*e^3) - (3*b*
d^2*p*Log[d + e*x])/(e*(b*d^3 - a*e^3)) - (a^(1/3)*b^(1/3)*(b^(1/3)*d + a^(
1/3)*e)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*(b*d^3 - a*e^3
)) + (b*d^2*p*Log[a + b*x^3])/(e*(b*d^3 - a*e^3)) - Log[c*(a + b*x^3)^p]/(e
*(d + e*x))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = -\frac{\log\left(c(a+bx^3)^p\right)}{e(d+ex)} + \frac{(3bp)\int \frac{x^2}{(d+ex)(a+bx^3)}\,dx}{e}$$

$$= -\frac{\log\left(c(a+bx^3)^p\right)}{e(d+ex)} + \frac{(3bp)\int \left(-\frac{d^2e}{(bd^3-ae^3)(d+ex)} + \frac{ade-ae^2x+bd^2x^2}{(bd^3-ae^3)(a+bx^3)}\right)dx}{e}$$

$$= -\frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)} - \frac{\log\left(c(a+bx^3)^p\right)}{e(d+ex)} + \frac{(3bp)\int \frac{ade-ae^2x+bd^2x^2}{a+bx^3}\,dx}{e\left(bd^3-ae^3\right)}$$

$$= -\frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)} - \frac{\log\left(c(a+bx^3)^p\right)}{e(d+ex)} + \frac{(3bp)\int \frac{ade-ae^2x}{a+bx^3}\,dx}{e\left(bd^3-ae^3\right)} + \frac{(3b^2d^2p)\int \frac{x^2}{a+bx^3}\,dx}{e\left(bd^3-ae^3\right)}$$

$$= -\frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)} + \frac{bd^2p\log\left(a+bx^3\right)}{e\left(bd^3-ae^3\right)} - \frac{\log\left(c(a+bx^3)^p\right)}{e(d+ex)} + \frac{\left(b^{2/3}p\right)\int \frac{\sqrt[3]{a}\left(2a\sqrt[3]{b}de-a^{4/3}e^2\right)+\sqrt[3]{b}\left(-a\sqrt[3]{b}de-a^{4/3}e^2\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{a^{2/3}e\left(bd^3-ae^3\right)} + \frac{\left(\sqrt[3]{a}b\left(d+\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)p\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{bd^3-ae^3}$$

$$= \frac{\sqrt[3]{a}b^{2/3}\left(d+\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{bd^3-ae^3} - \frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)} + \frac{bd^2p\log\left(a+bx^3\right)}{e\left(bd^3-ae^3\right)} - \frac{\log\left(c(a+bx^3)^p\right)}{e(d+ex)} + \frac{\left(3a^{2/3}b^{2/3}\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)p\right)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\left(bd^3-ae^3\right)} - \frac{\left(\sqrt[3]{a}b^{2/3}\left(d+\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)p\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\left(bd^3-ae^3\right)}$$

$$= \frac{\sqrt[3]{a}b^{2/3}\left(d+\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{bd^3-ae^3} - \frac{3bd^2p\log(d+ex)}{e\left(bd^3-ae^3\right)} - \frac{\sqrt[3]{a}b^{2/3}\left(d+\frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\left(bd^3-ae^3\right)} + \frac{bd^2p\log\left(a+bx^3\right)}{e\left(bd^3-ae^3\right)} - \frac{\log\left(c(a+bx^3)^p\right)}{e(d+ex)} + \frac{\left(3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)p\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{bd^3-ae^3}$$

$$= -\frac{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}d - \sqrt[3]{a}e\right)p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{bd^3 - ae^3}$$
$$+ \frac{\sqrt[3]{a}b^{2/3}\left(d + \frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)p\log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{bd^3 - ae^3} - \frac{3bd^2p\log(d+ex)}{e\left(bd^3 - ae^3\right)}$$
$$- \frac{\sqrt[3]{a}b^{2/3}\left(d + \frac{\sqrt[3]{a}e}{\sqrt[3]{b}}\right)p\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{2\left(bd^3 - ae^3\right)}$$
$$+ \frac{bd^2p\log\left(a + bx^3\right)}{e\left(bd^3 - ae^3\right)} - \frac{\log\left(c(a+bx^3)^p\right)}{e(d+ex)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.21 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.87

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{(d+ex)^2}\,dx =$$
$$-\frac{2\sqrt{3}\sqrt[3]{a}b^{2/3}dep(d+ex)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 3be^2px^2(d+ex)\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a}\right) - 2}{}$$

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(d + e*x)^2,x]
```

```
[Out] -1/2*(2*Sqrt[3]*a^(1/3)*b^(2/3)*d*e*p*(d + e*x)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + 3*b*e^2*p*x^2*(d + e*x)*Hypergeometric2F1[2/3, 1, 5/3, -
((b*x^3)/a)] - 2*a^(1/3)*b^(2/3)*d*e*p*(d + e*x)*Log[a^(1/3) + b^(1/3)*x] +
 6*b*d^2*p*(d + e*x)*Log[d + e*x] + a^(1/3)*b^(2/3)*d*e*p*(d + e*x)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*b*d^2*p*(d + e*x)*Log[a + b*x^3
] + 2*(b*d^3 - a*e^3)*Log[c*(a + b*x^3)^p])/(e*(b*d^3 - a*e^3)*(d + e*x))
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.95

method	result
parts	$-\frac{\ln(c(bx^3+a)^p)}{e(ex+d)}+\frac{3pb\left(\frac{d^2\ln(ex+d)}{a e^3-b d^3}+\frac{-ade\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+ae^2\left(-\frac{\ln\left(x+ \right.}{3b(}\right.}{ae^3-bd^3}\right.}{e}$
risch	Expression too large to display

[In] int(ln(c*(b*x^3+a)^p)/(e*x+d)^2,x,method=_RETURNVERBOSE)

[Out] -ln(c*(b*x^3+a)^p)/e/(e*x+d)+3*p*b/e*(d^2/(a*e^3-b*d^3)*ln(e*x+d)+(-a*d*e*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))+a*e^2*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-1/3*d^2*ln(b*x^3+a))/(a*e^3-b*d^3))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.25 (sec) , antiderivative size = 7010, normalized size of antiderivative = 24.01

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{(d+ex)^2}\,dx = \text{Too large to display}$$

[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^2,x, algorithm="fricas")

[Out] Too large to include

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{(d+ex)^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(b*x**3+a)**p)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.07

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{(d+ex)^2}\,dx = -\frac{\left(\frac{6\,d^2\log(ex+d)}{bd^3-ae^3} + \frac{2\sqrt{3}\left(ae^2\left(\frac{a}{b}\right)^{\frac{2}{3}} - ade\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(b^2d^3\left(\frac{a}{b}\right)^{\frac{2}{3}} - abe^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\left(2\,bd^2\left(\frac{a}{b}\right)^{\frac{2}{3}} - ae^2\left(\frac{a}{b}\right)^{\frac{1}{3}} - ade\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b^2d^3\left(\frac{a}{b}\right)^{\frac{2}{3}} - abe^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right.}{2\,e} - \frac{\log\left((bx^3+a)^p c\right)}{(ex+d)e}$$

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(6*d^2*log(e*x + d)/(b*d^3 - a*e^3) + 2*sqrt(3)*(a*e^2*(a/b)^(2/3) - a
*d*e*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2
*d^3*(a/b)^(2/3) - a*b*e^3*(a/b)^(2/3))*(a/b)^(1/3)) - (2*b*d^2*(a/b)^(2/3)
 - a*e^2*(a/b)^(1/3) - a*d*e)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*d
^3*(a/b)^(2/3) - a*b*e^3*(a/b)^(2/3)) - 2*(b*d^2*(a/b)^(2/3) + a*e^2*(a/b)^
(1/3) + a*d*e)*log(x + (a/b)^(1/3))/(b^2*d^3*(a/b)^(2/3) - a*b*e^3*(a/b)^(2
/3)))*b*p/e - log((b*x^3 + a)^p*c)/((e*x + d)*e)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.24

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{(d+ex)^2}\,dx$$
$$= -\frac{3\,bd^2p\log\left(ex+d\right)}{bd^3e-ae^4}+\frac{bd^2p\log\left(|bx^3+a|\right)}{bd^3e-ae^4}+\frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\,bp\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2d^2-(-ab^2)^{\frac{1}{3}}\,bde+(-ab^2)^{\frac{2}{3}}\,e^2}$$
$$+\frac{\left(ab^3d^3e^3p\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^2e^6p\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^3d^4e^2p+a^2b^2de^5p\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{ab^3d^6e^2-2\,a^2b^2d^3e^5+a^3be^8}$$
$$-\frac{p\log\left(bx^3+a\right)}{e^2x+de}$$
$$+\frac{\left((-ab^2)^{\frac{1}{3}}\,bdp-(-ab^2)^{\frac{2}{3}}\,ep\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\left(b^2d^3-abe^3\right)}-\frac{\log\left(c\right)}{e^2x+de}$$

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -3*b*d^2*p*log(e*x + d)/(b*d^3*e - a*e^4) + b*d^2*p*log(abs(b*x^3 + a))/(b*
d^3*e - a*e^4) + sqrt(3)*(-a*b^2)^(1/3)*b*p*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/(b^2*d^2 - (-a*b^2)^(1/3)*b*d*e + (-a*b^2)^(2/3)*e^2
) + (a*b^3*d^3*e^3*p*(-a/b)^(1/3) - a^2*b^2*e^6*p*(-a/b)^(1/3) - a*b^3*d^4*
e^2*p + a^2*b^2*d*e^5*p)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3*d^6
*e^2 - 2*a^2*b^2*d^3*e^5 + a^3*b*e^8) - p*log(b*x^3 + a)/(e^2*x + d*e) + 1/
2*((-a*b^2)^(1/3)*b*d*p - (-a*b^2)^(2/3)*e*p)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/(b^2*d^3 - a*b*e^3) - log(c)/(e^2*x + d*e)
```

Mupad [B] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 736, normalized size of antiderivative = 2.52

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{(d+ex)^2}\,dx$$
$$=\left(\sum_{k=1}^{3}\ln\left(-\frac{27\,a\,b^4\,d\,p^3+27\,a\,b^4\,e\,p^3\,x+\mathrm{root}(b\,d^3\,e^3\,z^3-a\,e^6\,z^3-3\,b\,d^2\,e^2\,p\,z^2+3\,b\,d\,e\,p^2\,z-b\,p^3,z,k)^3\,a\,b}{}\right.\right.$$
$$\left.\left.-a\,e^6\,z^3-3\,b\,d^2\,e^2\,p\,z^2+3\,b\,d\,e\,p^2\,z-b\,p^3,z,k\right)\right)-\frac{\ln\left(c\,(b\,x^3+a)^p\right)}{x\,e^2+d\,e}+\frac{3\,b\,d^2\,p\,\ln\left(d+e\,x\right)}{a\,e^4-b\,d^3\,e}$$

```
[In] int(log(c*(a + b*x^3)^p)/(d + e*x)^2,x)
```

```
[Out] symsum(log(-(27*a*b^4*d*p^3 + 27*a*b^4*e*p^3*x + 9*root(b*d^3*e^3*z^3 - a*e
^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z, k)^3*a*b^4*d^4*e^3 +
 45*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z - b*
p^3, z, k)^3*a^2*b^3*d*e^6 - 9*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2
*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z, k)^2*a^2*b^3*e^5*p + 36*root(b*d^3*e^3*z
^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z, k)^3*a^2*b^3
*e^7*x + 9*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2
*z - b*p^3, z, k)^2*a*b^4*d^3*e^2*p + 18*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3
*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z, k)^3*a*b^4*d^3*e^4*x - 45*root
(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z,
k)*a*b^4*d^2*e*p^2 - 72*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^2
+ 3*b*d*e*p^2*z - b*p^3, z, k)*a*b^4*d*e^2*p^2*x + 27*root(b*d^3*e^3*z^3 -
a*e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^2*z - b*p^3, z, k)^2*a*b^4*d^2*e^
3*p*x)/e^2)*root(b*d^3*e^3*z^3 - a*e^6*z^3 - 3*b*d^2*e^2*p*z^2 + 3*b*d*e*p^
2*z - b*p^3, z, k), k, 1, 3) - log(c*(a + b*x^3)^p)/(d*e + e^2*x) + (3*b*d^
2*p*log(d + e*x))/(a*e^4 - b*d^3*e)
```

3.197 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 391

$$
\begin{aligned}
\int \frac{\log\left(c(a+bx^3)^p\right)}{(d+ex)^3}\,dx
&= \frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)} - \frac{\sqrt{3}\sqrt[3]{a}b^{2/3}\left(2bd^3-3\sqrt[3]{a}b^{2/3}d^2e+ae^3\right)p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2\left(bd^3-ae^3\right)^2} \\
&\quad + \frac{\sqrt[3]{a}b^{2/3}\left(2bd^3+3\sqrt[3]{a}b^{2/3}d^2e+ae^3\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\left(bd^3-ae^3\right)^2} - \frac{3bd(bd^3+2ae^3)\,p\log(d+ex)}{2e\left(bd^3-ae^3\right)^2} \\
&\quad - \frac{\sqrt[3]{a}b^{2/3}\left(2bd^3+3\sqrt[3]{a}b^{2/3}d^2e+ae^3\right)p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4\left(bd^3-ae^3\right)^2} \\
&\quad + \frac{bd(bd^3+2ae^3)\,p\log\left(a+bx^3\right)}{2e\left(bd^3-ae^3\right)^2} - \frac{\log\left(c(a+bx^3)^p\right)}{2e(d+ex)^2}
\end{aligned}
$$

```
[Out] 3/2*b*d^2*p/e/(-a*e^3+b*d^3)/(e*x+d)+1/2*a^(1/3)*b^(2/3)*(2*b*d^3+3*a^(1/3)
*b^(2/3)*d^2*e+a*e^3)*p*ln(a^(1/3)+b^(1/3)*x)/(-a*e^3+b*d^3)^2-3/2*b*d*(2*a
*e^3+b*d^3)*p*ln(e*x+d)/e/(-a*e^3+b*d^3)^2-1/4*a^(1/3)*b^(2/3)*(2*b*d^3+3*a
^(1/3)*b^(2/3)*d^2*e+a*e^3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-a
*e^3+b*d^3)^2+1/2*b*d*(2*a*e^3+b*d^3)*p*ln(b*x^3+a)/e/(-a*e^3+b*d^3)^2-1/2*
ln(c*(b*x^3+a)^p)/e/(e*x+d)^2-1/2*a^(1/3)*b^(2/3)*(2*b*d^3-3*a^(1/3)*b^(2/3
)*d^2*e+a*e^3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/
(-a*e^3+b*d^3)^2
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2513, 6857, 1885, 1874, 31, 648, 631, 210, 642, 266}

$$\int \frac{\log \left(c\left(a+bx^3\right)^p\right)}{(d+ex)^3}\,dx = -\frac{\sqrt[3]{a}b^{2/3}p\left(3\sqrt[3]{a}b^{2/3}d^2e+ae^3+2bd^3\right)\log \left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4\left(bd^3-ae^3\right)^2} - \frac{\sqrt{3}\sqrt[3]{a}b^{2/3}p\arctan \left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)\left(-3\sqrt[3]{a}b^{2/3}d^2e+ae^3+2bd^3\right)}{2\left(bd^3-ae^3\right)^2} + \frac{\sqrt[3]{a}b^{2/3}p\left(3\sqrt[3]{a}b^{2/3}d^2e+ae^3+2bd^3\right)\log \left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\left(bd^3-ae^3\right)^2} - \frac{\log \left(c\left(a+bx^3\right)^p\right)}{2e(d+ex)^2} + \frac{bdp\left(2ae^3+bd^3\right)\log \left(a+bx^3\right)}{2e\left(bd^3-ae^3\right)^2} - \frac{3bdp\left(2ae^3+bd^3\right)\log(d+ex)}{2e\left(bd^3-ae^3\right)^2} + \frac{3bd^2p}{2e(d+ex)\left(bd^3-ae^3\right)}$$

```
[In] Int[Log[c*(a + b*x^3)^p]/(d + e*x)^3,x]
```

```
[Out] (3*b*d^2*p)/(2*e*(b*d^3 - a*e^3)*(d + e*x)) - (Sqrt[3]*a^(1/3)*b^(2/3)*(2*b
*d^3 - 3*a^(1/3)*b^(2/3)*d^2*e + a*e^3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(S
qrt[3]*a^(1/3))])/(2*(b*d^3 - a*e^3)^2) + (a^(1/3)*b^(2/3)*(2*b*d^3 + 3*a^(
1/3)*b^(2/3)*d^2*e + a*e^3)*p*Log[a^(1/3) + b^(1/3)*x])/(2*(b*d^3 - a*e^3)^
2) - (3*b*d*(b*d^3 + 2*a*e^3)*p*Log[d + e*x])/(2*e*(b*d^3 - a*e^3)^2) - (a^
(1/3)*b^(2/3)*(2*b*d^3 + 3*a^(1/3)*b^(2/3)*d^2*e + a*e^3)*p*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*(b*d^3 - a*e^3)^2) + (b*d*(b*d^3 + 2*a*
e^3)*p*Log[a + b*x^3])/(2*e*(b*d^3 - a*e^3)^2) - Log[c*(a + b*x^3)^p]/(2*e*
(d + e*x)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1874

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Dist[(-r)*((B*r - A*s)/(3*a
*s)), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B
*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1885

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2e(d+ex)^2}+\frac{(3bp)\int\frac{x^2}{(d+ex)^2\left(a+bx^3\right)}\,dx}{2e}\\
&= -\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2e(d+ex)^2}\\
&\quad+\frac{(3bp)\int\left(-\frac{d^2e}{\left(bd^3-ae^3\right)(d+ex)^2}-\frac{de\left(bd^3+2ae^3\right)}{\left(bd^3-ae^3\right)^2(d+ex)}+\frac{ae\left(2bd^3+ae^3\right)-3abd^2e^2x+bd\left(bd^3+2ae^3\right)x^2}{\left(bd^3-ae^3\right)^2\left(a+bx^3\right)}\right)dx}{2e}\\
&= \frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)}-\frac{3bd\left(bd^3+2ae^3\right)p\log(d+ex)}{2e\left(bd^3-ae^3\right)^2}\\
&\quad-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2e(d+ex)^2}+\frac{(3bp)\int\frac{ae\left(2bd^3+ae^3\right)-3abd^2e^2x+bd\left(bd^3+2ae^3\right)x^2}{a+bx^3}\,dx}{2e\left(bd^3-ae^3\right)^2}\\
&= \frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)}-\frac{3bd\left(bd^3+2ae^3\right)p\log(d+ex)}{2e\left(bd^3-ae^3\right)^2}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2e(d+ex)^2}\\
&\quad+\frac{(3bp)\int\frac{ae\left(2bd^3+ae^3\right)-3abd^2e^2x}{a+bx^3}\,dx}{2e\left(bd^3-ae^3\right)^2}+\frac{\left(3b^2d\left(bd^3+2ae^3\right)p\right)\int\frac{x^2}{a+bx^3}\,dx}{2e\left(bd^3-ae^3\right)^2}\\
&= \frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)}-\frac{3bd\left(bd^3+2ae^3\right)p\log(d+ex)}{2e\left(bd^3-ae^3\right)^2}\\
&\quad+\frac{bd\left(bd^3+2ae^3\right)p\log\left(a+bx^3\right)}{2e\left(bd^3-ae^3\right)^2}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{2e(d+ex)^2}\\
&\quad+\frac{\left(b^{2/3}p\right)\int\frac{\sqrt[3]{a}\left(-3a^{4/3}bd^2e^2+2a\sqrt[3]{b}e\left(2bd^3+ae^3\right)\right)+\sqrt[3]{b}\left(-3a^{4/3}bd^2e^2-a\sqrt[3]{b}e\left(2bd^3+ae^3\right)\right)x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^{2/3}e\left(bd^3-ae^3\right)^2}\\
&\quad+\frac{\left(\sqrt[3]{a}b\left(2bd^3+3\sqrt[3]{a}b^{2/3}d^2e+ae^3\right)p\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{2\left(bd^3-ae^3\right)^2}
\end{aligned}
$$

$$=\frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)}+\frac{\sqrt[3]{a}b^{2/3}\left(2bd^3+3\sqrt[3]{a}b^{2/3}d^2e+ae^3\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\left(bd^3-ae^3\right)^2}$$
$$-\frac{3bd(bd^3+2ae^3)\,p\log(d+ex)}{2e\left(bd^3-ae^3\right)^2}$$
$$+\frac{bd(bd^3+2ae^3)\,p\log\left(a+bx^3\right)}{2e\left(bd^3-ae^3\right)^2}-\frac{\log\left(c(a+bx^3)^p\right)}{2e(d+ex)^2}$$
$$+\frac{\left(3a^{2/3}b\left(2bd^3-3\sqrt[3]{a}b^{2/3}d^2e+ae^3\right)p\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{4\left(bd^3-ae^3\right)^2}$$
$$-\frac{\left(\sqrt[3]{a}b^{2/3}\left(2bd^3+3\sqrt[3]{a}b^{2/3}d^2e+ae^3\right)p\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{4\left(bd^3-ae^3\right)^2}$$

$$=\frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)}+\frac{\sqrt[3]{a}b^{2/3}\left(2bd^3+3\sqrt[3]{a}b^{2/3}d^2e+ae^3\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\left(bd^3-ae^3\right)^2}$$
$$-\frac{3bd(bd^3+2ae^3)\,p\log(d+ex)}{2e\left(bd^3-ae^3\right)^2}$$
$$-\frac{\sqrt[3]{a}b^{2/3}\left(2bd^3+3\sqrt[3]{a}b^{2/3}d^2e+ae^3\right)p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4\left(bd^3-ae^3\right)^2}$$
$$+\frac{bd(bd^3+2ae^3)\,p\log\left(a+bx^3\right)}{2e\left(bd^3-ae^3\right)^2}-\frac{\log\left(c(a+bx^3)^p\right)}{2e(d+ex)^2}$$
$$+\frac{\left(3\sqrt[3]{a}b^{2/3}\left(2bd^3-3\sqrt[3]{a}b^{2/3}d^2e+ae^3\right)p\right)\mathrm{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{2\left(bd^3-ae^3\right)^2}$$

$$=\frac{3bd^2p}{2e\left(bd^3-ae^3\right)(d+ex)}$$
$$-\frac{\sqrt{3}\sqrt[3]{a}b^{2/3}\left(2bd^3-3\sqrt[3]{a}b^{2/3}d^2e+ae^3\right)p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2\left(bd^3-ae^3\right)^2}$$
$$+\frac{\sqrt[3]{a}b^{2/3}\left(2bd^3+3\sqrt[3]{a}b^{2/3}d^2e+ae^3\right)p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\left(bd^3-ae^3\right)^2}$$
$$-\frac{3bd(bd^3+2ae^3)\,p\log(d+ex)}{2e\left(bd^3-ae^3\right)^2}$$
$$-\frac{\sqrt[3]{a}b^{2/3}\left(2bd^3+3\sqrt[3]{a}b^{2/3}d^2e+ae^3\right)p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4\left(bd^3-ae^3\right)^2}$$
$$+\frac{bd(bd^3+2ae^3)\,p\log\left(a+bx^3\right)}{2e\left(bd^3-ae^3\right)^2}-\frac{\log\left(c(a+bx^3)^p\right)}{2e(d+ex)^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.39 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.83

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{(d+ex)^3}\,dx$$

$$= \frac{b^{2/3}p(d+ex)\left(6\sqrt[3]{b}d^2(bd^3-ae^3)-2\sqrt{3}\sqrt[3]{a}e(2bd^3+ae^3)(d+ex)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-9b^{4/3}d^2e^2x^2(d+ex)\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a}\right)\cdots}{\cdots}$$

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(d + e*x)^3,x]
```

```
[Out] ((b^(2/3)*p*(d + e*x)*(6*b^(1/3)*d^2*(b*d^3 - a*e^3) - 2*Sqrt[3]*a^(1/3)*e*
(2*b*d^3 + a*e^3)*(d + e*x)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 9
*b^(4/3)*d^2*e^2*x^2*(d + e*x)*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)]
 + 2*a^(1/3)*e*(2*b*d^3 + a*e^3)*(d + e*x)*Log[a^(1/3) + b^(1/3)*x] - 6*b^(
1/3)*d*(b*d^3 + 2*a*e^3)*(d + e*x)*Log[d + e*x] - a^(1/3)*e*(2*b*d^3 + a*e^
3)*(d + e*x)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^(1/3)*d*(
b*d^3 + 2*a*e^3)*(d + e*x)*Log[a + b*x^3]))/(b*d^3 - a*e^3)^2 - 2*Log[c*(a
+ b*x^3)^p])/(4*e*(d + e*x)^2)
```

Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.89

method	result
parts	$-\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{2e\left(ex+d\right)^{2}}+\frac{3pb\left(-\frac{d^{2}}{\left(a\,e^{3}-b\,d^{3}\right)\left(ex+d\right)}-\frac{d\left(2a\,e^{3}+b\,d^{3}\right)\ln\left(ex+d\right)}{\left(a\,e^{3}-b\,d^{3}\right)^{2}}+\frac{\left(a^{2}e^{4}+2ab\,d^{3}e\right)\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\cdots\right.}{\cdots}\right.}{\cdots}$
risch	Expression too large to display

```
[In] int(ln(c*(b*x^3+a)^p)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(c*(b*x^3+a)^p)/e/(e*x+d)^2+3/2*p*b/e*(-d^2/(a*e^3-b*d^3)/(e*x+d)-d*
(2*a*e^3+b*d^3)/(a*e^3-b*d^3)^2*ln(e*x+d)+((a^2*e^4+2*a*b*d^3*e)*(1/3/b/(a/
b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)))-3*a*
b*d^2*e^2*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a
/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1)))+1/3*(2*a*b*d*e^3+b^2*d^4)/b*ln(b*x^3+a))/(a*e^3-b*d^3)^2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.42 (sec) , antiderivative size = 13236, normalized size of antiderivative = 33.85

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{(d+ex)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{(d+ex)^3}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(b*x**3+a)**p)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.32

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{(d+ex)^3}\,dx = -\frac{\left(\frac{2\sqrt{3}\left(3\,abd^2e^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,abd^3e\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^2e^4\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(b^3d^6\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,ab^2d^3e^3\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^2be^6\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{6\,d^2}{bd^4-ade^3+(bd^3e-ae^4)x}+\frac{6\,(bd^4+2\,ade^3)\log(ex+d)}{b^2d^6-2\,abd^3e^3+a^2e^6}\right.}{} - \frac{\log\left((bx^3+a)^pc\right)}{2\,(ex+d)^2e}$$

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(3)*(3*a*b*d^2*e^2*(a/b)^(2/3) - 2*a*b*d^3*e*(a/b)^(1/3) - a^2*
e^4*(a/b)^(1/3))*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^3*
d^6*(a/b)^(2/3) - 2*a*b^2*d^3*e^3*(a/b)^(2/3) + a^2*b*e^6*(a/b)^(2/3))*(a/b
)^(1/3)) - 6*d^2/(b*d^4 - a*d*e^3 + (b*d^3*e - a*e^4)*x) + 6*(b*d^4 + 2*a*d
*e^3)*log(e*x + d)/(b^2*d^6 - 2*a*b*d^3*e^3 + a^2*e^6) - (2*b^2*d^4*(a/b)^(
2/3) + 4*a*b*d*e^3*(a/b)^(2/3) - 3*a*b*d^2*e^2*(a/b)^(1/3) - 2*a*b*d^3*e -
a^2*e^4)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*d^6*(a/b)^(2/3) - 2*a*
b^2*d^3*e^3*(a/b)^(2/3) + a^2*b*e^6*(a/b)^(2/3)) - 2*(b^2*d^4*(a/b)^(2/3) +
 2*a*b*d*e^3*(a/b)^(2/3) + 3*a*b*d^2*e^2*(a/b)^(1/3) + 2*a*b*d^3*e + a^2*e^
4)*log(x + (a/b)^(1/3))/(b^3*d^6*(a/b)^(2/3) - 2*a*b^2*d^3*e^3*(a/b)^(2/3)
+ a^2*b*e^6*(a/b)^(2/3)))*b*p/e - 1/2*log((b*x^3 + a)^p*c)/((e*x + d)^2*e)
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.69

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{(d+ex)^3}\,dx$$
$$= \frac{\left(3\,ab^5d^8e^3p\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 6\,a^2b^4d^5e^6p\left(-\frac{a}{b}\right)^{\frac{1}{3}} + 3\,a^3b^3d^2e^9p\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 2\,ab^5d^9e^2p + 3\,a^2b^4d^6e^5p - a^4b^2e^{11}p\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{2\left(ab^5d^{12}e^2 - 4\,a^2b^4d^9e^5 + 6\,a^3b^3d^6e^8 - 4\,a^4b^2d^3e^{11} + a^5be^{14}\right)}$$
$$+ \frac{3\left(2\left(-ab^2\right)^{\frac{1}{3}}bdp - \left(-ab^2\right)^{\frac{2}{3}}ep\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{2\left(\sqrt{3}b^2d^4 + 2\sqrt{3}abde^3 - 2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}bd^3e - \sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}ae^4 + 3\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}d^2e^2\right)}$$
$$- \frac{p\log\left(bx^3+a\right)}{2\left(e^3x^2 + 2\,de^2x + d^2e\right)} - \frac{3\left(b^2d^4p + 2\,abde^3p\right)\log\left(ex+d\right)}{2\left(b^2d^6e - 2\,abd^3e^4 + a^2e^7\right)}$$
$$+ \frac{\left(2\left(-ab^2\right)^{\frac{1}{3}}bd^3p + \left(-ab^2\right)^{\frac{1}{3}}ae^3p - 3\left(-ab^2\right)^{\frac{2}{3}}d^2ep\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{4\left(b^2d^6 - 2\,abd^3e^3 + a^2e^6\right)}$$
$$+ \frac{\left(b^2d^4p + 2\,abde^3p\right)\log\left(|bx^3+a|\right)}{2\left(b^2d^6e - 2\,abd^3e^4 + a^2e^7\right)}$$
$$+ \frac{3\,bd^2epx + 3\,bd^3p - bd^3\log\left(c\right) + ae^3\log\left(c\right)}{2\left(bd^3e^3x^2 - ae^6x^2 + 2\,bd^4e^2x - 2\,ade^5x + bd^5e - ad^2e^4\right)}$$

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/2*(3*a*b^5*d^8*e^3*p*(-a/b)^(1/3) - 6*a^2*b^4*d^5*e^6*p*(-a/b)^(1/3) + 3*
a^3*b^3*d^2*e^9*p*(-a/b)^(1/3) - 2*a*b^5*d^9*e^2*p + 3*a^2*b^4*d^6*e^5*p -
a^4*b^2*e^11*p)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^5*d^12*e^2 - 4
*a^2*b^4*d^9*e^5 + 6*a^3*b^3*d^6*e^8 - 4*a^4*b^2*d^3*e^11 + a^5*b*e^14) + 3
/2*(2*(-a*b^2)^(1/3)*b*d*p - (-a*b^2)^(2/3)*e*p)*arctan(1/3*sqrt(3)*(2*x +
(-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^2*d^4 + 2*sqrt(3)*a*b*d*e^3 - 2*sqrt
(3)*(-a*b^2)^(1/3)*b*d^3*e - sqrt(3)*(-a*b^2)^(1/3)*a*e^4 + 3*sqrt(3)*(-a*b
^2)^(2/3)*d^2*e^2) - 1/2*p*log(b*x^3 + a)/(e^3*x^2 + 2*d*e^2*x + d^2*e) - 3
/2*(b^2*d^4*p + 2*a*b*d*e^3*p)*log(e*x + d)/(b^2*d^6*e - 2*a*b*d^3*e^4 + a^
2*e^7) + 1/4*(2*(-a*b^2)^(1/3)*b*d^3*p + (-a*b^2)^(1/3)*a*e^3*p - 3*(-a*b^2
)^(2/3)*d^2*e*p)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^2*d^6 - 2*a*b*
d^3*e^3 + a^2*e^6) + 1/2*(b^2*d^4*p + 2*a*b*d*e^3*p)*log(abs(b*x^3 + a))/(b
^2*d^6*e - 2*a*b*d^3*e^4 + a^2*e^7) + 1/2*(3*b*d^2*e*p*x + 3*b*d^3*p - b*d^
3*log(c) + a*e^3*log(c))/(b*d^3*e^3*x^2 - a*e^6*x^2 + 2*b*d^4*e^2*x - 2*a*d
*e^5*x + b*d^5*e - a*d^2*e^4)
```

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 2227, normalized size of antiderivative = 5.70

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{(d+ex)^3}\,dx = \text{Too large to display}$$

```
[In] int(log(c*(a + b*x^3)^p)/(d + e*x)^3,x)
```

```
[Out] symsum(log(-(27*a*b^6*d^4*p^3 + 216*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3
*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^
2*e*p^2*z + b^2*p^3, z, k)^3*a^2*b^5*d^7*e^6 - 648*root(16*a*b*d^3*e^6*z^3
- 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p
*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^3*a^3*b^4*d^4*e^9 + 72*root(16*a*
b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12
*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^3*a*b^6*d^10*e^3 +
360*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*
e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^3*a^4
*b^3*d*e^12 + 18*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^
3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3
, z, k)*a^3*b^4*e^7*p^2 + 288*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 -
 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^
2*z + b^2*p^3, z, k)^3*a^4*b^3*e^13*x + 27*a^2*b^5*d*e^3*p^3 - 27*a^2*b^5*e
^4*p^3*x + 36*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 +
 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z
, k)^2*a*b^6*d^8*e^2*p + 144*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 -
8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2
*z + b^2*p^3, z, k)^3*a*b^6*d^9*e^4*x - 90*root(16*a*b*d^3*e^6*z^3 - 8*b^2*
d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6
*b^2*d^2*e*p^2*z + b^2*p^3, z, k)*a^2*b^5*d^3*e^4*p^2 + 252*root(16*a*b*d^3
*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*
d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^2*a^2*b^5*d^5*e^5*p - 28
8*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^
5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^2*a^3*b
^4*d^2*e^8*p - 432*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*
z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p
^3, z, k)^3*a^3*b^4*d^3*e^10*x - 90*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3
*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^
2*e*p^2*z + b^2*p^3, z, k)*a*b^6*d^6*e*p^2 - 54*root(16*a*b*d^3*e^6*z^3 - 8
*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^
2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)*a^2*b^5*d^2*e^5*p^2*x + 360*root(16*
a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 +
12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^2*a^2*b^5*d^4*e^6
*p*x - 108*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24
*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*z + b^2*p^3, z, k
```

)*a*b^6*d^5*e^2*p^2*x + 144*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8
*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*
z + b^2*p^3, z, k)^2*a*b^6*d^7*e^3*p*x - 504*root(16*a*b*d^3*e^6*z^3 - 8*b^
2*d^6*e^3*z^3 - 8*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 -
6*b^2*d^2*e*p^2*z + b^2*p^3, z, k)^2*a^3*b^4*d*e^9*p*x)/(8*a^2*e^8 + 8*b^2
*d^6*e^2 - 16*a*b*d^3*e^5))*root(16*a*b*d^3*e^6*z^3 - 8*b^2*d^6*e^3*z^3 - 8
*a^2*e^9*z^3 + 24*a*b*d*e^5*p*z^2 + 12*b^2*d^4*e^2*p*z^2 - 6*b^2*d^2*e*p^2*
z + b^2*p^3, z, k), k, 1, 3) - log(c*(a + b*x^3)^p)/(2*(d^2*e + e^3*x^2 + 2
*d*e^2*x)) - (3*b*d^2*p)/(2*a*d*e^4 - 2*b*d^4*e + 2*a*e^5*x - 2*b*d^3*e^2*x
) - (3*b^2*d^4*p*log(d + e*x))/(2*a^2*e^7 + 2*b^2*d^6*e - 4*a*b*d^3*e^4) -
(6*a*b*d*e^3*p*log(d + e*x))/(2*a^2*e^7 + 2*b^2*d^6*e - 4*a*b*d^3*e^4)

3.198 $\int (d+ex)^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 139

$$\int (d+ex)^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx = \frac{be(6a^2d^2-4abde+b^2e^2)\,px}{4a^3} + \frac{be^2(4ad-be)px^2}{8a^2} + \frac{be^3px^3}{12a} + \frac{(d+ex)^4 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{4e} + \frac{d^4p\log(x)}{4e} - \frac{(ad-be)^4p\log(b+ax)}{4a^4e}$$

```
[Out] 1/4*b*e*(6*a^2*d^2-4*a*b*d*e+b^2*e^2)*p*x/a^3+1/8*b*e^2*(4*a*d-b*e)*p*x^2/a
^2+1/12*b*e^3*p*x^3/a+1/4*(e*x+d)^4*ln(c*(a+b/x)^p)/e+1/4*d^4*p*ln(x)/e-1/4
*(a*d-b*e)^4*p*ln(a*x+b)/a^4/e
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2513, 528, 84}

$$\int (d+ex)^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx = -\frac{p(ad-be)^4\log(ax+b)}{4a^4e} + \frac{be^2px^2(4ad-be)}{8a^2} + \frac{bepx(6a^2d^2-4abde+b^2e^2)}{4a^3} + \frac{(d+ex)^4 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{4e} + \frac{be^3px^3}{12a} + \frac{d^4p\log(x)}{4e}$$

```
[In] Int[(d + e*x)^3*Log[c*(a + b/x)^p],x]
```

```
[Out] (b*e*(6*a^2*d^2 - 4*a*b*d*e + b^2*e^2)*p*x)/(4*a^3) + (b*e^2*(4*a*d - b*e)*
p*x^2)/(8*a^2) + (b*e^3*p*x^3)/(12*a) + ((d + e*x)^4*Log[c*(a + b/x)^p])/(4
*e) + (d^4*p*Log[x])/(4*e) - ((a*d - b*e)^4*p*Log[b + a*x])/(4*a^4*e)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d+ex)^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e} + \frac{(bp)\int \frac{(d+ex)^4}{\left(a+\frac{b}{x}\right)x^2}\,dx}{4e} \\
&= \frac{(d+ex)^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e} + \frac{(bp)\int \frac{(d+ex)^4}{x(b+ax)}\,dx}{4e} \\
&= \frac{(d+ex)^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e} \\
&\quad + \frac{(bp)\int \left(\frac{e^2(6a^2d^2-4abde+b^2e^2)}{a^3}+\frac{d^4}{bx}+\frac{e^3(4ad-be)x}{a^2}+\frac{e^4x^2}{a}-\frac{(ad-be)^4}{a^3b(b+ax)}\right)dx}{4e} \\
&= \frac{be(6a^2d^2-4abde+b^2e^2)\,px}{4a^3} + \frac{be^2(4ad-be)px^2}{8a^2} + \frac{be^3px^3}{12a} \\
&\quad + \frac{(d+ex)^4 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4e} + \frac{d^4p\log(x)}{4e} - \frac{(ad-be)^4p\log(b+ax)}{4a^4e}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.82

$$\int (d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$$
$$= \frac{\frac{be^2px(6b^2e^2-3abe(8d+ex)+2a^2(18d^2+6dex+e^2x^2))}{6a^3} + (d+ex)^4\log\left(c\left(a+\frac{b}{x}\right)^p\right) + d^4p\log(x) - \frac{(ad-be)^4p\log(b+ax)}{a^4}}{4e}$$

```
[In] Integrate[(d + e*x)^3*Log[c*(a + b/x)^p],x]
```

```
[Out] ((b*e^2*p*x*(6*b^2*e^2 - 3*a*b*e*(8*d + e*x) + 2*a^2*(18*d^2 + 6*d*e*x + e^
2*x^2)))/(6*a^3) + (d + e*x)^4*Log[c*(a + b/x)^p] + d^4*p*Log[x] - ((a*d -
b*e)^4*p*Log[b + a*x])/a^4)/(4*e)
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.76

method	result
parts	$\frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)e^3x^4}{4} + \ln\left(c\left(a+\frac{b}{x}\right)^p\right)e^2d\,x^3 + \frac{3\ln\left(c\left(a+\frac{b}{x}\right)^p\right)e\,d^2x^2}{2} + \ln\left(c\left(a+\frac{b}{x}\right)^p\right)d^3x + \frac{\ln\left(c\left(a+\frac{b}{x}\right)\right.}{4e}$
parallelrisch	$-\frac{-6x^4\ln\left(c\left(\frac{ax+b}{x}\right)^p\right)a^4e^3-24x^3\ln\left(c\left(\frac{ax+b}{x}\right)^p\right)a^4d\,e^2-2x^3a^3b\,e^3p-36x^2\ln\left(c\left(\frac{ax+b}{x}\right)^p\right)a^4d^2e-12x^2a^3bd\,e^2p+3x^2a^2b^2e^3}{}$

```
[In] int((e*x+d)^3*ln(c*(a+b/x)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(c*(a+b/x)^p)*e^3*x^4+ln(c*(a+b/x)^p)*e^2*d*x^3+3/2*ln(c*(a+b/x)^p)*e
*d^2*x^2+ln(c*(a+b/x)^p)*d^3*x+1/4*ln(c*(a+b/x)^p)/e*d^4+1/4*p*b/e*(e^2/a^3
*(1/3*a^2*e^2*x^3+2*a^2*d*e*x^2-1/2*a*b*e^2*x^2+6*a^2*d^2*x-4*a*b*d*e*x+x*e
^2*b^2)+d^4/b*ln(x)+(-a^4*d^4+4*a^3*b*d^3*e-6*a^2*b^2*d^2*e^2+4*a*b^3*d*e^3
-b^4*e^4)/a^4/b*ln(a*x+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.72

$$\int (d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$$
$$= \frac{2\,a^3be^3px^3 + 3\,(4\,a^3bde^2 - a^2b^2e^3)px^2 + 6\,(6\,a^3bd^2e - 4\,a^2b^2de^2 + ab^3e^3)px + 6\,(4\,a^3bd^3 - 6\,a^2b^2d^2e + 4\,ab}{}$$

```
[In] integrate((e*x+d)^3*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/24*(2*a^3*b*e^3*p*x^3 + 3*(4*a^3*b*d*e^2 - a^2*b^2*e^3)*p*x^2 + 6*(6*a^3*
b*d^2*e - 4*a^2*b^2*d*e^2 + a*b^3*e^3)*p*x + 6*(4*a^3*b*d^3 - 6*a^2*b^2*d^2
*e + 4*a*b^3*d*e^2 - b^4*e^3)*p*log(a*x + b) + 6*(a^4*e^3*x^4 + 4*a^4*d*e^2
*x^3 + 6*a^4*d^2*e*x^2 + 4*a^4*d^3*x)*log(c) + 6*(a^4*e^3*p*x^4 + 4*a^4*d*e
^2*p*x^3 + 6*a^4*d^2*e*p*x^2 + 4*a^4*d^3*p*x)*log((a*x + b)/x))/a^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(128) = 256$.

Time = 1.66 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.55

$$\int (d+ex)^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx$$
$$= \begin{cases} d^3x\log\left(c\left(a+\frac{b}{x}\right)^p\right) + \frac{3d^2ex^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2} + de^2x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right) + \frac{e^3x^4\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{4} + \frac{bd^3p\log\left(x+\frac{b}{a}\right)}{a} + \frac{3bd^2}{2a} \\ d^3px + d^3x\log\left(c\left(\frac{b}{x}\right)^p\right) + \frac{3d^2epx^2}{4} + \frac{3d^2ex^2\log\left(c\left(\frac{b}{x}\right)^p\right)}{2} + \frac{de^2px^3}{3} + de^2x^3\log\left(c\left(\frac{b}{x}\right)^p\right) + \frac{e^3px^4}{16} + \frac{e^3x^4\log\left(c\left(\frac{b}{x}\right)^p\right)}{4} \end{cases}$$

```
[In] integrate((e*x+d)**3*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((d**3*x*log(c*(a + b/x)**p) + 3*d**2*e*x**2*log(c*(a + b/x)**p)/2
 + d*e**2*x**3*log(c*(a + b/x)**p) + e**3*x**4*log(c*(a + b/x)**p)/4 + b*d*
*3*p*log(x + b/a)/a + 3*b*d**2*e*p*x/(2*a) + b*d*e**2*p*x**2/(2*a) + b*e**3
*p*x**3/(12*a) - 3*b**2*d**2*e*p*log(x + b/a)/(2*a**2) - b**2*d*e**2*p*x/a*
*2 - b**2*e**3*p*x**2/(8*a**2) + b**3*d*e**2*p*log(x + b/a)/a**3 + b**3*e**
3*p*x/(4*a**3) - b**4*e**3*p*log(x + b/a)/(4*a**4), Ne(a, 0)), (d**3*p*x +
d**3*x*log(c*(b/x)**p) + 3*d**2*e*p*x**2/4 + 3*d**2*e*x**2*log(c*(b/x)**p)/
2 + d*e**2*p*x**3/3 + d*e**2*x**3*log(c*(b/x)**p) + e**3*p*x**4/16 + e**3*x
**4*log(c*(b/x)**p)/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.19

$$\int (d+ex)^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx$$
$$= \frac{1}{24}bp\left(\frac{2a^2e^3x^3 + 3(4a^2de^2 - abe^3)x^2 + 6(6a^2d^2e - 4abde^2 + b^2e^3)x}{a^3} + \frac{6(4a^3d^3 - 6a^2bd^2e + 4ab^2de^2 - }{a^4}\right.$$
$$+ \frac{1}{4}\left(e^3x^4 + 4de^2x^3 + 6d^2ex^2 + 4d^3x\right)\log\left(\left(a+\frac{b}{x}\right)^p c\right)$$

```
[In] integrate((e*x+d)^3*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/24*b*p*((2*a^2*e^3*x^3 + 3*(4*a^2*d*e^2 - a*b*e^3)*x^2 + 6*(6*a^2*d^2*e -
 4*a*b*d*e^2 + b^2*e^3)*x)/a^3 + 6*(4*a^3*d^3 - 6*a^2*b*d^2*e + 4*a*b^2*d*e
^2 - b^3*e^3)*log(a*x + b)/a^4) + 1/4*(e^3*x^4 + 4*d*e^2*x^3 + 6*d^2*e*x^2
+ 4*d^3*x)*log((a + b/x)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 847 vs. $2(127) = 254$.

Time = 0.33 (sec) , antiderivative size = 847, normalized size of antiderivative = 6.09

$$\int (d+ex)^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = -\frac{6\left(4a^3b^2d^3p-6a^2b^3d^2ep+4ab^4de^2p-b^5e^3p-\frac{12(ax+b)a^2b^2d^3p}{x}+\frac{12(ax+b)ab^3d^2ep}{x}-\frac{4(ax+b)b^4de^2p}{x}+\frac{12(ax+b)^2ab^2d^3p}{x^2}-\frac{6(ax+b)^2b^3d^2ep}{x^2}-\frac{4(a\cdots}{}\right.}{a^4-\frac{4(ax+b)a^3}{x}+\frac{6(ax+b)^2a^2}{x^2}-\frac{4(ax+b)^3a}{x^3}+\frac{(ax+b)^4}{x^4}}$$

```
[In] integrate((e*x+d)^3*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] -1/24*(6*(4*a^3*b^2*d^3*p - 6*a^2*b^3*d^2*e*p + 4*a*b^4*d*e^2*p - b^5*e^3*p
 - 12*(a*x + b)*a^2*b^2*d^3*p/x + 12*(a*x + b)*a*b^3*d^2*e*p/x - 4*(a*x + b
)*b^4*d*e^2*p/x + 12*(a*x + b)^2*a*b^2*d^3*p/x^2 - 6*(a*x + b)^2*b^3*d^2*e*
p/x^2 - 4*(a*x + b)^3*b^2*d^3*p/x^3)*log((a*x + b)/x)/(a^4 - 4*(a*x + b)*a^
3/x + 6*(a*x + b)^2*a^2/x^2 - 4*(a*x + b)^3*a/x^3 + (a*x + b)^4/x^4) + (36*
a^5*b^3*d^2*e*p - 36*a^4*b^4*d*e^2*p + 11*a^3*b^5*e^3*p + 24*a^6*b^2*d^3*lo
g(c) - 36*a^5*b^3*d^2*e*log(c) + 24*a^4*b^4*d*e^2*log(c) - 6*a^3*b^5*e^3*lo
g(c) - 108*(a*x + b)*a^4*b^3*d^2*e*p/x + 96*(a*x + b)*a^3*b^4*d*e^2*p/x - 2
6*(a*x + b)*a^2*b^5*e^3*p/x - 72*(a*x + b)*a^5*b^2*d^3*log(c)/x + 72*(a*x +
 b)*a^4*b^3*d^2*e*log(c)/x - 24*(a*x + b)*a^3*b^4*d*e^2*log(c)/x + 108*(a*x
 + b)^2*a^3*b^3*d^2*e*p/x^2 - 84*(a*x + b)^2*a^2*b^4*d*e^2*p/x^2 + 21*(a*x
+ b)^2*a*b^5*e^3*p/x^2 + 72*(a*x + b)^2*a^4*b^2*d^3*log(c)/x^2 - 36*(a*x +
b)^2*a^3*b^3*d^2*e*log(c)/x^2 - 36*(a*x + b)^3*a^2*b^3*d^2*e*p/x^3 + 24*(a*
x + b)^3*a*b^4*d*e^2*p/x^3 - 6*(a*x + b)^3*b^5*e^3*p/x^3 - 24*(a*x + b)^3*a
^3*b^2*d^3*log(c)/x^3)/(a^7 - 4*(a*x + b)*a^6/x + 6*(a*x + b)^2*a^5/x^2 - 4
*(a*x + b)^3*a^4/x^3 + (a*x + b)^4*a^3/x^4) + 6*(4*a^3*b^2*d^3*p - 6*a^2*b^
3*d^2*e*p + 4*a*b^4*d*e^2*p - b^5*e^3*p)*log(-a + (a*x + b)/x)/a^4 - 6*(4*a
^3*b^2*d^3*p - 6*a^2*b^3*d^2*e*p + 4*a*b^4*d*e^2*p - b^5*e^3*p)*log((a*x +
b)/x)/a^4)/b
```

Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.32

$$\int (d+ex)^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx$$
$$= x\left(\frac{b\left(\frac{b^2 e^3 p}{4 a^2}-\frac{b d e^2 p}{a}\right)}{a}+\frac{3 b d^2 e p}{2 a}\right)$$
$$+\ln\left(c\left(a+\frac{b}{x}\right)^p\right)\left(d^3 x+\frac{3 d^2 e x^2}{2}+d e^2 x^3+\frac{e^3 x^4}{4}\right)-x^2\left(\frac{b^2 e^3 p}{8 a^2}-\frac{b d e^2 p}{2 a}\right)$$
$$-\frac{\ln(b+a x)\left(-4 p a^3 b d^3+6 p a^2 b^2 d^2 e-4 p a b^3 d e^2+p b^4 e^3\right)}{4 a^4}+\frac{b e^3 p x^3}{12 a}$$

```
[In] int(log(c*(a + b/x)^p)*(d + e*x)^3,x)
```

```
[Out] x*((b*((b^2*e^3*p)/(4*a^2) - (b*d*e^2*p)/a))/a + (3*b*d^2*e*p)/(2*a)) + log
(c*(a + b/x)^p)*(d^3*x + (e^3*x^4)/4 + (3*d^2*e*x^2)/2 + d*e^2*x^3) - x^2*(
(b^2*e^3*p)/(8*a^2) - (b*d*e^2*p)/(2*a)) - (log(b + a*x)*(b^4*e^3*p - 4*a^3
*b*d^3*p - 4*a*b^3*d*e^2*p + 6*a^2*b^2*d^2*e*p))/(4*a^4) + (b*e^3*p*x^3)/(1
2*a)
```

3.199 $\int (d+ex)^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 102

$$\int (d+ex)^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx = \frac{be(3ad-be)px}{3a^2} + \frac{be^2px^2}{6a} + \frac{(d+ex)^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{d^3p\log(x)}{3e} - \frac{(ad-be)^3p\log(b+ax)}{3a^3e}$$

```
[Out] 1/3*b*e*(3*a*d-b*e)*p*x/a^2+1/6*b*e^2*p*x^2/a+1/3*(e*x+d)^3*ln(c*(a+b/x)^p)
/e+1/3*d^3*p*ln(x)/e-1/3*(a*d-b*e)^3*p*ln(a*x+b)/a^3/e
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2513, 528, 84}

$$\int (d+ex)^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx = -\frac{p(ad-be)^3\log(ax+b)}{3a^3e} + \frac{bepx(3ad-be)}{3a^2} + \frac{(d+ex)^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{be^2px^2}{6a} + \frac{d^3p\log(x)}{3e}$$

```
[In] Int[(d + e*x)^2*Log[c*(a + b/x)^p],x]
```

```
[Out] (b*e*(3*a*d - b*e)*p*x)/(3*a^2) + (b*e^2*p*x^2)/(6*a) + ((d + e*x)^3*Log[c*
(a + b/x)^p])/(3*e) + (d^3*p*Log[x])/(3*e) - ((a*d - b*e)^3*p*Log[b + a*x])
/(3*a^3*e)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\text{integral} = \frac{(d+ex)^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{(bp)\int \frac{(d+ex)^3}{\left(a+\frac{b}{x}\right)x^2}\,dx}{3e}$$

$$= \frac{(d+ex)^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{(bp)\int \frac{(d+ex)^3}{x(b+ax)}\,dx}{3e}$$

$$= \frac{(d+ex)^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{(bp)\int \left(\frac{e^2(3ad-be)}{a^2}+\frac{d^3}{bx}+\frac{e^3x}{a}-\frac{(ad-be)^3}{a^2b(b+ax)}\right)dx}{3e}$$

$$= \frac{be(3ad-be)px}{3a^2} + \frac{be^2px^2}{6a} + \frac{(d+ex)^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{d^3p\log(x)}{3e} - \frac{(ad-be)^3p\log(b+ax)}{3a^3e}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.84

$$\int (d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)dx$$
$$= \frac{2a^3(d+ex)^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)+p\left(abe^2x(6ad-2be+aex)+2a^3d^3\log(x)-2(ad-be)^3\log(b+ax)\right)}{6a^3e}$$

```
[In] Integrate[(d + e*x)^2*Log[c*(a + b/x)^p],x]
```

```
[Out] (2*a^3*(d + e*x)^3*Log[c*(a + b/x)^p] + p*(a*b*e^2*x*(6*a*d - 2*b*e + a*e*x
) + 2*a^3*d^3*Log[x] - 2*(a*d - b*e)^3*Log[b + a*x]))/(6*a^3*e)
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.65

method	result
parts	$\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)e^{2}x^{3}}{3}+\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)ed\,x^{2}+\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)d^{2}x+\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)d^{3}}{3e}+\frac{pb\left(\frac{e^{2}\left(\frac{1}{2}ae\,x^{2}+3xa\right.}{a^{2}}\right.}{}$
parallelrisch	$-\frac{-2x^{3}\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)a^{3}e^{2}-6x^{2}\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)a^{3}de-x^{2}a^{2}b\,e^{2}p+6\ln(x)a^{2}b\,d^{2}p-12\ln(ax+b)a^{2}b\,d^{2}p+6\ln(ax+b)a\,b^{2}dep-2}{6a^{3}}$

```
[In] int((e*x+d)^2*ln(c*(a+b/x)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(c*(a+b/x)^p)*e^2*x^3+ln(c*(a+b/x)^p)*e*d*x^2+ln(c*(a+b/x)^p)*d^2*x+1
/3*ln(c*(a+b/x)^p)/e*d^3+1/3*p*b/e*(e^2/a^2*(1/2*a*e*x^2+3*x*a*d-b*e*x)+d^3
/b*ln(x)+(-a^3*d^3+3*a^2*b*d^2*e-3*a*b^2*d*e^2+b^3*e^3)/a^3/b*ln(a*x+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.50

$$\int (d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$$
$$=\frac{a^2be^2px^2+2\,(3\,a^2bde-ab^2e^2)px+2\,(3\,a^2bd^2-3\,ab^2de+b^3e^2)p\log{(ax+b)}+2\,(a^3e^2x^3+3\,a^3dex^2+3\,a}{6\,a^3}$$

```
[In] integrate((e*x+d)^2*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/6*(a^2*b*e^2*p*x^2 + 2*(3*a^2*b*d*e - a*b^2*e^2)*p*x + 2*(3*a^2*b*d^2 - 3
*a*b^2*d*e + b^3*e^2)*p*log(a*x + b) + 2*(a^3*e^2*x^3 + 3*a^3*d*e*x^2 + 3*a
^3*d^2*x)*log(c) + 2*(a^3*e^2*p*x^3 + 3*a^3*d*e*p*x^2 + 3*a^3*d^2*p*x)*log(
(a*x + b)/x))/a^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(88) = 176.

Time = 0.95 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.12

$$\int (d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$$
$$=\begin{cases} d^2x\log\left(c\left(a+\frac{b}{x}\right)^p\right)+dex^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{e^2x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3}+\frac{bd^2p\log\left(x+\frac{b}{a}\right)}{a}+\frac{bdepx}{a}+\frac{be^2px^2}{6a}-\frac{b^2dep\log(}{a^2} \\ d^2px+d^2x\log\left(c\left(\frac{b}{x}\right)^p\right)+\frac{depx^2}{2}+dex^2\log\left(c\left(\frac{b}{x}\right)^p\right)+\frac{e^2px^3}{9}+\frac{e^2x^3\log\left(c\left(\frac{b}{x}\right)^p\right)}{3} \end{cases}$$

```
[In] integrate((e*x+d)**2*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((d**2*x*log(c*(a + b/x)**p) + d*e*x**2*log(c*(a + b/x)**p) + e**2
*x**3*log(c*(a + b/x)**p)/3 + b*d**2*p*log(x + b/a)/a + b*d*e*p*x/a + b*e**
2*p*x**2/(6*a) - b**2*d*e*p*log(x + b/a)/a**2 - b**2*e**2*p*x/(3*a**2) + b*
*3*e**2*p*log(x + b/a)/(3*a**3), Ne(a, 0)), (d**2*p*x + d**2*x*log(c*(b/x)*
*p) + d*e*p*x**2/2 + d*e*x**2*log(c*(b/x)**p) + e**2*p*x**3/9 + e**2*x**3*l
og(c*(b/x)**p)/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00

$$\int (d+ex)^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{1}{6}\,bp\left(\frac{ae^2x^2+2\,(3\,ade-be^2)x}{a^2}+\frac{2\,(3\,a^2d^2-3\,abde+b^2e^2)\log\left(ax+b\right)}{a^3}\right) + \frac{1}{3}\left(e^2x^3+3\,dex^2+3\,d^2x\right)\log\left(\left(a+\frac{b}{x}\right)^p c\right)$$

```
[In] integrate((e*x+d)^2*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/6*b*p*((a*e^2*x^2 + 2*(3*a*d*e - b*e^2)*x)/a^2 + 2*(3*a^2*d^2 - 3*a*b*d*e
 + b^2*e^2)*log(a*x + b)/a^3) + 1/3*(e^2*x^3 + 3*d*e*x^2 + 3*d^2*x)*log((a
+ b/x)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(92) = 184$.

Time = 0.32 (sec) , antiderivative size = 490, normalized size of antiderivative = 4.80

$$\int (d+ex)^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = -\frac{\frac{2\left(3\,a^2b^2d^2p-3\,ab^3dep+b^4e^2p-\frac{6\,(ax+b)ab^2d^2p}{x}+\frac{3\,(ax+b)b^3dep}{x}+\frac{3\,(ax+b)^2b^2d^2p}{x^2}\right)\log\left(\frac{ax+b}{x}\right)}{a^3-\frac{3\,(ax+b)a^2}{x}+\frac{3\,(ax+b)^2a}{x^2}-\frac{(ax+b)^3}{x^3}}+\frac{6\,a^3b^3dep-3\,a^2b^4e^2p+6\,a^4b^2d^2\log(c)-6\,a^3b^3}{}}{}$$

```
[In] integrate((e*x+d)^2*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] -1/6*(2*(3*a^2*b^2*d^2*p - 3*a*b^3*d*e*p + b^4*e^2*p - 6*(a*x + b)*a*b^2*d^
2*p/x + 3*(a*x + b)*b^3*d*e*p/x + 3*(a*x + b)^2*b^2*d^2*p/x^2)*log((a*x + b
)/x)/(a^3 - 3*(a*x + b)*a^2/x + 3*(a*x + b)^2*a/x^2 - (a*x + b)^3/x^3) + (6
*a^3*b^3*d*e*p - 3*a^2*b^4*e^2*p + 6*a^4*b^2*d^2*log(c) - 6*a^3*b^3*d*e*log
```

```
(c) + 2*a^2*b^4*e^2*log(c) - 12*(a*x + b)*a^2*b^3*d*e*p/x + 5*(a*x + b)*a*b
^4*e^2*p/x - 12*(a*x + b)*a^3*b^2*d^2*log(c)/x + 6*(a*x + b)*a^2*b^3*d*e*lo
g(c)/x + 6*(a*x + b)^2*a*b^3*d*e*p/x^2 - 2*(a*x + b)^2*b^4*e^2*p/x^2 + 6*(a
*x + b)^2*a^2*b^2*d^2*log(c)/x^2)/(a^5 - 3*(a*x + b)*a^4/x + 3*(a*x + b)^2*
a^3/x^2 - (a*x + b)^3*a^2/x^3) + 2*(3*a^2*b^2*d^2*p - 3*a*b^3*d*e*p + b^4*e
^2*p)*log(-a + (a*x + b)/x)/a^3 - 2*(3*a^2*b^2*d^2*p - 3*a*b^3*d*e*p + b^4*
e^2*p)*log((a*x + b)/x)/a^3)/b
```

Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09

$$\int (d+ex)^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \ln\left(c\left(a+\frac{b}{x}\right)^p\right)\left(d^2\,x+d\,e\,x^2+\frac{e^2\,x^3}{3}\right) - x\left(\frac{b^2\,e^2\,p}{3\,a^2}-\frac{b\,d\,e\,p}{a}\right) + \frac{\ln\left(b+a\,x\right)\,\left(3\,p\,a^2\,b\,d^2-3\,p\,a\,b^2\,d\,e+p\,b^3\,e^2\right)}{3\,a^3} + \frac{b\,e^2\,p\,x^2}{6\,a}$$

```
[In] int(log(c*(a + b/x)^p)*(d + e*x)^2,x)
```

```
[Out] log(c*(a + b/x)^p)*(d^2*x + (e^2*x^3)/3 + d*e*x^2) - x*((b^2*e^2*p)/(3*a^2)
 - (b*d*e*p)/a) + (log(b + a*x)*(b^3*e^2*p + 3*a^2*b*d^2*p - 3*a*b^2*d*e*p)
)/(3*a^3) + (b*e^2*p*x^2)/(6*a)
```

3.200 $\int (d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 78

$$\int (d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{bepx}{2a}+\frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}+\frac{d^2p\log(x)}{2e}-\frac{(ad-be)^2p\log(b+ax)}{2a^2e}$$

```
[Out] 1/2*b*e*p*x/a+1/2*(e*x+d)^2*ln(c*(a+b/x)^p)/e+1/2*d^2*p*ln(x)/e-1/2*(a*d-b*
e)^2*p*ln(a*x+b)/a^2/e
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2513, 528, 84}

$$\int (d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = -\frac{p(ad-be)^2\log(ax+b)}{2a^2e}+\frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}+\frac{bepx}{2a}+\frac{d^2p\log(x)}{2e}$$

```
[In] Int[(d + e*x)*Log[c*(a + b/x)^p],x]
```

```
[Out] (b*e*p*x)/(2*a) + ((d + e*x)^2*Log[c*(a + b/x)^p])/(2*e) + (d^2*p*Log[x])/(
2*e) - ((a*d - b*e)^2*p*Log[b + a*x])/(2*a^2*e)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
```

```
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}+\frac{(bp)\int\frac{(d+ex)^2}{\left(a+\frac{b}{x}\right)x^2}\,dx}{2e}\\
&= \frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}+\frac{(bp)\int\frac{(d+ex)^2}{x(b+ax)}\,dx}{2e}\\
&= \frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}+\frac{(bp)\int\left(\frac{e^2}{a}+\frac{d^2}{bx}-\frac{(ad-be)^2}{ab(b+ax)}\right)dx}{2e}\\
&= \frac{bepx}{2a}+\frac{(d+ex)^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}+\frac{d^2p\log(x)}{2e}-\frac{(ad-be)^2p\log(b+ax)}{2a^2e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.09

$$\begin{aligned}
\int(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)dx &= \frac{bdp\log\left(a+\frac{b}{x}\right)}{a}+dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)\\
&\quad+\frac{1}{2}ex^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bdp\log(x)}{a}\\
&\quad+\frac{1}{2}bep\left(\frac{x}{a}-\frac{b\log(b+ax)}{a^2}\right)
\end{aligned}$$

```
[In] Integrate[(d + e*x)*Log[c*(a + b/x)^p],x]
```

```
[Out] (b*d*p*Log[a + b/x])/a + d*x*Log[c*(a + b/x)^p] + (e*x^2*Log[c*(a + b/x)^p]
)/2 + (b*d*p*Log[x])/a + (b*e*p*(x/a - (b*Log[b + a*x])/a^2))/2
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

method	result
parts	$\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right) e\,x^{2}}{2}+\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right) dx+\frac{pb\left(\frac{ex}{a}+\frac{(2ad-be)\ln(ax+b)}{a^{2}}\right)}{2}$
parallelrisch	$-\frac{-x^{2}\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)a^{2}e+2\ln(x)abdp-4\ln(ax+b)abdp+\ln(ax+b)b^{2}ep-2x\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)a^{2}d-abepx+2\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)abd+b}{2a^{2}}$

```
[In] int((e*x+d)*ln(c*(a+b/x)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(c*(a+b/x)^p)*e*x^2+ln(c*(a+b/x)^p)*d*x+1/2*p*b*(e/a*x+(2*a*d-b*e)/a^
2*ln(a*x+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.03

$$\int (d+ex)\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)\,dx$$
$$=\frac{abepx+(2\,abd-b^{2}e)p\log(ax+b)+(a^{2}ex^{2}+2\,a^{2}dx)\log(c)+(a^{2}epx^{2}+2\,a^{2}dpx)\log\left(\frac{ax+b}{x}\right)}{2\,a^{2}}$$

```
[In] integrate((e*x+d)*log(c*(a+b/x)^p),x, algorithm="fricas")
```

```
[Out] 1/2*(a*b*e*p*x + (2*a*b*d - b^2*e)*p*log(a*x + b) + (a^2*e*x^2 + 2*a^2*d*x)
*log(c) + (a^2*e*p*x^2 + 2*a^2*d*p*x)*log((a*x + b)/x))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.44

$$\int (d+ex)\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)\,dx$$
$$=\begin{cases} dx\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)+\frac{ex^{2}\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{2}+\frac{bdp\log\left(x+\frac{b}{a}\right)}{a}+\frac{bepx}{2a}-\frac{b^{2}ep\log\left(x+\frac{b}{a}\right)}{2a^{2}} & \text{for } a\neq 0\\ dpx+dx\log\left(c\left(\frac{b}{x}\right)^{p}\right)+\frac{epx^{2}}{4}+\frac{ex^{2}\log\left(c\left(\frac{b}{x}\right)^{p}\right)}{2} & \text{otherwise}\end{cases}$$

```
[In] integrate((e*x+d)*ln(c*(a+b/x)**p),x)
```

```
[Out] Piecewise((d*x*log(c*(a + b/x)**p) + e*x**2*log(c*(a + b/x)**p)/2 + b*d*p*l
og(x + b/a)/a + b*e*p*x/(2*a) - b**2*e*p*log(x + b/a)/(2*a**2), Ne(a, 0)),
(d*p*x + d*x*log(c*(b/x)**p) + e*p*x**2/4 + e*x**2*log(c*(b/x)**p)/2, True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int (d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \frac{1}{2}\,bp\left(\frac{ex}{a}+\frac{(2\,ad-be)\log\left(ax+b\right)}{a^2}\right) + \frac{1}{2}\left(ex^2+2\,dx\right)\log\left(\left(a+\frac{b}{x}\right)^p c\right)$$

```
[In] integrate((e*x+d)*log(c*(a+b/x)^p),x, algorithm="maxima")
```

```
[Out] 1/2*b*p*(e*x/a + (2*a*d - b*e)*log(a*x + b)/a^2) + 1/2*(e*x^2 + 2*d*x)*log(
(a + b/x)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(70) = 140$.

Time = 0.45 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.99

$$\int (d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = -\frac{\frac{\left(2\,ab^2dp-b^3ep-\frac{2\,(ax+b)b^2dp}{x}\right)\log\left(\frac{ax+b}{x}\right)}{a^2-\frac{2\,(ax+b)a}{x}+\frac{(ax+b)^2}{x^2}}+\frac{ab^3ep+2\,a^2b^2d\log(c)-ab^3e\log(c)-\frac{(ax+b)b^3ep}{x}-\frac{2\,(ax+b)ab^2d\log(c)}{x}}{a^3-\frac{2\,(ax+b)a^2}{x}+\frac{(ax+b)^2a}{x^2}}+\frac{(2\,ab^2dp-b^3ep)\log\left(-\right.}{a^2}}{2\,b}$$

```
[In] integrate((e*x+d)*log(c*(a+b/x)^p),x, algorithm="giac")
```

```
[Out] -1/2*((2*a*b^2*d*p - b^3*e*p - 2*(a*x + b)*b^2*d*p/x)*log((a*x + b)/x)/(a^2
 - 2*(a*x + b)*a/x + (a*x + b)^2/x^2) + (a*b^3*e*p + 2*a^2*b^2*d*log(c) - a
*b^3*e*log(c) - (a*x + b)*b^3*e*p/x - 2*(a*x + b)*a*b^2*d*log(c)/x)/(a^3 -
2*(a*x + b)*a^2/x + (a*x + b)^2*a/x^2) + (2*a*b^2*d*p - b^3*e*p)*log(-a + (
a*x + b)/x)/a^2 - (2*a*b^2*d*p - b^3*e*p)*log((a*x + b)/x)/a^2)/b
```

Mupad [B] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.73

$$\int (d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \ln\left(c\,\left(a+\frac{b}{x}\right)^p\right)\,\left(\frac{e\,x^2}{2}+d\,x\right) - \frac{\ln\left(b+a\,x\right)\,\left(b^2\,e\,p-2\,a\,b\,d\,p\right)}{2\,a^2}+\frac{b\,e\,p\,x}{2\,a}$$

```
[In] int(log(c*(a + b/x)^p)*(d + e*x),x)
```

```
[Out] log(c*(a + b/x)^p)*(d*x + (e*x^2)/2) - (log(b + a*x)*(b^2*e*p - 2*a*b*d*p))
/(2*a^2) + (b*e*p*x)/(2*a)
```

3.201 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e} + \frac{p\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e}$$

```
[Out] ln(c*(a+b/x)^p)*ln(e*x+d)/e+p*ln(-e*x/d)*ln(e*x+d)/e-p*ln(-e*(a*x+b)/(a*d-b
*e))*ln(e*x+d)/e-p*polylog(2,a*(e*x+d)/(a*d-b*e))/e+p*polylog(2,1+e*x/d)/e
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2512, 266, 2463, 2441, 2352, 2440, 2438}

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e} - \frac{p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{e} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}$$

```
[In] Int[Log[c*(a + b/x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x)^p]*Log[d + e*x])/e + (p*Log[-((e*x)/d)]*Log[d + e*x])/e -
(p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e - (p*PolyLog[2, (a*(d
+ e*x))/(a*d - b*e)])/e + (p*PolyLog[2, 1 + (e*x)/d])/e
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$\text{integral} = \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{(bp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x}\right)x^2}\,dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{(bp)\int\left(\frac{\log(d+ex)}{bx} - \frac{a\log(d+ex)}{b(b+ax)}\right)dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\int \frac{\log(d+ex)}{x}\,dx}{e} - \frac{(ap)\int \frac{\log(d+ex)}{b+ax}\,dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e} - p\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx + p\int \frac{\log\left(\frac{e(b+ax)}{-ad+be}\right)}{d+ex}\,dx$$

$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e} + \frac{p\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{e} + \frac{p\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{ax}{-ad+be}\right)}{x}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e} - \frac{p\operatorname{Li}_2\left(\frac{a(d+ex)}{ad-be}\right)}{e} + \frac{p\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{e}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e} + \frac{p\operatorname{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e} - \frac{p\operatorname{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e}$$

```
[In] Integrate[Log[c*(a + b/x)^p]/(d + e*x),x]
[Out] (Log[c*(a + b/x)^p]*Log[d + e*x])/e + (p*Log[-((e*x)/d)]*Log[d + e*x])/e -
(p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e + (p*PolyLog[2, (d + e
*x)/d])/e - (p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/e
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.21

method	result
parts	$\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)\ln(ex+d)}{e}+pb\left(\frac{\operatorname{dilog}\left(-\frac{ex}{d}\right)+\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{be}-\frac{a\left(\frac{\operatorname{dilog}\left(\frac{-ad+a(ex+d)+be}{-ad+be}\right)}{a}+\frac{\ln(ex+d)\ln\left(\frac{-ad+a(ex+d)+be}{-ad+be}\right)}{a}\right.}{be}\right.$

```
[In] int(ln(c*(a+b/x)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] ln(c*(a+b/x)^p)*ln(e*x+d)/e+p*b*(1/b/e*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))
-a/b/e*(dilog((-a*d+a*(e*x+d)+b*e)/(-a*d+b*e))/a+ln(e*x+d)*ln((-a*d+a*(e*x+
d)+b*e)/(-a*d+b*e))/a))
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{d+ex}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)}{ex+d}\,dx$$

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x + b)/x)^p)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{d+ex}\,dx=\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{d+ex}\,dx$$

```
[In] integrate(ln(c*(a+b/x)**p)/(e*x+d),x)
```

```
[Out] Integral(log(c*(a + b/x)**p)/(d + e*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.41

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \frac{bp\left(\frac{\log(ex+d)\log\left(a+\frac{b}{x}\right)}{b}-\frac{\log(ex+d)\log\left(-\frac{aex+ad}{ad-be}+1\right)+\mathrm{Li}_2\left(\frac{aex+ad}{ad-be}\right)}{b}+\frac{\log(ex+d)\log\left(-\frac{ex+d}{d}+1\right)+\mathrm{Li}_2\left(\frac{ex+d}{d}\right)}{b}\right)}{e} - \frac{p\log(ex+d)\log\left(a+\frac{b}{x}\right)}{e}+\frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)\log(ex+d)}{e}$$

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*p*(log(e*x + d)*log(a + b/x)/b - (log(e*x + d)*log(-(a*e*x + a*d)/(a*d -
b*e) + 1) + dilog((a*e*x + a*d)/(a*d - b*e)))/b + (log(e*x + d)*log(-(e*x +
 d)/d + 1) + dilog((e*x + d)/d))/b)/e - p*log(e*x + d)*log(a + b/x)/e + log
((a + b/x)^p*c)*log(e*x + d)/e
```

Giac [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \frac{\ln\left(c\,\left(a+\frac{b}{x}\right)^p\right)}{d+e\,x}\,dx$$

```
[In] int(log(c*(a + b/x)^p)/(d + e*x),x)
```

```
[Out] int(log(c*(a + b/x)^p)/(d + e*x), x)
```

3.202 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2}\,dx = -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)} - \frac{p\log(x)}{de} + \frac{ap\log(b+ax)}{e(ad-be)} - \frac{bp\log(d+ex)}{d(ad-be)}$$

```
[Out] -ln(c*(a+b/x)^p)/e/(e*x+d)-p*ln(x)/d/e+a*p*ln(a*x+b)/e/(a*d-b*e)-b*p*ln(e*x
+d)/d/(a*d-b*e)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2513, 528, 84}

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2}\,dx = -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)} + \frac{ap\log(ax+b)}{e(ad-be)} - \frac{bp\log(d+ex)}{d(ad-be)} - \frac{p\log(x)}{de}$$

```
[In] Int[Log[c*(a + b/x)^p]/(d + e*x)^2,x]
```

```
[Out] -(Log[c*(a + b/x)^p]/(e*(d + e*x))) - (p*Log[x])/(d*e) + (a*p*Log[b + a*x])
/(e*(a*d - b*e)) - (b*p*Log[d + e*x])/(d*(a*d - b*e))
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)} - \frac{(bp)\int \frac{1}{\left(a+\frac{b}{x}\right)x^2(d+ex)}\,dx}{e}\\
&= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)} - \frac{(bp)\int \frac{1}{x(b+ax)(d+ex)}\,dx}{e}\\
&= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)} - \frac{(bp)\int \left(\frac{1}{bdx}+\frac{a^2}{b(-ad+be)(b+ax)}+\frac{e^2}{d(ad-be)(d+ex)}\right)dx}{e}\\
&= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)} - \frac{p\log(x)}{de} + \frac{ap\log(b+ax)}{e(ad-be)} - \frac{bp\log(d+ex)}{d(ad-be)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2}\,dx = -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)} - \frac{p\log(x)}{de} + \frac{ap\log(b+ax)}{e(ad-be)} - \frac{bp\log(d+ex)}{d(ad-be)}$$

```
[In] Integrate[Log[c*(a + b/x)^p]/(d + e*x)^2,x]
```

```
[Out] -(Log[c*(a + b/x)^p]/(e*(d + e*x))) - (p*Log[x])/(d*e) + (a*p*Log[b + a*x])
/(e*(a*d - b*e)) - (b*p*Log[d + e*x])/(d*(a*d - b*e))
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06

method	result	size
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{e(ex+d)}-\frac{pb\left(\frac{\ln(x)}{bd}+\frac{e\ln(ex+d)}{d(ad-be)}-\frac{a\ln(ax+b)}{b(ad-be)}\right)}{e}$	86
parallelrisch	$-\frac{-\ln(x)x\,b^{2}e\,p^{2}+\ln(ex+d)x\,b^{2}e\,p^{2}-\ln(x)b^{2}d\,p^{2}+\ln(ex+d)b^{2}d\,p^{2}-x\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)abdp-\ln\left(c\left(\frac{ax+b}{x}\right)^{p}\right)b^{2}dp}{(ex+d)pbd(ad-be)}$	124

```
[In] int(ln(c*(a+b/x)^p)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(c*(a+b/x)^p)/e/(e*x+d)-p*b/e*(1/b/d*ln(x)+e/d/(a*d-b*e)*ln(e*x+d)-a/b/(
a*d-b*e)*ln(a*x+b))
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.83

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{(d+ex)^{2}}\,dx = -\frac{(ad^{2}-bde)p\log\left(\frac{ax+b}{x}\right)-(adepx+ad^{2}p)\log(ax+b)+(be^{2}px+bdep)\log(ex+d)+(ad^{2}-bde)\log(}{ad^{3}e-bd^{2}e^{2}+(ad^{2}e^{2}-bde^{3})x}$$

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -((a*d^2 - b*d*e)*p*log((a*x + b)/x) - (a*d*e*p*x + a*d^2*p)*log(a*x + b) +
 (b*e^2*p*x + b*d*e*p)*log(e*x + d) + (a*d^2 - b*d*e)*log(c) + ((a*d*e - b*
e^2)*p*x + (a*d^2 - b*d*e)*p)*log(x))/(a*d^3*e - b*d^2*e^2 + (a*d^2*e^2 - b
*d*e^3)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(61) = 122.

Time = 3.72 (sec) , antiderivative size = 425, normalized size of antiderivative = 5.25

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2}\,dx$$
$$=\begin{cases} \frac{dp\log\left(\frac{d}{e}+x\right)}{d^2e+de^2x}+\frac{epx\log\left(\frac{d}{e}+x\right)}{d^2e+de^2x}+\frac{ex\log\left(c\left(\frac{b}{x}\right)^p\right)}{d^2e+de^2x} \\ \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)+\frac{bp\log(ax+b)}{a}}{d^2} \\ -\frac{dp}{d^2e+de^2x}+\frac{ex\log\left(c\left(\frac{b}{x}+\frac{be}{d}\right)^p\right)}{d^2e+de^2x} \\ \frac{-\frac{a\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b}+\frac{p}{x}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}}{e^2} \\ \frac{adx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{ad^3+ad^2ex-bd^2e-bde^2x}+\frac{bdp\log\left(x+\frac{b}{a}\right)}{ad^3+ad^2ex-bd^2e-bde^2x}-\frac{bdp\log\left(\frac{d}{e}+x\right)}{ad^3+ad^2ex-bd^2e-bde^2x}+\frac{bepx\log\left(x+\frac{b}{a}\right)}{ad^3+ad^2ex-bd^2e-bde^2x}-\frac{bepx\log\left(\frac{d}{e}+x\right)}{ad^3+ad^2ex-bd^2e-bde^2} \end{cases}$$

```
[In] integrate(ln(c*(a+b/x)**p)/(e*x+d)**2,x)
```

```
[Out] Piecewise((d*p*log(d/e + x)/(d**2*e + d*e**2*x) + e*p*x*log(d/e + x)/(d**2*
e + d*e**2*x) + e*x*log(c*(b/x)**p)/(d**2*e + d*e**2*x), Eq(a, 0)), ((x*log
(c*(a + b/x)**p) + b*p*log(a*x + b)/a)/d**2, Eq(e, 0)), (-d*p/(d**2*e + d*e
**2*x) + e*x*log(c*(b/x + b*e/d)**p)/(d**2*e + d*e**2*x), Eq(a, b*e/d)), ((
-a*log(c*(a + b/x)**p)/b + p/x - log(c*(a + b/x)**p)/x)/e**2, Eq(d, 0)), (a
*d*x*log(c*(a + b/x)**p)/(a*d**3 + a*d**2*e*x - b*d**2*e - b*d*e**2*x) + b*
d*p*log(x + b/a)/(a*d**3 + a*d**2*e*x - b*d**2*e - b*d*e**2*x) - b*d*p*log(
d/e + x)/(a*d**3 + a*d**2*e*x - b*d**2*e - b*d*e**2*x) + b*e*p*x*log(x + b/
a)/(a*d**3 + a*d**2*e*x - b*d**2*e - b*d*e**2*x) - b*e*p*x*log(d/e + x)/(a*
d**3 + a*d**2*e*x - b*d**2*e - b*d*e**2*x) - b*e*x*log(c*(a + b/x)**p)/(a*d
**3 + a*d**2*e*x - b*d**2*e - b*d*e**2*x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2}\,dx=\frac{bp\left(\frac{a\log(ax+b)}{abd-b^2e}-\frac{e\log(ex+d)}{ad^2-bde}-\frac{\log(x)}{bd}\right)}{e}-\frac{\log\left(\left(a+\frac{b}{x}\right)^pc\right)}{(ex+d)e}$$

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] b*p*(a*log(a*x + b)/(a*b*d - b^2*e) - e*log(e*x + d)/(a*d^2 - b*d*e) - log(
x)/(b*d))/e - log((a + b/x)^p*c)/((e*x + d)*e)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.79

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2}\,dx = -\frac{\frac{b^2p\log\left(-ad+be+\frac{(ax+b)d}{x}\right)}{ad^2-bde}+\frac{b^2p\log\left(\frac{ax+b}{x}\right)}{ad^2-bde-\frac{(ax+b)d^2}{x}}-\frac{b^2p\log\left(\frac{ax+b}{x}\right)}{ad^2-bde}+\frac{b^2\log(c)}{ad^2-bde-\frac{(ax+b)d^2}{x}}}{b}$$

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -(b^2*p*log(-a*d + b*e + (a*x + b)*d/x)/(a*d^2 - b*d*e) + b^2*p*log((a*x +
b)/x)/(a*d^2 - b*d*e - (a*x + b)*d^2/x) - b^2*p*log((a*x + b)/x)/(a*d^2 - b
*d*e) + b^2*log(c)/(a*d^2 - b*d*e - (a*x + b)*d^2/x))/b
```

Mupad [B] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2}\,dx = -\frac{\ln\left(c\left(\frac{b+a\,x}{x}\right)^p\right)}{x\,e^2+d\,e}-\frac{p\,\ln(x)}{d\,e}-\frac{a\,p\,\ln(b+a\,x)}{b\,e^2-a\,d\,e}-\frac{b\,p\,\ln(d+e\,x)}{a\,d^2-b\,d\,e}$$

```
[In] int(log(c*(a + b/x)^p)/(d + e*x)^2,x)
```

```
[Out] - log(c*((b + a*x)/x)^p)/(d*e + e^2*x) - (p*log(x))/(d*e) - (a*p*log(b + a*
x))/(b*e^2 - a*d*e) - (b*p*log(d + e*x))/(a*d^2 - b*d*e)
```

3.203 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 127

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}\,dx = \frac{bp}{2d(ad-be)(d+ex)} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2} - \frac{p\log(x)}{2d^2e} + \frac{a^2p\log(b+ax)}{2e(ad-be)^2} - \frac{b(2ad-be)p\log(d+ex)}{2d^2(ad-be)^2}$$

```
[Out] 1/2*b*p/d/(a*d-b*e)/(e*x+d)-1/2*ln(c*(a+b/x)^p)/e/(e*x+d)^2-1/2*p*ln(x)/d^2
/e+1/2*a^2*p*ln(a*x+b)/e/(a*d-b*e)^2-1/2*b*(2*a*d-b*e)*p*ln(e*x+d)/d^2/(a*d
-b*e)^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2513, 528, 84}

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}\,dx = \frac{a^2p\log(ax+b)}{2e(ad-be)^2} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2} - \frac{bp(2ad-be)\log(d+ex)}{2d^2(ad-be)^2} + \frac{bp}{2d(d+ex)(ad-be)} - \frac{p\log(x)}{2d^2e}$$

```
[In] Int[Log[c*(a + b/x)^p]/(d + e*x)^3,x]
```

```
[Out] (b*p)/(2*d*(a*d - b*e)*(d + e*x)) - Log[c*(a + b/x)^p]/(2*e*(d + e*x)^2) -
(p*Log[x])/(2*d^2*e) + (a^2*p*Log[b + a*x])/(2*e*(a*d - b*e)^2) - (b*(2*a*d
 - b*e)*p*Log[d + e*x])/(2*d^2*(a*d - b*e)^2)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2}-\frac{(bp)\int\frac{1}{\left(a+\frac{b}{x}\right)x^2(d+ex)^2}\,dx}{2e}\\
&= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2}-\frac{(bp)\int\frac{1}{x(b+ax)(d+ex)^2}\,dx}{2e}\\
&= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2}-\frac{(bp)\int\left(\frac{1}{bd^2x}-\frac{a^3}{b(-ad+be)^2(b+ax)}+\frac{e^2}{d(ad-be)(d+ex)^2}+\frac{e^2(2ad-be)}{d^2(ad-be)^2(d+ex)}\right)dx}{2e}\\
&= \frac{bp}{2d(ad-be)(d+ex)}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(d+ex)^2}-\frac{p\log(x)}{2d^2e}\\
&\quad+\frac{a^2p\log(b+ax)}{2e(ad-be)^2}-\frac{b(2ad-be)p\log(d+ex)}{2d^2(ad-be)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.89

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}\,dx$$
$$=\frac{\frac{bep}{d(ad-be)(d+ex)}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^2}-\frac{p\log(x)}{d^2}+\frac{a^2p\log(b+ax)}{(ad-be)^2}+\frac{be(-2ad+be)p\log(d+ex)}{d^2(ad-be)^2}}{2e}$$

```
[In] Integrate[Log[c*(a + b/x)^p]/(d + e*x)^3,x]
```

```
[Out] ((b*e*p)/(d*(a*d - b*e)*(d + e*x)) - Log[c*(a + b/x)^p]/(d + e*x)^2 - (p*Lo
g[x])/d^2 + (a^2*p*Log[b + a*x])/(a*d - b*e)^2 + (b*e*(-2*a*d + b*e)*p*Log[
d + e*x])/(d^2*(a*d - b*e)^2))/(2*e)
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94

method	result
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e(ex+d)^2}-\frac{pb\left(\frac{\ln(x)}{b\,d^2}-\frac{e}{d(ad-be)(ex+d)}+\frac{e(2ad-be)\ln(ex+d)}{d^2(ad-be)^2}-\frac{a^2\ln(ax+b)}{b(ad-be)^2}\right)}{2e}$
parallelrisch	$-\frac{-2\ln(x)x^2a^2bd\,e^4p^2+2\ln(ex+d)x^2a^2bd\,e^4p^2-4\ln(x)x\,a^2b\,d^2e^3p^2+2\ln(x)xa\,b^2d\,e^4p^2+4\ln(ex+d)x\,a^2b\,d^2e^3p^2-2\ln(ex+d)x}{}$

```
[In] int(ln(c*(a+b/x)^p)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(c*(a+b/x)^p)/e/(e*x+d)^2-1/2*p*b/e*(1/b/d^2*ln(x)-e/d/(a*d-b*e)/(e*
x+d)+e*(2*a*d-b*e)/d^2/(a*d-b*e)^2*ln(e*x+d)-a^2/b/(a*d-b*e)^2*ln(a*x+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. 2(117) = 234.

Time = 0.78 (sec) , antiderivative size = 428, normalized size of antiderivative = 3.37

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}\,dx$$
$$=\frac{(abd^2e^2-b^2de^3)px-(a^2d^4-2\,abd^3e+b^2d^2e^2)p\log\left(\frac{ax+b}{x}\right)+(abd^3e-b^2d^2e^2)p+(a^2d^2e^2px^2+2\,a^2d^3epx}{}$$

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*((a*b*d^2*e^2 - b^2*d*e^3)*p*x - (a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2)*
p*log((a*x + b)/x) + (a*b*d^3*e - b^2*d^2*e^2)*p + (a^2*d^2*e^2*p*x^2 + 2*a
```

```
^2*d^3*e*p*x + a^2*d^4*p)*log(a*x + b) - ((2*a*b*d*e^3 - b^2*e^4)*p*x^2 + 2
*(2*a*b*d^2*e^2 - b^2*d*e^3)*p*x + (2*a*b*d^3*e - b^2*d^2*e^2)*p)*log(e*x +
 d) - (a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2)*log(c) - ((a^2*d^2*e^2 - 2*a*b*
d*e^3 + b^2*e^4)*p*x^2 + 2*(a^2*d^3*e - 2*a*b*d^2*e^2 + b^2*d*e^3)*p*x + (a
^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2)*p)*log(x))/(a^2*d^6*e - 2*a*b*d^5*e^2 +
 b^2*d^4*e^3 + (a^2*d^4*e^3 - 2*a*b*d^3*e^4 + b^2*d^2*e^5)*x^2 + 2*(a^2*d^5
*e^2 - 2*a*b*d^4*e^3 + b^2*d^3*e^4)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(a+b/x)**p)/(e*x+d)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.26

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}\,dx = \frac{\left(\frac{a^2\log(ax+b)}{a^2bd^2-2\,ab^2de+b^3e^2} - \frac{(2\,ade-be^2)\log(ex+d)}{a^2d^4-2\,abd^3e+b^2d^2e^2} + \frac{e}{ad^3-bd^2e+(ad^2e-bde^2)x} - \frac{\log(x)}{bd^2}\right)bp}{2\,e} - \frac{\log\left(\left(a+\frac{b}{x}\right)^pc\right)}{2\,(ex+d)^2e}$$

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^3,x, algorithm="maxima")
[Out] 1/2*(a^2*log(a*x + b)/(a^2*b*d^2 - 2*a*b^2*d*e + b^3*e^2) - (2*a*d*e - b*e^
2)*log(e*x + d)/(a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2) + e/(a*d^3 - b*d^2*e
+ (a*d^2*e - b*d*e^2)*x) - log(x)/(b*d^2))*b*p/e - 1/2*log((a + b/x)^p*c)/(
(e*x + d)^2*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(117) = 234$.

Time = 0.32 (sec) , antiderivative size = 470, normalized size of antiderivative = 3.70

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}\,dx = -\frac{\frac{(2\,ab^2dp-b^3ep)\log\left(-ad+be+\frac{(ax+b)d}{x}\right)}{a^2d^4-2\,abd^3e+b^2d^2e^2}+\frac{\left(2\,ab^2dp-b^3ep-\frac{2\,(ax+b)b^2dp}{x}\right)\log\left(\frac{ax+b}{x}\right)}{a^2d^4-2\,abd^3e+b^2d^2e^2-\frac{2\,(ax+b)ad^4}{x}+\frac{2\,(ax+b)bd^3e}{x}+\frac{(ax+b)^2d^4}{x^2}}-\frac{(2\,ab^2dp-b^3ep)\log\left(\frac{ax+b}{x}\right)}{a^2d^4-2\,abd^3e+b^2d^2e^2}-}{2\,b}$$

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/2*((2*a*b^2*d*p - b^3*e*p)*log(-a*d + b*e + (a*x + b)*d/x)/(a^2*d^4 - 2*
a*b*d^3*e + b^2*d^2*e^2) + (2*a*b^2*d*p - b^3*e*p - 2*(a*x + b)*b^2*d*p/x)*
log((a*x + b)/x)/(a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 - 2*(a*x + b)*a*d^4/x
 + 2*(a*x + b)*b*d^3*e/x + (a*x + b)^2*d^4/x^2) - (2*a*b^2*d*p - b^3*e*p)*l
og((a*x + b)/x)/(a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2) - (a*b^3*d*e*p - b^4*
e^2*p - 2*a^2*b^2*d^2*log(c) + 3*a*b^3*d*e*log(c) - b^4*e^2*log(c) - (a*x +
 b)*b^3*d*e*p/x + 2*(a*x + b)*a*b^2*d^2*log(c)/x - 2*(a*x + b)*b^3*d*e*log(
c)/x)/(a^3*d^5 - 3*a^2*b*d^4*e + 3*a*b^2*d^3*e^2 - b^3*d^2*e^3 - 2*(a*x + b
)*a^2*d^5/x + 4*(a*x + b)*a*b*d^4*e/x - 2*(a*x + b)*b^2*d^3*e^2/x + (a*x +
b)^2*a*d^5/x^2 - (a*x + b)^2*b*d^4*e/x^2))/b
```

Mupad [B] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.71

$$\begin{aligned}\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}\,dx &= \frac{a^2\,p\,\ln\left(b+a\,x\right)}{2\,a^2\,d^2\,e-4\,a\,b\,d\,e^2+2\,b^2\,e^3}-\frac{\ln\left(c\,{\left(\frac{b+a\,x}{x}\right)}^p\right)}{2\,\left(d^2\,e+2\,d\,e^2\,x+e^3\,x^2\right)}\\ &\quad-\frac{p\,\ln\left(x\right)}{2\,d^2\,e}-\frac{b\,e\,p}{2\,b\,d^2\,e^2-2\,a\,d^3\,e+2\,b\,d\,e^3\,x-2\,a\,d^2\,e^2\,x}\\ &\quad+\frac{b^2\,e\,p\,\ln\left(d+e\,x\right)}{2\,a^2\,d^4-4\,a\,b\,d^3\,e+2\,b^2\,d^2\,e^2}-\frac{2\,a\,b\,d\,p\,\ln\left(d+e\,x\right)}{2\,a^2\,d^4-4\,a\,b\,d^3\,e+2\,b^2\,d^2\,e^2}\end{aligned}$$

```
[In] int(log(c*(a + b/x)^p)/(d + e*x)^3,x)
```

```
[Out] (a^2*p*log(b + a*x))/(2*b^2*e^3 + 2*a^2*d^2*e - 4*a*b*d*e^2) - log(c*((b +
a*x)/x)^p)/(2*(d^2*e + e^3*x^2 + 2*d*e^2*x)) - (p*log(x))/(2*d^2*e) - (b*e*
p)/(2*b*d^2*e^2 - 2*a*d^3*e + 2*b*d*e^3*x - 2*a*d^2*e^2*x) + (b^2*e*p*log(d
 + e*x))/(2*a^2*d^4 + 2*b^2*d^2*e^2 - 4*a*b*d^3*e) - (2*a*b*d*p*log(d + e*x
))/(2*a^2*d^4 + 2*b^2*d^2*e^2 - 4*a*b*d^3*e)
```

3.204 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 175

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4}\,dx = \frac{bp}{6d(ad-be)(d+ex)^2}+\frac{b(2ad-be)p}{3d^2(ad-be)^2(d+ex)}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3} -\frac{p\log(x)}{3d^3e}+\frac{a^3p\log(b+ax)}{3e(ad-be)^3}-\frac{b(3a^2d^2-3abde+b^2e^2)\,p\log(d+ex)}{3d^3(ad-be)^3}$$

```
[Out] 1/6*b*p/d/(a*d-b*e)/(e*x+d)^2+1/3*b*(2*a*d-b*e)*p/d^2/(a*d-b*e)^2/(e*x+d)-1
/3*ln(c*(a+b/x)^p)/e/(e*x+d)^3-1/3*p*ln(x)/d^3/e+1/3*a^3*p*ln(a*x+b)/e/(a*d
-b*e)^3-1/3*b*(3*a^2*d^2-3*a*b*d*e+b^2*e^2)*p*ln(e*x+d)/d^3/(a*d-b*e)^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2513, 528, 84}

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4}\,dx = \frac{a^3p\log(ax+b)}{3e(ad-be)^3}-\frac{bp(3a^2d^2-3abde+b^2e^2)\log(d+ex)}{3d^3(ad-be)^3} -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3}+\frac{bp(2ad-be)}{3d^2(d+ex)(ad-be)^2} +\frac{bp}{6d(d+ex)^2(ad-be)}-\frac{p\log(x)}{3d^3e}$$

```
[In] Int[Log[c*(a + b/x)^p]/(d + e*x)^4,x]
```

```
[Out] (b*p)/(6*d*(a*d - b*e)*(d + e*x)^2) + (b*(2*a*d - b*e)*p)/(3*d^2*(a*d - b*e
)^2*(d + e*x)) - Log[c*(a + b/x)^p]/(3*e*(d + e*x)^3) - (p*Log[x])/(3*d^3*e
```

```
) + (a^3*p*Log[b + a*x])/(3*e*(a*d - b*e)^3) - (b*(3*a^2*d^2 - 3*a*b*d*e +
b^2*e^2)*p*Log[d + e*x])/(3*d^3*(a*d - b*e)^3)
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3}-\frac{(bp)\int\frac{1}{\left(a+\frac{b}{x}\right)x^2(d+ex)^3}\,dx}{3e}\\
&= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3}-\frac{(bp)\int\frac{1}{x(b+ax)(d+ex)^3}\,dx}{3e}\\
&= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3}\\
&\quad-\frac{(bp)\int\left(\frac{1}{bd^3x}+\frac{a^4}{b(-ad+be)^3(b+ax)}+\frac{e^2}{d(ad-be)(d+ex)^3}+\frac{e^2(2ad-be)}{d^2(ad-be)^2(d+ex)^2}+\frac{e^2\left(3a^2d^2-3abde+b^2e^2\right)}{d^3(ad-be)^3(d+ex)}\right)dx}{3e}\\
&= \frac{bp}{6d(ad-be)(d+ex)^2}+\frac{b(2ad-be)p}{3d^2(ad-be)^2(d+ex)}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(d+ex)^3}\\
&\quad-\frac{p\log(x)}{3d^3e}+\frac{a^3p\log(b+ax)}{3e(ad-be)^3}-\frac{b\left(3a^2d^2-3abde+b^2e^2\right)p\log(d+ex)}{3d^3(ad-be)^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.94

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4}\,dx$$
$$=\frac{\frac{bep}{2d(ad-be)(d+ex)^2}+\frac{be(2ad-be)p}{d^2(ad-be)^2(d+ex)}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^3}-\frac{p\log(x)}{d^3}+\frac{a^3p\log(b+ax)}{(ad-be)^3}-\frac{be(3a^2d^2-3abde+b^2e^2)p\log(d+ex)}{d^3(ad-be)^3}}{3e}$$

```
[In] Integrate[Log[c*(a + b/x)^p]/(d + e*x)^4,x]
```

```
[Out] ((b*e*p)/(2*d*(a*d - b*e)*(d + e*x)^2) + (b*e*(2*a*d - b*e)*p)/(d^2*(a*d -
b*e)^2*(d + e*x)) - Log[c*(a + b/x)^p]/(d + e*x)^3 - (p*Log[x])/d^3 + (a^3*
p*Log[b + a*x])/(a*d - b*e)^3 - (b*e*(3*a^2*d^2 - 3*a*b*d*e + b^2*e^2)*p*Lo
g[d + e*x])/(d^3*(a*d - b*e)^3))/(3*e)
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.94

method	result
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e(ex+d)^3}-\frac{pb\left(\frac{\ln(x)}{d^3b}-\frac{e}{2d(ad-be)(ex+d)^2}-\frac{e(2ad-be)}{d^2(ad-be)^2(ex+d)}+\frac{e\left(3a^2d^2-3adeb+e^2b^2\right)\ln(ex+d)}{d^3(ad-be)^3}-\frac{a^3\ln(ax+b)}{b(ad-be)^3}\right)}{3e}$
parallelrisch	$-\frac{-2x^3\ln\left(c\left(\frac{ax+b}{x}\right)^p\right)a^3bd^3e^2p+18\ln(x)xab^3d^3e^2p^2+18\ln(ex+d)xa^2b^2d^4ep^2-18\ln(ex+d)xab^3d^3e^2p^2-6x^2\ln\left(c\left(\frac{ax+b}{x}\right.\right.}{}$

```
[In] int(ln(c*(a+b/x)^p)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*ln(c*(a+b/x)^p)/e/(e*x+d)^3-1/3*p*b/e*(1/d^3/b*ln(x)-1/2*e/d/(a*d-b*e)
/(e*x+d)^2-e*(2*a*d-b*e)/d^2/(a*d-b*e)^2/(e*x+d)+e*(3*a^2*d^2-3*a*b*d*e+b^2
*e^2)/d^3/(a*d-b*e)^3*ln(e*x+d)-a^3/b/(a*d-b*e)^3*ln(a*x+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. 2(163) = 326.

Time = 4.29 (sec) , antiderivative size = 818, normalized size of antiderivative = 4.67

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4}\,dx$$
$$=\frac{2\left(2a^2bd^3e^3-3ab^2d^2e^4+b^3de^5\right)px^2+\left(9a^2bd^4e^2-14ab^2d^3e^3+5b^3d^2e^4\right)px-2\left(a^3d^6-3a^2bd^5e+3ab^2\right.}{}$$

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(2*a^2*b*d^3*e^3 - 3*a*b^2*d^2*e^4 + b^3*d*e^5)*p*x^2 + (9*a^2*b*d^4
*e^2 - 14*a*b^2*d^3*e^3 + 5*b^3*d^2*e^4)*p*x - 2*(a^3*d^6 - 3*a^2*b*d^5*e +
 3*a*b^2*d^4*e^2 - b^3*d^3*e^3)*p*log((a*x + b)/x) + (5*a^2*b*d^5*e - 8*a*b
^2*d^4*e^2 + 3*b^3*d^3*e^3)*p + 2*(a^3*d^3*e^3*p*x^3 + 3*a^3*d^4*e^2*p*x^2
+ 3*a^3*d^5*e*p*x + a^3*d^6*p)*log(a*x + b) - 2*((3*a^2*b*d^2*e^4 - 3*a*b^2
*d*e^5 + b^3*e^6)*p*x^3 + 3*(3*a^2*b*d^3*e^3 - 3*a*b^2*d^2*e^4 + b^3*d*e^5)
*p*x^2 + 3*(3*a^2*b*d^4*e^2 - 3*a*b^2*d^3*e^3 + b^3*d^2*e^4)*p*x + (3*a^2*b
*d^5*e - 3*a*b^2*d^4*e^2 + b^3*d^3*e^3)*p)*log(e*x + d) - 2*(a^3*d^6 - 3*a^
2*b*d^5*e + 3*a*b^2*d^4*e^2 - b^3*d^3*e^3)*log(c) - 2*((a^3*d^3*e^3 - 3*a^2
*b*d^2*e^4 + 3*a*b^2*d*e^5 - b^3*e^6)*p*x^3 + 3*(a^3*d^4*e^2 - 3*a^2*b*d^3*
e^3 + 3*a*b^2*d^2*e^4 - b^3*d*e^5)*p*x^2 + 3*(a^3*d^5*e - 3*a^2*b*d^4*e^2 +
 3*a*b^2*d^3*e^3 - b^3*d^2*e^4)*p*x + (a^3*d^6 - 3*a^2*b*d^5*e + 3*a*b^2*d^
4*e^2 - b^3*d^3*e^3)*p)*log(x))/(a^3*d^9*e - 3*a^2*b*d^8*e^2 + 3*a*b^2*d^7*
e^3 - b^3*d^6*e^4 + (a^3*d^6*e^4 - 3*a^2*b*d^5*e^5 + 3*a*b^2*d^4*e^6 - b^3*
d^3*e^7)*x^3 + 3*(a^3*d^7*e^3 - 3*a^2*b*d^6*e^4 + 3*a*b^2*d^5*e^5 - b^3*d^4
*e^6)*x^2 + 3*(a^3*d^8*e^2 - 3*a^2*b*d^7*e^3 + 3*a*b^2*d^6*e^4 - b^3*d^5*e^
5)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(a+b/x)**p)/(e*x+d)**4,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.71

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4}\,dx = \frac{\left(\frac{2\,a^3\log(ax+b)}{a^3bd^3-3\,a^2b^2d^2e+3\,ab^3de^2-b^4e^3}-\frac{2\,(3\,a^2d^2e-3\,abde^2+b^2e^3)\log(ex+d)}{a^3d^6-3\,a^2bd^5e+3\,ab^2d^4e^2-b^3d^3e^3}+\frac{5\,ad^2e-3\,bde^2+2\,(2\,ade^2-be^3)x}{a^2d^6-2\,abd^5e+b^2d^4e^2+(a^2d^4e^2-2\,abd^3e^3+b^2d^2e^4)x^2+2\,(a^2d^5e}\right.}{6\,e} - \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{3\,(ex+d)^3e}$$

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^4,x, algorithm="maxima")
[Out] 1/6*(2*a^3*log(a*x + b)/(a^3*b*d^3 - 3*a^2*b^2*d^2*e + 3*a*b^3*d*e^2 - b^4*
e^3) - 2*(3*a^2*d^2*e - 3*a*b*d*e^2 + b^2*e^3)*log(e*x + d)/(a^3*d^6 - 3*a^
```

```
2*b*d^5*e + 3*a*b^2*d^4*e^2 - b^3*d^3*e^3) + (5*a*d^2*e - 3*b*d*e^2 + 2*(2*
a*d*e^2 - b*e^3)*x)/(a^2*d^6 - 2*a*b*d^5*e + b^2*d^4*e^2 + (a^2*d^4*e^2 - 2
*a*b*d^3*e^3 + b^2*d^2*e^4)*x^2 + 2*(a^2*d^5*e - 2*a*b*d^4*e^2 + b^2*d^3*e^
3)*x) - 2*log(x)/(b*d^3))*b*p/e - 1/3*log((a + b/x)^p*c)/((e*x + d)^3*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1097 vs. $2(163) = 326$.

Time = 0.32 (sec) , antiderivative size = 1097, normalized size of antiderivative = 6.27

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4}\,dx = \text{Too large to display}$$

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] -1/6*(2*(3*a^2*b^2*d^2*p - 3*a*b^3*d*e*p + b^4*e^2*p)*log(-a*d + b*e + (a*x
 + b)*d/x)/(a^3*d^6 - 3*a^2*b*d^5*e + 3*a*b^2*d^4*e^2 - b^3*d^3*e^3) + 2*(3
*a^2*b^2*d^2*p - 3*a*b^3*d*e*p + b^4*e^2*p - 6*(a*x + b)*a*b^2*d^2*p/x + 3*
(a*x + b)*b^3*d*e*p/x + 3*(a*x + b)^2*b^2*d^2*p/x^2)*log((a*x + b)/x)/(a^3*
d^6 - 3*a^2*b*d^5*e + 3*a*b^2*d^4*e^2 - b^3*d^3*e^3 - 3*(a*x + b)*a^2*d^6/x
 + 6*(a*x + b)*a*b*d^5*e/x - 3*(a*x + b)*b^2*d^4*e^2/x + 3*(a*x + b)^2*a*d^
6/x^2 - 3*(a*x + b)^2*b*d^5*e/x^2 - (a*x + b)^3*d^6/x^3) - 2*(3*a^2*b^2*d^2
*p - 3*a*b^3*d*e*p + b^4*e^2*p)*log((a*x + b)/x)/(a^3*d^6 - 3*a^2*b*d^5*e +
 3*a*b^2*d^4*e^2 - b^3*d^3*e^3) - (6*a^3*b^3*d^3*e*p - 15*a^2*b^4*d^2*e^2*p
 + 12*a*b^5*d*e^3*p - 3*b^6*e^4*p - 6*a^4*b^2*d^4*log(c) + 18*a^3*b^3*d^3*e
*log(c) - 20*a^2*b^4*d^2*e^2*log(c) + 10*a*b^5*d*e^3*log(c) - 2*b^6*e^4*log
(c) - 12*(a*x + b)*a^2*b^3*d^3*e*p/x + 19*(a*x + b)*a*b^4*d^2*e^2*p/x - 7*(
a*x + b)*b^5*d*e^3*p/x + 12*(a*x + b)*a^3*b^2*d^4*log(c)/x - 30*(a*x + b)*a
^2*b^3*d^3*e*log(c)/x + 24*(a*x + b)*a*b^4*d^2*e^2*log(c)/x - 6*(a*x + b)*b
^5*d*e^3*log(c)/x + 6*(a*x + b)^2*a*b^3*d^3*e*p/x^2 - 4*(a*x + b)^2*b^4*d^2
*e^2*p/x^2 - 6*(a*x + b)^2*a^2*b^2*d^4*log(c)/x^2 + 12*(a*x + b)^2*a*b^3*d^
3*e*log(c)/x^2 - 6*(a*x + b)^2*b^4*d^2*e^2*log(c)/x^2)/(a^5*d^8 - 5*a^4*b*d
^7*e + 10*a^3*b^2*d^6*e^2 - 10*a^2*b^3*d^5*e^3 + 5*a*b^4*d^4*e^4 - b^5*d^3*
e^5 - 3*(a*x + b)*a^4*d^8/x + 12*(a*x + b)*a^3*b*d^7*e/x - 18*(a*x + b)*a^2
*b^2*d^6*e^2/x + 12*(a*x + b)*a*b^3*d^5*e^3/x - 3*(a*x + b)*b^4*d^4*e^4/x +
 3*(a*x + b)^2*a^3*d^8/x^2 - 9*(a*x + b)^2*a^2*b*d^7*e/x^2 + 9*(a*x + b)^2*
a*b^2*d^6*e^2/x^2 - 3*(a*x + b)^2*b^3*d^5*e^3/x^2 - (a*x + b)^3*a^2*d^8/x^3
 + 2*(a*x + b)^3*a*b*d^7*e/x^3 - (a*x + b)^3*b^2*d^6*e^2/x^3))/b
```

Mupad [B] (verification not implemented)

Time = 3.00 (sec) , antiderivative size = 662, normalized size of antiderivative = 3.78

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{(d+ex)^4}\,dx = \frac{p\,\ln\left(d+e\,x\right)}{3\,d^3\,e}$$
$$-\frac{3\,b^2\,e^2\,p}{2\,\left(3\,a^2\,d^5\,e+6\,a^2\,d^4\,e^2\,x+3\,a^2\,d^3\,e^3\,x^2-6\,a\,b\,d^4\,e^2-12\,a\,b\,d^3\,e^3\,x-6\,a\,b\,d^2\,e^4\,x^2+3\,b^2\,d^3\,e^3+6\,b^2\,d^2\,e^4}$$
$$-\frac{p\,\ln\left(x\right)}{3\,d^3\,e}-\frac{a^3\,p\,\ln\left(b+a\,x\right)}{-3\,a^3\,d^3\,e+9\,a^2\,b\,d^2\,e^2-9\,a\,b^2\,d\,e^3+3\,b^3\,e^4}$$
$$-\frac{\ln\left(c\,{\left(\frac{b+a\,x}{x}\right)}^p\right)}{3\,\left(d^3\,e+3\,d^2\,e^2\,x+3\,d\,e^3\,x^2+e^4\,x^3\right)}$$
$$-\frac{b^2\,e^3\,p\,x}{3\,a^2\,d^6\,e+6\,a^2\,d^5\,e^2\,x+3\,a^2\,d^4\,e^3\,x^2-6\,a\,b\,d^5\,e^2-12\,a\,b\,d^4\,e^3\,x-6\,a\,b\,d^3\,e^4\,x^2+3\,b^2\,d^4\,e^3+6\,b^2\,d^3\,e^4\,x+}$$
$$-\frac{a^3\,d^3\,p\,\ln\left(d+e\,x\right)}{3\,a^3\,d^6\,e-9\,a^2\,b\,d^5\,e^2+9\,a\,b^2\,d^4\,e^3-3\,b^3\,d^3\,e^4}$$
$$+\frac{5\,a\,b\,d\,e\,p}{2\,\left(3\,a^2\,d^5\,e+6\,a^2\,d^4\,e^2\,x+3\,a^2\,d^3\,e^3\,x^2-6\,a\,b\,d^4\,e^2-12\,a\,b\,d^3\,e^3\,x-6\,a\,b\,d^2\,e^4\,x^2+3\,b^2\,d^3\,e^3+6\,b^2\,d^2\,e^4}$$
$$+\frac{2\,a\,b\,d\,e^2\,p\,x}{3\,a^2\,d^6\,e+6\,a^2\,d^5\,e^2\,x+3\,a^2\,d^4\,e^3\,x^2-6\,a\,b\,d^5\,e^2-12\,a\,b\,d^4\,e^3\,x-6\,a\,b\,d^3\,e^4\,x^2+3\,b^2\,d^4\,e^3+6\,b^2\,d^3\,e^4\,x+}$$

```
[In] int(log(c*(a + b/x)^p)/(d + e*x)^4,x)
```

```
[Out] (p*log(d + e*x))/(3*d^3*e) - (3*b^2*e^2*p)/(2*(3*a^2*d^5*e + 3*b^2*d^3*e^3
+ 6*a^2*d^4*e^2*x + 6*b^2*d^2*e^4*x + 3*b^2*d*e^5*x^2 + 3*a^2*d^3*e^3*x^2 -
 6*a*b*d^4*e^2 - 12*a*b*d^3*e^3*x - 6*a*b*d^2*e^4*x^2)) - (p*log(x))/(3*d^3
*e) - (a^3*p*log(b + a*x))/(3*b^3*e^4 - 3*a^3*d^3*e + 9*a^2*b*d^2*e^2 - 9*a
*b^2*d*e^3) - log(c*((b + a*x)/x)^p)/(3*(d^3*e + e^4*x^3 + 3*d^2*e^2*x + 3*
d*e^3*x^2)) - (b^2*e^3*p*x)/(3*a^2*d^6*e + 3*b^2*d^4*e^3 + 6*a^2*d^5*e^2*x
+ 6*b^2*d^3*e^4*x + 3*a^2*d^4*e^3*x^2 + 3*b^2*d^2*e^5*x^2 - 6*a*b*d^5*e^2 -
 12*a*b*d^4*e^3*x - 6*a*b*d^3*e^4*x^2) - (a^3*d^3*p*log(d + e*x))/(3*a^3*d^
6*e - 3*b^3*d^3*e^4 + 9*a*b^2*d^4*e^3 - 9*a^2*b*d^5*e^2) + (5*a*b*d*e*p)/(2
*(3*a^2*d^5*e + 3*b^2*d^3*e^3 + 6*a^2*d^4*e^2*x + 6*b^2*d^2*e^4*x + 3*b^2*d
*e^5*x^2 + 3*a^2*d^3*e^3*x^2 - 6*a*b*d^4*e^2 - 12*a*b*d^3*e^3*x - 6*a*b*d^2
*e^4*x^2)) + (2*a*b*d*e^2*p*x)/(3*a^2*d^6*e + 3*b^2*d^4*e^3 + 6*a^2*d^5*e^2
*x + 6*b^2*d^3*e^4*x + 3*a^2*d^4*e^3*x^2 + 3*b^2*d^2*e^5*x^2 - 6*a*b*d^5*e^
2 - 12*a*b*d^4*e^3*x - 6*a*b*d^3*e^4*x^2)
```

3.205 $\int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 105

$$\int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx} \, dx = \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d} + \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(-\frac{d(b+ax)}{ac-bd}\right)\log(c+dx)}{d} - \frac{\operatorname{PolyLog}\left(2,\frac{a(c+dx)}{ac-bd}\right)}{d} + \frac{\operatorname{PolyLog}\left(2,1+\frac{dx}{c}\right)}{d}$$

```
[Out] ln(a+b/x)*ln(d*x+c)/d+ln(-d*x/c)*ln(d*x+c)/d-ln(-d*(a*x+b)/(a*c-b*d))*ln(d*
x+c)/d-polylog(2,a*(d*x+c)/(a*c-b*d))/d+polylog(2,1+d*x/c)/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2512, 266, 2463, 2441, 2352, 2440, 2438}

$$\int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx} \, dx = -\frac{\operatorname{PolyLog}\left(2,\frac{a(c+dx)}{ac-bd}\right)}{d} + \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d} - \frac{\log(c+dx)\log\left(-\frac{d(ax+b)}{ac-bd}\right)}{d} + \frac{\operatorname{PolyLog}\left(2,\frac{dx}{c}+1\right)}{d} + \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}$$

```
[In] Int[Log[a + b/x]/(c + d*x),x]
```

```
[Out] (Log[a + b/x]*Log[c + d*x])/d + (Log[-((d*x)/c)]*Log[c + d*x])/d - (Log[-((
d*(b + a*x))/(a*c - b*d))]*Log[c + d*x])/d - PolyLog[2, (a*(c + d*x))/(a*c
- b*d)]/d + PolyLog[2, 1 + (d*x)/c]/d
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{b\int\frac{\log(c+dx)}{\left(a+\frac{b}{x}\right)x^2}\,dx}{d}\\
&= \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{b\int\left(\frac{\log(c+dx)}{bx}-\frac{a\log(c+dx)}{b(b+ax)}\right)dx}{d}\\
&= \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{\int\frac{\log(c+dx)}{x}\,dx}{d}-\frac{a\int\frac{\log(c+dx)}{b+ax}\,dx}{d}\\
&= \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}\\
&\quad-\frac{\log\left(-\frac{d(b+ax)}{ac-bd}\right)\log(c+dx)}{d}-\int\frac{\log\left(-\frac{dx}{c}\right)}{c+dx}\,dx+\int\frac{\log\left(\frac{d(b+ax)}{-ac+bd}\right)}{c+dx}\,dx\\
&= \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}-\frac{\log\left(-\frac{d(b+ax)}{ac-bd}\right)\log(c+dx)}{d}\\
&\quad+\frac{\mathrm{Li}_2\left(1+\frac{dx}{c}\right)}{d}+\frac{\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{ax}{-ac+bd}\right)}{x}\,dx,x,c+dx\right)}{d}\\
&= \frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}\\
&\quad-\frac{\log\left(-\frac{d(b+ax)}{ac-bd}\right)\log(c+dx)}{d}-\frac{\mathrm{Li}_2\left(\frac{a(c+dx)}{ac-bd}\right)}{d}+\frac{\mathrm{Li}_2\left(1+\frac{dx}{c}\right)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10

$$
\int\frac{\log\left(a+\frac{b}{x}\right)}{c+dx}\,dx
$$
$$
=\frac{\log\left(a+\frac{b}{x}\right)\log(c+dx)+\log(x)\log(c+dx)-\log\left(\frac{b}{a}+x\right)\log(c+dx)+\log\left(\frac{b}{a}+x\right)\log\left(\frac{a(c+dx)}{ac-bd}\right)-\log(x
}{d}
$$

```
[In] Integrate[Log[a + b/x]/(c + d*x),x]
```

```
[Out] (Log[a + b/x]*Log[c + d*x] + Log[x]*Log[c + d*x] - Log[b/a + x]*Log[c + d*x
] + Log[b/a + x]*Log[(a*(c + d*x))/(a*c - b*d)] - Log[x]*Log[1 + (d*x)/c] -
 PolyLog[2, -((d*x)/c)] + PolyLog[2, (d*(b + a*x))/(-(a*c) + b*d)])/d
```

Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.09

method	result
risch	$-\frac{\ln\left(a+\frac{b}{x}\right)\ln\left(-\frac{b}{ax}\right)}{d}-\frac{\operatorname{dilog}\left(-\frac{b}{ax}\right)}{d}+\frac{\operatorname{dilog}\left(\frac{-ca+bd+c\left(a+\frac{b}{x}\right)}{-ca+bd}\right)}{d}+\frac{\ln\left(a+\frac{b}{x}\right)\ln\left(\frac{-ca+bd+c\left(a+\frac{b}{x}\right)}{-ca+bd}\right)}{d}$
derivativedivides	$-b\left(\frac{\operatorname{dilog}\left(-\frac{b}{ax}\right)+\ln\left(a+\frac{b}{x}\right)\ln\left(-\frac{b}{ax}\right)}{db}-\frac{c\left(\frac{\operatorname{dilog}\left(\frac{-ca+bd+c\left(a+\frac{b}{x}\right)}{-ca+bd}\right)}{c}+\frac{\ln\left(a+\frac{b}{x}\right)\ln\left(\frac{-ca+bd+c\left(a+\frac{b}{x}\right)}{-ca+bd}\right)}{c}\right)}{db}\right)$
default	$-b\left(\frac{\operatorname{dilog}\left(-\frac{b}{ax}\right)+\ln\left(a+\frac{b}{x}\right)\ln\left(-\frac{b}{ax}\right)}{db}-\frac{c\left(\frac{\operatorname{dilog}\left(\frac{-ca+bd+c\left(a+\frac{b}{x}\right)}{-ca+bd}\right)}{c}+\frac{\ln\left(a+\frac{b}{x}\right)\ln\left(\frac{-ca+bd+c\left(a+\frac{b}{x}\right)}{-ca+bd}\right)}{c}\right)}{db}\right)$
parts	$\frac{\ln\left(a+\frac{b}{x}\right)\ln(dx+c)}{d}+b\left(\frac{\operatorname{dilog}\left(-\frac{xd}{c}\right)+\ln(dx+c)\ln\left(-\frac{xd}{c}\right)}{bd}-\frac{a\left(\frac{\operatorname{dilog}\left(\frac{-ca+a(dx+c)+bd}{-ca+bd}\right)}{a}+\frac{\ln(dx+c)\ln\left(\frac{-ca+a(dx+c)}{-ca+bd}\right.}{a}\right.}{bd}\right.$

```
[In] int(ln(a+b/x)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*ln(a+b/x)*ln(-b/a/x)-1/d*dilog(-b/a/x)+1/d*dilog((-c*a+b*d+c*(a+b/x))/
(-a*c+b*d))+1/d*ln(a+b/x)*ln((-c*a+b*d+c*(a+b/x))/(-a*c+b*d))
```

Fricas [F]

$$\int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx}\,dx=\int \frac{\log\left(a+\frac{b}{x}\right)}{dx+c}\,dx$$

```
[In] integrate(log(a+b/x)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(log((a*x + b)/x)/(d*x + c), x)
```

Sympy [F]

$$\int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx}\,dx = \int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx}\,dx$$

```
[In] integrate(ln(a+b/x)/(d*x+c),x)
[Out] Integral(log(a + b/x)/(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.78

$$\int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx}\,dx = -\frac{\log\left(\frac{dx}{c}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{dx}{c}\right)}{d} + \frac{\log\left(ax+b\right)\log\left(\frac{adx+bd}{ac-bd}+1\right)+\mathrm{Li}_2\left(-\frac{adx+bd}{ac-bd}\right)}{d}$$

```
[In] integrate(log(a+b/x)/(d*x+c),x, algorithm="maxima")
[Out] -(log(d*x/c + 1)*log(x) + dilog(-d*x/c))/d + (log(a*x + b)*log((a*d*x + b*d
)/(a*c - b*d) + 1) + dilog(-(a*d*x + b*d)/(a*c - b*d)))/d
```

Giac [F]

$$\int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx}\,dx = \int \frac{\log\left(a+\frac{b}{x}\right)}{dx+c}\,dx$$

```
[In] integrate(log(a+b/x)/(d*x+c),x, algorithm="giac")
[Out] integrate(log(a + b/x)/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(a+\frac{b}{x}\right)}{c+dx}\,dx = \int \frac{\ln\left(a+\frac{b}{x}\right)}{c+d\,x}\,dx$$

```
[In] int(log(a + b/x)/(c + d*x),x)
[Out] int(log(a + b/x)/(c + d*x), x)
```

3.206 $\int (d+ex)^m \log\left(c(a+bx^3)^p\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 301

$$\begin{aligned}
&\int (d+ex)^m \log\left(c(a+bx^3)^p\right) \, dx \\
&= \frac{\sqrt[3]{b}p(d+ex)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)(1+m)(2+m)} \\
&\quad + \frac{\sqrt[3]{b}p(d+ex)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e\left(\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e\right)(1+m)(2+m)} \\
&\quad + \frac{\sqrt[3]{b}p(d+ex)^{2+m}\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e\left(\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e\right)(1+m)(2+m)} \\
&\quad + \frac{(d+ex)^{1+m}\log\left(c(a+bx^3)^p\right)}{e(1+m)}
\end{aligned}$$

```
[Out] b^(1/3)*p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],b^(1/3)*(e*x+d)/(b^(1/3)*d
-a^(1/3)*e))/e/(b^(1/3)*d-a^(1/3)*e)/(1+m)/(2+m)+b^(1/3)*p*(e*x+d)^(2+m)*hy
pergeom([1, 2+m],[3+m],b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))/e/
(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e)/(1+m)/(2+m)+b^(1/3)*p*(e*x+d)^(2+m)*hyperg
eom([1, 2+m],[3+m],b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))/e/(b^(
1/3)*d-(-1)^(2/3)*a^(1/3)*e)/(1+m)/(2+m)+(e*x+d)^(1+m)*ln(c*(b*x^3+a)^p)/e/
(1+m)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2513, 6857, 70}

$$\begin{aligned}
&\int (d+ex)^m \log\left(c\left(a+bx^3\right)^p\right)\,dx \\
&= \frac{(d+ex)^{m+1}\log\left(c\left(a+bx^3\right)^p\right)}{e(m+1)} \\
&\quad+\frac{\sqrt[3]{b}p(d+ex)^{m+2}\,\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e(m+1)(m+2)\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)} \\
&\quad+\frac{\sqrt[3]{b}p(d+ex)^{m+2}\,\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e(m+1)(m+2)\left(\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d\right)} \\
&\quad+\frac{\sqrt[3]{b}p(d+ex)^{m+2}\,\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e(m+1)(m+2)\left(\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e\right)}
\end{aligned}$$

```
[In] Int[(d + e*x)^m*Log[c*(a + b*x^3)^p],x]
```

```
[Out] (b^(1/3)*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (b^(1/3)*(d
 + e*x))/(b^(1/3)*d - a^(1/3)*e)])/(e*(b^(1/3)*d - a^(1/3)*e)*(1 + m)*(2 +
m)) + (b^(1/3)*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (b^(1
/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/(e*(b^(1/3)*d + (-1)^(1
/3)*a^(1/3)*e)*(1 + m)*(2 + m)) + (b^(1/3)*p*(d + e*x)^(2 + m)*Hypergeometr
ic2F1[1, 2 + m, 3 + m, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*
e)])/(e*(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)*(1 + m)*(2 + m)) + ((d + e*x)^(1
 + m)*Log[c*(a + b*x^3)^p])/(e*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
```

```
&& (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^{1+m}\log\left(c(a+bx^3)^p\right)}{e(1+m)} - \frac{(3bp)\int\frac{x^2(d+ex)^{1+m}}{a+bx^3}\,dx}{e(1+m)} \\
&= \frac{(d+ex)^{1+m}\log\left(c(a+bx^3)^p\right)}{e(1+m)} \\
&\quad - \frac{(3bp)\int\left(\frac{(d+ex)^{1+m}}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{(d+ex)^{1+m}}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{(d+ex)^{1+m}}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx}{e(1+m)} \\
&= \frac{(d+ex)^{1+m}\log\left(c(a+bx^3)^p\right)}{e(1+m)} - \frac{\left(\sqrt[3]{b}p\right)\int\frac{(d+ex)^{1+m}}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e(1+m)} \\
&\quad - \frac{\left(\sqrt[3]{b}p\right)\int\frac{(d+ex)^{1+m}}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e(1+m)} - \frac{\left(\sqrt[3]{b}p\right)\int\frac{(d+ex)^{1+m}}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e(1+m)} \\
&= \frac{\sqrt[3]{b}p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e\left(\sqrt[3]{b}d-\sqrt[3]{a}e\right)(1+m)(2+m)} \\
&\quad + \frac{\sqrt[3]{b}p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e\left(\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e\right)(1+m)(2+m)} \\
&\quad + \frac{\sqrt[3]{b}p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e\left(\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e\right)(1+m)(2+m)} \\
&\quad + \frac{(d+ex)^{1+m}\log\left(c(a+bx^3)^p\right)}{e(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.79

$$\int (d+ex)^m \log \left(c\left(a+bx^3\right)^p\right)\, dx = \frac{(d+ex)^{1+m}\left(-\frac{\sqrt[3]{b}p(d+ex)\left(-\frac{\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}-\frac{\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}-\ldots\right)}{2+m}\ldots\right)}{e(1+m)}$$

```
[In] Integrate[(d + e*x)^m*Log[c*(a + b*x^3)^p],x]
[Out] ((d + e*x)^(1 + m)*(-((b^(1/3)*p*(d + e*x)*(-(Hypergeometric2F1[1, 2 + m, 3
 + m, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)]/(b^(1/3)*d - a^(1/3)*e))
 - Hypergeometric2F1[1, 2 + m, 3 + m, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)
^(1/3)*a^(1/3)*e)]/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e) - Hypergeometric2F1[1
, 2 + m, 3 + m, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)]/(b^
(1/3)*d - (-1)^(2/3)*a^(1/3)*e)))/(2 + m)) + Log[c*(a + b*x^3)^p]))/(e*(1 +
 m))
```

Maple [F]

$$\int \left(ex+d\right)^{m} \ln \left(c\left(b\,x^{3}+a\right)^{p}\right) dx$$

```
[In] int((e*x+d)^m*ln(c*(b*x^3+a)^p),x)
[Out] int((e*x+d)^m*ln(c*(b*x^3+a)^p),x)
```

Fricas [F]

$$\int (d+ex)^m \log \left(c\left(a+bx^3\right)^p\right)\, dx = \int (ex+d)^m \log \left(\left(bx^3+a\right)^p c\right)\, dx$$

```
[In] integrate((e*x+d)^m*log(c*(b*x^3+a)^p),x, algorithm="fricas")
[Out] integral((e*x + d)^m*log((b*x^3 + a)^p*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m \log \left(c\left(a+bx^3\right)^p\right) \, dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**m*ln(c*(b*x**3+a)**p),x)
[Out] Timed out
```

Maxima [F]

$$\int (d+ex)^m \log \left(c\left(a+bx^3\right)^p\right) \, dx = \int (ex+d)^m \log \left(\left(bx^3+a\right)^p c\right) \, dx$$

```
[In] integrate((e*x+d)^m*log(c*(b*x^3+a)^p),x, algorithm="maxima")
[Out] (e*x + d)*(e*x + d)^m*log((b*x^3 + a)^p)/(e*(m + 1)) + integrate(-(3*b*d*p*
x^2 - (e*(m + 1)*log(c) - 3*e*p)*b*x^3 - a*e*(m + 1)*log(c))*(e*x + d)^m/(b
*e*(m + 1)*x^3 + a*e*(m + 1)), x)
```

Giac [F]

$$\int (d+ex)^m \log \left(c\left(a+bx^3\right)^p\right) \, dx = \int (ex+d)^m \log \left(\left(bx^3+a\right)^p c\right) \, dx$$

```
[In] integrate((e*x+d)^m*log(c*(b*x^3+a)^p),x, algorithm="giac")
[Out] integrate((e*x + d)^m*log((b*x^3 + a)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \log \left(c\left(a+bx^3\right)^p\right) \, dx = \int \ln \left(c\,{\left(b\,x^3+a\right)}^p\right)\,{\left(d+e\,x\right)}^m \,d x$$

```
[In] int(log(c*(a + b*x^3)^p)*(d + e*x)^m,x)
[Out] int(log(c*(a + b*x^3)^p)*(d + e*x)^m, x)
```

3.207 $\int (d+ex)^m \log \left(c(a+bx^2)^p\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 205

$$\int (d+ex)^m \log \left(c(a+bx^2)^p\right) \, dx = \frac{\sqrt{b}p(d+ex)^{2+m} \text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e\left(\sqrt{b}d-\sqrt{-a}e\right)(1+m)(2+m)} + \frac{\sqrt{b}p(d+ex)^{2+m} \text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e\left(\sqrt{b}d+\sqrt{-a}e\right)(1+m)(2+m)} + \frac{(d+ex)^{1+m} \log \left(c(a+bx^2)^p\right)}{e(1+m)}$$

```
[Out] (e*x+d)^(1+m)*ln(c*(b*x^2+a)^p)/e/(1+m)+p*(e*x+d)^(2+m)*hypergeom([1, 2+m],
[3+m],(e*x+d)*b^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))*b^(1/2)/e/(1+m)/(2+m)/(-e*
(-a)^(1/2)+d*b^(1/2))+p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],(e*x+d)*b^(1
/2)/(e*(-a)^(1/2)+d*b^(1/2)))*b^(1/2)/e/(1+m)/(2+m)/(e*(-a)^(1/2)+d*b^(1/2)
)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used

$= \{2513, 845, 70\}$

$$\int (d+ex)^m \log \left(c\left(a+bx^2\right)^p\right) \, dx$$
$$= \frac{(d+ex)^{m+1} \log \left(c\left(a+bx^2\right)^p\right)}{e(m+1)}$$
$$+ \frac{\sqrt{b}p(d+ex)^{m+2} \text{Hypergeometric2F1}\left(1, m+2, m+3, \frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e(m+1)(m+2)\left(\sqrt{b}d-\sqrt{-a}e\right)}$$
$$+ \frac{\sqrt{b}p(d+ex)^{m+2} \text{Hypergeometric2F1}\left(1, m+2, m+3, \frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e(m+1)(m+2)\left(\sqrt{-a}e+\sqrt{b}d\right)}$$

```
[In] Int[(d + e*x)^m*Log[c*(a + b*x^2)^p],x]
```

```
[Out] (Sqrt[b]*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sqrt[b]*(d
 + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/(e*(Sqrt[b]*d - Sqrt[-a]*e)*(1 + m)*(2
+ m)) + (Sqrt[b]*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sq
rt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/(e*(Sqrt[b]*d + Sqrt[-a]*e)*(1
+ m)*(2 + m)) + ((d + e*x)^(1 + m)*Log[c*(a + b*x^2)^p])/(e*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d+ex)^{1+m}\log\left(c(a+bx^2)^p\right)}{e(1+m)} - \frac{(2bp)\int\frac{x(d+ex)^{1+m}}{a+bx^2}\,dx}{e(1+m)} \\
&= \frac{(d+ex)^{1+m}\log\left(c(a+bx^2)^p\right)}{e(1+m)} - \frac{(2bp)\int\left(-\frac{(d+ex)^{1+m}}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{(d+ex)^{1+m}}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{e(1+m)} \\
&= \frac{(d+ex)^{1+m}\log\left(c(a+bx^2)^p\right)}{e(1+m)} + \frac{\left(\sqrt{b}p\right)\int\frac{(d+ex)^{1+m}}{\sqrt{-a}-\sqrt{b}x}\,dx}{e(1+m)} - \frac{\left(\sqrt{b}p\right)\int\frac{(d+ex)^{1+m}}{\sqrt{-a}+\sqrt{b}x}\,dx}{e(1+m)} \\
&= \frac{\sqrt{b}p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e\left(\sqrt{b}d-\sqrt{-a}e\right)(1+m)(2+m)} \\
&\quad + \frac{\sqrt{b}p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e\left(\sqrt{b}d+\sqrt{-a}e\right)(1+m)(2+m)} \\
&\quad + \frac{(d+ex)^{1+m}\log\left(c(a+bx^2)^p\right)}{e(1+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.86

$$
\int (d+ex)^m\log\left(c(a+bx^2)^p\right)\,dx
$$

$$
= \frac{(d+ex)^{1+m}\left(\frac{\sqrt{b}p(d+ex)\left(\left(\sqrt{b}d+\sqrt{-a}e\right)\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)+\left(\sqrt{b}d-\sqrt{-a}e\right)\text{Hypergeometric2F1}\left(1,2+m,3+\right.\right.}{(bd^2+ae^2)(2+m)}\right.}{e(1+m)}
$$

```
[In] Integrate[(d + e*x)^m*Log[c*(a + b*x^2)^p],x]
```

```
[Out] ((d + e*x)^(1 + m)*((Sqrt[b]*p*(d + e*x)*((Sqrt[b]*d + Sqrt[-a]*e)*Hypergeo
metric2F1[1, 2 + m, 3 + m, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)] +
(Sqrt[b]*d - Sqrt[-a]*e)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sqrt[b]*(d + e
*x))/(Sqrt[b]*d + Sqrt[-a]*e)]))/((b*d^2 + a*e^2)*(2 + m)) + Log[c*(a + b*x
^2)^p]))/(e*(1 + m))
```

Maple [F]

$$\int (ex+d)^m \ln \left(c\left(b\,x^2+a\right)^p\right) dx$$

```
[In] int((e*x+d)^m*ln(c*(b*x^2+a)^p),x)
```

```
[Out] int((e*x+d)^m*ln(c*(b*x^2+a)^p),x)
```

Fricas [F]

$$\int (d+ex)^m \log \left(c\left(a+bx^2\right)^p\right)\,dx = \int (ex+d)^m \log \left(\left(bx^2+a\right)^p c\right)\,dx$$

```
[In] integrate((e*x+d)^m*log(c*(b*x^2+a)^p),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)^m*log((b*x^2 + a)^p*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m \log \left(c\left(a+bx^2\right)^p\right)\,dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**m*ln(c*(b*x**2+a)**p),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (d+ex)^m \log \left(c\left(a+bx^2\right)^p\right)\,dx = \int (ex+d)^m \log \left(\left(bx^2+a\right)^p c\right)\,dx$$

```
[In] integrate((e*x+d)^m*log(c*(b*x^2+a)^p),x, algorithm="maxima")
```

```
[Out] (e*p*x + d*p)*(e*x + d)^m*log(b*x^2 + a)/(e*(m + 1)) + integrate(-(2*b*d*p*
x - (e*(m + 1)*log(c) - 2*e*p)*b*x^2 - a*e*(m + 1)*log(c))*(e*x + d)^m/(b*e
*(m + 1)*x^2 + a*e*(m + 1)), x)
```

Giac [F]

$$\int (d+ex)^m \log \left(c(a+bx^2)^p\right) \, dx = \int (ex+d)^m \log \left((bx^2+a)^p c\right) \, dx$$

```
[In] integrate((e*x+d)^m*log(c*(b*x^2+a)^p),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^m*log((b*x^2 + a)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \log \left(c(a+bx^2)^p\right) \, dx = \int \ln \left(c \left(b\,x^2+a\right)^p\right) \left(d+e\,x\right)^m \, dx$$

```
[In] int(log(c*(a + b*x^2)^p)*(d + e*x)^m,x)
```

```
[Out] int(log(c*(a + b*x^2)^p)*(d + e*x)^m, x)
```

3.208 $\int (d+ex)^m \log \left(c(a+bx)^p\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 89

$$\int (d+ex)^m \log \left(c(a+bx)^p\right) \, dx = \frac{bp(d+ex)^{2+m} \text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{b(d+ex)}{bd-ae}\right)}{e(bd-ae)(1+m)(2+m)} + \frac{(d+ex)^{1+m} \log \left(c(a+bx)^p\right)}{e(1+m)}$$

```
[Out] b*p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],b*(e*x+d)/(-a*e+b*d))/e/(-a*e+b*
d)/(1+m)/(2+m)+(e*x+d)^(1+m)*ln(c*(b*x+a)^p)/e/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2442, 70}

$$\int (d+ex)^m \log \left(c(a+bx)^p\right) \, dx = \frac{(d+ex)^{m+1} \log \left(c(a+bx)^p\right)}{e(m+1)} + \frac{bp(d+ex)^{m+2} \text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{b(d+ex)}{bd-ae}\right)}{e(m+1)(m+2)(bd-ae)}$$

```
[In] Int[(d + e*x)^m*Log[c*(a + b*x)^p],x]
```

```
[Out] (b*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (b*(d + e*x))/(b*
d - a*e)])/(e*(b*d - a*e)*(1 + m)*(2 + m)) + ((d + e*x)^(1 + m)*Log[c*(a +
b*x)^p])/(e*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(d+ex)^{1+m}\log\left(c(a+bx)^p\right)}{e(1+m)} - \frac{(bp)\int\frac{(d+ex)^{1+m}}{a+bx}\,dx}{e(1+m)}\\ &= \frac{bp(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{b(d+ex)}{bd-ae}\right)}{e(bd-ae)(1+m)(2+m)} + \frac{(d+ex)^{1+m}\log\left(c(a+bx)^p\right)}{e(1+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.87

$$\int (d+ex)^m\log\left(c(a+bx)^p\right)\,dx = \frac{(d+ex)^{1+m}\left(\frac{bp(d+ex)\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{b(d+ex)}{bd-ae}\right)}{(bd-ae)(2+m)}+\log\left(c(a+bx)^p\right)\right)}{e(1+m)}$$

```
[In] Integrate[(d + e*x)^m*Log[c*(a + b*x)^p],x]
[Out] ((d + e*x)^(1 + m)*((b*p*(d + e*x)*Hypergeometric2F1[1, 2 + m, 3 + m, (b*(d
 + e*x))/(b*d - a*e)])/((b*d - a*e)*(2 + m)) + Log[c*(a + b*x)^p]))/(e*(1 +
 m))
```

Maple [F]

$$\int (ex+d)^m \ln \left(c(bx+a)^p\right) dx$$

```
[In] int((e*x+d)^m*ln(c*(b*x+a)^p),x)
[Out] int((e*x+d)^m*ln(c*(b*x+a)^p),x)
```

Fricas [F]

$$\int (d+ex)^m \log \left(c(a+bx)^p\right) \, dx = \int (ex+d)^m \log \left((bx+a)^p c\right) \, dx$$

```
[In] integrate((e*x+d)^m*log(c*(b*x+a)^p),x, algorithm="fricas")
[Out] integral((e*x + d)^m*log((b*x + a)^p*c), x)
```

Sympy [F(-2)]

Exception generated.

$$\int (d+ex)^m \log \left(c(a+bx)^p\right) \, dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((e*x+d)**m*ln(c*(b*x+a)**p),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (d+ex)^m \log \left(c(a+bx)^p\right) \, dx = \int (ex+d)^m \log \left((bx+a)^p c\right) \, dx$$

```
[In] integrate((e*x+d)^m*log(c*(b*x+a)^p),x, algorithm="maxima")
[Out] (e*x + d)*(e*x + d)^m*log((b*x + a)^p)/(e*(m + 1)) + integrate((a*e*(m + 1)
*log(c) - b*d*p + (e*(m + 1)*log(c) - e*p)*b*x)*(e*x + d)^m/(b*e*(m + 1)*x
+ a*e*(m + 1)), x)
```

Giac [F]

$$\int (d+ex)^m \log (c(a+bx)^p) \, dx = \int (ex+d)^m \log ((bx+a)^p c) \, dx$$

```
[In] integrate((e*x+d)^m*log(c*(b*x+a)^p),x, algorithm="giac")
[Out] integrate((e*x + d)^m*log((b*x + a)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \log (c(a+bx)^p) \, dx = \int \ln (c\, (a+b\,x)^p) \, (d+e\,x)^m \, dx$$

```
[In] int(log(c*(a + b*x)^p)*(d + e*x)^m,x)
[Out] int(log(c*(a + b*x)^p)*(d + e*x)^m, x)
```

3.209 $\int (d+ex)^m \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 135

$$\begin{aligned}\int (d+ex)^m \log \left(c\left(a+\frac{b}{x}\right)^p\right) dx &= \frac{ap(d+ex)^{2+m}\operatorname{Hypergeometric2F1}\left(1,2+m,3+m,\frac{a(d+ex)}{ad-be}\right)}{e(ad-be)(1+m)(2+m)} \\ &\quad - \frac{p(d+ex)^{2+m}\operatorname{Hypergeometric2F1}\left(1,2+m,3+m,1+\frac{ex}{d}\right)}{de\left(2+3m+m^2\right)} \\ &\quad + \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(1+m)}\end{aligned}$$

```
[Out] a*p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],a*(e*x+d)/(a*d-b*e))/e/(a*d-b*e)
/(1+m)/(2+m)-p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],1+e*x/d)/d/e/(m^2+3*m
+2)+(e*x+d)^(1+m)*ln(c*(a+b/x)^p)/e/(1+m)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

$= \{2513, 528, 88, 67, 70\}$

$$\int (d+ex)^m \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$$
$$= \frac{(d+ex)^{m+1}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(m+1)} + \frac{ap(d+ex)^{m+2}\,\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{a(d+ex)}{ad-be}\right)}{e(m+1)(m+2)(ad-be)} - \frac{p(d+ex)^{m+2}\,\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{ex}{d}+1\right)}{de\left(m^2+3m+2\right)}$$

```
[In] Int[(d + e*x)^m*Log[c*(a + b/x)^p],x]
```

```
[Out] (a*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (a*(d + e*x))/(a*
d - b*e)])/(e*(a*d - b*e)*(1 + m)*(2 + m)) - (p*(d + e*x)^(2 + m)*Hypergeom
etric2F1[1, 2 + m, 3 + m, 1 + (e*x)/d])/(d*e*(2 + 3*m + m^2)) + ((d + e*x)^
(1 + m)*Log[c*(a + b/x)^p])/(e*(1 + m))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(1+m)}+\frac{(bp)\int\frac{(d+ex)^{1+m}}{\left(a+\frac{b}{x}\right)x^2}\,dx}{e(1+m)}\\
&= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(1+m)}+\frac{(bp)\int\frac{(d+ex)^{1+m}}{x(b+ax)}\,dx}{e(1+m)}\\
&= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(1+m)}+\frac{p\int\frac{(d+ex)^{1+m}}{x}\,dx}{e(1+m)}-\frac{(ap)\int\frac{(d+ex)^{1+m}}{b+ax}\,dx}{e(1+m)}\\
&= \frac{ap(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{a(d+ex)}{ad-be}\right)}{e(ad-be)(1+m)(2+m)}\\
&\quad-\frac{p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;1+\frac{ex}{d}\right)}{de\left(2+3m+m^2\right)}+\frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.91

$$\int(d+ex)^m\log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx$$
$$=\frac{(d+ex)^{1+m}\left(-adp(d+ex)\,\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{a(d+ex)}{ad-be}\right)+(ad-be)\left(p(d+ex)\,\text{Hyperg}\right.\right.}{de(-ad+be)(1+m)(2+m)}$$

```
[In] Integrate[(d + e*x)^m*Log[c*(a + b/x)^p],x]
```

```
[Out] ((d + e*x)^(1 + m)*(-(a*d*p*(d + e*x)*Hypergeometric2F1[1, 2 + m, 3 + m, (a
*(d + e*x))/(a*d - b*e)]) + (a*d - b*e)*(p*(d + e*x)*Hypergeometric2F1[1, 2
 + m, 3 + m, 1 + (e*x)/d] - d*(2 + m)*Log[c*(a + b/x)^p])))/(d*e*(-(a*d) +
b*e)*(1 + m)*(2 + m))
```

Maple [F]

$$\int (ex+d)^m \ln\left(c\left(a+\frac{b}{x}\right)^p\right) dx$$

```
[In] int((e*x+d)^m*ln(c*(a+b/x)^p),x)
[Out] int((e*x+d)^m*ln(c*(a+b/x)^p),x)
```

Fricas [F]

$$\int (d+ex)^m \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx = \int (ex+d)^m \log\left(\left(a+\frac{b}{x}\right)^p c\right) dx$$

```
[In] integrate((e*x+d)^m*log(c*(a+b/x)^p),x, algorithm="fricas")
[Out] integral((e*x + d)^m*log(c*((a*x + b)/x)^p), x)
```

Sympy [F]

$$\int (d+ex)^m \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx = \int (d+ex)^m \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx$$

```
[In] integrate((e*x+d)**m*ln(c*(a+b/x)**p),x)
[Out] Integral((d + e*x)**m*log(c*(a + b/x)**p), x)
```

Maxima [F]

$$\int (d+ex)^m \log\left(c\left(a+\frac{b}{x}\right)^p\right) dx = \int (ex+d)^m \log\left(\left(a+\frac{b}{x}\right)^p c\right) dx$$

```
[In] integrate((e*x+d)^m*log(c*(a+b/x)^p),x, algorithm="maxima")
[Out] (e*x + d)*(e*x + d)^m*log((a*x + b)^p)/(e*(m + 1)) - integrate(-(b*e*(m + 1
)*log(c) - a*d*p + (e*(m + 1)*log(c) - e*p)*a*x - (a*e*(m + 1)*x + b*e*(m +
 1))*log(x^p))*(e*x + d)^m/(a*e*(m + 1)*x + b*e*(m + 1)), x)
```

Giac [F]

$$\int (d+ex)^m \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \int (ex+d)^m \log\left(\left(a+\frac{b}{x}\right)^p c\right)\,dx$$

```
[In] integrate((e*x+d)^m*log(c*(a+b/x)^p),x, algorithm="giac")
[Out] integrate((e*x + d)^m*log((a + b/x)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \log\left(c\left(a+\frac{b}{x}\right)^p\right)\,dx = \int \ln\left(c\left(a+\frac{b}{x}\right)^p\right)(d+e\,x)^m\,dx$$

```
[In] int(log(c*(a + b/x)^p)*(d + e*x)^m,x)
[Out] int(log(c*(a + b/x)^p)*(d + e*x)^m, x)
```

3.210 $\int (d+ex)^m \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 257

$$
\begin{aligned}
\int (d+ex)^m \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \, dx
&= \frac{\sqrt{-a}p(d+ex)^{2+m} \operatorname{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e\left(\sqrt{-a}d-\sqrt{b}e\right)(1+m)(2+m)} \\
&\quad + \frac{\sqrt{-a}p(d+ex)^{2+m} \operatorname{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e\left(\sqrt{-a}d+\sqrt{b}e\right)(1+m)(2+m)} \\
&\quad - \frac{2p(d+ex)^{2+m} \operatorname{Hypergeometric2F1}\left(1,2+m,3+m,1+\frac{ex}{d}\right)}{de(2+3m+m^2)} \\
&\quad + \frac{(d+ex)^{1+m} \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)}
\end{aligned}
$$

```
[Out] -2*p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],1+e*x/d)/d/e/(m^2+3*m+2)+(e*x+d
)^(1+m)*ln(c*(a+b/x^2)^p)/e/(1+m)+p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],
(e*x+d)*(-a)^(1/2)/(d*(-a)^(1/2)-e*b^(1/2)))*(-a)^(1/2)/e/(1+m)/(2+m)/(d*(-
a)^(1/2)-e*b^(1/2))+p*(e*x+d)^(2+m)*hypergeom([1, 2+m],[3+m],(e*x+d)*(-a)^(
1/2)/(d*(-a)^(1/2)+e*b^(1/2)))*(-a)^(1/2)/e/(1+m)/(2+m)/(d*(-a)^(1/2)+e*b^(
1/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2513, 1584, 975, 67, 845, 70}

$$\begin{aligned}
&\int (d+ex)^m \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \, dx \\
&= \frac{(d+ex)^{m+1}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(m+1)} \\
&\quad + \frac{\sqrt{-a}p(d+ex)^{m+2}\,\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e(m+1)(m+2)\left(\sqrt{-a}d-\sqrt{b}e\right)} \\
&\quad + \frac{\sqrt{-a}p(d+ex)^{m+2}\,\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e(m+1)(m+2)\left(\sqrt{-a}d+\sqrt{b}e\right)} \\
&\quad - \frac{2p(d+ex)^{m+2}\,\text{Hypergeometric2F1}\left(1,m+2,m+3,\frac{ex}{d}+1\right)}{de\left(m^2+3m+2\right)}
\end{aligned}$$

```
[In] Int[(d + e*x)^m*Log[c*(a + b/x^2)^p],x]
```

```
[Out] (Sqrt[-a]*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sqrt[-a]*
(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/(e*(Sqrt[-a]*d - Sqrt[b]*e)*(1 + m)*(
2 + m)) + (Sqrt[-a]*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m,
(Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/(e*(Sqrt[-a]*d + Sqrt[b]*e)
*(1 + m)*(2 + m)) - (2*p*(d + e*x)^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 +
m, 1 + (e*x)/d])/(d*e*(2 + 3*m + m^2)) + ((d + e*x)^(1 + m)*Log[c*(a + b/x^
2)^p])/(e*(1 + m))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 1584

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)}+\frac{(2bp)\int\frac{(d+ex)^{1+m}}{\left(a+\frac{b}{x^2}\right)x^3}\,dx}{e(1+m)}\\
&= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)}+\frac{(2bp)\int\frac{(d+ex)^{1+m}}{x(b+ax^2)}\,dx}{e(1+m)}\\
&= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)}+\frac{(2bp)\int\left(\frac{(d+ex)^{1+m}}{bx}-\frac{ax(d+ex)^{1+m}}{b(b+ax^2)}\right)dx}{e(1+m)}\\
&= \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)}+\frac{(2p)\int\frac{(d+ex)^{1+m}}{x}\,dx}{e(1+m)}-\frac{(2ap)\int\frac{x(d+ex)^{1+m}}{b+ax^2}\,dx}{e(1+m)}\\
&= -\frac{2p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;1+\frac{ex}{d}\right)}{de\left(2+3m+m^2\right)}+\frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)}\\
&\quad-\frac{(2ap)\int\left(-\frac{\sqrt{-a}(d+ex)^{1+m}}{2a\left(\sqrt{b}-\sqrt{-a}x\right)}+\frac{\sqrt{-a}(d+ex)^{1+m}}{2a\left(\sqrt{b}+\sqrt{-a}x\right)}\right)dx}{e(1+m)}
\end{aligned}
$$

$$= -\frac{2p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;1+\frac{ex}{d}\right)}{de\,(2+3m+m^2)} + \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)}$$
$$+ \frac{\left(\sqrt{-a}p\right)\int\frac{(d+ex)^{1+m}}{\sqrt{b}-\sqrt{-a}x}\,dx}{e(1+m)} - \frac{\left(\sqrt{-a}p\right)\int\frac{(d+ex)^{1+m}}{\sqrt{b}+\sqrt{-a}x}\,dx}{e(1+m)}$$
$$= \frac{\sqrt{-a}p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e\left(\sqrt{-a}d-\sqrt{b}e\right)(1+m)(2+m)}$$
$$+ \frac{\sqrt{-a}p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e\left(\sqrt{-a}d+\sqrt{b}e\right)(1+m)(2+m)}$$
$$- \frac{2p(d+ex)^{2+m}\,{}_2F_1\left(1,2+m;3+m;1+\frac{ex}{d}\right)}{de\,(2+3m+m^2)} + \frac{(d+ex)^{1+m}\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(1+m)}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.82

$$\int (d+ex)^m \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx$$
$$= \frac{(d+ex)^{1+m}\left(\frac{p(d+ex)\left(d\left(ad-\sqrt{-a}\sqrt{b}e\right)\text{Hypergeometric2F1}\left(1,2+m,3+m,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)+d\left(ad+\sqrt{-a}\sqrt{b}e\right)\text{Hypergeometric2F1}\left(1,2+m,3+\right.\right.}{d(ad^2+be^2)(2+m)}\right.}{e(1+m)}$$

```
[In] Integrate[(d + e*x)^m*Log[c*(a + b/x^2)^p],x]
```

```
[Out] ((d + e*x)^(1 + m)*((p*(d + e*x)*(d*(a*d - Sqrt[-a]*Sqrt[b]*e)*Hypergeometr
ic2F1[1, 2 + m, 3 + m, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)] + d*(
a*d + Sqrt[-a]*Sqrt[b]*e)*Hypergeometric2F1[1, 2 + m, 3 + m, (Sqrt[-a]*(d +
 e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] - 2*(a*d^2 + b*e^2)*Hypergeometric2F1[1, 2
 + m, 3 + m, 1 + (e*x)/d]))/(d*(a*d^2 + b*e^2)*(2 + m)) + Log[c*(a + b/x^2)
^p]))/(e*(1 + m))
```

Maple [F]

$$\int (ex+d)^m \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx$$

```
[In] int((e*x+d)^m*ln(c*(a+b/x^2)^p),x)
```

```
[Out] int((e*x+d)^m*ln(c*(a+b/x^2)^p),x)
```

Fricas [F]

$$\int (d+ex)^m \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \int (ex+d)^m \log \left(\left(a+\frac{b}{x^2}\right)^p c\right) dx$$

```
[In] integrate((e*x+d)^m*log(c*(a+b/x^2)^p),x, algorithm="fricas")
[Out] integral((e*x + d)^m*log(c*((a*x^2 + b)/x^2)^p), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d+ex)^m \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \text{Timed out}$$

```
[In] integrate((e*x+d)**m*ln(c*(a+b/x**2)**p),x)
[Out] Timed out
```

Maxima [F]

$$\int (d+ex)^m \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \int (ex+d)^m \log \left(\left(a+\frac{b}{x^2}\right)^p c\right) dx$$

```
[In] integrate((e*x+d)^m*log(c*(a+b/x^2)^p),x, algorithm="maxima")
[Out] (e*p*x + d*p)*(e*x + d)^m*log(a*x^2 + b)/(e*(m + 1)) - integrate((2*a*d*p*x
 - (e*(m + 1)*log(c) - 2*e*p)*a*x^2 - b*e*(m + 1)*log(c) + 2*(a*e*(m + 1)*x
^2 + b*e*(m + 1))*log(x^p))*(e*x + d)^m/(a*e*(m + 1)*x^2 + b*e*(m + 1)), x)
```

Giac [F]

$$\int (d+ex)^m \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) dx = \int (ex+d)^m \log \left(\left(a+\frac{b}{x^2}\right)^p c\right) dx$$

```
[In] integrate((e*x+d)^m*log(c*(a+b/x^2)^p),x, algorithm="giac")
[Out] integrate((e*x + d)^m*log((a + b/x^2)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \log \left(c\left(a+\frac{b}{x^2}\right)^p\right) \, dx = \int \ln \left(c\left(a+\frac{b}{x^2}\right)^p\right) (d+e\,x)^m \, dx$$

```
[In] int(log(c*(a + b/x^2)^p)*(d + e*x)^m,x)
[Out] int(log(c*(a + b/x^2)^p)*(d + e*x)^m, x)
```

3.211 $\int (f+gx)^m \log \left(c(d+ex^n)^p\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (f+gx)^m \log \left(c(d+ex^n)^p\right) \, dx = \text{Int}\left((f+gx)^m \log \left(c(d+ex^n)^p\right),x\right)$$

```
[Out] Unintegrable((g*x+f)^m*ln(c*(d+e*x^n)^p),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (f+gx)^m \log \left(c(d+ex^n)^p\right) \, dx = \int (f+gx)^m \log \left(c(d+ex^n)^p\right) \, dx$$

```
[In] Int[(f + g*x)^m*Log[c*(d + e*x^n)^p],x]
[Out] Defer[Int][(f + g*x)^m*Log[c*(d + e*x^n)^p], x]
```

Rubi steps

$$\text{integral} = \int (f+gx)^m \log \left(c(d+ex^n)^p\right) \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (f+gx)^m \log \left(c(d+ex^n)^p\right)\, dx=\int (f+gx)^m \log \left(c(d+ex^n)^p\right)\, dx$$

```
[In] Integrate[(f + g*x)^m*Log[c*(d + e*x^n)^p],x]
[Out] Integrate[(f + g*x)^m*Log[c*(d + e*x^n)^p], x]
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(gx+f\right)^{m} \ln \left(c\left(d+e\,x^{n}\right)^{p}\right) dx$$

```
[In] int((g*x+f)^m*ln(c*(d+e*x^n)^p),x)
[Out] int((g*x+f)^m*ln(c*(d+e*x^n)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (f+gx)^m \log \left(c(d+ex^n)^p\right)\, dx=\int (gx+f)^m \log \left((ex^n+d)^p c\right)\, dx$$

```
[In] integrate((g*x+f)^m*log(c*(d+e*x^n)^p),x, algorithm="fricas")
[Out] integral((g*x + f)^m*log((e*x^n + d)^p*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int (f+gx)^m \log \left(c(d+ex^n)^p\right)\, dx=\text{Timed out}$$

```
[In] integrate((g*x+f)**m*ln(c*(d+e*x**n)**p),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 100, normalized size of antiderivative = 5.00

$$\int (f+gx)^m \log \left(c(d+ex^n)^p\right) \, dx = \int (gx+f)^m \log \left((ex^n+d)^p c\right) \, dx$$

```
[In] integrate((g*x+f)^m*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] (g*x + f)*(g*x + f)^m*log((e*x^n + d)^p)/(g*(m + 1)) + integrate((d*g*(m +
1)*x*log(c) - (e*f*n*p + (e*g*n*p - e*g*(m + 1)*log(c))*x)*x^n)*(g*x + f)^m
/(e*g*(m + 1)*x*x^n + d*g*(m + 1)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int (f+gx)^m \log \left(c(d+ex^n)^p\right) \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*x+f)^m*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{-1,[0,
0,6,3,6,0,2,2,0,1,0]%%%}+%%%{1,[0,0,6,2,6,1,2,2,0,0,1]%%%}+%%%{1,[0,0,6,2,6
,0,2,2,0,
```

Mupad [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (f+gx)^m \log \left(c(d+ex^n)^p\right) \, dx = \int \ln \left(c\,{\left(d+e\,x^n\right)}^p\right)\,{\left(f+g\,x\right)}^m \,d x$$

```
[In] int(log(c*(d + e*x^n)^p)*(f + g*x)^m,x)
```

```
[Out] int(log(c*(d + e*x^n)^p)*(f + g*x)^m, x)
```

3.212 $\int (f+gx)^3 \log\left(c(d+ex^n)^p\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 234

$$\begin{aligned}\int (f+gx)^3 \log\left(c(d+ex^n)^p\right) \, dx &= -\frac{ef^3npx^{1+n}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(1+n)} \\ &\quad - \frac{3ef^2gnpx^{2+n}\,\text{Hypergeometric2F1}\left(1,\frac{2+n}{n},2\left(1+\frac{1}{n}\right),-\frac{ex^n}{d}\right)}{2d(2+n)} \\ &\quad - \frac{efg^2npx^{3+n}\,\text{Hypergeometric2F1}\left(1,\frac{3+n}{n},2+\frac{3}{n},-\frac{ex^n}{d}\right)}{d(3+n)} \\ &\quad - \frac{eg^3npx^{4+n}\,\text{Hypergeometric2F1}\left(1,\frac{4+n}{n},2\left(1+\frac{2}{n}\right),-\frac{ex^n}{d}\right)}{4d(4+n)} \\ &\quad - \frac{f^4p\log(d+ex^n)}{4g} + \frac{(f+gx)^4\log\left(c(d+ex^n)^p\right)}{4g}\end{aligned}$$

```
[Out] -e*f^3*n*p*x^(1+n)*hypergeom([1, 1+1/n],[2+1/n],-e*x^n/d)/d/(1+n)-3/2*e*f^2
*g*n*p*x^(2+n)*hypergeom([1, (2+n)/n],[2+2/n],-e*x^n/d)/d/(2+n)-e*f*g^2*n*p
*x^(3+n)*hypergeom([1, (3+n)/n],[2+3/n],-e*x^n/d)/d/(3+n)-1/4*e*g^3*n*p*x^(
4+n)*hypergeom([1, (4+n)/n],[2+4/n],-e*x^n/d)/d/(4+n)-1/4*f^4*p*ln(d+e*x^n)
/g+1/4*(g*x+f)^4*ln(c*(d+e*x^n)^p)/g
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2513, 1858, 266, 371}

$$
\begin{aligned}
&\int (f+gx)^3\log\left(c(d+ex^n)^p\right)\,dx\\
&=\frac{(f+gx)^4\log\left(c(d+ex^n)^p\right)}{4g}-\frac{f^4p\log(d+ex^n)}{4g}\\
&-\frac{ef^3npx^{n+1}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(n+1)}\\
&-\frac{3ef^2gnpx^{n+2}\,\text{Hypergeometric2F1}\left(1,\frac{n+2}{n},2\left(1+\frac{1}{n}\right),-\frac{ex^n}{d}\right)}{2d(n+2)}\\
&-\frac{efg^2npx^{n+3}\,\text{Hypergeometric2F1}\left(1,\frac{n+3}{n},2+\frac{3}{n},-\frac{ex^n}{d}\right)}{d(n+3)}\\
&-\frac{eg^3npx^{n+4}\,\text{Hypergeometric2F1}\left(1,\frac{n+4}{n},2\left(1+\frac{2}{n}\right),-\frac{ex^n}{d}\right)}{4d(n+4)}
\end{aligned}
$$

```
[In] Int[(f + g*x)^3*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*f^3*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^
n)/d)])/(d*(1 + n))) - (3*e*f^2*g*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n
)/n, 2*(1 + n^(-1)), -((e*x^n)/d)])/(2*d*(2 + n)) - (e*f*g^2*n*p*x^(3 + n)*
Hypergeometric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/(d*(3 + n)) - (e*g^
3*n*p*x^(4 + n)*Hypergeometric2F1[1, (4 + n)/n, 2*(1 + 2/n), -((e*x^n)/d)])
/(4*d*(4 + n)) - (f^4*p*Log[d + e*x^n])/(4*g) + ((f + g*x)^4*Log[c*(d + e*x
^n)^p])/(4*g)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1858

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
```

```
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) && !IGtQ[m, 0]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(f+gx)^4\log\left(c(d+ex^n)^p\right)}{4g} - \frac{(enp)\int \frac{x^{-1+n}(f+gx)^4}{d+ex^n}\,dx}{4g}\\
&= \frac{(f+gx)^4\log\left(c(d+ex^n)^p\right)}{4g} - \frac{(enp)\int \left(\frac{f^4x^{-1+n}}{d+ex^n}+\frac{4f^3gx^n}{d+ex^n}+\frac{6f^2g^2x^{1+n}}{d+ex^n}+\frac{4fg^3x^{2+n}}{d+ex^n}+\frac{g^4x^{3+n}}{d+ex^n}\right)dx}{4g}\\
&= \frac{(f+gx)^4\log\left(c(d+ex^n)^p\right)}{4g} - \left(ef^3np\right)\int \frac{x^n}{d+ex^n}\,dx - \frac{\left(ef^4np\right)\int \frac{x^{-1+n}}{d+ex^n}\,dx}{4g}\\
&\quad - \frac{1}{2}\left(3ef^2gnp\right)\int \frac{x^{1+n}}{d+ex^n}\,dx - \left(efg^2np\right)\int \frac{x^{2+n}}{d+ex^n}\,dx - \frac{1}{4}\left(eg^3np\right)\int \frac{x^{3+n}}{d+ex^n}\,dx\\
&= -\frac{ef^3npx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)}\\
&\quad - \frac{3ef^2gnpx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(2+n)}\\
&\quad - \frac{efg^2npx^{3+n}\,{}_2F_1\left(1,\frac{3+n}{n};2+\frac{3}{n};-\frac{ex^n}{d}\right)}{d(3+n)} - \frac{eg^3npx^{4+n}\,{}_2F_1\left(1,\frac{4+n}{n};2\left(1+\frac{2}{n}\right);-\frac{ex^n}{d}\right)}{4d(4+n)}\\
&\quad - \frac{f^4p\log\left(d+ex^n\right)}{4g} + \frac{(f+gx)^4\log\left(c(d+ex^n)^p\right)}{4g}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.96

$$\int (f+gx)^3\log\left(c(d+ex^n)^p\right)\,dx = \frac{-enp\left(\frac{4f^3gx^{1+n}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(1+n)}+\frac{6f^2g^2x^{2+n}\,\text{Hypergeometric2F1}\left(1,\frac{2+n}{n},2\left(1+\frac{1}{n}\right),-\frac{ex^n}{d}\right)}{d(2+n)}+\frac{4fg^3x^{3+n}\,\text{Hyperg}}{}\right.}{}$$

```
[In] Integrate[(f + g*x)^3*Log[c*(d + e*x^n)^p],x]
```

```
[Out] (-(e*n*p*((4*f^3*g*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -
((e*x^n)/d)])/(d*(1 + n)) + (6*f^2*g^2*x^(2 + n)*Hypergeometric2F1[1, (2 +
n)/n, 2*(1 + n^(-1)), -((e*x^n)/d)])/(d*(2 + n)) + (4*f*g^3*x^(3 + n)*Hyper
geometric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/(d*(3 + n)) + (g^4*x^(4
+ n)*Hypergeometric2F1[1, (4 + n)/n, 2 + 4/n, -((e*x^n)/d)])/(d*(4 + n)) +
(f^4*Log[d + e*x^n])/(e*n))) + (f + g*x)^4*Log[c*(d + e*x^n)^p])/(4*g)
```

Maple [F]

$$\int (gx+f)^3 \ln \left(c(d+e\,x^n)^p\right) dx$$

```
[In] int((g*x+f)^3*ln(c*(d+e*x^n)^p),x)
```

```
[Out] int((g*x+f)^3*ln(c*(d+e*x^n)^p),x)
```

Fricas [F]

$$\int (f+gx)^3 \log \left(c(d+ex^n)^p\right)\, dx = \int (gx+f)^3 \log \left((ex^n+d)^p c\right)\, dx$$

```
[In] integrate((g*x+f)^3*log(c*(d+e*x^n)^p),x, algorithm="fricas")
```

```
[Out] integral((g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3)*log((e*x^n + d)^p*c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.91 (sec) , antiderivative size = 515, normalized size of antiderivative = 2.20

$$\int (f+gx)^3 \log\left(c(d+ex^n)^p\right)\,dx = -\frac{d^{-2-\frac{4}{n}}d^{1+\frac{4}{n}}eg^3px^{n+4}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{4}{n}\right)\Gamma\left(1+\frac{4}{n}\right)}{4\Gamma\left(2+\frac{4}{n}\right)} - \frac{d^{-2-\frac{4}{n}}d^{1+\frac{4}{n}}eg^3px^{n+4}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{4}{n}\right)\Gamma\left(1+\frac{4}{n}\right)}{n\Gamma\left(2+\frac{4}{n}\right)} - \frac{d^{-2-\frac{3}{n}}d^{1+\frac{3}{n}}efg^2px^{n+3}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{3}{n}\right)\Gamma\left(1+\frac{3}{n}\right)}{\Gamma\left(2+\frac{3}{n}\right)} - \frac{3d^{-2-\frac{3}{n}}d^{1+\frac{3}{n}}efg^2px^{n+3}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{3}{n}\right)\Gamma\left(1+\frac{3}{n}\right)}{n\Gamma\left(2+\frac{3}{n}\right)} - \frac{3d^{-2-\frac{2}{n}}d^{1+\frac{2}{n}}ef^2gpx^{n+2}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{2\Gamma\left(2+\frac{2}{n}\right)} - \frac{3d^{-2-\frac{2}{n}}d^{1+\frac{2}{n}}ef^2gpx^{n+2}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{n\Gamma\left(2+\frac{2}{n}\right)} + f^3x\log\left(c(d+ex^n)^p\right) + \frac{3f^2gx^2\log\left(c(d+ex^n)^p\right)}{2} + fg^2x^3\log\left(c(d+ex^n)^p\right) + \frac{g^3x^4\log\left(c(d+ex^n)^p\right)}{4} + \frac{d^{-\frac{1}{n}}d^{1+\frac{1}{n}}ee^{\frac{1}{n}}e^{-1-\frac{1}{n}}f^3px\Phi\left(\frac{dx^{-n}e^{i\pi}}{e},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{dn\Gamma\left(1+\frac{1}{n}\right)}$$

```
[In] integrate((g*x+f)**3*ln(c*(d+e*x**n)**p),x)
```

```
[Out] -d**(-2 - 4/n)*d**(1 + 4/n)*e*g**3*p*x**(n + 4)*lerchphi(e*x**n*exp_polar(I
*pi)/d, 1, 1 + 4/n)*gamma(1 + 4/n)/(4*gamma(2 + 4/n)) - d**(-2 - 4/n)*d**(1
 + 4/n)*e*g**3*p*x**(n + 4)*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 4/n)*
gamma(1 + 4/n)/(n*gamma(2 + 4/n)) - d**(-2 - 3/n)*d**(1 + 3/n)*e*f*g**2*p*x
**(n + 3)*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 3/n)*gamma(1 + 3/n)/gam
ma(2 + 3/n) - 3*d**(-2 - 3/n)*d**(1 + 3/n)*e*f*g**2*p*x**(n + 3)*lerchphi(e
*x**n*exp_polar(I*pi)/d, 1, 1 + 3/n)*gamma(1 + 3/n)/(n*gamma(2 + 3/n)) - 3*
d**(-2 - 2/n)*d**(1 + 2/n)*e*f**2*g*p*x**(n + 2)*lerchphi(e*x**n*exp_polar(
I*pi)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(2*gamma(2 + 2/n)) - 3*d**(-2 - 2/n)*d*
*(1 + 2/n)*e*f**2*g*p*x**(n + 2)*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 +
2/n)*gamma(1 + 2/n)/(n*gamma(2 + 2/n)) + f**3*x*log(c*(d + e*x**n)**p) + 3*
f**2*g*x**2*log(c*(d + e*x**n)**p)/2 + f*g**2*x**3*log(c*(d + e*x**n)**p) +
 g**3*x**4*log(c*(d + e*x**n)**p)/4 + d**(1 + 1/n)*e*e**(1/n)*e**(-1 - 1/n)
```

```
*f**3*p*x*lerchphi(d*exp_polar(I*pi)/(e*x**n), 1, exp_polar(I*pi)/n)*gamma(
1/n)/(d*d**(1/n)*n*gamma(1 + 1/n))
```

Maxima [F]

$$\int (f+gx)^3 \log \left(c(d+ex^n)^p\right) \, dx = \int (gx+f)^3 \log \left((ex^n+d)^p c\right) \, dx$$

```
[In] integrate((g*x+f)^3*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] -1/16*(g^3*n*p - 4*g^3*log(c))*x^4 - 1/3*(f*g^2*n*p - 3*f*g^2*log(c))*x^3 -
 3/4*(f^2*g*n*p - 2*f^2*g*log(c))*x^2 - (f^3*n*p - f^3*log(c))*x + 1/4*(g^3
*x^4 + 4*f*g^2*x^3 + 6*f^2*g*x^2 + 4*f^3*x)*log((e*x^n + d)^p) + integrate(
1/4*(d*g^3*n*p*x^3 + 4*d*f*g^2*n*p*x^2 + 6*d*f^2*g*n*p*x + 4*d*f^3*n*p)/(e*
x^n + d), x)
```

Giac [F]

$$\int (f+gx)^3 \log \left(c(d+ex^n)^p\right) \, dx = \int (gx+f)^3 \log \left((ex^n+d)^p c\right) \, dx$$

```
[In] integrate((g*x+f)^3*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^3*log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3 \log \left(c(d+ex^n)^p\right) \, dx = \int \ln \left(c \left(d+e\,x^n\right)^p\right) \left(f+g\,x\right)^3 \, dx$$

```
[In] int(log(c*(d + e*x^n)^p)*(f + g*x)^3,x)
```

```
[Out] int(log(c*(d + e*x^n)^p)*(f + g*x)^3, x)
```

3.213 $\int (f+gx)^2 \log (c(d+ex^n)^p) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 181

$$\begin{aligned}\int (f+gx)^2 \log (c(d+ex^n)^p) \, dx &= -\frac{ef^2npx^{1+n} \text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(1+n)} \\ &\quad -\frac{efgnpx^{2+n} \text{Hypergeometric2F1}\left(1,\frac{2+n}{n},2\left(1+\frac{1}{n}\right),-\frac{ex^n}{d}\right)}{d(2+n)} \\ &\quad -\frac{eg^2npx^{3+n} \text{Hypergeometric2F1}\left(1,\frac{3+n}{n},2+\frac{3}{n},-\frac{ex^n}{d}\right)}{3d(3+n)} \\ &\quad -\frac{f^3p\log (d+ex^n)}{3g}+\frac{(f+gx)^3\log (c(d+ex^n)^p)}{3g}\end{aligned}$$

```
[Out] -e*f^2*n*p*x^(1+n)*hypergeom([1, 1+1/n],[2+1/n],-e*x^n/d)/d/(1+n)-e*f*g*n*p
*x^(2+n)*hypergeom([1, (2+n)/n],[2+2/n],-e*x^n/d)/d/(2+n)-1/3*e*g^2*n*p*x^(
3+n)*hypergeom([1, (3+n)/n],[2+3/n],-e*x^n/d)/d/(3+n)-1/3*f^3*p*ln(d+e*x^n)
/g+1/3*(g*x+f)^3*ln(c*(d+e*x^n)^p)/g
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used

= {2513, 1858, 266, 371}

$$\int (f+gx)^2\log\left(c(d+ex^n)^p\right)\,dx = \frac{(f+gx)^3\log\left(c(d+ex^n)^p\right)}{3g} - \frac{f^3p\log\left(d+ex^n\right)}{3g} - \frac{ef^2npx^{n+1}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(n+1)} - \frac{efgnpx^{n+2}\,\text{Hypergeometric2F1}\left(1,\frac{n+2}{n},2\left(1+\frac{1}{n}\right),-\frac{ex^n}{d}\right)}{d(n+2)} - \frac{eg^2npx^{n+3}\,\text{Hypergeometric2F1}\left(1,\frac{n+3}{n},2+\frac{3}{n},-\frac{ex^n}{d}\right)}{3d(n+3)}$$

```
[In] Int[(f + g*x)^2*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*f^2*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^
n)/d)])/(d*(1 + n))) - (e*f*g*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/n,
 2*(1 + n^(-1)), -((e*x^n)/d)])/(d*(2 + n)) - (e*g^2*n*p*x^(3 + n)*Hypergeo
metric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/(3*d*(3 + n)) - (f^3*p*Log[
d + e*x^n])/(3*g) + ((f + g*x)^3*Log[c*(d + e*x^n)^p])/(3*g)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1858

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$\text{integral} = \frac{(f+gx)^3\log\left(c(d+ex^n)^p\right)}{3g} - \frac{(enp)\int \frac{x^{-1+n}(f+gx)^3}{d+ex^n}\,dx}{3g}$$
$$= \frac{(f+gx)^3\log\left(c(d+ex^n)^p\right)}{3g} - \frac{(enp)\int\left(\frac{f^3x^{-1+n}}{d+ex^n}+\frac{3f^2gx^n}{d+ex^n}+\frac{3fg^2x^{1+n}}{d+ex^n}+\frac{g^3x^{2+n}}{d+ex^n}\right)dx}{3g}$$
$$= \frac{(f+gx)^3\log\left(c(d+ex^n)^p\right)}{3g} - \left(ef^2np\right)\int\frac{x^n}{d+ex^n}\,dx - \frac{(ef^3np)\int\frac{x^{-1+n}}{d+ex^n}\,dx}{3g} - (efgnp)\int\frac{x^{1+n}}{d+ex^n}\,dx - \frac{1}{3}\left(eg^2np\right)\int\frac{x^{2+n}}{d+ex^n}\,dx$$
$$= -\frac{ef^2npx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)} - \frac{efgnpx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{d(2+n)} - \frac{eg^2npx^{3+n}\,{}_2F_1\left(1,\frac{3+n}{n};2+\frac{3}{n};-\frac{ex^n}{d}\right)}{3d(3+n)} - \frac{f^3p\log(d+ex^n)}{3g} + \frac{(f+gx)^3\log\left(c(d+ex^n)^p\right)}{3g}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.98

$$\int (f+gx)^2\log\left(c(d+ex^n)^p\right)\,dx = \frac{-enp\left(\frac{3f^2gx^{1+n}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(1+n)}+\frac{3fg^2x^{2+n}\,\text{Hypergeometric2F1}\left(1,\frac{2+n}{n},2\left(1+\frac{1}{n}\right),-\frac{ex^n}{d}\right)}{d(2+n)}+\frac{g^3x^{3+n}\,\text{Hypergeom}}{}\right.}{3g}$$

```
[In] Integrate[(f + g*x)^2*Log[c*(d + e*x^n)^p],x]
```

```
[Out] (-(e*n*p*((3*f^2*g*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -
((e*x^n)/d)])/(d*(1 + n)) + (3*f*g^2*x^(2 + n)*Hypergeometric2F1[1, (2 + n)
/n, 2*(1 + n^(-1)), -((e*x^n)/d)])/(d*(2 + n)) + (g^3*x^(3 + n)*Hypergeomet
ric2F1[1, (3 + n)/n, 2 + 3/n, -((e*x^n)/d)])/(d*(3 + n)) + (f^3*Log[d + e*x
^n])/(e*n))) + (f + g*x)^3*Log[c*(d + e*x^n)^p])/(3*g)
```

Maple [F]

$$\int (gx+f)^2 \ln\left(c(d+e\,x^n)^p\right) dx$$

[In] int((g*x+f)^2*ln(c*(d+e*x^n)^p),x)

[Out] int((g*x+f)^2*ln(c*(d+e*x^n)^p),x)

Fricas [F]

$$\int (f+gx)^2 \log\left(c(d+ex^n)^p\right)\, dx = \int (gx+f)^2 \log\left((ex^n+d)^p c\right)\, dx$$

[In] integrate((g*x+f)^2*log(c*(d+e*x^n)^p),x, algorithm="fricas")

[Out] integral((g^2*x^2 + 2*f*g*x + f^2)*log((e*x^n + d)^p*c), x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.09 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.99

$$\begin{aligned}\int (f+gx)^2 \log\left(c(d+ex^n)^p\right)\, dx = &-\frac{d^{-2-\frac{3}{n}} d^{1+\frac{3}{n}} e g^2 p x^{n+3} \Phi\left(\frac{ex^n e^{i\pi}}{d}, 1, 1+\frac{3}{n}\right) \Gamma\left(1+\frac{3}{n}\right)}{3\Gamma\left(2+\frac{3}{n}\right)} \\ &- \frac{d^{-2-\frac{3}{n}} d^{1+\frac{3}{n}} e g^2 p x^{n+3} \Phi\left(\frac{ex^n e^{i\pi}}{d}, 1, 1+\frac{3}{n}\right) \Gamma\left(1+\frac{3}{n}\right)}{n\Gamma\left(2+\frac{3}{n}\right)} \\ &- \frac{d^{-2-\frac{2}{n}} d^{1+\frac{2}{n}} e f g p x^{n+2} \Phi\left(\frac{ex^n e^{i\pi}}{d}, 1, 1+\frac{2}{n}\right) \Gamma\left(1+\frac{2}{n}\right)}{\Gamma\left(2+\frac{2}{n}\right)} \\ &- \frac{2d^{-2-\frac{2}{n}} d^{1+\frac{2}{n}} e f g p x^{n+2} \Phi\left(\frac{ex^n e^{i\pi}}{d}, 1, 1+\frac{2}{n}\right) \Gamma\left(1+\frac{2}{n}\right)}{n\Gamma\left(2+\frac{2}{n}\right)} \\ &+ f^2 x \log\left(c(d+ex^n)^p\right) + f g x^2 \log\left(c(d+ex^n)^p\right) \\ &+ \frac{g^2 x^3 \log\left(c(d+ex^n)^p\right)}{3} \\ &+ \frac{d^{-\frac{1}{n}} d^{1+\frac{1}{n}} e e^{\frac{1}{n}} e^{-1-\frac{1}{n}} f^2 p x \Phi\left(\frac{dx^{-n} e^{i\pi}}{e}, 1, \frac{e^{i\pi}}{n}\right) \Gamma\left(\frac{1}{n}\right)}{dn\Gamma\left(1+\frac{1}{n}\right)}\end{aligned}$$

[In] integrate((g*x+f)**2*ln(c*(d+e*x**n)**p),x)

[Out] -d**(-2 - 3/n)*d**(1 + 3/n)*e*g**2*p*x**(n + 3)*lerchphi(e*x**n*exp_polar(I
*pi)/d, 1, 1 + 3/n)*gamma(1 + 3/n)/(3*gamma(2 + 3/n)) - d**(-2 - 3/n)*d**(1

```
 + 3/n)*e*g**2*p*x**(n + 3)*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 3/n)*
gamma(1 + 3/n)/(n*gamma(2 + 3/n)) - d**(-2 - 2/n)*d**(1 + 2/n)*e*f*g*p*x**(
n + 2)*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/gamma(
2 + 2/n) - 2*d**(-2 - 2/n)*d**(1 + 2/n)*e*f*g*p*x**(n + 2)*lerchphi(e*x**n*
exp_polar(I*pi)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(n*gamma(2 + 2/n)) + f**2*x*l
og(c*(d + e*x**n)**p) + f*g*x**2*log(c*(d + e*x**n)**p) + g**2*x**3*log(c*(
d + e*x**n)**p)/3 + d**(1 + 1/n)*e*e**(1/n)*e**(-1 - 1/n)*f**2*p*x*lerchphi
(d*exp_polar(I*pi)/(e*x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/(d*d**(1/n)*n
*gamma(1 + 1/n))
```

Maxima [F]

$$\int (f+gx)^2 \log\left(c(d+ex^n)^p\right)\,dx = \int (gx+f)^2 \log\left((ex^n+d)^p c\right)\,dx$$

```
[In] integrate((g*x+f)^2*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] -1/9*(g^2*n*p - 3*g^2*log(c))*x^3 - 1/2*(f*g*n*p - 2*f*g*log(c))*x^2 - (f^2
*n*p - f^2*log(c))*x + 1/3*(g^2*x^3 + 3*f*g*x^2 + 3*f^2*x)*log((e*x^n + d)^
p) + integrate(1/3*(d*g^2*n*p*x^2 + 3*d*f*g*n*p*x + 3*d*f^2*n*p)/(e*x^n + d
), x)
```

Giac [F]

$$\int (f+gx)^2 \log\left(c(d+ex^n)^p\right)\,dx = \int (gx+f)^2 \log\left((ex^n+d)^p c\right)\,dx$$

```
[In] integrate((g*x+f)^2*log(c*(d+e*x^n)^p),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^2*log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2 \log\left(c(d+ex^n)^p\right)\,dx = \int \ln\left(c\left(d+e\,x^n\right)^p\right)\,\left(f+g\,x\right)^2\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)*(f + g*x)^2,x)
```

```
[Out] int(log(c*(d + e*x^n)^p)*(f + g*x)^2, x)
```

3.214 $\int (f+gx)\log(c(d+ex^n)^p)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 132

$$\int (f+gx)\log(c(d+ex^n)^p)\,dx = -\frac{efnpx^{1+n}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(1+n)} - \frac{egnpx^{2+n}\,\text{Hypergeometric2F1}\left(1,\frac{2+n}{n},2\left(1+\frac{1}{n}\right),-\frac{ex^n}{d}\right)}{2d(2+n)} - \frac{f^2p\log(d+ex^n)}{2g} + \frac{(f+gx)^2\log(c(d+ex^n)^p)}{2g}$$

```
[Out] -e*f*n*p*x^(1+n)*hypergeom([1, 1+1/n],[2+1/n],-e*x^n/d)/d/(1+n)-1/2*e*g*n*p
*x^(2+n)*hypergeom([1, (2+n)/n],[2+2/n],-e*x^n/d)/d/(2+n)-1/2*f^2*p*ln(d+e*
x^n)/g+1/2*(g*x+f)^2*ln(c*(d+e*x^n)^p)/g
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2513, 1858, 266, 371}

$$\int (f+gx)\log(c(d+ex^n)^p)\,dx = \frac{(f+gx)^2\log(c(d+ex^n)^p)}{2g} - \frac{f^2p\log(d+ex^n)}{2g} - \frac{efnpx^{n+1}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(n+1)} - \frac{egnpx^{n+2}\,\text{Hypergeometric2F1}\left(1,\frac{n+2}{n},2\left(1+\frac{1}{n}\right),-\frac{ex^n}{d}\right)}{2d(n+2)}$$

```
[In] Int[(f + g*x)*Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*f*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)
/d)])/(d*(1 + n))) - (e*g*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/n, 2*(
1 + n^(-1)), -((e*x^n)/d)])/(2*d*(2 + n)) - (f^2*p*Log[d + e*x^n])/(2*g) +
((f + g*x)^2*Log[c*(d + e*x^n)^p])/(2*g)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1858

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(f+gx)^2\log\left(c(d+ex^n)^p\right)}{2g} - \frac{(enp)\int\frac{x^{-1+n}(f+gx)^2}{d+ex^n}\,dx}{2g} \\
&= \frac{(f+gx)^2\log\left(c(d+ex^n)^p\right)}{2g} - \frac{(enp)\int\left(\frac{f^2x^{-1+n}}{d+ex^n}+\frac{2fgx^n}{d+ex^n}+\frac{g^2x^{1+n}}{d+ex^n}\right)dx}{2g} \\
&= \frac{(f+gx)^2\log\left(c(d+ex^n)^p\right)}{2g} - (efnp)\int\frac{x^n}{d+ex^n}\,dx \\
&\quad - \frac{(ef^2np)\int\frac{x^{-1+n}}{d+ex^n}\,dx}{2g} - \frac{1}{2}(egnp)\int\frac{x^{1+n}}{d+ex^n}\,dx \\
&= -\frac{efnpx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)} - \frac{egnpx^{2+n}\,{}_2F_1\left(1,\frac{2+n}{n};2\left(1+\frac{1}{n}\right);-\frac{ex^n}{d}\right)}{2d(2+n)} \\
&\quad - \frac{f^2p\log(d+ex^n)}{2g} + \frac{(f+gx)^2\log\left(c(d+ex^n)^p\right)}{2g}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.98

$$\int (f+gx)\log\left(c(d+ex^n)^p\right)\,dx=-\frac{efnpx^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{n},1+\frac{1+n}{n},-\frac{ex^n}{d}\right)}{d(1+n)}-\frac{egnpx^{2+n}\,\text{Hypergeometric2F1}\left(1,\frac{2+n}{n},1+\frac{2+n}{n},-\frac{ex^n}{d}\right)}{2d(2+n)}+fx\log\left(c(d+ex^n)^p\right)+\frac{1}{2}gx^2\log\left(c(d+ex^n)^p\right)$$

```
[In] Integrate[(f + g*x)*Log[c*(d + e*x^n)^p],x]
[Out] -((e*f*n*p*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/n, 1 + (1 + n)/n, -((e*x^
n)/d)])/(d*(1 + n))) - (e*g*n*p*x^(2 + n)*Hypergeometric2F1[1, (2 + n)/n, 1
 + (2 + n)/n, -((e*x^n)/d)])/(2*d*(2 + n)) + f*x*Log[c*(d + e*x^n)^p] + (g*
x^2*Log[c*(d + e*x^n)^p])/2
```

Maple [F]

$$\int (gx+f)\ln\left(c(d+e\,x^n)^p\right)dx$$

```
[In] int((g*x+f)*ln(c*(d+e*x^n)^p),x)
[Out] int((g*x+f)*ln(c*(d+e*x^n)^p),x)
```

Fricas [F]

$$\int (f+gx)\log\left(c(d+ex^n)^p\right)\,dx=\int (gx+f)\log\left((ex^n+d)^pc\right)\,dx$$

```
[In] integrate((g*x+f)*log(c*(d+e*x^n)^p),x, algorithm="fricas")
[Out] integral((g*x + f)*log((e*x^n + d)^p*c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.78 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.62

$$\int (f+gx)\log\left(c(d+ex^n)^p\right)\,dx = -\frac{d^{-2-\frac{2}{n}}d^{1+\frac{2}{n}}egpx^{n+2}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{2\Gamma\left(2+\frac{2}{n}\right)} - \frac{d^{-2-\frac{2}{n}}d^{1+\frac{2}{n}}egpx^{n+2}\Phi\left(\frac{ex^ne^{i\pi}}{d},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{n\Gamma\left(2+\frac{2}{n}\right)} + fx\log\left(c(d+ex^n)^p\right) + \frac{gx^2\log\left(c(d+ex^n)^p\right)}{2} + \frac{d^{-\frac{1}{n}}d^{1+\frac{1}{n}}ee^{\frac{1}{n}}e^{-1-\frac{1}{n}}fpx\Phi\left(\frac{dx^{-n}e^{i\pi}}{e},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{dn\Gamma\left(1+\frac{1}{n}\right)}$$

```
[In] integrate((g*x+f)*ln(c*(d+e*x**n)**p),x)
```

```
[Out] -d**(-2 - 2/n)*d**(1 + 2/n)*e*g*p*x**(n + 2)*lerchphi(e*x**n*exp_polar(I*pi
)/d, 1, 1 + 2/n)*gamma(1 + 2/n)/(2*gamma(2 + 2/n)) - d**(-2 - 2/n)*d**(1 +
2/n)*e*g*p*x**(n + 2)*lerchphi(e*x**n*exp_polar(I*pi)/d, 1, 1 + 2/n)*gamma(
1 + 2/n)/(n*gamma(2 + 2/n)) + f*x*log(c*(d + e*x**n)**p) + g*x**2*log(c*(d
+ e*x**n)**p)/2 + d**(1 + 1/n)*e*e**(1/n)*e**(-1 - 1/n)*f*p*x*lerchphi(d*ex
p_polar(I*pi)/(e*x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/(d*d**(1/n)*n*gamm
a(1 + 1/n))
```

Maxima [F]

$$\int (f+gx)\log\left(c(d+ex^n)^p\right)\,dx = \int (gx+f)\log\left((ex^n+d)^pc\right)\,dx$$

```
[In] integrate((g*x+f)*log(c*(d+e*x^n)^p),x, algorithm="maxima")
```

```
[Out] -1/4*(g*n*p - 2*g*log(c))*x^2 - (f*n*p - f*log(c))*x + 1/2*(g*x^2 + 2*f*x)*
log((e*x^n + d)^p) + integrate(1/2*(d*g*n*p*x + 2*d*f*n*p)/(e*x^n + d), x)
```

Giac [F]

$$\int (f+gx)\log\left(c(d+ex^n)^p\right)\,dx=\int (gx+f)\log\left((ex^n+d)^pc\right)\,dx$$

```
[In] integrate((g*x+f)*log(c*(d+e*x^n)^p),x, algorithm="giac")
[Out] integrate((g*x + f)*log((e*x^n + d)^p*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx)\log\left(c(d+ex^n)^p\right)\,dx=\int \ln\left(c\,(d+e\,x^n)^p\right)\,(f+g\,x)\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)*(f + g*x),x)
[Out] int(log(c*(d + e*x^n)^p)*(f + g*x), x)
```

3.215 $\int \log \left(c(d+ex^n)^p\right) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 54

$$\int \log \left(c(d+ex^n)^p\right) \, dx = -\frac{enpx^{1+n} \operatorname{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(1+n)} + x\log \left(c(d+ex^n)^p\right)$$

```
[Out] -e*n*p*x^(1+n)*hypergeom([1, 1+1/n],[2+1/n],-e*x^n/d)/d/(1+n)+x*ln(c*(d+e*x
^n)^p)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2498, 371}

$$\int \log \left(c(d+ex^n)^p\right) \, dx = x\log \left(c(d+ex^n)^p\right) - \frac{enpx^{n+1} \operatorname{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(n+1)}$$

```
[In] Int[Log[c*(d + e*x^n)^p],x]
```

```
[Out] -((e*n*p*x^(1 + n)*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)/d
)])/(d*(1 + n))) + x*Log[c*(d + e*x^n)^p]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\log\left(c(d+ex^n)^p\right) - (enp)\int \frac{x^n}{d+ex^n}\,dx\\ &= -\frac{enpx^{1+n}\,{}_2F_1\left(1,1+\frac{1}{n};2+\frac{1}{n};-\frac{ex^n}{d}\right)}{d(1+n)} + x\log\left(c(d+ex^n)^p\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \log\left(c(d+ex^n)^p\right)\,dx = x\left(-\frac{enpx^n\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{n},2+\frac{1}{n},-\frac{ex^n}{d}\right)}{d(1+n)} + \log\left(c(d+ex^n)^p\right)\right)$$

```
[In] Integrate[Log[c*(d + e*x^n)^p],x]
[Out] x*(-((e*n*p*x^n*Hypergeometric2F1[1, 1 + n^(-1), 2 + n^(-1), -((e*x^n)/d)])
/(d*(1 + n))) + Log[c*(d + e*x^n)^p])
```

Maple [F]

$$\int \ln\left(c(d+e\,x^n)^p\right)dx$$

```
[In] int(ln(c*(d+e*x^n)^p),x)
[Out] int(ln(c*(d+e*x^n)^p),x)
```

Fricas [F]

$$\int \log \left(c(d+ex^n)^p\right) \, dx = \int \log \left((ex^n+d)^p c\right) \, dx$$

[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="fricas")

[Out] integral(log((e*x^n + d)^p*c), x)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.41

$$\int \log \left(c(d+ex^n)^p\right) \, dx = x \log \left(c(d+ex^n)^p\right) + \frac{d^{-\frac{1}{n}} d^{1+\frac{1}{n}} e e^{\frac{1}{n}} e^{-1-\frac{1}{n}} p x \Phi\left(\frac{dx^{-n}e^{i\pi}}{e}, 1, \frac{e^{i\pi}}{n}\right) \Gamma\left(\frac{1}{n}\right)}{dn\Gamma\left(1+\frac{1}{n}\right)}$$

[In] integrate(ln(c*(d+e*x**n)**p),x)

[Out] x*log(c*(d + e*x**n)**p) + d**(1 + 1/n)*e*e**(1/n)*e**(-1 - 1/n)*p*x*lerchphi(d*exp_polar(I*pi)/(e*x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/(d*d**(1/n)*n*gamma(1 + 1/n))

Maxima [F]

$$\int \log \left(c(d+ex^n)^p\right) \, dx = \int \log \left((ex^n+d)^p c\right) \, dx$$

[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="maxima")

[Out] d*n*p*integrate(1/(e*x^n + d), x) - (n*p - log(c))*x + x*log((e*x^n + d)^p)

Giac [F]

$$\int \log \left(c(d+ex^n)^p\right) \, dx = \int \log \left((ex^n+d)^p c\right) \, dx$$

[In] integrate(log(c*(d+e*x^n)^p),x, algorithm="giac")

[Out] integrate(log((e*x^n + d)^p*c), x)

Mupad [F(-1)]

Timed out.

$$\int \log \left(c(d+ex^n)^p\right)\, dx = \int \ln \left(c\,(d+e\,x^n)^p\right)\, dx$$

```
[In] int(log(c*(d + e*x^n)^p),x)
[Out] int(log(c*(d + e*x^n)^p), x)
```

3.216 $\int \frac{\log(c(d+ex^n)^p)}{f+gx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{f+gx}\,dx = \mathrm{Int}\left(\frac{\log\left(c(d+ex^n)^p\right)}{f+gx},x\right)$$

`[Out] Unintegrable(ln(c*(d+e*x^n)^p)/(g*x+f),x)`

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{f+gx}\,dx = \int \frac{\log\left(c(d+ex^n)^p\right)}{f+gx}\,dx$$

`[In] Int[Log[c*(d + e*x^n)^p]/(f + g*x),x]`

`[Out] Defer[Int][Log[c*(d + e*x^n)^p]/(f + g*x), x]`

Rubi steps

$$\text{integral} = \int \frac{\log\left(c(d+ex^n)^p\right)}{f+gx}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log (c(d+ex^n)^p)}{f+gx}\,dx=\int \frac{\log (c(d+ex^n)^p)}{f+gx}\,dx$$

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x),x]
[Out] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x), x]
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\ln (c(d+e\,x^n)^p)}{gx+f}dx$$

```
[In] int(ln(c*(d+e*x^n)^p)/(g*x+f),x)
[Out] int(ln(c*(d+e*x^n)^p)/(g*x+f),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log (c(d+ex^n)^p)}{f+gx}\,dx=\int \frac{\log ((ex^n+d)^pc)}{gx+f}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f),x, algorithm="fricas")
[Out] integral(log((e*x^n + d)^p*c)/(g*x + f), x)
```

Sympy [N/A]

Not integrable

Time = 3.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\log (c(d+ex^n)^p)}{f+gx}\,dx=\int \frac{\log (c(d+ex^n)^p)}{f+gx}\,dx$$

```
[In] integrate(ln(c*(d+e*x**n)**p)/(g*x+f),x)
[Out] Integral(log(c*(d + e*x**n)**p)/(f + g*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log(c(d+ex^n)^p)}{f+gx}\,dx = \int \frac{\log((ex^n+d)^pc)}{gx+f}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f),x, algorithm="maxima")
[Out] integrate(log((e*x^n + d)^p*c)/(g*x + f), x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log(c(d+ex^n)^p)}{f+gx}\,dx = \int \frac{\log((ex^n+d)^pc)}{gx+f}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f),x, algorithm="giac")
[Out] integrate(log((e*x^n + d)^p*c)/(g*x + f), x)
```

Mupad [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log(c(d+ex^n)^p)}{f+gx}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)}{f+g\,x}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)/(f + g*x),x)
[Out] int(log(c*(d + e*x^n)^p)/(f + g*x), x)
```

3.217 $\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{(f+gx)^2}\,dx = \mathrm{Int}\left(\frac{\log\left(c(d+ex^n)^p\right)}{(f+gx)^2},x\right)$$

```
[Out] Unintegrable(ln(c*(d+e*x^n)^p)/(g*x+f)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{(f+gx)^2}\,dx = \int \frac{\log\left(c(d+ex^n)^p\right)}{(f+gx)^2}\,dx$$

```
[In] Int[Log[c*(d + e*x^n)^p]/(f + g*x)^2,x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^n)^p]/(f + g*x)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\log\left(c(d+ex^n)^p\right)}{(f+gx)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^2}\,dx = \int \frac{\log(c(d+ex^n)^p)}{(f+gx)^2}\,dx$$

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x)^2,x]
[Out] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x)^2, x]
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\ln(c(d+e\,x^n)^p)}{(gx+f)^2}dx$$

```
[In] int(ln(c*(d+e*x^n)^p)/(g*x+f)^2,x)
[Out] int(ln(c*(d+e*x^n)^p)/(g*x+f)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^2}\,dx = \int \frac{\log((ex^n+d)^p c)}{(gx+f)^2}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^2,x, algorithm="fricas")
[Out] integral(log((e*x^n + d)^p*c)/(g^2*x^2 + 2*f*g*x + f^2), x)
```

Sympy [N/A]

Not integrable

Time = 43.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^2}\,dx = \int \frac{\log(c(d+ex^n)^p)}{(f+gx)^2}\,dx$$

```
[In] integrate(ln(c*(d+e*x**n)**p)/(g*x+f)**2,x)
[Out] Integral(log(c*(d + e*x**n)**p)/(f + g*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.25

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{(f+gx)^2}\,dx = \int \frac{\log \left((ex^n+d)^p c\right)}{(gx+f)^2}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^2,x, algorithm="maxima")
```

```
[Out] -d*n*p*integrate(1/(d*g^2*x^2 + d*f*g*x + (e*g^2*x^2 + e*f*g*x)*x^n), x) -
n*p*log(g*x + f)/(f*g) - (f*log((e*x^n + d)^p) + f*log(c) - (g*n*p*x + f*n*
p)*log(x))/(f*g^2*x + f^2*g)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{(f+gx)^2}\,dx = \int \frac{\log \left((ex^n+d)^p c\right)}{(gx+f)^2}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/(g*x + f)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{(f+gx)^2}\,dx = \int \frac{\ln \left(c\,(d+e\,x^n)^p\right)}{(f+g\,x)^2}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)/(f + g*x)^2,x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/(f + g*x)^2, x)
```

3.218 $\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{(f+gx)^3} dx = \operatorname{Int}\left(\frac{\log\left(c(d+ex^n)^p\right)}{(f+gx)^3}, x\right)$$

```
[Out] Unintegrable(ln(c*(d+e*x^n)^p)/(g*x+f)^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{(f+gx)^3} dx = \int \frac{\log\left(c(d+ex^n)^p\right)}{(f+gx)^3} dx$$

```
[In] Int[Log[c*(d + e*x^n)^p]/(f + g*x)^3,x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^n)^p]/(f + g*x)^3, x]
```

Rubi steps

$$\text{integral} = \int \frac{\log\left(c(d+ex^n)^p\right)}{(f+gx)^3} dx$$

Mathematica [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^3}\,dx = \int \frac{\log(c(d+ex^n)^p)}{(f+gx)^3}\,dx$$

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x)^3,x]
[Out] Integrate[Log[c*(d + e*x^n)^p]/(f + g*x)^3, x]
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\ln(c(d+e\,x^n)^p)}{(gx+f)^3}dx$$

```
[In] int(ln(c*(d+e*x^n)^p)/(g*x+f)^3,x)
[Out] int(ln(c*(d+e*x^n)^p)/(g*x+f)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^3}\,dx = \int \frac{\log((ex^n+d)^pc)}{(gx+f)^3}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^3,x, algorithm="fricas")
[Out] integral(log((e*x^n + d)^p*c)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c(d+ex^n)^p)}{(f+gx)^3}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(d+e*x**n)**p)/(g*x+f)**3,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 174, normalized size of antiderivative = 8.70

$$\int \frac{\log (c(d+ex^n)^p)}{(f+gx)^3}\,dx = \int \frac{\log ((ex^n+d)^p c)}{(gx+f)^3}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^3,x, algorithm="maxima")
```

```
[Out] -d*n*p*integrate(1/2/(d*g^3*x^3 + 2*d*f*g^2*x^2 + d*f^2*g*x + (e*g^3*x^3 +
2*e*f*g^2*x^2 + e*f^2*g*x)*x^n), x) + 1/2*(f*g*n*p*x + f^2*n*p - f^2*log((e
*x^n + d)^p) - f^2*log(c) + (g^2*n*p*x^2 + 2*f*g*n*p*x + f^2*n*p)*log(x))/(
f^2*g^3*x^2 + 2*f^3*g^2*x + f^4*g) - 1/2*n*p*log(g*x + f)/(f^2*g)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log (c(d+ex^n)^p)}{(f+gx)^3}\,dx = \int \frac{\log ((ex^n+d)^p c)}{(gx+f)^3}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/(g*x + f)^3, x)
```

Mupad [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\log (c(d+ex^n)^p)}{(f+gx)^3}\,dx = \int \frac{\ln \left(c\,{\left(d+e\,x^n\right)}^p\right)}{{\left(f+g\,x\right)}^3}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)/(f + g*x)^3,x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/(f + g*x)^3, x)
```

3.219 $\int \frac{x^3 \log(c(a+bx)^p)}{d+ex} dx$

Optimal result

Integrand size = 21, antiderivative size = 250

$$\begin{aligned}\int \frac{x^3 \log\left(c(a+bx)^p\right)}{d+ex} dx = & -\frac{d^2px}{e^3} - \frac{adpx}{2be^2} - \frac{a^2px}{3b^2e} + \frac{dpx^2}{4e^2} + \frac{apx^2}{6be} - \frac{px^3}{9e} \\ & + \frac{a^2dp\log(a+bx)}{2b^2e^2} + \frac{a^3p\log(a+bx)}{3b^3e} - \frac{dx^2\log\left(c(a+bx)^p\right)}{2e^2} \\ & + \frac{x^3\log\left(c(a+bx)^p\right)}{3e} + \frac{d^2(a+bx)\log\left(c(a+bx)^p\right)}{be^3} \\ & - \frac{d^3\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^4} - \frac{d^3p\,\mathrm{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e^4}\end{aligned}$$

```
[Out] -d^2*p*x/e^3-1/2*a*d*p*x/b/e^2-1/3*a^2*p*x/b^2/e+1/4*d*p*x^2/e^2+1/6*a*p*x^
2/b/e-1/9*p*x^3/e+1/2*a^2*d*p*ln(b*x+a)/b^2/e^2+1/3*a^3*p*ln(b*x+a)/b^3/e-1
/2*d*x^2*ln(c*(b*x+a)^p)/e^2+1/3*x^3*ln(c*(b*x+a)^p)/e+d^2*(b*x+a)*ln(c*(b*
x+a)^p)/b/e^3-d^3*ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d))/e^4-d^3*p*polylo
g(2,-e*(b*x+a)/(-a*e+b*d))/e^4
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used

$= \{45, 2463, 2436, 2332, 2442, 2441, 2440, 2438\}$

$$\int \frac{x^3 \log (c(a+bx)^p)}{d+ex}\,dx = \frac{a^3 p \log(a+bx)}{3b^3 e} + \frac{a^2 dp \log(a+bx)}{2b^2 e^2} - \frac{a^2 px}{3b^2 e} - \frac{d^3 \log (c(a+bx)^p) \log \left(\frac{b(d+ex)}{bd-ae}\right)}{e^4} + \frac{d^2 (a+bx) \log (c(a+bx)^p)}{be^3} - \frac{dx^2 \log (c(a+bx)^p)}{2e^2} + \frac{x^3 \log (c(a+bx)^p)}{3e} - \frac{d^3 p \operatorname{PolyLog}\left(2, -\frac{e(a+bx)}{bd-ae}\right)}{e^4} - \frac{adpx}{2be^2} + \frac{apx^2}{6be} - \frac{d^2 px}{e^3} + \frac{dpx^2}{4e^2} - \frac{px^3}{9e}$$

```
[In] Int[(x^3*Log[c*(a + b*x)^p])/(d + e*x),x]
```

```
[Out] -((d^2*p*x)/e^3) - (a*d*p*x)/(2*b*e^2) - (a^2*p*x)/(3*b^2*e) + (d*p*x^2)/(4
*e^2) + (a*p*x^2)/(6*b*e) - (p*x^3)/(9*e) + (a^2*d*p*Log[a + b*x])/(2*b^2*e
^2) + (a^3*p*Log[a + b*x])/(3*b^3*e) - (d*x^2*Log[c*(a + b*x)^p])/(2*e^2) +
 (x^3*Log[c*(a + b*x)^p])/(3*e) + (d^2*(a + b*x)*Log[c*(a + b*x)^p])/(b*e^3
) - (d^3*Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e^4 - (d^3*p*Po
lyLog[2, -((e*(a + b*x))/(b*d - a*e))])/e^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^2\log\left(c(a+bx)^p\right)}{e^3} - \frac{dx\log\left(c(a+bx)^p\right)}{e^2} + \frac{x^2\log\left(c(a+bx)^p\right)}{e} - \frac{d^3\log\left(c(a+bx)^p\right)}{e^3(d+ex)}\right) dx$$

$$= \frac{d^2\int\log\left(c(a+bx)^p\right)\,dx}{e^3} - \frac{d^3\int\frac{\log(c(a+bx)^p)}{d+ex}\,dx}{e^3} - \frac{d\int x\log\left(c(a+bx)^p\right)\,dx}{e^2} + \frac{\int x^2\log\left(c(a+bx)^p\right)\,dx}{e}$$

$$= -\frac{dx^2\log\left(c(a+bx)^p\right)}{2e^2} + \frac{x^3\log\left(c(a+bx)^p\right)}{3e} - \frac{d^3\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^4} + \frac{d^2\text{Subst}\left(\int\log\left(cx^p\right)\,dx, x, a+bx\right)}{be^3} + \frac{(bd^3p)\int\frac{\log\left(\frac{b(d+ex)}{bd-ae}\right)}{a+bx}\,dx}{e^4} + \frac{(bdp)\int\frac{x^2}{a+bx}\,dx}{2e^2} - \frac{(bp)\int\frac{x^3}{a+bx}\,dx}{3e}$$

$$
\begin{aligned}
&= -\frac{d^2px}{e^3} - \frac{dx^2\log\left(c(a+bx)^p\right)}{2e^2} + \frac{x^3\log\left(c(a+bx)^p\right)}{3e} + \frac{d^2(a+bx)\log\left(c(a+bx)^p\right)}{be^3} \\
&\quad - \frac{d^3\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^4} + \frac{(d^3p)\,\mathrm{Subst}\left(\int \frac{\log\left(1+\frac{ex}{bd-ae}\right)}{x}\,dx, x, a+bx\right)}{e^4} \\
&\quad + \frac{(bdp)\int\left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{2e^2} - \frac{(bp)\int\left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx}{3e} \\
&= -\frac{d^2px}{e^3} - \frac{adpx}{2be^2} - \frac{a^2px}{3b^2e} + \frac{dpx^2}{4e^2} + \frac{apx^2}{6be} - \frac{px^3}{9e} + \frac{a^2dp\log(a+bx)}{2b^2e^2} + \frac{a^3p\log(a+bx)}{3b^3e} \\
&\quad - \frac{dx^2\log\left(c(a+bx)^p\right)}{2e^2} + \frac{x^3\log\left(c(a+bx)^p\right)}{3e} + \frac{d^2(a+bx)\log\left(c(a+bx)^p\right)}{be^3} \\
&\quad - \frac{d^3\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^4} - \frac{d^3p\,\mathrm{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{e^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.73

$$
\int \frac{x^3\log\left(c(a+bx)^p\right)}{d+ex}\,dx
$$

$$
= \frac{6a^2e^2(3bd+2ae)p\log(a+bx) + b\Big(-epx(12a^2e^2 - 6abe(-3d+ex) + b^2(36d^2 - 9dex + 4e^2x^2)) + 6b\log(c}{36b^3e^4}
$$

```
[In] Integrate[(x^3*Log[c*(a + b*x)^p])/(d + e*x),x]
```

```
[Out] (6*a^2*e^2*(3*b*d + 2*a*e)*p*Log[a + b*x] + b*(-(e*p*x*(12*a^2*e^2 - 6*a*b*
e*(-3*d + e*x) + b^2*(36*d^2 - 9*d*e*x + 4*e^2*x^2))) + 6*b*Log[c*(a + b*x)
^p]*(6*a*d^2*e + b*e*x*(6*d^2 - 3*d*e*x + 2*e^2*x^2) - 6*b*d^3*Log[(b*(d +
e*x))/(b*d - a*e)])) - 36*b^3*d^3*p*PolyLog[2, (e*(a + b*x))/(-(b*d) + a*e)
])/(36*b^3*e^4)
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.19

method	result
parts	$\frac{x^3 \ln(c(bx+a)^p)}{3e} - \frac{d\,x^2 \ln(c(bx+a)^p)}{2e^2} + \frac{\ln(c(bx+a)^p)d^2 x}{e^3} - \frac{\ln(c(bx+a)^p)d^3 \ln(ex+d)}{e^4} - \frac{pb\left(-\frac{\frac{2(ex+d)^3 b^2}{3} - (ex+d)^2 abe - \frac{7(e}{}}{}\right.}{}$
risch	$\frac{\ln((bx+a)^p)x^3}{3e} - \frac{\ln((bx+a)^p)d\,x^2}{2e^2} + \frac{\ln((bx+a)^p)x\,d^2}{e^3} - \frac{\ln((bx+a)^p)d^3 \ln(ex+d)}{e^4} - \frac{p\,x^3}{9e} + \frac{dp\,x^2}{4e^2} - \frac{d^2 px}{e^3} - \frac{49p\,d^3}{36e^4} + \frac{ap}{6}$

```
[In] int(x^3*ln(c*(b*x+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*ln(c*(b*x+a)^p)/e-1/2*d*x^2*ln(c*(b*x+a)^p)/e^2+ln(c*(b*x+a)^p)/e^3
*d^2*x-ln(c*(b*x+a)^p)*d^3/e^4*ln(e*x+d)-p*b/e*(-1/6/e^3*(-1/b^3*(2/3*(e*x+
d)^3*b^2-(e*x+d)^2*a*b*e-7/2*(e*x+d)^2*b^2*d+2*(e*x+d)*a^2*e^2+5*(e*x+d)*a*
b*d*e+11*(e*x+d)*b^2*d^2)+a*e*(2*a^2*e^2+3*a*b*d*e+6*b^2*d^2)/b^4*ln((e*x+d
)*b+a*e-b*d))-1/e^3*d^3*(dilog(((e*x+d)*b+a*e-b*d)/(a*e-b*d))/b+ln(e*x+d)*l
n(((e*x+d)*b+a*e-b*d)/(a*e-b*d))/b))
```

Fricas [F]

$$\int \frac{x^3 \log \left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log \left((bx+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^3*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^3*log((b*x + a)^p*c)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{x^3 \log \left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log \left(c(a+bx)^p\right)}{d+ex}\,dx$$

```
[In] integrate(x**3*ln(c*(b*x+a)**p)/(e*x+d),x)
```

```
[Out] Integral(x**3*log(c*(a + b*x)**p)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x^3 \log \left(c(a+bx)^p\right)}{d+ex}\, dx = \int \frac{x^3 \log \left((bx+a)^p c\right)}{ex+d}\, dx$$

```
[In] integrate(x^3*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x^3*log((b*x + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^3 \log \left(c(a+bx)^p\right)}{d+ex}\, dx = \int \frac{x^3 \log \left((bx+a)^p c\right)}{ex+d}\, dx$$

```
[In] integrate(x^3*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x^3*log((b*x + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(c(a+bx)^p\right)}{d+ex}\, dx = \int \frac{x^3 \ln \left(c\,(a+b\,x)^p\right)}{d+e\,x}\, dx$$

```
[In] int((x^3*log(c*(a + b*x)^p))/(d + e*x),x)
[Out] int((x^3*log(c*(a + b*x)^p))/(d + e*x), x)
```

3.220 $\int \frac{x^2 \log(c(a+bx)^p)}{d+ex} dx$

Optimal result

Integrand size = 21, antiderivative size = 159

$$\int \frac{x^2 \log(c(a+bx)^p)}{d+ex} dx = \frac{dpx}{e^2} + \frac{apx}{2be} - \frac{px^2}{4e} - \frac{a^2 p \log(a+bx)}{2b^2 e} + \frac{x^2 \log(c(a+bx)^p)}{2e} - \frac{d(a+bx)\log(c(a+bx)^p)}{be^2} + \frac{d^2 \log(c(a+bx)^p) \log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^3} + \frac{d^2 p \operatorname{PolyLog}\left(2, -\frac{e(a+bx)}{bd-ae}\right)}{e^3}$$

```
[Out] d*p*x/e^2+1/2*a*p*x/b/e-1/4*p*x^2/e-1/2*a^2*p*ln(b*x+a)/b^2/e+1/2*x^2*ln(c*
(b*x+a)^p)/e-d*(b*x+a)*ln(c*(b*x+a)^p)/b/e^2+d^2*ln(c*(b*x+a)^p)*ln(b*(e*x+
d)/(-a*e+b*d))/e^3+d^2*p*polylog(2,-e*(b*x+a)/(-a*e+b*d))/e^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {45, 2463, 2436, 2332, 2442, 2441, 2440, 2438}

$$\int \frac{x^2 \log(c(a+bx)^p)}{d+ex} dx = -\frac{a^2 p \log(a+bx)}{2b^2 e} + \frac{d^2 \log(c(a+bx)^p) \log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^3} - \frac{d(a+bx)\log(c(a+bx)^p)}{be^2} + \frac{x^2 \log(c(a+bx)^p)}{2e} + \frac{d^2 p \operatorname{PolyLog}\left(2, -\frac{e(a+bx)}{bd-ae}\right)}{e^3} + \frac{apx}{2be} + \frac{dpx}{e^2} - \frac{px^2}{4e}$$

```
[In] Int[(x^2*Log[c*(a + b*x)^p])/(d + e*x),x]
```

```
[Out] (d*p*x)/e^2 + (a*p*x)/(2*b*e) - (p*x^2)/(4*e) - (a^2*p*Log[a + b*x])/(2*b^2
*e) + (x^2*Log[c*(a + b*x)^p])/(2*e) - (d*(a + b*x)*Log[c*(a + b*x)^p])/(b*
e^2) + (d^2*Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e^3 + (d^2*p
*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))])/e^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{d\log\left(c(a+bx)^p\right)}{e^2}+\frac{x\log\left(c(a+bx)^p\right)}{e}+\frac{d^2\log\left(c(a+bx)^p\right)}{e^2(d+ex)}\right)dx \\
&= -\frac{d\int\log\left(c(a+bx)^p\right)dx}{e^2}+\frac{d^2\int\frac{\log(c(a+bx)^p)}{d+ex}dx}{e^2}+\frac{\int x\log\left(c(a+bx)^p\right)dx}{e} \\
&= \frac{x^2\log\left(c(a+bx)^p\right)}{2e}+\frac{d^2\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^3} \\
&\quad -\frac{d\text{Subst}\left(\int\log\left(cx^p\right)dx,x,a+bx\right)}{be^2}-\frac{(bd^2p)\int\frac{\log\left(\frac{b(d+ex)}{bd-ae}\right)}{a+bx}dx}{e^3}-\frac{(bp)\int\frac{x^2}{a+bx}dx}{2e} \\
&= \frac{dpx}{e^2}+\frac{x^2\log\left(c(a+bx)^p\right)}{2e}-\frac{d(a+bx)\log\left(c(a+bx)^p\right)}{be^2}+\frac{d^2\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^3} \\
&\quad -\frac{(d^2p)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{ex}{bd-ae}\right)}{x}dx,x,a+bx\right)}{e^3}-\frac{(bp)\int\left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{2e} \\
&= \frac{dpx}{e^2}+\frac{apx}{2be}-\frac{px^2}{4e}-\frac{a^2p\log(a+bx)}{2b^2e}+\frac{x^2\log\left(c(a+bx)^p\right)}{2e} \\
&\quad -\frac{d(a+bx)\log\left(c(a+bx)^p\right)}{be^2}+\frac{d^2\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^3}+\frac{d^2p\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{e^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.80

$$
\int\frac{x^2\log\left(c(a+bx)^p\right)}{d+ex}dx
$$

$$
=\frac{bepx(4bd+2ae-bex)-2a^2e^2p\log(a+bx)+b\log\left(c(a+bx)^p\right)\left(-4ade+2bex(-2d+ex)+4bd^2\log\left(\frac{b}{\cdots}\right.\right.}{4b^2e^3}
$$

```
[In] Integrate[(x^2*Log[c*(a + b*x)^p])/(d + e*x),x]
```

```
[Out] (b*e*p*x*(4*b*d + 2*a*e - b*e*x) - 2*a^2*e^2*p*Log[a + b*x] + b*Log[c*(a +
b*x)^p]*(-4*a*d*e + 2*b*e*x*(-2*d + e*x) + 4*b*d^2*Log[(b*(d + e*x))/(b*d -
 a*e)]) + 4*b^2*d^2*p*PolyLog[2, (e*(a + b*x))/(-(b*d) + a*e)])/(4*b^2*e^3)
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.34

method	result
parts	$\frac{x^2 \ln(c(bx+a)^p)}{2e} - \frac{dx \ln(c(bx+a)^p)}{e^2} + \frac{\ln(c(bx+a)^p) d^2 \ln(ex+d)}{e^3} - \frac{pb\left(\frac{d^2\left(\frac{\operatorname{dilog}\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{b} + \frac{\ln(ex+d)\ln\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{b}\right.}{e^2}\right.}{}$
risch	$\frac{\ln((bx+a)^p)x^2}{2e} - \frac{\ln((bx+a)^p)dx}{e^2} + \frac{\ln((bx+a)^p)d^2\ln(ex+d)}{e^3} - \frac{p\,d^2 \operatorname{dilog}\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{e^3} - \frac{p\,d^2 \ln(ex+d)\ln\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{e^3}$

```
[In] int(x^2*ln(c*(b*x+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*ln(c*(b*x+a)^p)/e-d*x*ln(c*(b*x+a)^p)/e^2+ln(c*(b*x+a)^p)*d^2/e^3*l
n(e*x+d)-p*b/e*(d^2/e^2*(dilog(((e*x+d)*b+a*e-b*d)/(a*e-b*d))/b+ln(e*x+d)*l
n(((e*x+d)*b+a*e-b*d)/(a*e-b*d))/b)+1/2/e^2*(-1/b^2*((e*x+d)*a*e+3*d*(e*x+d
)*b-1/2*(e*x+d)^2*b)+a*e*(a*e+2*b*d)/b^3*ln((e*x+d)*b+a*e-b*d)))
```

Fricas [F]

$$\int \frac{x^2 \log \left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left((bx+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^2*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^2*log((b*x + a)^p*c)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{x^2 \log \left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left(c(a+bx)^p\right)}{d+ex}\,dx$$

```
[In] integrate(x**2*ln(c*(b*x+a)**p)/(e*x+d),x)
```

```
[Out] Integral(x**2*log(c*(a + b*x)**p)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x^2 \log \left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left((bx+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^2*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x^2*log((b*x + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^2 \log \left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left((bx+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^2*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x^2*log((b*x + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{x^2\ \ln\left(c\,{\left(a+b\,x\right)}^p\right)}{d+e\,x}\,dx$$

```
[In] int((x^2*log(c*(a + b*x)^p))/(d + e*x),x)
[Out] int((x^2*log(c*(a + b*x)^p))/(d + e*x), x)
```

3.221 $\int \frac{x\log(c(a+bx)^p)}{d+ex}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 91

$$\int \frac{x\log\left(c(a+bx)^p\right)}{d+ex}\,dx = -\frac{px}{e}+\frac{(a+bx)\log\left(c(a+bx)^p\right)}{be} - \frac{d\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^2}-\frac{dp\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e^2}$$

```
[Out] -p*x/e+(b*x+a)*ln(c*(b*x+a)^p)/b/e-d*ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d
))/e^2-d*p*polylog(2,-e*(b*x+a)/(-a*e+b*d))/e^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {45, 2463, 2436, 2332, 2441, 2440, 2438}

$$\int \frac{x\log\left(c(a+bx)^p\right)}{d+ex}\,dx = -\frac{d\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^2}+\frac{(a+bx)\log\left(c(a+bx)^p\right)}{be} - \frac{dp\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e^2}-\frac{px}{e}$$

```
[In] Int[(x*Log[c*(a + b*x)^p])/(d + e*x),x]
```

```
[Out] -((p*x)/e) + ((a + b*x)*Log[c*(a + b*x)^p])/(b*e) - (d*Log[c*(a + b*x)^p]*L
og[(b*(d + e*x))/(b*d - a*e)])/e^2 - (d*p*PolyLog[2, -((e*(a + b*x))/(b*d -
 a*e))])/e^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log \left(c(a+bx)^p\right)}{e} - \frac{d \log \left(c(a+bx)^p\right)}{e(d+ex)} \right) dx$$

$$= \frac{\int \log\left(c(a+bx)^p\right)\,dx}{e} - \frac{d\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx}{e}$$

$$= -\frac{d\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^2} + \frac{\text{Subst}\left(\int \log\left(cx^p\right)\,dx, x, a+bx\right)}{be} + \frac{(bdp)\int \frac{\log\left(\frac{b(d+ex)}{bd-ae}\right)}{a+bx}\,dx}{e^2}$$

$$= -\frac{px}{e} + \frac{(a+bx)\log\left(c(a+bx)^p\right)}{be} - \frac{d\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^2} + \frac{(dp)\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{bd-ae}\right)}{x}\,dx, x, a+bx\right)}{e^2}$$

$$= -\frac{px}{e} + \frac{(a+bx)\log\left(c(a+bx)^p\right)}{be} - \frac{d\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e^2} - \frac{dp\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{e^2}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.87

$$\int \frac{x\log\left(c(a+bx)^p\right)}{d+ex}\,dx = \frac{-bepx + \log\left(c(a+bx)^p\right)\left(ae+bex-bd\log\left(\frac{b(d+ex)}{bd-ae}\right)\right) - bdp\,\text{PolyLog}\left(2, \frac{e(a+bx)}{-bd+ae}\right)}{be^2}$$

```
[In] Integrate[(x*Log[c*(a + b*x)^p])/(d + e*x),x]
[Out] (-(b*e*p*x) + Log[c*(a + b*x)^p]*(a*e + b*e*x - b*d*Log[(b*(d + e*x))/(b*d
- a*e)]) - b*d*p*PolyLog[2, (e*(a + b*x))/(-(b*d) + a*e)])/(b*e^2)
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.71

method	result
parts	$\frac{x\ln(c(bx+a)^p)}{e} - \frac{\ln(c(bx+a)^p)d\ln(ex+d)}{e^2} - \frac{pb\left(\frac{ex+d}{eb} - \frac{a\ln((ex+d)b+ae-bd)}{b^2} - \frac{d\left(\frac{\mathrm{dilog}\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{b} + \frac{\ln(ex+d)\ln\left(\frac{(ex+d)b+ae-}{ae-bd}\right)}{b}\right)}{e}\right)}{e}$
risch	$\frac{\ln((bx+a)^p)x}{e} - \frac{\ln((bx+a)^p)d\ln(ex+d)}{e^2} - \frac{px}{e} - \frac{pd}{e^2} + \frac{pa\ln((ex+d)b+ae-bd)}{be} + \frac{pd\,\mathrm{dilog}\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{e^2} + \frac{pd\ln(ex+d)\ln(}{}$

```
[In] int(x*ln(c*(b*x+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(c*(b*x+a)^p)/e-ln(c*(b*x+a)^p)*d/e^2*ln(e*x+d)-p*b/e*(1/e*(e*x+d)/b-a/
b^2*ln((e*x+d)*b+a*e-b*d)-1/e*d*(dilog(((e*x+d)*b+a*e-b*d)/(a*e-b*d))/b+ln(
e*x+d)*ln(((e*x+d)*b+a*e-b*d)/(a*e-b*d))/b))
```

Fricas [F]

$$\int \frac{x \log \left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{x \log \left((bx+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(x*log((b*x + a)^p*c)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{x \log \left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{x \log \left(c(a+bx)^p\right)}{d+ex}\,dx$$

```
[In] integrate(x*ln(c*(b*x+a)**p)/(e*x+d),x)
[Out] Integral(x*log(c*(a + b*x)**p)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x \log \left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{x \log \left((bx+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x*log((b*x + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x \log \left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{x \log \left((bx+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x*log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x*log((b*x + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \log \left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{x\,\ln\left(c\,{\left(a+b\,x\right)}^p\right)}{d+e\,x} \,d x$$

```
[In] int((x*log(c*(a + b*x)^p))/(d + e*x),x)
[Out] int((x*log(c*(a + b*x)^p))/(d + e*x), x)
```

3.222 $\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 58

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx = \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e} + \frac{p\,\mathrm{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e}$$

```
[Out] ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d))/e+p*polylog(2,-e*(b*x+a)/(-a*e+b*d
))/e
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2441, 2440, 2438}

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx = \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e} + \frac{p\,\mathrm{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{e}$$

```
[In] Int[Log[c*(a + b*x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e + (p*PolyLog[2, -((e*
(a + b*x))/(b*d - a*e))])/e
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e} - \frac{(bp)\int \frac{\log\left(\frac{b(d+ex)}{bd-ae}\right)}{a+bx}\,dx}{e}\\
&= \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e} - \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{bd-ae}\right)}{x}\,dx, x, a+bx\right)}{e}\\
&= \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e} + \frac{p\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{e}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx = \frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{e} + \frac{p\,\text{PolyLog}\left(2, \frac{e(a+bx)}{-bd+ae}\right)}{e}$$

```
[In] Integrate[Log[c*(a + b*x)^p]/(d + e*x),x]
[Out] (Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)])/e + (p*PolyLog[2, (e*(a
 + b*x))/(-(b*d) + a*e)])/e
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.69

method	result
parts	$\frac{\ln\left(c(bx+a)^{p}\right)\ln(ex+d)}{e}-\frac{pb\left(\frac{\operatorname{dilog}\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{b}+\frac{\ln(ex+d)\ln\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{b}\right)}{e}$
risch	$\frac{\ln\left((bx+a)^{p}\right)\ln(ex+d)}{e}-\frac{p\operatorname{dilog}\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{e}-\frac{p\ln(ex+d)\ln\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{e}+\frac{\left(\frac{i\pi\operatorname{csgn}\left(i(bx+a)^{p}\right)\operatorname{csgn}\left(ic(bx+a)^{p}\right)^{2}}{2}-\frac{i\pi}{}\right.}{}$

[In] int(ln(c*(b*x+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)

[Out] ln(c*(b*x+a)^p)*ln(e*x+d)/e-p*b/e*(dilog(((e*x+d)*b+a*e-b*d)/(a*e-b*d))/b+l
n(e*x+d)*ln(((e*x+d)*b+a*e-b*d)/(a*e-b*d))/b)

Fricas [F]

$$\int\frac{\log\left(c(a+bx)^{p}\right)}{d+ex}\,dx=\int\frac{\log\left((bx+a)^{p}c\right)}{ex+d}\,dx$$

[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="fricas")

[Out] integral(log((b*x + a)^p*c)/(e*x + d), x)

Sympy [F]

$$\int\frac{\log\left(c(a+bx)^{p}\right)}{d+ex}\,dx=\int\frac{\log\left(c(a+bx)^{p}\right)}{d+ex}\,dx$$

[In] integrate(ln(c*(b*x+a)**p)/(e*x+d),x)

[Out] Integral(log(c*(a + b*x)**p)/(d + e*x), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(57) = 114.

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.03

$$\int\frac{\log\left(c(a+bx)^{p}\right)}{d+ex}\,dx=\frac{bp\left(\frac{\log(bx+a)\log(ex+d)}{b}-\frac{\log(ex+d)\log\left(-\frac{bex+bd}{bd-ae}+1\right)+\mathrm{Li}_2\left(\frac{bex+bd}{bd-ae}\right)}{b}\right)}{e}-\frac{p\log\left(bx+a\right)\log\left(ex+d\right)}{e}+\frac{\log\left((bx+a)^{p}c\right)\log\left(ex+d\right)}{e}$$

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*p*(log(b*x + a)*log(e*x + d)/b - (log(e*x + d)*log(-(b*e*x + b*d)/(b*d -
a*e) + 1) + dilog((b*e*x + b*d)/(b*d - a*e)))/b)/e - p*log(b*x + a)*log(e*x
 + d)/e + log((b*x + a)^p*c)*log(e*x + d)/e
```

Giac [F]

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{\log\left((bx+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(log(c*(b*x+a)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx)^p\right)}{d+ex}\,dx = \int \frac{\ln\left(c\,(a+b\,x)^p\right)}{d+e\,x}\,dx$$

```
[In] int(log(c*(a + b*x)^p)/(d + e*x),x)
```

```
[Out] int(log(c*(a + b*x)^p)/(d + e*x), x)
```

3.223 $\int \frac{\log(c(a+bx)^p)}{x(d+ex)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{\log\left(c(a+bx)^p\right)}{x(d+ex)}\,dx=\frac{\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d}-\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d}-\frac{p\,\mathrm{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d}+\frac{p\,\mathrm{PolyLog}\left(2,1+\frac{bx}{a}\right)}{d}$$

```
[Out] ln(-b*x/a)*ln(c*(b*x+a)^p)/d-ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d))/d-p*p
olylog(2,-e*(b*x+a)/(-a*e+b*d))/d+p*polylog(2,1+b*x/a)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {36, 29, 31, 2463, 2441, 2352, 2440, 2438}

$$\int \frac{\log\left(c(a+bx)^p\right)}{x(d+ex)}\,dx=-\frac{\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d}+\frac{\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d}-\frac{p\,\mathrm{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d}+\frac{p\,\mathrm{PolyLog}\left(2,\frac{bx}{a}+1\right)}{d}$$

```
[In] Int[Log[c*(a + b*x)^p]/(x*(d + e*x)),x]
```

```
[Out] (Log[-((b*x)/a)]*Log[c*(a + b*x)^p])/d - (Log[c*(a + b*x)^p]*Log[(b*(d + e*
x))/(b*d - a*e)])/d - (p*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))])/d + (p*P
olyLog[2, 1 + (b*x)/a])/d
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log(c(a+bx)^p)}{dx} - \frac{e\log(c(a+bx)^p)}{d(d+ex)}\right) dx$$

$$= \frac{\int \frac{\log(c(a+bx)^p)}{x} dx}{d} - \frac{e\int \frac{\log(c(a+bx)^p)}{d+ex} dx}{d}$$

$$= \frac{\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d} - \frac{\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{(bp)\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx} dx}{d} + \frac{(bp)\int \frac{\log\left(\frac{b(d+ex)}{bd-ae}\right)}{a+bx} dx}{d}$$

$$= \frac{\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d} - \frac{\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} + \frac{p\text{Li}_2\left(1+\frac{bx}{a}\right)}{d} + \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{ex}{bd-ae}\right)}{x} dx, x, a+bx\right)}{d}$$

$$= \frac{\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d} - \frac{\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} - \frac{p\text{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{d} + \frac{p\text{Li}_2\left(1+\frac{bx}{a}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01

$$\int \frac{\log(c(a+bx)^p)}{x(d+ex)} dx = \frac{\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d} - \frac{\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d} + \frac{p\,\text{PolyLog}\left(2,\frac{a+bx}{a}\right)}{d} - \frac{p\,\text{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d}$$

```
[In] Integrate[Log[c*(a + b*x)^p]/(x*(d + e*x)),x]
```

```
[Out] (Log[-((b*x)/a)]*Log[c*(a + b*x)^p])/d - (Log[c*(a + b*x)^p]*Log[(b*(d + e*
x))/(b*d - a*e)])/d + (p*PolyLog[2, (a + b*x)/a])/d - (p*PolyLog[2, -((e*(a
 + b*x))/(b*d - a*e))])/d
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.61

method	result
parts	$-\frac{\ln(c(bx+a)^p)\ln(ex+d)}{d}+\frac{\ln(c(bx+a)^p)\ln(x)}{d}-pb\left(\frac{\operatorname{dilog}\left(\frac{bx+a}{a}\right)}{db}+\frac{\ln(x)\ln\left(\frac{bx+a}{a}\right)}{db}-\frac{\operatorname{dilog}\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{db}-\frac{\ln(ex+d)}{}\right.$
risch	$-\frac{\ln((bx+a)^p)\ln(ex+d)}{d}+\frac{\ln((bx+a)^p)\ln(x)}{d}-\frac{p\operatorname{dilog}\left(\frac{bx+a}{a}\right)}{d}-\frac{p\ln(x)\ln\left(\frac{bx+a}{a}\right)}{d}+\frac{p\operatorname{dilog}\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{d}+\frac{p\ln(ex+d)}{}$

[In] int(ln(c*(b*x+a)^p)/x/(e*x+d),x,method=_RETURNVERBOSE)

[Out] -ln(c*(b*x+a)^p)/d*ln(e*x+d)+ln(c*(b*x+a)^p)/d*ln(x)-p*b*(1/d*dilog((b*x+a)/a)/b+1/d*ln(x)*ln((b*x+a)/a)/b-1/d*dilog(((e*x+d)*b+a*e-b*d)/(a*e-b*d))/b-1/d*ln(e*x+d)*ln(((e*x+d)*b+a*e-b*d)/(a*e-b*d))/b)

Fricas [F]

$$\int \frac{\log\left(c(a+bx)^p\right)}{x(d+ex)}\,dx = \int \frac{\log\left((bx+a)^p c\right)}{(ex+d)x}\,dx$$

[In] integrate(log(c*(b*x+a)^p)/x/(e*x+d),x, algorithm="fricas")

[Out] integral(log((b*x + a)^p*c)/(e*x^2 + d*x), x)

Sympy [F]

$$\int \frac{\log\left(c(a+bx)^p\right)}{x(d+ex)}\,dx = \int \frac{\log\left(c(a+bx)^p\right)}{x\,(d+ex)}\,dx$$

[In] integrate(ln(c*(b*x+a)**p)/x/(e*x+d),x)

[Out] Integral(log(c*(a + b*x)**p)/(x*(d + e*x)), x)

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.27

$$\int \frac{\log(c(a+bx)^p)}{x(d+ex)}\,dx$$
$$= -bp\left(\frac{\log\left(\frac{bx}{a}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)}{bd} - \frac{\log(ex+d)\log\left(-\frac{bex+bd}{bd-ae}+1\right)+\mathrm{Li}_2\left(\frac{bex+bd}{bd-ae}\right)}{bd}\right)$$
$$- \left(\frac{\log(ex+d)}{d} - \frac{\log(x)}{d}\right)\log\left((bx+a)^pc\right)$$

```
[In] integrate(log(c*(b*x+a)^p)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -b*p*((log(b*x/a + 1)*log(x) + dilog(-b*x/a))/(b*d) - (log(e*x + d)*log(-(b
*e*x + b*d)/(b*d - a*e) + 1) + dilog((b*e*x + b*d)/(b*d - a*e)))/(b*d)) - (
log(e*x + d)/d - log(x)/d)*log((b*x + a)^p*c)
```

Giac [F]

$$\int \frac{\log(c(a+bx)^p)}{x(d+ex)}\,dx = \int \frac{\log\left((bx+a)^pc\right)}{(ex+d)x}\,dx$$

```
[In] integrate(log(c*(b*x+a)^p)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^p*c)/((e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c(a+bx)^p)}{x(d+ex)}\,dx = \int \frac{\ln\left(c\,(a+b\,x)^p\right)}{x\,(d+e\,x)}\,dx$$

```
[In] int(log(c*(a + b*x)^p)/(x*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b*x)^p)/(x*(d + e*x)), x)
```

3.224 $\int \frac{\log(c(a+bx)^p)}{x^2(d+ex)} dx$

Optimal result

Integrand size = 21, antiderivative size = 146

$$\int \frac{\log(c(a+bx)^p)}{x^2(d+ex)} dx = \frac{bp\log(x)}{ad} - \frac{bp\log(a+bx)}{ad} - \frac{\log(c(a+bx)^p)}{dx} - \frac{e\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d^2} + \frac{e\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^2} + \frac{ep\operatorname{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d^2} - \frac{ep\operatorname{PolyLog}\left(2,1+\frac{bx}{a}\right)}{d^2}$$

```
[Out] b*p*ln(x)/a/d-b*p*ln(b*x+a)/a/d-ln(c*(b*x+a)^p)/d/x-e*ln(-b*x/a)*ln(c*(b*x+
a)^p)/d^2+e*ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*e+b*d))/d^2+e*p*polylog(2,-e*(
b*x+a)/(-a*e+b*d))/d^2-e*p*polylog(2,1+b*x/a)/d^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {46, 2463, 2442, 36, 29, 31, 2441, 2352, 2440, 2438}

$$\int \frac{\log(c(a+bx)^p)}{x^2(d+ex)} dx = -\frac{e\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d^2} + \frac{e\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^2} - \frac{\log(c(a+bx)^p)}{dx} + \frac{ep\operatorname{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d^2} - \frac{ep\operatorname{PolyLog}\left(2,\frac{bx}{a}+1\right)}{d^2} + \frac{bp\log(x)}{ad} - \frac{bp\log(a+bx)}{ad}$$

```
[In] Int[Log[c*(a + b*x)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (b*p*Log[x])/(a*d) - (b*p*Log[a + b*x])/(a*d) - Log[c*(a + b*x)^p]/(d*x) -
(e*Log[-((b*x)/a)]*Log[c*(a + b*x)^p])/d^2 + (e*Log[c*(a + b*x)^p]*Log[(b*(
d + e*x))/(b*d - a*e)])/d^2 + (e*p*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))]
)/d^2 - (e*p*PolyLog[2, 1 + (b*x)/a])/d^2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\log\left(c(a+bx)^p\right)}{dx^2} - \frac{e\log\left(c(a+bx)^p\right)}{d^2x} + \frac{e^2\log\left(c(a+bx)^p\right)}{d^2(d+ex)}\right) dx \\
&= \frac{\int \frac{\log(c(a+bx)^p)}{x^2}\,dx}{d} - \frac{e\int \frac{\log(c(a+bx)^p)}{x}\,dx}{d^2} + \frac{e^2\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx}{d^2} \\
&= -\frac{\log\left(c(a+bx)^p\right)}{dx} - \frac{e\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d^2} + \frac{e\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^2} \\
&\quad + \frac{(bp)\int \frac{1}{x(a+bx)}\,dx}{d} + \frac{(bep)\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx}{d^2} - \frac{(bep)\int \frac{\log\left(\frac{b(d+ex)}{bd-ae}\right)}{a+bx}\,dx}{d^2} \\
&= -\frac{\log\left(c(a+bx)^p\right)}{dx} - \frac{e\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d^2} + \frac{e\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^2} \\
&\quad - \frac{ep\mathrm{Li}_2\left(1+\frac{bx}{a}\right)}{d^2} + \frac{(bp)\int \frac{1}{x}\,dx}{ad} - \frac{(b^2p)\int \frac{1}{a+bx}\,dx}{ad} - \frac{(ep)\mathrm{Subst}\left(\int \frac{\log\left(1+\frac{ex}{bd-ae}\right)}{x}\,dx, x, a+bx\right)}{d^2} \\
&= \frac{bp\log(x)}{ad} - \frac{bp\log(a+bx)}{ad} - \frac{\log\left(c(a+bx)^p\right)}{dx} - \frac{e\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d^2} \\
&\quad + \frac{e\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^2} + \frac{ep\mathrm{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{d^2} - \frac{ep\mathrm{Li}_2\left(1+\frac{bx}{a}\right)}{d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01

$$\int \frac{\log \left(c(a+bx)^p\right)}{x^2(d+ex)}\,dx = \frac{bp\log(x)}{ad}-\frac{bp\log(a+bx)}{ad}-\frac{\log \left(c(a+bx)^p\right)}{dx}-\frac{e\log \left(-\frac{bx}{a}\right)\log \left(c(a+bx)^p\right)}{d^2}+\frac{e\log \left(c(a+bx)^p\right)\log \left(\frac{b(d+ex)}{bd-ae}\right)}{d^2}-\frac{ep\,\mathrm{PolyLog}\left(2,\frac{a+bx}{a}\right)}{d^2}+\frac{ep\,\mathrm{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d^2}$$

```
[In] Integrate[Log[c*(a + b*x)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (b*p*Log[x])/(a*d) - (b*p*Log[a + b*x])/(a*d) - Log[c*(a + b*x)^p]/(d*x) -
(e*Log[-((b*x)/a)]*Log[c*(a + b*x)^p])/d^2 + (e*Log[c*(a + b*x)^p]*Log[(b*(
d + e*x))/(b*d - a*e)])/d^2 - (e*p*PolyLog[2, (a + b*x)/a])/d^2 + (e*p*Poly
Log[2, -((e*(a + b*x))/(b*d - a*e))])/d^2
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.38

method	result
parts	$\frac{\ln(c(bx+a)^p)e\ln(ex+d)}{d^2}-\frac{\ln(c(bx+a)^p)}{dx}-\frac{\ln(c(bx+a)^p)e\ln(x)}{d^2}-pb\Bigg(\frac{e\Big(\frac{\mathrm{dilog}\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{b}+\frac{\ln(ex+d)\ln\big(\frac{(ex+d)b+ae-bd}{ae-bd}}{b}}{d^2}$
risch	$\frac{\ln((bx+a)^p)e\ln(ex+d)}{d^2}-\frac{\ln((bx+a)^p)}{dx}-\frac{\ln((bx+a)^p)e\ln(x)}{d^2}-\frac{pe\,\mathrm{dilog}\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{d^2}-\frac{pe\ln(ex+d)\ln\left(\frac{(ex+d)b+ae-bd}{ae-bd}\right)}{d^2}$

```
[In] int(ln(c*(b*x+a)^p)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] ln(c*(b*x+a)^p)*e/d^2*ln(e*x+d)-ln(c*(b*x+a)^p)/d/x-ln(c*(b*x+a)^p)*e/d^2*l
n(x)-p*b*(e/d^2*(dilog(((e*x+d)*b+a*e-b*d)/(a*e-b*d))/b+ln(e*x+d)*ln(((e*x+
d)*b+a*e-b*d)/(a*e-b*d))/b)+1/d/a*ln(b*x+a)-1/d/a*ln(x)-e/d^2*dilog((b*x+a)
/a)/b-e/d^2*ln(x)*ln((b*x+a)/a)/b)
```

Fricas [F]

$$\int \frac{\log(c(a+bx)^p)}{x^2(d+ex)}\,dx = \int \frac{\log\left((bx+a)^p c\right)}{(ex+d)x^2}\,dx$$

```
[In] integrate(log(c*(b*x+a)^p)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x + a)^p*c)/(e*x^3 + d*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log(c(a+bx)^p)}{x^2(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(b*x+a)**p)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.07

$$\begin{aligned}&\int \frac{\log(c(a+bx)^p)}{x^2(d+ex)}\,dx\\&= bp\left(\frac{\left(\log\left(\frac{bx}{a}+1\right)\log(x)+\operatorname{Li}_2\left(-\frac{bx}{a}\right)\right)e}{bd^2} - \frac{\left(\log(ex+d)\log\left(-\frac{bex+bd}{bd-ae}+1\right)+\operatorname{Li}_2\left(\frac{bex+bd}{bd-ae}\right)\right)e}{bd^2} - \frac{\log(bx+a)}{ad} +\right.\\&\quad + \left(\frac{e\log(ex+d)}{d^2} - \frac{e\log(x)}{d^2} - \frac{1}{dx}\right)\log\left((bx+a)^p c\right)\end{aligned}$$

```
[In] integrate(log(c*(b*x+a)^p)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*p*((log(b*x/a + 1)*log(x) + dilog(-b*x/a))*e/(b*d^2) - (log(e*x + d)*log(
-(b*e*x + b*d)/(b*d - a*e) + 1) + dilog((b*e*x + b*d)/(b*d - a*e)))*e/(b*d^
2) - log(b*x + a)/(a*d) + log(x)/(a*d)) + (e*log(e*x + d)/d^2 - e*log(x)/d^
2 - 1/(d*x))*log((b*x + a)^p*c)
```

Giac [F]

$$\int \frac{\log(c(a+bx)^p)}{x^2(d+ex)}\,dx = \int \frac{\log((bx+a)^p c)}{(ex+d)x^2}\,dx$$

```
[In] integrate(log(c*(b*x+a)^p)/x^2/(e*x+d),x, algorithm="giac")
[Out] integrate(log((b*x + a)^p*c)/((e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c(a+bx)^p)}{x^2(d+ex)}\,dx = \int \frac{\ln(c\,(a+b\,x)^p)}{x^2\,(d+e\,x)}\,dx$$

```
[In] int(log(c*(a + b*x)^p)/(x^2*(d + e*x)),x)
[Out] int(log(c*(a + b*x)^p)/(x^2*(d + e*x)), x)
```

3.225 $\int \frac{\log(c(a+bx)^p)}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 227

$$\begin{aligned}\int \frac{\log(c(a+bx)^p)}{x^3(d+ex)}\,dx = &-\frac{bp}{2adx}-\frac{b^2p\log(x)}{2a^2d}-\frac{bep\log(x)}{ad^2}+\frac{b^2p\log(a+bx)}{2a^2d}\\ &+\frac{bep\log(a+bx)}{ad^2}-\frac{\log(c(a+bx)^p)}{2dx^2}+\frac{e\log(c(a+bx)^p)}{d^2x}\\ &+\frac{e^2\log\left(-\frac{bx}{a}\right)\log(c(a+bx)^p)}{d^3}-\frac{e^2\log(c(a+bx)^p)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^3}\\ &-\frac{e^2p\,\mathrm{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d^3}+\frac{e^2p\,\mathrm{PolyLog}\left(2,1+\frac{bx}{a}\right)}{d^3}\end{aligned}$$

```
[Out] -1/2*b*p/a/d/x-1/2*b^2*p*ln(x)/a^2/d-b*e*p*ln(x)/a/d^2+1/2*b^2*p*ln(b*x+a)/
a^2/d+b*e*p*ln(b*x+a)/a/d^2-1/2*ln(c*(b*x+a)^p)/d/x^2+e*ln(c*(b*x+a)^p)/d^2
/x+e^2*ln(-b*x/a)*ln(c*(b*x+a)^p)/d^3-e^2*ln(c*(b*x+a)^p)*ln(b*(e*x+d)/(-a*
e+b*d))/d^3-e^2*p*polylog(2,-e*(b*x+a)/(-a*e+b*d))/d^3+e^2*p*polylog(2,1+b*
x/a)/d^3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules

used = {46, 2463, 2442, 36, 29, 31, 2441, 2352, 2440, 2438}

$$\int \frac{\log\left(c(a+bx)^p\right)}{x^3(d+ex)}\,dx = -\frac{b^2p\log(x)}{2a^2d} + \frac{b^2p\log(a+bx)}{2a^2d} + \frac{e^2\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d^3} - \frac{e^2\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^3} + \frac{e\log\left(c(a+bx)^p\right)}{d^2x} - \frac{\log\left(c(a+bx)^p\right)}{2dx^2} - \frac{e^2p\,\mathrm{PolyLog}\left(2,-\frac{e(a+bx)}{bd-ae}\right)}{d^3} + \frac{e^2p\,\mathrm{PolyLog}\left(2,\frac{bx}{a}+1\right)}{d^3} - \frac{bep\log(x)}{ad^2} + \frac{bep\log(a+bx)}{ad^2} - \frac{bp}{2adx}$$

```
[In] Int[Log[c*(a + b*x)^p]/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*(b*p)/(a*d*x) - (b^2*p*Log[x])/(2*a^2*d) - (b*e*p*Log[x])/(a*d^2) + (b
^2*p*Log[a + b*x])/(2*a^2*d) + (b*e*p*Log[a + b*x])/(a*d^2) - Log[c*(a + b*
x)^p]/(2*d*x^2) + (e*Log[c*(a + b*x)^p])/(d^2*x) + (e^2*Log[-((b*x)/a)]*Log
[c*(a + b*x)^p])/d^3 - (e^2*Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e
)])/d^3 - (e^2*p*PolyLog[2, -((e*(a + b*x))/(b*d - a*e))])/d^3 + (e^2*p*Pol
yLog[2, 1 + (b*x)/a])/d^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\log\left(c(a+bx)^p\right)}{dx^3} - \frac{e\log\left(c(a+bx)^p\right)}{d^2x^2} + \frac{e^2\log\left(c(a+bx)^p\right)}{d^3x} - \frac{e^3\log\left(c(a+bx)^p\right)}{d^3(d+ex)}\right) dx \\
&= \frac{\int \frac{\log(c(a+bx)^p)}{x^3}\,dx}{d} - \frac{e\int \frac{\log(c(a+bx)^p)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{\log(c(a+bx)^p)}{x}\,dx}{d^3} - \frac{e^3\int \frac{\log(c(a+bx)^p)}{d+ex}\,dx}{d^3}
\end{aligned}$$

$$= -\frac{\log\left(c(a+bx)^p\right)}{2dx^2} + \frac{e\log\left(c(a+bx)^p\right)}{d^2x} + \frac{e^2\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d^3} - \frac{e^2\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^3} + \frac{(bp)\int\frac{1}{x^2(a+bx)}\,dx}{2d} - \frac{(bep)\int\frac{1}{x(a+bx)}\,dx}{d^2} - \frac{(be^2p)\int\frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx}{d^3} + \frac{(be^2p)\int\frac{\log\left(\frac{b(d+ex)}{bd-ae}\right)}{a+bx}\,dx}{d^3}$$

$$= -\frac{\log\left(c(a+bx)^p\right)}{2dx^2} + \frac{e\log\left(c(a+bx)^p\right)}{d^2x} + \frac{e^2\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d^3} - \frac{e^2\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^3} + \frac{e^2p\mathrm{Li}_2\left(1+\frac{bx}{a}\right)}{d^3} + \frac{(bp)\int\left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx}{2d} - \frac{(bep)\int\frac{1}{x}\,dx}{ad^2} + \frac{(b^2ep)\int\frac{1}{a+bx}\,dx}{ad^2} + \frac{(e^2p)\,\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{ex}{bd-ae}\right)}{x}\,dx,x,a+bx\right)}{d^3}$$

$$= -\frac{bp}{2adx} - \frac{b^2p\log(x)}{2a^2d} - \frac{bep\log(x)}{ad^2} + \frac{b^2p\log(a+bx)}{2a^2d} + \frac{bep\log(a+bx)}{ad^2} - \frac{\log\left(c(a+bx)^p\right)}{2dx^2} + \frac{e\log\left(c(a+bx)^p\right)}{d^2x} + \frac{e^2\log\left(-\frac{bx}{a}\right)\log\left(c(a+bx)^p\right)}{d^3} - \frac{e^2\log\left(c(a+bx)^p\right)\log\left(\frac{b(d+ex)}{bd-ae}\right)}{d^3} - \frac{e^2p\mathrm{Li}_2\left(-\frac{e(a+bx)}{bd-ae}\right)}{d^3} + \frac{e^2p\mathrm{Li}_2\left(1+\frac{bx}{a}\right)}{d^3}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.85

$$\int\frac{\log\left(c(a+bx)^p\right)}{x^3(d+ex)}\,dx = -\frac{\frac{2bdep\log(x)}{a} - \frac{2bdep\log(a+bx)}{a} + \frac{bd^2p(a+bx\log(x)-bx\log(a+bx))}{a^2x} + \frac{d^2\log(c(a+bx)^p)}{x^2} - \frac{2de\log(c(a+bx)^p)}{x} - 2e^2\log\left(-\frac{bx}{a}\right)\mathrm{lo}\cdots}{\cdots}$$

```
[In] Integrate[Log[c*(a + b*x)^p]/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*((2*b*d*e*p*Log[x])/a - (2*b*d*e*p*Log[a + b*x])/a + (b*d^2*p*(a + b*x
*Log[x] - b*x*Log[a + b*x]))/(a^2*x) + (d^2*Log[c*(a + b*x)^p])/x^2 - (2*d*
e*Log[c*(a + b*x)^p])/x - 2*e^2*Log[-((b*x)/a)]*Log[c*(a + b*x)^p] + 2*e^2*
Log[c*(a + b*x)^p]*Log[(b*(d + e*x))/(b*d - a*e)] + 2*e^2*p*PolyLog[2, (e*(
a + b*x))/(-(b*d) + a*e)] - 2*e^2*p*PolyLog[2, 1 + (b*x)/a])/d^3
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.15

method	result
parts	$-\frac{\ln(c(bx+a)^p)e^2\ln(ex+d)}{d^3}-\frac{\ln(c(bx+a)^p)}{2d\,x^2}+\frac{\ln(c(bx+a)^p)e^2\ln(x)}{d^3}+\frac{e\ln(c(bx+a)^p)}{d^2x}-\frac{pb\left(\frac{2e^2\operatorname{dilog}\left(\frac{bx+a}{a}\right)}{d^3b}+\frac{2e^2\ln(x)\ln\left(\frac{bx+a}{a}\right)}{d^3b}\right.}{}$
risch	$-\frac{\ln((bx+a)^p)e^2\ln(ex+d)}{d^3}-\frac{\ln((bx+a)^p)}{2d\,x^2}+\frac{\ln((bx+a)^p)e^2\ln(x)}{d^3}+\frac{\ln((bx+a)^p)e}{d^2x}+\frac{bep\ln(bx+a)}{a\,d^2}+\frac{b^2p\ln(bx+a)}{2a^2d}-\frac{bep\ln(x}{a\,d^2}$

```
[In] int(ln(c*(b*x+a)^p)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(c*(b*x+a)^p)*e^2/d^3*ln(e*x+d)-1/2*ln(c*(b*x+a)^p)/d/x^2+ln(c*(b*x+a)^p
)*e^2/d^3*ln(x)+e*ln(c*(b*x+a)^p)/d^2/x-1/2*p*b*(2*e^2/d^3*dilog((b*x+a)/a)
/b+2*e^2/d^3*ln(x)*ln((b*x+a)/a)/b-2*e^2/d^3*dilog(((e*x+d)*b+a*e-b*d)/(a*e
-b*d))/b-2*e^2/d^3*ln(e*x+d)*ln(((e*x+d)*b+a*e-b*d)/(a*e-b*d))/b-1/d^2*((2*
a*e+b*d)/a^2*ln(b*x+a)-1/a*d/x+1/a^2*(-2*a*e-b*d)*ln(x)))
```

Fricas [F]

$$\int \frac{\log\left(c(a+bx)^p\right)}{x^3(d+ex)}\,dx=\int \frac{\log\left((bx+a)^pc\right)}{(ex+d)x^3}\,dx$$

```
[In] integrate(log(c*(b*x+a)^p)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x + a)^p*c)/(e*x^4 + d*x^3), x)
```

Sympy [F]

$$\int \frac{\log\left(c(a+bx)^p\right)}{x^3(d+ex)}\,dx=\int \frac{\log\left(c(a+bx)^p\right)}{x^3\left(d+ex\right)}\,dx$$

```
[In] integrate(ln(c*(b*x+a)**p)/x**3/(e*x+d),x)
```

```
[Out] Integral(log(c*(a + b*x)**p)/(x**3*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.95

$$\int \frac{\log(c(a+bx)^p)}{x^3(d+ex)}\,dx$$
$$= \frac{1}{2}\left(2\,e\left(\frac{\log(bx+a)}{ad^2} - \frac{\log(x)}{ad^2}\right) - \frac{2\left(\log\left(\frac{bx}{a}+1\right)\log(x) + \mathrm{Li}_2\left(-\frac{bx}{a}\right)\right)e^2}{bd^3} + \frac{2\left(\log(ex+d)\log\left(-\frac{bex+bd}{bd-ae} + \right.\right.}{bd^3}\right.$$
$$- \frac{1}{2}\left(\frac{2\,e^2\log(ex+d)}{d^3} - \frac{2\,e^2\log(x)}{d^3} - \frac{2\,ex-d}{d^2x^2}\right)\log\left((bx+a)^pc\right)$$

```
[In] integrate(log(c*(b*x+a)^p)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*(2*e*(log(b*x + a)/(a*d^2) - log(x)/(a*d^2)) - 2*(log(b*x/a + 1)*log(x)
 + dilog(-b*x/a))*e^2/(b*d^3) + 2*(log(e*x + d)*log(-(b*e*x + b*d)/(b*d - a
*e) + 1) + dilog((b*e*x + b*d)/(b*d - a*e)))*e^2/(b*d^3) + b*log(b*x + a)/(
a^2*d) - b*log(x)/(a^2*d) - 1/(a*d*x))*b*p - 1/2*(2*e^2*log(e*x + d)/d^3 -
2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2*x^2))*log((b*x + a)^p*c)
```

Giac [F]

$$\int \frac{\log(c(a+bx)^p)}{x^3(d+ex)}\,dx = \int \frac{\log\left((bx+a)^pc\right)}{(ex+d)x^3}\,dx$$

```
[In] integrate(log(c*(b*x+a)^p)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)^p*c)/((e*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c(a+bx)^p)}{x^3(d+ex)}\,dx = \int \frac{\ln\left(c\,(a+b\,x)^p\right)}{x^3\,(d+e\,x)}\,dx$$

```
[In] int(log(c*(a + b*x)^p)/(x^3*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b*x)^p)/(x^3*(d + e*x)), x)
```

3.226 $\int \frac{x^3 \log\left(c(a+bx^2)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 394

$$
\begin{aligned}
\int \frac{x^3 \log\left(c(a+bx^2)^p\right)}{d+ex}\,dx &= -\frac{2d^2px}{e^3}+\frac{2apx}{3be}+\frac{dpx^2}{2e^2}-\frac{2px^3}{9e}+\frac{2\sqrt{a}d^2p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^3} \\
&\quad-\frac{2a^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}e}+\frac{d^3p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e^4} \\
&\quad+\frac{d^3p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e^4}+\frac{d^2x\log\left(c(a+bx^2)^p\right)}{e^3} \\
&\quad+\frac{x^3\log\left(c(a+bx^2)^p\right)}{3e}-\frac{d(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2be^2} \\
&\quad-\frac{d^3\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e^4} \\
&\quad+\frac{d^3p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^4}+\frac{d^3p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e^4}
\end{aligned}
$$

```
[Out] -2*d^2*p*x/e^3+2/3*a*p*x/b/e+1/2*d*p*x^2/e^2-2/9*p*x^3/e-2/3*a^(3/2)*p*arct
an(x*b^(1/2)/a^(1/2))/b^(3/2)/e+d^2*x*ln(c*(b*x^2+a)^p)/e^3+1/3*x^3*ln(c*(b
*x^2+a)^p)/e-1/2*d*(b*x^2+a)*ln(c*(b*x^2+a)^p)/b/e^2-d^3*ln(e*x+d)*ln(c*(b*
x^2+a)^p)/e^4+d^3*p*ln(e*x+d)*ln(e*((-a)^(1/2)-x*b^(1/2))/(e*(-a)^(1/2)+d*b
^(1/2)))/e^4+d^3*p*ln(e*x+d)*ln(-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(1/2)+d*
b^(1/2)))/e^4+d^3*p*polylog(2,(e*x+d)*b^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/e^
4+d^3*p*polylog(2,(e*x+d)*b^(1/2)/(e*(-a)^(1/2)+d*b^(1/2)))/e^4+2*d^2*p*arc
tan(x*b^(1/2)/a^(1/2))*a^(1/2)/e^3/b^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {2516, 2498, 327, 211, 2504, 2436, 2332, 2505, 308, 2512, 266, 2463, 2441, 2440, 2438}

$$\int \frac{x^3 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = -\frac{2a^{3/2}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}e}+\frac{2\sqrt{a}d^2p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^3}-\frac{d^3\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e^4}+\frac{d^2x\log\left(c(a+bx^2)^p\right)}{e^3}-\frac{d(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2be^2}+\frac{x^3\log\left(c(a+bx^2)^p\right)}{3e}+\frac{d^3p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^4}+\frac{d^3p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e^4}+\frac{d^3p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e^4}+\frac{d^3p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^4}+\frac{2apx}{3be}-\frac{2d^2px}{e^3}+\frac{dpx^2}{2e^2}-\frac{2px^3}{9e}$$

```
[In] Int[(x^3*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

```
[Out] (-2*d^2*p*x)/e^3 + (2*a*p*x)/(3*b*e) + (d*p*x^2)/(2*e^2) - (2*p*x^3)/(9*e)
+ (2*Sqrt[a]*d^2*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*e^3) - (2*a^(3/2)*
p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(3*b^(3/2)*e) + (d^3*p*Log[(e*(Sqrt[-a] - Sq
rt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*(Sq
rt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e*x])/e^4 + (d^2*x*
Log[c*(a + b*x^2)^p])/e^3 + (x^3*Log[c*(a + b*x^2)^p])/(3*e) - (d*(a + b*x^
2)*Log[c*(a + b*x^2)^p])/(2*b*e^2) - (d^3*Log[d + e*x]*Log[c*(a + b*x^2)^p]
)/e^4 + (d^3*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e^
4 + (d^3*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/e^4
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^2 \log \left(c(a+bx^2)^p\right)}{e^3} - \frac{dx \log \left(c(a+bx^2)^p\right)}{e^2} + \frac{x^2 \log \left(c(a+bx^2)^p\right)}{e} - \frac{d^3 \log \left(c(a+bx^2)^p\right)}{e^3(d+ex)}\right) dx$$

$$= \frac{d^2 \int \log\left(c(a+bx^2)^p\right) dx}{e^3} - \frac{d^3 \int \frac{\log(c(a+bx^2)^p)}{d+ex} dx}{e^3} - \frac{d \int x \log\left(c(a+bx^2)^p\right) dx}{e^2} + \frac{\int x^2 \log\left(c(a+bx^2)^p\right) dx}{e}$$

$$= \frac{d^2 x \log\left(c(a+bx^2)^p\right)}{e^3} + \frac{x^3 \log\left(c(a+bx^2)^p\right)}{3e} - \frac{d^3 \log(d+ex) \log\left(c(a+bx^2)^p\right)}{e^4} - \frac{d\text{Subst}\left(\int \log\left(c(a+bx)^p\right) dx, x, x^2\right)}{2e^2} + \frac{(2bd^3p) \int \frac{x \log(d+ex)}{a+bx^2} dx}{e^4} - \frac{(2bd^2p) \int \frac{x^2}{a+bx^2} dx}{e^3} - \frac{(2bp) \int \frac{x^4}{a+bx^2} dx}{3e}$$

$$= -\frac{2d^2px}{e^3} + \frac{d^2 x \log\left(c(a+bx^2)^p\right)}{e^3} + \frac{x^3 \log\left(c(a+bx^2)^p\right)}{3e} - \frac{d^3 \log(d+ex) \log\left(c(a+bx^2)^p\right)}{e^4} - \frac{d\text{Subst}\left(\int \log\left(cx^p\right) dx, x, a+bx^2\right)}{2be^2} + \frac{(2bd^3p) \int \left(-\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right) dx}{e^4} + \frac{(2ad^2p) \int \frac{1}{a+bx^2} dx}{e^3} - \frac{(2bp) \int \left(-\frac{a}{b^2} + \frac{x^2}{b} + \frac{a^2}{b^2(a+bx^2)}\right) dx}{3e}$$

$$= -\frac{2d^2px}{e^3} + \frac{2apx}{3be} + \frac{dpx^2}{2e^2} - \frac{2px^3}{9e} + \frac{2\sqrt{a}d^2p \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^3} + \frac{d^2 x \log\left(c(a+bx^2)^p\right)}{e^3} + \frac{x^3 \log\left(c(a+bx^2)^p\right)}{3e} - \frac{d(a+bx^2) \log\left(c(a+bx^2)^p\right)}{2be^2} - \frac{d^3 \log(d+ex) \log\left(c(a+bx^2)^p\right)}{e^4} - \frac{\left(\sqrt{b}d^3p\right) \int \frac{\log(d+ex)}{\sqrt{-a}-\sqrt{b}x} dx}{e^4} + \frac{\left(\sqrt{b}d^3p\right) \int \frac{\log(d+ex)}{\sqrt{-a}+\sqrt{b}x} dx}{e^4} - \frac{(2a^2p) \int \frac{1}{a+bx^2} dx}{3be}$$

$$= -\frac{2d^2px}{e^3} + \frac{2apx}{3be} + \frac{dpx^2}{2e^2} - \frac{2px^3}{9e} + \frac{2\sqrt{a}d^2p \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^3} - \frac{2a^{3/2}p \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}e} + \frac{d^3p \log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right) \log(d+ex)}{e^4} + \frac{d^3p \log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right) \log(d+ex)}{e^4} + \frac{d^2 x \log\left(c(a+bx^2)^p\right)}{e^3} + \frac{x^3 \log\left(c(a+bx^2)^p\right)}{3e} - \frac{d(a+bx^2) \log\left(c(a+bx^2)^p\right)}{2be^2} - \frac{d^3 \log(d+ex) \log\left(c(a+bx^2)^p\right)}{e^4} - \frac{(d^3p) \int \frac{\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)}{d+ex} dx}{e^3} - \frac{(d^3p) \int \frac{\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{-\sqrt{b}d+\sqrt{-a}e}\right)}{d+ex} dx}{e^3}$$

$$= -\frac{2d^2px}{e^3} + \frac{2apx}{3be} + \frac{dpx^2}{2e^2} - \frac{2px^3}{9e} + \frac{2\sqrt{a}d^2p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^3} - \frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}e} + \frac{d^3p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e^4} + \frac{d^3p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e^4} + \frac{d^2x\log\left(c(a+bx^2)^p\right)}{e^3} + \frac{x^3\log\left(c(a+bx^2)^p\right)}{3e} - \frac{d(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2be^2} - \frac{d^3\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e^4} - \frac{(d^3p)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{b}x}{-\sqrt{b}d+\sqrt{-a}e}\right)}{x}dx,x,d+ex\right)}{e^4} - \frac{(d^3p)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{b}x}{\sqrt{b}d+\sqrt{-a}e}\right)}{x}dx,x,d+ex\right)}{e^4}$$

$$= -\frac{2d^2px}{e^3} + \frac{2apx}{3be} + \frac{dpx^2}{2e^2} - \frac{2px^3}{9e} + \frac{2\sqrt{a}d^2p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^3} - \frac{2a^{3/2}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3b^{3/2}e} + \frac{d^3p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e^4} + \frac{d^3p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e^4} + \frac{d^2x\log\left(c(a+bx^2)^p\right)}{e^3} + \frac{x^3\log\left(c(a+bx^2)^p\right)}{3e} - \frac{d(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2be^2} - \frac{d^3\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e^4} + \frac{d^3p\operatorname{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^4} + \frac{d^3p\operatorname{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e^4}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.94

$$\int\frac{x^3\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = \frac{-36d^2epx + \frac{12ae^3px}{b} - 4e^3px^3 + \frac{36\sqrt{a}d^2ep\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} - \frac{12a^{3/2}e^3p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}} + 18d^3p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d}{}$$

```
[In] Integrate[(x^3*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

```
[Out] (-36*d^2*e*p*x + (12*a*e^3*p*x)/b - 4*e^3*p*x^3 + (36*Sqrt[a]*d^2*e*p*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] - (12*a^(3/2)*e^3*p*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/b^(3/2) + 18*d^3*p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-
a]*e)]*Log[d + e*x] + 18*d^3*p*Log[(e*(Sqrt[-a] + Sqrt[b]*x))/(-(Sqrt[b]*d)
 + Sqrt[-a]*e)]*Log[d + e*x] + 18*d^2*e*x*Log[c*(a + b*x^2)^p] + 6*e^3*x^3*
Log[c*(a + b*x^2)^p] - 18*d^3*Log[d + e*x]*Log[c*(a + b*x^2)^p] + 9*d*e^2*(
p*x^2 - ((a + b*x^2)*Log[c*(a + b*x^2)^p])/b) + 18*d^3*p*PolyLog[2, (Sqrt[b
```

]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)] + 18*d^3*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/(18*e^4)

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.05

method	result
parts	$\frac{x^3 \ln(c(b x^2+a)^p)}{3e} - \frac{\ln(c(b x^2+a)^p) d x^2}{2e^2} + \frac{d^2 x \ln(c(b x^2+a)^p)}{e^3} - \frac{d^3 \ln(ex+d) \ln(c(b x^2+a)^p)}{e^4} - \frac{2pb\left(\frac{d^3\left(-\frac{\ln(ex+d)\left(\ln\left(\frac{e\sqrt{-a}}{e}\right.\right.}{}\right.}{}\right.}{}$
risch	$\frac{\ln((b x^2+a)^p) x^3}{3e} - \frac{\ln((b x^2+a)^p) d x^2}{2e^2} + \frac{\ln((b x^2+a)^p) x d^2}{e^3} - \frac{\ln((b x^2+a)^p) d^3 \ln(ex+d)}{e^4} - \frac{2p x^3}{9e} + \frac{dp x^2}{2e^2} - \frac{2d^2 px}{e^3} - \frac{49p}{18e}$

[In] int(x^3*ln(c*(b*x^2+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)

[Out] 1/3*x^3*ln(c*(b*x^2+a)^p)/e-1/2*ln(c*(b*x^2+a)^p)/e^2*d*x^2+d^2*x*ln(c*(b*x^2+a)^p)/e^3-d^3*ln(e*x+d)*ln(c*(b*x^2+a)^p)/e^4-2*p*b/e^2*(d^3/e^2*(-1/2*ln(e*x+d)*(ln((e*(-a*b)^(1/2)-(e*x+d)*b+b*d)/(e*(-a*b)^(1/2)+b*d))+ln((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1/2)-b*d)))/b-1/2*(dilog((e*(-a*b)^(1/2)-(e*x+d)*b+b*d)/(e*(-a*b)^(1/2)+b*d))+dilog((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1/2)-b*d)))/b)+1/6/e^2*(-1/b^2*(2*(e*x+d)*a*e^2-11*(e*x+d)*b*d^2+7/2*d*(e*x+d)^2*b-2/3*(e*x+d)^3*b)+1/b^2*a*e^2*(3/2*d*ln((e*x+d)^2*b-2*d*(e*x+d)*b+a*e^2+b*d^2)+(2*a*e^2-6*b*d^2)/e/(a*b)^(1/2)*arctan(1/2*(2*(e*x+d)*b-2*b*d)/e/(a*b)^(1/2)))))

Fricas [F]

$$\int \frac{x^3 \log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log\left((bx^2+a)^p c\right)}{ex+d}\,dx$$

[In] integrate(x^3*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")

[Out] integral(x^3*log((b*x^2 + a)^p*c)/(e*x + d), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*ln(c*(b*x**2+a)**p)/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log \left((bx^2+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x^3*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^3 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log \left((bx^2+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^3*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x^3*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{x^3\,\ln \left(c\,{\left(b\,x^2+a\right)}^p\right)}{d+e\,x}\,dx$$

```
[In] int((x^3*log(c*(a + b*x^2)^p))/(d + e*x),x)
[Out] int((x^3*log(c*(a + b*x^2)^p))/(d + e*x), x)
```

3.227 $\int \frac{x^2 \log\left(c(a+bx^2)^p\right)}{d+ex} dx$

Optimal result

Integrand size = 23, antiderivative size = 313

$$\begin{aligned}\int \frac{x^2 \log\left(c(a+bx^2)^p\right)}{d+ex} dx &= \frac{2dpx}{e^2} - \frac{px^2}{2e} - \frac{2\sqrt{a}dp\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^2} \\ &\quad - \frac{d^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e^3} \\ &\quad - \frac{d^2p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e^3} - \frac{dx\log\left(c(a+bx^2)^p\right)}{e^2} \\ &\quad + \frac{(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2be} + \frac{d^2\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e^3} \\ &\quad - \frac{d^2p\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^3} - \frac{d^2p\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e^3}\end{aligned}$$

```
[Out] 2*d*p*x/e^2-1/2*p*x^2/e-d*x*ln(c*(b*x^2+a)^p)/e^2+1/2*(b*x^2+a)*ln(c*(b*x^2
+a)^p)/b/e+d^2*ln(e*x+d)*ln(c*(b*x^2+a)^p)/e^3-d^2*p*ln(e*x+d)*ln(e*((-a)^(
1/2)-x*b^(1/2))/(e*(-a)^(1/2)+d*b^(1/2)))/e^3-d^2*p*ln(e*x+d)*ln(-e*((-a)^(
1/2)+x*b^(1/2))/(-e*(-a)^(1/2)+d*b^(1/2)))/e^3-d^2*p*polylog(2,(e*x+d)*b^(1
/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/e^3-d^2*p*polylog(2,(e*x+d)*b^(1/2)/(e*(-a)^
(1/2)+d*b^(1/2)))/e^3-2*d*p*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/e^2/b^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {2516, 2498, 327, 211, 2504, 2436, 2332, 2512, 266, 2463, 2441, 2440, 2438}

$$\int \frac{x^2\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = -\frac{2\sqrt{a}dp\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^2}+\frac{d^2\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e^3} - \frac{dx\log\left(c(a+bx^2)^p\right)}{e^2}+\frac{(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2be} - \frac{d^2p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^3}-\frac{d^2p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e^3} - \frac{d^2p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e^3} - \frac{d^2p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^3}+\frac{2dpx}{e^2}-\frac{px^2}{2e}$$

```
[In] Int[(x^2*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

```
[Out] (2*d*p*x)/e^2 - (p*x^2)/(2*e) - (2*Sqrt[a]*d*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(Sqrt[b]*e^2) - (d^2*p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a
]*e)]*Log[d + e*x])/e^3 - (d^2*p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*
d - Sqrt[-a]*e))]*Log[d + e*x])/e^3 - (d*x*Log[c*(a + b*x^2)^p])/e^2 + ((a
+ b*x^2)*Log[c*(a + b*x^2)^p])/(2*b*e) + (d^2*Log[d + e*x]*Log[c*(a + b*x^2
)^p])/e^3 - (d^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)]
)/e^3 - (d^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/e^
3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{d\log\left(c(a+bx^2)^p\right)}{e^2}+\frac{x\log\left(c(a+bx^2)^p\right)}{e}+\frac{d^2\log\left(c(a+bx^2)^p\right)}{e^2(d+ex)}\right)dx \\
&= -\frac{d\int\log\left(c(a+bx^2)^p\right)\,dx}{e^2}+\frac{d^2\int\frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx}{e^2}+\frac{\int x\log\left(c(a+bx^2)^p\right)\,dx}{e} \\
&= -\frac{dx\log\left(c(a+bx^2)^p\right)}{e^2}+\frac{d^2\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e^3} \\
&\quad+\frac{\operatorname{Subst}\left(\int\log\left(c(a+bx)^p\right)\,dx,x,x^2\right)}{2e}-\frac{(2bd^2p)\int\frac{x\log(d+ex)}{a+bx^2}\,dx}{e^3}+\frac{(2bdp)\int\frac{x^2}{a+bx^2}\,dx}{e^2} \\
&= \frac{2dpx}{e^2}-\frac{dx\log\left(c(a+bx^2)^p\right)}{e^2}+\frac{d^2\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e^3} \\
&\quad+\frac{\operatorname{Subst}\left(\int\log\left(cx^p\right)\,dx,x,a+bx^2\right)}{2be} \\
&\quad-\frac{(2bd^2p)\int\left(-\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{e^3}-\frac{(2adp)\int\frac{1}{a+bx^2}\,dx}{e^2}
\end{aligned}
$$

$$=\frac{2dpx}{e^2}-\frac{px^2}{2e}-\frac{2\sqrt{a}dp\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^2}-\frac{dx\log\left(c(a+bx^2)^p\right)}{e^2}+\frac{(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2be}+\frac{d^2\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e^3}+\frac{\left(\sqrt{b}d^2p\right)\int\frac{\log(d+ex)}{\sqrt{-a}-\sqrt{b}x}\,dx}{e^3}-\frac{\left(\sqrt{b}d^2p\right)\int\frac{\log(d+ex)}{\sqrt{-a}+\sqrt{b}x}\,dx}{e^3}$$

$$=\frac{2dpx}{e^2}-\frac{px^2}{2e}-\frac{2\sqrt{a}dp\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^2}-\frac{d^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e^3}-\frac{d^2p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e^3}-\frac{dx\log\left(c(a+bx^2)^p\right)}{e^2}+\frac{(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2be}+\frac{d^2\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e^3}+\frac{(d^2p)\int\frac{\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)}{d+ex}\,dx}{e^2}+\frac{(d^2p)\int\frac{\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{-\sqrt{b}d+\sqrt{-a}e}\right)}{d+ex}\,dx}{e^2}$$

$$=\frac{2dpx}{e^2}-\frac{px^2}{2e}-\frac{2\sqrt{a}dp\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^2}-\frac{d^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e^3}-\frac{d^2p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e^3}-\frac{dx\log\left(c(a+bx^2)^p\right)}{e^2}+\frac{(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2be}+\frac{d^2\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e^3}+\frac{(d^2p)\,\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{b}x}{-\sqrt{b}d+\sqrt{-a}e}\right)}{x}\,dx,x,d+ex\right)}{e^3}+\frac{(d^2p)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{b}x}{\sqrt{b}d+\sqrt{-a}e}\right)}{x}\,dx,x,d+ex\right)}{e^3}$$

$$=\frac{2dpx}{e^2}-\frac{px^2}{2e}-\frac{2\sqrt{a}dp\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^2}-\frac{d^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e^3}-\frac{d^2p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e^3}-\frac{dx\log\left(c(a+bx^2)^p\right)}{e^2}+\frac{(a+bx^2)\log\left(c(a+bx^2)^p\right)}{2be}+\frac{d^2\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e^3}-\frac{d^2p\,\mathrm{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^3}-\frac{d^2p\,\mathrm{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e^3}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.99

$$\int \frac{x^2 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \frac{2dpx}{e^2} - \frac{2\sqrt{a}dp\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e^2} - \frac{d^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e^3} - \frac{dx\log\left(c(a+bx^2)^p\right)}{e^2} + \frac{d^2\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e^3} - \frac{px^2-\frac{(a+bx^2)\log\left(c(a+bx^2)^p\right)}{b}}{2e} - \frac{d^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^3} - \frac{d^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e^3}$$

```
[In] Integrate[(x^2*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

```
[Out] (2*d*p*x)/e^2 - (2*Sqrt[a]*d*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*e^2) -
 (d^2*p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*
x])/e^3 - (d^2*p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))
]*Log[d + e*x])/e^3 - (d*x*Log[c*(a + b*x^2)^p])/e^2 + (d^2*Log[d + e*x]*Lo
g[c*(a + b*x^2)^p])/e^3 - (p*x^2 - ((a + b*x^2)*Log[c*(a + b*x^2)^p])/b)/(2
*e) - (d^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e^3
- (d^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/e^3
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.11

method	result
parts	$\frac{x^2\ln\left(c\left(b\,x^2+a\right)^p\right)}{2e}-\frac{dx\ln\left(c\left(b\,x^2+a\right)^p\right)}{e^2}+\frac{d^2\ln(ex+d)\ln\left(c\left(b\,x^2+a\right)^p\right)}{e^3}-\frac{2pb\left(\frac{(ex+d)^2}{4eb}-\frac{3d(ex+d)}{2eb}-\frac{ea\ln\left((ex+d)^2b-2d(ex+d)b+\right.}{4b^2}\right.}{}$
risch	$\frac{\ln\left(\left(b\,x^2+a\right)^p\right)x^2}{2e}-\frac{\ln\left(\left(b\,x^2+a\right)^p\right)dx}{e^2}+\frac{\ln\left(\left(b\,x^2+a\right)^p\right)d^2\ln(ex+d)}{e^3}-\frac{p\,d^2\ln(ex+d)\ln\left(\frac{e\sqrt{-ab}-(ex+d)b+bd}{e\sqrt{-ab}+bd}\right)}{e^3}-\frac{p\,d^2\ln(ex+d)\,\mathrm{l}}{}$

```
[In] int(x^2*ln(c*(b*x^2+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*ln(c*(b*x^2+a)^p)/e-d*x*ln(c*(b*x^2+a)^p)/e^2+d^2*ln(e*x+d)*ln(c*(b
*x^2+a)^p)/e^3-2*p*b/e^2*(1/4/e/b*(e*x+d)^2-3/2/e/b*d*(e*x+d)-1/4*e*a/b^2*l
n((e*x+d)^2*b-2*d*(e*x+d)*b+a*e^2+b*d^2)+a/b*d/(a*b)^(1/2)*arctan(1/2*(2*(e
*x+d)*b-2*b*d)/e/(a*b)^(1/2))-d^2/e*(-1/2*ln(e*x+d)*(ln((e*(-a*b)^(1/2)-(e*
```

```
x+d)*b+b*d)/(e*(-a*b)^(1/2)+b*d))+ln((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*
b)^(1/2)-b*d)))/b-1/2*(dilog((e*(-a*b)^(1/2)-(e*x+d)*b+b*d)/(e*(-a*b)^(1/2)
+b*d))+dilog((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1/2)-b*d)))/b))
```

Fricas [F]

$$\int \frac{x^2 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left((bx^2+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(x^2*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*ln(c*(b*x**2+a)**p)/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left((bx^2+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x^2*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^2 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left((bx^2+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^2*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x^2*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{x^2\,\ln\left(c\,(b\,x^2+a)^p\right)}{d+e\,x}\,dx$$

```
[In] int((x^2*log(c*(a + b*x^2)^p))/(d + e*x),x)
[Out] int((x^2*log(c*(a + b*x^2)^p))/(d + e*x), x)
```

3.228 $\int \frac{x\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 256

$$\begin{aligned}\int \frac{x\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = &-\frac{2px}{e}+\frac{2\sqrt{a}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e}+\frac{dp\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e^2}\\&+\frac{dp\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e^2}\\&+\frac{x\log\left(c(a+bx^2)^p\right)}{e}-\frac{d\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e^2}\\&+\frac{dp\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^2}+\frac{dp\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e^2}\end{aligned}$$

```
[Out] -2*p*x/e+x*ln(c*(b*x^2+a)^p)/e-d*ln(e*x+d)*ln(c*(b*x^2+a)^p)/e^2+d*p*ln(e*x
+d)*ln(e*((-a)^(1/2)-x*b^(1/2))/(e*(-a)^(1/2)+d*b^(1/2)))/e^2+d*p*ln(e*x+d)
*ln(-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(1/2)+d*b^(1/2)))/e^2+d*p*polylog(2,
(e*x+d)*b^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/e^2+d*p*polylog(2,(e*x+d)*b^(1/2
)/(e*(-a)^(1/2)+d*b^(1/2)))/e^2+2*p*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/e/b^(
1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2516, 2498, 327, 211, 2512, 266, 2463, 2441, 2440, 2438}

$$\int \frac{x\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = \frac{2\sqrt{a}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e}-\frac{d\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e^2}+\frac{x\log\left(c(a+bx^2)^p\right)}{e}+\frac{dp\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^2}+\frac{dp\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e^2}+\frac{dp\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e^2}+\frac{dp\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^2}-\frac{2px}{e}$$

```
[In] Int[(x*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

```
[Out] (-2*p*x)/e + (2*Sqrt[a]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*e) + (d*p*L
og[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/e^2 +
 (d*p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e
*x])/e^2 + (x*Log[c*(a + b*x^2)^p])/e - (d*Log[d + e*x]*Log[c*(a + b*x^2)^p
])/e^2 + (d*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e^2
 + (d*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/e^2
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{d\log\left(c\left(a+bx^2\right)^p\right)}{e(d+ex)}\right) dx$$

$$= \frac{\int \log\left(c\left(a+bx^2\right)^p\right)\,dx}{e} - \frac{d\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx}{e}$$

$$= \frac{x\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{d\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^2} + \frac{(2bdp)\int \frac{x\log(d+ex)}{a+bx^2}\,dx}{e^2} - \frac{(2bp)\int \frac{x^2}{a+bx^2}\,dx}{e}$$

$$= -\frac{2px}{e} + \frac{x\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{d\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^2} + \frac{(2bdp)\int \left(-\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right) dx}{e^2} + \frac{(2ap)\int \frac{1}{a+bx^2}\,dx}{e}$$

$$= -\frac{2px}{e} + \frac{2\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e} + \frac{x\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{d\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^2} - \frac{\left(\sqrt{b}dp\right)\int \frac{\log(d+ex)}{\sqrt{-a}-\sqrt{b}x}\,dx}{e^2} + \frac{\left(\sqrt{b}dp\right)\int \frac{\log(d+ex)}{\sqrt{-a}+\sqrt{b}x}\,dx}{e^2}$$

$$= -\frac{2px}{e} + \frac{2\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e} + \frac{dp\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e^2} + \frac{x\log\left(c\left(a+bx^2\right)^p\right)}{e} - \frac{d\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{e^2} - \frac{(dp)\int \frac{\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)}{d+ex}\,dx}{e} - \frac{(dp)\int \frac{\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{-\sqrt{b}d+\sqrt{-a}e}\right)}{d+ex}\,dx}{e}$$

$$= -\frac{2px}{e} + \frac{2\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e} + \frac{dp\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e^2} + \frac{x\log\left(c(a+bx^2)^p\right)}{e} - \frac{d\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e^2} - \frac{(dp)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{b}x}{-\sqrt{b}d+\sqrt{-a}e}\right)}{x}dx,x,d+ex\right)}{e^2} - \frac{(dp)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{b}x}{\sqrt{b}d+\sqrt{-a}e}\right)}{x}dx,x,d+ex\right)}{e^2}$$

$$= -\frac{2px}{e} + \frac{2\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}e} + \frac{dp\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e^2} + \frac{x\log\left(c(a+bx^2)^p\right)}{e} - \frac{d\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e^2} + \frac{dp\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e^2} + \frac{dp\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e^2}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.93

$$\int\frac{x\log\left(c(a+bx^2)^p\right)}{d+ex}dx = \frac{-2epx + \frac{2\sqrt{a}ep\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} + dp\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex) + dp\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{-\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex) + ex\log}{e^2}$$

```
[In] Integrate[(x*Log[c*(a + b*x^2)^p])/(d + e*x),x]
```

```
[Out] (-2*e*p*x + (2*Sqrt[a]*e*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + d*p*Log[(
e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x] + d*p*Log[
(e*(Sqrt[-a] + Sqrt[b]*x))/(-(Sqrt[b]*d) + Sqrt[-a]*e)]*Log[d + e*x] + e*x*
Log[c*(a + b*x^2)^p] - d*Log[d + e*x]*Log[c*(a + b*x^2)^p] + d*p*PolyLog[2,
 (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)] + d*p*PolyLog[2, (Sqrt[b]*(d
 + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/e^2
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.03

method	result
parts	$\frac{x\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)}{e}-\frac{d\ln\left(ex+d\right)\ln\left(c\left(b\,x^{2}+a\right)^{p}\right)}{e^{2}}-\frac{2pb\left(\frac{ex+d}{b}-\frac{ae\arctan\left(\frac{2(ex+d)b-2bd}{2e\sqrt{ab}}\right)}{b\sqrt{ab}}+d\left(-\frac{\ln(ex+d)\left(\ln\left(\frac{e\sqrt{-ab}-(ex+d)b+bd}{e\sqrt{-ab}+bd}\right)\right.}{2b}\right.\right.}{}$
risch	$\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)x}{e}-\frac{\ln\left(\left(b\,x^{2}+a\right)^{p}\right)d\ln\left(ex+d\right)}{e^{2}}-\frac{2px}{e}-\frac{2pd}{e^{2}}+\frac{2pa\arctan\left(\frac{2(ex+d)b-2bd}{2e\sqrt{ab}}\right)}{e\sqrt{ab}}+\frac{pd\ln(ex+d)\ln\left(\frac{e\sqrt{-ab}-(ex+d)b+}{e\sqrt{-ab}+bd}\right.}{e^{2}}$

```
[In] int(x*ln(c*(b*x^2+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(c*(b*x^2+a)^p)/e-d*ln(e*x+d)*ln(c*(b*x^2+a)^p)/e^2-2*p*b/e^2*(1/b*(e*x
+d)-a*e/b/(a*b)^(1/2)*arctan(1/2*(2*(e*x+d)*b-2*b*d)/e/(a*b)^(1/2))+d*(-1/2
*ln(e*x+d)*(ln((e*(-a*b)^(1/2)-(e*x+d)*b+b*d)/(e*(-a*b)^(1/2)+b*d))+ln((e*(
-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1/2)-b*d)))/b-1/2*(dilog((e*(-a*b)^(1
/2)-(e*x+d)*b+b*d)/(e*(-a*b)^(1/2)+b*d))+dilog((e*(-a*b)^(1/2)+(e*x+d)*b-b*
d)/(e*(-a*b)^(1/2)-b*d)))/b))
```

Fricas [F]

$$\int \frac{x\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx=\int \frac{x\log\left((bx^2+a)^pc\right)}{ex+d}\,dx$$

```
[In] integrate(x*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{x\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx=\int \frac{x\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx$$

```
[In] integrate(x*ln(c*(b*x**2+a)**p)/(e*x+d),x)
```

```
[Out] Integral(x*log(c*(a + b*x**2)**p)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{x \log (c(a+bx^2)^p)}{d+ex} \, dx = \int \frac{x \log ((bx^2+a)^p c)}{ex+d} \, dx$$

```
[In] integrate(x*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x \log (c(a+bx^2)^p)}{d+ex} \, dx = \int \frac{x \log ((bx^2+a)^p c)}{ex+d} \, dx$$

```
[In] integrate(x*log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x*log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \log (c(a+bx^2)^p)}{d+ex} \, dx = \int \frac{x \ln (c (b x^2+a)^p)}{d+e x} \, dx$$

```
[In] int((x*log(c*(a + b*x^2)^p))/(d + e*x),x)
[Out] int((x*log(c*(a + b*x^2)^p))/(d + e*x), x)
```

3.229 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 201

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e}$$

```
[Out] ln(e*x+d)*ln(c*(b*x^2+a)^p)/e-p*ln(e*x+d)*ln(e*((-a)^(1/2)-x*b^(1/2))/(e*(-
a)^(1/2)+d*b^(1/2)))/e-p*ln(e*x+d)*ln(-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(1
/2)+d*b^(1/2)))/e-p*polylog(2,(e*x+d)*b^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/e-
p*polylog(2,(e*x+d)*b^(1/2)/(e*(-a)^(1/2)+d*b^(1/2)))/e
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used

$= \{2512, 266, 2463, 2441, 2440, 2438\}$

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e} - \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e} - \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e} - \frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)}{e} - \frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e}$$

```
[In] Int[Log[c*(a + b*x^2)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])
/e) - (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d
+ e*x])/e + (Log[d + e*x]*Log[c*(a + b*x^2)^p])/e - (p*PolyLog[2, (Sqrt[b]*
(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e - (p*PolyLog[2, (Sqrt[b]*(d + e*x))
/(Sqrt[b]*d + Sqrt[-a]*e)])/e
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e} - \frac{(2bp)\int \frac{x\log(d+ex)}{a+bx^2}\,dx}{e} \\
&= \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e} - \frac{(2bp)\int \left(-\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{e} \\
&= \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e} + \frac{\left(\sqrt{b}p\right)\int \frac{\log(d+ex)}{\sqrt{-a}-\sqrt{b}x}\,dx}{e} - \frac{\left(\sqrt{b}p\right)\int \frac{\log(d+ex)}{\sqrt{-a}+\sqrt{b}x}\,dx}{e} \\
&= -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e} \\
&\quad - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e} \\
&\quad + p\int \frac{\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)}{d+ex}\,dx + p\int \frac{\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{-\sqrt{b}d+\sqrt{-a}e}\right)}{d+ex}\,dx \\
&= -\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e} \\
&\quad + \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e} + \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{b}x}{-\sqrt{b}d+\sqrt{-a}e}\right)}{x}\,dx, x, d+ex\right)}{e} \\
&\quad + \frac{p\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{b}x}{\sqrt{b}d+\sqrt{-a}e}\right)}{x}\,dx, x, d+ex\right)}{e}
\end{aligned}
$$

$$= -\frac{p\log\left(\frac{e(\sqrt{-a}-\sqrt{b}x)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(\sqrt{-a}+\sqrt{b}x)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e} - \frac{p\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e} - \frac{p\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\,dx = -\frac{p\log\left(\frac{e(\sqrt{-a}-\sqrt{b}x)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(\sqrt{-a}+\sqrt{b}x)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{e} - \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{e} - \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{e}$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])
/e) - (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d
+ e*x])/e + (Log[d + e*x]*Log[c*(a + b*x^2)^p])/e - (p*PolyLog[2, (Sqrt[b]*
(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/e - (p*PolyLog[2, (Sqrt[b]*(d + e*x))
/(Sqrt[b]*d + Sqrt[-a]*e)])/e
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.98

method	result
parts	$\frac{\ln(ex+d)\ln\left(c\left(b\,x^2+a\right)^p\right)}{e} - \frac{2pb\left(\frac{\ln(ex+d)\left(\ln\left(\frac{e\sqrt{-ab}-(ex+d)b+bd}{e\sqrt{-ab}+bd}\right)+\ln\left(\frac{e\sqrt{-ab}+(ex+d)b-bd}{e\sqrt{-ab}-bd}\right)\right)}{2b} + \frac{\text{dilog}\left(\frac{e\sqrt{-ab}-(ex+d)b+bd}{e\sqrt{-ab}+bd}\right)+\text{dilog}(e\cdots}{2b}\cdots\right)}{e}$
risch	$\frac{\ln\left(\left(b\,x^2+a\right)^p\right)\ln(ex+d)}{e} - \frac{p\ln(ex+d)\ln\left(\frac{e\sqrt{-ab}-(ex+d)b+bd}{e\sqrt{-ab}+bd}\right)}{e} - \frac{p\ln(ex+d)\ln\left(\frac{e\sqrt{-ab}+(ex+d)b-bd}{e\sqrt{-ab}-bd}\right)}{e} - \frac{p\,\text{dilog}\left(\frac{e\sqrt{-ab}-(ex+d)b+\cdots}{e\sqrt{-ab}+bd}\right)}{e}$

```
[In] int(ln(c*(b*x^2+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] ln(e*x+d)*ln(c*(b*x^2+a)^p)/e-2*p*b/e*(1/2*ln(e*x+d)*(ln((e*(-a*b)^(1/2)-(e
*x+d)*b+b*d)/(e*(-a*b)^(1/2)+b*d))+ln((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a
*b)^(1/2)-b*d)))/b+1/2*(dilog((e*(-a*b)^(1/2)-(e*x+d)*b+b*d)/(e*(-a*b)^(1/2
)+b*d))+dilog((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1/2)-b*d)))/b)
```

Fricas [F]

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{\log \left((bx^2+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{\log \left(c(a+bx^2)^p\right)}{d+ex}\,dx$$

```
[In] integrate(ln(c*(b*x**2+a)**p)/(e*x+d),x)
[Out] Integral(log(c*(a + b*x**2)**p)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{\log \left((bx^2+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{\log \left((bx^2+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{d+ex}\,dx = \int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{d+e\,x} \,dx$$

```
[In] int(log(c*(a + b*x^2)^p)/(d + e*x),x)
[Out] int(log(c*(a + b*x^2)^p)/(d + e*x), x)
```

3.230 $\int \frac{\log\left(c(a+bx^2)^p\right)}{x(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 247

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x(d+ex)}\,dx = \frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d} + \frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2d} - \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{d} + \frac{p\,\mathrm{PolyLog}\left(2,1+\frac{bx^2}{a}\right)}{2d}$$

```
[Out] 1/2*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)/d-ln(e*x+d)*ln(c*(b*x^2+a)^p)/d+p*ln(e*x
+d)*ln(e*((-a)^(1/2)-x*b^(1/2))/(e*(-a)^(1/2)+d*b^(1/2)))/d+p*ln(e*x+d)*ln(
-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(1/2)+d*b^(1/2)))/d+1/2*p*polylog(2,1+b*
x^2/a)/d+p*polylog(2,(e*x+d)*b^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/d+p*polylog
(2,(e*x+d)*b^(1/2)/(e*(-a)^(1/2)+d*b^(1/2)))/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2516, 2504, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x(d+ex)}\,dx = -\frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d} + \frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2d} + \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d} + \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{d} + \frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)}{d} + \frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d} + \frac{p\,\text{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2d}$$

```
[In] Int[Log[c*(a + b*x^2)^p]/(x*(d + e*x)),x]
```

```
[Out] (p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/d
 + (p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e
*x])/d + (Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(2*d) - (Log[d + e*x]*Log
[c*(a + b*x^2)^p])/d + (p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[
-a]*e)])/d + (p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/d
 + (p*PolyLog[2, 1 + (b*x^2)/a])/(2*d)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\log\left(c(a+bx^2)^p\right)}{dx} - \frac{e\log\left(c(a+bx^2)^p\right)}{d(d+ex)}\right) dx \\
&= \frac{\int \frac{\log\left(c(a+bx^2)^p\right)}{x}\, dx}{d} - \frac{e\int \frac{\log\left(c(a+bx^2)^p\right)}{d+ex}\, dx}{d}
\end{aligned}$$

$$=-\frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d}+\frac{\text{Subst}\left(\int\frac{\log(c(a+bx)^p)}{x}\,dx,x,x^2\right)}{2d}+\frac{(2bp)\int\frac{x\log(d+ex)}{a+bx^2}\,dx}{d}$$

$$=\frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2d}-\frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d}-\frac{(bp)\text{Subst}\left(\int\frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx,x,x^2\right)}{2d}+\frac{(2bp)\int\left(-\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{d}$$

$$=\frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2d}-\frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d}+\frac{p\text{Li}_2\left(1+\frac{bx^2}{a}\right)}{2d}-\frac{\left(\sqrt{b}p\right)\int\frac{\log(d+ex)}{\sqrt{-a}-\sqrt{b}x}\,dx}{d}+\frac{\left(\sqrt{b}p\right)\int\frac{\log(d+ex)}{\sqrt{-a}+\sqrt{b}x}\,dx}{d}$$

$$=\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d}+\frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d}+\frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2d}-\frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d}+\frac{p\text{Li}_2\left(1+\frac{bx^2}{a}\right)}{2d}-\frac{(ep)\int\frac{\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)}{d+ex}\,dx}{d}-\frac{(ep)\int\frac{\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{-\sqrt{b}d+\sqrt{-a}e}\right)}{d+ex}\,dx}{d}$$

$$=\frac{p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d}+\frac{p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d}+\frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2d}-\frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d}+\frac{p\text{Li}_2\left(1+\frac{bx^2}{a}\right)}{2d}-\frac{p\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{b}x}{-\sqrt{b}d+\sqrt{-a}e}\right)}{x}\,dx,x,d+ex\right)}{d}-\frac{p\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{b}x}{\sqrt{b}d+\sqrt{-a}e}\right)}{x}\,dx,x,d+ex\right)}{d}$$

$$= \frac{p\log\left(\frac{e(\sqrt{-a}-\sqrt{b}x)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e(\sqrt{-a}+\sqrt{b}x)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d} + \frac{\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2d} - \frac{\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d} + \frac{p\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d} + \frac{p\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{d} + \frac{p\text{Li}_2\left(1+\frac{bx^2}{a}\right)}{2d}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.93

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x(d+ex)}\,dx = \frac{2p\log\left(\frac{e(\sqrt{-a}-\sqrt{b}x)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex) + 2p\log\left(\frac{e(\sqrt{-a}+\sqrt{b}x)}{-\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex) + \log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right) - 2}{}$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(x*(d + e*x)),x]
```

```
[Out] (2*p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x]
+ 2*p*Log[(e*(Sqrt[-a] + Sqrt[b]*x))/(-(Sqrt[b]*d) + Sqrt[-a]*e)]*Log[d + e
*x] + Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] - 2*Log[d + e*x]*Log[c*(a + b*
x^2)^p] + 2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)] + 2*
p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)] + p*PolyLog[2, 1
 + (b*x^2)/a])/(2*d)
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.26

method	result
parts	$-\frac{\ln(ex+d)\ln\left(c\left(b\,x^2+a\right)^p\right)}{d}+\frac{\ln\left(c\left(b\,x^2+a\right)^p\right)\ln(x)}{d}-2pb\left(\frac{\frac{\ln(x)\left(\ln\left(\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}\right)+\ln\left(\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}\right)\right)}{2b}+\frac{\text{dilog}\left(\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}\right)+\text{dilog}(}{2b}}{d}\right.$
risch	$-\frac{\ln\left(\left(b\,x^2+a\right)^p\right)\ln(ex+d)}{d}+\frac{\ln\left(\left(b\,x^2+a\right)^p\right)\ln(x)}{d}-\frac{p\ln(x)\ln\left(\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}\right)}{d}-\frac{p\ln(x)\ln\left(\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}\right)}{d}-\frac{p\,\text{dilog}\left(\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}\right)}{d}$

```
[In] int(ln(c*(b*x^2+a)^p)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(e*x+d)*ln(c*(b*x^2+a)^p)/d+ln(c*(b*x^2+a)^p)/d*ln(x)-2*p*b*(1/d*(1/2*ln
(x)*(ln((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+ln((b*x+(-a*b)^(1/2))/(-a*b)^(1/2
)))/b+1/2*(dilog((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+dilog((b*x+(-a*b)^(1/2))
```

```
/(-a*b)^(1/2)))/b)-1/d*(1/2*ln(e*x+d)*(ln((e*(-a*b)^(1/2)-(e*x+d)*b+b*d)/(e
*(-a*b)^(1/2)+b*d))+ln((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1/2)-b*d))
)/b+1/2*(dilog((e*(-a*b)^(1/2)-(e*x+d)*b+b*d)/(e*(-a*b)^(1/2)+b*d))+dilog((
e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1/2)-b*d)))/b))
```

Fricas [F]

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x(d+ex)}\,dx = \int \frac{\log \left((bx^2+a)^p c\right)}{(ex+d)x}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x/(e*x+d),x, algorithm="fricas")
[Out] integral(log((b*x^2 + a)^p*c)/(e*x^2 + d*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(b*x**2+a)**p)/x/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x(d+ex)}\,dx = \int \frac{\log \left((bx^2+a)^p c\right)}{(ex+d)x}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((b*x^2 + a)^p*c)/((e*x + d)*x), x)
```

Giac [F]

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x(d+ex)}\,dx = \int \frac{\log \left((bx^2+a)^p c\right)}{(ex+d)x}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x/(e*x+d),x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)/((e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x(d+ex)}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)}{x\,\left(d+e\,x\right)}\,dx$$

```
[In] int(log(c*(a + b*x^2)^p)/(x*(d + e*x)),x)
[Out] int(log(c*(a + b*x^2)^p)/(x*(d + e*x)), x)
```

3.231 $\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 306

$$\begin{aligned}\int \frac{\log\left(c(a+bx^2)^p\right)}{x^2(d+ex)}\,dx &= \frac{2\sqrt{b}p\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d}-\frac{ep\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d^2}\\&\quad-\frac{ep\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d^2}\\&\quad-\frac{\log\left(c(a+bx^2)^p\right)}{dx}-\frac{e\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2d^2}\\&\quad+\frac{e\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d^2}-\frac{ep\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d^2}\\&\quad-\frac{ep\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{d^2}-\frac{ep\operatorname{PolyLog}\left(2,1+\frac{bx^2}{a}\right)}{2d^2}\end{aligned}$$

```
[Out] -ln(c*(b*x^2+a)^p)/d/x-1/2*e*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)/d^2+e*ln(e*x+d)
*ln(c*(b*x^2+a)^p)/d^2-e*p*ln(e*x+d)*ln(e*((-a)^(1/2)-x*b^(1/2))/(e*(-a)^(1
/2)+d*b^(1/2)))/d^2-e*p*ln(e*x+d)*ln(-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(1/
2)+d*b^(1/2)))/d^2-1/2*e*p*polylog(2,1+b*x^2/a)/d^2-e*p*polylog(2,(e*x+d)*b
^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/d^2-e*p*polylog(2,(e*x+d)*b^(1/2)/(e*(-a)
^(1/2)+d*b^(1/2)))/d^2+2*p*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/d/a^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {2516, 2505, 211, 2504, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$\begin{aligned}\int \frac{\log \left(c\left(a+b x^2\right)^p\right)}{x^2(d+e x)} dx = &\frac{2 \sqrt{b} p \arctan \left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{\sqrt{a} d}-\frac{e \log \left(-\frac{b x^2}{a}\right) \log \left(c\left(a+b x^2\right)^p\right)}{2 d^2} \\ &+\frac{e \log (d+e x) \log \left(c\left(a+b x^2\right)^p\right)}{d^2}-\frac{\log \left(c\left(a+b x^2\right)^p\right)}{d x} \\ &-\frac{e p \operatorname{PolyLog}\left(2, \frac{b x^2}{a}+1\right)}{2 d^2}-\frac{e p \operatorname{PolyLog}\left(2, \frac{\sqrt{b}(d+e x)}{\sqrt{b} d-\sqrt{-a} e}\right)}{d^2} \\ &-\frac{e p \operatorname{PolyLog}\left(2, \frac{\sqrt{b}(d+e x)}{\sqrt{b} d+\sqrt{-a} e}\right)}{d^2}-\frac{e p \log (d+e x) \log \left(\frac{e\left(\sqrt{-a}-\sqrt{b} x\right)}{\sqrt{-a} e+\sqrt{b} d}\right)}{d^2} \\ &-\frac{e p \log (d+e x) \log \left(-\frac{e\left(\sqrt{-a}+\sqrt{b} x\right)}{\sqrt{b} d-\sqrt{-a} e}\right)}{d^2}\end{aligned}$$

```
[In] Int[Log[c*(a + b*x^2)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*d) - (e*p*Log[(e*(Sqrt[-
a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x])/d^2 - (e*p*Log[-((
e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-a]*e))]*Log[d + e*x])/d^2 - Lo
g[c*(a + b*x^2)^p]/(d*x) - (e*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p])/(2*d^
2) + (e*Log[d + e*x]*Log[c*(a + b*x^2)^p])/d^2 - (e*p*PolyLog[2, (Sqrt[b]*(
d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/d^2 - (e*p*PolyLog[2, (Sqrt[b]*(d + e*
x))/(Sqrt[b]*d + Sqrt[-a]*e)])/d^2 - (e*p*PolyLog[2, 1 + (b*x^2)/a])/(2*d^2
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\log\left(c(a+bx^2)^p\right)}{dx^2}-\frac{e\log\left(c(a+bx^2)^p\right)}{d^2x}+\frac{e^2\log\left(c(a+bx^2)^p\right)}{d^2(d+ex)}\right)dx\\
&= \frac{\int \frac{\log(c(a+bx^2)^p)}{x^2}\,dx}{d}-\frac{e\int \frac{\log(c(a+bx^2)^p)}{x}\,dx}{d^2}+\frac{e^2\int \frac{\log(c(a+bx^2)^p)}{d+ex}\,dx}{d^2}\\
&= -\frac{\log\left(c(a+bx^2)^p\right)}{dx}+\frac{e\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d^2}\\
&\quad -\frac{e\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx,x,x^2\right)}{2d^2}+\frac{(2bp)\int \frac{1}{a+bx^2}\,dx}{d}-\frac{(2bep)\int \frac{x\log(d+ex)}{a+bx^2}\,dx}{d^2}\\
&= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d}-\frac{\log\left(c(a+bx^2)^p\right)}{dx}-\frac{e\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2d^2}\\
&\quad +\frac{e\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d^2}+\frac{(bep)\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx,x,x^2\right)}{2d^2}\\
&\quad -\frac{(2bep)\int \left(-\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{d^2}\\
&= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d}-\frac{\log\left(c(a+bx^2)^p\right)}{dx}-\frac{e\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2d^2}\\
&\quad +\frac{e\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d^2}-\frac{ep\text{Li}_2\left(1+\frac{bx^2}{a}\right)}{2d^2}\\
&\quad +\frac{\left(\sqrt{b}ep\right)\int \frac{\log(d+ex)}{\sqrt{-a}-\sqrt{b}x}\,dx}{d^2}-\frac{\left(\sqrt{b}ep\right)\int \frac{\log(d+ex)}{\sqrt{-a}+\sqrt{b}x}\,dx}{d^2}
\end{aligned}$$

$$= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{ep\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d^2} - \frac{\log\left(c(a+bx^2)^p\right)}{dx} - \frac{e\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2d^2} + \frac{e\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d^2} - \frac{ep\text{Li}_2\left(1+\frac{bx^2}{a}\right)}{2d^2} + \frac{(e^2p)\int\frac{\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)}{d+ex}dx}{d^2} + \frac{(e^2p)\int\frac{\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{-\sqrt{b}d+\sqrt{-a}e}\right)}{d+ex}dx}{d^2}$$

$$= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{ep\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d^2} - \frac{\log\left(c(a+bx^2)^p\right)}{dx} - \frac{e\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2d^2} + \frac{e\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d^2} - \frac{ep\text{Li}_2\left(1+\frac{bx^2}{a}\right)}{2d^2} + \frac{(ep)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{b}x}{-\sqrt{b}d+\sqrt{-a}e}\right)}{x}dx,x,d+ex\right)}{d^2} + \frac{(ep)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{b}x}{\sqrt{b}d+\sqrt{-a}e}\right)}{x}dx,x,d+ex\right)}{d^2}$$

$$= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{ep\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d^2} - \frac{\log\left(c(a+bx^2)^p\right)}{dx} - \frac{e\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2d^2} + \frac{e\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d^2} - \frac{ep\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d^2} - \frac{ep\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{d^2} - \frac{ep\text{Li}_2\left(1+\frac{bx^2}{a}\right)}{2d^2}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.92

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^2(d+ex)}\,dx = -\frac{-\frac{4\sqrt{b}dp\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}} + 2ep\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex) + 2ep\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{-\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex) + \frac{2d\log(c(a-}{x}}{}$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(x^2*(d + e*x)),x]
```

```
[Out] -1/2*((-4*Sqrt[b]*d*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + 2*e*p*Log[(e*(
Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[d + e*x] + 2*e*p*Log[(
e*(Sqrt[-a] + Sqrt[b]*x))/(-(Sqrt[b]*d) + Sqrt[-a]*e)]*Log[d + e*x] + (2*d*
Log[c*(a + b*x^2)^p])/x + e*Log[-((b*x^2)/a)]*Log[c*(a + b*x^2)^p] - 2*e*Lo
g[d + e*x]*Log[c*(a + b*x^2)^p] + 2*e*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqr
t[b]*d - Sqrt[-a]*e)] + 2*e*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + S
qrt[-a]*e)] + e*p*PolyLog[2, 1 + (b*x^2)/a])/d^2
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.16

method	result
parts	$\frac{e\ln(ex+d)\ln\left(c\left(b\,x^2+a\right)^p\right)}{d^2} - \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)}{dx} - \frac{\ln\left(c\left(b\,x^2+a\right)^p\right)e\ln(x)}{d^2} - 2pb\left(\frac{e\left(\frac{\ln(ex+d)\left(\ln\left(\frac{e\sqrt{-ab}-(ex+d)b+bd}{e\sqrt{-ab}+bd}\right)+\ln\left(\frac{e\sqrt{-}}{}\right.\right.}{2b}\right.}{}\right.$
risch	$\frac{\ln\left(\left(b\,x^2+a\right)^p\right)e\ln(ex+d)}{d^2} - \frac{\ln\left(\left(b\,x^2+a\right)^p\right)}{dx} - \frac{\ln\left(\left(b\,x^2+a\right)^p\right)e\ln(x)}{d^2} - \frac{pe\ln(ex+d)\ln\left(\frac{e\sqrt{-ab}-(ex+d)b+bd}{e\sqrt{-ab}+bd}\right)}{d^2} - \frac{pe\ln(ex+d)\ln\left(\right.}{}$

```
[In] int(ln(c*(b*x^2+a)^p)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] e*ln(e*x+d)*ln(c*(b*x^2+a)^p)/d^2-ln(c*(b*x^2+a)^p)/d/x-ln(c*(b*x^2+a)^p)*e
/d^2*ln(x)-2*p*b*(e/d^2*(1/2*ln(e*x+d)*(ln((e*(-a*b)^(1/2)-(e*x+d)*b+b*d)/(
e*(-a*b)^(1/2)+b*d))+ln((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1/2)-b*d)
))/b+1/2*(dilog((e*(-a*b)^(1/2)-(e*x+d)*b+b*d)/(e*(-a*b)^(1/2)+b*d))+dilog(
(e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1/2)-b*d)))/b)-1/d/(a*b)^(1/2)*ar
ctan(b*x/(a*b)^(1/2))-e/d^2*(1/2*ln(x)*(ln((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2)
)+ln((b*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/b+1/2*(dilog((-b*x+(-a*b)^(1/2))/(-a
*b)^(1/2))+dilog((b*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/b))
```

Fricas [F]

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\log \left((bx^2+a)^p c\right)}{(ex+d)x^2}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2/(e*x+d),x, algorithm="fricas")
[Out] integral(log((b*x^2 + a)^p*c)/(e*x^3 + d*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^2(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(b*x**2+a)**p)/x**2/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\log \left((bx^2+a)^p c\right)}{(ex+d)x^2}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((b*x^2 + a)^p*c)/((e*x + d)*x^2), x)
```

Giac [F]

$$\int \frac{\log \left(c(a+bx^2)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\log \left((bx^2+a)^p c\right)}{(ex+d)x^2}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x^2/(e*x+d),x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)/((e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\ln\left(c\,{\left(b\,x^2+a\right)}^p\right)}{x^2\,\left(d+e\,x\right)} \,d x$$

```
[In] int(log(c*(a + b*x^2)^p)/(x^2*(d + e*x)),x)
[Out] int(log(c*(a + b*x^2)^p)/(x^2*(d + e*x)), x)
```

3.232 $\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 371

$$\begin{aligned}\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3(d+ex)}\,dx = & -\frac{2\sqrt{b}ep\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d^2}+\frac{bp\log(x)}{ad} \\ & +\frac{e^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d^3} \\ & +\frac{e^2p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d^3} \\ & -\frac{bp\log\left(a+bx^2\right)}{2ad}-\frac{\log\left(c(a+bx^2)^p\right)}{2dx^2} \\ & +\frac{e\log\left(c(a+bx^2)^p\right)}{d^2x}+\frac{e^2\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2d^3} \\ & -\frac{e^2\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d^3}+\frac{e^2p\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d^3} \\ & +\frac{e^2p\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{d^3}+\frac{e^2p\operatorname{PolyLog}\left(2,1+\frac{bx^2}{a}\right)}{2d^3}\end{aligned}$$

```
[Out] b*p*ln(x)/a/d-1/2*b*p*ln(b*x^2+a)/a/d-1/2*ln(c*(b*x^2+a)^p)/d/x^2+e*ln(c*(b
*x^2+a)^p)/d^2/x+1/2*e^2*ln(-b*x^2/a)*ln(c*(b*x^2+a)^p)/d^3-e^2*ln(e*x+d)*l
n(c*(b*x^2+a)^p)/d^3+e^2*p*ln(e*x+d)*ln(e*((-a)^(1/2)-x*b^(1/2))/(e*(-a)^(1
/2)+d*b^(1/2)))/d^3+e^2*p*ln(e*x+d)*ln(-e*((-a)^(1/2)+x*b^(1/2))/(-e*(-a)^(
1/2)+d*b^(1/2)))/d^3+1/2*e^2*p*polylog(2,1+b*x^2/a)/d^3+e^2*p*polylog(2,(e*
x+d)*b^(1/2)/(-e*(-a)^(1/2)+d*b^(1/2)))/d^3+e^2*p*polylog(2,(e*x+d)*b^(1/2)
```

```
/(e*(-a)^(1/2)+d*b^(1/2)))/d^3-2*e*p*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/d^2/
a^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {2516, 2504, 2442, 36, 29, 31, 2505, 211, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$\begin{aligned}\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3(d+ex)}\,dx = &-\frac{2\sqrt{b}ep\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d^2}+\frac{e^2\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2d^3}\\ &-\frac{e^2\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d^3}+\frac{e\log\left(c(a+bx^2)^p\right)}{d^2x}\\ &-\frac{\log\left(c(a+bx^2)^p\right)}{2dx^2}+\frac{e^2p\,\mathrm{PolyLog}\left(2,\frac{bx^2}{a}+1\right)}{2d^3}\\ &+\frac{e^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d^3}+\frac{e^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{d^3}\\ &+\frac{e^2p\log(d+ex)\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}e+\sqrt{b}d}\right)}{d^3}\\ &+\frac{e^2p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d^3}\\ &-\frac{bp\log\left(a+bx^2\right)}{2ad}+\frac{bp\log(x)}{ad}\end{aligned}$$

```
[In] Int[Log[c*(a + b*x^2)^p]/(x^3*(d + e*x)),x]
```

```
[Out] (-2*Sqrt[b]*e*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*d^2) + (b*p*Log[x])/(
a*d) + (e^2*p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[
d + e*x])/d^3 + (e^2*p*Log[-((e*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*d - Sqrt[-
a]*e))]*Log[d + e*x])/d^3 - (b*p*Log[a + b*x^2])/(2*a*d) - Log[c*(a + b*x^2
)^p]/(2*d*x^2) + (e*Log[c*(a + b*x^2)^p])/(d^2*x) + (e^2*Log[-((b*x^2)/a)]*
Log[c*(a + b*x^2)^p])/(2*d^3) - (e^2*Log[d + e*x]*Log[c*(a + b*x^2)^p])/d^3
 + (e^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)])/d^3 + (
e^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)])/d^3 + (e^2*
p*PolyLog[2, 1 + (b*x^2)/a])/(2*d^3)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
```

```
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log\left(c(a+bx^2)^p\right)}{dx^3} - \frac{e\log\left(c(a+bx^2)^p\right)}{d^2x^2} + \frac{e^2\log\left(c(a+bx^2)^p\right)}{d^3x} - \frac{e^3\log\left(c(a+bx^2)^p\right)}{d^3(d+ex)}\right) dx$$

$$=\frac{\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^3}\,dx}{d}-\frac{e\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x^2}\,dx}{d^2}+\frac{e^2\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{x}\,dx}{d^3}-\frac{e^3\int \frac{\log\left(c\left(a+bx^2\right)^p\right)}{d+ex}\,dx}{d^3}$$

$$=\frac{e\log\left(c(a+bx^2)^p\right)}{d^2x}-\frac{e^2\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d^3}+\frac{\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x^2}\,dx,x,x^2\right)}{2d}+\frac{e^2\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx,x,x^2\right)}{2d^3}-\frac{(2bep)\int \frac{1}{a+bx^2}\,dx}{d^2}+\frac{(2be^2p)\int \frac{x\log(d+ex)}{a+bx^2}\,dx}{d^3}$$

$$=-\frac{2\sqrt{b}ep\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d^2}-\frac{\log\left(c(a+bx^2)^p\right)}{2dx^2}+\frac{e\log\left(c(a+bx^2)^p\right)}{d^2x}+\frac{e^2\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2d^3}-\frac{e^2\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d^3}+\frac{(bp)\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx,x,x^2\right)}{2d}-\frac{(be^2p)\,\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx,x,x^2\right)}{2d^3}+\frac{(2be^2p)\int \left(-\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\log(d+ex)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{d^3}$$

$$=-\frac{2\sqrt{b}ep\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d^2}-\frac{\log\left(c(a+bx^2)^p\right)}{2dx^2}+\frac{e\log\left(c(a+bx^2)^p\right)}{d^2x}+\frac{e^2\log\left(-\frac{bx^2}{a}\right)\log\left(c(a+bx^2)^p\right)}{2d^3}-\frac{e^2\log(d+ex)\log\left(c(a+bx^2)^p\right)}{d^3}+\frac{e^2p\text{Li}_2\left(1+\frac{bx^2}{a}\right)}{2d^3}+\frac{(bp)\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2ad}-\frac{(b^2p)\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx,x,x^2\right)}{2ad}-\frac{\left(\sqrt{b}e^2p\right)\int \frac{\log(d+ex)}{\sqrt{-a}-\sqrt{b}x}\,dx}{d^3}+\frac{\left(\sqrt{b}e^2p\right)\int \frac{\log(d+ex)}{\sqrt{-a}+\sqrt{b}x}\,dx}{d^3}$$

$$= -\frac{2\sqrt{b}ep\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d^2} + \frac{bp\log(x)}{ad} + \frac{e^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d^3}$$
$$+ \frac{e^2p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d^3} - \frac{bp\log\left(a+bx^2\right)}{2ad}$$
$$- \frac{\log\left(c\left(a+bx^2\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+bx^2\right)^p\right)}{d^2x} + \frac{e^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d^3}$$
$$- \frac{e^2\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d^3} + \frac{e^2p\text{Li}_2\left(1+\frac{bx^2}{a}\right)}{2d^3}$$
$$- \frac{(e^3p)\int\frac{\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)}{d+ex}dx}{d^3} - \frac{(e^3p)\int\frac{\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{-\sqrt{b}d+\sqrt{-a}e}\right)}{d+ex}dx}{d^3}$$

$$= -\frac{2\sqrt{b}ep\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d^2} + \frac{bp\log(x)}{ad} + \frac{e^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d^3}$$
$$+ \frac{e^2p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d^3} - \frac{bp\log\left(a+bx^2\right)}{2ad}$$
$$- \frac{\log\left(c\left(a+bx^2\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+bx^2\right)^p\right)}{d^2x} + \frac{e^2\log\left(-\frac{bx^2}{a}\right)\log\left(c\left(a+bx^2\right)^p\right)}{2d^3}$$
$$- \frac{e^2\log(d+ex)\log\left(c\left(a+bx^2\right)^p\right)}{d^3} + \frac{e^2p\text{Li}_2\left(1+\frac{bx^2}{a}\right)}{2d^3}$$
$$- \frac{(e^2p)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{b}x}{-\sqrt{b}d+\sqrt{-a}e}\right)}{x}dx, x, d+ex\right)}{d^3}$$
$$- \frac{(e^2p)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{b}x}{\sqrt{b}d+\sqrt{-a}e}\right)}{x}dx, x, d+ex\right)}{d^3}$$

$$= -\frac{2\sqrt{b}ep\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d^2} + \frac{bp\log(x)}{ad} + \frac{e^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{d^3}$$
$$+ \frac{e^2p\log\left(-\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}d-\sqrt{-a}e}\right)\log(d+ex)}{d^3} - \frac{bp\log(a+bx^2)}{2ad}$$
$$- \frac{\log(c(a+bx^2)^p)}{2dx^2} + \frac{e\log(c(a+bx^2)^p)}{d^2x}$$
$$+ \frac{e^2\log\left(-\frac{bx^2}{a}\right)\log(c(a+bx^2)^p)}{2d^3} - \frac{e^2\log(d+ex)\log(c(a+bx^2)^p)}{d^3}$$
$$+ \frac{e^2p\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d-\sqrt{-a}e}\right)}{d^3} + \frac{e^2p\text{Li}_2\left(\frac{\sqrt{b}(d+ex)}{\sqrt{b}d+\sqrt{-a}e}\right)}{d^3} + \frac{e^2p\text{Li}_2\left(1+\frac{bx^2}{a}\right)}{2d^3}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.94

$$\int \frac{\log(c(a+bx^2)^p)}{x^3(d+ex)}\,dx$$
$$= \frac{-\frac{4\sqrt{b}dep\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{2bd^2p\log(x)}{a} + 2e^2p\log\left(\frac{e\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex) + 2e^2p\log\left(\frac{e\left(\sqrt{-a}+\sqrt{b}x\right)}{-\sqrt{b}d+\sqrt{-a}e}\right)\log(d+ex)}{}$$

```
[In] Integrate[Log[c*(a + b*x^2)^p]/(x^3*(d + e*x)),x]
```

```
[Out] ((-4*Sqrt[b]*d*e*p*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + (2*b*d^2*p*Log[x]
)/a + 2*e^2*p*Log[(e*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*d + Sqrt[-a]*e)]*Log[
d + e*x] + 2*e^2*p*Log[(e*(Sqrt[-a] + Sqrt[b]*x))/(-(Sqrt[b]*d) + Sqrt[-a]*
e)]*Log[d + e*x] - (b*d^2*p*Log[a + b*x^2])/a - (d^2*Log[c*(a + b*x^2)^p])/
x^2 + (2*d*e*Log[c*(a + b*x^2)^p])/x - 2*e^2*Log[d + e*x]*Log[c*(a + b*x^2)
^p] + 2*e^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d - Sqrt[-a]*e)] + 2*
e^2*p*PolyLog[2, (Sqrt[b]*(d + e*x))/(Sqrt[b]*d + Sqrt[-a]*e)] + e^2*(Log[-
((b*x^2)/a)]*Log[c*(a + b*x^2)^p] + p*PolyLog[2, 1 + (b*x^2)/a]))/(2*d^3)
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.11

method	result
parts	$-\frac{e^{2}\ln(ex+d)\ln(c(b\,x^{2}+a)^{p})}{d^{3}}-\frac{\ln(c(b\,x^{2}+a)^{p})}{2d\,x^{2}}+\frac{\ln(c(b\,x^{2}+a)^{p})e^{2}\ln(x)}{d^{3}}+\frac{e\ln(c(b\,x^{2}+a)^{p})}{d^{2}x}-pb\left(-\frac{\ln(x)}{da}+\frac{\ln(b\,x^{2}+\right.$
risch	$-\frac{\ln((b\,x^{2}+a)^{p})e^{2}\ln(ex+d)}{d^{3}}-\frac{\ln((b\,x^{2}+a)^{p})}{2d\,x^{2}}+\frac{\ln((b\,x^{2}+a)^{p})e^{2}\ln(x)}{d^{3}}+\frac{\ln((b\,x^{2}+a)^{p})e}{d^{2}x}+\frac{bp\ln(x)}{ad}-\frac{bp\ln(b\,x^{2}+a)}{2ad}-$

[In] int(ln(c*(b*x^2+a)^p)/x^3/(e*x+d),x,method=_RETURNVERBOSE)

[Out] -e^2*ln(e*x+d)*ln(c*(b*x^2+a)^p)/d^3-1/2*ln(c*(b*x^2+a)^p)/d/x^2+ln(c*(b*x^2+a)^p)*e^2/d^3*ln(x)+e*ln(c*(b*x^2+a)^p)/d^2/x-p*b*(-1/d/a*ln(x)+1/2/d/a*ln(b*x^2+a)+2/d^2*e/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+2*e^2/d^3*(1/2*ln(x)*(ln((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+ln((b*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/b+1/2*(dilog((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))+dilog((b*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/b)-2*e^2/d^3*(1/2*ln(e*x+d)*(ln((e*(-a*b)^(1/2)-(e*x+d)*b+b*d)/(e*(-a*b)^(1/2)+b*d))+ln((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1/2)-b*d)))/b+1/2*(dilog((e*(-a*b)^(1/2)-(e*x+d)*b+b*d)/(e*(-a*b)^(1/2)+b*d))+dilog((e*(-a*b)^(1/2)+(e*x+d)*b-b*d)/(e*(-a*b)^(1/2)-b*d)))/b))

Fricas [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3(d+ex)}\,dx=\int \frac{\log\left((bx^2+a)^pc\right)}{(ex+d)x^3}\,dx$$

[In] integrate(log(c*(b*x^2+a)^p)/x^3/(e*x+d),x, algorithm="fricas")

[Out] integral(log((b*x^2 + a)^p*c)/(e*x^4 + d*x^3), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3(d+ex)}\,dx=\text{Timed out}$$

[In] integrate(ln(c*(b*x**2+a)**p)/x**3/(e*x+d),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3(d+ex)}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)}{(ex+d)x^3}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x^3/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((b*x^2 + a)^p*c)/((e*x + d)*x^3), x)
```

Giac [F]

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3(d+ex)}\,dx = \int \frac{\log\left((bx^2+a)^p c\right)}{(ex+d)x^3}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^p)/x^3/(e*x+d),x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^p*c)/((e*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^2)^p\right)}{x^3(d+ex)}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^p\right)}{x^3\,\left(d+e\,x\right)}\,dx$$

```
[In] int(log(c*(a + b*x^2)^p)/(x^3*(d + e*x)),x)
[Out] int(log(c*(a + b*x^2)^p)/(x^3*(d + e*x)), x)
```

3.233 $\int \frac{x^3 \log \left(c \left(a+b x^3\right)^p\right)}{d+e x} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 692

$$\int \frac{x^3 \log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = -\frac{3d^2px}{e^3}+\frac{3dpx^2}{4e^2}-\frac{px^3}{3e}-\frac{\sqrt{3}\sqrt[3]{a}d^2p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}e^3}$$
$$+\frac{\sqrt{3}a^{2/3}dp\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}e^2}$$
$$+\frac{\sqrt[3]{a}d^2p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e^3}+\frac{a^{2/3}dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}e^2}$$
$$+\frac{d^3p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e^4}$$
$$+\frac{d^3p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e^4}$$
$$+\frac{d^3p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e^4}$$
$$-\frac{\sqrt[3]{a}d^2p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}e^3}$$
$$-\frac{a^{2/3}dp\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}e^2}$$
$$+\frac{d^2x\log\left(c(a+bx^3)^p\right)}{e^3}-\frac{dx^2\log\left(c(a+bx^3)^p\right)}{2e^2}$$
$$+\frac{(a+bx^3)\log\left(c(a+bx^3)^p\right)}{3be}-\frac{d^3\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e^4}$$
$$+\frac{d^3p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^4}$$
$$+\frac{d^3p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e^4}$$
$$+\frac{d^3p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^4}$$

```
[Out] -3*d^2*p*x/e^3+3/4*d*p*x^2/e^2-1/3*p*x^3/e+a^(1/3)*d^2*p*ln(a^(1/3)+b^(1/3)
*x)/b^(1/3)/e^3+1/2*a^(2/3)*d*p*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)/e^2+d^3*p*ln(
-e*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/e^4+d^3*p*ln(-e*((-
1)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/e^4
+d^3*p*ln((-1)^(1/3)*e*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)
*a^(1/3)*e))*ln(e*x+d)/e^4-1/2*a^(1/3)*d^2*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/b^(1/3)/e^3-1/4*a^(2/3)*d*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/b^(2/3)/e^2+d^2*x*ln(c*(b*x^3+a)^p)/e^3-1/2*d*x^2*ln(c*(b*x^3+a)^p)/
e^2+1/3*(b*x^3+a)*ln(c*(b*x^3+a)^p)/b/e-d^3*ln(e*x+d)*ln(c*(b*x^3+a)^p)/e^4
+d^3*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-a^(1/3)*e))/e^4+d^3*p*polylog(2
,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))/e^4+d^3*p*polylog(2,b^(1
/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))/e^4-a^(1/3)*d^2*p*arctan(1/3*
(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)/e^3+1/2*a^(2/3)*d*p*
arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(2/3)/e^2
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {2516, 2498, 327, 206, 31, 648, 631, 210, 642, 2505, 298, 2504, 2436, 2332, 2512, 266,

2463, 2441, 2440, 2438}

$$\int \frac{x^3 \log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = \frac{\sqrt{3}a^{2/3}dp\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}e^2} - \frac{\sqrt[3]{a}d^2p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}e^3} - \frac{a^{2/3}dp\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}e^2} + \frac{a^{2/3}dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}e^2} - \frac{\sqrt{3}\sqrt[3]{a}d^2p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}e^3} - \frac{d^3\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e^4} + \frac{d^2x\log\left(c(a+bx^3)^p\right)}{e^3} - \frac{dx^2\log\left(c(a+bx^3)^p\right)}{2e^2} + \frac{(a+bx^3)\log\left(c(a+bx^3)^p\right)}{3be} + \frac{d^3p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^4} + \frac{d^3p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e^4} + \frac{d^3p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^4} + \frac{d^3p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^4} + \frac{d^3p\log(d+ex)\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^4} + \frac{d^3p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e^4} + \frac{\sqrt[3]{a}d^2p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e^3} - \frac{3d^2px}{e^3} + \frac{3dpx^2}{4e^2} - \frac{px^3}{3e}$$

```
[In] Int[(x^3*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

```
[Out] (-3*d^2*p*x)/e^3 + (3*d*p*x^2)/(4*e^2) - (p*x^3)/(3*e) - (Sqrt[3]*a^(1/3)*d
^2*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(b^(1/3)*e^3) + (Sq
rt[3]*a^(2/3)*d*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2*b^(
2/3)*e^2) + (a^(1/3)*d^2*p*Log[a^(1/3) + b^(1/3)*x])/(b^(1/3)*e^3) + (a^(2/
3)*d*p*Log[a^(1/3) + b^(1/3)*x])/(2*b^(2/3)*e^2) + (d^3*p*Log[-((e*(a^(1/3)
 + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/e^4 + (d^3*p*Log[-((
e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e))]*Lo
g[d + e*x])/e^4 + (d^3*p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)
)/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e^4 - (a^(1/3)*d^2*p*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)*e^3) - (a^(2/3)*d*
p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*b^(2/3)*e^2) + (d^2*x*
Log[c*(a + b*x^3)^p])/e^3 - (d*x^2*Log[c*(a + b*x^3)^p])/(2*e^2) + ((a + b*
x^3)*Log[c*(a + b*x^3)^p])/(3*b*e) - (d^3*Log[d + e*x]*Log[c*(a + b*x^3)^p]
)/e^4 + (d^3*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/e^4
 + (d^3*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)
])/e^4 + (d^3*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1
/3)*e)])/e^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^2\log\left(c(a+bx^3)^p\right)}{e^3} - \frac{dx\log\left(c(a+bx^3)^p\right)}{e^2} + \frac{x^2\log\left(c(a+bx^3)^p\right)}{e} - \frac{d^3\log\left(c(a+bx^3)^p\right)}{e^3(d+ex)}\right)dx$$

$$= \frac{d^2\int\log\left(c(a+bx^3)^p\right)\,dx}{e^3} - \frac{d^3\int\frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx}{e^3} - \frac{d\int x\log\left(c(a+bx^3)^p\right)\,dx}{e^2} + \frac{\int x^2\log\left(c(a+bx^3)^p\right)\,dx}{e}$$

$$= \frac{d^2x\log\left(c(a+bx^3)^p\right)}{e^3} - \frac{dx^2\log\left(c(a+bx^3)^p\right)}{2e^2} - \frac{d^3\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e^4} + \frac{\operatorname{Subst}\left(\int\log\left(c(a+bx)^p\right)\,dx,x,x^3\right)}{3e} + \frac{(3bd^3p)\int\frac{x^2\log(d+ex)}{a+bx^3}\,dx}{e^4} - \frac{(3bd^2p)\int\frac{x^3}{a+bx^3}\,dx}{e^3} + \frac{(3bdp)\int\frac{x^4}{a+bx^3}\,dx}{2e^2}$$

$$= -\frac{3d^2px}{e^3} + \frac{3dpx^2}{4e^2} + \frac{d^2x\log\left(c(a+bx^3)^p\right)}{e^3} - \frac{dx^2\log\left(c(a+bx^3)^p\right)}{2e^2} - \frac{d^3\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e^4} + \frac{\operatorname{Subst}\left(\int\log\left(cx^p\right)\,dx,x,a+bx^3\right)}{3be} + \frac{(3bd^3p)\int\left(\frac{\log(d+ex)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\log(d+ex)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\log(d+ex)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx}{e^4} + \frac{(3ad^2p)\int\frac{1}{a+bx^3}\,dx}{e^3} - \frac{(3adp)\int\frac{x}{a+bx^3}\,dx}{2e^2}$$

$$
\begin{aligned}
&= -\frac{3d^2px}{e^3} + \frac{3dpx^2}{4e^2} - \frac{px^3}{3e} + \frac{d^2x\log\left(c(a+bx^3)^p\right)}{e^3} \\
&\quad - \frac{dx^2\log\left(c(a+bx^3)^p\right)}{2e^2} + \frac{(a+bx^3)\log\left(c(a+bx^3)^p\right)}{3be} \\
&\quad - \frac{d^3\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e^4} + \frac{\left(\sqrt[3]{b}d^3p\right)\int\frac{\log(d+ex)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e^4} \\
&\quad + \frac{\left(\sqrt[3]{b}d^3p\right)\int\frac{\log(d+ex)}{-\sqrt[3]{-1}\,\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e^4} + \frac{\left(\sqrt[3]{b}d^3p\right)\int\frac{\log(d+ex)}{(-1)^{2/3}\,\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e^4} \\
&\quad + \frac{\left(\sqrt[3]{a}d^2p\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e^3} + \frac{\left(\sqrt[3]{a}d^2p\right)\int\frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{e^3} \\
&\quad + \frac{\left(a^{2/3}dp\right)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{2\sqrt[3]{b}e^2} - \frac{\left(a^{2/3}dp\right)\int\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{b}e^2}
\end{aligned}
$$

$$= -\frac{3d^2px}{e^3} + \frac{3dpx^2}{4e^2} - \frac{px^3}{3e} + \frac{\sqrt[3]{a}d^2p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e^3}$$

$$+ \frac{a^{2/3}dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}e^2} + \frac{d^3p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e^4}$$

$$+ \frac{d^3p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e^4}$$

$$+ \frac{d^3p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e^4} + \frac{d^2x\log\left(c(a+bx^3)^p\right)}{e^3}$$

$$- \frac{dx^2\log\left(c(a+bx^3)^p\right)}{2e^2} + \frac{(a+bx^3)\log\left(c(a+bx^3)^p\right)}{3be}$$

$$- \frac{d^3\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e^4} + \frac{\left(3a^{2/3}d^2p\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2e^3}$$

$$- \frac{\left(\sqrt[3]{a}d^2p\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{b}e^3} - \frac{\left(d^3p\right)\int\frac{\log\left(\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d+\sqrt[3]{a}e}\right)}{d+ex}\,dx}{e^3}$$

$$- \frac{\left(d^3p\right)\int\frac{\log\left(\frac{e\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d-\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d+ex}\,dx}{e^3} - \frac{\left(d^3p\right)\int\frac{\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d+(-1)^{2/3}\sqrt[3]{a}e}\right)}{d+ex}\,dx}{e^3}$$

$$- \frac{\left(a^{2/3}dp\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{4b^{2/3}e^2} - \frac{(3adp)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{4\sqrt[3]{b}e^2}$$

$$
\begin{aligned}
&= -\frac{3d^2px}{e^3} + \frac{3dpx^2}{4e^2} - \frac{px^3}{3e} + \frac{\sqrt[3]{a}d^2p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e^3} \\
&+ \frac{a^{2/3}dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}e^2} + \frac{d^3p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e^4} \\
&+ \frac{d^3p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e^4} \\
&+ \frac{d^3p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e^4} \\
&- \frac{\sqrt[3]{a}d^2p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}e^3} - \frac{a^{2/3}dp\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}e^2} \\
&+ \frac{d^2x\log\left(c(a+bx^3)^p\right)}{e^3} - \frac{dx^2\log\left(c(a+bx^3)^p\right)}{2e^2} \\
&+ \frac{(a+bx^3)\log\left(c(a+bx^3)^p\right)}{3be} - \frac{d^3\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e^4} \\
&- \frac{(d^3p)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d+\sqrt[3]{a}e}\right)}{x}\,dx,x,d+ex\right)}{e^4} \\
&- \frac{(d^3p)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d-\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{x}\,dx,x,d+ex\right)}{e^4} \\
&- \frac{(d^3p)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d+(-1)^{2/3}\sqrt[3]{a}e}\right)}{x}\,dx,x,d+ex\right)}{e^4} \\
&+ \frac{\left(3\sqrt[3]{a}d^2p\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}e^3} \\
&- \frac{\left(3a^{2/3}dp\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{2b^{2/3}e^2}
\end{aligned}
$$

$$= -\frac{3d^2px}{e^3} + \frac{3dpx^2}{4e^2} - \frac{px^3}{3e} - \frac{\sqrt{3}\sqrt[3]{a}d^2p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}e^3}$$
$$+ \frac{\sqrt{3}a^{2/3}dp\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}e^2} + \frac{\sqrt[3]{a}d^2p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e^3}$$
$$+ \frac{a^{2/3}dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}e^2} + \frac{d^3p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e^4}$$
$$+ \frac{d^3p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e^4}$$
$$+ \frac{d^3p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e^4}$$
$$- \frac{\sqrt[3]{a}d^2p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}e^3} - \frac{a^{2/3}dp\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}e^2}$$
$$+ \frac{d^2x\log\left(c(a+bx^3)^p\right)}{e^3} - \frac{dx^2\log\left(c(a+bx^3)^p\right)}{2e^2} + \frac{(a+bx^3)\log\left(c(a+bx^3)^p\right)}{3be}$$
$$- \frac{d^3\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e^4} + \frac{d^3p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^4}$$
$$+ \frac{d^3p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e^4} + \frac{d^3p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^4}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.27 (sec) , antiderivative size = 581, normalized size of antiderivative = 0.84

$$\int \frac{x^3\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx$$
$$= \frac{-36bd^2epx + 9bde^2px^2 - 4be^3px^3 - 12\sqrt{3}\sqrt[3]{a}b^{2/3}d^2ep\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 9bde^2px^2\,\text{Hypergeometric2F1}}{}$$

```
[In] Integrate[(x^3*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

```
[Out] (-36*b*d^2*e*p*x + 9*b*d*e^2*p*x^2 - 4*b*e^3*p*x^3 - 12*Sqrt[3]*a^(1/3)*b^(
2/3)*d^2*e*p*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 9*b*d*e^2*p*x^2*
Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)] + 12*a^(1/3)*b^(2/3)*d^2*e*p*L
og[a^(1/3) + b^(1/3)*x] + 12*b*d^3*p*Log[(e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x
))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x] + 12*b*d^3*p*Log[(e*(a^
(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + a^(1/3)*e)]*Log[d + e*x] + 12*b*d^3*p*L
og[(e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + (-1)^(2/3)*a^(1/3)*
e)]*Log[d + e*x] - 6*a^(1/3)*b^(2/3)*d^2*e*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2] + 4*a*e^3*Log[c*(a + b*x^3)^p] + 12*b*d^2*e*x*Log[c*(a + b
*x^3)^p] - 6*b*d*e^2*x^2*Log[c*(a + b*x^3)^p] + 4*b*e^3*x^3*Log[c*(a + b*x^
3)^p] - 12*b*d^3*Log[d + e*x]*Log[c*(a + b*x^3)^p] + 12*b*d^3*p*PolyLog[2,
(b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)] + 12*b*d^3*p*PolyLog[2, (b^(1/
3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)] + 12*b*d^3*p*PolyLog[2, (
b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/(12*b*e^4)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.78 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.44

method	result
parts	$\frac{\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)x^{3}}{3e}-\frac{d\,x^{2}\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{2e^{2}}+\frac{d^{2}x\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{e^{3}}-\frac{d^{3}\ln\left(ex+d\right)\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{e^{4}}-\frac{3pb\left(-\frac{\frac{2(ex+d)^{3}}{3}-\frac{7d(ex+\cdots}{2}}{b}\cdots\right.}{\cdots}$
risch	$\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)x^{3}}{3e}-\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)d\,x^{2}}{2e^{2}}+\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)x\,d^{2}}{e^{3}}-\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)d^{3}\ln\left(ex+d\right)}{e^{4}}-\frac{p\,x^{3}}{3e}+\frac{3dp\,x^{2}}{4e^{2}}-\frac{3d^{2}px}{e^{3}}-\cdots$

```
[In] int(x^3*ln(c*(b*x^3+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(c*(b*x^3+a)^p)/e*x^3-1/2*d*x^2*ln(c*(b*x^3+a)^p)/e^2+d^2*x*ln(c*(b*x
^3+a)^p)/e^3-d^3*ln(e*x+d)*ln(c*(b*x^3+a)^p)/e^4-3*p*b/e^3*(-1/6/e*(-1/b*(2
/3*(e*x+d)^3-7/2*d*(e*x+d)^2+11*d^2*(e*x+d))+1/3/b^2*sum((2*_R^2-7*_R*d+11*
d^2)/(_R^2-2*_R*d+d^2)*ln(e*x-_R+d),_R=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+
a*e^3-b*d^3))*a*e^3)-1/3*d^3/e/b*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((
-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3)))
```

Fricas [F]

$$\int \frac{x^3 \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log \left((bx^3+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^3*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(x^3*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*ln(c*(b*x**3+a)**p)/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3 \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log \left((bx^3+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^3*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x^3*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^3 \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log \left((bx^3+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^3*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x^3*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{x^3\,\ln\left(c\,(b\,x^3+a)^p\right)}{d+e\,x}\,dx$$

```
[In] int((x^3*log(c*(a + b*x^3)^p))/(d + e*x),x)
[Out] int((x^3*log(c*(a + b*x^3)^p))/(d + e*x), x)
```

3.234 $\int \frac{x^2 \log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 643

$$\int \frac{x^2\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = \frac{3dpx}{e^2}-\frac{3px^2}{4e}+\frac{\sqrt{3}\sqrt[3]{a}dp\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}e^2}-\frac{\sqrt{3}a^{2/3}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}e}-\frac{\sqrt[3]{a}dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e^2}-\frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}e}-\frac{d^2p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e^3}-\frac{d^2p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e^3}-\frac{d^2p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e^3}+\frac{\sqrt[3]{a}dp\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}e^2}+\frac{a^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}e}-\frac{dx\log\left(c(a+bx^3)^p\right)}{e^2}+\frac{x^2\log\left(c(a+bx^3)^p\right)}{2e}+\frac{d^2\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e^3}-\frac{d^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^3}-\frac{d^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e^3}-\frac{d^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^3}$$

```
[Out] 3*d*p*x/e^2-3/4*p*x^2/e-a^(1/3)*d*p*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)/e^2-1/2*a
^(2/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)/e-d^2*p*ln(-e*(a^(1/3)+b^(1/3)*x)/(b
```

```
^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/e^3-d^2*p*ln(-e*((-1)^(2/3)*a^(1/3)+b^(1/3)*
x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/e^3-d^2*p*ln((-1)^(1/3)*e*(a
^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))*ln(e*x+d)/e^
3+1/2*a^(1/3)*d*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(1/3)/e^2+1/4
*a^(2/3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(2/3)/e-d*x*ln(c*(b*
x^3+a)^p)/e^2+1/2*x^2*ln(c*(b*x^3+a)^p)/e+d^2*ln(e*x+d)*ln(c*(b*x^3+a)^p)/e
^3-d^2*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-a^(1/3)*e))/e^3-d^2*p*polylog
(2,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))/e^3-d^2*p*polylog(2,b^
(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))/e^3+a^(1/3)*d*p*arctan(1/3*
(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)/e^2-1/2*a^(2/3)*p*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/b^(2/3)/e
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {2516, 2498, 327, 206, 31, 648, 631, 210, 642, 2505, 298, 2512, 266, 2463, 2441, 2440,

2438}

$$\int \frac{x^2 \log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = -\frac{\sqrt{3}a^{2/3}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}e} + \frac{\sqrt[3]{a}dp\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}e^2} + \frac{a^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}e} - \frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}e} + \frac{\sqrt{3}\sqrt[3]{a}dp\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}e^2} + \frac{d^2\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e^3} - \frac{dx\log\left(c(a+bx^3)^p\right)}{e^2} + \frac{x^2\log\left(c(a+bx^3)^p\right)}{2e} - \frac{d^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^3} - \frac{d^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e^3} - \frac{d^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^3} - \frac{d^2p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^3} - \frac{d^2p\log(d+ex)\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^3} - \frac{d^2p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e^3} - \frac{\sqrt[3]{a}dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e^2} + \frac{3dpx}{e^2} - \frac{3px^2}{4e}$$

```
[In] Int[(x^2*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

```
[Out] (3*d*p*x)/e^2 - (3*p*x^2)/(4*e) + (Sqrt[3]*a^(1/3)*d*p*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(b^(1/3)*e^2) - (Sqrt[3]*a^(2/3)*p*ArcTan[(a
```

```
^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2*b^(2/3)*e) - (a^(1/3)*d*p*Log[
a^(1/3) + b^(1/3)*x])/(b^(1/3)*e^2) - (a^(2/3)*p*Log[a^(1/3) + b^(1/3)*x])/
(2*b^(2/3)*e) - (d^2*p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)
*e))]*Log[d + e*x])/e^3 - (d^2*p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))
/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e))]*Log[d + e*x])/e^3 - (d^2*p*Log[((-1)^
(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e
)]*Log[d + e*x])/e^3 + (a^(1/3)*d*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(2*b^(1/3)*e^2) + (a^(2/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(4*b^(2/3)*e) - (d*x*Log[c*(a + b*x^3)^p])/e^2 + (x^2*Log[c*(a +
 b*x^3)^p])/(2*e) + (d^2*Log[d + e*x]*Log[c*(a + b*x^3)^p])/e^3 - (d^2*p*Po
lyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/e^3 - (d^2*p*PolyLog
[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/e^3 - (d^2*p*P
olyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/e^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{d\log\left(c\left(a+bx^3\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+bx^3\right)^p\right)}{e}+\frac{d^2\log\left(c\left(a+bx^3\right)^p\right)}{e^2(d+ex)}\right)dx\\
&= -\frac{d\int\log\left(c\left(a+bx^3\right)^p\right)\,dx}{e^2}+\frac{d^2\int\frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx}{e^2}+\frac{\int x\log\left(c\left(a+bx^3\right)^p\right)\,dx}{e}\\
&= -\frac{dx\log\left(c\left(a+bx^3\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+bx^3\right)^p\right)}{2e}+\frac{d^2\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e^3}\\
&\quad -\frac{\left(3bd^2p\right)\int\frac{x^2\log(d+ex)}{a+bx^3}\,dx}{e^3}+\frac{(3bdp)\int\frac{x^3}{a+bx^3}\,dx}{e^2}-\frac{(3bp)\int\frac{x^4}{a+bx^3}\,dx}{2e}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3dpx}{e^2} - \frac{3px^2}{4e} - \frac{dx \log\left(c(a+bx^3)^p\right)}{e^2} \\
&\quad + \frac{x^2 \log\left(c(a+bx^3)^p\right)}{2e} + \frac{d^2 \log(d+ex) \log\left(c(a+bx^3)^p\right)}{e^3} \\
&\quad - \frac{(3bd^2p) \int \left(\frac{\log(d+ex)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\log(d+ex)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\log(d+ex)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right) dx}{e^3} \\
&\quad - \frac{(3adp) \int \frac{1}{a+bx^3}\, dx}{e^2} + \frac{(3ap) \int \frac{x}{a+bx^3}\, dx}{2e} \\
&= \frac{3dpx}{e^2} - \frac{3px^2}{4e} - \frac{dx \log\left(c(a+bx^3)^p\right)}{e^2} + \frac{x^2 \log\left(c(a+bx^3)^p\right)}{2e} \\
&\quad + \frac{d^2 \log(d+ex) \log\left(c(a+bx^3)^p\right)}{e^3} - \frac{\left(\sqrt[3]{b}d^2p\right) \int \frac{\log(d+ex)}{\sqrt[3]{a}+\sqrt[3]{b}x}\, dx}{e^3} \\
&\quad - \frac{\left(\sqrt[3]{b}d^2p\right) \int \frac{\log(d+ex)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\, dx}{e^3} - \frac{\left(\sqrt[3]{b}d^2p\right) \int \frac{\log(d+ex)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\, dx}{e^3} \\
&\quad - \frac{\left(\sqrt[3]{a}dp\right) \int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\, dx}{e^2} - \frac{\left(\sqrt[3]{a}dp\right) \int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\, dx}{e^2} \\
&\quad - \frac{\left(a^{2/3}p\right) \int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\, dx}{2\sqrt[3]{b}e} + \frac{\left(a^{2/3}p\right) \int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\, dx}{2\sqrt[3]{b}e}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3dpx}{e^2} - \frac{3px^2}{4e} - \frac{\sqrt[3]{a}dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e^2} - \frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}e} \\
&\quad - \frac{d^2p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e^3} \\
&\quad - \frac{d^2p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e^3} \\
&\quad - \frac{d^2p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e^3} \\
&\quad - \frac{dx\log\left(c(a+bx^3)^p\right)}{e^2} + \frac{x^2\log\left(c(a+bx^3)^p\right)}{2e} \\
&\quad + \frac{d^2\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e^3} - \frac{\left(3a^{2/3}dp\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2e^2} \\
&\quad + \frac{\left(\sqrt[3]{a}dp\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{b}e^2} + \frac{\left(d^2p\right)\int\frac{\log\left(\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d+\sqrt[3]{a}e}\right)}{d+ex}\,dx}{e^2} \\
&\quad + \frac{\left(d^2p\right)\int\frac{\log\left(\frac{e\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d-\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d+ex}\,dx}{e^2} + \frac{\left(d^2p\right)\int\frac{\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d+(-1)^{2/3}\sqrt[3]{a}e}\right)}{d+ex}\,dx}{e^2} \\
&\quad + \frac{\left(a^{2/3}p\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{4b^{2/3}e} + \frac{(3ap)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{4\sqrt[3]{b}e}
\end{aligned}
$$

$$= \frac{3dpx}{e^2} - \frac{3px^2}{4e} - \frac{\sqrt[3]{a} dp \log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{\sqrt[3]{b} e^2} - \frac{a^{2/3} p \log\left(\sqrt[3]{a} + \sqrt[3]{b}x\right)}{2b^{2/3} e}$$

$$- \frac{d^2 p \log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right) \log(d+ex)}{e^3}$$

$$- \frac{d^2 p \log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right) \log(d+ex)}{e^3}$$

$$- \frac{d^2 p \log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right) \log(d+ex)}{e^3}$$

$$+ \frac{\sqrt[3]{a} dp \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{2\sqrt[3]{b} e^2} + \frac{a^{2/3} p \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2\right)}{4b^{2/3} e}$$

$$- \frac{dx \log\left(c(a+bx^3)^p\right)}{e^2} + \frac{x^2 \log\left(c(a+bx^3)^p\right)}{2e} + \frac{d^2 \log(d+ex) \log\left(c(a+bx^3)^p\right)}{e^3}$$

$$+ \frac{(d^2 p) \operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d+\sqrt[3]{a}e}\right)}{x} \, dx, x, d+ex\right)}{e^3}$$

$$+ \frac{(d^2 p) \operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d-\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{x} \, dx, x, d+ex\right)}{e^3}$$

$$+ \frac{(d^2 p) \operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d+(-1)^{2/3}\sqrt[3]{a}e}\right)}{x} \, dx, x, d+ex\right)}{e^3}$$

$$- \frac{\left(3\sqrt[3]{a} dp\right) \operatorname{Subst}\left(\int \frac{1}{-3-x^2} \, dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b} e^2}$$

$$+ \frac{\left(3a^{2/3} p\right) \operatorname{Subst}\left(\int \frac{1}{-3-x^2} \, dx, x, 1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{2b^{2/3} e}$$

$$
\begin{aligned}
&= \frac{3dpx}{e^2} - \frac{3px^2}{4e} + \frac{\sqrt{3}\sqrt[3]{a}dp\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}e^2} \\
&- \frac{\sqrt{3}a^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2b^{2/3}e} - \frac{\sqrt[3]{a}dp\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e^2} \\
&- \frac{a^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2b^{2/3}e} - \frac{d^2p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e^3} \\
&- \frac{d^2p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e^3} \\
&- \frac{d^2p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e^3} \\
&+ \frac{\sqrt[3]{a}dp\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}e^2} + \frac{a^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4b^{2/3}e} \\
&- \frac{dx\log\left(c(a+bx^3)^p\right)}{e^2} + \frac{x^2\log\left(c(a+bx^3)^p\right)}{2e} + \frac{d^2\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e^3} \\
&- \frac{d^2p\operatorname{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^3} - \frac{d^2p\operatorname{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e^3} - \frac{d^2p\operatorname{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^3}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.26 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.78

$$
\int \frac{x^2\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx =
$$

$$
-\frac{-12depx + 3e^2px^2 - \frac{4\sqrt{3}\sqrt[3]{a}dep\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} - 3e^2px^2\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a}\right) + \frac{4\sqrt[3]{a}dep\,\text{lo}}{}}{}
$$

```
[In] Integrate[(x^2*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

```
[Out] -1/4*(-12*d*e*p*x + 3*e^2*p*x^2 - (4*Sqrt[3]*a^(1/3)*d*e*p*ArcTan[(1 - (2*b
^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) - 3*e^2*p*x^2*Hypergeometric2F1[2/3, 1
, 5/3, -((b*x^3)/a)] + (4*a^(1/3)*d*e*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) +
 4*d^2*p*Log[(e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a
^(1/3)*e)]*Log[d + e*x] + 4*d^2*p*Log[(e*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*
d) + a^(1/3)*e)]*Log[d + e*x] + 4*d^2*p*Log[(e*((-1)^(2/3)*a^(1/3) + b^(1/3
)*x))/(-(b^(1/3)*d) + (-1)^(2/3)*a^(1/3)*e)]*Log[d + e*x] - (2*a^(1/3)*d*e*
p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) + 4*d*e*x*Log[c*(
a + b*x^3)^p] - 2*e^2*x^2*Log[c*(a + b*x^3)^p] - 4*d^2*Log[d + e*x]*Log[c*(
a + b*x^3)^p] + 4*d^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)
*e)] + 4*d^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/
3)*e)] + 4*d^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(
1/3)*e)])/e^3
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.53 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.39

method	result
parts	$\frac{x^{2} \ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{2e}-\frac{dx\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{e^{2}}+\frac{d^{2}\ln(ex+d)\ln\left(c\left(b\,x^{3}+a\right)^{p}\right)}{e^{3}}-\frac{3pb\left(\frac{d^{2}\left(\sum_{\_R1=\operatorname{RootOf}\left(\_Z^{3}b-3bd\_Z^{2}+3b\,d^{2}\_Z+a\,e^{3}\ldots\right.}\right.}{\ldots}\right.}{\ldots}$
risch	$\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)x^{2}}{2e}-\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)dx}{e^{2}}+\frac{\ln\left(\left(b\,x^{3}+a\right)^{p}\right)d^{2}\ln(ex+d)}{e^{3}}-\frac{p\,d^{2}\left(\sum_{\_R1=\operatorname{RootOf}\left(\_Z^{3}b-3bd\_Z^{2}+3b\,d^{2}\_Z+a\,e^{3}-b\,d^{3}\right)}\ldots\right.}{\ldots}$

```
[In] int(x^2*ln(c*(b*x^3+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*ln(c*(b*x^3+a)^p)/e-d*x*ln(c*(b*x^3+a)^p)/e^2+d^2*ln(e*x+d)*ln(c*(b
*x^3+a)^p)/e^3-3*p*b/e^3*(1/3*d^2/b*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilo
g((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))+1
/4/b*(e*x+d)^2-3/2/b*(e*x+d)*d+1/6/b^2*sum((-_R+3*d)/(_R^2-2*_R*d+d^2)*ln(e
*x-_R+d),_R=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))*a*e^3)
```

Fricas [F]

$$\int \frac{x^2 \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left((bx^3+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^2*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(x^2*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*ln(c*(b*x**3+a)**p)/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2 \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left((bx^3+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^2*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x^2*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^2 \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left((bx^3+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^2*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x^2*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{x^2\ \ln\left(c\,(b\,x^3+a)^p\right)}{d+e\,x}\,dx$$

```
[In] int((x^2*log(c*(a + b*x^3)^p))/(d + e*x),x)
[Out] int((x^2*log(c*(a + b*x^3)^p))/(d + e*x), x)
```

3.235 $\int \frac{x \log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 457

$$\int \frac{x\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = -\frac{3px}{e}-\frac{\sqrt{3}\sqrt[3]{a}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}e}+\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e}+\frac{dp\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e^2}+\frac{dp\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e^2}+\frac{dp\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e^2}-\frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}e}+\frac{x\log\left(c(a+bx^3)^p\right)}{e}-\frac{d\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e^2}+\frac{dp\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^2}+\frac{dp\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e^2}+\frac{dp\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^2}$$

```
[Out] -3*p*x/e+a^(1/3)*p*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)/e+d*p*ln(-e*(a^(1/3)+b^(1/
3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/e^2+d*p*ln(-e*((-1)^(2/3)*a^(1/3)+b^
(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/e^2+d*p*ln((-1)^(1/3)*
e*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))*ln(e*x+d
)/e^2-1/2*a^(1/3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(1/3)/e+x*l
n(c*(b*x^3+a)^p)/e-d*ln(e*x+d)*ln(c*(b*x^3+a)^p)/e^2+d*p*polylog(2,b^(1/3)*
(e*x+d)/(b^(1/3)*d-a^(1/3)*e))/e^2+d*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d
+(-1)^(1/3)*a^(1/3)*e))/e^2+d*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(
2/3)*a^(1/3)*e))/e^2-a^(1/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(
1/2))*3^(1/2)/b^(1/3)/e
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {2516, 2498, 327, 206, 31, 648, 631, 210, 642, 2512, 266, 2463, 2441, 2440, 2438}

$$\int \frac{x\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx = -\frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}e} - \frac{\sqrt{3}\sqrt[3]{a}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}e} - \frac{d\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e^2} + \frac{x\log\left(c\left(a+bx^3\right)^p\right)}{e} + \frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^2} + \frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e^2} + \frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^2} + \frac{dp\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^2} + \frac{dp\log(d+ex)\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^2} + \frac{dp\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e^2} + \frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e} - \frac{3px}{e}$$

```
[In] Int[(x*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

```
[Out] (-3*p*x)/e - (Sqrt[3]*a^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(
1/3))])/(b^(1/3)*e) + (a^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/(b^(1/3)*e) + (d
*p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/
e^2 + (d*p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/
3)*a^(1/3)*e))]*Log[d + e*x])/e^2 + (d*p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^
(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e^2 - (
```

```
a^(1/3)*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*b^(1/3)*e) + (
x*Log[c*(a + b*x^3)^p])/e - (d*Log[d + e*x]*Log[c*(a + b*x^3)^p])/e^2 + (d*
p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/e^2 + (d*p*PolyL
og[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/e^2 + (d*p*P
olyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/e^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\log\left(c\left(a+bx^3\right)^p\right)}{e}-\frac{d\log\left(c\left(a+bx^3\right)^p\right)}{e(d+ex)}\right)dx\\
&= \frac{\int \log\left(c\left(a+bx^3\right)^p\right)\,dx}{e}-\frac{d\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{d+ex}\,dx}{e}\\
&= \frac{x\log\left(c\left(a+bx^3\right)^p\right)}{e}-\frac{d\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e^2}\\
&\quad+\frac{(3bdp)\int \frac{x^2\log(d+ex)}{a+bx^3}\,dx}{e^2}-\frac{(3bp)\int \frac{x^3}{a+bx^3}\,dx}{e}\\
&= -\frac{3px}{e}+\frac{x\log\left(c\left(a+bx^3\right)^p\right)}{e}-\frac{d\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e^2}\\
&\quad+\frac{(3bdp)\int \left(\frac{\log(d+ex)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\log(d+ex)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\log(d+ex)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx}{e^2}\\
&\quad+\frac{(3ap)\int \frac{1}{a+bx^3}\,dx}{e}\\
&= -\frac{3px}{e}+\frac{x\log\left(c\left(a+bx^3\right)^p\right)}{e}-\frac{d\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e^2}\\
&\quad+\frac{\left(\sqrt[3]{b}dp\right)\int \frac{\log(d+ex)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e^2}+\frac{\left(\sqrt[3]{b}dp\right)\int \frac{\log(d+ex)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e^2}\\
&\quad+\frac{\left(\sqrt[3]{b}dp\right)\int \frac{\log(d+ex)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e^2}+\frac{\left(\sqrt[3]{a}p\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e}\\
&\quad+\frac{\left(\sqrt[3]{a}p\right)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{e}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3px}{e} + \frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e} + \frac{dp\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e^2} \\
&+ \frac{dp\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e^2} \\
&+ \frac{dp\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e^2} + \frac{x\log\left(c(a+bx^3)^p\right)}{e} \\
&- \frac{d\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e^2} + \frac{\left(3a^{2/3}p\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2e} \\
&- \frac{\left(\sqrt[3]{a}p\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{b}e} - \frac{(dp)\int\frac{\log\left(\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d+\sqrt[3]{a}e}\right)}{d+ex}\,dx}{e} \\
&- \frac{(dp)\int\frac{\log\left(\frac{e\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d-\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d+ex}\,dx}{e} - \frac{(dp)\int\frac{\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d+(-1)^{2/3}\sqrt[3]{a}e}\right)}{d+ex}\,dx}{e}
\end{aligned}
$$

$$= -\frac{3px}{e} + \frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e} + \frac{dp\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e^2}$$

$$+ \frac{dp\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e^2}$$

$$+ \frac{dp\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e^2}$$

$$- \frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}e}$$

$$+ \frac{x\log\left(c\left(a+bx^3\right)^p\right)}{e} - \frac{d\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{e^2}$$

$$- \frac{(dp)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d+\sqrt[3]{a}e}\right)}{x}\,dx,x,d+ex\right)}{e^2}$$

$$- \frac{(dp)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d-\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{x}\,dx,x,d+ex\right)}{e^2}$$

$$- \frac{(dp)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d+(-1)^{2/3}\sqrt[3]{a}e}\right)}{x}\,dx,x,d+ex\right)}{e^2}$$

$$+ \frac{\left(3\sqrt[3]{a}p\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}e}$$

$$= -\frac{3px}{e} - \frac{\sqrt{3}\sqrt[3]{a}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}e} + \frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}e}$$
$$+ \frac{dp\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e^2}$$
$$+ \frac{dp\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e^2}$$
$$+ \frac{dp\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e^2}$$
$$- \frac{\sqrt[3]{a}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}e} + \frac{x\log\left(c(a+bx^3)^p\right)}{e}$$
$$- \frac{d\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e^2} + \frac{dp\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e^2}$$
$$+ \frac{dp\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e^2} + \frac{dp\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e^2}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 430, normalized size of antiderivative = 0.94

$$\int \frac{x\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx$$
$$= \frac{-6epx - \frac{2\sqrt{3}\sqrt[3]{a}ep\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}} + \frac{2\sqrt[3]{a}ep\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}} + 2dp\log\left(\frac{e\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex) + 2}{}$$

```
[In] Integrate[(x*Log[c*(a + b*x^3)^p])/(d + e*x),x]
```

```
[Out] (-6*e*p*x - (2*Sqrt[3]*a^(1/3)*e*p*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]])/b^(1/3) + (2*a^(1/3)*e*p*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + 2*d*p*Log
[(e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*L
og[d + e*x] + 2*d*p*Log[(e*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + a^(1/3)*e
```

```
)]*Log[d + e*x] + 2*d*p*Log[(e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(-(b^(1/3)
*d) + (-1)^(2/3)*a^(1/3)*e)]*Log[d + e*x] - (a^(1/3)*e*p*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3) + 2*e*x*Log[c*(a + b*x^3)^p] - 2*d*Lo
g[d + e*x]*Log[c*(a + b*x^3)^p] + 2*d*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(
1/3)*d - a^(1/3)*e)] + 2*d*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-
1)^(1/3)*a^(1/3)*e)] + 2*d*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-
1)^(2/3)*a^(1/3)*e)])/(2*e^2)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.37 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.45

method	result
parts	$\frac{x \ln\left(c \left(b \,x^{3}+a \right)^{p}\right)}{e}-\frac{d \ln\left(e x +d \right) \ln\left(c \left(b \,x^{3}+a \right)^{p}\right)}{e^{2}}-\frac{3 p b \left(\frac{\left(e x +d \right) e}{b}-\frac{\left(\sum_{\_R =\operatorname{RootOf}\left(\_Z^{3} b -3 b d \,\_Z^{2}+3 b \,d^{2} \_Z +a \,e^{3}-b \,d^{3}\right)}\frac{\ln\left(e x -\_R \ldots\right.}{\_R^{2}-2 \_R \ldots}\right.}{3 b^{2}}\ldots\right.}{\ldots}$
risch	$\frac{\ln\left(\left(b \,x^{3}+a \right)^{p}\right) x}{e}-\frac{\ln\left(\left(b \,x^{3}+a \right)^{p}\right) d \ln\left(e x +d \right)}{e^{2}}-\frac{3 p x}{e}-\frac{3 p d}{e^{2}}+\frac{p e \left(\sum_{\_R =\operatorname{RootOf}\left(\_Z^{3} b -3 b d \,\_Z^{2}+3 b \,d^{2} \_Z +a \,e^{3}-b \,d^{3}\right)}\frac{\ln\left(e x -\ldots\right.}{\_R^{2}-2 \ldots}\right.}{b}\ldots$

```
[In] int(x*ln(c*(b*x^3+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(c*(b*x^3+a)^p)/e-d*ln(e*x+d)*ln(c*(b*x^3+a)^p)/e^2-3*p*b/e^3*(1/b*(e*x
+d)*e-1/3/b^2*sum(1/(_R^2-2*_R*d+d^2)*ln(e*x-_R+d),_R=RootOf(_Z^3*b-3*_Z^2*
b*d+3*_Z*b*d^2+a*e^3-b*d^3))*a*e^4-1/3*d*e/b*sum(ln(e*x+d)*ln((-e*x+_R1-d)/
_R1)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-
b*d^3)))
```

Fricas [F]

$$\int \frac{x \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{x \log \left((bx^3+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate(x*ln(c*(b*x**3+a)**p)/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{x \log \left((bx^3+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{x \log \left((bx^3+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x*log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x*log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{x\,\ln\left(c\,{\left(b\,x^3+a\right)}^p\right)}{d+e\,x}\,dx$$

```
[In] int((x*log(c*(a + b*x^3)^p))/(d + e*x),x)
[Out] int((x*log(c*(a + b*x^3)^p))/(d + e*x), x)
```

3.236 $\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 308

$$\begin{aligned}\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = & -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} \\ & -\frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e} \\ & -\frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e} \\ & +\frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e}-\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e} \\ & -\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e} \\ & -\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e}\end{aligned}$$

```
[Out] -p*ln(-e*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/e-p*ln(-e*((-
1)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/e-p
*ln((-1)^(1/3)*e*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/
3)*e))*ln(e*x+d)/e+ln(e*x+d)*ln(c*(b*x^3+a)^p)/e-p*polylog(2,b^(1/3)*(e*x+d
```

```
)/(b^(1/3)*d-a^(1/3)*e))/e-p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3
)*a^(1/3)*e))/e-p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e
))/e
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2512, 266, 2463, 2441, 2440, 2438}

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{d+ex}\,dx=\frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e}-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e}-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e}-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e}-\frac{p\log(d+ex)\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e}-\frac{p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{e}$$

```
[In] Int[Log[c*(a + b*x^3)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x]
)/e) - (p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3
)*a^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)
*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e + (Log[d +
 e*x]*Log[c*(a + b*x^3)^p])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d - a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1
/3)*a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2
/3)*a^(1/3)*e)])/e
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e} - \frac{(3bp)\int\frac{x^2\log(d+ex)}{a+bx^3}\,dx}{e} \\
&= \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e} \\
&\quad - \frac{(3bp)\int\left(\frac{\log(d+ex)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\log(d+ex)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\log(d+ex)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx}{e}
\end{aligned}$$

$$= \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e} - \frac{\left(\sqrt[3]{b}p\right)\int \frac{\log(d+ex)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e} - \frac{\left(\sqrt[3]{b}p\right)\int \frac{\log(d+ex)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e} - \frac{\left(\sqrt[3]{b}p\right)\int \frac{\log(d+ex)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{e}$$

$$= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e} + p\int \frac{\log\left(\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d+\sqrt[3]{a}e}\right)}{d+ex}\,dx + p\int \frac{\log\left(\frac{e\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d-\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d+ex}\,dx + p\int \frac{\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d+(-1)^{2/3}\sqrt[3]{a}e}\right)}{d+ex}\,dx$$

$$= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e}$$

$$-\frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e}$$

$$+\frac{p\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d+\sqrt[3]{a}e}\right)}{x}dx,x,d+ex\right)}{e}$$

$$+\frac{p\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d-\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{x}dx,x,d+ex\right)}{e}$$

$$+\frac{p\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d+(-1)^{2/3}\sqrt[3]{a}e}\right)}{x}dx,x,d+ex\right)}{e}$$

$$= -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e}$$

$$-\frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e}$$

$$-\frac{p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e} - \frac{p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e} - \frac{p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.02

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{d+ex}\,dx = -\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{e} + \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{e}$$

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(d + e*x),x]
```

```
[Out] -((p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x]
)/e) - (p*Log[-(((-1)^(2/3)*e*(a^(1/3) - (-1)^(1/3)*b^(1/3)*x))/(b^(1/3)*d
- (-1)^(2/3)*a^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(a^(1/3) +
 (-1)^(2/3)*b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/e
 + (Log[d + e*x]*Log[c*(a + b*x^3)^p])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x)
)/(b^(1/3)*d - a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d + (-1)^(1/3)*a^(1/3)*e)])/e - (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*
d - (-1)^(2/3)*a^(1/3)*e)])/e
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.33

method	result
parts	$\frac{\ln(ex+d)\ln\left(c\left(b\,x^3+a\right)^p\right)}{e}-\frac{p\left(\sum_{\_R1=\mathrm{RootOf}\left(\_Z^3b-3bd\,\_Z^2+3b\,d^2\,\_Z+a\,e^3-b\,d^3\right)}\left(\ln(ex+d)\ln\left(\frac{-ex+\_R1-d}{\_R1}\right)+\mathrm{dilog}\left(\frac{-ex+\_R1-d}{\_R1}\right)\right)\right)}{e}$
risch	$\frac{\ln\left(\left(b\,x^3+a\right)^p\right)\ln(ex+d)}{e}-\frac{p\left(\sum_{\_R1=\mathrm{RootOf}\left(\_Z^3b-3bd\,\_Z^2+3b\,d^2\,\_Z+a\,e^3-b\,d^3\right)}\left(\ln(ex+d)\ln\left(\frac{-ex+\_R1-d}{\_R1}\right)+\mathrm{dilog}\left(\frac{-ex+\_R1-d}{\_R1}\right)\right)\right)}{e}$

```
[In] int(ln(c*(b*x^3+a)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] ln(e*x+d)*ln(c*(b*x^3+a)^p)/e-p/e*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog(
(-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))
```

Fricas [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx=\int \frac{\log\left((bx^3+a)^pc\right)}{ex+d}\,dx$$

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx=\text{Timed out}$$

```
[In] integrate(ln(c*(b*x**3+a)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{\log\left((bx^3+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{\log\left((bx^3+a)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(log(c*(b*x^3+a)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(log((b*x^3 + a)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx = \int \frac{\ln\left(c\,(b\,x^3+a)^p\right)}{d+e\,x}\,dx$$

```
[In] int(log(c*(a + b*x^3)^p)/(d + e*x),x)
[Out] int(log(c*(a + b*x^3)^p)/(d + e*x), x)
```

3.237 $\int \frac{\log\left(c(a+bx^3)^p\right)}{x(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 352

$$\begin{aligned}\int \frac{\log\left(c(a+bx^3)^p\right)}{x(d+ex)}\,dx=&\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d}\\&+\frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{d}\\&+\frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{d}\\&+\frac{\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d}-\frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d}\\&+\frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d}+\frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d}\\&+\frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d}+\frac{p\operatorname{PolyLog}\left(2,1+\frac{bx^3}{a}\right)}{3d}\end{aligned}$$

```
[Out] p*ln(-e*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/d+p*ln(-e*((-1
)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/d+p*
ln((-1)^(1/3)*e*(a^(1/3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3
)*e))*ln(e*x+d)/d+1/3*ln(-b*x^3/a)*ln(c*(b*x^3+a)^p)/d-ln(e*x+d)*ln(c*(b*x^
```

```
3+a)^p)/d+p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-a^(1/3)*e))/d+p*polylog(2,
b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))/d+p*polylog(2,b^(1/3)*(e*
x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))/d+1/3*p*polylog(2,1+b*x^3/a)/d
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2516, 2504, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$\begin{aligned}\int \frac{\log \left(c(a+bx^3)^p\right)}{x(d+ex)}\,dx = &-\frac{\log(d+ex)\log \left(c(a+bx^3)^p\right)}{d}+\frac{\log \left(-\frac{bx^3}{a}\right)\log \left(c(a+bx^3)^p\right)}{3d}\\ &+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d}+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d}\\ &+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d}\\ &+\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d}\\ &+\frac{p\log(d+ex)\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d}\\ &+\frac{p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{d}\\ &+\frac{p\,\text{PolyLog}\left(2,\frac{bx^3}{a}+1\right)}{3d}\end{aligned}$$

```
[In] Int[Log[c*(a + b*x^3)^p]/(x*(d + e*x)),x]
```

```
[Out] (p*Log[-((e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/
d + (p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a
^(1/3)*e))]*Log[d + e*x])/d + (p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^
(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d + (Log[-((b*x
^3)/a)]*Log[c*(a + b*x^3)^p])/(3*d) - (Log[d + e*x]*Log[c*(a + b*x^3)^p])/d
 + (p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)])/d + (p*PolyL
og[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)])/d + (p*PolyL
og[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/d + (p*PolyL
og[2, 1 + (b*x^3)/a])/(3*d)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
```

```
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log\left(c(a+bx^3)^p\right)}{dx} - \frac{e\log\left(c(a+bx^3)^p\right)}{d(d+ex)}\right) dx$$

$$= \frac{\int \frac{\log\left(c(a+bx^3)^p\right)}{x}\,dx}{d} - \frac{e\int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex}\,dx}{d}$$

$$= -\frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d} + \frac{\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x, x^3\right)}{3d} + \frac{(3bp)\int \frac{x^2\log(d+ex)}{a+bx^3}\,dx}{d}$$

$$= \frac{\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d} - \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d} - \frac{(bp)\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx, x, x^3\right)}{3d} + \frac{(3bp)\int \left(\frac{\log(d+ex)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\log(d+ex)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\log(d+ex)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right) dx}{d}$$

$$= \frac{\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d} - \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d} + \frac{p\text{Li}_2\left(1+\frac{bx^3}{a}\right)}{3d} + \frac{\left(\sqrt[3]{b}p\right)\int \frac{\log(d+ex)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{d} + \frac{\left(\sqrt[3]{b}p\right)\int \frac{\log(d+ex)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{d} + \frac{\left(\sqrt[3]{b}p\right)\int \frac{\log(d+ex)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{d}$$

$$=\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d}+\frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{d}+\frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{d}+\frac{\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d}-\frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d}+\frac{p\text{Li}_2\left(1+\frac{bx^3}{a}\right)}{3d}-\frac{(ep)\int\frac{\log\left(\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d+\sqrt[3]{a}e}\right)}{d+ex}dx}{d}-\frac{(ep)\int\frac{\log\left(\frac{e\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d-\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d+ex}dx}{d}-\frac{(ep)\int\frac{\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d+(-1)^{2/3}\sqrt[3]{a}e}\right)}{d+ex}dx}{d}$$

$$=\frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d}+\frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{d}+\frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{d}+\frac{\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d}-\frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d}+\frac{p\text{Li}_2\left(1+\frac{bx^3}{a}\right)}{3d}-\frac{p\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d+\sqrt[3]{a}e}\right)}{x}dx,x,d+ex\right)}{d}-\frac{p\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d-\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{x}dx,x,d+ex\right)}{d}-\frac{p\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d+(-1)^{2/3}\sqrt[3]{a}e}\right)}{x}dx,x,d+ex\right)}{d}$$

$$
\begin{aligned}
&= \frac{p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{d} \\
&+ \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{d} + \frac{\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d} \\
&- \frac{\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d} + \frac{p\mathrm{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d} \\
&+ \frac{p\mathrm{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d} + \frac{p\mathrm{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d} + \frac{p\mathrm{Li}_2\left(1+\frac{bx^3}{a}\right)}{3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.93

$$
\int \frac{\log\left(c(a+bx^3)^p\right)}{x(d+ex)}\,dx
= \frac{3p\log\left(\frac{e\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex) + 3p\log\left(\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d+\sqrt[3]{a}e}\right)\log(d+ex) + 3p\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d+(-1)^{2/3}\sqrt[3]{a}e}\right)}{}
$$

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(x*(d + e*x)),x]
```

```
[Out] (3*p*Log[(e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/
3)*e)]*Log[d + e*x] + 3*p*Log[(e*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + a^(
1/3)*e)]*Log[d + e*x] + 3*p*Log[(e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(-(b^(
1/3)*d) + (-1)^(2/3)*a^(1/3)*e)]*Log[d + e*x] + Log[-((b*x^3)/a)]*Log[c*(a
+ b*x^3)^p] - 3*Log[d + e*x]*Log[c*(a + b*x^3)^p] + 3*p*PolyLog[2, (b^(1/3)
*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)] + 3*p*PolyLog[2, (b^(1/3)*(d + e*x))/(
b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)] + 3*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^
(1/3)*d - (-1)^(2/3)*a^(1/3)*e)] + p*PolyLog[2, 1 + (b*x^3)/a])/(3*d)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.04 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.48

method	result
parts	$-\frac{\ln(ex+d)\ln(c(b\,x^3+a)^p)}{d}+\frac{\ln(c(b\,x^3+a)^p)\ln(x)}{d}-3pb\left(\frac{\sum_{\_R1=\mathrm{RootOf}(\_Z^3b+a)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)}{3db}\right.$
risch	$-\frac{\ln((b\,x^3+a)^p)\ln(ex+d)}{d}+\frac{\ln((b\,x^3+a)^p)\ln(x)}{d}-\frac{p\left(\sum_{\_R1=\mathrm{RootOf}(\_Z^3b+a)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)}{d}+$

```
[In] int(ln(c*(b*x^3+a)^p)/x/(e*x+d),x,method=_RETURNVERBOSE)
[Out] -ln(e*x+d)*ln(c*(b*x^3+a)^p)/d+ln(c*(b*x^3+a)^p)/d*ln(x)-3*p*b*(1/3/d/b*sum
(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=RootOf(_Z^3*b+a))-1/3/d/b*sum
(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3
*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3)))
```

Fricas [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x(d+ex)}\,dx=\int \frac{\log\left((bx^3+a)^p c\right)}{(ex+d)x}\,dx$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x/(e*x+d),x, algorithm="fricas")
[Out] integral(log((b*x^3 + a)^p*c)/(e*x^2 + d*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x(d+ex)}\,dx=\text{Timed out}$$

```
[In] integrate(ln(c*(b*x**3+a)**p)/x/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x(d+ex)}\,dx = \int \frac{\log\left((bx^3+a)^p c\right)}{(ex+d)x}\,dx$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((b*x^3 + a)^p*c)/((e*x + d)*x), x)
```

Giac [F]

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x(d+ex)}\,dx = \int \frac{\log\left((bx^3+a)^p c\right)}{(ex+d)x}\,dx$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x/(e*x+d),x, algorithm="giac")
[Out] integrate(log((b*x^3 + a)^p*c)/((e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x(d+ex)}\,dx = \int \frac{\ln\left(c\,(b\,x^3+a)^p\right)}{x\,\left(d+e\,x\right)}\,dx$$

```
[In] int(log(c*(a + b*x^3)^p)/(x*(d + e*x)),x)
[Out] int(log(c*(a + b*x^3)^p)/(x*(d + e*x)), x)
```

3.238 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 510

$$
\begin{aligned}
\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^2(d+ex)}\,dx = & -\frac{\sqrt{3}\sqrt[3]{b}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}d}-\frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d} \\
& -\frac{ep\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d^2} \\
& -\frac{ep\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{d^2} \\
& -\frac{ep\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{d^2} \\
& +\frac{\sqrt[3]{b}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}d}-\frac{\log\left(c\left(a+bx^3\right)^p\right)}{dx} \\
& -\frac{e\log\left(-\frac{bx^3}{a}\right)\log\left(c\left(a+bx^3\right)^p\right)}{3d^2}+\frac{e\log(d+ex)\log\left(c\left(a+bx^3\right)^p\right)}{d^2} \\
& -\frac{ep\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^2}-\frac{ep\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d^2} \\
& -\frac{ep\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d^2}-\frac{ep\operatorname{PolyLog}\left(2,1+\frac{bx^3}{a}\right)}{3d^2}
\end{aligned}
$$

```
[Out] -b^(1/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/d-e*p*ln(-e*(a^(1/3)+b^(1/3)*x)/(b
^(1/3)*d-a^(1/3)*e))*ln(e*x+d)/d^2-e*p*ln(-e*((-1)^(2/3)*a^(1/3)+b^(1/3)*x)
/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))*ln(e*x+d)/d^2-e*p*ln((-1)^(1/3)*e*(a^(1/
3)+(-1)^(2/3)*b^(1/3)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))*ln(e*x+d)/d^2+1/
2*b^(1/3)*p*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/d-ln(c*(b*x^3
+a)^p)/d/x-1/3*e*ln(-b*x^3/a)*ln(c*(b*x^3+a)^p)/d^2+e*ln(e*x+d)*ln(c*(b*x^3
+a)^p)/d^2-e*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-a^(1/3)*e))/d^2-e*p*pol
ylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))/d^2-e*p*polylog(2,
b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))/d^2-1/3*e*p*polylog(2,1+b
*x^3/a)/d^2-b^(1/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))*3^(
1/2)/a^(1/3)/d
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {2516, 2505, 298, 31, 648, 631, 210, 642, 2504, 2441, 2352, 2512, 266, 2463, 2440,

2438}

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2(d+ex)}\,dx = \frac{\sqrt[3]{b}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}d} - \frac{\sqrt{3}\sqrt[3]{b}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}d} - \frac{e\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d^2} + \frac{e\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d^2} - \frac{\log\left(c(a+bx^3)^p\right)}{dx} - \frac{ep\,\text{PolyLog}\left(2,\frac{bx^3}{a}+1\right)}{3d^2} - \frac{ep\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^2} - \frac{ep\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d^2} - \frac{ep\,\text{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d^2} - \frac{ep\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^2} - \frac{ep\log(d+ex)\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d^2} - \frac{ep\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{d^2} - \frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d}$$

```
[In] Int[Log[c*(a + b*x^3)^p]/(x^2*(d + e*x)),x]
```

```
[Out] -((Sqrt[3]*b^(1/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(a^
(1/3)*d)) - (b^(1/3)*p*Log[a^(1/3) + b^(1/3)*x])/(a^(1/3)*d) - (e*p*Log[-((
e*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/d^2 - (e*p
*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)
*e))]*Log[d + e*x])/d^2 - (e*p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b^(1
/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d^2 + (b^(1/3)*p*
Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*a^(1/3)*d) - Log[c*(a +
b*x^3)^p]/(d*x) - (e*Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p])/(3*d^2) + (e*L
og[d + e*x]*Log[c*(a + b*x^3)^p])/d^2 - (e*p*PolyLog[2, (b^(1/3)*(d + e*x))
/(b^(1/3)*d - a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/
3)*d + (-1)^(1/3)*a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b
```

```
^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)])/d^2 - (e*p*PolyLog[2, 1 + (b*x^3)/a])/(3
*d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\log \left(c(a+bx^3)^p\right)}{dx^2} - \frac{e \log \left(c(a+bx^3)^p\right)}{d^2x} + \frac{e^2 \log \left(c(a+bx^3)^p\right)}{d^2(d+ex)}\right) dx \\
&= \frac{\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2} dx}{d} - \frac{e \int \frac{\log\left(c(a+bx^3)^p\right)}{x} dx}{d^2} + \frac{e^2 \int \frac{\log\left(c(a+bx^3)^p\right)}{d+ex} dx}{d^2} \\
&= -\frac{\log \left(c(a+bx^3)^p\right)}{dx} + \frac{e \log(d+ex) \log \left(c(a+bx^3)^p\right)}{d^2} \\
&\quad - \frac{e\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x} dx, x, x^3\right)}{3d^2} + \frac{(3bp) \int \frac{x}{a+bx^3} dx}{d} - \frac{(3bep) \int \frac{x^2 \log(d+ex)}{a+bx^3} dx}{d^2} \\
&= -\frac{\log \left(c(a+bx^3)^p\right)}{dx} - \frac{e \log \left(-\frac{bx^3}{a}\right) \log \left(c(a+bx^3)^p\right)}{3d^2} \\
&\quad + \frac{e \log(d+ex) \log \left(c(a+bx^3)^p\right)}{d^2} - \frac{\left(b^{2/3}p\right) \int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x} dx}{\sqrt[3]{a}d} \\
&\quad + \frac{\left(b^{2/3}p\right) \int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2} dx}{\sqrt[3]{a}d} + \frac{(bep)\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx} dx, x, x^3\right)}{3d^2} \\
&\quad - \frac{(3bep) \int \left(\frac{\log(d+ex)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\log(d+ex)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\log(d+ex)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right) dx}{d^2}
\end{aligned}$$

$$= -\frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d} - \frac{\log\left(c(a+bx^3)^p\right)}{dx} - \frac{e\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d^2} + \frac{e\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d^2} - \frac{ep\mathrm{Li}_2\left(1+\frac{bx^3}{a}\right)}{3d^2} + \frac{\left(\sqrt[3]{b}p\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{2\sqrt[3]{a}d} + \frac{\left(3b^{2/3}p\right)\int\frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}dx}{2d} - \frac{\left(\sqrt[3]{b}ep\right)\int\frac{\log(d+ex)}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{d^2} - \frac{\left(\sqrt[3]{b}ep\right)\int\frac{\log(d+ex)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}dx}{d^2} - \frac{\left(\sqrt[3]{b}ep\right)\int\frac{\log(d+ex)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}dx}{d^2}$$

$$= -\frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d} - \frac{ep\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{d^2} + \frac{\sqrt[3]{b}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}d} - \frac{\log\left(c(a+bx^3)^p\right)}{dx} - \frac{e\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d^2} + \frac{e\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d^2} - \frac{ep\mathrm{Li}_2\left(1+\frac{bx^3}{a}\right)}{3d^2} + \frac{\left(3\sqrt[3]{b}p\right)\mathrm{Subst}\left(\int\frac{1}{-3-x^2}dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}d} + \frac{(e^2p)\int\frac{\log\left(\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d+\sqrt[3]{a}e}\right)}{d+ex}dx}{d^2} + \frac{(e^2p)\int\frac{\log\left(\frac{e\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d-\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d+ex}dx}{d^2} + \frac{(e^2p)\int\frac{\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d+(-1)^{2/3}\sqrt[3]{a}e}\right)}{d+ex}dx}{d^2}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}d} - \frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d}$$
$$- \frac{ep\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d^2}$$
$$- \frac{ep\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{d^2}$$
$$- \frac{ep\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{d^2}$$
$$+ \frac{\sqrt[3]{b}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}d} - \frac{\log\left(c(a+bx^3)^p\right)}{dx}$$
$$- \frac{e\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d^2} + \frac{e\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d^2}$$
$$- \frac{ep\text{Li}_2\left(1+\frac{bx^3}{a}\right)}{3d^2} + \frac{(ep)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d+\sqrt[3]{a}e}\right)}{x}dx,x,d+ex\right)}{d^2}$$
$$+ \frac{(ep)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d-\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{x}dx,x,d+ex\right)}{d^2}$$
$$+ \frac{(ep)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d+(-1)^{2/3}\sqrt[3]{a}e}\right)}{x}dx,x,d+ex\right)}{d^2}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}d} - \frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d}$$
$$-\frac{ep\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d^2}$$
$$-\frac{ep\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{d^2}$$
$$-\frac{ep\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{d^2}$$
$$+\frac{\sqrt[3]{b}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}d} - \frac{\log\left(c(a+bx^3)^p\right)}{dx}$$
$$-\frac{e\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d^2} + \frac{e\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d^2}$$
$$-\frac{ep\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^2} - \frac{ep\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d^2}$$
$$-\frac{ep\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d^2} - \frac{ep\text{Li}_2\left(1+\frac{bx^3}{a}\right)}{3d^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 395, normalized size of antiderivative = 0.77

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^2(d+ex)}\,dx$$
$$= \frac{9bdpx^3\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a}\right)-2a\left(3epx\log\left(\frac{e\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)+3epx\log\right.}{}$$

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (9*b*d*p*x^3*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)] - 2*a*(3*e*p*x*Lo
g[(e*((-1)^(1/3)*a^(1/3) - b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*
```

```
Log[d + e*x] + 3*e*p*x*Log[(e*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + a^(1/3
)*e)]*Log[d + e*x] + 3*e*p*x*Log[(e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(-(b^
(1/3)*d) + (-1)^(2/3)*a^(1/3)*e)]*Log[d + e*x] + 3*d*Log[c*(a + b*x^3)^p] +
 e*x*Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p] - 3*e*x*Log[d + e*x]*Log[c*(a +
 b*x^3)^p] + 3*e*p*x*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)
] + 3*e*p*x*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*
e)] + 3*e*p*x*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3
)*e)] + e*p*x*PolyLog[2, 1 + (b*x^3)/a]))/(6*a*d^2*x)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.19 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.57

method	result
parts	$\frac{e\ln(ex+d)\ln(c(b\,x^3+a)^p)}{d^2}-\frac{\ln(c(b\,x^3+a)^p)}{dx}-\frac{\ln(c(b\,x^3+a)^p)e\ln(x)}{d^2}-3pb\left(\frac{e\left(\sum_{\_R1=\mathrm{RootOf}\left(\_Z^3b-3bd\_Z^2+3b\,d^2\_Z+a\,e^3-\right.}\right.}{}\right.$
risch	$\frac{\ln((b\,x^3+a)^p)e\ln(ex+d)}{d^2}-\frac{\ln((b\,x^3+a)^p)}{dx}-\frac{\ln((b\,x^3+a)^p)e\ln(x)}{d^2}-\frac{pe\left(\sum_{\_R1=\mathrm{RootOf}\left(\_Z^3b-3bd\_Z^2+3b\,d^2\_Z+a\,e^3-b\,d^3\right)}\left(\ln(e\right.\right.}{d^2}$

```
[In] int(ln(c*(b*x^3+a)^p)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] e*ln(e*x+d)*ln(c*(b*x^3+a)^p)/d^2-ln(c*(b*x^3+a)^p)/d/x-ln(c*(b*x^3+a)^p)*e
/d^2*ln(x)-3*p*b*(1/3*e/d^2/b*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*
x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3))+1/3/d/b
/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-1/6/d/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/
b)^(2/3))-1/3/d*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))-1/3*e/d^2/b*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=RootOf(_Z^3
*b+a))
```

Fricas [F]

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\log \left((bx^3+a)^p c\right)}{(ex+d)x^2}\,dx$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2/(e*x+d),x, algorithm="fricas")
[Out] integral(log((b*x^3 + a)^p*c)/(e*x^3 + d*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^2(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**2/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\log \left((bx^3+a)^p c\right)}{(ex+d)x^2}\,dx$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((b*x^3 + a)^p*c)/((e*x + d)*x^2), x)
```

Giac [F]

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\log \left((bx^3+a)^p c\right)}{(ex+d)x^2}\,dx$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x^2/(e*x+d),x, algorithm="giac")
[Out] integrate(log((b*x^3 + a)^p*c)/((e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\ln \left(c\,{\left(b\,x^3+a\right)}^p\right)}{x^2\,\left(d+e\,x\right)} \,d x$$

```
[In] int(log(c*(a + b*x^3)^p)/(x^2*(d + e*x)),x)
[Out] int(log(c*(a + b*x^3)^p)/(x^2*(d + e*x)), x)
```

3.239 $\int \frac{\log\left(c\left(a+bx^3\right)^p\right)}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 674

$$\int \frac{\log(c(a+bx^3)^p)}{x^3(d+ex)}\,dx = -\frac{\sqrt{3}b^{2/3}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}d}+\frac{\sqrt{3}\sqrt[3]{b}ep\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}d^2}+\frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}d}+\frac{\sqrt[3]{b}ep\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d^2}+\frac{e^2p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d^3}+\frac{e^2p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{d^3}+\frac{e^2p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{d^3}-\frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4a^{2/3}d}-\frac{\sqrt[3]{b}ep\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}d^2}-\frac{\log(c(a+bx^3)^p)}{2dx^2}+\frac{e\log(c(a+bx^3)^p)}{d^2x}+\frac{e^2\log\left(-\frac{bx^3}{a}\right)\log(c(a+bx^3)^p)}{3d^3}-\frac{e^2\log(d+ex)\log(c(a+bx^3)^p)}{d^3}+\frac{e^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^3}+\frac{e^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d^3}+\frac{e^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d^3}+\frac{e^2p\,\mathrm{PolyLog}\left(2,1+\frac{bx^3}{a}\right)}{3d^3}$$

```
[Out] 1/2*b^(2/3)*p*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/d+b^(1/3)*e*p*ln(a^(1/3)+b^(1/3
)*x)/a^(1/3)/d^2+e^2*p*ln(-e*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-a^(1/3)*e))*ln(
e*x+d)/d^3+e^2*p*ln(-e*((-1)^(2/3)*a^(1/3)+b^(1/3)*x)/(b^(1/3)*d-(-1)^(2/3)
*a^(1/3)*e))*ln(e*x+d)/d^3+e^2*p*ln((-1)^(1/3)*e*(a^(1/3)+(-1)^(2/3)*b^(1/3
)*x)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))*ln(e*x+d)/d^3-1/4*b^(2/3)*p*ln(a^(2/
```

```
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/d-1/2*b^(1/3)*e*p*ln(a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/d^2-1/2*ln(c*(b*x^3+a)^p)/d/x^2+e*ln(c*(
b*x^3+a)^p)/d^2/x+1/3*e^2*ln(-b*x^3/a)*ln(c*(b*x^3+a)^p)/d^3-e^2*ln(e*x+d)*
ln(c*(b*x^3+a)^p)/d^3+e^2*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-a^(1/3)*e)
)/d^3+e^2*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d+(-1)^(1/3)*a^(1/3)*e))/d^3
+e^2*p*polylog(2,b^(1/3)*(e*x+d)/(b^(1/3)*d-(-1)^(2/3)*a^(1/3)*e))/d^3+1/3*
e^2*p*polylog(2,1+b*x^3/a)/d^3-1/2*b^(2/3)*p*arctan(1/3*(a^(1/3)-2*b^(1/3)*
x)/a^(1/3)*3^(1/2))*3^(1/2)/a^(2/3)/d+b^(1/3)*e*p*arctan(1/3*(a^(1/3)-2*b^(
1/3)*x)/a^(1/3)*3^(1/2))*3^(1/2)/a^(1/3)/d^2
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 674, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {2516, 2505, 206, 31, 648, 631, 210, 642, 298, 2504, 2441, 2352, 2512, 266, 2463,

2440, 2438}

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^3(d+ex)}\,dx = -\frac{\sqrt{3}b^{2/3}p\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}d} - \frac{\sqrt[3]{b}ep\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}d^2} - \frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4a^{2/3}d} + \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}d} + \frac{\sqrt{3}\sqrt[3]{b}ep\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}d^2} + \frac{e^2\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d^3} - \frac{e^2\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d^3} + \frac{e\log\left(c(a+bx^3)^p\right)}{d^2x} - \frac{\log\left(c(a+bx^3)^p\right)}{2dx^2} + \frac{e^2p\,\mathrm{PolyLog}\left(2,\frac{bx^3}{a}+1\right)}{3d^3} + \frac{e^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^3} + \frac{e^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d^3} + \frac{e^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d^3} + \frac{e^2p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^3} + \frac{e^2p\log(d+ex)\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d^3} + \frac{e^2p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{-1}\sqrt[3]{a}e+\sqrt[3]{b}d}\right)}{d^3} + \frac{\sqrt[3]{b}ep\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d^2}$$

```
[In] Int[Log[c*(a + b*x^3)^p]/(x^3*(d + e*x)),x]
```

```
[Out] -1/2*(Sqrt[3]*b^(2/3)*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(a^(2/3)*d) + (Sqrt[3]*b^(1/3)*e*p*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(a^(1/3)*d^2) + (b^(2/3)*p*Log[a^(1/3) + b^(1/3)*x])/(2*a^(2/3)*
d) + (b^(1/3)*e*p*Log[a^(1/3) + b^(1/3)*x])/(a^(1/3)*d^2) + (e^2*p*Log[-((e
*(a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - a^(1/3)*e))]*Log[d + e*x])/d^3 + (e^2*
p*Log[-((e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3
)*e))]*Log[d + e*x])/d^3 + (e^2*p*Log[((-1)^(1/3)*e*(a^(1/3) + (-1)^(2/3)*b
^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x])/d^3 - (b^(2/3)
*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(4*a^(2/3)*d) - (b^(1/3)
*e*p*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(2*a^(1/3)*d^2) - Log[
c*(a + b*x^3)^p]/(2*d*x^2) + (e*Log[c*(a + b*x^3)^p])/(d^2*x) + (e^2*Log[-(
(b*x^3)/a)]*Log[c*(a + b*x^3)^p])/(3*d^3) - (e^2*Log[d + e*x]*Log[c*(a + b*
x^3)^p])/d^3 + (e^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - a^(1/3)*e
)])/d^3 + (e^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d + (-1)^(1/3)*a^(
1/3)*e)])/d^3 + (e^2*p*PolyLog[2, (b^(1/3)*(d + e*x))/(b^(1/3)*d - (-1)^(2/
3)*a^(1/3)*e)])/d^3 + (e^2*p*PolyLog[2, 1 + (b*x^3)/a])/(3*d^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log\left(c(a+bx^3)^p\right)}{dx^3} - \frac{e\log\left(c(a+bx^3)^p\right)}{d^2x^2} + \frac{e^2\log\left(c(a+bx^3)^p\right)}{d^3x} - \frac{e^3\log\left(c(a+bx^3)^p\right)}{d^3(d+ex)}\right) dx$$

$$= \frac{\int \frac{\log(c(a+bx^3)^p)}{x^3}\,dx}{d} - \frac{e\int \frac{\log(c(a+bx^3)^p)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{\log(c(a+bx^3)^p)}{x}\,dx}{d^3} - \frac{e^3\int \frac{\log(c(a+bx^3)^p)}{d+ex}\,dx}{d^3}$$

$$= -\frac{\log\left(c(a+bx^3)^p\right)}{2dx^2} + \frac{e\log\left(c(a+bx^3)^p\right)}{d^2x} - \frac{e^2\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d^3} + \frac{e^2\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x, x^3\right)}{3d^3} + \frac{(3bp)\int \frac{1}{a+bx^3}\,dx}{2d} - \frac{(3bep)\int \frac{x}{a+bx^3}\,dx}{d^2} + \frac{(3be^2p)\int \frac{x^2\log(d+ex)}{a+bx^3}\,dx}{d^3}$$

$$= -\frac{\log\left(c(a+bx^3)^p\right)}{2dx^2} + \frac{e\log\left(c(a+bx^3)^p\right)}{d^2x} + \frac{e^2\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d^3} - \frac{e^2\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d^3} + \frac{(bp)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{2a^{2/3}d} + \frac{(bp)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^{2/3}d} + \frac{(b^{2/3}ep)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{\sqrt[3]{a}d^2} - \frac{(b^{2/3}ep)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{\sqrt[3]{a}d^2} - \frac{(be^2p)\,\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx, x, x^3\right)}{3d^3} + \frac{(3be^2p)\int \left(\frac{\log(d+ex)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\log(d+ex)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\log(d+ex)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx}{d^3}$$

$$= \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}d} + \frac{\sqrt[3]{b}ep\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d^2} - \frac{\log\left(c(a+bx^3)^p\right)}{2dx^2} + \frac{e\log\left(c(a+bx^3)^p\right)}{d^2x} + \frac{e^2\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d^3} - \frac{e^2\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d^3} + \frac{e^2p\text{Li}_2\left(1+\frac{bx^3}{a}\right)}{3d^3} - \frac{(b^{2/3}p)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{4a^{2/3}d} + \frac{(3bp)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{4\sqrt[3]{a}d} - \frac{\left(\sqrt[3]{b}ep\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{a}d^2} - \frac{(3b^{2/3}ep)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2d^2} + \frac{\left(\sqrt[3]{b}e^2p\right)\int \frac{\log(d+ex)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{d^3} + \frac{\left(\sqrt[3]{b}e^2p\right)\int \frac{\log(d+ex)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{d^3} + \frac{\left(\sqrt[3]{b}e^2p\right)\int \frac{\log(d+ex)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{d^3}$$

$$
\begin{aligned}
&= \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}d} + \frac{\sqrt[3]{b}ep\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d^2} \\
&+ \frac{e^2p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d^3} \\
&+ \frac{e^2p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{d^3} \\
&+ \frac{e^2p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{d^3} \\
&- \frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4a^{2/3}d} - \frac{\sqrt[3]{b}ep\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}d^2} \\
&- \frac{\log\left(c(a+bx^3)^p\right)}{2dx^2} + \frac{e\log\left(c(a+bx^3)^p\right)}{d^2x} + \frac{e^2\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d^3} \\
&- \frac{e^2\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d^3} + \frac{e^2p\text{Li}_2\left(1+\frac{bx^3}{a}\right)}{3d^3} \\
&+ \frac{\left(3b^{2/3}p\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{2a^{2/3}d} \\
&- \frac{\left(3\sqrt[3]{b}ep\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}d^2} - \frac{(e^3p)\int\frac{\log\left(\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d+\sqrt[3]{a}e}\right)}{d+ex}\,dx}{d^3} \\
&- \frac{(e^3p)\int\frac{\log\left(\frac{e\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d-\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d+ex}\,dx}{d^3} - \frac{(e^3p)\int\frac{\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{-\sqrt[3]{b}d+(-1)^{2/3}\sqrt[3]{a}e}\right)}{d+ex}\,dx}{d^3}
\end{aligned}
$$

$$
= -\frac{\sqrt{3}b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}d}+\frac{\sqrt{3}\sqrt[3]{b}ep\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}d^2}
+\frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}d}+\frac{\sqrt[3]{b}ep\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d^2}
+\frac{e^2p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d^3}
+\frac{e^2p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{d^3}
+\frac{e^2p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{d^3}
-\frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4a^{2/3}d}-\frac{\sqrt[3]{b}ep\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}d^2}
-\frac{\log\left(c(a+bx^3)^p\right)}{2dx^2}+\frac{e\log\left(c(a+bx^3)^p\right)}{d^2x}+\frac{e^2\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d^3}
-\frac{e^2\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d^3}+\frac{e^2p\operatorname{Li}_2\left(1+\frac{bx^3}{a}\right)}{3d^3}
-\frac{(e^2p)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d+\sqrt[3]{a}e}\right)}{x}dx,x,d+ex\right)}{d^3}
-\frac{(e^2p)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d-\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{x}dx,x,d+ex\right)}{d^3}
-\frac{(e^2p)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{b}x}{-\sqrt[3]{b}d+(-1)^{2/3}\sqrt[3]{a}e}\right)}{x}dx,x,d+ex\right)}{d^3}
$$

$$= -\frac{\sqrt{3}b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}d} + \frac{\sqrt{3}\sqrt[3]{b}ep\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}d^2}$$
$$+ \frac{b^{2/3}p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}d} + \frac{\sqrt[3]{b}ep\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}d^2}$$
$$+ \frac{e^2p\log\left(-\frac{e\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)\log(d+ex)}{d^3}$$
$$+ \frac{e^2p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)\log(d+ex)}{d^3}$$
$$+ \frac{e^2p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)\log(d+ex)}{d^3}$$
$$- \frac{b^{2/3}p\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{4a^{2/3}d} - \frac{\sqrt[3]{b}ep\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{a}d^2}$$
$$- \frac{\log\left(c(a+bx^3)^p\right)}{2dx^2} + \frac{e\log\left(c(a+bx^3)^p\right)}{d^2x} + \frac{e^2\log\left(-\frac{bx^3}{a}\right)\log\left(c(a+bx^3)^p\right)}{3d^3}$$
$$- \frac{e^2\log(d+ex)\log\left(c(a+bx^3)^p\right)}{d^3} + \frac{e^2p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-\sqrt[3]{a}e}\right)}{d^3}$$
$$+ \frac{e^2p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d+\sqrt[3]{-1}\sqrt[3]{a}e}\right)}{d^3} + \frac{e^2p\text{Li}_2\left(\frac{\sqrt[3]{b}(d+ex)}{\sqrt[3]{b}d-(-1)^{2/3}\sqrt[3]{a}e}\right)}{d^3} + \frac{e^2p\text{Li}_2\left(1+\frac{bx^3}{a}\right)}{3d^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.25 (sec) , antiderivative size = 542, normalized size of antiderivative = 0.80

$$\int \frac{\log\left(c(a+bx^3)^p\right)}{x^3(d+ex)}\,dx$$
$$= \frac{-\frac{6\sqrt{3}b^{2/3}d^2p\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{18bdepx^2\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{bx^3}{a}\right)}{a} + \frac{6b^{2/3}d^2p\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} + 12e^2p\log\left(\frac{e\left(\sqrt[3]{\ }\right.}{\sqrt[3]{\ }}\right.}{}$$

```
[In] Integrate[Log[c*(a + b*x^3)^p]/(x^3*(d + e*x)),x]
```

```
[Out] ((-6*Sqrt[3]*b^(2/3)*d^2*p*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(
2/3) - (18*b*d*e*p*x^2*Hypergeometric2F1[2/3, 1, 5/3, -((b*x^3)/a)])/a + (6
*b^(2/3)*d^2*p*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + 12*e^2*p*Log[(e*((-1)^(1
/3)*a^(1/3) - b^(1/3)*x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)]*Log[d + e*x]
+ 12*e^2*p*Log[(e*(a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) + a^(1/3)*e)]*Log[d
+ e*x] + 12*e^2*p*Log[(e*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(-(b^(1/3)*d) +
(-1)^(2/3)*a^(1/3)*e)]*Log[d + e*x] - (3*b^(2/3)*d^2*p*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3) - (6*d^2*Log[c*(a + b*x^3)^p])/x^2 + (1
2*d*e*Log[c*(a + b*x^3)^p])/x + 4*e^2*Log[-((b*x^3)/a)]*Log[c*(a + b*x^3)^p
] - 12*e^2*Log[d + e*x]*Log[c*(a + b*x^3)^p] + 12*e^2*p*PolyLog[2, (b^(1/3)
*(d + e*x))/(b^(1/3)*d - a^(1/3)*e)] + 12*e^2*p*PolyLog[2, (b^(1/3)*(d + e*
x))/(b^(1/3)*d + (-1)^(1/3)*a^(1/3)*e)] + 12*e^2*p*PolyLog[2, (b^(1/3)*(d +
 e*x))/(b^(1/3)*d - (-1)^(2/3)*a^(1/3)*e)] + 4*e^2*p*PolyLog[2, 1 + (b*x^3)
/a])/(12*d^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.92 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.62

method	result
parts	$-\frac{e^{2} \ln\left(e x+d\right) \ln\left(c \left(b \,x^{3}+a\right)^{p}\right)}{d^{3}}-\frac{\ln\left(c \left(b \,x^{3}+a\right)^{p}\right)}{2 d \,x^{2}}+\frac{\ln\left(c \left(b \,x^{3}+a\right)^{p}\right) e^{2} \ln\left(x \right)}{d^{3}}+\frac{e \ln\left(c \left(b \,x^{3}+a\right)^{p}\right)}{d^{2} x}-\frac{3 p b \left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 d b \left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^{2}\right.}{}\right.}{}$
risch	$-\frac{\ln\left(\left(b \,x^{3}+a\right)^{p}\right) e^{2} \ln\left(e x+d\right)}{d^{3}}-\frac{\ln\left(\left(b \,x^{3}+a\right)^{p}\right)}{2 d \,x^{2}}+\frac{\ln\left(\left(b \,x^{3}+a\right)^{p}\right) e^{2} \ln\left(x \right)}{d^{3}}+\frac{\ln\left(\left(b \,x^{3}+a\right)^{p}\right) e}{d^{2} x}+\frac{p \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{2 d \left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{p \ln\left(x^{2}-\left(\frac{a}{b}\right)\right.}{4 d \left(\right.}$

```
[In] int(ln(c*(b*x^3+a)^p)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -e^2*ln(e*x+d)*ln(c*(b*x^3+a)^p)/d^3-1/2*ln(c*(b*x^3+a)^p)/d/x^2+ln(c*(b*x^
3+a)^p)*e^2/d^3*ln(x)+e*ln(c*(b*x^3+a)^p)/d^2/x-3/2*p*b*(-1/3/d/b/(a/b)^(2/
3)*ln(x+(a/b)^(1/3))+1/6/d/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-
1/3/d/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-2/3/d^2
*e/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/3/d^2*e/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/
```

```
3)*x+(a/b)^(2/3))+2/3/d^2*e*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/
b)^(1/3)*x-1))+2/3*e^2/d^3/b*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),_
R1=RootOf(_Z^3*b+a))-2/3*e^2/d^3/b*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog
((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*b-3*_Z^2*b*d+3*_Z*b*d^2+a*e^3-b*d^3)))
```

Fricas [F]

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^3(d+ex)}\,dx = \int \frac{\log \left((bx^3+a)^pc\right)}{(ex+d)x^3}\,dx$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x^3/(e*x+d),x, algorithm="fricas")
[Out] integral(log((b*x^3 + a)^p*c)/(e*x^4 + d*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^3(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(b*x**3+a)**p)/x**3/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^3(d+ex)}\,dx = \int \frac{\log \left((bx^3+a)^pc\right)}{(ex+d)x^3}\,dx$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x^3/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((b*x^3 + a)^p*c)/((e*x + d)*x^3), x)
```

Giac [F]

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^3(d+ex)}\,dx = \int \frac{\log \left((bx^3+a)^pc\right)}{(ex+d)x^3}\,dx$$

```
[In] integrate(log(c*(b*x^3+a)^p)/x^3/(e*x+d),x, algorithm="giac")
[Out] integrate(log((b*x^3 + a)^p*c)/((e*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(a+bx^3)^p\right)}{x^3(d+ex)} \, dx = \int \frac{\ln \left(c\,{\left(b\,x^3+a\right)}^p\right)}{x^3\,\left(d+e\,x\right)} \,d x$$

```
[In] int(log(c*(a + b*x^3)^p)/(x^3*(d + e*x)),x)
[Out] int(log(c*(a + b*x^3)^p)/(x^3*(d + e*x)), x)
```

3.240 $\int \frac{x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 297

$$\begin{aligned}\int \frac{x^3 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx =& -\frac{bdpx}{2ae^2}-\frac{b^2px}{3a^2e}+\frac{bpx^2}{6ae}+\frac{d^2x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3}-\frac{dx^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2}\\&+\frac{x^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e}+\frac{bd^2p\log(b+ax)}{ae^3}+\frac{b^2dp\log(b+ax)}{2a^2e^2}\\&+\frac{b^3p\log(b+ax)}{3a^3e}-\frac{d^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^4}\\&-\frac{d^3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^4}+\frac{d^3p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e^4}\\&+\frac{d^3p\,\mathrm{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^4}-\frac{d^3p\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e^4}\end{aligned}$$

```
[Out] -1/2*b*d*p*x/a/e^2-1/3*b^2*p*x/a^2/e+1/6*b*p*x^2/a/e+d^2*x*ln(c*(a+b/x)^p)/
e^3-1/2*d*x^2*ln(c*(a+b/x)^p)/e^2+1/3*x^3*ln(c*(a+b/x)^p)/e+b*d^2*p*ln(a*x+
b)/a/e^3+1/2*b^2*d*p*ln(a*x+b)/a^2/e^2+1/3*b^3*p*ln(a*x+b)/a^3/e-d^3*ln(c*(
a+b/x)^p)*ln(e*x+d)/e^4-d^3*p*ln(-e*x/d)*ln(e*x+d)/e^4+d^3*p*ln(-e*(a*x+b)/
(a*d-b*e))*ln(e*x+d)/e^4+d^3*p*polylog(2,a*(e*x+d)/(a*d-b*e))/e^4-d^3*p*pol
ylog(2,1+e*x/d)/e^4
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {2516, 2498, 269, 31, 2505, 199, 45, 2512, 266, 2463, 2441, 2352, 2440, 2438}

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex} \, dx = \frac{b^3 p \log(ax+b)}{3a^3 e} + \frac{b^2 dp \log(ax+b)}{2a^2 e^2} - \frac{b^2 px}{3a^2 e} - \frac{d^3 \log(d+ex) \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^4} + \frac{d^2 x \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2} + \frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{d^3 p \operatorname{PolyLog}\left(2, \frac{a(d+ex)}{ad-be}\right)}{e^4} + \frac{d^3 p \log(d+ex) \log \left(-\frac{e(ax+b)}{ad-be}\right)}{e^4} + \frac{bd^2 p \log(ax+b)}{ae^3} - \frac{bdpx}{2ae^2} + \frac{bpx^2}{6ae} - \frac{d^3 p \operatorname{PolyLog}\left(2, \frac{ex}{d}+1\right)}{e^4} - \frac{d^3 p \log\left(-\frac{ex}{d}\right) \log(d+ex)}{e^4}$$

```
[In] Int[(x^3*Log[c*(a + b/x)^p])/(d + e*x),x]
```

```
[Out] -1/2*(b*d*p*x)/(a*e^2) - (b^2*p*x)/(3*a^2*e) + (b*p*x^2)/(6*a*e) + (d^2*x*L
og[c*(a + b/x)^p])/e^3 - (d*x^2*Log[c*(a + b/x)^p])/(2*e^2) + (x^3*Log[c*(a
 + b/x)^p])/(3*e) + (b*d^2*p*Log[b + a*x])/(a*e^3) + (b^2*d*p*Log[b + a*x])
/(2*a^2*e^2) + (b^3*p*Log[b + a*x])/(3*a^3*e) - (d^3*Log[c*(a + b/x)^p]*Log
[d + e*x])/e^4 - (d^3*p*Log[-((e*x)/d)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e
*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e^4 + (d^3*p*PolyLog[2, (a*(d + e*x
))/(a*d - b*e)])/e^4 - (d^3*p*PolyLog[2, 1 + (e*x)/d])/e^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3}-\frac{dx \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e}\right. \\
&\qquad \left.-\frac{d^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3(d+ex)}\right) dx \\
&= \frac{d^2 \int \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx}{e^3}-\frac{d^3 \int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex} \, dx}{e^3} \\
&\quad -\frac{d \int x \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx}{e^2}+\frac{\int x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \, dx}{e} \\
&= \frac{d^2 x \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3}-\frac{dx^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2}+\frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} \\
&\quad -\frac{d^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \log(d+ex)}{e^4}-\frac{(bd^3 p) \int \frac{\log(d+ex)}{\left(a+\frac{b}{x}\right)x^2} \, dx}{e^4} \\
&\quad +\frac{(bd^2 p) \int \frac{1}{\left(a+\frac{b}{x}\right)x} \, dx}{e^3}-\frac{(bdp) \int \frac{1}{a+\frac{b}{x}} \, dx}{2e^2}+\frac{(bp) \int \frac{x}{a+\frac{b}{x}} \, dx}{3e}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{d^2 x \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{d x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2 e^2} + \frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3 e} \\
&\quad - \frac{d^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \log(d+e x)}{e^4} - \frac{\left(b d^3 p\right) \int \left(\frac{\log(d+e x)}{b x} - \frac{a \log(d+e x)}{b(b+a x)}\right) dx}{e^4} \\
&\quad + \frac{\left(b d^2 p\right) \int \frac{1}{b+a x} \, dx}{e^3} - \frac{(b d p) \int \frac{x}{b+a x} \, dx}{2 e^2} + \frac{(b p) \int \frac{x^2}{b+a x} \, dx}{3 e} \\
&= \frac{d^2 x \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{d x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2 e^2} + \frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3 e} + \frac{b d^2 p \log(b+a x)}{a e^3} \\
&\quad - \frac{d^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \log(d+e x)}{e^4} - \frac{\left(d^3 p\right) \int \frac{\log(d+e x)}{x} \, dx}{e^4} + \frac{\left(a d^3 p\right) \int \frac{\log(d+e x)}{b+a x} \, dx}{e^4} \\
&\quad - \frac{(b d p) \int \left(\frac{1}{a} - \frac{b}{a(b+a x)}\right) dx}{2 e^2} + \frac{(b p) \int \left(-\frac{b}{a^2} + \frac{x}{a} + \frac{b^2}{a^2(b+a x)}\right) dx}{3 e} \\
&= -\frac{b d p x}{2 a e^2} - \frac{b^2 p x}{3 a^2 e} + \frac{b p x^2}{6 a e} + \frac{d^2 x \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{d x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2 e^2} \\
&\quad + \frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3 e} + \frac{b d^2 p \log(b+a x)}{a e^3} + \frac{b^2 d p \log(b+a x)}{2 a^2 e^2} + \frac{b^3 p \log(b+a x)}{3 a^3 e} \\
&\quad - \frac{d^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \log(d+e x)}{e^4} - \frac{d^3 p \log \left(-\frac{e x}{d}\right) \log(d+e x)}{e^4} \\
&\quad + \frac{d^3 p \log \left(-\frac{e(b+a x)}{a d-b e}\right) \log(d+e x)}{e^4} + \frac{\left(d^3 p\right) \int \frac{\log\left(-\frac{e x}{d}\right)}{d+e x} \, dx}{e^3} - \frac{\left(d^3 p\right) \int \frac{\log\left(\frac{e(b+a x)}{-a d+b e}\right)}{d+e x} \, dx}{e^3} \\
&= -\frac{b d p x}{2 a e^2} - \frac{b^2 p x}{3 a^2 e} + \frac{b p x^2}{6 a e} + \frac{d^2 x \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{d x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2 e^2} \\
&\quad + \frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3 e} + \frac{b d^2 p \log(b+a x)}{a e^3} + \frac{b^2 d p \log(b+a x)}{2 a^2 e^2} \\
&\quad + \frac{b^3 p \log(b+a x)}{3 a^3 e} - \frac{d^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \log(d+e x)}{e^4} \\
&\quad - \frac{d^3 p \log \left(-\frac{e x}{d}\right) \log(d+e x)}{e^4} + \frac{d^3 p \log \left(-\frac{e(b+a x)}{a d-b e}\right) \log(d+e x)}{e^4} \\
&\quad - \frac{d^3 p \operatorname{Li}_2\left(1+\frac{e x}{d}\right)}{e^4} - \frac{\left(d^3 p\right) \text{Subst}\left(\int \frac{\log\left(1+\frac{a x}{-a d+b e}\right)}{x} \, dx, x, d+e x\right)}{e^4} \\
&= -\frac{b d p x}{2 a e^2} - \frac{b^2 p x}{3 a^2 e} + \frac{b p x^2}{6 a e} + \frac{d^2 x \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{d x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2 e^2} \\
&\quad + \frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3 e} + \frac{b d^2 p \log(b+a x)}{a e^3} + \frac{b^2 d p \log(b+a x)}{2 a^2 e^2} + \frac{b^3 p \log(b+a x)}{3 a^3 e} \\
&\quad - \frac{d^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \log(d+e x)}{e^4} - \frac{d^3 p \log \left(-\frac{e x}{d}\right) \log(d+e x)}{e^4} \\
&\quad + \frac{d^3 p \log \left(-\frac{e(b+a x)}{a d-b e}\right) \log(d+e x)}{e^4} + \frac{d^3 p \operatorname{Li}_2\left(\frac{a(d+e x)}{a d-b e}\right)}{e^4} - \frac{d^3 p \operatorname{Li}_2\left(1+\frac{e x}{d}\right)}{e^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.04

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex} \, dx = \frac{bd^2 p \log \left(a+\frac{b}{x}\right)}{ae^3}+\frac{d^2 x \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2}+\frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{3e} + \frac{bd^2 p \log(x)}{ae^3}-\frac{bp\left(\frac{2bx}{a^2}-\frac{x^2}{a}-\frac{2b^2 \log\left(a+\frac{b}{x}\right)}{a^3}-\frac{2b^2 \log(x)}{a^3}\right)}{6e} - \frac{bdp\left(\frac{x}{a}-\frac{b \log(b+ax)}{a^2}\right)}{2e^2}-\frac{d^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right) \log(d+ex)}{e^4} - \frac{d^3 p \log \left(-\frac{ex}{d}\right) \log(d+ex)}{e^4}+\frac{d^3 p \log \left(-\frac{e(b+ax)}{ad-be}\right) \log(d+ex)}{e^4} - \frac{d^3 p \operatorname{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e^4}+\frac{d^3 p \operatorname{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^4}$$

```
[In] Integrate[(x^3*Log[c*(a + b/x)^p])/(d + e*x),x]
```

```
[Out] (b*d^2*p*Log[a + b/x])/(a*e^3) + (d^2*x*Log[c*(a + b/x)^p])/e^3 - (d*x^2*Lo
g[c*(a + b/x)^p])/(2*e^2) + (x^3*Log[c*(a + b/x)^p])/(3*e) + (b*d^2*p*Log[x
])/(a*e^3) - (b*p*((2*b*x)/a^2 - x^2/a - (2*b^2*Log[a + b/x])/a^3 - (2*b^2*
Log[x])/a^3))/(6*e) - (b*d*p*(x/a - (b*Log[b + a*x])/a^2))/(2*e^2) - (d^3*L
og[c*(a + b/x)^p]*Log[d + e*x])/e^4 - (d^3*p*Log[-((e*x)/d)]*Log[d + e*x])/
e^4 + (d^3*p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e^4 - (d^3*p*P
olyLog[2, (d + e*x)/d])/e^4 + (d^3*p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])
/e^4
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.01

method	result
parts	$\frac{x^3 \ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{3e}-\frac{d\,x^2 \ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e^2}+\frac{d^2 x \ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3}-\frac{d^3 \ln\left(c\left(a+\frac{b}{x}\right)^p\right) \ln(ex+d)}{e^4}+pbe\left(-\frac{5ad(ex+d)-a(ex+d)^2+2}{a^2}\right.$

```
[In] int(x^3*ln(c*(a+b/x)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*ln(c*(a+b/x)^p)/e-1/2*d*x^2*ln(c*(a+b/x)^p)/e^2+d^2*x*ln(c*(a+b/x)^
p)/e^3-d^3*ln(c*(a+b/x)^p)*ln(e*x+d)/e^4+p*b*e*(-1/6/e^4*(1/a^2*(5*a*d*(e*x
+d)-a*(e*x+d)^2+2*(e*x+d)*b*e)+(-6*a^2*d^2-3*a*b*d*e-2*b^2*e^2)/a^3*ln(a*d-
```

```
a*(e*x+d)-b*e))+1/e^5*d^3/b*dilog((-a*d+a*(e*x+d)+b*e)/(-a*d+b*e))+1/e^5*d^
3/b*ln(e*x+d)*ln((-a*d+a*(e*x+d)+b*e)/(-a*d+b*e))-1/e^5*d^3/b*ln(e*x+d)*ln(
-e*x/d)-1/e^5*d^3/b*dilog(-e*x/d))
```

Fricas [F]

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx=\int \frac{x^3 \log \left(\left(a+\frac{b}{x}\right)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^3*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(x^3*log(c*((a*x + b)/x)^p)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx=\text{Timed out}$$

```
[In] integrate(x**3*ln(c*(a+b/x)**p)/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx=\int \frac{x^3 \log \left(\left(a+\frac{b}{x}\right)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^3*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x^3*log((a + b/x)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx=\int \frac{x^3 \log \left(\left(a+\frac{b}{x}\right)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^3*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x^3*log((a + b/x)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \frac{x^3\, \ln \left(c\,\left(a+\frac{b}{x}\right)^p\right)}{d+e\,x}\,dx$$

```
[In] int((x^3*log(c*(a + b/x)^p))/(d + e*x),x)
[Out] int((x^3*log(c*(a + b/x)^p))/(d + e*x), x)
```

3.241 $\int \frac{x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex} dx$

Optimal result

Integrand size = 23, antiderivative size = 219

$$\begin{aligned}\int \frac{x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex} dx &= \frac{bpx}{2ae} - \frac{dx \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} - \frac{bdp\log(b+ax)}{ae^2} \\ &\quad - \frac{b^2p\log(b+ax)}{2a^2e} + \frac{d^2 \log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^3} \\ &\quad + \frac{d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e^3} \\ &\quad - \frac{d^2p\,\mathrm{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^3} + \frac{d^2p\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e^3}\end{aligned}$$

```
[Out] 1/2*b*p*x/a/e-d*x*ln(c*(a+b/x)^p)/e^2+1/2*x^2*ln(c*(a+b/x)^p)/e-b*d*p*ln(a*
x+b)/a/e^2-1/2*b^2*p*ln(a*x+b)/a^2/e+d^2*ln(c*(a+b/x)^p)*ln(e*x+d)/e^3+d^2*
p*ln(-e*x/d)*ln(e*x+d)/e^3-d^2*p*ln(-e*(a*x+b)/(a*d-b*e))*ln(e*x+d)/e^3-d^2
*p*polylog(2,a*(e*x+d)/(a*d-b*e))/e^3+d^2*p*polylog(2,1+e*x/d)/e^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules

used = {2516, 2498, 269, 31, 2505, 199, 45, 2512, 266, 2463, 2441, 2352, 2440, 2438}

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = -\frac{b^2 p \log(ax+b)}{2a^2 e}+\frac{d^2 \log(d+ex) \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^3} - \frac{dx \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} - \frac{d^2 p \operatorname{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^3}-\frac{d^2 p \log(d+ex) \log \left(-\frac{e(ax+b)}{ad-be}\right)}{e^3} - \frac{bdp \log(ax+b)}{ae^2}+\frac{bpx}{2ae}+\frac{d^2 p \operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^3} + \frac{d^2 p \log\left(-\frac{ex}{d}\right) \log(d+ex)}{e^3}$$

```
[In] Int[(x^2*Log[c*(a + b/x)^p])/(d + e*x),x]
```

```
[Out] (b*p*x)/(2*a*e) - (d*x*Log[c*(a + b/x)^p])/e^2 + (x^2*Log[c*(a + b/x)^p])/(
2*e) - (b*d*p*Log[b + a*x])/(a*e^2) - (b^2*p*Log[b + a*x])/(2*a^2*e) + (d^2
*Log[c*(a + b/x)^p]*Log[d + e*x])/e^3 + (d^2*p*Log[-((e*x)/d)]*Log[d + e*x]
)/e^3 - (d^2*p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e^3 - (d^2*p
*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/e^3 + (d^2*p*PolyLog[2, 1 + (e*x)/d
])/e^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}+\frac{d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2(d+ex)}\right)dx\\
&= -\frac{d\int\log\left(c\left(a+\frac{b}{x}\right)^p\right)dx}{e^2}+\frac{d^2\int\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx}{e^2}+\frac{\int x\log\left(c\left(a+\frac{b}{x}\right)^p\right)dx}{e}\\
&= -\frac{dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}+\frac{d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^3}\\
&\quad+\frac{(bd^2p)\int\frac{\log(d+ex)}{\left(a+\frac{b}{x}\right)x^2}\,dx}{e^3}-\frac{(bdp)\int\frac{1}{\left(a+\frac{b}{x}\right)x}\,dx}{e^2}+\frac{(bp)\int\frac{1}{a+\frac{b}{x}}\,dx}{2e}\\
&= -\frac{dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}+\frac{d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^3}\\
&\quad+\frac{(bd^2p)\int\left(\frac{\log(d+ex)}{bx}-\frac{a\log(d+ex)}{b(b+ax)}\right)dx}{e^3}-\frac{(bdp)\int\frac{1}{b+ax}\,dx}{e^2}+\frac{(bp)\int\frac{x}{b+ax}\,dx}{2e}\\
&= -\frac{dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}-\frac{bdp\log(b+ax)}{ae^2}\\
&\quad+\frac{d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^3}+\frac{(d^2p)\int\frac{\log(d+ex)}{x}\,dx}{e^3}\\
&\quad-\frac{(ad^2p)\int\frac{\log(d+ex)}{b+ax}\,dx}{e^3}+\frac{(bp)\int\left(\frac{1}{a}-\frac{b}{a(b+ax)}\right)dx}{2e}\\
&= \frac{bpx}{2ae}-\frac{dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}-\frac{bdp\log(b+ax)}{ae^2}\\
&\quad-\frac{b^2p\log(b+ax)}{2a^2e}+\frac{d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^3}+\frac{d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3}\\
&\quad-\frac{d^2p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e^3}-\frac{(d^2p)\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{e^2}+\frac{(d^2p)\int\frac{\log\left(\frac{e(b+ax)}{-ad+be}\right)}{d+ex}\,dx}{e^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{bpx}{2ae} - \frac{dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} - \frac{bdp\log(b+ax)}{ae^2} \\
&\quad - \frac{b^2p\log(b+ax)}{2a^2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^3} \\
&\quad + \frac{d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e^3} \\
&\quad + \frac{d^2p\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{e^3} + \frac{(d^2p)\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{ax}{-ad+be}\right)}{x}\,dx, x, d+ex\right)}{e^3} \\
&= \frac{bpx}{2ae} - \frac{dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} - \frac{bdp\log(b+ax)}{ae^2} \\
&\quad - \frac{b^2p\log(b+ax)}{2a^2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^3} + \frac{d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3} \\
&\quad - \frac{d^2p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e^3} - \frac{d^2p\operatorname{Li}_2\left(\frac{a(d+ex)}{ad-be}\right)}{e^3} + \frac{d^2p\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{e^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.05

$$
\begin{aligned}
\int \frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx &= -\frac{bdp\log\left(a+\frac{b}{x}\right)}{ae^2} - \frac{dx\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} \\
&\quad + \frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e} - \frac{bdp\log(x)}{ae^2} \\
&\quad + \frac{bp\left(\frac{x}{a} - \frac{b\log(b+ax)}{a^2}\right)}{2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^3} \\
&\quad + \frac{d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e^3} \\
&\quad + \frac{d^2p\operatorname{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e^3} - \frac{d^2p\operatorname{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^3}
\end{aligned}
$$

```
[In] Integrate[(x^2*Log[c*(a + b/x)^p])/(d + e*x),x]
```

```
[Out] -((b*d*p*Log[a + b/x])/(a*e^2)) - (d*x*Log[c*(a + b/x)^p])/e^2 + (x^2*Log[c
*(a + b/x)^p])/(2*e) - (b*d*p*Log[x])/(a*e^2) + (b*p*(x/a - (b*Log[b + a*x]
)/a^2))/(2*e) + (d^2*Log[c*(a + b/x)^p]*Log[d + e*x])/e^3 + (d^2*p*Log[-((e
*x)/d)]*Log[d + e*x])/e^3 - (d^2*p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d
+ e*x])/e^3 + (d^2*p*PolyLog[2, (d + e*x)/d])/e^3 - (d^2*p*PolyLog[2, (a*(d
 + e*x))/(a*d - b*e)])/e^3
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.10

method	result
parts	$\frac{x^2\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{2e}-\frac{dx\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2}+\frac{d^2\ln\left(c\left(a+\frac{b}{x}\right)^p\right)\ln(ex+d)}{e^3}+pbe\left(-\frac{d^2\operatorname{dilog}\left(\frac{-ad+a(ex+d)+be}{-ad+be}\right)}{e^4b}-\frac{d^2\ln(ex+d)\ln\left(\right.}{}\right.$

[In] int(x^2*ln(c*(a+b/x)^p)/(e*x+d),x,method=_RETURNVERBOSE)

[Out] 1/2*x^2*ln(c*(a+b/x)^p)/e-d*x*ln(c*(a+b/x)^p)/e^2+d^2*ln(c*(a+b/x)^p)*ln(e*x+d)/e^3+p*b*e*(-1/e^4*d^2/b*dilog((-a*d+a*(e*x+d)+b*e)/(-a*d+b*e))-1/e^4*d^2/b*ln(e*x+d)*ln((-a*d+a*(e*x+d)+b*e)/(-a*d+b*e))+1/e^4*d^2/b*ln(e*x+d)*ln(-e*x/d)+1/e^4*d^2/b*dilog(-e*x/d)+1/2/e^3*((e*x+d)/a+(-2*a*d-b*e)/a^2*ln(a*d-a*(e*x+d)-b*e)))

Fricas [F]

$$\int \frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx=\int \frac{x^2\log\left(\left(a+\frac{b}{x}\right)^pc\right)}{ex+d}\,dx$$

[In] integrate(x^2*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")

[Out] integral(x^2*log(c*((a*x + b)/x)^p)/(e*x + d), x)

Sympy [F]

$$\int \frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx=\int \frac{x^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$$

[In] integrate(x**2*ln(c*(a+b/x)**p)/(e*x+d),x)

[Out] Integral(x**2*log(c*(a + b/x)**p)/(d + e*x), x)

Maxima [F]

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left(\left(a+\frac{b}{x}\right)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^2*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x^2*log((a + b/x)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left(\left(a+\frac{b}{x}\right)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^2*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x^2*log((a + b/x)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \frac{x^2 \ln \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+e\,x}\,dx$$

```
[In] int((x^2*log(c*(a + b/x)^p))/(d + e*x),x)
[Out] int((x^2*log(c*(a + b/x)^p))/(d + e*x), x)
```

3.242 $\int \frac{x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex} dx$

Optimal result

Integrand size = 21, antiderivative size = 151

$$\int \frac{x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex} dx = \frac{x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} + \frac{bp \log(b+ax)}{ae} - \frac{d \log\left(c\left(a+\frac{b}{x}\right)^p\right) \log(d+ex)}{e^2} - \frac{dp \log\left(-\frac{ex}{d}\right) \log(d+ex)}{e^2} + \frac{dp \log\left(-\frac{e(b+ax)}{ad-be}\right) \log(d+ex)}{e^2} + \frac{dp \operatorname{PolyLog}\left(2, \frac{a(d+ex)}{ad-be}\right)}{e^2} - \frac{dp \operatorname{PolyLog}\left(2, 1+\frac{ex}{d}\right)}{e^2}$$

```
[Out] x*ln(c*(a+b/x)^p)/e+b*p*ln(a*x+b)/a/e-d*ln(c*(a+b/x)^p)*ln(e*x+d)/e^2-d*p*l
n(-e*x/d)*ln(e*x+d)/e^2+d*p*ln(-e*(a*x+b)/(a*d-b*e))*ln(e*x+d)/e^2+d*p*poly
log(2,a*(e*x+d)/(a*d-b*e))/e^2-d*p*polylog(2,1+e*x/d)/e^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {2516, 2498, 269, 31, 2512, 266, 2463, 2441, 2352, 2440, 2438}

$$\int \frac{x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex} dx = -\frac{d \log(d+ex) \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e^2} + \frac{x \log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} + \frac{dp \operatorname{PolyLog}\left(2, \frac{a(d+ex)}{ad-be}\right)}{e^2} + \frac{dp \log(d+ex) \log\left(-\frac{e(ax+b)}{ad-be}\right)}{e^2} + \frac{bp \log(ax+b)}{ae} - \frac{dp \operatorname{PolyLog}\left(2, \frac{ex}{d}+1\right)}{e^2} - \frac{dp \log\left(-\frac{ex}{d}\right) \log(d+ex)}{e^2}$$

```
[In] Int[(x*Log[c*(a + b/x)^p])/(d + e*x),x]
```

```
[Out] (x*Log[c*(a + b/x)^p])/e + (b*p*Log[b + a*x])/(a*e) - (d*Log[c*(a + b/x)^p]
*Log[d + e*x])/e^2 - (d*p*Log[-((e*x)/d)]*Log[d + e*x])/e^2 + (d*p*Log[-((e
*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e^2 + (d*p*PolyLog[2, (a*(d + e*x))
/(a*d - b*e)])/e^2 - (d*p*PolyLog[2, 1 + (e*x)/d])/e^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}-\frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e(d+ex)}\right)dx\\
&= \frac{\int \log\left(c\left(a+\frac{b}{x}\right)^p\right)dx}{e}-\frac{d\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx}{e}\\
&= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}-\frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2}-\frac{(bdp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x}\right)x^2}\,dx}{e^2}+\frac{(bp)\int \frac{1}{\left(a+\frac{b}{x}\right)x}\,dx}{e}\\
&= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}-\frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2}\\
&\quad -\frac{(bdp)\int \left(\frac{\log(d+ex)}{bx}-\frac{a\log(d+ex)}{b(b+ax)}\right)dx}{e^2}+\frac{(bp)\int \frac{1}{b+ax}\,dx}{e}\\
&= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}+\frac{bp\log(b+ax)}{ae}-\frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2}\\
&\quad -\frac{(dp)\int \frac{\log(d+ex)}{x}\,dx}{e^2}+\frac{(adp)\int \frac{\log(d+ex)}{b+ax}\,dx}{e^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}+\frac{bp\log(b+ax)}{ae}-\frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2} \\
&\quad -\frac{dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2}+\frac{dp\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e^2} \\
&\quad +\frac{(dp)\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}dx}{e}-\frac{(dp)\int\frac{\log\left(\frac{e(b+ax)}{-ad+be}\right)}{d+ex}dx}{e} \\
&= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}+\frac{bp\log(b+ax)}{ae}-\frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2} \\
&\quad -\frac{dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2}+\frac{dp\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e^2} \\
&\quad -\frac{dp\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{e^2}-\frac{(dp)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{ax}{-ad+be}\right)}{x}dx,x,d+ex\right)}{e^2} \\
&= \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}+\frac{bp\log(b+ax)}{ae}-\frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2} \\
&\quad -\frac{dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2}+\frac{dp\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e^2} \\
&\quad +\frac{dp\operatorname{Li}_2\left(\frac{a(d+ex)}{ad-be}\right)}{e^2}-\frac{dp\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{e^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.09

$$
\begin{aligned}
\int\frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}dx &= \frac{bp\log\left(a+\frac{b}{x}\right)}{ae}+\frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e} \\
&\quad +\frac{bp\log(x)}{ae}-\frac{d\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e^2} \\
&\quad -\frac{dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2}+\frac{dp\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e^2} \\
&\quad -\frac{dp\operatorname{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e^2}+\frac{dp\operatorname{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e^2}
\end{aligned}
$$

```
[In] Integrate[(x*Log[c*(a + b/x)^p])/(d + e*x),x]
```

```
[Out] (b*p*Log[a + b/x])/(a*e) + (x*Log[c*(a + b/x)^p])/e + (b*p*Log[x])/(a*e) -
(d*Log[c*(a + b/x)^p]*Log[d + e*x])/e^2 - (d*p*Log[-((e*x)/d)]*Log[d + e*x]
)/e^2 + (d*p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e^2 - (d*p*Pol
yLog[2, (d + e*x)/d])/e^2 + (d*p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/e^2
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.25

method	result
parts	$\frac{x\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{e}-\frac{d\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)\ln(ex+d)}{e^{2}}+pbe\left(\frac{\ln(ad-a(ex+d)-be)}{e^{2}a}-\frac{d\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{e^{3}b}-\frac{d\,\mathrm{dilog}\left(-\frac{ex}{d}\right)}{e^{3}b}+\frac{d\,\mathrm{dilog}}{}\right.$

[In] int(x*ln(c*(a+b/x)^p)/(e*x+d),x,method=_RETURNVERBOSE)

[Out] x*ln(c*(a+b/x)^p)/e-d*ln(c*(a+b/x)^p)*ln(e*x+d)/e^2+p*b*e*(1/e^2*ln(a*d-a*(e*x+d)-b*e)/a-1/e^3*d/b*ln(e*x+d)*ln(-e*x/d)-1/e^3*d/b*dilog(-e*x/d)+1/e^3*d/b*dilog((-a*d+a*(e*x+d)+b*e)/(-a*d+b*e))+1/e^3*d/b*ln(e*x+d)*ln((-a*d+a*(e*x+d)+b*e)/(-a*d+b*e)))

Fricas [F]

$$\int \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx=\int \frac{x\log\left(\left(a+\frac{b}{x}\right)^pc\right)}{ex+d}\,dx$$

[In] integrate(x*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")

[Out] integral(x*log(c*((a*x + b)/x)^p)/(e*x + d), x)

Sympy [F]

$$\int \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx=\int \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$$

[In] integrate(x*ln(c*(a+b/x)**p)/(e*x+d),x)

[Out] Integral(x*log(c*(a + b/x)**p)/(d + e*x), x)

Maxima [F]

$$\int \frac{x\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx=\int \frac{x\log\left(\left(a+\frac{b}{x}\right)^pc\right)}{ex+d}\,dx$$

[In] integrate(x*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")

[Out] integrate(x*log((a + b/x)^p*c)/(e*x + d), x)

Giac [F]

$$\int \frac{x \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \frac{x \log \left(\left(a+\frac{b}{x}\right)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x*log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x*log((a + b/x)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \frac{x \ln \left(c \left(a+\frac{b}{x}\right)^p\right)}{d+e\,x}\,dx$$

```
[In] int((x*log(c*(a + b/x)^p))/(d + e*x),x)
[Out] int((x*log(c*(a + b/x)^p))/(d + e*x), x)
```

3.243 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e}+\frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e} - \frac{p\operatorname{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e}+\frac{p\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e}$$

```
[Out] ln(c*(a+b/x)^p)*ln(e*x+d)/e+p*ln(-e*x/d)*ln(e*x+d)/e-p*ln(-e*(a*x+b)/(a*d-b
*e))*ln(e*x+d)/e-p*polylog(2,a*(e*x+d)/(a*d-b*e))/e+p*polylog(2,1+e*x/d)/e
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2512, 266, 2463, 2441, 2352, 2440, 2438}

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{e}-\frac{p\operatorname{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e} - \frac{p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{e} + \frac{p\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e}+\frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}$$

```
[In] Int[Log[c*(a + b/x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x)^p]*Log[d + e*x])/e + (p*Log[-((e*x)/d)]*Log[d + e*x])/e -
(p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e - (p*PolyLog[2, (a*(d
+ e*x))/(a*d - b*e)])/e + (p*PolyLog[2, 1 + (e*x)/d])/e
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$\text{integral} = \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{(bp)\int\frac{\log(d+ex)}{\left(a+\frac{b}{x}\right)x^2}\,dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{(bp)\int\left(\frac{\log(d+ex)}{bx}-\frac{a\log(d+ex)}{b(b+ax)}\right)dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\int\frac{\log(d+ex)}{x}\,dx}{e} - \frac{(ap)\int\frac{\log(d+ex)}{b+ax}\,dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e} - p\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx + p\int\frac{\log\left(\frac{e(b+ax)}{-ad+be}\right)}{d+ex}\,dx$$

$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e} + \frac{p\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{e} + \frac{p\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{ax}{-ad+be}\right)}{x}\,dx,x,d+ex\right)}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e} - \frac{p\mathrm{Li}_2\left(\frac{a(d+ex)}{ad-be}\right)}{e} + \frac{p\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{e}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01

$$\int\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{e} + \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{e} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{e}$$

```
[In] Integrate[Log[c*(a + b/x)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x)^p]*Log[d + e*x])/e + (p*Log[-((e*x)/d)]*Log[d + e*x])/e -
(p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/e + (p*PolyLog[2, (d + e
*x)/d])/e - (p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/e
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.21

method	result
parts	$\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)\ln(ex+d)}{e}+pb\left(\frac{\operatorname{dilog}\left(-\frac{ex}{d}\right)+\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{be}-\frac{a\left(\frac{\operatorname{dilog}\left(\frac{-ad+a(ex+d)+be}{-ad+be}\right)}{a}+\frac{\ln(ex+d)\ln\left(\frac{-ad+a(ex+d)+be}{-ad+be}\right)}{a}\right.}{be}\right.$

[In] int(ln(c*(a+b/x)^p)/(e*x+d),x,method=_RETURNVERBOSE)

[Out] ln(c*(a+b/x)^p)*ln(e*x+d)/e+p*b*(1/b/e*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))-a/b/e*(dilog((-a*d+a*(e*x+d)+b*e)/(-a*d+b*e))/a+ln(e*x+d)*ln((-a*d+a*(e*x+d)+b*e)/(-a*d+b*e))/a))

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{d+ex}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)}{ex+d}\,dx$$

[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="fricas")

[Out] integral(log(c*((a*x + b)/x)^p)/(e*x + d), x)

Sympy [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{d+ex}\,dx=\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{d+ex}\,dx$$

[In] integrate(ln(c*(a+b/x)**p)/(e*x+d),x)

[Out] Integral(log(c*(a + b/x)**p)/(d + e*x), x)

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.41

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \frac{bp\left(\frac{\log(ex+d)\log\left(a+\frac{b}{x}\right)}{b} - \frac{\log(ex+d)\log\left(-\frac{aex+ad}{ad-be}+1\right)+\mathrm{Li}_2\left(\frac{aex+ad}{ad-be}\right)}{b} + \frac{\log(ex+d)\log\left(-\frac{ex+d}{d}+1\right)+\mathrm{Li}_2\left(\frac{ex+d}{d}\right)}{b}\right)}{e} - \frac{p\log(ex+d)\log\left(a+\frac{b}{x}\right)}{e} + \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)\log(ex+d)}{e}$$

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="maxima")
```

```
[Out] b*p*(log(e*x + d)*log(a + b/x)/b - (log(e*x + d)*log(-(a*e*x + a*d)/(a*d -
b*e) + 1) + dilog((a*e*x + a*d)/(a*d - b*e)))/b + (log(e*x + d)*log(-(e*x +
 d)/d + 1) + dilog((e*x + d)/d))/b)/e - p*log(e*x + d)*log(a + b/x)/e + log
((a + b/x)^p*c)*log(e*x + d)/e
```

Giac [F]

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(log(c*(a+b/x)^p)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx = \int \frac{\ln\left(c\,{\left(a+\frac{b}{x}\right)}^p\right)}{d+e\,x}\,dx$$

```
[In] int(log(c*(a + b/x)^p)/(d + e*x),x)
```

```
[Out] int(log(c*(a + b/x)^p)/(d + e*x), x)
```

3.244 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 159

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x(d+ex)}\,dx = -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{d} - \frac{p\,\mathrm{PolyLog}\left(2,1+\frac{b}{ax}\right)}{d} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d} - \frac{p\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{d}$$

```
[Out] -ln(c*(a+b/x)^p)*ln(-b/a/x)/d-ln(c*(a+b/x)^p)*ln(e*x+d)/d-p*ln(-e*x/d)*ln(e
*x+d)/d+p*ln(-e*(a*x+b)/(a*d-b*e))*ln(e*x+d)/d-p*polylog(2,1+b/a/x)/d+p*pol
ylog(2,a*(e*x+d)/(a*d-b*e))/d-p*polylog(2,1+e*x/d)/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used

$= \{2516, 2504, 2441, 2352, 2512, 266, 2463, 2440, 2438\}$

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x(d+ex)}\,dx = -\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d} - \frac{\log\left(-\frac{b}{ax}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d} + \frac{p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{d} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{b}{ax}+1\right)}{d} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d} - \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d}$$

```
[In] Int[Log[c*(a + b/x)^p]/(x*(d + e*x)),x]
```

```
[Out] -((Log[c*(a + b/x)^p]*Log[-(b/(a*x))])/d) - (Log[c*(a + b/x)^p]*Log[d + e*x
])/d - (p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[-((e*(b + a*x))/(a*d - b
*e))]*Log[d + e*x])/d - (p*PolyLog[2, 1 + b/(a*x)])/d + (p*PolyLog[2, (a*(d
 + e*x))/(a*d - b*e)])/d - (p*PolyLog[2, 1 + (e*x)/d])/d
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{dx}-\frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d(d+ex)}\right)dx\\
&= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx}{d}-\frac{e\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx}{d}\\
&= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d}-\frac{\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx,x,\frac{1}{x}\right)}{d}-\frac{(bp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x}\right)x^2}\,dx}{d}\\
&= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d}\\
&\quad -\frac{(bp)\int \left(\frac{\log(d+ex)}{bx}-\frac{a\log(d+ex)}{b(b+ax)}\right)dx}{d}+\frac{(bp)\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx,x,\frac{1}{x}\right)}{d}
\end{aligned}
$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{p\text{Li}_2\left(1+\frac{b}{ax}\right)}{d} - \frac{p\int\frac{\log(d+ex)}{x}\,dx}{d} + \frac{(ap)\int\frac{\log(d+ex)}{b+ax}\,dx}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{d} - \frac{p\text{Li}_2\left(1+\frac{b}{ax}\right)}{d} + \frac{(ep)\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{d} - \frac{(ep)\int\frac{\log\left(\frac{e(b+ax)}{-ad+be}\right)}{d+ex}\,dx}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{d} - \frac{p\text{Li}_2\left(1+\frac{b}{ax}\right)}{d} - \frac{p\text{Li}_2\left(1+\frac{ex}{d}\right)}{d} - \frac{p\text{Subst}\left(\int\frac{\log\left(1+\frac{ax}{-ad+be}\right)}{x}\,dx, x, d+ex\right)}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{d} - \frac{p\text{Li}_2\left(1+\frac{b}{ax}\right)}{d} + \frac{p\text{Li}_2\left(\frac{a(d+ex)}{ad-be}\right)}{d} - \frac{p\text{Li}_2\left(1+\frac{ex}{d}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.01

$$\int\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x(d+ex)}\,dx = -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d} - \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d} - \frac{p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{d} - \frac{p\,\text{PolyLog}\left(2,\frac{a+\frac{b}{x}}{a}\right)}{d} - \frac{p\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{d} + \frac{p\,\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d}$$

```
[In] Integrate[Log[c*(a + b/x)^p]/(x*(d + e*x)),x]
```

```
[Out] -((Log[c*(a + b/x)^p]*Log[-(b/(a*x))])/d) - (Log[c*(a + b/x)^p]*Log[d + e*x
])/d - (p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[-((e*(b + a*x))/(a*d - b
*e))]*Log[d + e*x])/d - (p*PolyLog[2, (a + b/x)/a])/d - (p*PolyLog[2, (d +
e*x)/d])/d + (p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/d
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.31

method	result
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)\ln(ex+d)}{d}+\frac{\ln\left(c\left(a+\frac{b}{x}\right)^{p}\right)\ln(x)}{d}+pb\left(\frac{\ln(x)^{2}}{2db}-\frac{\operatorname{dilog}\left(\frac{ax+b}{b}\right)}{db}-\frac{\ln(x)\ln\left(\frac{ax+b}{b}\right)}{db}-\frac{\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{db}\right.$

```
[In] int(ln(c*(a+b/x)^p)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(c*(a+b/x)^p)*ln(e*x+d)/d+ln(c*(a+b/x)^p)/d*ln(x)+p*b*(1/2/d/b*ln(x)^2-1
/d/b*dilog((a*x+b)/b)-1/d/b*ln(x)*ln((a*x+b)/b)-1/d/b*ln(e*x+d)*ln(-e*x/d)-
1/d/b*dilog(-e*x/d)+1/d/b*dilog((-a*d+a*(e*x+d)+b*e)/(-a*d+b*e))+1/d/b*ln(e
*x+d)*ln((-a*d+a*(e*x+d)+b*e)/(-a*d+b*e)))
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x(d+ex)}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)}{(ex+d)x}\,dx$$

```
[In] integrate(log(c*(a+b/x)^p)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x + b)/x)^p)/(e*x^2 + d*x), x)
```

Sympy [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x(d+ex)}\,dx=\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x\,(d+ex)}\,dx$$

```
[In] integrate(ln(c*(a+b/x)**p)/x/(e*x+d),x)
```

```
[Out] Integral(log(c*(a + b/x)**p)/(x*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.13

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x(d+ex)}\,dx=$$
$$-\frac{1}{2}bp\left(\frac{2\log(ex+d)\log(x)-\log(x)^{2}}{bd}+\frac{2\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\mathrm{Li}_{2}\left(-\frac{ax}{b}\right)\right)}{bd}-\frac{2\left(\log\left(\frac{ex}{d}+1\right)\log(x)\right.}{bd}\right.$$
$$-\left(\frac{\log(ex+d)}{d}-\frac{\log(x)}{d}\right)\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)$$

```
[In] integrate(log(c*(a+b/x)^p)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*b*p*((2*log(e*x + d)*log(x) - log(x)^2)/(b*d) + 2*(log(a*x/b + 1)*log(
x) + dilog(-a*x/b))/(b*d) - 2*(log(e*x/d + 1)*log(x) + dilog(-e*x/d))/(b*d)
 - 2*(log(e*x + d)*log(-(a*e*x + a*d)/(a*d - b*e) + 1) + dilog((a*e*x + a*d
)/(a*d - b*e)))/(b*d)) - (log(e*x + d)/d - log(x)/d)*log((a + b/x)^p*c)
```

Giac [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x}\right)^{p}c\right)}{(ex+d)x}\,dx$$

```
[In] integrate(log(c*(a+b/x)^p)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(log((a + b/x)^p*c)/((e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^{p}\right)}{x(d+ex)}\,dx = \int \frac{\ln\left(c\,\left(a+\frac{b}{x}\right)^{p}\right)}{x\,\left(d+e\,x\right)}\,dx$$

```
[In] int(log(c*(a + b/x)^p)/(x*(d + e*x)),x)
```

```
[Out] int(log(c*(a + b/x)^p)/(x*(d + e*x)), x)
```

3.245 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 198

$$\begin{aligned}\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)}\,dx &= \frac{p}{dx}-\frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd}+\frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2}\\&\quad+\frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^2}+\frac{ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2}\\&\quad-\frac{ep\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{d^2}+\frac{ep\operatorname{PolyLog}\left(2,1+\frac{b}{ax}\right)}{d^2}\\&\quad-\frac{ep\operatorname{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d^2}+\frac{ep\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{d^2}\end{aligned}$$

```
[Out] p/d/x-(a+b/x)*ln(c*(a+b/x)^p)/b/d+e*ln(c*(a+b/x)^p)*ln(-b/a/x)/d^2+e*ln(c*(
a+b/x)^p)*ln(e*x+d)/d^2+e*p*ln(-e*x/d)*ln(e*x+d)/d^2-e*p*ln(-e*(a*x+b)/(a*d
-b*e))*ln(e*x+d)/d^2+e*p*polylog(2,1+b/a/x)/d^2-e*p*polylog(2,a*(e*x+d)/(a*
d-b*e))/d^2+e*p*polylog(2,1+e*x/d)/d^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules

used = {2516, 2504, 2436, 2332, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)}\,dx = \frac{e\log\left(-\frac{b}{ax}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2} + \frac{e\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd} + \frac{ep\,\text{PolyLog}\left(2,\frac{b}{ax}+1\right)}{d^2} - \frac{ep\,\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d^2} - \frac{ep\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{d^2} + \frac{ep\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^2} + \frac{ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2} + \frac{p}{dx}$$

```
[In] Int[Log[c*(a + b/x)^p]/(x^2*(d + e*x)),x]
```

```
[Out] p/(d*x) - ((a + b/x)*Log[c*(a + b/x)^p])/(b*d) + (e*Log[c*(a + b/x)^p]*Log[
-(b/(a*x))])/d^2 + (e*Log[c*(a + b/x)^p]*Log[d + e*x])/d^2 + (e*p*Log[-((e*
x)/d)]*Log[d + e*x])/d^2 - (e*p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e
*x])/d^2 + (e*p*PolyLog[2, 1 + b/(a*x)])/d^2 - (e*p*PolyLog[2, (a*(d + e*x)
)/(a*d - b*e)])/d^2 + (e*p*PolyLog[2, 1 + (e*x)/d])/d^2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{dx^2} - \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2x} + \frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2(d+ex)} \right) dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx}{d^2}$$

$$= \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^2} - \frac{\text{Subst}\left(\int \log\left(c(a+bx)^p\right)\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(bep)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x}\right)x^2}\,dx}{d^2}$$

$$= \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^2} - \frac{\text{Subst}\left(\int \log\left(cx^p\right)\,dx, x, a+\frac{b}{x}\right)}{bd} + \frac{(bep)\int \left(\frac{\log(d+ex)}{bx} - \frac{a\log(d+ex)}{b(b+ax)}\right)\,dx}{d^2} - \frac{(bep)\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx, x, \frac{1}{x}\right)}{d^2}$$

$$= \frac{p}{dx} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^2} + \frac{ep\text{Li}_2\left(1+\frac{b}{ax}\right)}{d^2} + \frac{(ep)\int \frac{\log(d+ex)}{x}\,dx}{d^2} - \frac{(aep)\int \frac{\log(d+ex)}{b+ax}\,dx}{d^2}$$

$$= \frac{p}{dx} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^2} + \frac{ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{d^2} + \frac{ep\text{Li}_2\left(1+\frac{b}{ax}\right)}{d^2} - \frac{(e^2p)\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{d^2} + \frac{(e^2p)\int \frac{\log\left(\frac{e(b+ax)}{-ad+be}\right)}{d+ex}\,dx}{d^2}$$

$$= \frac{p}{dx} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^2} + \frac{ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{d^2} + \frac{ep\text{Li}_2\left(1+\frac{b}{ax}\right)}{d^2} + \frac{ep\text{Li}_2\left(1+\frac{ex}{d}\right)}{d^2} + \frac{(ep)\text{Subst}\left(\int \frac{\log\left(1+\frac{ax}{-ad+be}\right)}{x}\,dx, x, d+ex\right)}{d^2}$$

$$= \frac{p}{dx} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^2} + \frac{ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{d^2} + \frac{ep\text{Li}_2\left(1+\frac{b}{ax}\right)}{d^2} - \frac{ep\text{Li}_2\left(\frac{a(d+ex)}{ad-be}\right)}{d^2} + \frac{ep\text{Li}_2\left(1+\frac{ex}{d}\right)}{d^2}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.01

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)}\,dx = \frac{p}{dx} - \frac{\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^2} + \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^2} + \frac{ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{d^2} + \frac{ep\,\text{PolyLog}\left(2,\frac{a+\frac{b}{x}}{a}\right)}{d^2} + \frac{ep\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{d^2} - \frac{ep\,\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d^2}$$

```
[In] Integrate[Log[c*(a + b/x)^p]/(x^2*(d + e*x)),x]
```

```
[Out] p/(d*x) - ((a + b/x)*Log[c*(a + b/x)^p])/(b*d) + (e*Log[c*(a + b/x)^p]*Log[
-(b/(a*x))])/d^2 + (e*Log[c*(a + b/x)^p]*Log[d + e*x])/d^2 + (e*p*Log[-((e*
x)/d)]*Log[d + e*x])/d^2 - (e*p*Log[-((e*(b + a*x))/(a*d - b*e))]*Log[d + e
*x])/d^2 + (e*p*PolyLog[2, (a + b/x)/a])/d^2 + (e*p*PolyLog[2, (d + e*x)/d]
)/d^2 - (e*p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/d^2
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.33

method	result
parts	$\frac{e\ln\left(c\left(a+\frac{b}{x}\right)^p\right)\ln(ex+d)}{d^2} - \frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{dx} - \frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)e\ln(x)}{d^2} + pb\left(\frac{e\left(\frac{\text{dilog}\left(-\frac{ex}{d}\right)+\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{b} - \frac{a\left(\text{dilog}\left(\frac{-ad+\cdots}{\cdots}\right)\cdots\right)}{\cdots}\right)}{d^2\cdots}\right.$

[In] int(ln(c*(a+b/x)^p)/x^2/(e*x+d),x,method=_RETURNVERBOSE)

[Out] e*ln(c*(a+b/x)^p)*ln(e*x+d)/d^2-ln(c*(a+b/x)^p)/d/x-ln(c*(a+b/x)^p)*e/d^2*ln(x)+p*b*(e/d^2*(1/b*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))-a/b*(dilog((-a*d+a*(e*x+d)+b*e)/(-a*d+b*e))/a+ln(e*x+d)*ln((-a*d+a*(e*x+d)+b*e)/(-a*d+b*e))/a))+1/d/b/x+1/d*a/b^2*ln(x)-1/d*a/b^2*ln(a*x+b)-1/2*e/d^2/b*ln(x)^2+e/d^2/b*dilog((a*x+b)/b)+e/d^2/b*ln(x)*ln((a*x+b)/b))

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{(ex+d)x^2}\,dx$$

[In] integrate(log(c*(a+b/x)^p)/x^2/(e*x+d),x, algorithm="fricas")

[Out] integral(log(c*((a*x + b)/x)^p)/(e*x^3 + d*x^2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)}\,dx=\text{Timed out}$$

[In] integrate(ln(c*(a+b/x)**p)/x**2/(e*x+d),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.16

$$\begin{aligned}&\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)}\,dx\\&=\frac{1}{2}\,bp\left(\frac{2\left(\log\left(\frac{ax}{b}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)e}{bd^2}-\frac{2\left(\log\left(\frac{ex}{d}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{ex}{d}\right)\right)e}{bd^2}-\frac{2\left(\log\left(ex+d\right)\log\left(-\right.\right.}{}\right.\\&\quad+\left(\frac{e\log\left(ex+d\right)}{d^2}-\frac{e\log\left(x\right)}{d^2}-\frac{1}{dx}\right)\log\left(\left(a+\frac{b}{x}\right)^pc\right)\end{aligned}$$

[In] integrate(log(c*(a+b/x)^p)/x^2/(e*x+d),x, algorithm="maxima")

[Out] 1/2*b*p*(2*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*e/(b*d^2) - 2*(log(e*x/d + 1)*log(x) + dilog(-e*x/d))*e/(b*d^2) - 2*(log(e*x + d)*log(-(a*e*x + a*d

```
)/(a*d - b*e) + 1) + dilog((a*e*x + a*d)/(a*d - b*e)))*e/(b*d^2) - 2*a*log(
a*x + b)/(b^2*d) + 2*a*log(x)/(b^2*d) + (2*e*log(e*x + d)*log(x) - e*log(x)
^2)/(b*d^2) + 2/(b*d*x)) + (e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x))*lo
g((a + b/x)^p*c)
```

Giac [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{(ex+d)x^2}\,dx$$

```
[In] integrate(log(c*(a+b/x)^p)/x^2/(e*x+d),x, algorithm="giac")
[Out] integrate(log((a + b/x)^p*c)/((e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2(d+ex)}\,dx=\int \frac{\ln\left(c\,\left(a+\frac{b}{x}\right)^p\right)}{x^2\,\left(d+e\,x\right)}\,dx$$

```
[In] int(log(c*(a + b/x)^p)/(x^2*(d + e*x)),x)
[Out] int(log(c*(a + b/x)^p)/(x^2*(d + e*x)), x)
```

3.246 $\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 287

$$\begin{aligned}\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx &= \frac{p}{4dx^2}-\frac{ap}{2bdx}-\frac{ep}{d^2x}+\frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2d}+\frac{e\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd^2}\\&-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2dx^2}-\frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^3}\\&-\frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^3}-\frac{e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3}\\&+\frac{e^2p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{d^3}-\frac{e^2p\,\mathrm{PolyLog}\left(2,1+\frac{b}{ax}\right)}{d^3}\\&+\frac{e^2p\,\mathrm{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d^3}-\frac{e^2p\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{d^3}\end{aligned}$$

```
[Out] 1/4*p/d/x^2-1/2*a*p/b/d/x-e*p/d^2/x+1/2*a^2*p*ln(a+b/x)/b^2/d+e*(a+b/x)*ln(
c*(a+b/x)^p)/b/d^2-1/2*ln(c*(a+b/x)^p)/d/x^2-e^2*ln(c*(a+b/x)^p)*ln(-b/a/x)
/d^3-e^2*ln(c*(a+b/x)^p)*ln(e*x+d)/d^3-e^2*p*ln(-e*x/d)*ln(e*x+d)/d^3+e^2*p
*ln(-e*(a*x+b)/(a*d-b*e))*ln(e*x+d)/d^3-e^2*p*polylog(2,1+b/a/x)/d^3+e^2*p*
polylog(2,a*(e*x+d)/(a*d-b*e))/d^3-e^2*p*polylog(2,1+e*x/d)/d^3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {2516, 2504, 2442, 45, 2436, 2332, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx = \frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2d}-\frac{e^2\log\left(-\frac{b}{ax}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^3}-\frac{e^2\log(d+ex)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^3}+\frac{e\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2dx^2}-\frac{e^2p\,\text{PolyLog}\left(2,\frac{b}{ax}+1\right)}{d^3}+\frac{e^2p\,\text{PolyLog}\left(2,\frac{a(d+ex)}{ad-be}\right)}{d^3}+\frac{e^2p\log(d+ex)\log\left(-\frac{e(ax+b)}{ad-be}\right)}{d^3}-\frac{ap}{2bdx}-\frac{e^2p\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^3}-\frac{e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3}-\frac{ep}{d^2x}+\frac{p}{4dx^2}$$

```
[In] Int[Log[c*(a + b/x)^p]/(x^3*(d + e*x)),x]
```

```
[Out] p/(4*d*x^2) - (a*p)/(2*b*d*x) - (e*p)/(d^2*x) + (a^2*p*Log[a + b/x])/(2*b^2
*d) + (e*(a + b/x)*Log[c*(a + b/x)^p])/(b*d^2) - Log[c*(a + b/x)^p]/(2*d*x^
2) - (e^2*Log[c*(a + b/x)^p]*Log[-(b/(a*x))])/d^3 - (e^2*Log[c*(a + b/x)^p]
*Log[d + e*x])/d^3 - (e^2*p*Log[-((e*x)/d)]*Log[d + e*x])/d^3 + (e^2*p*Log[
-((e*(b + a*x))/(a*d - b*e))]*Log[d + e*x])/d^3 - (e^2*p*PolyLog[2, 1 + b/(
a*x)])/d^3 + (e^2*p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)])/d^3 - (e^2*p*Pol
yLog[2, 1 + (e*x)/d])/d^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{dx^3} - \frac{e\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^2x^2} + \frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^3x} - \frac{e^3\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d^3(d+ex)}\right) dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x}\,dx}{d^3} - \frac{e^3\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{d+ex}\,dx}{d^3}$$

$$= -\frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^3} - \frac{\text{Subst}\left(\int x\log\left(c(a+bx)^p\right)\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\text{Subst}\left(\int \log\left(c(a+bx)^p\right)\,dx, x, \frac{1}{x}\right)}{d^2} - \frac{e^2\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x, \frac{1}{x}\right)}{d^3} - \frac{(be^2p)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x}\right)x^2}\,dx}{d^3}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2dx^2} - \frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^3} - \frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^3} + \frac{e\text{Subst}\left(\int \log\left(cx^p\right)\,dx, x, a+\frac{b}{x}\right)}{bd^2} + \frac{(bp)\text{Subst}\left(\int \frac{x^2}{a+bx}\,dx, x, \frac{1}{x}\right)}{2d} - \frac{(be^2p)\int \left(\frac{\log(d+ex)}{bx} - \frac{a\log(d+ex)}{b(b+ax)}\right)dx}{d^3} + \frac{(be^2p)\,\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx, x, \frac{1}{x}\right)}{d^3}$$

$$=-\frac{ep}{d^2x}+\frac{e\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2dx^2}-\frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^3}-\frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^3}-\frac{e^2p\text{Li}_2\left(1+\frac{b}{ax}\right)}{d^3}+\frac{(bp)\text{Subst}\left(\int\left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)\,dx,x,\frac{1}{x}\right)}{2d}-\frac{(e^2p)\int\frac{\log(d+ex)}{x}\,dx}{d^3}+\frac{(ae^2p)\int\frac{\log(d+ex)}{b+ax}\,dx}{d^3}$$

$$=\frac{p}{4dx^2}-\frac{ap}{2bdx}-\frac{ep}{d^2x}+\frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2d}+\frac{e\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2dx^2}-\frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^3}-\frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^3}-\frac{e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3}+\frac{e^2p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{d^3}-\frac{e^2p\text{Li}_2\left(1+\frac{b}{ax}\right)}{d^3}+\frac{(e^3p)\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{d^3}-\frac{(e^3p)\int\frac{\log\left(\frac{e(b+ax)}{-ad+be}\right)}{d+ex}\,dx}{d^3}$$

$$=\frac{p}{4dx^2}-\frac{ap}{2bdx}-\frac{ep}{d^2x}+\frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2d}+\frac{e\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2dx^2}-\frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^3}-\frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^3}-\frac{e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3}+\frac{e^2p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{d^3}-\frac{e^2p\text{Li}_2\left(1+\frac{b}{ax}\right)}{d^3}-\frac{e^2p\text{Li}_2\left(1+\frac{ex}{d}\right)}{d^3}-\frac{(e^2p)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{ax}{-ad+be}\right)}{x}\,dx,x,d+ex\right)}{d^3}$$

$$=\frac{p}{4dx^2}-\frac{ap}{2bdx}-\frac{ep}{d^2x}+\frac{a^2p\log\left(a+\frac{b}{x}\right)}{2b^2d}+\frac{e\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{bd^2}-\frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{2dx^2}-\frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log\left(-\frac{b}{ax}\right)}{d^3}-\frac{e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log(d+ex)}{d^3}-\frac{e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3}+\frac{e^2p\log\left(-\frac{e(b+ax)}{ad-be}\right)\log(d+ex)}{d^3}-\frac{e^2p\text{Li}_2\left(1+\frac{b}{ax}\right)}{d^3}+\frac{e^2p\text{Li}_2\left(\frac{a(d+ex)}{ad-be}\right)}{d^3}-\frac{e^2p\text{Li}_2\left(1+\frac{ex}{d}\right)}{d^3}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.92

$$\int \frac{\log \left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx = -\frac{-\frac{d^2p}{x^2}+\frac{2ad^2p}{bx}+\frac{4dep}{x}-\frac{2a^2d^2p\log\left(a+\frac{b}{x}\right)}{b^2}-\frac{4de\left(a+\frac{b}{x}\right)\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{b}+\frac{2d^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^2}+4e^2\log\left(c\left(a+\frac{b}{x}\right)^p\right)\log}{}$$

```
[In] Integrate[Log[c*(a + b/x)^p]/(x^3*(d + e*x)),x]
```

```
[Out] -1/4*(-((d^2*p)/x^2) + (2*a*d^2*p)/(b*x) + (4*d*e*p)/x - (2*a^2*d^2*p*Log[a
 + b/x])/b^2 - (4*d*e*(a + b/x)*Log[c*(a + b/x)^p])/b + (2*d^2*Log[c*(a + b
/x)^p])/x^2 + 4*e^2*Log[c*(a + b/x)^p]*Log[-(b/(a*x))] + 4*e^2*Log[c*(a + b
/x)^p]*Log[d + e*x] + 4*e^2*p*Log[-((e*x)/d)]*Log[d + e*x] - 4*e^2*p*Log[(e
*(b + a*x))/(-(a*d) + b*e)]*Log[d + e*x] + 4*e^2*p*PolyLog[2, 1 + b/(a*x)]
- 4*e^2*p*PolyLog[2, (a*(d + e*x))/(a*d - b*e)] + 4*e^2*p*PolyLog[2, 1 + (e
*x)/d])/d^3
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.20

method	result
parts	$-\frac{e^2\ln\left(c\left(a+\frac{b}{x}\right)^p\right)\ln(ex+d)}{d^3}-\frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)}{2d\,x^2}+\frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)e^2\ln(x)}{d^3}+\frac{\ln\left(c\left(a+\frac{b}{x}\right)^p\right)e}{d^2x}+\frac{pb\left(-\frac{-\frac{d}{2b\,x^2}-\frac{-ad-2be}{b^2x}+\frac{(ad+}{}}{}\right.}{}$

```
[In] int(ln(c*(a+b/x)^p)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -e^2*ln(c*(a+b/x)^p)*ln(e*x+d)/d^3-1/2*ln(c*(a+b/x)^p)/d/x^2+ln(c*(a+b/x)^p
)*e^2/d^3*ln(x)+ln(c*(a+b/x)^p)*e/d^2/x+1/2*p*b*(-1/d^2*(-1/2*d/b/x^2-(-a*d
-2*b*e)/b^2/x+(a*d+2*b*e)/b^3*a*ln(x)-(a*d+2*b*e)/b^3*a*ln(a*x+b))+e^2/d^3/
b*ln(x)^2-2*e^2/d^3/b*dilog((a*x+b)/b)-2*e^2/d^3/b*ln(x)*ln((a*x+b)/b)-2*e^
2/d^3/b*ln(e*x+d)*ln(-e*x/d)-2*e^2/d^3/b*dilog(-e*x/d)+2*e^2/d^3/b*dilog((-
a*d+a*(e*x+d)+b*e)/(-a*d+b*e))+2*e^2/d^3/b*ln(e*x+d)*ln((-a*d+a*(e*x+d)+b*e
)/(-a*d+b*e)))
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{(ex+d)x^3}\,dx$$

[In] integrate(log(c*(a+b/x)^p)/x^3/(e*x+d),x, algorithm="fricas")

[Out] integral(log(c*((a*x + b)/x)^p)/(e*x^4 + d*x^3), x)

Sympy [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx=\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3\,(d+ex)}\,dx$$

[In] integrate(ln(c*(a+b/x)**p)/x**3/(e*x+d),x)

[Out] Integral(log(c*(a + b/x)**p)/(x**3*(d + e*x)), x)

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.07

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx$$
$$=\frac{1}{4}\left(4\,e\left(\frac{a\log(ax+b)}{b^2d^2}-\frac{a\log(x)}{b^2d^2}-\frac{1}{bd^2x}\right)-\frac{4\left(\log\left(\frac{ax}{b}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{ax}{b}\right)\right)e^2}{bd^3}+\frac{4\left(\log\left(\frac{ex}{d}+1\right)\log(\right.}{bd}\right.$$
$$-\frac{1}{2}\left(\frac{2\,e^2\log(ex+d)}{d^3}-\frac{2\,e^2\log(x)}{d^3}-\frac{2\,ex-d}{d^2x^2}\right)\log\left(\left(a+\frac{b}{x}\right)^p c\right)$$

[In] integrate(log(c*(a+b/x)^p)/x^3/(e*x+d),x, algorithm="maxima")

[Out] 1/4*(4*e*(a*log(a*x + b)/(b^2*d^2) - a*log(x)/(b^2*d^2) - 1/(b*d^2*x)) - 4*(log(a*x/b + 1)*log(x) + dilog(-a*x/b))*e^2/(b*d^3) + 4*(log(e*x/d + 1)*log(x) + dilog(-e*x/d))*e^2/(b*d^3) + 4*(log(e*x + d)*log(-(a*e*x + a*d)/(a*d - b*e) + 1) + dilog((a*e*x + a*d)/(a*d - b*e)))*e^2/(b*d^3) + 2*a^2*log(a*x + b)/(b^3*d) - 2*a^2*log(x)/(b^3*d) - 2*(2*e^2*log(e*x + d)*log(x) - e^2*log(x)^2)/(b*d^3) - (2*a*x - b)/(b^2*d*x^2))*b*p - 1/2*(2*e^2*log(e*x + d)/d^3 - 2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2*x^2))*log((a + b/x)^p*c)

Giac [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x}\right)^p c\right)}{(ex+d)x^3}\,dx$$

```
[In] integrate(log(c*(a+b/x)^p)/x^3/(e*x+d),x, algorithm="giac")
[Out] integrate(log((a + b/x)^p*c)/((e*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x}\right)^p\right)}{x^3(d+ex)}\,dx = \int \frac{\ln\left(c\,{\left(a+\frac{b}{x}\right)}^p\right)}{x^3\,\left(d+e\,x\right)}\,dx$$

```
[In] int(log(c*(a + b/x)^p)/(x^3*(d + e*x)),x)
[Out] int(log(c*(a + b/x)^p)/(x^3*(d + e*x)), x)
```

3.247 $\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} dx$

Optimal result

Integrand size = 23, antiderivative size = 421

$$\begin{aligned}
\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} dx &= \frac{2bpx}{3ae} + \frac{2\sqrt{b}d^2p\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^3} - \frac{2b^{3/2}p\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3a^{3/2}e} \\
&+ \frac{d^2x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2} \\
&+ \frac{x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3e} - \frac{d^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^4} \\
&- \frac{2d^3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^4} \\
&+ \frac{d^3p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e^4} \\
&+ \frac{d^3p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{e^4} \\
&- \frac{bdp\log\left(b+ax^2\right)}{2ae^2} + \frac{d^3p\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^4} \\
&+ \frac{d^3p\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^4} - \frac{2d^3p\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e^4}
\end{aligned}$$

```
[Out] 2/3*b*p*x/a/e-2/3*b^(3/2)*p*arctan(x*a^(1/2)/b^(1/2))/a^(3/2)/e+d^2*x*ln(c*
(a+b/x^2)^p)/e^3-1/2*d*x^2*ln(c*(a+b/x^2)^p)/e^2+1/3*x^3*ln(c*(a+b/x^2)^p)/
e-d^3*ln(c*(a+b/x^2)^p)*ln(e*x+d)/e^4-2*d^3*p*ln(-e*x/d)*ln(e*x+d)/e^4-1/2*
b*d*p*ln(a*x^2+b)/a/e^2+d^3*p*ln(e*x+d)*ln(-e*(x*(-a)^(1/2)+b^(1/2))/(d*(-a
)^(1/2)-e*b^(1/2)))/e^4+d^3*p*ln(e*x+d)*ln(e*(-x*(-a)^(1/2)+b^(1/2))/(d*(-a
```

```
)^(1/2)+e*b^(1/2)))/e^4-2*d^3*p*polylog(2,1+e*x/d)/e^4+d^3*p*polylog(2,(e*x
+d)*(-a)^(1/2)/(d*(-a)^(1/2)-e*b^(1/2)))/e^4+d^3*p*polylog(2,(e*x+d)*(-a)^(
1/2)/(d*(-a)^(1/2)+e*b^(1/2)))/e^4+2*d^2*p*arctan(x*a^(1/2)/b^(1/2))*b^(1/2
)/e^3/a^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {2516, 2498, 269, 211, 2505, 266, 199, 327, 2512, 2463, 2441, 2352, 2440, 2438}

$$\begin{aligned}\int \frac{x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} dx = &-\frac{2b^{3/2}p \arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3a^{3/2}e}+\frac{2\sqrt{b}d^2p\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^3} \\ &-\frac{d^3\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^4}+\frac{d^2x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} \\ &-\frac{dx^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2}+\frac{x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3e} \\ &+\frac{d^3p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^4}+\frac{d^3p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^4} \\ &+\frac{d^3p\log(d+ex)\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^4} \\ &+\frac{d^3p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}x+\sqrt{b}\right)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^4}-\frac{bdp\log\left(ax^2+b\right)}{2ae^2} \\ &+\frac{2bpx}{3ae}-\frac{2d^3p\,\mathrm{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^4}-\frac{2d^3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^4}\end{aligned}$$

```
[In] Int[(x^3*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

```
[Out] (2*b*p*x)/(3*a*e) + (2*Sqrt[b]*d^2*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[a]*
e^3) - (2*b^(3/2)*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(3*a^(3/2)*e) + (d^2*x*Log
[c*(a + b/x^2)^p])/e^3 - (d*x^2*Log[c*(a + b/x^2)^p])/(2*e^2) + (x^3*Log[c*
(a + b/x^2)^p])/(3*e) - (d^3*Log[c*(a + b/x^2)^p]*Log[d + e*x])/e^4 - (2*d^
3*p*Log[-((e*x)/d)]*Log[d + e*x])/e^4 + (d^3*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x
))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*(Sqrt[b] +
 Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[d + e*x])/e^4 - (b*d*p*Log[b +
 a*x^2])/(2*a*e^2) + (d^3*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - S
qrt[b]*e)])/e^4 + (d^3*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt
[b]*e)])/e^4 - (2*d^3*p*PolyLog[2, 1 + (e*x)/d])/e^4
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3}-\frac{dx \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+\frac{x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} -\frac{d^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3(d+ex)}\right) dx$$

$$= \frac{d^2 \int \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx}{e^3} - \frac{d^3 \int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx}{e^3} - \frac{d\int x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx}{e^2} + \frac{\int x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx}{e}$$

$$= \frac{d^2x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2} + \frac{x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3e} - \frac{d^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^4} - \frac{(2bd^3p)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^2}\right)x^3}\,dx}{e^4} + \frac{(2bd^2p)\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^2}\,dx}{e^3} - \frac{(bdp)\int \frac{1}{\left(a+\frac{b}{x^2}\right)x}\,dx}{e^2} + \frac{(2bp)\int \frac{1}{a+\frac{b}{x^2}}\,dx}{3e}$$

$$= \frac{d^2x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2} + \frac{x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3e} - \frac{d^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^4} - \frac{(2bd^3p)\int \left(\frac{\log(d+ex)}{bx} - \frac{ax\log(d+ex)}{b(b+ax^2)}\right)dx}{e^4} + \frac{(2bd^2p)\int \frac{1}{b+ax^2}\,dx}{e^3} - \frac{(bdp)\int \frac{x}{b+ax^2}\,dx}{e^2} + \frac{(2bp)\int \frac{x^2}{b+ax^2}\,dx}{3e}$$

$$= \frac{2bpx}{3ae} + \frac{2\sqrt{b}d^2p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^3} + \frac{d^2x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2} + \frac{x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3e} - \frac{d^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^4} - \frac{bdp\log\left(b+ax^2\right)}{2ae^2} - \frac{(2d^3p)\int \frac{\log(d+ex)}{x}\,dx}{e^4} + \frac{(2ad^3p)\int \frac{x\log(d+ex)}{b+ax^2}\,dx}{e^4} - \frac{(2b^2p)\int \frac{1}{b+ax^2}\,dx}{3ae}$$

$$= \frac{2bpx}{3ae} + \frac{2\sqrt{b}d^2p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^3} - \frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3a^{3/2}e} + \frac{d^2x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2} + \frac{x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3e} - \frac{d^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^4} - \frac{2d^3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^4} - \frac{bdp\log\left(b+ax^2\right)}{2ae^2} + \frac{(2ad^3p)\int \left(-\frac{\sqrt{-a}\log(d+ex)}{2a\left(\sqrt{b}-\sqrt{-a}x\right)} + \frac{\sqrt{-a}\log(d+ex)}{2a\left(\sqrt{b}+\sqrt{-a}x\right)}\right)dx}{e^4} + \frac{(2d^3p)\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{e^3}$$

$$= \frac{2bpx}{3ae} + \frac{2\sqrt{b}d^2p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^3} - \frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3a^{3/2}e} + \frac{d^2x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2} + \frac{x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3e} - \frac{d^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^4} - \frac{2d^3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^4} - \frac{bdp\log\left(b+ax^2\right)}{2ae^2} - \frac{2d^3p\text{Li}_2\left(1+\frac{ex}{d}\right)}{e^4} - \frac{\left(\sqrt{-a}d^3p\right)\int\frac{\log(d+ex)}{\sqrt{b}-\sqrt{-a}x}\,dx}{e^4} + \frac{\left(\sqrt{-a}d^3p\right)\int\frac{\log(d+ex)}{\sqrt{b}+\sqrt{-a}x}\,dx}{e^4}$$

$$= \frac{2bpx}{3ae} + \frac{2\sqrt{b}d^2p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^3} - \frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3a^{3/2}e} + \frac{d^2x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2} + \frac{x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3e} - \frac{d^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^4} - \frac{2d^3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^4} + \frac{d^3p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e^4} + \frac{d^3p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{e^4} - \frac{bdp\log\left(b+ax^2\right)}{2ae^2} - \frac{2d^3p\text{Li}_2\left(1+\frac{ex}{d}\right)}{e^4} - \frac{\left(d^3p\right)\int\frac{\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d+ex}\,dx}{e^3} - \frac{\left(d^3p\right)\int\frac{\log\left(\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{-\sqrt{-a}d+\sqrt{b}e}\right)}{d+ex}\,dx}{e^3}$$

$$= \frac{2bpx}{3ae} + \frac{2\sqrt{b}d^2p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^3} - \frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3a^{3/2}e} + \frac{d^2x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2} + \frac{x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3e} - \frac{d^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^4} - \frac{2d^3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^4} + \frac{d^3p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e^4} + \frac{d^3p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{e^4} - \frac{bdp\log\left(b+ax^2\right)}{2ae^2} - \frac{2d^3p\text{Li}_2\left(1+\frac{ex}{d}\right)}{e^4} - \frac{\left(d^3p\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-a}x}{-\sqrt{-a}d+\sqrt{b}e}\right)}{x}\,dx,x,d+ex\right)}{e^4} - \frac{\left(d^3p\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-a}x}{\sqrt{-a}d+\sqrt{b}e}\right)}{x}\,dx,x,d+ex\right)}{e^4}$$

$$
\begin{aligned}
&= \frac{2bpx}{3ae} + \frac{2\sqrt{b}d^2p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^3} - \frac{2b^{3/2}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{3a^{3/2}e} + \frac{d^2x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} \\
&- \frac{dx^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2} + \frac{x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3e} - \frac{d^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^4} \\
&- \frac{2d^3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^4} + \frac{d^3p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e^4} \\
&+ \frac{d^3p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{e^4} - \frac{bdp\log\left(b+ax^2\right)}{2ae^2} \\
&+ \frac{d^3p\operatorname{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^4} + \frac{d^3p\operatorname{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^4} - \frac{2d^3p\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{e^4}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.96

$$
\int \frac{x^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx
$$

$$
= \frac{-12\sqrt{a}\sqrt{b}d^2ep\arctan\left(\frac{\sqrt{b}}{\sqrt{a}x}\right) + 4be^3px\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{b}{ax^2}\right) - 3bde^2p\log\left(a+\frac{b}{x^2}\right) + 6ad^2e
$$

```
[In] Integrate[(x^3*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

```
[Out] (-12*Sqrt[a]*Sqrt[b]*d^2*e*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)] + 4*b*e^3*p*x*Hype
rgeometric2F1[-1/2, 1, 1/2, -(b/(a*x^2))] - 3*b*d*e^2*p*Log[a + b/x^2] + 6*
a*d^2*e*x*Log[c*(a + b/x^2)^p] - 3*a*d*e^2*x^2*Log[c*(a + b/x^2)^p] + 2*a*e
^3*x^3*Log[c*(a + b/x^2)^p] - 6*b*d*e^2*p*Log[x] - 6*a*d^3*Log[c*(a + b/x^2
)^p]*Log[d + e*x] - 12*a*d^3*p*Log[-((e*x)/d)]*Log[d + e*x] + 6*a*d^3*p*Log
[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x] + 6*a*d^
3*p*Log[(e*(Sqrt[b] + Sqrt[-a]*x))/(-(Sqrt[-a]*d) + Sqrt[b]*e)]*Log[d + e*x
] + 6*a*d^3*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)] + 6
*a*d^3*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] - 12*a*d
^3*p*PolyLog[2, 1 + (e*x)/d])/(6*a*e^4)
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.98

method	result
parts	$\frac{x^3 \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{3e}-\frac{d\,x^2 \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e^2}+\frac{d^2 x \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3}-\frac{d^3 \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)\ln(ex+d)}{e^4}+2pb\,e^2\left(\frac{d^3\left(-\frac{\mathrm{dilog}\left(-\frac{e}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.$

[In] int(x^3*ln(c*(a+b/x^2)^p)/(e*x+d),x,method=_RETURNVERBOSE)

[Out] 1/3*x^3*ln(c*(a+b/x^2)^p)/e-1/2*d*x^2*ln(c*(a+b/x^2)^p)/e^2+d^2*x*ln(c*(a+b/x^2)^p)/e^3-d^3*ln(c*(a+b/x^2)^p)*ln(e*x+d)/e^4+2*p*b*e^2*(1/e^4*d^3*(-1/b/e^2*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))-a/b/e^2*(-1/2*ln(e*x+d)*(ln((e*(-a*b)^(1/2)+a*d-a*(e*x+d))/(e*(-a*b)^(1/2)+a*d))+ln((e*(-a*b)^(1/2)-a*d+a*(e*x+d))/(e*(-a*b)^(1/2)-a*d)))/a-1/2*(dilog((e*(-a*b)^(1/2)+a*d-a*(e*x+d))/(e*(-a*b)^(1/2)+a*d))+dilog((e*(-a*b)^(1/2)-a*d+a*(e*x+d))/(e*(-a*b)^(1/2)-a*d)))/a))+1/6/e^4*(2*(e*x+d)/a+1/a*(-3/2*d*ln(a*d^2-2*a*d*(e*x+d)+a*(e*x+d)^2+e^2*b)+(6*a*d^2-2*b*e^2)/e/(a*b)^(1/2)*arctan(1/2*(-2*a*d+2*a*(e*x+d))/e/(a*b)^(1/2)))))

Fricas [F]

$$\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx=\int \frac{x^3 \log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d}\,dx$$

[In] integrate(x^3*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="fricas")

[Out] integral(x^3*log(c*((a*x^2 + b)/x^2)^p)/(e*x + d), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*ln(c*(a+b/x**2)**p)/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log \left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^3*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x^3*log((a + b/x^2)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log \left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^3*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x^3*log((a + b/x^2)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \frac{x^3 \ln \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+e\,x}\,dx$$

```
[In] int((x^3*log(c*(a + b/x^2)^p))/(d + e*x),x)
[Out] int((x^3*log(c*(a + b/x^2)^p))/(d + e*x), x)
```

3.248 $\int \frac{x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} dx$

Optimal result

Integrand size = 23, antiderivative size = 353

$$\begin{aligned}\int \frac{x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} dx = & -\frac{2\sqrt{b}dp \arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^2} - \frac{dx \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} \\ & + \frac{x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} + \frac{d^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^3} \\ & + \frac{2d^2 p \log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3} \\ & - \frac{d^2 p \log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e^3} \\ & - \frac{d^2 p \log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{e^3} \\ & + \frac{bp\log(b+ax^2)}{2ae} - \frac{d^2 p \operatorname{PolyLog}\left(2, \frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^3} \\ & - \frac{d^2 p \operatorname{PolyLog}\left(2, \frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^3} + \frac{2d^2 p \operatorname{PolyLog}\left(2, 1+\frac{ex}{d}\right)}{e^3}\end{aligned}$$

```
[Out] -d*x*ln(c*(a+b/x^2)^p)/e^2+1/2*x^2*ln(c*(a+b/x^2)^p)/e+d^2*ln(c*(a+b/x^2)^p
)*ln(e*x+d)/e^3+2*d^2*p*ln(-e*x/d)*ln(e*x+d)/e^3+1/2*b*p*ln(a*x^2+b)/a/e-d^
2*p*ln(e*x+d)*ln(-e*(x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)-e*b^(1/2)))/e^3-d^
2*p*ln(e*x+d)*ln(e*(-x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)+e*b^(1/2)))/e^3+2*
d^2*p*polylog(2,1+e*x/d)/e^3-d^2*p*polylog(2,(e*x+d)*(-a)^(1/2)/(d*(-a)^(1/
2)-e*b^(1/2)))/e^3-d^2*p*polylog(2,(e*x+d)*(-a)^(1/2)/(d*(-a)^(1/2)+e*b^(1/
2)))/e^3-2*d*p*arctan(x*a^(1/2)/b^(1/2))*b^(1/2)/e^2/a^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {2516, 2498, 269, 211, 2505, 266, 2512, 2463, 2441, 2352, 2440, 2438}

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} \, dx = -\frac{2\sqrt{b}dp \arctan \left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^2}+\frac{d^2 \log(d+ex) \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^3} -\frac{dx \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+\frac{x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} -\frac{d^2p \operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^3}-\frac{d^2p \operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^3} -\frac{d^2p \log(d+ex) \log \left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^3} -\frac{d^2p \log(d+ex) \log \left(-\frac{e\left(\sqrt{-a}x+\sqrt{b}\right)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^3}+\frac{bp \log \left(ax^2+b\right)}{2ae} +\frac{2d^2p \operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^3}+\frac{2d^2p \log \left(-\frac{ex}{d}\right) \log(d+ex)}{e^3}$$

```
[In] Int[(x^2*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

```
[Out] (-2*Sqrt[b]*d*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[a]*e^2) - (d*x*Log[c*(a
+ b/x^2)^p])/e^2 + (x^2*Log[c*(a + b/x^2)^p])/(2*e) + (d^2*Log[c*(a + b/x^2
)^p]*Log[d + e*x])/e^3 + (2*d^2*p*Log[-((e*x)/d)]*Log[d + e*x])/e^3 - (d^2*
p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x])/e^
3 - (d^2*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[
d + e*x])/e^3 + (b*p*Log[b + a*x^2])/(2*a*e) - (d^2*p*PolyLog[2, (Sqrt[-a]*
(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/e^3 - (d^2*p*PolyLog[2, (Sqrt[-a]*(d
+ e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/e^3 + (2*d^2*p*PolyLog[2, 1 + (e*x)/d])/
e^3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e}+\frac{d^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2(d+ex)}\right)dx \\
&= -\frac{d\int\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx}{e^2}+\frac{d^2\int\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}dx}{e^2}+\frac{\int x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)dx}{e} \\
&= -\frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e}+\frac{d^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^3} \\
&\quad +\frac{(2bd^2p)\int\frac{\log(d+ex)}{\left(a+\frac{b}{x^2}\right)x^3}dx}{e^3}-\frac{(2bdp)\int\frac{1}{\left(a+\frac{b}{x^2}\right)x^2}dx}{e^2}+\frac{(bp)\int\frac{1}{\left(a+\frac{b}{x^2}\right)x}dx}{e} \\
&= -\frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e}+\frac{d^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^3} \\
&\quad +\frac{(2bd^2p)\int\left(\frac{\log(d+ex)}{bx}-\frac{ax\log(d+ex)}{b(b+ax^2)}\right)dx}{e^3}-\frac{(2bdp)\int\frac{1}{b+ax^2}dx}{e^2}+\frac{(bp)\int\frac{x}{b+ax^2}dx}{e} \\
&= -\frac{2\sqrt{b}dp\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^2}-\frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} \\
&\quad +\frac{d^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^3}+\frac{bp\log(b+ax^2)}{2ae} \\
&\quad +\frac{(2d^2p)\int\frac{\log(d+ex)}{x}dx}{e^3}-\frac{(2ad^2p)\int\frac{x\log(d+ex)}{b+ax^2}dx}{e^3}
\end{aligned}$$

$$= -\frac{2\sqrt{b}dp\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^2} - \frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^3} + \frac{2d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3} + \frac{bp\log\left(b+ax^2\right)}{2ae} - \frac{(2ad^2p)\int\left(-\frac{\sqrt{-a}\log(d+ex)}{2a\left(\sqrt{b}-\sqrt{-a}x\right)} + \frac{\sqrt{-a}\log(d+ex)}{2a\left(\sqrt{b}+\sqrt{-a}x\right)}\right)dx}{e^3} - \frac{(2d^2p)\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}dx}{e^2}$$

$$= -\frac{2\sqrt{b}dp\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^2} - \frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^3} + \frac{2d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3} + \frac{bp\log\left(b+ax^2\right)}{2ae} + \frac{2d^2p\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{e^3} + \frac{\left(\sqrt{-a}d^2p\right)\int\frac{\log(d+ex)}{\sqrt{b}-\sqrt{-a}x}dx}{e^3} - \frac{\left(\sqrt{-a}d^2p\right)\int\frac{\log(d+ex)}{\sqrt{b}+\sqrt{-a}x}dx}{e^3}$$

$$= -\frac{2\sqrt{b}dp\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^2} - \frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^3} + \frac{2d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{e^3} + \frac{bp\log\left(b+ax^2\right)}{2ae} + \frac{2d^2p\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{e^3} + \frac{(d^2p)\int\frac{\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d+ex}dx}{e^2} + \frac{(d^2p)\int\frac{\log\left(\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{-\sqrt{-a}d+\sqrt{b}e}\right)}{d+ex}dx}{e^2}$$

$$= -\frac{2\sqrt{b}dp\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^2} - \frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^3} + \frac{2d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{e^3} + \frac{bp\log(b+ax^2)}{2ae} + \frac{2d^2p\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{e^3} + \frac{(d^2p)\,\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-a}x}{-\sqrt{-a}d+\sqrt{b}e}\right)}{x}\,dx, x, d+ex\right)}{e^3} + \frac{(d^2p)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-a}x}{\sqrt{-a}d+\sqrt{b}e}\right)}{x}\,dx, x, d+ex\right)}{e^3}$$

$$= -\frac{2\sqrt{b}dp\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e^2} - \frac{dx\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^3} + \frac{2d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{e^3} + \frac{bp\log(b+ax^2)}{2ae} - \frac{d^2p\mathrm{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^3} - \frac{d^2p\mathrm{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^3} + \frac{2d^2p\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{e^3}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.99

$$\int\frac{x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \frac{4\sqrt{a}\sqrt{b}dep\arctan\left(\frac{\sqrt{b}}{\sqrt{a}x}\right) + be^2p\log\left(a+\frac{b}{x^2}\right) - 2adex\log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + ae^2x^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right) + 2be^2p\log$$

```
[In] Integrate[(x^2*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

```
[Out] (4*Sqrt[a]*Sqrt[b]*d*e*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)] + b*e^2*p*Log[a + b/x^
2] - 2*a*d*e*x*Log[c*(a + b/x^2)^p] + a*e^2*x^2*Log[c*(a + b/x^2)^p] + 2*b*
e^2*p*Log[x] + 2*a*d^2*Log[c*(a + b/x^2)^p]*Log[d + e*x] + 4*a*d^2*p*Log[-(
```

(e*x)/d)]*Log[d + e*x] - 2*a*d^2*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x] - 2*a*d^2*p*Log[(e*(Sqrt[b] + Sqrt[-a]*x))/(-(Sqrt[-a]*d) + Sqrt[b]*e)]*Log[d + e*x] - 2*a*d^2*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)] - 2*a*d^2*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] + 4*a*d^2*p*PolyLog[2, 1 + (e*x)/d])/(2*a*e^3)

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.03

method	result
parts	$\frac{x^2 \ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2e}-\frac{dx\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2}+\frac{d^2\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)\ln(ex+d)}{e^3}+2pb\,e^2\left(\frac{\ln\left(a\,d^2-2ad(ex+d)+a(ex+d)^2+e^2b\right)}{4e^3a}-\right.$

[In] int(x^2*ln(c*(a+b/x^2)^p)/(e*x+d),x,method=_RETURNVERBOSE)

[Out] 1/2*x^2*ln(c*(a+b/x^2)^p)/e-d*x*ln(c*(a+b/x^2)^p)/e^2+d^2*ln(c*(a+b/x^2)^p)*ln(e*x+d)/e^3+2*p*b*e^2*(1/4/e^3/a*ln(a*d^2-2*a*d*(e*x+d)+a*(e*x+d)^2+e^2*b)-1/e^4*d/(a*b)^(1/2)*arctan(1/2*(-2*a*d+2*a*(e*x+d))/e/(a*b)^(1/2))-1/e^3*d^2*(-1/b/e^2*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))-a/b/e^2*(-1/2*ln(e*x+d)*(ln((e*(-a*b)^(1/2)+a*d-a*(e*x+d))/(e*(-a*b)^(1/2)+a*d))+ln((e*(-a*b)^(1/2)-a*d+a*(e*x+d))/(e*(-a*b)^(1/2)-a*d)))/a-1/2*(dilog((e*(-a*b)^(1/2)+a*d-a*(e*x+d))/(e*(-a*b)^(1/2)+a*d))+dilog((e*(-a*b)^(1/2)-a*d+a*(e*x+d))/(e*(-a*b)^(1/2)-a*d)))/a)))

Fricas [F]

$$\int \frac{x^2 \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx=\int \frac{x^2 \log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d}\,dx$$

[In] integrate(x^2*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="fricas")

[Out] integral(x^2*log(c*((a*x^2 + b)/x^2)^p)/(e*x + d), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*ln(c*(a+b/x**2)**p)/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^2*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x^2*log((a + b/x^2)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^2*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x^2*log((a + b/x^2)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \frac{x^2 \ln \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+e\,x}\,dx$$

```
[In] int((x^2*log(c*(a + b/x^2)^p))/(d + e*x),x)
[Out] int((x^2*log(c*(a + b/x^2)^p))/(d + e*x), x)
```

3.249 $\int \frac{x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 291

$$
\begin{aligned}
\int \frac{x \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx =& \frac{2\sqrt{b}p\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e} + \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} \\
&- \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{2dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2} \\
&+ \frac{dp\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e^2} \\
&+ \frac{dp\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{e^2} \\
&+ \frac{dp\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^2} \\
&+ \frac{dp\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^2} - \frac{2dp\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e^2}
\end{aligned}
$$

```
[Out] x*ln(c*(a+b/x^2)^p)/e-d*ln(c*(a+b/x^2)^p)*ln(e*x+d)/e^2-2*d*p*ln(-e*x/d)*ln
(e*x+d)/e^2+d*p*ln(e*x+d)*ln(-e*(x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)-e*b^(1
/2)))/e^2+d*p*ln(e*x+d)*ln(e*(-x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)+e*b^(1/2
)))/e^2-2*d*p*polylog(2,1+e*x/d)/e^2+d*p*polylog(2,(e*x+d)*(-a)^(1/2)/(d*(-
a)^(1/2)-e*b^(1/2)))/e^2+d*p*polylog(2,(e*x+d)*(-a)^(1/2)/(d*(-a)^(1/2)+e*b
^(1/2)))/e^2+2*p*arctan(x*a^(1/2)/b^(1/2))*b^(1/2)/e/a^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {2516, 2498, 269, 211, 2512, 266, 2463, 2441, 2352, 2440, 2438}

$$\int \frac{x \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} \, dx = \frac{2\sqrt{b}p \arctan \left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e} - \frac{d \log(d+ex) \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e^2} + \frac{x \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} + \frac{dp \operatorname{PolyLog}\left(2, \frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^2} + \frac{dp \operatorname{PolyLog}\left(2, \frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^2} + \frac{dp \log(d+ex) \log \left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^2} + \frac{dp \log(d+ex) \log \left(-\frac{e\left(\sqrt{-a}x+\sqrt{b}\right)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^2} - \frac{2dp \operatorname{PolyLog}\left(2, \frac{ex}{d}+1\right)}{e^2} - \frac{2dp \log \left(-\frac{ex}{d}\right) \log(d+ex)}{e^2}$$

```
[In] Int[(x*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

```
[Out] (2*Sqrt[b]*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[a]*e) + (x*Log[c*(a + b/x^2
)^p])/e - (d*Log[c*(a + b/x^2)^p]*Log[d + e*x])/e^2 - (2*d*p*Log[-((e*x)/d)
]*Log[d + e*x])/e^2 + (d*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqr
t[b]*e)]*Log[d + e*x])/e^2 + (d*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a
]*d - Sqrt[b]*e))]*Log[d + e*x])/e^2 + (d*p*PolyLog[2, (Sqrt[-a]*(d + e*x))
/(Sqrt[-a]*d - Sqrt[b]*e)])/e^2 + (d*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqr
t[-a]*d + Sqrt[b]*e)])/e^2 - (2*d*p*PolyLog[2, 1 + (e*x)/d])/e^2
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
```

```
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e}-\frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e(d+ex)}\right)dx\\
&= \frac{\int \log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\,dx}{e}-\frac{d\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx}{e}\\
&= \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e}-\frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2}\\
&\quad -\frac{(2bdp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^2}\right)x^3}\,dx}{e^2}+\frac{(2bp)\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^2}\,dx}{e}\\
&= \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e}-\frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2}\\
&\quad -\frac{(2bdp)\int \left(\frac{\log(d+ex)}{bx}-\frac{ax\log(d+ex)}{b(b+ax^2)}\right)dx}{e^2}+\frac{(2bp)\int \frac{1}{b+ax^2}\,dx}{e}\\
&= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e}+\frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e}-\frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2}\\
&\quad -\frac{(2dp)\int \frac{\log(d+ex)}{x}\,dx}{e^2}+\frac{(2adp)\int \frac{x\log(d+ex)}{b+ax^2}\,dx}{e^2}\\
&= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e}+\frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e}\\
&\quad -\frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2}-\frac{2dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2}\\
&\quad +\frac{(2adp)\int \left(-\frac{\sqrt{-a}\log(d+ex)}{2a\left(\sqrt{b}-\sqrt{-a}x\right)}+\frac{\sqrt{-a}\log(d+ex)}{2a\left(\sqrt{b}+\sqrt{-a}x\right)}\right)dx}{e^2}+\frac{(2dp)\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{e}\\
&= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e}+\frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e}-\frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2}\\
&\quad -\frac{2dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2}-\frac{2dp\text{Li}_2\left(1+\frac{ex}{d}\right)}{e^2}\\
&\quad -\frac{\left(\sqrt{-a}dp\right)\int \frac{\log(d+ex)}{\sqrt{b}-\sqrt{-a}x}\,dx}{e^2}+\frac{\left(\sqrt{-a}dp\right)\int \frac{\log(d+ex)}{\sqrt{b}+\sqrt{-a}x}\,dx}{e^2}
\end{aligned}
$$

$$= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e} + \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{2dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{e^2} - \frac{2dp\text{Li}_2\left(1+\frac{ex}{d}\right)}{e^2} - \frac{(dp)\int\frac{\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d+ex}dx}{e} - \frac{(dp)\int\frac{\log\left(\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{-\sqrt{-a}d+\sqrt{b}e}\right)}{d+ex}dx}{e}$$

$$= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e} + \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{2dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{e^2} - \frac{2dp\text{Li}_2\left(1+\frac{ex}{d}\right)}{e^2} - \frac{(dp)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-a}x}{-\sqrt{-a}d+\sqrt{b}e}\right)}{x}dx,x,d+ex\right)}{e^2} - \frac{(dp)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-a}x}{\sqrt{-a}d+\sqrt{b}e}\right)}{x}dx,x,d+ex\right)}{e^2}$$

$$= \frac{2\sqrt{b}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{a}e} + \frac{x\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e^2} - \frac{2dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{e^2} + \frac{dp\text{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e^2} + \frac{dp\text{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e^2} - \frac{2dp\text{Li}_2\left(1+\frac{ex}{d}\right)}{e^2}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.93

$$\int \frac{x \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx$$
$$= \frac{-\frac{2\sqrt{b}ep\arctan\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{\sqrt{a}} + ex\log\left(c\left(a+\frac{b}{x^2}\right)^p\right) - d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex) - 2dp\log\left(-\frac{ex}{d}\right)\log(d+ex) + dp}{}$$

```
[In] Integrate[(x*Log[c*(a + b/x^2)^p])/(d + e*x),x]
```

```
[Out] ((-2*Sqrt[b]*e*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[a] + e*x*Log[c*(a + b/x^
2)^p] - d*Log[c*(a + b/x^2)^p]*Log[d + e*x] - 2*d*p*Log[-((e*x)/d)]*Log[d +
 e*x] + d*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d
+ e*x] + d*p*Log[(e*(Sqrt[b] + Sqrt[-a]*x))/(-(Sqrt[-a]*d) + Sqrt[b]*e)]*Lo
g[d + e*x] + d*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)]
+ d*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)] - 2*d*p*Pol
yLog[2, 1 + (e*x)/d])/e^2
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.02

method	result
parts	$\frac{x\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e} - \frac{d\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)\ln(ex+d)}{e^2} + 2pb\,e^2\left(\frac{\arctan\left(\frac{-2ad+2a(ex+d)}{2e\sqrt{ab}}\right)}{e^3\sqrt{ab}} + \frac{d\left(-\frac{\mathrm{dilog}\left(-\frac{ex}{d}\right)+\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{b\,e^2} - \frac{a}{}\right.}{}\right.$

```
[In] int(x*ln(c*(a+b/x^2)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(c*(a+b/x^2)^p)/e-d*ln(c*(a+b/x^2)^p)*ln(e*x+d)/e^2+2*p*b*e^2*(1/e^3/(a
*b)^(1/2)*arctan(1/2*(-2*a*d+2*a*(e*x+d))/e/(a*b)^(1/2))+1/e^2*d*(-1/b/e^2*
(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))-a/b/e^2*(-1/2*ln(e*x+d)*(ln((e*(-a*b)^
(1/2)+a*d-a*(e*x+d))/(e*(-a*b)^(1/2)+a*d))+ln((e*(-a*b)^(1/2)-a*d+a*(e*x+d)
)/(e*(-a*b)^(1/2)-a*d)))/a-1/2*(dilog((e*(-a*b)^(1/2)+a*d-a*(e*x+d))/(e*(-a
*b)^(1/2)+a*d))+dilog((e*(-a*b)^(1/2)-a*d+a*(e*x+d))/(e*(-a*b)^(1/2)-a*d)))
/a))
```

Fricas [F]

$$\int \frac{x \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} \, dx = \int \frac{x \log \left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d} \, dx$$

```
[In] integrate(x*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="fricas")
[Out] integral(x*log(c*((a*x^2 + b)/x^2)^p)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} \, dx = \text{Timed out}$$

```
[In] integrate(x*ln(c*(a+b/x**2)**p)/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} \, dx = \int \frac{x \log \left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d} \, dx$$

```
[In] integrate(x*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x*log((a + b/x^2)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x \log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} \, dx = \int \frac{x \log \left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d} \, dx$$

```
[In] integrate(x*log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x*log((a + b/x^2)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \log \left(c \left(a+\frac{b}{x^2}\right)^p\right)}{d+ex} \, dx = \int \frac{x\, \ln \left(c\, \left(a+\frac{b}{x^2}\right)^p\right)}{d+e\,x} \,dx$$

```
[In] int((x*log(c*(a + b/x^2)^p))/(d + e*x),x)
[Out] int((x*log(c*(a + b/x^2)^p))/(d + e*x), x)
```

3.250 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 241

$$\begin{aligned}\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx &= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e}+\frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}\\&\quad-\frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e}\\&\quad-\frac{p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{e}-\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e}\\&\quad-\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e}+\frac{2p\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e}\end{aligned}$$

```
[Out] ln(c*(a+b/x^2)^p)*ln(e*x+d)/e+2*p*ln(-e*x/d)*ln(e*x+d)/e-p*ln(e*x+d)*ln(-e*
(x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)-e*b^(1/2)))/e-p*ln(e*x+d)*ln(e*(-x*(-a
)^(1/2)+b^(1/2))/(d*(-a)^(1/2)+e*b^(1/2)))/e+2*p*polylog(2,1+e*x/d)/e-p*pol
ylog(2,(e*x+d)*(-a)^(1/2)/(d*(-a)^(1/2)-e*b^(1/2)))/e-p*polylog(2,(e*x+d)*(
-a)^(1/2)/(d*(-a)^(1/2)+e*b^(1/2)))/e
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2512, 266, 2463, 2441, 2352, 2440, 2438}

$$\begin{aligned}\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = &\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{e}-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e}\\&-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e}-\frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e}\\&-\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}x+\sqrt{b}\right)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e}\\&+\frac{2p\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e}+\frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}\end{aligned}$$

```
[In] Int[Log[c*(a + b/x^2)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x^2)^p]*Log[d + e*x])/e + (2*p*Log[-((e*x)/d)]*Log[d + e*x])/
e - (p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x
])/e - (p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[d
 + e*x])/e - (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/
e - (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/e + (2*p*
PolyLog[2, 1 + (e*x)/d])/e
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{(2bp)\int\frac{\log(d+ex)}{\left(a+\frac{b}{x^2}\right)x^3}\,dx}{e} \\
&= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{(2bp)\int\left(\frac{\log(d+ex)}{bx}-\frac{ax\log(d+ex)}{b(b+ax^2)}\right)dx}{e} \\
&= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{(2p)\int\frac{\log(d+ex)}{x}\,dx}{e} - \frac{(2ap)\int\frac{x\log(d+ex)}{b+ax^2}\,dx}{e} \\
&= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} \\
&\quad - (2p)\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx - \frac{(2ap)\int\left(-\frac{\sqrt{-a}\log(d+ex)}{2a\left(\sqrt{b}-\sqrt{-a}x\right)}+\frac{\sqrt{-a}\log(d+ex)}{2a\left(\sqrt{b}+\sqrt{-a}x\right)}\right)dx}{e} \\
&= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e} + \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} \\
&\quad + \frac{2p\text{Li}_2\left(1+\frac{ex}{d}\right)}{e} + \frac{\left(\sqrt{-a}p\right)\int\frac{\log(d+ex)}{\sqrt{b}-\sqrt{-a}x}\,dx}{e} - \frac{\left(\sqrt{-a}p\right)\int\frac{\log(d+ex)}{\sqrt{b}+\sqrt{-a}x}\,dx}{e}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e}+\frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} \\
&\quad -\frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{e} \\
&\quad +\frac{2p\text{Li}_2\left(1+\frac{ex}{d}\right)}{e}+p\int\frac{\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d+ex}dx+p\int\frac{\log\left(\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{-\sqrt{-a}d+\sqrt{b}e}\right)}{d+ex}dx \\
&= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e}+\frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} \\
&\quad -\frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{e} \\
&\quad +\frac{2p\text{Li}_2\left(1+\frac{ex}{d}\right)}{e}+\frac{p\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-a}x}{-\sqrt{-a}d+\sqrt{b}e}\right)}{x}dx,x,d+ex\right)}{e} \\
&\quad +\frac{p\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-a}x}{\sqrt{-a}d+\sqrt{b}e}\right)}{x}dx,x,d+ex\right)}{e} \\
&= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e}+\frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} \\
&\quad -\frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{e} \\
&\quad -\frac{p\text{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e}+\frac{2p\text{Li}_2\left(1+\frac{ex}{d}\right)}{e}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}dx &= \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{e}+\frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} \\
&\quad -\frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{e} \\
&\quad -\frac{p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{e}+\frac{2p\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e} \\
&\quad -\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{e}-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{e}
\end{aligned}
$$

```
[In] Integrate[Log[c*(a + b/x^2)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x^2)^p]*Log[d + e*x])/e + (2*p*Log[-((e*x)/d)]*Log[d + e*x])/
e - (p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x
])/e - (p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[d
 + e*x])/e + (2*p*PolyLog[2, (d + e*x)/d])/e - (p*PolyLog[2, (Sqrt[-a]*(d +
 e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/e - (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(S
qrt[-a]*d + Sqrt[b]*e)])/e
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.97

method	result
parts	$\frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)\ln(ex+d)}{e}+2pbe\left(\frac{a\left(-\frac{\ln(ex+d)\left(\ln\left(\frac{e\sqrt{-ab}+ad-a(ex+d)}{e\sqrt{-ab}+ad}\right)+\ln\left(\frac{e\sqrt{-ab}-ad+a(ex+d)}{e\sqrt{-ab}-ad}\right)\right)}{2a}-\frac{\operatorname{dilog}\left(\frac{e\sqrt{-ab}+ad-a(ex+d)}{e\sqrt{-ab}+ad}\right.}{}\right.}{be^2}\right.$

```
[In] int(ln(c*(a+b/x^2)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] ln(c*(a+b/x^2)^p)*ln(e*x+d)/e+2*p*b*e*(a/b/e^2*(-1/2*ln(e*x+d)*(ln((e*(-a*b
)^(1/2)+a*d-a*(e*x+d))/(e*(-a*b)^(1/2)+a*d))+ln((e*(-a*b)^(1/2)-a*d+a*(e*x+
d))/(e*(-a*b)^(1/2)-a*d)))/a-1/2*(dilog((e*(-a*b)^(1/2)+a*d-a*(e*x+d))/(e*(
-a*b)^(1/2)+a*d))+dilog((e*(-a*b)^(1/2)-a*d+a*(e*x+d))/(e*(-a*b)^(1/2)-a*d)
))/a)+1/b/e^2*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d)))
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x^2 + b)/x^2)^p)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx$$

```
[In] integrate(ln(c*(a+b/x**2)**p)/(e*x+d),x)
[Out] Integral(log(c*(a + b/x**2)**p)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \frac{\log \left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((a + b/x^2)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \frac{\log \left(\left(a+\frac{b}{x^2}\right)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(log(c*(a+b/x^2)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(log((a + b/x^2)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx = \int \frac{\ln \left(c\,\left(a+\frac{b}{x^2}\right)^p\right)}{d+e\,x}\,dx$$

```
[In] int(log(c*(a + b/x^2)^p)/(d + e*x),x)
[Out] int(log(c*(a + b/x^2)^p)/(d + e*x), x)
```

3.251 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 287

$$\begin{aligned}\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}\,dx = &-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d}\\&-\frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d}+\frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{d}\\&+\frac{p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{d}\\&-\frac{p\operatorname{PolyLog}\left(2,1+\frac{b}{ax^2}\right)}{2d}+\frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d}\\&+\frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d}-\frac{2p\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{d}\end{aligned}$$

```
[Out] -1/2*ln(c*(a+b/x^2)^p)*ln(-b/a/x^2)/d-ln(c*(a+b/x^2)^p)*ln(e*x+d)/d-2*p*ln(
-e*x/d)*ln(e*x+d)/d+p*ln(e*x+d)*ln(-e*(x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)-
e*b^(1/2)))/d+p*ln(e*x+d)*ln(e*(-x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)+e*b^(1
/2)))/d-1/2*p*polylog(2,1+b/a/x^2)/d-2*p*polylog(2,1+e*x/d)/d+p*polylog(2,(
e*x+d)*(-a)^(1/2)/(d*(-a)^(1/2)-e*b^(1/2)))/d+p*polylog(2,(e*x+d)*(-a)^(1/2
)/(d*(-a)^(1/2)+e*b^(1/2)))/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2516, 2504, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$\begin{aligned}\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}\,dx = &-\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d}-\frac{\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2d}\\&+\frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d}+\frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d}\\&+\frac{p\log(d+ex)\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d}\\&+\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}x+\sqrt{b}\right)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d}-\frac{p\operatorname{PolyLog}\left(2,\frac{b}{ax^2}+1\right)}{2d}\\&-\frac{2p\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d}-\frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d}\end{aligned}$$

```
[In] Int[Log[c*(a + b/x^2)^p]/(x*(d + e*x)),x]
```

```
[Out] -1/2*(Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/d - (Log[c*(a + b/x^2)^p]*Log
[d + e*x])/d - (2*p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[(e*(Sqrt[b] -
Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x])/d + (p*Log[-((e*(Sqrt[
b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[d + e*x])/d - (p*PolyLog[2
, 1 + b/(a*x^2)])/(2*d) + (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d -
Sqrt[b]*e)])/d + (p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e
)])/d - (2*p*PolyLog[2, 1 + (e*x)/d])/d
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} - \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d(d+ex)}\right)dx$$

$$=\frac{\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx}{d}-\frac{e\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx}{d}$$

$$=-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d}-\frac{\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx,x,\frac{1}{x^2}\right)}{2d}-\frac{(2bp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^2}\right)x^3}\,dx}{d}$$

$$=-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d}+\frac{(bp)\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx,x,\frac{1}{x^2}\right)}{2d}-\frac{(2bp)\int \left(\frac{\log(d+ex)}{bx}-\frac{ax\log(d+ex)}{b(b+ax^2)}\right)dx}{d}$$

$$=-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d}-\frac{p\text{Li}_2\left(1+\frac{b}{ax^2}\right)}{2d}-\frac{(2p)\int \frac{\log(d+ex)}{x}\,dx}{d}+\frac{(2ap)\int \frac{x\log(d+ex)}{b+ax^2}\,dx}{d}$$

$$=-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d}-\frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d}-\frac{p\text{Li}_2\left(1+\frac{b}{ax^2}\right)}{2d}+\frac{(2ap)\int \left(-\frac{\sqrt{-a}\log(d+ex)}{2a\left(\sqrt{b}-\sqrt{-a}x\right)}+\frac{\sqrt{-a}\log(d+ex)}{2a\left(\sqrt{b}+\sqrt{-a}x\right)}\right)dx}{d}+\frac{(2ep)\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{d}$$

$$=-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d}-\frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d}-\frac{p\text{Li}_2\left(1+\frac{b}{ax^2}\right)}{2d}-\frac{2p\text{Li}_2\left(1+\frac{ex}{d}\right)}{d}-\frac{\left(\sqrt{-a}p\right)\int \frac{\log(d+ex)}{\sqrt{b}-\sqrt{-a}x}\,dx}{d}+\frac{\left(\sqrt{-a}p\right)\int \frac{\log(d+ex)}{\sqrt{b}+\sqrt{-a}x}\,dx}{d}$$

$$=-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d}-\frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d}+\frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{d}+\frac{p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{d}-\frac{p\text{Li}_2\left(1+\frac{b}{ax^2}\right)}{2d}-\frac{2p\text{Li}_2\left(1+\frac{ex}{d}\right)}{d}-\frac{(ep)\int \frac{\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d+ex}\,dx}{d}-\frac{(ep)\int \frac{\log\left(\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{-\sqrt{-a}d+\sqrt{b}e}\right)}{d+ex}\,dx}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{d} - \frac{p\mathrm{Li}_2\left(1+\frac{b}{ax^2}\right)}{2d} - \frac{2p\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{d} - \frac{p\mathrm{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-a}x}{-\sqrt{-a}d+\sqrt{b}e}\right)}{x}dx, x, d+ex\right)}{d} - \frac{p\mathrm{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-a}x}{\sqrt{-a}d+\sqrt{b}e}\right)}{x}dx, x, d+ex\right)}{d}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{d} - \frac{p\mathrm{Li}_2\left(1+\frac{b}{ax^2}\right)}{2d} + \frac{p\mathrm{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d} + \frac{p\mathrm{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d} - \frac{2p\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.01

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}dx = -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d} - \frac{2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{d} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{a+\frac{b}{x^2}}{a}\right)}{2d} - \frac{2p\,\mathrm{PolyLog}\left(2,\frac{d+ex}{d}\right)}{d} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d}$$

```
[In] Integrate[Log[c*(a + b/x^2)^p]/(x*(d + e*x)),x]
```

```
[Out] -1/2*(Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/d - (Log[c*(a + b/x^2)^p]*Log
[d + e*x])/d - (2*p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[(e*(Sqrt[b] -
Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x])/d + (p*Log[-((e*(Sqrt[
b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sqrt[b]*e))]*Log[d + e*x])/d - (p*PolyLog[2
, (a + b/x^2)/a])/(2*d) - (2*p*PolyLog[2, (d + e*x)/d])/d + (p*PolyLog[2, (
Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/d + (p*PolyLog[2, (Sqrt[-a]*
(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/d
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.26

method	result
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)\ln(ex+d)}{d}+\frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)\ln(x)}{d}+2pb\left(\frac{\frac{\ln(x)^2}{2b}-\frac{a\left(\frac{\ln(x)\left(\ln\left(\frac{-ax+\sqrt{-ab}}{\sqrt{-ab}}\right)+\ln\left(\frac{ax+\sqrt{-ab}}{\sqrt{-ab}}\right)\right)}{2a}+\frac{\mathrm{dilog}\left(\frac{-ax+\sqrt{-}}{\sqrt{-ab}}\right.}{}\right.}{b}}{d}\right.$

```
[In] int(ln(c*(a+b/x^2)^p)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(c*(a+b/x^2)^p)*ln(e*x+d)/d+ln(c*(a+b/x^2)^p)/d*ln(x)+2*p*b*(1/d*(1/2/b*
ln(x)^2-a/b*(1/2*ln(x)*(ln((-a*x+(-a*b)^(1/2))/(-a*b)^(1/2))+ln((a*x+(-a*b)
^(1/2))/(-a*b)^(1/2)))/a+1/2*(dilog((-a*x+(-a*b)^(1/2))/(-a*b)^(1/2))+dilog
((a*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/a))-1/d*(-a/b*(1/2*ln(e*x+d)*(ln((e*(-a*
b)^(1/2)+a*d-a*(e*x+d))/(e*(-a*b)^(1/2)+a*d))+ln((e*(-a*b)^(1/2)-a*d+a*(e*x
+d))/(e*(-a*b)^(1/2)-a*d)))/a+1/2*(dilog((e*(-a*b)^(1/2)+a*d-a*(e*x+d))/(e*
(-a*b)^(1/2)+a*d))+dilog((e*(-a*b)^(1/2)-a*d+a*(e*x+d))/(e*(-a*b)^(1/2)-a*d
)))/a)+1/b*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))))
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x}\,dx$$

```
[In] integrate(log(c*(a+b/x^2)^p)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x^2 + b)/x^2)^p)/(e*x^2 + d*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(a+b/x**2)**p)/x/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x}\,dx$$

```
[In] integrate(log(c*(a+b/x^2)^p)/x/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((a + b/x^2)^p*c)/((e*x + d)*x), x)
```

Giac [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x}\,dx$$

```
[In] integrate(log(c*(a+b/x^2)^p)/x/(e*x+d),x, algorithm="giac")
[Out] integrate(log((a + b/x^2)^p*c)/((e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x(d+ex)}\,dx = \int \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x\,\left(d+e\,x\right)}\,dx$$

```
[In] int(log(c*(a + b/x^2)^p)/(x*(d + e*x)),x)
[Out] int(log(c*(a + b/x^2)^p)/(x*(d + e*x)), x)
```

3.252 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 357

$$
\begin{aligned}
\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx ={}& \frac{2p}{dx}+\frac{2\sqrt{a}p\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} \\
&+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^2} \\
&+\frac{2ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2}-\frac{ep\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{d^2} \\
&-\frac{ep\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{d^2} \\
&+\frac{ep\operatorname{PolyLog}\left(2,1+\frac{b}{ax^2}\right)}{2d^2}-\frac{ep\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d^2} \\
&-\frac{ep\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d^2}+\frac{2ep\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{d^2}
\end{aligned}
$$

```
[Out] 2*p/d/x-ln(c*(a+b/x^2)^p)/d/x+1/2*e*ln(c*(a+b/x^2)^p)*ln(-b/a/x^2)/d^2+e*ln
(c*(a+b/x^2)^p)*ln(e*x+d)/d^2+2*e*p*ln(-e*x/d)*ln(e*x+d)/d^2-e*p*ln(e*x+d)*
ln(-e*(x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)-e*b^(1/2)))/d^2-e*p*ln(e*x+d)*ln
(e*(-x*(-a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)+e*b^(1/2)))/d^2+1/2*e*p*polylog(2,
1+b/a/x^2)/d^2+2*e*p*polylog(2,1+e*x/d)/d^2-e*p*polylog(2,(e*x+d)*(-a)^(1/2
)/(d*(-a)^(1/2)-e*b^(1/2)))/d^2-e*p*polylog(2,(e*x+d)*(-a)^(1/2)/(d*(-a)^(1
/2)+e*b^(1/2)))/d^2+2*p*arctan(x*a^(1/2)/b^(1/2))*a^(1/2)/d/b^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {2516, 2505, 269, 331, 211, 2504, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$\int \frac{\log \left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx = \frac{2\sqrt{a}p\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d}+\frac{e\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2d^2}+\frac{e\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx}+\frac{ep\operatorname{PolyLog}\left(2,\frac{b}{ax^2}+1\right)}{2d^2}-\frac{ep\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d^2}-\frac{ep\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d^2}-\frac{ep\log(d+ex)\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d^2}-\frac{ep\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}x+\sqrt{b}\right)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d^2}+\frac{2ep\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^2}+\frac{2ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2}+\frac{2p}{dx}$$

```
[In] Int[Log[c*(a + b/x^2)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (2*p)/(d*x) + (2*Sqrt[a]*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])/(Sqrt[b]*d) - Log[c
*(a + b/x^2)^p]/(d*x) + (e*Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/(2*d^2)
+ (e*Log[c*(a + b/x^2)^p]*Log[d + e*x])/d^2 + (2*e*p*Log[-((e*x)/d)]*Log[d
+ e*x])/d^2 - (e*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]
*Log[d + e*x])/d^2 - (e*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a]*d - Sq
rt[b]*e))]*Log[d + e*x])/d^2 + (e*p*PolyLog[2, 1 + b/(a*x^2)])/(2*d^2) - (e
*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/d^2 - (e*p*Po
lyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/d^2 + (2*e*p*PolyL
og[2, 1 + (e*x)/d])/d^2
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx^2}-\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x}+\frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2(d+ex)}\right)dx \\
&= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx}{d}-\frac{e\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx}{d^2}+\frac{e^2\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx}{d^2} \\
&= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^2} \\
&\quad +\frac{e\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx,x,\frac{1}{x^2}\right)}{2d^2}-\frac{(2bp)\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^4}\,dx}{d}+\frac{(2bep)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^2}\right)x^3}\,dx}{d^2} \\
&= -\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} \\
&\quad +\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^2}-\frac{(2bp)\int \frac{1}{x^2(b+ax^2)}\,dx}{d} \\
&\quad -\frac{(bep)\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx,x,\frac{1}{x^2}\right)}{2d^2}+\frac{(2bep)\int \left(\frac{\log(d+ex)}{bx}-\frac{ax\log(d+ex)}{b(b+ax^2)}\right)dx}{d^2}
\end{aligned}
$$

$$=\frac{2p}{dx}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^2}+\frac{ep\text{Li}_2\left(1+\frac{b}{ax^2}\right)}{2d^2}+\frac{(2ap)\int\frac{1}{b+ax^2}\,dx}{d}+\frac{(2ep)\int\frac{\log(d+ex)}{x}\,dx}{d^2}-\frac{(2aep)\int\frac{x\log(d+ex)}{b+ax^2}\,dx}{d^2}$$

$$=\frac{2p}{dx}+\frac{2\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^2}+\frac{2ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2}+\frac{ep\text{Li}_2\left(1+\frac{b}{ax^2}\right)}{2d^2}-\frac{(2aep)\int\left(-\frac{\sqrt{-a}\log(d+ex)}{2a\left(\sqrt{b}-\sqrt{-a}x\right)}+\frac{\sqrt{-a}\log(d+ex)}{2a\left(\sqrt{b}+\sqrt{-a}x\right)}\right)dx}{d^2}-\frac{(2e^2p)\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{d^2}$$

$$=\frac{2p}{dx}+\frac{2\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^2}+\frac{2ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2}+\frac{ep\text{Li}_2\left(1+\frac{b}{ax^2}\right)}{2d^2}+\frac{2ep\text{Li}_2\left(1+\frac{ex}{d}\right)}{d^2}+\frac{\left(\sqrt{-a}ep\right)\int\frac{\log(d+ex)}{\sqrt{b}-\sqrt{-a}x}\,dx}{d^2}-\frac{\left(\sqrt{-a}ep\right)\int\frac{\log(d+ex)}{\sqrt{b}+\sqrt{-a}x}\,dx}{d^2}$$

$$=\frac{2p}{dx}+\frac{2\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d}-\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^2}+\frac{2ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2}-\frac{ep\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{d^2}-\frac{ep\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{d^2}+\frac{ep\text{Li}_2\left(1+\frac{b}{ax^2}\right)}{2d^2}+\frac{2ep\text{Li}_2\left(1+\frac{ex}{d}\right)}{d^2}+\frac{(e^2p)\int\frac{\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d+ex}\,dx}{d^2}+\frac{(e^2p)\int\frac{\log\left(\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{-\sqrt{-a}d+\sqrt{b}e}\right)}{d+ex}\,dx}{d^2}$$

$$= \frac{2p}{dx} + \frac{2\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^2} + \frac{2ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{d^2} + \frac{ep\text{Li}_2\left(1+\frac{b}{ax^2}\right)}{2d^2} + \frac{2ep\text{Li}_2\left(1+\frac{ex}{d}\right)}{d^2} + \frac{(ep)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-a}x}{-\sqrt{-a}d+\sqrt{b}e}\right)}{x}dx,x,d+ex\right)}{d^2} + \frac{(ep)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-a}x}{\sqrt{-a}d+\sqrt{b}e}\right)}{x}dx,x,d+ex\right)}{d^2}$$

$$= \frac{2p}{dx} + \frac{2\sqrt{a}p\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d} - \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^2} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^2} + \frac{2ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{d^2} + \frac{ep\text{Li}_2\left(1+\frac{b}{ax^2}\right)}{2d^2} - \frac{ep\text{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d^2} - \frac{ep\text{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d^2} + \frac{2ep\text{Li}_2\left(1+\frac{ex}{d}\right)}{d^2}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.92

$$\int\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx = \frac{\frac{4dp}{x} - \frac{4\sqrt{a}dp\arctan\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{\sqrt{b}} - \frac{2d\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x} + e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right) + 2e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{}$$

```
[In] Integrate[Log[c*(a + b/x^2)^p]/(x^2*(d + e*x)),x]
```

```
[Out] ((4*d*p)/x - (4*Sqrt[a]*d*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[b] - (2*d*Log
[c*(a + b/x^2)^p])/x + e*Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))] + 2*e*Log[c
*(a + b/x^2)^p]*Log[d + e*x] + 4*e*p*Log[-((e*x)/d)]*Log[d + e*x] - 2*e*p*L
og[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x] - 2*e*
p*Log[(e*(Sqrt[b] + Sqrt[-a]*x))/(-(Sqrt[-a]*d) + Sqrt[b]*e)]*Log[d + e*x]
+ e*p*PolyLog[2, 1 + b/(a*x^2)] - 2*e*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sq
rt[-a]*d - Sqrt[b]*e)] - 2*e*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d
+ Sqrt[b]*e)] + 4*e*p*PolyLog[2, 1 + (e*x)/d])/(2*d^2)
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.17

method	result
parts	$\frac{e\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)\ln(ex+d)}{d^2}-\frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx}-\frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)e\ln(x)}{d^2}+2pb\left(\frac{e\left(-\frac{a\left(\frac{\ln(ex+d)\left(\ln\left(\frac{e\sqrt{-ab}+ad-a(ex+d)}{e\sqrt{-ab}+ad}\right)+\ln\left(\right.\right.}{2a}\right.}{}\right.}{}\right.$

[In] int(ln(c*(a+b/x^2)^p)/x^2/(e*x+d),x,method=_RETURNVERBOSE)

[Out] e*ln(c*(a+b/x^2)^p)*ln(e*x+d)/d^2-ln(c*(a+b/x^2)^p)/d/x-ln(c*(a+b/x^2)^p)*e/d^2*ln(x)+2*p*b*(e/d^2*(-a/b*(1/2*ln(e*x+d)*(ln((e*(-a*b)^(1/2)+a*d-a*(e*x+d))/(e*(-a*b)^(1/2)+a*d))+ln((e*(-a*b)^(1/2)-a*d+a*(e*x+d))/(e*(-a*b)^(1/2)-a*d)))/a+1/2*(dilog((e*(-a*b)^(1/2)+a*d-a*(e*x+d))/(e*(-a*b)^(1/2)+a*d))+dilog((e*(-a*b)^(1/2)-a*d+a*(e*x+d))/(e*(-a*b)^(1/2)-a*d)))/a)+1/b*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d)))+1/d/b/x+1/d*a/b/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2))-e/d^2*(1/2/b*ln(x)^2-a/b*(1/2*ln(x)*(ln((-a*x+(-a*b)^(1/2))/(-a*b)^(1/2))+ln((a*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/a+1/2*(dilog((-a*x+(-a*b)^(1/2))/(-a*b)^(1/2))+dilog((a*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/a)))

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x^2}\,dx$$

[In] integrate(log(c*(a+b/x^2)^p)/x^2/(e*x+d),x, algorithm="fricas")

[Out] integral(log(c*((a*x^2 + b)/x^2)^p)/(e*x^3 + d*x^2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(a+b/x**2)**p)/x**2/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x^2}\,dx$$

```
[In] integrate(log(c*(a+b/x^2)^p)/x^2/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((a + b/x^2)^p*c)/((e*x + d)*x^2), x)
```

Giac [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x^2}\,dx$$

```
[In] integrate(log(c*(a+b/x^2)^p)/x^2/(e*x+d),x, algorithm="giac")
[Out] integrate(log((a + b/x^2)^p*c)/((e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\ln\left(c\,\left(a+\frac{b}{x^2}\right)^p\right)}{x^2\,\left(d+e\,x\right)}\,dx$$

```
[In] int(log(c*(a + b/x^2)^p)/(x^2*(d + e*x)),x)
[Out] int(log(c*(a + b/x^2)^p)/(x^2*(d + e*x)), x)
```

3.253 $\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 414

$$\begin{aligned}\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx &= \frac{p}{2dx^2}-\frac{2ep}{d^2x}-\frac{2\sqrt{a}ep\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd}\\
&+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x}-\frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^3}\\
&-\frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^3}-\frac{2e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3}\\
&+\frac{e^2p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{d^3}\\
&+\frac{e^2p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{d^3}\\
&-\frac{e^2p\operatorname{PolyLog}\left(2,1+\frac{b}{ax^2}\right)}{2d^3}+\frac{e^2p\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d^3}\\
&+\frac{e^2p\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d^3}-\frac{2e^2p\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{d^3}\end{aligned}$$

```
[Out] 1/2*p/d/x^2-2*e*p/d^2/x-1/2*(a+b/x^2)*ln(c*(a+b/x^2)^p)/b/d+e*ln(c*(a+b/x^2
)^p)/d^2/x-1/2*e^2*ln(c*(a+b/x^2)^p)*ln(-b/a/x^2)/d^3-e^2*ln(c*(a+b/x^2)^p)
*ln(e*x+d)/d^3-2*e^2*p*ln(-e*x/d)*ln(e*x+d)/d^3+e^2*p*ln(e*x+d)*ln(-e*(x*(-
a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)-e*b^(1/2)))/d^3+e^2*p*ln(e*x+d)*ln(e*(-x*(-
a)^(1/2)+b^(1/2))/(d*(-a)^(1/2)+e*b^(1/2)))/d^3-1/2*e^2*p*polylog(2,1+b/a/x
^2)/d^3-2*e^2*p*polylog(2,1+e*x/d)/d^3+e^2*p*polylog(2,(e*x+d)*(-a)^(1/2)/(
d*(-a)^(1/2)-e*b^(1/2)))/d^3+e^2*p*polylog(2,(e*x+d)*(-a)^(1/2)/(d*(-a)^(1/
2)+e*b^(1/2)))/d^3-2*e*p*arctan(x*a^(1/2)/b^(1/2))*a^(1/2)/d^2/b^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {2516, 2504, 2436, 2332, 2505, 269, 331, 211, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$\begin{aligned}\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx = &-\frac{2\sqrt{a}ep\arctan\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d^2}-\frac{e^2\log\left(-\frac{b}{ax^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2d^3}\\&-\frac{e^2\log(d+ex)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^3}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x}\\&-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd}-\frac{e^2p\operatorname{PolyLog}\left(2,\frac{b}{ax^2}+1\right)}{2d^3}\\&+\frac{e^2p\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d^3}+\frac{e^2p\operatorname{PolyLog}\left(2,\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d^3}\\&+\frac{e^2p\log(d+ex)\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d^3}\\&+\frac{e^2p\log(d+ex)\log\left(-\frac{e\left(\sqrt{-a}x+\sqrt{b}\right)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d^3}-\frac{2e^2p\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^3}\\&-\frac{2e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3}-\frac{2ep}{d^2x}+\frac{p}{2dx^2}\end{aligned}$$

```
[In] Int[Log[c*(a + b/x^2)^p]/(x^3*(d + e*x)),x]
```

```
[Out] p/(2*d*x^2) - (2*e*p)/(d^2*x) - (2*Sqrt[a]*e*p*ArcTan[(Sqrt[a]*x)/Sqrt[b]])
/(Sqrt[b]*d^2) - ((a + b/x^2)*Log[c*(a + b/x^2)^p])/(2*b*d) + (e*Log[c*(a +
 b/x^2)^p])/(d^2*x) - (e^2*Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))])/(2*d^3)
- (e^2*Log[c*(a + b/x^2)^p]*Log[d + e*x])/d^3 - (2*e^2*p*Log[-((e*x)/d)]*Lo
g[d + e*x])/d^3 + (e^2*p*Log[(e*(Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[
b]*e)]*Log[d + e*x])/d^3 + (e^2*p*Log[-((e*(Sqrt[b] + Sqrt[-a]*x))/(Sqrt[-a
]*d - Sqrt[b]*e))]*Log[d + e*x])/d^3 - (e^2*p*PolyLog[2, 1 + b/(a*x^2)])/(2
*d^3) + (e^2*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d - Sqrt[b]*e)])/d
^3 + (e^2*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d + Sqrt[b]*e)])/d^3
- (2*e^2*p*PolyLog[2, 1 + (e*x)/d])/d^3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{dx^3} - \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x^2} + \frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^3x} - \frac{e^3\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^3(d+ex)}\right) dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x}\,dx}{d^3} - \frac{e^3\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d+ex}\,dx}{d^3}$$

$$= \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x} - \frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^3} - \frac{\text{Subst}\left(\int \log\left(c(a+bx)^p\right)\,dx,x,\frac{1}{x^2}\right)}{2d} - \frac{e^2\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx,x,\frac{1}{x^2}\right)}{2d^3} + \frac{(2bep)\int \frac{1}{\left(a+\frac{b}{x^2}\right)x^4}\,dx}{d^2} - \frac{(2be^2p)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^2}\right)x^3}\,dx}{d^3}$$

$$= \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x} - \frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^3} - \frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^3} - \frac{\text{Subst}\left(\int \log\left(cx^p\right)\,dx,x,a+\frac{b}{x^2}\right)}{2bd} + \frac{(2bep)\int \frac{1}{x^2(b+ax^2)}\,dx}{d^2} + \frac{(be^2p)\,\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx,x,\frac{1}{x^2}\right)}{2d^3} - \frac{(2be^2p)\int \left(\frac{\log(d+ex)}{bx} - \frac{ax\log(d+ex)}{b(b+ax^2)}\right)\,dx}{d^3}$$

$$= \frac{p}{2dx^2} - \frac{2ep}{d^2x} - \frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x} - \frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^3} - \frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^3} - \frac{e^2p\text{Li}_2\left(1+\frac{b}{ax^2}\right)}{2d^3} - \frac{(2aep)\int \frac{1}{b+ax^2}\,dx}{d^2} - \frac{(2e^2p)\int \frac{\log(d+ex)}{x}\,dx}{d^3} + \frac{(2ae^2p)\int \frac{x\log(d+ex)}{b+ax^2}\,dx}{d^3}$$

$$= \frac{p}{2dx^2} - \frac{2ep}{d^2x} - \frac{2\sqrt{a}ep\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d^2} - \frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x} - \frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^3} - \frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^3} - \frac{2e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3} - \frac{e^2p\text{Li}_2\left(1+\frac{b}{ax^2}\right)}{2d^3} + \frac{(2ae^2p)\int \left(-\frac{\sqrt{-a}\log(d+ex)}{2a\left(\sqrt{b}-\sqrt{-a}x\right)} + \frac{\sqrt{-a}\log(d+ex)}{2a\left(\sqrt{b}+\sqrt{-a}x\right)}\right)\,dx}{d^3} + \frac{(2e^3p)\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{d^3}$$

$$=\frac{p}{2dx^2}-\frac{2ep}{d^2x}-\frac{2\sqrt{a}ep\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x}-\frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^3}-\frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^3}-\frac{2e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3}-\frac{e^2p\text{Li}_2\left(1+\frac{b}{ax^2}\right)}{2d^3}-\frac{2e^2p\text{Li}_2\left(1+\frac{ex}{d}\right)}{d^3}-\frac{\left(\sqrt{-a}e^2p\right)\int\frac{\log(d+ex)}{\sqrt{b}-\sqrt{-a}x}\,dx}{d^3}+\frac{\left(\sqrt{-a}e^2p\right)\int\frac{\log(d+ex)}{\sqrt{b}+\sqrt{-a}x}\,dx}{d^3}$$

$$=\frac{p}{2dx^2}-\frac{2ep}{d^2x}-\frac{2\sqrt{a}ep\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x}-\frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^3}-\frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^3}-\frac{2e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3}+\frac{e^2p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{d^3}+\frac{e^2p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{d^3}-\frac{e^2p\text{Li}_2\left(1+\frac{b}{ax^2}\right)}{2d^3}-\frac{2e^2p\text{Li}_2\left(1+\frac{ex}{d}\right)}{d^3}-\frac{\left(e^3p\right)\int\frac{\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d+ex}\,dx}{d^3}-\frac{\left(e^3p\right)\int\frac{\log\left(\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{-\sqrt{-a}d+\sqrt{b}e}\right)}{d+ex}\,dx}{d^3}$$

$$=\frac{p}{2dx^2}-\frac{2ep}{d^2x}-\frac{2\sqrt{a}ep\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d^2}-\frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd}+\frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x}-\frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^3}-\frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^3}-\frac{2e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3}+\frac{e^2p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{d^3}+\frac{e^2p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{d^3}-\frac{e^2p\text{Li}_2\left(1+\frac{b}{ax^2}\right)}{2d^3}-\frac{2e^2p\text{Li}_2\left(1+\frac{ex}{d}\right)}{d^3}-\frac{\left(e^2p\right)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{-a}x}{-\sqrt{-a}d+\sqrt{b}e}\right)}{x}\,dx,x,d+ex\right)}{d^3}-\frac{\left(e^2p\right)\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{-a}x}{\sqrt{-a}d+\sqrt{b}e}\right)}{x}\,dx,x,d+ex\right)}{d^3}$$

$$= \frac{p}{2dx^2} - \frac{2ep}{d^2x} - \frac{2\sqrt{a}ep\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{b}}\right)}{\sqrt{b}d^2} - \frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{2bd} + \frac{e\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{d^2x}$$
$$- \frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{b}{ax^2}\right)}{2d^3} - \frac{e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log(d+ex)}{d^3}$$
$$- \frac{2e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3} + \frac{e^2p\log\left(\frac{e\left(\sqrt{b}-\sqrt{-a}x\right)}{\sqrt{-a}d+\sqrt{b}e}\right)\log(d+ex)}{d^3}$$
$$+ \frac{e^2p\log\left(-\frac{e\left(\sqrt{b}+\sqrt{-a}x\right)}{\sqrt{-a}d-\sqrt{b}e}\right)\log(d+ex)}{d^3} - \frac{e^2p\mathrm{Li}_2\left(1+\frac{b}{ax^2}\right)}{2d^3}$$
$$+ \frac{e^2p\mathrm{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d-\sqrt{b}e}\right)}{d^3} + \frac{e^2p\mathrm{Li}_2\left(\frac{\sqrt{-a}(d+ex)}{\sqrt{-a}d+\sqrt{b}e}\right)}{d^3} - \frac{2e^2p\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{d^3}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.93

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx$$
$$= \frac{-\frac{4dep}{x} + \frac{4\sqrt{a}dep\arctan\left(\frac{\sqrt{b}}{\sqrt{a}x}\right)}{\sqrt{b}} + \frac{2de\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x} + d^2\left(\frac{p}{x^2} - \frac{\left(a+\frac{b}{x^2}\right)\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{b}\right) - e^2\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)\log\left(-\frac{}{a}\right.}{}$$

```
[In] Integrate[Log[c*(a + b/x^2)^p]/(x^3*(d + e*x)),x]
```

```
[Out] ((-4*d*e*p)/x + (4*Sqrt[a]*d*e*p*ArcTan[Sqrt[b]/(Sqrt[a]*x)])/Sqrt[b] + (2*
d*e*Log[c*(a + b/x^2)^p])/x + d^2*(p/x^2 - ((a + b/x^2)*Log[c*(a + b/x^2)^p
])/b) - e^2*Log[c*(a + b/x^2)^p]*Log[-(b/(a*x^2))] - 2*e^2*Log[c*(a + b/x^2
)^p]*Log[d + e*x] - 4*e^2*p*Log[-((e*x)/d)]*Log[d + e*x] + 2*e^2*p*Log[(e*(
Sqrt[b] - Sqrt[-a]*x))/(Sqrt[-a]*d + Sqrt[b]*e)]*Log[d + e*x] + 2*e^2*p*Log
[(e*(Sqrt[b] + Sqrt[-a]*x))/(-(Sqrt[-a]*d) + Sqrt[b]*e)]*Log[d + e*x] - e^2
*p*PolyLog[2, 1 + b/(a*x^2)] + 2*e^2*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqr
t[-a]*d - Sqrt[b]*e)] + 2*e^2*p*PolyLog[2, (Sqrt[-a]*(d + e*x))/(Sqrt[-a]*d
 + Sqrt[b]*e)] - 4*e^2*p*PolyLog[2, 1 + (e*x)/d])/(2*d^3)
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.18

method	result
parts	$-\frac{e^{2} \ln\left(c\left(a+\frac{b}{x^{2}}\right)^{p}\right) \ln(ex+d)}{d^{3}}-\frac{\ln\left(c\left(a+\frac{b}{x^{2}}\right)^{p}\right)}{2d\,x^{2}}+\frac{\ln\left(c\left(a+\frac{b}{x^{2}}\right)^{p}\right) e^{2} \ln(x)}{d^{3}}+\frac{e \ln\left(c\left(a+\frac{b}{x^{2}}\right)^{p}\right)}{d^{2} x}+pb\left(\frac{1}{2db\,x^{2}}-\frac{2e}{d^{2}bx}+\right.$

[In] int(ln(c*(a+b/x^2)^p)/x^3/(e*x+d),x,method=_RETURNVERBOSE)

[Out] -e^2*ln(c*(a+b/x^2)^p)*ln(e*x+d)/d^3-1/2*ln(c*(a+b/x^2)^p)/d/x^2+ln(c*(a+b/x^2)^p)*e^2/d^3*ln(x)+e*ln(c*(a+b/x^2)^p)/d^2/x+p*b*(1/2/d/b/x^2-2/d^2/b*e/x+1/d*a/b^2*ln(x)-1/2/d/b^2*a*ln(a*x^2+b)-2/d^2/b*a*e/(a*b)^(1/2)*arctan(a*x/(a*b)^(1/2))+2*e^2/d^3*(1/2/b*ln(x)^2-a/b*(1/2*ln(x)*(ln((-a*x+(-a*b)^(1/2))/(-a*b)^(1/2))+ln((a*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/a+1/2*(dilog((-a*x+(-a*b)^(1/2))/(-a*b)^(1/2))+dilog((a*x+(-a*b)^(1/2))/(-a*b)^(1/2)))/a))-2*e^2/d^3*(-a/b*(1/2*ln(e*x+d)*(ln((e*(-a*b)^(1/2)+a*d-a*(e*x+d))/(e*(-a*b)^(1/2)+a*d))+ln((e*(-a*b)^(1/2)-a*d+a*(e*x+d))/(e*(-a*b)^(1/2)-a*d)))/a+1/2*(dilog((e*(-a*b)^(1/2)+a*d-a*(e*x+d))/(e*(-a*b)^(1/2)+a*d))+dilog((e*(-a*b)^(1/2)-a*d+a*(e*x+d))/(e*(-a*b)^(1/2)-a*d)))/a)+1/b*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))))

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x^3}\,dx$$

[In] integrate(log(c*(a+b/x^2)^p)/x^3/(e*x+d),x, algorithm="fricas")

[Out] integral(log(c*((a*x^2 + b)/x^2)^p)/(e*x^4 + d*x^3), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(a+b/x**2)**p)/x**3/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x^3}\,dx$$

```
[In] integrate(log(c*(a+b/x^2)^p)/x^3/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((a + b/x^2)^p*c)/((e*x + d)*x^3), x)
```

Giac [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^2}\right)^p c\right)}{(ex+d)x^3}\,dx$$

```
[In] integrate(log(c*(a+b/x^2)^p)/x^3/(e*x+d),x, algorithm="giac")
[Out] integrate(log((a + b/x^2)^p*c)/((e*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3(d+ex)}\,dx = \int \frac{\ln\left(c\left(a+\frac{b}{x^2}\right)^p\right)}{x^3\,(d+e\,x)}\,dx$$

```
[In] int(log(c*(a + b/x^2)^p)/(x^3*(d + e*x)),x)
[Out] int(log(c*(a + b/x^2)^p)/(x^3*(d + e*x)), x)
```

3.254 $\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 714

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = -\frac{\sqrt{3}\sqrt[3]{b}d^2p\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e^3} + \frac{\sqrt{3}b^{2/3}dp\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3}e^2} + \frac{d^2x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} + \frac{\sqrt[3]{b}d^2p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e^3} + \frac{b^{2/3}dp\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{2a^{2/3}e^2} - \frac{d^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^4} - \frac{3d^3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^4} + \frac{d^3p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e^4} + \frac{d^3p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e^4} + \frac{d^3p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e^4} - \frac{\sqrt[3]{b}d^2p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{a}e^3} - \frac{b^{2/3}dp\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{4a^{2/3}e^2} + \frac{bp\log\left(b+ax^3\right)}{3ae} + \frac{d^3p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^4} + \frac{d^3p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^4} + \frac{d^3p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^4} - \frac{3d^3p\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e^4}$$

```
[Out] d^2*x*ln(c*(a+b/x^3)^p)/e^3-1/2*d*x^2*ln(c*(a+b/x^3)^p)/e^2+1/3*x^3*ln(c*(a
+b/x^3)^p)/e+b^(1/3)*d^2*p*ln(b^(1/3)+a^(1/3)*x)/a^(1/3)/e^3+1/2*b^(2/3)*d*
p*ln(b^(1/3)+a^(1/3)*x)/a^(2/3)/e^2-d^3*ln(c*(a+b/x^3)^p)*ln(e*x+d)/e^4-3*d
^3*p*ln(-e*x/d)*ln(e*x+d)/e^4+d^3*p*ln(-e*(b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-b^
(1/3)*e))*ln(e*x+d)/e^4+d^3*p*ln(-e*((-1)^(2/3)*b^(1/3)+a^(1/3)*x)/(a^(1/3)
*d-(-1)^(2/3)*b^(1/3)*e))*ln(e*x+d)/e^4+d^3*p*ln((-1)^(1/3)*e*(b^(1/3)+(-1)
^(2/3)*a^(1/3)*x)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))*ln(e*x+d)/e^4-1/2*b^(1/
3)*d^2*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(1/3)/e^3-1/4*b^(2/3)*
d*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(2/3)/e^2+1/3*b*p*ln(a*x^3+
b)/a/e+d^3*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-b^(1/3)*e))/e^4+d^3*p*pol
ylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))/e^4+d^3*p*polylog(
2,a^(1/3)*(e*x+d)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))/e^4-3*d^3*p*polylog(2,1
+e*x/d)/e^4-b^(1/3)*d^2*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(1/3)*3^(1/2))
*3^(1/2)/a^(1/3)/e^3+1/2*b^(2/3)*d*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(1/
3)*3^(1/2))*3^(1/2)/a^(2/3)/e^2
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 714, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {2516, 2498, 269, 206, 31, 648, 631, 210, 642, 2505, 298, 266, 2512, 2463, 2441, 2352,

2440, 2438}

$$\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \frac{\sqrt{3}b^{2/3}dp\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3}e^2} - \frac{\sqrt[3]{b}d^2p\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{a}e^3} - \frac{b^{2/3}dp\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{4a^{2/3}e^2} + \frac{b^{2/3}dp\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{2a^{2/3}e^2} - \frac{\sqrt{3}\sqrt[3]{b}d^2p\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e^3} - \frac{d^3\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^4} + \frac{d^2x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} + \frac{d^3p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^4} + \frac{d^3p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^4} + \frac{d^3p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^4} + \frac{d^3p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^4} + \frac{d^3p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+(-1)^{2/3}\sqrt[3]{b}\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^4} + \frac{d^3p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left((-1)^{2/3}\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^4} + \frac{\sqrt[3]{b}d^2p\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}e^3} + \frac{bp\log\left(ax^3+b\right)}{3ae} - \frac{3d^3p\,\mathrm{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^4} - \frac{3d^3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^4}$$

```
[In] Int[(x^3*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

```
[Out] -((Sqrt[3]*b^(1/3)*d^2*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])
/(a^(1/3)*e^3)) + (Sqrt[3]*b^(2/3)*d*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt
[3]*b^(1/3))])/(2*a^(2/3)*e^2) + (d^2*x*Log[c*(a + b/x^3)^p])/e^3 - (d*x^2*
Log[c*(a + b/x^3)^p])/(2*e^2) + (x^3*Log[c*(a + b/x^3)^p])/(3*e) + (b^(1/3)
*d^2*p*Log[b^(1/3) + a^(1/3)*x])/(a^(1/3)*e^3) + (b^(2/3)*d*p*Log[b^(1/3) +
 a^(1/3)*x])/(2*a^(2/3)*e^2) - (d^3*Log[c*(a + b/x^3)^p]*Log[d + e*x])/e^4
- (3*d^3*p*Log[-((e*x)/d)]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*(b^(1/3) + a
^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/e^4 + (d^3*p*Log[-((e*((
-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d
+ e*x])/e^4 + (d^3*p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a
^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/e^4 - (b^(1/3)*d^2*p*Log[b^
(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(2*a^(1/3)*e^3) - (b^(2/3)*d*p*Lo
g[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(4*a^(2/3)*e^2) + (b*p*Log[b
+ a*x^3])/(3*a*e) + (d^3*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1
/3)*e)])/e^4 + (d^3*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3
)*b^(1/3)*e)])/e^4 + (d^3*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1
)^(2/3)*b^(1/3)*e)])/e^4 - (3*d^3*p*PolyLog[2, 1 + (e*x)/d])/e^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\text{integral} = \int \left(\frac{d^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3}-\frac{dx \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} - \frac{d^3 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3(d+ex)}\right) dx$$

$$= \frac{d^2 \int \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\,dx}{e^3} - \frac{d^3 \int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx}{e^3} - \frac{d \int x \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\,dx}{e^2} + \frac{\int x^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\,dx}{e}$$

$$= \frac{d^2 x \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{d x^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} - \frac{d^3 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^4} - \frac{(3bd^3p)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^3}\right)x^4}\,dx}{e^4} + \frac{(3bd^2p)\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^3}\,dx}{e^3} - \frac{(3bdp)\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^2}\,dx}{2e^2} + \frac{(bp)\int \frac{1}{\left(a+\frac{b}{x^3}\right)x}\,dx}{e}$$

$$= \frac{d^2 x \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{d x^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} - \frac{d^3 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^4} - \frac{(3bd^3p)\int \left(\frac{\log(d+ex)}{bx} - \frac{ax^2\log(d+ex)}{b(b+ax^3)}\right)dx}{e^4} + \frac{(3bd^2p)\int \frac{1}{b+ax^3}\,dx}{e^3} - \frac{(3bdp)\int \frac{x}{b+ax^3}\,dx}{2e^2} + \frac{(bp)\int \frac{x^2}{b+ax^3}\,dx}{e}$$

$$= \frac{d^2 x \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{d x^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} - \frac{d^3 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^4} + \frac{bp\log\left(b+ax^3\right)}{3ae} - \frac{(3d^3p)\int \frac{\log(d+ex)}{x}\,dx}{e^4} + \frac{(3ad^3p)\int \frac{x^2\log(d+ex)}{b+ax^3}\,dx}{e^4} + \frac{\left(\sqrt[3]{b}d^2p\right)\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}x}\,dx}{e^3} + \frac{\left(\sqrt[3]{b}d^2p\right)\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx}{e^3} + \frac{\left(b^{2/3}dp\right)\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}x}\,dx}{2\sqrt[3]{a}e^2} - \frac{\left(b^{2/3}dp\right)\int \frac{\sqrt[3]{b}+\sqrt[3]{a}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx}{2\sqrt[3]{a}e^2}$$

$$
\begin{aligned}
&= \frac{d^2x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} \\
&+ \frac{\sqrt[3]{b}d^2p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e^3} + \frac{b^{2/3}dp\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{2a^{2/3}e^2} \\
&- \frac{d^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^4} - \frac{3d^3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^4} + \frac{bp\log\left(b+ax^3\right)}{3ae} \\
&+ \frac{(3ad^3p)\int\left(\frac{\log(d+ex)}{3a^{2/3}\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)} + \frac{\log(d+ex)}{3a^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{b}+\sqrt[3]{a}x\right)} + \frac{\log(d+ex)}{3a^{2/3}\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}\right)dx}{e^4} \\
&- \frac{\left(\sqrt[3]{b}d^2p\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx}{2\sqrt[3]{a}e^3} \\
&+ \frac{\left(3b^{2/3}d^2p\right)\int\frac{1}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx}{2e^3} + \frac{(3d^3p)\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{e^3} \\
&- \frac{\left(b^{2/3}dp\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx}{4a^{2/3}e^2} - \frac{(3bdp)\int\frac{1}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx}{4\sqrt[3]{a}e^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{d^2x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} \\
&+ \frac{\sqrt[3]{b}d^2p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e^3} + \frac{b^{2/3}dp\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{2a^{2/3}e^2} \\
&- \frac{d^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^4} - \frac{3d^3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^4} \\
&- \frac{\sqrt[3]{b}d^2p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{a}e^3} - \frac{b^{2/3}dp\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{4a^{2/3}e^2} \\
&+ \frac{bp\log\left(b+ax^3\right)}{3ae} - \frac{3d^3p\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{e^4} + \frac{\left(\sqrt[3]{a}d^3p\right)\int\frac{\log(d+ex)}{\sqrt[3]{b}+\sqrt[3]{a}x}\,dx}{e^4} \\
&+ \frac{\left(\sqrt[3]{a}d^3p\right)\int\frac{\log(d+ex)}{-\sqrt[3]{-1}\sqrt[3]{b}+\sqrt[3]{a}x}\,dx}{e^4} + \frac{\left(\sqrt[3]{a}d^3p\right)\int\frac{\log(d+ex)}{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x}\,dx}{e^4} \\
&+ \frac{\left(3\sqrt[3]{b}d^2p\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}e^3} \\
&- \frac{\left(3b^{2/3}dp\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)}{2a^{2/3}e^2}
\end{aligned}
$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}d^2p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e^3} + \frac{\sqrt{3}b^{2/3}dp\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3}e^2}$$
$$+ \frac{d^2x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2}$$
$$+ \frac{x^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} + \frac{\sqrt[3]{b}d^2p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e^3}$$
$$+ \frac{b^{2/3}dp\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{2a^{2/3}e^2} - \frac{d^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^4}$$
$$- \frac{3d^3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^4} + \frac{d^3p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e^4}$$
$$+ \frac{d^3p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e^4}$$
$$+ \frac{d^3p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e^4}$$
$$- \frac{\sqrt[3]{b}d^2p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{a}e^3} - \frac{b^{2/3}dp\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{4a^{2/3}e^2}$$
$$+ \frac{bp\log\left(b+ax^3\right)}{3ae} - \frac{3d^3p\text{Li}_2\left(1+\frac{ex}{d}\right)}{e^4} - \frac{(d^3p)\int\frac{\log\left(\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{-\sqrt[3]{a}d+\sqrt[3]{b}e}\right)}{d+ex}dx}{e^3}$$
$$- \frac{(d^3p)\int\frac{\log\left(\frac{e\left(-\sqrt[3]{-1}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{-\sqrt[3]{a}d-\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d+ex}dx}{e^3} - \frac{(d^3p)\int\frac{\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{-\sqrt[3]{a}d+(-1)^{2/3}\sqrt[3]{b}e}\right)}{d+ex}dx}{e^3}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}d^2p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e^3} + \frac{\sqrt{3}b^{2/3}dp\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3}e^2}$$

$$+ \frac{d^2x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2}$$

$$+ \frac{x^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} + \frac{\sqrt[3]{b}d^2p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e^3}$$

$$+ \frac{b^{2/3}dp\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{2a^{2/3}e^2} - \frac{d^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^4}$$

$$- \frac{3d^3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^4} + \frac{d^3p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e^4}$$

$$+ \frac{d^3p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e^4}$$

$$+ \frac{d^3p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e^4}$$

$$- \frac{\sqrt[3]{b}d^2p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{a}e^3}$$

$$- \frac{b^{2/3}dp\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{4a^{2/3}e^2} + \frac{bp\log\left(b+ax^3\right)}{3ae}$$

$$- \frac{3d^3p\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{e^4} - \frac{(d^3p)\,\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{a}x}{-\sqrt[3]{a}d+\sqrt[3]{b}e}\right)}{x}dx, x, d+ex\right)}{e^4}$$

$$- \frac{(d^3p)\,\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{a}x}{-\sqrt[3]{a}d-\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{x}dx, x, d+ex\right)}{e^4}$$

$$- \frac{(d^3p)\,\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{a}x}{-\sqrt[3]{a}d+(-1)^{2/3}\sqrt[3]{b}e}\right)}{x}dx, x, d+ex\right)}{e^4}$$

$$
\begin{aligned}
&= -\frac{\sqrt{3}\sqrt[3]{b}d^2p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e^3} + \frac{\sqrt{3}b^{2/3}dp\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3}e^2} \\
&+ \frac{d^2x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} \\
&+ \frac{x^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} + \frac{\sqrt[3]{b}d^2p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e^3} \\
&+ \frac{b^{2/3}dp\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{2a^{2/3}e^2} - \frac{d^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^4} \\
&- \frac{3d^3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^4} + \frac{d^3p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e^4} \\
&+ \frac{d^3p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e^4} \\
&+ \frac{d^3p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e^4} \\
&- \frac{\sqrt[3]{b}d^2p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{a}e^3} - \frac{b^{2/3}dp\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{4a^{2/3}e^2} \\
&+ \frac{bp\log\left(b+ax^3\right)}{3ae} + \frac{d^3p\operatorname{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^4} + \frac{d^3p\operatorname{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^4} \\
&+ \frac{d^3p\operatorname{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^4} - \frac{3d^3p\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{e^4}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.21 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.75

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex} \, dx = \frac{3bdp \,\text{Hypergeometric2F1}\left(\frac{1}{3},1,\frac{4}{3},-\frac{b}{ax^3}\right)}{2ae^2x} - \frac{3bd^2p \,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{b}{ax^3}\right)}{2ae^3x^2} + \frac{bp\log\left(a+\frac{b}{x^3}\right)}{3ae} + \frac{d^2x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{x^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} + \frac{bp\log(x)}{ae} - \frac{d^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^4} - \frac{3d^3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^4} + \frac{d^3p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e^4} + \frac{d^3p\log\left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e^4} + \frac{d^3p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e^4} - \frac{3d^3p\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e^4} + \frac{d^3p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^4} + \frac{d^3p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^4} + \frac{d^3p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^4}$$

```
[In] Integrate[(x^3*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

```
[Out] (3*b*d*p*Hypergeometric2F1[1/3, 1, 4/3, -(b/(a*x^3))])/(2*a*e^2*x) - (3*b*d
^2*p*Hypergeometric2F1[2/3, 1, 5/3, -(b/(a*x^3))])/(2*a*e^3*x^2) + (b*p*Log
[a + b/x^3])/(3*a*e) + (d^2*x*Log[c*(a + b/x^3)^p])/e^3 - (d*x^2*Log[c*(a +
 b/x^3)^p])/(2*e^2) + (x^3*Log[c*(a + b/x^3)^p])/(3*e) + (b*p*Log[x])/(a*e)
 - (d^3*Log[c*(a + b/x^3)^p]*Log[d + e*x])/e^4 - (3*d^3*p*Log[-((e*x)/d)]*L
```

```
og[d + e*x])/e^4 + (d^3*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1
/3)*e))]*Log[d + e*x])/e^4 + (d^3*p*Log[-(((-1)^(2/3)*e*(b^(1/3) - (-1)^(1/
3)*a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/e^4 + (d^
3*p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(
1/3)*b^(1/3)*e)]*Log[d + e*x])/e^4 - (3*d^3*p*PolyLog[2, (d + e*x)/d])/e^4
+ (d^3*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/e^4 + (d^
3*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/e^4
 + (d^3*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)
])/e^4
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.47 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.42

method	result
parts	$\frac{x^3 \ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3e} - \frac{d\,x^2 \ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e^2} + \frac{d^2 x \ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{d^3 \ln\left(c\left(a+\frac{b}{x^3}\right)^p\right) \ln(ex+d)}{e^4} + 3pb\,e^3 \left(-\frac{\_R=\mathrm{RootOf}(\_}{}\right.$

```
[In] int(x^3*ln(c*(a+b/x^3)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*ln(c*(a+b/x^3)^p)/e-1/2*d*x^2*ln(c*(a+b/x^3)^p)/e^2+d^2*x*ln(c*(a+b
/x^3)^p)/e^3-d^3*ln(c*(a+b/x^3)^p)*ln(e*x+d)/e^4+3*p*b*e^3*(-1/18/e^4/a*sum
((2*_R^2-7*_R*d+11*d^2)/(-_R^2+2*_R*d-d^2)*ln(e*x-_R+d),_R=RootOf(_Z^3*a-3*
_Z^2*a*d+3*_Z*a*d^2-a*d^3+b*e^3))-1/e^4*d^3*(1/b/e^3*(dilog(-e*x/d)+ln(e*x+
d)*ln(-e*x/d))-1/3/b/e^3*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1
-d)/_R1),_R1=RootOf(_Z^3*a-3*_Z^2*a*d+3*_Z*a*d^2-a*d^3+b*e^3))))
```

Fricas [F]

$$\int \frac{x^3 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \frac{x^3 \log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^3*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^3*log(c*((a*x^3 + b)/x^3)^p)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex} \, dx = \text{Timed out}$$

```
[In] integrate(x**3*ln(c*(a+b/x**3)**p)/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex} \, dx = \int \frac{x^3 \log \left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d} \, dx$$

```
[In] integrate(x^3*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x^3*log((a + b/x^3)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex} \, dx = \int \frac{x^3 \log \left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d} \, dx$$

```
[In] integrate(x^3*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x^3*log((a + b/x^3)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex} \, dx = \int \frac{x^3 \ln \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+e\,x} \, dx$$

```
[In] int((x^3*log(c*(a + b/x^3)^p))/(d + e*x),x)
[Out] int((x^3*log(c*(a + b/x^3)^p))/(d + e*x), x)
```

3.255 $\int \frac{x^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 666

$$\int \frac{x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \frac{\sqrt{3}\sqrt[3]{b}dp\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e^2}-\frac{\sqrt{3}b^{2/3}p\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3}e}-\frac{dx\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e}-\frac{\sqrt[3]{b}dp\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e^2}-\frac{b^{2/3}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{2a^{2/3}e}+\frac{d^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^3}+\frac{3d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3}-\frac{d^2p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e^3}-\frac{d^2p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e^3}-\frac{d^2p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e^3}+\frac{\sqrt[3]{b}dp\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{a}e^2}+\frac{b^{2/3}p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{4a^{2/3}e}-\frac{d^2p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^3}-\frac{d^2p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^3}-\frac{d^2p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^3}+\frac{3d^2p\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e^3}$$

```
[Out] -d*x*ln(c*(a+b/x^3)^p)/e^2+1/2*x^2*ln(c*(a+b/x^3)^p)/e-b^(1/3)*d*p*ln(b^(1/
3)+a^(1/3)*x)/a^(1/3)/e^2-1/2*b^(2/3)*p*ln(b^(1/3)+a^(1/3)*x)/a^(2/3)/e+d^2
*ln(c*(a+b/x^3)^p)*ln(e*x+d)/e^3+3*d^2*p*ln(-e*x/d)*ln(e*x+d)/e^3-d^2*p*ln(
```

```
-e*(b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-b^(1/3)*e))*ln(e*x+d)/e^3-d^2*p*ln(-e*((-
1)^(2/3)*b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))*ln(e*x+d)/e^3
-d^2*p*ln((-1)^(1/3)*e*(b^(1/3)+(-1)^(2/3)*a^(1/3)*x)/(a^(1/3)*d+(-1)^(1/3)
*b^(1/3)*e))*ln(e*x+d)/e^3+1/2*b^(1/3)*d*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(
2/3)*x^2)/a^(1/3)/e^2+1/4*b^(2/3)*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^
2)/a^(2/3)/e-d^2*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-b^(1/3)*e))/e^3-d^2
*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))/e^3-d^2*p*po
lylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))/e^3+3*d^2*p*polyl
og(2,1+e*x/d)/e^3+b^(1/3)*d*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(1/3)*3^(1
/2))*3^(1/2)/a^(1/3)/e^2-1/2*b^(2/3)*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(
1/3)*3^(1/2))*3^(1/2)/a^(2/3)/e
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 666, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {2516, 2498, 269, 206, 31, 648, 631, 210, 642, 2505, 298, 2512, 266, 2463, 2441, 2352,

2440, 2438}

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex} dx = -\frac{\sqrt{3}b^{2/3}p \arctan \left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3}e} + \frac{\sqrt[3]{b}dp \log \left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{a}e^2} + \frac{b^{2/3}p \log \left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{4a^{2/3}e} - \frac{b^{2/3}p \log \left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{2a^{2/3}e} + \frac{\sqrt{3}\sqrt[3]{b}dp \arctan \left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e^2} + \frac{d^2 \log(d+ex) \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^3} - \frac{dx \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \frac{x^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e} - \frac{d^2p \operatorname{PolyLog}\left(2, \frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^3} - \frac{d^2p \operatorname{PolyLog}\left(2, \frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^3} - \frac{d^2p \operatorname{PolyLog}\left(2, \frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^3} - \frac{d^2p \log(d+ex) \log \left(-\frac{e\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^3} - \frac{d^2p \log(d+ex) \log \left(-\frac{e\left(\sqrt[3]{a}x+(-1)^{2/3}\sqrt[3]{b}\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^3} - \frac{d^2p \log(d+ex) \log \left(\frac{\sqrt[3]{-1}e\left((-1)^{2/3}\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^3} - \frac{\sqrt[3]{b}dp \log \left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}e^2} + \frac{3d^2p \operatorname{PolyLog}\left(2, \frac{ex}{d}+1\right)}{e^3} + \frac{3d^2p \log \left(-\frac{ex}{d}\right) \log(d+ex)}{e^3}$$

```
[In] Int[(x^2*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

```
[Out] (Sqrt[3]*b^(1/3)*d*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(a^
(1/3)*e^2) - (Sqrt[3]*b^(2/3)*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(
1/3))])/(2*a^(2/3)*e) - (d*x*Log[c*(a + b/x^3)^p])/e^2 + (x^2*Log[c*(a + b/
x^3)^p])/(2*e) - (b^(1/3)*d*p*Log[b^(1/3) + a^(1/3)*x])/(a^(1/3)*e^2) - (b^
(2/3)*p*Log[b^(1/3) + a^(1/3)*x])/(2*a^(2/3)*e) + (d^2*Log[c*(a + b/x^3)^p]
*Log[d + e*x])/e^3 + (3*d^2*p*Log[-((e*x)/d)]*Log[d + e*x])/e^3 - (d^2*p*Lo
g[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/e^3 -
 (d^2*p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*
b^(1/3)*e))]*Log[d + e*x])/e^3 - (d^2*p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(
2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/e^3 + (b
^(1/3)*d*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(2*a^(1/3)*e^2)
+ (b^(2/3)*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(4*a^(2/3)*e)
- (d^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/e^3 - (d^
2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/e^3
 - (d^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)
])/e^3 + (3*d^2*p*PolyLog[2, 1 + (e*x)/d])/e^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e}+\frac{d^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2(d+ex)}\right)dx$$

$$= -\frac{d\int\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)dx}{e^2}+\frac{d^2\int\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}dx}{e^2}+\frac{\int x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)dx}{e}$$

$$= -\frac{dx\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^3} + \frac{(3bd^2p)\int\frac{\log(d+ex)}{\left(a+\frac{b}{x^3}\right)x^4}dx}{e^3} - \frac{(3bdp)\int\frac{1}{\left(a+\frac{b}{x^3}\right)x^3}dx}{e^2} + \frac{(3bp)\int\frac{1}{\left(a+\frac{b}{x^3}\right)x^2}dx}{2e}$$

$$= -\frac{dx\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^3} + \frac{(3bd^2p)\int\left(\frac{\log(d+ex)}{bx} - \frac{ax^2\log(d+ex)}{b(b+ax^3)}\right)dx}{e^3} - \frac{(3bdp)\int\frac{1}{b+ax^3}dx}{e^2} + \frac{(3bp)\int\frac{x}{b+ax^3}dx}{2e}$$

$$= -\frac{dx\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^3} + \frac{(3d^2p)\int\frac{\log(d+ex)}{x}dx}{e^3} - \frac{(3ad^2p)\int\frac{x^2\log(d+ex)}{b+ax^3}dx}{e^3} - \frac{\left(\sqrt[3]{b}dp\right)\int\frac{1}{\sqrt[3]{b}+\sqrt[3]{a}x}dx}{e^2} - \frac{\left(\sqrt[3]{b}dp\right)\int\frac{2\sqrt[3]{b}-\sqrt[3]{a}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}dx}{e^2} - \frac{(b^{2/3}p)\int\frac{1}{\sqrt[3]{b}+\sqrt[3]{a}x}dx}{2\sqrt[3]{a}e} + \frac{(b^{2/3}p)\int\frac{\sqrt[3]{b}+\sqrt[3]{a}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}dx}{2\sqrt[3]{a}e}$$

$$= -\frac{dx\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e} - \frac{\sqrt[3]{b}dp\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e^2} - \frac{b^{2/3}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{2a^{2/3}e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^3} + \frac{3d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3} - \frac{(3ad^2p)\int\left(\frac{\log(d+ex)}{3a^{2/3}\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)} + \frac{\log(d+ex)}{3a^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{b}+\sqrt[3]{a}x\right)} + \frac{\log(d+ex)}{3a^{2/3}\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}\right)dx}{e^3} + \frac{\left(\sqrt[3]{b}dp\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}dx}{2\sqrt[3]{a}e^2} - \frac{(3b^{2/3}dp)\int\frac{1}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}dx}{2e^2} - \frac{(3d^2p)\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}dx}{e^2} + \frac{(b^{2/3}p)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}dx}{4a^{2/3}e} + \frac{(3bp)\int\frac{1}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}dx}{4\sqrt[3]{a}e}$$

$$= -\frac{dx\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e}-\frac{\sqrt[3]{b}dp\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e^2}$$
$$-\frac{b^{2/3}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{2a^{2/3}e}+\frac{d^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^3}$$
$$+\frac{3d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3}+\frac{\sqrt[3]{b}dp\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{a}e^2}$$
$$+\frac{b^{2/3}p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{4a^{2/3}e}+\frac{3d^2p\text{Li}_2\left(1+\frac{ex}{d}\right)}{e^3}$$
$$-\frac{\left(\sqrt[3]{a}d^2p\right)\int\frac{\log(d+ex)}{\sqrt[3]{b}+\sqrt[3]{a}x}\,dx}{e^3}-\frac{\left(\sqrt[3]{a}d^2p\right)\int\frac{\log(d+ex)}{-\sqrt[3]{-1}\sqrt[3]{b}+\sqrt[3]{a}x}\,dx}{e^3}$$
$$-\frac{\left(\sqrt[3]{a}d^2p\right)\int\frac{\log(d+ex)}{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x}\,dx}{e^3}-\frac{\left(3\sqrt[3]{b}dp\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}e^2}$$
$$+\frac{\left(3b^{2/3}p\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)}{2a^{2/3}e}$$

$$
\begin{aligned}
&= \frac{\sqrt{3}\sqrt[3]{b}dp\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e^2} - \frac{\sqrt{3}b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3}e} \\
&\quad - \frac{dx\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e} - \frac{\sqrt[3]{b}dp\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e^2} \\
&\quad - \frac{b^{2/3}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{2a^{2/3}e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^3} \\
&\quad + \frac{3d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e^3} \\
&\quad - \frac{d^2p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e^3} \\
&\quad - \frac{d^2p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e^3} \\
&\quad + \frac{\sqrt[3]{b}dp\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{a}e^2} + \frac{b^{2/3}p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{4a^{2/3}e} \\
&\quad + \frac{3d^2p\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{e^3} + \frac{(d^2p)\int\frac{\log\left(\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{-\sqrt[3]{a}d+\sqrt[3]{b}e}\right)}{d+ex}dx}{e^2} \\
&\quad + \frac{(d^2p)\int\frac{\log\left(\frac{e\left(-\sqrt[3]{-1}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{-\sqrt[3]{a}d-\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d+ex}dx}{e^2} + \frac{(d^2p)\int\frac{\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{-\sqrt[3]{a}d+(-1)^{2/3}\sqrt[3]{b}e}\right)}{d+ex}dx}{e^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\sqrt{3}\sqrt[3]{b} d p \tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a} e^2} - \frac{\sqrt{3} b^{2/3} p \tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3} e} \\
&- \frac{dx \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \frac{x^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e} - \frac{\sqrt[3]{b} d p \log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a} e^2} \\
&- \frac{b^{2/3} p \log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{2a^{2/3} e} + \frac{d^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right) \log(d+ex)}{e^3} \\
&+ \frac{3d^2 p \log\left(-\frac{ex}{d}\right) \log(d+ex)}{e^3} - \frac{d^2 p \log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right) \log(d+ex)}{e^3} \\
&- \frac{d^2 p \log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right) \log(d+ex)}{e^3} \\
&- \frac{d^2 p \log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right) \log(d+ex)}{e^3} \\
&+ \frac{\sqrt[3]{b} d p \log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{a} e^2} + \frac{b^{2/3} p \log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{4a^{2/3} e} \\
&+ \frac{3d^2 p \operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{e^3} + \frac{(d^2 p) \operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[3]{a}x}{-\sqrt[3]{a}d+\sqrt[3]{b}e}\right)}{x} dx, x, d+ex\right)}{e^3} \\
&+ \frac{(d^2 p) \operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[3]{a}x}{-\sqrt[3]{a}d-\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{x} dx, x, d+ex\right)}{e^3} \\
&+ \frac{(d^2 p) \operatorname{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[3]{a}x}{-\sqrt[3]{a}d+(-1)^{2/3}\sqrt[3]{b}e}\right)}{x} dx, x, d+ex\right)}{e^3}
\end{aligned}
$$

$$
= \frac{\sqrt{3}\sqrt[3]{b}dp\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e^2} - \frac{\sqrt{3}b^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2a^{2/3}e}
$$

$$
- \frac{dx\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e} - \frac{\sqrt[3]{b}dp\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e^2}
$$

$$
- \frac{b^{2/3}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{2a^{2/3}e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^3}
$$

$$
+ \frac{3d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e^3}
$$

$$
- \frac{d^2p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e^3}
$$

$$
- \frac{d^2p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e^3}
$$

$$
+ \frac{\sqrt[3]{b}dp\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{a}e^2} + \frac{b^{2/3}p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{4a^{2/3}e}
$$

$$
- \frac{d^2p\operatorname{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^3} - \frac{d^2p\operatorname{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^3}
$$

$$
- \frac{d^2p\operatorname{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^3} + \frac{3d^2p\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{e^3}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.13 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.73

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = -\frac{3bp\,\text{Hypergeometric2F1}\left(\frac{1}{3},1,\frac{4}{3},-\frac{b}{ax^3}\right)}{2aex} + \frac{3bdp\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{b}{ax^3}\right)}{2ae^2x^2} - \frac{dx\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \frac{x^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e} + \frac{d^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^3} + \frac{3d^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e^3} - \frac{d^2p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e^3} + \frac{3d^2p\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e^3} - \frac{d^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^3} - \frac{d^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^3} - \frac{d^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^3}$$

```
[In] Integrate[(x^2*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

```
[Out] (-3*b*p*Hypergeometric2F1[1/3, 1, 4/3, -(b/(a*x^3))])/(2*a*e*x) + (3*b*d*p*
Hypergeometric2F1[2/3, 1, 5/3, -(b/(a*x^3))])/(2*a*e^2*x^2) - (d*x*Log[c*(a
 + b/x^3)^p])/e^2 + (x^2*Log[c*(a + b/x^3)^p])/(2*e) + (d^2*Log[c*(a + b/x^
3)^p]*Log[d + e*x])/e^3 + (3*d^2*p*Log[-((e*x)/d)]*Log[d + e*x])/e^3 - (d^2
*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/
e^3 - (d^2*p*Log[-(((-1)^(2/3)*e*(b^(1/3) - (-1)^(1/3)*a^(1/3)*x))/(a^(1/3)
*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/e^3 - (d^2*p*Log[((-1)^(1/3)*e*(
```

```
b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d
+ e*x])/e^3 + (3*d^2*p*PolyLog[2, (d + e*x)/d])/e^3 - (d^2*p*PolyLog[2, (a^
(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/e^3 - (d^2*p*PolyLog[2, (a^(1/3)
*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/e^3 - (d^2*p*PolyLog[2, (a
^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/e^3
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.95 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.40

method	result
parts	$\frac{x^2 \ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2e} - \frac{dx \ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} + \frac{d^2 \ln\left(c\left(a+\frac{b}{x^3}\right)^p\right) \ln(ex+d)}{e^3} + 3pb\, e^3 \left(\frac{d^2 \left(\frac{\mathrm{dilog}\left(-\frac{ex}{d}\right)+\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{b\, e^3} - \_R1 \right.}{} \right.$

```
[In] int(x^2*ln(c*(a+b/x^3)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*ln(c*(a+b/x^3)^p)/e-d*x*ln(c*(a+b/x^3)^p)/e^2+d^2*ln(c*(a+b/x^3)^p)
*ln(e*x+d)/e^3+3*p*b*e^3*(1/e^3*d^2*(1/b/e^3*(dilog(-e*x/d)+ln(e*x+d)*ln(-e
*x/d))-1/3/b/e^3*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1)
,_R1=RootOf(_Z^3*a-3*_Z^2*a*d+3*_Z*a*d^2-a*d^3+b*e^3)))+1/6/e^3/a*sum((-_R+
3*d)/(-_R^2+2*_R*d-d^2)*ln(e*x-_R+d),_R=RootOf(_Z^3*a-3*_Z^2*a*d+3*_Z*a*d^2
-a*d^3+b*e^3)))
```

Fricas [F]

$$\int \frac{x^2 \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\, dx = \int \frac{x^2 \log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\, dx$$

```
[In] integrate(x^2*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(x^2*log(c*((a*x^3 + b)/x^3)^p)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*ln(c*(a+b/x**3)**p)/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^2*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x^2*log((a + b/x^3)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \frac{x^2 \log \left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x^2*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x^2*log((a + b/x^3)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \frac{x^2 \ln \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+e\,x}\,dx$$

```
[In] int((x^2*log(c*(a + b/x^3)^p))/(d + e*x),x)
[Out] int((x^2*log(c*(a + b/x^3)^p))/(d + e*x), x)
```

3.256 $\int \frac{x \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 488

$$\int \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = -\frac{\sqrt{3}\sqrt[3]{b}p\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e} + \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} + \frac{\sqrt[3]{b}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2} - \frac{3dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e^2} - \frac{\sqrt[3]{b}p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{a}e} + \frac{dp\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^2} + \frac{dp\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^2} + \frac{dp\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^2} - \frac{3dp\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e^2}$$

```
[Out] x*ln(c*(a+b/x^3)^p)/e+b^(1/3)*p*ln(b^(1/3)+a^(1/3)*x)/a^(1/3)/e-d*ln(c*(a+b
/x^3)^p)*ln(e*x+d)/e^2-3*d*p*ln(-e*x/d)*ln(e*x+d)/e^2+d*p*ln(-e*(b^(1/3)+a^
(1/3)*x)/(a^(1/3)*d-b^(1/3)*e))*ln(e*x+d)/e^2+d*p*ln(-e*((-1)^(2/3)*b^(1/3)
+a^(1/3)*x)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))*ln(e*x+d)/e^2+d*p*ln((-1)^(1/
3)*e*(b^(1/3)+(-1)^(2/3)*a^(1/3)*x)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))*ln(e*
x+d)/e^2-1/2*b^(1/3)*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/a^(1/3)/e+
d*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-b^(1/3)*e))/e^2+d*p*polylog(2,a^(1
/3)*(e*x+d)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))/e^2+d*p*polylog(2,a^(1/3)*(e*
```

```
x+d)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))/e^2-3*d*p*polylog(2,1+e*x/d)/e^2-b^(
1/3)*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(1/3)*3^(1/2))*3^(1/2)/a^(1/3)/e
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {2516, 2498, 269, 206, 31, 648, 631, 210, 642, 2512, 266, 2463, 2441, 2352, 2440, 2438}

$$
\begin{aligned}
\int \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = & -\frac{\sqrt[3]{b}p\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{a}e} \\
& -\frac{\sqrt{3}\sqrt[3]{b}p\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e}-\frac{d\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e^2} \\
& +\frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e}+\frac{dp\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^2} \\
& +\frac{dp\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^2} \\
& +\frac{dp\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^2} \\
& +\frac{dp\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^2} \\
& +\frac{dp\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+(-1)^{2/3}\sqrt[3]{b}\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^2} \\
& +\frac{dp\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left((-1)^{2/3}\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^2} \\
& +\frac{\sqrt[3]{b}p\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}e}-\frac{3dp\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e^2} \\
& -\frac{3dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2}
\end{aligned}
$$

```
[In] Int[(x*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

```
[Out] -((Sqrt[3]*b^(1/3)*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(a^
(1/3)*e)) + (x*Log[c*(a + b/x^3)^p])/e + (b^(1/3)*p*Log[b^(1/3) + a^(1/3)*x
])/(a^(1/3)*e) - (d*Log[c*(a + b/x^3)^p]*Log[d + e*x])/e^2 - (3*d*p*Log[-((
e*x)/d)]*Log[d + e*x])/e^2 + (d*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*
d - b^(1/3)*e))]*Log[d + e*x])/e^2 + (d*p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^
(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/e^2 + (d*p*Log
[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^
(1/3)*e)]*Log[d + e*x])/e^2 - (b^(1/3)*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x +
a^(2/3)*x^2])/(2*a^(1/3)*e) + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*
d - b^(1/3)*e)])/e^2 + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1
)^(1/3)*b^(1/3)*e)])/e^2 + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d -
 (-1)^(2/3)*b^(1/3)*e)])/e^2 - (3*d*p*PolyLog[2, 1 + (e*x)/d])/e^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
```

```
e, n, p}, x]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e(d+ex)}\right)dx$$

$$= \frac{\int \log\left(c\left(a+\frac{b}{x^3}\right)^p\right)dx}{e} - \frac{d\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}dx}{e}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2} - \frac{(3bdp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^3}\right)x^4}dx}{e^2} + \frac{(3bp)\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^3}dx}{e}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2} - \frac{(3bdp)\int \left(\frac{\log(d+ex)}{bx} - \frac{ax^2\log(d+ex)}{b(b+ax^3)}\right)dx}{e^2} + \frac{(3bp)\int \frac{1}{b+ax^3}dx}{e}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2} - \frac{(3dp)\int \frac{\log(d+ex)}{x}dx}{e^2} + \frac{(3adp)\int \frac{x^2\log(d+ex)}{b+ax^3}dx}{e^2} + \frac{\left(\sqrt[3]{b}p\right)\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}x}dx}{e} + \frac{\left(\sqrt[3]{b}p\right)\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}dx}{e}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} + \frac{\sqrt[3]{b}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2} - \frac{3dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2} + \frac{(3adp)\int\left(\frac{\log(d+ex)}{3a^{2/3}\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)} + \frac{\log(d+ex)}{3a^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{b}+\sqrt[3]{a}x\right)} + \frac{\log(d+ex)}{3a^{2/3}\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}\right)dx}{e^2} - \frac{\left(\sqrt[3]{b}p\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}dx}{2\sqrt[3]{a}e} + \frac{\left(3b^{2/3}p\right)\int\frac{1}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}dx}{2e} + \frac{(3dp)\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}dx}{e}$$

$$= \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} + \frac{\sqrt[3]{b}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2} - \frac{3dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2} - \frac{\sqrt[3]{b}p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{a}e} - \frac{3dp\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{e^2} + \frac{\left(\sqrt[3]{a}dp\right)\int\frac{\log(d+ex)}{\sqrt[3]{b}+\sqrt[3]{a}x}dx}{e^2} + \frac{\left(\sqrt[3]{a}dp\right)\int\frac{\log(d+ex)}{-\sqrt[3]{-1}\sqrt[3]{b}+\sqrt[3]{a}x}dx}{e^2} + \frac{\left(\sqrt[3]{a}dp\right)\int\frac{\log(d+ex)}{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x}dx}{e^2} + \frac{\left(3\sqrt[3]{b}p\right)\mathrm{Subst}\left(\int\frac{1}{-3-x^2}dx, x, 1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}e}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e} + \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e}$$

$$+ \frac{\sqrt[3]{b}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2}$$

$$- \frac{3dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e^2}$$

$$+ \frac{dp\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e^2}$$

$$+ \frac{dp\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e^2}$$

$$- \frac{\sqrt[3]{b}p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{a}e} - \frac{3dp\text{Li}_2\left(1+\frac{ex}{d}\right)}{e^2}$$

$$- \frac{(dp)\int\frac{\log\left(\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{-\sqrt[3]{a}d+\sqrt[3]{b}e}\right)}{d+ex}dx}{e} - \frac{(dp)\int\frac{\log\left(\frac{e\left(-\sqrt[3]{-1}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{-\sqrt[3]{a}d-\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d+ex}dx}{e}$$

$$- \frac{(dp)\int\frac{\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{-\sqrt[3]{a}d+(-1)^{2/3}\sqrt[3]{b}e}\right)}{d+ex}dx}{e}$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e} + \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e}$$
$$+ \frac{\sqrt[3]{b}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2}$$
$$- \frac{3dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e^2}$$
$$+ \frac{dp\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e^2}$$
$$+ \frac{dp\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e^2}$$
$$- \frac{\sqrt[3]{b}p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{a}e} - \frac{3dp\text{Li}_2\left(1+\frac{ex}{d}\right)}{e^2}$$
$$- \frac{(dp)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{a}x}{-\sqrt[3]{a}d+\sqrt[3]{b}e}\right)}{x}dx, x, d+ex\right)}{e^2}$$
$$- \frac{(dp)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{a}x}{-\sqrt[3]{a}d-\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{x}dx, x, d+ex\right)}{e^2}$$
$$- \frac{(dp)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{a}x}{-\sqrt[3]{a}d+(-1)^{2/3}\sqrt[3]{b}e}\right)}{x}dx, x, d+ex\right)}{e^2}$$

$$
\begin{aligned}
&= -\frac{\sqrt{3}\sqrt[3]{b}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{a}e} + \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} \\
&+ \frac{\sqrt[3]{b}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2} \\
&- \frac{3dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e^2} \\
&+ \frac{dp\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e^2} \\
&+ \frac{dp\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e^2} \\
&- \frac{\sqrt[3]{b}p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{a}e} + \frac{dp\operatorname{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^2} \\
&+ \frac{dp\operatorname{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^2} + \frac{dp\operatorname{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^2} - \frac{3dp\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{e^2}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.11 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.83

$$\int \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = -\frac{3bp\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{b}{ax^3}\right)}{2aex^2}+\frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e} - \frac{d\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e^2}-\frac{3dp\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e^2} + \frac{dp\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e^2} - \frac{3dp\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e^2}+\frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e^2} + \frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e^2} + \frac{dp\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e^2}$$

```
[In] Integrate[(x*Log[c*(a + b/x^3)^p])/(d + e*x),x]
```

```
[Out] (-3*b*p*Hypergeometric2F1[2/3, 1, 5/3, -(b/(a*x^3))])/(2*a*e*x^2) + (x*Log[
c*(a + b/x^3)^p])/e - (d*Log[c*(a + b/x^3)^p]*Log[d + e*x])/e^2 - (3*d*p*Lo
g[-((e*x)/d)]*Log[d + e*x])/e^2 + (d*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(
1/3)*d - b^(1/3)*e))]*Log[d + e*x])/e^2 + (d*p*Log[-(((-1)^(2/3)*e*(b^(1/3)
 - (-1)^(1/3)*a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x]
)/e^2 + (d*p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d
 + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/e^2 - (3*d*p*PolyLog[2, (d + e*x)/d
])/e^2 + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/e^2
+ (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/
e^2 + (d*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e
)])/e^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.64 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.49

method	result
parts	$\frac{x\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e}-\frac{d\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)\ln(ex+d)}{e^2}+3pb\,e^3\left(-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^3a-3\_Z^2ad+3\_Z a\,d^2-a\,d^3+e^3b\right)}\frac{\ln\left(ex-\_R+d\right)}{-\_R^2+2\_R d-}}{3e^2a}\right.$

[In] int(x*ln(c*(a+b/x^3)^p)/(e*x+d),x,method=_RETURNVERBOSE)

[Out] x*ln(c*(a+b/x^3)^p)/e-d*ln(c*(a+b/x^3)^p)*ln(e*x+d)/e^2+3*p*b*e^3*(-1/3/e^2/a*sum(1/(-_R^2+2*_R*d-d^2)*ln(e*x-_R+d),_R=RootOf(_Z^3*a-3*_Z^2*a*d+3*_Z*a*d^2-a*d^3+b*e^3))-1/e^2*d*(1/b/e^3*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d))-1/3/b/e^3*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*a-3*_Z^2*a*d+3*_Z*a*d^2-a*d^3+b*e^3))))

Fricas [F]

$$\int \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx=\int \frac{x\log\left(\left(a+\frac{b}{x^3}\right)^pc\right)}{ex+d}\,dx$$

[In] integrate(x*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="fricas")

[Out] integral(x*log(c*((a*x^3 + b)/x^3)^p)/(e*x + d), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{x\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx=\text{Timed out}$$

[In] integrate(x*ln(c*(a+b/x**3)**p)/(e*x+d),x)

[Out] Timed out

Maxima [F]

$$\int \frac{x \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \frac{x \log \left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(x*log((a + b/x^3)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{x \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \frac{x \log \left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(x*log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(x*log((a + b/x^3)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \frac{x\,\ln \left(c\,{\left(a+\frac{b}{x^3}\right)}^p\right)}{d+e\,x}\,dx$$

```
[In] int((x*log(c*(a + b/x^3)^p))/(d + e*x),x)
[Out] int((x*log(c*(a + b/x^3)^p))/(d + e*x), x)
```

3.257 $\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 344

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e}+\frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}$$
$$-\frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e}$$
$$-\frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e}$$
$$-\frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e}$$
$$-\frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e}-\frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e}$$
$$-\frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e}+\frac{3p\operatorname{PolyLog}\left(2,1+\frac{ex}{d}\right)}{e}$$

```
[Out] ln(c*(a+b/x^3)^p)*ln(e*x+d)/e+3*p*ln(-e*x/d)*ln(e*x+d)/e-p*ln(-e*(b^(1/3)+a
^(1/3)*x)/(a^(1/3)*d-b^(1/3)*e))*ln(e*x+d)/e-p*ln(-e*((-1)^(2/3)*b^(1/3)+a^
(1/3)*x)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))*ln(e*x+d)/e-p*ln((-1)^(1/3)*e*(b
^(1/3)+(-1)^(2/3)*a^(1/3)*x)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))*ln(e*x+d)/e-
p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-b^(1/3)*e))/e-p*polylog(2,a^(1/3)*(e
```

```
*x+d)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))/e-p*polylog(2,a^(1/3)*(e*x+d)/(a^(1
/3)*d-(-1)^(2/3)*b^(1/3)*e))/e+3*p*polylog(2,1+e*x/d)/e
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2512, 266, 2463, 2441, 2352, 2440, 2438}

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx=\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{e}-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e}$$
$$-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e}$$
$$-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e}$$
$$-\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e}$$
$$-\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+(-1)^{2/3}\sqrt[3]{b}\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e}$$
$$-\frac{p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left((-1)^{2/3}\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e}$$
$$+\frac{3p\,\text{PolyLog}\left(2,\frac{ex}{d}+1\right)}{e}+\frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}$$

```
[In] Int[Log[c*(a + b/x^3)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x^3)^p]*Log[d + e*x])/e + (3*p*Log[-((e*x)/d)]*Log[d + e*x])/
e - (p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*
x])/e - (p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/
3)*b^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3
)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/e - (p*Poly
Log[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/e - (p*PolyLog[2, (a^(
1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/e - (p*PolyLog[2, (a^(
1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/e + (3*p*PolyLog[2, 1
+ (e*x)/d])/e
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rubi steps

$$\text{integral} = \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{(3bp)\int\frac{\log(d+ex)}{\left(a+\frac{b}{x^3}\right)x^4}\,dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{(3bp)\int\left(\frac{\log(d+ex)}{bx}-\frac{ax^2\log(d+ex)}{b(b+ax^3)}\right)dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{(3p)\int\frac{\log(d+ex)}{x}\,dx}{e} - \frac{(3ap)\int\frac{x^2\log(d+ex)}{b+ax^3}\,dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - (3p)\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx$$

$$- \frac{(3ap)\int\left(\frac{\log(d+ex)}{3a^{2/3}\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}+\frac{\log(d+ex)}{3a^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{b}+\sqrt[3]{a}x\right)}+\frac{\log(d+ex)}{3a^{2/3}\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}\right)dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} + \frac{3p\text{Li}_2\left(1+\frac{ex}{d}\right)}{e}$$

$$- \frac{\left(\sqrt[3]{a}p\right)\int\frac{\log(d+ex)}{\sqrt[3]{b}+\sqrt[3]{a}x}\,dx}{e} - \frac{\left(\sqrt[3]{a}p\right)\int\frac{\log(d+ex)}{-\sqrt[3]{-1}\sqrt[3]{b}+\sqrt[3]{a}x}\,dx}{e}$$

$$- \frac{\left(\sqrt[3]{a}p\right)\int\frac{\log(d+ex)}{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x}\,dx}{e}$$

$$= \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}$$

$$- \frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e}$$

$$- \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e}$$

$$+ \frac{3p\text{Li}_2\left(1+\frac{ex}{d}\right)}{e} + p\int\frac{\log\left(\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{-\sqrt[3]{a}d+\sqrt[3]{b}e}\right)}{d+ex}\,dx$$

$$+ p\int\frac{\log\left(\frac{e\left(-\sqrt[3]{-1}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{-\sqrt[3]{a}d-\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d+ex}\,dx + p\int\frac{\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{-\sqrt[3]{a}d+(-1)^{2/3}\sqrt[3]{b}e}\right)}{d+ex}\,dx$$

$$=\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e}+\frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e}-\frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e}+\frac{3p\text{Li}_2\left(1+\frac{ex}{d}\right)}{e}+\frac{p\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{a}x}{-\sqrt[3]{a}d+\sqrt[3]{b}e}\right)}{x}dx,x,d+ex\right)}{e}+\frac{p\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{a}x}{-\sqrt[3]{a}d-\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{x}dx,x,d+ex\right)}{e}+\frac{p\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{a}x}{-\sqrt[3]{a}d+(-1)^{2/3}\sqrt[3]{b}e}\right)}{x}dx,x,d+ex\right)}{e}$$

$$=\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e}+\frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e}-\frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e}-\frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e}-\frac{p\text{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e}+\frac{3p\text{Li}_2\left(1+\frac{ex}{d}\right)}{e}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.02

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{e} + \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{e} - \frac{p\log\left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{e} - \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{e} + \frac{3p\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{e} - \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{e} - \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{e} - \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{e}$$

```
[In] Integrate[Log[c*(a + b/x^3)^p]/(d + e*x),x]
```

```
[Out] (Log[c*(a + b/x^3)^p]*Log[d + e*x])/e + (3*p*Log[-((e*x)/d)]*Log[d + e*x])/
e - (p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*
x])/e - (p*Log[-(((-1)^(2/3)*e*(b^(1/3) - (-1)^(1/3)*a^(1/3)*x))/(a^(1/3)*d
 - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/e - (p*Log[((-1)^(1/3)*e*(b^(1/3)
+ (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/
e + (3*p*PolyLog[2, (d + e*x)/d])/e - (p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^
(1/3)*d - b^(1/3)*e)])/e - (p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (
-1)^(1/3)*b^(1/3)*e)])/e - (p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (
-1)^(2/3)*b^(1/3)*e)])/e
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.57 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.41

method	result
parts	$\frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)\ln(ex+d)}{e}+3pb\,e^2\left(\frac{\operatorname{dilog}\left(-\frac{ex}{d}\right)+\ln(ex+d)\ln\left(-\frac{ex}{d}\right)}{b\,e^3}-\frac{\sum_{\_R1=\operatorname{RootOf}\left(\_Z^3a-3\_Z^2ad+3\_Za\,d^2-a\,d^3+e^3b\right)}\left(\ln(ex\right.}{3b\,e^3}\right.$

```
[In] int(ln(c*(a+b/x^3)^p)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] ln(c*(a+b/x^3)^p)*ln(e*x+d)/e+3*p*b*e^2*(1/b/e^3*(dilog(-e*x/d)+ln(e*x+d)*l
n(-e*x/d))-1/3/b/e^3*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/
_R1),_R1=RootOf(_Z^3*a-3*_Z^2*a*d+3*_Z*a*d^2-a*d^3+b*e^3)))
```

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(log(c*((a*x^3 + b)/x^3)^p)/(e*x + d), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx=\text{Timed out}$$

```
[In] integrate(ln(c*(a+b/x**3)**p)/(e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((a + b/x^3)^p*c)/(e*x + d), x)
```

Giac [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{ex+d}\,dx$$

```
[In] integrate(log(c*(a+b/x^3)^p)/(e*x+d),x, algorithm="giac")
[Out] integrate(log((a + b/x^3)^p*c)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx = \int \frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+e\,x}\,dx$$

```
[In] int(log(c*(a + b/x^3)^p)/(d + e*x),x)
[Out] int(log(c*(a + b/x^3)^p)/(d + e*x), x)
```

3.258 $\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 388

$$
\begin{aligned}
\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)}\,dx &= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} \\
&\quad - \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{d} \\
&\quad + \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{d} \\
&\quad + \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{d} \\
&\quad - \frac{p\,\mathrm{PolyLog}\left(2,1+\frac{b}{ax^3}\right)}{3d} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d} \\
&\quad + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d} \\
&\quad + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d} - \frac{3p\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{d}
\end{aligned}
$$

```
[Out] -1/3*ln(c*(a+b/x^3)^p)*ln(-b/a/x^3)/d-ln(c*(a+b/x^3)^p)*ln(e*x+d)/d-3*p*ln(
-e*x/d)*ln(e*x+d)/d+p*ln(-e*(b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-b^(1/3)*e))*ln(e
```

```
*x+d)/d+p*ln(-e*((-1)^(2/3)*b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3
)*e))*ln(e*x+d)/d+p*ln((-1)^(1/3)*e*(b^(1/3)+(-1)^(2/3)*a^(1/3)*x)/(a^(1/3)
*d+(-1)^(1/3)*b^(1/3)*e))*ln(e*x+d)/d-1/3*p*polylog(2,1+b/a/x^3)/d+p*polylo
g(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-b^(1/3)*e))/d+p*polylog(2,a^(1/3)*(e*x+d)/(a
^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))/d+p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-(-
1)^(2/3)*b^(1/3)*e))/d-3*p*polylog(2,1+e*x/d)/d
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2516, 2504, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$\begin{aligned}\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)}\,dx = &-\frac{\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d}-\frac{\log\left(-\frac{b}{ax^3}\right)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3d}\\&+\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d}+\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d}\\&+\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d}\\&+\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d}\\&+\frac{p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+(-1)^{2/3}\sqrt[3]{b}\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d}\\&+\frac{p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left((-1)^{2/3}\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d}\\&-\frac{p\,\mathrm{PolyLog}\left(2,\frac{b}{ax^3}+1\right)}{3d}-\frac{3p\,\mathrm{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d}\\&-\frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d}\end{aligned}$$

```
[In] Int[Log[c*(a + b/x^3)^p]/(x*(d + e*x)),x]
```

```
[Out] -1/3*(Log[c*(a + b/x^3)^p]*Log[-(b/(a*x^3))])/d - (Log[c*(a + b/x^3)^p]*Log
[d + e*x])/d - (3*p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[-((e*(b^(1/3)
+ a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/d + (p*Log[-((e*((-1)
^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e
```

```
*x])/d + (p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d
+ (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/d - (p*PolyLog[2, 1 + b/(a*x^3)])/(3
*d) + (p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d + (p*Po
lyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/d + (p*Po
lyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/d - (3*p*
PolyLog[2, 1 + (e*x)/d])/d
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} - \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d(d+ex)}\right) dx \\
&= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx}{d} \\
&= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} - \frac{\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x, \frac{1}{x^3}\right)}{3d} - \frac{(3bp)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^3}\right)x^4}\,dx}{d} \\
&= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} \\
&\quad + \frac{(bp)\text{Subst}\left(\int \frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx, x, \frac{1}{x^3}\right)}{3d} - \frac{(3bp)\int \left(\frac{\log(d+ex)}{bx} - \frac{ax^2\log(d+ex)}{b(b+ax^3)}\right) dx}{d} \\
&= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} \\
&\quad - \frac{p\text{Li}_2\left(1+\frac{b}{ax^3}\right)}{3d} - \frac{(3p)\int \frac{\log(d+ex)}{x}\,dx}{d} + \frac{(3ap)\int \frac{x^2\log(d+ex)}{b+ax^3}\,dx}{d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} \\
&- \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} - \frac{p\mathrm{Li}_2\left(1+\frac{b}{ax^3}\right)}{3d} \\
&+ \frac{(3ap)\int\left(\frac{\log(d+ex)}{3a^{2/3}\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)} + \frac{\log(d+ex)}{3a^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{b}+\sqrt[3]{a}x\right)} + \frac{\log(d+ex)}{3a^{2/3}\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}\right)dx}{d} \\
&+ \frac{(3ep)\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}dx}{d} \\
&= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} \\
&- \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} - \frac{p\mathrm{Li}_2\left(1+\frac{b}{ax^3}\right)}{3d} \\
&- \frac{3p\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{d} + \frac{\left(\sqrt[3]{a}p\right)\int\frac{\log(d+ex)}{\sqrt[3]{b}+\sqrt[3]{a}x}dx}{d} \\
&+ \frac{\left(\sqrt[3]{a}p\right)\int\frac{\log(d+ex)}{-\sqrt[3]{-1}\sqrt[3]{b}+\sqrt[3]{a}x}dx}{d} + \frac{\left(\sqrt[3]{a}p\right)\int\frac{\log(d+ex)}{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x}dx}{d} \\
&= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} \\
&- \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{d} \\
&+ \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{d} \\
&+ \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{d} \\
&- \frac{p\mathrm{Li}_2\left(1+\frac{b}{ax^3}\right)}{3d} - \frac{3p\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{d} - \frac{(ep)\int\frac{\log\left(\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{-\sqrt[3]{a}d+\sqrt[3]{b}e}\right)}{d+ex}dx}{d} \\
&- \frac{(ep)\int\frac{\log\left(\frac{e\left(-\sqrt[3]{-1}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{-\sqrt[3]{a}d-\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d+ex}dx}{d} - \frac{(ep)\int\frac{\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{-\sqrt[3]{a}d+(-1)^{2/3}\sqrt[3]{b}e}\right)}{d+ex}dx}{d}
\end{aligned}
$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d}$$

$$- \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{d}$$

$$+ \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{d}$$

$$+ \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{d} - \frac{p\text{Li}_2\left(1+\frac{b}{ax^3}\right)}{3d}$$

$$- \frac{3p\text{Li}_2\left(1+\frac{ex}{d}\right)}{d} - \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[3]{a}x}{-\sqrt[3]{a}d+\sqrt[3]{b}e}\right)}{x}dx, x, d+ex\right)}{d}$$

$$- \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[3]{a}x}{-\sqrt[3]{a}d-\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{x}dx, x, d+ex\right)}{d}$$

$$- \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[3]{a}x}{-\sqrt[3]{a}d+(-1)^{2/3}\sqrt[3]{b}e}\right)}{x}dx, x, d+ex\right)}{d}$$

$$
\begin{aligned}
&= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d} \\
&- \frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d} + \frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{d} \\
&+ \frac{p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{d} \\
&+ \frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{d} \\
&- \frac{p\mathrm{Li}_2\left(1+\frac{b}{ax^3}\right)}{3d} + \frac{p\mathrm{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d} + \frac{p\mathrm{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d} \\
&+ \frac{p\mathrm{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d} - \frac{3p\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.02

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)}\,dx=-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d}-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d}-\frac{3p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d}+\frac{p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{d}+\frac{p\log\left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{d}+\frac{p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{d}-\frac{p\operatorname{PolyLog}\left(2,\frac{a+\frac{b}{x^3}}{a}\right)}{3d}-\frac{3p\operatorname{PolyLog}\left(2,\frac{d+ex}{d}\right)}{d}+\frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d}+\frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d}+\frac{p\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d}$$

```
[In] Integrate[Log[c*(a + b/x^3)^p]/(x*(d + e*x)),x]
```

```
[Out] -1/3*(Log[c*(a + b/x^3)^p]*Log[-(b/(a*x^3))])/d - (Log[c*(a + b/x^3)^p]*Log
[d + e*x])/d - (3*p*Log[-((e*x)/d)]*Log[d + e*x])/d + (p*Log[-((e*(b^(1/3)
+ a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/d + (p*Log[-(((-1)^(2
/3)*e*(b^(1/3) - (-1)^(1/3)*a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))
]*Log[d + e*x])/d + (p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/
(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/d - (p*PolyLog[2, (a + b/
x^3)/a])/(3*d) - (3*p*PolyLog[2, (d + e*x)/d])/d + (p*PolyLog[2, (a^(1/3)*(
d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d + (p*PolyLog[2, (a^(1/3)*(d + e*x))/(
a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/d + (p*PolyLog[2, (a^(1/3)*(d + e*x))/(
a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/d
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.55 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.57

method	result
parts	$-\frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)\ln(ex+d)}{d}+\frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)\ln(x)}{d}+3pb\left(\frac{\ln(x)^2}{2db}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(\_Z^3a+b\right)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(-\right.\right.}{3db}\right.$

[In] int(ln(c*(a+b/x^3)^p)/x/(e*x+d),x,method=_RETURNVERBOSE)

[Out] -ln(c*(a+b/x^3)^p)*ln(e*x+d)/d+ln(c*(a+b/x^3)^p)/d*ln(x)+3*p*b*(1/2/d/b*ln(x)^2-1/3/d/b*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=RootOf(_Z^3*a+b))+1/3/d/b*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*a-3*_Z^2*a*d+3*_Z*a*d^2-a*d^3+b*e^3))-1/d/b*ln(e*x+d)*ln(-e*x/d)-1/d/b*dilog(-e*x/d))

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x}\,dx$$

[In] integrate(log(c*(a+b/x^3)^p)/x/(e*x+d),x, algorithm="fricas")

[Out] integral(log(c*((a*x^3 + b)/x^3)^p)/(e*x^2 + d*x), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)}\,dx=\text{Timed out}$$

[In] integrate(ln(c*(a+b/x**3)**p)/x/(e*x+d),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x}\,dx$$

```
[In] integrate(log(c*(a+b/x^3)^p)/x/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((a + b/x^3)^p*c)/((e*x + d)*x), x)
```

Giac [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x}\,dx$$

```
[In] integrate(log(c*(a+b/x^3)^p)/x/(e*x+d),x, algorithm="giac")
[Out] integrate(log((a + b/x^3)^p*c)/((e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x(d+ex)}\,dx = \int \frac{\ln\left(c\,\left(a+\frac{b}{x^3}\right)^p\right)}{x\,\left(d+e\,x\right)}\,dx$$

```
[In] int(log(c*(a + b/x^3)^p)/(x*(d + e*x)),x)
[Out] int(log(c*(a + b/x^3)^p)/(x*(d + e*x)), x)
```

3.259 $\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 557

$$\int \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx = \frac{3p}{dx} - \frac{\sqrt{3}\sqrt[3]{a}p\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{b}d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} - \frac{\sqrt[3]{a}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{b}d} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^2} + \frac{3ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{d^2} + \frac{\sqrt[3]{a}p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{b}d} + \frac{ep\,\mathrm{PolyLog}\left(2,1+\frac{b}{ax^3}\right)}{3d^2} - \frac{ep\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^2} - \frac{ep\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^2} - \frac{ep\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^2} + \frac{3ep\,\mathrm{PolyLog}\left(2,1+\frac{ex}{d}\right)}{d^2}$$

```
[Out] 3*p/d/x-ln(c*(a+b/x^3)^p)/d/x+1/3*e*ln(c*(a+b/x^3)^p)*ln(-b/a/x^3)/d^2-a^(1
/3)*p*ln(b^(1/3)+a^(1/3)*x)/b^(1/3)/d+e*ln(c*(a+b/x^3)^p)*ln(e*x+d)/d^2+3*e
*p*ln(-e*x/d)*ln(e*x+d)/d^2-e*p*ln(-e*(b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-b^(1/3
)*e))*ln(e*x+d)/d^2-e*p*ln(-e*((-1)^(2/3)*b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-(-1
)^(2/3)*b^(1/3)*e))*ln(e*x+d)/d^2-e*p*ln((-1)^(1/3)*e*(b^(1/3)+(-1)^(2/3)*a
^(1/3)*x)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))*ln(e*x+d)/d^2+1/2*a^(1/3)*p*ln(
b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/b^(1/3)/d+1/3*e*p*polylog(2,1+b/a/x^
3)/d^2-e*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-b^(1/3)*e))/d^2-e*p*polylog
(2,a^(1/3)*(e*x+d)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*e))/d^2-e*p*polylog(2,a^(1
/3)*(e*x+d)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))/d^2+3*e*p*polylog(2,1+e*x/d)/
d^2-a^(1/3)*p*arctan(1/3*(b^(1/3)-2*a^(1/3)*x)/b^(1/3)*3^(1/2))*3^(1/2)/b^(
1/3)/d
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.783, Rules used = {2516, 2505, 269, 331, 298, 31, 648, 631, 210, 642, 2504, 2441, 2352, 2512, 266, 2463, 2440, 2438}

$$\begin{aligned}\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx &= \frac{\sqrt[3]{a}p\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{b}d}-\frac{\sqrt{3}\sqrt[3]{a}p\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{b}d}\\
&\quad+\frac{e\log\left(-\frac{b}{ax^3}\right)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3d^2}+\frac{e\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2}\\
&\quad-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx}+\frac{ep\operatorname{PolyLog}\left(2,\frac{b}{ax^3}+1\right)}{3d^2}\\
&\quad-\frac{ep\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^2}-\frac{ep\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^2}\\
&\quad-\frac{ep\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^2}\\
&\quad-\frac{ep\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^2}\\
&\quad-\frac{ep\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+(-1)^{2/3}\sqrt[3]{b}\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^2}\\
&\quad-\frac{ep\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left((-1)^{2/3}\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^2}\\
&\quad-\frac{\sqrt[3]{a}p\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{b}d}+\frac{3ep\operatorname{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^2}\\
&\quad+\frac{3ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2}+\frac{3p}{dx}\end{aligned}$$

```
[In] Int[Log[c*(a + b/x^3)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (3*p)/(d*x) - (Sqrt[3]*a^(1/3)*p*ArcTan[(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^
(1/3))])/(b^(1/3)*d) - Log[c*(a + b/x^3)^p]/(d*x) + (e*Log[c*(a + b/x^3)^p]
*Log[-(b/(a*x^3))])/(3*d^2) - (a^(1/3)*p*Log[b^(1/3) + a^(1/3)*x])/(b^(1/3)
*d) + (e*Log[c*(a + b/x^3)^p]*Log[d + e*x])/d^2 + (3*e*p*Log[-((e*x)/d)]*Lo
```

```
g[d + e*x])/d^2 - (e*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)
*e))]*Log[d + e*x])/d^2 - (e*p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(
a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/d^2 - (e*p*Log[((-1)^(1/3
)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*L
og[d + e*x])/d^2 + (a^(1/3)*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2
])/(2*b^(1/3)*d) + (e*p*PolyLog[2, 1 + b/(a*x^3)])/(3*d^2) - (e*p*PolyLog[2
, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (a^(1
/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (
a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/d^2 + (3*e*p*PolyLo
g[2, 1 + (e*x)/d])/d^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx^2} - \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} + \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2(d+ex)}\right) dx$$

$$= \frac{\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2}\,dx}{d} - \frac{e\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x}\,dx}{d^2} + \frac{e^2\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}\,dx}{d^2}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^2} + \frac{e\text{Subst}\left(\int \frac{\log(c(a+bx)^p)}{x}\,dx, x, \frac{1}{x^3}\right)}{3d^2} - \frac{(3bp)\int \frac{1}{\left(a+\frac{b}{x^3}\right)x^5}\,dx}{d} + \frac{(3bep)\int \frac{\log(d+ex)}{\left(a+\frac{b}{x^3}\right)x^4}\,dx}{d^2}$$

$$= -\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^2} - \frac{(3bp)\int\frac{1}{x^2(b+ax^3)}\,dx}{d} - \frac{(bep)\text{Subst}\left(\int\frac{\log\left(-\frac{bx}{a}\right)}{a+bx}\,dx, x, \frac{1}{x^3}\right)}{3d^2} + \frac{(3bep)\int\left(\frac{\log(d+ex)}{bx} - \frac{ax^2\log(d+ex)}{b(b+ax^3)}\right)dx}{d^2}$$

$$= \frac{3p}{dx} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^2} + \frac{ep\text{Li}_2\left(1+\frac{b}{ax^3}\right)}{3d^2} + \frac{(3ap)\int\frac{x}{b+ax^3}\,dx}{d} + \frac{(3ep)\int\frac{\log(d+ex)}{x}\,dx}{d^2} - \frac{(3aep)\int\frac{x^2\log(d+ex)}{b+ax^3}\,dx}{d^2}$$

$$= \frac{3p}{dx} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^2} + \frac{3ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2} + \frac{ep\text{Li}_2\left(1+\frac{b}{ax^3}\right)}{3d^2} - \frac{(a^{2/3}p)\int\frac{1}{\sqrt[3]{b}+\sqrt[3]{a}x}\,dx}{\sqrt[3]{b}d} + \frac{(a^{2/3}p)\int\frac{\sqrt[3]{b}+\sqrt[3]{a}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx}{\sqrt[3]{b}d} - \frac{(3aep)\int\left(\frac{\log(d+ex)}{3a^{2/3}\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)} + \frac{\log(d+ex)}{3a^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{b}+\sqrt[3]{a}x\right)} + \frac{\log(d+ex)}{3a^{2/3}\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}\right)dx}{d^2} - \frac{(3e^2p)\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{d^2}$$

$$= \frac{3p}{dx} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} - \frac{\sqrt[3]{a}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{b}d} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^2} + \frac{3ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2} + \frac{ep\text{Li}_2\left(1+\frac{b}{ax^3}\right)}{3d^2} + \frac{3ep\text{Li}_2\left(1+\frac{ex}{d}\right)}{d^2} + \frac{(3a^{2/3}p)\int\frac{1}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx}{2d} + \frac{(\sqrt[3]{a}p)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx}{2\sqrt[3]{b}d} - \frac{(\sqrt[3]{a}ep)\int\frac{\log(d+ex)}{\sqrt[3]{b}+\sqrt[3]{a}x}\,dx}{d^2} - \frac{(\sqrt[3]{a}ep)\int\frac{\log(d+ex)}{-\sqrt[3]{-1}\sqrt[3]{b}+\sqrt[3]{a}x}\,dx}{d^2} - \frac{(\sqrt[3]{a}ep)\int\frac{\log(d+ex)}{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x}\,dx}{d^2}$$

$$
\begin{aligned}
&= \frac{3p}{dx} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} \\
&\quad - \frac{\sqrt[3]{a}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{b}d} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^2} \\
&\quad + \frac{3ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{d^2} \\
&\quad - \frac{ep\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{d^2} \\
&\quad - \frac{ep\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{d^2} \\
&\quad + \frac{\sqrt[3]{a}p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{b}d} + \frac{ep\text{Li}_2\left(1+\frac{b}{ax^3}\right)}{3d^2} + \frac{3ep\text{Li}_2\left(1+\frac{ex}{d}\right)}{d^2} \\
&\quad + \frac{\left(3\sqrt[3]{a}p\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)}{\sqrt[3]{b}d} + \frac{(e^2p)\int\frac{\log\left(\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{-\sqrt[3]{a}d+\sqrt[3]{b}e}\right)}{d+ex}\,dx}{d^2} \\
&\quad + \frac{(e^2p)\int\frac{\log\left(\frac{e\left(-\sqrt[3]{-1}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{-\sqrt[3]{a}d-\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d+ex}\,dx}{d^2} + \frac{(e^2p)\int\frac{\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{-\sqrt[3]{a}d+(-1)^{2/3}\sqrt[3]{b}e}\right)}{d+ex}\,dx}{d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3p}{dx} - \frac{\sqrt{3}\sqrt[3]{a}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{b}d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} \\
&+ \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} - \frac{\sqrt[3]{a}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{b}d} \\
&+ \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^2} + \frac{3ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2} \\
&- \frac{ep\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{d^2} \\
&- \frac{ep\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{d^2} \\
&- \frac{ep\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{d^2} \\
&+ \frac{\sqrt[3]{a}p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{b}d} + \frac{ep\text{Li}_2\left(1+\frac{b}{ax^3}\right)}{3d^2} \\
&+ \frac{3ep\text{Li}_2\left(1+\frac{ex}{d}\right)}{d^2} + \frac{(ep)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{a}x}{-\sqrt[3]{a}d+\sqrt[3]{b}e}\right)}{x}dx,x,d+ex\right)}{d^2} \\
&+ \frac{(ep)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{a}x}{-\sqrt[3]{a}d-\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{x}dx,x,d+ex\right)}{d^2} \\
&+ \frac{(ep)\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{a}x}{-\sqrt[3]{a}d+(-1)^{2/3}\sqrt[3]{b}e}\right)}{x}dx,x,d+ex\right)}{d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3p}{dx} - \frac{\sqrt{3}\sqrt[3]{a}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{b}d} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} \\
&+ \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} - \frac{\sqrt[3]{a}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{b}d} \\
&+ \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^2} + \frac{3ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2} \\
&- \frac{ep\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{d^2} \\
&- \frac{ep\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{d^2} \\
&- \frac{ep\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{d^2} \\
&+ \frac{\sqrt[3]{a}p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{b}d} + \frac{ep\mathrm{Li}_2\left(1+\frac{b}{ax^3}\right)}{3d^2} - \frac{ep\mathrm{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^2} \\
&- \frac{ep\mathrm{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^2} - \frac{ep\mathrm{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^2} + \frac{3ep\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{d^2}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 463, normalized size of antiderivative = 0.83

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx = \frac{3bp\,\text{Hypergeometric2F1}\left(1,\frac{4}{3},\frac{7}{3},-\frac{b}{ax^3}\right)}{4adx^4} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^2} + \frac{3ep\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{d^2} - \frac{ep\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{d^2} + \frac{ep\,\text{PolyLog}\left(2,\frac{a+\frac{b}{x^3}}{a}\right)}{3d^2} + \frac{3ep\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{d^2} - \frac{ep\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^2} - \frac{ep\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^2} - \frac{ep\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^2}$$

```
[In] Integrate[Log[c*(a + b/x^3)^p]/(x^2*(d + e*x)),x]
```

```
[Out] (3*b*p*Hypergeometric2F1[1, 4/3, 7/3, -(b/(a*x^3))])/(4*a*d*x^4) - Log[c*(a
 + b/x^3)^p]/(d*x) + (e*Log[c*(a + b/x^3)^p]*Log[-(b/(a*x^3))])/(3*d^2) + (
e*Log[c*(a + b/x^3)^p]*Log[d + e*x])/d^2 + (3*e*p*Log[-((e*x)/d)]*Log[d + e
*x])/d^2 - (e*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*L
og[d + e*x])/d^2 - (e*p*Log[-(((-1)^(2/3)*e*(b^(1/3) - (-1)^(1/3)*a^(1/3)*x
))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/d^2 - (e*p*Log[((-1)^
(1/3)*e*(b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e
)]*Log[d + e*x])/d^2 + (e*p*PolyLog[2, (a + b/x^3)/a])/(3*d^2) + (3*e*p*Pol
yLog[2, (d + e*x)/d])/d^2 - (e*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d
- b^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^
(1/3)*b^(1/3)*e)])/d^2 - (e*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (
-1)^(2/3)*b^(1/3)*e)])/d^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.97 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.62

method	result
parts	$\frac{e\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)\ln(ex+d)}{d^2}-\frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx}-\frac{\ln\left(c\left(a+\frac{b}{x^3}\right)^p\right)e\ln(x)}{d^2}+3pb\left(\frac{e\left(-\frac{\sum_{\_R1=\mathrm{RootOf}\left(\_Z^3a-3\_Z^2ad+3\_Za\,d^2-\right.}}{}\right.}{}\right.$

[In] int(ln(c*(a+b/x^3)^p)/x^2/(e*x+d),x,method=_RETURNVERBOSE)

[Out] e*ln(c*(a+b/x^3)^p)*ln(e*x+d)/d^2-ln(c*(a+b/x^3)^p)/d/x-ln(c*(a+b/x^3)^p)*e/d^2*ln(x)+3*p*b*(e/d^2*(-1/3/b*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*a-3*_Z^2*a*d+3*_Z*a*d^2-a*d^3+b*e^3))+1/b*(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d)))+1/d/b/x-1/3/d/b/(1/a*b)^(1/3)*ln(x+(1/a*b)^(1/3))+1/6/d/b/(1/a*b)^(1/3)*ln(x^2-(1/a*b)^(1/3)*x+(1/a*b)^(2/3))+1/3/d/b*3^(1/2)/(1/a*b)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*x-1))-1/2*e/d^2/b*ln(x)^2+1/3*e/d^2/b*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=RootOf(_Z^3*a+b)))

Fricas [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx=\int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x^2}\,dx$$

[In] integrate(log(c*(a+b/x^3)^p)/x^2/(e*x+d),x, algorithm="fricas")

[Out] integral(log(c*((a*x^3 + b)/x^3)^p)/(e*x^3 + d*x^2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(a+b/x**3)**p)/x**2/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x^2}\,dx$$

```
[In] integrate(log(c*(a+b/x^3)^p)/x^2/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((a + b/x^3)^p*c)/((e*x + d)*x^2), x)
```

Giac [F]

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\log\left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x^2}\,dx$$

```
[In] integrate(log(c*(a+b/x^3)^p)/x^2/(e*x+d),x, algorithm="giac")
[Out] integrate(log((a + b/x^3)^p*c)/((e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2(d+ex)}\,dx = \int \frac{\ln\left(c\,\left(a+\frac{b}{x^3}\right)^p\right)}{x^2\,\left(d+e\,x\right)}\,dx$$

```
[In] int(log(c*(a + b/x^3)^p)/(x^2*(d + e*x)),x)
[Out] int(log(c*(a + b/x^3)^p)/(x^2*(d + e*x)), x)
```

3.260 $\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 737

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx = \frac{3p}{4dx^2}-\frac{3ep}{d^2x}-\frac{\sqrt{3}a^{2/3}p\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2b^{2/3}d}$$
$$+\frac{\sqrt{3}\sqrt[3]{a}ep\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{b}d^2}-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2}$$
$$+\frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x}-\frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^3}$$
$$+\frac{a^{2/3}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{2b^{2/3}d}+\frac{\sqrt[3]{a}ep\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{b}d^2}$$
$$-\frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^3}-\frac{3e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3}$$
$$+\frac{e^2p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{d^3}$$
$$+\frac{e^2p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{d^3}$$
$$+\frac{e^2p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{d^3}$$
$$-\frac{a^{2/3}p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{4b^{2/3}d}$$
$$-\frac{\sqrt[3]{a}ep\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{b}d^2}$$
$$-\frac{e^2p\,\text{PolyLog}\left(2,1+\frac{b}{ax^3}\right)}{3d^3}+\frac{e^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^3}$$
$$+\frac{e^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^3}$$
$$+\frac{e^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^3}-\frac{3e^2p\,\text{PolyLog}\left(2,1+\frac{ex}{d}\right)}{d^3}$$

```
[Out] 3/4*p/d/x^2-3*e*p/d^2/x-1/2*ln(c*(a+b/x^3)^p)/d/x^2+e*ln(c*(a+b/x^3)^p)/d^2
/x-1/3*e^2*ln(c*(a+b/x^3)^p)*ln(-b/a/x^3)/d^3+1/2*a^(2/3)*p*ln(b^(1/3)+a^(1
/3)*x)/b^(2/3)/d+a^(1/3)*e*p*ln(b^(1/3)+a^(1/3)*x)/b^(1/3)/d^2-e^2*ln(c*(a+
b/x^3)^p)*ln(e*x+d)/d^3-3*e^2*p*ln(-e*x/d)*ln(e*x+d)/d^3+e^2*p*ln(-e*(b^(1/
3)+a^(1/3)*x)/(a^(1/3)*d-b^(1/3)*e))*ln(e*x+d)/d^3+e^2*p*ln(-e*((-1)^(2/3)*
b^(1/3)+a^(1/3)*x)/(a^(1/3)*d-(-1)^(2/3)*b^(1/3)*e))*ln(e*x+d)/d^3+e^2*p*ln
((-1)^(1/3)*e*(b^(1/3)+(-1)^(2/3)*a^(1/3)*x)/(a^(1/3)*d+(-1)^(1/3)*b^(1/3)*
e))*ln(e*x+d)/d^3-1/4*a^(2/3)*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/b
^(2/3)/d-1/2*a^(1/3)*e*p*ln(b^(2/3)-a^(1/3)*b^(1/3)*x+a^(2/3)*x^2)/b^(1/3)/
d^2-1/3*e^2*p*polylog(2,1+b/a/x^3)/d^3+e^2*p*polylog(2,a^(1/3)*(e*x+d)/(a^(
1/3)*d-b^(1/3)*e))/d^3+e^2*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d+(-1)^(1/3
)*b^(1/3)*e))/d^3+e^2*p*polylog(2,a^(1/3)*(e*x+d)/(a^(1/3)*d-(-1)^(2/3)*b^(
1/3)*e))/d^3-3*e^2*p*polylog(2,1+e*x/d)/d^3-1/2*a^(2/3)*p*arctan(1/3*(b^(1/
3)-2*a^(1/3)*x)/b^(1/3)*3^(1/2))*3^(1/2)/b^(2/3)/d+a^(1/3)*e*p*arctan(1/3*(
b^(1/3)-2*a^(1/3)*x)/b^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)/d^2
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 737, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.826$, Rules used = {2516, 2505, 269, 331, 206, 31, 648, 631, 210, 642, 298, 2504, 2441, 2352, 2512, 266,

2463, 2440, 2438}

$$\int \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx = -\frac{\sqrt{3}a^{2/3}p\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2b^{2/3}d} - \frac{\sqrt[3]{a}ep\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{2\sqrt[3]{b}d^2} - \frac{a^{2/3}p\log\left(a^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{4b^{2/3}d} + \frac{a^{2/3}p\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{2b^{2/3}d} + \frac{\sqrt{3}\sqrt[3]{a}ep\arctan\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{b}d^2} - \frac{e^2\log\left(-\frac{b}{ax^3}\right)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{3d^3} - \frac{e^2\log(d+ex)\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^3} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2} - \frac{e^2p\,\mathrm{PolyLog}\left(2,\frac{b}{ax^3}+1\right)}{3d^3} + \frac{e^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^3} + \frac{e^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^3} + \frac{e^2p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^3} + \frac{e^2p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^3} + \frac{e^2p\log(d+ex)\log\left(-\frac{e\left(\sqrt[3]{a}x+(-1)^{2/3}\sqrt[3]{b}\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^3} + \frac{e^2p\log(d+ex)\log\left(\frac{\sqrt[3]{-1}e\left((-1)^{2/3}\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^3} + \frac{\sqrt[3]{a}ep\log\left(\sqrt[3]{a}x+\sqrt[3]{b}\right)}{\sqrt[3]{b}d^2} - \frac{3e^2p\,\mathrm{PolyLog}\left(2,\frac{ex}{d}+1\right)}{d^3} - \frac{3e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3} - \frac{3ep}{d^2x} + \frac{3p}{4dx^2}$$

```
[In] Int[Log[c*(a + b/x^3)^p]/(x^3*(d + e*x)),x]
```

```
[Out] (3*p)/(4*d*x^2) - (3*e*p)/(d^2*x) - (Sqrt[3]*a^(2/3)*p*ArcTan[(b^(1/3) - 2*
a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(2*b^(2/3)*d) + (Sqrt[3]*a^(1/3)*e*p*ArcTan[
(b^(1/3) - 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))])/(b^(1/3)*d^2) - Log[c*(a + b/x^
3)^p]/(2*d*x^2) + (e*Log[c*(a + b/x^3)^p])/(d^2*x) - (e^2*Log[c*(a + b/x^3)
^p]*Log[-(b/(a*x^3))])/(3*d^3) + (a^(2/3)*p*Log[b^(1/3) + a^(1/3)*x])/(2*b^
(2/3)*d) + (a^(1/3)*e*p*Log[b^(1/3) + a^(1/3)*x])/(b^(1/3)*d^2) - (e^2*Log[
c*(a + b/x^3)^p]*Log[d + e*x])/d^3 - (3*e^2*p*Log[-((e*x)/d)]*Log[d + e*x])
/d^3 + (e^2*p*Log[-((e*(b^(1/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log
[d + e*x])/d^3 + (e^2*p*Log[-((e*((-1)^(2/3)*b^(1/3) + a^(1/3)*x))/(a^(1/3)
*d - (-1)^(2/3)*b^(1/3)*e))]*Log[d + e*x])/d^3 + (e^2*p*Log[((-1)^(1/3)*e*(
b^(1/3) + (-1)^(2/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d
+ e*x])/d^3 - (a^(2/3)*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(4
*b^(2/3)*d) - (a^(1/3)*e*p*Log[b^(2/3) - a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/
(2*b^(1/3)*d^2) - (e^2*p*PolyLog[2, 1 + b/(a*x^3)])/(3*d^3) + (e^2*p*PolyLo
g[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d^3 + (e^2*p*PolyLog[2,
(a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])/d^3 + (e^2*p*PolyL
og[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3)*e)])/d^3 - (3*e^2
*p*PolyLog[2, 1 + (e*x)/d])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
```

```
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2516

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log
```

```
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{dx^3}-\frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x^2}+\frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^3x}\right.\\
&\qquad\left.-\frac{e^3\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^3(d+ex)}\right)dx\\
&=\frac{\int\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3}dx}{d}-\frac{e\int\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^2}dx}{d^2}+\frac{e^2\int\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x}dx}{d^3}-\frac{e^3\int\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d+ex}dx}{d^3}\\
&=-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2}+\frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x}-\frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^3}\\
&\quad-\frac{e^2\text{Subst}\left(\int\frac{\log(c(a+bx)^p)}{x}dx,x,\frac{1}{x^3}\right)}{3d^3}-\frac{(3bp)\int\frac{1}{\left(a+\frac{b}{x^3}\right)x^6}dx}{2d}\\
&\quad+\frac{(3bep)\int\frac{1}{\left(a+\frac{b}{x^3}\right)x^5}dx}{d^2}-\frac{(3be^2p)\int\frac{\log(d+ex)}{\left(a+\frac{b}{x^3}\right)x^4}dx}{d^3}\\
&=-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2}+\frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x}-\frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^3}\\
&\quad-\frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^3}-\frac{(3bp)\int\frac{1}{x^3(b+ax^3)}dx}{2d}+\frac{(3bep)\int\frac{1}{x^2(b+ax^3)}dx}{d^2}\\
&\quad+\frac{(be^2p)\text{Subst}\left(\int\frac{\log\left(-\frac{bx}{a}\right)}{a+bx}dx,x,\frac{1}{x^3}\right)}{3d^3}-\frac{(3be^2p)\int\left(\frac{\log(d+ex)}{bx}-\frac{ax^2\log(d+ex)}{b(b+ax^3)}\right)dx}{d^3}\\
&=\frac{3p}{4dx^2}-\frac{3ep}{d^2x}-\frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2}+\frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x}-\frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^3}\\
&\quad-\frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^3}-\frac{e^2p\text{Li}_2\left(1+\frac{b}{ax^3}\right)}{3d^3}+\frac{(3ap)\int\frac{1}{b+ax^3}dx}{2d}\\
&\quad-\frac{(3aep)\int\frac{x}{b+ax^3}dx}{d^2}-\frac{(3e^2p)\int\frac{\log(d+ex)}{x}dx}{d^3}+\frac{(3ae^2p)\int\frac{x^2\log(d+ex)}{b+ax^3}dx}{d^3}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{3p}{4dx^2} - \frac{3ep}{d^2x} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} - \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^3} \\
&\quad - \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^3} - \frac{3e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3} \\
&\quad - \frac{e^2p\mathrm{Li}_2\left(1+\frac{b}{ax^3}\right)}{3d^3} + \frac{(ap)\int\frac{1}{\sqrt[3]{b}+\sqrt[3]{a}x}\,dx}{2b^{2/3}d} + \frac{(ap)\int\frac{2\sqrt[3]{b}-\sqrt[3]{a}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx}{2b^{2/3}d} \\
&\quad + \frac{\left(a^{2/3}ep\right)\int\frac{1}{\sqrt[3]{b}+\sqrt[3]{a}x}\,dx}{\sqrt[3]{b}d^2} - \frac{\left(a^{2/3}ep\right)\int\frac{\sqrt[3]{b}+\sqrt[3]{a}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx}{\sqrt[3]{b}d^2} \\
&\quad + \frac{(3ae^2p)\int\left(\frac{\log(d+ex)}{3a^{2/3}\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}+\frac{\log(d+ex)}{3a^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{b}+\sqrt[3]{a}x\right)}+\frac{\log(d+ex)}{3a^{2/3}\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}\right)dx}{d^3} \\
&\quad + \frac{(3e^3p)\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx}{d^3} \\
&= \frac{3p}{4dx^2} - \frac{3ep}{d^2x} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} - \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^3} \\
&\quad + \frac{a^{2/3}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{2b^{2/3}d} + \frac{\sqrt[3]{a}ep\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{b}d^2} \\
&\quad - \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^3} - \frac{3e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3} \\
&\quad - \frac{e^2p\mathrm{Li}_2\left(1+\frac{b}{ax^3}\right)}{3d^3} - \frac{3e^2p\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{d^3} - \frac{\left(a^{2/3}p\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx}{4b^{2/3}d} \\
&\quad + \frac{(3ap)\int\frac{1}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx}{4\sqrt[3]{b}d} - \frac{\left(3a^{2/3}ep\right)\int\frac{1}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx}{2d^2} \\
&\quad - \frac{\left(\sqrt[3]{a}ep\right)\int\frac{-\sqrt[3]{a}\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx}{2\sqrt[3]{b}d^2} + \frac{\left(\sqrt[3]{a}e^2p\right)\int\frac{\log(d+ex)}{\sqrt[3]{b}+\sqrt[3]{a}x}\,dx}{d^3} \\
&\quad + \frac{\left(\sqrt[3]{a}e^2p\right)\int\frac{\log(d+ex)}{-\sqrt[3]{-1}\sqrt[3]{b}+\sqrt[3]{a}x}\,dx}{d^3} + \frac{\left(\sqrt[3]{a}e^2p\right)\int\frac{\log(d+ex)}{(-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x}\,dx}{d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3p}{4dx^2} - \frac{3ep}{d^2x} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} \\
&\quad - \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^3} + \frac{a^{2/3}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{2b^{2/3}d} \\
&\quad + \frac{\sqrt[3]{a}ep\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{b}d^2} - \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^3} \\
&\quad - \frac{3e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3} + \frac{e^2p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{d^3} \\
&\quad + \frac{e^2p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{d^3} \\
&\quad + \frac{e^2p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{d^3} \\
&\quad - \frac{a^{2/3}p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{4b^{2/3}d} - \frac{\sqrt[3]{a}ep\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{b}d^2} \\
&\quad - \frac{e^2p\mathrm{Li}_2\left(1+\frac{b}{ax^3}\right)}{3d^3} - \frac{3e^2p\mathrm{Li}_2\left(1+\frac{ex}{d}\right)}{d^3} + \frac{\left(3a^{2/3}p\right)\mathrm{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)}{2b^{2/3}d} \\
&\quad - \frac{\left(3\sqrt[3]{a}ep\right)\mathrm{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)}{\sqrt[3]{b}d^2} - \frac{\left(e^3p\right)\int\frac{\log\left(\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{-\sqrt[3]{a}d+\sqrt[3]{b}e}\right)}{d+ex}\,dx}{d^3} \\
&\quad - \frac{\left(e^3p\right)\int\frac{\log\left(\frac{e\left(-\sqrt[3]{-1}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{-\sqrt[3]{a}d-\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d+ex}\,dx}{d^3} - \frac{\left(e^3p\right)\int\frac{\log\left(\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{-\sqrt[3]{a}d+(-1)^{2/3}\sqrt[3]{b}e}\right)}{d+ex}\,dx}{d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3p}{4dx^2} - \frac{3ep}{d^2x} - \frac{\sqrt{3}a^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2b^{2/3}d} \\
&+ \frac{\sqrt{3}\sqrt[3]{a}ep\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{b}d^2} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} \\
&- \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^3} + \frac{a^{2/3}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{2b^{2/3}d} \\
&+ \frac{\sqrt[3]{a}ep\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{b}d^2} - \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^3} \\
&- \frac{3e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3} + \frac{e^2p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{d^3} \\
&+ \frac{e^2p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{d^3} \\
&+ \frac{e^2p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{d^3} \\
&- \frac{a^{2/3}p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{4b^{2/3}d} \\
&- \frac{\sqrt[3]{a}ep\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{b}d^2} - \frac{e^2p\operatorname{Li}_2\left(1+\frac{b}{ax^3}\right)}{3d^3} \\
&- \frac{3e^2p\operatorname{Li}_2\left(1+\frac{ex}{d}\right)}{d^3} - \frac{(e^2p)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{a}x}{-\sqrt[3]{a}d+\sqrt[3]{b}e}\right)}{x}\,dx,x,d+ex\right)}{d^3} \\
&- \frac{(e^2p)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{a}x}{-\sqrt[3]{a}d-\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{x}\,dx,x,d+ex\right)}{d^3} \\
&- \frac{(e^2p)\operatorname{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt[3]{a}x}{-\sqrt[3]{a}d+(-1)^{2/3}\sqrt[3]{b}e}\right)}{x}\,dx,x,d+ex\right)}{d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3p}{4dx^2} - \frac{3ep}{d^2x} - \frac{\sqrt{3}a^{2/3}p\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{2b^{2/3}d} \\
&+ \frac{\sqrt{3}\sqrt[3]{a}ep\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt[3]{b}d^2} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} \\
&- \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^3} + \frac{a^{2/3}p\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{2b^{2/3}d} \\
&+ \frac{\sqrt[3]{a}ep\log\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{b}d^2} - \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^3} \\
&- \frac{3e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3} + \frac{e^2p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{d^3} \\
&+ \frac{e^2p\log\left(-\frac{e\left((-1)^{2/3}\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{d^3} \\
&+ \frac{e^2p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{d^3} \\
&- \frac{a^{2/3}p\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{4b^{2/3}d} - \frac{\sqrt[3]{a}ep\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{2\sqrt[3]{b}d^2} \\
&- \frac{e^2p\text{Li}_2\left(1+\frac{b}{ax^3}\right)}{3d^3} + \frac{e^2p\text{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^3} + \frac{e^2p\text{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^3} \\
&+ \frac{e^2p\text{Li}_2\left(\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^3} - \frac{3e^2p\text{Li}_2\left(1+\frac{ex}{d}\right)}{d^3}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.13 (sec) , antiderivative size = 535, normalized size of antiderivative = 0.73

$$\int \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx = -\frac{3bep\,\text{Hypergeometric2F1}\left(1,\frac{4}{3},\frac{7}{3},-\frac{b}{ax^3}\right)}{4ad^2x^4} + \frac{3bp\,\text{Hypergeometric2F1}\left(1,\frac{5}{3},\frac{8}{3},-\frac{b}{ax^3}\right)}{10adx^5} - \frac{\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{2dx^2} + \frac{e\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)}{d^2x} - \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log\left(-\frac{b}{ax^3}\right)}{3d^3} - \frac{e^2\log\left(c\left(a+\frac{b}{x^3}\right)^p\right)\log(d+ex)}{d^3} - \frac{3e^2p\log\left(-\frac{ex}{d}\right)\log(d+ex)}{d^3} + \frac{e^2p\log\left(-\frac{e\left(\sqrt[3]{b}+\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)\log(d+ex)}{d^3} + \frac{e^2p\log\left(-\frac{(-1)^{2/3}e\left(\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)\log(d+ex)}{d^3} + \frac{e^2p\log\left(\frac{\sqrt[3]{-1}e\left(\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}x\right)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)\log(d+ex)}{d^3} - \frac{e^2p\,\text{PolyLog}\left(2,\frac{a+\frac{b}{x^3}}{a}\right)}{3d^3} - \frac{3e^2p\,\text{PolyLog}\left(2,\frac{d+ex}{d}\right)}{d^3} + \frac{e^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-\sqrt[3]{b}e}\right)}{d^3} + \frac{e^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d+\sqrt[3]{-1}\sqrt[3]{b}e}\right)}{d^3} + \frac{e^2p\,\text{PolyLog}\left(2,\frac{\sqrt[3]{a}(d+ex)}{\sqrt[3]{a}d-(-1)^{2/3}\sqrt[3]{b}e}\right)}{d^3}$$

```
[In] Integrate[Log[c*(a + b/x^3)^p]/(x^3*(d + e*x)),x]
```

```
[Out] (-3*b*e*p*Hypergeometric2F1[1, 4/3, 7/3, -(b/(a*x^3))])/(4*a*d^2*x^4) + (3*
b*p*Hypergeometric2F1[1, 5/3, 8/3, -(b/(a*x^3))])/(10*a*d*x^5) - Log[c*(a +
 b/x^3)^p]/(2*d*x^2) + (e*Log[c*(a + b/x^3)^p])/(d^2*x) - (e^2*Log[c*(a + b
/x^3)^p]*Log[-(b/(a*x^3))])/(3*d^3) - (e^2*Log[c*(a + b/x^3)^p]*Log[d + e*x
```

```
])/d^3 - (3*e^2*p*Log[-((e*x)/d)]*Log[d + e*x])/d^3 + (e^2*p*Log[-((e*(b^(1
/3) + a^(1/3)*x))/(a^(1/3)*d - b^(1/3)*e))]*Log[d + e*x])/d^3 + (e^2*p*Log[
-(((-1)^(2/3)*e*(b^(1/3) - (-1)^(1/3)*a^(1/3)*x))/(a^(1/3)*d - (-1)^(2/3)*b
^(1/3)*e))]*Log[d + e*x])/d^3 + (e^2*p*Log[((-1)^(1/3)*e*(b^(1/3) + (-1)^(2
/3)*a^(1/3)*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)]*Log[d + e*x])/d^3 - (e^
2*p*PolyLog[2, (a + b/x^3)/a])/(3*d^3) - (3*e^2*p*PolyLog[2, (d + e*x)/d])/
d^3 + (e^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - b^(1/3)*e)])/d^3 +
 (e^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d + (-1)^(1/3)*b^(1/3)*e)])
/d^3 + (e^2*p*PolyLog[2, (a^(1/3)*(d + e*x))/(a^(1/3)*d - (-1)^(2/3)*b^(1/3
)*e)])/d^3
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.66 (sec) , antiderivative size = 489, normalized size of antiderivative = 0.66

method	result
parts	$-\frac{e^{2} \ln\left(c\left(a+\frac{b}{x^{3}}\right)^{p}\right) \ln(ex+d)}{d^{3}}-\frac{\ln\left(c\left(a+\frac{b}{x^{3}}\right)^{p}\right)}{2d\,x^{2}}+\frac{\ln\left(c\left(a+\frac{b}{x^{3}}\right)^{p}\right) e^{2} \ln(x)}{d^{3}}+\frac{e \ln\left(c\left(a+\frac{b}{x^{3}}\right)^{p}\right)}{d^{2} x}+\frac{3pb\left(\frac{1}{2db\,x^{2}}-\frac{2e}{d^{2}bx}+\frac{\ln\left(x+\left(\cdots\right.\right.}{3db\left(\cdots\right.}\cdots\right.}{\cdots}$

```
[In] int(ln(c*(a+b/x^3)^p)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -e^2*ln(c*(a+b/x^3)^p)*ln(e*x+d)/d^3-1/2*ln(c*(a+b/x^3)^p)/d/x^2+ln(c*(a+b/
x^3)^p)*e^2/d^3*ln(x)+e*ln(c*(a+b/x^3)^p)/d^2/x+3/2*p*b*(1/2/d/b/x^2-2/d^2/
b*e/x+1/3/d/b/(1/a*b)^(2/3)*ln(x+(1/a*b)^(1/3))-1/6/d/b/(1/a*b)^(2/3)*ln(x^
2-(1/a*b)^(1/3)*x+(1/a*b)^(2/3))+1/3/d/b/(1/a*b)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(1/a*b)^(1/3)*x-1))+2/3/d^2/b*e/(1/a*b)^(1/3)*ln(x+(1/a*b)^(1/3))
-1/3/d^2/b*e/(1/a*b)^(1/3)*ln(x^2-(1/a*b)^(1/3)*x+(1/a*b)^(2/3))-2/3/d^2/b*
e*3^(1/2)/(1/a*b)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/a*b)^(1/3)*x-1))+e^2/d^3/b
*ln(x)^2-2/3*e^2/d^3/b*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/_R1),_R1=Roo
tOf(_Z^3*a+b))-2*e^2/d^3*(-1/3/b*sum(ln(e*x+d)*ln((-e*x+_R1-d)/_R1)+dilog((
-e*x+_R1-d)/_R1),_R1=RootOf(_Z^3*a-3*_Z^2*a*d+3*_Z*a*d^2-a*d^3+b*e^3))+1/b*
(dilog(-e*x/d)+ln(e*x+d)*ln(-e*x/d)))
```

Fricas [F]

$$\int \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx = \int \frac{\log \left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x^3}\,dx$$

```
[In] integrate(log(c*(a+b/x^3)^p)/x^3/(e*x+d),x, algorithm="fricas")
[Out] integral(log(c*((a*x^3 + b)/x^3)^p)/(e*x^4 + d*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(a+b/x**3)**p)/x**3/(e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx = \int \frac{\log \left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x^3}\,dx$$

```
[In] integrate(log(c*(a+b/x^3)^p)/x^3/(e*x+d),x, algorithm="maxima")
[Out] integrate(log((a + b/x^3)^p*c)/((e*x + d)*x^3), x)
```

Giac [F]

$$\int \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx = \int \frac{\log \left(\left(a+\frac{b}{x^3}\right)^p c\right)}{(ex+d)x^3}\,dx$$

```
[In] integrate(log(c*(a+b/x^3)^p)/x^3/(e*x+d),x, algorithm="giac")
[Out] integrate(log((a + b/x^3)^p*c)/((e*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c\left(a+\frac{b}{x^3}\right)^p\right)}{x^3(d+ex)}\,dx=\int \frac{\ln \left(c\,{\left(a+\frac{b}{x^3}\right)}^p\right)}{x^3\,\left(d+e\,x\right)} \,d x$$

```
[In] int(log(c*(a + b/x^3)^p)/(x^3*(d + e*x)),x)
[Out] int(log(c*(a + b/x^3)^p)/(x^3*(d + e*x)), x)
```

3.261 $\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{f+gx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 749

$$\int \frac{\log\left(c(d+ex^3)^p\right)}{f+gx^2}\,dx = \frac{3p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(i\sqrt[3]{e}\sqrt{f}+\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2i\sqrt{f}\sqrt{g}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt[3]{e}\sqrt{f}+\sqrt[6]{-1}\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2(-1)^{5/6}\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\left(\sqrt[3]{e}\sqrt{f}+(-1)^{5/6}\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} + \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^3)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{3ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2\sqrt{f}\sqrt{g}} + \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(i\sqrt[3]{e}\sqrt{f}+\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{f}\sqrt{g}} + \frac{ip\,\mathrm{PolyLog}\left(2,1+\frac{2i\sqrt{f}\sqrt{g}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt[3]{e}\sqrt{f}+\sqrt[6]{-1}\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{f}\sqrt{g}} + \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2(-1)^{5/6}\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\left(\sqrt[3]{e}\sqrt{f}+(-1)^{5/6}\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{f}\sqrt{g}}$$

```
[Out] arctan(x*g^(1/2)/f^(1/2))*ln(c*(e*x^3+d)^p)/f^(1/2)/g^(1/2)+3*p*arctan(x*g^
(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)-p*arctan
(x*g^(1/2)/f^(1/2))*ln(2*(d^(1/3)+e^(1/3)*x)*f^(1/2)*g^(1/2)/(I*e^(1/3)*f^(
1/2)+d^(1/3)*g^(1/2))/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)-p*arctan(x*g^(
1/2)/f^(1/2))*ln(-2*I*((-1)^(2/3)*d^(1/3)+e^(1/3)*x)*f^(1/2)*g^(1/2)/(e^(1/
3)*f^(1/2)+(-1)^(1/6)*d^(1/3)*g^(1/2))/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/
2)-p*arctan(x*g^(1/2)/f^(1/2))*ln(2*(-1)^(5/6)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*
x)*f^(1/2)*g^(1/2)/(e^(1/3)*f^(1/2)+(-1)^(5/6)*d^(1/3)*g^(1/2))/(f^(1/2)-I*
```

```
x*g^(1/2)))/f^(1/2)/g^(1/2)-3/2*I*p*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1
/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,1-2*(d^(1/3)+e^(1/3)*x)*f^(1/2)*g^(
1/2)/(I*e^(1/3)*f^(1/2)+d^(1/3)*g^(1/2))/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(
1/2)+1/2*I*p*polylog(2,1+2*I*((-1)^(2/3)*d^(1/3)+e^(1/3)*x)*f^(1/2)*g^(1/2)
/(e^(1/3)*f^(1/2)+(-1)^(1/6)*d^(1/3)*g^(1/2))/(f^(1/2)-I*x*g^(1/2)))/f^(1/2
)/g^(1/2)+1/2*I*p*polylog(2,1-2*(-1)^(5/6)*(d^(1/3)+(-1)^(2/3)*e^(1/3)*x)*f
^(1/2)*g^(1/2)/(e^(1/3)*f^(1/2)+(-1)^(5/6)*d^(1/3)*g^(1/2))/(f^(1/2)-I*x*g^
(1/2)))/f^(1/2)/g^(1/2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 749, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used

$= \{211, 2520, 12, 266, 6857, 4966, 2449, 2352, 2497\}$

$$\int \frac{\log \left(c(d+ex^3)^p\right)}{f+gx^2}\,dx = \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^3)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{(\sqrt{f}-i\sqrt{g}x)\left(\sqrt[3]{d}\sqrt{g}+i\sqrt[3]{e}\sqrt{f}\right)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2i\sqrt{f}\sqrt{g}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}{(\sqrt{f}-i\sqrt{g}x)\left(\sqrt[6]{-1}\sqrt[3]{d}\sqrt{g}+\sqrt[3]{e}\sqrt{f}\right)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2(-1)^{5/6}\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{(\sqrt{f}-i\sqrt{g}x)\left((-1)^{5/6}\sqrt[3]{d}\sqrt{g}+\sqrt[3]{e}\sqrt{f}\right)}\right)}{\sqrt{f}\sqrt{g}} + \frac{3p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} + \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(i\sqrt[3]{e}\sqrt{f}+\sqrt[3]{d}\sqrt{g}\right)(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}} + \frac{ip\,\mathrm{PolyLog}\left(2,\frac{2i\sqrt{f}\sqrt{g}\left(\sqrt[3]{e}x+(-1)^{2/3}\sqrt[3]{d}\right)}{\left(\sqrt[3]{e}\sqrt{f}+\sqrt[6]{-1}\sqrt[3]{d}\sqrt{g}\right)(\sqrt{f}-i\sqrt{g}x)}+1\right)}{2\sqrt{f}\sqrt{g}} + \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2(-1)^{5/6}\sqrt{f}\sqrt{g}\left((-1)^{2/3}\sqrt[3]{e}x+\sqrt[3]{d}\right)}{\left(\sqrt[3]{e}\sqrt{f}+(-1)^{5/6}\sqrt[3]{d}\sqrt{g}\right)(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}} - \frac{3ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2\sqrt{f}\sqrt{g}}$$

```
[In] Int[Log[c*(d + e*x^3)^p]/(f + g*x^2),x]
```

```
[Out] (3*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/
(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(
d^(1/3) + e^(1/3)*x))/((I*e^(1/3)*Sqrt[f] + d^(1/3)*Sqrt[g])*(Sqrt[f] - I*S
qrt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[((-2*I)
*Sqrt[f]*Sqrt[g]*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((e^(1/3)*Sqrt[f] + (-1)
```

```
^(1/6)*d^(1/3)*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*A
rcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*(-1)^(5/6)*Sqrt[f]*Sqrt[g]*(d^(1/3) + (-1
)^(2/3)*e^(1/3)*x))/((e^(1/3)*Sqrt[f] + (-1)^(5/6)*d^(1/3)*Sqrt[g])*(Sqrt[f
] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*
(d + e*x^3)^p])/(Sqrt[f]*Sqrt[g]) - (((3*I)/2)*p*PolyLog[2, 1 - (2*Sqrt[f])
/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 - (2*S
qrt[f]*Sqrt[g]*(d^(1/3) + e^(1/3)*x))/((I*e^(1/3)*Sqrt[f] + d^(1/3)*Sqrt[g]
)*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + ((
2*I)*Sqrt[f]*Sqrt[g]*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/((e^(1/3)*Sqrt[f] +
(-1)^(1/6)*d^(1/3)*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) +
((I/2)*p*PolyLog[2, 1 - (2*(-1)^(5/6)*Sqrt[f]*Sqrt[g]*(d^(1/3) + (-1)^(2/3)
*e^(1/3)*x))/((e^(1/3)*Sqrt[f] + (-1)^(5/6)*d^(1/3)*Sqrt[g])*(Sqrt[f] - I*S
qrt[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^3)^p\right)}{\sqrt{f}\sqrt{g}} - (3ep)\int \frac{x^2\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\,(d+ex^3)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^3)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{(3ep)\int \frac{x^2\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{d+ex^3}\,dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^3)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{(3ep)\int \left(\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{3e^{2/3}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{3e^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{3e^{2/3}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}\right)dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^3)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{\left(\sqrt[3]{e}p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{\sqrt{f}\sqrt{g}} - \frac{\left(\sqrt[3]{e}p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{\sqrt{f}\sqrt{g}} - \frac{\left(\sqrt[3]{e}p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{(-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{3p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(i\sqrt[3]{e}\sqrt{f}+\sqrt[3]{d}\sqrt{g}\right)(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}}$$

$$- \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2i\sqrt{f}\sqrt{g}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt[3]{e}\sqrt{f}+\sqrt[6]{-1}\sqrt[3]{d}\sqrt{g}\right)(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}}$$

$$- \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2(-1)^{5/6}\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\left(\sqrt[3]{e}\sqrt{f}+(-1)^{5/6}\sqrt[3]{d}\sqrt{g}\right)(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}}$$

$$+ \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^3)^p\right)}{\sqrt{f}\sqrt{g}} - 3\frac{p\int\frac{\log\left(\frac{2}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{1+\frac{gx^2}{f}}dx}{f}$$

$$+ \frac{p\int\frac{\log\left(\frac{2\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\sqrt{f}\left(i\sqrt[3]{e}+\frac{\sqrt[3]{d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{f} + \frac{p\int\frac{\log\left(\frac{2\sqrt{g}\left(-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\sqrt{f}\left(i\sqrt[3]{e}-\frac{\sqrt[3]{-1}\sqrt[3]{d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{f}$$

$$+ \frac{p\int\frac{\log\left(\frac{2\sqrt{g}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\sqrt{f}\left(i\sqrt[3]{e}+\frac{(-1)^{2/3}\sqrt[3]{d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{f}$$

$$
\begin{aligned}
&= \frac{3p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(i\sqrt[3]{e}\sqrt{f}+\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} \\
&- \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2i\sqrt{f}\sqrt{g}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt[3]{e}\sqrt{f}+\sqrt[6]{-1}\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} \\
&- \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2(-1)^{5/6}\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\left(\sqrt[3]{e}\sqrt{f}+(-1)^{5/6}\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} \\
&+ \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{ip\operatorname{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(i\sqrt[3]{e}\sqrt{f}+\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{f}\sqrt{g}} \\
&+ \frac{ip\operatorname{Li}_2\left(1+\frac{2i\sqrt{f}\sqrt{g}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt[3]{e}\sqrt{f}+\sqrt[6]{-1}\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{f}\sqrt{g}} \\
&+ \frac{ip\operatorname{Li}_2\left(1-\frac{2(-1)^{5/6}\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\left(\sqrt[3]{e}\sqrt{f}+(-1)^{5/6}\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{f}\sqrt{g}} \\
&- 3\frac{(ip)\operatorname{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{\sqrt{f}\sqrt{g}}
\end{aligned}
$$

$$= \frac{3p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(i\sqrt[3]{e}\sqrt{f}+\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}}$$
$$- \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2i\sqrt{f}\sqrt{g}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt[3]{e}\sqrt{f}+\sqrt[6]{-1}\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}}$$
$$- \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2(-1)^{5/6}\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\left(\sqrt[3]{e}\sqrt{f}+(-1)^{5/6}\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}}$$
$$+ \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^3\right)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{3ip\operatorname{Li}_2\left(1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2\sqrt{f}\sqrt{g}}$$
$$+ \frac{ip\operatorname{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(i\sqrt[3]{e}\sqrt{f}+\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{f}\sqrt{g}}$$
$$+ \frac{ip\operatorname{Li}_2\left(1+\frac{2i\sqrt{f}\sqrt{g}\left((-1)^{2/3}\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\sqrt[3]{e}\sqrt{f}+\sqrt[6]{-1}\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{f}\sqrt{g}}$$
$$+ \frac{ip\operatorname{Li}_2\left(1-\frac{2(-1)^{5/6}\sqrt{f}\sqrt{g}\left(\sqrt[3]{d}+(-1)^{2/3}\sqrt[3]{e}x\right)}{\left(\sqrt[3]{e}\sqrt{f}+(-1)^{5/6}\sqrt[3]{d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{f}\sqrt{g}}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 867, normalized size of antiderivative = 1.16

$$\int \frac{\log\left(c\left(d+ex^3\right)^p\right)}{f+gx^2}\,dx$$
$$= \frac{-p\log\left(\frac{\sqrt{g}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\sqrt[3]{e}\sqrt{-f}+\sqrt[3]{d}\sqrt{g}}\right)\log\left(\sqrt{-f}-\sqrt{g}x\right)-p\log\left(\frac{\sqrt{g}\left(-\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\sqrt[3]{e}\sqrt{-f}-\sqrt[3]{-1}\sqrt[3]{d}\sqrt{g}}\right)\log\left(\sqrt{-f}-\sqrt{g}x\right)-p\log}{}$$

```
[In] Integrate[Log[c*(d + e*x^3)^p]/(f + g*x^2),x]
```

```
[Out] (-(p*Log[(Sqrt[g]*(d^(1/3) + e^(1/3)*x))/(e^(1/3)*Sqrt[-f] + d^(1/3)*Sqrt[g
])]*Log[Sqrt[-f] - Sqrt[g]*x]) - p*Log[(Sqrt[g]*(-((-1)^(1/3)*d^(1/3)) + e^
```

(1/3)*x))/(e^(1/3)*Sqrt[-f] - (-1)^(1/3)*d^(1/3)*Sqrt[g])]*Log[Sqrt[-f] - S
qrt[g]*x] - p*Log[(Sqrt[g]*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/(e^(1/3)*Sqrt[
-f] + (-1)^(2/3)*d^(1/3)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*x] + p*Log[-((Sqr
t[g]*(d^(1/3) + e^(1/3)*x))/(e^(1/3)*Sqrt[-f] - d^(1/3)*Sqrt[g]))]*Log[Sqrt
[-f] + Sqrt[g]*x] + p*Log[(Sqrt[g]*((-1)^(2/3)*d^(1/3) + e^(1/3)*x))/(-(e^(
1/3)*Sqrt[-f]) + (-1)^(2/3)*d^(1/3)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*x] + p
*Log[((-1)^(1/3)*Sqrt[g]*(d^(1/3) + (-1)^(2/3)*e^(1/3)*x))/(e^(1/3)*Sqrt[-f
] + (-1)^(1/3)*d^(1/3)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*x] + Log[Sqrt[-f] -
 Sqrt[g]*x]*Log[c*(d + e*x^3)^p] - Log[Sqrt[-f] + Sqrt[g]*x]*Log[c*(d + e*x
^3)^p] - p*PolyLog[2, (e^(1/3)*(Sqrt[-f] - Sqrt[g]*x))/(e^(1/3)*Sqrt[-f] +
d^(1/3)*Sqrt[g])] - p*PolyLog[2, (e^(1/3)*(Sqrt[-f] - Sqrt[g]*x))/(e^(1/3)*
Sqrt[-f] - (-1)^(1/3)*d^(1/3)*Sqrt[g])] - p*PolyLog[2, (e^(1/3)*(Sqrt[-f] -
 Sqrt[g]*x))/(e^(1/3)*Sqrt[-f] + (-1)^(2/3)*d^(1/3)*Sqrt[g])] + p*PolyLog[2
, (e^(1/3)*(Sqrt[-f] + Sqrt[g]*x))/(e^(1/3)*Sqrt[-f] - d^(1/3)*Sqrt[g])] +
p*PolyLog[2, (e^(1/3)*(Sqrt[-f] + Sqrt[g]*x))/(e^(1/3)*Sqrt[-f] + (-1)^(1/3
)*d^(1/3)*Sqrt[g])] + p*PolyLog[2, (e^(1/3)*(Sqrt[-f] + Sqrt[g]*x))/(e^(1/3
)*Sqrt[-f] - (-1)^(2/3)*d^(1/3)*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g])

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.45 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.36

method	result
risch	$\frac{\left(\ln\left(\left(e\,x^{3}+d\right)^{p}\right)-p\ln\left(e\,x^{3}+d\right)\right)\arctan\left(\frac{g x}{\sqrt{f g}}\right)}{\sqrt{f g}}+\frac{p\left(\sum_{\_\alpha=\mathrm{RootOf}\left(g\_Z^{2}+f\right)}\frac{\ln\left(x-\_\alpha\right)\ln\left(e\,x^{3}+d\right)-\left(\sum_{\_R1=\mathrm{RootOf}\left(\_Z^{3}eg+3\_\alpha\_Z^{2}eg-\right.}\right.}{}\right.}{2g}$

[In] int(ln(c*(e*x^3+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)

[Out] (ln((e*x^3+d)^p)-p*ln(e*x^3+d))/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2))+1/2*p/g
sum(1/_alpha(ln(x-_alpha)*ln(e*x^3+d)-sum(ln(x-_alpha)*ln((_R1-x+_alpha)/
_R1)+dilog((_R1-x+_alpha)/_R1),_R1=RootOf(_Z^3*e*g+3*_Z^2*_alpha*e*g-3*_Z*e
*f-_alpha*e*f+d*g))),_alpha=RootOf(_Z^2*g+f))+(1/2*I*Pi*csgn(I*(e*x^3+d)^p)
*csgn(I*c*(e*x^3+d)^p)^2-1/2*I*Pi*csgn(I*(e*x^3+d)^p)*csgn(I*c*(e*x^3+d)^p)
*csgn(I*c)-1/2*I*Pi*csgn(I*c*(e*x^3+d)^p)^3+1/2*I*Pi*csgn(I*c*(e*x^3+d)^p)^
2*csgn(I*c)+ln(c))/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2))

Fricas [F]

$$\int \frac{\log\left(c(d+ex^3)^p\right)}{f+gx^2}\,dx=\int \frac{\log\left((ex^3+d)^p c\right)}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(e*x^3+d)^p)/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log((e*x^3 + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^3)^p\right)}{f+gx^2}\,dx=\text{Timed out}$$

```
[In] integrate(ln(c*(e*x**3+d)**p)/(g*x**2+f),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(c(d+ex^3)^p\right)}{f+gx^2}\,dx=\int \frac{\log\left((ex^3+d)^p c\right)}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(e*x^3+d)^p)/(g*x^2+f),x, algorithm="maxima")
[Out] integrate(log((e*x^3 + d)^p*c)/(g*x^2 + f), x)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^3)^p\right)}{f+gx^2}\,dx=\int \frac{\log\left((ex^3+d)^p c\right)}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(e*x^3+d)^p)/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log((e*x^3 + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^3)^p\right)}{f+gx^2}\,dx=\int \frac{\ln \left(c\,{\left(e\,x^3+d\right)}^p\right)}{g\,x^2+f} \,dx$$

```
[In] int(log(c*(d + e*x^3)^p)/(f + g*x^2),x)
[Out] int(log(c*(d + e*x^3)^p)/(f + g*x^2), x)
```

3.262 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 533

$$\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx = \frac{2p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} + \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} + \frac{ip\,\mathrm{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}} + \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}}$$

```
[Out] arctan(x*g^(1/2)/f^(1/2))*ln(c*(e*x^2+d)^p)/f^(1/2)/g^(1/2)+2*p*arctan(x*g^
(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)-p*arctan
(x*g^(1/2)/f^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)-p*arcta
n(x*g^(1/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)-I*p*pol
ylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2
,1+2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2
```

```
)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,1-2*((-d)^
(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(
-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {211, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$\begin{aligned}\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = & \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\sqrt{g}} \\ & -\frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{\sqrt{f}\sqrt{g}} \\ & -\frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{\sqrt{f}\sqrt{g}} \\ & +\frac{2p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} \\ & +\frac{ip\,\mathrm{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}+1\right)}{2\sqrt{f}\sqrt{g}} \\ & +\frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}} \\ & -\frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}}\end{aligned}$$

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

```
[Out] (2*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/
(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))]])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*S
qrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[
g])*(Sqrt[f] - I*Sqrt[g]*x))]])/(Sqrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt
[f]]*Log[c*(d + e*x^2)^p])/(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, 1 - (2*Sqrt[
f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + (
2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sq
rt[g])*(Sqrt[f] - I*Sqrt[g]*x))]])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1
 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d
]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))]])/(Sqrt[f]*Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
```

```
/; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\text{integral} = \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\sqrt{g}} - (2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\,(d+ex^2)}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{(2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{(2ep)\int \left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}(\sqrt{-d}-\sqrt{e}x)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}(\sqrt{-d}+\sqrt{e}x)}\right)dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{(\sqrt{e}p)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{\sqrt{f}\sqrt{g}} - \frac{(\sqrt{e}p)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{\sqrt{f}\sqrt{g}}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}}$$
$$- \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}}$$
$$+ \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\sqrt{g}} - 2\frac{p\int \frac{\log\left(\frac{2}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{1+\frac{gx^2}{f}}\,dx}{f}$$
$$+ \frac{p\int \frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{f}\left(-i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}\,dx}{f} + \frac{p\int \frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{f}\left(i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}\,dx}{f}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}}$$
$$- \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}}$$
$$+ \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{ip\mathrm{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}}$$
$$+ \frac{ip\mathrm{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}} - 2\frac{(ip)\mathrm{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{\sqrt{f}\sqrt{g}}$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}}$$
$$- \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}}$$
$$+ \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{ip\text{Li}_2\left(1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}}$$
$$+ \frac{ip\text{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}} + \frac{ip\text{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.06

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx =$$
$$-\frac{i\left(p\log\left(\frac{\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)+p\log\left(\frac{\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{-i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)-p\log\left(\frac{\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{-i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right.\right.}{}$$

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

```
[Out] ((-1/2*I)*(p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt
[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + p*Log[(Sqrt[g]*(Sqrt[-d] +
Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x
)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] +
 Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-
d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]
*x)/Sqrt[f]] + (2*I)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p] + p*P
olyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*S
qrt[g])] + p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f]
+ I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sq
rt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*S
qrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]))/(Sqrt[f]*Sqrt[g])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.34 (sec) , antiderivative size = 449, normalized size of antiderivative = 0.84

method	result
risch	$\frac{(\ln((e\,x^2+d)^p)-p\ln(e\,x^2+d))\arctan\left(\frac{gx}{\sqrt{fg}}\right)}{\sqrt{fg}}+\frac{p\left(\sum_{\_\alpha=\mathrm{RootOf}\left(g\_Z^2+f\right)}\frac{\ln(x-\_\alpha)\ln\left(e\,x^2+d\right)-\ln(x-\_\alpha)\left(\ln\left(\frac{\mathrm{RootOf}\left(e\_Z^2g+2\_\alpha\right.}{\mathrm{RootOf}\left(e\_Z^2g+\right.}\right.\right.}{}\right.}{}$

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] (ln((e*x^2+d)^p)-p*ln(e*x^2+d))/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2))+1/2*p/g
*sum(1/_alpha*(ln(x-_alpha)*ln(e*x^2+d)-ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2
*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d
*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_al
pha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))-dilog((RootOf(_Z^2*
e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*
e*g+d*g-e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=
2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2))),_alpha=Root
Of(_Z^2*g+f))+(1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I*P
i*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(e*
x^2+d)^p)^3+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+ln(c))/(f*g)^(1/2)*a
rctan(g*x/(f*g)^(1/2))
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx=\int \frac{\log\left((ex^2+d)^pc\right)}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
[Out] Integral(log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

Maxima [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)}{g\,x^2+f}\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)/(f + g*x^2),x)
[Out] int(log(c*(d + e*x^2)^p)/(f + g*x^2), x)
```

3.263 $\int \frac{\log(c(d+ex)^p)}{f+gx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 229

$$\begin{aligned}\int \frac{\log(c(d+ex)^p)}{f+gx^2}\,dx =& \frac{\log(c(d+ex)^p)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} \\ &- \frac{\log(c(d+ex)^p)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} \\ &- \frac{p\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}\end{aligned}$$

```
[Out] 1/2*ln(c*(e*x+d)^p)*ln(e*((-f)^(1/2)-x*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/(
-f)^(1/2)/g^(1/2)-1/2*ln(c*(e*x+d)^p)*ln(e*((-f)^(1/2)+x*g^(1/2))/(e*(-f)^(
1/2)-d*g^(1/2)))/(-f)^(1/2)/g^(1/2)-1/2*p*polylog(2,-(e*x+d)*g^(1/2)/(e*(-f
)^(1/2)-d*g^(1/2)))/(-f)^(1/2)/g^(1/2)+1/2*p*polylog(2,(e*x+d)*g^(1/2)/(e*(
-f)^(1/2)+d*g^(1/2)))/(-f)^(1/2)/g^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{2456, 2441, 2440, 2438\}$

$$\int \frac{\log(c(d+ex)^p)}{f+gx^2}\,dx = \frac{\log(c(d+ex)^p)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log(c(d+ex)^p)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{p\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2\sqrt{-f}\sqrt{g}}$$

```
[In] Int[Log[c*(d + e*x)^p]/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e*x)^p]*Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g])
])/(2*Sqrt[-f]*Sqrt[g]) - (Log[c*(d + e*x)^p]*Log[(e*(Sqrt[-f] + Sqrt[g]*x)
)/(e*Sqrt[-f] - d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - (p*PolyLog[2, -((Sqrt[g
]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))])/(2*Sqrt[-f]*Sqrt[g]) + (p*PolyLog[
2, (Sqrt[g]*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2456

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sqrt{-f}\log\left(c(d+ex)^p\right)}{2f\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\sqrt{-f}\log\left(c(d+ex)^p\right)}{2f\left(\sqrt{-f}+\sqrt{g}x\right)}\right)dx$$

$$= -\frac{\int \frac{\log(c(d+ex)^p)}{\sqrt{-f}-\sqrt{g}x}\,dx}{2\sqrt{-f}} - \frac{\int \frac{\log(c(d+ex)^p)}{\sqrt{-f}+\sqrt{g}x}\,dx}{2\sqrt{-f}}$$

$$= \frac{\log\left(c(d+ex)^p\right)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log\left(c(d+ex)^p\right)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{(ep)\int \frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx}{2\sqrt{-f}\sqrt{g}} + \frac{(ep)\int \frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx}{2\sqrt{-f}\sqrt{g}}$$

$$= \frac{\log\left(c(d+ex)^p\right)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log\left(c(d+ex)^p\right)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{2\sqrt{-f}\sqrt{g}} - \frac{p\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx, x, d+ex\right)}{2\sqrt{-f}\sqrt{g}}$$

$$= \frac{\log\left(c(d+ex)^p\right)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log\left(c(d+ex)^p\right)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{p\text{Li}_2\left(-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{p\text{Li}_2\left(\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.78

$$\int \frac{\log\left(c(d+ex)^p\right)}{f+gx^2}\,dx$$
$$= \frac{\log\left(c(d+ex)^p\right)\left(\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)-\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)\right)-p\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}-d\sqrt{g}}\right)+p\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}$$

```
[In] Integrate[Log[c*(d + e*x)^p]/(f + g*x^2),x]
[Out] (Log[c*(d + e*x)^p]*(Log[(e*(Sqrt[-f] - Sqrt[g]*x))/(e*Sqrt[-f] + d*Sqrt[g]
)] - Log[(e*(Sqrt[-f] + Sqrt[g]*x))/(e*Sqrt[-f] - d*Sqrt[g])]) - p*PolyLog[
2, -((Sqrt[g]*(d + e*x))/(e*Sqrt[-f] - d*Sqrt[g]))] + p*PolyLog[2, (Sqrt[g]
*(d + e*x))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.92 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.71

method	result
risch	$-\frac{\arctan\left(\frac{2g(ex+d)-2dg}{2e\sqrt{fg}}\right)p\ln(ex+d)}{\sqrt{fg}}+\frac{\arctan\left(\frac{2g(ex+d)-2dg}{2e\sqrt{fg}}\right)\ln((ex+d)^p)}{\sqrt{fg}}+\frac{p\ln(ex+d)\ln\left(\frac{e\sqrt{-fg}-g(ex+d)+dg}{e\sqrt{-fg}+dg}\right)}{2\sqrt{-fg}}-\frac{p\ln(ex+c}{}$

```
[In] int(ln(c*(e*x+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))*p*ln(e*x+d)+1/
(f*g)^(1/2)*arctan(1/2*(2*g*(e*x+d)-2*d*g)/e/(f*g)^(1/2))*ln((e*x+d)^p)+1/2
*p*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e*(-f*g)^(1/2)
+d*g))-1/2*p*ln(e*x+d)/(-f*g)^(1/2)*ln((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)/(e*(-
f*g)^(1/2)-d*g))+1/2*p/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/2)-g*(e*x+d)+d*g)/(e
*(-f*g)^(1/2)+d*g))-1/2*p/(-f*g)^(1/2)*dilog((e*(-f*g)^(1/2)+g*(e*x+d)-d*g)
/(e*(-f*g)^(1/2)-d*g))+(1/2*I*Pi*csgn(I*(e*x+d)^p)*csgn(I*c*(e*x+d)^p)^2-1/
2*I*Pi*csgn(I*(e*x+d)^p)*csgn(I*c*(e*x+d)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(e
*x+d)^p)^3+1/2*I*Pi*csgn(I*c*(e*x+d)^p)^2*csgn(I*c)+ln(c))/(f*g)^(1/2)*arct
an(g*x/(f*g)^(1/2))
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex)^p\right)}{f+gx^2}\,dx=\int \frac{\log\left((ex+d)^pc\right)}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(e*x+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F]

$$\int \frac{\log\left(c(d+ex)^p\right)}{f+gx^2}\,dx=\int \frac{\log\left(c(d+ex)^p\right)}{f+gx^2}\,dx$$

```
[In] integrate(ln(c*(e*x+d)**p)/(g*x**2+f),x)
```

```
[Out] Integral(log(c*(d + e*x)**p)/(f + g*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.35

$$\int \frac{\log(c(d+ex)^p)}{f+gx^2}\,dx$$

$$= \frac{ep\left(\frac{2\arctan\left(\frac{gx}{\sqrt{fg}}\right)\log(ex+d)}{e} + \frac{\arctan\left(\frac{(e^2x+de)\sqrt{f}\sqrt{g}}{e^2f+d^2g}, \frac{degx+d^2g}{e^2f+d^2g}\right)\log(gx^2+f) - \arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{e^2gx^2+2\,degx+d^2g}{e^2f+d^2g}\right) - i\,\mathrm{Li}_2\left(\frac{degx+e^2f-}{e^2f+2i}\right.}{e}\right.}{2\sqrt{fg}}$$

$$- \frac{p\arctan\left(\frac{gx}{\sqrt{fg}}\right)\log(ex+d)}{\sqrt{fg}} + \frac{\arctan\left(\frac{gx}{\sqrt{fg}}\right)\log((ex+d)^pc)}{\sqrt{fg}}$$

```
[In] integrate(log(c*(e*x+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/2*e*p*(2*arctan(g*x/sqrt(f*g))*log(e*x + d)/e + (arctan2((e^2*x + d*e)*sq
rt(f)*sqrt(g)/(e^2*f + d^2*g), (d*e*g*x + d^2*g)/(e^2*f + d^2*g))*log(g*x^2
 + f) - arctan(sqrt(g)*x/sqrt(f))*log((e^2*g*x^2 + 2*d*e*g*x + d^2*g)/(e^2*
f + d^2*g)) - I*dilog((d*e*g*x + e^2*f - (I*e^2*x - I*d*e)*sqrt(f)*sqrt(g))
/(e^2*f + 2*I*d*e*sqrt(f)*sqrt(g) - d^2*g)) + I*dilog((d*e*g*x + e^2*f + (I
*e^2*x - I*d*e)*sqrt(f)*sqrt(g))/(e^2*f - 2*I*d*e*sqrt(f)*sqrt(g) - d^2*g))
)/e)/sqrt(f*g) - p*arctan(g*x/sqrt(f*g))*log(e*x + d)/sqrt(f*g) + arctan(g*
x/sqrt(f*g))*log((e*x + d)^p*c)/sqrt(f*g)
```

Giac [F]

$$\int \frac{\log(c(d+ex)^p)}{f+gx^2}\,dx = \int \frac{\log((ex+d)^pc)}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(e*x+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log((e*x + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex)^p\right)}{f+gx^2}\,dx=\int \frac{\ln\left(c\,{\left(d+e\,x\right)}^p\right)}{g\,x^2+f}\,dx$$

```
[In] int(log(c*(d + e*x)^p)/(f + g*x^2),x)
[Out] int(log(c*(d + e*x)^p)/(f + g*x^2), x)
```

3.264 $\int \frac{\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 360

$$
\begin{aligned}
\int \frac{\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^2}\,dx &= \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} \\
&\quad - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(e+dx)}{(id\sqrt{f}+e\sqrt{g})\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} \\
&\quad + \frac{ip\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}} - \frac{ip\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}} \\
&\quad - \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2\sqrt{f}\sqrt{g}} \\
&\quad + \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(e+dx)}{(id\sqrt{f}+e\sqrt{g})\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{f}\sqrt{g}}
\end{aligned}
$$

```
[Out] arctan(x*g^(1/2)/f^(1/2))*ln(c*(d+e/x)^p)/f^(1/2)/g^(1/2)+p*arctan(x*g^(1/2
)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)-p*arctan(x*g
^(1/2)/f^(1/2))*ln(2*(d*x+e)*f^(1/2)*g^(1/2)/(I*d*f^(1/2)+e*g^(1/2))/(f^(1/
2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,-I*x*g^(1/2)/f^(1/2))/f^
(1/2)/g^(1/2)-1/2*I*p*polylog(2,I*x*g^(1/2)/f^(1/2))/f^(1/2)/g^(1/2)-1/2*I*
p*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*poly
log(2,1-2*(d*x+e)*f^(1/2)*g^(1/2)/(I*d*f^(1/2)+e*g^(1/2))/(f^(1/2)-I*x*g^(1
/2)))/f^(1/2)/g^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {211, 2520, 12, 266, 6820, 4996, 4940, 2438, 4966, 2449, 2352, 2497}

$$\int \frac{\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^2}\,dx = \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(dx+e)}{(\sqrt{f}-i\sqrt{g}x)(e\sqrt{g}+id\sqrt{f})}\right)}{\sqrt{f}\sqrt{g}} + \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} + \frac{ip\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(e+dx)}{(i\sqrt{f}d+e\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}} + \frac{ip\,\text{PolyLog}\left(2,-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}} - \frac{ip\,\text{PolyLog}\left(2,\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}} - \frac{ip\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2\sqrt{f}\sqrt{g}}$$

```
[In] Int[Log[c*(d + e/x)^p]/(f + g*x^2),x]
```

```
[Out] (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e/x)^p])/(Sqrt[f]*Sqrt[g]) + (p*Arc
Tan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]
*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(e + d*x)
)/((I*d*Sqrt[f] + e*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) +
 ((I/2)*p*PolyLog[2, ((-I)*Sqrt[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) - ((I/2)*
p*PolyLog[2, (I*Sqrt[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) - ((I/2)*p*PolyLog[2
, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*Po
lyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(e + d*x))/((I*d*Sqrt[f] + e*Sqrt[g])*(Sqrt
[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4996

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + (ep)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\left(d+\frac{e}{x}\right)x^2}\,dx \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{(ep)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\left(d+\frac{e}{x}\right)x^2}\,dx}{\sqrt{f}\sqrt{g}} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{(ep)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x(e+dx)}\,dx}{\sqrt{f}\sqrt{g}} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{(ep)\int \left(\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{ex} - \frac{d\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{e(e+dx)}\right)dx}{\sqrt{f}\sqrt{g}} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{p\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x}\,dx}{\sqrt{f}\sqrt{g}} - \frac{(dp)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{e+dx}\,dx}{\sqrt{f}\sqrt{g}} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} \\
&\quad - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(e+dx)}{\left(id\sqrt{f}+e\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\int \frac{\log\left(\frac{2}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{1+\frac{gx^2}{f}}\,dx}{f} \\
&\quad + \frac{p\int \frac{\log\left(\frac{2\sqrt{g}(e+dx)}{\sqrt{f}\left(id+\frac{e\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}\,dx}{f} + \frac{(ip)\int \frac{\log\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{x}\,dx}{2\sqrt{f}\sqrt{g}} - \frac{(ip)\int \frac{\log\left(1+\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{x}\,dx}{2\sqrt{f}\sqrt{g}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} \\
&\quad - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(e+dx)}{\left(id\sqrt{f}+e\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} + \frac{ip\operatorname{Li}_2\left(-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}} - \frac{ip\operatorname{Li}_2\left(\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}} \\
&\quad + \frac{ip\operatorname{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(e+dx)}{\left(id\sqrt{f}+e\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{f}\sqrt{g}} - \frac{(ip)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{\sqrt{f}\sqrt{g}} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} \\
&\quad - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(e+dx)}{\left(id\sqrt{f}+e\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}} + \frac{ip\operatorname{Li}_2\left(-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}} \\
&\quad - \frac{ip\operatorname{Li}_2\left(\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{f}\sqrt{g}} - \frac{ip\operatorname{Li}_2\left(1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2\sqrt{f}\sqrt{g}} + \frac{ip\operatorname{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(e+dx)}{\left(id\sqrt{f}+e\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{f}\sqrt{g}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.04

$$
\int \frac{\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^2}\,dx
$$

$$
= \frac{\log\left(c\left(d+\frac{e}{x}\right)^p\right)\log\left(\sqrt{-f}-\sqrt{g}x\right) + p\log\left(\frac{\sqrt{g}x}{\sqrt{-f}}\right)\log\left(\sqrt{-f}-\sqrt{g}x\right) - p\log\left(\frac{\sqrt{g}(e+dx)}{d\sqrt{-f}+e\sqrt{g}}\right)\log\left(\sqrt{-f}-\sqrt{g}x\right.}{}
$$

```
[In] Integrate[Log[c*(d + e/x)^p]/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e/x)^p]*Log[Sqrt[-f] - Sqrt[g]*x] + p*Log[(Sqrt[g]*x)/Sqrt[-f]]
*Log[Sqrt[-f] - Sqrt[g]*x] - p*Log[(Sqrt[g]*(e + d*x))/(d*Sqrt[-f] + e*Sqrt
[g])]*Log[Sqrt[-f] - Sqrt[g]*x] - Log[c*(d + e/x)^p]*Log[Sqrt[-f] + Sqrt[g]
*x] - p*Log[(f*Sqrt[g]*x)/(-f)^(3/2)]*Log[Sqrt[-f] + Sqrt[g]*x] + p*Log[-((
Sqrt[g]*(e + d*x))/(d*Sqrt[-f] - e*Sqrt[g]))]*Log[Sqrt[-f] + Sqrt[g]*x] - p
*PolyLog[2, (d*(Sqrt[-f] - Sqrt[g]*x))/(d*Sqrt[-f] + e*Sqrt[g])] + p*PolyLo
g[2, (d*(Sqrt[-f] + Sqrt[g]*x))/(d*Sqrt[-f] - e*Sqrt[g])] - p*PolyLog[2, 1
+ (Sqrt[g]*x)/Sqrt[-f]] + p*PolyLog[2, 1 + (f*Sqrt[g]*x)/(-f)^(3/2)])/(2*Sq
rt[-f]*Sqrt[g])
```

Maple [F]

$$\int \frac{\ln\left(c\left(d+\frac{e}{x}\right)^{p}\right)}{g\,x^{2}+f}dx$$

```
[In] int(ln(c*(d+e/x)^p)/(g*x^2+f),x)
[Out] int(ln(c*(d+e/x)^p)/(g*x^2+f),x)
```

Fricas [F]

$$\int \frac{\log\left(c\left(d+\frac{e}{x}\right)^{p}\right)}{f+gx^{2}}\,dx = \int \frac{\log\left(c\left(d+\frac{e}{x}\right)^{p}\right)}{gx^{2}+f}\,dx$$

```
[In] integrate(log(c*(d+e/x)^p)/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log(c*((d*x + e)/x)^p)/(g*x^2 + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(d+\frac{e}{x}\right)^{p}\right)}{f+gx^{2}}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(d+e/x)**p)/(g*x**2+f),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.05

$$\int \frac{\log\left(c\left(d+\frac{e}{x}\right)^{p}\right)}{f+gx^{2}}\,dx$$
$$= \frac{ep\Bigg(\frac{4\arctan\left(\frac{gx}{\sqrt{fg}}\right)\log\left(d+\frac{e}{x}\right)}{e} - \frac{\left(\pi-2\arctan\left(\frac{\left(d^{2}x+de\right)\sqrt{f}\sqrt{g}}{d^{2}f+e^{2}g},\frac{degx+e^{2}g}{d^{2}f+e^{2}g}\right)\right)\log\left(gx^{2}+f\right)-4\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)+2\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log}{}}{4}$$
$$- \frac{p\arctan\left(\frac{gx}{\sqrt{fg}}\right)\log\left(d+\frac{e}{x}\right)}{\sqrt{fg}} + \frac{\arctan\left(\frac{gx}{\sqrt{fg}}\right)\log\left(c\left(d+\frac{e}{x}\right)^{p}\right)}{\sqrt{fg}}$$

```
[In] integrate(log(c*(d+e/x)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] 1/4*e*p*(4*arctan(g*x/sqrt(f*g))*log(d + e/x)/e - ((pi - 2*arctan2((d^2*x +
 d*e)*sqrt(f)*sqrt(g)/(d^2*f + e^2*g), (d*e*g*x + e^2*g)/(d^2*f + e^2*g)))*
log(g*x^2 + f) - 4*arctan(sqrt(g)*x/sqrt(f))*log(sqrt(g)*x/sqrt(f)) + 2*arc
tan(sqrt(g)*x/sqrt(f))*log((d^2*g*x^2 + 2*d*e*g*x + e^2*g)/(d^2*f + e^2*g))
 + 2*I*dilog((I*sqrt(g)*x + sqrt(f))/sqrt(f)) - 2*I*dilog(-(I*sqrt(g)*x - s
qrt(f))/sqrt(f)) + 2*I*dilog((d*e*g*x + d^2*f - (I*d^2*x - I*d*e)*sqrt(f)*s
qrt(g))/(d^2*f + 2*I*d*e*sqrt(f)*sqrt(g) - e^2*g)) - 2*I*dilog((d*e*g*x + d
^2*f + (I*d^2*x - I*d*e)*sqrt(f)*sqrt(g))/(d^2*f - 2*I*d*e*sqrt(f)*sqrt(g)
- e^2*g)))/e)/sqrt(f*g) - p*arctan(g*x/sqrt(f*g))*log(d + e/x)/sqrt(f*g) +
arctan(g*x/sqrt(f*g))*log(c*(d + e/x)^p)/sqrt(f*g)
```

Giac [F]

$$\int \frac{\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^2}\,dx=\int \frac{\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(d+e/x)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(log(c*(d + e/x)^p)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^2}\,dx=\int \frac{\ln\left(c\,\left(d+\frac{e}{x}\right)^p\right)}{g\,x^2+f}\,dx$$

```
[In] int(log(c*(d + e/x)^p)/(f + g*x^2),x)
```

```
[Out] int(log(c*(d + e/x)^p)/(f + g*x^2), x)
```

3.265 $\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 597

$$
\begin{aligned}
\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx &= \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{2p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} \\
&- \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}-\sqrt{-d}x)}{(i\sqrt{-d}\sqrt{f}-\sqrt{e}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} \\
&- \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}+\sqrt{-d}x)}{(i\sqrt{-d}\sqrt{f}+\sqrt{e}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} \\
&+ \frac{ip\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}} - \frac{ip\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}} \\
&- \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} \\
&+ \frac{ip\,\mathrm{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}-\sqrt{-d}x)}{(i\sqrt{-d}\sqrt{f}-\sqrt{e}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}} \\
&+ \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}+\sqrt{-d}x)}{(i\sqrt{-d}\sqrt{f}+\sqrt{e}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}}
\end{aligned}
$$

```
[Out] arctan(x*g^(1/2)/f^(1/2))*ln(c*(d+e/x^2)^p)/f^(1/2)/g^(1/2)+2*p*arctan(x*g^
(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)-p*arctan
(x*g^(1/2)/f^(1/2))*ln(-2*(-x*(-d)^(1/2)+e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-
I*x*g^(1/2))/(I*(-d)^(1/2)*f^(1/2)-e^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)-p*arct
an(x*g^(1/2)/f^(1/2))*ln(2*(x*(-d)^(1/2)+e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-
```

```
I*x*g^(1/2))/(I*(-d)^(1/2)*f^(1/2)+e^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)+I*p*po
lylog(2,-I*x*g^(1/2)/f^(1/2))/f^(1/2)/g^(1/2)-I*p*polylog(2,I*x*g^(1/2)/f^(
1/2))/f^(1/2)/g^(1/2)-I*p*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1
/2)/g^(1/2)+1/2*I*p*polylog(2,1+2*(-x*(-d)^(1/2)+e^(1/2))*f^(1/2)*g^(1/2)/(
f^(1/2)-I*x*g^(1/2))/(I*(-d)^(1/2)*f^(1/2)-e^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2
)+1/2*I*p*polylog(2,1-2*(x*(-d)^(1/2)+e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x
*g^(1/2))/(I*(-d)^(1/2)*f^(1/2)+e^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {211, 2520, 12, 266, 6820, 5048, 4940, 2438, 4966, 2449, 2352, 2497}

$$\begin{aligned}\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx=&\frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}}\\&-\frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{e}-\sqrt{-d}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{e}\sqrt{g}+i\sqrt{-d}\sqrt{f}\right)}\right)}{\sqrt{f}\sqrt{g}}\\&-\frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}x+\sqrt{e}\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{e}\sqrt{g}+i\sqrt{-d}\sqrt{f}\right)}\right)}{\sqrt{f}\sqrt{g}}\\&+\frac{2p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}}\\&+\frac{ip\operatorname{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{e}-\sqrt{-d}x\right)}{\left(i\sqrt{-d}\sqrt{f}-\sqrt{e}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}+1\right)}{2\sqrt{f}\sqrt{g}}\\&+\frac{ip\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}x+\sqrt{e}\right)}{\left(i\sqrt{-d}\sqrt{f}+\sqrt{e}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{f}\sqrt{g}}\\&+\frac{ip\operatorname{PolyLog}\left(2,-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}-\frac{ip\operatorname{PolyLog}\left(2,\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\\&-\frac{ip\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}}\end{aligned}$$

```
[In] Int[Log[c*(d + e/x^2)^p]/(f + g*x^2),x]
```

```
[Out] (ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e/x^2)^p])/(Sqrt[f]*Sqrt[g]) + (2*p
*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqr
t[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqr
t[e] - Sqrt[-d]*x))/((I*Sqrt[-d]*Sqrt[f] - Sqrt[e]*Sqrt[g])*(Sqrt[f] - I*Sq
rt[g]*x))])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[
```

```
f]*Sqrt[g]*(Sqrt[e] + Sqrt[-d]*x))/((I*Sqrt[-d]*Sqrt[f] + Sqrt[e]*Sqrt[g])*
(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + (I*p*PolyLog[2, ((-I)*Sqrt[g
]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, (I*Sqrt[g]*x)/Sqrt[f]])/
(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)
])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[e]
- Sqrt[-d]*x))/((I*Sqrt[-d]*Sqrt[f] - Sqrt[e]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]
*x))])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt
[e] + Sqrt[-d]*x))/((I*Sqrt[-d]*Sqrt[f] + Sqrt[e]*Sqrt[g])*(Sqrt[f] - I*Sqr
t[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}}+(2ep)\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\left(d+\frac{e}{x^2}\right)x^3}\,dx\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}}+\frac{(2ep)\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\left(d+\frac{e}{x^2}\right)x^3}\,dx}{\sqrt{f}\sqrt{g}}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}}+\frac{(2ep)\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x(e+dx^2)}\,dx}{\sqrt{f}\sqrt{g}}
\end{aligned}$$

$$=\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}}+\frac{(2ep)\int\left(\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{ex}-\frac{dx\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{e(e+dx^2)}\right)dx}{\sqrt{f}\sqrt{g}}$$

$$=\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}}+\frac{(2p)\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{x}dx}{\sqrt{f}\sqrt{g}}-\frac{(2dp)\int\frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{e+dx^2}dx}{\sqrt{f}\sqrt{g}}$$

$$=\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}}+\frac{(ip)\int\frac{\log\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{x}dx}{\sqrt{f}\sqrt{g}}$$

$$-\frac{(ip)\int\frac{\log\left(1+\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{x}dx}{\sqrt{f}\sqrt{g}}-\frac{(2dp)\int\left(-\frac{\sqrt{-d}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2d(\sqrt{e}-\sqrt{-d}x)}+\frac{\sqrt{-d}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2d(\sqrt{e}+\sqrt{-d}x)}\right)dx}{\sqrt{f}\sqrt{g}}$$

$$=\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}}+\frac{ip\text{Li}_2\left(-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}-\frac{ip\text{Li}_2\left(\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}$$

$$+\frac{\left(\sqrt{-d}p\right)\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{e}-\sqrt{-d}x}dx}{\sqrt{f}\sqrt{g}}-\frac{\left(\sqrt{-d}p\right)\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{e}+\sqrt{-d}x}dx}{\sqrt{f}\sqrt{g}}$$

$$=\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}}+\frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}}$$

$$-\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}-\sqrt{-d}x)}{(i\sqrt{-d}\sqrt{f}-\sqrt{e}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}}$$

$$-\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}+\sqrt{-d}x)}{(i\sqrt{-d}\sqrt{f}+\sqrt{e}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}}$$

$$+\frac{ip\text{Li}_2\left(-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}-\frac{ip\text{Li}_2\left(\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}-2\frac{p\int\frac{\log\left(\frac{2}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{1+\frac{gx^2}{f}}dx}{f}$$

$$+\frac{p\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{e}-\sqrt{-d}x)}{\sqrt{f}\left(-i\sqrt{-d}+\frac{\sqrt{e}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{f}+\frac{p\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{e}+\sqrt{-d}x)}{\sqrt{f}\left(i\sqrt{-d}+\frac{\sqrt{e}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}-\sqrt{-d}x)}{(i\sqrt{-d}\sqrt{f}-\sqrt{e}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}+\sqrt{-d}x)}{(i\sqrt{-d}\sqrt{f}+\sqrt{e}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} + \frac{ip\mathrm{Li}_2\left(-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}} - \frac{ip\mathrm{Li}_2\left(\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}} + \frac{ip\mathrm{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}-\sqrt{-d}x)}{(i\sqrt{-d}\sqrt{f}-\sqrt{e}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}} + \frac{ip\mathrm{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}+\sqrt{-d}x)}{(i\sqrt{-d}\sqrt{f}+\sqrt{e}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}} - 2\frac{(ip)\mathrm{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{\sqrt{f}\sqrt{g}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}-\sqrt{-d}x)}{(i\sqrt{-d}\sqrt{f}-\sqrt{e}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}+\sqrt{-d}x)}{(i\sqrt{-d}\sqrt{f}+\sqrt{e}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} + \frac{ip\mathrm{Li}_2\left(-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}} - \frac{ip\mathrm{Li}_2\left(\frac{i\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}} - \frac{ip\mathrm{Li}_2\left(1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} + \frac{ip\mathrm{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}-\sqrt{-d}x)}{(i\sqrt{-d}\sqrt{f}-\sqrt{e}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}} + \frac{ip\mathrm{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}+\sqrt{-d}x)}{(i\sqrt{-d}\sqrt{f}+\sqrt{e}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.18

$$\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx = \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)\log\left(\sqrt{-f}-\sqrt{g}x\right)+2p\log\left(\frac{\sqrt{g}x}{\sqrt{-f}}\right)\log\left(\sqrt{-f}-\sqrt{g}x\right)-p\log\left(\frac{\sqrt{g}(-\sqrt{e}+\sqrt{-d}x)}{\sqrt{-d}\sqrt{-f}-\sqrt{e}\sqrt{g}}\right)\log\left(\sqrt{-f}-\right.}{}$$

```
[In] Integrate[Log[c*(d + e/x^2)^p]/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e/x^2)^p]*Log[Sqrt[-f] - Sqrt[g]*x] + 2*p*Log[(Sqrt[g]*x)/Sqrt[
-f]]*Log[Sqrt[-f] - Sqrt[g]*x] - p*Log[(Sqrt[g]*(-Sqrt[e] + Sqrt[-d]*x))/(S
```

```
qrt[-d]*Sqrt[-f] - Sqrt[e]*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*x] - p*Log[(Sqr
t[g]*(Sqrt[e] + Sqrt[-d]*x))/(Sqrt[-d]*Sqrt[-f] + Sqrt[e]*Sqrt[g])]*Log[Sqr
t[-f] - Sqrt[g]*x] - Log[c*(d + e/x^2)^p]*Log[Sqrt[-f] + Sqrt[g]*x] - 2*p*L
og[(f*Sqrt[g]*x)/(-f)^(3/2)]*Log[Sqrt[-f] + Sqrt[g]*x] + p*Log[(Sqrt[g]*(Sq
rt[e] - Sqrt[-d]*x))/(Sqrt[-d]*Sqrt[-f] + Sqrt[e]*Sqrt[g])]*Log[Sqrt[-f] +
Sqrt[g]*x] + p*Log[-((Sqrt[g]*(Sqrt[e] + Sqrt[-d]*x))/(Sqrt[-d]*Sqrt[-f] -
Sqrt[e]*Sqrt[g]))]*Log[Sqrt[-f] + Sqrt[g]*x] - p*PolyLog[2, (Sqrt[-d]*(Sqrt
[-f] - Sqrt[g]*x))/(Sqrt[-d]*Sqrt[-f] - Sqrt[e]*Sqrt[g])] - p*PolyLog[2, (S
qrt[-d]*(Sqrt[-f] - Sqrt[g]*x))/(Sqrt[-d]*Sqrt[-f] + Sqrt[e]*Sqrt[g])] + p*
PolyLog[2, (Sqrt[-d]*(Sqrt[-f] + Sqrt[g]*x))/(Sqrt[-d]*Sqrt[-f] - Sqrt[e]*S
qrt[g])] + p*PolyLog[2, (Sqrt[-d]*(Sqrt[-f] + Sqrt[g]*x))/(Sqrt[-d]*Sqrt[-f
] + Sqrt[e]*Sqrt[g])] - 2*p*PolyLog[2, 1 + (Sqrt[g]*x)/Sqrt[-f]] + 2*p*Poly
Log[2, 1 + (f*Sqrt[g]*x)/(-f)^(3/2)])/(2*Sqrt[-f]*Sqrt[g])
```

Maple [F]

$$\int \frac{\ln\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{g\,x^2+f}dx$$

```
[In] int(ln(c*(d+e/x^2)^p)/(g*x^2+f),x)
[Out] int(ln(c*(d+e/x^2)^p)/(g*x^2+f),x)
```

Fricas [F]

$$\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(d+e/x^2)^p)/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log(c*((d*x^2 + e)/x^2)^p)/(g*x^2 + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(d+e/x**2)**p)/(g*x**2+f),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(d+e/x^2)^p)/(g*x^2+f),x, algorithm="maxima")
[Out] integrate(log(c*(d + e/x^2)^p)/(g*x^2 + f), x)
```

Giac [F]

$$\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(d+e/x^2)^p)/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log(c*(d + e/x^2)^p)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(d+\frac{e}{x^2}\right)^p\right)}{f+gx^2}\,dx = \int \frac{\ln\left(c\,\left(d+\frac{e}{x^2}\right)^p\right)}{g\,x^2+f}\,dx$$

```
[In] int(log(c*(d + e/x^2)^p)/(f + g*x^2),x)
[Out] int(log(c*(d + e/x^2)^p)/(f + g*x^2), x)
```

3.266 $\int \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f+gx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 541

$$\begin{aligned}\int \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f+gx^2}\,dx = & -\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} \\
& +\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} \\
& -\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}+\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} \\
& +\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}+\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} \\
& -\frac{p\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} \\
& +\frac{p\,\mathrm{PolyLog}\left(2,-\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{e\sqrt[4]{-f}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} \\
& -\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{e\sqrt[4]{-f}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}\end{aligned}$$

```
[Out] 1/2*ln(c*(d+e*x^(1/2))^p)*ln(e*((-f)^(1/4)-g^(1/4)*x^(1/2))/(e*(-f)^(1/4)+d
*g^(1/4)))/(-f)^(1/2)/g^(1/2)+1/2*ln(c*(d+e*x^(1/2))^p)*ln(e*((-f)^(1/4)+g^
(1/4)*x^(1/2))/(e*(-f)^(1/4)-d*g^(1/4)))/(-f)^(1/2)/g^(1/2)-1/2*ln(c*(d+e*x
^(1/2))^p)*ln(e*(g^(1/4)*x^(1/2)+(-(-f)^(1/2))^(1/2))/(-d*g^(1/4)+e*(-(-f)^
(1/2))^(1/2)))/(-f)^(1/2)/g^(1/2)-1/2*ln(c*(d+e*x^(1/2))^p)*ln(e*(-g^(1/4)*
x^(1/2)+(-(-f)^(1/2))^(1/2))/(d*g^(1/4)+e*(-(-f)^(1/2))^(1/2)))/(-f)^(1/2)/
g^(1/2)+1/2*p*polylog(2,-g^(1/4)*(d+e*x^(1/2))/(e*(-f)^(1/4)-d*g^(1/4)))/(-
f)^(1/2)/g^(1/2)+1/2*p*polylog(2,g^(1/4)*(d+e*x^(1/2))/(e*(-f)^(1/4)+d*g^(1
/4)))/(-f)^(1/2)/g^(1/2)-1/2*p*polylog(2,-g^(1/4)*(d+e*x^(1/2))/(-d*g^(1/4)
+e*(-(-f)^(1/2))^(1/2)))/(-f)^(1/2)/g^(1/2)-1/2*p*polylog(2,g^(1/4)*(d+e*x^
(1/2))/(d*g^(1/4)+e*(-(-f)^(1/2))^(1/2)))/(-f)^(1/2)/g^(1/2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2522, 281, 211, 2463, 266, 2441, 2440, 2438}

$$\begin{aligned}\int \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f+gx^2}\,dx = & -\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}\sqrt{x}\right)}{d\sqrt[4]{g}+e\sqrt{-\sqrt{-f}}}\right)}{2\sqrt{-f}\sqrt{g}} \\ & +\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)}{d\sqrt[4]{g}+e\sqrt[4]{-f}}\right)}{2\sqrt{-f}\sqrt{g}} \\ & -\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}+\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} \\ & +\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}+\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} \\ & -\frac{p\,\text{PolyLog}\left(2,-\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} \\ & +\frac{p\,\text{PolyLog}\left(2,-\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{e\sqrt[4]{-f}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} \\ & -\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{\sqrt[4]{g}d+e\sqrt{-\sqrt{-f}}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt[4]{g}\left(d+e\sqrt{x}\right)}{\sqrt[4]{g}d+e\sqrt[4]{-f}}\right)}{2\sqrt{-f}\sqrt{g}}\end{aligned}$$

```
[In] Int[Log[c*(d + e*Sqrt[x])^p]/(f + g*x^2),x]
```

```
[Out] -1/2*(Log[c*(d + e*Sqrt[x])^p]*Log[(e*(Sqrt[-Sqrt[-f]] - g^(1/4)*Sqrt[x]))/
(e*Sqrt[-Sqrt[-f]] + d*g^(1/4))])/(Sqrt[-f]*Sqrt[g]) + (Log[c*(d + e*Sqrt[x
])^p]*Log[(e*((-f)^(1/4) - g^(1/4)*Sqrt[x]))/(e*(-f)^(1/4) + d*g^(1/4))])/(
2*Sqrt[-f]*Sqrt[g]) - (Log[c*(d + e*Sqrt[x])^p]*Log[(e*(Sqrt[-Sqrt[-f]] + g
^(1/4)*Sqrt[x]))/(e*Sqrt[-Sqrt[-f]] - d*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) + (
Log[c*(d + e*Sqrt[x])^p]*Log[(e*((-f)^(1/4) + g^(1/4)*Sqrt[x]))/(e*(-f)^(1/
4) - d*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) - (p*PolyLog[2, -((g^(1/4)*(d + e*Sq
rt[x]))/(e*Sqrt[-Sqrt[-f]] - d*g^(1/4)))])/(2*Sqrt[-f]*Sqrt[g]) + (p*PolyLo
g[2, -((g^(1/4)*(d + e*Sqrt[x]))/(e*(-f)^(1/4) - d*g^(1/4)))])/(2*Sqrt[-f]*
Sqrt[g]) - (p*PolyLog[2, (g^(1/4)*(d + e*Sqrt[x]))/(e*Sqrt[-Sqrt[-f]] + d*g
^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) + (p*PolyLog[2, (g^(1/4)*(d + e*Sqrt[x]))/(e
*(-f)^(1/4) + d*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2522

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Su
bst[Int[x^(k - 1)*(f + g*x^(k*s))^r*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x],
 x, x^(1/k)], x] /; IntegerQ[k*s]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q,
r, s}, x] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x\log\left(c(d+ex)^p\right)}{f+gx^4}\,dx,x,\sqrt{x}\right)\\
&= 2\text{Subst}\left(\int\left(-\frac{\sqrt{g}x\log\left(c(d+ex)^p\right)}{2\sqrt{-f}\left(\sqrt{-f}\sqrt{g}-gx^2\right)}-\frac{\sqrt{g}x\log\left(c(d+ex)^p\right)}{2\sqrt{-f}\left(\sqrt{-f}\sqrt{g}+gx^2\right)}\right)dx,x,\sqrt{x}\right)\\
&= -\frac{\sqrt{g}\text{Subst}\left(\int\frac{x\log(c(d+ex)^p)}{\sqrt{-f}\sqrt{g}-gx^2}\,dx,x,\sqrt{x}\right)}{\sqrt{-f}}-\frac{\sqrt{g}\text{Subst}\left(\int\frac{x\log(c(d+ex)^p)}{\sqrt{-f}\sqrt{g}+gx^2}\,dx,x,\sqrt{x}\right)}{\sqrt{-f}}\\
&= -\frac{\sqrt{g}\text{Subst}\left(\int\left(-\frac{\log(c(d+ex)^p)}{2g^{3/4}\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}x\right)}+\frac{\log(c(d+ex)^p)}{2g^{3/4}\left(\sqrt{-\sqrt{-f}}+\sqrt[4]{g}x\right)}\right)dx,x,\sqrt{x}\right)}{\sqrt{-f}}\\
&\quad-\frac{\sqrt{g}\text{Subst}\left(\int\left(\frac{\log(c(d+ex)^p)}{2g^{3/4}\left(\sqrt[4]{-f}-\sqrt[4]{g}x\right)}-\frac{\log(c(d+ex)^p)}{2g^{3/4}\left(\sqrt[4]{-f}+\sqrt[4]{g}x\right)}\right)dx,x,\sqrt{x}\right)}{\sqrt{-f}}\\
&= \frac{\text{Subst}\left(\int\frac{\log(c(d+ex)^p)}{\sqrt{-\sqrt{-f}}-\sqrt[4]{g}x}\,dx,x,\sqrt{x}\right)}{2\sqrt{-f}\sqrt[4]{g}}-\frac{\text{Subst}\left(\int\frac{\log(c(d+ex)^p)}{\sqrt[4]{-f}-\sqrt[4]{g}x}\,dx,x,\sqrt{x}\right)}{2\sqrt{-f}\sqrt[4]{g}}\\
&\quad-\frac{\text{Subst}\left(\int\frac{\log(c(d+ex)^p)}{\sqrt{-\sqrt{-f}}+\sqrt[4]{g}x}\,dx,x,\sqrt{x}\right)}{2\sqrt{-f}\sqrt[4]{g}}+\frac{\text{Subst}\left(\int\frac{\log(c(d+ex)^p)}{\sqrt[4]{-f}+\sqrt[4]{g}x}\,dx,x,\sqrt{x}\right)}{2\sqrt{-f}\sqrt[4]{g}}
\end{aligned}$$

$$= -\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}+\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}+\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{(ep)\mathrm{Subst}\left(\int\frac{\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}x\right)}{e\sqrt{-\sqrt{-f}}+d\sqrt[4]{g}}\right)}{d+ex}dx,x,\sqrt{x}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{(ep)\mathrm{Subst}\left(\int\frac{\log\left(\frac{e\left(\sqrt[4]{-f}-\sqrt[4]{g}x\right)}{e\sqrt[4]{-f}+d\sqrt[4]{g}}\right)}{d+ex}dx,x,\sqrt{x}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{(ep)\mathrm{Subst}\left(\int\frac{\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}+\sqrt[4]{g}x\right)}{e\sqrt{-\sqrt{-f}}-d\sqrt[4]{g}}\right)}{d+ex}dx,x,\sqrt{x}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{(ep)\mathrm{Subst}\left(\int\frac{\log\left(\frac{e\left(\sqrt[4]{-f}+\sqrt[4]{g}x\right)}{e\sqrt[4]{-f}-d\sqrt[4]{g}}\right)}{d+ex}dx,x,\sqrt{x}\right)}{2\sqrt{-f}\sqrt{g}}$$

$$= -\frac{\log\left(c(d+e\sqrt{x})^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c(d+e\sqrt{x})^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log\left(c(d+e\sqrt{x})^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}+\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c(d+e\sqrt{x})^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}+\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[4]{g}x}{e\sqrt{-\sqrt{-f}}-d\sqrt[4]{g}}\right)}{x}dx, x, d+e\sqrt{x}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[4]{g}x}{e\sqrt[4]{-f}-d\sqrt[4]{g}}\right)}{x}dx, x, d+e\sqrt{x}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{p\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt[4]{g}x}{e\sqrt{-\sqrt{-f}}+d\sqrt[4]{g}}\right)}{x}dx, x, d+e\sqrt{x}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{p\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt[4]{g}x}{e\sqrt[4]{-f}+d\sqrt[4]{g}}\right)}{x}dx, x, d+e\sqrt{x}\right)}{2\sqrt{-f}\sqrt{g}}$$

$$= -\frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt{-\sqrt{-f}}+\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt{-\sqrt{-f}}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}+\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{p\operatorname{Li}_2\left(-\frac{\sqrt[4]{g}(d+e\sqrt{x})}{e\sqrt{-\sqrt{-f}}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{p\operatorname{Li}_2\left(-\frac{\sqrt[4]{g}(d+e\sqrt{x})}{e\sqrt[4]{-f}-d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{p\operatorname{Li}_2\left(\frac{\sqrt[4]{g}(d+e\sqrt{x})}{e\sqrt{-\sqrt{-f}}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{p\operatorname{Li}_2\left(\frac{\sqrt[4]{g}(d+e\sqrt{x})}{e\sqrt[4]{-f}+d\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.78

$$\int \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)}{f+gx^2}\,dx = \frac{\log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}+d\sqrt[4]{g}}\right) - \log\left(c\left(d+e\sqrt{x}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{-f}-i\sqrt[4]{g}\sqrt{x}\right)}{e\sqrt[4]{-f}+id\sqrt[4]{g}}\right) - \log\left(c\left(d+e\right.\right.}{}$$

```
[In] Integrate[Log[c*(d + e*Sqrt[x])^p]/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e*Sqrt[x])^p]*Log[(e*((-f)^(1/4) - g^(1/4)*Sqrt[x]))/(e*(-f)^(1
/4) + d*g^(1/4))] - Log[c*(d + e*Sqrt[x])^p]*Log[(e*((-f)^(1/4) - I*g^(1/4)
*Sqrt[x]))/(e*(-f)^(1/4) + I*d*g^(1/4))] - Log[c*(d + e*Sqrt[x])^p]*Log[(e*
((-f)^(1/4) + I*g^(1/4)*Sqrt[x]))/(e*(-f)^(1/4) - I*d*g^(1/4))] + Log[c*(d
+ e*Sqrt[x])^p]*Log[(e*((-f)^(1/4) + g^(1/4)*Sqrt[x]))/(e*(-f)^(1/4) - d*g^
(1/4))] + p*PolyLog[2, -((g^(1/4)*(d + e*Sqrt[x]))/(e*(-f)^(1/4) - d*g^(1/4
)))] - p*PolyLog[2, (I*g^(1/4)*(d + e*Sqrt[x]))/(e*(-f)^(1/4) + I*d*g^(1/4)
)] - p*PolyLog[2, (g^(1/4)*(d + e*Sqrt[x]))/(I*e*(-f)^(1/4) + d*g^(1/4))] +
 p*PolyLog[2, (g^(1/4)*(d + e*Sqrt[x]))/(e*(-f)^(1/4) + d*g^(1/4))])/(2*Sqr
t[-f]*Sqrt[g])
```

Maple [F]

$$\int \frac{\ln \left(c{\left(d+e\sqrt{x}\right)}^{p}\right)}{g\,x^{2}+f}dx$$

```
[In] int(ln(c*(d+e*x^(1/2))^p)/(g*x^2+f),x)
[Out] int(ln(c*(d+e*x^(1/2))^p)/(g*x^2+f),x)
```

Fricas [F]

$$\int \frac{\log\left(c(d+e\sqrt{x})^{p}\right)}{f+gx^{2}}\,dx = \int \frac{\log\left((e\sqrt{x}+d)^{p}c\right)}{gx^{2}+f}\,dx$$

```
[In] integrate(log(c*(d+e*x^(1/2))^p)/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log((e*sqrt(x) + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+e\sqrt{x})^{p}\right)}{f+gx^{2}}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(d+e*x**(1/2))**p)/(g*x**2+f),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(c(d+e\sqrt{x})^{p}\right)}{f+gx^{2}}\,dx = \int \frac{\log\left((e\sqrt{x}+d)^{p}c\right)}{gx^{2}+f}\,dx$$

```
[In] integrate(log(c*(d+e*x^(1/2))^p)/(g*x^2+f),x, algorithm="maxima")
[Out] integrate(log((e*sqrt(x) + d)^p*c)/(g*x^2 + f), x)
```

Giac [F]

$$\int \frac{\log \left(c\left(d+e\sqrt{x}\right)^p\right)}{f+gx^2}\,dx = \int \frac{\log \left(\left(e\sqrt{x}+d\right)^p c\right)}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(d+e*x^(1/2))^p)/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log((e*sqrt(x) + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c\left(d+e\sqrt{x}\right)^p\right)}{f+gx^2}\,dx = \int \frac{\ln \left(c\,{\left(d+e\,\sqrt{x}\right)}^p\right)}{g\,x^2+f}\,dx$$

```
[In] int(log(c*(d + e*x^(1/2))^p)/(f + g*x^2),x)
[Out] int(log(c*(d + e*x^(1/2))^p)/(f + g*x^2), x)
```

3.267 $\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f+gx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 561

$$\begin{aligned}
\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f+gx^2}\,dx &= -\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} \\
&\quad - \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\frac{e\left(\sqrt[4]{g}+\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} \\
&\quad + \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt[4]{-f}}{\sqrt{x}}\right)}{d\sqrt[4]{-f}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} \\
&\quad + \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\frac{e\left(\sqrt[4]{g}+\frac{\sqrt[4]{-f}}{\sqrt{x}}\right)}{d\sqrt[4]{-f}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} \\
&\quad - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-\sqrt{-f}}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{-f}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt[4]{-f}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} \\
&\quad - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-\sqrt{-f}}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{-f}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt[4]{-f}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}
\end{aligned}$$

```
[Out] 1/2*ln(c*(d+e/x^(1/2))^p)*ln(e*(g^(1/4)-(-f)^(1/4)/x^(1/2))/(d*(-f)^(1/4)+e
*g^(1/4)))/(-f)^(1/2)/g^(1/2)+1/2*ln(c*(d+e/x^(1/2))^p)*ln(-e*(g^(1/4)+(-f)
^(1/4)/x^(1/2))/(d*(-f)^(1/4)-e*g^(1/4)))/(-f)^(1/2)/g^(1/2)-1/2*ln(c*(d+e/
x^(1/2))^p)*ln(e*(g^(1/4)-(-(-f)^(1/2))^(1/2)/x^(1/2))/(e*g^(1/4)+d*(-(-f)^
(1/2))^(1/2)))/(-f)^(1/2)/g^(1/2)-1/2*ln(c*(d+e/x^(1/2))^p)*ln(-e*(g^(1/4)+
(-(-f)^(1/2))^(1/2)/x^(1/2))/(-e*g^(1/4)+d*(-(-f)^(1/2))^(1/2)))/(-f)^(1/2)
/g^(1/2)+1/2*p*polylog(2,(-f)^(1/4)*(d+e/x^(1/2))/(d*(-f)^(1/4)-e*g^(1/4)))
/(-f)^(1/2)/g^(1/2)+1/2*p*polylog(2,(-f)^(1/4)*(d+e/x^(1/2))/(d*(-f)^(1/4)+
e*g^(1/4)))/(-f)^(1/2)/g^(1/2)-1/2*p*polylog(2,(d+e/x^(1/2))*(-(-f)^(1/2))^
(1/2)/(-e*g^(1/4)+d*(-(-f)^(1/2))^(1/2)))/(-f)^(1/2)/g^(1/2)-1/2*p*polylog(
2,(d+e/x^(1/2))*(-(-f)^(1/2))^(1/2)/(e*g^(1/4)+d*(-(-f)^(1/2))^(1/2)))/(-f)
^(1/2)/g^(1/2)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules

used = {2522, 2525, 269, 281, 211, 2463, 266, 2441, 2440, 2438}

$$\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f+gx^2}\,dx = -\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\frac{e\left(\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}+\sqrt[4]{g}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt[4]{-f}}{\sqrt{x}}\right)}{d\sqrt[4]{-f}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\frac{e\left(\frac{\sqrt[4]{-f}}{\sqrt{x}}+\sqrt[4]{g}\right)}{d\sqrt[4]{-f}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-\sqrt{-f}}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{-f}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt[4]{-f}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-\sqrt{-f}}\left(d+\frac{e}{\sqrt{x}}\right)}{\sqrt{-\sqrt{-f}}d+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt[4]{-f}\left(d+\frac{e}{\sqrt{x}}\right)}{\sqrt[4]{-f}d+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}$$

```
[In] Int[Log[c*(d + e/Sqrt[x])^p]/(f + g*x^2),x]
```

```
[Out] -1/2*(Log[c*(d + e/Sqrt[x])^p]*Log[(e*(g^(1/4) - Sqrt[-Sqrt[-f]]/Sqrt[x]))/
(d*Sqrt[-Sqrt[-f]] + e*g^(1/4))])/(Sqrt[-f]*Sqrt[g]) - (Log[c*(d + e/Sqrt[x
])^p]*Log[-((e*(g^(1/4) + Sqrt[-Sqrt[-f]]/Sqrt[x]))/(d*Sqrt[-Sqrt[-f]] - e*
g^(1/4)))])/(2*Sqrt[-f]*Sqrt[g]) + (Log[c*(d + e/Sqrt[x])^p]*Log[(e*(g^(1/4
) - (-f)^(1/4)/Sqrt[x]))/(d*(-f)^(1/4) + e*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g])
+ (Log[c*(d + e/Sqrt[x])^p]*Log[-((e*(g^(1/4) + (-f)^(1/4)/Sqrt[x]))/(d*(-f
)^(1/4) - e*g^(1/4)))])/(2*Sqrt[-f]*Sqrt[g]) - (p*PolyLog[2, (Sqrt[-Sqrt[-f
]]*(d + e/Sqrt[x]))/(d*Sqrt[-Sqrt[-f]] - e*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g])
+ (p*PolyLog[2, ((-f)^(1/4)*(d + e/Sqrt[x]))/(d*(-f)^(1/4) - e*g^(1/4))])/(
2*Sqrt[-f]*Sqrt[g]) - (p*PolyLog[2, (Sqrt[-Sqrt[-f]]*(d + e/Sqrt[x]))/(d*Sq
rt[-Sqrt[-f]] + e*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g]) + (p*PolyLog[2, ((-f)^(1/
4)*(d + e/Sqrt[x]))/(d*(-f)^(1/4) + e*g^(1/4))])/(2*Sqrt[-f]*Sqrt[g])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2522

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Su
bst[Int[x^(k - 1)*(f + g*x^(k*s))^r*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x],
 x, x^(1/k)], x] /; IntegerQ[k*s]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q,
r, s}, x] && FractionQ[n]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{f+gx^4}\,dx, x, \sqrt{x}\right)\\
&= -\left(2\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{\left(f+\frac{g}{x^4}\right)x^3}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= -\left(2\text{Subst}\left(\int \left(-\frac{fx\log\left(c(d+ex)^p\right)}{2\sqrt{-f}\sqrt{g}\left(\sqrt{-f}\sqrt{g}-fx^2\right)}-\frac{fx\log\left(c(d+ex)^p\right)}{2\sqrt{-f}\sqrt{g}\left(\sqrt{-f}\sqrt{g}+fx^2\right)}\right)dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= -\frac{\sqrt{-f}\,\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{\sqrt{-f}\sqrt{g}-fx^2}\,dx, x, \frac{1}{\sqrt{x}}\right)}{\sqrt{g}}-\frac{\sqrt{-f}\,\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{\sqrt{-f}\sqrt{g}+fx^2}\,dx, x, \frac{1}{\sqrt{x}}\right)}{\sqrt{g}}\\
&= -\frac{\sqrt{-f}\,\text{Subst}\left(\int \left(\frac{\sqrt{-\sqrt{-f}}\log(c(d+ex)^p)}{2f\left(\sqrt[4]{g}-\sqrt{-\sqrt{-f}}x\right)}-\frac{\sqrt{-\sqrt{-f}}\log(c(d+ex)^p)}{2f\left(\sqrt[4]{g}+\sqrt{-\sqrt{-f}}x\right)}\right)dx, x, \frac{1}{\sqrt{x}}\right)}{\sqrt{g}}\\
&\quad-\frac{\sqrt{-f}\,\text{Subst}\left(\int \left(-\frac{\sqrt[4]{-f}\log(c(d+ex)^p)}{2f\left(\sqrt[4]{g}-\sqrt[4]{-f}x\right)}+\frac{\sqrt[4]{-f}\log(c(d+ex)^p)}{2f\left(\sqrt[4]{g}+\sqrt[4]{-f}x\right)}\right)dx, x, \frac{1}{\sqrt{x}}\right)}{\sqrt{g}}\\
&= -\frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt[4]{g}-\sqrt{-\sqrt{-f}}x}\,dx, x, \frac{1}{\sqrt{x}}\right)}{2\sqrt{-\sqrt{-f}}\sqrt{g}}+\frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt[4]{g}+\sqrt{-\sqrt{-f}}x}\,dx, x, \frac{1}{\sqrt{x}}\right)}{2\sqrt{-\sqrt{-f}}\sqrt{g}}\\
&\quad-\frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt[4]{g}-\sqrt[4]{-f}x}\,dx, x, \frac{1}{\sqrt{x}}\right)}{2\sqrt[4]{-f}\sqrt{g}}+\frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt[4]{g}+\sqrt[4]{-f}x}\,dx, x, \frac{1}{\sqrt{x}}\right)}{2\sqrt[4]{-f}\sqrt{g}}
\end{aligned}
$$

$$= -\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\frac{e\left(\sqrt[4]{g}+\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt[4]{-f}}{\sqrt{x}}\right)}{d\sqrt[4]{-f}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\frac{e\left(\sqrt[4]{g}+\frac{\sqrt[4]{-f}}{\sqrt{x}}\right)}{d\sqrt[4]{-f}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{(ep)\text{Subst}\left(\int \frac{\log\left(\frac{e\left(\sqrt[4]{g}-\sqrt{-\sqrt{-f}}x\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{d+ex}dx, x, \frac{1}{\sqrt{x}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{(ep)\text{Subst}\left(\int \frac{\log\left(\frac{e\left(\sqrt[4]{g}+\sqrt{-\sqrt{-f}}x\right)}{-d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{d+ex}dx, x, \frac{1}{\sqrt{x}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{(ep)\text{Subst}\left(\int \frac{\log\left(\frac{e\left(\sqrt[4]{g}-\sqrt[4]{-f}x\right)}{d\sqrt[4]{-f}+e\sqrt[4]{g}}\right)}{d+ex}dx, x, \frac{1}{\sqrt{x}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{(ep)\text{Subst}\left(\int \frac{\log\left(\frac{e\left(\sqrt[4]{g}+\sqrt[4]{-f}x\right)}{-d\sqrt[4]{-f}+e\sqrt[4]{g}}\right)}{d+ex}dx, x, \frac{1}{\sqrt{x}}\right)}{2\sqrt{-f}\sqrt{g}}$$

$$= -\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\frac{e\left(\sqrt[4]{g}+\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt[4]{-f}}{\sqrt{x}}\right)}{d\sqrt[4]{-f}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\frac{e\left(\sqrt[4]{g}+\frac{\sqrt[4]{-f}}{\sqrt{x}}\right)}{d\sqrt[4]{-f}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-\sqrt{-f}}x}{-d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{x}\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{p\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-\sqrt{-f}}x}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{x}\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt[4]{-f}x}{-d\sqrt[4]{-f}+e\sqrt[4]{g}}\right)}{x}\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{p\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt[4]{-f}x}{d\sqrt[4]{-f}+e\sqrt[4]{g}}\right)}{x}\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{2\sqrt{-f}\sqrt{g}}$$

$$= -\frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\frac{e\left(\sqrt[4]{g}+\frac{\sqrt{-\sqrt{-f}}}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(\frac{e\left(\sqrt[4]{g}-\frac{\sqrt[4]{-f}}{\sqrt{x}}\right)}{d\sqrt[4]{-f}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\frac{e\left(\sqrt[4]{g}+\frac{\sqrt[4]{-f}}{\sqrt{x}}\right)}{d\sqrt[4]{-f}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{p\operatorname{Li}_2\left(\frac{\sqrt{-\sqrt{-f}}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{p\operatorname{Li}_2\left(\frac{\sqrt[4]{-f}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt[4]{-f}-e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} - \frac{p\operatorname{Li}_2\left(\frac{\sqrt{-\sqrt{-f}}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt{-\sqrt{-f}}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}} + \frac{p\operatorname{Li}_2\left(\frac{\sqrt[4]{-f}\left(d+\frac{e}{\sqrt{x}}\right)}{d\sqrt[4]{-f}+e\sqrt[4]{g}}\right)}{2\sqrt{-f}\sqrt{g}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 912, normalized size of antiderivative = 1.63

$$\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f+gx^2}\,dx$$

$$= \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)\log\left(-\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right) - p\log\left(-\frac{\sqrt[4]{g}\left(e+d\sqrt{x}\right)}{d\sqrt[4]{-f}-e\sqrt[4]{g}}\right)\log\left(-\sqrt[4]{-f}-\sqrt[4]{g}\sqrt{x}\right) - \log\left(c\left(d\right.\right.}{}$$

```
[In] Integrate[Log[c*(d + e/Sqrt[x])^p]/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e/Sqrt[x])^p]*Log[-(-f)^(1/4) - g^(1/4)*Sqrt[x]] - p*Log[-((g^(
1/4)*(e + d*Sqrt[x]))/(d*(-f)^(1/4) - e*g^(1/4)))]*Log[-(-f)^(1/4) - g^(1/4
)*Sqrt[x]] - Log[c*(d + e/Sqrt[x])^p]*Log[(-I)*(-f)^(1/4) - g^(1/4)*Sqrt[x]
] + p*Log[(I*g^(1/4)*(e + d*Sqrt[x]))/(d*(-f)^(1/4) + I*e*g^(1/4))]*Log[(-I
)*(-f)^(1/4) - g^(1/4)*Sqrt[x]] - Log[c*(d + e/Sqrt[x])^p]*Log[I*(-f)^(1/4)
 - g^(1/4)*Sqrt[x]] + p*Log[(g^(1/4)*(e + d*Sqrt[x]))/(I*d*(-f)^(1/4) + e*g
```

```
^(1/4))]*Log[I*(-f)^(1/4) - g^(1/4)*Sqrt[x]] + Log[c*(d + e/Sqrt[x])^p]*Log
[(-f)^(1/4) - g^(1/4)*Sqrt[x]] - p*Log[(g^(1/4)*(e + d*Sqrt[x]))/(d*(-f)^(1
/4) + e*g^(1/4))]*Log[(-f)^(1/4) - g^(1/4)*Sqrt[x]] - p*Log[I*(-f)^(1/4) -
g^(1/4)*Sqrt[x]]*Log[((-I)*g^(1/4)*Sqrt[x])/(-f)^(1/4)] - p*Log[(-I)*(-f)^(
1/4) - g^(1/4)*Sqrt[x]]*Log[(I*g^(1/4)*Sqrt[x])/(-f)^(1/4)] + p*Log[(-f)^(1
/4) - g^(1/4)*Sqrt[x]]*Log[(g^(1/4)*Sqrt[x])/(-f)^(1/4)] + p*Log[-(-f)^(1/4
) - g^(1/4)*Sqrt[x]]*Log[(f*g^(1/4)*Sqrt[x])/(-f)^(5/4)] - p*PolyLog[2, (d*
((-f)^(1/4) - g^(1/4)*Sqrt[x]))/(d*(-f)^(1/4) + e*g^(1/4))] + p*PolyLog[2,
(d*((-f)^(1/4) - I*g^(1/4)*Sqrt[x]))/(d*(-f)^(1/4) + I*e*g^(1/4))] + p*Poly
Log[2, (d*((-f)^(1/4) + I*g^(1/4)*Sqrt[x]))/(d*(-f)^(1/4) - I*e*g^(1/4))] -
 p*PolyLog[2, (d*((-f)^(1/4) + g^(1/4)*Sqrt[x]))/(d*(-f)^(1/4) - e*g^(1/4))
] - p*PolyLog[2, 1 - (I*g^(1/4)*Sqrt[x])/(-f)^(1/4)] - p*PolyLog[2, 1 + (I*
g^(1/4)*Sqrt[x])/(-f)^(1/4)] + p*PolyLog[2, 1 + (g^(1/4)*Sqrt[x])/(-f)^(1/4
)] + p*PolyLog[2, 1 + (f*g^(1/4)*Sqrt[x])/(-f)^(5/4)])/(2*Sqrt[-f]*Sqrt[g])
```

Maple [F]

$$\int \frac{\ln \left(c \left(d+\frac{e}{\sqrt{x}}\right)^{p}\right)}{g\,x^{2}+f}dx$$

```
[In] int(ln(c*(d+e/x^(1/2))^p)/(g*x^2+f),x)
[Out] int(ln(c*(d+e/x^(1/2))^p)/(g*x^2+f),x)
```

Fricas [F]

$$\int \frac{\log \left(c \left(d+\frac{e}{\sqrt{x}}\right)^{p}\right)}{f+gx^{2}}\,dx=\int \frac{\log \left(c \left(d+\frac{e}{\sqrt{x}}\right)^{p}\right)}{gx^{2}+f}\,dx$$

```
[In] integrate(log(c*(d+e/x^(1/2))^p)/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log(c*((d*x + e*sqrt(x))/x)^p)/(g*x^2 + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c \left(d+\frac{e}{\sqrt{x}}\right)^{p}\right)}{f+gx^{2}}\,dx=\text{Timed out}$$

```
[In] integrate(ln(c*(d+e/x**(1/2))**p)/(g*x**2+f),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(d+e/x^(1/2))^p)/(g*x^2+f),x, algorithm="maxima")
[Out] integrate(log(c*(d + e/sqrt(x))^p)/(g*x^2 + f), x)
```

Giac [F]

$$\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(d+e/x^(1/2))^p)/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log(c*(d + e/sqrt(x))^p)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{f+gx^2}\,dx = \int \frac{\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^p\right)}{g\,x^2+f}\,dx$$

```
[In] int(log(c*(d + e/x^(1/2))^p)/(f + g*x^2),x)
[Out] int(log(c*(d + e/x^(1/2))^p)/(f + g*x^2), x)
```

3.268 $\int (f+gx^2)^3 \log\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 338

$$\begin{aligned}\int (f+gx^2)^3\log\left(c(d+ex^2)^p\right)\,dx &= -2f^3px+\frac{2df^2gpx}{e}-\frac{6d^2fg^2px}{5e^2}+\frac{2d^3g^3px}{7e^3}-\frac{2}{3}f^2gpx^3\\&+\frac{2dfg^2px^3}{5e}-\frac{2d^2g^3px^3}{21e^2}-\frac{6}{25}fg^2px^5+\frac{2dg^3px^5}{35e}-\frac{2}{49}g^3px^7+\frac{2\sqrt{d}f^3p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}\\&-\frac{2d^{3/2}f^2gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}}+\frac{6d^{5/2}fg^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}}-\frac{2d^{7/2}g^3p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}}\\&+f^3x\log\left(c(d+ex^2)^p\right)+f^2gx^3\log\left(c(d+ex^2)^p\right)+\frac{3}{5}fg^2x^5\log\left(c(d+ex^2)^p\right)+\frac{1}{7}g^3x^7\log\left(c(d+ex^2)^p\right)\end{aligned}$$

```
[Out] -2*f^3*p*x+2*d*f^2*g*p*x/e-6/5*d^2*f*g^2*p*x/e^2+2/7*d^3*g^3*p*x/e^3-2/3*f^
2*g*p*x^3+2/5*d*f*g^2*p*x^3/e-2/21*d^2*g^3*p*x^3/e^2-6/25*f*g^2*p*x^5+2/35*
d*g^3*p*x^5/e-2/49*g^3*p*x^7-2*d^(3/2)*f^2*g*p*arctan(x*e^(1/2)/d^(1/2))/e^
(3/2)+6/5*d^(5/2)*f*g^2*p*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)-2/7*d^(7/2)*g^3
*p*arctan(x*e^(1/2)/d^(1/2))/e^(7/2)+f^3*x*ln(c*(e*x^2+d)^p)+f^2*g*x^3*ln(c
*(e*x^2+d)^p)+3/5*f*g^2*x^5*ln(c*(e*x^2+d)^p)+1/7*g^3*x^7*ln(c*(e*x^2+d)^p)
+2*f^3*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2521, 2498, 327, 211, 2505, 308}

$$\int (f+gx^2)^3 \log\left(c(d+ex^2)^p\right)\,dx = -\frac{2d^{3/2}f^2gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}} + \frac{6d^{5/2}fg^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}} - \frac{2d^{7/2}g^3p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}} + \frac{2\sqrt{d}f^3p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + f^3x\log\left(c(d+ex^2)^p\right) + f^2gx^3\log\left(c(d+ex^2)^p\right) + \frac{3}{5}fg^2x^5\log\left(c(d+ex^2)^p\right) + \frac{1}{7}g^3x^7\log\left(c(d+ex^2)^p\right) + \frac{2d^3}{7}$$

```
[In] Int[(f + g*x^2)^3*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f^3*p*x + (2*d*f^2*g*p*x)/e - (6*d^2*f*g^2*p*x)/(5*e^2) + (2*d^3*g^3*p*x
)/(7*e^3) - (2*f^2*g*p*x^3)/3 + (2*d*f*g^2*p*x^3)/(5*e) - (2*d^2*g^3*p*x^3)
/(21*e^2) - (6*f*g^2*p*x^5)/25 + (2*d*g^3*p*x^5)/(35*e) - (2*g^3*p*x^7)/49
+ (2*Sqrt[d]*f^3*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*f^2*g*
p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2) + (6*d^(5/2)*f*g^2*p*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/(5*e^(5/2)) - (2*d^(7/2)*g^3*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(
7*e^(7/2)) + f^3*x*Log[c*(d + e*x^2)^p] + f^2*g*x^3*Log[c*(d + e*x^2)^p] +
(3*f*g^2*x^5*Log[c*(d + e*x^2)^p])/5 + (g^3*x^7*Log[c*(d + e*x^2)^p])/7
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(f^3 \log \left(c\left(d+ex^2\right)^p\right)+3f^2gx^2 \log \left(c\left(d+ex^2\right)^p\right)+3fg^2x^4 \log \left(c\left(d+ex^2\right)^p\right) + g^3x^6 \log \left(c\left(d+ex^2\right)^p\right)\right) dx \\
&= f^3 \int \log \left(c\left(d+ex^2\right)^p\right) dx+\left(3f^2g\right) \int x^2 \log \left(c\left(d+ex^2\right)^p\right) dx + \left(3fg^2\right) \int x^4 \log \left(c\left(d+ex^2\right)^p\right) dx+g^3 \int x^6 \log \left(c\left(d+ex^2\right)^p\right) dx \\
&= f^3x \log \left(c\left(d+ex^2\right)^p\right)+f^2gx^3 \log \left(c\left(d+ex^2\right)^p\right)+\frac{3}{5}fg^2x^5 \log \left(c\left(d+ex^2\right)^p\right) + \frac{1}{7}g^3x^7 \log \left(c\left(d+ex^2\right)^p\right)-\left(2ef^3p\right) \int \frac{x^2}{d+ex^2} dx-\left(2ef^2gp\right) \int \frac{x^4}{d+ex^2} dx \\
&\quad - \frac{1}{5}\left(6efg^2p\right) \int \frac{x^6}{d+ex^2} dx-\frac{1}{7}\left(2eg^3p\right) \int \frac{x^8}{d+ex^2} dx \\
&= -2f^3px+f^3x \log \left(c\left(d+ex^2\right)^p\right)+f^2gx^3 \log \left(c\left(d+ex^2\right)^p\right) + \frac{3}{5}fg^2x^5 \log \left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^3x^7 \log \left(c\left(d+ex^2\right)^p\right) \\
&\quad + \left(2df^3p\right) \int \frac{1}{d+ex^2} dx-\left(2ef^2gp\right) \int \left(-\frac{d}{e^2}+\frac{x^2}{e}+\frac{d^2}{e^2\left(d+ex^2\right)}\right) dx \\
&\quad - \frac{1}{5}\left(6efg^2p\right) \int \left(\frac{d^2}{e^3}-\frac{dx^2}{e^2}+\frac{x^4}{e}-\frac{d^3}{e^3\left(d+ex^2\right)}\right) dx \\
&\quad - \frac{1}{7}\left(2eg^3p\right) \int \left(-\frac{d^3}{e^4}+\frac{d^2x^2}{e^3}-\frac{dx^4}{e^2}+\frac{x^6}{e}+\frac{d^4}{e^4\left(d+ex^2\right)}\right) dx
\end{aligned}$$

$$
\begin{aligned}
&= -2f^3px + \frac{2df^2gpx}{e} - \frac{6d^2fg^2px}{5e^2} + \frac{2d^3g^3px}{7e^3} - \frac{2}{3}f^2gpx^3 + \frac{2dfg^2px^3}{5e} - \frac{2d^2g^3px^3}{21e^2} \\
&\quad - \frac{6}{25}fg^2px^5 + \frac{2dg^3px^5}{35e} - \frac{2}{49}g^3px^7 + \frac{2\sqrt{d}f^3p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + f^3x\log\left(c\left(d+ex^2\right)^p\right) \\
&\quad + f^2gx^3\log\left(c\left(d+ex^2\right)^p\right) + \frac{3}{5}fg^2x^5\log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{7}g^3x^7\log\left(c\left(d+ex^2\right)^p\right) \\
&\quad - \frac{(2d^2f^2gp)\int\frac{1}{d+ex^2}\,dx}{e} + \frac{(6d^3fg^2p)\int\frac{1}{d+ex^2}\,dx}{5e^2} - \frac{(2d^4g^3p)\int\frac{1}{d+ex^2}\,dx}{7e^3} \\
&= -2f^3px + \frac{2df^2gpx}{e} - \frac{6d^2fg^2px}{5e^2} + \frac{2d^3g^3px}{7e^3} - \frac{2}{3}f^2gpx^3 + \frac{2dfg^2px^3}{5e} \\
&\quad - \frac{2d^2g^3px^3}{21e^2} - \frac{6}{25}fg^2px^5 + \frac{2dg^3px^5}{35e} - \frac{2}{49}g^3px^7 + \frac{2\sqrt{d}f^3p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} \\
&\quad - \frac{2d^{3/2}f^2gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}} + \frac{6d^{5/2}fg^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}} - \frac{2d^{7/2}g^3p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}} \\
&\quad + f^3x\log\left(c\left(d+ex^2\right)^p\right) + f^2gx^3\log\left(c\left(d+ex^2\right)^p\right) + \frac{3}{5}fg^2x^5\log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{7}g^3x^7\log\left(c\left(d+ex^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.64

$$
\begin{aligned}
&\int \left(f+gx^2\right)^3\log\left(c\left(d+ex^2\right)^p\right)\,dx = \\
&-\frac{2px(-525d^3g^3 + 35d^2eg^2(63f+5gx^2) - 105de^2g(35f^2+7fgx^2+g^2x^4) + e^3(3675f^3+1225f^2gx^2+4}{3675e^3} \\
&-\frac{2\sqrt{d}(-35e^3f^3 + 35de^2f^2g - 21d^2efg^2 + 5d^3g^3)\,p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{35e^{7/2}} \\
&+\frac{1}{35}x\left(35f^3+35f^2gx^2+21fg^2x^4+5g^3x^6\right)\log\left(c\left(d+ex^2\right)^p\right)
\end{aligned}
$$

```
[In] Integrate[(f + g*x^2)^3*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (-2*p*x*(-525*d^3*g^3 + 35*d^2*e*g^2*(63*f + 5*g*x^2) - 105*d*e^2*g*(35*f^2
 + 7*f*g*x^2 + g^2*x^4) + e^3*(3675*f^3 + 1225*f^2*g*x^2 + 441*f*g^2*x^4 +
75*g^3*x^6)))/(3675*e^3) - (2*Sqrt[d]*(-35*e^3*f^3 + 35*d*e^2*f^2*g - 21*d^
2*e*f*g^2 + 5*d^3*g^3)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(35*e^(7/2)) + (x*(35
*f^3 + 35*f^2*g*x^2 + 21*f*g^2*x^4 + 5*g^3*x^6)*Log[c*(d + e*x^2)^p])/35
```

Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.76

method	result
parts	$\frac{g^3x^7\ln(c(e\,x^2+d)^p)}{7}+\frac{3f\,g^2x^5\ln(c(e\,x^2+d)^p)}{5}+f^2g\,x^3\ln\left(c(e\,x^2+d)^p\right)+f^3x\ln\left(c(e\,x^2+d)^p\right)-\frac{2pe\left(-\frac{-\frac{5}{7}e\cdots}{}\right.}{}$
risch	Expression too large to display

```
[In] int((g*x^2+f)^3*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*g^3*x^7*ln(c*(e*x^2+d)^p)+3/5*f*g^2*x^5*ln(c*(e*x^2+d)^p)+f^2*g*x^3*ln(
c*(e*x^2+d)^p)+f^3*x*ln(c*(e*x^2+d)^p)-2/35*p*e*(-1/e^4*(-5/7*e^3*g^3*x^7+d
*e^2*g^3*x^5-21/5*e^3*f*g^2*x^5-5/3*d^2*e*g^3*x^3+7*d*e^2*f*g^2*x^3-35/3*e^
3*f^2*g*x^3+5*d^3*x*g^3-21*d^2*e*x*f*g^2+35*d*e^2*x*f^2*g-35*x*e^3*f^3)+d*(
5*d^3*g^3-21*d^2*e*f*g^2+35*d*e^2*f^2*g-35*e^3*f^3)/e^4/(d*e)^(1/2)*arctan(
x*e/(d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.76

$$\int \left(f+gx^2\right)^3\log\left(c\left(d+ex^2\right)^p\right)\,dx$$

$$= \left[-\frac{150\,e^3g^3px^7+42\,(21\,e^3fg^2-5\,de^2g^3)px^5+70\,(35\,e^3f^2g-21\,de^2fg^2+5\,d^2eg^3)px^3+105\,(35\,e^3f^3-35\cdots}{}\right.$$

$$-\frac{150\,e^3g^3px^7+42\,(21\,e^3fg^2-5\,de^2g^3)px^5+70\,(35\,e^3f^2g-21\,de^2fg^2+5\,d^2eg^3)px^3-210\,(35\,e^3f^3-35\cdots}{}$$

```
[In] integrate((g*x^2+f)^3*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/3675*(150*e^3*g^3*p*x^7 + 42*(21*e^3*f*g^2 - 5*d*e^2*g^3)*p*x^5 + 70*(3
5*e^3*f^2*g - 21*d*e^2*f*g^2 + 5*d^2*e*g^3)*p*x^3 + 105*(35*e^3*f^3 - 35*d*
e^2*f^2*g + 21*d^2*e*f*g^2 - 5*d^3*g^3)*p*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqr
t(-d/e) - d)/(e*x^2 + d)) + 210*(35*e^3*f^3 - 35*d*e^2*f^2*g + 21*d^2*e*f*g
^2 - 5*d^3*g^3)*p*x - 105*(5*e^3*g^3*p*x^7 + 21*e^3*f*g^2*p*x^5 + 35*e^3*f^
2*g*p*x^3 + 35*e^3*f^3*p*x)*log(e*x^2 + d) - 105*(5*e^3*g^3*x^7 + 21*e^3*f*
g^2*x^5 + 35*e^3*f^2*g*x^3 + 35*e^3*f^3*x)*log(c))/e^3, -1/3675*(150*e^3*g^
```

```
3*p*x^7 + 42*(21*e^3*f*g^2 - 5*d*e^2*g^3)*p*x^5 + 70*(35*e^3*f^2*g - 21*d*e
^2*f*g^2 + 5*d^2*e*g^3)*p*x^3 - 210*(35*e^3*f^3 - 35*d*e^2*f^2*g + 21*d^2*e
*f*g^2 - 5*d^3*g^3)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 210*(35*e^3*f^3 -
 35*d*e^2*f^2*g + 21*d^2*e*f*g^2 - 5*d^3*g^3)*p*x - 105*(5*e^3*g^3*p*x^7 +
21*e^3*f*g^2*p*x^5 + 35*e^3*f^2*g*p*x^3 + 35*e^3*f^3*p*x)*log(e*x^2 + d) -
105*(5*e^3*g^3*x^7 + 21*e^3*f*g^2*x^5 + 35*e^3*f^2*g*x^3 + 35*e^3*f^3*x)*lo
g(c))/e^3]
```

Sympy [A] (verification not implemented)

Time = 124.72 (sec) , antiderivative size = 697, normalized size of antiderivative = 2.06

$$\int \left(f+gx^2\right)^3 \log\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$= \begin{cases} \left(f^3x+f^2gx^3+\frac{3fg^2x^5}{5}+\frac{g^3x^7}{7}\right)\log\left(0^pc\right) \\ \left(f^3x+f^2gx^3+\frac{3fg^2x^5}{5}+\frac{g^3x^7}{7}\right)\log\left(cd^p\right) \\ -2f^3px+f^3x\log\left(c\left(ex^2\right)^p\right)-\frac{2f^2gpx^3}{3}+f^2gx^3\log\left(c\left(ex^2\right)^p\right)-\frac{6fg^2px^5}{25}+\frac{3fg^2x^5\log\left(c\left(ex^2\right)^p\right)}{5}-\frac{2g^3px^7}{49}+ \\ -\frac{2d^4g^3p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{7e^4\sqrt{-\frac{d}{e}}}+\frac{d^4g^3\log\left(c\left(d+ex^2\right)^p\right)}{7e^4\sqrt{-\frac{d}{e}}}+\frac{6d^3fg^2p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{5e^3\sqrt{-\frac{d}{e}}}-\frac{3d^3fg^2\log\left(c\left(d+ex^2\right)^p\right)}{5e^3\sqrt{-\frac{d}{e}}}+\frac{2d^3g^3px}{7e^3}- \end{cases}$$

```
[In] integrate((g*x**2+f)**3*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise(((f**3*x + f**2*g*x**3 + 3*f*g**2*x**5/5 + g**3*x**7/7)*log(0**p*
c), Eq(d, 0) & Eq(e, 0)), ((f**3*x + f**2*g*x**3 + 3*f*g**2*x**5/5 + g**3*x
**7/7)*log(c*d**p), Eq(e, 0)), (-2*f**3*p*x + f**3*x*log(c*(e*x**2)**p) - 2
*f**2*g*p*x**3/3 + f**2*g*x**3*log(c*(e*x**2)**p) - 6*f*g**2*p*x**5/25 + 3*
f*g**2*x**5*log(c*(e*x**2)**p)/5 - 2*g**3*p*x**7/49 + g**3*x**7*log(c*(e*x*
*2)**p)/7, Eq(d, 0)), (-2*d**4*g**3*p*log(x - sqrt(-d/e))/(7*e**4*sqrt(-d/e
)) + d**4*g**3*log(c*(d + e*x**2)**p)/(7*e**4*sqrt(-d/e)) + 6*d**3*f*g**2*p
*log(x - sqrt(-d/e))/(5*e**3*sqrt(-d/e)) - 3*d**3*f*g**2*log(c*(d + e*x**2)
**p)/(5*e**3*sqrt(-d/e)) + 2*d**3*g**3*p*x/(7*e**3) - 2*d**2*f**2*g*p*log(x
 - sqrt(-d/e))/(e**2*sqrt(-d/e)) + d**2*f**2*g*log(c*(d + e*x**2)**p)/(e**2
*sqrt(-d/e)) - 6*d**2*f*g**2*p*x/(5*e**2) - 2*d**2*g**3*p*x**3/(21*e**2) +
2*d*f**3*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) - d*f**3*log(c*(d + e*x**2)**
p)/(e*sqrt(-d/e)) + 2*d*f**2*g*p*x/e + 2*d*f*g**2*p*x**3/(5*e) + 2*d*g**3*p
*x**5/(35*e) - 2*f**3*p*x + f**3*x*log(c*(d + e*x**2)**p) - 2*f**2*g*p*x**3
/3 + f**2*g*x**3*log(c*(d + e*x**2)**p) - 6*f*g**2*p*x**5/25 + 3*f*g**2*x**
5*log(c*(d + e*x**2)**p)/5 - 2*g**3*p*x**7/49 + g**3*x**7*log(c*(d + e*x**2
)**p)/7, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(f+gx^2\right)^3 \log\left(c\left(d+ex^2\right)^p\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x^2+f)^3*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.79

$$\begin{aligned}
&\int \left(f+gx^2\right)^3 \log\left(c\left(d+ex^2\right)^p\right)\,dx \\
&= -\frac{1}{49}\left(2\,g^3p-7\,g^3\log\left(c\right)\right)x^7 - \frac{\left(42\,efg^2p-10\,dg^3p-105\,efg^2\log\left(c\right)\right)x^5}{175\,e} \\
&\quad - \frac{\left(70\,e^2f^2gp-42\,defg^2p+10\,d^2g^3p-105\,e^2f^2g\log\left(c\right)\right)x^3}{105\,e^2} \\
&\quad + \frac{1}{35}\left(5\,g^3px^7+21\,fg^2px^5+35\,f^2gpx^3+35\,f^3px\right)\log\left(ex^2+d\right) \\
&\quad - \frac{\left(70\,e^3f^3p-70\,de^2f^2gp+42\,d^2efg^2p-10\,d^3g^3p-35\,e^3f^3\log\left(c\right)\right)x}{35\,e^3} \\
&\quad + \frac{2\left(35\,de^3f^3p-35\,d^2e^2f^2gp+21\,d^3efg^2p-5\,d^4g^3p\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{35\,\sqrt{de}e^3}
\end{aligned}$$

```
[In] integrate((g*x^2+f)^3*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -1/49*(2*g^3*p - 7*g^3*log(c))*x^7 - 1/175*(42*e*f*g^2*p - 10*d*g^3*p - 105
*e*f*g^2*log(c))*x^5/e - 1/105*(70*e^2*f^2*g*p - 42*d*e*f*g^2*p + 10*d^2*g^
3*p - 105*e^2*f^2*g*log(c))*x^3/e^2 + 1/35*(5*g^3*p*x^7 + 21*f*g^2*p*x^5 +
35*f^2*g*p*x^3 + 35*f^3*p*x)*log(e*x^2 + d) - 1/35*(70*e^3*f^3*p - 70*d*e^2
*f^2*g*p + 42*d^2*e*f*g^2*p - 10*d^3*g^3*p - 35*e^3*f^3*log(c))*x/e^3 + 2/3
5*(35*d*e^3*f^3*p - 35*d^2*e^2*f^2*g*p + 21*d^3*e*f*g^2*p - 5*d^4*g^3*p)*ar
ctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^3)
```

Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.88

$$\int \left(f+gx^2\right)^3\log\left(c\left(d+ex^2\right)^p\right)\,dx = x^3\left(\frac{d\left(\frac{6\,f\,g^2\,p}{5}-\frac{2\,d\,g^3\,p}{7\,e}\right)}{3\,e}-\frac{2\,f^2\,g\,p}{3}\right) - x\left(2\,f^3\,p+\frac{d\left(\frac{d\left(\frac{6\,f\,g^2\,p}{5}-\frac{2\,d\,g^3\,p}{7\,e}\right)}{e}-2\,f^2\,g\,p\right)}{e}\right) - x^5\left(\frac{6\,f\,g^2\,p}{25}-\frac{2\,d\,g^3\,p}{35\,e}\right) + \ln\left(c\left(e\,x^2+d\right)^p\right)\left(f^3\,x+f^2\,g\,x^3+\frac{3\,f\,g^2\,x^5}{5}+\frac{g^3\,x^7}{7}\right)-\frac{2\,g^3\,p\,x^7}{49} - \frac{2\,\sqrt{d}\,p\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e}\,p\,x\,\left(5\,d^3\,g^3-21\,d^2\,e\,f\,g^2+35\,d\,e^2\,f^2\,g-35\,e^3\,f^3\right)}{5\,p\,d^4\,g^3-21\,p\,d^3\,e\,f\,g^2+35\,p\,d^2\,e^2\,f^2\,g-35\,p\,d\,e^3\,f^3}\right)\,\left(5\,d^3\,g^3-21\,d^2\,e\,f\,g^2+35\,d\,e^2\,f^2\,g-35\,e^3\,f^3\right)}{35\,e^{7/2}}$$

```
[In] int(log(c*(d + e*x^2)^p)*(f + g*x^2)^3,x)
```

```
[Out] x^3*((d*((6*f*g^2*p)/5 - (2*d*g^3*p)/(7*e)))/(3*e) - (2*f^2*g*p)/3) - x*(2*
f^3*p + (d*((d*((6*f*g^2*p)/5 - (2*d*g^3*p)/(7*e)))/e - 2*f^2*g*p))/e) - x^
5*((6*f*g^2*p)/25 - (2*d*g^3*p)/(35*e)) + log(c*(d + e*x^2)^p)*(f^3*x + (g^
3*x^7)/7 + f^2*g*x^3 + (3*f*g^2*x^5)/5) - (2*g^3*p*x^7)/49 - (2*d^(1/2)*p*a
tan((d^(1/2)*e^(1/2)*p*x*(5*d^3*g^3 - 35*e^3*f^3 + 35*d*e^2*f^2*g - 21*d^2*
e*f*g^2))/(5*d^4*g^3*p - 35*d*e^3*f^3*p - 21*d^3*e*f*g^2*p + 35*d^2*e^2*f^2
*g*p))*(5*d^3*g^3 - 35*e^3*f^3 + 35*d*e^2*f^2*g - 21*d^2*e*f*g^2))/(35*e^(7
/2))
```

3.269 $\int (f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 221

$$\begin{aligned}\int (f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx &= -2f^2px+\frac{4dfgpx}{3e}-\frac{2d^2g^2px}{5e^2}-\frac{4}{9}fgpx^3+\frac{2dg^2px^3}{15e}-\frac{2}{25}g^2px^5\\&\quad+\frac{2\sqrt{d}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{4d^{3/2}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2d^{5/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}}\\&\quad+f^2x\log\left(c(d+ex^2)^p\right)+\frac{2}{3}fgx^3\log\left(c(d+ex^2)^p\right)+\frac{1}{5}g^2x^5\log\left(c(d+ex^2)^p\right)\end{aligned}$$

```
[Out] -2*f^2*p*x+4/3*d*f*g*p*x/e-2/5*d^2*g^2*p*x/e^2-4/9*f*g*p*x^3+2/15*d*g^2*p*x
^3/e-2/25*g^2*p*x^5-4/3*d^(3/2)*f*g*p*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)+2/5
*d^(5/2)*g^2*p*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)+f^2*x*ln(c*(e*x^2+d)^p)+2/
3*f*g*x^3*ln(c*(e*x^2+d)^p)+1/5*g^2*x^5*ln(c*(e*x^2+d)^p)+2*f^2*p*arctan(x*
e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2521, 2498, 327, 211, 2505, 308}

$$\begin{aligned}&\int (f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx\\&=-\frac{4d^{3/2}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2d^{5/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}}+\frac{2\sqrt{d}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}\\&\quad+f^2x\log\left(c(d+ex^2)^p\right)+\frac{2}{3}fgx^3\log\left(c(d+ex^2)^p\right)+\frac{1}{5}g^2x^5\log\left(c(d+ex^2)^p\right)-\frac{2d^2g^2px}{5e^2}+\frac{4dfgpx}{3e}+\frac{2dg^2px^3}{15e}-\end{aligned}$$

```
[In] Int[(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f^2*p*x + (4*d*f*g*p*x)/(3*e) - (2*d^2*g^2*p*x)/(5*e^2) - (4*f*g*p*x^3)/
9 + (2*d*g^2*p*x^3)/(15*e) - (2*g^2*p*x^5)/25 + (2*Sqrt[d]*f^2*p*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/Sqrt[e] - (4*d^(3/2)*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/
(3*e^(3/2)) + (2*d^(5/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*e^(5/2)) + f
^2*x*Log[c*(d + e*x^2)^p] + (2*f*g*x^3*Log[c*(d + e*x^2)^p])/3 + (g^2*x^5*L
og[c*(d + e*x^2)^p])/5
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
```

```
0] && LtQ[r, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(f^2\log\left(c\left(d+ex^2\right)^p\right)+2fgx^2\log\left(c\left(d+ex^2\right)^p\right)+g^2x^4\log\left(c\left(d+ex^2\right)^p\right)\right)\,dx\\
&= f^2\int \log\left(c\left(d+ex^2\right)^p\right)\,dx+(2fg)\int x^2\log\left(c\left(d+ex^2\right)^p\right)\,dx+g^2\int x^4\log\left(c\left(d+ex^2\right)^p\right)\,dx\\
&= f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log\left(c\left(d+ex^2\right)^p\right)\\
&\quad-\left(2ef^2p\right)\int \frac{x^2}{d+ex^2}\,dx-\frac{1}{3}(4efgp)\int \frac{x^4}{d+ex^2}\,dx-\frac{1}{5}\left(2eg^2p\right)\int \frac{x^6}{d+ex^2}\,dx\\
&= -2f^2px+f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log\left(c\left(d+ex^2\right)^p\right)\\
&\quad+\left(2df^2p\right)\int \frac{1}{d+ex^2}\,dx-\frac{1}{3}(4efgp)\int \left(-\frac{d}{e^2}+\frac{x^2}{e}+\frac{d^2}{e^2\left(d+ex^2\right)}\right)\,dx\\
&\quad-\frac{1}{5}\left(2eg^2p\right)\int \left(\frac{d^2}{e^3}-\frac{dx^2}{e^2}+\frac{x^4}{e}-\frac{d^3}{e^3\left(d+ex^2\right)}\right)\,dx\\
&= -2f^2px+\frac{4dfgpx}{3e}-\frac{2d^2g^2px}{5e^2}-\frac{4}{9}fgpx^3+\frac{2dg^2px^3}{15e}-\frac{2}{25}g^2px^5\\
&\quad+\frac{2\sqrt{d}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log\left(c\left(d+ex^2\right)^p\right)\\
&\quad+\frac{1}{5}g^2x^5\log\left(c\left(d+ex^2\right)^p\right)-\frac{\left(4d^2fgp\right)\int \frac{1}{d+ex^2}\,dx}{3e}+\frac{\left(2d^3g^2p\right)\int \frac{1}{d+ex^2}\,dx}{5e^2}\\
&= -2f^2px+\frac{4dfgpx}{3e}-\frac{2d^2g^2px}{5e^2}-\frac{4}{9}fgpx^3+\frac{2dg^2px^3}{15e}-\frac{2}{25}g^2px^5\\
&\quad+\frac{2\sqrt{d}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{4d^{3/2}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2d^{5/2}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}}\\
&\quad+f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log\left(c\left(d+ex^2\right)^p\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.68

$$\int (f+gx^2)^2 \log \left(c(d+ex^2)^p\right) \, dx$$

$$= \frac{30\sqrt{d}(15e^2f^2 - 10defg + 3d^2g^2)\, p \arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) + \sqrt{e}x\left(-2p(45d^2g^2 - 15deg(10f + gx^2) + e^2(225f^2 + 5\right.}{225e^{5/2}}$$

[In] Integrate[(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]

[Out] (30*Sqrt[d]*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[e]*x*(-2*p*(45*d^2*g^2 - 15*d*e*g*(10*f + g*x^2) + e^2*(225*f^2 + 50*f*g*x^2 + 9*g^2*x^4)) + 15*e^2*(15*f^2 + 10*f*g*x^2 + 3*g^2*x^4)*Log[c*(d + e*x^2)^p]))/(225*e^(5/2))

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.76

method	result
parts	$\frac{g^2x^5 \ln\left(c(e\,x^2+d)^p\right)}{5} + \frac{2fg\,x^3 \ln\left(c(e\,x^2+d)^p\right)}{3} + f^2x \ln\left(c(e\,x^2+d)^p\right) - \frac{2pe\left(\frac{\frac{3}{5}e^2g^2x^5 - de\,g^2x^3 + \frac{10}{3}e^2fg\,x^3 + 3d^2g^2x - 10d}{e^3}\right.}{}$
risch	$\frac{\ln(c)g^2x^5}{5} + x\ln(c)\,f^2 - \frac{i\pi\,g^2x^5 \operatorname{csgn}\left(i(e\,x^2+d)^p\right) \operatorname{csgn}\left(ic(e\,x^2+d)^p\right) \operatorname{csgn}(ic)}{10} + \frac{i\pi fg\,x^3 \operatorname{csgn}\left(i(e\,x^2+d)^p\right)\operatorname{csgn}\left(ic(e\,x^2+d)\right.}{3}$

[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)

[Out] 1/5*g^2*x^5*ln(c*(e*x^2+d)^p)+2/3*f*g*x^3*ln(c*(e*x^2+d)^p)+f^2*x*ln(c*(e*x^2+d)^p)-2/15*p*e*(1/e^3*(3/5*e^2*g^2*x^5-d*e*g^2*x^3+10/3*e^2*f*g*x^3+3*d^2*g^2*x-10*d*e*f*g*x+15*e^2*f^2*x)-d*(3*d^2*g^2-10*d*e*f*g+15*e^2*f^2)/e^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.83

$$\int \left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$= \left[-\frac{18\,e^2g^2px^5+10\,(10\,e^2fg-3\,deg^2)px^3-15\,(15\,e^2f^2-10\,defg+3\,d^2g^2)p\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2+2\,ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right)+\cdots}{\cdots}\right.$$
$$\left.-\frac{18\,e^2g^2px^5+10\,(10\,e^2fg-3\,deg^2)px^3-30\,(15\,e^2f^2-10\,defg+3\,d^2g^2)p\sqrt{\frac{d}{e}}\arctan\left(\frac{ex\sqrt{\frac{d}{e}}}{d}\right)+30\,(15\,e\cdots}{\cdots}\right.$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/225*(18*e^2*g^2*p*x^5 + 10*(10*e^2*f*g - 3*d*e*g^2)*p*x^3 - 15*(15*e^2*
f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) -
d)/(e*x^2 + d)) + 30*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*x - 15*(3*e^2*
g^2*p*x^5 + 10*e^2*f*g*p*x^3 + 15*e^2*f^2*p*x)*log(e*x^2 + d) - 15*(3*e^2*g
^2*x^5 + 10*e^2*f*g*x^3 + 15*e^2*f^2*x)*log(c))/e^2, -1/225*(18*e^2*g^2*p*x
^5 + 10*(10*e^2*f*g - 3*d*e*g^2)*p*x^3 - 30*(15*e^2*f^2 - 10*d*e*f*g + 3*d^
2*g^2)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 30*(15*e^2*f^2 - 10*d*e*f*g +
3*d^2*g^2)*p*x - 15*(3*e^2*g^2*p*x^5 + 10*e^2*f*g*p*x^3 + 15*e^2*f^2*p*x)*l
og(e*x^2 + d) - 15*(3*e^2*g^2*x^5 + 10*e^2*f*g*x^3 + 15*e^2*f^2*x)*log(c))/
e^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(231) = 462.

Time = 33.10 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.16

$$\int \left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$= \begin{cases} \left(f^2x+\frac{2fgx^3}{3}+\frac{g^2x^5}{5}\right)\log\left(0^pc\right) \\ \left(f^2x+\frac{2fgx^3}{3}+\frac{g^2x^5}{5}\right)\log\left(cd^p\right) \\ -2f^2px+f^2x\log\left(c\left(ex^2\right)^p\right)-\frac{4fgpx^3}{9}+\frac{2fgx^3\log\left(c\left(ex^2\right)^p\right)}{3}-\frac{2g^2px^5}{25}+\frac{g^2x^5\log\left(c\left(ex^2\right)^p\right)}{5} \\ \frac{2d^3g^2p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{5e^3\sqrt{-\frac{d}{e}}}-\frac{d^3g^2\log\left(c\left(d+ex^2\right)^p\right)}{5e^3\sqrt{-\frac{d}{e}}}-\frac{4d^2fgp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{3e^2\sqrt{-\frac{d}{e}}}+\frac{2d^2fg\log\left(c\left(d+ex^2\right)^p\right)}{3e^2\sqrt{-\frac{d}{e}}}-\frac{2d^2g^2px}{5e^2}+\frac{2df^2p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{e\sqrt{-\frac{d}{e}}}\cdots \end{cases}$$

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((((f**2*x + 2*f*g*x**3/3 + g**2*x**5/5)*log(0**p*c), Eq(d, 0) & Eq
(e, 0)), ((f**2*x + 2*f*g*x**3/3 + g**2*x**5/5)*log(c*d**p), Eq(e, 0)), (-2
*f**2*p*x + f**2*x*log(c*(e*x**2)**p) - 4*f*g*p*x**3/9 + 2*f*g*x**3*log(c*(
e*x**2)**p)/3 - 2*g**2*p*x**5/25 + g**2*x**5*log(c*(e*x**2)**p)/5, Eq(d, 0)
), (2*d**3*g**2*p*log(x - sqrt(-d/e))/(5*e**3*sqrt(-d/e)) - d**3*g**2*log(c
*(d + e*x**2)**p)/(5*e**3*sqrt(-d/e)) - 4*d**2*f*g*p*log(x - sqrt(-d/e))/(3
*e**2*sqrt(-d/e)) + 2*d**2*f*g*log(c*(d + e*x**2)**p)/(3*e**2*sqrt(-d/e)) -
 2*d**2*g**2*p*x/(5*e**2) + 2*d*f**2*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) -
 d*f**2*log(c*(d + e*x**2)**p)/(e*sqrt(-d/e)) + 4*d*f*g*p*x/(3*e) + 2*d*g**
2*p*x**3/(15*e) - 2*f**2*p*x + f**2*x*log(c*(d + e*x**2)**p) - 4*f*g*p*x**3
/9 + 2*f*g*x**3*log(c*(d + e*x**2)**p)/3 - 2*g**2*p*x**5/25 + g**2*x**5*log
(c*(d + e*x**2)**p)/5, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.79

$$\begin{aligned}\int \left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)\,dx=&-\frac{1}{25}\left(2\,g^2p-5\,g^2\log\left(c\right)\right)x^5\\&-\frac{2\left(10\,efgp-3\,dg^2p-15\,efg\log\left(c\right)\right)x^3}{45\,e}\\&+\frac{1}{15}\left(3\,g^2px^5+10\,fgpx^3+15\,f^2px\right)\log\left(ex^2+d\right)\\&-\frac{\left(30\,e^2f^2p-20\,defgp+6\,d^2g^2p-15\,e^2f^2\log\left(c\right)\right)x}{15\,e^2}\\&+\frac{2\left(15\,de^2f^2p-10\,d^2efgp+3\,d^3g^2p\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{15\,\sqrt{de}e^2}\end{aligned}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -1/25*(2*g^2*p - 5*g^2*log(c))*x^5 - 2/45*(10*e*f*g*p - 3*d*g^2*p - 15*e*f*
g*log(c))*x^3/e + 1/15*(3*g^2*p*x^5 + 10*f*g*p*x^3 + 15*f^2*p*x)*log(e*x^2
+ d) - 1/15*(30*e^2*f^2*p - 20*d*e*f*g*p + 6*d^2*g^2*p - 15*e^2*f^2*log(c))
*x/e^2 + 2/15*(15*d*e^2*f^2*p - 10*d^2*e*f*g*p + 3*d^3*g^2*p)*arctan(e*x/sq
rt(d*e))/(sqrt(d*e)*e^2)
```

Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.87

$$\begin{aligned}&\int (f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx\\&=\ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(f^2\,x+\frac{2\,f\,g\,x^3}{3}+\frac{g^2\,x^5}{5}\right)\\&\quad-x\left(2\,f^2\,p-\frac{d\left(\frac{4\,f\,g\,p}{3}-\frac{2\,d\,g^2\,p}{5\,e}\right)}{e}\right)-x^3\left(\frac{4\,f\,g\,p}{9}-\frac{2\,d\,g^2\,p}{15\,e}\right)-\frac{2\,g^2\,p\,x^5}{25}\\&\quad+\frac{2\,\sqrt{d}\,p\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e}\,p\,x\,\left(3\,d^2\,g^2-10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{3\,p\,d^3\,g^2-10\,p\,d^2\,e\,f\,g+15\,p\,d\,e^2\,f^2}\right)\,\left(3\,d^2\,g^2-10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{15\,e^{5/2}}\end{aligned}$$

```
[In] int(log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

```
[Out] log(c*(d + e*x^2)^p)*(f^2*x + (g^2*x^5)/5 + (2*f*g*x^3)/3) - x*(2*f^2*p - (
d*((4*f*g*p)/3 - (2*d*g^2*p)/(5*e)))/e) - x^3*((4*f*g*p)/9 - (2*d*g^2*p)/(1
5*e)) - (2*g^2*p*x^5)/25 + (2*d^(1/2)*p*atan((d^(1/2)*e^(1/2)*p*x*(3*d^2*g^
2 + 15*e^2*f^2 - 10*d*e*f*g))/(3*d^3*g^2*p + 15*d*e^2*f^2*p - 10*d^2*e*f*g*
p))*(3*d^2*g^2 + 15*e^2*f^2 - 10*d*e*f*g))/(15*e^(5/2))
```

3.270 $\int (f + gx^2) \log\left(c(d + ex^2)^p\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int (f + gx^2) \log\left(c(d + ex^2)^p\right) \, dx = -2fpx + \frac{2dgpx}{3e} - \frac{2}{9}gpx^3 + \frac{2\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{2d^{3/2}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}} + fx\log\left(c(d + ex^2)^p\right) + \frac{1}{3}gx^3\log\left(c(d + ex^2)^p\right)$$

```
[Out] -2*f*p*x+2/3*d*g*p*x/e-2/9*g*p*x^3-2/3*d^(3/2)*g*p*arctan(x*e^(1/2)/d^(1/2)
)/e^(3/2)+f*x*ln(c*(e*x^2+d)^p)+1/3*g*x^3*ln(c*(e*x^2+d)^p)+2*f*p*arctan(x*
e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2521, 2498, 327, 211, 2505, 308}

$$\int (f + gx^2) \log\left(c(d + ex^2)^p\right) \, dx = -\frac{2d^{3/2}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}} + \frac{2\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + fx\log\left(c(d + ex^2)^p\right) + \frac{1}{3}gx^3\log\left(c(d + ex^2)^p\right) + \frac{2dgpx}{3e} - 2fpx - \frac{2}{9}gpx^3$$

```
[In] Int[(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f*p*x + (2*d*g*p*x)/(3*e) - (2*g*p*x^3)/9 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^
(3/2)) + f*x*Log[c*(d + e*x^2)^p] + (g*x^3*Log[c*(d + e*x^2)^p])/3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(f \log \left(c\left(d+ex^2\right)^p\right)+gx^2 \log \left(c\left(d+ex^2\right)^p\right)\right)\,dx\\
&= f\int \log \left(c\left(d+ex^2\right)^p\right)\,dx+g\int x^2 \log \left(c\left(d+ex^2\right)^p\right)\,dx\\
&= fx \log \left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3 \log \left(c\left(d+ex^2\right)^p\right)-(2efp)\int \frac{x^2}{d+ex^2}\,dx-\frac{1}{3}(2egp)\int \frac{x^4}{d+ex^2}\,dx\\
&= -2fpx+fx \log \left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3 \log \left(c\left(d+ex^2\right)^p\right)\\
&\quad +(2dfp)\int \frac{1}{d+ex^2}\,dx-\frac{1}{3}(2egp)\int \left(-\frac{d}{e^2}+\frac{x^2}{e}+\frac{d^2}{e^2\left(d+ex^2\right)}\right)\,dx\\
&= -2fpx+\frac{2dgpx}{3e}-\frac{2}{9}gpx^3+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}\\
&\quad +fx \log \left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3 \log \left(c\left(d+ex^2\right)^p\right)-\frac{\left(2d^2gp\right)\int \frac{1}{d+ex^2}\,dx}{3e}\\
&= -2fpx+\frac{2dgpx}{3e}-\frac{2}{9}gpx^3+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{2d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}\\
&\quad +fx \log \left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3 \log \left(c\left(d+ex^2\right)^p\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int \left(f+gx^2\right)\log \left(c\left(d+ex^2\right)^p\right)\,dx &= -2fpx+\frac{2dgpx}{3e}-\frac{2}{9}gpx^3\\
&\quad +\frac{2\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{2d^{3/2}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}\\
&\quad +fx \log \left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3 \log \left(c\left(d+ex^2\right)^p\right)
\end{aligned}
$$

```
[In] Integrate[(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f*p*x + (2*d*g*p*x)/(3*e) - (2*g*p*x^3)/9 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^
(3/2)) + f*x*Log[c*(d + e*x^2)^p] + (g*x^3*Log[c*(d + e*x^2)^p])/3
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.76

method	result
parts	$\frac{g\,x^{3}\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{3}+f x\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)-\frac{2pe\left(-\frac{-\frac{1}{3}eg\,x^{3}+dgx-3efx}{e^{2}}+\frac{d(dg-3ef)\arctan\left(\frac{xe}{\sqrt{de}}\right)}{e^{2}\sqrt{de}}\right)}{3}$
risch	$\left(\frac{1}{3}g\,x^{3}+fx\right)\ln\left(\left(e\,x^{2}+d\right)^{p}\right)+\frac{ix\pi f\ \mathrm{csgn}\left(i\left(e\,x^{2}+d\right)^{p}\right)\mathrm{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right)^{2}}{2}-\frac{ix\pi f\ \mathrm{csgn}\left(i\left(e\,x^{2}+d\right)^{p}\right)\mathrm{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right.}{2}$

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*g*x^3*ln(c*(e*x^2+d)^p)+f*x*ln(c*(e*x^2+d)^p)-2/3*p*e*(-1/e^2*(-1/3*e*g
*x^3+d*g*x-3*e*f*x)+d*(d*g-3*e*f)/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.88

$$\int\left(f+gx^{2}\right)\log\left(c\left(d+ex^{2}\right)^{p}\right)\,dx$$

$$=\left[-\frac{2\,egpx^{3}+3\,(3\,ef-dg)p\sqrt{-\frac{d}{e}}\log\left(\frac{ex^{2}-2\,ex\sqrt{-\frac{d}{e}}-d}{ex^{2}+d}\right)+6\,(3\,ef-dg)px-3\,(egpx^{3}+3\,efpx)\log\left(ex^{2}+d\right.}{9\,e}\right.$$

$$-\frac{2\,egpx^{3}-6\,(3\,ef-dg)p\sqrt{\frac{d}{e}}\arctan\left(\frac{ex\sqrt{\frac{d}{e}}}{d}\right)+6\,(3\,ef-dg)px-3\,(egpx^{3}+3\,efpx)\log\left(ex^{2}+d\right)-3\,(e}{9\,e}$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/9*(2*e*g*p*x^3 + 3*(3*e*f - d*g)*p*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-
d/e) - d)/(e*x^2 + d)) + 6*(3*e*f - d*g)*p*x - 3*(e*g*p*x^3 + 3*e*f*p*x)*lo
g(e*x^2 + d) - 3*(e*g*x^3 + 3*e*f*x)*log(c))/e, -1/9*(2*e*g*p*x^3 - 6*(3*e*
f - d*g)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 6*(3*e*f - d*g)*p*x - 3*(e*g
*p*x^3 + 3*e*f*p*x)*log(e*x^2 + d) - 3*(e*g*x^3 + 3*e*f*x)*log(c))/e]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(121) = 242$.

Time = 8.50 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.22

$$\int (f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$= \begin{cases} \left(fx+\frac{gx^3}{3}\right)\log\left(0^pc\right) \\ \left(fx+\frac{gx^3}{3}\right)\log\left(cd^p\right) \\ -2fpx+fx\log\left(c\left(ex^2\right)^p\right)-\frac{2gpx^3}{9}+\frac{gx^3\log\left(c\left(ex^2\right)^p\right)}{3} \\ -\frac{2d^2gp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{3e^2\sqrt{-\frac{d}{e}}}+\frac{d^2g\log\left(c\left(d+ex^2\right)^p\right)}{3e^2\sqrt{-\frac{d}{e}}}+\frac{2dfp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{e\sqrt{-\frac{d}{e}}}-\frac{df\log\left(c\left(d+ex^2\right)^p\right)}{e\sqrt{-\frac{d}{e}}}+\frac{2dgpx}{3e}-2fpx+fx\log\left(c(d\right. \end{cases}$$

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((((f*x + g*x**3/3)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), ((f*x + g*x*
*3/3)*log(c*d**p), Eq(e, 0)), (-2*f*p*x + f*x*log(c*(e*x**2)**p) - 2*g*p*x*
*3/9 + g*x**3*log(c*(e*x**2)**p)/3, Eq(d, 0)), (-2*d**2*g*p*log(x - sqrt(-d
/e))/(3*e**2*sqrt(-d/e)) + d**2*g*log(c*(d + e*x**2)**p)/(3*e**2*sqrt(-d/e)
) + 2*d*f*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) - d*f*log(c*(d + e*x**2)**p)
/(e*sqrt(-d/e)) + 2*d*g*p*x/(3*e) - 2*f*p*x + f*x*log(c*(d + e*x**2)**p) -
2*g*p*x**3/9 + g*x**3*log(c*(d + e*x**2)**p)/3, True))
```

Maxima [F(-2)]

Exception generated.

$$\int (f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

$$\int (f+gx^2) \log \left(c(d+ex^2)^p\right) dx = -\frac{1}{9} (2 gp - 3 g \log (c))x^3 + \frac{1}{3} (gpx^3 + 3 fpx) \log (ex^2+d) - \frac{(6 efp - 2 dgp - 3 ef \log (c))x}{3 e} + \frac{2 (3 defp - d^2gp) \arctan \left(\frac{ex}{\sqrt{de}}\right)}{3 \sqrt{de}e}$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -1/9*(2*g*p - 3*g*log(c))*x^3 + 1/3*(g*p*x^3 + 3*f*p*x)*log(e*x^2 + d) - 1/
3*(6*e*f*p - 2*d*g*p - 3*e*f*log(c))*x/e + 2/3*(3*d*e*f*p - d^2*g*p)*arctan
(e*x/sqrt(d*e))/(sqrt(d*e)*e)
```

Mupad [B] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.83

$$\int (f + gx^2) \log \left(c(d + ex^2)^p\right) dx = \ln \left(c \left(e x^2 + d\right)^p\right) \left(\frac{g x^3}{3} + f x\right) - x \left(2 f p - \frac{2 d g p}{3 e}\right) - \frac{2 g p x^3}{9} - \frac{2 \sqrt{d} p \operatorname{atan}\left(\frac{\sqrt{d} \sqrt{e} p x (d g - 3 e f)}{d^2 g p - 3 d e f p}\right) (d g - 3 e f)}{3 e^{3/2}}$$

```
[In] int(log(c*(d + e*x^2)^p)*(f + g*x^2),x)
```

```
[Out] log(c*(d + e*x^2)^p)*(f*x + (g*x^3)/3) - x*(2*f*p - (2*d*g*p)/(3*e)) - (2*g
*p*x^3)/9 - (2*d^(1/2)*p*atan((d^(1/2)*e^(1/2)*p*x*(d*g - 3*e*f))/(d^2*g*p
- 3*d*e*f*p))*(d*g - 3*e*f))/(3*e^(3/2))
```

3.271 $\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 533

$$\begin{aligned}\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx &= \frac{2p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} \\ &- \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} \\ &- \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} \\ &+ \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} \\ &+ \frac{ip\,\mathrm{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}} \\ &+ \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}}\end{aligned}$$

```
[Out] arctan(x*g^(1/2)/f^(1/2))*ln(c*(e*x^2+d)^p)/f^(1/2)/g^(1/2)+2*p*arctan(x*g^
(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)-p*arctan
(x*g^(1/2)/f^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)-p*arcta
n(x*g^(1/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)-I*p*pol
ylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2
,1+2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2
```

```
)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,1-2*((-d)^
(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(
-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {211, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$\begin{aligned}\int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx=&\frac{\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \log \left(c(d+ex^2)^p\right)}{\sqrt{f}\sqrt{g}}\\&-\frac{p\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \log \left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{\sqrt{f}\sqrt{g}}\\&-\frac{p\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \log \left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{\sqrt{f}\sqrt{g}}\\&+\frac{2p\arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \log \left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}}\\&+\frac{ip\,\text{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}+1\right)}{2\sqrt{f}\sqrt{g}}\\&+\frac{ip\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}}\\&-\frac{ip\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}}\end{aligned}$$

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

```
[Out] (2*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/
(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))]])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*S
qrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[
g])*(Sqrt[f] - I*Sqrt[g]*x))]])/(Sqrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt
[f]]*Log[c*(d + e*x^2)^p])/(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, 1 - (2*Sqrt[
f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + (
2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sq
rt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1
 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d
]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
```

```
/; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}}-(2ep)\int\frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\left(d+ex^2\right)}\,dx\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}}-\frac{(2ep)\int\frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{\sqrt{f}\sqrt{g}}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}}-\frac{(2ep)\int\left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{\sqrt{f}\sqrt{g}}\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}}+\frac{\left(\sqrt{e}p\right)\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{\sqrt{f}\sqrt{g}}-\frac{\left(\sqrt{e}p\right)\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{\sqrt{f}\sqrt{g}}\\
&= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}}-\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}}\\
&\quad-\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}}\\
&\quad+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}}-2\frac{p\int\frac{\log\left(\frac{2}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{1+\frac{gx^2}{f}}\,dx}{f}\\
&\quad+\frac{p\int\frac{\log\left(\frac{2\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt{f}\left(-i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}\,dx}{f}+\frac{p\int\frac{\log\left(\frac{2\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\sqrt{f}\left(i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}\,dx}{f}\\
&= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}}-\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}}\\
&\quad-\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{\sqrt{f}\sqrt{g}}\\
&\quad+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{f}\sqrt{g}}+\frac{ip\,\mathrm{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{f}\sqrt{g}}\\
&\quad+\frac{ip\,\mathrm{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2\sqrt{f}\sqrt{g}}-2\frac{(ip)\mathrm{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{\sqrt{f}\sqrt{g}}
\end{aligned}
$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}}$$
$$- \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}}$$
$$+ \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{ip\mathrm{Li}_2\left(1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}}$$
$$+ \frac{ip\mathrm{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}} + \frac{ip\mathrm{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.06

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx =$$
$$-\frac{i\left(p\log\left(\frac{\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)+p\log\left(\frac{\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{-i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)-p\log\left(\frac{\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{-i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right.\right.}{}$$

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

```
[Out] ((-1/2*I)*(p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt
[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + p*Log[(Sqrt[g]*(Sqrt[-d] +
Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x
)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] +
 Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-
d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]
*x)/Sqrt[f]] + (2*I)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p] + p*P
olyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*S
qrt[g])] + p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f]
+ I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sq
rt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*S
qrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]))/(Sqrt[f]*Sqrt[g])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.88 (sec) , antiderivative size = 449, normalized size of antiderivative = 0.84

method	result
risch	$\frac{(\ln((e\,x^2+d)^p)-p\ln(e\,x^2+d))\arctan\left(\frac{gx}{\sqrt{fg}}\right)}{\sqrt{fg}}+\frac{p\left(\sum_{\_\alpha=\mathrm{RootOf}\left(g\_Z^2+f\right)}\frac{\ln(x-\_\alpha)\ln\left(e\,x^2+d\right)-\ln(x-\_\alpha)\left(\ln\left(\frac{\mathrm{RootOf}\left(e\_Z^2 g+2\_\alpha\right.}{\mathrm{RootOf}\left(e\_Z^2 g+\right.}\right.\right.}{}\right.}{}$

[In] int(ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)

[Out] (ln((e*x^2+d)^p)-p*ln(e*x^2+d))/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2))+1/2*p/g *sum(1/_alpha*(ln(x-_alpha)*ln(e*x^2+d)-ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2 *_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d *g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_al pha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))-dilog((RootOf(_Z^2* e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha* e*g+d*g-e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index= 2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2))),_alpha=Root Of(_Z^2*g+f))+(1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I*P i*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(e* x^2+d)^p)^3+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+ln(c))/(f*g)^(1/2)*a rctan(g*x/(f*g)^(1/2))

Fricas [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx=\int \frac{\log\left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")

[Out] integral(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)

Sympy [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
[Out] Integral(log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

Maxima [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)}{g\,x^2+f}\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)/(f + g*x^2),x)
[Out] int(log(c*(d + e*x^2)^p)/(f + g*x^2), x)
```

3.272 $\int \frac{\log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 751

$$
\begin{aligned}
\int \frac{\log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx &= \frac{\sqrt{d}\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)} - \frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} \\
&+ \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}} \\
&- \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}\sqrt{g}} \\
&- \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}\sqrt{g}} \\
&+ \frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} - \frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} \\
&+ \frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2f^{3/2}\sqrt{g}} \\
&- \frac{ip\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2f^{3/2}\sqrt{g}} \\
&+ \frac{ip\operatorname{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{3/2}\sqrt{g}} \\
&+ \frac{ip\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{3/2}\sqrt{g}}
\end{aligned}
$$

```
[Out] p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/f/(-d*g+e*f)+1/2*arctan(x*g^(1/
2)/f^(1/2))*ln(c*(e*x^2+d)^p)/f^(3/2)/g^(1/2)+p*arctan(x*g^(1/2)/f^(1/2))*l
n(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*p*arctan(x*g^(1/2)/f
^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/
(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*p*arctan(x*g^(1
/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/
2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*I*p*polylog
(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(3/2)/g^(1/2)+1/4*I*p*polylog(2,1+2
*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^
(1/2)-(-d)^(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)+1/4*I*p*polylog(2,1-2*((-d)^(1/2
)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^
(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*e*p*ln((-f)^(1/2)-x*g^(1/2))/(-d*g+e*f)
/(-f)^(1/2)/g^(1/2)+1/2*e*p*ln((-f)^(1/2)+x*g^(1/2))/(-d*g+e*f)/(-f)^(1/2)/
g^(1/2)-1/4*ln(c*(e*x^2+d)^p)/f/g^(1/2)/((-f)^(1/2)-x*g^(1/2))+1/4*ln(c*(e*
x^2+d)^p)/f/g^(1/2)/((-f)^(1/2)+x*g^(1/2))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 751, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules

used = {2521, 2513, 815, 649, 211, 266, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2f^{3/2}\sqrt{g}}$$
$$-\frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2f^{3/2}\sqrt{g}}$$
$$-\frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{2f^{3/2}\sqrt{g}}$$
$$+\frac{\sqrt{d}\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)}+\frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}}$$
$$-\frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}$$
$$+\frac{ip\,\mathrm{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)(\sqrt{f}-i\sqrt{g}x)}+1\right)}{4f^{3/2}\sqrt{g}}$$
$$+\frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{\left(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}\right)(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{3/2}\sqrt{g}}$$
$$-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}+\frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}$$
$$-\frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2f^{3/2}\sqrt{g}}$$

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^2)^2,x]
```

```
[Out] (Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(f*(e*f - d*g)) - (e*p*Log[
Sqrt[-f] - Sqrt[g]*x])/(2*Sqrt[-f]*Sqrt[g]*(e*f - d*g)) + (p*ArcTan[(Sqrt[g
]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3/2)*Sqrt[g]) -
 (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]
*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(2*
f^(3/2)*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(S
qrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*
Sqrt[g]*x))])/(2*f^(3/2)*Sqrt[g]) + (e*p*Log[Sqrt[-f] + Sqrt[g]*x])/(2*Sqrt
[-f]*Sqrt[g]*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqrt[-f] - S
qrt[g]*x)) + Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqrt[-f] + Sqrt[g]*x)) + (A
rcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/(2*f^(3/2)*Sqrt[g]) - ((I/
2)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3/2)*Sqrt[g])
 + ((I/4)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*S
qrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(f^(3/2)*Sqrt
[g]) + ((I/4)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(
```

```
(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(f^(3/2)*
Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\text{integral} = \int \left(-\frac{g\log\left(c(d+ex^2)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}-gx\right)^2} - \frac{g\log\left(c(d+ex^2)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}+gx\right)^2} - \frac{g\log\left(c(d+ex^2)^p\right)}{2f\left(-fg-g^2x^2\right)}\right) dx$$

$$= -\frac{g\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(\sqrt{-f}\sqrt{g}-gx\right)^2}\,dx}{4f} - \frac{g\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(\sqrt{-f}\sqrt{g}+gx\right)^2}\,dx}{4f} - \frac{g\int \frac{\log\left(c(d+ex^2)^p\right)}{-fg-g^2x^2}\,dx}{2f}$$

$$= -\frac{\log \left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{\log \left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log \left(c(d+ex^2)^p\right)}{2f^{3/2}\sqrt{g}}$$
$$+ \frac{(ep)\int \frac{x}{\left(\sqrt{-f}\sqrt{g}-gx\right)(d+ex^2)}\,dx}{2f} - \frac{(ep)\int \frac{x}{\left(\sqrt{-f}\sqrt{g}+gx\right)(d+ex^2)}\,dx}{2f} - \frac{(egp)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}g^{3/2}(d+ex^2)}\,dx}{f}$$

$$= -\frac{\log \left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{\log \left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log \left(c(d+ex^2)^p\right)}{2f^{3/2}\sqrt{g}}$$
$$- \frac{(ep)\int \left(\frac{\sqrt{-f}}{(ef-dg)\left(\sqrt{-f}+\sqrt{g}x\right)} - \frac{-d\sqrt{g}-e\sqrt{-f}x}{\sqrt{g}(-ef+dg)(d+ex^2)}\right)dx}{2f}$$
$$+ \frac{(ep)\int \left(\frac{\sqrt{-f}}{(ef-dg)\left(-\sqrt{-f}+\sqrt{g}x\right)} - \frac{d\sqrt{g}-e\sqrt{-f}x}{\sqrt{g}(-ef+dg)(d+ex^2)}\right)dx}{2f} - \frac{(ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{f^{3/2}\sqrt{g}}$$

$$= -\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} + \frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} - \frac{\log \left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}$$
$$+ \frac{\log \left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log \left(c(d+ex^2)^p\right)}{2f^{3/2}\sqrt{g}}$$
$$- \frac{(ep)\int \left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{f^{3/2}\sqrt{g}}$$
$$- \frac{(ep)\int \frac{-d\sqrt{g}-e\sqrt{-f}x}{d+ex^2}\,dx}{2f\sqrt{g}(ef-dg)} + \frac{(ep)\int \frac{d\sqrt{g}-e\sqrt{-f}x}{d+ex^2}\,dx}{2f\sqrt{g}(ef-dg)}$$

$$= -\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} + \frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} - \frac{\log \left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}$$
$$+ \frac{\log \left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log \left(c(d+ex^2)^p\right)}{2f^{3/2}\sqrt{g}}$$
$$+ \frac{\left(\sqrt{e}p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2f^{3/2}\sqrt{g}} - \frac{\left(\sqrt{e}p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{2f^{3/2}\sqrt{g}} + 2\frac{(dep)\int \frac{1}{d+ex^2}\,dx}{2f(ef-dg)}$$

$$=\frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)}-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}+\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}}$$
$$-\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}\sqrt{g}}$$
$$-\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}\sqrt{g}}+\frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}$$
$$-\frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}$$
$$+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2f^{3/2}\sqrt{g}}-2\frac{p\int\frac{\log\left(\frac{2}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{1+\frac{gx^2}{f}}dx}{2f^2}$$
$$+\frac{p\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{f}\left(-i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{2f^2}+\frac{p\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{f}\left(i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{2f^2}$$

$$=\frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)}-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}+\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}}$$
$$-\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}\sqrt{g}}$$
$$-\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}\sqrt{g}}+\frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}$$
$$-\frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}$$
$$+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2f^{3/2}\sqrt{g}}+\frac{ip\text{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{3/2}\sqrt{g}}$$
$$+\frac{ip\text{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{3/2}\sqrt{g}}-2\frac{(ip)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}dx,x,\frac{1}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{2f^{3/2}\sqrt{g}}$$

$$
\begin{aligned}
&= \frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)} - \frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}} \\
&- \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}\sqrt{g}} \\
&- \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}\sqrt{g}} + \frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} \\
&- \frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} \\
&+ \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2f^{3/2}\sqrt{g}} - \frac{ip\operatorname{Li}_2\left(1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2f^{3/2}\sqrt{g}} \\
&+ \frac{ip\operatorname{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{3/2}\sqrt{g}} + \frac{ip\operatorname{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{3/2}\sqrt{g}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.17

$$
\int \frac{\log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx
$$

$$
= \frac{\frac{2\sqrt{-d}\sqrt{e}\sqrt{f}p\log(\sqrt{-d}-\sqrt{e}x)}{-ef+dg} + \frac{2\sqrt{-d}\sqrt{e}\sqrt{f}p\log(\sqrt{-d}+\sqrt{e}x)}{ef-dg} + \frac{2e\sqrt{-f^2}p\log(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}(ef-dg)} + \frac{2e\sqrt{-f^2}p\log(\sqrt{-f}+\sqrt{g}x)}{\sqrt{g}(-ef+dg)} - \frac{ip\log\left(\frac{\sqrt{g}\ldots}{i\sqrt{e}\ldots}\right.}{}}{}
$$

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2)^2,x]
```

```
[Out] ((2*Sqrt[-d]*Sqrt[e]*Sqrt[f]*p*Log[Sqrt[-d] - Sqrt[e]*x])/(-(e*f) + d*g) +
(2*Sqrt[-d]*Sqrt[e]*Sqrt[f]*p*Log[Sqrt[-d] + Sqrt[e]*x])/(e*f - d*g) + (2*e
*Sqrt[-f^2]*p*Log[Sqrt[-f] - Sqrt[g]*x])/(Sqrt[g]*(e*f - d*g)) + (2*e*Sqrt[
-f^2]*p*Log[Sqrt[-f] + Sqrt[g]*x])/(Sqrt[g]*(-(e*f) + d*g)) - (I*p*Log[(Sqr
t[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1
- (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[g] - (I*p*Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x
))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]
])/Sqrt[g] + (I*p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f
] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[g] + (I*p*Log[(
Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log
[1 + (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[g] + (Sqrt[f]*Log[c*(d + e*x^2)^p])/(-(Sq
rt[-f]*Sqrt[g]) + g*x) + (Sqrt[f]*Log[c*(d + e*x^2)^p])/(Sqrt[-f]*Sqrt[g] +
 g*x) + (2*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/Sqrt[g] - (I*p
*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]
*Sqrt[g])]])/Sqrt[g] - (I*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sq
```

```
rt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])])/Sqrt[g] + (I*p*PolyLog[2, (Sqrt[e]*(S
qrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])])/Sqrt[g] + (
I*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[
-d]*Sqrt[g])])/Sqrt[g])/(4*f^(3/2))
```

Maple [F]

$$\int \frac{\ln\left(c(e\,x^2+d)^p\right)}{\left(g\,x^2+f\right)^2}dx$$

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
[Out] int(ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral(log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx=\int \frac{\log\left((ex^2+d)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx=\int \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{{\left(g\,x^2+f\right)}^2}\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)/(f + g*x^2)^2,x)
[Out] int(log(c*(d + e*x^2)^p)/(f + g*x^2)^2, x)
```

3.273 $\int (f+gx^2)^2 \log^2\left(c(d+ex^2)^p\right) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 945

$$\begin{aligned}\int (f+gx^2)^2 \log^2\left(c(d+ex^2)^p\right) \, dx &= 8f^2p^2x - \frac{64dfgp^2x}{9e} + \frac{184d^2g^2p^2x}{75e^2} + \frac{16}{27}fgp^2x^3 \\ &- \frac{64dg^2p^2x^3}{225e} + \frac{8}{125}g^2p^2x^5 - \frac{8\sqrt{d}f^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{64d^{3/2}fgp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}} \\ &- \frac{184d^{5/2}g^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{75e^{5/2}} + \frac{4i\sqrt{d}f^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} - \frac{8id^{3/2}fgp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{3e^{3/2}} \\ &+ \frac{4id^{5/2}g^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{5e^{5/2}} + \frac{8\sqrt{d}f^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} \\ &- \frac{16d^{3/2}fgp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{3e^{3/2}} + \frac{8d^{5/2}g^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{5e^{5/2}} \\ &- 4f^2px\log\left(c(d+ex^2)^p\right) + \frac{8dfgpx\log\left(c(d+ex^2)^p\right)}{3e} - \frac{4d^2g^2px\log\left(c(d+ex^2)^p\right)}{5e^2} - \frac{8}{9}fgpx^3\log\left(c(d+ex^2)^p\right.\end{aligned}$$

```
[Out] -8/9*f*g*p*x^3*ln(c*(e*x^2+d)^p)-8*f^2*p^2*arctan(x*e^(1/2)/d^(1/2))*d^(1/2
)/e^(1/2)-184/75*d^(5/2)*g^2*p^2*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)+8*f^2*p^
2*x+8/125*g^2*p^2*x^5+1/5*g^2*x^5*ln(c*(e*x^2+d)^p)^2+184/75*d^2*g^2*p^2*x/
e^2-64/225*d*g^2*p^2*x^3/e+8/3*d*f*g*p*x*ln(c*(e*x^2+d)^p)/e+16/27*f*g*p^2*
x^3-4*f^2*p*x*ln(c*(e*x^2+d)^p)-4/25*g^2*p*x^5*ln(c*(e*x^2+d)^p)+2/3*f*g*x^
3*ln(c*(e*x^2+d)^p)^2-64/9*d*f*g*p^2*x/e+8/5*d^(5/2)*g^2*p^2*arctan(x*e^(1/
2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(5/2)+4*f^2*p*arctan(x*e^
(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)*d^(1/2)/e^(1/2)+8*f^2*p^2*arctan(x*e^(1/2)
/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)+64/9*d^(3/2)*
f*g*p^2*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)-4/5*d^2*g^2*p*x*ln(c*(e*x^2+d)^p)
```

```
/e^2+4/15*d*g^2*p*x^3*ln(c*(e*x^2+d)^p)/e+4/5*d^(5/2)*g^2*p*arctan(x*e^(1/2
)/d^(1/2))*ln(c*(e*x^2+d)^p)/e^(5/2)+f^2*x*ln(c*(e*x^2+d)^p)^2+4/5*I*d^(5/2
)*g^2*p^2*arctan(x*e^(1/2)/d^(1/2))^2/e^(5/2)+4/5*I*d^(5/2)*g^2*p^2*polylog
(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(5/2)+4*I*f^2*p^2*arctan(x*e^(1/2)/
d^(1/2))^2*d^(1/2)/e^(1/2)+4*I*f^2*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e
^(1/2)))*d^(1/2)/e^(1/2)-8/3*d^(3/2)*f*g*p*arctan(x*e^(1/2)/d^(1/2))*ln(c*(
e*x^2+d)^p)/e^(3/2)-16/3*d^(3/2)*f*g*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(
1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(3/2)-8/3*I*d^(3/2)*f*g*p^2*arctan(x*e^(1/2)/
d^(1/2))^2/e^(3/2)-8/3*I*d^(3/2)*f*g*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x
*e^(1/2)))/e^(3/2)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 945, normalized size of antiderivative = 1.00, number of steps used = 50, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules

used = {2521, 2500, 2526, 2498, 327, 211, 2520, 12, 5040, 4964, 2449, 2352, 2507, 2505, 308}

$$
\begin{aligned}
\int \left(f+gx^2\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)\,dx =& \frac{8}{125}g^2p^2x^5 + \frac{1}{5}g^2\log^2\left(c\left(ex^2+d\right)^p\right)x^5 \\
&- \frac{4}{25}g^2p\log\left(c\left(ex^2+d\right)^p\right)x^5 - \frac{64dg^2p^2x^3}{225e} \\
&+ \frac{16}{27}fgp^2x^3 + \frac{2}{3}fg\log^2\left(c\left(ex^2+d\right)^p\right)x^3 \\
&+ \frac{4dg^2p\log\left(c\left(ex^2+d\right)^p\right)x^3}{15e} \\
&- \frac{8}{9}fgp\log\left(c\left(ex^2+d\right)^p\right)x^3 \\
&+ 8f^2p^2x + \frac{184d^2g^2p^2x}{75e^2} - \frac{64dfgp^2x}{9e} \\
&+ f^2\log^2\left(c\left(ex^2+d\right)^p\right)x - 4f^2p\log\left(c\left(ex^2+d\right)^p\right)x \\
&- \frac{4d^2g^2p\log\left(c\left(ex^2+d\right)^p\right)x}{5e^2} \\
&+ \frac{8dfgp\log\left(c\left(ex^2+d\right)^p\right)x}{3e} \\
&+ \frac{4i\sqrt{d}f^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} \\
&+ \frac{4id^{5/2}g^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{5e^{5/2}} \\
&- \frac{8id^{3/2}fgp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{3e^{3/2}} - \frac{8\sqrt{d}f^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} \\
&- \frac{184d^{5/2}g^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{75e^{5/2}} \\
&+ \frac{64d^{3/2}fgp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}} \\
&+ \frac{8\sqrt{d}f^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{\sqrt{e}} \\
&+ \frac{8d^{5/2}g^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{5e^{5/2}} \\
&- \frac{16d^{3/2}fgp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{3e^{3/2}} \\
&+ \frac{4\sqrt{d}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(ex^2+d\right)^p\right)}{\sqrt{e}} \\
&+ \frac{4d^{5/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(ex^2+d\right)^p\right)}{5e^{5/2}} \\
&- \frac{8d^{3/2}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(ex^2+d\right)^p\right)}{3e^{3/2}} \\
&\quad 4i\sqrt{d}f^2p^2\,\mathrm{PolyLog}\left(2, 1-\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)
\end{aligned}
$$

```
[In] Int[(f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] 8*f^2*p^2*x - (64*d*f*g*p^2*x)/(9*e) + (184*d^2*g^2*p^2*x)/(75*e^2) + (16*f
*g*p^2*x^3)/27 - (64*d*g^2*p^2*x^3)/(225*e) + (8*g^2*p^2*x^5)/125 - (8*Sqrt
[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + (64*d^(3/2)*f*g*p^2*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/(9*e^(3/2)) - (184*d^(5/2)*g^2*p^2*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/(75*e^(5/2)) + ((4*I)*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt
[d]]^2)/Sqrt[e] - (((8*I)/3)*d^(3/2)*f*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)
/e^(3/2) + (((4*I)/5)*d^(5/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(5/2
) + (8*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d]
 + I*Sqrt[e]*x)])/Sqrt[e] - (16*d^(3/2)*f*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(3*e^(3/2)) + (8*d^(5/2)*g^2*p^2
*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(5*e
^(5/2)) - 4*f^2*p*x*Log[c*(d + e*x^2)^p] + (8*d*f*g*p*x*Log[c*(d + e*x^2)^p
])/(3*e) - (4*d^2*g^2*p*x*Log[c*(d + e*x^2)^p])/(5*e^2) - (8*f*g*p*x^3*Log[
c*(d + e*x^2)^p])/9 + (4*d*g^2*p*x^3*Log[c*(d + e*x^2)^p])/(15*e) - (4*g^2*
p*x^5*Log[c*(d + e*x^2)^p])/25 + (4*Sqrt[d]*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d
]]*Log[c*(d + e*x^2)^p])/Sqrt[e] - (8*d^(3/2)*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]]*Log[c*(d + e*x^2)^p])/(3*e^(3/2)) + (4*d^(5/2)*g^2*p*ArcTan[(Sqrt[e]*x
)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(5*e^(5/2)) + f^2*x*Log[c*(d + e*x^2)^p]^2
 + (2*f*g*x^3*Log[c*(d + e*x^2)^p]^2)/3 + (g^2*x^5*Log[c*(d + e*x^2)^p]^2)/
5 + ((4*I)*Sqrt[d]*f^2*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*
x)])/Sqrt[e] - (((8*I)/3)*d^(3/2)*f*g*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[
d] + I*Sqrt[e]*x)])/e^(3/2) + (((4*I)/5)*d^(5/2)*g^2*p^2*PolyLog[2, 1 - (2*
Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/e^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2500

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[x^n*(
(a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
```

```
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(f^2\log^2\left(c\left(d+ex^2\right)^p\right)+2fgx^2\log^2\left(c\left(d+ex^2\right)^p\right)+g^2x^4\log^2\left(c\left(d+ex^2\right)^p\right)\right)\,dx\\
&= f^2\int \log^2\left(c\left(d+ex^2\right)^p\right)\,dx+(2fg)\int x^2\log^2\left(c\left(d+ex^2\right)^p\right)\,dx\\
&\quad +g^2\int x^4\log^2\left(c\left(d+ex^2\right)^p\right)\,dx
\end{aligned}$$

$$= f^2x\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log^2\left(c\left(d+ex^2\right)^p\right)$$
$$+\frac{1}{5}g^2x^5\log^2\left(c\left(d+ex^2\right)^p\right)-\left(4ef^2p\right)\int\frac{x^2\log\left(c(d+ex^2)^p\right)}{d+ex^2}\,dx$$
$$-\frac{1}{3}(8efgp)\int\frac{x^4\log\left(c(d+ex^2)^p\right)}{d+ex^2}\,dx-\frac{1}{5}\left(4eg^2p\right)\int\frac{x^6\log\left(c(d+ex^2)^p\right)}{d+ex^2}\,dx$$

$$= f^2x\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5\log^2\left(c\left(d+ex^2\right)^p\right)$$
$$-\left(4ef^2p\right)\int\left(\frac{\log\left(c(d+ex^2)^p\right)}{e}-\frac{d\log\left(c(d+ex^2)^p\right)}{e\left(d+ex^2\right)}\right)dx$$
$$-\frac{1}{3}(8efgp)\int\left(-\frac{d\log\left(c(d+ex^2)^p\right)}{e^2}+\frac{x^2\log\left(c(d+ex^2)^p\right)}{e}\right.$$
$$\left.+\frac{d^2\log\left(c(d+ex^2)^p\right)}{e^2\left(d+ex^2\right)}\right)dx-\frac{1}{5}\left(4eg^2p\right)\int\left(\frac{d^2\log\left(c(d+ex^2)^p\right)}{e^3}\right.$$
$$\left.-\frac{dx^2\log\left(c(d+ex^2)^p\right)}{e^2}+\frac{x^4\log\left(c(d+ex^2)^p\right)}{e}-\frac{d^3\log\left(c(d+ex^2)^p\right)}{e^3\left(d+ex^2\right)}\right)dx$$

$$= f^2x\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3\log^2\left(c\left(d+ex^2\right)^p\right)$$
$$+\frac{1}{5}g^2x^5\log^2\left(c\left(d+ex^2\right)^p\right)-\left(4f^2p\right)\int\log\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$+\left(4df^2p\right)\int\frac{\log\left(c(d+ex^2)^p\right)}{d+ex^2}\,dx-\frac{1}{3}(8fgp)\int x^2\log\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$+\frac{(8dfgp)\int\log\left(c\left(d+ex^2\right)^p\right)\,dx}{3e}-\frac{\left(8d^2fgp\right)\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx}{3e}$$
$$-\frac{1}{5}\left(4g^2p\right)\int x^4\log\left(c\left(d+ex^2\right)^p\right)\,dx-\frac{\left(4d^2g^2p\right)\int\log\left(c\left(d+ex^2\right)^p\right)\,dx}{5e^2}$$
$$+\frac{\left(4d^3g^2p\right)\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx}{5e^2}+\frac{\left(4dg^2p\right)\int x^2\log\left(c\left(d+ex^2\right)^p\right)\,dx}{5e}$$

$$= -4f^2px\log\left(c\left(d+ex^2\right)^p\right) + \frac{8dfgpx\log\left(c\left(d+ex^2\right)^p\right)}{3e} - \frac{4d^2g^2px\log\left(c\left(d+ex^2\right)^p\right)}{5e^2} - \frac{8}{9}fgpx^3\log\left(c\left(d+ex^2\right)^p\right) + \frac{4dg^2px^3\log\left(c\left(d+ex^2\right)^p\right)}{15e} - \frac{4}{25}g^2px^5\log\left(c\left(d+ex^2\right)^p\right) + \frac{4\sqrt{d}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{e}} - \frac{8d^{3/2}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{3e^{3/2}} + \frac{4d^{5/2}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{5e^{5/2}} + f^2x\log^2\left(c\left(d+ex^2\right)^p\right) + \frac{2}{3}fgx^3\log^2\left(c\left(d+ex^2\right)^p\right) + \frac{1}{5}g^2x^5\log^2\left(c\left(d+ex^2\right)^p\right) + \left(8ef^2p^2\right)\int\frac{x^2}{d+ex}$$

$$= 8f^2p^2x - \frac{16dfgp^2x}{3e} + \frac{8d^2g^2p^2x}{5e^2} - 4f^2px\log\left(c\left(d+ex^2\right)^p\right) + \frac{8dfgpx\log\left(c\left(d+ex^2\right)^p\right)}{3e} - \frac{4d^2g^2px\log\left(c\left(d+ex^2\right)^p\right)}{5e^2} - \frac{8}{9}fgpx^3\log\left(c\left(d+ex^2\right)^p\right) + \frac{4dg^2px^3\log\left(c\left(d+ex^2\right)^p\right)}{15e} - \frac{4}{25}g^2px^5\log\left(c\left(d+ex^2\right)^p\right) + \frac{4\sqrt{d}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{e}} - \frac{8d^{3/2}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{3e^{3/2}} + \frac{4d^{5/2}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{5e^{5/2}} + f^2x\log^2\left(c\left(d+ex^2\right)^p\right) + \frac{2}{3}fgx^3\log^2\left(c\left(d+ex^2\right)^p\right) + \frac{1}{5}g^2x^5\log^2\left(c\left(d+ex^2\right)^p\right) - \left(8df^2p^2\right)\int\frac{1}{d+ex}$$

$$= 8f^2p^2x - \frac{64dfgp^2x}{9e} + \frac{184d^2g^2p^2x}{75e^2} + \frac{16}{27}fgp^2x^3 - \frac{64dg^2p^2x^3}{225e} + \frac{8}{125}g^2p^2x^5 - \frac{8\sqrt{d}f^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{16d^{3/2}fgp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}} - \frac{8d^{5/2}g^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}} + \frac{4i\sqrt{d}f^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} - \frac{8id^{3/2}fgp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{3e^{3/2}} + \frac{4id^{5/2}g^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{5e^{5/2}} - 4f^2px\log\left(c\left(d+ex^2\right)^p\right) + \frac{8dfgpx\log\left(c\left(d+ex^2\right)^p\right)}{3e} - \frac{4d^2g^2px\log\left(c\left(d+ex^2\right)^p\right)}{5e^2} - \frac{8}{9}fgpx^3\log\left(c\left(\right.\right.$$

$$= 8f^2p^2x - \frac{64dfgp^2x}{9e} + \frac{184d^2g^2p^2x}{75e^2} + \frac{16}{27}fgp^2x^3 - \frac{64dg^2p^2x^3}{225e}$$
$$+ \frac{8}{125}g^2p^2x^5 - \frac{8\sqrt{d}f^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{64d^{3/2}fgp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}}$$
$$- \frac{184d^{5/2}g^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{75e^{5/2}} + \frac{4i\sqrt{d}f^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} - \frac{8id^{3/2}fgp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{3e^{3/2}}$$
$$+ \frac{4id^{5/2}g^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{5e^{5/2}} + \frac{8\sqrt{d}f^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}$$
$$- \frac{16d^{3/2}fgp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{3e^{3/2}} + \frac{8d^{5/2}g^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{5e^{5/2}}$$

$$-4f^2px\log\left(c\left(d+ex^2\right)^p\right) + \frac{8dfgpx\log\left(c\left(d+ex^2\right)^p\right)}{3e} - \frac{4d^2g^2px\log\left(c\left(d+ex^2\right)^p\right)}{5e^2} - \frac{8}{9}fgpx^3\log\left(c(d\right.$$

$$= 8f^2p^2x - \frac{64dfgp^2x}{9e} + \frac{184d^2g^2p^2x}{75e^2} + \frac{16}{27}fgp^2x^3 - \frac{64dg^2p^2x^3}{225e}$$
$$+ \frac{8}{125}g^2p^2x^5 - \frac{8\sqrt{d}f^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{64d^{3/2}fgp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}}$$
$$- \frac{184d^{5/2}g^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{75e^{5/2}} + \frac{4i\sqrt{d}f^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} - \frac{8id^{3/2}fgp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{3e^{3/2}}$$
$$+ \frac{4id^{5/2}g^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{5e^{5/2}} + \frac{8\sqrt{d}f^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}$$
$$- \frac{16d^{3/2}fgp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{3e^{3/2}} + \frac{8d^{5/2}g^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{5e^{5/2}}$$

$$-4f^2px\log\left(c\left(d+ex^2\right)^p\right) + \frac{8dfgpx\log\left(c\left(d+ex^2\right)^p\right)}{3e} - \frac{4d^2g^2px\log\left(c\left(d+ex^2\right)^p\right)}{5e^2} - \frac{8}{9}fgpx^3\log\left(c(d\right.$$

$$= 8f^2p^2x - \frac{64dfgp^2x}{9e} + \frac{184d^2g^2p^2x}{75e^2} + \frac{16}{27}fgp^2x^3 - \frac{64dg^2p^2x^3}{225e} + \frac{8}{125}g^2p^2x^5 - \frac{8\sqrt{d}f^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{64d^{3/2}fgp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}} - \frac{184d^{5/2}g^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{75e^{5/2}} + \frac{4i\sqrt{d}f^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} - \frac{8id^{3/2}fgp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{3e^{3/2}} + \frac{4id^{5/2}g^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{5e^{5/2}} + \frac{8\sqrt{d}f^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} - \frac{16d^{3/2}fgp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{3e^{3/2}} + \frac{8d^{5/2}g^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{5e^{5/2}} - 4f^2px\log\left(c(d+ex^2)^p\right) + \frac{8dfgpx\log\left(c(d+ex^2)^p\right)}{3e} - \frac{4d^2g^2px\log\left(c(d+ex^2)^p\right)}{5e^2} - \frac{8}{9}fgpx^3\log\left(c(\right.$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 435, normalized size of antiderivative = 0.46

$$\int \left(f+gx^2\right)^2\log^2\left(c\left(d+ex^2\right)^p\right)\,dx = \frac{900i\sqrt{d}(15e^2f^2-10defg+3d^2g^2)\,p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2 + 60\sqrt{d}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-2(225e^2f^2-200defg+6\right.}{}$$

```
[In] Integrate[(f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] ((900*I)*Sqrt[d]*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p^2*ArcTan[(Sqrt[e]*
x)/Sqrt[d]]^2 + 60*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-2*(225*e^2*f^2 -
 200*d*e*f*g + 69*d^2*g^2)*p + 30*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*L
og[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + 15*(15*e^2*f^2 - 10*d*e*f*g + 3*d
^2*g^2)*Log[c*(d + e*x^2)^p]) + Sqrt[e]*x*(8*p^2*(1035*d^2*g^2 - 120*d*e*g*
(25*f + g*x^2) + e^2*(3375*f^2 + 250*f*g*x^2 + 27*g^2*x^4)) - 60*p*(45*d^2*
g^2 - 15*d*e*g*(10*f + g*x^2) + e^2*(225*f^2 + 50*f*g*x^2 + 9*g^2*x^4))*Log
[c*(d + e*x^2)^p] + 225*e^2*(15*f^2 + 10*f*g*x^2 + 3*g^2*x^4)*Log[c*(d + e*
x^2)^p]^2) + (900*I)*Sqrt[d]*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p^2*Poly
Log[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)])/(3375*e^(5/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.22 (sec) , antiderivative size = 1077, normalized size of antiderivative = 1.14

method	result	size
risch	Expression too large to display	1077

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4/25*p*g^2*x^5*ln((e*x^2+d)^p)-4*p*f^2*x*ln((e*x^2+d)^p)-184/75*p^2/e^2*g^
2*d^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-4*p^2*d/(d*e)^(1/2)*arctan(x*e/(d
*e)^(1/2))*f^2*ln(e*x^2+d)+4/15*p/e*g^2*d*x^3*ln((e*x^2+d)^p)-4/5*p/e^2*g^2
*d^2*x*ln((e*x^2+d)^p)+4*p*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*f^2*ln((e*
x^2+d)^p)+2/3*ln((e*x^2+d)^p)^2*f*g*x^3+8/3*p/e*d*f*g*x*ln((e*x^2+d)^p)-8*p
^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*f^2-8/9*p*f*g*x^3*ln((e*x^2+d)^p)-
8/3*p/e*d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*f*g*ln((e*x^2+d)^p)+8/3*p^2
/e*d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*f*g*ln(e*x^2+d)+1/5*ln((e*x^2+d)
^p)^2*g^2*x^5+ln((e*x^2+d)^p)^2*x*f^2-4/15*p^2*e*Sum(-1/2*(ln(x-_alpha)*ln(
e*x^2+d)-2*e*(1/4/_alpha/e*ln(x-_alpha)^2+1/2*_alpha/d*ln(x-_alpha)*ln(1/2*
(x+_alpha)/_alpha)+1/2*_alpha/d*dilog(1/2*(x+_alpha)/_alpha)))*d*(3*d^2*g^2
-10*d*e*f*g+15*e^2*f^2)/e^4/_alpha,_alpha=RootOf(_Z^2*e+d))+64/9*p^2/e*d^2/
(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*f*g-4/5*p^2/e^2*g^2*d^3/(d*e)^(1/2)*arc
tan(x*e/(d*e)^(1/2))*ln(e*x^2+d)+(I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+
d)^p)^2-I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-I*Pi*csgn(
I*c*(e*x^2+d)^p)^3+I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+2*ln(c))*(1/5*ln(
(e*x^2+d)^p)*g^2*x^5+2/3*ln((e*x^2+d)^p)*f*g*x^3+ln((e*x^2+d)^p)*x*f^2-2/15
*p*e*(1/e^3*(3/5*e^2*g^2*x^5-d*e*g^2*x^3+10/3*e^2*f*g*x^3+3*d^2*g^2*x-10*d*
e*f*g*x+15*e^2*f^2*x)-d*(3*d^2*g^2-10*d*e*f*g+15*e^2*f^2)/e^3/(d*e)^(1/2)*a
rctan(x*e/(d*e)^(1/2))))+4/5*p/e^2*g^2*d^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/
2))*ln((e*x^2+d)^p)+1/4*(I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-I
*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^
2+d)^p)^3+I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+2*ln(c))^2*(1/5*g^2*x^5+2/
3*f*g*x^3+f^2*x)+8*f^2*p^2*x+8/125*g^2*p^2*x^5-64/9*d*f*g*p^2*x/e+184/75*d^
2*g^2*p^2*x/e^2-64/225*d*g^2*p^2*x^3/e+16/27*f*g*p^2*x^3
```

Fricas [F]

$$\int (f+gx^2)^2 \log^2 \left(c(d+ex^2)^p\right) \,dx = \int (gx^2+f)^2 \log \left((ex^2+d)^p c\right)^2 \,dx$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)^2, x)
```

Sympy [F]

$$\int (f+gx^2)^2 \log^2 \left(c(d+ex^2)^p\right) \,dx = \int (f+gx^2)^2 \log \left(c(d+ex^2)^p\right)^2 \,dx$$

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f + g*x**2)**2*log(c*(d + e*x**2)**p)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (f+gx^2)^2 \log^2 \left(c(d+ex^2)^p\right) \,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int (f+gx^2)^2 \log^2 \left(c(d+ex^2)^p\right) \,dx = \int (gx^2+f)^2 \log \left((ex^2+d)^p c\right)^2 \,dx$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(f+gx^2\right)^2\log^2\left(c\left(d+ex^2\right)^p\right)\,dx=\int \ln\left(c\left(e\,x^2+d\right)^p\right)^2\left(g\,x^2+f\right)^2\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)^2*(f + g*x^2)^2,x)
[Out] int(log(c*(d + e*x^2)^p)^2*(f + g*x^2)^2, x)
```

3.274 $\int (f+gx^2)\log^2\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 548

$$\int (f+gx^2)\log^2\left(c(d+ex^2)^p\right)\,dx = 8fp^2x-\frac{32dgp^2x}{9e}+\frac{8}{27}gp^2x^3-\frac{8\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$
$$+\frac{32d^{3/2}gp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}}+\frac{4i\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}}-\frac{4id^{3/2}gp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{3e^{3/2}}$$
$$+\frac{8\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}-\frac{8d^{3/2}gp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{3e^{3/2}}$$
$$-4fpx\log\left(c(d+ex^2)^p\right)+\frac{4dgpx\log\left(c(d+ex^2)^p\right)}{3e}-\frac{4}{9}gpx^3\log\left(c(d+ex^2)^p\right)+\frac{4\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(c}{\sqrt{e}}$$

```
[Out] 8*f*p^2*x-32/9*d*g*p^2*x/e+8/27*g*p^2*x^3+32/9*d^(3/2)*g*p^2*arctan(x*e^(1/
2)/d^(1/2))/e^(3/2)+4*I*f*p^2*arctan(x*e^(1/2)/d^(1/2))^2*d^(1/2)/e^(1/2)-4
*f*p*x*ln(c*(e*x^2+d)^p)+4/3*d*g*p*x*ln(c*(e*x^2+d)^p)/e-4/9*g*p*x^3*ln(c*(
e*x^2+d)^p)-4/3*d^(3/2)*g*p*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)/e^(
3/2)+f*x*ln(c*(e*x^2+d)^p)^2+1/3*g*x^3*ln(c*(e*x^2+d)^p)^2-8/3*d^(3/2)*g*p^
2*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(3/2)-4/3
*I*d^(3/2)*g*p^2*arctan(x*e^(1/2)/d^(1/2))^2/e^(3/2)-8*f*p^2*arctan(x*e^(1/
2)/d^(1/2))*d^(1/2)/e^(1/2)-4/3*I*d^(3/2)*g*p^2*polylog(2,1-2*d^(1/2)/(d^(1
/2)+I*x*e^(1/2)))/e^(3/2)+4*f*p*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)
*d^(1/2)/e^(1/2)+8*f*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*
x*e^(1/2)))*d^(1/2)/e^(1/2)+4*I*f*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^
(1/2)))*d^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {2521, 2500, 2526, 2498, 327, 211, 2520, 12, 5040, 4964, 2449, 2352, 2507, 2505, 308}

$$\int (f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)\,dx = -\frac{4d^{3/2}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{3e^{3/2}} + \frac{4\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{e}} - \frac{4id^{3/2}gp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{3e^{3/2}} + \frac{32d^{3/2}gp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}} - \frac{8d^{3/2}gp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{3e^{3/2}} + \frac{4i\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} - \frac{8\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{8\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} + fx\log^2\left(c\left(d+ex^2\right)^p\right) - 4fpx\log\left(c\left(d+ex^2\right)^p\right) + \frac{4dgpx\log\left(c(d+ex^2)^p\right)}{3e} + \frac{1}{3}gx^3\log^2\left(c\left(d+ex^2\right)^p\right) - \frac{4}{9}gpx^3$$

```
[In] Int[(f + g*x^2)*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] 8*f*p^2*x - (32*d*g*p^2*x)/(9*e) + (8*g*p^2*x^3)/27 - (8*Sqrt[d]*f*p^2*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + (32*d^(3/2)*g*p^2*ArcTan[(Sqrt[e]*x)/Sqr
t[d]])/(9*e^(3/2)) + ((4*I)*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sq
rt[e] - (((4*I)/3)*d^(3/2)*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(3/2) + (
8*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x)])/Sqrt[e] - (8*d^(3/2)*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sq
rt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(3*e^(3/2)) - 4*f*p*x*Log[c*(d + e*x^2)^p]
 + (4*d*g*p*x*Log[c*(d + e*x^2)^p])/(3*e) - (4*g*p*x^3*Log[c*(d + e*x^2)^p]
)/9 + (4*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt
[e] - (4*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(3*e
^(3/2)) + f*x*Log[c*(d + e*x^2)^p]^2 + (g*x^3*Log[c*(d + e*x^2)^p]^2)/3 + (
(4*I)*Sqrt[d]*f*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sq
rt[e] - (((4*I)/3)*d^(3/2)*g*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x)])/e^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2500

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[x^n*(
(a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(f\log^2\left(c\left(d+ex^2\right)^p\right)+gx^2\log^2\left(c\left(d+ex^2\right)^p\right)\right)\,dx\\
&= f\int \log^2\left(c\left(d+ex^2\right)^p\right)\,dx+g\int x^2\log^2\left(c\left(d+ex^2\right)^p\right)\,dx\\
&= fx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^2\left(c\left(d+ex^2\right)^p\right)\\
&\quad-(4efp)\int \frac{x^2\log\left(c(d+ex^2)^p\right)}{d+ex^2}\,dx-\frac{1}{3}(4egp)\int \frac{x^4\log\left(c(d+ex^2)^p\right)}{d+ex^2}\,dx\\
&= fx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^2\left(c\left(d+ex^2\right)^p\right)\\
&\quad-(4efp)\int \left(\frac{\log\left(c(d+ex^2)^p\right)}{e}-\frac{d\log\left(c(d+ex^2)^p\right)}{e\left(d+ex^2\right)}\right)dx\\
&\quad-\frac{1}{3}(4egp)\int \left(-\frac{d\log\left(c(d+ex^2)^p\right)}{e^2}+\frac{x^2\log\left(c(d+ex^2)^p\right)}{e}\right.\\
&\qquad\left.+\frac{d^2\log\left(c(d+ex^2)^p\right)}{e^2\left(d+ex^2\right)}\right)dx\\
&= fx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^2\left(c\left(d+ex^2\right)^p\right)-(4fp)\int \log\left(c\left(d+ex^2\right)^p\right)\,dx\\
&\quad+(4dfp)\int \frac{\log\left(c(d+ex^2)^p\right)}{d+ex^2}\,dx-\frac{1}{3}(4gp)\int x^2\log\left(c\left(d+ex^2\right)^p\right)\,dx\\
&\quad+\frac{(4dgp)\int \log\left(c\left(d+ex^2\right)^p\right)\,dx}{3e}-\frac{(4d^2gp)\int \frac{\log\left(c(d+ex^2)^p\right)}{d+ex^2}\,dx}{3e}\\
&= -4fpx\log\left(c\left(d+ex^2\right)^p\right)+\frac{4dgpx\log\left(c\left(d+ex^2\right)^p\right)}{3e}-\frac{4}{9}gpx^3\log\left(c\left(d+ex^2\right)^p\right)\\
&\quad+\frac{4\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{e}}-\frac{4d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{3e^{3/2}}\\
&\quad+fx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^2\left(c\left(d+ex^2\right)^p\right)+\left(8efp^2\right)\int \frac{x^2}{d+ex^2}\,dx\\
&\quad-\left(8defp^2\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}\left(d+ex^2\right)}\,dx-\frac{1}{3}\left(8dgp^2\right)\int \frac{x^2}{d+ex^2}\,dx\\
&\quad+\frac{1}{3}\left(8d^2gp^2\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}\left(d+ex^2\right)}\,dx+\frac{1}{9}\left(8egp^2\right)\int \frac{x^4}{d+ex^2}\,dx
\end{aligned}
$$

$$= 8fp^2x - \frac{8dgp^2x}{3e} - 4fpx\log\left(c\left(d+ex^2\right)^p\right) + \frac{4dgpx\log\left(c\left(d+ex^2\right)^p\right)}{3e} - \frac{4}{9}gpx^3\log\left(c\left(d+ex^2\right)^p\right) + \frac{4\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{e}} - \frac{4d^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{3e^{3/2}} + fx\log^2\left(c\left(d+ex^2\right)^p\right) + \frac{1}{3}gx^3\log^2\left(c\left(d+ex^2\right)^p\right) - \left(8dfp^2\right)\int\frac{1}{d+ex^2}\,dx - \left(8\sqrt{d}\sqrt{e}fp^2\right)\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d+ex^2}\,dx + \frac{\left(8d^2gp^2\right)\int\frac{1}{d+ex^2}\,dx}{3e} + \frac{\left(8d^{3/2}gp^2\right)\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d+ex^2}\,dx}{3\sqrt{e}} + \frac{1}{9}\left(8egp^2\right)\int\left(-\frac{d}{e^2}+\frac{x^2}{e}+\frac{d^2}{e^2\left(d+ex^2\right)}\right)dx$$

$$= 8fp^2x - \frac{32dgp^2x}{9e} + \frac{8}{27}gp^2x^3 - \frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{8d^{3/2}gp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}} + \frac{4i\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} - \frac{4id^{3/2}gp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{3e^{3/2}}$$

$$-4fpx\log\left(c\left(d+ex^2\right)^p\right) + \frac{4dgpx\log\left(c\left(d+ex^2\right)^p\right)}{3e} - \frac{4}{9}gpx^3\log\left(c\left(d+ex^2\right)^p\right) + \frac{4\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$

$$= 8fp^2x - \frac{32dgp^2x}{9e} + \frac{8}{27}gp^2x^3 - \frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{32d^{3/2}gp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}} + \frac{4i\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} - \frac{4id^{3/2}gp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{3e^{3/2}} + \frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} - \frac{8d^{3/2}gp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{3e^{3/2}}$$

$$-4fpx\log\left(c\left(d+ex^2\right)^p\right) + \frac{4dgpx\log\left(c\left(d+ex^2\right)^p\right)}{3e} - \frac{4}{9}gpx^3\log\left(c\left(d+ex^2\right)^p\right) + \frac{4\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$

$$= 8fp^2x - \frac{32dgp^2x}{9e} + \frac{8}{27}gp^2x^3 - \frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$
$$+ \frac{32d^{3/2}gp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}} + \frac{4i\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} - \frac{4id^{3/2}gp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{3e^{3/2}}$$
$$+ \frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} - \frac{8d^{3/2}gp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{3e^{3/2}}$$
$$-4fpx\log\left(c\left(d+ex^2\right)^p\right) + \frac{4dgpx\log\left(c\left(d+ex^2\right)^p\right)}{3e} - \frac{4}{9}gpx^3\log\left(c\left(d+ex^2\right)^p\right) + \frac{4\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{}}$$

$$= 8fp^2x - \frac{32dgp^2x}{9e} + \frac{8}{27}gp^2x^3 - \frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}$$
$$+ \frac{32d^{3/2}gp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}} + \frac{4i\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} - \frac{4id^{3/2}gp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{3e^{3/2}}$$
$$+ \frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} - \frac{8d^{3/2}gp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{3e^{3/2}}$$
$$-4fpx\log\left(c\left(d+ex^2\right)^p\right) + \frac{4dgpx\log\left(c\left(d+ex^2\right)^p\right)}{3e} - \frac{4}{9}gpx^3\log\left(c\left(d+ex^2\right)^p\right) + \frac{4\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.51

$$\int \left(f+gx^2\right)\log^2\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$= \frac{-36i\sqrt{d}(-3ef+dg)p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2 - 12\sqrt{d}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(2(9ef-4dg)p+6(-3ef+dg)p\log\left(\frac{2\sqrt{}}{\sqrt{d}+}\right.\right.}{}$$

```
[In] Integrate[(f + g*x^2)*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] ((-36*I)*Sqrt[d]*(-3*e*f + d*g)*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 - 12*Sqrt
[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(2*(9*e*f - 4*d*g)*p + 6*(-3*e*f + d*g)*p
*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + (-9*e*f + 3*d*g)*Log[c*(d + e*x
^2)^p]) + Sqrt[e]*x*(8*p^2*(27*e*f - 12*d*g + e*g*x^2) - 12*p*(9*e*f - 3*d*
g + e*g*x^2)*Log[c*(d + e*x^2)^p] + 9*e*(3*f + g*x^2)*Log[c*(d + e*x^2)^p]^
2) - (36*I)*Sqrt[d]*(-3*e*f + d*g)*p^2*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/(
(-I)*Sqrt[d] + Sqrt[e]*x)])/(27*e^(3/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.83 (sec) , antiderivative size = 729, normalized size of antiderivative = 1.33

method	result	size
risch	Expression too large to display	729

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln((e*x^2+d)^p)^2*g*x^3+ln((e*x^2+d)^p)^2*x*f-4/9*p*g*x^3*ln((e*x^2+d)^
p)+4/3*p/e*g*d*x*ln((e*x^2+d)^p)-4*p*f*x*ln((e*x^2+d)^p)+4/3*p^2/e*g*d^2/(d
*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*ln(e*x^2+d)-4/3*p/e*g*d^2/(d*e)^(1/2)*arc
tan(x*e/(d*e)^(1/2))*ln((e*x^2+d)^p)-4*p^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(
1/2))*f*ln(e*x^2+d)+4*p*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*f*ln((e*x^2+d
)^p)+8/27*g*p^2*x^3-32/9*d*g*p^2*x/e+32/9*p^2/e*g*d^2/(d*e)^(1/2)*arctan(x*
e/(d*e)^(1/2))+8*f*p^2*x-8*p^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*f-4/3*
p^2*e*Sum(1/2*(ln(x-_alpha)*ln(e*x^2+d)-2*e*(1/4/_alpha/e*ln(x-_alpha)^2+1/
2*_alpha/d*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+1/2*_alpha/d*dilog(1/2*(x
+_alpha)/_alpha)))*d*(d*g-3*e*f)/e^3/_alpha,_alpha=RootOf(_Z^2*e+d))+(I*Pi*
csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I
*c*(e*x^2+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^2+d)^p)^3+I*Pi*csgn(I*c*(e*x^2
+d)^p)^2*csgn(I*c)+2*ln(c))*(1/3*ln((e*x^2+d)^p)*g*x^3+ln((e*x^2+d)^p)*x*f-
2/3*p*e*(1/e^2*(1/3*e*g*x^3-d*g*x+3*e*f*x)+d*(d*g-3*e*f)/e^2/(d*e)^(1/2)*ar
ctan(x*e/(d*e)^(1/2))))+1/4*(I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)
^2-I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(
e*x^2+d)^p)^3+I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+2*ln(c))^2*(1/3*g*x^3+
f*x)
```

Fricas [F]

$$\int (f+gx^2)\log^2\left(c(d+ex^2)^p\right)\,dx=\int (gx^2+f)\log\left((ex^2+d)^pc\right)^2\,dx$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g*x^2 + f)*log((e*x^2 + d)^p*c)^2, x)
```

Sympy [F]

$$\int (f+gx^2)\log^2\left(c(d+ex^2)^p\right)\,dx=\int (f+gx^2)\log\left(c(d+ex^2)^p\right)^2\,dx$$

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f + g*x**2)*log(c*(d + e*x**2)**p)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (f+gx^2)\log^2\left(c(d+ex^2)^p\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int (f+gx^2)\log^2\left(c(d+ex^2)^p\right)\,dx=\int (gx^2+f)\log\left((ex^2+d)^pc\right)^2\,dx$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)*log((e*x^2 + d)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (f+gx^2)\log^2\left(c(d+ex^2)^p\right)\,dx=\int \ln\left(c\,(e\,x^2+d)^p\right)^2\,(g\,x^2+f)\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)^2*(f + g*x^2),x)
```

```
[Out] int(log(c*(d + e*x^2)^p)^2*(f + g*x^2), x)
```

3.275 $\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx = \text{Int}\left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}, x\right)$$

```
[Out] Unintegrable(ln(c*(e*x^2+d)^p)^2/(g*x^2+f),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx = \int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx$$

```
[In] Int[Log[c*(d + e*x^2)^p]^2/(f + g*x^2),x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^2/(f + g*x^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} dx$$

Mathematica [N/A]

Not integrable

Time = 2.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

```
[In] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^2),x]
[Out] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^2), x]
```

Maple [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c(e\,x^2+d)^p\right)^2}{g\,x^2+f}dx$$

```
[In] int(ln(c*(e*x^2+d)^p)^2/(g*x^2+f),x)
[Out] int(ln(c*(e*x^2+d)^p)^2/(g*x^2+f),x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^2}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log((e*x^2 + d)^p*c)^2/(g*x^2 + f), x)
```

Sympy [N/A]

Not integrable

Time = 10.87 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left(c(d+ex^2)^p\right)^2}{f+gx^2}\,dx$$

```
[In] integrate(ln(c*(e*x**2+d)**p)**2/(g*x**2+f),x)
[Out] Integral(log(c*(d + e*x**2)**p)**2/(f + g*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^2}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f),x, algorithm="maxima")
[Out] integrate(log((e*x^2 + d)^p*c)^2/(g*x^2 + f), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^2}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log((e*x^2 + d)^p*c)^2/(g*x^2 + f), x)
```

Mupad [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)^2}{g\,x^2+f}\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)^2/(f + g*x^2),x)
[Out] int(log(c*(d + e*x^2)^p)^2/(f + g*x^2), x)
```

3.276 $\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx=\text{Int}\left(\frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^2)^2},x\right)$$

```
[Out] Unintegrable(ln(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx=\int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx$$

```
[In] Int[Log[c*(d + e*x^2)^p]^2/(f + g*x^2)^2,x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^2/(f + g*x^2)^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.69 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx$$

```
[In] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^2)^2,x]
[Out] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^2)^2, x]
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c(e\,x^2+d)^p\right)^2}{(g\,x^2+f)^2}dx$$

```
[In] int(ln(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x)
[Out] int(ln(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{\log\left((ex^2+d)^pc\right)^2}{(gx^2+f)^2}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral(log((e*x^2 + d)^p*c)^2/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(e*x**2+d)**p)**2/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^2}{(gx^2+f)^2}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)^2/(g*x^2 + f)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)^2}{(g\,x^2+f)^2}\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)^2/(f + g*x^2)^2,x)
[Out] int(log(c*(d + e*x^2)^p)^2/(f + g*x^2)^2, x)
```

3.277 $\int (f+gx^2) \log^3\left(c(d+ex^2)^p\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (f+gx^2) \log^3\left(c(d+ex^2)^p\right) \, dx = -48fp^3x + \frac{208dgp^3x}{9e} - \frac{16}{27}gp^3x^3 + \frac{48\sqrt{d}fp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{208d^{3/2}gp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}} - \frac{24i\sqrt{d}fp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} + \frac{32id^{3/2}gp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{3e^{3/2}} - \frac{48\sqrt{d}fp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} + \frac{64d^{3/2}gp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{3e^{3/2}} + 24fp^2x\log\left(c(d+ex^2)^p\right) - \frac{32dgp^2x\log\left(c(d+ex^2)^p\right)}{3e} + \frac{8}{9}gp^2x^3\log\left(c(d+ex^2)^p\right) - \frac{24\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right.}{\sqrt{e}}$$

```
[Out] -48*f*p^3*x+208/9*d*g*p^3*x/e-16/27*g*p^3*x^3-208/9*d^(3/2)*g*p^3*arctan(x*
e^(1/2)/d^(1/2))/e^(3/2)-24*I*f*p^3*arctan(x*e^(1/2)/d^(1/2))^2*d^(1/2)/e^(
1/2)+24*f*p^2*x*ln(c*(e*x^2+d)^p)-32/3*d*g*p^2*x*ln(c*(e*x^2+d)^p)/e+8/9*g*
p^2*x^3*ln(c*(e*x^2+d)^p)+32/3*d^(3/2)*g*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(c
*(e*x^2+d)^p)/e^(3/2)-6*f*p*x*ln(c*(e*x^2+d)^p)^2+2*d*g*p*x*ln(c*(e*x^2+d)^
p)^2/e-2/3*g*p*x^3*ln(c*(e*x^2+d)^p)^2+f*x*ln(c*(e*x^2+d)^p)^3+1/3*g*x^3*ln
(c*(e*x^2+d)^p)^3+64/3*d^(3/2)*g*p^3*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)
/(d^(1/2)+I*x*e^(1/2)))/e^(3/2)+32/3*I*d^(3/2)*g*p^3*arctan(x*e^(1/2)/d^(1/
2))^2/e^(3/2)+48*f*p^3*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)+32/3*I*d^(
3/2)*g*p^3*polylog(2,(-d^(1/2)+I*x*e^(1/2))/(d^(1/2)+I*x*e^(1/2)))/e^(3/2)-
24*f*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)*d^(1/2)/e^(1/2)-48*f*p
^3*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^
(1/2)-24*I*f*p^3*polylog(2,(-d^(1/2)+I*x*e^(1/2))/(d^(1/2)+I*x*e^(1/2)))*d^
```

```
(1/2)/e^(1/2)-2*d*(d*g-3*e*f)*p*Unintegrable(ln(c*(e*x^2+d)^p)^2/(e*x^2+d),
x)/e
```

Rubi [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (f+gx^2)\log^3\left(c\left(d+ex^2\right)^p\right)\,dx=\int (f+gx^2)\log^3\left(c\left(d+ex^2\right)^p\right)\,dx$$

```
[In] Int[(f + g*x^2)*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] -48*f*p^3*x + (208*d*g*p^3*x)/(9*e) - (16*g*p^3*x^3)/27 + (48*Sqrt[d]*f*p^3
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (208*d^(3/2)*g*p^3*ArcTan[(Sqrt[e]*
x)/Sqrt[d]])/(9*e^(3/2)) - ((24*I)*Sqrt[d]*f*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]
]^2)/Sqrt[e] + (((32*I)/3)*d^(3/2)*g*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(
3/2) - (48*Sqrt[d]*f*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[
d] + I*Sqrt[e]*x)])/Sqrt[e] + (64*d^(3/2)*g*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(3*e^(3/2)) + 24*f*p^2*x*Log[c*(
d + e*x^2)^p] - (32*d*g*p^2*x*Log[c*(d + e*x^2)^p])/(3*e) + (8*g*p^2*x^3*Lo
g[c*(d + e*x^2)^p])/9 - (24*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c
*(d + e*x^2)^p])/Sqrt[e] + (32*d^(3/2)*g*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Lo
g[c*(d + e*x^2)^p])/(3*e^(3/2)) - 6*f*p*x*Log[c*(d + e*x^2)^p]^2 + (2*d*g*p
*x*Log[c*(d + e*x^2)^p]^2)/e - (2*g*p*x^3*Log[c*(d + e*x^2)^p]^2)/3 + f*x*L
og[c*(d + e*x^2)^p]^3 + (g*x^3*Log[c*(d + e*x^2)^p]^3)/3 - ((24*I)*Sqrt[d]*
f*p^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + (((32*
I)/3)*d^(3/2)*g*p^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/e^
(3/2) + 6*d*f*p*Defer[Int][Log[c*(d + e*x^2)^p]^2/(d + e*x^2), x] - (2*d^2*
g*p*Defer[Int][Log[c*(d + e*x^2)^p]^2/(d + e*x^2), x])/e
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(f\log^3\left(c\left(d+ex^2\right)^p\right)+gx^2\log^3\left(c\left(d+ex^2\right)^p\right)\right)\,dx\\
&= f\int \log^3\left(c\left(d+ex^2\right)^p\right)\,dx+g\int x^2\log^3\left(c\left(d+ex^2\right)^p\right)\,dx\\
&= fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^3\left(c\left(d+ex^2\right)^p\right)\\
&\quad -(6efp)\int \frac{x^2\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx-(2egp)\int \frac{x^4\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^3\left(c\left(d+ex^2\right)^p\right)\\
&\quad-(6efp)\int\left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{e}-\frac{d\log^2\left(c\left(d+ex^2\right)^p\right)}{e\left(d+ex^2\right)}\right)dx\\
&\quad-(2egp)\int\left(-\frac{d\log^2\left(c\left(d+ex^2\right)^p\right)}{e^2}+\frac{x^2\log^2\left(c\left(d+ex^2\right)^p\right)}{e}\right.\\
&\qquad\left.+\frac{d^2\log^2\left(c\left(d+ex^2\right)^p\right)}{e^2\left(d+ex^2\right)}\right)dx\\
&= fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^3\left(c\left(d+ex^2\right)^p\right)-(6fp)\int\log^2\left(c\left(d+ex^2\right)^p\right)dx\\
&\quad+(6dfp)\int\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}dx-(2gp)\int x^2\log^2\left(c\left(d+ex^2\right)^p\right)dx\\
&\quad+\frac{(2dgp)\int\log^2\left(c\left(d+ex^2\right)^p\right)dx}{e}-\frac{(2d^2gp)\int\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}dx}{e}\\
&= -6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{2dgpx\log^2\left(c\left(d+ex^2\right)^p\right)}{e}-\frac{2}{3}gpx^3\log^2\left(c\left(d+ex^2\right)^p\right)\\
&\quad+fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^3\left(c\left(d+ex^2\right)^p\right)+(6dfp)\int\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}dx\\
&\quad-\frac{(2d^2gp)\int\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}dx}{e}+\left(24efp^2\right)\int\frac{x^2\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}dx\\
&\quad-\left(8dgp^2\right)\int\frac{x^2\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}dx+\frac{1}{3}\left(8egp^2\right)\int\frac{x^4\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}dx\\
&= -6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{2dgpx\log^2\left(c\left(d+ex^2\right)^p\right)}{e}\\
&\quad-\frac{2}{3}gpx^3\log^2\left(c\left(d+ex^2\right)^p\right)+fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^3\left(c\left(d+ex^2\right)^p\right)\\
&\quad+(6dfp)\int\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}dx-\frac{(2d^2gp)\int\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}dx}{e}\\
&\quad+\left(24efp^2\right)\int\left(\frac{\log\left(c\left(d+ex^2\right)^p\right)}{e}-\frac{d\log\left(c\left(d+ex^2\right)^p\right)}{e\left(d+ex^2\right)}\right)dx\\
&\quad-\left(8dgp^2\right)\int\left(\frac{\log\left(c\left(d+ex^2\right)^p\right)}{e}-\frac{d\log\left(c\left(d+ex^2\right)^p\right)}{e\left(d+ex^2\right)}\right)dx\\
&\quad+\frac{1}{3}\left(8egp^2\right)\int\left(-\frac{d\log\left(c\left(d+ex^2\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(d+ex^2\right)^p\right)}{e}\right.\\
&\qquad\left.+\frac{d^2\log\left(c\left(d+ex^2\right)^p\right)}{e^2\left(d+ex^2\right)}\right)dx
\end{aligned}
$$

$$
\begin{aligned}
&= -6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{2dgpx\log^2\left(c\left(d+ex^2\right)^p\right)}{e}-\frac{2}{3}gpx^3\log^2\left(c\left(d+ex^2\right)^p\right)\\
&\quad+fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^3\left(c\left(d+ex^2\right)^p\right)+(6dfp)\int\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx\\
&\quad-\frac{(2d^2gp)\int\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx}{e}+\left(24fp^2\right)\int\log\left(c\left(d+ex^2\right)^p\right)\,dx\\
&\quad-\left(24dfp^2\right)\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx+\frac{1}{3}\left(8gp^2\right)\int x^2\log\left(c\left(d+ex^2\right)^p\right)\,dx\\
&\quad-\frac{(8dgp^2)\int\log\left(c\left(d+ex^2\right)^p\right)\,dx}{3e}-\frac{(8dgp^2)\int\log\left(c\left(d+ex^2\right)^p\right)\,dx}{e}\\
&\quad+\frac{(8d^2gp^2)\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx}{3e}+\frac{(8d^2gp^2)\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx}{e}\\
&= 24fp^2x\log\left(c\left(d+ex^2\right)^p\right)-\frac{32dgp^2x\log\left(c\left(d+ex^2\right)^p\right)}{3e}\\
&\quad+\frac{8}{9}gp^2x^3\log\left(c\left(d+ex^2\right)^p\right)-\frac{24\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{e}}\\
&\quad+\frac{32d^{3/2}gp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{3e^{3/2}}\\
&\quad-6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{2dgpx\log^2\left(c\left(d+ex^2\right)^p\right)}{e}-\frac{2}{3}gpx^3\log^2\left(c\left(d+ex^2\right)^p\right)\\
&\quad+fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}gx^3\log^3\left(c\left(d+ex^2\right)^p\right)+(6dfp)\int\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx\\
&\quad-\frac{(2d^2gp)\int\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx}{e}-\left(48efp^3\right)\int\frac{x^2}{d+ex^2}\,dx\\
&\quad+\left(48defp^3\right)\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}\left(d+ex^2\right)}\,dx+\frac{1}{3}\left(16dgp^3\right)\int\frac{x^2}{d+ex^2}\,dx\\
&\quad+\left(16dgp^3\right)\int\frac{x^2}{d+ex^2}\,dx-\frac{1}{3}\left(16d^2gp^3\right)\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}\left(d+ex^2\right)}\,dx\\
&\quad-\left(16d^2gp^3\right)\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}\left(d+ex^2\right)}\,dx-\frac{1}{9}\left(16egp^3\right)\int\frac{x^4}{d+ex^2}\,dx
\end{aligned}
$$

$$= -48fp^3x + \frac{64dgp^3x}{3e} + 24fp^2x\log\left(c\left(d+ex^2\right)^p\right) - \frac{32dgp^2x\log\left(c(d+ex^2)^p\right)}{3e} + \frac{8}{9}gp^2x^3\log\left(c\left(d+ex^2\right)^p\right) - \frac{24\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{e}} + \frac{32d^{3/2}gp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{3e^{3/2}} - 6fpx\log^2\left(c\left(d+ex^2\right)^p\right) + \frac{2dgpx\log^2\left(c(d+ex^2)^p\right)}{e} - \frac{2}{3}gpx^3\log^2\left(c\left(d+ex^2\right)^p\right) + fx\log^3\left(c\left(d+ex^2\right)^p\right) + \frac{1}{3}gx^3\log^3\left(c\left(d+ex^2\right)^p\right) + (6dfp)\int\frac{\log^2\left(c(d+ex^2)^p\right)}{d+ex^2}\,dx - \frac{(2d^2gp)\int\frac{\log^2(c(d+ex^2)^p)}{d+ex^2}\,dx}{e} + \left(48dfp^3\right)\int\frac{1}{d+ex^2}\,dx + \left(48\sqrt{d}\sqrt{e}fp^3\right)\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d+ex^2}\,dx - \frac{(16d^2gp^3)\int\frac{1}{d+ex^2}\,dx}{3e} - \frac{(16d^2gp^3)\int\frac{1}{d+ex^2}\,dx}{e} - \frac{(16d^{3/2}gp^3)\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d+ex^2}\,dx}{3\sqrt{e}} - \frac{(16d^{3/2}gp^3)\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d+ex^2}\,dx}{\sqrt{e}} - \frac{1}{9}(16egp^3)\int\left(-\frac{d}{e^2} + \frac{x^2}{e} + \frac{d^2}{e^2(d+ex^2)}\right)dx$$

$$= -48fp^3x + \frac{208dgp^3x}{9e} - \frac{16}{27}gp^3x^3 + \frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{64d^{3/2}gp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}} - \frac{24i\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} + \frac{32id^{3/2}gp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{3e^{3/2}}$$

$$+24fp^2x\log\left(c\left(d+ex^2\right)^p\right) - \frac{32dgp^2x\log\left(c(d+ex^2)^p\right)}{3e} + \frac{8}{9}gp^2x^3\log\left(c\left(d+ex^2\right)^p\right) - \frac{24\sqrt{d}fp^2\tan^{-1}}{}$$

$$= -48fp^3x + \frac{208dgp^3x}{9e} - \frac{16}{27}gp^3x^3 + \frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{208d^{3/2}gp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}} - \frac{24i\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} + \frac{32id^{3/2}gp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{3e^{3/2}} - \frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} + \frac{64d^{3/2}gp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{3e^{3/2}}$$

$$+24fp^2x\log\left(c\left(d+ex^2\right)^p\right) - \frac{32dgp^2x\log\left(c(d+ex^2)^p\right)}{3e} + \frac{8}{9}gp^2x^3\log\left(c\left(d+ex^2\right)^p\right) - \frac{24\sqrt{d}fp^2\tan^{-1}}{}$$

$$
\begin{aligned}
&= -48fp^3x + \frac{208dgp^3x}{9e} - \frac{16}{27}gp^3x^3 + \frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} \\
&\quad - \frac{208d^{3/2}gp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}} - \frac{24i\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} + \frac{32id^{3/2}gp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{3e^{3/2}} \\
&\quad - \frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} + \frac{64d^{3/2}gp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{3e^{3/2}} \\
&\quad + 24fp^2x\log\left(c(d+ex^2)^p\right) - \frac{32dgp^2x\log\left(c(d+ex^2)^p\right)}{3e} + \frac{8}{9}gp^2x^3\log\left(c(d+ex^2)^p\right) - \frac{24\sqrt{d}fp^2\tan^{-1}}{}
\end{aligned}
$$

$$
\begin{aligned}
&= -48fp^3x + \frac{208dgp^3x}{9e} - \frac{16}{27}gp^3x^3 + \frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} \\
&\quad - \frac{208d^{3/2}gp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9e^{3/2}} - \frac{24i\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} + \frac{32id^{3/2}gp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{3e^{3/2}} \\
&\quad - \frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} + \frac{64d^{3/2}gp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{3e^{3/2}} \\
&\quad + 24fp^2x\log\left(c(d+ex^2)^p\right) - \frac{32dgp^2x\log\left(c(d+ex^2)^p\right)}{3e} + \frac{8}{9}gp^2x^3\log\left(c(d+ex^2)^p\right) - \frac{24\sqrt{d}fp^2\tan^{-1}}{}
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1772 vs. $2(683) = 1366$.

Time = 9.07 (sec) , antiderivative size = 1772, normalized size of antiderivative = 80.55

$$\int (f+gx^2)\log^3\left(c(d+ex^2)^p\right)\,dx = \frac{2dgpx\left(-p\log\left(d+ex^2\right)+\log\left(c(d+ex^2)^p\right)\right)^2}{e}$$
$$+\frac{6\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-p\log\left(d+ex^2\right)+\log\left(c(d+ex^2)^p\right)\right)^2}{\sqrt{e}}$$
$$-\frac{2d^{3/2}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-p\log\left(d+ex^2\right)+\log\left(c(d+ex^2)^p\right)\right)^2}{e^{3/2}}$$
$$+3fpx\log\left(d+ex^2\right)\left(-p\log\left(d+ex^2\right)+\log\left(c(d+ex^2)^p\right)\right)^2$$
$$+gpx^3\log\left(d+ex^2\right)\left(-p\log\left(d+ex^2\right)+\log\left(c(d+ex^2)^p\right)\right)^2$$
$$+fx\left(-p\log\left(d+ex^2\right)+\log\left(c(d+ex^2)^p\right)\right)^2\left(-6p-p\log\left(d+ex^2\right)+\log\left(c(d+ex^2)^p\right)\right)$$
$$+\frac{1}{3}gx^3\left(-p\log\left(d+ex^2\right)+\log\left(c(d+ex^2)^p\right)\right)^2\left(-2p-p\log\left(d+ex^2\right)+\log\left(c(d+ex^2)^p\right)\right)$$
$$+3fp^2\left(-p\log\left(d+ex^2\right)+\log\left(c(d+ex^2)^p\right)\right)\left(x\log^2\left(d+ex^2\right)-\frac{4\left(-i\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2+\sqrt{e}x(-2+\log\right.}{}\right.$$
$$+3gp^2\left(-p\log\left(d+ex^2\right)+\log\left(c(d+ex^2)^p\right)\right)\left(\frac{1}{3}x^3\log^2\left(d+ex^2\right)-\frac{4\left(9id^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2+3d^{3/2}\arctan\right.}{}\right.$$

```
[In] Integrate[(f + g*x^2)*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] (2*d*g*p*x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/e + (6*Sqrt[d]*f
*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])
^2)/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-(p*Log[d + e*x^2
]) + Log[c*(d + e*x^2)^p])^2)/e^(3/2) + 3*f*p*x*Log[d + e*x^2]*(-(p*Log[d +
 e*x^2]) + Log[c*(d + e*x^2)^p])^2 + g*p*x^3*Log[d + e*x^2]*(-(p*Log[d + e*
x^2]) + Log[c*(d + e*x^2)^p])^2 + f*x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x
^2)^p])^2*(-6*p - p*Log[d + e*x^2] + Log[c*(d + e*x^2)^p]) + (g*x^3*(-(p*Lo
g[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2*(-2*p - p*Log[d + e*x^2] + Log[c*(d
 + e*x^2)^p]))/3 + 3*f*p^2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])*(x*
Log[d + e*x^2]^2 - (4*((-I)*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 + Sqrt[e]
*x*(-2 + Log[d + e*x^2]) - Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-2 + 2*Log[
(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + Log[d + e*x^2]) - I*Sqrt[d]*PolyLog[
2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)]))/Sqrt[e]) + 3*g*p^2
*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])*((x^3*Log[d + e*x^2]^2)/3 - (
4*((9*I)*d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 + 3*d^(3/2)*ArcTan[(Sqrt[e]*
x)/Sqrt[d]]*(-8 + 6*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + 3*Log[d + e*
x^2]) + Sqrt[e]*x*(24*d - 2*e*x^2 + (-9*d + 3*e*x^2)*Log[d + e*x^2]) + (9*I
)*d^(3/2)*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)]))/
```

```
(27*e^(3/2))) + (g*p^3*(416*Sqrt[-d]*d^(3/2)*Sqrt[d + e*x^2]*Sqrt[1 - d/(d
+ e*x^2)]*ArcSin[Sqrt[d]/Sqrt[d + e*x^2]] + 36*Sqrt[-d]*d^(3/2)*Sqrt[1 - d/
(d + e*x^2)]*(8*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2,
3/2}, d/(d + e*x^2)] + 4*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3
/2}, d/(d + e*x^2)]*Log[d + e*x^2] + Sqrt[d + e*x^2]*ArcSin[Sqrt[d]/Sqrt[d
+ e*x^2]]*Log[d + e*x^2]^2) - (2*Sqrt[-d]*e*x^2*((d + e*x^2)*(16 - 24*Log[d
 + e*x^2] + 18*Log[d + e*x^2]^2 - 9*Log[d + e*x^2]^3) + d*(-640 + 312*Log[d
 + e*x^2] - 72*Log[d + e*x^2]^2 + 9*Log[d + e*x^2]^3)))/3 - 48*d^2*(4*Sqrt[
e*x^2]*ArcTanh[Sqrt[e*x^2]/Sqrt[-d]]*(Log[d + e*x^2] - Log[(d + e*x^2)/d])
- Sqrt[-d]*Sqrt[1 - (d + e*x^2)/d]*(Log[(d + e*x^2)/d]^2 - 4*Log[(d + e*x^2
)/d]*Log[(1 + Sqrt[1 - (d + e*x^2)/d])/2] + 2*Log[(1 + Sqrt[1 - (d + e*x^2)
/d])/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[1 - (d + e*x^2)/d]/2])))))/(18*Sqrt[-d]*
e^2*x) + (f*p^3*(-48*Sqrt[-d^2]*Sqrt[d + e*x^2]*Sqrt[1 - d/(d + e*x^2)]*Arc
Sin[Sqrt[d]/Sqrt[d + e*x^2]] - 6*Sqrt[-d^2]*Sqrt[1 - d/(d + e*x^2)]*(8*Sqrt
[d]*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, d/(d + e*x^2)]
 + 4*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2}, d/(d + e*x^2)]*
Log[d + e*x^2] + Sqrt[d + e*x^2]*ArcSin[Sqrt[d]/Sqrt[d + e*x^2]]*Log[d + e*
x^2]^2) + Sqrt[-d]*e*x^2*(-48 + 24*Log[d + e*x^2] - 6*Log[d + e*x^2]^2 + Lo
g[d + e*x^2]^3) + 24*d*Sqrt[e*x^2]*ArcTanh[Sqrt[e*x^2]/Sqrt[-d]]*(Log[d + e
*x^2] - Log[(d + e*x^2)/d]) + 6*(-d)^(3/2)*Sqrt[1 - (d + e*x^2)/d]*(Log[(d
+ e*x^2)/d]^2 - 4*Log[(d + e*x^2)/d]*Log[(1 + Sqrt[1 - (d + e*x^2)/d])/2] +
 2*Log[(1 + Sqrt[1 - (d + e*x^2)/d])/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[1 - (d
+ e*x^2)/d]/2])))/(Sqrt[-d]*e*x)
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \left(g\,x^{2}+f\right) \ln \left(c\left(e\,x^{2}+d\right)^{p}\right)^{3}dx$$

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)^3,x)
[Out] int((g*x^2+f)*ln(c*(e*x^2+d)^p)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (f+gx^2)\log^3\left(c\left(d+ex^2\right)^p\right)\,dx = \int \left(gx^2+f\right)\log\left(\left(ex^2+d\right)^pc\right)^3\,dx$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="fricas")
[Out] integral((g*x^2 + f)*log((e*x^2 + d)^p*c)^3, x)
```

Sympy [N/A]

Not integrable

Time = 10.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int (f+gx^2)\log^3\left(c(d+ex^2)^p\right)\,dx=\int (f+gx^2)\log\left(c(d+ex^2)^p\right)^3\,dx$$

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)**3,x)
```

```
[Out] Integral((f + g*x**2)*log(c*(d + e*x**2)**p)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (f+gx^2)\log^3\left(c(d+ex^2)^p\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (f+gx^2)\log^3\left(c(d+ex^2)^p\right)\,dx=\int (gx^2+f)\log\left((ex^2+d)^pc\right)^3\,dx$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)*log((e*x^2 + d)^p*c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (f + gx^2) \log^3 \left(c(d + ex^2)^p\right) dx = \int \ln \left(c \left(e x^2 + d\right)^p\right)^3 \left(g x^2 + f\right) dx$$

```
[In] int(log(c*(d + e*x^2)^p)^3*(f + g*x^2),x)
[Out] int(log(c*(d + e*x^2)^p)^3*(f + g*x^2), x)
```

3.278 $\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx=\operatorname{Int}\left(\frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^2},x\right)$$

```
[Out] Unintegrable(ln(c*(e*x^2+d)^p)^3/(g*x^2+f),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx=\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

```
[In] Int[Log[c*(d + e*x^2)^p]^3/(f + g*x^2),x]
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^3/(f + g*x^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.98 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx=\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

```
[In] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^2),x]
[Out] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^2), x]
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c(e\,x^2+d)^p\right)^3}{g\,x^2+f}dx$$

```
[In] int(ln(c*(e*x^2+d)^p)^3/(g*x^2+f),x)
[Out] int(ln(c*(e*x^2+d)^p)^3/(g*x^2+f),x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx=\int \frac{\log\left((ex^2+d)^p c\right)^3}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log((e*x^2 + d)^p*c)^3/(g*x^2 + f), x)
```

Sympy [N/A]

Not integrable

Time = 17.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx=\int \frac{\log\left(c(d+ex^2)^p\right)^3}{f+gx^2}\,dx$$

```
[In] integrate(ln(c*(e*x**2+d)**p)**3/(g*x**2+f),x)
[Out] Integral(log(c*(d + e*x**2)**p)**3/(f + g*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 2.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^3}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f),x, algorithm="maxima")
[Out] integrate(log((e*x^2 + d)^p*c)^3/(g*x^2 + f), x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^3}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log((e*x^2 + d)^p*c)^3/(g*x^2 + f), x)
```

Mupad [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)^3}{g\,x^2+f}\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)^3/(f + g*x^2),x)
[Out] int(log(c*(d + e*x^2)^p)^3/(f + g*x^2), x)
```

3.279 $\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^2)^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^2)^2} \, dx = \operatorname{Int}\left(\frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}, x\right)$$

```
[Out] Unintegrable(ln(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^2)^2} \, dx = \int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^2)^2} \, dx$$

```
[In] Int[Log[c*(d + e*x^2)^p]^3/(f + g*x^2)^2,x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^3/(f + g*x^2)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^2)^2} \, dx$$

Mathematica [N/A]

Not integrable

Time = 6.57 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx$$

```
[In] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^2)^2,x]
[Out] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^2)^2, x]
```

Maple [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c(e\,x^2+d)^p\right)^3}{(g\,x^2+f)^2}dx$$

```
[In] int(ln(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x)
[Out] int(ln(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^3}{(gx^2+f)^2}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral(log((e*x^2 + d)^p*c)^3/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(e*x**2+d)**p)**3/(g*x**2+f)**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time $= 0.36$ (sec) , antiderivative size $= 26$, normalized size of antiderivative $= 1.08$

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{\log\left((ex^2+d)^pc\right)^3}{(gx^2+f)^2}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate(log((e*x^2 + d)^p*c)^3/(g*x^2 + f)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx=\int \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)^3}{{\left(g\,x^2+f\right)}^2}\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)^3/(f + g*x^2)^2,x)
[Out] int(log(c*(d + e*x^2)^p)^3/(f + g*x^2)^2, x)
```

3.280 $\int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx = \text{Int}\left(\frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)},x\right)$$

```
[Out] Unintegrable((g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx = \int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx$$

```
[In] Int[(f + g*x^2)^2/Log[c*(d + e*x^2)^p],x]
```

```
[Out] Defer[Int][(f + g*x^2)^2/Log[c*(d + e*x^2)^p], x]
```

Rubi steps

$$\text{integral} = \int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx=\int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx$$

```
[In] Integrate[(f + g*x^2)^2/Log[c*(d + e*x^2)^p],x]
[Out] Integrate[(f + g*x^2)^2/Log[c*(d + e*x^2)^p], x]
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(g\,x^2+f)^2}{\ln(c\,(e\,x^2+d)^p)}dx$$

```
[In] int((g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
[Out] int((g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx=\int \frac{(gx^2+f)^2}{\log((ex^2+d)^pc)}\,dx$$

```
[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="fricas")
[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)/log((e*x^2 + d)^p*c), x)
```

Sympy [N/A]

Not integrable

Time = 8.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx=\int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx$$

```
[In] integrate((g*x**2+f)**2/ln(c*(e*x**2+d)**p),x)
[Out] Integral((f + g*x**2)**2/log(c*(d + e*x**2)**p), x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx = \int \frac{(gx^2+f)^2}{\log((ex^2+d)^pc)}\,dx$$

```
[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] integrate((g*x^2 + f)^2/log((e*x^2 + d)^p*c), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx = \int \frac{(gx^2+f)^2}{\log((ex^2+d)^pc)}\,dx$$

```
[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="giac")
[Out] integrate((g*x^2 + f)^2/log((e*x^2 + d)^p*c), x)
```

Mupad [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^2)^2}{\log(c(d+ex^2)^p)}\,dx = \int \frac{(g\,x^2+f)^2}{\ln(c\,(e\,x^2+d)^p)}\,dx$$

```
[In] int((f + g*x^2)^2/log(c*(d + e*x^2)^p),x)
[Out] int((f + g*x^2)^2/log(c*(d + e*x^2)^p), x)
```

3.281 $\int \frac{f+gx^2}{\log(c(d+ex^2)^p)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\operatorname{Int}\left(\frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable((g*x^2+f)/ln(c*(e*x^2+d)^p),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] Int[(f + g*x^2)/Log[c*(d + e*x^2)^p],x]
```

```
[Out] Defer[Int][(f + g*x^2)/Log[c*(d + e*x^2)^p], x]
```

Rubi steps

$$\text{integral}=\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^2}{\log \left(c \left(d+ex^2\right)^p\right)} \, dx = \int \frac{f+gx^2}{\log \left(c \left(d+ex^2\right)^p\right)} \, dx$$

```
[In] Integrate[(f + g*x^2)/Log[c*(d + e*x^2)^p],x]
[Out] Integrate[(f + g*x^2)/Log[c*(d + e*x^2)^p], x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{g\,x^{2}+f}{\ln \left(c \left(e\,x^{2}+d\right)^{p}\right)}dx$$

```
[In] int((g*x^2+f)/ln(c*(e*x^2+d)^p),x)
[Out] int((g*x^2+f)/ln(c*(e*x^2+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^2}{\log \left(c \left(d+ex^2\right)^p\right)} \, dx = \int \frac{gx^2+f}{\log \left(\left(ex^2+d\right)^p c\right)} \, dx$$

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="fricas")
[Out] integral((g*x^2 + f)/log((e*x^2 + d)^p*c), x)
```

Sympy [N/A]

Not integrable

Time = 4.49 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{f+gx^2}{\log \left(c \left(d+ex^2\right)^p\right)} \, dx = \int \frac{f+gx^2}{\log \left(c \left(d+ex^2\right)^p\right)} \, dx$$

```
[In] integrate((g*x**2+f)/ln(c*(e*x**2+d)**p),x)
[Out] Integral((f + g*x**2)/log(c*(d + e*x**2)**p), x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{gx^2+f}{\log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] integrate((g*x^2 + f)/log((e*x^2 + d)^p*c), x)
```

Giac [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{gx^2+f}{\log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="giac")
[Out] integrate((g*x^2 + f)/log((e*x^2 + d)^p*c), x)
```

Mupad [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{g\,x^2+f}{\ln\left(c\,\left(e\,x^2+d\right)^p\right)}\,dx$$

```
[In] int((f + g*x^2)/log(c*(d + e*x^2)^p),x)
[Out] int((f + g*x^2)/log(c*(d + e*x^2)^p), x)
```

3.282 $\int \frac{1}{(f+gx^2)\log(c(d+ex^2)^p)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx^2)\log(c(d+ex^2)^p)}\,dx = \mathrm{Int}\left(\frac{1}{(f+gx^2)\log(c(d+ex^2)^p)},x\right)$$

```
[Out] Unintegrable(1/(g*x^2+f)/ln(c*(e*x^2+d)^p),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx^2)\log(c(d+ex^2)^p)}\,dx = \int \frac{1}{(f+gx^2)\log(c(d+ex^2)^p)}\,dx$$

```
[In] Int[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]),x]
[Out] Defer[Int][1/((f + g*x^2)*Log[c*(d + e*x^2)^p]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(f+gx^2)\log(c(d+ex^2)^p)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] Integrate[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]),x]
[Out] Integrate[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]), x]
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g\,x^2+f)\ln\left(c\left(e\,x^2+d\right)^p\right)}dx$$

```
[In] int(1/(g*x^2+f)/ln(c*(e*x^2+d)^p),x)
[Out] int(1/(g*x^2+f)/ln(c*(e*x^2+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(gx^2+f)\log\left((ex^2+d)^p c\right)}\,dx$$

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="fricas")
[Out] integral(1/((g*x^2 + f)*log((e*x^2 + d)^p*c)), x)
```

Sympy [N/A]

Not integrable

Time = 15.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] integrate(1/(g*x**2+f)/ln(c*(e*x**2+d)**p),x)
[Out] Integral(1/((f + g*x**2)*log(c*(d + e*x**2)**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)\log(c(d+ex^2)^p)}\,dx = \int \frac{1}{(gx^2+f)\log((ex^2+d)^pc)}\,dx$$

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] integrate(1/((g*x^2 + f)*log((e*x^2 + d)^p*c)), x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)\log(c(d+ex^2)^p)}\,dx = \int \frac{1}{(gx^2+f)\log((ex^2+d)^pc)}\,dx$$

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p),x, algorithm="giac")
[Out] integrate(1/((g*x^2 + f)*log((e*x^2 + d)^p*c)), x)
```

Mupad [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)\log(c(d+ex^2)^p)}\,dx = \int \frac{1}{\ln(c(e\,x^2+d)^p)\,(g\,x^2+f)}\,dx$$

```
[In] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^2)),x)
[Out] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^2)), x)
```

3.283 $\int \frac{1}{(f+gx^2)^2 \log(c(d+ex^2)^p)} dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)} \, dx = \text{Int}\left(\frac{1}{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)}, x\right)$$

```
[Out] Unintegrable(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)} \, dx = \int \frac{1}{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)} \, dx$$

```
[In] Int[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]),x]
[Out] Defer[Int][1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(f+gx^2)^2 \log\left(c\left(d+ex^2\right)^p\right)} \, dx$$

Mathematica [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)^2\log(c(d+ex^2)^p)}\,dx = \int \frac{1}{(f+gx^2)^2\log(c(d+ex^2)^p)}\,dx$$

```
[In] Integrate[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]),x]
[Out] Integrate[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]), x]
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g\,x^2+f)^2\ln(c\,(e\,x^2+d)^p)}dx$$

```
[In] int(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
[Out] int(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{1}{(f+gx^2)^2\log(c(d+ex^2)^p)}\,dx = \int \frac{1}{(gx^2+f)^2\log((ex^2+d)^pc)}\,dx$$

```
[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="fricas")
[Out] integral(1/((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)), x)
```

Sympy [N/A]

Not integrable

Time = 167.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(f+gx^2)^2\log(c(d+ex^2)^p)}\,dx = \int \frac{1}{(f+gx^2)^2\log(c(d+ex^2)^p)}\,dx$$

```
[In] integrate(1/(g*x**2+f)**2/ln(c*(e*x**2+d)**p),x)
[Out] Integral(1/((f + g*x**2)**2*log(c*(d + e*x**2)**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{\left(gx^2+f\right)^2 \log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

```
[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] integrate(1/((g*x^2 + f)^2*log((e*x^2 + d)^p*c)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{\left(gx^2+f\right)^2 \log\left(\left(ex^2+d\right)^p c\right)}\,dx$$

```
[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p),x, algorithm="giac")
[Out] integrate(1/((g*x^2 + f)^2*log((e*x^2 + d)^p*c)), x)
```

Mupad [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{\ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(g\,x^2+f\right)^2}\,dx$$

```
[In] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^2)^2),x)
[Out] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^2)^2), x)
```

3.284 $\int \frac{(f+gx^2)^2}{\log^2(c(d+ex^2)^p)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(f+gx^2)^2}{\log^2(c(d+ex^2)^p)}\,dx = \operatorname{Int}\left(\frac{(f+gx^2)^2}{\log^2(c(d+ex^2)^p)},x\right)$$

```
[Out] Unintegrable((g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(f+gx^2)^2}{\log^2(c(d+ex^2)^p)}\,dx = \int \frac{(f+gx^2)^2}{\log^2(c(d+ex^2)^p)}\,dx$$

```
[In] Int[(f + g*x^2)^2/Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] Defer[Int][(f + g*x^2)^2/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(f+gx^2)^2}{\log^2(c(d+ex^2)^p)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] Integrate[(f + g*x^2)^2/Log[c*(d + e*x^2)^p]^2,x]
[Out] Integrate[(f + g*x^2)^2/Log[c*(d + e*x^2)^p]^2, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(g\,x^{2}+f\right)^{2}}{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{2}}dx$$

```
[In] int((g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
[Out] int((g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(gx^2+f)^2}{\log\left((ex^2+d)^pc\right)^2}\,dx$$

```
[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)/log((e*x^2 + d)^p*c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 11.74 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(f+gx^2)^2}{\log\left(c\left(d+ex^2\right)^p\right)^2}\,dx$$

```
[In] integrate((g*x**2+f)**2/ln(c*(e*x**2+d)**p)**2,x)
[Out] Integral((f + g*x**2)**2/log(c*(d + e*x**2)**p)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 6.17

$$\int \frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(gx^2+f)^2}{\log\left((ex^2+d)^pc\right)^2}\,dx$$

```
[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
[Out] -1/2*(e*g^2*x^6 + (2*e*f*g + d*g^2)*x^4 + d*f^2 + (e*f^2 + 2*d*f*g)*x^2)/(e
*p*x*log((e*x^2 + d)^p) + e*p*x*log(c)) + integrate(1/2*(5*e*g^2*x^6 + 3*(2
*e*f*g + d*g^2)*x^4 - d*f^2 + (e*f^2 + 2*d*f*g)*x^2)/(e*p*x^2*log((e*x^2 +
d)^p) + e*p*x^2*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^2)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(gx^2+f)^2}{\log\left((ex^2+d)^pc\right)^2}\,dx$$

```
[In] integrate((g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
[Out] integrate((g*x^2 + f)^2/log((e*x^2 + d)^p*c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^2)^2}{\log^2(c(d+ex^2)^p)}\,dx = \int \frac{(g\,x^2+f)^2}{\ln\left(c\,(e\,x^2+d)^p\right)^2}\,dx$$

```
[In] int((f + g*x^2)^2/log(c*(d + e*x^2)^p)^2,x)
[Out] int((f + g*x^2)^2/log(c*(d + e*x^2)^p)^2, x)
```

3.285 $\int \frac{f+gx^2}{\log^2(c(d+ex^2)^p)} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\operatorname{Int}\left(\frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable((g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] Int[(f + g*x^2)/Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] Defer[Int][(f + g*x^2)/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] Integrate[(f + g*x^2)/Log[c*(d + e*x^2)^p]^2,x]
[Out] Integrate[(f + g*x^2)/Log[c*(d + e*x^2)^p]^2, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{g\,x^2+f}{\ln\left(c\left(e\,x^2+d\right)^p\right)^2}dx$$

```
[In] int((g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
[Out] int((g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{gx^2+f}{\log\left(\left(ex^2+d\right)^pc\right)^2}\,dx$$

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
[Out] integral((g*x^2 + f)/log((e*x^2 + d)^p*c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 7.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{f+gx^2}{\log\left(c\left(d+ex^2\right)^p\right)^2}\,dx$$

```
[In] integrate((g*x**2+f)/ln(c*(e*x**2+d)**p)**2,x)
[Out] Integral((f + g*x**2)/log(c*(d + e*x**2)**p)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 101, normalized size of antiderivative = 4.59

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{gx^2+f}{\log\left(\left(ex^2+d\right)^p c\right)^2}\,dx$$

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*g*x^4 + (e*f + d*g)*x^2 + d*f)/(e*p*x*log((e*x^2 + d)^p) + e*p*x*lo
g(c)) + integrate(1/2*(3*e*g*x^4 + (e*f + d*g)*x^2 - d*f)/(e*p*x^2*log((e*x
^2 + d)^p) + e*p*x^2*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{gx^2+f}{\log\left(\left(ex^2+d\right)^p c\right)^2}\,dx$$

```
[In] integrate((g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^2 + f)/log((e*x^2 + d)^p*c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{g\,x^2+f}{\ln\left(c\,\left(e\,x^2+d\right)^p\right)^2}\,dx$$

```
[In] int((f + g*x^2)/log(c*(d + e*x^2)^p)^2,x)
```

```
[Out] int((f + g*x^2)/log(c*(d + e*x^2)^p)^2, x)
```

3.286 $\int \frac{1}{(f+gx^2)\log^2(c(d+ex^2)^p)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \operatorname{Int}\left(\frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] Int[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]^2),x]
```

```
[Out] Defer[Int][1/((f + g*x^2)*Log[c*(d + e*x^2)^p]^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] Integrate[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]^2),x]
[Out] Integrate[1/((f + g*x^2)*Log[c*(d + e*x^2)^p]^2), x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(g\,x^2+f\right)\ln\left(c\left(e\,x^2+d\right)^p\right)^2}dx$$

```
[In] int(1/(g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
[Out] int(1/(g*x^2+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{(gx^2+f)\log\left(\left(ex^2+d\right)^p c\right)^2}\,dx$$

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
[Out] integral(1/((g*x^2 + f)*log((e*x^2 + d)^p*c)^2), x)
```

Sympy [N/A]

Not integrable

Time = 22.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{(f+gx^2)\log\left(c\left(d+ex^2\right)^p\right)^2}\,dx$$

```
[In] integrate(1/(g*x**2+f)/ln(c*(e*x**2+d)**p)**2,x)
[Out] Integral(1/((f + g*x**2)*log(c*(d + e*x**2)**p)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 6.54

$$\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{(gx^2+f)\log\left((ex^2+d)^p c\right)^2}\,dx$$

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*x^2 + d)/(e*g*p*x^3*log(c) + e*f*p*x*log(c) + (e*g*p*x^3 + e*f*p*x)
*log((e*x^2 + d)^p)) - integrate(1/2*(e*g*x^4 - (e*f - 3*d*g)*x^2 + d*f)/(e
*g^2*p*x^6*log(c) + 2*e*f*g*p*x^4*log(c) + e*f^2*p*x^2*log(c) + (e*g^2*p*x^
6 + 2*e*f*g*p*x^4 + e*f^2*p*x^2)*log((e*x^2 + d)^p)), x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{(gx^2+f)\log\left((ex^2+d)^p c\right)^2}\,dx$$

```
[In] integrate(1/(g*x^2+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x^2 + f)*log((e*x^2 + d)^p*c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{\ln\left(c\left(e\,x^2+d\right)^p\right)^2\,\left(g\,x^2+f\right)}\,dx$$

```
[In] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^2)),x)
```

```
[Out] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^2)), x)
```

3.287 $\int \frac{1}{(f+gx^2)^2 \log^2(c(d+ex^2)^p)} dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx^2)^2 \log^2(c(d+ex^2)^p)} dx = \operatorname{Int}\left(\frac{1}{(f+gx^2)^2 \log^2(c(d+ex^2)^p)}, x\right)$$

```
[Out] Unintegrable(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx^2)^2 \log^2(c(d+ex^2)^p)} dx = \int \frac{1}{(f+gx^2)^2 \log^2(c(d+ex^2)^p)} dx$$

```
[In] Int[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2),x]
```

```
[Out] Defer[Int][1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(f+gx^2)^2 \log^2(c(d+ex^2)^p)} dx$$

Mathematica [N/A]

Not integrable

Time = 4.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)^2\log^2(c(d+ex^2)^p)}\,dx = \int \frac{1}{(f+gx^2)^2\log^2(c(d+ex^2)^p)}\,dx$$

```
[In] Integrate[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2),x]
[Out] Integrate[1/((f + g*x^2)^2*Log[c*(d + e*x^2)^p]^2), x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g\,x^2+f)^2\ln(c\,(e\,x^2+d)^p)^2}dx$$

```
[In] int(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
[Out] int(1/(g*x^2+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{1}{(f+gx^2)^2\log^2(c(d+ex^2)^p)}\,dx = \int \frac{1}{(gx^2+f)^2\log((ex^2+d)^pc)^2}\,dx$$

```
[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
[Out] integral(1/((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx^2)^2\log^2(c(d+ex^2)^p)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(g*x**2+f)**2/ln(c*(e*x**2+d)**p)**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 8.92

$$\int \frac{1}{(f+gx^2)^2 \log^2 (c (d+ex^2)^p)} dx = \int \frac{1}{(gx^2+f)^2 \log ((ex^2+d)^p c)^2} dx$$

```
[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*x^2 + d)/(e*g^2*p*x^5*log(c) + 2*e*f*g*p*x^3*log(c) + e*f^2*p*x*log
(c) + (e*g^2*p*x^5 + 2*e*f*g*p*x^3 + e*f^2*p*x)*log((e*x^2 + d)^p)) - integ
rate(1/2*(3*e*g*x^4 - (e*f - 5*d*g)*x^2 + d*f)/(e*g^3*p*x^8*log(c) + 3*e*f*
g^2*p*x^6*log(c) + 3*e*f^2*g*p*x^4*log(c) + e*f^3*p*x^2*log(c) + (e*g^3*p*x
^8 + 3*e*f*g^2*p*x^6 + 3*e*f^2*g*p*x^4 + e*f^3*p*x^2)*log((e*x^2 + d)^p)),
x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)^2 \log^2 (c (d+ex^2)^p)} dx = \int \frac{1}{(gx^2+f)^2 \log ((ex^2+d)^p c)^2} dx$$

```
[In] integrate(1/(g*x^2+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x^2 + f)^2*log((e*x^2 + d)^p*c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^2)^2 \log^2 (c (d+ex^2)^p)} dx = \int \frac{1}{\ln (c (e\,x^2+d)^p)^2 \,(g\,x^2+f)^2} dx$$

```
[In] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^2)^2),x)
```

```
[Out] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^2)^2), x)
```

3.288 $\int (f+gx^3)^3 \log \left(c(d+ex^2)^p\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 366

$$\begin{aligned}\int (f+gx^3)^3 \log \left(c(d+ex^2)^p\right) \, dx &= -2f^3px + \frac{6d^3fg^2px}{7e^3} + \frac{3df^2gpx^2}{4e} - \frac{d^4g^3px^2}{10e^4} \\ &\quad - \frac{2d^2fg^2px^3}{7e^2} - \frac{3}{8}f^2gpx^4 + \frac{d^3g^3px^4}{20e^3} + \frac{6dfg^2px^5}{35e} \\ &\quad - \frac{d^2g^3px^6}{30e^2} - \frac{6}{49}fg^2px^7 + \frac{dg^3px^8}{40e} - \frac{1}{50}g^3px^{10} \\ &\quad + \frac{2\sqrt{d}f^3p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{6d^{7/2}fg^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}} \\ &\quad - \frac{3d^2f^2gp\log(d+ex^2)}{4e^2} + \frac{d^5g^3p\log(d+ex^2)}{10e^5} \\ &\quad + f^3x\log\left(c(d+ex^2)^p\right) + \frac{3}{4}f^2gx^4\log\left(c(d+ex^2)^p\right) \\ &\quad + \frac{3}{7}fg^2x^7\log\left(c(d+ex^2)^p\right) + \frac{1}{10}g^3x^{10}\log\left(c(d+ex^2)^p\right)\end{aligned}$$

```
[Out] -2*f^3*p*x+6/7*d^3*f*g^2*p*x/e^3+3/4*d*f^2*g*p*x^2/e-1/10*d^4*g^3*p*x^2/e^4
-2/7*d^2*f*g^2*p*x^3/e^2-3/8*f^2*g*p*x^4+1/20*d^3*g^3*p*x^4/e^3+6/35*d*f*g^
2*p*x^5/e-1/30*d^2*g^3*p*x^6/e^2-6/49*f*g^2*p*x^7+1/40*d*g^3*p*x^8/e-1/50*g
^3*p*x^10-6/7*d^(7/2)*f*g^2*p*arctan(x*e^(1/2)/d^(1/2))/e^(7/2)-3/4*d^2*f^2
*g*p*ln(e*x^2+d)/e^2+1/10*d^5*g^3*p*ln(e*x^2+d)/e^5+f^3*x*ln(c*(e*x^2+d)^p)
+3/4*f^2*g*x^4*ln(c*(e*x^2+d)^p)+3/7*f*g^2*x^7*ln(c*(e*x^2+d)^p)+1/10*g^3*x
^10*ln(c*(e*x^2+d)^p)+2*f^3*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {2521, 2498, 327, 211, 2504, 2442, 45, 2505, 308}

$$\begin{aligned}\int (f+gx^3)^3\log\left(c\left(d+ex^2\right)^p\right)\,dx = &-\frac{6d^{7/2}fg^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}}+\frac{2\sqrt{d}f^3p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}\\&+f^3x\log\left(c\left(d+ex^2\right)^p\right)+\frac{3}{4}f^2gx^4\log\left(c\left(d+ex^2\right)^p\right)\\&+\frac{3}{7}fg^2x^7\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{10}g^3x^{10}\log\left(c\left(d+ex^2\right)^p\right)\\&+\frac{d^5g^3p\log\left(d+ex^2\right)}{10e^5}-\frac{d^4g^3px^2}{10e^4}+\frac{6d^3fg^2px}{7e^3}\\&+\frac{d^3g^3px^4}{20e^3}-\frac{3d^2f^2gp\log\left(d+ex^2\right)}{4e^2}-\frac{2d^2fg^2px^3}{7e^2}\\&-\frac{d^2g^3px^6}{30e^2}+\frac{3df^2gpx^2}{4e}+\frac{6dfg^2px^5}{35e}+\frac{dg^3px^8}{40e}\\&-2f^3px-\frac{3}{8}f^2gpx^4-\frac{6}{49}fg^2px^7-\frac{1}{50}g^3px^{10}\end{aligned}$$

```
[In] Int[(f + g*x^3)^3*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f^3*p*x + (6*d^3*f*g^2*p*x)/(7*e^3) + (3*d*f^2*g*p*x^2)/(4*e) - (d^4*g^3
*p*x^2)/(10*e^4) - (2*d^2*f*g^2*p*x^3)/(7*e^2) - (3*f^2*g*p*x^4)/8 + (d^3*g
^3*p*x^4)/(20*e^3) + (6*d*f*g^2*p*x^5)/(35*e) - (d^2*g^3*p*x^6)/(30*e^2) -
(6*f*g^2*p*x^7)/49 + (d*g^3*p*x^8)/(40*e) - (g^3*p*x^10)/50 + (2*Sqrt[d]*f^
3*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (6*d^(7/2)*f*g^2*p*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(7*e^(7/2)) - (3*d^2*f^2*g*p*Log[d + e*x^2])/(4*e^2) + (d^5
*g^3*p*Log[d + e*x^2])/(10*e^5) + f^3*x*Log[c*(d + e*x^2)^p] + (3*f^2*g*x^4
*Log[c*(d + e*x^2)^p])/4 + (3*f*g^2*x^7*Log[c*(d + e*x^2)^p])/7 + (g^3*x^10
*Log[c*(d + e*x^2)^p])/10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
```

```
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(f^3 \log\left(c\left(d+ex^2\right)^p\right)+3f^2gx^3\log\left(c\left(d+ex^2\right)^p\right)+3fg^2x^6\log\left(c\left(d+ex^2\right)^p\right)\right. \\
&\qquad \left.+g^3x^9\log\left(c\left(d+ex^2\right)^p\right)\right)\,dx \\
&= f^3\int \log\left(c\left(d+ex^2\right)^p\right)\,dx+\left(3f^2g\right)\int x^3\log\left(c\left(d+ex^2\right)^p\right)\,dx \\
&\quad +\left(3fg^2\right)\int x^6\log\left(c\left(d+ex^2\right)^p\right)\,dx+g^3\int x^9\log\left(c\left(d+ex^2\right)^p\right)\,dx \\
&= f^3x\log\left(c\left(d+ex^2\right)^p\right)+\frac{3}{7}fg^2x^7\log\left(c\left(d+ex^2\right)^p\right) \\
&\quad +\frac{1}{2}\left(3f^2g\right)\text{Subst}\left(\int x\log\left(c(d+ex)^p\right)\,dx,x,x^2\right) \\
&\quad +\frac{1}{2}g^3\text{Subst}\left(\int x^4\log\left(c(d+ex)^p\right)\,dx,x,x^2\right) \\
&\quad -\left(2ef^3p\right)\int \frac{x^2}{d+ex^2}\,dx-\frac{1}{7}\left(6efg^2p\right)\int \frac{x^8}{d+ex^2}\,dx \\
&= -2f^3px+f^3x\log\left(c\left(d+ex^2\right)^p\right)+\frac{3}{4}f^2gx^4\log\left(c\left(d+ex^2\right)^p\right) \\
&\quad +\frac{3}{7}fg^2x^7\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{10}g^3x^{10}\log\left(c\left(d+ex^2\right)^p\right) \\
&\quad +\left(2df^3p\right)\int \frac{1}{d+ex^2}\,dx-\frac{1}{4}\left(3ef^2gp\right)\text{Subst}\left(\int \frac{x^2}{d+ex}\,dx,x,x^2\right) \\
&\quad -\frac{1}{7}\left(6efg^2p\right)\int \left(-\frac{d^3}{e^4}+\frac{d^2x^2}{e^3}-\frac{dx^4}{e^2}+\frac{x^6}{e}+\frac{d^4}{e^4\left(d+ex^2\right)}\right)\,dx \\
&\quad -\frac{1}{10}\left(eg^3p\right)\text{Subst}\left(\int \frac{x^5}{d+ex}\,dx,x,x^2\right) \\
&= -2f^3px+\frac{6d^3fg^2px}{7e^3}-\frac{2d^2fg^2px^3}{7e^2}+\frac{6dfg^2px^5}{35e}-\frac{6}{49}fg^2px^7 \\
&\quad +\frac{2\sqrt{d}f^3p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+f^3x\log\left(c\left(d+ex^2\right)^p\right)+\frac{3}{4}f^2gx^4\log\left(c\left(d+ex^2\right)^p\right) \\
&\quad +\frac{3}{7}fg^2x^7\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{10}g^3x^{10}\log\left(c\left(d+ex^2\right)^p\right) \\
&\quad -\frac{1}{4}\left(3ef^2gp\right)\text{Subst}\left(\int \left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)\,dx,x,x^2\right)-\frac{\left(6d^4fg^2p\right)\int \frac{1}{d+ex^2}\,dx}{7e^3} \\
&\quad -\frac{1}{10}\left(eg^3p\right)\text{Subst}\left(\int \left(\frac{d^4}{e^5}-\frac{d^3x}{e^4}+\frac{d^2x^2}{e^3}-\frac{dx^3}{e^2}+\frac{x^4}{e}-\frac{d^5}{e^5(d+ex)}\right)\,dx,x,x^2\right)
\end{aligned}$$

$$= -2f^3px + \frac{6d^3fg^2px}{7e^3} + \frac{3df^2gpx^2}{4e} - \frac{d^4g^3px^2}{10e^4} - \frac{2d^2fg^2px^3}{7e^2} - \frac{3}{8}f^2gpx^4 + \frac{d^3g^3px^4}{20e^3} + \frac{6dfg^2px^5}{35e} - \frac{d^2g^3px^6}{30e^2} - \frac{6}{49}fg^2px^7 + \frac{dg^3px^8}{40e} - \frac{1}{50}g^3px^{10} + \frac{2\sqrt{d}f^3p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{6d^{7/2}fg^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}} - \frac{3d^2f^2gp\log(d+ex^2)}{4e^2} + \frac{d^5g^3p\log(d+ex^2)}{10e^5} + f^3x\log\left(c(d+ex^2)^p\right) + \frac{3}{4}f^2gx^4\log\left(c(d+ex^2)^p\right) + \frac{3}{7}fg^2x^7\log\left(c(d+ex^2)^p\right) + \frac{1}{10}g^3x^{10}\log\left(c(d+ex^2)^p\right)$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.70

$$\int \left(f+gx^3\right)^3\log\left(c\left(d+ex^2\right)^p\right)\,dx$$

$$= \frac{-epx(2940d^4g^3x + 140d^2e^2g^2x^2(60f+7gx^3) - 210d^3eg^2(120f+7gx^3) - 105de^3gx(210f^2+48fgx^3+7}{}$$

```
[In] Integrate[(f + g*x^3)^3*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (-(e*p*x*(2940*d^4*g^3*x + 140*d^2*e^2*g^2*x^2*(60*f + 7*g*x^3) - 210*d^3*e
*g^2*(120*f + 7*g*x^3) - 105*d*e^3*g*x*(210*f^2 + 48*f*g*x^3 + 7*g^2*x^6) +
 3*e^4*(19600*f^3 + 3675*f^2*g*x^3 + 1200*f*g^2*x^6 + 196*g^3*x^9))) - 8400
*Sqrt[d]*e^(3/2)*f*(-7*e^3*f^2 + 3*d^3*g^2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]] +
 1470*d^2*g*(-15*e^3*f^2 + 2*d^3*g^2)*p*Log[d + e*x^2] + 210*e^5*x*(140*f^3
 + 105*f^2*g*x^3 + 60*f*g^2*x^6 + 14*g^3*x^9)*Log[c*(d + e*x^2)^p])/(29400*
e^5)
```

Maple [A] (verified)

Time = 7.72 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.84

method	result
parts	$\frac{g^3x^{10}\ln\left(c\left(e\,x^2+d\right)^p\right)}{10} + \frac{3f\,g^2x^7\ln\left(c\left(e\,x^2+d\right)^p\right)}{7} + \frac{3f^2g\,x^4\ln\left(c\left(e\,x^2+d\right)^p\right)}{4} + f^3x\ln\left(c\left(e\,x^2+d\right)^p\right) - \frac{pe\left(\frac{7}{5}e^4g^3x^{10} - \ldots\right.}{}$
risch	Expression too large to display

```
[In] int((g*x^3+f)^3*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*g^3*x^10*ln(c*(e*x^2+d)^p)+3/7*f*g^2*x^7*ln(c*(e*x^2+d)^p)+3/4*f^2*g*x
^4*ln(c*(e*x^2+d)^p)+f^3*x*ln(c*(e*x^2+d)^p)-1/70*p*e*(1/e^5*(7/5*e^4*g^3*x
^10-7/4*d*e^3*g^3*x^8+60/7*e^4*f*g^2*x^7+7/3*d^2*e^2*g^3*x^6-12*d*e^3*f*g^2
*x^5-7/2*d^3*e*g^3*x^4+105/4*e^4*f^2*g*x^4+20*d^2*e^2*f*g^2*x^3+7*d^4*g^3*x
^2-105/2*d*f^2*g*x^2*e^3-60*x*d^3*f*g^2*e+140*x*e^4*f^3)-d/e^5*(1/2*(14*d^4
*g^3-105*d*e^3*f^2*g)/e*ln(e*x^2+d)+(-60*d^3*e*f*g^2+140*e^4*f^3)/(d*e)^(1/
2)*arctan(x*e/(d*e)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.93

$$\int \left(f+gx^3\right)^3\log\left(c\left(d+ex^2\right)^p\right)\,dx$$

$$=\left[-\frac{588\,e^5g^3px^{10}-735\,de^4g^3px^8+3600\,e^5fg^2px^7+980\,d^2e^3g^3px^6-5040\,de^4fg^2px^5+8400\,d^2e^3fg^2px^3+}{}\right.$$

$$-\frac{588\,e^5g^3px^{10}-735\,de^4g^3px^8+3600\,e^5fg^2px^7+980\,d^2e^3g^3px^6-5040\,de^4fg^2px^5+8400\,d^2e^3fg^2px^3+7}{}$$

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/29400*(588*e^5*g^3*p*x^10 - 735*d*e^4*g^3*p*x^8 + 3600*e^5*f*g^2*p*x^7
+ 980*d^2*e^3*g^3*p*x^6 - 5040*d*e^4*f*g^2*p*x^5 + 8400*d^2*e^3*f*g^2*p*x^3
 + 735*(15*e^5*f^2*g - 2*d^3*e^2*g^3)*p*x^4 - 1470*(15*d*e^4*f^2*g - 2*d^4*
e*g^3)*p*x^2 + 4200*(7*e^5*f^3 - 3*d^3*e^2*f*g^2)*p*sqrt(-d/e)*log((e*x^2 -
 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) + 8400*(7*e^5*f^3 - 3*d^3*e^2*f*g^2)*p*
x - 210*(14*e^5*g^3*p*x^10 + 60*e^5*f*g^2*p*x^7 + 105*e^5*f^2*g*p*x^4 + 140
*e^5*f^3*p*x - 7*(15*d^2*e^3*f^2*g - 2*d^5*g^3)*p)*log(e*x^2 + d) - 210*(14
*e^5*g^3*x^10 + 60*e^5*f*g^2*x^7 + 105*e^5*f^2*g*x^4 + 140*e^5*f^3*x)*log(c
))/e^5, -1/29400*(588*e^5*g^3*p*x^10 - 735*d*e^4*g^3*p*x^8 + 3600*e^5*f*g^2
*p*x^7 + 980*d^2*e^3*g^3*p*x^6 - 5040*d*e^4*f*g^2*p*x^5 + 8400*d^2*e^3*f*g^
2*p*x^3 + 735*(15*e^5*f^2*g - 2*d^3*e^2*g^3)*p*x^4 - 1470*(15*d*e^4*f^2*g -
 2*d^4*e*g^3)*p*x^2 - 8400*(7*e^5*f^3 - 3*d^3*e^2*f*g^2)*p*sqrt(d/e)*arctan
(e*x*sqrt(d/e)/d) + 8400*(7*e^5*f^3 - 3*d^3*e^2*f*g^2)*p*x - 210*(14*e^5*g^
3*p*x^10 + 60*e^5*f*g^2*p*x^7 + 105*e^5*f^2*g*p*x^4 + 140*e^5*f^3*p*x - 7*(
15*d^2*e^3*f^2*g - 2*d^5*g^3)*p)*log(e*x^2 + d) - 210*(14*e^5*g^3*x^10 + 60
*e^5*f*g^2*x^7 + 105*e^5*f^2*g*x^4 + 140*e^5*f^3*x)*log(c))/e^5]
```

Sympy [F(-1)]

Timed out.

$$\int \left(f+gx^3\right)^3 \log\left(c\left(d+ex^2\right)^p\right)\,dx = \text{Timed out}$$

```
[In] integrate((g*x**3+f)**3*ln(c*(e*x**2+d)**p),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \left(f+gx^3\right)^3 \log\left(c\left(d+ex^2\right)^p\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \left(f+gx^3\right)^3 \log\left(c\left(d+ex^2\right)^p\right)\,dx &= \frac{dg^3px^8}{40\,e} - \frac{1}{50}\left(g^3p-5\,g^3\log\left(c\right)\right)x^{10} - \frac{d^2g^3px^6}{30\,e^2} + \frac{6\,dfg^2px^5}{35\,e} - \frac{3}{49}\left(2\,fg^2p-7\,fg^2\log\left(c\right)\right)x^7 \\ &\quad - \frac{2\,d^2fg^2px^3}{7\,e^2} - \frac{\left(15\,e^3f^2gp-2\,d^3g^3p-30\,e^3f^2g\log\left(c\right)\right)x^4}{40\,e^3} \\ &\quad + \frac{1}{140}\left(14\,g^3px^{10}+60\,fg^2px^7+105\,f^2gpx^4+140\,f^3px\right)\log\left(ex^2+d\right) \\ &\quad - \frac{\left(14\,e^3f^3p-6\,d^3fg^2p-7\,e^3f^3\log\left(c\right)\right)x}{7\,e^3} + \frac{\left(15\,de^3f^2gp-2\,d^4g^3p\right)x^2}{20\,e^4} \\ &\quad + \frac{2\left(7\,de^3f^3p-3\,d^4fg^2p\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{7\,\sqrt{de}e^3} - \frac{\left(15\,d^2e^3f^2gp-2\,d^5g^3p\right)\log\left(ex^2+d\right)}{20\,e^5}\end{aligned}$$

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/40*d*g^3*p*x^8/e - 1/50*(g^3*p - 5*g^3*log(c))*x^10 - 1/30*d^2*g^3*p*x^6/
e^2 + 6/35*d*f*g^2*p*x^5/e - 3/49*(2*f*g^2*p - 7*f*g^2*log(c))*x^7 - 2/7*d^
2*f*g^2*p*x^3/e^2 - 1/40*(15*e^3*f^2*g*p - 2*d^3*g^3*p - 30*e^3*f^2*g*log(c
))*x^4/e^3 + 1/140*(14*g^3*p*x^10 + 60*f*g^2*p*x^7 + 105*f^2*g*p*x^4 + 140*
f^3*p*x)*log(e*x^2 + d) - 1/7*(14*e^3*f^3*p - 6*d^3*f*g^2*p - 7*e^3*f^3*log
(c))*x/e^3 + 1/20*(15*d*e^3*f^2*g*p - 2*d^4*g^3*p)*x^2/e^4 + 2/7*(7*d*e^3*f
^3*p - 3*d^4*f*g^2*p)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^3) - 1/20*(15*d^2*
e^3*f^2*g*p - 2*d^5*g^3*p)*log(e*x^2 + d)/e^5
```

Mupad [B] (verification not implemented)

Time = 4.73 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.86

$$\int (f+gx^3)^3 \log \left(c\left(d+ex^2\right)^p\right)\,dx = \frac{g^3\,x^{10}\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{10}-2\,f^3\,p\,x-\frac{g^3\,p\,x^{10}}{50}+f^3\,x\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)+\frac{3\,f^2\,g\,x^4\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{4}+\frac{3\,f\,g^2\,x^7\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{7}-\frac{3\,f^2\,g\,p\,x^4}{8}-\frac{6\,f\,g^2\,p\,x^7}{49}+\frac{d\,g^3\,p\,x^8}{40\,e}+\frac{2\,\sqrt{d}\,f^3\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{d^5\,g^3\,p\,\ln\left(e\,x^2+d\right)}{10\,e^5}-\frac{d^2\,g^3\,p\,x^6}{30\,e^2}+\frac{d^3\,g^3\,p\,x^4}{20\,e^3}-\frac{d^4\,g^3\,p\,x^2}{10\,e^4}-\frac{6\,d^{7/2}\,f\,g^2\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{7\,e^{7/2}}-\frac{3\,d^2\,f^2\,g\,p\,\ln\left(e\,x^2+d\right)}{4\,e^2}-\frac{2\,d^2\,f\,g^2\,p\,x^3}{7\,e^2}+\frac{3\,d\,f^2\,g\,p\,x^2}{4\,e}+\frac{6\,d\,f\,g^2\,p\,x^5}{35\,e}+\frac{6\,d^3\,f\,g^2\,p\,x}{7\,e^3}$$

```
[In] int(log(c*(d + e*x^2)^p)*(f + g*x^3)^3,x)
```

```
[Out] (g^3*x^10*log(c*(d + e*x^2)^p))/10 - 2*f^3*p*x - (g^3*p*x^10)/50 + f^3*x*lo
g(c*(d + e*x^2)^p) + (3*f^2*g*x^4*log(c*(d + e*x^2)^p))/4 + (3*f*g^2*x^7*lo
g(c*(d + e*x^2)^p))/7 - (3*f^2*g*p*x^4)/8 - (6*f*g^2*p*x^7)/49 + (d*g^3*p*x
^8)/(40*e) + (2*d^(1/2)*f^3*p*atan((e^(1/2)*x)/d^(1/2)))/e^(1/2) + (d^5*g^3
*p*log(d + e*x^2))/(10*e^5) - (d^2*g^3*p*x^6)/(30*e^2) + (d^3*g^3*p*x^4)/(2
0*e^3) - (d^4*g^3*p*x^2)/(10*e^4) - (6*d^(7/2)*f*g^2*p*atan((e^(1/2)*x)/d^(
1/2)))/(7*e^(7/2)) - (3*d^2*f^2*g*p*log(d + e*x^2))/(4*e^2) - (2*d^2*f*g^2*
p*x^3)/(7*e^2) + (3*d*f^2*g*p*x^2)/(4*e) + (6*d*f*g^2*p*x^5)/(35*e) + (6*d^
3*f*g^2*p*x)/(7*e^3)
```

3.289 $\int (f+gx^3)^2 \log(c(d+ex^2)^p)\, dx$

Optimal result

Integrand size = 22, antiderivative size = 231

$$\int (f+gx^3)^2 \log\left(c(d+ex^2)^p\right) dx = -2f^2px + \frac{2d^3g^2px}{7e^3} + \frac{dfgpx^2}{2e} - \frac{2d^2g^2px^3}{21e^2} - \frac{1}{4}fgpx^4 + \frac{2dg^2px^5}{35e} - \frac{2}{49}g^2px^7 + \frac{2\sqrt{d}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{2d^{7/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}} - \frac{d^2fgp\log(d+ex^2)}{2e^2} + f^2x\log\left(c(d+ex^2)^p\right) + \frac{1}{2}fgx^4\log\left(c(d+ex^2)^p\right) + \frac{1}{7}g^2x^7\log\left(c(d+ex^2)^p\right)$$

```
[Out] -2*f^2*p*x+2/7*d^3*g^2*p*x/e^3+1/2*d*f*g*p*x^2/e-2/21*d^2*g^2*p*x^3/e^2-1/4
*f*g*p*x^4+2/35*d*g^2*p*x^5/e-2/49*g^2*p*x^7-2/7*d^(7/2)*g^2*p*arctan(x*e^(
1/2)/d^(1/2))/e^(7/2)-1/2*d^2*f*g*p*ln(e*x^2+d)/e^2+f^2*x*ln(c*(e*x^2+d)^p)
+1/2*f*g*x^4*ln(c*(e*x^2+d)^p)+1/7*g^2*x^7*ln(c*(e*x^2+d)^p)+2*f^2*p*arctan
(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used

$= \{2521, 2498, 327, 211, 2504, 2442, 45, 2505, 308\}$

$$\int \left(f+gx^3\right)^2 \log\left(c\left(d+ex^2\right)^p\right)\,dx = -\frac{2d^{7/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}} + \frac{2\sqrt{d}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + f^2x\log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{2}fgx^4\log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{7}g^2x^7\log\left(c\left(d+ex^2\right)^p\right) + \frac{2d^3g^2px}{7e^3} - \frac{d^2fgp\log\left(d+ex^2\right)}{2e^2} - \frac{2d^2g^2px^3}{21e^2} + \frac{dfgpx^2}{2e} + \frac{2dg^2px^5}{35e} - 2f^2px - \frac{1}{4}fgpx^4 - \frac{2}{49}g^2px^7$$

```
[In] Int[(f + g*x^3)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f^2*p*x + (2*d^3*g^2*p*x)/(7*e^3) + (d*f*g*p*x^2)/(2*e) - (2*d^2*g^2*p*x
^3)/(21*e^2) - (f*g*p*x^4)/4 + (2*d*g^2*p*x^5)/(35*e) - (2*g^2*p*x^7)/49 +
(2*Sqrt[d]*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(7/2)*g^2*p*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(7*e^(7/2)) - (d^2*f*g*p*Log[d + e*x^2])/(2*e^2)
 + f^2*x*Log[c*(d + e*x^2)^p] + (f*g*x^4*Log[c*(d + e*x^2)^p])/2 + (g^2*x^7
*Log[c*(d + e*x^2)^p])/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\text{integral} = \int \left(f^2\log\left(c\left(d+ex^2\right)^p\right)+2fgx^3\log\left(c\left(d+ex^2\right)^p\right)+g^2x^6\log\left(c\left(d+ex^2\right)^p\right)\right)\,dx$$

$$=f^2\int\log\left(c\left(d+ex^2\right)^p\right)\,dx+(2fg)\int x^3\log\left(c\left(d+ex^2\right)^p\right)\,dx+g^2\int x^6\log\left(c\left(d+ex^2\right)^p\right)\,dx$$

$$
\begin{aligned}
&= f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log\left(c\left(d+ex^2\right)^p\right) \\
&\quad +(fg)\text{Subst}\left(\int x\log\left(c(d+ex)^p\right)\,dx,x,x^2\right) \\
&\quad -\left(2ef^2p\right)\int\frac{x^2}{d+ex^2}\,dx-\frac{1}{7}\left(2eg^2p\right)\int\frac{x^8}{d+ex^2}\,dx \\
&= -2f^2px+f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}fgx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log\left(c\left(d+ex^2\right)^p\right) \\
&\quad +\left(2df^2p\right)\int\frac{1}{d+ex^2}\,dx-\frac{1}{2}(efgp)\text{Subst}\left(\int\frac{x^2}{d+ex}\,dx,x,x^2\right) \\
&\quad -\frac{1}{7}\left(2eg^2p\right)\int\left(-\frac{d^3}{e^4}+\frac{d^2x^2}{e^3}-\frac{dx^4}{e^2}+\frac{x^6}{e}+\frac{d^4}{e^4\left(d+ex^2\right)}\right)\,dx \\
&= -2f^2px+\frac{2d^3g^2px}{7e^3}-\frac{2d^2g^2px^3}{21e^2}+\frac{2dg^2px^5}{35e}-\frac{2}{49}g^2px^7+\frac{2\sqrt{d}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} \\
&\quad +f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}fgx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log\left(c\left(d+ex^2\right)^p\right) \\
&\quad -\frac{1}{2}(efgp)\text{Subst}\left(\int\left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)\,dx,x,x^2\right)-\frac{\left(2d^4g^2p\right)\int\frac{1}{d+ex^2}\,dx}{7e^3} \\
&= -2f^2px+\frac{2d^3g^2px}{7e^3}+\frac{dfgpx^2}{2e}-\frac{2d^2g^2px^3}{21e^2}-\frac{1}{4}fgpx^4+\frac{2dg^2px^5}{35e}-\frac{2}{49}g^2px^7 \\
&\quad +\frac{2\sqrt{d}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{2d^{7/2}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}}-\frac{d^2fgp\log\left(d+ex^2\right)}{2e^2} \\
&\quad +f^2x\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}fgx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log\left(c\left(d+ex^2\right)^p\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.77

$$
\begin{aligned}
&\int\left(f+gx^3\right)^2\log\left(c\left(d+ex^2\right)^p\right)\,dx \\
&= \frac{px(840d^3g^2-280d^2eg^2x^2+42de^2gx(35f+4gx^3)-15e^3(392f^2+49fgx^3+8g^2x^6))}{2940e^3} \\
&\quad -\frac{2\sqrt{d}(-7e^3f^2+d^3g^2)\,p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}}-\frac{d^2fgp\log\left(d+ex^2\right)}{2e^2} \\
&\quad +\frac{1}{14}x\left(14f^2+7fgx^3+2g^2x^6\right)\log\left(c\left(d+ex^2\right)^p\right)
\end{aligned}
$$

```
[In] Integrate[(f + g*x^3)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (p*x*(840*d^3*g^2 - 280*d^2*e*g^2*x^2 + 42*d*e^2*g*x*(35*f + 4*g*x^3) - 15*
e^3*(392*f^2 + 49*f*g*x^3 + 8*g^2*x^6)))/(2940*e^3) - (2*Sqrt[d]*(-7*e^3*f^
```

```
2 + d^3*g^2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(7*e^(7/2)) - (d^2*f*g*p*Log[d
+ e*x^2])/(2*e^2) + (x*(14*f^2 + 7*f*g*x^3 + 2*g^2*x^6)*Log[c*(d + e*x^2)^p
])/14
```

Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.84

method	result
parts	$\frac{g^{2} x^{7} \ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{7}+\frac{f g\,x^{4} \ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{2}+f^{2} x \ln\left(c\left(e\,x^{2}+d\right)^{p}\right)-\frac{pe\left(-\frac{-\frac{2}{7} e^{3} g^{2} x^{7}+\frac{2}{5} d\,e^{2} g^{2} x^{5}-\frac{7}{4} e^{3} f g\,x^{4}-\frac{2}{3} d^{2} e\,g^{2} x}{e^{4}}\right.}{}$
risch	$x \ln\left(c\right) f^{2}+\frac{i \pi\,g^{2} x^{7} \operatorname{csgn}\left(i c\left(e\,x^{2}+d\right)^{p}\right)^{2} \operatorname{csgn}\left(i c\right)}{14}-\frac{i \pi f g\,x^{4} \operatorname{csgn}\left(i c\left(e\,x^{2}+d\right)^{p}\right)^{3}}{4}-\frac{p \ln\left(-d^{4} g^{2}+7 d\,e^{3} f^{2}-\sqrt{-d^{7} e\,g^{4}+14 d^{4} e^{4}}\right.}{2 e^{2}}$

```
[In] int((g*x^3+f)^2*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*g^2*x^7*ln(c*(e*x^2+d)^p)+1/2*f*g*x^4*ln(c*(e*x^2+d)^p)+f^2*x*ln(c*(e*x
^2+d)^p)-1/7*p*e*(-1/e^4*(-2/7*e^3*g^2*x^7+2/5*d*e^2*g^2*x^5-7/4*e^3*f*g*x^
4-2/3*d^2*e*g^2*x^3+7/2*d*f*g*x^2*e^2+2*x*d^3*g^2-14*x*e^3*f^2)+d/e^4*(7/2*
d*e*f*g*ln(e*x^2+d)+(2*d^3*g^2-14*e^3*f^2)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/
2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.97

$$\int \left(f+gx^3\right)^2 \log\left(c\left(d+ex^2\right)^p\right)\,dx$$

$$= \left[-\frac{120\,e^3g^2px^7 - 168\,de^2g^2px^5 + 735\,e^3fgpx^4 + 280\,d^2eg^2px^3 - 1470\,de^2fgpx^2 + 420\,(7\,e^3f^2 - d^3g^2)p\sqrt{}}{} \right.$$

$$-\frac{120\,e^3g^2px^7 - 168\,de^2g^2px^5 + 735\,e^3fgpx^4 + 280\,d^2eg^2px^3 - 1470\,de^2fgpx^2 - 840\,(7\,e^3f^2 - d^3g^2)p\sqrt{\frac{d}{e}}}{}$$

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/2940*(120*e^3*g^2*p*x^7 - 168*d*e^2*g^2*p*x^5 + 735*e^3*f*g*p*x^4 + 280
*d^2*e*g^2*p*x^3 - 1470*d*e^2*f*g*p*x^2 + 420*(7*e^3*f^2 - d^3*g^2)*p*sqrt(
-d/e)*log((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) + 840*(7*e^3*f^2 - d^
3*g^2)*p*x - 210*(2*e^3*g^2*p*x^7 + 7*e^3*f*g*p*x^4 + 14*e^3*f^2*p*x - 7*d^
2*e*f*g*p)*log(e*x^2 + d) - 210*(2*e^3*g^2*x^7 + 7*e^3*f*g*x^4 + 14*e^3*f^2
*x)*log(c))/e^3, -1/2940*(120*e^3*g^2*p*x^7 - 168*d*e^2*g^2*p*x^5 + 735*e^3
*f*g*p*x^4 + 280*d^2*e*g^2*p*x^3 - 1470*d*e^2*f*g*p*x^2 - 840*(7*e^3*f^2 -
d^3*g^2)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 840*(7*e^3*f^2 - d^3*g^2)*p*
x - 210*(2*e^3*g^2*p*x^7 + 7*e^3*f*g*p*x^4 + 14*e^3*f^2*p*x - 7*d^2*e*f*g*p
)*log(e*x^2 + d) - 210*(2*e^3*g^2*x^7 + 7*e^3*f*g*x^4 + 14*e^3*f^2*x)*log(c
))/e^3]
```

Sympy [A] (verification not implemented)

Time = 120.70 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.90

$$\int (f+gx^3)^2 \log \left(c(d+ex^2)^p\right)\,dx$$
$$= \begin{cases} \left(f^2x+\frac{fgx^4}{2}+\frac{g^2x^7}{7}\right)\log(0^pc) \\ \left(f^2x+\frac{fgx^4}{2}+\frac{g^2x^7}{7}\right)\log(cd^p) \\ -2f^2px+f^2x\log\left(c(ex^2)^p\right)-\frac{fgpx^4}{4}+\frac{fgx^4\log\left(c(ex^2)^p\right)}{2}-\frac{2g^2px^7}{49}+\frac{g^2x^7\log\left(c(ex^2)^p\right)}{7} \\ -\frac{2d^4g^2p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{7e^4\sqrt{-\frac{d}{e}}}+\frac{d^4g^2\log\left(c(d+ex^2)^p\right)}{7e^4\sqrt{-\frac{d}{e}}}+\frac{2d^3g^2px}{7e^3}-\frac{d^2fg\log\left(c(d+ex^2)^p\right)}{2e^2}-\frac{2d^2g^2px^3}{21e^2}+\frac{2df^2p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{e\sqrt{-\frac{d}{e}}}-\frac{df^2\log}{} \end{cases}$$

```
[In] integrate((g*x**3+f)**2*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((((f**2*x + f*g*x**4/2 + g**2*x**7/7)*log(0**p*c), Eq(d, 0) & Eq(e
, 0)), ((f**2*x + f*g*x**4/2 + g**2*x**7/7)*log(c*d**p), Eq(e, 0)), (-2*f**
2*p*x + f**2*x*log(c*(e*x**2)**p) - f*g*p*x**4/4 + f*g*x**4*log(c*(e*x**2)*
*p)/2 - 2*g**2*p*x**7/49 + g**2*x**7*log(c*(e*x**2)**p)/7, Eq(d, 0)), (-2*d
**4*g**2*p*log(x - sqrt(-d/e))/(7*e**4*sqrt(-d/e)) + d**4*g**2*log(c*(d + e
*x**2)**p)/(7*e**4*sqrt(-d/e)) + 2*d**3*g**2*p*x/(7*e**3) - d**2*f*g*log(c*
(d + e*x**2)**p)/(2*e**2) - 2*d**2*g**2*p*x**3/(21*e**2) + 2*d*f**2*p*log(x
 - sqrt(-d/e))/(e*sqrt(-d/e)) - d*f**2*log(c*(d + e*x**2)**p)/(e*sqrt(-d/e)
) + d*f*g*p*x**2/(2*e) + 2*d*g**2*p*x**5/(35*e) - 2*f**2*p*x + f**2*x*log(c
*(d + e*x**2)**p) - f*g*p*x**4/4 + f*g*x**4*log(c*(d + e*x**2)**p)/2 - 2*g*
*2*p*x**7/49 + g**2*x**7*log(c*(d + e*x**2)**p)/7, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(f+gx^3\right)^2 \log \left(c\left(d+ex^2\right)^p\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \left(f+gx^3\right)^2 \log \left(c\left(d+ex^2\right)^p\right)\,dx &= \frac{2\,dg^2px^5}{35\,e} - \frac{1}{49}\,\left(2\,g^2p - 7\,g^2\log\left(c\right)\right)x^7 \\ &\quad - \frac{2\,d^2g^2px^3}{21\,e^2} + \frac{dfgpx^2}{2\,e} \\ &\quad - \frac{1}{4}\,\left(fgp - 2\,fg\log\left(c\right)\right)x^4 - \frac{d^2fgp\log\left(ex^2+d\right)}{2\,e^2} \\ &\quad + \frac{1}{14}\,\left(2\,g^2px^7 + 7\,fgpx^4 + 14\,f^2px\right)\log\left(ex^2+d\right) \\ &\quad - \frac{\left(14\,e^3f^2p - 2\,d^3g^2p - 7\,e^3f^2\log\left(c\right)\right)x}{7\,e^3} \\ &\quad + \frac{2\,\left(7\,de^3f^2p - d^4g^2p\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{7\,\sqrt{de}e^3}\end{aligned}$$

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 2/35*d*g^2*p*x^5/e - 1/49*(2*g^2*p - 7*g^2*log(c))*x^7 - 2/21*d^2*g^2*p*x^3
/e^2 + 1/2*d*f*g*p*x^2/e - 1/4*(f*g*p - 2*f*g*log(c))*x^4 - 1/2*d^2*f*g*p*l
og(e*x^2 + d)/e^2 + 1/14*(2*g^2*p*x^7 + 7*f*g*p*x^4 + 14*f^2*p*x)*log(e*x^2
 + d) - 1/7*(14*e^3*f^2*p - 2*d^3*g^2*p - 7*e^3*f^2*log(c))*x/e^3 + 2/7*(7*
d*e^3*f^2*p - d^4*g^2*p)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^3)
```

Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.37

$$\int \left(f+gx^3\right)^2\log \left(c\left(d+ex^2\right)^p\right)\,dx=\frac{g^2\,x^7\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{7}-2\,f^2\,p\,x-\frac{2\,g^2\,p\,x^7}{49}+f^2\,x\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)+\frac{f\,g\,x^4\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{2}-\frac{f\,g\,p\,x^4}{4}+\frac{2\,d\,g^2\,p\,x^5}{35\,e}+\frac{2\,d^3\,g^2\,p\,x}{7\,e^3}-\frac{2\,\sqrt{d}\,f^2\,p\,\mathrm{atan}\left(\frac{7\,\sqrt{d}\,e^{7/2}\,f^2\,p\,x}{d^4\,g^2\,p-7\,d\,e^3\,f^2\,p}-\frac{d^{7/2}\,\sqrt{e}\,g^2\,p\,x}{d^4\,g^2\,p-7\,d\,e^3\,f^2\,p}\right)}{\sqrt{e}}+\frac{2\,d^{7/2}\,g^2\,p\,\mathrm{atan}\left(\frac{7\,\sqrt{d}\,e^{7/2}\,f^2\,p\,x}{d^4\,g^2\,p-7\,d\,e^3\,f^2\,p}-\frac{d^{7/2}\,\sqrt{e}\,g^2\,p\,x}{d^4\,g^2\,p-7\,d\,e^3\,f^2\,p}\right)}{7\,e^{7/2}}-\frac{2\,d^2\,g^2\,p\,x^3}{21\,e^2}+\frac{d\,f\,g\,p\,x^2}{2\,e}-\frac{d^2\,f\,g\,p\,\ln\left(e\,x^2+d\right)}{2\,e^2}$$

```
[In] int(log(c*(d + e*x^2)^p)*(f + g*x^3)^2,x)
```

```
[Out] (g^2*x^7*log(c*(d + e*x^2)^p))/7 - 2*f^2*p*x - (2*g^2*p*x^7)/49 + f^2*x*log
(c*(d + e*x^2)^p) + (f*g*x^4*log(c*(d + e*x^2)^p))/2 - (f*g*p*x^4)/4 + (2*d
*g^2*p*x^5)/(35*e) + (2*d^3*g^2*p*x)/(7*e^3) - (2*d^(1/2)*f^2*p*atan((7*d^(
1/2)*e^(7/2)*f^2*p*x)/(d^4*g^2*p - 7*d*e^3*f^2*p) - (d^(7/2)*e^(1/2)*g^2*p*
x)/(d^4*g^2*p - 7*d*e^3*f^2*p)))/e^(1/2) + (2*d^(7/2)*g^2*p*atan((7*d^(1/2)
*e^(7/2)*f^2*p*x)/(d^4*g^2*p - 7*d*e^3*f^2*p) - (d^(7/2)*e^(1/2)*g^2*p*x)/(
d^4*g^2*p - 7*d*e^3*f^2*p)))/(7*e^(7/2)) - (2*d^2*g^2*p*x^3)/(21*e^2) + (d*
f*g*p*x^2)/(2*e) - (d^2*f*g*p*log(d + e*x^2))/(2*e^2)
```

3.290 $\int (f+gx^3)\log\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 110

$$\int (f+gx^3)\log\left(c(d+ex^2)^p\right)\,dx = -2fpx+\frac{dgpx^2}{4e}-\frac{1}{8}gpx^4+\frac{2\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{d^2gp\log(d+ex^2)}{4e^2}+fx\log\left(c(d+ex^2)^p\right)+\frac{1}{4}gx^4\log\left(c(d+ex^2)^p\right)$$

```
[Out] -2*f*p*x+1/4*d*g*p*x^2/e-1/8*g*p*x^4-1/4*d^2*g*p*ln(e*x^2+d)/e^2+f*x*ln(c*(
e*x^2+d)^p)+1/4*g*x^4*ln(c*(e*x^2+d)^p)+2*f*p*arctan(x*e^(1/2)/d^(1/2))*d^(
1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2521, 2498, 327, 211, 2504, 2442, 45}

$$\int (f+gx^3)\log\left(c(d+ex^2)^p\right)\,dx = \frac{2\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+fx\log\left(c(d+ex^2)^p\right)+\frac{1}{4}gx^4\log\left(c(d+ex^2)^p\right)-\frac{d^2gp\log(d+ex^2)}{4e^2}+\frac{dgpx^2}{4e}-2fpx-\frac{1}{8}gpx^4$$

```
[In] Int[(f + g*x^3)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f*p*x + (d*g*p*x^2)/(4*e) - (g*p*x^4)/8 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/Sqrt[e] - (d^2*g*p*Log[d + e*x^2])/(4*e^2) + f*x*Log[c*(d + e
*x^2)^p] + (g*x^4*Log[c*(d + e*x^2)^p])/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(f \log \left(c\left(d+ex^2\right)^p\right)+gx^3 \log \left(c\left(d+ex^2\right)^p\right)\right)\,dx\\
&= f\int \log \left(c\left(d+ex^2\right)^p\right)\,dx+g\int x^3 \log \left(c\left(d+ex^2\right)^p\right)\,dx\\
&= fx\log \left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}g\text{Subst}\left(\int x\log \left(c(d+ex)^p\right)\,dx,x,x^2\right)-(2efp)\int \frac{x^2}{d+ex^2}\,dx\\
&= -2fpx+fx\log \left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^4\log \left(c\left(d+ex^2\right)^p\right)\\
&\quad +(2dfp)\int \frac{1}{d+ex^2}\,dx-\frac{1}{4}(egp)\text{Subst}\left(\int \frac{x^2}{d+ex}\,dx,x,x^2\right)\\
&= -2fpx+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+fx\log \left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^4\log \left(c\left(d+ex^2\right)^p\right)\\
&\quad -\frac{1}{4}(egp)\text{Subst}\left(\int \left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)\,dx,x,x^2\right)\\
&= -2fpx+\frac{dgpx^2}{4e}-\frac{1}{8}gpx^4+\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{d^2gp\log \left(d+ex^2\right)}{4e^2}\\
&\quad +fx\log \left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^4\log \left(c\left(d+ex^2\right)^p\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int \left(f+gx^3\right)\log \left(c\left(d+ex^2\right)^p\right)\,dx &= -2fpx+\frac{dgpx^2}{4e}-\frac{1}{8}gpx^4\\
&\quad +\frac{2\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{d^2gp\log \left(d+ex^2\right)}{4e^2}\\
&\quad +fx\log \left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}gx^4\log \left(c\left(d+ex^2\right)^p\right)
\end{aligned}$$

```
[In] Integrate[(f + g*x^3)*Log[c*(d + e*x^2)^p],x]
```

[Out] -2*f*p*x + (d*g*p*x^2)/(4*e) - (g*p*x^4)/8 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (d^2*g*p*Log[d + e*x^2])/(4*e^2) + f*x*Log[c*(d + e*x^2)^p] + (g*x^4*Log[c*(d + e*x^2)^p])/4

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.95

method	result
parts	$\frac{g\,x^4\ln\left(c\left(e\,x^2+d\right)^p\right)}{4}+f x\ln\left(c\left(e\,x^2+d\right)^p\right)-\frac{pe\left(-\frac{-\frac{1}{4}eg\,x^4+\frac{1}{2}dg\,x^2-4efx}{e^2}+\frac{d\left(\frac{dg\ln\left(e\,x^2+d\right)}{2e}-\frac{4ef\arctan\left(\frac{xe}{\sqrt{de}}\right)}{\sqrt{de}}\right)}{e^2}\right)}{2}$
risch	$\left(\frac{1}{4}g\,x^4+fx\right)\ln\left(\left(e\,x^2+d\right)^p\right)+\frac{i\,\mathrm{csgn}(ic)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^2x^4g\pi}{8}-\frac{i\pi g\,x^4\,\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)\mathrm{csgn}(ic}{8}$

[In] int((g*x^3+f)*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)

[Out] 1/4*g*x^4*ln(c*(e*x^2+d)^p)+f*x*ln(c*(e*x^2+d)^p)-1/2*p*e*(-1/e^2*(-1/4*e*g*x^4+1/2*d*g*x^2-4*e*f*x)+d/e^2*(1/2*d*g/e*ln(e*x^2+d)-4*e*f/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.27

$$\int \left(f+gx^3\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$=\left[-\frac{e^2gpx^4-2\,degpx^2-8\,e^2fp\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2+2\,ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right)+16\,e^2fpx-2\left(e^2gpx^4+4\,e^2fpx-d^2gp\right)\log\left(e\right.}{8\,e^2}\right.$$
$$-\frac{e^2gpx^4-2\,degpx^2-16\,e^2fp\sqrt{\frac{d}{e}}\arctan\left(\frac{ex\sqrt{\frac{d}{e}}}{d}\right)+16\,e^2fpx-2\left(e^2gpx^4+4\,e^2fpx-d^2gp\right)\log\left(ex^2+d\right.}{8\,e^2}$$

[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")

[Out] [-1/8*(e^2*g*p*x^4 - 2*d*e*g*p*x^2 - 8*e^2*f*p*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) + 16*e^2*f*p*x - 2*(e^2*g*p*x^4 + 4*e^2*f*p*x - d^2*g*p)*log(e*x^2 + d) - 2*(e^2*g*x^4 + 4*e^2*f*x)*log(c))/e^2, -1/8*(

```
e^2*g*p*x^4 - 2*d*e*g*p*x^2 - 16*e^2*f*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d)
+ 16*e^2*f*p*x - 2*(e^2*g*p*x^4 + 4*e^2*f*p*x - d^2*g*p)*log(e*x^2 + d) - 2
*(e^2*g*x^4 + 4*e^2*f*x)*log(c))/e^2]
```

Sympy [A] (verification not implemented)

Time = 16.50 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.95

$$\int (f+gx^3)\log\left(c(d+ex^2)^p\right)\,dx$$
$$=\begin{cases}\left(fx+\frac{gx^4}{4}\right)\log\left(0^pc\right)\\ \left(fx+\frac{gx^4}{4}\right)\log\left(cd^p\right)\\ -2fpx+fx\log\left(c(ex^2)^p\right)-\frac{gpx^4}{8}+\frac{gx^4\log\left(c(ex^2)^p\right)}{4}\\ -\frac{d^2g\log\left(c(d+ex^2)^p\right)}{4e^2}+\frac{2dfp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{e\sqrt{-\frac{d}{e}}}-\frac{df\log\left(c(d+ex^2)^p\right)}{e\sqrt{-\frac{d}{e}}}+\frac{dgpx^2}{4e}-2fpx+fx\log\left(c(d+ex^2)^p\right)-\frac{gpx^4}{8}+\frac{g}{}\end{cases}$$

```
[In] integrate((g*x**3+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise(((f*x + g*x**4/4)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), ((f*x + g*x*
*4/4)*log(c*d**p), Eq(e, 0)), (-2*f*p*x + f*x*log(c*(e*x**2)**p) - g*p*x**4
/8 + g*x**4*log(c*(e*x**2)**p)/4, Eq(d, 0)), (-d**2*g*log(c*(d + e*x**2)**p
)/(4*e**2) + 2*d*f*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) - d*f*log(c*(d + e*
x**2)**p)/(e*sqrt(-d/e)) + d*g*p*x**2/(4*e) - 2*f*p*x + f*x*log(c*(d + e*x*
*2)**p) - g*p*x**4/8 + g*x**4*log(c*(d + e*x**2)**p)/4, True))
```

Maxima [F(-2)]

Exception generated.

$$\int (f+gx^3)\log\left(c(d+ex^2)^p\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.89

$$\int \left(f+gx^3\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx = -\frac{1}{8}\,(gp-2\,g\log{(c)})x^4+\frac{dgpx^2}{4\,e}+\frac{2\,dfp\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} - \frac{d^2gp\log{(ex^2+d)}}{4\,e^2}-(2\,fp-f\log{(c)})x + \frac{1}{4}\,\left(gpx^4+4\,fpx\right)\log\left(ex^2+d\right)$$

[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")

[Out] -1/8*(g*p - 2*g*log(c))*x^4 + 1/4*d*g*p*x^2/e + 2*d*f*p*arctan(e*x/sqrt(d*e))/sqrt(d*e) - 1/4*d^2*g*p*log(e*x^2 + d)/e^2 - (2*f*p - f*log(c))*x + 1/4*(g*p*x^4 + 4*f*p*x)*log(e*x^2 + d)

Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \left(f+gx^3\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx = f\,x\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)-\frac{g\,p\,x^4}{8}-2\,f\,p\,x + \frac{g\,x^4\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{4}+\frac{d\,g\,p\,x^2}{4\,e} + \frac{2\,\sqrt{d}\,f\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{d^2\,g\,p\,\ln\left(e\,x^2+d\right)}{4\,e^2}$$

[In] int(log(c*(d + e*x^2)^p)*(f + g*x^3),x)

[Out] f*x*log(c*(d + e*x^2)^p) - (g*p*x^4)/8 - 2*f*p*x + (g*x^4*log(c*(d + e*x^2)^p))/4 + (d*g*p*x^2)/(4*e) + (2*d^(1/2)*f*p*atan((e^(1/2)*x)/d^(1/2)))/e^(1/2) - (d^2*g*p*log(d + e*x^2))/(4*e^2)

3.291 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 1165

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = -\frac{p\log\left(\frac{\sqrt[3]{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}+\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)}{3f^{2/3}\sqrt[3]{g}}$$
$$-\frac{p\log\left(-\frac{\sqrt[3]{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}-\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)}{3f^{2/3}\sqrt[3]{g}}$$
$$-\frac{(-1)^{2/3}p\log\left(-\frac{\sqrt[3]{-1}\sqrt[3]{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}-\sqrt[3]{-1}\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)}{3f^{2/3}\sqrt[3]{g}}$$
$$-\frac{(-1)^{2/3}p\log\left(\frac{\sqrt[3]{-1}\sqrt[3]{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}+\sqrt[3]{-1}\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)}{3f^{2/3}\sqrt[3]{g}}$$
$$+\frac{\sqrt[3]{-1}p\log\left(\frac{(-1)^{2/3}\sqrt[3]{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}+(-1)^{2/3}\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}x\right)}{3f^{2/3}\sqrt[3]{g}}$$
$$+\frac{\sqrt[3]{-1}p\log\left(-\frac{(-1)^{2/3}\sqrt[3]{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}-(-1)^{2/3}\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}x\right)}{3f^{2/3}\sqrt[3]{g}}$$
$$+\frac{\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{3f^{2/3}\sqrt[3]{g}}$$
$$+\frac{(-1)^{2/3}\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{3f^{2/3}\sqrt[3]{g}}$$
$$-\frac{\sqrt[3]{-1}\log\left(-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{3f^{2/3}\sqrt[3]{g}}$$
$$-\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}{\sqrt{e}\sqrt[3]{f}-\sqrt{-d}\sqrt[3]{g}}\right)}{3f^{2/3}\sqrt[3]{g}}$$
$$-\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}{\sqrt{e}\sqrt[3]{f}+\sqrt{-d}\sqrt[3]{g}}\right)}{3f^{2/3}\sqrt[3]{g}}$$
$$-\frac{(-1)^{2/3}p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt[3]{f}-\sqrt[3]{-1}\sqrt[3]{g}x\right)}{\sqrt{e}\sqrt[3]{f}-\sqrt[3]{-1}\sqrt{-d}\sqrt[3]{g}}\right)}{3f^{2/3}\sqrt[3]{g}}$$
$$-\frac{(-1)^{2/3}p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt[3]{f}-\sqrt[3]{-1}\sqrt[3]{g}x\right)}{\sqrt{e}\sqrt[3]{f}+\sqrt[3]{-1}\sqrt{-d}\sqrt[3]{g}}\right)}{3f^{2/3}\sqrt[3]{g}}$$

```
[Out] 1/3*ln(-f^(1/3)-g^(1/3)*x)*ln(c*(e*x^2+d)^p)/f^(2/3)/g^(1/3)+1/3*(-1)^(2/3)
*ln(-f^(1/3)+(-1)^(1/3)*g^(1/3)*x)*ln(c*(e*x^2+d)^p)/f^(2/3)/g^(1/3)-1/3*(-
1)^(1/3)*ln(-f^(1/3)-(-1)^(2/3)*g^(1/3)*x)*ln(c*(e*x^2+d)^p)/f^(2/3)/g^(1/3
)-1/3*p*ln(-f^(1/3)-g^(1/3)*x)*ln(g^(1/3)*((-d)^(1/2)-x*e^(1/2))/(g^(1/3)*(
-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)-1/3*(-1)^(2/3)*p*ln(-f^(1/3)+(-
1)^(1/3)*g^(1/3)*x)*ln(-(-1)^(1/3)*g^(1/3)*((-d)^(1/2)-x*e^(1/2))/(-(-1)^(1
/3)*g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)+1/3*(-1)^(1/3)*p*l
n(-f^(1/3)-(-1)^(2/3)*g^(1/3)*x)*ln((-1)^(2/3)*g^(1/3)*((-d)^(1/2)-x*e^(1/2
))/((-1)^(2/3)*g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)-1/3*p*l
n(-f^(1/3)-g^(1/3)*x)*ln(-g^(1/3)*((-d)^(1/2)+x*e^(1/2))/(-g^(1/3)*(-d)^(1/
2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)-1/3*(-1)^(2/3)*p*ln(-f^(1/3)+(-1)^(1/3
)*g^(1/3)*x)*ln((-1)^(1/3)*g^(1/3)*((-d)^(1/2)+x*e^(1/2))/((-1)^(1/3)*g^(1/
3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)+1/3*(-1)^(1/3)*p*ln(-f^(1/3
)-(-1)^(2/3)*g^(1/3)*x)*ln(-(-1)^(2/3)*g^(1/3)*((-d)^(1/2)+x*e^(1/2))/(-(-1
)^(2/3)*g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)-1/3*p*polylog(
2,(f^(1/3)+g^(1/3)*x)*e^(1/2)/(-g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3
)/g^(1/3)-1/3*p*polylog(2,(f^(1/3)+g^(1/3)*x)*e^(1/2)/(g^(1/3)*(-d)^(1/2)+f
^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)-1/3*(-1)^(2/3)*p*polylog(2,(f^(1/3)-(-1)^(
1/3)*g^(1/3)*x)*e^(1/2)/(-(-1)^(1/3)*g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f
^(2/3)/g^(1/3)-1/3*(-1)^(2/3)*p*polylog(2,(f^(1/3)-(-1)^(1/3)*g^(1/3)*x)*e^
(1/2)/((-1)^(1/3)*g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)+1/3*
(-1)^(1/3)*p*polylog(2,(f^(1/3)+(-1)^(2/3)*g^(1/3)*x)*e^(1/2)/(-(-1)^(2/3)*
g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)+1/3*(-1)^(1/3)*p*polyl
og(2,(f^(1/3)+(-1)^(2/3)*g^(1/3)*x)*e^(1/2)/((-1)^(2/3)*g^(1/3)*(-d)^(1/2)+
f^(1/3)*e^(1/2)))/f^(2/3)/g^(1/3)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 1165, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used

$= \{2521, 2512, 266, 2463, 2441, 2440, 2438\}$

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = -\frac{p\log\left(\frac{\sqrt[3]{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)\log\left(-\sqrt[3]{g}x-\sqrt[3]{f}\right)}{3f^{2/3}\sqrt[3]{g}}$$

$$-\frac{p\log\left(-\frac{\sqrt[3]{g}(\sqrt{e}x+\sqrt{-d})}{\sqrt{e}\sqrt[3]{f}-\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{g}x-\sqrt[3]{f}\right)}{3f^{2/3}\sqrt[3]{g}}$$

$$+\frac{\log\left(c(ex^2+d)^p\right)\log\left(-\sqrt[3]{g}x-\sqrt[3]{f}\right)}{3f^{2/3}\sqrt[3]{g}}$$

$$-\frac{(-1)^{2/3}p\log\left(-\frac{\sqrt[3]{-1}\sqrt[3]{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}-\sqrt[3]{-1}\sqrt{-d}\sqrt[3]{g}}\right)\log\left(\sqrt[3]{-1}\sqrt[3]{g}x-\sqrt[3]{f}\right)}{3f^{2/3}\sqrt[3]{g}}$$

$$-\frac{(-1)^{2/3}p\log\left(\frac{\sqrt[3]{-1}\sqrt[3]{g}(\sqrt{e}x+\sqrt{-d})}{\sqrt[3]{-1}\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)\log\left(\sqrt[3]{-1}\sqrt[3]{g}x-\sqrt[3]{f}\right)}{3f^{2/3}\sqrt[3]{g}}$$

$$+\frac{\sqrt[3]{-1}p\log\left(\frac{(-1)^{2/3}\sqrt[3]{g}(\sqrt{-d}-\sqrt{e}x)}{(-1)^{2/3}\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)\log\left(-(-1)^{2/3}\sqrt[3]{g}x-\sqrt[3]{f}\right)}{3f^{2/3}\sqrt[3]{g}}$$

$$+\frac{\sqrt[3]{-1}p\log\left(-\frac{(-1)^{2/3}\sqrt[3]{g}(\sqrt{e}x+\sqrt{-d})}{\sqrt{e}\sqrt[3]{f}-(-1)^{2/3}\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-(-1)^{2/3}\sqrt[3]{g}x-\sqrt[3]{f}\right)}{3f^{2/3}\sqrt[3]{g}}$$

$$+\frac{(-1)^{2/3}\log\left(\sqrt[3]{-1}\sqrt[3]{g}x-\sqrt[3]{f}\right)\log\left(c(ex^2+d)^p\right)}{3f^{2/3}\sqrt[3]{g}}$$

$$-\frac{\sqrt[3]{-1}\log\left(-(-1)^{2/3}\sqrt[3]{g}x-\sqrt[3]{f}\right)\log\left(c(ex^2+d)^p\right)}{3f^{2/3}\sqrt[3]{g}}$$

$$-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt[3]{g}x+\sqrt[3]{f}\right)}{\sqrt{e}\sqrt[3]{f}-\sqrt{-d}\sqrt[3]{g}}\right)}{3f^{2/3}\sqrt[3]{g}}$$

$$-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt[3]{g}x+\sqrt[3]{f}\right)}{\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)}{3f^{2/3}\sqrt[3]{g}}$$

$$-\frac{(-1)^{2/3}p\,\text{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt[3]{f}-\sqrt[3]{-1}\sqrt[3]{g}x\right)}{\sqrt{e}\sqrt[3]{f}-\sqrt[3]{-1}\sqrt{-d}\sqrt[3]{g}}\right)}{3f^{2/3}\sqrt[3]{g}}$$

$$-\frac{(-1)^{2/3}p\,\text{PolyLog}\left(2,\frac{\sqrt{e}\left(\sqrt[3]{f}-\sqrt[3]{-1}\sqrt[3]{g}x\right)}{\sqrt[3]{-1}\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)}{3f^{2/3}\sqrt[3]{g}}$$

$$\sqrt[3]{-1}p\,\text{PolyLog}\left(2,\frac{\sqrt{e}\left((-1)^{2/3}\sqrt[3]{g}x+\sqrt[3]{f}\right)}{\ldots}\right.$$

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^3),x]
```

```
[Out] -1/3*(p*Log[(g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^
(1/3))]*Log[-f^(1/3) - g^(1/3)*x])/(f^(2/3)*g^(1/3)) - (p*Log[-((g^(1/3)*(S
qrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - Sqrt[-d]*g^(1/3)))]*Log[-f^(1/3) -
 g^(1/3)*x])/(3*f^(2/3)*g^(1/3)) - ((-1)^(2/3)*p*Log[-(((-1)^(1/3)*g^(1/3)*
(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - (-1)^(1/3)*Sqrt[-d]*g^(1/3)))]*L
og[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x])/(3*f^(2/3)*g^(1/3)) - ((-1)^(2/3)*p*Lo
g[((-1)^(1/3)*g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + (-1)^(1/3)
*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x])/(3*f^(2/3)*g^(1/3
)) + ((-1)^(1/3)*p*Log[((-1)^(2/3)*g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]
*f^(1/3) + (-1)^(2/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) - (-1)^(2/3)*g^(1/3)*
x])/(3*f^(2/3)*g^(1/3)) + ((-1)^(1/3)*p*Log[-(((-1)^(2/3)*g^(1/3)*(Sqrt[-d]
 + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - (-1)^(2/3)*Sqrt[-d]*g^(1/3)))]*Log[-f^(1/
3) - (-1)^(2/3)*g^(1/3)*x])/(3*f^(2/3)*g^(1/3)) + (Log[-f^(1/3) - g^(1/3)*x
]*Log[c*(d + e*x^2)^p])/(3*f^(2/3)*g^(1/3)) + ((-1)^(2/3)*Log[-f^(1/3) + (-
1)^(1/3)*g^(1/3)*x]*Log[c*(d + e*x^2)^p])/(3*f^(2/3)*g^(1/3)) - ((-1)^(1/3)
*Log[-f^(1/3) - (-1)^(2/3)*g^(1/3)*x]*Log[c*(d + e*x^2)^p])/(3*f^(2/3)*g^(1
/3)) - (p*PolyLog[2, (Sqrt[e]*(f^(1/3) + g^(1/3)*x))/(Sqrt[e]*f^(1/3) - Sqr
t[-d]*g^(1/3))])/(3*f^(2/3)*g^(1/3)) - (p*PolyLog[2, (Sqrt[e]*(f^(1/3) + g^
(1/3)*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1/3))])/(3*f^(2/3)*g^(1/3)) - ((-1
)^(2/3)*p*PolyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/3)*g^(1/3)*x))/(Sqrt[e]*f^
(1/3) - (-1)^(1/3)*Sqrt[-d]*g^(1/3))])/(3*f^(2/3)*g^(1/3)) - ((-1)^(2/3)*p*
PolyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-
1)^(1/3)*Sqrt[-d]*g^(1/3))])/(3*f^(2/3)*g^(1/3)) + ((-1)^(1/3)*p*PolyLog[2,
 (Sqrt[e]*(f^(1/3) + (-1)^(2/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) - (-1)^(2/3)*S
qrt[-d]*g^(1/3))])/(3*f^(2/3)*g^(1/3)) + ((-1)^(1/3)*p*PolyLog[2, (Sqrt[e]*
(f^(1/3) + (-1)^(2/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(2/3)*Sqrt[-d]*g^
(1/3))])/(3*f^(2/3)*g^(1/3))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{\log\left(c(d+ex^2)^p\right)}{3f^{2/3}\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)}-\frac{\log\left(c(d+ex^2)^p\right)}{3f^{2/3}\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)}-\frac{\log\left(c(d+ex^2)^p\right)}{3f^{2/3}\left(-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}x\right)}\right)dx\\
&= -\frac{\int \frac{\log(c(d+ex^2)^p)}{-\sqrt[3]{f}-\sqrt[3]{g}x}\,dx}{3f^{2/3}}-\frac{\int \frac{\log(c(d+ex^2)^p)}{-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x}\,dx}{3f^{2/3}}-\frac{\int \frac{\log(c(d+ex^2)^p)}{-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}x}\,dx}{3f^{2/3}}
\end{aligned}
$$

$$= \frac{\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{3f^{2/3}\sqrt[3]{g}}+\frac{(-1)^{2/3}\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{3f^{2/3}\sqrt[3]{g}}$$
$$-\frac{\sqrt[3]{-1}\log\left(-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{3f^{2/3}\sqrt[3]{g}}-\frac{(2ep)\int\frac{x\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)}{d+ex^2}\,dx}{3f^{2/3}\sqrt[3]{g}}$$
$$+\frac{(2\sqrt[3]{-1}ep)\int\frac{x\log\left(-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}x\right)}{d+ex^2}\,dx}{3f^{2/3}\sqrt[3]{g}}-\frac{(2(-1)^{2/3}ep)\int\frac{x\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)}{d+ex^2}\,dx}{3f^{2/3}\sqrt[3]{g}}$$

$$= \frac{\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{3f^{2/3}\sqrt[3]{g}}$$
$$+\frac{(-1)^{2/3}\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{3f^{2/3}\sqrt[3]{g}}$$
$$-\frac{\sqrt[3]{-1}\log\left(-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{3f^{2/3}\sqrt[3]{g}}$$
$$-\frac{(2ep)\int\left(-\frac{\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)}{2\sqrt{e}(\sqrt{-d}-\sqrt{e}x)}+\frac{\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)}{2\sqrt{e}(\sqrt{-d}+\sqrt{e}x)}\right)dx}{3f^{2/3}\sqrt[3]{g}}$$
$$+\frac{(2\sqrt[3]{-1}ep)\int\left(-\frac{\log\left(-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}x\right)}{2\sqrt{e}(\sqrt{-d}-\sqrt{e}x)}+\frac{\log\left(-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}x\right)}{2\sqrt{e}(\sqrt{-d}+\sqrt{e}x)}\right)dx}{3f^{2/3}\sqrt[3]{g}}$$
$$-\frac{(2(-1)^{2/3}ep)\int\left(-\frac{\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)}{2\sqrt{e}(\sqrt{-d}-\sqrt{e}x)}+\frac{\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)}{2\sqrt{e}(\sqrt{-d}+\sqrt{e}x)}\right)dx}{3f^{2/3}\sqrt[3]{g}}$$

$$= \frac{\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{3f^{2/3}\sqrt[3]{g}}$$
$$+ \frac{(-1)^{2/3}\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{3f^{2/3}\sqrt[3]{g}}$$
$$- \frac{\sqrt[3]{-1}\log\left(-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{3f^{2/3}\sqrt[3]{g}}$$
$$+ \frac{\left(\sqrt{e}p\right)\int\frac{\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{3f^{2/3}\sqrt[3]{g}} - \frac{\left(\sqrt{e}p\right)\int\frac{\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{3f^{2/3}\sqrt[3]{g}}$$
$$- \frac{\left(\sqrt[3]{-1}\sqrt{e}p\right)\int\frac{\log\left(-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}x\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{3f^{2/3}\sqrt[3]{g}}$$
$$+ \frac{\left(\sqrt[3]{-1}\sqrt{e}p\right)\int\frac{\log\left(-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}x\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{3f^{2/3}\sqrt[3]{g}}$$
$$+ \frac{\left((-1)^{2/3}\sqrt{e}p\right)\int\frac{\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{3f^{2/3}\sqrt[3]{g}}$$
$$- \frac{\left((-1)^{2/3}\sqrt{e}p\right)\int\frac{\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{3f^{2/3}\sqrt[3]{g}}$$

$$
\begin{aligned}
&= -\frac{p\log\left(\frac{\sqrt[3]{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}+\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)}{3f^{2/3}\sqrt[3]{g}} \\
&- \frac{p\log\left(-\frac{\sqrt[3]{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}-\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)}{3f^{2/3}\sqrt[3]{g}} \\
&- \frac{(-1)^{2/3}p\log\left(-\frac{\sqrt[3]{-1}\sqrt[3]{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}-\sqrt[3]{-1}\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)}{3f^{2/3}\sqrt[3]{g}} \\
&- \frac{(-1)^{2/3}p\log\left(\frac{\sqrt[3]{-1}\sqrt[3]{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}+\sqrt[3]{-1}\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)}{3f^{2/3}\sqrt[3]{g}} \\
&+ \frac{\sqrt[3]{-1}p\log\left(\frac{(-1)^{2/3}\sqrt[3]{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}+(-1)^{2/3}\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}x\right)}{3f^{2/3}\sqrt[3]{g}} \\
&+ \frac{\sqrt[3]{-1}p\log\left(-\frac{(-1)^{2/3}\sqrt[3]{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}-(-1)^{2/3}\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}x\right)}{3f^{2/3}\sqrt[3]{g}} \\
&+ \frac{\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{3f^{2/3}\sqrt[3]{g}} \\
&+ \frac{(-1)^{2/3}\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{3f^{2/3}\sqrt[3]{g}} \\
&- \frac{\sqrt[3]{-1}\log\left(-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{3f^{2/3}\sqrt[3]{g}} \\
&- \frac{p\int\frac{\log\left(-\frac{\sqrt[3]{g}(\sqrt{-d}-\sqrt{e}x)}{-\sqrt{e}\sqrt[3]{f}-\sqrt{-d}\sqrt[3]{g}}\right)}{-\sqrt[3]{f}-\sqrt[3]{g}x}dx}{3f^{2/3}} - \frac{p\int\frac{\log\left(\frac{\sqrt[3]{-1}\sqrt[3]{g}(\sqrt{-d}-\sqrt{e}x)}{-\sqrt{e}\sqrt[3]{f}+\sqrt[3]{-1}\sqrt{-d}\sqrt[3]{g}}\right)}{-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x}dx}{3f^{2/3}} \\
&- \frac{p\int\frac{\log\left(-\frac{(-1)^{2/3}\sqrt[3]{g}(\sqrt{-d}-\sqrt{e}x)}{-\sqrt{e}\sqrt[3]{f}-(-1)^{2/3}\sqrt{-d}\sqrt[3]{g}}\right)}{-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}x}dx}{3f^{2/3}} - \frac{p\int\frac{\log\left(-\frac{\sqrt[3]{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}-\sqrt{-d}\sqrt[3]{g}}\right)}{-\sqrt[3]{f}-\sqrt[3]{g}x}dx}{3f^{2/3}} \\
&- \frac{p\int\frac{\log\left(\frac{\sqrt[3]{-1}\sqrt[3]{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}+\sqrt[3]{-1}\sqrt{-d}\sqrt[3]{g}}\right)}{-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x}dx}{3f^{2/3}} - \frac{p\int\frac{\log\left(-\frac{(-1)^{2/3}\sqrt[3]{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}-(-1)^{2/3}\sqrt{-d}\sqrt[3]{g}}\right)}{-\sqrt[3]{f}-(-1)^{2/3}\sqrt[3]{g}x}dx}{3f^{2/3}}
\end{aligned}
$$

$= \text{Too large to display}$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 990, normalized size of antiderivative = 0.85

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^3}\,dx$$

$$= \frac{-p\log\left(\frac{\sqrt[3]{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}+\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)-p\log\left(\frac{\sqrt[3]{g}(\sqrt{-d}+\sqrt{e}x)}{-\sqrt{e}\sqrt[3]{f}+\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}-\sqrt[3]{g}x\right)-(-1)^{2/3}p\,\mathrm{l}}{}$$

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^3),x]
```

```
[Out] (-(p*Log[(g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1/
3))]*Log[-f^(1/3) - g^(1/3)*x]) - p*Log[(g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(-
(Sqrt[e]*f^(1/3)) + Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) - g^(1/3)*x] - (-1)^(2/
3)*p*Log[((-1)^(1/3)*g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(-(Sqrt[e]*f^(1/3)) +
(-1)^(1/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x] - (-1)^(
2/3)*p*Log[((-1)^(1/3)*g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + (
-1)^(1/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x] + (-1)^(1
/3)*p*Log[((-1)^(2/3)*g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + (-
1)^(2/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) - (-1)^(2/3)*g^(1/3)*x] + (-1)^(1/
3)*p*Log[((-1)^(2/3)*g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(-(Sqrt[e]*f^(1/3)) +
(-1)^(2/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) - (-1)^(2/3)*g^(1/3)*x] + Log[-f
^(1/3) - g^(1/3)*x]*Log[c*(d + e*x^2)^p] + (-1)^(2/3)*Log[-f^(1/3) + (-1)^(
1/3)*g^(1/3)*x]*Log[c*(d + e*x^2)^p] - (-1)^(1/3)*Log[-f^(1/3) - (-1)^(2/3)
*g^(1/3)*x]*Log[c*(d + e*x^2)^p] - p*PolyLog[2, (Sqrt[e]*(f^(1/3) + g^(1/3)
*x))/(Sqrt[e]*f^(1/3) - Sqrt[-d]*g^(1/3))] - p*PolyLog[2, (Sqrt[e]*(f^(1/3)
 + g^(1/3)*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1/3))] - (-1)^(2/3)*p*PolyLog
[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) - (-1)^(1/3
)*Sqrt[-d]*g^(1/3))] - (-1)^(2/3)*p*PolyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/
3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(1/3)*Sqrt[-d]*g^(1/3))] + (-1)^(1/3
)*p*PolyLog[2, (Sqrt[e]*(f^(1/3) + (-1)^(2/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3)
- (-1)^(2/3)*Sqrt[-d]*g^(1/3))] + (-1)^(1/3)*p*PolyLog[2, (Sqrt[e]*(f^(1/3)
 + (-1)^(2/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(2/3)*Sqrt[-d]*g^(1/3))])
/(3*f^(2/3)*g^(1/3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.54 (sec) , antiderivative size = 577, normalized size of antiderivative = 0.50

method	result	size
risch	Expression too large to display	577

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^3+f),x,method=_RETURNVERBOSE)
```

```
[Out] (ln((e*x^2+d)^p)-p*ln(e*x^2+d))*(1/3/g/(f/g)^(2/3)*ln(x+(f/g)^(1/3))-1/6/g/
(f/g)^(2/3)*ln(x^2-(f/g)^(1/3)*x+(f/g)^(2/3))+1/3/g/(f/g)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(f/g)^(1/3)*x-1)))+1/3*p/g*sum(1/_alpha^2*(ln(x-_alpha)*
ln(e*x^2+d)-ln(x-_alpha)*(ln((RootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,inde
x=1)-x+_alpha)/RootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,index=1))+ln((RootO
f(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,index=2)-x+_alpha)/RootOf(_Z^2*e+2*_Z*_
alpha*e+_alpha^2*e+d,index=2)))-dilog((RootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2
*e+d,index=1)-x+_alpha)/RootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,index=1))-
dilog((RootOf(_Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,index=2)-x+_alpha)/RootOf(_
Z^2*e+2*_Z*_alpha*e+_alpha^2*e+d,index=2))),_alpha=RootOf(_Z^3*g+f))+(1/2*I
*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I*Pi*csgn(I*(e*x^2+d)^p
)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3+1/2*I*Pi
*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+ln(c))*(1/3/g/(f/g)^(2/3)*ln(x+(f/g)^(1/
3))-1/6/g/(f/g)^(2/3)*ln(x^2-(f/g)^(1/3)*x+(f/g)^(2/3))+1/3/g/(f/g)^(2/3)*3
^(1/2)*arctan(1/3*3^(1/2)*(2/(f/g)^(1/3)*x-1)))
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{gx^3+f}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g*x^3 + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**3+f),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \frac{\log \left((ex^2+d)^p c\right)}{gx^3+f}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f),x, algorithm="maxima")
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^3 + f), x)
```

Giac [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \frac{\log \left((ex^2+d)^p c\right)}{gx^3+f}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f),x, algorithm="giac")
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^3 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \frac{\ln \left(c\,{\left(e\,x^2+d\right)}^p\right)}{g\,x^3+f}\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)/(f + g*x^3),x)
[Out] int(log(c*(d + e*x^2)^p)/(f + g*x^3), x)
```

3.292 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 1861

$$
\begin{aligned}
\int \frac{\log\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx &= \frac{2\sqrt{d}\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9f^{4/3}\left(ef^{2/3}+dg^{2/3}\right)}+\frac{2(-1)^{2/3}\sqrt{d}\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^4f^{4/3}\left(ef^{2/3}+(-1)^{2/3}dg^{2/3}\right)} \\
&+\frac{4\sqrt{d}\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9f^{4/3}\left(2ef^{2/3}-\left(1+i\sqrt{3}\right)dg^{2/3}\right)}-\frac{2ep\log\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}{9f\left(ef^{2/3}+dg^{2/3}\right)\sqrt[3]{g}} \\
&-\frac{2p\log\left(\frac{\sqrt[3]{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}+\sqrt{-d}\sqrt[3]{g}}\right)\log\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}{9f^{5/3}\sqrt[3]{g}} \\
&-\frac{2p\log\left(-\frac{\sqrt[3]{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}-\sqrt{-d}\sqrt[3]{g}}\right)\log\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}{9f^{5/3}\sqrt[3]{g}} \\
&+\frac{2\sqrt[3]{-1}ep\log\left(\sqrt[3]{f}-\sqrt[3]{-1}\sqrt[3]{g}x\right)}{\left(1+\sqrt[3]{-1}\right)^4f\left(ef^{2/3}+(-1)^{2/3}dg^{2/3}\right)\sqrt[3]{g}} \\
&+\frac{2i\sqrt{3}p\log\left(-\frac{\sqrt[3]{-1}\sqrt[3]{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}-\sqrt[3]{-1}\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)}{\left(1+\sqrt[3]{-1}\right)^5f^{5/3}\sqrt[3]{g}} \\
&+\frac{2i\sqrt{3}p\log\left(\frac{\sqrt[3]{-1}\sqrt[3]{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}+\sqrt[3]{-1}\sqrt{-d}\sqrt[3]{g}}\right)\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)}{\left(1+\sqrt[3]{-1}\right)^5f^{5/3}\sqrt[3]{g}} \\
&+\frac{4\sqrt[3]{-1}ep\log\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\right)}{9f\left(2ef^{2/3}-\left(1+i\sqrt{3}\right)dg^{2/3}\right)\sqrt[3]{g}} \\
&-\frac{2p\log\left(\frac{(-1)^{2/3}\sqrt[3]{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}+(-1)^{2/3}\sqrt{-d}\sqrt[3]{g}}\right)\log\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\right)}{\left(1+\sqrt[3]{-1}\right)^4f^{5/3}\sqrt[3]{g}} \\
&-\frac{2p\log\left(-\frac{(-1)^{2/3}\sqrt[3]{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}-(-1)^{2/3}\sqrt{-d}\sqrt[3]{g}}\right)\log\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\right)}{\left(1+\sqrt[3]{-1}\right)^4f^{5/3}\sqrt[3]{g}} \\
&+\frac{ep\log\left(d+ex^2\right)}{9f\left(ef^{2/3}+dg^{2/3}\right)\sqrt[3]{g}} \\
&-\frac{\sqrt[3]{-1}ep\log\left(d+ex^2\right)}{\left(1+\sqrt[3]{-1}\right)^4f\left(ef^{2/3}+(-1)^{2/3}dg^{2/3}\right)\sqrt[3]{g}} \\
&-\frac{2\sqrt[3]{-1}ep\log\left(d+ex^2\right)}{9f\left(2ef^{2/3}-\left(1+i\sqrt{3}\right)dg^{2/3}\right)\sqrt[3]{g}}-\frac{\log\left(c(d+ex^2)^p\right)}{9f^{4/3}\sqrt[3]{g}\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)} \\
&-\frac{\log\left(c(d+ex^2)^p\right)}{\left(1+\sqrt[3]{-1}\right)^4f^{4/3}\sqrt[3]{g}\left((-1)^{2/3}\sqrt[3]{f}+\sqrt[3]{g}x\right)} \\
&+\frac{\sqrt[3]{-1}\log\left(c(d+ex^2)^p\right)}{9f^{4/3}\sqrt[3]{g}\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}\ldots\right)}
\end{aligned}
$$

```
[Out] 2*(-1)^(2/3)*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/(1+(-1)^(1/3))^4/f
^(4/3)/(e*f^(2/3)+(-1)^(2/3)*d*g^(2/3))+2*(-1)^(1/3)*e*p*ln(f^(1/3)-(-1)^(1
/3)*g^(1/3)*x)/(1+(-1)^(1/3))^4/f/(e*f^(2/3)+(-1)^(2/3)*d*g^(2/3))/g^(1/3)-
ln(c*(e*x^2+d)^p)/(1+(-1)^(1/3))^4/f^(4/3)/g^(1/3)/((-1)^(2/3)*f^(1/3)+g^(1
/3)*x)+1/9*(-1)^(1/3)*ln(c*(e*x^2+d)^p)/f^(4/3)/g^(1/3)/(f^(1/3)+(-1)^(2/3)
*g^(1/3)*x)-2/9*p*ln(f^(1/3)+g^(1/3)*x)*ln(g^(1/3)*((-d)^(1/2)-x*e^(1/2))/(
g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(5/3)/g^(1/3)-2/9*p*ln(f^(1/3)+g^(1/
3)*x)*ln(-g^(1/3)*((-d)^(1/2)+x*e^(1/2))/(-g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/
2)))/f^(5/3)/g^(1/3)-4/9*ln(c*(e*x^2+d)^p)*ln(2*f^(1/3)-g^(1/3)*x*(1-I*3^(1
/2)))/f^(5/3)/g^(1/3)/(1-I*3^(1/2))+4/9*p*polylog(2,(2*f^(1/3)-g^(1/3)*x*(1
-I*3^(1/2)))*e^(1/2)/(g^(1/3)*(1-I*3^(1/2))*(-d)^(1/2)+2*f^(1/3)*e^(1/2)))/
f^(5/3)/g^(1/3)/(1-I*3^(1/2))+4/9*p*polylog(2,(2*f^(1/3)-g^(1/3)*x*(1-I*3^(
1/2)))*e^(1/2)/(I*g^(1/3)*(3^(1/2)+I)*(-d)^(1/2)+2*f^(1/3)*e^(1/2)))/f^(5/3
)/g^(1/3)/(1-I*3^(1/2))-4/9*ln(c*(e*x^2+d)^p)*ln(2*f^(1/3)-g^(1/3)*x*(1+I*3
^(1/2)))/f^(5/3)/g^(1/3)/(1+I*3^(1/2))+4/9*p*polylog(2,(2*f^(1/3)-g^(1/3)*x
*(1+I*3^(1/2)))*e^(1/2)/(-g^(1/3)*(1+I*3^(1/2))*(-d)^(1/2)+2*f^(1/3)*e^(1/2
)))/f^(5/3)/g^(1/3)/(1+I*3^(1/2))+4/9*p*polylog(2,(2*f^(1/3)-g^(1/3)*x*(1+I
*3^(1/2)))*e^(1/2)/(g^(1/3)*(1+I*3^(1/2))*(-d)^(1/2)+2*f^(1/3)*e^(1/2)))/f^
(5/3)/g^(1/3)/(1+I*3^(1/2))-(-1)^(1/3)*e*p*ln(e*x^2+d)/(1+(-1)^(1/3))^4/f/(
e*f^(2/3)+(-1)^(2/3)*d*g^(2/3))/g^(1/3)-2/9*e*p*ln(f^(1/3)+g^(1/3)*x)/f/(e*
f^(2/3)+d*g^(2/3))/g^(1/3)+1/9*e*p*ln(e*x^2+d)/f/(e*f^(2/3)+d*g^(2/3))/g^(1
/3)+4/9*p*ln(2*f^(1/3)-g^(1/3)*x*(1-I*3^(1/2)))*ln(-g^(1/3)*(3^(1/2)+I)*((-
d)^(1/2)-x*e^(1/2))/(-g^(1/3)*(3^(1/2)+I)*(-d)^(1/2)+2*I*f^(1/3)*e^(1/2)))/
f^(5/3)/g^(1/3)/(1-I*3^(1/2))+4/9*p*ln(2*f^(1/3)-g^(1/3)*x*(1-I*3^(1/2)))*l
n(g^(1/3)*(3^(1/2)+I)*((-d)^(1/2)+x*e^(1/2))/(g^(1/3)*(3^(1/2)+I)*(-d)^(1/2
)+2*I*f^(1/3)*e^(1/2)))/f^(5/3)/g^(1/3)/(1-I*3^(1/2))+4/9*p*ln(2*f^(1/3)-g^
(1/3)*x*(1+I*3^(1/2)))*ln(-g^(1/3)*(1+I*3^(1/2))*((-d)^(1/2)-x*e^(1/2))/(-g
^(1/3)*(1+I*3^(1/2))*(-d)^(1/2)+2*f^(1/3)*e^(1/2)))/f^(5/3)/g^(1/3)/(1+I*3^
(1/2))+4/9*p*ln(2*f^(1/3)-g^(1/3)*x*(1+I*3^(1/2)))*ln(g^(1/3)*(1+I*3^(1/2))
*((-d)^(1/2)+x*e^(1/2))/(g^(1/3)*(1+I*3^(1/2))*(-d)^(1/2)+2*f^(1/3)*e^(1/2)
))/f^(5/3)/g^(1/3)/(1+I*3^(1/2))+2/9*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^
(1/2)/f^(4/3)/(e*f^(2/3)+d*g^(2/3))+4/9*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)
*e^(1/2)/f^(4/3)/(2*e*f^(2/3)-d*g^(2/3)*(1+I*3^(1/2)))-1/9*ln(c*(e*x^2+d)^p
)/f^(4/3)/g^(1/3)/(f^(1/3)+g^(1/3)*x)+2/9*ln(f^(1/3)+g^(1/3)*x)*ln(c*(e*x^2
+d)^p)/f^(5/3)/g^(1/3)-2/9*p*polylog(2,(f^(1/3)+g^(1/3)*x)*e^(1/2)/(-g^(1/3
)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(5/3)/g^(1/3)-2/9*p*polylog(2,(f^(1/3)+g^(
1/3)*x)*e^(1/2)/(g^(1/3)*(-d)^(1/2)+f^(1/3)*e^(1/2)))/f^(5/3)/g^(1/3)+4/9*(
-1)^(1/3)*e*p*ln(f^(1/3)+(-1)^(2/3)*g^(1/3)*x)/f/g^(1/3)/(2*e*f^(2/3)-d*g^(
2/3)*(1+I*3^(1/2)))-2/9*(-1)^(1/3)*e*p*ln(e*x^2+d)/f/g^(1/3)/(2*e*f^(2/3)-d
*g^(2/3)*(1+I*3^(1/2)))
```

Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 1863, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules

used = {2521, 2513, 815, 649, 211, 266, 2512, 2463, 2441, 2440, 2438}

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(f+gx^3\right)^2}\,dx = \frac{2\sqrt{d}\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9f^{4/3}\left(g^{2/3}d+ef^{2/3}\right)} + \frac{2(-1)^{2/3}\sqrt{d}\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(1+\sqrt[3]{-1}\right)^4 f^{4/3}\left((-1)^{2/3}g^{2/3}d+ef^{2/3}\right)}$$
$$+\frac{4\sqrt{d}\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{9f^{4/3}\left(i\left(i-\sqrt{3}\right)g^{2/3}d+2ef^{2/3}\right)} - \frac{2ep\log\left(\sqrt[3]{g}x+\sqrt[3]{f}\right)}{9f\left(g^{2/3}d+ef^{2/3}\right)\sqrt[3]{g}}$$
$$-\frac{2p\log\left(\frac{\sqrt[3]{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)\log\left(\sqrt[3]{g}x+\sqrt[3]{f}\right)}{9f^{5/3}\sqrt[3]{g}}$$
$$-\frac{2p\log\left(-\frac{\sqrt[3]{g}(\sqrt{e}x+\sqrt{-d})}{\sqrt{e}\sqrt[3]{f}-\sqrt{-d}\sqrt[3]{g}}\right)\log\left(\sqrt[3]{g}x+\sqrt[3]{f}\right)}{9f^{5/3}\sqrt[3]{g}}$$
$$+\frac{2\sqrt[3]{-1}ep\log\left(\sqrt[3]{f}-\sqrt[3]{-1}\sqrt[3]{g}x\right)}{\left(1+\sqrt[3]{-1}\right)^4 f\left((-1)^{2/3}g^{2/3}d+ef^{2/3}\right)\sqrt[3]{g}}$$
$$+\frac{2i\sqrt{3}p\log\left(-\frac{\sqrt[3]{-1}\sqrt[3]{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{e}\sqrt[3]{f}-\sqrt[3]{-1}\sqrt{-d}\sqrt[3]{g}}\right)\log\left(\sqrt[3]{-1}\sqrt[3]{g}x-\sqrt[3]{f}\right)}{\left(1+\sqrt[3]{-1}\right)^5 f^{5/3}\sqrt[3]{g}}$$
$$+\frac{2i\sqrt{3}p\log\left(\frac{\sqrt[3]{-1}\sqrt[3]{g}(\sqrt{e}x+\sqrt{-d})}{\sqrt[3]{-1}\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)\log\left(\sqrt[3]{-1}\sqrt[3]{g}x-\sqrt[3]{f}\right)}{\left(1+\sqrt[3]{-1}\right)^5 f^{5/3}\sqrt[3]{g}}$$
$$+\frac{4\sqrt[3]{-1}ep\log\left((-1)^{2/3}\sqrt[3]{g}x+\sqrt[3]{f}\right)}{9f\left(2ef^{2/3}-\left(1+i\sqrt{3}\right)dg^{2/3}\right)\sqrt[3]{g}}$$
$$-\frac{2p\log\left(\frac{(-1)^{2/3}\sqrt[3]{g}(\sqrt{-d}-\sqrt{e}x)}{(-1)^{2/3}\sqrt[3]{g}\sqrt{-d}+\sqrt{e}\sqrt[3]{f}}\right)\log\left((-1)^{2/3}\sqrt[3]{g}x+\sqrt[3]{f}\right)}{\left(1+\sqrt[3]{-1}\right)^4 f^{5/3}\sqrt[3]{g}}$$
$$-\frac{2p\log\left(-\frac{(-1)^{2/3}\sqrt[3]{g}(\sqrt{e}x+\sqrt{-d})}{\sqrt{e}\sqrt[3]{f}-(-1)^{2/3}\sqrt{-d}\sqrt[3]{g}}\right)\log\left((-1)^{2/3}\sqrt[3]{g}x+\sqrt[3]{f}\right)}{\left(1+\sqrt[3]{-1}\right)^4 f^{5/3}\sqrt[3]{g}}$$
$$+\frac{ep\log\left(ex^2+d\right)}{9f\left(g^{2/3}d+ef^{2/3}\right)\sqrt[3]{g}}$$
$$-\frac{\sqrt[3]{-1}ep\log\left(ex^2+d\right)}{\left(1+\sqrt[3]{-1}\right)^4 f\left((-1)^{2/3}g^{2/3}d+ef^{2/3}\right)\sqrt[3]{g}}$$
$$-\frac{2\sqrt[3]{-1}ep\log\left(ex^2+d\right)}{9f\left(2ef^{2/3}-\left(1+i\sqrt{3}\right)dg^{2/3}\right)\sqrt[3]{g}}$$
$$+\frac{2\log\left(\sqrt[3]{g}x+\sqrt[3]{f}\right)\log\left(c(ex^2+d)^p\right)}{9f^{5/3}\sqrt[3]{g}}$$
$$-\frac{2i\sqrt{3}\log\left(\sqrt[3]{-1}\sqrt[3]{g}x-\sqrt[3]{f}\right)\log\left(c(ex^2+d)^p\right)}{\left(1+\sqrt[3]{-1}\right)^5 f^{5/3}\sqrt[3]{g}}$$

$2\log\left((-1)^{2/3}\sqrt[3]{g}x+\sqrt[3]{f}\right)\log\left(c(ex^2+d)^p\right)$

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^3)^2,x]
```

```
[Out] (2*Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(9*f^(4/3)*(e*f^(2/3) + d
*g^(2/3))) + (2*(-1)^(2/3)*Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(
(1 + (-1)^(1/3))^4*f^(4/3)*(e*f^(2/3) + (-1)^(2/3)*d*g^(2/3))) + (4*Sqrt[d]
*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(9*f^(4/3)*(2*e*f^(2/3) + I*(I - Sq
rt[3])*d*g^(2/3))) - (2*e*p*Log[f^(1/3) + g^(1/3)*x])/(9*f*(e*f^(2/3) + d*g
^(2/3))*g^(1/3)) - (2*p*Log[(g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/
3) + Sqrt[-d]*g^(1/3))]*Log[f^(1/3) + g^(1/3)*x])/(9*f^(5/3)*g^(1/3)) - (2*
p*Log[-((g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - Sqrt[-d]*g^(1/3
)))]*Log[f^(1/3) + g^(1/3)*x])/(9*f^(5/3)*g^(1/3)) + (2*(-1)^(1/3)*e*p*Log[
f^(1/3) - (-1)^(1/3)*g^(1/3)*x])/((1 + (-1)^(1/3))^4*f*(e*f^(2/3) + (-1)^(2
/3)*d*g^(2/3))*g^(1/3)) + ((2*I)*Sqrt[3]*p*Log[-(((-1)^(1/3)*g^(1/3)*(Sqrt[
-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - (-1)^(1/3)*Sqrt[-d]*g^(1/3)))]*Log[-f^
(1/3) + (-1)^(1/3)*g^(1/3)*x])/((1 + (-1)^(1/3))^5*f^(5/3)*g^(1/3)) + ((2*I
)*Sqrt[3]*p*Log[((-1)^(1/3)*g^(1/3)*(Sqrt[-d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3
) + (-1)^(1/3)*Sqrt[-d]*g^(1/3))]*Log[-f^(1/3) + (-1)^(1/3)*g^(1/3)*x])/((1
 + (-1)^(1/3))^5*f^(5/3)*g^(1/3)) + (4*(-1)^(1/3)*e*p*Log[f^(1/3) + (-1)^(2
/3)*g^(1/3)*x])/(9*f*(2*e*f^(2/3) - (1 + I*Sqrt[3])*d*g^(2/3))*g^(1/3)) - (
2*p*Log[((-1)^(2/3)*g^(1/3)*(Sqrt[-d] - Sqrt[e]*x))/(Sqrt[e]*f^(1/3) + (-1)
^(2/3)*Sqrt[-d]*g^(1/3))]*Log[f^(1/3) + (-1)^(2/3)*g^(1/3)*x])/((1 + (-1)^(
1/3))^4*f^(5/3)*g^(1/3)) - (2*p*Log[-(((-1)^(2/3)*g^(1/3)*(Sqrt[-d] + Sqrt[
e]*x))/(Sqrt[e]*f^(1/3) - (-1)^(2/3)*Sqrt[-d]*g^(1/3)))]*Log[f^(1/3) + (-1)
^(2/3)*g^(1/3)*x])/((1 + (-1)^(1/3))^4*f^(5/3)*g^(1/3)) + (e*p*Log[d + e*x^
2])/(9*f*(e*f^(2/3) + d*g^(2/3))*g^(1/3)) - ((-1)^(1/3)*e*p*Log[d + e*x^2])
/((1 + (-1)^(1/3))^4*f*(e*f^(2/3) + (-1)^(2/3)*d*g^(2/3))*g^(1/3)) - (2*(-1
)^(1/3)*e*p*Log[d + e*x^2])/(9*f*(2*e*f^(2/3) - (1 + I*Sqrt[3])*d*g^(2/3))*
g^(1/3)) - Log[c*(d + e*x^2)^p]/(9*f^(4/3)*g^(1/3)*(f^(1/3) + g^(1/3)*x)) -
 Log[c*(d + e*x^2)^p]/((1 + (-1)^(1/3))^4*f^(4/3)*g^(1/3)*((-1)^(2/3)*f^(1/
3) + g^(1/3)*x)) + ((-1)^(1/3)*Log[c*(d + e*x^2)^p])/(9*f^(4/3)*g^(1/3)*(f^
(1/3) + (-1)^(2/3)*g^(1/3)*x)) + (2*Log[f^(1/3) + g^(1/3)*x]*Log[c*(d + e*x
^2)^p])/(9*f^(5/3)*g^(1/3)) - ((2*I)*Sqrt[3]*Log[-f^(1/3) + (-1)^(1/3)*g^(1
/3)*x]*Log[c*(d + e*x^2)^p])/((1 + (-1)^(1/3))^5*f^(5/3)*g^(1/3)) + (2*Log[
f^(1/3) + (-1)^(2/3)*g^(1/3)*x]*Log[c*(d + e*x^2)^p])/((1 + (-1)^(1/3))^4*f
^(5/3)*g^(1/3)) - (2*p*PolyLog[2, (Sqrt[e]*(f^(1/3) + g^(1/3)*x))/(Sqrt[e]*
f^(1/3) - Sqrt[-d]*g^(1/3))])/(9*f^(5/3)*g^(1/3)) - (2*p*PolyLog[2, (Sqrt[e
]*(f^(1/3) + g^(1/3)*x))/(Sqrt[e]*f^(1/3) + Sqrt[-d]*g^(1/3))])/(9*f^(5/3)*
g^(1/3)) + ((2*I)*Sqrt[3]*p*PolyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(1/3)*g^(1/
3)*x))/(Sqrt[e]*f^(1/3) - (-1)^(1/3)*Sqrt[-d]*g^(1/3))])/((1 + (-1)^(1/3))^
5*f^(5/3)*g^(1/3)) + ((2*I)*Sqrt[3]*p*PolyLog[2, (Sqrt[e]*(f^(1/3) - (-1)^(
1/3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(1/3)*Sqrt[-d]*g^(1/3))])/((1 + (-
1)^(1/3))^5*f^(5/3)*g^(1/3)) - (2*p*PolyLog[2, (Sqrt[e]*(f^(1/3) + (-1)^(2/
3)*g^(1/3)*x))/(Sqrt[e]*f^(1/3) - (-1)^(2/3)*Sqrt[-d]*g^(1/3))])/((1 + (-1)
^(1/3))^4*f^(5/3)*g^(1/3)) - (2*p*PolyLog[2, (Sqrt[e]*(f^(1/3) + (-1)^(2/3)
*g^(1/3)*x))/(Sqrt[e]*f^(1/3) + (-1)^(2/3)*Sqrt[-d]*g^(1/3))])/((1 + (-1)^(
```

```
1/3))^4*f^(5/3)*g^(1/3))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
```

```
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log\left(c(d+ex^2)^p\right)}{9f^{4/3}\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)^2} + \frac{2\log\left(c(d+ex^2)^p\right)}{9f^{5/3}\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)} + \frac{(-1)^{2/3}\log\left(c(d+ex^2)^p\right)}{\left(1+\sqrt[3]{-1}\right)^4 f^{4/3}\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)^2} - \frac{2(-1)^{5/6}\sqrt{3}\log\left(c(d+ex^2)^p\right)}{\left(1+\sqrt[3]{-1}\right)^5 f^{5/3}\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)} + \frac{\log\left(c(d+ex^2)^p\right)}{\left(-1+\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4 f^{4/3}\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\right)^2} + \frac{2(-1)^{2/3}\log\left(c(d+ex^2)^p\right)}{\left(1+\sqrt[3]{-1}\right)^4 f^{5/3}\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\right)} \right) dx$$

$$= \frac{2\int \frac{\log(c(d+ex^2)^p)}{\sqrt[3]{f}+\sqrt[3]{g}x}\,dx}{9f^{5/3}} + \frac{2\int \frac{\log(c(d+ex^2)^p)}{\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x}\,dx}{9f^{5/3}} - \frac{\left(2(-1)^{5/6}\sqrt{3}\right)\int \frac{\log(c(d+ex^2)^p)}{-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x}\,dx}{\left(1+\sqrt[3]{-1}\right)^5 f^{5/3}}$$

$$+ \frac{\int \frac{\log(c(d+ex^2)^p)}{\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)^2}\,dx}{9f^{4/3}} + \frac{\int \frac{\log(c(d+ex^2)^p)}{\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)^2}\,dx}{9f^{4/3}} + \frac{\int \frac{\log(c(d+ex^2)^p)}{\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\right)^2}\,dx}{9f^{4/3}}$$

$$= -\frac{\log\left(c(d+ex^2)^p\right)}{9f^{4/3}\sqrt[3]{g}\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)} + \frac{\sqrt[3]{-1}\log\left(c(d+ex^2)^p\right)}{9f^{4/3}\sqrt[3]{g}\left((-1)^{2/3}\sqrt[3]{f}+\sqrt[3]{g}x\right)}$$

$$+ \frac{\sqrt[3]{-1}\log\left(c(d+ex^2)^p\right)}{9f^{4/3}\sqrt[3]{g}\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\right)} + \frac{2\log\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{9f^{5/3}\sqrt[3]{g}}$$

$$- \frac{2i\sqrt{3}\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{\left(1+\sqrt[3]{-1}\right)^5 f^{5/3}\sqrt[3]{g}}$$

$$- \frac{2\sqrt[3]{-1}\log\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{9f^{5/3}\sqrt[3]{g}} - \frac{(4ep)\int \frac{x\log\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}{d+ex^2}\,dx}{9f^{5/3}\sqrt[3]{g}}$$

$$+ \frac{\left(4\sqrt[3]{-1}ep\right)\int \frac{x\log\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\right)}{d+ex^2}\,dx}{9f^{5/3}\sqrt[3]{g}} + \frac{\left(4i\sqrt{3}ep\right)\int \frac{x\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)}{d+ex^2}\,dx}{\left(1+\sqrt[3]{-1}\right)^5 f^{5/3}\sqrt[3]{g}}$$

$$+ \frac{(2ep)\int \frac{x}{\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)(d+ex^2)}\,dx}{9f^{4/3}\sqrt[3]{g}} - \frac{\left(2\sqrt[3]{-1}ep\right)\int \frac{x}{\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\right)(d+ex^2)}\,dx}{9f^{4/3}\sqrt[3]{g}}$$

$$- \frac{\left(2(-1)^{2/3}ep\right)\int \frac{x}{\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)(d+ex^2)}\,dx}{9f^{4/3}\sqrt[3]{g}}$$

$$= -\frac{\log\left(c(d+ex^2)^p\right)}{9f^{4/3}\sqrt[3]{g}\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)} + \frac{\sqrt[3]{-1}\log\left(c(d+ex^2)^p\right)}{9f^{4/3}\sqrt[3]{g}\left((-1)^{2/3}\sqrt[3]{f}+\sqrt[3]{g}x\right)}$$

$$+ \frac{\sqrt[3]{-1}\log\left(c(d+ex^2)^p\right)}{9f^{4/3}\sqrt[3]{g}\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\right)} + \frac{2\log\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{9f^{5/3}\sqrt[3]{g}}$$

$$- \frac{2i\sqrt{3}\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{\left(1+\sqrt[3]{-1}\right)^5 f^{5/3}\sqrt[3]{g}}$$

$$- \frac{2\sqrt[3]{-1}\log\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{9f^{5/3}\sqrt[3]{g}}$$

$$- \frac{(4ep)\int\left(-\frac{\log\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\log\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{9f^{5/3}\sqrt[3]{g}}$$

$$+ \frac{\left(4\sqrt[3]{-1}ep\right)\int\left(-\frac{\log\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\right)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\log\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\right)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{9f^{5/3}\sqrt[3]{g}}$$

$$+ \frac{\left(4i\sqrt{3}ep\right)\int\left(-\frac{\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{\left(1+\sqrt[3]{-1}\right)^5 f^{5/3}\sqrt[3]{g}}$$

$$+ \frac{(2ep)\int\left(-\frac{\sqrt[3]{f}\sqrt[3]{g}}{\left(ef^{2/3}+dg^{2/3}\right)\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}+\frac{d\sqrt[3]{g}+e\sqrt[3]{f}x}{\left(ef^{2/3}+dg^{2/3}\right)(d+ex^2)}\right)dx}{9f^{4/3}\sqrt[3]{g}}$$

$$- \frac{\left(2\sqrt[3]{-1}ep\right)\int\left(-\frac{(-1)^{2/3}\sqrt[3]{f}\sqrt[3]{g}}{\left(ef^{2/3}-\sqrt[3]{-1}dg^{2/3}\right)\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\right)}+\frac{(-1)^{2/3}d\sqrt[3]{g}+e\sqrt[3]{f}x}{\left(ef^{2/3}-\sqrt[3]{-1}dg^{2/3}\right)(d+ex^2)}\right)dx}{9f^{4/3}\sqrt[3]{g}}$$

$$- \frac{\left(2(-1)^{2/3}ep\right)\int\left(-\frac{\sqrt[3]{-1}\sqrt[3]{f}\sqrt[3]{g}}{\left(ef^{2/3}+(-1)^{2/3}dg^{2/3}\right)\left(\sqrt[3]{f}-\sqrt[3]{-1}\sqrt[3]{g}x\right)}+\frac{\sqrt[3]{-1}d\sqrt[3]{g}-e\sqrt[3]{f}x}{\left(ef^{2/3}+(-1)^{2/3}dg^{2/3}\right)(d+ex^2)}\right)dx}{9f^{4/3}\sqrt[3]{g}}$$

$$= -\frac{2ep\log\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}{9f\left(ef^{2/3}+dg^{2/3}\right)\sqrt[3]{g}} - \frac{2(-1)^{2/3}ep\log\left(\sqrt[3]{f}-\sqrt[3]{-1}\sqrt[3]{g}x\right)}{9f\left(ef^{2/3}+(-1)^{2/3}dg^{2/3}\right)\sqrt[3]{g}}$$
$$+ \frac{2\sqrt[3]{-1}ep\log\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\right)}{9f\left(ef^{2/3}-\sqrt[3]{-1}dg^{2/3}\right)\sqrt[3]{g}} - \frac{\log\left(c(d+ex^2)^p\right)}{9f^{4/3}\sqrt[3]{g}\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}$$
$$+ \frac{\sqrt[3]{-1}\log\left(c(d+ex^2)^p\right)}{9f^{4/3}\sqrt[3]{g}\left((-1)^{2/3}\sqrt[3]{f}+\sqrt[3]{g}x\right)} + \frac{\sqrt[3]{-1}\log\left(c(d+ex^2)^p\right)}{9f^{4/3}\sqrt[3]{g}\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\right)}$$
$$+ \frac{2\log\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{9f^{5/3}\sqrt[3]{g}}$$
$$- \frac{2i\sqrt{3}\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{\left(1+\sqrt[3]{-1}\right)^5 f^{5/3}\sqrt[3]{g}}$$
$$- \frac{2\sqrt[3]{-1}\log\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\right)\log\left(c(d+ex^2)^p\right)}{9f^{5/3}\sqrt[3]{g}} + \frac{\left(2\sqrt{e}p\right)\int\frac{\log\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{9f^{5/3}\sqrt[3]{g}}$$
$$- \frac{\left(2\sqrt{e}p\right)\int\frac{\log\left(\sqrt[3]{f}+\sqrt[3]{g}x\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{9f^{5/3}\sqrt[3]{g}} - \frac{\left(2\sqrt[3]{-1}\sqrt{e}p\right)\int\frac{\log\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{9f^{5/3}\sqrt[3]{g}}$$
$$+ \frac{\left(2\sqrt[3]{-1}\sqrt{e}p\right)\int\frac{\log\left(\sqrt[3]{f}+(-1)^{2/3}\sqrt[3]{g}x\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{9f^{5/3}\sqrt[3]{g}}$$
$$- \frac{\left(2i\sqrt{3}\sqrt{e}p\right)\int\frac{\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{\left(1+\sqrt[3]{-1}\right)^5 f^{5/3}\sqrt[3]{g}}$$
$$+ \frac{\left(2i\sqrt{3}\sqrt{e}p\right)\int\frac{\log\left(-\sqrt[3]{f}+\sqrt[3]{-1}\sqrt[3]{g}x\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{\left(1+\sqrt[3]{-1}\right)^5 f^{5/3}\sqrt[3]{g}} + \frac{(2ep)\int\frac{d\sqrt[3]{g}+e\sqrt[3]{f}x}{d+ex^2}\,dx}{9f^{4/3}\left(ef^{2/3}+dg^{2/3}\right)\sqrt[3]{g}}$$
$$- \frac{\left(2\sqrt[3]{-1}ep\right)\int\frac{(-1)^{2/3}d\sqrt[3]{g}+e\sqrt[3]{f}x}{d+ex^2}\,dx}{9f^{4/3}\left(ef^{2/3}-\sqrt[3]{-1}dg^{2/3}\right)\sqrt[3]{g}} - \frac{\left(2(-1)^{2/3}ep\right)\int\frac{\sqrt[3]{-1}d\sqrt[3]{g}-e\sqrt[3]{f}x}{d+ex^2}\,dx}{9f^{4/3}\left(ef^{2/3}+(-1)^{2/3}dg^{2/3}\right)\sqrt[3]{g}}$$

= Too large to display

Mathematica [A] (warning: unable to verify)

Time = 6.81 (sec) , antiderivative size = 2168, normalized size of antiderivative = 1.16

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{\left(f+gx^3\right)^2} \, dx = \text{Result too large to show}$$

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^3)^2,x]
```

```
[Out] (x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(3*f*(f + g*x^3)) + (2*Arc
Tan[(-f^(1/3) + 2*g^(1/3)*x)/(Sqrt[3]*f^(1/3))]*(-(p*Log[d + e*x^2]) + Log[
c*(d + e*x^2)^p]))/(3*Sqrt[3]*f^(5/3)*g^(1/3)) + (2*Log[f^(1/3) + g^(1/3)*x
]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]))/(9*f^(5/3)*g^(1/3)) - ((-(p
*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])*Log[f^(2/3) - f^(1/3)*g^(1/3)*x +
g^(2/3)*x^2])/(9*f^(5/3)*g^(1/3)) + p*(-1/3*((-1 + (-1)^(1/3))*(-(Log[((-I)
*Sqrt[d])/Sqrt[e] + x]/((-1)^(2/3)*f^(1/3) + g^(1/3)*x)) + (Sqrt[e]*(Log[I*
Sqrt[d] - Sqrt[e]*x] - Log[-((-1)^(2/3)*f^(1/3)) - g^(1/3)*x]))/((-1)^(2/3)
*Sqrt[e]*f^(1/3) + I*Sqrt[d]*g^(1/3))))/((1 + (-1)^(1/3))^2*f^(4/3)*g^(1/3)
) - ((-1 + (-1)^(1/3))*(-(Log[(I*Sqrt[d])/Sqrt[e] + x]/((-1)^(2/3)*f^(1/3)
+ g^(1/3)*x)) + (Sqrt[e]*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[-((-1)^(2/3)*f^(
1/3)) - g^(1/3)*x]))/((-1)^(2/3)*Sqrt[e]*f^(1/3) - I*Sqrt[d]*g^(1/3))))/(3*
(1 + (-1)^(1/3))^2*f^(4/3)*g^(1/3)) + ((-1)^(1/3)*(-(Log[((-I)*Sqrt[d])/Sqr
t[e] + x]/(f^(1/3) + g^(1/3)*x)) + (Sqrt[e]*(Log[I*Sqrt[d] - Sqrt[e]*x] - L
og[f^(1/3) + g^(1/3)*x]))/(Sqrt[e]*f^(1/3) + I*Sqrt[d]*g^(1/3))))/(3*(1 + (
-1)^(1/3))^2*f^(4/3)*g^(1/3)) + ((-1)^(1/3)*(-(Log[(I*Sqrt[d])/Sqrt[e] + x]
/(f^(1/3) + g^(1/3)*x)) + (Sqrt[e]*(Log[I*Sqrt[d] + Sqrt[e]*x] - Log[f^(1/3
) + g^(1/3)*x]))/(Sqrt[e]*f^(1/3) - I*Sqrt[d]*g^(1/3))))/(3*(1 + (-1)^(1/3)
)^2*f^(4/3)*g^(1/3)) - (Log[((-I)*Sqrt[d])/Sqrt[e] + x]/((-1)^(1/3)*f^(1/3)
 - g^(1/3)*x) + (Sqrt[e]*(-Log[I*Sqrt[d] - Sqrt[e]*x] + Log[f^(1/3) + (-1)^
(2/3)*g^(1/3)*x]))/((-1)^(1/3)*Sqrt[e]*f^(1/3) - I*Sqrt[d]*g^(1/3)))/(3*(1
+ (-1)^(1/3))^2*f^(4/3)*g^(1/3)) - (Log[(I*Sqrt[d])/Sqrt[e] + x]/((-1)^(1/3
)*f^(1/3) - g^(1/3)*x) + (Sqrt[e]*(-Log[I*Sqrt[d] + Sqrt[e]*x] + Log[f^(1/3
) + (-1)^(2/3)*g^(1/3)*x]))/((-1)^(1/3)*Sqrt[e]*f^(1/3) + I*Sqrt[d]*g^(1/3)
))/(3*(1 + (-1)^(1/3))^2*f^(4/3)*g^(1/3)) + ((-Log[((-I)*Sqrt[d])/Sqrt[e] +
 x] - Log[(I*Sqrt[d])/Sqrt[e] + x] + Log[d + e*x^2])*((3*f^(2/3)*x)/(f + g*
x^3) - (2*Sqrt[3]*ArcTan[(1 - (2*g^(1/3)*x)/f^(1/3))/Sqrt[3]])/g^(1/3) + (2
*Log[f^(1/3) + g^(1/3)*x])/g^(1/3) - Log[f^(2/3) - f^(1/3)*g^(1/3)*x + g^(2
/3)*x^2]/g^(1/3)))/(9*f^(5/3)) - (2*(Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[((-1)
^(2/3)*f^(1/3) + g^(1/3)*x)/((-1)^(2/3)*f^(1/3) - (I*Sqrt[d]*g^(1/3))/Sqrt[
e])] + PolyLog[2, -((g^(1/3)*(Sqrt[d] - I*Sqrt[e]*x))/((-1)^(1/6)*Sqrt[e]*f
^(1/3) - Sqrt[d]*g^(1/3)))]))/(3*(1 + (-1)^(1/3))^2*f^(5/3)*g^(1/3)) - (2*(
-1 + (-1)^(1/3))*(Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[-((-((-1)^(1/3)*f^(1/3))
 + g^(1/3)*x)/((-1)^(1/3)*f^(1/3) + (I*Sqrt[d]*g^(1/3))/Sqrt[e]))] + PolyLo
g[2, -((g^(1/3)*(Sqrt[d] - I*Sqrt[e]*x))/((-1)^(5/6)*Sqrt[e]*f^(1/3) - Sqrt
[d]*g^(1/3)))]))/(3*(1 + (-1)^(1/3))^2*f^(5/3)*g^(1/3)) + (2*(-1)^(1/3)*(Lo
```

```
g[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(f^(1/3) + g^(1/3)*x)/(f^(1/3) + (I*Sqrt[
d]*g^(1/3))/Sqrt[e])] + PolyLog[2, (I*g^(1/3)*(Sqrt[d] + I*Sqrt[e]*x))/(Sqr
t[e]*f^(1/3) + I*Sqrt[d]*g^(1/3))]))/(3*(1 + (-1)^(1/3))^2*f^(5/3)*g^(1/3))
 - (2*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[((-1)^(2/3)*f^(1/3) + g^(1/3)*x)
/((-1)^(2/3)*f^(1/3) + (I*Sqrt[d]*g^(1/3))/Sqrt[e])] + PolyLog[2, (g^(1/3)*
(Sqrt[d] + I*Sqrt[e]*x))/((-1)^(1/6)*Sqrt[e]*f^(1/3) + Sqrt[d]*g^(1/3))]))/
(3*(1 + (-1)^(1/3))^2*f^(5/3)*g^(1/3)) - (2*(-1 + (-1)^(1/3))*(Log[((-I)*Sq
rt[d])/Sqrt[e] + x]*Log[(-((-1)^(1/3)*f^(1/3)) + g^(1/3)*x)/(-((-1)^(1/3)*f
^(1/3)) + (I*Sqrt[d]*g^(1/3))/Sqrt[e])] + PolyLog[2, (g^(1/3)*(Sqrt[d] + I*
Sqrt[e]*x))/((-1)^(5/6)*Sqrt[e]*f^(1/3) + Sqrt[d]*g^(1/3))]))/(3*(1 + (-1)^
(1/3))^2*f^(5/3)*g^(1/3)) + (2*(-1)^(1/3)*(Log[(I*Sqrt[d])/Sqrt[e] + x]*Log
[(f^(1/3) + g^(1/3)*x)/(f^(1/3) - (I*Sqrt[d]*g^(1/3))/Sqrt[e])] + PolyLog[2
, -((g^(1/3)*(I*Sqrt[d] + Sqrt[e]*x))/(Sqrt[e]*f^(1/3) - I*Sqrt[d]*g^(1/3))
)]))/(3*(1 + (-1)^(1/3))^2*f^(5/3)*g^(1/3)))
```

Maple [F]

$$\int \frac{\ln \left(c(e\,x^{2}+d)^{p}\right)}{\left(g\,x^{3}+f\right)^{2}}dx$$

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^3+f)^2,x)
[Out] int(ln(c*(e*x^2+d)^p)/(g*x^3+f)^2,x)
```

Fricas [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{\left(f+gx^3\right)^2}\,dx=\int \frac{\log \left((ex^2+d)^pc\right)}{\left(gx^3+f\right)^2}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f)^2,x, algorithm="fricas")
[Out] integral(log((e*x^2 + d)^p*c)/(g^2*x^6 + 2*f*g*x^3 + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{\left(f+gx^3\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**3+f)**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{(gx^3+f)^2}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^3+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^3 + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)}{(g\,x^3+f)^2}\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)/(f + g*x^3)^2,x)
```

```
[Out] int(log(c*(d + e*x^2)^p)/(f + g*x^3)^2, x)
```

3.293 $\int (f+gx^3)^3 \log^2\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1221

$$
\begin{aligned}
\int (f+gx^3)^3\log^2\left(c(d+ex^2)^p\right)\,dx &= 8f^3p^2x-\frac{1408d^3fg^2p^2x}{245e^3}-\frac{3df^2gp^2x^2}{e} \\
&+\frac{d^4g^3p^2x^2}{e^4}+\frac{568d^2fg^2p^2x^3}{735e^2}-\frac{288dfg^2p^2x^5}{1225e}+\frac{24}{343}fg^2p^2x^7+\frac{3f^2gp^2(d+ex^2)^2}{8e^2} \\
&-\frac{d^3g^3p^2(d+ex^2)^2}{2e^5}+\frac{2d^2g^3p^2(d+ex^2)^3}{9e^5}-\frac{dg^3p^2(d+ex^2)^4}{16e^5}+\frac{g^3p^2(d+ex^2)^5}{125e^5} \\
&-\frac{8\sqrt{d}f^3p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{1408d^{7/2}fg^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{245e^{7/2}}+\frac{4i\sqrt{d}f^3p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} \\
&-\frac{12id^{7/2}fg^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{7e^{7/2}}+\frac{8\sqrt{d}f^3p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} \\
&-\frac{24d^{7/2}fg^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{7e^{7/2}}-\frac{d^5g^3p^2\log^2(d+ex^2)}{10e^5} \\
&-4f^3px\log\left(c(d+ex^2)^p\right)+\frac{12d^3fg^2px\log\left(c(d+ex^2)^p\right)}{7e^3}-\frac{4d^2fg^2px^3\log\left(c(d+ex^2)^p\right)}{7e^2}+\frac{12dfg^2px^5\log (c}{35e}
\end{aligned}
$$

```
[Out] 8*f^3*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(
1/2)/e^(1/2)-d^4*g^3*p*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e^5+d^3*g^3*p*(e*x^2+d)^
2*ln(c*(e*x^2+d)^p)/e^5+d^4*g^3*p^2*x^2/e^4-1/10*d^5*g^3*p^2*ln(e*x^2+d)^2/
e^5-12/49*f*g^2*p*x^7*ln(c*(e*x^2+d)^p)-1/25*g^3*p*(e*x^2+d)^5*ln(c*(e*x^2+
d)^p)/e^5+3/4*f^2*g*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)^2/e^2-8*f^3*p^2*arctan(x*
e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)+4*I*f^3*p^2*arctan(x*e^(1/2)/d^(1/2))^2*d^
(1/2)/e^(1/2)+4*I*f^3*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1
/2)/e^(1/2)+8*f^3*p^2*x+1/10*g^3*x^10*ln(c*(e*x^2+d)^p)^2-1408/245*d^3*f*g^
2*p^2*x/e^3-3*d*f^2*g*p^2*x^2/e+568/735*d^2*f*g^2*p^2*x^3/e^2-288/1225*d*f*
```

```
g^2*p^2*x^5/e+1408/245*d^(7/2)*f*g^2*p^2*arctan(x*e^(1/2)/d^(1/2))/e^(7/2)-
3/4*f^2*g*p*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)/e^2-2/3*d^2*g^3*p*(e*x^2+d)^3*ln(
c*(e*x^2+d)^p)/e^5+1/4*d*g^3*p*(e*x^2+d)^4*ln(c*(e*x^2+d)^p)/e^5+1/5*d^5*g^
3*p*ln(e*x^2+d)*ln(c*(e*x^2+d)^p)/e^5-3/2*d*f^2*g*(e*x^2+d)*ln(c*(e*x^2+d)^
p)^2/e^2+4*f^3*p*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)*d^(1/2)/e^(1/2
)+24/343*f*g^2*p^2*x^7+1/125*g^3*p^2*(e*x^2+d)^5/e^5-4*f^3*p*x*ln(c*(e*x^2+
d)^p)+3/7*f*g^2*x^7*ln(c*(e*x^2+d)^p)^2-4/7*d^2*f*g^2*p*x^3*ln(c*(e*x^2+d)^
p)/e^2+12/35*d*f*g^2*p*x^5*ln(c*(e*x^2+d)^p)/e+3*d*f^2*g*p*(e*x^2+d)*ln(c*(
e*x^2+d)^p)/e^2-12/7*d^(7/2)*f*g^2*p*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+
d)^p)/e^(7/2)-24/7*d^(7/2)*f*g^2*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)
/(d^(1/2)+I*x*e^(1/2)))/e^(7/2)+12/7*d^3*f*g^2*p*x*ln(c*(e*x^2+d)^p)/e^3-12
/7*I*d^(7/2)*f*g^2*p^2*arctan(x*e^(1/2)/d^(1/2))^2/e^(7/2)-12/7*I*d^(7/2)*f
*g^2*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(7/2)+3/8*f^2*g*p^2
*(e*x^2+d)^2/e^2-1/2*d^3*g^3*p^2*(e*x^2+d)^2/e^5+2/9*d^2*g^3*p^2*(e*x^2+d)^
3/e^5-1/16*d*g^3*p^2*(e*x^2+d)^4/e^5+f^3*x*ln(c*(e*x^2+d)^p)^2
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 1221, normalized size of antiderivative = 1.00, number of steps used = 55, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 1.208$, Rules used = {2521, 2500, 2526, 2498, 327, 211, 2520, 12, 5040, 4964, 2449, 2352, 2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341, 2507, 2505, 308, 2445, 2458, 45, 2372, 14, 2338}

$$\int \left(f+gx^3\right)^3 \log^2\left(c\left(d+ex^2\right)^p\right)\,dx = \frac{1}{10}g^3\log^2\left(c\left(ex^2+d\right)^p\right)x^{10}+\frac{24}{343}fg^2p^2x^7 + \frac{3}{7}fg^2\log^2\left(c\left(ex^2+d\right)^p\right)x^7 - \frac{12}{49}fg^2p\log\left(c\left(ex^2+d\right)^p\right)x^7 - \frac{288dfg^2p^2x^5}{1225e} + \frac{12dfg^2p\log\left(c\left(ex^2+d\right)^p\right)x^5}{35e} + \frac{568d^2fg^2p^2x^3}{735e^2} - \frac{4d^2fg^2p\log\left(c\left(ex^2+d\right)^p\right)x^3}{7e^2} + \frac{d^4g^3p^2x^2}{e^4} - \frac{3df^2gp^2x^2}{e} + 8f^3p^2x - \frac{1408d^3fg^2p^2x}{245e^3} + f^3\log^2\left(c\left(ex^2+d\right)^p\right)x - 4f^3p\log\left(c\left(ex^2+d\right)^p\right)x + \frac{12d^3fg^2p\log\left(c\left(ex^2+d\right)^p\right)x}{7e^3} + \frac{g^3p^2\left(ex^2+d\right)^5}{125e^5} - \frac{dg^3p^2\left(ex^2+d\right)^4}{16e^5} + \frac{2d^2g^3p^2\left(ex^2+d\right)^3}{9e^5} - \frac{d^3g^3p^2\left(ex^2+d\right)^2}{2e^5} + \frac{3f^2gp^2\left(ex^2+d\right)^2}{8e^2} + \frac{4i\sqrt{d}f^3p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} - \frac{12id^{7/2}fg^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{7e^{7/2}} - \frac{d^5g^3p^2\log^2\left(ex^2+d\right)}{10e^5} + \frac{3f^2g\left(ex^2+d\right)^2\log^2\left(c\left(ex^2+d\right)^p\right)}{4e^2} - \frac{3df^2g\left(ex^2+d\right)\log^2\left(c\left(ex^2+d\right)^p\right)}{2e^2} - \frac{8\sqrt{d}f^3p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{1408d^{7/2}fg^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{245e^{7/2}} + \frac{8\sqrt{d}f^3p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{\sqrt{e}} - \frac{24d^{7/2}fg^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{7e^{7/2}} - \frac{g^3p\left(ex^2+d\right)^5\log\left(c\left(ex^2+d\right)^p\right)}{25e^5} + \frac{dg^3p\left(ex^2+d\right)^4\log\left(c\left(ex^2+d\right)^p\right)}{4e^5} - \frac{2d^2g^3p\left(ex^2+d\right)^3\log\left(c\left(ex^2+d\right)^p\right)}{3e^5} + \frac{d^3g^3p\left(ex^2+d\right)^2\log\left(c\left(ex^2+d\right)^p\right)}{}$$

```
[In] Int[(f + g*x^3)^3*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] 8*f^3*p^2*x - (1408*d^3*f*g^2*p^2*x)/(245*e^3) - (3*d*f^2*g*p^2*x^2)/e + (d
^4*g^3*p^2*x^2)/e^4 + (568*d^2*f*g^2*p^2*x^3)/(735*e^2) - (288*d*f*g^2*p^2*
x^5)/(1225*e) + (24*f*g^2*p^2*x^7)/343 + (3*f^2*g*p^2*(d + e*x^2)^2)/(8*e^2
) - (d^3*g^3*p^2*(d + e*x^2)^2)/(2*e^5) + (2*d^2*g^3*p^2*(d + e*x^2)^3)/(9*
e^5) - (d*g^3*p^2*(d + e*x^2)^4)/(16*e^5) + (g^3*p^2*(d + e*x^2)^5)/(125*e^
5) - (8*Sqrt[d]*f^3*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + (1408*d^(7/2
)*f*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(245*e^(7/2)) + ((4*I)*Sqrt[d]*f^3
*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sqrt[e] - (((12*I)/7)*d^(7/2)*f*g^2*p^2
*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(7/2) + (8*Sqrt[d]*f^3*p^2*ArcTan[(Sqrt[e
]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] - (24*d^(7/
2)*f*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[
e]*x)])/(7*e^(7/2)) - (d^5*g^3*p^2*Log[d + e*x^2]^2)/(10*e^5) - 4*f^3*p*x*L
og[c*(d + e*x^2)^p] + (12*d^3*f*g^2*p*x*Log[c*(d + e*x^2)^p])/(7*e^3) - (4*
d^2*f*g^2*p*x^3*Log[c*(d + e*x^2)^p])/(7*e^2) + (12*d*f*g^2*p*x^5*Log[c*(d
+ e*x^2)^p])/(35*e) - (12*f*g^2*p*x^7*Log[c*(d + e*x^2)^p])/49 + (3*d*f^2*g
*p*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e^2 - (d^4*g^3*p*(d + e*x^2)*Log[c*(d
+ e*x^2)^p])/e^5 - (3*f^2*g*p*(d + e*x^2)^2*Log[c*(d + e*x^2)^p])/(4*e^2) +
 (d^3*g^3*p*(d + e*x^2)^2*Log[c*(d + e*x^2)^p])/e^5 - (2*d^2*g^3*p*(d + e*x
^2)^3*Log[c*(d + e*x^2)^p])/(3*e^5) + (d*g^3*p*(d + e*x^2)^4*Log[c*(d + e*x
^2)^p])/(4*e^5) - (g^3*p*(d + e*x^2)^5*Log[c*(d + e*x^2)^p])/(25*e^5) + (4*
Sqrt[d]*f^3*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt[e] - (
12*d^(7/2)*f*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(7*e^(
7/2)) + (d^5*g^3*p*Log[d + e*x^2]*Log[c*(d + e*x^2)^p])/(5*e^5) + f^3*x*Log
[c*(d + e*x^2)^p]^2 + (3*f*g^2*x^7*Log[c*(d + e*x^2)^p]^2)/7 + (g^3*x^10*Lo
g[c*(d + e*x^2)^p]^2)/10 - (3*d*f^2*g*(d + e*x^2)*Log[c*(d + e*x^2)^p]^2)/(
2*e^2) + (3*f^2*g*(d + e*x^2)^2*Log[c*(d + e*x^2)^p]^2)/(4*e^2) + ((4*I)*Sq
rt[d]*f^3*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e]
- (((12*I)/7)*d^(7/2)*f*g^2*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqr
t[e]*x)])/e^(7/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2500

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[x^n*(
(a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
```

```
b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(f^3 \log^2\left(c\left(d+ex^2\right)^p\right) + 3f^2gx^3\log^2\left(c\left(d+ex^2\right)^p\right) + 3fg^2x^6\log^2\left(c\left(d+ex^2\right)^p\right) + g^3x^9\log^2\left(c\left(d+ex^2\right)^p\right)\right) dx$$

$$= f^3 \int \log^2\left(c\left(d+ex^2\right)^p\right) dx + \left(3f^2g\right)\int x^3\log^2\left(c\left(d+ex^2\right)^p\right) dx + \left(3fg^2\right)\int x^6\log^2\left(c\left(d+ex^2\right)^p\right) dx + g^3\int x^9\log^2\left(c\left(d+ex^2\right)^p\right) dx$$

$$= f^3x\log^2\left(c\left(d+ex^2\right)^p\right) + \frac{3}{7}fg^2x^7\log^2\left(c\left(d+ex^2\right)^p\right) + \frac{1}{2}\left(3f^2g\right)\text{Subst}\left(\int x\log^2\left(c(d+ex)^p\right) dx, x, x^2\right) + \frac{1}{2}g^3\text{Subst}\left(\int x^4\log^2\left(c(d+ex)^p\right) dx, x, x^2\right) - \left(4ef^3p\right)\int \frac{x^2\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2} dx - \frac{1}{7}\left(12efg^2p\right)\int \frac{x^8\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2} dx$$

$$= f^3x\log^2\left(c\left(d+ex^2\right)^p\right) + \frac{3}{7}fg^2x^7\log^2\left(c\left(d+ex^2\right)^p\right) + \frac{1}{10}g^3x^{10}\log^2\left(c\left(d+ex^2\right)^p\right) + \frac{1}{2}\left(3f^2g\right)\text{Subst}\left(\int \left(-\frac{d\log^2\left(c(d+ex)^p\right)}{e} + \frac{(d+ex)\log^2\left(c(d+ex)^p\right)}{e}\right) dx, x, x^2\right) - \left(4ef^3p\right)\int \left(\frac{\log\left(c\left(d+ex^2\right)^p\right)}{e} - \frac{d\log\left(c\left(d+ex^2\right)^p\right)}{e\left(d+ex^2\right)}\right) dx - \frac{1}{7}\left(12efg^2p\right)\int \left(-\frac{d^3\log\left(c\left(d+ex^2\right)^p\right)}{e^4} + \frac{d^2x^2\log\left(c\left(d+ex^2\right)^p\right)}{e^3} - \frac{dx^4\log\left(c\left(d+ex^2\right)^p\right)}{e^2} + \frac{x^6\log\left(c\left(d+ex^2\right)^p\right)}{e} + \frac{d^4\log\left(c\left(d+ex^2\right)^p\right)}{e^4\left(d+ex^2\right)}\right) dx - \frac{1}{5}\left(eg^3p\right)\text{Subst}\left(\int \frac{x^5\log\left(c(d+ex)^p\right)}{d+ex} dx, x, x^2\right)$$

$$= f^3 x \log^2\left(c\left(d+ex^2\right)^p\right) + \frac{3}{7} f g^2 x^7 \log^2\left(c\left(d+ex^2\right)^p\right) + \frac{1}{10} g^3 x^{10} \log^2\left(c\left(d+ex^2\right)^p\right)$$
$$+ \frac{(3f^2g)\,\text{Subst}\left(\int (d+ex)\log^2\left(c(d+ex)^p\right)\,dx, x, x^2\right)}{2e}$$
$$- \frac{(3df^2g)\,\text{Subst}\left(\int \log^2\left(c(d+ex)^p\right)\,dx, x, x^2\right)}{2e} - (4f^3p)\int \log\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$+ (4df^3p)\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx - \frac{1}{7}(12fg^2p)\int x^6\log\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$+ \frac{(12d^3fg^2p)\int \log\left(c\left(d+ex^2\right)^p\right)\,dx}{7e^3} - \frac{(12d^4fg^2p)\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx}{7e^3}$$
$$- \frac{(12d^2fg^2p)\int x^2\log\left(c\left(d+ex^2\right)^p\right)\,dx}{7e^2} + \frac{(12dfg^2p)\int x^4\log\left(c\left(d+ex^2\right)^p\right)\,dx}{7e}$$
$$- \frac{1}{5}(g^3p)\,\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^5\log\left(cx^p\right)}{x}\,dx, x, d+ex^2\right)$$

$$
\begin{aligned}
&= -4f^3px\log\left(c\left(d+ex^2\right)^p\right)+\frac{12d^3fg^2px\log\left(c\left(d+ex^2\right)^p\right)}{7e^3}\\
&-\frac{4d^2fg^2px^3\log\left(c\left(d+ex^2\right)^p\right)}{7e^2}+\frac{12dfg^2px^5\log\left(c\left(d+ex^2\right)^p\right)}{35e}\\
&-\frac{12}{49}fg^2px^7\log\left(c\left(d+ex^2\right)^p\right)-\frac{d^4g^3p(d+ex^2)\log\left(c\left(d+ex^2\right)^p\right)}{e^5}\\
&+\frac{d^3g^3p(d+ex^2)^2\log\left(c\left(d+ex^2\right)^p\right)}{e^5}-\frac{2d^2g^3p(d+ex^2)^3\log\left(c\left(d+ex^2\right)^p\right)}{3e^5}\\
&+\frac{dg^3p(d+ex^2)^4\log\left(c\left(d+ex^2\right)^p\right)}{4e^5}-\frac{g^3p(d+ex^2)^5\log\left(c\left(d+ex^2\right)^p\right)}{25e^5}\\
&+\frac{4\sqrt{d}f^3p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{e}}\\
&-\frac{12d^{7/2}fg^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{7e^{7/2}}+\frac{d^5g^3p\log\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{5e^5}\\
&+f^3x\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{3}{7}fg^2x^7\log^2\left(c\left(d+ex^2\right)^p\right)\\
&+\frac{1}{10}g^3x^{10}\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{(3f^2g)\,\mathrm{Subst}\left(\int x\log^2\left(cx^p\right)\,dx,x,d+ex^2\right)}{2e^2}\\
&-\frac{(3df^2g)\,\mathrm{Subst}\left(\int \log^2\left(cx^p\right)\,dx,x,d+ex^2\right)}{2e^2}+\left(8ef^3p^2\right)\int\frac{x^2}{d+ex^2}\,dx\\
&-\left(8def^3p^2\right)\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}\left(d+ex^2\right)}\,dx-\frac{1}{35}\left(24dfg^2p^2\right)\int\frac{x^6}{d+ex^2}\,dx\\
&-\frac{(24d^3fg^2p^2)\int\frac{x^2}{d+ex^2}\,dx}{7e^2}+\frac{(24d^4fg^2p^2)\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}(d+ex^2)}\,dx}{7e^2}\\
&+\frac{(8d^2fg^2p^2)\int\frac{x^4}{d+ex^2}\,dx}{7e}+\frac{1}{49}\left(24efg^2p^2\right)\int\frac{x^8}{d+ex^2}\,dx\\
&+\frac{1}{5}\left(g^3p^2\right)\mathrm{Subst}\left(\int\frac{300d^4x-300d^3x^2+200d^2x^3-75dx^4+12x^5-60d^5\log(x)}{60e^5x}\,dx,x,d+ex^2\right)
\end{aligned}
$$

$$
\begin{aligned}
&= 8f^3p^2x - \frac{24d^3fg^2p^2x}{7e^3} - 4f^3px\log\left(c\left(d+ex^2\right)^p\right) \\
&+ \frac{12d^3fg^2px\log\left(c\left(d+ex^2\right)^p\right)}{7e^3} - \frac{4d^2fg^2px^3\log\left(c\left(d+ex^2\right)^p\right)}{7e^2} \\
&+ \frac{12dfg^2px^5\log\left(c\left(d+ex^2\right)^p\right)}{35e} - \frac{12}{49}fg^2px^7\log\left(c\left(d+ex^2\right)^p\right) \\
&- \frac{d^4g^3p(d+ex^2)\log\left(c\left(d+ex^2\right)^p\right)}{e^5} + \frac{d^3g^3p(d+ex^2)^2\log\left(c\left(d+ex^2\right)^p\right)}{e^5} \\
&- \frac{2d^2g^3p(d+ex^2)^3\log\left(c\left(d+ex^2\right)^p\right)}{3e^5} + \frac{dg^3p(d+ex^2)^4\log\left(c\left(d+ex^2\right)^p\right)}{4e^5} \\
&- \frac{g^3p(d+ex^2)^5\log\left(c\left(d+ex^2\right)^p\right)}{25e^5} + \frac{4\sqrt{d}f^3p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{e}} \\
&- \frac{12d^{7/2}fg^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{7e^{7/2}} + \frac{d^5g^3p\log\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{5e^5} \\
&+ f^3x\log^2\left(c\left(d+ex^2\right)^p\right) + \frac{3}{7}fg^2x^7\log^2\left(c\left(d+ex^2\right)^p\right) \\
&+ \frac{1}{10}g^3x^{10}\log^2\left(c\left(d+ex^2\right)^p\right) - \frac{3df^2g(d+ex^2)\log^2\left(c\left(d+ex^2\right)^p\right)}{2e^2} \\
&+ \frac{3f^2g(d+ex^2)^2\log^2\left(c\left(d+ex^2\right)^p\right)}{4e^2} - \frac{(3f^2gp)\,\mathrm{Subst}\left(\int x\log\left(cx^p\right)\,dx, x, d+ex^2\right)}{2e^2} \\
&+ \frac{(3df^2gp)\,\mathrm{Subst}\left(\int \log\left(cx^p\right)\,dx, x, d+ex^2\right)}{e^2} \\
&- \left(8df^3p^2\right)\int \frac{1}{d+ex^2}\,dx - \left(8\sqrt{d}\sqrt{e}f^3p^2\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d+ex^2}\,dx \\
&- \frac{1}{35}\left(24dfg^2p^2\right)\int\left(\frac{d^2}{e^3} - \frac{dx^2}{e^2} + \frac{x^4}{e} - \frac{d^3}{e^3\left(d+ex^2\right)}\right)dx + \frac{\left(24d^4fg^2p^2\right)\int\frac{1}{d+ex^2}\,dx}{7e^3} \\
&+ \frac{\left(24d^{7/2}fg^2p^2\right)\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d+ex^2}\,dx}{7e^{5/2}} + \frac{\left(8d^2fg^2p^2\right)\int\left(-\frac{d}{e^2} + \frac{x^2}{e} + \frac{d^2}{e^2(d+ex^2)}\right)dx}{7e} \\
&+ \frac{1}{49}\left(24efg^2p^2\right)\int\left(-\frac{d^3}{e^4} + \frac{d^2x^2}{e^3} - \frac{dx^4}{e^2} + \frac{x^6}{e} + \frac{d^4}{e^4\left(d+ex^2\right)}\right)dx + \frac{\left(g^3p^2\right)\mathrm{Subst}\left(\int \frac{300d^4x-300d^3x^2+200d \cdots}{\cdots}\right.}{\cdots}
\end{aligned}
$$

$$= 8f^3p^2x - \frac{1408d^3fg^2p^2x}{245e^3} - \frac{3df^2gp^2x^2}{e} + \frac{568d^2fg^2p^2x^3}{735e^2} - \frac{288dfg^2p^2x^5}{1225e} + \frac{24}{343}fg^2p^2x^7 + \frac{3f^2gp^2(d+ex^2)^2}{8e^2} - \frac{8\sqrt{d}f^3p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{24d^{7/2}fg^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}} + \frac{4i\sqrt{d}f^3p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} - \frac{12id^{7/2}fg^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{7e^{7/2}}$$

$$-4f^3px\log\left(c(d+ex^2)^p\right) + \frac{12d^3fg^2px\log\left(c(d+ex^2)^p\right)}{7e^3} - \frac{4d^2fg^2px^3\log\left(c(d+ex^2)^p\right)}{7e^2} + \frac{12dfg^2px}{}$$

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Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 780, normalized size of antiderivative = 0.64

$$\int (f+gx^3)^3\log^2\left(c(d+ex^2)^p\right)\,dx = f^3x\log^2\left(c(d+ex^2)^p\right) + \frac{3}{4}f^2gx^4\log^2\left(c(d+ex^2)^p\right) + \frac{3}{7}fg^2x^7\log^2\left(c(d+ex^2)^p\right) + \frac{1}{10}g^3x^{10}\log^2\left(c(d+ex^2)^p\right)$$
$$+ \frac{3f^2g\left(ep^2x^2(-6d+ex^2) + 2d^2p^2\log(d+ex^2) + 2p(2d^2+2dex^2-e^2x^4)\log\left(c(d+ex^2)^p\right) - 2d^2\log^2\left(c\right.\right.}{8e^2}$$
$$+ \frac{g^3\left(ep^2x^2(8220d^4 - 2310d^3ex^2 + 940d^2e^2x^4 - 405de^3x^6 + 144e^4x^8) - 4620d^5p^2\log(d+ex^2) - 60p(60d\right.}{180}$$
$$+ \frac{4f^3p\left(i\sqrt{d}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2 + \sqrt{e}x(2p - \log\left(c(d+ex^2)^p\right)) + \sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-2p + 2p\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right.\right.\right.}{\sqrt{e}}$$
$$+ \frac{4fg^2p\left(-11025id^{7/2}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2 - 105d^{7/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-352p + 210p\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right) + 105\log\left(\right.\right.\right.}{}$$

```
[In] Integrate[(f + g*x^3)^3*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] f^3*x*Log[c*(d + e*x^2)^p]^2 + (3*f^2*g*x^4*Log[c*(d + e*x^2)^p]^2)/4 + (3*
f*g^2*x^7*Log[c*(d + e*x^2)^p]^2)/7 + (g^3*x^10*Log[c*(d + e*x^2)^p]^2)/10
+ (3*f^2*g*(e*p^2*x^2*(-6*d + e*x^2) + 2*d^2*p^2*Log[d + e*x^2] + 2*p*(2*d^
2 + 2*d*e*x^2 - e^2*x^4)*Log[c*(d + e*x^2)^p] - 2*d^2*Log[c*(d + e*x^2)^p]^
2))/(8*e^2) + (g^3*(e*p^2*x^2*(8220*d^4 - 2310*d^3*e*x^2 + 940*d^2*e^2*x^4
- 405*d*e^3*x^6 + 144*e^4*x^8) - 4620*d^5*p^2*Log[d + e*x^2] - 60*p*(60*d^5
 + 60*d^4*e*x^2 - 30*d^3*e^2*x^4 + 20*d^2*e^3*x^6 - 15*d*e^4*x^8 + 12*e^5*x
^10)*Log[c*(d + e*x^2)^p] + 1800*d^5*Log[c*(d + e*x^2)^p]^2))/(18000*e^5) +
 (4*f^3*p*(I*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 + Sqrt[e]*x*(2*p - Log
[c*(d + e*x^2)^p]) + Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-2*p + 2*p*Log[(2
```

```
*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + Log[c*(d + e*x^2)^p]) + I*Sqrt[d]*p*Po
lyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)]))/Sqrt[e] + (4
*f*g^2*p*((-11025*I)*d^(7/2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2 - 105*d^(7/2)*
ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-352*p + 210*p*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x)] + 105*Log[c*(d + e*x^2)^p]) + Sqrt[e]*x*(2*p*(-18480*d^3 + 2485*d
^2*e*x^2 - 756*d*e^2*x^4 + 225*e^3*x^6) + 105*(105*d^3 - 35*d^2*e*x^2 + 21*
d*e^2*x^4 - 15*e^3*x^6)*Log[c*(d + e*x^2)^p]) - (11025*I)*d^(7/2)*p*PolyLog
[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)]))/(25725*e^(7/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 10.28 (sec) , antiderivative size = 1584, normalized size of antiderivative = 1.30

method	result	size
risch	Expression too large to display	1584

```
[In] int((g*x^3+f)^3*ln(c*(e*x^2+d)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 47/900*p^2/e^2*d^2*g^3*x^6-77/600*p^2/e^3*x^4*d^3*g^3+3/8*p^2*x^4*f^2*g-137
/300*p^2/e^5*d^5*ln(e*x^2+d)*g^3-12/49*p*f*g^2*x^7*ln((e*x^2+d)^p)-3/4*p*f^
2*g*x^4*ln((e*x^2+d)^p)-8*p^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*f^3-12/
7*p/e^3*d^4/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*f*g^2*ln((e*x^2+d)^p)+12/7*
p^2/e^3*d^4/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*f*g^2*ln(e*x^2+d)-9/400*p^2
/e*d*g^3*x^8+1/10*ln((e*x^2+d)^p)^2*g^3*x^10+ln((e*x^2+d)^p)^2*x*f^3+3/7*ln
((e*x^2+d)^p)^2*g^2*f*x^7+3/4*ln((e*x^2+d)^p)^2*f^2*g*x^4+1/125*p^2*g^3*x^1
0-1/25*p*g^3*x^10*ln((e*x^2+d)^p)-4*p*x*f^3*ln((e*x^2+d)^p)+12/35*p/e*d*f*g
^2*x^5*ln((e*x^2+d)^p)-4/7*p/e^2*d^2*f*g^2*x^3*ln((e*x^2+d)^p)+3/2*p/e*d*f^
2*g*x^2*ln((e*x^2+d)^p)+12/7*p/e^3*x*d^3*f*g^2*ln((e*x^2+d)^p)-3/2*p/e^2*d^
2*ln(e*x^2+d)*f^2*g*ln((e*x^2+d)^p)+1408/245*p^2/e^3*f*g^2*d^4/(d*e)^(1/2)*
arctan(x*e/(d*e)^(1/2))-1/35*p^2*e*Sum(-1/2*(ln(x-_alpha)*ln(e*x^2+d)-2*e*(
1/4/_alpha/e*ln(x-_alpha)^2+1/2*_alpha/d*ln(x-_alpha)*ln(1/2*(x+_alpha)/_al
pha)+1/2*_alpha/d*dilog(1/2*(x+_alpha)/_alpha)))*d*(14*_alpha*d^4*g^3-105*_
alpha*d*e^3*f^2*g-60*d^3*e*f*g^2+140*e^4*f^3)/e^6/_alpha,_alpha=RootOf(_Z^2
*e+d))-4*p^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*f^3*ln(e*x^2+d)+4*p*d/(d
*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*f^3*ln((e*x^2+d)^p)+1/20*p/e*d*g^3*x^8*ln
((e*x^2+d)^p)-1/15*p/e^2*d^2*g^3*x^6*ln((e*x^2+d)^p)+1/10*p/e^3*d^3*g^3*x^4
*ln((e*x^2+d)^p)-1/5*p/e^4*d^4*g^3*x^2*ln((e*x^2+d)^p)+1/4*(I*Pi*csgn(I*(e*
x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+
d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^2+d)^p)^3+I*Pi*csgn(I*c*(e*x^2+d)^p)^2*c
sgn(I*c)+2*ln(c))^2*(1/10*g^3*x^10+3/7*f*g^2*x^7+3/4*f^2*g*x^4+f^3*x)+1/5*p
/e^5*d^5*ln(e*x^2+d)*g^3*ln((e*x^2+d)^p)+3/2*p^2/e^2*d^2*ln(e*x^2+d)^2*f^2*
g+9/4*p^2/e^2*d^2*ln(e*x^2+d)*f^2*g+8*f^3*p^2*x+(I*Pi*csgn(I*(e*x^2+d)^p)*c
sgn(I*c*(e*x^2+d)^p)^2-I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(
I*c)-I*Pi*csgn(I*c*(e*x^2+d)^p)^3+I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+2*
```

```
ln(c))*(1/10*ln((e*x^2+d)^p)*g^3*x^10+3/7*ln((e*x^2+d)^p)*g^2*f*x^7+3/4*ln(
(e*x^2+d)^p)*f^2*g*x^4+ln((e*x^2+d)^p)*x*f^3-1/70*p*e*(1/e^5*(7/5*e^4*g^3*x
^10-7/4*d*e^3*g^3*x^8+60/7*e^4*f*g^2*x^7+7/3*d^2*e^2*g^3*x^6-12*d*e^3*f*g^2
*x^5-7/2*d^3*e*g^3*x^4+105/4*e^4*f^2*g*x^4+20*d^2*e^2*f*g^2*x^3+7*d^4*g^3*x
^2-105/2*d*f^2*g*x^2*e^3-60*x*d^3*f*g^2*e+140*x*e^4*f^3)-d/e^5*(1/2*(14*d^4
*g^3-105*d*e^3*f^2*g)/e*ln(e*x^2+d)+(-60*d^3*e*f*g^2+140*e^4*f^3)/(d*e)^(1/
2)*arctan(x*e/(d*e)^(1/2)))))-1408/245*d^3*f*g^2*p^2*x/e^3-9/4*d*f^2*g*p^2*
x^2/e+568/735*d^2*f*g^2*p^2*x^3/e^2-288/1225*d*f*g^2*p^2*x^5/e-1/5*d^5*g^3*
p^2*ln(e*x^2+d)^2/e^5+24/343*f*g^2*p^2*x^7+137/300*d^4*g^3*p^2*x^2/e^4
```

Fricas [F]

$$\int \left(f+gx^3\right)^3\log^2\left(c\left(d+ex^2\right)^p\right)\,dx=\int \left(gx^3+f\right)^3\log\left(\left(ex^2+d\right)^pc\right)^2\,dx$$

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g^3*x^9 + 3*f*g^2*x^6 + 3*f^2*g*x^3 + f^3)*log((e*x^2 + d)^p*c)^2
, x)
```

Sympy [F]

$$\int \left(f+gx^3\right)^3\log^2\left(c\left(d+ex^2\right)^p\right)\,dx=\int \left(f+gx^3\right)^3\log\left(c\left(d+ex^2\right)^p\right)^2\,dx$$

```
[In] integrate((g*x**3+f)**3*ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f + g*x**3)**3*log(c*(d + e*x**2)**p)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(f+gx^3\right)^3\log^2\left(c\left(d+ex^2\right)^p\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \left(f+gx^3\right)^3 \log^2\left(c\left(d+ex^2\right)^p\right)\,dx = \int \left(gx^3+f\right)^3 \log\left(\left(ex^2+d\right)^p c\right)^2\,dx$$

```
[In] integrate((g*x^3+f)^3*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
[Out] integrate((g*x^3 + f)^3*log((e*x^2 + d)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(f+gx^3\right)^3 \log^2\left(c\left(d+ex^2\right)^p\right)\,dx = \int \ln\left(c\left(e\,x^2+d\right)^p\right)^2 \left(g\,x^3+f\right)^3\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^3,x)
[Out] int(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^3, x)
```

3.294 $\int (f+gx^3)^2 \log^2\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 835

$$\int (f+gx^3)^2 \log^2\left(c(d+ex^2)^p\right)\,dx = 8f^2p^2x - \frac{1408d^3g^2p^2x}{735e^3} - \frac{2dfgp^2x^2}{e} + \frac{568d^2g^2p^2x^3}{2205e^2} - \frac{96dg^2p^2x^5}{1225e} + \frac{8}{343}g^2p^2x^7 + \frac{fgp^2(d+ex^2)^2}{4e^2} - \frac{8\sqrt{d}f^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{1408d^{7/2}g^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{735e^{7/2}} + \frac{4i\sqrt{d}f^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} - \frac{4id^{7/2}g^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{7e^{7/2}} + \frac{8\sqrt{d}f^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} - \frac{8d^{7/2}g^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{7e^{7/2}} - 4f^2px\log\left(c(d+ex^2)^p\right) + \frac{4d^3g^2px\log\left(c(d+ex^2)^p\right)}{7e^3} - \frac{4d^2g^2px^3\log\left(c(d+ex^2)^p\right)}{21e^2} + \frac{4dg^2px^5\log\left(c(d+e\right.}{35e}$$

```
[Out] -2*d*f*g*p^2*x^2/e+4/7*d^3*g^2*p*x*ln(c*(e*x^2+d)^p)/e^3-4/21*d^2*g^2*p*x^3
*ln(c*(e*x^2+d)^p)/e^2+4/35*d*g^2*p*x^5*ln(c*(e*x^2+d)^p)/e-1/2*f*g*p*(e*x^
2+d)^2*ln(c*(e*x^2+d)^p)/e^2-4/7*d^(7/2)*g^2*p*arctan(x*e^(1/2)/d^(1/2))*ln
(c*(e*x^2+d)^p)/e^(7/2)-8/7*d^(7/2)*g^2*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(2*
d^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(7/2)-4/7*I*d^(7/2)*g^2*p^2*arctan(x*e^(1/
2)/d^(1/2))^2/e^(7/2)-4/7*I*d^(7/2)*g^2*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+
I*x*e^(1/2)))/e^(7/2)-d*f*g*(e*x^2+d)*ln(c*(e*x^2+d)^p)^2/e^2-8*f^2*p^2*arc
tan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)+8*f^2*p^2*x+8/343*g^2*p^2*x^7+1/7*g^
2*x^7*ln(c*(e*x^2+d)^p)^2-4*f^2*p*x*ln(c*(e*x^2+d)^p)-4/49*g^2*p*x^7*ln(c*(
e*x^2+d)^p)+4*f^2*p*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)*d^(1/2)/e^(
1/2)+8*f^2*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2))
)*d^(1/2)/e^(1/2)+f^2*x*ln(c*(e*x^2+d)^p)^2+4*I*f^2*p^2*arctan(x*e^(1/2)/d^
(1/2))^2*d^(1/2)/e^(1/2)+4*I*f^2*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(
```

```
1/2)))*d^(1/2)/e^(1/2)+2*d*f*g*p*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e^2+1408/735*d
^(7/2)*g^2*p^2*arctan(x*e^(1/2)/d^(1/2))/e^(7/2)+1/2*f*g*(e*x^2+d)^2*ln(c*(
e*x^2+d)^p)^2/e^2-1408/735*d^3*g^2*p^2*x/e^3+568/2205*d^2*g^2*p^2*x^3/e^2-9
6/1225*d*g^2*p^2*x^5/e+1/4*f*g*p^2*(e*x^2+d)^2/e^2
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 835, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.958$, Rules used = {2521, 2500, 2526, 2498, 327, 211, 2520, 12, 5040, 4964, 2449, 2352, 2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341, 2507, 2505, 308}

$$\int \left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)\,dx = \frac{8}{343}g^2p^2x^7+\frac{1}{7}g^2\log^2\left(c\left(ex^2+d\right)^p\right)x^7 - \frac{4}{49}g^2p\log\left(c\left(ex^2+d\right)^p\right)x^7-\frac{96dg^2p^2x^5}{1225e} + \frac{4dg^2p\log\left(c\left(ex^2+d\right)^p\right)x^5}{35e}+\frac{568d^2g^2p^2x^3}{2205e^2} - \frac{4d^2g^2p\log\left(c\left(ex^2+d\right)^p\right)x^3}{21e^2} - \frac{2dfgp^2x^2}{e}+8f^2p^2x-\frac{1408d^3g^2p^2x}{735e^3} + f^2\log^2\left(c\left(ex^2+d\right)^p\right)x-4f^2p\log\left(c\left(ex^2+d\right)^p\right)x + \frac{4d^3g^2p\log\left(c\left(ex^2+d\right)^p\right)x}{7e^3} + \frac{fgp^2\left(ex^2+d\right)^2}{4e^2}+\frac{4i\sqrt{d}f^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} - \frac{4id^{7/2}g^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{7e^{7/2}} + \frac{fg\left(ex^2+d\right)^2\log^2\left(c\left(ex^2+d\right)^p\right)}{2e^2} - \frac{dfg\left(ex^2+d\right)\log^2\left(c\left(ex^2+d\right)^p\right)}{e^2} - \frac{8\sqrt{d}f^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{1408d^{7/2}g^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{735e^{7/2}} + \frac{8\sqrt{d}f^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{\sqrt{e}} - \frac{8d^{7/2}g^2p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{7e^{7/2}} - \frac{fgp\left(ex^2+d\right)^2\log\left(c\left(ex^2+d\right)^p\right)}{2e^2} + \frac{2dfgp\left(ex^2+d\right)\log\left(c\left(ex^2+d\right)^p\right)}{e^2} + \frac{4\sqrt{d}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(ex^2+d\right)^p\right)}{\sqrt{e}} - \frac{4d^{7/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(ex^2+d\right)^p\right)}{7e^{7/2}} + \frac{4i\sqrt{d}f^2p^2\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{\sqrt{e}}$$

$$4id^{7/2}g^2p^2\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)$$

```
[In] Int[(f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] 8*f^2*p^2*x - (1408*d^3*g^2*p^2*x)/(735*e^3) - (2*d*f*g*p^2*x^2)/e + (568*d
^2*g^2*p^2*x^3)/(2205*e^2) - (96*d*g^2*p^2*x^5)/(1225*e) + (8*g^2*p^2*x^7)/
343 + (f*g*p^2*(d + e*x^2)^2)/(4*e^2) - (8*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*
x)/Sqrt[d]])/Sqrt[e] + (1408*d^(7/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(
735*e^(7/2)) + ((4*I)*Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sqrt[e
] - (((4*I)/7)*d^(7/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e^(7/2) + (8*
Sqrt[d]*f^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x)])/Sqrt[e] - (8*d^(7/2)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*
Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(7*e^(7/2)) - 4*f^2*p*x*Log[c*(d + e*x^2
)^p] + (4*d^3*g^2*p*x*Log[c*(d + e*x^2)^p])/(7*e^3) - (4*d^2*g^2*p*x^3*Log[
c*(d + e*x^2)^p])/(21*e^2) + (4*d*g^2*p*x^5*Log[c*(d + e*x^2)^p])/(35*e) -
(4*g^2*p*x^7*Log[c*(d + e*x^2)^p])/49 + (2*d*f*g*p*(d + e*x^2)*Log[c*(d + e
*x^2)^p])/e^2 - (f*g*p*(d + e*x^2)^2*Log[c*(d + e*x^2)^p])/(2*e^2) + (4*Sqr
t[d]*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt[e] - (4*d
^(7/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(7*e^(7/2))
+ f^2*x*Log[c*(d + e*x^2)^p]^2 + (g^2*x^7*Log[c*(d + e*x^2)^p]^2)/7 - (d*f*
g*(d + e*x^2)*Log[c*(d + e*x^2)^p]^2)/e^2 + (f*g*(d + e*x^2)^2*Log[c*(d + e
*x^2)^p]^2)/(2*e^2) + ((4*I)*Sqrt[d]*f^2*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sq
rt[d] + I*Sqrt[e]*x)])/Sqrt[e] - (((4*I)/7)*d^(7/2)*g^2*p^2*PolyLog[2, 1 -
(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/e^(7/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
```

```
d*g, 0] && IGtQ[q, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2500

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[x^n*(
(a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(f^2 \log^2\left(c\left(d+ex^2\right)^p\right)+2fgx^3\log^2\left(c\left(d+ex^2\right)^p\right)+g^2x^6\log^2\left(c\left(d+ex^2\right)^p\right)\right)\,dx \\
&= f^2\int \log^2\left(c\left(d+ex^2\right)^p\right)\,dx+(2fg)\int x^3\log^2\left(c\left(d+ex^2\right)^p\right)\,dx \\
&\quad +g^2\int x^6\log^2\left(c\left(d+ex^2\right)^p\right)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= f^2 x\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log^2\left(c\left(d+ex^2\right)^p\right)\\
&\quad+(fg)\mathrm{Subst}\left(\int x\log^2\left(c(d+ex)^p\right)\,dx,x,x^2\right)\\
&\quad-\left(4ef^2p\right)\int\frac{x^2\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx-\frac{1}{7}\left(4eg^2p\right)\int\frac{x^8\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx\\
&= f^2 x\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log^2\left(c\left(d+ex^2\right)^p\right)\\
&\quad+(fg)\mathrm{Subst}\left(\int\left(-\frac{d\log^2\left(c(d+ex)^p\right)}{e}+\frac{(d+ex)\log^2\left(c(d+ex)^p\right)}{e}\right)\,dx,x,x^2\right)\\
&\quad-\left(4ef^2p\right)\int\left(\frac{\log\left(c\left(d+ex^2\right)^p\right)}{e}-\frac{d\log\left(c\left(d+ex^2\right)^p\right)}{e\left(d+ex^2\right)}\right)\,dx\\
&\quad-\frac{1}{7}\left(4eg^2p\right)\int\left(-\frac{d^3\log\left(c\left(d+ex^2\right)^p\right)}{e^4}+\frac{d^2x^2\log\left(c\left(d+ex^2\right)^p\right)}{e^3}\right.\\
&\qquad\left.-\frac{dx^4\log\left(c\left(d+ex^2\right)^p\right)}{e^2}+\frac{x^6\log\left(c\left(d+ex^2\right)^p\right)}{e}+\frac{d^4\log\left(c\left(d+ex^2\right)^p\right)}{e^4\left(d+ex^2\right)}\right)\,dx\\
&= f^2 x\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log^2\left(c\left(d+ex^2\right)^p\right)\\
&\quad+\frac{(fg)\mathrm{Subst}\left(\int(d+ex)\log^2\left(c(d+ex)^p\right)\,dx,x,x^2\right)}{e}\\
&\quad-\frac{(dfg)\mathrm{Subst}\left(\int\log^2\left(c(d+ex)^p\right)\,dx,x,x^2\right)}{e}-\left(4f^2p\right)\int\log\left(c\left(d+ex^2\right)^p\right)\,dx\\
&\quad+\left(4df^2p\right)\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx-\frac{1}{7}\left(4g^2p\right)\int x^6\log\left(c\left(d+ex^2\right)^p\right)\,dx\\
&\quad+\frac{\left(4d^3g^2p\right)\int\log\left(c\left(d+ex^2\right)^p\right)\,dx}{7e^3}-\frac{\left(4d^4g^2p\right)\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx}{7e^3}\\
&\quad-\frac{\left(4d^2g^2p\right)\int x^2\log\left(c\left(d+ex^2\right)^p\right)\,dx}{7e^2}+\frac{\left(4dg^2p\right)\int x^4\log\left(c\left(d+ex^2\right)^p\right)\,dx}{7e}
\end{aligned}
$$

$$
\begin{aligned}
&= -4f^2px\log\left(c\left(d+ex^2\right)^p\right)+\frac{4d^3g^2px\log\left(c\left(d+ex^2\right)^p\right)}{7e^3}\\
&-\frac{4d^2g^2px^3\log\left(c\left(d+ex^2\right)^p\right)}{21e^2}+\frac{4dg^2px^5\log\left(c\left(d+ex^2\right)^p\right)}{35e}\\
&-\frac{4}{49}g^2px^7\log\left(c\left(d+ex^2\right)^p\right)+\frac{4\sqrt{d}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{e}}\\
&-\frac{4d^{7/2}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{7e^{7/2}}+f^2x\log^2\left(c\left(d+ex^2\right)^p\right)\\
&+\frac{1}{7}g^2x^7\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{(fg)\text{Subst}\left(\int x\log^2\left(cx^p\right)\,dx,x,d+ex^2\right)}{e^2}\\
&-\frac{(dfg)\text{Subst}\left(\int \log^2\left(cx^p\right)\,dx,x,d+ex^2\right)}{e^2}+\left(8ef^2p^2\right)\int\frac{x^2}{d+ex^2}\,dx\\
&-\left(8def^2p^2\right)\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}\left(d+ex^2\right)}\,dx-\frac{1}{35}\left(8dg^2p^2\right)\int\frac{x^6}{d+ex^2}\,dx\\
&-\frac{\left(8d^3g^2p^2\right)\int\frac{x^2}{d+ex^2}\,dx}{7e^2}+\frac{\left(8d^4g^2p^2\right)\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}(d+ex^2)}\,dx}{7e^2}\\
&+\frac{\left(8d^2g^2p^2\right)\int\frac{x^4}{d+ex^2}\,dx}{21e}+\frac{1}{49}\left(8eg^2p^2\right)\int\frac{x^8}{d+ex^2}\,dx
\end{aligned}
$$

$$= 8f^2p^2x - \frac{8d^3g^2p^2x}{7e^3} - 4f^2px\log\left(c\left(d+ex^2\right)^p\right) + \frac{4d^3g^2px\log\left(c\left(d+ex^2\right)^p\right)}{7e^3} - \frac{4d^2g^2px^3\log\left(c\left(d+ex^2\right)^p\right)}{21e^2} + \frac{4dg^2px^5\log\left(c\left(d+ex^2\right)^p\right)}{35e} - \frac{4}{49}g^2px^7\log\left(c\left(d+ex^2\right)^p\right) + \frac{4\sqrt{d}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{e}} - \frac{4d^{7/2}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{7e^{7/2}} + f^2x\log^2\left(c\left(d+ex^2\right)^p\right) + \frac{1}{7}g^2x^7\log^2\left(c\left(d+ex^2\right)^p\right) - \frac{dfg(d+ex^2)\log^2\left(c\left(d+ex^2\right)^p\right)}{e^2} + \frac{fg(d+ex^2)^2\log^2\left(c\left(d+ex^2\right)^p\right)}{2e^2} - \frac{(fgp)\text{Subst}\left(\int x\log\left(cx^p\right)\,dx, x, d+ex^2\right)}{e^2} + \frac{(2dfgp)\text{Subst}\left(\int \log\left(cx^p\right)\,dx, x, d+ex^2\right)}{e^2} - \left(8df^2p^2\right)\int\frac{1}{d+ex^2}\,dx - \left(8\sqrt{d}\sqrt{e}f^2p^2\right)\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d+ex^2}\,dx - \frac{1}{35}\left(8dg^2p^2\right)\int\left(\frac{d^2}{e^3} - \frac{dx^2}{e^2} + \frac{x^4}{e} - \frac{d^3}{e^3\left(d+ex^2\right)}\right)dx + \frac{\left(8d^4g^2p^2\right)\int\frac{1}{d+ex^2}\,dx}{7e^3} + \frac{\left(8d^{7/2}g^2p^2\right)\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d+ex^2}\,dx}{7e^{5/2}} + \frac{\left(8d^2g^2p^2\right)\int\left(-\frac{d}{e^2} + \frac{x^2}{e} + \frac{d^2}{e^2(d+ex^2)}\right)dx}{21e} + \frac{1}{49}\left(8eg^2p^2\right)\int\left(-\frac{d^3}{e^4} + \frac{d^2x^2}{e^3} - \frac{dx^4}{e^2} + \frac{x^6}{e} + \frac{d^4}{e^4\left(d+ex^2\right)}\right)dx$$

$$= 8f^2p^2x - \frac{1408d^3g^2p^2x}{735e^3} - \frac{2dfgp^2x^2}{e} + \frac{568d^2g^2p^2x^3}{2205e^2} - \frac{96dg^2p^2x^5}{1225e} + \frac{8}{343}g^2p^2x^7 + \frac{fgp^2(d+ex^2)^2}{4e^2} - \frac{8\sqrt{d}f^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{8d^{7/2}g^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}} + \frac{4i\sqrt{d}f^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} - \frac{4id^{7/2}g^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{7e^{7/2}}$$

$$-4f^2px\log\left(c\left(d+ex^2\right)^p\right) + \frac{4d^3g^2px\log\left(c\left(d+ex^2\right)^p\right)}{7e^3} - \frac{4d^2g^2px^3\log\left(c\left(d+ex^2\right)^p\right)}{21e^2} + \frac{4dg^2px^5\log\left(c\right.}{35e}$$

$$
\begin{aligned}
&= 8f^2p^2x - \frac{1408d^3g^2p^2x}{735e^3} - \frac{2dfgp^2x^2}{e} + \frac{568d^2g^2p^2x^3}{2205e^2} - \frac{96dg^2p^2x^5}{1225e} \\
&+ \frac{8}{343}g^2p^2x^7 + \frac{fgp^2(d+ex^2)^2}{4e^2} - \frac{8\sqrt{d}f^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} \\
&+ \frac{1408d^{7/2}g^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{735e^{7/2}} + \frac{4i\sqrt{d}f^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} - \frac{4id^{7/2}g^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{7e^{7/2}} \\
&+ \frac{8\sqrt{d}f^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} - \frac{8d^{7/2}g^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{7e^{7/2}}
\end{aligned}
$$

$$
-4f^2px\log\left(c\left(d+ex^2\right)^p\right) + \frac{4d^3g^2px\log\left(c\left(d+ex^2\right)^p\right)}{7e^3} - \frac{4d^2g^2px^3\log\left(c\left(d+ex^2\right)^p\right)}{21e^2} + \frac{4dg^2px^5\log\left(\right.}{35}
$$

$$
\begin{aligned}
&= 8f^2p^2x - \frac{1408d^3g^2p^2x}{735e^3} - \frac{2dfgp^2x^2}{e} + \frac{568d^2g^2p^2x^3}{2205e^2} - \frac{96dg^2p^2x^5}{1225e} \\
&+ \frac{8}{343}g^2p^2x^7 + \frac{fgp^2(d+ex^2)^2}{4e^2} - \frac{8\sqrt{d}f^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} \\
&+ \frac{1408d^{7/2}g^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{735e^{7/2}} + \frac{4i\sqrt{d}f^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} - \frac{4id^{7/2}g^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{7e^{7/2}} \\
&+ \frac{8\sqrt{d}f^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} - \frac{8d^{7/2}g^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{7e^{7/2}}
\end{aligned}
$$

$$
-4f^2px\log\left(c\left(d+ex^2\right)^p\right) + \frac{4d^3g^2px\log\left(c\left(d+ex^2\right)^p\right)}{7e^3} - \frac{4d^2g^2px^3\log\left(c\left(d+ex^2\right)^p\right)}{21e^2} + \frac{4dg^2px^5\log\left(\right.}{35}
$$

$$
\begin{aligned}
&= 8f^2p^2x - \frac{1408d^3g^2p^2x}{735e^3} - \frac{2dfgp^2x^2}{e} + \frac{568d^2g^2p^2x^3}{2205e^2} - \frac{96dg^2p^2x^5}{1225e} \\
&+ \frac{8}{343}g^2p^2x^7 + \frac{fgp^2(d+ex^2)^2}{4e^2} - \frac{8\sqrt{d}f^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} \\
&+ \frac{1408d^{7/2}g^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{735e^{7/2}} + \frac{4i\sqrt{d}f^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} - \frac{4id^{7/2}g^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{7e^{7/2}} \\
&+ \frac{8\sqrt{d}f^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} - \frac{8d^{7/2}g^2p^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{7e^{7/2}}
\end{aligned}
$$

$$
-4f^2px\log\left(c\left(d+ex^2\right)^p\right) + \frac{4d^3g^2px\log\left(c\left(d+ex^2\right)^p\right)}{7e^3} - \frac{4d^2g^2px^3\log\left(c\left(d+ex^2\right)^p\right)}{21e^2} + \frac{4dg^2px^5\log\left(\right.}{35}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 475, normalized size of antiderivative = 0.57

$$\int \left(f+gx^3\right)^2 \log^2\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$= \frac{-176400i\sqrt{d}(-7e^3f^2+d^3g^2)\,p^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2-1680\sqrt{d}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(2(735e^3f^2-176d^3g^2)\,p-210(7\right.}{}$$

```
[In] Integrate[(f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] ((-176400*I)*Sqrt[d]*(-7*e^3*f^2 + d^3*g^2)*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
^2 - 1680*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(2*(735*e^3*f^2 - 176*d^3*g
^2)*p - 210*(7*e^3*f^2 - d^3*g^2)*p*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)
] - 105*(7*e^3*f^2 - d^3*g^2)*Log[c*(d + e*x^2)^p]) + Sqrt[e]*(p^2*x*(-5913
60*d^3*g^2 + 79520*d^2*e*g^2*x^2 - 378*d*e^2*g*x*(1225*f + 64*g*x^3) + 225*
e^3*(10976*f^2 + 343*f*g*x^3 + 32*g^2*x^6)) + 154350*d^2*e*f*g*p^2*Log[d +
e*x^2] - 210*p*(-840*d^3*g^2*x + 70*d^2*e*g*(-21*f + 4*g*x^3) - 42*d*e^2*g*
x^2*(35*f + 4*g*x^3) + 15*e^3*x*(392*f^2 + 49*f*g*x^3 + 8*g^2*x^6))*Log[c*(
d + e*x^2)^p] + 22050*(-7*d^2*e*f*g + e^3*x*(14*f^2 + 7*f*g*x^3 + 2*g^2*x^6
))*Log[c*(d + e*x^2)^p]^2) - (176400*I)*Sqrt[d]*(-7*e^3*f^2 + d^3*g^2)*p^2*
PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)])/(308700*e^(
7/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.75 (sec) , antiderivative size = 1127, normalized size of antiderivative = 1.35

method	result	size
risch	Expression too large to display	1127

```
[In] int((g*x^3+f)^2*ln(c*(e*x^2+d)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4/49*p*g^2*x^7*ln((e*x^2+d)^p)-4/21*p/e^2*d^2*g^2*x^3*ln((e*x^2+d)^p)+4/7*
p/e^3*x*d^3*g^2*ln((e*x^2+d)^p)+p^2/e^2*d^2*f*g*ln(e*x^2+d)^2+3/2*p^2/e^2*d
^2*f*g*ln(e*x^2+d)+1408/735*p^2/e^3*g^2*d^4/(d*e)^(1/2)*arctan(x*e/(d*e)^(1
/2))-4*p*f^2*x*ln((e*x^2+d)^p)+1/4*(I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^
2+d)^p)^2-I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-I*Pi*csg
n(I*c*(e*x^2+d)^p)^3+I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+2*ln(c))^2*(1/7
*g^2*x^7+1/2*f*g*x^4+f^2*x)+1/4*p^2*f*g*x^4-1/2*p*f*g*x^4*ln((e*x^2+d)^p)-4
*p^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*f^2*ln(e*x^2+d)+4*p*d/(d*e)^(1/2
)*arctan(x*e/(d*e)^(1/2))*f^2*ln((e*x^2+d)^p)+4/35*p/e*d*g^2*x^5*ln((e*x^2+
d)^p)-8*p^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*f^2+ln((e*x^2+d)^p)^2*x*f
```

```
^2+1/2*ln((e*x^2+d)^p)^2*f*g*x^4+1/7*ln((e*x^2+d)^p)^2*g^2*x^7-2/7*p^2*e*Su
m(1/2*(ln(x-_alpha)*ln(e*x^2+d)-2*e*(1/4/_alpha/e*ln(x-_alpha)^2+1/2*_alpha
/d*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+1/2*_alpha/d*dilog(1/2*(x+_alpha)
/_alpha)))*d*(7*_alpha*d*e^2*f*g+2*d^3*g^2-14*e^3*f^2)/e^5/_alpha,_alpha=Ro
otOf(_Z^2*e+d))+(I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-I*Pi*csgn
(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^2+d)^p)^
3+I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+2*ln(c))*(1/7*ln((e*x^2+d)^p)*g^2*
x^7+1/2*ln((e*x^2+d)^p)*f*g*x^4+ln((e*x^2+d)^p)*x*f^2-1/7*p*e*(1/e^4*(2/7*e
^3*g^2*x^7-2/5*d*e^2*g^2*x^5+7/4*e^3*f*g*x^4+2/3*d^2*e*g^2*x^3-7/2*d*f*g*x^
2*e^2-2*x*d^3*g^2+14*x*e^3*f^2)+d/e^4*(7/2*d*e*f*g*ln(e*x^2+d)+(2*d^3*g^2-1
4*e^3*f^2)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)))))+p/e*d*f*g*x^2*ln((e*x^2+d
)^p)-p/e^2*d^2*f*g*ln(e*x^2+d)*ln((e*x^2+d)^p)-4/7*p/e^3*d^4/(d*e)^(1/2)*ar
ctan(x*e/(d*e)^(1/2))*g^2*ln((e*x^2+d)^p)+4/7*p^2/e^3*d^4/(d*e)^(1/2)*arcta
n(x*e/(d*e)^(1/2))*g^2*ln(e*x^2+d)-3/2*d*f*g*p^2*x^2/e+8*f^2*p^2*x+8/343*g^
2*p^2*x^7-1408/735*d^3*g^2*p^2*x/e^3+568/2205*d^2*g^2*p^2*x^3/e^2-96/1225*d
*g^2*p^2*x^5/e
```

Fricas [F]

$$\int \left(f+gx^3\right)^2\log^2\left(c\left(d+ex^2\right)^p\right)\,dx=\int \left(gx^3+f\right)^2\log\left(\left(ex^2+d\right)^pc\right)^2\,dx$$

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
[Out] integral((g^2*x^6 + 2*f*g*x^3 + f^2)*log((e*x^2 + d)^p*c)^2, x)
```

Sympy [F]

$$\int \left(f+gx^3\right)^2\log^2\left(c\left(d+ex^2\right)^p\right)\,dx=\int \left(f+gx^3\right)^2\log\left(c\left(d+ex^2\right)^p\right)^2\,dx$$

```
[In] integrate((g*x**3+f)**2*ln(c*(e*x**2+d)**p)**2,x)
[Out] Integral((f + g*x**3)**2*log(c*(d + e*x**2)**p)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(f+gx^3\right)^2 \log^2 \left(c\left(d+ex^2\right)^p\right) \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \left(f+gx^3\right)^2 \log^2 \left(c\left(d+ex^2\right)^p\right) \, dx = \int \left(gx^3+f\right)^2 \log \left(\left(ex^2+d\right)^p c\right)^2 \, dx$$

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)^2*log((e*x^2 + d)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(f+gx^3\right)^2 \log^2 \left(c\left(d+ex^2\right)^p\right) \, dx = \int \ln \left(c\left(e\,x^2+d\right)^p\right)^2 \left(g\,x^3+f\right)^2 \, dx$$

```
[In] int(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^2,x)
```

```
[Out] int(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^2, x)
```

3.295 $\int (f+gx^3)\log^2\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 395

$$\begin{aligned}\int (f+gx^3)\log^2\left(c(d+ex^2)^p\right)\,dx &= 8fp^2x-\frac{dgp^2x^2}{e}+\frac{gp^2(d+ex^2)^2}{8e^2}\\&\quad-\frac{8\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{4i\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}}\\&\quad+\frac{8\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}\\&\quad-4fpx\log\left(c(d+ex^2)^p\right)\\&\quad+\frac{dgp(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e^2}\\&\quad-\frac{gp(d+ex^2)^2\log\left(c(d+ex^2)^p\right)}{4e^2}\\&\quad+\frac{4\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{e}}\\&\quad+fx\log^2\left(c(d+ex^2)^p\right)\\&\quad-\frac{dg(d+ex^2)\log^2\left(c(d+ex^2)^p\right)}{2e^2}\\&\quad+\frac{g(d+ex^2)^2\log^2\left(c(d+ex^2)^p\right)}{4e^2}\\&\quad+\frac{4i\sqrt{d}fp^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}\end{aligned}$$

```
[Out] 8*f*p^2*x-d*g*p^2*x^2/e+1/8*g*p^2*(e*x^2+d)^2/e^2-4*f*p*x*ln(c*(e*x^2+d)^p)
+d*g*p*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e^2-1/4*g*p*(e*x^2+d)^2*ln(c*(e*x^2+d)^p
```

```
)/e^2+f*x*ln(c*(e*x^2+d)^p)^2-1/2*d*g*(e*x^2+d)*ln(c*(e*x^2+d)^p)^2/e^2+1/4
*g*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)^2/e^2-8*f*p^2*arctan(x*e^(1/2)/d^(1/2))*d^
(1/2)/e^(1/2)+4*I*f*p^2*arctan(x*e^(1/2)/d^(1/2))^2*d^(1/2)/e^(1/2)+4*f*p*a
rctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)*d^(1/2)/e^(1/2)+8*f*p^2*arctan(x
*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)+4*I*f
*p^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {2521, 2500, 2526, 2498, 327, 211, 2520, 12, 5040, 4964, 2449, 2352, 2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\int (f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)\,dx=\frac{4\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{e}}+\frac{4i\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}}-\frac{8\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{8\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}+\frac{g(d+ex^2)^2\log^2\left(c\left(d+ex^2\right)^p\right)}{4e^2}-\frac{dg(d+ex^2)\log^2\left(c\left(d+ex^2\right)^p\right)}{2e^2}-\frac{gp(d+ex^2)^2\log\left(c\left(d+ex^2\right)^p\right)}{4e^2}+\frac{dgp(d+ex^2)\log\left(c\left(d+ex^2\right)^p\right)}{e^2}+fx\log^2\left(c\left(d+ex^2\right)^p\right)-4fpx\log\left(c\left(d+ex^2\right)^p\right)+\frac{gp^2(d+ex^2)^2}{8e^2}+\frac{4i\sqrt{d}fp^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{i\sqrt{e}x+\sqrt{d}}\right)}{\sqrt{e}}-\frac{dgp^2x^2}{e}+8fp^2x$$

```
[In] Int[(f + g*x^3)*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] 8*f*p^2*x - (d*g*p^2*x^2)/e + (g*p^2*(d + e*x^2)^2)/(8*e^2) - (8*Sqrt[d]*f*
p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + ((4*I)*Sqrt[d]*f*p^2*ArcTan[(Sqr
t[e]*x)/Sqrt[d]]^2)/Sqrt[e] + (8*Sqrt[d]*f*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*
Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] - 4*f*p*x*Log[c*(d + e*x^
2)^p] + (d*g*p*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e^2 - (g*p*(d + e*x^2)^2*L
```

```
og[c*(d + e*x^2)^p])/(4*e^2) + (4*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*L
og[c*(d + e*x^2)^p])/Sqrt[e] + f*x*Log[c*(d + e*x^2)^p]^2 - (d*g*(d + e*x^2
)*Log[c*(d + e*x^2)^p]^2)/(2*e^2) + (g*(d + e*x^2)^2*Log[c*(d + e*x^2)^p]^2
)/(4*e^2) + ((4*I)*Sqrt[d]*f*p^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x)])/Sqrt[e]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2500

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[x^n*(
(a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(f\log^2\left(c\left(d+ex^2\right)^p\right)+gx^3\log^2\left(c\left(d+ex^2\right)^p\right)\right)\,dx$$

$$= f\int \log^2\left(c\left(d+ex^2\right)^p\right)\,dx + g\int x^3\log^2\left(c\left(d+ex^2\right)^p\right)\,dx$$

$$= fx\log^2\left(c\left(d+ex^2\right)^p\right) + \frac{1}{2}g\text{Subst}\left(\int x\log^2\left(c(d+ex)^p\right)\,dx, x, x^2\right)$$
$$- (4efp)\int \frac{x^2\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx$$

$$= fx\log^2\left(c\left(d+ex^2\right)^p\right)$$
$$+ \frac{1}{2}g\text{Subst}\left(\int \left(-\frac{d\log^2\left(c(d+ex)^p\right)}{e} + \frac{(d+ex)\log^2\left(c(d+ex)^p\right)}{e}\right)\,dx, x, x^2\right)$$
$$- (4efp)\int \left(\frac{\log\left(c\left(d+ex^2\right)^p\right)}{e} - \frac{d\log\left(c\left(d+ex^2\right)^p\right)}{e\left(d+ex^2\right)}\right)\,dx$$

$$= fx\log^2\left(c\left(d+ex^2\right)^p\right) + \frac{g\text{Subst}\left(\int (d+ex)\log^2\left(c(d+ex)^p\right)\,dx, x, x^2\right)}{2e}$$
$$- \frac{(dg)\text{Subst}\left(\int \log^2\left(c(d+ex)^p\right)\,dx, x, x^2\right)}{2e}$$
$$- (4fp)\int \log\left(c\left(d+ex^2\right)^p\right)\,dx + (4dfp)\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx$$

$$= -4fpx\log\left(c\left(d+ex^2\right)^p\right) + \frac{4\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{e}}$$
$$+ fx\log^2\left(c\left(d+ex^2\right)^p\right) + \frac{g\text{Subst}\left(\int x\log^2\left(cx^p\right)\,dx, x, d+ex^2\right)}{2e^2}$$
$$- \frac{(dg)\text{Subst}\left(\int \log^2\left(cx^p\right)\,dx, x, d+ex^2\right)}{2e^2}$$
$$+ \left(8efp^2\right)\int \frac{x^2}{d+ex^2}\,dx - \left(8defp^2\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}\left(d+ex^2\right)}\,dx$$

$$= 8fp^2x - 4fpx\log\left(c\left(d+ex^2\right)^p\right) + \frac{4\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{\sqrt{e}}$$
$$+ fx\log^2\left(c\left(d+ex^2\right)^p\right) - \frac{dg\left(d+ex^2\right)\log^2\left(c\left(d+ex^2\right)^p\right)}{2e^2}$$
$$+ \frac{g\left(d+ex^2\right)^2\log^2\left(c\left(d+ex^2\right)^p\right)}{4e^2} - \frac{(gp)\text{Subst}\left(\int x\log\left(cx^p\right)\,dx, x, d+ex^2\right)}{2e^2}$$
$$+ \frac{(dgp)\text{Subst}\left(\int \log\left(cx^p\right)\,dx, x, d+ex^2\right)}{e^2}$$
$$- \left(8dfp^2\right)\int \frac{1}{d+ex^2}\,dx - \left(8\sqrt{d}\sqrt{e}fp^2\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d+ex^2}\,dx$$

$$
\begin{aligned}
&= 8fp^2x - \frac{dgp^2x^2}{e} + \frac{gp^2(d+ex^2)^2}{8e^2} - \frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{4i\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} \\
&\quad - 4fpx\log\left(c(d+ex^2)^p\right) + \frac{dgp(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e^2} \\
&\quad - \frac{gp(d+ex^2)^2\log\left(c(d+ex^2)^p\right)}{4e^2} + \frac{4\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{e}} \\
&\quad + fx\log^2\left(c(d+ex^2)^p\right) - \frac{dg(d+ex^2)\log^2\left(c(d+ex^2)^p\right)}{2e^2} \\
&\quad + \frac{g(d+ex^2)^2\log^2\left(c(d+ex^2)^p\right)}{4e^2} + (8fp^2)\int \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{i - \frac{\sqrt{e}x}{\sqrt{d}}}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= 8fp^2x - \frac{dgp^2x^2}{e} + \frac{gp^2(d+ex^2)^2}{8e^2} - \frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} \\
&\quad + \frac{4i\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} + \frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} \\
&\quad - 4fpx\log\left(c(d+ex^2)^p\right) + \frac{dgp(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e^2} \\
&\quad - \frac{gp(d+ex^2)^2\log\left(c(d+ex^2)^p\right)}{4e^2} + \frac{4\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{e}} \\
&\quad + fx\log^2\left(c(d+ex^2)^p\right) - \frac{dg(d+ex^2)\log^2\left(c(d+ex^2)^p\right)}{2e^2} \\
&\quad + \frac{g(d+ex^2)^2\log^2\left(c(d+ex^2)^p\right)}{4e^2} - (8fp^2)\int \frac{\log\left(\frac{2}{1+\frac{i\sqrt{e}x}{\sqrt{d}}}\right)}{1+\frac{ex^2}{d}}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= 8fp^2x - \frac{dgp^2x^2}{e} + \frac{gp^2(d+ex^2)^2}{8e^2} - \frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} \\
&\quad + \frac{4i\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} + \frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} \\
&\quad - 4fpx\log\left(c\left(d+ex^2\right)^p\right) + \frac{dgp(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e^2} \\
&\quad - \frac{gp(d+ex^2)^2\log\left(c(d+ex^2)^p\right)}{4e^2} + \frac{4\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{e}} \\
&\quad + fx\log^2\left(c\left(d+ex^2\right)^p\right) - \frac{dg(d+ex^2)\log^2\left(c(d+ex^2)^p\right)}{2e^2} \\
&\quad + \frac{g(d+ex^2)^2\log^2\left(c(d+ex^2)^p\right)}{4e^2} + \frac{\left(8i\sqrt{d}fp^2\right)\mathrm{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+\frac{i\sqrt{e}x}{\sqrt{d}}}\right)}{\sqrt{e}}
\end{aligned}
$$

$$
\begin{aligned}
&= 8fp^2x - \frac{dgp^2x^2}{e} + \frac{gp^2(d+ex^2)^2}{8e^2} - \frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} \\
&\quad + \frac{4i\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} + \frac{8\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} \\
&\quad - 4fpx\log\left(c\left(d+ex^2\right)^p\right) + \frac{dgp(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e^2} \\
&\quad - \frac{gp(d+ex^2)^2\log\left(c(d+ex^2)^p\right)}{4e^2} + \frac{4\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{e}} \\
&\quad + fx\log^2\left(c\left(d+ex^2\right)^p\right) - \frac{dg(d+ex^2)\log^2\left(c(d+ex^2)^p\right)}{2e^2} \\
&\quad + \frac{g(d+ex^2)^2\log^2\left(c(d+ex^2)^p\right)}{4e^2} + \frac{4i\sqrt{d}fp^2\mathrm{Li}_2\left(1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.01

$$\int (f+gx^3)\log^2\left(c(d+ex^2)^p\right)\,dx$$
$$= fx\log^2\left(c(d+ex^2)^p\right)+\frac{1}{4}gx^4\log^2\left(c(d+ex^2)^p\right)$$
$$-\frac{1}{2}gp\left(\frac{3dpx^2}{2e}-\frac{px^4}{4}-\frac{d^2p\log(d+ex^2)}{2e^2}-\frac{d^2\log\left(c(d+ex^2)^p\right)}{e^2}-\frac{dx^2\log\left(c(d+ex^2)^p\right)}{e}\right.$$
$$\left.+\frac{1}{2}x^4\log\left(c(d+ex^2)^p\right)+\frac{d^2\log^2\left(c(d+ex^2)^p\right)}{2e^2p}\right)-4efp\left(-\frac{2px}{e}+\frac{2\sqrt{d}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}}\right.$$
$$+\frac{x\log\left(c(d+ex^2)^p\right)}{e}-\frac{\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{e^{3/2}}$$
$$\left.-\frac{\sqrt{d}p\left(\frac{i\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{e}+\frac{2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2i\sqrt{d}}{i\sqrt{d}-\sqrt{e}x}\right)}{e}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{d}+\sqrt{e}x}{i\sqrt{d}-\sqrt{e}x}\right)}{e}\right)}{\sqrt{e}}\right)$$

```
[In] Integrate[(f + g*x^3)*Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] f*x*Log[c*(d + e*x^2)^p]^2 + (g*x^4*Log[c*(d + e*x^2)^p]^2)/4 - (g*p*((3*d*
p*x^2)/(2*e) - (p*x^4)/4 - (d^2*p*Log[d + e*x^2])/(2*e^2) - (d^2*Log[c*(d +
 e*x^2)^p])/e^2 - (d*x^2*Log[c*(d + e*x^2)^p])/e + (x^4*Log[c*(d + e*x^2)^p
])/2 + (d^2*Log[c*(d + e*x^2)^p]^2)/(2*e^2*p)))/2 - 4*e*f*p*((-2*p*x)/e + (
2*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2) + (x*Log[c*(d + e*x^2)^p])
/e - (Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/e^(3/2) - (
Sqrt[d]*p*((I*ArcTan[(Sqrt[e]*x)/Sqrt[d]]^2)/e + (2*ArcTan[(Sqrt[e]*x)/Sqrt
[d]]*Log[((2*I)*Sqrt[d])/(I*Sqrt[d] - Sqrt[e]*x)])/e + (I*PolyLog[2, -((I*S
qrt[d] + Sqrt[e]*x)/(I*Sqrt[d] - Sqrt[e]*x))])/e))/Sqrt[e])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.02 (sec) , antiderivative size = 724, normalized size of antiderivative = 1.83

method	result	size
risch	Expression too large to display	724

```
[In] int((g*x^3+f)*ln(c*(e*x^2+d)^p)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln((e*x^2+d)^p)^2*g*x^4+ln((e*x^2+d)^p)^2*x*f+1/2*p^2/e^2*d^2*g*ln(e*x^
2+d)^2-1/2*p/e^2*d^2*g*ln(e*x^2+d)*ln((e*x^2+d)^p)-4*p^2*d/(d*e)^(1/2)*arct
an(x*e/(d*e)^(1/2))*f*ln(e*x^2+d)+4*p*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
*f*ln((e*x^2+d)^p)-1/4*p*g*x^4*ln((e*x^2+d)^p)+1/2*p/e*d*g*x^2*ln((e*x^2+d)
^p)-4*p*f*x*ln((e*x^2+d)^p)+1/8*p^2*x^4*g-3/4*d*g*p^2*x^2/e+3/4*p^2/e^2*d^2
*g*ln(e*x^2+d)+8*f*p^2*x-8*p^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))*f-p^2*
e*Sum(1/2*(ln(x-_alpha)*ln(e*x^2+d)-2*e*(1/4/_alpha/e*ln(x-_alpha)^2+1/2*_a
lpha/d*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+1/2*_alpha/d*dilog(1/2*(x+_al
pha)/_alpha)))*d*(_alpha*d*g-4*e*f)/e^3/_alpha,_alpha=RootOf(_Z^2*e+d))+(I*
Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-I*Pi*csgn(I*(e*x^2+d)^p)*csg
n(I*c*(e*x^2+d)^p)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^2+d)^p)^3+I*Pi*csgn(I*c*(e*
x^2+d)^p)^2*csgn(I*c)+2*ln(c))*(1/4*ln((e*x^2+d)^p)*g*x^4+ln((e*x^2+d)^p)*x
*f-1/2*p*e*(d/e^2*(1/2*d*g/e*ln(e*x^2+d)-4*e*f/(d*e)^(1/2)*arctan(x*e/(d*e)
^(1/2)))+1/e^2*(1/4*e*g*x^4-1/2*d*g*x^2+4*e*f*x)))+1/4*(I*Pi*csgn(I*(e*x^2+
d)^p)*csgn(I*c*(e*x^2+d)^p)^2-I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p
)*csgn(I*c)-I*Pi*csgn(I*c*(e*x^2+d)^p)^3+I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(
I*c)+2*ln(c))^2*(1/4*g*x^4+f*x)
```

Fricas [F]

$$\int (f+gx^3)\log^2\left(c(d+ex^2)^p\right)\,dx=\int \left(gx^3+f\right)\log\left(\left(ex^2+d\right)^pc\right)^2\,dx$$

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
```

```
[Out] integral((g*x^3 + f)*log((e*x^2 + d)^p*c)^2, x)
```

Sympy [F]

$$\int (f + gx^3)\log^2\left(c(d + ex^2)^p\right)\,dx = \int (f + gx^3)\log\left(c(d + ex^2)^p\right)^2\,dx$$

```
[In] integrate((g*x**3+f)*ln(c*(e*x**2+d)**p)**2,x)
```

```
[Out] Integral((f + g*x**3)*log(c*(d + e*x**2)**p)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (f + gx^3)\log^2\left(c(d + ex^2)^p\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int (f + gx^3)\log^2\left(c(d + ex^2)^p\right)\,dx = \int (gx^3 + f)\log\left((ex^2 + d)^p c\right)^2\,dx$$

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)*log((e*x^2 + d)^p*c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (f + gx^3)\log^2\left(c(d + ex^2)^p\right)\,dx = \int \ln\left(c\left(e\,x^2 + d\right)^p\right)^2\left(g\,x^3 + f\right)\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)^2*(f + g*x^3),x)
```

```
[Out] int(log(c*(d + e*x^2)^p)^2*(f + g*x^3), x)
```

3.296 $\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^3} dx = \text{Int}\left(\frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^3}, x\right)$$

```
[Out] Unintegrable(ln(c*(e*x^2+d)^p)^2/(g*x^3+f),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^3} dx = \int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^3} dx$$

```
[In] Int[Log[c*(d + e*x^2)^p]^2/(f + g*x^3),x]
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^2/(f + g*x^3), x]
```

Rubi steps

$$\text{integral} = \int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^3} dx$$

Mathematica [N/A]

Not integrable

Time = 17.72 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx=\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx$$

```
[In] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^3),x]
[Out] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^3), x]
```

Maple [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c(e\,x^2+d)^p\right)^2}{g\,x^3+f}dx$$

```
[In] int(ln(c*(e*x^2+d)^p)^2/(g*x^3+f),x)
[Out] int(ln(c*(e*x^2+d)^p)^2/(g*x^3+f),x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx=\int \frac{\log\left((ex^2+d)^pc\right)^2}{gx^3+f}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f),x, algorithm="fricas")
[Out] integral(log((e*x^2 + d)^p*c)^2/(g*x^3 + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx=\text{Timed out}$$

```
[In] integrate(ln(c*(e*x**2+d)**p)**2/(g*x**3+f),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^2}{gx^3+f}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f),x, algorithm="maxima")
[Out] integrate(log((e*x^2 + d)^p*c)^2/(g*x^3 + f), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^2}{gx^3+f}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f),x, algorithm="giac")
[Out] integrate(log((e*x^2 + d)^p*c)^2/(g*x^3 + f), x)
```

Mupad [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)^2}{g\,x^3+f}\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)^2/(f + g*x^3),x)
[Out] int(log(c*(d + e*x^2)^p)^2/(f + g*x^3), x)
```

3.297 $\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx=\mathrm{Int}\left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2},x\right)$$

```
[Out] Unintegrable(ln(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx=\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx$$

```
[In] Int[Log[c*(d + e*x^2)^p]^2/(f + g*x^3)^2,x]
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^2/(f + g*x^3)^2, x]
```

Rubi steps

$$\text{integral}=\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 26.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx = \int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx$$

```
[In] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^3)^2,x]
[Out] Integrate[Log[c*(d + e*x^2)^p]^2/(f + g*x^3)^2, x]
```

Maple [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c(e\,x^2+d)^p\right)^2}{(g\,x^3+f)^2}dx$$

```
[In] int(ln(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x)
[Out] int(ln(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^2}{(gx^3+f)^2}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x, algorithm="fricas")
[Out] integral(log((e*x^2 + d)^p*c)^2/(g^2*x^6 + 2*f*g*x^3 + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{\left(f+gx^3\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(e*x**2+d)**p)**2/(g*x**3+f)**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{\left(f+gx^3\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{\left(f+gx^3\right)^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^2}{\left(gx^3+f\right)^2}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)^2/(g*x^3+f)^2,x, algorithm="giac")
[Out] integrate(log((e*x^2 + d)^p*c)^2/(g*x^3 + f)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^2\left(c(d+ex^2)^p\right)}{\left(f+gx^3\right)^2}\,dx=\int \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)^2}{{\left(g\,x^3+f\right)}^2} \,dx$$

```
[In] int(log(c*(d + e*x^2)^p)^2/(f + g*x^3)^2,x)
[Out] int(log(c*(d + e*x^2)^p)^2/(f + g*x^3)^2, x)
```

3.298 $\int (f+gx^3)^2 \log^3\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$
\begin{aligned}
&\int (f+gx^3)^2 \log^3\left(c(d+ex^2)^p\right)\,dx \\
&= -48f^2p^3x+\frac{351136d^3g^2p^3x}{25725e^3}+\frac{6dfgp^3x^2}{e}-\frac{55456d^2g^2p^3x^3}{77175e^2}+\frac{5232dg^2p^3x^5}{42875e}-\frac{48g^2p^3x^7}{2401} \\
&\quad -\frac{3fgp^3(d+ex^2)^2}{8e^2}+\frac{48\sqrt{d}f^2p^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{351136d^{7/2}g^2p^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{25725e^{7/2}} \\
&\quad -\frac{24i\sqrt{d}f^2p^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}}+\frac{1408id^{7/2}g^2p^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{245e^{7/2}} \\
&\quad -\frac{48\sqrt{d}f^2p^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}}+\frac{2816d^{7/2}g^2p^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{245e^{7/2}} \\
&\quad +24f^2p^2x\log\left(c(d+ex^2)^p\right)-\frac{1408d^3g^2p^2x\log\left(c(d+ex^2)^p\right)}{245e^3}+\frac{568d^2g^2p^2x^3\log\left(c(d+ex^2)^p\right)}{735e^2}-\frac{288dg^2p^2x
\end{aligned}
$$

```
[Out] 6*d*f*g*p^3*x^2/e-1408/245*d^3*g^2*p^2*x*ln(c*(e*x^2+d)^p)/e^3+568/735*d^2*
g^2*p^2*x^3*ln(c*(e*x^2+d)^p)/e^2-288/1225*d*g^2*p^2*x^5*ln(c*(e*x^2+d)^p)/
e+3/4*f*g*p^2*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)/e^2+1408/245*d^(7/2)*g^2*p^2*ar
ctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)/e^(7/2)+6/7*d^3*g^2*p*x*ln(c*(e*x
^2+d)^p)^2/e^3-2/7*d^2*g^2*p*x^3*ln(c*(e*x^2+d)^p)^2/e^2+6/35*d*g^2*p*x^5*l
n(c*(e*x^2+d)^p)^2/e-3/4*f*g*p*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)^2/e^2+2816/245
*d^(7/2)*g^2*p^3*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2
)))/e^(7/2)-24*f^2*p^2*arctan(x*e^(1/2)/d^(1/2))*ln(c*(e*x^2+d)^p)*d^(1/2)/
e^(1/2)-48*f^2*p^3*arctan(x*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1
/2)))*d^(1/2)/e^(1/2)-24*I*f^2*p^3*arctan(x*e^(1/2)/d^(1/2))^2*d^(1/2)/e^(1
/2)-24*I*f^2*p^3*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/
```

```
2)-d*f*g*(e*x^2+d)*ln(c*(e*x^2+d)^p)^3/e^2-48*f^2*p^3*x-48/2401*g^2*p^3*x^7
+1/7*g^2*x^7*ln(c*(e*x^2+d)^p)^3+1408/245*I*d^(7/2)*g^2*p^3*polylog(2,1-2*d
^(1/2)/(d^(1/2)+I*x*e^(1/2)))/e^(7/2)+1408/245*I*d^(7/2)*g^2*p^3*arctan(x*e
^(1/2)/d^(1/2))^2/e^(7/2)+24*f^2*p^2*x*ln(c*(e*x^2+d)^p)+24/343*g^2*p^2*x^7
*ln(c*(e*x^2+d)^p)-6*f^2*p*x*ln(c*(e*x^2+d)^p)^2-6/49*g^2*p*x^7*ln(c*(e*x^2
+d)^p)^2+6*d*f^2*p*Unintegrable(ln(c*(e*x^2+d)^p)^2/(e*x^2+d),x)+f^2*x*ln(c
*(e*x^2+d)^p)^3-6*d*f*g*p^2*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e^2+3*d*f*g*p*(e*x^
2+d)*ln(c*(e*x^2+d)^p)^2/e^2-351136/25725*d^(7/2)*g^2*p^3*arctan(x*e^(1/2)/
d^(1/2))/e^(7/2)+1/2*f*g*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)^3/e^2+48*f^2*p^3*arc
tan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)-6/7*d^4*g^2*p*Unintegrable(ln(c*(e*x
^2+d)^p)^2/(e*x^2+d),x)/e^3+351136/25725*d^3*g^2*p^3*x/e^3-55456/77175*d^2*
g^2*p^3*x^3/e^2+5232/42875*d*g^2*p^3*x^5/e-3/8*f*g*p^3*(e*x^2+d)^2/e^2
```

Rubi [N/A]

Not integrable

Time = 1.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(f+gx^3\right)^2 \log^3\left(c\left(d+ex^2\right)^p\right)\,dx=\int \left(f+gx^3\right)^2 \log^3\left(c\left(d+ex^2\right)^p\right)\,dx$$

```
[In] Int[(f + g*x^3)^2*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] -48*f^2*p^3*x + (351136*d^3*g^2*p^3*x)/(25725*e^3) + (6*d*f*g*p^3*x^2)/e -
(55456*d^2*g^2*p^3*x^3)/(77175*e^2) + (5232*d*g^2*p^3*x^5)/(42875*e) - (48*
g^2*p^3*x^7)/2401 - (3*f*g*p^3*(d + e*x^2)^2)/(8*e^2) + (48*Sqrt[d]*f^2*p^3
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (351136*d^(7/2)*g^2*p^3*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(25725*e^(7/2)) - ((24*I)*Sqrt[d]*f^2*p^3*ArcTan[(Sqrt[e]
*x)/Sqrt[d]]^2)/Sqrt[e] + (((1408*I)/245)*d^(7/2)*g^2*p^3*ArcTan[(Sqrt[e]*x
)/Sqrt[d]]^2)/e^(7/2) - (48*Sqrt[d]*f^2*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log
[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + (2816*d^(7/2)*g^2*p^3*ArcT
an[(Sqrt[e]*x)/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/(245*e^(7
/2)) + 24*f^2*p^2*x*Log[c*(d + e*x^2)^p] - (1408*d^3*g^2*p^2*x*Log[c*(d + e
*x^2)^p])/(245*e^3) + (568*d^2*g^2*p^2*x^3*Log[c*(d + e*x^2)^p])/(735*e^2)
- (288*d*g^2*p^2*x^5*Log[c*(d + e*x^2)^p])/(1225*e) + (24*g^2*p^2*x^7*Log[c
*(d + e*x^2)^p])/343 - (6*d*f*g*p^2*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e^2 +
 (3*f*g*p^2*(d + e*x^2)^2*Log[c*(d + e*x^2)^p])/(4*e^2) - (24*Sqrt[d]*f^2*p
^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt[e] + (1408*d^(7/2
)*g^2*p^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/(245*e^(7/2)) -
 6*f^2*p*x*Log[c*(d + e*x^2)^p]^2 + (6*d^3*g^2*p*x*Log[c*(d + e*x^2)^p]^2)/
(7*e^3) - (2*d^2*g^2*p*x^3*Log[c*(d + e*x^2)^p]^2)/(7*e^2) + (6*d*g^2*p*x^5
*Log[c*(d + e*x^2)^p]^2)/(35*e) - (6*g^2*p*x^7*Log[c*(d + e*x^2)^p]^2)/49 +
 (3*d*f*g*p*(d + e*x^2)*Log[c*(d + e*x^2)^p]^2)/e^2 - (3*f*g*p*(d + e*x^2)^
2*Log[c*(d + e*x^2)^p]^2)/(4*e^2) + f^2*x*Log[c*(d + e*x^2)^p]^3 + (g^2*x^7
```

```
*Log[c*(d + e*x^2)^p]^3)/7 - (d*f*g*(d + e*x^2)*Log[c*(d + e*x^2)^p]^3)/e^2
 + (f*g*(d + e*x^2)^2*Log[c*(d + e*x^2)^p]^3)/(2*e^2) - ((24*I)*Sqrt[d]*f^2
*p^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + (((1408
*I)/245)*d^(7/2)*g^2*p^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)
])/e^(7/2) + 6*d*f^2*p*Defer[Int][Log[c*(d + e*x^2)^p]^2/(d + e*x^2), x] -
(6*d^4*g^2*p*Defer[Int][Log[c*(d + e*x^2)^p]^2/(d + e*x^2), x])/(7*e^3)
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(f^2\log^3\left(c\left(d+ex^2\right)^p\right)+2fgx^3\log^3\left(c\left(d+ex^2\right)^p\right)+g^2x^6\log^3\left(c\left(d+ex^2\right)^p\right)\right)\,dx\\
&= f^2\int \log^3\left(c\left(d+ex^2\right)^p\right)\,dx+(2fg)\int x^3\log^3\left(c\left(d+ex^2\right)^p\right)\,dx\\
&\quad +g^2\int x^6\log^3\left(c\left(d+ex^2\right)^p\right)\,dx\\
&= f^2x\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log^3\left(c\left(d+ex^2\right)^p\right)\\
&\quad +(fg)\text{Subst}\left(\int x\log^3\left(c(d+ex)^p\right)\,dx,x,x^2\right)\\
&\quad -\left(6ef^2p\right)\int \frac{x^2\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx-\frac{1}{7}\left(6eg^2p\right)\int \frac{x^8\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx\\
&= f^2x\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log^3\left(c\left(d+ex^2\right)^p\right)\\
&\quad +(fg)\text{Subst}\left(\int \left(-\frac{d\log^3\left(c(d+ex)^p\right)}{e}+\frac{(d+ex)\log^3\left(c(d+ex)^p\right)}{e}\right)\,dx,x,x^2\right)\\
&\quad -\left(6ef^2p\right)\int \left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{e}-\frac{d\log^2\left(c\left(d+ex^2\right)^p\right)}{e\left(d+ex^2\right)}\right)\,dx\\
&\quad -\frac{1}{7}\left(6eg^2p\right)\int \left(-\frac{d^3\log^2\left(c\left(d+ex^2\right)^p\right)}{e^4}+\frac{d^2x^2\log^2\left(c\left(d+ex^2\right)^p\right)}{e^3}\right.\\
&\qquad \left.-\frac{dx^4\log^2\left(c\left(d+ex^2\right)^p\right)}{e^2}+\frac{x^6\log^2\left(c\left(d+ex^2\right)^p\right)}{e}+\frac{d^4\log^2\left(c\left(d+ex^2\right)^p\right)}{e^4\left(d+ex^2\right)}\right)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= f^2 x \log^3\left(c\left(d+ex^2\right)^p\right) + \frac{1}{7} g^2 x^7 \log^3\left(c\left(d+ex^2\right)^p\right) \\
&\quad + \frac{(fg)\text{Subst}\left(\int (d+ex)\log^3\left(c(d+ex)^p\right)\,dx, x, x^2\right)}{e} \\
&\quad - \frac{(dfg)\text{Subst}\left(\int \log^3\left(c(d+ex)^p\right)\,dx, x, x^2\right)}{e} - \left(6f^2p\right)\int \log^2\left(c\left(d+ex^2\right)^p\right)\,dx \\
&\quad + \left(6df^2p\right)\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx - \frac{1}{7}\left(6g^2p\right)\int x^6 \log^2\left(c\left(d+ex^2\right)^p\right)\,dx \\
&\quad + \frac{\left(6d^3g^2p\right)\int \log^2\left(c\left(d+ex^2\right)^p\right)\,dx}{7e^3} - \frac{\left(6d^4g^2p\right)\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx}{7e^3} \\
&\quad - \frac{\left(6d^2g^2p\right)\int x^2\log^2\left(c\left(d+ex^2\right)^p\right)\,dx}{7e^2} + \frac{\left(6dg^2p\right)\int x^4\log^2\left(c\left(d+ex^2\right)^p\right)\,dx}{7e}
\end{aligned}
$$

$$
\begin{aligned}
&= -6f^2 p x \log^2\left(c\left(d+ex^2\right)^p\right) + \frac{6d^3g^2px\log^2\left(c\left(d+ex^2\right)^p\right)}{7e^3} \\
&\quad - \frac{2d^2g^2px^3\log^2\left(c\left(d+ex^2\right)^p\right)}{7e^2} + \frac{6dg^2px^5\log^2\left(c\left(d+ex^2\right)^p\right)}{35e} \\
&\quad - \frac{6}{49} g^2 p x^7 \log^2\left(c\left(d+ex^2\right)^p\right) + f^2 x \log^3\left(c\left(d+ex^2\right)^p\right) \\
&\quad + \frac{1}{7} g^2 x^7 \log^3\left(c\left(d+ex^2\right)^p\right) + \frac{(fg)\text{Subst}\left(\int x\log^3\left(cx^p\right)\,dx, x, d+ex^2\right)}{e^2} \\
&\quad - \frac{(dfg)\text{Subst}\left(\int \log^3\left(cx^p\right)\,dx, x, d+ex^2\right)}{e^2} + \left(6df^2p\right)\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx \\
&\quad - \frac{\left(6d^4g^2p\right)\int \frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx}{7e^3} + \left(24ef^2p^2\right)\int \frac{x^2\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx \\
&\quad - \frac{1}{35}\left(24dg^2p^2\right)\int \frac{x^6\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx - \frac{\left(24d^3g^2p^2\right)\int \frac{x^2\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx}{7e^2} \\
&\quad + \frac{\left(8d^2g^2p^2\right)\int \frac{x^4\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx}{7e} + \frac{1}{49}\left(24eg^2p^2\right)\int \frac{x^8\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -6f^2px\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{6d^3g^2px\log^2\left(c\left(d+ex^2\right)^p\right)}{7e^3}\\
&\quad -\frac{2d^2g^2px^3\log^2\left(c\left(d+ex^2\right)^p\right)}{7e^2}+\frac{6dg^2px^5\log^2\left(c\left(d+ex^2\right)^p\right)}{35e}\\
&\quad -\frac{6}{49}g^2px^7\log^2\left(c\left(d+ex^2\right)^p\right)+f^2x\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log^3\left(c\left(d+ex^2\right)^p\right)\\
&\quad -\frac{dfg(d+ex^2)\log^3\left(c\left(d+ex^2\right)^p\right)}{e^2}+\frac{fg(d+ex^2)^2\log^3\left(c\left(d+ex^2\right)^p\right)}{2e^2}\\
&\quad +\left(6df^2p\right)\int\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx-\frac{(3fgp)\text{Subst}\left(\int x\log^2\left(cx^p\right)\,dx,x,d+ex^2\right)}{2e^2}\\
&\quad +\frac{(3dfgp)\text{Subst}\left(\int\log^2\left(cx^p\right)\,dx,x,d+ex^2\right)}{e^2}-\frac{\left(6d^4g^2p\right)\int\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx}{7e^3}\\
&\quad +\left(24ef^2p^2\right)\int\left(\frac{\log\left(c\left(d+ex^2\right)^p\right)}{e}-\frac{d\log\left(c\left(d+ex^2\right)^p\right)}{e\left(d+ex^2\right)}\right)dx\\
&\quad -\frac{1}{35}\left(24dg^2p^2\right)\int\left(\frac{d^2\log\left(c\left(d+ex^2\right)^p\right)}{e^3}-\frac{dx^2\log\left(c\left(d+ex^2\right)^p\right)}{e^2}\right.\\
&\qquad\qquad\left.+\frac{x^4\log\left(c\left(d+ex^2\right)^p\right)}{e}-\frac{d^3\log\left(c\left(d+ex^2\right)^p\right)}{e^3\left(d+ex^2\right)}\right)dx\\
&\quad -\frac{\left(24d^3g^2p^2\right)\int\left(\frac{\log\left(c\left(d+ex^2\right)^p\right)}{e}-\frac{d\log\left(c\left(d+ex^2\right)^p\right)}{e\left(d+ex^2\right)}\right)dx}{7e^2}\\
&\quad +\frac{\left(8d^2g^2p^2\right)\int\left(-\frac{d\log\left(c\left(d+ex^2\right)^p\right)}{e^2}+\frac{x^2\log\left(c\left(d+ex^2\right)^p\right)}{e}+\frac{d^2\log\left(c\left(d+ex^2\right)^p\right)}{e^2\left(d+ex^2\right)}\right)dx}{7e}\\
&\quad +\frac{1}{49}\left(24eg^2p^2\right)\int\left(-\frac{d^3\log\left(c\left(d+ex^2\right)^p\right)}{e^4}+\frac{d^2x^2\log\left(c\left(d+ex^2\right)^p\right)}{e^3}\right.\\
&\qquad\qquad\left.-\frac{dx^4\log\left(c\left(d+ex^2\right)^p\right)}{e^2}+\frac{x^6\log\left(c\left(d+ex^2\right)^p\right)}{e}+\frac{d^4\log\left(c\left(d+ex^2\right)^p\right)}{e^4\left(d+ex^2\right)}\right)dx
\end{aligned}
$$

$$
\begin{aligned}
&= -6f^2px\log^2\left(c(d+ex^2)^p\right)+\frac{6d^3g^2px\log^2\left(c(d+ex^2)^p\right)}{7e^3} \\
&\quad -\frac{2d^2g^2px^3\log^2\left(c(d+ex^2)^p\right)}{7e^2}+\frac{6dg^2px^5\log^2\left(c(d+ex^2)^p\right)}{35e} \\
&\quad -\frac{6}{49}g^2px^7\log^2\left(c(d+ex^2)^p\right)+\frac{3dfgp(d+ex^2)\log^2\left(c(d+ex^2)^p\right)}{e^2} \\
&\quad -\frac{3fgp(d+ex^2)^2\log^2\left(c(d+ex^2)^p\right)}{4e^2}+f^2x\log^3\left(c(d+ex^2)^p\right) \\
&\quad +\frac{1}{7}g^2x^7\log^3\left(c(d+ex^2)^p\right)-\frac{dfg(d+ex^2)\log^3\left(c(d+ex^2)^p\right)}{e^2} \\
&\quad +\frac{fg(d+ex^2)^2\log^3\left(c(d+ex^2)^p\right)}{2e^2}+\left(6df^2p\right)\int\frac{\log^2\left(c(d+ex^2)^p\right)}{d+ex^2}\,dx \\
&\quad -\frac{\left(6d^4g^2p\right)\int\frac{\log^2\left(c(d+ex^2)^p\right)}{d+ex^2}\,dx}{7e^3}+\left(24f^2p^2\right)\int\log\left(c(d+ex^2)^p\right)\,dx \\
&\quad -\left(24df^2p^2\right)\int\frac{\log\left(c(d+ex^2)^p\right)}{d+ex^2}\,dx+\frac{\left(3fgp^2\right)\mathrm{Subst}\left(\int x\log\left(cx^p\right)\,dx,x,d+ex^2\right)}{2e^2} \\
&\quad -\frac{\left(6dfgp^2\right)\mathrm{Subst}\left(\int\log\left(cx^p\right)\,dx,x,d+ex^2\right)}{e^2} \\
&\quad +\frac{1}{49}\left(24g^2p^2\right)\int x^6\log\left(c(d+ex^2)^p\right)\,dx-\frac{\left(24d^3g^2p^2\right)\int\log\left(c(d+ex^2)^p\right)\,dx}{49e^3} \\
&\quad -\frac{\left(24d^3g^2p^2\right)\int\log\left(c(d+ex^2)^p\right)\,dx}{35e^3}-\frac{\left(8d^3g^2p^2\right)\int\log\left(c(d+ex^2)^p\right)\,dx}{7e^3} \\
&\quad -\frac{\left(24d^3g^2p^2\right)\int\log\left(c(d+ex^2)^p\right)\,dx}{7e^3}+\frac{\left(24d^4g^2p^2\right)\int\frac{\log\left(c(d+ex^2)^p\right)}{d+ex^2}\,dx}{49e^3} \\
&\quad +\frac{\left(24d^4g^2p^2\right)\int\frac{\log\left(c(d+ex^2)^p\right)}{d+ex^2}\,dx}{35e^3}+\frac{\left(8d^4g^2p^2\right)\int\frac{\log\left(c(d+ex^2)^p\right)}{d+ex^2}\,dx}{7e^3} \\
&\quad +\frac{\left(24d^4g^2p^2\right)\int\frac{\log\left(c(d+ex^2)^p\right)}{d+ex^2}\,dx}{7e^3}+\frac{\left(24d^2g^2p^2\right)\int x^2\log\left(c(d+ex^2)^p\right)\,dx}{49e^2} \\
&\quad +\frac{\left(24d^2g^2p^2\right)\int x^2\log\left(c(d+ex^2)^p\right)\,dx}{35e^2}+\frac{\left(8d^2g^2p^2\right)\int x^2\log\left(c(d+ex^2)^p\right)\,dx}{7e^2} \\
&\quad -\frac{\left(24dg^2p^2\right)\int x^4\log\left(c(d+ex^2)^p\right)\,dx}{49e}-\frac{\left(24dg^2p^2\right)\int x^4\log\left(c(d+ex^2)^p\right)\,dx}{35e}
\end{aligned}
$$

$= \text{Too large to display}$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2385 vs. $2(1126) = 2252$.

Time = 8.70 (sec) , antiderivative size = 2385, normalized size of antiderivative = 99.38

$$\int \left(f+gx^3\right)^2 \log^3\left(c\left(d+ex^2\right)^p\right)\,dx = \text{Result too large to show}$$

```
[In] Integrate[(f + g*x^3)^2*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] (f*g*p^3*(d + e*x^2)*(-8*d*(-6 + 6*Log[d + e*x^2] - 3*Log[d + e*x^2]^2 + Lo
g[d + e*x^2]^3) + (d + e*x^2)*(-3 + 6*Log[d + e*x^2] - 6*Log[d + e*x^2]^2 +
 4*Log[d + e*x^2]^3)))/(8*e^2) + 6*f*g*p^2*((x^4*Log[d + e*x^2]^2)/4 - e*((
3*d*x^2)/(4*e^2) - x^4/(8*e) - (3*d^2*Log[d + e*x^2])/(4*e^3) - (d*x^2*Log[
d + e*x^2])/(2*e^2) + (x^4*Log[d + e*x^2])/(4*e) + (d^2*Log[d + e*x^2]^2)/(
4*e^3)))*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p]) + (3*d*f*g*p*x^2*(-(p
*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(2*e) - (2*d^2*g^2*p*x^3*(-(p*L
og[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(7*e^2) + (6*d*g^2*p*x^5*(-(p*Log
[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(35*e) - (3*d^2*f*g*p*Log[d + e*x^2
]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/(2*e^2) + (3*p*x*(14*f^2
+ 7*f*g*x^3 + 2*g^2*x^6)*Log[d + e*x^2]*(-(p*Log[d + e*x^2]) + Log[c*(d + e
*x^2)^p])^2)/14 + (f*g*x^4*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2*(
-3*p + 2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])))/4 + (g^2*x^7*(-(p*L
og[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2*(-6*p + 7*(-(p*Log[d + e*x^2]) + L
og[c*(d + e*x^2)^p])))/49 + (x*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])
^2*(-42*e^3*f^2*p + 6*d^3*g^2*p + 7*e^3*f^2*(-(p*Log[d + e*x^2]) + Log[c*(d
 + e*x^2)^p])))/(7*e^3) - (6*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-7*d*e^3*f^2*p*(-
(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2 + d^4*g^2*p*(-(p*Log[d + e*x^2
]) + Log[c*(d + e*x^2)^p])^2))/(7*Sqrt[d]*e^(7/2)) + 3*f^2*p^2*(-(p*Log[d +
 e*x^2]) + Log[c*(d + e*x^2)^p])*(x*Log[d + e*x^2]^2 - (4*((-I)*Sqrt[d]*Arc
Tan[(Sqrt[e]*x)/Sqrt[d]]^2 + Sqrt[e]*x*(-2 + Log[d + e*x^2]) - Sqrt[d]*ArcT
an[(Sqrt[e]*x)/Sqrt[d]]*(-2 + 2*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] +
Log[d + e*x^2]) - I*Sqrt[d]*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d
] + Sqrt[e]*x)]))/Sqrt[e]) + 3*g^2*p^2*(-(p*Log[d + e*x^2]) + Log[c*(d + e*
x^2)^p])*((x^7*Log[d + e*x^2]^2)/7 - (4*((11025*I)*d^(7/2)*ArcTan[(Sqrt[e]*
x)/Sqrt[d]]^2 + 105*d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]*(-352 + 210*Log[(2*
Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + 105*Log[d + e*x^2]) + Sqrt[e]*x*(36960*
d^3 - 4970*d^2*e*x^2 + 1512*d*e^2*x^4 - 450*e^3*x^6 - 105*(105*d^3 - 35*d^2
*e*x^2 + 21*d*e^2*x^4 - 15*e^3*x^6)*Log[d + e*x^2]) + (11025*I)*d^(7/2)*Pol
yLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] + Sqrt[e]*x)]))/(77175*e^(7/2
))) + (g^2*p^3*(702272*Sqrt[-d]*d^(7/2)*Sqrt[d + e*x^2]*Sqrt[1 - d/(d + e*x
^2)]*ArcSin[Sqrt[d]/Sqrt[d + e*x^2]] + 44100*Sqrt[-d]*d^(7/2)*Sqrt[1 - d/(d
 + e*x^2)]*(8*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/
2}, d/(d + e*x^2)] + 4*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2
```

```
}, d/(d + e*x^2)]*Log[d + e*x^2] + Sqrt[d + e*x^2]*ArcSin[Sqrt[d]/Sqrt[d +
e*x^2]]*Log[d + e*x^2]^2) - (2*Sqrt[-d]*e*x^2*(-1125*(d + e*x^2)^3*(-48 + 1
68*Log[d + e*x^2] - 294*Log[d + e*x^2]^2 + 343*Log[d + e*x^2]^3) + 27*d*(d
+ e*x^2)^2*(-18208 + 44520*Log[d + e*x^2] - 53900*Log[d + e*x^2]^2 + 42875*
Log[d + e*x^2]^3) + d^3*(-39193856 + 18434640*Log[d + e*x^2] - 3880800*Log[
d + e*x^2]^2 + 385875*Log[d + e*x^2]^3) - d^2*(d + e*x^2)*(-2762192 + 39244
80*Log[d + e*x^2] - 2690100*Log[d + e*x^2]^2 + 1157625*Log[d + e*x^2]^3)))/
105 - 73920*d^4*(4*Sqrt[e*x^2]*ArcTanh[Sqrt[e*x^2]/Sqrt[-d]]*(Log[d + e*x^2
] - Log[(d + e*x^2)/d]) - Sqrt[-d]*Sqrt[1 - (d + e*x^2)/d]*(Log[(d + e*x^2)
/d]^2 - 4*Log[(d + e*x^2)/d]*Log[(1 + Sqrt[1 - (d + e*x^2)/d])/2] + 2*Log[(
1 + Sqrt[1 - (d + e*x^2)/d])/2]^2 - 4*PolyLog[2, 1/2 - Sqrt[1 - (d + e*x^2)
/d]/2]))))/(51450*Sqrt[-d]*e^4*x) + (f^2*p^3*(-48*Sqrt[-d^2]*Sqrt[d + e*x^2
]*Sqrt[1 - d/(d + e*x^2)]*ArcSin[Sqrt[d]/Sqrt[d + e*x^2]] - 6*Sqrt[-d^2]*Sq
rt[1 - d/(d + e*x^2)]*(8*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3
/2, 3/2, 3/2}, d/(d + e*x^2)] + 4*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2, 1/2}
, {3/2, 3/2}, d/(d + e*x^2)]*Log[d + e*x^2] + Sqrt[d + e*x^2]*ArcSin[Sqrt[d
]/Sqrt[d + e*x^2]]*Log[d + e*x^2]^2) + Sqrt[-d]*e*x^2*(-48 + 24*Log[d + e*x
^2] - 6*Log[d + e*x^2]^2 + Log[d + e*x^2]^3) + 24*d*Sqrt[e*x^2]*ArcTanh[Sqr
t[e*x^2]/Sqrt[-d]]*(Log[d + e*x^2] - Log[(d + e*x^2)/d]) + 6*(-d)^(3/2)*Sqr
t[1 - (d + e*x^2)/d]*(Log[(d + e*x^2)/d]^2 - 4*Log[(d + e*x^2)/d]*Log[(1 +
Sqrt[1 - (d + e*x^2)/d])/2] + 2*Log[(1 + Sqrt[1 - (d + e*x^2)/d])/2]^2 - 4*
PolyLog[2, 1/2 - Sqrt[1 - (d + e*x^2)/d]/2])))/(Sqrt[-d]*e*x)
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \left(g\,x^{3}+f\right)^{2} \ln \left(c\left(e\,x^{2}+d\right)^{p}\right)^{3}dx$$

```
[In] int((g*x^3+f)^2*ln(c*(e*x^2+d)^p)^3,x)
[Out] int((g*x^3+f)^2*ln(c*(e*x^2+d)^p)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \left(f+gx^3\right)^2 \log^3\left(c\left(d+ex^2\right)^p\right)\,dx = \int \left(gx^{3}+f\right)^{2} \log \left(\left(ex^{2}+d\right)^{p} c\right)^{3}\,dx$$

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^3,x, algorithm="fricas")
[Out] integral((g^2*x^6 + 2*f*g*x^3 + f^2)*log((e*x^2 + d)^p*c)^3, x)
```

Sympy [N/A]

Not integrable

Time = 32.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \left(f+gx^3\right)^2 \log^3\left(c\left(d+ex^2\right)^p\right)\,dx=\int \left(f+gx^3\right)^2 \log\left(c\left(d+ex^2\right)^p\right)^3\,dx$$

```
[In] integrate((g*x**3+f)**2*ln(c*(e*x**2+d)**p)**3,x)
[Out] Integral((f + g*x**3)**2*log(c*(d + e*x**2)**p)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(f+gx^3\right)^2 \log^3\left(c\left(d+ex^2\right)^p\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \left(f+gx^3\right)^2 \log^3\left(c\left(d+ex^2\right)^p\right)\,dx=\int \left(gx^3+f\right)^2 \log\left(\left(ex^2+d\right)^p c\right)^3\,dx$$

```
[In] integrate((g*x^3+f)^2*log(c*(e*x^2+d)^p)^3,x, algorithm="giac")
[Out] integrate((g*x^3 + f)^2*log((e*x^2 + d)^p*c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (f+gx^3)^2 \log^3 \left(c(d+ex^2)^p\right) \, dx = \int \ln \left(c\left(e\,x^2+d\right)^p\right)^3 \left(g\,x^3+f\right)^2 dx$$

```
[In] int(log(c*(d + e*x^2)^p)^3*(f + g*x^3)^2,x)
[Out] int(log(c*(d + e*x^2)^p)^3*(f + g*x^3)^2, x)
```

3.299 $\int (f+gx^3) \log^3 \left(c(d+ex^2)^p\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (f+gx^3)\log^3\left(c(d+ex^2)^p\right)\,dx = -48fp^3x+\frac{3dgp^3x^2}{e}-\frac{3gp^3(d+ex^2)^2}{16e^2} + \frac{48\sqrt{d}fp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{24i\sqrt{d}fp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} - \frac{48\sqrt{d}fp^3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} + 24fp^2x\log\left(c(d+ex^2)^p\right) - \frac{3dgp^2(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e^2} + \frac{3gp^2(d+ex^2)^2\log\left(c(d+ex^2)^p\right)}{8e^2} - \frac{24\sqrt{d}fp^2\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{e}} - 6fpx\log^2\left(c(d+ex^2)^p\right) + \frac{3dgp(d+ex^2)\log^2\left(c(d+ex^2)^p\right)}{2e^2} - \frac{3gp(d+ex^2)^2\log^2\left(c(d+ex^2)^p\right)}{8e^2} + fx\log^3\left(c(d+ex^2)^p\right) - \frac{dg(d+ex^2)\log^3\left(c(d+ex^2)^p\right)}{2e^2} + \frac{g(d+ex^2)^2\log^3\left(c(d+ex^2)^p\right)}{4e^2} - \frac{24i\sqrt{d}fp^3\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} + 6dfp\operatorname{Int}\left(\frac{\log^2\left(c(d+ex^2)^p\right)}{d+ex^2},x\right)$$

```
[Out] -48*f*p^3*x+3*d*g*p^3*x^2/e-3/16*g*p^3*(e*x^2+d)^2/e^2+24*f*p^2*x*ln(c*(e*x
^2+d)^p)-3*d*g*p^2*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e^2+3/8*g*p^2*(e*x^2+d)^2*ln
(c*(e*x^2+d)^p)/e^2-6*f*p*x*ln(c*(e*x^2+d)^p)^2+3/2*d*g*p*(e*x^2+d)*ln(c*(e
*x^2+d)^p)^2/e^2-3/8*g*p*(e*x^2+d)^2*ln(c*(e*x^2+d)^p)^2/e^2+f*x*ln(c*(e*x^
2+d)^p)^3-1/2*d*g*(e*x^2+d)*ln(c*(e*x^2+d)^p)^3/e^2+1/4*g*(e*x^2+d)^2*ln(c*
(e*x^2+d)^p)^3/e^2+48*f*p^3*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)-24*I*
f*p^3*arctan(x*e^(1/2)/d^(1/2))^2*d^(1/2)/e^(1/2)-24*f*p^2*arctan(x*e^(1/2)
```

```
/d^(1/2))*ln(c*(e*x^2+d)^p)*d^(1/2)/e^(1/2)-48*f*p^3*arctan(x*e^(1/2)/d^(1/
2))*ln(2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)-24*I*f*p^3*polylog(
2,1-2*d^(1/2)/(d^(1/2)+I*x*e^(1/2)))*d^(1/2)/e^(1/2)+6*d*f*p*Unintegrable(l
n(c*(e*x^2+d)^p)^2/(e*x^2+d),x)
```

Rubi [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(f+gx^3\right) \log^3\left(c\left(d+ex^2\right)^p\right) \, dx=\int \left(f+gx^3\right) \log^3\left(c\left(d+ex^2\right)^p\right) \, dx$$

```
[In] Int[(f + g*x^3)*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] -48*f*p^3*x + (3*d*g*p^3*x^2)/e - (3*g*p^3*(d + e*x^2)^2)/(16*e^2) + (48*Sq
rt[d]*f*p^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - ((24*I)*Sqrt[d]*f*p^3*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]]^2)/Sqrt[e] - (48*Sqrt[d]*f*p^3*ArcTan[(Sqrt[e]*x)
/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + 24*f*p^2*x*Lo
g[c*(d + e*x^2)^p] - (3*d*g*p^2*(d + e*x^2)*Log[c*(d + e*x^2)^p])/e^2 + (3*
g*p^2*(d + e*x^2)^2*Log[c*(d + e*x^2)^p])/(8*e^2) - (24*Sqrt[d]*f*p^2*ArcTa
n[(Sqrt[e]*x)/Sqrt[d]]*Log[c*(d + e*x^2)^p])/Sqrt[e] - 6*f*p*x*Log[c*(d + e
*x^2)^p]^2 + (3*d*g*p*(d + e*x^2)*Log[c*(d + e*x^2)^p]^2)/(2*e^2) - (3*g*p*
(d + e*x^2)^2*Log[c*(d + e*x^2)^p]^2)/(8*e^2) + f*x*Log[c*(d + e*x^2)^p]^3
- (d*g*(d + e*x^2)*Log[c*(d + e*x^2)^p]^3)/(2*e^2) + (g*(d + e*x^2)^2*Log[c
*(d + e*x^2)^p]^3)/(4*e^2) - ((24*I)*Sqrt[d]*f*p^3*PolyLog[2, 1 - (2*Sqrt[d
])/(Sqrt[d] + I*Sqrt[e]*x)])/Sqrt[e] + 6*d*f*p*Defer[Int][Log[c*(d + e*x^2)
^p]^2/(d + e*x^2), x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(f \log^3\left(c\left(d+ex^2\right)^p\right)+gx^3 \log^3\left(c\left(d+ex^2\right)^p\right)\right) \, dx \\
&= f \int \log^3\left(c\left(d+ex^2\right)^p\right) \, dx+g \int x^3 \log^3\left(c\left(d+ex^2\right)^p\right) \, dx \\
&= fx \log^3\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}g\text{Subst}\left(\int x \log^3\left(c(d+ex)^p\right) \, dx,x,x^2\right) \\
&\quad -(6efp) \int \frac{x^2 \log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2} \, dx \\
&= fx \log^3\left(c\left(d+ex^2\right)^p\right) \\
&\quad +\frac{1}{2}g\text{Subst}\left(\int \left(-\frac{d \log^3\left(c(d+ex)^p\right)}{e}+\frac{(d+ex) \log^3\left(c(d+ex)^p\right)}{e}\right) \, dx,x,x^2\right) \\
&\quad -(6efp) \int \left(\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{e}-\frac{d \log^2\left(c\left(d+ex^2\right)^p\right)}{e\left(d+ex^2\right)}\right) \, dx
\end{aligned}$$

$$= fx\log^3\left(c\left(d+ex^2\right)^p\right)+\frac{g\text{Subst}\left(\int(d+ex)\log^3\left(c(d+ex)^p\right)\,dx,x,x^2\right)}{2e}$$
$$-\frac{(dg)\text{Subst}\left(\int\log^3\left(c(d+ex)^p\right)\,dx,x,x^2\right)}{2e}$$
$$-(6fp)\int\log^2\left(c\left(d+ex^2\right)^p\right)\,dx+(6dfp)\int\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx$$
$$= -6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+fx\log^3\left(c\left(d+ex^2\right)^p\right)$$
$$+\frac{g\text{Subst}\left(\int x\log^3\left(cx^p\right)\,dx,x,d+ex^2\right)}{2e^2}-\frac{(dg)\text{Subst}\left(\int\log^3\left(cx^p\right)\,dx,x,d+ex^2\right)}{2e^2}$$
$$+(6dfp)\int\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx+\left(24efp^2\right)\int\frac{x^2\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx$$
$$= -6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+fx\log^3\left(c\left(d+ex^2\right)^p\right)$$
$$-\frac{dg\left(d+ex^2\right)\log^3\left(c\left(d+ex^2\right)^p\right)}{2e^2}+\frac{g\left(d+ex^2\right)^2\log^3\left(c\left(d+ex^2\right)^p\right)}{4e^2}$$
$$+(6dfp)\int\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx-\frac{(3gp)\text{Subst}\left(\int x\log^2\left(cx^p\right)\,dx,x,d+ex^2\right)}{4e^2}$$
$$+\frac{(3dgp)\text{Subst}\left(\int\log^2\left(cx^p\right)\,dx,x,d+ex^2\right)}{2e^2}$$
$$+\left(24efp^2\right)\int\left(\frac{\log\left(c\left(d+ex^2\right)^p\right)}{e}-\frac{d\log\left(c\left(d+ex^2\right)^p\right)}{e\left(d+ex^2\right)}\right)\,dx$$
$$= -6fpx\log^2\left(c\left(d+ex^2\right)^p\right)+\frac{3dgp\left(d+ex^2\right)\log^2\left(c\left(d+ex^2\right)^p\right)}{2e^2}$$
$$-\frac{3gp\left(d+ex^2\right)^2\log^2\left(c\left(d+ex^2\right)^p\right)}{8e^2}+fx\log^3\left(c\left(d+ex^2\right)^p\right)$$
$$-\frac{dg\left(d+ex^2\right)\log^3\left(c\left(d+ex^2\right)^p\right)}{2e^2}+\frac{g\left(d+ex^2\right)^2\log^3\left(c\left(d+ex^2\right)^p\right)}{4e^2}$$
$$+(6dfp)\int\frac{\log^2\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx+\left(24fp^2\right)\int\log\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$-\left(24dfp^2\right)\int\frac{\log\left(c\left(d+ex^2\right)^p\right)}{d+ex^2}\,dx+\frac{\left(3gp^2\right)\text{Subst}\left(\int x\log\left(cx^p\right)\,dx,x,d+ex^2\right)}{4e^2}$$
$$-\frac{\left(3dgp^2\right)\text{Subst}\left(\int\log\left(cx^p\right)\,dx,x,d+ex^2\right)}{e^2}$$

$$
\begin{aligned}
&= \frac{3dgp^3x^2}{e} - \frac{3gp^3(d+ex^2)^2}{16e^2} + 24fp^2x\log\left(c(d+ex^2)^p\right) \\
&\quad - \frac{3dgp^2(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e^2} + \frac{3gp^2(d+ex^2)^2\log\left(c(d+ex^2)^p\right)}{8e^2} \\
&\quad - \frac{24\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{e}} - 6fpx\log^2\left(c(d+ex^2)^p\right) \\
&\quad + \frac{3dgp(d+ex^2)\log^2\left(c(d+ex^2)^p\right)}{2e^2} - \frac{3gp(d+ex^2)^2\log^2\left(c(d+ex^2)^p\right)}{8e^2} \\
&\quad + fx\log^3\left(c(d+ex^2)^p\right) - \frac{dg(d+ex^2)\log^3\left(c(d+ex^2)^p\right)}{2e^2} \\
&\quad + \frac{g(d+ex^2)^2\log^3\left(c(d+ex^2)^p\right)}{4e^2} + (6dfp)\int\frac{\log^2\left(c(d+ex^2)^p\right)}{d+ex^2}\,dx \\
&\quad - \left(48efp^3\right)\int\frac{x^2}{d+ex^2}\,dx + \left(48defp^3\right)\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}\,(d+ex^2)}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -48fp^3x + \frac{3dgp^3x^2}{e} - \frac{3gp^3(d+ex^2)^2}{16e^2} + 24fp^2x\log\left(c(d+ex^2)^p\right) \\
&\quad - \frac{3dgp^2(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e^2} + \frac{3gp^2(d+ex^2)^2\log\left(c(d+ex^2)^p\right)}{8e^2} \\
&\quad - \frac{24\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{e}} - 6fpx\log^2\left(c(d+ex^2)^p\right) \\
&\quad + \frac{3dgp(d+ex^2)\log^2\left(c(d+ex^2)^p\right)}{2e^2} - \frac{3gp(d+ex^2)^2\log^2\left(c(d+ex^2)^p\right)}{8e^2} \\
&\quad + fx\log^3\left(c(d+ex^2)^p\right) - \frac{dg(d+ex^2)\log^3\left(c(d+ex^2)^p\right)}{2e^2} \\
&\quad + \frac{g(d+ex^2)^2\log^3\left(c(d+ex^2)^p\right)}{4e^2} + (6dfp)\int\frac{\log^2\left(c(d+ex^2)^p\right)}{d+ex^2}\,dx \\
&\quad + \left(48dfp^3\right)\int\frac{1}{d+ex^2}\,dx + \left(48\sqrt{d}\sqrt{e}fp^3\right)\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d+ex^2}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -48fp^3x + \frac{3dgp^3x^2}{e} - \frac{3gp^3(d+ex^2)^2}{16e^2} + \frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} \\
&\quad - \frac{24i\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} + 24fp^2x\log\left(c\left(d+ex^2\right)^p\right) \\
&\quad - \frac{3dgp^2(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e^2} + \frac{3gp^2(d+ex^2)^2\log\left(c(d+ex^2)^p\right)}{8e^2} \\
&\quad - \frac{24\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{e}} \\
&\quad - 6fpx\log^2\left(c\left(d+ex^2\right)^p\right) + \frac{3dgp(d+ex^2)\log^2\left(c(d+ex^2)^p\right)}{2e^2} \\
&\quad - \frac{3gp(d+ex^2)^2\log^2\left(c(d+ex^2)^p\right)}{8e^2} + fx\log^3\left(c\left(d+ex^2\right)^p\right) \\
&\quad - \frac{dg(d+ex^2)\log^3\left(c(d+ex^2)^p\right)}{2e^2} + \frac{g(d+ex^2)^2\log^3\left(c(d+ex^2)^p\right)}{4e^2} \\
&\quad + (6dfp)\int \frac{\log^2\left(c(d+ex^2)^p\right)}{d+ex^2}\,dx - \left(48fp^3\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{i-\frac{\sqrt{e}x}{\sqrt{d}}}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -48fp^3x + \frac{3dgp^3x^2}{e} - \frac{3gp^3(d+ex^2)^2}{16e^2} + \frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} \\
&\quad - \frac{24i\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} - \frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} \\
&\quad + 24fp^2x\log\left(c\left(d+ex^2\right)^p\right) - \frac{3dgp^2(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e^2} \\
&\quad + \frac{3gp^2(d+ex^2)^2\log\left(c(d+ex^2)^p\right)}{8e^2} - \frac{24\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{e}} \\
&\quad - 6fpx\log^2\left(c\left(d+ex^2\right)^p\right) + \frac{3dgp(d+ex^2)\log^2\left(c(d+ex^2)^p\right)}{2e^2} \\
&\quad - \frac{3gp(d+ex^2)^2\log^2\left(c(d+ex^2)^p\right)}{8e^2} + fx\log^3\left(c\left(d+ex^2\right)^p\right) \\
&\quad - \frac{dg(d+ex^2)\log^3\left(c(d+ex^2)^p\right)}{2e^2} + \frac{g(d+ex^2)^2\log^3\left(c(d+ex^2)^p\right)}{4e^2} \\
&\quad + (6dfp)\int \frac{\log^2\left(c(d+ex^2)^p\right)}{d+ex^2}\,dx + \left(48fp^3\right)\int \frac{\log\left(\frac{2}{1+\frac{i\sqrt{e}x}{\sqrt{d}}}\right)}{1+\frac{ex^2}{d}}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -48fp^3x + \frac{3dgp^3x^2}{e} - \frac{3gp^3(d+ex^2)^2}{16e^2} + \frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} \\
&\quad - \frac{24i\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} - \frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} \\
&\quad + 24fp^2x\log\left(c(d+ex^2)^p\right) - \frac{3dgp^2(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e^2} \\
&\quad + \frac{3gp^2(d+ex^2)^2\log\left(c(d+ex^2)^p\right)}{8e^2} - \frac{24\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{e}} \\
&\quad - 6fpx\log^2\left(c(d+ex^2)^p\right) + \frac{3dgp(d+ex^2)\log^2\left(c(d+ex^2)^p\right)}{2e^2} \\
&\quad - \frac{3gp(d+ex^2)^2\log^2\left(c(d+ex^2)^p\right)}{8e^2} + fx\log^3\left(c(d+ex^2)^p\right) \\
&\quad - \frac{dg(d+ex^2)\log^3\left(c(d+ex^2)^p\right)}{2e^2} + \frac{g(d+ex^2)^2\log^3\left(c(d+ex^2)^p\right)}{4e^2} \\
&\quad + (6dfp)\int\frac{\log^2\left(c(d+ex^2)^p\right)}{d+ex^2}\,dx - \frac{\left(48i\sqrt{d}fp^3\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+\frac{i\sqrt{e}x}{\sqrt{d}}}\right)}{\sqrt{e}}
\end{aligned}
$$

$$
\begin{aligned}
&= -48fp^3x + \frac{3dgp^3x^2}{e} - \frac{3gp^3(d+ex^2)^2}{16e^2} + \frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} \\
&\quad - \frac{24i\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{\sqrt{e}} - \frac{48\sqrt{d}fp^3\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} \\
&\quad + 24fp^2x\log\left(c(d+ex^2)^p\right) - \frac{3dgp^2(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e^2} \\
&\quad + \frac{3gp^2(d+ex^2)^2\log\left(c(d+ex^2)^p\right)}{8e^2} - \frac{24\sqrt{d}fp^2\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{e}} \\
&\quad - 6fpx\log^2\left(c(d+ex^2)^p\right) + \frac{3dgp(d+ex^2)\log^2\left(c(d+ex^2)^p\right)}{2e^2} \\
&\quad - \frac{3gp(d+ex^2)^2\log^2\left(c(d+ex^2)^p\right)}{8e^2} + fx\log^3\left(c(d+ex^2)^p\right) \\
&\quad - \frac{dg(d+ex^2)\log^3\left(c(d+ex^2)^p\right)}{2e^2} + \frac{g(d+ex^2)^2\log^3\left(c(d+ex^2)^p\right)}{4e^2} \\
&\quad - \frac{24i\sqrt{d}fp^3\text{Li}_2\left(1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)}{\sqrt{e}} + (6dfp)\int\frac{\log^2\left(c(d+ex^2)^p\right)}{d+ex^2}\,dx
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1051 vs. 2(518) = 1036.

Time = 1.30 (sec) , antiderivative size = 1051, normalized size of antiderivative = 47.77

$$\int (f+gx^3)\log^3\left(c(d+ex^2)^p\right)\,dx$$

$$\begin{aligned}
&= \frac{1}{4}gx^4\log^3\left(c(d+ex^2)^p\right) + \frac{6\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-p\log(d+ex^2)+\log\left(c(d+ex^2)^p\right)\right)^2}{\sqrt{e}} \\
&\quad + 3fpx\log(d+ex^2)\left(-p\log(d+ex^2)+\log\left(c(d+ex^2)^p\right)\right)^2 \\
&\quad + fx\left(-p\log(d+ex^2)+\log\left(c(d+ex^2)^p\right)\right)^2\left(-6p-p\log(d+ex^2)+\log\left(c(d+ex^2)^p\right)\right) \\
&\quad - \frac{3}{4}gp\left(-\frac{7dp^2x^2}{2e}+\frac{p^2x^4}{4}+\frac{d^2p^2\log(d+ex^2)}{2e^2}+\frac{3d^2p\log\left(c(d+ex^2)^p\right)}{e^2}\right. \\
&\qquad + \frac{3dpx^2\log\left(c(d+ex^2)^p\right)}{e} - \frac{1}{2}px^4\log\left(c(d+ex^2)^p\right) - \frac{3d^2\log^2\left(c(d+ex^2)^p\right)}{2e^2} \\
&\qquad \left. - \frac{dx^2\log^2\left(c(d+ex^2)^p\right)}{e} + \frac{1}{2}x^4\log^2\left(c(d+ex^2)^p\right) + \frac{d^2\log^3\left(c(d+ex^2)^p\right)}{3e^2p}\right) \\
&\quad + 3fp^2\left(-p\log(d+ex^2)+\log\left(c(d+ex^2)^p\right)\right)\left(x\log^2(d+ex^2)\right. \\
&\quad - \frac{4\left(-i\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2+\sqrt{e}x(-2+\log(d+ex^2))-\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-2+2\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}x}\right)+\log(d\ldots\right.\right.}{\sqrt{e}} \\
&\quad + \frac{fp^3\left(-48\sqrt{-d^2}\sqrt{d+ex^2}\sqrt{1-\frac{d}{d+ex^2}}\arcsin\left(\frac{\sqrt{d}}{\sqrt{d+ex^2}}\right)-6\sqrt{-d^2}\sqrt{1-\frac{d}{d+ex^2}}\left(8\sqrt{d}\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2},\frac{3}{2};\frac{d}{\ldots}\right.\right.\right.}{\ldots}
\end{aligned}$$

```
[In] Integrate[(f + g*x^3)*Log[c*(d + e*x^2)^p]^3,x]
```

```
[Out] (g*x^4*Log[c*(d + e*x^2)^p]^3)/4 + (6*Sqrt[d]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d
]]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2)/Sqrt[e] + 3*f*p*x*Log[d
+ e*x^2]*(-(p*Log[d + e*x^2]) + Log[c*(d + e*x^2)^p])^2 + f*x*(-(p*Log[d +
e*x^2]) + Log[c*(d + e*x^2)^p])^2*(-6*p - p*Log[d + e*x^2] + Log[c*(d + e*x
^2)^p]) - (3*g*p*((-7*d*p^2*x^2)/(2*e) + (p^2*x^4)/4 + (d^2*p^2*Log[d + e*x
^2])/(2*e^2) + (3*d^2*p*Log[c*(d + e*x^2)^p])/e^2 + (3*d*p*x^2*Log[c*(d + e
*x^2)^p])/e - (p*x^4*Log[c*(d + e*x^2)^p])/2 - (3*d^2*Log[c*(d + e*x^2)^p]^
2)/(2*e^2) - (d*x^2*Log[c*(d + e*x^2)^p]^2)/e + (x^4*Log[c*(d + e*x^2)^p]^2
)/2 + (d^2*Log[c*(d + e*x^2)^p]^3)/(3*e^2*p)))/4 + 3*f*p^2*(-(p*Log[d + e*x
```

```
^2]) + Log[c*(d + e*x^2)^p])*(x*Log[d + e*x^2]^2 - (4*((-I)*Sqrt[d]*ArcTan[
(Sqrt[e]*x)/Sqrt[d]]^2 + Sqrt[e]*x*(-2 + Log[d + e*x^2]) - Sqrt[d]*ArcTan[(
Sqrt[e]*x)/Sqrt[d]]*(-2 + 2*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x)] + Log[
d + e*x^2]) - I*Sqrt[d]*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x)/((-I)*Sqrt[d] +
Sqrt[e]*x)]))/Sqrt[e]) + (f*p^3*(-48*Sqrt[-d^2]*Sqrt[d + e*x^2]*Sqrt[1 - d/
(d + e*x^2)]*ArcSin[Sqrt[d]/Sqrt[d + e*x^2]] - 6*Sqrt[-d^2]*Sqrt[1 - d/(d +
 e*x^2)]*(8*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}
, d/(d + e*x^2)] + 4*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3/2},
 d/(d + e*x^2)]*Log[d + e*x^2] + Sqrt[d + e*x^2]*ArcSin[Sqrt[d]/Sqrt[d + e*
x^2]]*Log[d + e*x^2]^2) + Sqrt[-d]*e*x^2*(-48 + 24*Log[d + e*x^2] - 6*Log[d
 + e*x^2]^2 + Log[d + e*x^2]^3) + 24*d*Sqrt[e*x^2]*ArcTanh[Sqrt[e*x^2]/Sqrt
[-d]]*(Log[d + e*x^2] - Log[(d + e*x^2)/d]) + 6*(-d)^(3/2)*Sqrt[1 - (d + e*
x^2)/d]*(Log[(d + e*x^2)/d]^2 - 4*Log[(d + e*x^2)/d]*Log[(1 + Sqrt[1 - (d +
 e*x^2)/d])/2] + 2*Log[(1 + Sqrt[1 - (d + e*x^2)/d])/2]^2 - 4*PolyLog[2, 1/
2 - Sqrt[1 - (d + e*x^2)/d]/2])))/(Sqrt[-d]*e*x)
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \left(g\,x^{3}+f\right) \ln \left(c\left(e\,x^{2}+d\right)^{p}\right)^{3}dx$$

```
[In] int((g*x^3+f)*ln(c*(e*x^2+d)^p)^3,x)
```

```
[Out] int((g*x^3+f)*ln(c*(e*x^2+d)^p)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \left(f+gx^3\right) \log^3\left(c\left(d+ex^2\right)^p\right)\,dx=\int \left(gx^{3}+f\right) \log \left(\left(ex^{2}+d\right)^{p}c\right)^{3}\,dx$$

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="fricas")
```

```
[Out] integral((g*x^3 + f)*log((e*x^2 + d)^p*c)^3, x)
```

Sympy [N/A]

Not integrable

Time = 10.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \left(f+gx^3\right)\log^3\left(c\left(d+ex^2\right)^p\right)\,dx=\int \left(f+gx^3\right)\log\left(c\left(d+ex^2\right)^p\right)^3\,dx$$

```
[In] integrate((g*x**3+f)*ln(c*(e*x**2+d)**p)**3,x)
[Out] Integral((f + g*x**3)*log(c*(d + e*x**2)**p)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(f+gx^3\right)\log^3\left(c\left(d+ex^2\right)^p\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \left(f+gx^3\right)\log^3\left(c\left(d+ex^2\right)^p\right)\,dx=\int \left(gx^3+f\right)\log\left(\left(ex^2+d\right)^pc\right)^3\,dx$$

```
[In] integrate((g*x^3+f)*log(c*(e*x^2+d)^p)^3,x, algorithm="giac")
[Out] integrate((g*x^3 + f)*log((e*x^2 + d)^p*c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (f + gx^3) \log^3 \left(c(d + ex^2)^p\right) \, dx = \int \ln \left(c \left(e\,x^2 + d\right)^p\right)^3 \left(g\,x^3 + f\right) \, dx$$

```
[In] int(log(c*(d + e*x^2)^p)^3*(f + g*x^3),x)
[Out] int(log(c*(d + e*x^2)^p)^3*(f + g*x^3), x)
```

3.300 $\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx=\text{Int}\left(\frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3},x\right)$$

```
[Out] Unintegrable(ln(c*(e*x^2+d)^p)^3/(g*x^3+f),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx=\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx$$

```
[In] Int[Log[c*(d + e*x^2)^p]^3/(f + g*x^3),x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^3/(f + g*x^3), x]
```

Rubi steps

$$\text{integral}=\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{f+gx^3}\,dx$$

Mathematica [N/A]

Not integrable

Time = 27.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx$$

```
[In] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^3),x]
[Out] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^3), x]
```

Maple [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c(e\,x^2+d)^p\right)^3}{g\,x^3+f}dx$$

```
[In] int(ln(c*(e*x^2+d)^p)^3/(g*x^3+f),x)
[Out] int(ln(c*(e*x^2+d)^p)^3/(g*x^3+f),x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^3}{gx^3+f}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f),x, algorithm="fricas")
[Out] integral(log((e*x^2 + d)^p*c)^3/(g*x^3 + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(e*x**2+d)**p)**3/(g*x**3+f),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^3}{gx^3+f}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f),x, algorithm="maxima")
[Out] integrate(log((e*x^2 + d)^p*c)^3/(g*x^3 + f), x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)^3}{gx^3+f}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f),x, algorithm="giac")
[Out] integrate(log((e*x^2 + d)^p*c)^3/(g*x^3 + f), x)
```

Mupad [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{f+gx^3}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)^3}{g\,x^3+f}\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)^3/(f + g*x^3),x)
[Out] int(log(c*(d + e*x^2)^p)^3/(f + g*x^3), x)
```

3.301 $\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2} dx = \operatorname{Int}\left(\frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2}, x\right)$$

```
[Out] Unintegrable(ln(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2} dx = \int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2} dx$$

```
[In] Int[Log[c*(d + e*x^2)^p]^3/(f + g*x^3)^2,x]
[Out] Defer[Int][Log[c*(d + e*x^2)^p]^3/(f + g*x^3)^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\log^3\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^3\right)^2} dx$$

Mathematica [N/A]

Not integrable

Time = 29.94 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx = \int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx$$

```
[In] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^3)^2,x]
[Out] Integrate[Log[c*(d + e*x^2)^p]^3/(f + g*x^3)^2, x]
```

Maple [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c(e\,x^2+d)^p\right)^3}{(g\,x^3+f)^2}dx$$

```
[In] int(ln(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x)
[Out] int(ln(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx = \int \frac{\log\left((ex^2+d)^pc\right)^3}{(gx^3+f)^2}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x, algorithm="fricas")
[Out] integral(log((e*x^2 + d)^p*c)^3/(g^2*x^6 + 2*f*g*x^3 + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{\left(f+gx^3\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(e*x**2+d)**p)**3/(g*x**3+f)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{\left(f+gx^3\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{\left(f+gx^3\right)^2}\,dx = \int \frac{\log\left(\left(ex^2+d\right)^p c\right)^3}{\left(gx^3+f\right)^2}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)^3/(g*x^3+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)^3/(g*x^3 + f)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\log^3\left(c(d+ex^2)^p\right)}{(f+gx^3)^2}\,dx=\int \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)^3}{{\left(g\,x^3+f\right)}^2}\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)^3/(f + g*x^3)^2,x)
[Out] int(log(c*(d + e*x^2)^p)^3/(f + g*x^3)^2, x)
```

3.302 $\int \frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)}\,dx=\operatorname{Int}\left(\frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)},x\right)$$

```
[Out] Unintegrable((g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)}\,dx=\int \frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)}\,dx$$

```
[In] Int[(f + g*x^3)^2/Log[c*(d + e*x^2)^p],x]
[Out] Defer[Int][(f + g*x^3)^2/Log[c*(d + e*x^2)^p], x]
```

Rubi steps

$$\text{integral}=\int \frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^3)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(f+gx^3)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] Integrate[(f + g*x^3)^2/Log[c*(d + e*x^2)^p],x]
[Out] Integrate[(f + g*x^3)^2/Log[c*(d + e*x^2)^p], x]
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(g\,x^{3}+f\right)^{2}}{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}dx$$

```
[In] int((g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
[Out] int((g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{(f+gx^3)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(gx^3+f)^2}{\log\left((ex^2+d)^pc\right)}\,dx$$

```
[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="fricas")
[Out] integral((g^2*x^6 + 2*f*g*x^3 + f^2)/log((e*x^2 + d)^p*c), x)
```

Sympy [N/A]

Not integrable

Time = 14.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{(f+gx^3)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(f+gx^3)^2}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] integrate((g*x**3+f)**2/ln(c*(e*x**2+d)**p),x)
[Out] Integral((f + g*x**3)**2/log(c*(d + e*x**2)**p), x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)}\,dx = \int \frac{(gx^3+f)^2}{\log((ex^2+d)^pc)}\,dx$$

```
[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] integrate((g*x^3 + f)^2/log((e*x^2 + d)^p*c), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)}\,dx = \int \frac{(gx^3+f)^2}{\log((ex^2+d)^pc)}\,dx$$

```
[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="giac")
[Out] integrate((g*x^3 + f)^2/log((e*x^2 + d)^p*c), x)
```

Mupad [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)}\,dx = \int \frac{(g\,x^3+f)^2}{\ln(c\,(e\,x^2+d)^p)}\,dx$$

```
[In] int((f + g*x^3)^2/log(c*(d + e*x^2)^p),x)
[Out] int((f + g*x^3)^2/log(c*(d + e*x^2)^p), x)
```

3.303 $\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\text{Int}\left(\frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable((g*x^3+f)/ln(c*(e*x^2+d)^p),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] Int[(f + g*x^3)/Log[c*(d + e*x^2)^p],x]
```

```
[Out] Defer[Int][(f + g*x^3)/Log[c*(d + e*x^2)^p], x]
```

Rubi steps

$$\text{integral}=\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] Integrate[(f + g*x^3)/Log[c*(d + e*x^2)^p],x]
[Out] Integrate[(f + g*x^3)/Log[c*(d + e*x^2)^p], x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{g\,x^{3}+f}{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}dx$$

```
[In] int((g*x^3+f)/ln(c*(e*x^2+d)^p),x)
[Out] int((g*x^3+f)/ln(c*(e*x^2+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{gx^3+f}{\log\left(\left(ex^2+d\right)^pc\right)}\,dx$$

```
[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="fricas")
[Out] integral((g*x^3 + f)/log((e*x^2 + d)^p*c), x)
```

Sympy [N/A]

Not integrable

Time = 5.66 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] integrate((g*x**3+f)/ln(c*(e*x**2+d)**p),x)
[Out] Integral((f + g*x**3)/log(c*(d + e*x**2)**p), x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f + gx^3}{\log\left(c\left(d + ex^2\right)^p\right)}\,dx = \int \frac{gx^3 + f}{\log\left(\left(ex^2 + d\right)^p c\right)}\,dx$$

```
[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] integrate((g*x^3 + f)/log((e*x^2 + d)^p*c), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f + gx^3}{\log\left(c\left(d + ex^2\right)^p\right)}\,dx = \int \frac{gx^3 + f}{\log\left(\left(ex^2 + d\right)^p c\right)}\,dx$$

```
[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="giac")
[Out] integrate((g*x^3 + f)/log((e*x^2 + d)^p*c), x)
```

Mupad [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f + gx^3}{\log\left(c\left(d + ex^2\right)^p\right)}\,dx = \int \frac{g\,x^3 + f}{\ln\left(c\left(e\,x^2 + d\right)^p\right)}\,dx$$

```
[In] int((f + g*x^3)/log(c*(d + e*x^2)^p),x)
[Out] int((f + g*x^3)/log(c*(d + e*x^2)^p), x)
```

3.304 $\int \frac{1}{(f+gx^3)\log(c(d+ex^2)^p)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx = \text{Int}\left(\frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x^3+f)/ln(c*(e*x^2+d)^p),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] Int[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]),x]
```

```
[Out] Defer[Int][1/((f + g*x^3)*Log[c*(d + e*x^2)^p]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 7.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] Integrate[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]),x]
[Out] Integrate[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]), x]
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(g\,x^{3}+f\right)\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}dx$$

```
[In] int(1/(g*x^3+f)/ln(c*(e*x^2+d)^p),x)
[Out] int(1/(g*x^3+f)/ln(c*(e*x^2+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(gx^3+f)\log\left((ex^2+d)^p c\right)}\,dx$$

```
[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="fricas")
[Out] integral(1/((g*x^3 + f)*log((e*x^2 + d)^p*c)), x)
```

Sympy [N/A]

Not integrable

Time = 101.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] integrate(1/(g*x**3+f)/ln(c*(e*x**2+d)**p),x)
[Out] Integral(1/((f + g*x**3)*log(c*(d + e*x**2)**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{(gx^3+f)\log\left((ex^2+d)^pc\right)}\,dx$$

```
[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] integrate(1/((g*x^3 + f)*log((e*x^2 + d)^p*c)), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{(gx^3+f)\log\left((ex^2+d)^pc\right)}\,dx$$

```
[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p),x, algorithm="giac")
[Out] integrate(1/((g*x^3 + f)*log((e*x^2 + d)^p*c)), x)
```

Mupad [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{\ln\left(c\,(e\,x^2+d)^p\right)\,\left(g\,x^3+f\right)}\,dx$$

```
[In] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^3)),x)
[Out] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^3)), x)
```

3.305 $\int \frac{1}{(f+gx^3)^2\log(c(d+ex^2)^p)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx^3)^2\log(c(d+ex^2)^p)}\,dx=\text{Int}\left(\frac{1}{(f+gx^3)^2\log(c(d+ex^2)^p)},x\right)$$

```
[Out] Unintegrable(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx^3)^2\log(c(d+ex^2)^p)}\,dx=\int \frac{1}{(f+gx^3)^2\log(c(d+ex^2)^p)}\,dx$$

```
[In] Int[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]),x]
```

```
[Out] Defer[Int][1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{(f+gx^3)^2\log(c(d+ex^2)^p)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 9.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)^2 \log(c(d+ex^2)^p)} \, dx = \int \frac{1}{(f+gx^3)^2 \log(c(d+ex^2)^p)} \, dx$$

```
[In] Integrate[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]),x]
[Out] Integrate[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]), x]
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g\,x^3+f)^2 \ln(c\,(e\,x^2+d)^p)}dx$$

```
[In] int(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
[Out] int(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{1}{(f+gx^3)^2 \log(c(d+ex^2)^p)} \, dx = \int \frac{1}{(gx^3+f)^2 \log((ex^2+d)^p c)} \, dx$$

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="fricas")
[Out] integral(1/((g^2*x^6 + 2*f*g*x^3 + f^2)*log((e*x^2 + d)^p*c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx^3)^2 \log(c(d+ex^2)^p)} \, dx = \text{Timed out}$$

```
[In] integrate(1/(g*x**3+f)**2/ln(c*(e*x**2+d)**p),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)^2 \log(c(d+ex^2)^p)}\,dx = \int \frac{1}{(gx^3+f)^2 \log((ex^2+d)^p c)}\,dx$$

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] integrate(1/((g*x^3 + f)^2*log((e*x^2 + d)^p*c)), x)
```

Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)^2 \log(c(d+ex^2)^p)}\,dx = \int \frac{1}{(gx^3+f)^2 \log((ex^2+d)^p c)}\,dx$$

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p),x, algorithm="giac")
[Out] integrate(1/((g*x^3 + f)^2*log((e*x^2 + d)^p*c)), x)
```

Mupad [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)^2 \log(c(d+ex^2)^p)}\,dx = \int \frac{1}{\ln\left(c\,(e\,x^2+d)^p\right)\,(g\,x^3+f)^2}\,dx$$

```
[In] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^3)^2),x)
[Out] int(1/(log(c*(d + e*x^2)^p)*(f + g*x^3)^2), x)
```

3.306 $\int \frac{(f+gx^3)^2}{\log^2(c(d+ex^2)^p)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(f+gx^3)^2}{\log^2(c(d+ex^2)^p)}\,dx = \operatorname{Int}\left(\frac{(f+gx^3)^2}{\log^2(c(d+ex^2)^p)},x\right)$$

```
[Out] Unintegrable((g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(f+gx^3)^2}{\log^2(c(d+ex^2)^p)}\,dx = \int \frac{(f+gx^3)^2}{\log^2(c(d+ex^2)^p)}\,dx$$

```
[In] Int[(f + g*x^3)^2/Log[c*(d + e*x^2)^p]^2,x]
```

```
[Out] Defer[Int][(f + g*x^3)^2/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(f+gx^3)^2}{\log^2(c(d+ex^2)^p)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^3)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{(f+gx^3)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] Integrate[(f + g*x^3)^2/Log[c*(d + e*x^2)^p]^2,x]
[Out] Integrate[(f + g*x^3)^2/Log[c*(d + e*x^2)^p]^2, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(g\,x^{3}+f\right)^{2}}{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{2}}dx$$

```
[In] int((g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
[Out] int((g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{(f+gx^3)^2}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{\left(gx^3+f\right)^2}{\log\left(\left(ex^2+d\right)^pc\right)^2}\,dx$$

```
[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
[Out] integral((g^2*x^6 + 2*f*g*x^3 + f^2)/log((e*x^2 + d)^p*c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 18.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(f+gx^3)^2}{\log^2(c(d+ex^2)^p)}\,dx = \int \frac{(f+gx^3)^2}{\log(c(d+ex^2)^p)^2}\,dx$$

```
[In] integrate((g*x**3+f)**2/ln(c*(e*x**2+d)**p)**2,x)
[Out] Integral((f + g*x**3)**2/log(c*(d + e*x**2)**p)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 152, normalized size of antiderivative = 6.33

$$\int \frac{(f+gx^3)^2}{\log^2(c(d+ex^2)^p)}\,dx = \int \frac{(gx^3+f)^2}{\log((ex^2+d)^pc)^2}\,dx$$

```
[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
[Out] -1/2*(e*g^2*x^8 + d*g^2*x^6 + 2*e*f*g*x^5 + 2*d*f*g*x^3 + e*f^2*x^2 + d*f^2
)/(e*p*x*log((e*x^2 + d)^p) + e*p*x*log(c)) + integrate(1/2*(7*e*g^2*x^8 +
5*d*g^2*x^6 + 8*e*f*g*x^5 + 4*d*f*g*x^3 + e*f^2*x^2 - d*f^2)/(e*p*x^2*log((
e*x^2 + d)^p) + e*p*x^2*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^3)^2}{\log^2(c(d+ex^2)^p)}\,dx = \int \frac{(gx^3+f)^2}{\log((ex^2+d)^pc)^2}\,dx$$

```
[In] integrate((g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
[Out] integrate((g*x^3 + f)^2/log((e*x^2 + d)^p*c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx^3)^2}{\log^2(c(d+ex^2)^p)}\,dx=\int \frac{(g\,x^3+f)^2}{\ln\left(c\,(e\,x^2+d)^p\right)^2}\,dx$$

```
[In] int((f + g*x^3)^2/log(c*(d + e*x^2)^p)^2,x)
[Out] int((f + g*x^3)^2/log(c*(d + e*x^2)^p)^2, x)
```

3.307 $\int \frac{f+gx^3}{\log^2(c(d+ex^2)^p)} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx = \text{Int}\left(\frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable((g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx = \int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

```
[In] Int[(f + g*x^3)/Log[c*(d + e*x^2)^p]^2,x]
[Out] Defer[Int][(f + g*x^3)/Log[c*(d + e*x^2)^p]^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Mathematica [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] Integrate[(f + g*x^3)/Log[c*(d + e*x^2)^p]^2,x]
[Out] Integrate[(f + g*x^3)/Log[c*(d + e*x^2)^p]^2, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{g\,x^{3}+f}{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{2}}dx$$

```
[In] int((g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
[Out] int((g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{gx^3+f}{\log\left(\left(ex^2+d\right)^p c\right)^2}\,dx$$

```
[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
[Out] integral((g*x^3 + f)/log((e*x^2 + d)^p*c)^2, x)
```

Sympy [N/A]

Not integrable

Time = 8.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{f+gx^3}{\log\left(c\left(d+ex^2\right)^p\right)^2}\,dx$$

```
[In] integrate((g*x**3+f)/ln(c*(e*x**2+d)**p)**2,x)
[Out] Integral((f + g*x**3)/log(c*(d + e*x**2)**p)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 4.73

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{gx^3+f}{\log\left(\left(ex^2+d\right)^pc\right)^2}\,dx$$

```
[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*g*x^5 + d*g*x^3 + e*f*x^2 + d*f)/(e*p*x*log((e*x^2 + d)^p) + e*p*x*
log(c)) + integrate(1/2*(4*e*g*x^5 + 2*d*g*x^3 + e*f*x^2 - d*f)/(e*p*x^2*lo
g((e*x^2 + d)^p) + e*p*x^2*log(c)), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{gx^3+f}{\log\left(\left(ex^2+d\right)^pc\right)^2}\,dx$$

```
[In] integrate((g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f)/log((e*x^2 + d)^p*c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{f+gx^3}{\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{g\,x^3+f}{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)^2}\,dx$$

```
[In] int((f + g*x^3)/log(c*(d + e*x^2)^p)^2,x)
```

```
[Out] int((f + g*x^3)/log(c*(d + e*x^2)^p)^2, x)
```

3.308 $\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)} dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)} dx = \text{Int}\left(\frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)} dx = \int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

```
[In] Int[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]^2),x]
```

```
[Out] Defer[Int][1/((f + g*x^3)*Log[c*(d + e*x^2)^p]^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)} dx$$

Mathematica [N/A]

Not integrable

Time = 9.47 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] Integrate[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]^2),x]
[Out] Integrate[1/((f + g*x^3)*Log[c*(d + e*x^2)^p]^2), x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(g\,x^{3}+f\right)\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)^{2}}dx$$

```
[In] int(1/(g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
[Out] int(1/(g*x^3+f)/ln(c*(e*x^2+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(gx^3+f)\log\left((ex^2+d)^pc\right)^2}\,dx$$

```
[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
[Out] integral(1/((g*x^3 + f)*log((e*x^2 + d)^p*c)^2), x)
```

Sympy [N/A]

Not integrable

Time = 139.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(f+gx^3)\log\left(c\left(d+ex^2\right)^p\right)^2}\,dx$$

```
[In] integrate(1/(g*x**3+f)/ln(c*(e*x**2+d)**p)**2,x)
[Out] Integral(1/((f + g*x**3)*log(c*(d + e*x**2)**p)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 6.62

$$\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(gx^3+f)\log\left(\left(ex^2+d\right)^pc\right)^2}\,dx$$

```
[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*x^2 + d)/(e*g*p*x^4*log(c) + e*f*p*x*log(c) + (e*g*p*x^4 + e*f*p*x)
*log((e*x^2 + d)^p)) - integrate(1/2*(2*e*g*x^5 + 4*d*g*x^3 - e*f*x^2 + d*f
)/(e*g^2*p*x^8*log(c) + 2*e*f*g*p*x^5*log(c) + e*f^2*p*x^2*log(c) + (e*g^2*
p*x^8 + 2*e*f*g*p*x^5 + e*f^2*p*x^2)*log((e*x^2 + d)^p)), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{(gx^3+f)\log\left(\left(ex^2+d\right)^pc\right)^2}\,dx$$

```
[In] integrate(1/(g*x^3+f)/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x^3 + f)*log((e*x^2 + d)^p*c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx=\int \frac{1}{\ln\left(c\left(e\,x^2+d\right)^p\right)^2\,\left(g\,x^3+f\right)}\,dx$$

```
[In] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^3)),x)
```

```
[Out] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^3)), x)
```

3.309 $\int \frac{1}{(f+gx^3)^2 \log^2(c(d+ex^2)^p)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{(f+gx^3)^2 \log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \text{Int}\left(\frac{1}{(f+gx^3)^2 \log^2\left(c\left(d+ex^2\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(f+gx^3)^2 \log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{(f+gx^3)^2 \log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

```
[In] Int[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2),x]
[Out] Defer[Int][1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(f+gx^3)^2 \log^2\left(c\left(d+ex^2\right)^p\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 10.86 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)^2\log^2(c(d+ex^2)^p)}\,dx=\int \frac{1}{(f+gx^3)^2\log^2(c(d+ex^2)^p)}\,dx$$

```
[In] Integrate[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2),x]
[Out] Integrate[1/((f + g*x^3)^2*Log[c*(d + e*x^2)^p]^2), x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{(g\,x^3+f)^2\ln(c\,(e\,x^2+d)^p)^2}dx$$

```
[In] int(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
[Out] int(1/(g*x^3+f)^2/ln(c*(e*x^2+d)^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{1}{(f+gx^3)^2\log^2(c(d+ex^2)^p)}\,dx=\int \frac{1}{(gx^3+f)^2\log\left((ex^2+d)^pc\right)^2}\,dx$$

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="fricas")
[Out] integral(1/((g^2*x^6 + 2*f*g*x^3 + f^2)*log((e*x^2 + d)^p*c)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx^3)^2\log^2(c(d+ex^2)^p)}\,dx=\text{Timed out}$$

```
[In] integrate(1/(g*x**3+f)**2/ln(c*(e*x**2+d)**p)**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 215, normalized size of antiderivative = 8.96

$$\int \frac{1}{(f+gx^3)^2\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{(gx^3+f)^2\log\left((ex^2+d)^pc\right)^2}\,dx$$

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e*x^2 + d)/(e*g^2*p*x^7*log(c) + 2*e*f*g*p*x^4*log(c) + e*f^2*p*x*log
(c) + (e*g^2*p*x^7 + 2*e*f*g*p*x^4 + e*f^2*p*x)*log((e*x^2 + d)^p)) - integ
rate(1/2*(5*e*g*x^5 + 7*d*g*x^3 - e*f*x^2 + d*f)/(e*g^3*p*x^11*log(c) + 3*e
*f*g^2*p*x^8*log(c) + 3*e*f^2*g*p*x^5*log(c) + e*f^3*p*x^2*log(c) + (e*g^3*
p*x^11 + 3*e*f*g^2*p*x^8 + 3*e*f^2*g*p*x^5 + e*f^3*p*x^2)*log((e*x^2 + d)^p
)), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)^2\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{(gx^3+f)^2\log\left((ex^2+d)^pc\right)^2}\,dx$$

```
[In] integrate(1/(g*x^3+f)^2/log(c*(e*x^2+d)^p)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x^3 + f)^2*log((e*x^2 + d)^p*c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(f+gx^3)^2\log^2\left(c\left(d+ex^2\right)^p\right)}\,dx = \int \frac{1}{\ln\left(c\left(e\,x^2+d\right)^p\right)^2\,\left(g\,x^3+f\right)^2}\,dx$$

```
[In] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^2),x)
```

```
[Out] int(1/(log(c*(d + e*x^2)^p)^2*(f + g*x^3)^2), x)
```

3.310 $\int x^5(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 142

$$\int x^5(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = -\frac{d^2(4ef-3dg)px^2}{24e^3}+\frac{d(4ef-3dg)px^4}{48e^2} - \frac{(4ef-3dg)px^6}{72e}-\frac{1}{32}gpx^8 + \frac{d^3(4ef-3dg)p\log(d+ex^2)}{24e^4} + \frac{1}{6}fx^6\log\left(c(d+ex^2)^p\right)+\frac{1}{8}gx^8\log\left(c(d+ex^2)^p\right)$$

```
[Out] -1/24*d^2*(-3*d*g+4*e*f)*p*x^2/e^3+1/48*d*(-3*d*g+4*e*f)*p*x^4/e^2-1/72*(-3
*d*g+4*e*f)*p*x^6/e-1/32*g*p*x^8+1/24*d^3*(-3*d*g+4*e*f)*p*ln(e*x^2+d)/e^4+
1/6*f*x^6*ln(c*(e*x^2+d)^p)+1/8*g*x^8*ln(c*(e*x^2+d)^p)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2525, 45, 2461, 12, 78}

$$\int x^5(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = \frac{1}{6}fx^6\log\left(c(d+ex^2)^p\right)+\frac{1}{8}gx^8\log\left(c(d+ex^2)^p\right) + \frac{d^3p(4ef-3dg)\log(d+ex^2)}{24e^4}-\frac{d^2px^2(4ef-3dg)}{24e^3} + \frac{dpx^4(4ef-3dg)}{48e^2}-\frac{px^6(4ef-3dg)}{72e}-\frac{1}{32}gpx^8$$

```
[In] Int[x^5*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -1/24*(d^2*(4*e*f - 3*d*g)*p*x^2)/e^3 + (d*(4*e*f - 3*d*g)*p*x^4)/(48*e^2)
- ((4*e*f - 3*d*g)*p*x^6)/(72*e) - (g*p*x^8)/32 + (d^3*(4*e*f - 3*d*g)*p*Lo
g[d + e*x^2])/(24*e^4) + (f*x^6*Log[c*(d + e*x^2)^p])/6 + (g*x^8*Log[c*(d +
 e*x^2)^p])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int x^2(f+gx)\log\left(c(d+ex)^p\right)\,dx, x, x^2\right)$$

$$= \frac{1}{6}fx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}gx^8\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{2}(ep)\text{Subst}\left(\int\frac{x^3(4f+3gx)}{12(d+ex)}\,dx,x,x^2\right)$$

$$= \frac{1}{6}fx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}gx^8\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{24}(ep)\text{Subst}\left(\int\frac{x^3(4f+3gx)}{d+ex}\,dx,x,x^2\right)$$

$$= \frac{1}{6}fx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}gx^8\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{24}(ep)\text{Subst}\left(\int\left(-\frac{d^2(-4ef+3dg)}{e^4}+\frac{d(-4ef+3dg)x}{e^3}+\frac{(4ef-3dg)x^2}{e^2}+\frac{3gx^3}{e}+\frac{d^3(-4ef+3dg)}{e^4(d+ex)}\right)\,dx,x,x^2\right)$$

$$= -\frac{d^2(4ef-3dg)px^2}{24e^3}+\frac{d(4ef-3dg)px^4}{48e^2}-\frac{(4ef-3dg)px^6}{72e}-\frac{1}{32}gpx^8+\frac{d^3(4ef-3dg)p\log\left(d+ex^2\right)}{24e^4}+\frac{1}{6}fx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}gx^8\log\left(c\left(d+ex^2\right)^p\right)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.20

$$\int x^5\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx = -\frac{d^2fpx^2}{6e^2}+\frac{d^3gpx^2}{8e^3}+\frac{dfpx^4}{12e}-\frac{d^2gpx^4}{16e^2}-\frac{1}{18}fpx^6+\frac{dgpx^6}{24e}-\frac{1}{32}gpx^8+\frac{d^3fp\log\left(d+ex^2\right)}{6e^3}-\frac{d^4gp\log\left(d+ex^2\right)}{8e^4}+\frac{1}{6}fx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}gx^8\log\left(c\left(d+ex^2\right)^p\right)$$

```
[In] Integrate[x^5*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -1/6*(d^2*f*p*x^2)/e^2 + (d^3*g*p*x^2)/(8*e^3) + (d*f*p*x^4)/(12*e) - (d^2*
g*p*x^4)/(16*e^2) - (f*p*x^6)/18 + (d*g*p*x^6)/(24*e) - (g*p*x^8)/32 + (d^3
*f*p*Log[d + e*x^2])/(6*e^3) - (d^4*g*p*Log[d + e*x^2])/(8*e^4) + (f*x^6*Lo
g[c*(d + e*x^2)^p])/6 + (g*x^8*Log[c*(d + e*x^2)^p])/8
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

method	result
parts	$\frac{g\,x^8\ln\left(c\left(e\,x^2+d\right)^p\right)}{8}+\frac{f\,x^6\ln\left(c\left(e\,x^2+d\right)^p\right)}{6}-\frac{pe\left(-\frac{-\frac{3}{4}g\,x^8e^3+dg\,x^6e^2-\frac{4}{3}x^6e^3f-\frac{3}{2}x^4d^2eg+2x^4d\,e^2f+3d^3g\,x^2-4d^2ef\,x^2}{2e^4}+\frac{d^3(3d}{}\right.}{12}$
parallelrisch	$-\frac{-36x^8\ln\left(c\left(e\,x^2+d\right)^p\right)e^4g+9e^4gp\,x^8-48x^6\ln\left(c\left(e\,x^2+d\right)^p\right)e^4f-12d\,e^3gp\,x^6+16e^4fp\,x^6+18d^2e^2gp\,x^4-24d\,e^3fp\,x^4-36d^3egp}{288e^4}$
risch	$\left(\frac{1}{8}g\,x^8+\frac{1}{6}f\,x^6\right)\ln\left(\left(e\,x^2+d\right)^p\right)-\frac{i\pi g\,x^8\,\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)\mathrm{csgn}(ic)}{16}+\frac{i\pi g\,x^8\,\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right.}{16}$

```
[In] int(x^5*(g*x^2+f)*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*g*x^8*ln(c*(e*x^2+d)^p)+1/6*f*x^6*ln(c*(e*x^2+d)^p)-1/12*p*e*(-1/2/e^4*
(-3/4*g*x^8*e^3+d*g*x^6*e^2-4/3*x^6*e^3*f-3/2*x^4*d^2*e*g+2*x^4*d*e^2*f+3*d
^3*g*x^2-4*d^2*e*f*x^2)+1/2*d^3*(3*d*g-4*e*f)/e^5*ln(e*x^2+d))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.07

$$\int x^5(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = -\frac{9\,e^4gpx^8+4\,(4\,e^4f-3\,de^3g)px^6-6\,(4\,de^3f-3\,d^2e^2g)px^4+12\,(4\,d^2e^2f-3\,d^3eg)px^2-12\,(3\,e^4gpx^8+}{288\,e^4}$$

```
[In] integrate(x^5*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] -1/288*(9*e^4*g*p*x^8 + 4*(4*e^4*f - 3*d*e^3*g)*p*x^6 - 6*(4*d*e^3*f - 3*d^
2*e^2*g)*p*x^4 + 12*(4*d^2*e^2*f - 3*d^3*e*g)*p*x^2 - 12*(3*e^4*g*p*x^8 + 4
*e^4*f*p*x^6 + (4*d^3*e*f - 3*d^4*g)*p)*log(e*x^2 + d) - 12*(3*e^4*g*x^8 +
4*e^4*f*x^6)*log(c))/e^4
```

Sympy [F(-1)]

Timed out.

$$\int x^5(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = \text{Timed out}$$

```
[In] integrate(x**5*(g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.93

$$\int x^5(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = -\frac{1}{288}\,ep\left(\frac{9\,e^3gx^8+4\,(4\,e^3f-3\,de^2g)x^6-6\,(4\,de^2f-3\,d^2eg)x^4+12\,(4\,d^2ef-3\,d^3g)x^2}{e^4}-\frac{12\,(4\,d^3ef-\right. + \frac{1}{24}\,(3\,gx^8+4\,fx^6)\log\left((ex^2+d)^pc\right)$$

```
[In] integrate(x^5*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] -1/288*e*p*((9*e^3*g*x^8 + 4*(4*e^3*f - 3*d*e^2*g)*x^6 - 6*(4*d*e^2*f - 3*d
^2*e*g)*x^4 + 12*(4*d^2*e*f - 3*d^3*g)*x^2)/e^4 - 12*(4*d^3*e*f - 3*d^4*g)*
log(e*x^2 + d)/e^5) + 1/24*(3*g*x^8 + 4*f*x^6)*log((e*x^2 + d)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(128) = 256$.

Time = 0.33 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.81

$$
\begin{aligned}
&\int x^5(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx \\
&= \frac{(ex^2+d)^3 fp\log(ex^2+d)}{6e^3} - \frac{(ex^2+d)^2 dfp\log(ex^2+d)}{2e^3} + \frac{(ex^2+d)^4 gp\log(ex^2+d)}{8e^4} \\
&\quad - \frac{(ex^2+d)^3 dgp\log(ex^2+d)}{2e^4} + \frac{3(ex^2+d)^2 d^2gp\log(ex^2+d)}{4e^4} - \frac{(ex^2+d)^3 fp}{18e^3} \\
&\quad + \frac{(ex^2+d)^2 dfp}{4e^3} - \frac{(ex^2+d)^4 gp}{32e^4} + \frac{(ex^2+d)^3 dgp}{6e^4} - \frac{3(ex^2+d)^2 d^2gp}{8e^4} + \frac{(ex^2+d)^3 f\log(c)}{6e^3} \\
&\quad - \frac{(ex^2+d)^2 df\log(c)}{2e^3} + \frac{(ex^2+d)^4 g\log(c)}{8e^4} - \frac{(ex^2+d)^3 dg\log(c)}{2e^4} + \frac{3(ex^2+d)^2 d^2g\log(c)}{4e^4} \\
&\quad - \frac{(ex^2-(ex^2+d)\log(ex^2+d)+d)d^2efp - (ex^2-(ex^2+d)\log(ex^2+d)+d)d^3gp - (ex^2+d)d^2ef\log}{2e^4}
\end{aligned}
$$

```
[In] integrate(x^5*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/6*(e*x^2 + d)^3*f*p*log(e*x^2 + d)/e^3 - 1/2*(e*x^2 + d)^2*d*f*p*log(e*x^
2 + d)/e^3 + 1/8*(e*x^2 + d)^4*g*p*log(e*x^2 + d)/e^4 - 1/2*(e*x^2 + d)^3*d
*g*p*log(e*x^2 + d)/e^4 + 3/4*(e*x^2 + d)^2*d^2*g*p*log(e*x^2 + d)/e^4 - 1/
18*(e*x^2 + d)^3*f*p/e^3 + 1/4*(e*x^2 + d)^2*d*f*p/e^3 - 1/32*(e*x^2 + d)^4
*g*p/e^4 + 1/6*(e*x^2 + d)^3*d*g*p/e^4 - 3/8*(e*x^2 + d)^2*d^2*g*p/e^4 + 1/
6*(e*x^2 + d)^3*f*log(c)/e^3 - 1/2*(e*x^2 + d)^2*d*f*log(c)/e^3 + 1/8*(e*x^
2 + d)^4*g*log(c)/e^4 - 1/2*(e*x^2 + d)^3*d*g*log(c)/e^4 + 3/4*(e*x^2 + d)^
2*d^2*g*log(c)/e^4 - 1/2*((e*x^2 - (e*x^2 + d)*log(e*x^2 + d) + d)*d^2*e*f*
p - (e*x^2 - (e*x^2 + d)*log(e*x^2 + d) + d)*d^3*g*p - (e*x^2 + d)*d^2*e*f*
log(c) + (e*x^2 + d)*d^3*g*log(c))/e^4
```

Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89

$$\int x^5 (f+gx^2) \log \left(c(d+ex^2)^p\right) \, dx = \ln \left(c\left(e\,x^2+d\right)^p\right) \left(\frac{g\,x^8}{8}+\frac{f\,x^6}{6}\right)-x^6\left(\frac{f\,p}{18}-\frac{d\,g\,p}{24\,e}\right) - \frac{g\,p\,x^8}{32}-\frac{\ln\left(e\,x^2+d\right)\,\left(3\,d^4\,g\,p-4\,d^3\,e\,f\,p\right)}{24\,e^4} + \frac{d\,x^4\,\left(\frac{f\,p}{3}-\frac{d\,g\,p}{4\,e}\right)}{4\,e}-\frac{d^2\,x^2\,\left(\frac{f\,p}{3}-\frac{d\,g\,p}{4\,e}\right)}{2\,e^2}$$

```
[In] int(x^5*log(c*(d + e*x^2)^p)*(f + g*x^2),x)
```

```
[Out] log(c*(d + e*x^2)^p)*((f*x^6)/6 + (g*x^8)/8) - x^6*((f*p)/18 - (d*g*p)/(24*
e)) - (g*p*x^8)/32 - (log(d + e*x^2)*(3*d^4*g*p - 4*d^3*e*f*p))/(24*e^4) +
(d*x^4*((f*p)/3 - (d*g*p)/(4*e)))/(4*e) - (d^2*x^2*((f*p)/3 - (d*g*p)/(4*e)
))/(2*e^2)
```

3.311 $\int x^3(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 119

$$\int x^3(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx=\frac{d(3ef-2dg)px^2}{12e^2}-\frac{(3ef-2dg)px^4}{24e}-\frac{1}{18}gpx^6-\frac{d^2(3ef-2dg)p\log(d+ex^2)}{12e^3}+\frac{1}{4}fx^4\log\left(c(d+ex^2)^p\right)+\frac{1}{6}gx^6\log\left(c(d+ex^2)^p\right)$$

```
[Out] 1/12*d*(-2*d*g+3*e*f)*p*x^2/e^2-1/24*(-2*d*g+3*e*f)*p*x^4/e-1/18*g*p*x^6-1/
12*d^2*(-2*d*g+3*e*f)*p*ln(e*x^2+d)/e^3+1/4*f*x^4*ln(c*(e*x^2+d)^p)+1/6*g*x
^6*ln(c*(e*x^2+d)^p)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2525, 45, 2461, 12, 78}

$$\int x^3(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx=\frac{1}{4}fx^4\log\left(c(d+ex^2)^p\right)+\frac{1}{6}gx^6\log\left(c(d+ex^2)^p\right)-\frac{d^2p(3ef-2dg)\log(d+ex^2)}{12e^3}+\frac{dpx^2(3ef-2dg)}{12e^2}-\frac{px^4(3ef-2dg)}{24e}-\frac{1}{18}gpx^6$$

```
[In] Int[x^3*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (d*(3*e*f - 2*d*g)*p*x^2)/(12*e^2) - ((3*e*f - 2*d*g)*p*x^4)/(24*e) - (g*p*
x^6)/18 - (d^2*(3*e*f - 2*d*g)*p*Log[d + e*x^2])/(12*e^3) + (f*x^4*Log[c*(d
 + e*x^2)^p])/4 + (g*x^6*Log[c*(d + e*x^2)^p])/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int x(f+gx)\log\left(c(d+ex)^p\right)\,dx, x, x^2\right)$$

$$= \frac{1}{4}fx^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{6}gx^6\log\left(c\left(d+ex^2\right)^p\right)-\frac{1}{2}(ep)\text{Subst}\left(\int\frac{x^2(3f+2gx)}{6(d+ex)}\,dx, x, x^2\right)$$

$$
\begin{aligned}
&= \frac{1}{4} f x^4 \log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{6} g x^6 \log\left(c\left(d+ex^2\right)^p\right) - \frac{1}{12}(ep)\text{Subst}\left(\int \frac{x^2(3f+2gx)}{d+ex}\,dx, x, x^2\right) \\
&= \frac{1}{4} f x^4 \log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{6} g x^6 \log\left(c\left(d+ex^2\right)^p\right) - \frac{1}{12}(ep)\text{Subst}\left(\int \left(\frac{d(-3ef+2dg)}{e^3}\right.\right. \\
&\qquad \left.\left. + \frac{(3ef-2dg)x}{e^2} + \frac{2gx^2}{e} - \frac{d^2(-3ef+2dg)}{e^3(d+ex)}\right) dx, x, x^2\right) \\
&= \frac{d(3ef-2dg)px^2}{12e^2} - \frac{(3ef-2dg)px^4}{24e} - \frac{1}{18} g p x^6 - \frac{d^2(3ef-2dg)p\log\left(d+ex^2\right)}{12e^3} \\
&\quad + \frac{1}{4} f x^4 \log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{6} g x^6 \log\left(c\left(d+ex^2\right)^p\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.18

$$
\begin{aligned}
\int x^3\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx = {} & \frac{dfpx^2}{4e} - \frac{d^2gpx^2}{6e^2} - \frac{1}{8} f p x^4 + \frac{dgpx^4}{12e} - \frac{1}{18} g p x^6 \\
& - \frac{d^2 f p \log\left(d+ex^2\right)}{4e^2} + \frac{d^3 g p \log\left(d+ex^2\right)}{6e^3} \\
& + \frac{1}{4} f x^4 \log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{6} g x^6 \log\left(c\left(d+ex^2\right)^p\right)
\end{aligned}
$$

```
[In] Integrate[x^3*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (d*f*p*x^2)/(4*e) - (d^2*g*p*x^2)/(6*e^2) - (f*p*x^4)/8 + (d*g*p*x^4)/(12*e
) - (g*p*x^6)/18 - (d^2*f*p*Log[d + e*x^2])/(4*e^2) + (d^3*g*p*Log[d + e*x^
2])/(6*e^3) + (f*x^4*Log[c*(d + e*x^2)^p])/4 + (g*x^6*Log[c*(d + e*x^2)^p])
/6
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

method	result
parts	$\frac{g\,x^{6} \ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{6}+\frac{f\,x^{4} \ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{4}-\frac{p e \left(\frac{\frac{2}{3} e^{2} g\,x^{6}-d g\,x^{4} e+\frac{3}{2} f\,x^{4} e^{2}+2 d^{2} g\,x^{2}-3 d e f\,x^{2}}{2 e^{3}}-\frac{d^{2} \left(2 d g-3 e f\right) \ln\left(e\,x^{2}+d\right)}{2 e^{4}}\right)}{6}$
parallelrisch	$\frac{12 x^{6} \ln\left(c\left(e\,x^{2}+d\right)^{p}\right) e^{3} g-4 x^{6} e^{3} g p+18 x^{4} \ln\left(c\left(e\,x^{2}+d\right)^{p}\right) e^{3} f+6 x^{4} d\,e^{2} g p-9 x^{4} e^{3} f p-12 x^{2} d^{2} e g p+18 x^{2} d\,e^{2} f p+12 \ln\left(e\,x^{2}+d\right) d^{3} g}{72 e^{3}}$
risch	$\left(\frac{1}{6} g\,x^{6}+\frac{1}{4} f\,x^{4}\right) \ln\left(\left(e\,x^{2}+d\right)^{p}\right)-\frac{i \pi g\,x^{6} \operatorname{csgn}\left(i\left(e\,x^{2}+d\right)^{p}\right) \operatorname{csgn}\left(i c\left(e\,x^{2}+d\right)^{p}\right) \operatorname{csgn}\left(i c\right)}{12}-\frac{i \pi f\,x^{4} \operatorname{csgn}\left(i c\left(e\,x^{2}+d\right)\right.}{8}$

```
[In] int(x^3*(g*x^2+f)*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*g*x^6*ln(c*(e*x^2+d)^p)+1/4*f*x^4*ln(c*(e*x^2+d)^p)-1/6*p*e*(1/2/e^3*(2
/3*e^2*g*x^6-d*g*x^4*e+3/2*f*x^4*e^2+2*d^2*g*x^2-3*d*e*f*x^2)-1/2*d^2*(2*d*
g-3*e*f)/e^4*ln(e*x^2+d))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08

$$\int x^3(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = -\frac{4\,e^3gpx^6+3\,(3\,e^3f-2\,de^2g)px^4-6\,(3\,de^2f-2\,d^2eg)px^2-6\,(2\,e^3gpx^6+3\,e^3fpx^4-(3\,d^2ef-2\,d^3g)p}{72\,e^3}$$

```
[In] integrate(x^3*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] -1/72*(4*e^3*g*p*x^6 + 3*(3*e^3*f - 2*d*e^2*g)*p*x^4 - 6*(3*d*e^2*f - 2*d^2
*e*g)*p*x^2 - 6*(2*e^3*g*p*x^6 + 3*e^3*f*p*x^4 - (3*d^2*e*f - 2*d^3*g)*p)*l
og(e*x^2 + d) - 6*(2*e^3*g*x^6 + 3*e^3*f*x^4)*log(c))/e^3
```

Sympy [A] (verification not implemented)

Time = 61.46 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.31

$$\int x^3(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = \begin{cases} \frac{d^3g\log\left(c(d+ex^2)^p\right)}{6e^3} - \frac{d^2f\log\left(c(d+ex^2)^p\right)}{4e^2} - \frac{d^2gpx^2}{6e^2} + \frac{dfpx^2}{4e} + \frac{dgpx^4}{12e} - \frac{fpx^4}{8} + \frac{fx^4\log\left(c(d+ex^2)^p\right)}{4} - \frac{gpx^6}{18} + \frac{gx^6\log\left(c(d+\right.}{6} \\ \left(\frac{fx^4}{4}+\frac{gx^6}{6}\right)\log\left(cd^p\right) \end{cases}$$

```
[In] integrate(x**3*(g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((d**3*g*log(c*(d + e*x**2)**p)/(6*e**3) - d**2*f*log(c*(d + e*x**
2)**p)/(4*e**2) - d**2*g*p*x**2/(6*e**2) + d*f*p*x**2/(4*e) + d*g*p*x**4/(1
2*e) - f*p*x**4/8 + f*x**4*log(c*(d + e*x**2)**p)/4 - g*p*x**6/18 + g*x**6*
log(c*(d + e*x**2)**p)/6, Ne(e, 0)), ((f*x**4/4 + g*x**6/6)*log(c*d**p), Tr
ue))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.91

$$\int x^3\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx = -\frac{1}{72}\,ep\left(\frac{4\,e^2gx^6+3\,(3\,e^2f-2\,deg)x^4-6\,(3\,def-2\,d^2g)x^2}{e^3}+\frac{6\,(3\,d^2ef-2\,d^3g)\log\left(ex^2+d\right)}{e^4}\right) + \frac{1}{12}\left(2\,gx^6+3\,fx^4\right)\log\left(\left(ex^2+d\right)^pc\right)$$

```
[In] integrate(x^3*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] -1/72*e*p*((4*e^2*g*x^6 + 3*(3*e^2*f - 2*d*e*g)*x^4 - 6*(3*d*e*f - 2*d^2*g)
*x^2)/e^3 + 6*(3*d^2*e*f - 2*d^3*g)*log(e*x^2 + d)/e^4) + 1/12*(2*g*x^6 + 3
*f*x^4)*log((e*x^2 + d)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(107) = 214$.

Time = 0.31 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.26

$$\int x^3\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx = \frac{\left(ex^2+d\right)^2fp\log\left(ex^2+d\right)}{4\,e^2} + \frac{\left(ex^2+d\right)^3gp\log\left(ex^2+d\right)}{6\,e^3} - \frac{\left(ex^2+d\right)^2dgp\log\left(ex^2+d\right)}{2\,e^3} - \frac{\left(ex^2+d\right)^2fp}{8\,e^2} - \frac{\left(ex^2+d\right)^3gp}{18\,e^3} + \frac{\left(ex^2+d\right)^2dgp}{4\,e^3} + \frac{\left(ex^2+d\right)^2f\log\left(c\right)}{4\,e^2} + \frac{\left(ex^2+d\right)^3g\log\left(c\right)}{6\,e^3} - \frac{\left(ex^2+d\right)^2dg\log\left(c\right)}{2\,e^3} + \frac{(ex^2-(ex^2+d)\log\left(ex^2+d\right)+d)defp-(ex^2-(ex^2+d)\log\left(ex^2+d\right)+d)d^2gp-(ex^2+d)def\log\left(c\right.}{2\,e^3}$$

```
[In] integrate(x^3*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/4*(e*x^2 + d)^2*f*p*log(e*x^2 + d)/e^2 + 1/6*(e*x^2 + d)^3*g*p*log(e*x^2
+ d)/e^3 - 1/2*(e*x^2 + d)^2*d*g*p*log(e*x^2 + d)/e^3 - 1/8*(e*x^2 + d)^2*f
*p/e^2 - 1/18*(e*x^2 + d)^3*g*p/e^3 + 1/4*(e*x^2 + d)^2*d*g*p/e^3 + 1/4*(e*
x^2 + d)^2*f*log(c)/e^2 + 1/6*(e*x^2 + d)^3*g*log(c)/e^3 - 1/2*(e*x^2 + d)^
2*d*g*log(c)/e^3 + 1/2*((e*x^2 - (e*x^2 + d)*log(e*x^2 + d) + d)*d*e*f*p -
(e*x^2 - (e*x^2 + d)*log(e*x^2 + d) + d)*d^2*g*p - (e*x^2 + d)*d*e*f*log(c)
 + (e*x^2 + d)*d^2*g*log(c))/e^3
```

Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87

$$\int x^3(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = \ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(\frac{g\,x^6}{6}+\frac{f\,x^4}{4}\right)-x^4\left(\frac{f\,p}{8}-\frac{d\,g\,p}{12\,e}\right) - \frac{g\,p\,x^6}{18}+\frac{\ln\left(e\,x^2+d\right)\,\left(2\,d^3\,g\,p-3\,d^2\,e\,f\,p\right)}{12\,e^3} + \frac{d\,x^2\,\left(\frac{f\,p}{2}-\frac{d\,g\,p}{3\,e}\right)}{2\,e}$$

```
[In] int(x^3*log(c*(d + e*x^2)^p)*(f + g*x^2),x)
```

```
[Out] log(c*(d + e*x^2)^p)*((f*x^4)/4 + (g*x^6)/6) - x^4*((f*p)/8 - (d*g*p)/(12*e
)) - (g*p*x^6)/18 + (log(d + e*x^2)*(2*d^3*g*p - 3*d^2*e*f*p))/(12*e^3) + (
d*x^2*((f*p)/2 - (d*g*p)/(3*e)))/(2*e)
```

3.312 $\int x(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 94

$$\int x(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = -\frac{(ef-dg)px^2}{4e}-\frac{p(f+gx^2)^2}{8g}-\frac{(ef-dg)^2p\log(d+ex^2)}{4e^2g}+\frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{4g}$$

```
[Out] -1/4*(-d*g+e*f)*p*x^2/e-1/8*p*(g*x^2+f)^2/g-1/4*(-d*g+e*f)^2*p*ln(e*x^2+d)/
e^2/g+1/4*(g*x^2+f)^2*ln(c*(e*x^2+d)^p)/g
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2525, 2442, 45}

$$\int x(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{4g}-\frac{p(ef-dg)^2\log(d+ex^2)}{4e^2g}-\frac{px^2(ef-dg)}{4e}-\frac{p(f+gx^2)^2}{8g}$$

```
[In] Int[x*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -1/4*((e*f - d*g)*p*x^2)/e - (p*(f + g*x^2)^2)/(8*g) - ((e*f - d*g)^2*p*Log
[d + e*x^2])/(4*e^2*g) + ((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/(4*g)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int (f+gx)\log\left(c(d+ex)^p\right)\,dx,x,x^2\right)\\
&= \frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{4g}-\frac{(ep)\text{Subst}\left(\int\frac{(f+gx)^2}{d+ex}\,dx,x,x^2\right)}{4g}\\
&= \frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{4g}-\frac{(ep)\text{Subst}\left(\int\left(\frac{g(ef-dg)}{e^2}+\frac{(ef-dg)^2}{e^2(d+ex)}+\frac{g(f+gx)}{e}\right)\,dx,x,x^2\right)}{4g}\\
&= -\frac{(ef-dg)px^2}{4e}-\frac{p\left(f+gx^2\right)^2}{8g}-\frac{(ef-dg)^2p\log\left(d+ex^2\right)}{4e^2g}+\frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{4g}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04

$$\int x(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = \frac{dgpx^2}{4e}-\frac{1}{8}gpx^4-\frac{d^2gp\log(d+ex^2)}{4e^2}+\frac{1}{4}gx^4\log\left(c(d+ex^2)^p\right)+\frac{1}{2}f\left(-px^2+\frac{(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e}\right)$$

```
[In] Integrate[x*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (d*g*p*x^2)/(4*e) - (g*p*x^4)/8 - (d^2*g*p*Log[d + e*x^2])/(4*e^2) + (g*x^4
*Log[c*(d + e*x^2)^p])/4 + (f*(-(p*x^2) + ((d + e*x^2)*Log[c*(d + e*x^2)^p]
)/e))/2
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.35

method	result
parts	$\frac{g\,x^4\ln\left(c\left(e\,x^2+d\right)^p\right)}{4}+\frac{\ln\left(c\left(e\,x^2+d\right)^p\right)f\,x^2}{2}+\frac{\ln\left(c\left(e\,x^2+d\right)^p\right)f^2}{4g}-\frac{pe\left(-\frac{g\left(-\frac{1}{2}eg\,x^4+dg\,x^2-2fe\,x^2\right)}{2e^2}+\frac{\left(g^2d^2-2defg+e^2f^2\right)\ln}{2e^3}\right.}{2g}$
parallelrisch	$-\frac{-2x^4\ln\left(c\left(e\,x^2+d\right)^p\right)e^2g+gp\,x^4e^2-4x^2\ln\left(c\left(e\,x^2+d\right)^p\right)e^2f-2dgp\,x^2e+4x^2e^2fp+2\ln\left(e\,x^2+d\right)d^2gp-8\ln\left(e\,x^2+d\right)defp+4\ln(}{8e^2}$
risch	Expression too large to display

```
[In] int(x*(g*x^2+f)*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*g*x^4*ln(c*(e*x^2+d)^p)+1/2*ln(c*(e*x^2+d)^p)*f*x^2+1/4*ln(c*(e*x^2+d)^
p)/g*f^2-1/2/g*p*e*(-1/2*g/e^2*(-1/2*e*g*x^4+d*g*x^2-2*f*e*x^2)+1/2*(d^2*g^
2-2*d*e*f*g+e^2*f^2)/e^3*ln(e*x^2+d))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int x(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = -\frac{e^2gpx^4+2\,(2\,e^2f-deg)px^2-2\,(e^2gpx^4+2\,e^2fpx^2+(2\,def-d^2g)p)\log\left(ex^2+d\right)-2\,(e^2gx^4+2\,e^2f}{8\,e^2}$$

```
[In] integrate(x*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] -1/8*(e^2*g*p*x^4 + 2*(2*e^2*f - d*e*g)*p*x^2 - 2*(e^2*g*p*x^4 + 2*e^2*f*p*
x^2 + (2*d*e*f - d^2*g)*p)*log(e*x^2 + d) - 2*(e^2*g*x^4 + 2*e^2*f*x^2)*log
(c))/e^2
```

Sympy [A] (verification not implemented)

Time = 15.77 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.34

$$\int x(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = \begin{cases} -\frac{d^2g\log\left(c(d+ex^2)^p\right)}{4e^2}+\frac{df\log\left(c(d+ex^2)^p\right)}{2e}+\frac{dgpx^2}{4e}-\frac{fpx^2}{2}+\frac{fx^2\log\left(c(d+ex^2)^p\right)}{2}-\frac{gpx^4}{8}+\frac{gx^4\log\left(c(d+ex^2)^p\right)}{4} & \text{for } e\neq \\ \left(\frac{fx^2}{2}+\frac{gx^4}{4}\right)\log\left(cd^p\right) & \text{otherwi} \end{cases}$$

```
[In] integrate(x*(g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((-d**2*g*log(c*(d + e*x**2)**p)/(4*e**2) + d*f*log(c*(d + e*x**2)
**p)/(2*e) + d*g*p*x**2/(4*e) - f*p*x**2/2 + f*x**2*log(c*(d + e*x**2)**p)/
2 - g*p*x**4/8 + g*x**4*log(c*(d + e*x**2)**p)/4, Ne(e, 0)), ((f*x**2/2 + g
*x**4/4)*log(c*d**p), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int x(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = -\frac{ep\left(\frac{eg^2x^4+2\,(2\,efg-dg^2)x^2}{e^2}+\frac{2\,(e^2f^2-2\,defg+d^2g^2)\log\left(ex^2+d\right)}{e^3}\right)}{8\,g} + \frac{(gx^2+f)^2\log\left((ex^2+d)^pc\right)}{4\,g}$$

[In] integrate(x*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")

[Out] -1/8*e*p*((e*g^2*x^4 + 2*(2*e*f*g - d*g^2)*x^2)/e^2 + 2*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(e*x^2 + d)/e^3)/g + 1/4*(g*x^2 + f)^2*log((e*x^2 + d)^p*c)/g

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.52

$$\int x(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$$
$$=\frac{2\left(ex^2+d\right)^2gp\log\left(ex^2+d\right)-\left(ex^2+d\right)^2gp+2\left(ex^2+d\right)^2g\log\left(c\right)}{8\,e^2}$$
$$-\frac{(ex^2-(ex^2+d)\log\left(ex^2+d\right)+d)efp-(ex^2-(ex^2+d)\log\left(ex^2+d\right)+d)dgp-(ex^2+d)ef\log\left(c\right)+}{2\,e^2}$$

[In] integrate(x*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")

[Out] 1/8*(2*(e*x^2 + d)^2*g*p*log(e*x^2 + d) - (e*x^2 + d)^2*g*p + 2*(e*x^2 + d)^2*g*log(c))/e^2 - 1/2*((e*x^2 - (e*x^2 + d)*log(e*x^2 + d) + d)*e*f*p - (e*x^2 - (e*x^2 + d)*log(e*x^2 + d) + d)*d*g*p - (e*x^2 + d)*e*f*log(c) + (e*x^2 + d)*d*g*log(c))/e^2

Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

$$\int x(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx=\ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(\frac{g\,x^4}{4}+\frac{f\,x^2}{2}\right)-x^2\,\left(\frac{f\,p}{2}-\frac{d\,g\,p}{4\,e}\right)-\frac{g\,p\,x^4}{8}-\frac{\ln\left(e\,x^2+d\right)\,\left(d^2\,g\,p-2\,d\,e\,f\,p\right)}{4\,e^2}$$

[In] int(x*log(c*(d + e*x^2)^p)*(f + g*x^2),x)

[Out] log(c*(d + e*x^2)^p)*((f*x^2)/2 + (g*x^4)/4) - x^2*((f*p)/2 - (d*g*p)/(4*e)) - (g*p*x^4)/8 - (log(d + e*x^2)*(d^2*g*p - 2*d*e*f*p))/(4*e^2)

3.313 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx = -\frac{1}{2}gpx^2+\frac{g(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2e}+\frac{1}{2}f\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)+\frac{1}{2}fp\,\text{PolyLog}\left(2,1+\frac{ex^2}{d}\right)$$

```
[Out] -1/2*g*p*x^2+1/2*g*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e+1/2*f*ln(-e*x^2/d)*ln(c*(e
*x^2+d)^p)+1/2*f*p*polylog(2,1+e*x^2/d)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2525, 45, 2463, 2436, 2332, 2441, 2352}

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx = \frac{1}{2}f\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)+\frac{g(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2e}+\frac{1}{2}fp\,\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)-\frac{1}{2}gpx^2$$

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x,x]
```

```
[Out] -1/2*(g*p*x^2) + (g*(d + e*x^2)*Log[c*(d + e*x^2)^p])/(2*e) + (f*Log[-((e*x
^2)/d)]*Log[c*(d + e*x^2)^p])/2 + (f*p*PolyLog[2, 1 + (e*x^2)/d])/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
```

```
|| IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)\log\left(c(d+ex)^p\right)}{x}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(g\log\left(c(d+ex)^p\right)+\frac{f\log\left(c(d+ex)^p\right)}{x}\right)\,dx, x, x^2\right)\\
&= \frac{1}{2}f\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x}\,dx, x, x^2\right)+\frac{1}{2}g\text{Subst}\left(\int \log\left(c(d+ex)^p\right)\,dx, x, x^2\right)\\
&= \frac{1}{2}f\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{g\text{Subst}\left(\int \log\left(cx^p\right)\,dx, x, d+ex^2\right)}{2e}\\
&\quad -\frac{1}{2}(efp)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^2\right)\\
&= -\frac{1}{2}gpx^2+\frac{g\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2e}\\
&\quad +\frac{1}{2}f\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}fp\text{Li}_2\left(1+\frac{ex^2}{d}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$
\begin{aligned}
\int \frac{\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{x}\,dx &= \frac{1}{2}g\left(-px^2+\frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{e}\right)\\
&\quad +\frac{1}{2}f\left(\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)\right.\\
&\quad \left.+p\,\text{PolyLog}\left(2,\frac{d+ex^2}{d}\right)\right)
\end{aligned}
$$

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x,x]
```

```
[Out] (g*(-(p*x^2) + ((d + e*x^2)*Log[c*(d + e*x^2)^p])/e))/2 + (f*(Log[-((e*x^2)
/d)]*Log[c*(d + e*x^2)^p] + p*PolyLog[2, (d + e*x^2)/d]))/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(74) = 148$.

Time = 0.31 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.91

method	result
parts	$\frac{\ln(c(e\,x^2+d)^p) g\,x^2}{2} + \ln\left(c(e\,x^2+d)^p\right) f \ln(x) - pe\left(\frac{g\,x^2}{2e} - \frac{gd\ln(e\,x^2+d)}{2e^2} + 2f\left(\frac{\ln(x)\left(\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)+\ln\left(\frac{ex+\sqrt{}}{\sqrt{-}}\right.\right.}{2e}\right.\right.$
risch	$\frac{\ln((e\,x^2+d)^p) g\,x^2}{2} + \ln\left((e\,x^2+d)^p\right) f \ln(x) - \frac{gp\,x^2}{2} + \frac{pgd\ln(e\,x^2+d)}{2e} - pf\ln(x)\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right) - pf\ln($

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(c*(e*x^2+d)^p)*g*x^2+ln(c*(e*x^2+d)^p)*f*ln(x)-p*e*(1/2*g*x^2/e-1/2*
g*d/e^2*ln(e*x^2+d)+2*f*(1/2*ln(x)*(ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+ln
((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)))/e+1/2*(dilog((-e*x+(-d*e)^(1/2))/(-d*e)^
(1/2))+dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)))/e))
```

Fricas [F]

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx = \int \frac{(gx^2+f)\log\left((ex^2+d)^p c\right)}{x}\,dx$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x,x, algorithm="fricas")
```

```
[Out] integral((g*x^2 + f)*log((e*x^2 + d)^p*c)/x, x)
```

Sympy [F]

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx = \int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx$$

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x,x)
```

```
[Out] Integral((f + g*x**2)*log(c*(d + e*x**2)**p)/x, x)
```

Maxima [F]

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx = \int \frac{(gx^2+f)\log\left((ex^2+d)^p c\right)}{x}\,dx$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x,x, algorithm="maxima")
[Out] integrate((g*x^2 + f)*log((e*x^2 + d)^p*c)/x, x)
```

Giac [F]

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx = \int \frac{(gx^2+f)\log\left((ex^2+d)^p c\right)}{x}\,dx$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x,x, algorithm="giac")
[Out] integrate((g*x^2 + f)*log((e*x^2 + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)\,\left(g\,x^2+f\right)}{x}\,dx$$

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x,x)
[Out] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x, x)
```

3.314 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 93

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}\,dx = \frac{efp\log(x)}{d}-\frac{efp\log\left(d+ex^2\right)}{2d}-\frac{f\log\left(c(d+ex^2)^p\right)}{2x^2}+\frac{1}{2}g\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)+\frac{1}{2}gp\,\mathrm{PolyLog}\left(2,1+\frac{ex^2}{d}\right)$$

```
[Out] e*f*p*ln(x)/d-1/2*e*f*p*ln(e*x^2+d)/d-1/2*f*ln(c*(e*x^2+d)^p)/x^2+1/2*g*ln(
-e*x^2/d)*ln(c*(e*x^2+d)^p)+1/2*g*p*polylog(2,1+e*x^2/d)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2525, 45, 2463, 2442, 36, 29, 31, 2441, 2352}

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}\,dx = -\frac{f\log\left(c(d+ex^2)^p\right)}{2x^2}+\frac{1}{2}g\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)-\frac{efp\log\left(d+ex^2\right)}{2d}+\frac{efp\log(x)}{d}+\frac{1}{2}gp\,\mathrm{PolyLog}\left(2,\frac{ex^2}{d}+1\right)$$

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^3,x]
```

```
[Out] (e*f*p*Log[x])/d - (e*f*p*Log[d + e*x^2])/(2*d) - (f*Log[c*(d + e*x^2)^p])/
(2*x^2) + (g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/2 + (g*p*PolyLog[2, 1
+ (e*x^2)/d])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)\log\left(c(d+ex)^p\right)}{x^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{f\log\left(c(d+ex)^p\right)}{x^2}+\frac{g\log\left(c(d+ex)^p\right)}{x}\right)\,dx, x, x^2\right)\\
&= \frac{1}{2}f\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x^2}\,dx, x, x^2\right)+\frac{1}{2}g\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x}\,dx, x, x^2\right)\\
&= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2x^2}+\frac{1}{2}g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)\\
&\quad+\frac{1}{2}(efp)\text{Subst}\left(\int \frac{1}{x(d+ex)}\,dx, x, x^2\right)-\frac{1}{2}(egp)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^2\right)\\
&= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2x^2}+\frac{1}{2}g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}gp\text{Li}_2\left(1+\frac{ex^2}{d}\right)\\
&\quad+\frac{(efp)\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2d}-\frac{\left(e^2fp\right)\text{Subst}\left(\int \frac{1}{d+ex}\,dx, x, x^2\right)}{2d}\\
&= \frac{efp\log(x)}{d}-\frac{efp\log\left(d+ex^2\right)}{2d}-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{2x^2}\\
&\quad+\frac{1}{2}g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}gp\text{Li}_2\left(1+\frac{ex^2}{d}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}\,dx = \frac{efp\log(x)}{d} - \frac{efp\log\left(d+ex^2\right)}{2d} - \frac{f\log\left(c(d+ex^2)^p\right)}{2x^2} + \frac{1}{2}g\left(\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right) + p\,\mathrm{PolyLog}\left(2,\frac{d+ex^2}{d}\right)\right)$$

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^3,x]
```

```
[Out] (e*f*p*Log[x])/d - (e*f*p*Log[d + e*x^2])/(2*d) - (f*Log[c*(d + e*x^2)^p])/
(2*x^2) + (g*(Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + p*PolyLog[2, (d + e*
x^2)/d]))/2
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.67

method	result
parts	$-\frac{f\ln\left(c\left(e\,x^2+d\right)^p\right)}{2x^2}+\ln\left(c\left(e\,x^2+d\right)^p\right)g\ln(x)-pe\left(\frac{f\ln\left(e\,x^2+d\right)}{2d}-\frac{f\ln(x)}{d}+2g\left(\frac{\ln(x)\left(\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)+\ln\left(\frac{ex}{}\right.\right.}{2e}\right.\right.$
risch	$\ln\left(\left(e\,x^2+d\right)^p\right)g\ln(x)-\frac{\ln\left(\left(e\,x^2+d\right)^p\right)f}{2x^2}-\frac{efp\ln\left(e\,x^2+d\right)}{2d}+\frac{efp\ln(x)}{d}-pg\ln(x)\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)-pg\ln$

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*f*ln(c*(e*x^2+d)^p)/x^2+ln(c*(e*x^2+d)^p)*g*ln(x)-p*e*(1/2*f/d*ln(e*x^
2+d)-f/d*ln(x)+2*g*(1/2*ln(x)*(ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+ln((e*x
+(-d*e)^(1/2))/(-d*e)^(1/2)))/e+1/2*(dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2)
)+dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)))/e))
```

Fricas [F]

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}\,dx = \int \frac{(gx^2+f)\log\left((ex^2+d)^pc\right)}{x^3}\,dx$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^3,x, algorithm="fricas")
```

```
[Out] integral((g*x^2 + f)*log((e*x^2 + d)^p*c)/x^3, x)
```

Sympy [F]

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}\,dx = \int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}\,dx$$

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**3,x)
[Out] Integral((f + g*x**2)*log(c*(d + e*x**2)**p)/x**3, x)
```

Maxima [F]

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}\,dx = \int \frac{(gx^2+f)\log\left((ex^2+d)^p c\right)}{x^3}\,dx$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^3,x, algorithm="maxima")
[Out] integrate((g*x^2 + f)*log((e*x^2 + d)^p*c)/x^3, x)
```

Giac [F]

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}\,dx = \int \frac{(gx^2+f)\log\left((ex^2+d)^p c\right)}{x^3}\,dx$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^3,x, algorithm="giac")
[Out] integrate((g*x^2 + f)*log((e*x^2 + d)^p*c)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^3}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)\,(g\,x^2+f)}{x^3}\,dx$$

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^3,x)
[Out] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^3, x)
```

3.315 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^5}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 93

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^5}\,dx = -\frac{efp}{4dx^2}-\frac{e(ef-2dg)p\log(x)}{2d^2}+\frac{(ef-dg)^2p\log(d+ex^2)}{4d^2f}-\frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{4fx^4}$$

```
[Out] -1/4*e*f*p/d/x^2-1/2*e*(-2*d*g+e*f)*p*ln(x)/d^2+1/4*(-d*g+e*f)^2*p*ln(e*x^2
+d)/d^2/f-1/4*(g*x^2+f)^2*ln(c*(e*x^2+d)^p)/f/x^4
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2525, 37, 2461, 12, 90}

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^5}\,dx = -\frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{4fx^4}+\frac{p(ef-dg)^2\log(d+ex^2)}{4d^2f}-\frac{ep\log(x)(ef-2dg)}{2d^2}-\frac{efp}{4dx^2}$$

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^5,x]
```

```
[Out] -1/4*(e*f*p)/(d*x^2) - (e*(e*f - 2*d*g)*p*Log[x])/(2*d^2) + ((e*f - d*g)^2*
p*Log[d + e*x^2])/(4*d^2*f) - ((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/(4*f*x^4
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)\log\left(c(d+ex)^p\right)}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{4fx^4} - \frac{1}{2}(ep)\text{Subst}\left(\int -\frac{(f+gx)^2}{2fx^2(d+ex)}\,dx, x, x^2\right)$$

$$
= -\frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{4fx^4} + \frac{(ep)\text{Subst}\left(\int \frac{(f+gx)^2}{x^2(d+ex)}\,dx, x, x^2\right)}{4f}
$$

$$
= -\frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{4fx^4} + \frac{(ep)\text{Subst}\left(\int \left(\frac{f^2}{dx^2} + \frac{f(-ef+2dg)}{d^2x} + \frac{(-ef+dg)^2}{d^2(d+ex)}\right)\,dx, x, x^2\right)}{4f}
$$

$$
= -\frac{efp}{4dx^2} - \frac{e(ef-2dg)p\log(x)}{2d^2} + \frac{(ef-dg)^2p\log(d+ex^2)}{4d^2f} - \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{4fx^4}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.13

$$
\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^5}\,dx = \frac{egp\log(x)}{d} - \frac{egp\log(d+ex^2)}{2d} + \frac{1}{4}efp\left(-\frac{1}{dx^2} - \frac{2e\log(x)}{d^2} + \frac{e\log(d+ex^2)}{d^2}\right) - \frac{f\log\left(c(d+ex^2)^p\right)}{4x^4} - \frac{g\log\left(c(d+ex^2)^p\right)}{2x^2}
$$

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^5,x]
```

```
[Out] (e*g*p*Log[x])/d - (e*g*p*Log[d + e*x^2])/(2*d) + (e*f*p*(-(1/(d*x^2)) - (2
*e*Log[x])/d^2 + (e*Log[d + e*x^2])/d^2))/4 - (f*Log[c*(d + e*x^2)^p])/(4*x
^4) - (g*Log[c*(d + e*x^2)^p])/(2*x^2)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

method	result
parts	$-\frac{\ln\left(c\left(e\,x^2+d\right)^p\right)g}{2x^2}-\frac{\ln\left(c\left(e\,x^2+d\right)^p\right)f}{4x^4}-\frac{pe\left(\frac{(-2dg+ef)\ln(x)}{d^2}+\frac{f}{2d\,x^2}+\frac{(2dg-ef)\ln\left(e\,x^2+d\right)}{2d^2}\right)}{2}$
parallelrisch	$\frac{4\ln(x)x^4deg\,p^2-2\ln(x)x^4e^2f\,p^2-2x^4\ln\left(c\left(e\,x^2+d\right)^p\right)degp+x^4\ln\left(c\left(e\,x^2+d\right)^p\right)e^2fp+x^4e^2f\,p^2-2x^2\ln\left(c\left(e\,x^2+d\right)^p\right)d^2gp-x^2}{4x^4p\,d^2}$
risch	$-\frac{\left(2g\,x^2+f\right)\ln\left(\left(e\,x^2+d\right)^p\right)}{4x^4}-\frac{2i\pi\,d^2g\,x^2\,\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^2-2i\pi\,d^2g\,x^2\,\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^2+\right.\right.}{}$

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(c*(e*x^2+d)^p)*g/x^2-1/4*ln(c*(e*x^2+d)^p)*f/x^4-1/2*p*e*(1/d^2*(-2
*d*g+e*f)*ln(x)+1/2*f/d/x^2+1/2*(2*d*g-e*f)/d^2*ln(e*x^2+d))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^5}\,dx = -\frac{2\,(e^2f-2\,deg)px^4\log(x)+defpx^2+(2\,d^2gpx^2-(e^2f-2\,deg)px^4+d^2fp)\log(ex^2+d)+(2\,d^2gx^2+d}{4\,d^2x^4}$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^5,x, algorithm="fricas")
[Out] -1/4*(2*(e^2*f - 2*d*e*g)*p*x^4*log(x) + d*e*f*p*x^2 + (2*d^2*g*p*x^2 - (e^
2*f - 2*d*e*g)*p*x^4 + d^2*f*p)*log(e*x^2 + d) + (2*d^2*g*x^2 + d^2*f)*log(
c))/(d^2*x^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(83) = 166.

Time = 74.45 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.80

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^5}\,dx = \begin{cases} -\frac{f\log\left(c(d+ex^2)^p\right)}{4x^4}-\frac{g\log\left(c(d+ex^2)^p\right)}{2x^2}-\frac{efp}{4dx^2}+\frac{egp\log(x)}{d}-\frac{eg\log\left(c(d+ex^2)^p\right)}{2d}-\frac{e^2fp\log(x)}{2d^2}+\frac{e^2f\log\left(c(d+ex^2)^p\right)}{4d^2} & \text{for } d \\ -\frac{fp}{8x^4}-\frac{f\log\left(c(ex^2)^p\right)}{4x^4}-\frac{gp}{2x^2}-\frac{g\log\left(c(ex^2)^p\right)}{2x^2} & \text{othe} \end{cases}$$

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**5,x)
[Out] Piecewise((-f*log(c*(d + e*x**2)**p)/(4*x**4) - g*log(c*(d + e*x**2)**p)/(2
*x**2) - e*f*p/(4*d*x**2) + e*g*p*log(x)/d - e*g*log(c*(d + e*x**2)**p)/(2*
d) - e**2*f*p*log(x)/(2*d**2) + e**2*f*log(c*(d + e*x**2)**p)/(4*d**2), Ne(
d, 0)), (-f*p/(8*x**4) - f*log(c*(e*x**2)**p)/(4*x**4) - g*p/(2*x**2) - g*l
og(c*(e*x**2)**p)/(2*x**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^5}\,dx = \frac{1}{4}\,ep\left(\frac{(ef-2\,dg)\log(ex^2+d)}{d^2}-\frac{(ef-2\,dg)\log(x^2)}{d^2}-\frac{f}{dx^2}\right) - \frac{(2\,gx^2+f)\log\left((ex^2+d)^pc\right)}{4\,x^4}$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*e*p*((e*f - 2*d*g)*log(e*x^2 + d)/d^2 - (e*f - 2*d*g)*log(x^2)/d^2 - f/
(d*x^2)) - 1/4*(2*g*x^2 + f)*log((e*x^2 + d)^p*c)/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(85) = 170$.

Time = 0.31 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.25

$$\int \frac{(f + gx^2)\log\left(c(d + ex^2)^p\right)}{x^5}\,dx = -\frac{\frac{(e^3fp + 2\,(ex^2+d)e^2gp - 2\,de^2gp)\log(ex^2+d)}{(ex^2+d)^2 - 2\,(ex^2+d)d + d^2} + \frac{(ex^2+d)e^3fp - de^3fp + de^3f\log(c) + 2\,(ex^2+d)de^2g\log(c) - 2\,d^2e^2g\log(c)}{(ex^2+d)^2d - 2\,(ex^2+d)d^2 + d^3} - \frac{(e^3fp - 2\,de^2g \cdots}{\cdots}}{4\,e}$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^5,x, algorithm="giac")
```

```
[Out] -1/4*((e^3*f*p + 2*(e*x^2 + d)*e^2*g*p - 2*d*e^2*g*p)*log(e*x^2 + d)/((e*x^
2 + d)^2 - 2*(e*x^2 + d)*d + d^2) + ((e*x^2 + d)*e^3*f*p - d*e^3*f*p + d*e^
3*f*log(c) + 2*(e*x^2 + d)*d*e^2*g*log(c) - 2*d^2*e^2*g*log(c))/((e*x^2 + d
)^2*d - 2*(e*x^2 + d)*d^2 + d^3) - (e^3*f*p - 2*d*e^2*g*p)*log(e*x^2 + d)/d
^2 + (e^3*f*p - 2*d*e^2*g*p)*log(e*x^2)/d^2)/e
```

Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91

$$\int \frac{(f + gx^2)\log\left(c(d + ex^2)^p\right)}{x^5}\,dx = \frac{\ln\left(e\,x^2 + d\right)\,\left(e^2\,f\,p - 2\,d\,e\,g\,p\right)}{4\,d^2} - \frac{\ln\left(c\,{\left(e\,x^2 + d\right)}^p\right)\,\left(\frac{g\,x^2}{2} + \frac{f}{4}\right)}{x^4} - \frac{\ln\left(x\right)\,\left(e^2\,f\,p - 2\,d\,e\,g\,p\right)}{2\,d^2} - \frac{e\,f\,p}{4\,d\,x^2}$$

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^5,x)
```

```
[Out] (log(d + e*x^2)*(e^2*f*p - 2*d*e*g*p))/(4*d^2) - (log(c*(d + e*x^2)^p)*(f/4
 + (g*x^2)/2))/x^4 - (log(x)*(e^2*f*p - 2*d*e*g*p))/(2*d^2) - (e*f*p)/(4*d*
x^2)
```

3.316 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^7}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 125

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^7}\,dx = -\frac{efp}{12dx^4}+\frac{e(2ef-3dg)p}{12d^2x^2}+\frac{e^2(2ef-3dg)p\log(x)}{6d^3} - \frac{e^2(2ef-3dg)p\log(d+ex^2)}{12d^3} - \frac{f\log\left(c(d+ex^2)^p\right)}{6x^6}-\frac{g\log\left(c(d+ex^2)^p\right)}{4x^4}$$

```
[Out] -1/12*e*f*p/d/x^4+1/12*e*(-3*d*g+2*e*f)*p/d^2/x^2+1/6*e^2*(-3*d*g+2*e*f)*p*
ln(x)/d^3-1/12*e^2*(-3*d*g+2*e*f)*p*ln(e*x^2+d)/d^3-1/6*f*ln(c*(e*x^2+d)^p)
/x^6-1/4*g*ln(c*(e*x^2+d)^p)/x^4
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2525, 45, 2461, 12, 78}

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^7}\,dx = -\frac{f\log\left(c(d+ex^2)^p\right)}{6x^6}-\frac{g\log\left(c(d+ex^2)^p\right)}{4x^4} - \frac{e^2p(2ef-3dg)\log(d+ex^2)}{12d^3} + \frac{e^2p\log(x)(2ef-3dg)}{6d^3}+\frac{ep(2ef-3dg)}{12d^2x^2}-\frac{efp}{12dx^4}$$

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^7,x]
```

```
[Out] -1/12*(e*f*p)/(d*x^4) + (e*(2*e*f - 3*d*g)*p)/(12*d^2*x^2) + (e^2*(2*e*f -
3*d*g)*p*Log[x])/(6*d^3) - (e^2*(2*e*f - 3*d*g)*p*Log[d + e*x^2])/(12*d^3)
- (f*Log[c*(d + e*x^2)^p])/(6*x^6) - (g*Log[c*(d + e*x^2)^p])/(4*x^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)\log\left(c(d+ex)^p\right)}{x^4}\,dx, x, x^2\right)$$

$$
\begin{aligned}
&= -\frac{f\log\left(c(d+ex^2)^p\right)}{6x^6} - \frac{g\log\left(c(d+ex^2)^p\right)}{4x^4} - \frac{1}{2}(ep)\text{Subst}\left(\int \frac{-2f-3gx}{6x^3(d+ex)}\,dx, x, x^2\right) \\
&= -\frac{f\log\left(c(d+ex^2)^p\right)}{6x^6} - \frac{g\log\left(c(d+ex^2)^p\right)}{4x^4} - \frac{1}{12}(ep)\text{Subst}\left(\int \frac{-2f-3gx}{x^3(d+ex)}\,dx, x, x^2\right) \\
&= -\frac{f\log\left(c(d+ex^2)^p\right)}{6x^6} - \frac{g\log\left(c(d+ex^2)^p\right)}{4x^4} - \frac{1}{12}(ep)\text{Subst}\left(\int \left(-\frac{2f}{dx^3}\right.\right. \\
&\qquad \left.\left. + \frac{2ef-3dg}{d^2x^2} + \frac{e(-2ef+3dg)}{d^3x} - \frac{e^2(-2ef+3dg)}{d^3(d+ex)}\right)\,dx, x, x^2\right) \\
&= -\frac{efp}{12dx^4} + \frac{e(2ef-3dg)p}{12d^2x^2} + \frac{e^2(2ef-3dg)p\log(x)}{6d^3} \\
&\quad - \frac{e^2(2ef-3dg)p\log\left(d+ex^2\right)}{12d^3} - \frac{f\log\left(c(d+ex^2)^p\right)}{6x^6} - \frac{g\log\left(c(d+ex^2)^p\right)}{4x^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.07

$$
\begin{aligned}
\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^7}\,dx &= \frac{1}{4}egp\left(-\frac{1}{dx^2} - \frac{2e\log(x)}{d^2} + \frac{e\log\left(d+ex^2\right)}{d^2}\right) \\
&\quad + \frac{1}{3}efp\left(-\frac{1}{4dx^4} + \frac{e}{2d^2x^2} + \frac{e^2\log(x)}{d^3} - \frac{e^2\log\left(d+ex^2\right)}{2d^3}\right) \\
&\quad - \frac{f\log\left(c(d+ex^2)^p\right)}{6x^6} - \frac{g\log\left(c(d+ex^2)^p\right)}{4x^4}
\end{aligned}
$$

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^7,x]
```

```
[Out] (e*g*p*(-(1/(d*x^2)) - (2*e*Log[x])/d^2 + (e*Log[d + e*x^2])/d^2))/4 + (e*f
*p*(-1/4*1/(d*x^4) + e/(2*d^2*x^2) + (e^2*Log[x])/d^3 - (e^2*Log[d + e*x^2]
)/(2*d^3)))/3 - (f*Log[c*(d + e*x^2)^p])/(6*x^6) - (g*Log[c*(d + e*x^2)^p])
/(4*x^4)
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

method	result
parts	$-\frac{g\ln\left(c\left(e\,x^2+d\right)^p\right)}{4x^4}-\frac{f\ln\left(c\left(e\,x^2+d\right)^p\right)}{6x^6}-\frac{pe\left(-\frac{-3dg+2ef}{2d^2x^2}+\frac{(3dg-2ef)e\ln(x)}{d^3}+\frac{f}{2d\,x^4}-\frac{e(3dg-2ef)\ln\left(e\,x^2+d\right)}{2d^3}\right)}{6}$
parallelrisch	$-\frac{6\ln(x)x^6d\,e^2g\,p^2-4\ln(x)x^6e^3f\,p^2-3x^6\ln\left(c\left(e\,x^2+d\right)^p\right)d\,e^2gp+2x^6\ln\left(c\left(e\,x^2+d\right)^p\right)e^3fp-3x^6d\,e^2g\,p^2+2x^6e^3f\,p^2+3x^4d^2eg\,p}{12x^6p\,d^3}$
risch	$-\frac{\left(3g\,x^2+2f\right)\ln\left(\left(e\,x^2+d\right)^p\right)}{12x^6}-\frac{12\ln(x)d\,e^2gp\,x^6-8\ln(x)e^3fp\,x^6-6\ln\left(-e\,x^2-d\right)d\,e^2gp\,x^6+4\ln\left(-e\,x^2-d\right)e^3fp\,x^6+3i\pi\,d^3g\,x}{}$

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*g*ln(c*(e*x^2+d)^p)/x^4-1/6*f*ln(c*(e*x^2+d)^p)/x^6-1/6*p*e*(-1/2*(-3*
d*g+2*e*f)/d^2/x^2+(3*d*g-2*e*f)/d^3*e*ln(x)+1/2*f/d/x^4-1/2*e*(3*d*g-2*e*f
)/d^3*ln(e*x^2+d))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.03

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^7}\,dx = \frac{2\,(2\,e^3f-3\,de^2g)px^6\log(x)-d^2efpx^2+(2\,de^2f-3\,d^2eg)px^4-((2\,e^3f-3\,de^2g)px^6+3\,d^3gpx^2+2\,d^3f}{12\,d^3x^6}$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(2*(2*e^3*f - 3*d*e^2*g)*p*x^6*log(x) - d^2*e*f*p*x^2 + (2*d*e^2*f - 3
*d^2*e*g)*p*x^4 - ((2*e^3*f - 3*d*e^2*g)*p*x^6 + 3*d^3*g*p*x^2 + 2*d^3*f*p)
*log(e*x^2 + d) - (3*d^3*g*x^2 + 2*d^3*f)*log(c))/(d^3*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^7}\,dx = \text{Timed out}$$

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**7,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^7}\,dx = -\frac{1}{12}\,ep\left(\frac{(2\,e^2f-3\,deg)\log(ex^2+d)}{d^3}-\frac{(2\,e^2f-3\,deg)\log(x^2)}{d^3}-\frac{(2\,ef-3\,dg)x^2-df}{d^2x^4}\right) - \frac{(3\,gx^2+2\,f)\log\left((ex^2+d)^pc\right)}{12\,x^6}$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*e*p*((2*e^2*f - 3*d*e*g)*log(e*x^2 + d)/d^3 - (2*e^2*f - 3*d*e*g)*log
(x^2)/d^3 - ((2*e*f - 3*d*g)*x^2 - d*f)/(d^2*x^4)) - 1/12*(3*g*x^2 + 2*f)*l
og((e*x^2 + d)^p*c)/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(113) = 226$.

Time = 0.33 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.53

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^7}\,dx = -\frac{\frac{(2e^4fp+3(ex^2+d)e^3gp-3de^3gp)\log(ex^2+d)}{(ex^2+d)^3-3(ex^2+d)^2d+3(ex^2+d)d^2-d^3} - \frac{2(ex^2+d)^2e^4fp-5(ex^2+d)de^4fp+3d^2e^4fp-3(ex^2+d)^2de^3gp+6(ex^2+d)d^2e^3gp-3d^3e^3}{(ex^2+d)^3d^2-3(ex^2+d)^2d^3+3(ex^2+d)d}}{12e}$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^7,x, algorithm="giac")
```

```
[Out] -1/12*((2*e^4*f*p + 3*(e*x^2 + d)*e^3*g*p - 3*d*e^3*g*p)*log(e*x^2 + d)/((e
*x^2 + d)^3 - 3*(e*x^2 + d)^2*d + 3*(e*x^2 + d)*d^2 - d^3) - (2*(e*x^2 + d)
^2*e^4*f*p - 5*(e*x^2 + d)*d*e^4*f*p + 3*d^2*e^4*f*p - 3*(e*x^2 + d)^2*d*e^
3*g*p + 6*(e*x^2 + d)*d^2*e^3*g*p - 3*d^3*e^3*g*p - 2*d^2*e^4*f*log(c) - 3*
(e*x^2 + d)*d^2*e^3*g*log(c) + 3*d^3*e^3*g*log(c))/((e*x^2 + d)^3*d^2 - 3*(
e*x^2 + d)^2*d^3 + 3*(e*x^2 + d)*d^4 - d^5) + (2*e^4*f*p - 3*d*e^3*g*p)*log
(e*x^2 + d)/d^3 - (2*e^4*f*p - 3*d*e^3*g*p)*log(e*x^2)/d^3)/e
```

Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^7}\,dx = \frac{\ln(x)\ (2e^3fp-3de^2gp)}{6d^3} - \frac{\ln\left(c\,(e\,x^2+d)^p\right)\ \left(\frac{g\,x^2}{4}+\frac{f}{6}\right)}{x^6} - \frac{\ln(e\,x^2+d)\ (2e^3fp-3de^2gp)}{12d^3} - \frac{\frac{e\,f\,p}{2\,d}+\frac{e\,p\,x^2\,(3\,d\,g-2\,e\,f)}{2\,d^2}}{6\,x^4}$$

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^7,x)
```

```
[Out] (log(x)*(2*e^3*f*p - 3*d*e^2*g*p))/(6*d^3) - (log(c*(d + e*x^2)^p)*(f/6 + (
g*x^2)/4))/x^6 - (log(d + e*x^2)*(2*e^3*f*p - 3*d*e^2*g*p))/(12*d^3) - ((e*
f*p)/(2*d) + (e*p*x^2*(3*d*g - 2*e*f))/(2*d^2))/(6*x^4)
```

3.317 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^9}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^9}\,dx = -\frac{efp}{24dx^6}+\frac{e(3ef-4dg)p}{48d^2x^4}-\frac{e^2(3ef-4dg)p}{24d^3x^2} - \frac{e^3(3ef-4dg)p\log(x)}{12d^4}+\frac{e^3(3ef-4dg)p\log\left(d+ex^2\right)}{24d^4} - \frac{f\log\left(c(d+ex^2)^p\right)}{8x^8}-\frac{g\log\left(c(d+ex^2)^p\right)}{6x^6}$$

```
[Out] -1/24*e*f*p/d/x^6+1/48*e*(-4*d*g+3*e*f)*p/d^2/x^4-1/24*e^2*(-4*d*g+3*e*f)*p
/d^3/x^2-1/12*e^3*(-4*d*g+3*e*f)*p*ln(x)/d^4+1/24*e^3*(-4*d*g+3*e*f)*p*ln(e
*x^2+d)/d^4-1/8*f*ln(c*(e*x^2+d)^p)/x^8-1/6*g*ln(c*(e*x^2+d)^p)/x^6
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2525, 45, 2461, 12, 78}

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^9}\,dx = -\frac{f\log\left(c(d+ex^2)^p\right)}{8x^8}-\frac{g\log\left(c(d+ex^2)^p\right)}{6x^6} +\frac{e^3p(3ef-4dg)\log\left(d+ex^2\right)}{24d^4}-\frac{e^3p\log(x)(3ef-4dg)}{12d^4} -\frac{e^2p(3ef-4dg)}{24d^3x^2}+\frac{ep(3ef-4dg)}{48d^2x^4}-\frac{efp}{24dx^6}$$

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^9,x]
```

```
[Out] -1/24*(e*f*p)/(d*x^6) + (e*(3*e*f - 4*d*g)*p)/(48*d^2*x^4) - (e^2*(3*e*f -
4*d*g)*p)/(24*d^3*x^2) - (e^3*(3*e*f - 4*d*g)*p*Log[x])/(12*d^4) + (e^3*(3*
e*f - 4*d*g)*p*Log[d + e*x^2])/(24*d^4) - (f*Log[c*(d + e*x^2)^p])/(8*x^8)
- (g*Log[c*(d + e*x^2)^p])/(6*x^6)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)\log\left(c(d+ex)^p\right)}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{f\log\left(c(d+ex^2)^p\right)}{8x^8} - \frac{g\log\left(c(d+ex^2)^p\right)}{6x^6} - \frac{1}{2}(ep)\text{Subst}\left(\int\frac{-3f-4gx}{12x^4(d+ex)}\,dx,x,x^2\right)$$

$$= -\frac{f\log\left(c(d+ex^2)^p\right)}{8x^8} - \frac{g\log\left(c(d+ex^2)^p\right)}{6x^6} - \frac{1}{24}(ep)\text{Subst}\left(\int\frac{-3f-4gx}{x^4(d+ex)}\,dx,x,x^2\right)$$

$$= -\frac{f\log\left(c(d+ex^2)^p\right)}{8x^8} - \frac{g\log\left(c(d+ex^2)^p\right)}{6x^6} - \frac{1}{24}(ep)\text{Subst}\left(\int\left(-\frac{3f}{dx^4} + \frac{3ef-4dg}{d^2x^3} + \frac{e(-3ef+4dg)}{d^3x^2} - \frac{e^2(-3ef+4dg)}{d^4x} + \frac{e^3(-3ef+4dg)}{d^4(d+ex)}\right)dx,x,x^2\right)$$

$$= -\frac{efp}{24dx^6} + \frac{e(3ef-4dg)p}{48d^2x^4} - \frac{e^2(3ef-4dg)p}{24d^3x^2} - \frac{e^3(3ef-4dg)p\log(x)}{12d^4} + \frac{e^3(3ef-4dg)p\log(d+ex^2)}{24d^4} - \frac{f\log\left(c(d+ex^2)^p\right)}{8x^8} - \frac{g\log\left(c(d+ex^2)^p\right)}{6x^6}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.09

$$\int\frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^9}\,dx = \frac{1}{3}egp\left(-\frac{1}{4dx^4} + \frac{e}{2d^2x^2} + \frac{e^2\log(x)}{d^3} - \frac{e^2\log(d+ex^2)}{2d^3}\right) + \frac{1}{8}efp\left(-\frac{1}{3dx^6} + \frac{e}{2d^2x^4} - \frac{e^2}{d^3x^2} - \frac{2e^3\log(x)}{d^4} + \frac{e^3\log(d+ex^2)}{d^4}\right) - \frac{f\log\left(c(d+ex^2)^p\right)}{8x^8} - \frac{g\log\left(c(d+ex^2)^p\right)}{6x^6}$$

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^9,x]
```

```
[Out] (e*g*p*(-1/4*1/(d*x^4) + e/(2*d^2*x^2) + (e^2*Log[x])/d^3 - (e^2*Log[d + e*
x^2])/(2*d^3)))/3 + (e*f*p*(-1/3*1/(d*x^6) + e/(2*d^2*x^4) - e^2/(d^3*x^2)
- (2*e^3*Log[x])/d^4 + (e^3*Log[d + e*x^2])/d^4))/8 - (f*Log[c*(d + e*x^2)^
p])/(8*x^8) - (g*Log[c*(d + e*x^2)^p])/(6*x^6)
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.89

method	result
parts	$-\frac{g\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{6x^{6}}-\frac{f\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{8x^{8}}-\frac{pe\left(-\frac{-4dg+3ef}{4d^{2}x^{4}}-\frac{(4dg-3ef)e}{2d^{3}x^{2}}+\frac{f}{2d\,x^{6}}-\frac{(4dg-3ef)e^{2}\ln(x)}{d^{4}}+\frac{e^{2}(4dg-3ef)\ln\left(e\,x^{2}+d\right)}{2d^{4}}\right.}{12}$
parallelrisch	$\frac{16\ln(x)x^{8}d\,e^{3}g\,p^{2}-12\ln(x)x^{8}e^{4}f\,p^{2}-8x^{8}\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)d\,e^{3}gp+6x^{8}\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)e^{4}fp-8x^{8}d\,e^{3}g\,p^{2}+6x^{8}e^{4}f\,p^{2}+8x^{6}d^{2}e^{2}g}{48x^{8}d^{4}p}$
risch	$-\frac{\left(4g\,x^{2}+3f\right)\ln\left(\left(e\,x^{2}+d\right)^{p}\right)}{24x^{8}}-\frac{8\ln\left(e\,x^{2}+d\right)d\,e^{3}gp\,x^{8}-6\ln\left(e\,x^{2}+d\right)e^{4}fp\,x^{8}-16\ln(x)d\,e^{3}gp\,x^{8}+12\ln(x)e^{4}fp\,x^{8}+4i\pi\,d^{4}g\,x^{2}c}{}$

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*g*ln(c*(e*x^2+d)^p)/x^6-1/8*f*ln(c*(e*x^2+d)^p)/x^8-1/12*p*e*(-1/4*(-4
*d*g+3*e*f)/d^2/x^4-1/2*(4*d*g-3*e*f)/d^3*e/x^2+1/2*f/d/x^6-(4*d*g-3*e*f)/d
^4*e^2*ln(x)+1/2*e^2*(4*d*g-3*e*f)/d^4*ln(e*x^2+d))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^9}\,dx = -\frac{4\,(3\,e^4f-4\,de^3g)px^8\log(x)+2\,d^3efpx^2+2\,(3\,de^3f-4\,d^2e^2g)px^6-(3\,d^2e^2f-4\,d^3eg)px^4-2\,((3\,e^4f-}{48\,d^4x^8}$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^9,x, algorithm="fricas")
```

```
[Out] -1/48*(4*(3*e^4*f - 4*d*e^3*g)*p*x^8*log(x) + 2*d^3*e*f*p*x^2 + 2*(3*d*e^3*
f - 4*d^2*e^2*g)*p*x^6 - (3*d^2*e^2*f - 4*d^3*e*g)*p*x^4 - 2*((3*e^4*f - 4*
d*e^3*g)*p*x^8 - 4*d^4*g*p*x^2 - 3*d^4*f*p)*log(e*x^2 + d) + 2*(4*d^4*g*x^2
 + 3*d^4*f)*log(c))/(d^4*x^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^9}\,dx = \text{Timed out}$$

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**9,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.89

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^9}\,dx$$
$$=\frac{1}{48}\,ep\left(\frac{2\,(3\,e^3f-4\,de^2g)\log\left(ex^2+d\right)}{d^4}-\frac{2\,(3\,e^3f-4\,de^2g)\log\left(x^2\right)}{d^4}-\frac{2\,(3\,e^2f-4\,deg)x^4+2\,d^2f-(3\,de\right.$$
$$-\frac{(4\,gx^2+3\,f)\log\left((ex^2+d)^pc\right)}{24\,x^8}$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^9,x, algorithm="maxima")
```

```
[Out] 1/48*e*p*(2*(3*e^3*f - 4*d*e^2*g)*log(e*x^2 + d)/d^4 - 2*(3*e^3*f - 4*d*e^2
*g)*log(x^2)/d^4 - (2*(3*e^2*f - 4*d*e*g)*x^4 + 2*d^2*f - (3*d*e*f - 4*d^2*
g)*x^2)/(d^3*x^6)) - 1/24*(4*g*x^2 + 3*f)*log((e*x^2 + d)^p*c)/x^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(134) = 268.

Time = 0.32 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.56

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^9}\,dx =$$
$$-\frac{\frac{2\,(3\,e^5fp+4\,(ex^2+d)e^4gp-4\,de^4gp)\log(ex^2+d)}{(ex^2+d)^4-4\,(ex^2+d)^3d+6\,(ex^2+d)^2d^2-4\,(ex^2+d)d^3+d^4}+\frac{6\,(ex^2+d)^3e^5fp-21\,(ex^2+d)^2de^5fp+26\,(ex^2+d)d^2e^5fp-11\,d^3e^5fp-8\,(ex^2+}{(ex^2+d)^4d}}{}$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^9,x, algorithm="giac")
```

```
[Out] -1/48*(2*(3*e^5*f*p + 4*(e*x^2 + d)*e^4*g*p - 4*d*e^4*g*p)*log(e*x^2 + d)/(
(e*x^2 + d)^4 - 4*(e*x^2 + d)^3*d + 6*(e*x^2 + d)^2*d^2 - 4*(e*x^2 + d)*d^3
 + d^4) + (6*(e*x^2 + d)^3*e^5*f*p - 21*(e*x^2 + d)^2*d*e^5*f*p + 26*(e*x^2
 + d)*d^2*e^5*f*p - 11*d^3*e^5*f*p - 8*(e*x^2 + d)^3*d*e^4*g*p + 28*(e*x^2
+ d)^2*d^2*e^4*g*p - 32*(e*x^2 + d)*d^3*e^4*g*p + 12*d^4*e^4*g*p + 6*d^3*e^
5*f*log(c) + 8*(e*x^2 + d)*d^3*e^4*g*log(c) - 8*d^4*e^4*g*log(c))/((e*x^2 +
 d)^4*d^3 - 4*(e*x^2 + d)^3*d^4 + 6*(e*x^2 + d)^2*d^5 - 4*(e*x^2 + d)*d^6 +
 d^7) - 2*(3*e^5*f*p - 4*d*e^4*g*p)*log(e*x^2 + d)/d^4 + 2*(3*e^5*f*p - 4*d
*e^4*g*p)*log(e*x^2)/d^4)/e
```

Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.91

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^9}\,dx = \frac{\ln\left(e\,x^2+d\right)\,\left(3\,e^4\,f\,p-4\,d\,e^3\,g\,p\right)}{24\,d^4} - \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{g\,x^2}{6}+\frac{f}{8}\right)}{x^8} - \frac{\frac{e\,f\,p}{2\,d}+\frac{e\,p\,x^2\,\left(4\,d\,g-3\,e\,f\right)}{4\,d^2}-\frac{e^2\,p\,x^4\,\left(4\,d\,g-3\,e\,f\right)}{2\,d^3}}{12\,x^6} - \frac{\ln\left(x\right)\,\left(3\,e^4\,f\,p-4\,d\,e^3\,g\,p\right)}{12\,d^4}$$

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^9,x)
```

```
[Out] (log(d + e*x^2)*(3*e^4*f*p - 4*d*e^3*g*p))/(24*d^4) - (log(c*(d + e*x^2)^p)
*(f/8 + (g*x^2)/6))/x^8 - ((e*f*p)/(2*d) + (e*p*x^2*(4*d*g - 3*e*f))/(4*d^2
) - (e^2*p*x^4*(4*d*g - 3*e*f))/(2*d^3))/(12*x^6) - (log(x)*(3*e^4*f*p - 4*
d*e^3*g*p))/(12*d^4)
```

3.318 $\int x^2(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 154

$$\int x^2(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx=\frac{2dfpx}{3e}-\frac{2d^2gpx}{5e^2}-\frac{2}{9}fpx^3+\frac{2dgpx^3}{15e}-\frac{2}{25}gpx^5-\frac{2d^{3/2}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2d^{5/2}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}}+\frac{1}{3}fx^3\log\left(c(d+ex^2)^p\right)+\frac{1}{5}gx^5\log\left(c(d+ex^2)^p\right)$$

```
[Out] 2/3*d*f*p*x/e-2/5*d^2*g*p*x/e^2-2/9*f*p*x^3+2/15*d*g*p*x^3/e-2/25*g*p*x^5-2
/3*d^(3/2)*f*p*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)+2/5*d^(5/2)*g*p*arctan(x*e
^(1/2)/d^(1/2))/e^(5/2)+1/3*f*x^3*ln(c*(e*x^2+d)^p)+1/5*g*x^5*ln(c*(e*x^2+d
)^p)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2526, 2505, 308, 211}

$$\int x^2(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx=-\frac{2d^{3/2}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2d^{5/2}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}}+\frac{1}{3}fx^3\log\left(c(d+ex^2)^p\right)+\frac{1}{5}gx^5\log\left(c(d+ex^2)^p\right)-\frac{2d^2gpx}{5e^2}+\frac{2dfpx}{3e}+\frac{2dgpx^3}{15e}-\frac{2}{9}fpx^3-\frac{2}{25}gpx^5$$

```
[In] Int[x^2*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (2*d*f*p*x)/(3*e) - (2*d^2*g*p*x)/(5*e^2) - (2*f*p*x^3)/9 + (2*d*g*p*x^3)/(
15*e) - (2*g*p*x^5)/25 - (2*d^(3/2)*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^(
```

```
3/2)) + (2*d^(5/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*e^(5/2)) + (f*x^3*Lo
g[c*(d + e*x^2)^p])/3 + (g*x^5*Log[c*(d + e*x^2)^p])/5
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(fx^2\log\left(c\left(d+ex^2\right)^p\right)+gx^4\log\left(c\left(d+ex^2\right)^p\right)\right)\,dx\\
&= f\int x^2\log\left(c\left(d+ex^2\right)^p\right)\,dx+g\int x^4\log\left(c\left(d+ex^2\right)^p\right)\,dx\\
&= \frac{1}{3}fx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}gx^5\log\left(c\left(d+ex^2\right)^p\right)\\
&\quad-\frac{1}{3}(2efp)\int\frac{x^4}{d+ex^2}\,dx-\frac{1}{5}(2egp)\int\frac{x^6}{d+ex^2}\,dx\\
&= \frac{1}{3}fx^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}gx^5\log\left(c\left(d+ex^2\right)^p\right)\\
&\quad-\frac{1}{3}(2efp)\int\left(-\frac{d}{e^2}+\frac{x^2}{e}+\frac{d^2}{e^2\left(d+ex^2\right)}\right)\,dx\\
&\quad-\frac{1}{5}(2egp)\int\left(\frac{d^2}{e^3}-\frac{dx^2}{e^2}+\frac{x^4}{e}-\frac{d^3}{e^3\left(d+ex^2\right)}\right)\,dx
\end{aligned}
$$

$$= \frac{2dfpx}{3e} - \frac{2d^2gpx}{5e^2} - \frac{2}{9}fpx^3 + \frac{2dgpx^3}{15e} - \frac{2}{25}gpx^5 + \frac{1}{3}fx^3\log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{5}gx^5\log\left(c\left(d+ex^2\right)^p\right) - \frac{(2d^2fp)\int\frac{1}{d+ex^2}\,dx}{3e} + \frac{(2d^3gp)\int\frac{1}{d+ex^2}\,dx}{5e^2}$$

$$= \frac{2dfpx}{3e} - \frac{2d^2gpx}{5e^2} - \frac{2}{9}fpx^3 + \frac{2dgpx^3}{15e} - \frac{2}{25}gpx^5 - \frac{2d^{3/2}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}} + \frac{2d^{5/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}} + \frac{1}{3}fx^3\log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{5}gx^5\log\left(c\left(d+ex^2\right)^p\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.77

$$\int x^2\left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx = \frac{30d^{3/2}(-5ef+3dg)p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right) + \sqrt{e}x\left(-2p(45d^2g-15de(5f+gx^2)+e^2x^2(25f+9gx^2)) + 15e^2x^2(\right.}{225e^{5/2}}$$

```
[In] Integrate[x^2*(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (30*d^(3/2)*(-5*e*f + 3*d*g)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[e]*x*(-2*
p*(45*d^2*g - 15*d*e*(5*f + g*x^2) + e^2*x^2*(25*f + 9*g*x^2)) + 15*e^2*x^2
*(5*f + 3*g*x^2)*Log[c*(d + e*x^2)^p]))/(225*e^(5/2))
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.77

method	result
parts	$\frac{g\,x^5\ln\left(c\left(e\,x^2+d\right)^p\right)}{5} + \frac{f\,x^3\ln\left(c\left(e\,x^2+d\right)^p\right)}{3} - \frac{2pe\left(\frac{\frac{3}{5}e^2g\,x^5-dg\,x^3e+\frac{5}{3}f\,x^3e^2+3d^2gx-5defx}{e^3} - \frac{d^2(3dg-5ef)\arctan\left(\frac{xe}{\sqrt{de}}\right)}{e^3\sqrt{de}}\right)}{15}$
risch	$\left(\frac{1}{5}g\,x^5+\frac{1}{3}f\,x^3\right)\ln\left(\left(e\,x^2+d\right)^p\right) - \frac{i\pi g\,x^5\,\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)\mathrm{csgn}(ic)}{10} + \frac{i\pi g\,x^5\,\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csg}}{10}$

```
[In] int(x^2*(g*x^2+f)*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*g*x^5*ln(c*(e*x^2+d)^p)+1/3*f*x^3*ln(c*(e*x^2+d)^p)-2/15*p*e*(1/e^3*(3/
5*e^2*g*x^5-d*g*x^3*e+5/3*f*x^3*e^2+3*d^2*g*x-5*d*e*f*x)-d^2*(3*d*g-5*e*f)/
e^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.95

$$\int x^2(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$$
$$=\left[-\frac{18\,e^2gpx^5+10\,(5\,e^2f-3\,deg)px^3+15\,(5\,def-3\,d^2g)p\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2+2\,ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right)-30\,(5\,def-3\,d^2g)}{225\,e^2}\right.$$
$$-\frac{18\,e^2gpx^5+10\,(5\,e^2f-3\,deg)px^3+30\,(5\,def-3\,d^2g)p\sqrt{\frac{d}{e}}\arctan\left(\frac{ex\sqrt{\frac{d}{e}}}{d}\right)-30\,(5\,def-3\,d^2g)px-15}{225\,e^2}$$

```
[In] integrate(x^2*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/225*(18*e^2*g*p*x^5 + 10*(5*e^2*f - 3*d*e*g)*p*x^3 + 15*(5*d*e*f - 3*d^
2*g)*p*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) - 30*(5*d
*e*f - 3*d^2*g)*p*x - 15*(3*e^2*g*p*x^5 + 5*e^2*f*p*x^3)*log(e*x^2 + d) - 1
5*(3*e^2*g*x^5 + 5*e^2*f*x^3)*log(c))/e^2, -1/225*(18*e^2*g*p*x^5 + 10*(5*e
^2*f - 3*d*e*g)*p*x^3 + 30*(5*d*e*f - 3*d^2*g)*p*sqrt(d/e)*arctan(e*x*sqrt(
d/e)/d) - 30*(5*d*e*f - 3*d^2*g)*p*x - 15*(3*e^2*g*p*x^5 + 5*e^2*f*p*x^3)*l
og(e*x^2 + d) - 15*(3*e^2*g*x^5 + 5*e^2*f*x^3)*log(c))/e^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(158) = 316.

Time = 31.34 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.08

$$\int x^2(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$$
$$=\begin{cases}\left(\frac{fx^3}{3}+\frac{gx^5}{5}\right)\log(0^pc)\\ \left(\frac{fx^3}{3}+\frac{gx^5}{5}\right)\log(cd^p)\\ -\frac{2fpx^3}{9}+\frac{fx^3\log\left(c(ex^2)^p\right)}{3}-\frac{2gpx^5}{25}+\frac{gx^5\log\left(c(ex^2)^p\right)}{5}\\ \frac{2d^3gp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{5e^3\sqrt{-\frac{d}{e}}}-\frac{d^3g\log\left(c(d+ex^2)^p\right)}{5e^3\sqrt{-\frac{d}{e}}}-\frac{2d^2fp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{3e^2\sqrt{-\frac{d}{e}}}+\frac{d^2f\log\left(c(d+ex^2)^p\right)}{3e^2\sqrt{-\frac{d}{e}}}-\frac{2d^2gpx}{5e^2}+\frac{2dfpx}{3e}+\frac{2dgpx^3}{15e}-\frac{2fpx}{9}\end{cases}$$

```
[In] integrate(x**2*(g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise(((f*x**3/3 + g*x**5/5)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), ((f*x**
3/3 + g*x**5/5)*log(c*d**p), Eq(e, 0)), (-2*f*p*x**3/9 + f*x**3*log(c*(e*x*
*2)**p)/3 - 2*g*p*x**5/25 + g*x**5*log(c*(e*x**2)**p)/5, Eq(d, 0)), (2*d**3
*g*p*log(x - sqrt(-d/e))/(5*e**3*sqrt(-d/e)) - d**3*g*log(c*(d + e*x**2)**p
)/(5*e**3*sqrt(-d/e)) - 2*d**2*f*p*log(x - sqrt(-d/e))/(3*e**2*sqrt(-d/e))
+ d**2*f*log(c*(d + e*x**2)**p)/(3*e**2*sqrt(-d/e)) - 2*d**2*g*p*x/(5*e**2)
 + 2*d*f*p*x/(3*e) + 2*d*g*p*x**3/(15*e) - 2*f*p*x**3/9 + f*x**3*log(c*(d +
 e*x**2)**p)/3 - 2*g*p*x**5/25 + g*x**5*log(c*(d + e*x**2)**p)/5, True))
```

Maxima [F(-2)]

Exception generated.

$$\int x^2(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

$$\begin{aligned}\int x^2(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = & -\frac{1}{25}\,(2\,gp-5\,g\log\left(c\right))x^5 \\ & -\frac{(10\,efp-6\,dgp-15\,ef\log\left(c\right))x^3}{45\,e} \\ & +\frac{1}{15}\left(3\,gpx^5+5\,fpx^3\right)\log\left(ex^2+d\right) \\ & +\frac{2\,(5\,defp-3\,d^2gp)x}{15\,e^2} \\ & -\frac{2\,(5\,d^2efp-3\,d^3gp)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{15\,\sqrt{de}e^2}\end{aligned}$$

```
[In] integrate(x^2*(g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -1/25*(2*g*p - 5*g*log(c))*x^5 - 1/45*(10*e*f*p - 6*d*g*p - 15*e*f*log(c))*
x^3/e + 1/15*(3*g*p*x^5 + 5*f*p*x^3)*log(e*x^2 + d) + 2/15*(5*d*e*f*p - 3*d
^2*g*p)*x/e^2 - 2/15*(5*d^2*e*f*p - 3*d^3*g*p)*arctan(e*x/sqrt(d*e))/(sqrt(
d*e)*e^2)
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.82

$$\int x^2(f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx=\ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(\frac{g\,x^5}{5}+\frac{f\,x^3}{3}\right)-x^3\left(\frac{2\,f\,p}{9}-\frac{2\,d\,g\,p}{15\,e}\right)-\frac{2\,g\,p\,x^5}{25}+\frac{d\,x\,\left(\frac{2\,f\,p}{3}-\frac{2\,d\,g\,p}{5\,e}\right)}{e}+\frac{2\,d^{3/2}\,p\,\mathrm{atan}\left(\frac{d^{3/2}\,\sqrt{e}\,p\,x\,(3\,d\,g-5\,e\,f)}{3\,d^3\,g\,p-5\,d^2\,e\,f\,p}\right)\,(3\,d\,g-5\,e\,f)}{15\,e^{5/2}}$$

```
[In] int(x^2*log(c*(d + e*x^2)^p)*(f + g*x^2),x)
```

```
[Out] log(c*(d + e*x^2)^p)*((f*x^3)/3 + (g*x^5)/5) - x^3*((2*f*p)/9 - (2*d*g*p)/(
15*e)) - (2*g*p*x^5)/25 + (d*x*((2*f*p)/3 - (2*d*g*p)/(5*e)))/e + (2*d^(3/2
)*p*atan((d^(3/2)*e^(1/2)*p*x*(3*d*g - 5*e*f))/(3*d^3*g*p - 5*d^2*e*f*p))*(
3*d*g - 5*e*f))/(15*e^(5/2))
```

3.319 $\int (f+gx^2) \log(c(d+ex^2)^p) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int (f+gx^2) \log\left(c(d+ex^2)^p\right) \, dx = -2fpx + \frac{2dgpx}{3e} - \frac{2}{9}gpx^3 + \frac{2\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{2d^{3/2}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}} + fx\log\left(c(d+ex^2)^p\right) + \frac{1}{3}gx^3\log\left(c(d+ex^2)^p\right)$$

```
[Out] -2*f*p*x+2/3*d*g*p*x/e-2/9*g*p*x^3-2/3*d^(3/2)*g*p*arctan(x*e^(1/2)/d^(1/2)
)/e^(3/2)+f*x*ln(c*(e*x^2+d)^p)+1/3*g*x^3*ln(c*(e*x^2+d)^p)+2*f*p*arctan(x*
e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2521, 2498, 327, 211, 2505, 308}

$$\int (f+gx^2) \log\left(c(d+ex^2)^p\right) \, dx = -\frac{2d^{3/2}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}} + \frac{2\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + fx\log\left(c(d+ex^2)^p\right) + \frac{1}{3}gx^3\log\left(c(d+ex^2)^p\right) + \frac{2dgpx}{3e} - 2fpx - \frac{2}{9}gpx^3$$

```
[In] Int[(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f*p*x + (2*d*g*p*x)/(3*e) - (2*g*p*x^3)/9 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^
(3/2)) + f*x*Log[c*(d + e*x^2)^p] + (g*x^3*Log[c*(d + e*x^2)^p])/3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(f \log\left(c\left(d+ex^2\right)^p\right) + gx^2 \log\left(c\left(d+ex^2\right)^p\right)\right) dx \\
&= f \int \log\left(c\left(d+ex^2\right)^p\right) dx + g \int x^2 \log\left(c\left(d+ex^2\right)^p\right) dx \\
&= fx \log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{3}gx^3 \log\left(c\left(d+ex^2\right)^p\right) - (2efp) \int \frac{x^2}{d+ex^2} dx - \frac{1}{3}(2egp) \int \frac{x^4}{d+ex^2} dx \\
&= -2fpx + fx \log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{3}gx^3 \log\left(c\left(d+ex^2\right)^p\right) \\
&\quad + (2dfp) \int \frac{1}{d+ex^2} dx - \frac{1}{3}(2egp) \int \left(-\frac{d}{e^2} + \frac{x^2}{e} + \frac{d^2}{e^2\left(d+ex^2\right)}\right) dx \\
&= -2fpx + \frac{2dgpx}{3e} - \frac{2}{9}gpx^3 + \frac{2\sqrt{d}fp \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} \\
&\quad + fx \log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{3}gx^3 \log\left(c\left(d+ex^2\right)^p\right) - \frac{\left(2d^2gp\right) \int \frac{1}{d+ex^2} dx}{3e} \\
&= -2fpx + \frac{2dgpx}{3e} - \frac{2}{9}gpx^3 + \frac{2\sqrt{d}fp \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{2d^{3/2}gp \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}} \\
&\quad + fx \log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{3}gx^3 \log\left(c\left(d+ex^2\right)^p\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int \left(f+gx^2\right) \log\left(c\left(d+ex^2\right)^p\right) dx = {} & -2fpx + \frac{2dgpx}{3e} - \frac{2}{9}gpx^3 \\
& + \frac{2\sqrt{d}fp \arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{2d^{3/2}gp \arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}} \\
& + fx \log\left(c\left(d+ex^2\right)^p\right) + \frac{1}{3}gx^3 \log\left(c\left(d+ex^2\right)^p\right)
\end{aligned}
$$

```
[In] Integrate[(f + g*x^2)*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f*p*x + (2*d*g*p*x)/(3*e) - (2*g*p*x^3)/9 + (2*Sqrt[d]*f*p*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/Sqrt[e] - (2*d^(3/2)*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^
(3/2)) + f*x*Log[c*(d + e*x^2)^p] + (g*x^3*Log[c*(d + e*x^2)^p])/3
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.76

method	result
parts	$\frac{g\,x^{3} \ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{3}+f x \ln\left(c\left(e\,x^{2}+d\right)^{p}\right)-\frac{2 p e \left(-\frac{-\frac{1}{3} e g\,x^{3}+d g x-3 e f x}{e^{2}}+\frac{d\left(d g-3 e f\right) \arctan\left(\frac{x e}{\sqrt{d e}}\right)}{e^{2} \sqrt{d e}}\right)}{3}$
risch	$\left(\frac{1}{3} g\,x^{3}+f x\right) \ln\left(\left(e\,x^{2}+d\right)^{p}\right)+\frac{i x \pi f\ \mathrm{csgn}\left(i\left(e\,x^{2}+d\right)^{p}\right) \mathrm{csgn}\left(i c\left(e\,x^{2}+d\right)^{p}\right)^{2}}{2}-\frac{i x \pi f\ \mathrm{csgn}\left(i\left(e\,x^{2}+d\right)^{p}\right) \mathrm{csgn}\left(i c\left(e\,x^{2}+d\right)^{p}\right.}{2}$

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*g*x^3*ln(c*(e*x^2+d)^p)+f*x*ln(c*(e*x^2+d)^p)-2/3*p*e*(-1/e^2*(-1/3*e*g
*x^3+d*g*x-3*e*f*x)+d*(d*g-3*e*f)/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.88

$$\int \left(f+gx^2\right)\log\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$=\left[-\frac{2\,egpx^3+3\,(3\,ef-dg)p\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2-2\,ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right)+6\,(3\,ef-dg)px-3\,(egpx^3+3\,efpx)\log\left(ex^2+d\right.}{9\,e}\right.$$
$$-\frac{2\,egpx^3-6\,(3\,ef-dg)p\sqrt{\frac{d}{e}}\arctan\left(\frac{ex\sqrt{\frac{d}{e}}}{d}\right)+6\,(3\,ef-dg)px-3\,(egpx^3+3\,efpx)\log\left(ex^2+d\right)-3\,(e}{9\,e}$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/9*(2*e*g*p*x^3 + 3*(3*e*f - d*g)*p*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-
d/e) - d)/(e*x^2 + d)) + 6*(3*e*f - d*g)*p*x - 3*(e*g*p*x^3 + 3*e*f*p*x)*lo
g(e*x^2 + d) - 3*(e*g*x^3 + 3*e*f*x)*log(c))/e, -1/9*(2*e*g*p*x^3 - 6*(3*e*
f - d*g)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 6*(3*e*f - d*g)*p*x - 3*(e*g
*p*x^3 + 3*e*f*p*x)*log(e*x^2 + d) - 3*(e*g*x^3 + 3*e*f*x)*log(c))/e]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(121) = 242$.

Time = 7.80 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.22

$$\int (f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx$$
$$=\begin{cases}\left(fx+\frac{gx^3}{3}\right)\log\left(0^pc\right)\\ \left(fx+\frac{gx^3}{3}\right)\log\left(cd^p\right)\\ -2fpx+fx\log\left(c(ex^2)^p\right)-\frac{2gpx^3}{9}+\frac{gx^3\log\left(c(ex^2)^p\right)}{3}\\ -\frac{2d^2gp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{3e^2\sqrt{-\frac{d}{e}}}+\frac{d^2g\log\left(c(d+ex^2)^p\right)}{3e^2\sqrt{-\frac{d}{e}}}+\frac{2dfp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{e\sqrt{-\frac{d}{e}}}-\frac{df\log\left(c(d+ex^2)^p\right)}{e\sqrt{-\frac{d}{e}}}+\frac{2dgpx}{3e}-2fpx+fx\log\left(c(d\right.\end{cases}$$

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise(((f*x + g*x**3/3)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), ((f*x + g*x*
*3/3)*log(c*d**p), Eq(e, 0)), (-2*f*p*x + f*x*log(c*(e*x**2)**p) - 2*g*p*x*
*3/9 + g*x**3*log(c*(e*x**2)**p)/3, Eq(d, 0)), (-2*d**2*g*p*log(x - sqrt(-d
/e))/(3*e**2*sqrt(-d/e)) + d**2*g*log(c*(d + e*x**2)**p)/(3*e**2*sqrt(-d/e)
) + 2*d*f*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) - d*f*log(c*(d + e*x**2)**p)
/(e*sqrt(-d/e)) + 2*d*g*p*x/(3*e) - 2*f*p*x + f*x*log(c*(d + e*x**2)**p) -
2*g*p*x**3/9 + g*x**3*log(c*(d + e*x**2)**p)/3, True))
```

Maxima [F(-2)]

Exception generated.

$$\int (f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

$$\int (f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = -\frac{1}{9}\,(2\,gp-3\,g\log(c))x^3+\frac{1}{3}\,(gpx^3+3\,fpx)\log\left(ex^2+d\right) - \frac{(6\,efp-2\,dgp-3\,ef\log(c))x}{3\,e} + \frac{2\,(3\,defp-d^2gp)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{3\,\sqrt{de}e}$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -1/9*(2*g*p - 3*g*log(c))*x^3 + 1/3*(g*p*x^3 + 3*f*p*x)*log(e*x^2 + d) - 1/
3*(6*e*f*p - 2*d*g*p - 3*e*f*log(c))*x/e + 2/3*(3*d*e*f*p - d^2*g*p)*arctan
(e*x/sqrt(d*e))/(sqrt(d*e)*e)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.83

$$\int (f+gx^2)\log\left(c(d+ex^2)^p\right)\,dx = \ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(\frac{g\,x^3}{3}+f\,x\right)-x\,\left(2\,f\,p-\frac{2\,d\,g\,p}{3\,e}\right) - \frac{2\,g\,p\,x^3}{9}-\frac{2\,\sqrt{d}\,p\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e}\,p\,x\,(d\,g-3\,e\,f)}{d^2\,g\,p-3\,d\,e\,f\,p}\right)\,(d\,g-3\,e\,f)}{3\,e^{3/2}}$$

```
[In] int(log(c*(d + e*x^2)^p)*(f + g*x^2),x)
```

```
[Out] log(c*(d + e*x^2)^p)*(f*x + (g*x^3)/3) - x*(2*f*p - (2*d*g*p)/(3*e)) - (2*g
*p*x^3)/9 - (2*d^(1/2)*p*atan((d^(1/2)*e^(1/2)*p*x*(d*g - 3*e*f))/(d^2*g*p
- 3*d*e*f*p))*(d*g - 3*e*f))/(3*e^(3/2))
```

3.320 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^2}\,dx = -2gpx + \frac{2(ef+dg)p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}} - \frac{f\log\left(c(d+ex^2)^p\right)}{x} + gx\log\left(c(d+ex^2)^p\right)$$

```
[Out] -2*g*p*x-f*ln(c*(e*x^2+d)^p)/x+g*x*ln(c*(e*x^2+d)^p)+2*(d*g+e*f)*p*arctan(x
*e^(1/2)/d^(1/2))/d^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2526, 2498, 327, 211, 2505}

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^2}\,dx = \frac{2\sqrt{e}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{d}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{f\log\left(c(d+ex^2)^p\right)}{x} + gx\log\left(c(d+ex^2)^p\right) - 2gpx$$

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^2,x]
```

```
[Out] -2*g*p*x + (2*Sqrt[e]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (2*Sqrt[d]
*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (f*Log[c*(d + e*x^2)^p])/x + g*
x*Log[c*(d + e*x^2)^p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(g\log\left(c\left(d+ex^2\right)^p\right)+\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\right)\,dx\\
&= f\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\,dx+g\int \log\left(c\left(d+ex^2\right)^p\right)\,dx\\
&= -\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{x}+gx\log\left(c\left(d+ex^2\right)^p\right)+(2efp)\int \frac{1}{d+ex^2}\,dx-(2egp)\int \frac{x^2}{d+ex^2}\,dx\\
&= -2gpx+\frac{2\sqrt{e}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{f\log\left(c\left(d+ex^2\right)^p\right)}{x}\\
&\quad +gx\log\left(c\left(d+ex^2\right)^p\right)+(2dgp)\int \frac{1}{d+ex^2}\,dx
\end{aligned}
$$

$$= -2gpx + \frac{2\sqrt{e}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{d}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{f\log\left(c(d+ex^2)^p\right)}{x} + gx\log\left(c(d+ex^2)^p\right)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^2}\,dx = -2gpx + \frac{2(ef+dg)p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}} + \left(-\frac{f}{x}+gx\right)\log\left(c(d+ex^2)^p\right)$$

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^2,x]
```

```
[Out] -2*g*p*x + (2*(e*f + d*g)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e])
+ (-(f/x) + g*x)*Log[c*(d + e*x^2)^p]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

method	result
parts	$gx\ln\left(c(e\,x^2+d)^p\right)-\frac{f\ln(c(e\,x^2+d)^p)}{x}-2pe\left(\frac{gx}{e}+\frac{(-dg-ef)\arctan\left(\frac{xe}{\sqrt{de}}\right)}{e\sqrt{de}}\right)$
risch	$-\frac{(-g\,x^2+f)\ln((e\,x^2+d)^p)}{x}+\frac{i\pi g\,x^2\operatorname{csgn}(i(e\,x^2+d)^p)\operatorname{csgn}(ic(e\,x^2+d)^p)^2de-i\pi g\,x^2\operatorname{csgn}(i(e\,x^2+d)^p)\operatorname{csgn}(ic(e\,x^2+d)^p)\operatorname{csgn}($

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] g*x*ln(c*(e*x^2+d)^p)-f*ln(c*(e*x^2+d)^p)/x-2*p*e*(g*x/e+(-d*g-e*f)/e/(d*e)
^(1/2)*arctan(x*e/(d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.76

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^2}\,dx$$
$$= \left[-\frac{2\,degpx^2+\sqrt{-de}(ef+dg)px\log\left(\frac{ex^2-2\sqrt{-de}x-d}{ex^2+d}\right)-(degpx^2-defp)\log\left(ex^2+d\right)-(degx^2-def)\log(c)}{dex}, -\frac{2\,degpx^2-2\sqrt{de}(ef+dg)px\arctan\left(\frac{\sqrt{de}x}{d}\right)-(degpx^2-defp)\log\left(ex^2+d\right)-(degx^2-def)\log(c)}{dex}\right]$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^2,x, algorithm="fricas")
```

```
[Out] [-(2*d*e*g*p*x^2 + sqrt(-d*e)*(e*f + d*g)*p*x*log((e*x^2 - 2*sqrt(-d*e)*x -
 d)/(e*x^2 + d)) - (d*e*g*p*x^2 - d*e*f*p)*log(e*x^2 + d) - (d*e*g*x^2 - d*
e*f)*log(c))/(d*e*x), -(2*d*e*g*p*x^2 - 2*sqrt(d*e)*(e*f + d*g)*p*x*arctan(
sqrt(d*e)*x/d) - (d*e*g*p*x^2 - d*e*f*p)*log(e*x^2 + d) - (d*e*g*x^2 - d*e*
f)*log(c))/(d*e*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(71) = 142$.

Time = 15.37 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.83

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^2}\,dx$$
$$= \begin{cases} \left(-\frac{f}{x}+gx\right)\log\left(0^pc\right) \\ \left(-\frac{f}{x}+gx\right)\log\left(cd^p\right) \\ -\frac{2fp}{x}-\frac{f\log\left(c(ex^2)^p\right)}{x}-2gpx+gx\log\left(c(ex^2)^p\right) \\ \frac{2dgp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{e\sqrt{-\frac{d}{e}}}-\frac{dg\log\left(c(d+ex^2)^p\right)}{e\sqrt{-\frac{d}{e}}}+\frac{2fp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{\sqrt{-\frac{d}{e}}}-\frac{f\log\left(c(d+ex^2)^p\right)}{\sqrt{-\frac{d}{e}}}-\frac{f\log\left(c(d+ex^2)^p\right)}{x}-2gpx+gx\log\left(c(d+ex^2)^p\right) \end{cases}$$

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**2,x)
```

```
[Out] Piecewise(((-f/x + g*x)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), ((-f/x + g*x)*lo
g(c*d**p), Eq(e, 0)), (-2*f*p/x - f*log(c*(e*x**2)**p)/x - 2*g*p*x + g*x*lo
g(c*(e*x**2)**p), Eq(d, 0)), (2*d*g*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) -
d*g*log(c*(d + e*x**2)**p)/(e*sqrt(-d/e)) + 2*f*p*log(x - sqrt(-d/e))/sqrt(
-d/e) - f*log(c*(d + e*x**2)**p)/sqrt(-d/e) - f*log(c*(d + e*x**2)**p)/x -
2*g*p*x + g*x*log(c*(d + e*x**2)**p), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^2}\,dx = -(2\,gp-g\log(c))x + \left(gpx-\frac{fp}{x}\right)\log\left(ex^2+d\right) + \frac{2\,(efp+dgp)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} - \frac{f\log(c)}{x}$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^2,x, algorithm="giac")
```

```
[Out] -(2*g*p - g*log(c))*x + (g*p*x - f*p/x)*log(e*x^2 + d) + 2*(e*f*p + d*g*p)*
arctan(e*x/sqrt(d*e))/sqrt(d*e) - f*log(c)/x
```

Mupad [B] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.15

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^2}\,dx = \ln\left(c\left(e\,x^2+d\right)^p\right)\left(2\,g\,x-\frac{g\,x^2+f}{x}\right) - 2\,g\,p\,x + \frac{2\,p\,\mathrm{atan}\left(\frac{2\,\sqrt{e}\,p\,x\,(d\,g+e\,f)}{\sqrt{d}\,(2\,d\,g\,p+2\,e\,f\,p)}\right)\,(d\,g+e\,f)}{\sqrt{d}\,\sqrt{e}}$$

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^2,x)
```

```
[Out] log(c*(d + e*x^2)^p)*(2*g*x - (f + g*x^2)/x) - 2*g*p*x + (2*p*atan((2*e^(1/
2)*p*x*(d*g + e*f))/(d^(1/2)*(2*d*g*p + 2*e*f*p)))*(d*g + e*f))/(d^(1/2)*e^
(1/2))
```

3.321 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^4}\,dx = -\frac{2efp}{3dx} - \frac{2e^{3/2}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}} + \frac{2\sqrt{e}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{f\log\left(c(d+ex^2)^p\right)}{3x^3} - \frac{g\log\left(c(d+ex^2)^p\right)}{x}$$

```
[Out] -2/3*e*f*p/d/x-2/3*e^(3/2)*f*p*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)-1/3*f*ln(c
*(e*x^2+d)^p)/x^3-g*ln(c*(e*x^2+d)^p)/x+2*g*p*arctan(x*e^(1/2)/d^(1/2))*e^(
1/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2526, 2505, 331, 211}

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^4}\,dx = -\frac{2e^{3/2}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}} + \frac{2\sqrt{e}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{f\log\left(c(d+ex^2)^p\right)}{3x^3} - \frac{g\log\left(c(d+ex^2)^p\right)}{x} - \frac{2efp}{3dx}$$

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^4,x]
```

```
[Out] (-2*e*f*p)/(3*d*x) - (2*e^(3/2)*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*d^(3/2)
) + (2*Sqrt[e]*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] - (f*Log[c*(d + e*x
^2)^p])/(3*x^3) - (g*Log[c*(d + e*x^2)^p])/x
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{f \log \left(c(d+ex^2)^p\right)}{x^4}+\frac{g \log \left(c(d+ex^2)^p\right)}{x^2}\right) dx \\
&= f \int \frac{\log \left(c(d+ex^2)^p\right)}{x^4} \, dx+g \int \frac{\log \left(c(d+ex^2)^p\right)}{x^2} \, dx \\
&= -\frac{f \log \left(c(d+ex^2)^p\right)}{3x^3}-\frac{g \log \left(c(d+ex^2)^p\right)}{x} \\
&\quad +\frac{1}{3}(2efp) \int \frac{1}{x^2\left(d+ex^2\right)} \, dx+(2egp) \int \frac{1}{d+ex^2} \, dx \\
&= -\frac{2efp}{3dx}+\frac{2\sqrt{e}gp \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{f \log \left(c(d+ex^2)^p\right)}{3x^3} \\
&\quad -\frac{g \log \left(c(d+ex^2)^p\right)}{x}-\frac{(2e^2fp) \int \frac{1}{d+ex^2} \, dx}{3d}
\end{aligned}
$$

$$= -\frac{2efp}{3dx} - \frac{2e^{3/2}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}} + \frac{2\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{f\log\left(c(d+ex^2)^p\right)}{3x^3} - \frac{g\log\left(c(d+ex^2)^p\right)}{x}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^4}\,dx = \frac{2\sqrt{e}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{2efp\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{ex^2}{d}\right)}{3dx} - \frac{f\log\left(c(d+ex^2)^p\right)}{3x^3} - \frac{g\log\left(c(d+ex^2)^p\right)}{x}$$

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^4,x]
```

```
[Out] (2*Sqrt[e]*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] - (2*e*f*p*Hypergeometr
ic2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) - (f*Log[c*(d + e*x^2)^p])/(3*x^
3) - (g*Log[c*(d + e*x^2)^p])/x
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.71

method	result
parts	$-\frac{g\ln\left(c\left(e\,x^2+d\right)^p\right)}{x}-\frac{f\ln\left(c\left(e\,x^2+d\right)^p\right)}{3x^3}-\frac{2pe\left(\frac{(-3dg+ef)\arctan\left(\frac{xe}{\sqrt{de}}\right)}{d\sqrt{de}}+\frac{f}{dx}\right)}{3}$
risch	$-\frac{(3g\,x^2+f)\ln\left(\left(e\,x^2+d\right)^p\right)}{3x^3}-\frac{3i\pi\,d^2g\,x^2\,\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^2-3i\pi\,d^2g\,x^2\,\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)\mathrm{csg}}{}$

```
[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -g*ln(c*(e*x^2+d)^p)/x-1/3*f*ln(c*(e*x^2+d)^p)/x^3-2/3*p*e*((-3*d*g+e*f)/d/
(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+f/d/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.77

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^4}\,dx$$
$$= \left[-\frac{(ef-3\,dg)px^3\sqrt{-\frac{e}{d}}\log\left(\frac{ex^2+2\,dx\sqrt{-\frac{e}{d}}-d}{ex^2+d}\right)+2\,efpx^2+(3\,dgpx^2+dfp)\log\left(ex^2+d\right)+(3\,dgx^2+df)\,\mathrm{l}}{3\,dx^3}\right.$$
$$\left.-\frac{2\,(ef-3\,dg)px^3\sqrt{\frac{e}{d}}\arctan\left(x\sqrt{\frac{e}{d}}\right)+2\,efpx^2+(3\,dgpx^2+dfp)\log\left(ex^2+d\right)+(3\,dgx^2+df)\log\left(c\right)}{3\,dx^3}\right]$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/3*((e*f - 3*d*g)*p*x^3*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e/d) - d)/(e
*x^2 + d)) + 2*e*f*p*x^2 + (3*d*g*p*x^2 + d*f*p)*log(e*x^2 + d) + (3*d*g*x^
2 + d*f)*log(c))/(d*x^3), -1/3*(2*(e*f - 3*d*g)*p*x^3*sqrt(e/d)*arctan(x*sq
rt(e/d)) + 2*e*f*p*x^2 + (3*d*g*p*x^2 + d*f*p)*log(e*x^2 + d) + (3*d*g*x^2
+ d*f)*log(c))/(d*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 901 vs. 2(104) = 208.

Time = 36.67 (sec) , antiderivative size = 901, normalized size of antiderivative = 8.34

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^4}\,dx$$
$$= \begin{cases} \left(-\frac{f}{3x^3}-\frac{g}{x}\right)\log\left(0^pc\right) \\ \left(-\frac{f}{3x^3}-\frac{g}{x}\right)\log\left(cd^p\right) \\ -\frac{2fp}{9x^3}-\frac{f\log\left(c(ex^2)^p\right)}{3x^3}-\frac{2gp}{x}-\frac{g\log\left(c(ex^2)^p\right)}{x} \\ \left(-\frac{f}{3x^3}-\frac{g}{x}\right)\log\left(0^pc\right) \\ -\frac{d^2f\sqrt{-\frac{d}{e}}\log\left(c(d+ex^2)^p\right)}{3d^2x^3\sqrt{-\frac{d}{e}}+3dex^5\sqrt{-\frac{d}{e}}}+\frac{6d^2gpx^3\log\left(x-\sqrt{-\frac{d}{e}}\right)}{3d^2x^3\sqrt{-\frac{d}{e}}+3dex^5\sqrt{-\frac{d}{e}}}-\frac{3d^2gx^3\log\left(c(d+ex^2)^p\right)}{3d^2x^3\sqrt{-\frac{d}{e}}+3dex^5\sqrt{-\frac{d}{e}}}-\frac{3d^2gx^2\sqrt{-\frac{d}{e}}\log\left(c(d+ex^2)^p\right)}{3d^2x^3\sqrt{-\frac{d}{e}}+3dex^5\sqrt{-\frac{d}{e}}}-\frac{2dfpx^3\log\left(\right.}{\frac{3d^2x^3\sqrt{-\frac{d}{e}}}{e}+} \end{cases}$$

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**4,x)
```

```
[Out] Piecewise(((-f/(3*x**3) - g/x)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), ((-f/(3*x
**3) - g/x)*log(c*d**p), Eq(e, 0)), (-2*f*p/(9*x**3) - f*log(c*(e*x**2)**p)
/(3*x**3) - 2*g*p/x - g*log(c*(e*x**2)**p)/x, Eq(d, 0)), ((-f/(3*x**3) - g/
x)*log(0**p*c), Eq(d, -e*x**2)), (-d**2*f*sqrt(-d/e)*log(c*(d + e*x**2)**p)
/(3*d**2*x**3*sqrt(-d/e) + 3*d*e*x**5*sqrt(-d/e)) + 6*d**2*g*p*x**3*log(x -
 sqrt(-d/e))/(3*d**2*x**3*sqrt(-d/e) + 3*d*e*x**5*sqrt(-d/e)) - 3*d**2*g*x*
*3*log(c*(d + e*x**2)**p)/(3*d**2*x**3*sqrt(-d/e) + 3*d*e*x**5*sqrt(-d/e))
- 3*d**2*g*x**2*sqrt(-d/e)*log(c*(d + e*x**2)**p)/(3*d**2*x**3*sqrt(-d/e) +
 3*d*e*x**5*sqrt(-d/e)) - 2*d*f*p*x**3*log(x - sqrt(-d/e))/(3*d**2*x**3*sqr
t(-d/e)/e + 3*d*x**5*sqrt(-d/e)) - 2*d*f*p*x**2*sqrt(-d/e)/(3*d**2*x**3*sqr
t(-d/e)/e + 3*d*x**5*sqrt(-d/e)) + d*f*x**3*log(c*(d + e*x**2)**p)/(3*d**2*
x**3*sqrt(-d/e)/e + 3*d*x**5*sqrt(-d/e)) - d*f*x**2*sqrt(-d/e)*log(c*(d + e
*x**2)**p)/(3*d**2*x**3*sqrt(-d/e)/e + 3*d*x**5*sqrt(-d/e)) + 6*d*g*p*x**5*
log(x - sqrt(-d/e))/(3*d**2*x**3*sqrt(-d/e)/e + 3*d*x**5*sqrt(-d/e)) - 3*d*
g*x**5*log(c*(d + e*x**2)**p)/(3*d**2*x**3*sqrt(-d/e)/e + 3*d*x**5*sqrt(-d/
e)) - 3*d*g*x**4*sqrt(-d/e)*log(c*(d + e*x**2)**p)/(3*d**2*x**3*sqrt(-d/e)/
e + 3*d*x**5*sqrt(-d/e)) - 2*e*f*p*x**5*log(x - sqrt(-d/e))/(3*d**2*x**3*sq
rt(-d/e)/e + 3*d*x**5*sqrt(-d/e)) - 2*e*f*p*x**4*sqrt(-d/e)/(3*d**2*x**3*sq
rt(-d/e)/e + 3*d*x**5*sqrt(-d/e)) + e*f*x**5*log(c*(d + e*x**2)**p)/(3*d**2
*x**3*sqrt(-d/e)/e + 3*d*x**5*sqrt(-d/e)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.81

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^4}\,dx = -\frac{2\,(e^2fp-3\,degp)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{3\,\sqrt{de}d} - \frac{(3\,gpx^2+fp)\log\left(ex^2+d\right)}{3\,x^3} - \frac{2\,efpx^2+3\,dgx^2\log\left(c\right)+df\log\left(c\right)}{3\,dx^3}$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^4,x, algorithm="giac")
```

```
[Out] -2/3*(e^2*f*p - 3*d*e*g*p)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d) - 1/3*(3*g*p
*x^2 + f*p)*log(e*x^2 + d)/x^3 - 1/3*(2*e*f*p*x^2 + 3*d*g*x^2*log(c) + d*f*
log(c))/(d*x^3)
```

Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.60

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^4}\,dx = \frac{2\,\sqrt{e}\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(3\,d\,g-e\,f\right)}{3\,d^{3/2}} - \frac{2\,e\,f\,p}{3\,d\,x} - \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(g\,x^2+\frac{f}{3}\right)}{x^3}$$

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^4,x)
```

```
[Out] (2*e^(1/2)*p*atan((e^(1/2)*x)/d^(1/2))*(3*d*g - e*f))/(3*d^(3/2)) - (2*e*f*
p)/(3*d*x) - (log(c*(d + e*x^2)^p)*(f/3 + g*x^2))/x^3
```

3.322 $\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^6}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^6}\,dx = -\frac{2efp}{15dx^3}+\frac{2e^2fp}{5d^2x}-\frac{2egp}{3dx}+\frac{2e^{5/2}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{2e^{3/2}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}-\frac{f\log\left(c(d+ex^2)^p\right)}{5x^5}-\frac{g\log\left(c(d+ex^2)^p\right)}{3x^3}$$

```
[Out] -2/15*e*f*p/d/x^3+2/5*e^2*f*p/d^2/x-2/3*e*g*p/d/x+2/5*e^(5/2)*f*p*arctan(x*
e^(1/2)/d^(1/2))/d^(5/2)-2/3*e^(3/2)*g*p*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)-
1/5*f*ln(c*(e*x^2+d)^p)/x^5-1/3*g*ln(c*(e*x^2+d)^p)/x^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2526, 2505, 331, 211}

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^6}\,dx = \frac{2e^{5/2}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{2e^{3/2}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}-\frac{f\log\left(c(d+ex^2)^p\right)}{5x^5}-\frac{g\log\left(c(d+ex^2)^p\right)}{3x^3}+\frac{2e^2fp}{5d^2x}-\frac{2efp}{15dx^3}-\frac{2egp}{3dx}$$

```
[In] Int[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^6,x]
```

```
[Out] (-2*e*f*p)/(15*d*x^3) + (2*e^2*f*p)/(5*d^2*x) - (2*e*g*p)/(3*d*x) + (2*e^(5
/2)*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*d^(5/2)) - (2*e^(3/2)*g*p*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(3*d^(3/2)) - (f*Log[c*(d + e*x^2)^p])/(5*x^5) - (g*Log
[c*(d + e*x^2)^p])/(3*x^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{f\log\left(c(d+ex^2)^p\right)}{x^6}+\frac{g\log\left(c(d+ex^2)^p\right)}{x^4}\right)dx\\
&= f\int \frac{\log\left(c(d+ex^2)^p\right)}{x^6}\,dx+g\int \frac{\log\left(c(d+ex^2)^p\right)}{x^4}\,dx\\
&= -\frac{f\log\left(c(d+ex^2)^p\right)}{5x^5}-\frac{g\log\left(c(d+ex^2)^p\right)}{3x^3}\\
&\quad+\frac{1}{5}(2efp)\int \frac{1}{x^4\left(d+ex^2\right)}\,dx+\frac{1}{3}(2egp)\int \frac{1}{x^2\left(d+ex^2\right)}\,dx
\end{aligned}
$$

$$= -\frac{2efp}{15dx^3} - \frac{2egp}{3dx} - \frac{f\log\left(c(d+ex^2)^p\right)}{5x^5} - \frac{g\log\left(c(d+ex^2)^p\right)}{3x^3} - \frac{(2e^2fp)\int\frac{1}{x^2(d+ex^2)}\,dx}{5d} - \frac{(2e^2gp)\int\frac{1}{d+ex^2}\,dx}{3d}$$

$$= -\frac{2efp}{15dx^3} + \frac{2e^2fp}{5d^2x} - \frac{2egp}{3dx} - \frac{2e^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}} - \frac{f\log\left(c(d+ex^2)^p\right)}{5x^5} - \frac{g\log\left(c(d+ex^2)^p\right)}{3x^3} + \frac{(2e^3fp)\int\frac{1}{d+ex^2}\,dx}{5d^2}$$

$$= -\frac{2efp}{15dx^3} + \frac{2e^2fp}{5d^2x} - \frac{2egp}{3dx} + \frac{2e^{5/2}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}} - \frac{2e^{3/2}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}} - \frac{f\log\left(c(d+ex^2)^p\right)}{5x^5} - \frac{g\log\left(c(d+ex^2)^p\right)}{3x^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.72

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^6}\,dx = -\frac{2efp\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\frac{ex^2}{d}\right)}{15dx^3} - \frac{2egp\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{ex^2}{d}\right)}{3dx} - \frac{f\log\left(c(d+ex^2)^p\right)}{5x^5} - \frac{g\log\left(c(d+ex^2)^p\right)}{3x^3}$$

```
[In] Integrate[((f + g*x^2)*Log[c*(d + e*x^2)^p])/x^6,x]
```

```
[Out] (-2*e*f*p*Hypergeometric2F1[-3/2, 1, -1/2, -((e*x^2)/d)])/(15*d*x^3) - (2*e
*g*p*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) - (f*Log[c*(d +
 e*x^2)^p])/(5*x^5) - (g*Log[c*(d + e*x^2)^p])/(3*x^3)
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.69

method	result
parts	$-\frac{g\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{3x^{3}}-\frac{f\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{5x^{5}}-\frac{2pe\left(-\frac{-5dg+3ef}{d^{2}x}+\frac{f}{d\,x^{3}}+\frac{e(5dg-3ef)\arctan\left(\frac{xe}{\sqrt{de}}\right)}{d^{2}\sqrt{de}}\right)}{15}$
risch	$-\frac{\left(5g\,x^{2}+3f\right)\ln\left(\left(e\,x^{2}+d\right)^{p}\right)}{15x^{5}}+\frac{-5i\pi\,d^{2}g\,x^{2}\operatorname{csgn}\left(i\left(e\,x^{2}+d\right)^{p}\right)\operatorname{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right)^{2}+5i\pi\,d^{2}g\,x^{2}\operatorname{csgn}\left(i\left(e\,x^{2}+d\right)^{p}\right)\operatorname{csgn}\left(ic\left(e\,x^{2}+d\right)^{}\right.}{}$

[In] int((g*x^2+f)*ln(c*(e*x^2+d)^p)/x^6,x,method=_RETURNVERBOSE)

[Out] -1/3*g*ln(c*(e*x^2+d)^p)/x^3-1/5*f*ln(c*(e*x^2+d)^p)/x^5-2/15*p*e*(-1/d^2*(
-5*d*g+3*e*f)/x+f/d/x^3+e*(5*d*g-3*e*f)/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1
/2)))

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.85

$$\int\frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^6}\,dx$$
$$=\left[-\frac{(3\,e^2f-5\,deg)px^5\sqrt{-\frac{e}{d}}\log\left(\frac{ex^2-2\,dx\sqrt{-\frac{e}{d}}-d}{ex^2+d}\right)+2\,defpx^2-2\,(3\,e^2f-5\,deg)px^4+(5\,d^2gpx^2+3\,d^2fp}{15\,d^2x^5}\right.$$

[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^6,x, algorithm="fricas")

[Out] [-1/15*((3*e^2*f - 5*d*e*g)*p*x^5*sqrt(-e/d)*log((e*x^2 - 2*d*x*sqrt(-e/d)
- d)/(e*x^2 + d)) + 2*d*e*f*p*x^2 - 2*(3*e^2*f - 5*d*e*g)*p*x^4 + (5*d^2*g*
p*x^2 + 3*d^2*f*p)*log(e*x^2 + d) + (5*d^2*g*x^2 + 3*d^2*f)*log(c))/(d^2*x^
5), 1/15*(2*(3*e^2*f - 5*d*e*g)*p*x^5*sqrt(e/d)*arctan(x*sqrt(e/d)) - 2*d*e
*f*p*x^2 + 2*(3*e^2*f - 5*d*e*g)*p*x^4 - (5*d^2*g*p*x^2 + 3*d^2*f*p)*log(e*
x^2 + d) - (5*d^2*g*x^2 + 3*d^2*f)*log(c))/(d^2*x^5)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. 2(138) = 276.

Time = 155.65 (sec) , antiderivative size = 1134, normalized size of antiderivative = 8.10

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^6}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x**2+f)*ln(c*(e*x**2+d)**p)/x**6,x)
```

```
[Out] Piecewise((((-f/(5*x**5) - g/(3*x**3))*log(0**p*c), Eq(d, 0) & Eq(e, 0)), ((
-f/(5*x**5) - g/(3*x**3))*log(c*d**p), Eq(e, 0)), (-2*f*p/(25*x**5) - f*log
(c*(e*x**2)**p)/(5*x**5) - 2*g*p/(9*x**3) - g*log(c*(e*x**2)**p)/(3*x**3),
Eq(d, 0)), ((-f/(5*x**5) - g/(3*x**3))*log(0**p*c), Eq(d, -e*x**2)), (-3*d*
*3*f*sqrt(-d/e)*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e) + 15*d**2*e
*x**7*sqrt(-d/e)) - 5*d**3*g*x**2*sqrt(-d/e)*log(c*(d + e*x**2)**p)/(15*d**
3*x**5*sqrt(-d/e) + 15*d**2*e*x**7*sqrt(-d/e)) - 2*d**2*f*p*x**2*sqrt(-d/e)
/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 3*d**2*f*x**2*sqrt
(-d/e)*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqr
t(-d/e)) - 10*d**2*g*p*x**5*log(x - sqrt(-d/e))/(15*d**3*x**5*sqrt(-d/e)/e
+ 15*d**2*x**7*sqrt(-d/e)) - 10*d**2*g*p*x**4*sqrt(-d/e)/(15*d**3*x**5*sqrt
(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) + 5*d**2*g*x**5*log(c*(d + e*x**2)**p)/
(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 5*d**2*g*x**4*sqrt(
-d/e)*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt
(-d/e)) + 6*d*e*f*p*x**5*log(x - sqrt(-d/e))/(15*d**3*x**5*sqrt(-d/e)/e + 1
5*d**2*x**7*sqrt(-d/e)) + 4*d*e*f*p*x**4*sqrt(-d/e)/(15*d**3*x**5*sqrt(-d/e
)/e + 15*d**2*x**7*sqrt(-d/e)) - 3*d*e*f*x**5*log(c*(d + e*x**2)**p)/(15*d*
*3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 10*d*e*g*p*x**7*log(x - s
qrt(-d/e))/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 10*d*e*g
*p*x**6*sqrt(-d/e)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) +
5*d*e*g*x**7*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x*
*7*sqrt(-d/e)) + 6*e**2*f*p*x**7*log(x - sqrt(-d/e))/(15*d**3*x**5*sqrt(-d/
e)/e + 15*d**2*x**7*sqrt(-d/e)) + 6*e**2*f*p*x**6*sqrt(-d/e)/(15*d**3*x**5*
sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 3*e**2*f*x**7*log(c*(d + e*x**2)*
*p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.84

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^6}\,dx = \frac{2\,(3\,e^3fp-5\,de^2gp)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{15\,\sqrt{de}d^2} - \frac{(5\,gpx^2+3\,fp)\log\left(ex^2+d\right)}{15\,x^5} + \frac{6\,e^2fpx^4-10\,degpx^4-2\,defpx^2-5\,d^2gx^2\log\left(c\right)-3\,d^2f\log\left(c\right)}{15\,d^2x^5}$$

```
[In] integrate((g*x^2+f)*log(c*(e*x^2+d)^p)/x^6,x, algorithm="giac")
```

```
[Out] 2/15*(3*e^3*f*p - 5*d*e^2*g*p)*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2) - 1/15
*(5*g*p*x^2 + 3*f*p)*log(e*x^2 + d)/x^5 + 1/15*(6*e^2*f*p*x^4 - 10*d*e*g*p*
x^4 - 2*d*e*f*p*x^2 - 5*d^2*g*x^2*log(c) - 3*d^2*f*log(c))/(d^2*x^5)
```

Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.63

$$\int \frac{(f+gx^2)\log\left(c(d+ex^2)^p\right)}{x^6}\,dx = -\frac{\frac{2\,e\,f\,p}{d}+\frac{2\,e\,p\,x^2\,(5\,d\,g-3\,e\,f)}{d^2}}{15\,x^3} - \frac{\ln\left(c\,{(e\,x^2+d)}^p\right)\,\left(\frac{g\,x^2}{3}+\frac{f}{5}\right)}{x^5} - \frac{2\,e^{3/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(5\,d\,g-3\,e\,f)}{15\,d^{5/2}}$$

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2))/x^6,x)
```

```
[Out] - ((2*e*f*p)/d + (2*e*p*x^2*(5*d*g - 3*e*f))/d^2)/(15*x^3) - (log(c*(d + e*
x^2)^p)*(f/5 + (g*x^2)/3))/x^5 - (2*e^(3/2)*p*atan((e^(1/2)*x)/d^(1/2))*(5*
d*g - 3*e*f))/(15*d^(5/2))
```

3.323 $\int x^5(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 251

$$\begin{aligned}\int x^5(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx = &-\frac{d^2(ef-dg)^2px^2}{2e^4}\\ &+\frac{d(ef-2dg)(ef-dg)p(d+ex^2)^2}{4e^5}\\ &-\frac{(e^2f^2-6defg+6d^2g^2)\,p(d+ex^2)^3}{18e^5}\\ &-\frac{g(ef-2dg)p(d+ex^2)^4}{16e^5}-\frac{g^2p(d+ex^2)^5}{50e^5}\\ &+\frac{d^3(10e^2f^2-15defg+6d^2g^2)\,p\log(d+ex^2)}{60e^5}\\ &+\frac{1}{6}f^2x^6\log\left(c(d+ex^2)^p\right)+\frac{1}{4}fgx^8\log\left(c(d+ex^2)^p\right)\\ &+\frac{1}{10}g^2x^{10}\log\left(c(d+ex^2)^p\right)\end{aligned}$$

```
[Out] -1/2*d^2*(-d*g+e*f)^2*p*x^2/e^4+1/4*d*(-2*d*g+e*f)*(-d*g+e*f)*p*(e*x^2+d)^2
/e^5-1/18*(6*d^2*g^2-6*d*e*f*g+e^2*f^2)*p*(e*x^2+d)^3/e^5-1/16*g*(-2*d*g+e*
f)*p*(e*x^2+d)^4/e^5-1/50*g^2*p*(e*x^2+d)^5/e^5+1/60*d^3*(6*d^2*g^2-15*d*e*
f*g+10*e^2*f^2)*p*ln(e*x^2+d)/e^5+1/6*f^2*x^6*ln(c*(e*x^2+d)^p)+1/4*f*g*x^8
*ln(c*(e*x^2+d)^p)+1/10*g^2*x^10*ln(c*(e*x^2+d)^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2525, 45, 2461, 12, 907}

$$\begin{aligned}\int x^5(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx &= \frac{1}{6}f^2x^6\log\left(c(d+ex^2)^p\right)+\frac{1}{4}fgx^8\log\left(c(d+ex^2)^p\right)\\ &+\frac{1}{10}g^2x^{10}\log\left(c(d+ex^2)^p\right)-\frac{d^2px^2(ef-dg)^2}{2e^4}\\ &-\frac{p(d+ex^2)^3\left(6d^2g^2-6defg+e^2f^2\right)}{18e^5}\\ &+\frac{d^3p(6d^2g^2-15defg+10e^2f^2)\log\left(d+ex^2\right)}{60e^5}\\ &-\frac{gp(d+ex^2)^4\left(ef-2dg\right)}{16e^5}\\ &+\frac{dp(d+ex^2)^2\left(ef-2dg\right)(ef-dg)}{4e^5}\\ &-\frac{g^2p(d+ex^2)^5}{50e^5}\end{aligned}$$

```
[In] Int[x^5*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -1/2*(d^2*(e*f - d*g)^2*p*x^2)/e^4 + (d*(e*f - 2*d*g)*(e*f - d*g)*p*(d + e*
x^2)^2)/(4*e^5) - ((e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2)*p*(d + e*x^2)^3)/(18*e
^5) - (g*(e*f - 2*d*g)*p*(d + e*x^2)^4)/(16*e^5) - (g^2*p*(d + e*x^2)^5)/(5
0*e^5) + (d^3*(10*e^2*f^2 - 15*d*e*f*g + 6*d^2*g^2)*p*Log[d + e*x^2])/(60*e
^5) + (f^2*x^6*Log[c*(d + e*x^2)^p])/6 + (f*g*x^8*Log[c*(d + e*x^2)^p])/4 +
 (g^2*x^10*Log[c*(d + e*x^2)^p])/10
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
```

```
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x^2(f+gx)^2\log\left(c(d+ex)^p\right)\,dx,x,x^2\right)\\
&= \frac{1}{6}f^2x^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}fgx^8\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{10}g^2x^{10}\log\left(c\left(d+ex^2\right)^p\right)\\
&\quad-\frac{1}{2}(ep)\text{Subst}\left(\int\frac{x^3(10f^2+15fgx+6g^2x^2)}{30(d+ex)}\,dx,x,x^2\right)\\
&= \frac{1}{6}f^2x^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}fgx^8\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{10}g^2x^{10}\log\left(c\left(d+ex^2\right)^p\right)\\
&\quad-\frac{1}{60}(ep)\text{Subst}\left(\int\frac{x^3(10f^2+15fgx+6g^2x^2)}{d+ex}\,dx,x,x^2\right)\\
&= \frac{1}{6}f^2x^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{4}fgx^8\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{10}g^2x^{10}\log\left(c\left(d+ex^2\right)^p\right)\\
&\quad-\frac{1}{60}(ep)\text{Subst}\left(\int\left(\frac{30d^2(-ef+dg)^2}{e^5}-\frac{d^3(10e^2f^2-15defg+6d^2g^2)}{e^5(d+ex)}\right.\right.\\
&\qquad+\frac{30d(ef-2dg)(-ef+dg)(d+ex)}{e^5}+\frac{10(e^2f^2-6defg+6d^2g^2)(d+ex)^2}{e^5}\\
&\qquad\left.\left.+\frac{15g(ef-2dg)(d+ex)^3}{e^5}+\frac{6g^2(d+ex)^4}{e^5}\right)\,dx,x,x^2\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{d^2(ef-dg)^2px^2}{2e^4} + \frac{d(ef-2dg)(ef-dg)p(d+ex^2)^2}{4e^5} \\
&\quad - \frac{(e^2f^2-6defg+6d^2g^2)\,p(d+ex^2)^3}{18e^5} - \frac{g(ef-2dg)p(d+ex^2)^4}{16e^5} \\
&\quad - \frac{g^2p(d+ex^2)^5}{50e^5} + \frac{d^3(10e^2f^2-15defg+6d^2g^2)\,p\log(d+ex^2)}{60e^5} \\
&\quad + \frac{1}{6}f^2x^6\log\left(c(d+ex^2)^p\right) + \frac{1}{4}fgx^8\log\left(c(d+ex^2)^p\right) + \frac{1}{10}g^2x^{10}\log\left(c(d+ex^2)^p\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.82

$$
\int x^5(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx
$$
$$
= \frac{-epx^2(360d^4g^2-180d^3eg(5f+gx^2)-30de^3x^2(10f^2+10fgx^2+3g^2x^4)+30d^2e^2(20f^2+15fgx^2+4g^2}{}
$$

```
[In] Integrate[x^5*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (-(e*p*x^2*(360*d^4*g^2 - 180*d^3*e*g*(5*f + g*x^2) - 30*d*e^3*x^2*(10*f^2
+ 10*f*g*x^2 + 3*g^2*x^4) + 30*d^2*e^2*(20*f^2 + 15*f*g*x^2 + 4*g^2*x^4) +
e^4*x^4*(200*f^2 + 225*f*g*x^2 + 72*g^2*x^4))) + 60*d^3*(10*e^2*f^2 - 15*d*
e*f*g + 6*d^2*g^2)*p*Log[d + e*x^2] + 60*e^5*x^6*(10*f^2 + 15*f*g*x^2 + 6*g
^2*x^4)*Log[c*(d + e*x^2)^p])/(3600*e^5)
```

Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.01

method	result
parts	$\frac{g^2x^{10}\ln\left(c\left(e\,x^2+d\right)^p\right)}{10}+\frac{fg\,x^8\ln\left(c\left(e\,x^2+d\right)^p\right)}{4}+\frac{f^2x^6\ln\left(c\left(e\,x^2+d\right)^p\right)}{6}-\frac{pe\left(\frac{\frac{6}{5}e^4g^2x^{10}-\frac{3}{2}x^8d\,e^3g^2+\frac{15}{4}e^4fg\,x^8+2x^6d^2e^2g^2}{}\right.}{}$
parallelrisch	$\frac{360x^{10}\ln\left(c\left(e\,x^2+d\right)^p\right)e^5g^2-72e^5g^2p\,x^{10}+900x^8\ln\left(c\left(e\,x^2+d\right)^p\right)e^5fg+90d\,e^4g^2p\,x^8-225e^5fgp\,x^8+600x^6\ln\left(c\left(e\,x^2+d\right)^p\right)e^5f}{}$
risch	$\frac{dfgp\,x^6}{12e}+\frac{\ln(c)g^2x^{10}}{10}+\frac{\ln(c)f^2x^6}{6}+\frac{i\pi\,g^2x^{10}\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^2\mathrm{csgn}(ic)}{20}+\frac{i\pi\,g^2x^{10}\,\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right.\right.}{20}$

```
[In] int(x^5*(g*x^2+f)^2*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*g^2*x^10*ln(c*(e*x^2+d)^p)+1/4*f*g*x^8*ln(c*(e*x^2+d)^p)+1/6*f^2*x^6*l
n(c*(e*x^2+d)^p)-1/30*p*e*(1/2/e^5*(6/5*e^4*g^2*x^10-3/2*x^8*d*e^3*g^2+15/4
*e^4*f*g*x^8+2*x^6*d^2*e^2*g^2-5*x^6*d*e^3*f*g+10/3*e^4*f^2*x^6-3*x^4*d^3*e
*g^2+15/2*x^4*d^2*e^2*f*g-5*x^4*d*e^3*f^2+6*d^4*g^2*x^2-15*d^3*e*f*g*x^2+10
```

```
*d^2*e^2*f^2*x^2)-1/2*d^3*(6*d^2*g^2-15*d*e*f*g+10*e^2*f^2)/e^6*ln(e*x^2+d)
)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.04

$$\int x^5 (f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx = -\frac{72\,e^5g^2px^{10}+45\,(5\,e^5fg-2\,de^4g^2)px^8+20\,(10\,e^5f^2-15\,de^4fg+6\,d^2e^3g^2)px^6-30\,(10\,de^4f^2-15\,d^2e^3}{}$$

```
[In] integrate(x^5*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] -1/3600*(72*e^5*g^2*p*x^10 + 45*(5*e^5*f*g - 2*d*e^4*g^2)*p*x^8 + 20*(10*e^
5*f^2 - 15*d*e^4*f*g + 6*d^2*e^3*g^2)*p*x^6 - 30*(10*d*e^4*f^2 - 15*d^2*e^3
*f*g + 6*d^3*e^2*g^2)*p*x^4 + 60*(10*d^2*e^3*f^2 - 15*d^3*e^2*f*g + 6*d^4*e
*g^2)*p*x^2 - 60*(6*e^5*g^2*p*x^10 + 15*e^5*f*g*p*x^8 + 10*e^5*f^2*p*x^6 +
(10*d^3*e^2*f^2 - 15*d^4*e*f*g + 6*d^5*g^2)*p)*log(e*x^2 + d) - 60*(6*e^5*g
^2*x^10 + 15*e^5*f*g*x^8 + 10*e^5*f^2*x^6)*log(c))/e^5
```

Sympy [F(-1)]

Timed out.

$$\int x^5 (f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx = \text{Timed out}$$

```
[In] integrate(x**5*(g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.89

$$\int x^5 (f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx = -\frac{1}{3600}\,ep\left(\frac{72\,e^4g^2x^{10}+45\,(5\,e^4fg-2\,de^3g^2)x^8+20\,(10\,e^4f^2-15\,de^3fg+6\,d^2e^2g^2)x^6-30\,(10\,de^3f^2-1}{e^5}\right. + \frac{1}{60}\left(6\,g^2x^{10}+15\,fgx^8+10\,f^2x^6\right)\log\left((ex^2+d)^pc\right)$$

```
[In] integrate(x^5*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] -1/3600*e*p*((72*e^4*g^2*x^10 + 45*(5*e^4*f*g - 2*d*e^3*g^2)*x^8 + 20*(10*e
^4*f^2 - 15*d*e^3*f*g + 6*d^2*e^2*g^2)*x^6 - 30*(10*d*e^3*f^2 - 15*d^2*e^2*
f*g + 6*d^3*e*g^2)*x^4 + 60*(10*d^2*e^2*f^2 - 15*d^3*e*f*g + 6*d^4*g^2)*x^2
)/e^5 - 60*(10*d^3*e^2*f^2 - 15*d^4*e*f*g + 6*d^5*g^2)*log(e*x^2 + d)/e^6)
+ 1/60*(6*g^2*x^10 + 15*f*g*x^8 + 10*f^2*x^6)*log((e*x^2 + d)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 753 vs. $2(233) = 466$.

Time = 0.33 (sec) , antiderivative size = 753, normalized size of antiderivative = 3.00

$$
\begin{aligned}
&\int x^5 (f + gx^2)^2 \log \left(c(d + ex^2)^p\right) \, dx \\
&= \frac{(ex^2+d)^3 f^2 p \log(ex^2+d)}{6\,e^3} - \frac{(ex^2+d)^2 d f^2 p \log(ex^2+d)}{2\,e^3} + \frac{(ex^2+d)^4 f g p \log(ex^2+d)}{4\,e^4} \\
&\quad - \frac{(ex^2+d)^3 d f g p \log(ex^2+d)}{e^4} + \frac{3\,(ex^2+d)^2 d^2 f g p \log(ex^2+d)}{2\,e^4} \\
&\quad + \frac{(ex^2+d)^5 g^2 p \log(ex^2+d)}{10\,e^5} - \frac{(ex^2+d)^4 d g^2 p \log(ex^2+d)}{2\,e^5} \\
&\quad + \frac{(ex^2+d)^3 d^2 g^2 p \log(ex^2+d)}{e^5} - \frac{(ex^2+d)^2 d^3 g^2 p \log(ex^2+d)}{e^5} - \frac{(ex^2+d)^3 f^2 p}{18\,e^3} \\
&\quad + \frac{(ex^2+d)^2 d f^2 p}{4\,e^3} - \frac{(ex^2+d)^4 f g p}{16\,e^4} + \frac{(ex^2+d)^3 d f g p}{3\,e^4} - \frac{3\,(ex^2+d)^2 d^2 f g p}{4\,e^4} \\
&\quad - \frac{(ex^2+d)^5 g^2 p}{50\,e^5} + \frac{(ex^2+d)^4 d g^2 p}{8\,e^5} - \frac{(ex^2+d)^3 d^2 g^2 p}{3\,e^5} + \frac{(ex^2+d)^2 d^3 g^2 p}{2\,e^5} \\
&\quad + \frac{(ex^2+d)^3 f^2 \log(c)}{6\,e^3} - \frac{(ex^2+d)^2 d f^2 \log(c)}{2\,e^3} + \frac{(ex^2+d)^4 f g \log(c)}{4\,e^4} \\
&\quad - \frac{(ex^2+d)^3 d f g \log(c)}{e^4} + \frac{3\,(ex^2+d)^2 d^2 f g \log(c)}{2\,e^4} + \frac{(ex^2+d)^5 g^2 \log(c)}{10\,e^5} \\
&\quad - \frac{(ex^2+d)^4 d g^2 \log(c)}{2\,e^5} + \frac{(ex^2+d)^3 d^2 g^2 \log(c)}{e^5} - \frac{(ex^2+d)^2 d^3 g^2 \log(c)}{e^5} \\
&\quad - \frac{(ex^2 - (ex^2+d)\log(ex^2+d) + d) d^2 e^2 f^2 p - 2\,(ex^2 - (ex^2+d)\log(ex^2+d) + d) d^3 e f g p + (ex^2 - (ex
\end{aligned}
$$

```
[In] integrate(x^5*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/6*(e*x^2 + d)^3*f^2*p*log(e*x^2 + d)/e^3 - 1/2*(e*x^2 + d)^2*d*f^2*p*log(
e*x^2 + d)/e^3 + 1/4*(e*x^2 + d)^4*f*g*p*log(e*x^2 + d)/e^4 - (e*x^2 + d)^3
*d*f*g*p*log(e*x^2 + d)/e^4 + 3/2*(e*x^2 + d)^2*d^2*f*g*p*log(e*x^2 + d)/e^
4 + 1/10*(e*x^2 + d)^5*g^2*p*log(e*x^2 + d)/e^5 - 1/2*(e*x^2 + d)^4*d*g^2*p
*log(e*x^2 + d)/e^5 + (e*x^2 + d)^3*d^2*g^2*p*log(e*x^2 + d)/e^5 - (e*x^2 +
 d)^2*d^3*g^2*p*log(e*x^2 + d)/e^5 - 1/18*(e*x^2 + d)^3*f^2*p/e^3 + 1/4*(e*
x^2 + d)^2*d*f^2*p/e^3 - 1/16*(e*x^2 + d)^4*f*g*p/e^4 + 1/3*(e*x^2 + d)^3*d
```

```
*f*g*p/e^4 - 3/4*(e*x^2 + d)^2*d^2*f*g*p/e^4 - 1/50*(e*x^2 + d)^5*g^2*p/e^5
 + 1/8*(e*x^2 + d)^4*d*g^2*p/e^5 - 1/3*(e*x^2 + d)^3*d^2*g^2*p/e^5 + 1/2*(e
*x^2 + d)^2*d^3*g^2*p/e^5 + 1/6*(e*x^2 + d)^3*f^2*log(c)/e^3 - 1/2*(e*x^2 +
 d)^2*d*f^2*log(c)/e^3 + 1/4*(e*x^2 + d)^4*f*g*log(c)/e^4 - (e*x^2 + d)^3*d
*f*g*log(c)/e^4 + 3/2*(e*x^2 + d)^2*d^2*f*g*log(c)/e^4 + 1/10*(e*x^2 + d)^5
*g^2*log(c)/e^5 - 1/2*(e*x^2 + d)^4*d*g^2*log(c)/e^5 + (e*x^2 + d)^3*d^2*g^
2*log(c)/e^5 - (e*x^2 + d)^2*d^3*g^2*log(c)/e^5 - 1/2*((e*x^2 - (e*x^2 + d)
*log(e*x^2 + d) + d)*d^2*e^2*f^2*p - 2*(e*x^2 - (e*x^2 + d)*log(e*x^2 + d)
+ d)*d^3*e*f*g*p + (e*x^2 - (e*x^2 + d)*log(e*x^2 + d) + d)*d^4*g^2*p - (e*
x^2 + d)*d^2*e^2*f^2*log(c) + 2*(e*x^2 + d)*d^3*e*f*g*log(c) - (e*x^2 + d)*
d^4*g^2*log(c))/e^5
```

Mupad [B] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.89

$$\int x^5(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx$$
$$=\ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(\frac{f^2\,x^6}{6}+\frac{f\,g\,x^8}{4}+\frac{g^2\,x^{10}}{10}\right)-x^6\left(\frac{f^2\,p}{18}-\frac{d\left(\frac{f\,g\,p}{2}-\frac{d\,g^2\,p}{5\,e}\right)}{6\,e}\right)$$
$$-x^8\left(\frac{f\,g\,p}{16}-\frac{d\,g^2\,p}{40\,e}\right)-\frac{g^2\,p\,x^{10}}{50}+\frac{\ln\left(e\,x^2+d\right)\,\left(6\,p\,d^5\,g^2-15\,p\,d^4\,e\,f\,g+10\,p\,d^3\,e^2\,f^2\right)}{60\,e^5}$$
$$+\frac{d\,x^4\left(\frac{f^2\,p}{3}-\frac{d\left(\frac{f\,g\,p}{2}-\frac{d\,g^2\,p}{5\,e}\right)}{e}\right)}{4\,e}-\frac{d^2\,x^2\left(\frac{f^2\,p}{3}-\frac{d\left(\frac{f\,g\,p}{2}-\frac{d\,g^2\,p}{5\,e}\right)}{e}\right)}{2\,e^2}$$

```
[In] int(x^5*log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

```
[Out] log(c*(d + e*x^2)^p)*((f^2*x^6)/6 + (g^2*x^10)/10 + (f*g*x^8)/4) - x^6*((f^
2*p)/18 - (d*((f*g*p)/2 - (d*g^2*p)/(5*e)))/(6*e)) - x^8*((f*g*p)/16 - (d*g
^2*p)/(40*e)) - (g^2*p*x^10)/50 + (log(d + e*x^2)*(6*d^5*g^2*p + 10*d^3*e^2
*f^2*p - 15*d^4*e*f*g*p))/(60*e^5) + (d*x^4*((f^2*p)/3 - (d*((f*g*p)/2 - (d
*g^2*p)/(5*e)))/e))/(4*e) - (d^2*x^2*((f^2*p)/3 - (d*((f*g*p)/2 - (d*g^2*p)
/(5*e)))/e))/(2*e^2)
```

3.324 $\int x^3(f+gx^2)^2 \log\left(c(d+ex^2)^p\right) dx$

Optimal result

Integrand size = 25, antiderivative size = 210

$$\int x^3(f+gx^2)^2 \log\left(c(d+ex^2)^p\right) dx = \frac{d(ef-dg)^2px^2}{2e^3} - \frac{(ef-3dg)(ef-dg)p(d+ex^2)^2}{8e^4} - \frac{g(2ef-3dg)p(d+ex^2)^3}{18e^4} - \frac{g^2p(d+ex^2)^4}{32e^4} - \frac{d^2(6e^2f^2-8defg+3d^2g^2)\,p\log(d+ex^2)}{24e^4} + \frac{1}{4}f^2x^4\log\left(c(d+ex^2)^p\right) + \frac{1}{3}fgx^6\log\left(c(d+ex^2)^p\right) + \frac{1}{8}g^2x^8\log\left(c(d+ex^2)^p\right)$$

```
[Out] 1/2*d*(-d*g+e*f)^2*p*x^2/e^3-1/8*(-3*d*g+e*f)*(-d*g+e*f)*p*(e*x^2+d)^2/e^4-
1/18*g*(-3*d*g+2*e*f)*p*(e*x^2+d)^3/e^4-1/32*g^2*p*(e*x^2+d)^4/e^4-1/24*d^2
*(3*d^2*g^2-8*d*e*f*g+6*e^2*f^2)*p*ln(e*x^2+d)/e^4+1/4*f^2*x^4*ln(c*(e*x^2+
d)^p)+1/3*f*g*x^6*ln(c*(e*x^2+d)^p)+1/8*g^2*x^8*ln(c*(e*x^2+d)^p)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used

$= \{2525, 45, 2461, 12, 907\}$

$$\int x^3(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx = \frac{1}{4}f^2x^4\log\left(c(d+ex^2)^p\right)+\frac{1}{3}fgx^6\log\left(c(d+ex^2)^p\right) + \frac{1}{8}g^2x^8\log\left(c(d+ex^2)^p\right) - \frac{d^2p(3d^2g^2-8defg+6e^2f^2)\log(d+ex^2)}{24e^4} - \frac{gp(d+ex^2)^3(2ef-3dg)}{18e^4} - \frac{p(d+ex^2)^2(ef-3dg)(ef-dg)}{8e^4} - \frac{g^2p(d+ex^2)^4}{32e^4} + \frac{dpx^2(ef-dg)^2}{2e^3}$$

```
[In] Int[x^3*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (d*(e*f - d*g)^2*p*x^2)/(2*e^3) - ((e*f - 3*d*g)*(e*f - d*g)*p*(d + e*x^2)^
2)/(8*e^4) - (g*(2*e*f - 3*d*g)*p*(d + e*x^2)^3)/(18*e^4) - (g^2*p*(d + e*x
^2)^4)/(32*e^4) - (d^2*(6*e^2*f^2 - 8*d*e*f*g + 3*d^2*g^2)*p*Log[d + e*x^2]
)/(24*e^4) + (f^2*x^4*Log[c*(d + e*x^2)^p])/4 + (f*g*x^6*Log[c*(d + e*x^2)^
p])/3 + (g^2*x^8*Log[c*(d + e*x^2)^p])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
```

```
x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x(f+gx)^2\log\left(c(d+ex)^p\right)\,dx,x,x^2\right)\\
&= \frac{1}{4}f^2x^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}fgx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}g^2x^8\log\left(c\left(d+ex^2\right)^p\right)\\
&\quad-\frac{1}{2}(ep)\text{Subst}\left(\int\frac{x^2(6f^2+8fgx+3g^2x^2)}{12(d+ex)}\,dx,x,x^2\right)\\
&= \frac{1}{4}f^2x^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}fgx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}g^2x^8\log\left(c\left(d+ex^2\right)^p\right)\\
&\quad-\frac{1}{24}(ep)\text{Subst}\left(\int\frac{x^2(6f^2+8fgx+3g^2x^2)}{d+ex}\,dx,x,x^2\right)\\
&= \frac{1}{4}f^2x^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}fgx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}g^2x^8\log\left(c\left(d+ex^2\right)^p\right)\\
&\quad-\frac{1}{24}(ep)\text{Subst}\left(\int\left(-\frac{12d(-ef+dg)^2}{e^4}+\frac{d^2(6e^2f^2-8defg+3d^2g^2)}{e^4(d+ex)}\right.\right.\\
&\qquad+\frac{6(ef-3dg)(ef-dg)(d+ex)}{e^4}+\frac{4g(2ef-3dg)(d+ex)^2}{e^4}\\
&\qquad\left.\left.+\frac{3g^2(d+ex)^3}{e^4}\right)\,dx,x,x^2\right)\\
&= \frac{d(ef-dg)^2px^2}{2e^3}-\frac{(ef-3dg)(ef-dg)p\left(d+ex^2\right)^2}{8e^4}-\frac{g(2ef-3dg)p\left(d+ex^2\right)^3}{18e^4}\\
&\quad-\frac{g^2p\left(d+ex^2\right)^4}{32e^4}-\frac{d^2(6e^2f^2-8defg+3d^2g^2)\,p\log\left(d+ex^2\right)}{24e^4}\\
&\quad+\frac{1}{4}f^2x^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}fgx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}g^2x^8\log\left(c\left(d+ex^2\right)^p\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.22

$$\int x^3\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)\,dx=\frac{df^2px^2}{4e}-\frac{d^2fgpx^2}{3e^2}+\frac{d^3g^2px^2}{8e^3}-\frac{1}{8}f^2px^4+\frac{dfgpx^4}{6e}-\frac{d^2g^2px^4}{16e^2}-\frac{1}{9}fgpx^6+\frac{dg^2px^6}{24e}-\frac{1}{32}g^2px^8-\frac{d^2f^2p\log\left(d+ex^2\right)}{4e^2}+\frac{d^3fgp\log\left(d+ex^2\right)}{3e^3}-\frac{d^4g^2p\log\left(d+ex^2\right)}{8e^4}+\frac{1}{4}f^2x^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}fgx^6\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{8}g^2x^8\log\left(c\left(d+ex^2\right)^p\right)$$

```
[In] Integrate[x^3*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (d*f^2*p*x^2)/(4*e) - (d^2*f*g*p*x^2)/(3*e^2) + (d^3*g^2*p*x^2)/(8*e^3) - (
f^2*p*x^4)/8 + (d*f*g*p*x^4)/(6*e) - (d^2*g^2*p*x^4)/(16*e^2) - (f*g*p*x^6)
/9 + (d*g^2*p*x^6)/(24*e) - (g^2*p*x^8)/32 - (d^2*f^2*p*Log[d + e*x^2])/(4*
e^2) + (d^3*f*g*p*Log[d + e*x^2])/(3*e^3) - (d^4*g^2*p*Log[d + e*x^2])/(8*e
^4) + (f^2*x^4*Log[c*(d + e*x^2)^p])/4 + (f*g*x^6*Log[c*(d + e*x^2)^p])/3 +
 (g^2*x^8*Log[c*(d + e*x^2)^p])/8
```

Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00

method	result
parts	$\frac{g^2x^8\ln\left(c\left(e\,x^2+d\right)^p\right)}{8}+\frac{fg\,x^6\ln\left(c\left(e\,x^2+d\right)^p\right)}{3}+\frac{f^2x^4\ln\left(c\left(e\,x^2+d\right)^p\right)}{4}-pe\Bigg(-\frac{-\frac{3}{4}e^3g^2x^8+d\,e^2g^2x^6-\frac{8}{3}e^3fg\,x^6-\frac{3}{2}d^2e\,g^2x^4+4}{}$
parallelrisch	$-\frac{-36x^8\ln\left(c\left(e\,x^2+d\right)^p\right)e^4g^2+9x^8e^4g^2p-96x^6\ln\left(c\left(e\,x^2+d\right)^p\right)e^4fg-12x^6d\,e^3g^2p+32x^6e^4fgp-72x^4\ln\left(c\left(e\,x^2+d\right)^p\right)e^4f^2+18x}{}$
risch	$\frac{\ln(c)g^2x^8}{8}+\frac{\ln(c)f^2x^4}{4}+\frac{i\pi fg\,x^6\operatorname{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^2\operatorname{csgn}(ic)}{6}-\frac{i\pi\,f^2x^4\operatorname{csgn}\left(i\left(e\,x^2+d\right)^p\right)\operatorname{csgn}\left(ic\left(e\,x^2+d\right)^p\right)\operatorname{csgn}(ic)}{8}-$

```
[In] int(x^3*(g*x^2+f)^2*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*g^2*x^8*ln(c*(e*x^2+d)^p)+1/3*f*g*x^6*ln(c*(e*x^2+d)^p)+1/4*f^2*x^4*ln(
c*(e*x^2+d)^p)-1/12*p*e*(-1/2/e^4*(-3/4*e^3*g^2*x^8+d*e^2*g^2*x^6-8/3*e^3*f
*g*x^6-3/2*d^2*e*g^2*x^4+4*d*f*g*x^4*e^2-3*x^4*e^3*f^2+3*x^2*d^3*g^2-8*d^2*
e*f*g*x^2+6*d*e^2*f^2*x^2)+1/2*d^2*(3*d^2*g^2-8*d*e*f*g+6*e^2*f^2)/e^5*ln(e
*x^2+d))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.07

$$\int x^3 (f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx = -\frac{9\,e^4 g^2 p x^8 + 4\,(8\,e^4 fg - 3\,de^3 g^2)px^6 + 6\,(6\,e^4 f^2 - 8\,de^3 fg + 3\,d^2e^2g^2)px^4 - 12\,(6\,de^3 f^2 - 8\,d^2e^2 fg + 3\,d}{}$$

```
[In] integrate(x^3*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
[Out] -1/288*(9*e^4*g^2*p*x^8 + 4*(8*e^4*f*g - 3*d*e^3*g^2)*p*x^6 + 6*(6*e^4*f^2
 - 8*d*e^3*f*g + 3*d^2*e^2*g^2)*p*x^4 - 12*(6*d*e^3*f^2 - 8*d^2*e^2*f*g + 3*
d^3*e*g^2)*p*x^2 - 12*(3*e^4*g^2*p*x^8 + 8*e^4*f*g*p*x^6 + 6*e^4*f^2*p*x^4
 - (6*d^2*e^2*f^2 - 8*d^3*e*f*g + 3*d^4*g^2)*p)*log(e*x^2 + d) - 12*(3*e^4*g
^2*x^8 + 8*e^4*f*g*x^6 + 6*e^4*f^2*x^4)*log(c))/e^4
```

Sympy [F(-1)]

Timed out.

$$\int x^3 (f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.88

$$\int x^3 (f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx = -\frac{1}{288}\,ep\left(\frac{9\,e^3 g^2 x^8 + 4\,(8\,e^3 fg - 3\,de^2 g^2)x^6 + 6\,(6\,e^3 f^2 - 8\,de^2 fg + 3\,d^2 eg^2)x^4 - 12\,(6\,de^2 f^2 - 8\,d^2 efg +}{e^4}\right. \\ + \frac{1}{24}\left(3\,g^2x^8 + 8\,fgx^6 + 6\,f^2x^4\right)\log\left((ex^2+d)^p c\right)$$

```
[In] integrate(x^3*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
[Out] -1/288*e*p*((9*e^3*g^2*x^8 + 4*(8*e^3*f*g - 3*d*e^2*g^2)*x^6 + 6*(6*e^3*f^2
 - 8*d*e^2*f*g + 3*d^2*e*g^2)*x^4 - 12*(6*d*e^2*f^2 - 8*d^2*e*f*g + 3*d^3*g
^2)*x^2)/e^4 + 12*(6*d^2*e^2*f^2 - 8*d^3*e*f*g + 3*d^4*g^2)*log(e*x^2 + d)/
e^5) + 1/24*(3*g^2*x^8 + 8*f*g*x^6 + 6*f^2*x^4)*log((e*x^2 + d)^p*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. 2(194) = 388.

Time = 0.32 (sec) , antiderivative size = 544, normalized size of antiderivative = 2.59

$$\int x^3(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx=\frac{(ex^2+d)^2f^2p\log(ex^2+d)}{4\,e^2}$$
$$+\frac{(ex^2+d)^3fgp\log(ex^2+d)}{3\,e^3}-\frac{(ex^2+d)^2dfgp\log(ex^2+d)}{e^3}+\frac{(ex^2+d)^4g^2p\log(ex^2+d)}{8\,e^4}$$
$$-\frac{(ex^2+d)^3dg^2p\log(ex^2+d)}{2\,e^4}+\frac{3\,(ex^2+d)^2d^2g^2p\log(ex^2+d)}{4\,e^4}-\frac{(ex^2+d)^2f^2p}{8\,e^2}$$
$$-\frac{(ex^2+d)^3fgp}{9\,e^3}+\frac{(ex^2+d)^2dfgp}{2\,e^3}-\frac{(ex^2+d)^4g^2p}{32\,e^4}+\frac{(ex^2+d)^3dg^2p}{6\,e^4}$$
$$-\frac{3\,(ex^2+d)^2d^2g^2p}{8\,e^4}+\frac{(ex^2+d)^2f^2\log(c)}{4\,e^2}+\frac{(ex^2+d)^3fg\log(c)}{3\,e^3}-\frac{(ex^2+d)^2dfg\log(c)}{e^3}$$
$$+\frac{(ex^2+d)^4g^2\log(c)}{8\,e^4}-\frac{(ex^2+d)^3dg^2\log(c)}{2\,e^4}+\frac{3\,(ex^2+d)^2d^2g^2\log(c)}{4\,e^4}$$
$$+\frac{(ex^2-(ex^2+d)\log(ex^2+d)+d)de^2f^2p-2\,(ex^2-(ex^2+d)\log(ex^2+d)+d)d^2efgp+(ex^2-(ex^2+\cdots}{2\,e^{\cdots}}$$

```
[In] integrate(x^3*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/4*(e*x^2 + d)^2*f^2*p*log(e*x^2 + d)/e^2 + 1/3*(e*x^2 + d)^3*f*g*p*log(e*
x^2 + d)/e^3 - (e*x^2 + d)^2*d*f*g*p*log(e*x^2 + d)/e^3 + 1/8*(e*x^2 + d)^4
*g^2*p*log(e*x^2 + d)/e^4 - 1/2*(e*x^2 + d)^3*d*g^2*p*log(e*x^2 + d)/e^4 +
3/4*(e*x^2 + d)^2*d^2*g^2*p*log(e*x^2 + d)/e^4 - 1/8*(e*x^2 + d)^2*f^2*p/e^
2 - 1/9*(e*x^2 + d)^3*f*g*p/e^3 + 1/2*(e*x^2 + d)^2*d*f*g*p/e^3 - 1/32*(e*x
^2 + d)^4*g^2*p/e^4 + 1/6*(e*x^2 + d)^3*d*g^2*p/e^4 - 3/8*(e*x^2 + d)^2*d^2
*g^2*p/e^4 + 1/4*(e*x^2 + d)^2*f^2*log(c)/e^2 + 1/3*(e*x^2 + d)^3*f*g*log(c
)/e^3 - (e*x^2 + d)^2*d*f*g*log(c)/e^3 + 1/8*(e*x^2 + d)^4*g^2*log(c)/e^4 -
 1/2*(e*x^2 + d)^3*d*g^2*log(c)/e^4 + 3/4*(e*x^2 + d)^2*d^2*g^2*log(c)/e^4
+ 1/2*((e*x^2 - (e*x^2 + d)*log(e*x^2 + d) + d)*d*e^2*f^2*p - 2*(e*x^2 - (e
*x^2 + d)*log(e*x^2 + d) + d)*d^2*e*f*g*p + (e*x^2 - (e*x^2 + d)*log(e*x^2
+ d) + d)*d^3*g^2*p - (e*x^2 + d)*d*e^2*f^2*log(c) + 2*(e*x^2 + d)*d^2*e*f*
g*log(c) - (e*x^2 + d)*d^3*g^2*log(c))/e^4
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.88

$$\int x^3(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx=\ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(\frac{f^2\,x^4}{4}+\frac{f\,g\,x^6}{3}+\frac{g^2\,x^8}{8}\right)-x^4\left(\frac{f^2\,p}{8}-\frac{d\left(\frac{2\,f\,g\,p}{3}-\frac{d\,g^2\,p}{4\,e}\right)}{4\,e}\right)-x^6\left(\frac{f\,g\,p}{9}-\frac{d\,g^2\,p}{24\,e}\right)-\frac{g^2\,p\,x^8}{32}-\frac{\ln\left(e\,x^2+d\right)\,\left(3\,p\,d^4\,g^2-8\,p\,d^3\,e\,f\,g+6\,p\,d^2\,e^2\,f^2\right)}{24\,e^4}+\frac{d\,x^2\left(\frac{f^2\,p}{2}-\frac{d\left(\frac{2\,f\,g\,p}{3}-\frac{d\,g^2\,p}{4\,e}\right)}{e}\right)}{2\,e}$$

```
[In] int(x^3*log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

```
[Out] log(c*(d + e*x^2)^p)*((f^2*x^4)/4 + (g^2*x^8)/8 + (f*g*x^6)/3) - x^4*((f^2*
p)/8 - (d*((2*f*g*p)/3 - (d*g^2*p)/(4*e)))/(4*e)) - x^6*((f*g*p)/9 - (d*g^2
*p)/(24*e)) - (g^2*p*x^8)/32 - (log(d + e*x^2)*(3*d^4*g^2*p + 6*d^2*e^2*f^2
*p - 8*d^3*e*f*g*p))/(24*e^4) + (d*x^2*((f^2*p)/2 - (d*((2*f*g*p)/3 - (d*g^
2*p)/(4*e)))/e))/(2*e)
```

3.325 $\int x(f+gx^2)^2 \log\left(c(d+ex^2)^p\right) dx$

Optimal result

Integrand size = 23, antiderivative size = 124

$$\int x(f+gx^2)^2 \log\left(c(d+ex^2)^p\right) dx = -\frac{(ef-dg)^2px^2}{6e^2} - \frac{(ef-dg)p(f+gx^2)^2}{12eg} - \frac{p(f+gx^2)^3}{18g} - \frac{(ef-dg)^3p\log(d+ex^2)}{6e^3g} + \frac{(f+gx^2)^3\log\left(c(d+ex^2)^p\right)}{6g}$$

```
[Out] -1/6*(-d*g+e*f)^2*p*x^2/e^2-1/12*(-d*g+e*f)*p*(g*x^2+f)^2/e/g-1/18*p*(g*x^2
+f)^3/g-1/6*(-d*g+e*f)^3*p*ln(e*x^2+d)/e^3/g+1/6*(g*x^2+f)^3*ln(c*(e*x^2+d)
^p)/g
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2525, 2442, 45}

$$\int x(f+gx^2)^2 \log\left(c(d+ex^2)^p\right) dx = \frac{(f+gx^2)^3\log\left(c(d+ex^2)^p\right)}{6g} - \frac{p(ef-dg)^3\log(d+ex^2)}{6e^3g} - \frac{px^2(ef-dg)^2}{6e^2} - \frac{p(f+gx^2)^2(ef-dg)}{12eg} - \frac{p(f+gx^2)^3}{18g}$$

```
[In] Int[x*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -1/6*((e*f - d*g)^2*p*x^2)/e^2 - ((e*f - d*g)*p*(f + g*x^2)^2)/(12*e*g) - (
p*(f + g*x^2)^3)/(18*g) - ((e*f - d*g)^3*p*Log[d + e*x^2])/(6*e^3*g) + ((f
+ g*x^2)^3*Log[c*(d + e*x^2)^p])/(6*g)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int (f+gx)^2\log\left(c(d+ex)^p\right)\,dx,x,x^2\right)\\
&= \frac{(f+gx^2)^3\log\left(c(d+ex^2)^p\right)}{6g}-\frac{(ep)\text{Subst}\left(\int\frac{(f+gx)^3}{d+ex}\,dx,x,x^2\right)}{6g}\\
&= \frac{(f+gx^2)^3\log\left(c(d+ex^2)^p\right)}{6g}\\
&\quad-\frac{(ep)\text{Subst}\left(\int\left(\frac{g(ef-dg)^2}{e^3}+\frac{(ef-dg)^3}{e^3(d+ex)}+\frac{g(ef-dg)(f+gx)}{e^2}+\frac{g(f+gx)^2}{e}\right)\,dx,x,x^2\right)}{6g}\\
&= -\frac{(ef-dg)^2px^2}{6e^2}-\frac{(ef-dg)p(f+gx^2)^2}{12eg}-\frac{p(f+gx^2)^3}{18g}\\
&\quad-\frac{(ef-dg)^3p\log(d+ex^2)}{6e^3g}+\frac{(f+gx^2)^3\log\left(c(d+ex^2)^p\right)}{6g}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.09

$$\int x\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$=\frac{6d^2g(-3ef+dg)p\log\left(d+ex^2\right)+e\left(-px^2(6d^2g^2-3deg(6f+gx^2)+e^2(18f^2+9fgx^2+2g^2x^4))+6e(3df\cdots\right.}{36e^3}$$

```
[In] Integrate[x*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (6*d^2*g*(-3*e*f + d*g)*p*Log[d + e*x^2] + e*(-(p*x^2*(6*d^2*g^2 - 3*d*e*g*
(6*f + g*x^2) + e^2*(18*f^2 + 9*f*g*x^2 + 2*g^2*x^4))) + 6*e*(3*d*f^2 + e*x
^2*(3*f^2 + 3*f*g*x^2 + g^2*x^4))*Log[c*(d + e*x^2)^p]))/(36*e^3)
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.66

method	result
parts	$\frac{\ln\left(c\left(e\,x^2+d\right)^p\right)g^2x^6}{6}+\frac{fg\,x^4\ln\left(c\left(e\,x^2+d\right)^p\right)}{2}+\frac{\ln\left(c\left(e\,x^2+d\right)^p\right)f^2x^2}{2}+\frac{\ln\left(c\left(e\,x^2+d\right)^p\right)f^3}{6g}-\frac{pe\left(\frac{g\left(\frac{1}{3}e^2g^2x^6-\frac{1}{2}de\,g^2x^4+\frac{3}{2}\cdots\right.}{}\right.}{}$
parallelrisch	$\frac{6x^6\ln\left(c\left(e\,x^2+d\right)^p\right)e^3g^2-2x^6e^3g^2p+18x^4\ln\left(c\left(e\,x^2+d\right)^p\right)e^3fg+3x^4d\,e^2g^2p-9x^4e^3fgp+18x^2\ln\left(c\left(e\,x^2+d\right)^p\right)e^3f^2-6x^2d^2e\,g^2p\cdots}{}$
risch	$\frac{\ln(c)f^2x^2}{2}+\frac{g^2\ln(c)x^6}{6}+\frac{g^2dp\,x^4}{12e}+\frac{ig^2\pi\,x^6\,\mathrm{csgn}\left(i\left(e\,x^2+d\right)^p\right)\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^2}{12}+\frac{ig^2\pi\,x^6\mathrm{csgn}\left(ic\left(e\,x^2+d\right)^p\right)^2\,\mathrm{csgn}(ic)\cdots}{12}$

```
[In] int(x*(g*x^2+f)^2*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*ln(c*(e*x^2+d)^p)*g^2*x^6+1/2*f*g*x^4*ln(c*(e*x^2+d)^p)+1/2*ln(c*(e*x^2
+d)^p)*f^2*x^2+1/6*ln(c*(e*x^2+d)^p)/g*f^3-1/3/g*p*e*(1/2*g/e^3*(1/3*e^2*g^
2*x^6-1/2*d*e*g^2*x^4+3/2*e^2*f*g*x^4+d^2*g^2*x^2-3*d*e*f*g*x^2+3*e^2*f^2*x
^2)+1/2*(-d^3*g^3+3*d^2*e*f*g^2-3*d*e^2*f^2*g+e^3*f^3)/e^4*ln(e*x^2+d))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.45

$$\int x\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)\,dx=$$
$$-\frac{2\,e^3g^2px^6+3\,(3\,e^3fg-de^2g^2)px^4+6\,(3\,e^3f^2-3\,de^2fg+d^2eg^2)px^2-6\,(e^3g^2px^6+3\,e^3fgpx^4+3\,e^3f^2p\cdots}{36\,e^3}$$

```
[In] integrate(x*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] -1/36*(2*e^3*g^2*p*x^6 + 3*(3*e^3*f*g - d*e^2*g^2)*p*x^4 + 6*(3*e^3*f^2 - 3
*d*e^2*f*g + d^2*e*g^2)*p*x^2 - 6*(e^3*g^2*p*x^6 + 3*e^3*f*g*p*x^4 + 3*e^3*
f^2*p*x^2 + (3*d*e^2*f^2 - 3*d^2*e*f*g + d^3*g^2)*p)*log(e*x^2 + d) - 6*(e^
3*g^2*x^6 + 3*e^3*f*g*x^4 + 3*e^3*f^2*x^2)*log(c))/e^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(104) = 208$.

Time = 60.08 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.90

$$\int x(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx$$
$$= \begin{cases} \frac{d^3g^2\log\left(c(d+ex^2)^p\right)}{6e^3} - \frac{d^2fg\log\left(c(d+ex^2)^p\right)}{2e^2} - \frac{d^2g^2px^2}{6e^2} + \frac{df^2\log\left(c(d+ex^2)^p\right)}{2e} + \frac{dfgpx^2}{2e} + \frac{dg^2px^4}{12e} - \frac{f^2px^2}{2} + \frac{f^2x^2\log\left(c(d+e\right.}{2} \\ \left(\frac{f^2x^2}{2} + \frac{fgx^4}{2} + \frac{g^2x^6}{6}\right)\log\left(cd^p\right) \end{cases}$$

```
[In] integrate(x*(g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((d**3*g**2*log(c*(d + e*x**2)**p)/(6*e**3) - d**2*f*g*log(c*(d +
e*x**2)**p)/(2*e**2) - d**2*g**2*p*x**2/(6*e**2) + d*f**2*log(c*(d + e*x**2
)**p)/(2*e) + d*f*g*p*x**2/(2*e) + d*g**2*p*x**4/(12*e) - f**2*p*x**2/2 + f
**2*x**2*log(c*(d + e*x**2)**p)/2 - f*g*p*x**4/4 + f*g*x**4*log(c*(d + e*x*
*2)**p)/2 - g**2*p*x**6/18 + g**2*x**6*log(c*(d + e*x**2)**p)/6, Ne(e, 0)),
 ((f**2*x**2/2 + f*g*x**4/2 + g**2*x**6/6)*log(c*d**p), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.23

$$\int x(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx = \frac{(gx^2+f)^3\log\left((ex^2+d)^pc\right)}{6\,g} - \frac{ep\left(\frac{2\,e^2g^3x^6+3\,(3\,e^2fg^2-deg^3)x^4+6\,(3\,e^2f^2g-3\,defg^2+d^2g^3)x^2}{e^3} + \frac{6\,(e^3f^3-3\,de^2f^2g+3\,d^2efg^2-d^3g^3)\log(ex^2+d)}{e^4}\right)}{36\,g}$$

```
[In] integrate(x*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] 1/6*(g*x^2 + f)^3*log((e*x^2 + d)^p*c)/g - 1/36*e*p*((2*e^2*g^3*x^6 + 3*(3*
e^2*f*g^2 - d*e*g^3)*x^4 + 6*(3*e^2*f^2*g - 3*d*e*f*g^2 + d^2*g^3)*x^2)/e^3
 + 6*(e^3*f^3 - 3*d*e^2*f^2*g + 3*d^2*e*f*g^2 - d^3*g^3)*log(e*x^2 + d)/e^4
)/g
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(114) = 228$.

Time = 0.30 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.74

$$\int x(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx$$
$$= \frac{(ex^2+d)^2fgp\log(ex^2+d)}{2e^2} + \frac{(ex^2+d)^3g^2p\log(ex^2+d)}{6e^3} - \frac{(ex^2+d)^2dg^2p\log(ex^2+d)}{2e^3} - \frac{(ex^2+d)^2fgp}{4e^2} - \frac{(ex^2+d)^3g^2p}{18e^3} + \frac{(ex^2+d)^2dg^2p}{4e^3} + \frac{(ex^2+d)^2fg\log(c)}{2e^2} + \frac{(ex^2+d)^3g^2\log(c)}{6e^3} - \frac{(ex^2+d)^2dg^2\log(c)}{2e^3} - \frac{(ex^2-(ex^2+d)\log(ex^2+d)+d)e^2f^2p - 2(ex^2-(ex^2+d)\log(ex^2+d)+d)defgp + (ex^2-(ex^2+d}{2e^3}$$

```
[In] integrate(x*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] 1/2*(e*x^2 + d)^2*f*g*p*log(e*x^2 + d)/e^2 + 1/6*(e*x^2 + d)^3*g^2*p*log(e*
x^2 + d)/e^3 - 1/2*(e*x^2 + d)^2*d*g^2*p*log(e*x^2 + d)/e^3 - 1/4*(e*x^2 +
d)^2*f*g*p/e^2 - 1/18*(e*x^2 + d)^3*g^2*p/e^3 + 1/4*(e*x^2 + d)^2*d*g^2*p/e
^3 + 1/2*(e*x^2 + d)^2*f*g*log(c)/e^2 + 1/6*(e*x^2 + d)^3*g^2*log(c)/e^3 -
1/2*(e*x^2 + d)^2*d*g^2*log(c)/e^3 - 1/2*((e*x^2 - (e*x^2 + d)*log(e*x^2 +
d) + d)*e^2*f^2*p - 2*(e*x^2 - (e*x^2 + d)*log(e*x^2 + d) + d)*d*e*f*g*p +
(e*x^2 - (e*x^2 + d)*log(e*x^2 + d) + d)*d^2*g^2*p - (e*x^2 + d)*e^2*f^2*lo
g(c) + 2*(e*x^2 + d)*d*e*f*g*log(c) - (e*x^2 + d)*d^2*g^2*log(c))/e^3
```

Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15

$$\int x(f+gx^2)^2\log\left(c(d+ex^2)^p\right)\,dx = \ln\left(c\,(e\,x^2+d)^p\right)\,\left(\frac{f^2\,x^2}{2}+\frac{f\,g\,x^4}{2}+\frac{g^2\,x^6}{6}\right) - x^2\left(\frac{f^2\,p}{2}-\frac{d\left(f\,g\,p-\frac{d\,g^2\,p}{3\,e}\right)}{2\,e}\right) - x^4\left(\frac{f\,g\,p}{4}-\frac{d\,g^2\,p}{12\,e}\right) + \frac{\ln\left(e\,x^2+d\right)\,\left(p\,d^3\,g^2-3\,p\,d^2\,e\,f\,g+3\,p\,d\,e^2\,f^2\right)}{6\,e^3} - \frac{g^2\,p\,x^6}{18}$$

```
[In] int(x*log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

```
[Out] log(c*(d + e*x^2)^p)*((f^2*x^2)/2 + (g^2*x^6)/6 + (f*g*x^4)/2) - x^2*((f^2*
p)/2 - (d*(f*g*p - (d*g^2*p)/(3*e)))/(2*e)) - x^4*((f*g*p)/4 - (d*g^2*p)/(1
2*e)) + (log(d + e*x^2)*(d^3*g^2*p + 3*d*e^2*f^2*p - 3*d^2*e*f*g*p))/(6*e^3
) - (g^2*p*x^6)/18
```

3.326 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 153

$$\begin{aligned}\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x} \, dx = & -fgpx^2 + \frac{dg^2px^2}{4e} - \frac{1}{8}g^2px^4 \\ & - \frac{d^2g^2p\log(d+ex^2)}{4e^2} + \frac{1}{4}g^2x^4\log\left(c(d+ex^2)^p\right) \\ & + \frac{fg(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e} \\ & + \frac{1}{2}f^2\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right) \\ & + \frac{1}{2}f^2p\,\text{PolyLog}\left(2,1+\frac{ex^2}{d}\right)\end{aligned}$$

```
[Out] -f*g*p*x^2+1/4*d*g^2*p*x^2/e-1/8*g^2*p*x^4-1/4*d^2*g^2*p*ln(e*x^2+d)/e^2+1/
4*g^2*x^4*ln(c*(e*x^2+d)^p)+f*g*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e+1/2*f^2*ln(-e
*x^2/d)*ln(c*(e*x^2+d)^p)+1/2*f^2*p*polylog(2,1+e*x^2/d)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used

= {2525, 45, 2463, 2436, 2332, 2441, 2352, 2442}

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x}\,dx = \frac{1}{2}f^2\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right) + \frac{fg(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e} + \frac{1}{4}g^2x^4\log\left(c(d+ex^2)^p\right) - \frac{d^2g^2p\log\left(d+ex^2\right)}{4e^2} + \frac{1}{2}f^2p\,\mathrm{PolyLog}\left(2,\frac{ex^2}{d}+1\right) + \frac{dg^2px^2}{4e} - fgpx^2 - \frac{1}{8}g^2px^4$$

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x,x]
```

```
[Out] -(f*g*p*x^2) + (d*g^2*p*x^2)/(4*e) - (g^2*p*x^4)/8 - (d^2*g^2*p*Log[d + e*x
^2])/(4*e^2) + (g^2*x^4*Log[c*(d + e*x^2)^p])/4 + (f*g*(d + e*x^2)*Log[c*(d
 + e*x^2)^p])/e + (f^2*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/2 + (f^2*p*P
olyLog[2, 1 + (e*x^2)/d])/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)^2\log\left(c(d+ex)^p\right)}{x}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(2fg\log\left(c(d+ex)^p\right)+\frac{f^2\log\left(c(d+ex)^p\right)}{x}+g^2x\log\left(c(d+ex)^p\right)\right)\,dx,x,x^2\right)\\
&= \frac{1}{2}f^2\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x}\,dx,x,x^2\right)+(fg)\text{Subst}\left(\int \log\left(c(d+ex)^p\right)\,dx,x,x^2\right)\\
&\quad+\frac{1}{2}g^2\text{Subst}\left(\int x\log\left(c(d+ex)^p\right)\,dx,x,x^2\right)\\
&= \frac{1}{4}g^2x^4\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{2}f^2\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)\\
&\quad+\frac{(fg)\text{Subst}\left(\int \log\left(cx^p\right)\,dx,x,d+ex^2\right)}{e}\\
&\quad-\frac{1}{2}\left(ef^2p\right)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx,x,x^2\right)-\frac{1}{4}\left(eg^2p\right)\text{Subst}\left(\int \frac{x^2}{d+ex}\,dx,x,x^2\right)
\end{aligned}
$$

$$= -fgpx^2 + \frac{1}{4}g^2x^4\log\left(c(d+ex^2)^p\right) + \frac{fg(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e} + \frac{1}{2}f^2\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right) + \frac{1}{2}f^2p\text{Li}_2\left(1+\frac{ex^2}{d}\right) - \frac{1}{4}(eg^2p)\,\text{Subst}\left(\int\left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)dx, x, x^2\right)$$

$$= -fgpx^2 + \frac{dg^2px^2}{4e} - \frac{1}{8}g^2px^4 - \frac{d^2g^2p\log(d+ex^2)}{4e^2} + \frac{1}{4}g^2x^4\log\left(c(d+ex^2)^p\right) + \frac{fg(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e} + \frac{1}{2}f^2\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right) + \frac{1}{2}f^2p\text{Li}_2\left(1+\frac{ex^2}{d}\right)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

$$\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x}\,dx = \frac{-egpx^2(8ef-2dg+egx^2) - 2d^2g^2p\log(d+ex^2) + 2e\left(g(4df+4efx^2+egx^4)+2ef^2\log\left(-\frac{ex^2}{d}\right)\right)\log(}{8e^2}$$

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x,x]
```

```
[Out] (-(e*g*p*x^2*(8*e*f - 2*d*g + e*g*x^2)) - 2*d^2*g^2*p*Log[d + e*x^2] + 2*e*
(g*(4*d*f + 4*e*f*x^2 + e*g*x^4) + 2*e*f^2*Log[-((e*x^2)/d)])*Log[c*(d + e*
x^2)^p] + 4*e^2*f^2*p*PolyLog[2, 1 + (e*x^2)/d])/(8*e^2)
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.37

method	result
parts	$\frac{g^2x^4\ln(c(e\,x^2+d)^p)}{4} + \ln\left(c(e\,x^2+d)^p\right)fg\,x^2 + \ln\left(c(e\,x^2+d)^p\right)f^2\ln(x) - \frac{pe\left(g\left(\frac{\frac{1}{2}eg\,x^4-dg\,x^2+4fe\,x^2}{2e^2}+\frac{d(d}{}\right.\right.}{}$
risch	$\frac{\ln((e\,x^2+d)^p)g^2x^4}{4} + \ln\left((e\,x^2+d)^p\right)fg\,x^2 + \ln\left((e\,x^2+d)^p\right)f^2\ln(x) - \frac{g^2p\,x^4}{8} + \frac{d\,g^2p\,x^2}{4e} - fgp\,x^2 -$

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*g^2*x^4*ln(c*(e*x^2+d)^p)+ln(c*(e*x^2+d)^p)*f*g*x^2+ln(c*(e*x^2+d)^p)*f
^2*ln(x)-1/2*p*e*(g*(1/2/e^2*(1/2*e*g*x^4-d*g*x^2+4*f*e*x^2)+1/2*d*(d*g-4*e
```

```
*f)/e^3*ln(e*x^2+d))+4*f^2*(1/2*ln(x)*(ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))
+ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)))/e+1/2*(dilog((-e*x+(-d*e)^(1/2))/(-d*
e)^(1/2))+dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)))/e))
```

Fricas [F]

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x}\, dx = \int \frac{(gx^2+f)^2 \log \left((ex^2+d)^p c\right)}{x}\, dx$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x,x, algorithm="fricas")
[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)/x, x)
```

Sympy [F]

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x}\, dx = \int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x}\, dx$$

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x,x)
[Out] Integral((f + g*x**2)**2*log(c*(d + e*x**2)**p)/x, x)
```

Maxima [F]

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x}\, dx = \int \frac{(gx^2+f)^2 \log \left((ex^2+d)^p c\right)}{x}\, dx$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x,x, algorithm="maxima")
[Out] integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)/x, x)
```

Giac [F]

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x}\, dx = \int \frac{(gx^2+f)^2 \log \left((ex^2+d)^p c\right)}{x}\, dx$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x,x, algorithm="giac")
[Out] integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(f+gx^2\right)^2\log\left(c\left(d+ex^2\right)^p\right)}{x}\,dx=\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(g\,x^2+f\right)^2}{x}\,dx$$

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x,x)
[Out] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x, x)
```

3.327 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 135

$$\begin{aligned}\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^3} dx &= -\frac{1}{2}g^2px^2 + \frac{ef^2p\log(x)}{d} - \frac{ef^2p\log(d+ex^2)}{2d} \\ &\quad - \frac{f^2\log\left(c(d+ex^2)^p\right)}{2x^2} + \frac{g^2(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2e} \\ &\quad + fg\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right) \\ &\quad + fgp\,\mathrm{PolyLog}\left(2,1+\frac{ex^2}{d}\right)\end{aligned}$$

```
[Out] -1/2*g^2*p*x^2+e*f^2*p*ln(x)/d-1/2*e*f^2*p*ln(e*x^2+d)/d-1/2*f^2*ln(c*(e*x^
2+d)^p)/x^2+1/2*g^2*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e+f*g*ln(-e*x^2/d)*ln(c*(e*
x^2+d)^p)+f*g*p*polylog(2,1+e*x^2/d)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2525, 45, 2463, 2436, 2332, 2442, 36, 29, 31, 2441, 2352}

$$\begin{aligned}\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^3} dx &= -\frac{f^2\log\left(c(d+ex^2)^p\right)}{2x^2} + fg\log\left(-\frac{ex^2}{d}\right)\log\left(c(d + ex^2)^p\right) \\ &\quad + \frac{g^2(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2e} - \frac{ef^2p\log(d+ex^2)}{2d} \\ &\quad + \frac{ef^2p\log(x)}{d} + fgp\,\mathrm{PolyLog}\left(2,\frac{ex^2}{d}+1\right) - \frac{1}{2}g^2px^2\end{aligned}$$

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^3,x]
```

```
[Out] -1/2*(g^2*p*x^2) + (e*f^2*p*Log[x])/d - (e*f^2*p*Log[d + e*x^2])/(2*d) - (f
^2*Log[c*(d + e*x^2)^p])/(2*x^2) + (g^2*(d + e*x^2)*Log[c*(d + e*x^2)^p])/(
2*e) + f*g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + f*g*p*PolyLog[2, 1 + (e
*x^2)/d]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)^2\log\left(c(d+ex)^p\right)}{x^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(g^2\log\left(c(d+ex)^p\right)+\frac{f^2\log\left(c(d+ex)^p\right)}{x^2}+\frac{2fg\log\left(c(d+ex)^p\right)}{x}\right)\,dx, x, x^2\right) \\
&= \frac{1}{2}f^2\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x^2}\,dx, x, x^2\right) \\
&\quad + (fg)\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x}\,dx, x, x^2\right) \\
&\quad + \frac{1}{2}g^2\text{Subst}\left(\int \log\left(c(d+ex)^p\right)\,dx, x, x^2\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{f^2\log\left(c(d+ex^2)^p\right)}{2x^2} + fg\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right) \\
&\quad + \frac{g^2\text{Subst}\left(\int\log\left(cx^p\right)\,dx,x,d+ex^2\right)}{2e} + \frac{1}{2}\left(ef^2p\right)\text{Subst}\left(\int\frac{1}{x(d+ex)}\,dx,x,x^2\right) \\
&\quad - (efgp)\text{Subst}\left(\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx,x,x^2\right) \\
&= -\frac{1}{2}g^2px^2 - \frac{f^2\log\left(c(d+ex^2)^p\right)}{2x^2} + \frac{g^2(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2e} \\
&\quad + fg\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right) + fgp\text{Li}_2\left(1+\frac{ex^2}{d}\right) \\
&\quad + \frac{\left(ef^2p\right)\text{Subst}\left(\int\frac{1}{x}\,dx,x,x^2\right)}{2d} - \frac{\left(e^2f^2p\right)\text{Subst}\left(\int\frac{1}{d+ex}\,dx,x,x^2\right)}{2d} \\
&= -\frac{1}{2}g^2px^2 + \frac{ef^2p\log(x)}{d} - \frac{ef^2p\log\left(d+ex^2\right)}{2d} \\
&\quad - \frac{f^2\log\left(c(d+ex^2)^p\right)}{2x^2} + \frac{g^2(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2e} \\
&\quad + fg\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right) + fgp\text{Li}_2\left(1+\frac{ex^2}{d}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

$$
\begin{aligned}
\int\frac{\left(f+gx^2\right)^2\log\left(c(d+ex^2)^p\right)}{x^3}\,dx &= \frac{ef^2p\log(x)}{d} - \frac{ef^2p\log\left(d+ex^2\right)}{2d} - \frac{f^2\log\left(c(d+ex^2)^p\right)}{2x^2} \\
&\quad - \frac{1}{2}g^2\left(px^2 - \frac{(d+ex^2)\log\left(c(d+ex^2)^p\right)}{e}\right) \\
&\quad + fg\left(\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)\right. \\
&\qquad \left. + p\,\text{PolyLog}\left(2,\frac{d+ex^2}{d}\right)\right)
\end{aligned}
$$

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^3,x]
```

```
[Out] (e*f^2*p*Log[x])/d - (e*f^2*p*Log[d + e*x^2])/(2*d) - (f^2*Log[c*(d + e*x^2
)^p])/(2*x^2) - (g^2*(p*x^2 - ((d + e*x^2)*Log[c*(d + e*x^2)^p])/e))/2 + f*
g*(Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + p*PolyLog[2, (d + e*x^2)/d])
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.56

method	result
parts	$\frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right) g^{2} x^{2}}{2}-\frac{f^{2} \ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{2 x^{2}}+2 \ln\left(c\left(e\,x^{2}+d\right)^{p}\right) f g \ln\left(x\right)-p e\left(\frac{x^{2} g^{2}}{2 e}-\frac{\left(g^{2} d^{2}-e^{2} f^{2}\right) \ln\left(e\,x^{2}+d\right)}{2 d\,e^{2}}-\right.$
risch	$\frac{\ln\left(\left(e\,x^{2}+d\right)^{p}\right) x^{2} g^{2}}{2}+2 \ln\left(\left(e\,x^{2}+d\right)^{p}\right) f g \ln\left(x\right)-\frac{\ln\left(\left(e\,x^{2}+d\right)^{p}\right) f^{2}}{2 x^{2}}-\frac{g^{2} p\,x^{2}}{2}+\frac{p d \ln\left(e\,x^{2}+d\right) g^{2}}{2 e}-\frac{e\,f^{2} p \ln\left(e\,x^{2}+d\right)}{2 d}$

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(c*(e*x^2+d)^p)*g^2*x^2-1/2*f^2*ln(c*(e*x^2+d)^p)/x^2+2*ln(c*(e*x^2+d
)^p)*f*g*ln(x)-p*e*(1/2*x^2*g^2/e-1/2*(d^2*g^2-e^2*f^2)/d/e^2*ln(e*x^2+d)-f
^2/d*ln(x)+4*f*g*(1/2*ln(x)*(ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+ln((e*x+(
-d*e)^(1/2))/(-d*e)^(1/2)))/e+1/2*(dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+
dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)))/e))
```

Fricas [F]

$$\int \frac{\left(f+gx^{2}\right)^{2}\log\left(c\left(d+ex^{2}\right)^{p}\right)}{x^{3}}\,dx=\int \frac{\left(gx^{2}+f\right)^{2}\log\left(\left(ex^{2}+d\right)^{p}c\right)}{x^{3}}\,dx$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^3,x, algorithm="fricas")
```

```
[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)/x^3, x)
```

Sympy [F]

$$\int \frac{\left(f+gx^{2}\right)^{2}\log\left(c\left(d+ex^{2}\right)^{p}\right)}{x^{3}}\,dx=\int \frac{\left(f+gx^{2}\right)^{2}\log\left(c\left(d+ex^{2}\right)^{p}\right)}{x^{3}}\,dx$$

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**3,x)
```

```
[Out] Integral((f + g*x**2)**2*log(c*(d + e*x**2)**p)/x**3, x)
```

Maxima [F]

$$\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x^3}\,dx = \int \frac{(gx^2+f)^2 \log((ex^2+d)^p c)}{x^3}\,dx$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^3,x, algorithm="maxima")
[Out] integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)/x^3, x)
```

Giac [F]

$$\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x^3}\,dx = \int \frac{(gx^2+f)^2 \log((ex^2+d)^p c)}{x^3}\,dx$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^3,x, algorithm="giac")
[Out] integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x^3}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)\,(g\,x^2+f)^2}{x^3}\,dx$$

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^3,x)
[Out] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^3, x)
```

3.328 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^5} dx$

Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^5} dx = -\frac{ef^2p}{4dx^2} - \frac{e^2f^2p\log(x)}{2d^2} + \frac{2efgp\log(x)}{d} + \frac{e^2f^2p\log(d+ex^2)}{4d^2} - \frac{efgp\log(d+ex^2)}{d} - \frac{f^2\log\left(c(d+ex^2)^p\right)}{4x^4} - \frac{fg\log\left(c(d+ex^2)^p\right)}{x^2} + \frac{1}{2}g^2\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right) + \frac{1}{2}g^2p\,\mathrm{PolyLog}\left(2,1+\frac{ex^2}{d}\right)$$

```
[Out] -1/4*e*f^2*p/d/x^2-1/2*e^2*f^2*p*ln(x)/d^2+2*e*f*g*p*ln(x)/d+1/4*e^2*f^2*p*
ln(e*x^2+d)/d^2-e*f*g*p*ln(e*x^2+d)/d-1/4*f^2*ln(c*(e*x^2+d)^p)/x^4-f*g*ln(
c*(e*x^2+d)^p)/x^2+1/2*g^2*ln(-e*x^2/d)*ln(c*(e*x^2+d)^p)+1/2*g^2*p*polylog
(2,1+e*x^2/d)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules

used = {2525, 45, 2463, 2442, 46, 36, 29, 31, 2441, 2352}

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^5}\,dx = -\frac{f^2\log\left(c(d+ex^2)^p\right)}{4x^4} - \frac{fg\log\left(c(d+ex^2)^p\right)}{x^2} + \frac{1}{2}g^2\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right) + \frac{e^2f^2p\log\left(d+ex^2\right)}{4d^2} - \frac{e^2f^2p\log(x)}{2d^2} - \frac{ef^2p}{4dx^2} - \frac{efgp\log\left(d+ex^2\right)}{d} + \frac{2efgp\log(x)}{d} + \frac{1}{2}g^2p\,\mathrm{PolyLog}\left(2,\frac{ex^2}{d}+1\right)$$

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^5,x]
```

```
[Out] -1/4*(e*f^2*p)/(d*x^2) - (e^2*f^2*p*Log[x])/(2*d^2) + (2*e*f*g*p*Log[x])/d
+ (e^2*f^2*p*Log[d + e*x^2])/(4*d^2) - (e*f*g*p*Log[d + e*x^2])/d - (f^2*Lo
g[c*(d + e*x^2)^p])/(4*x^4) - (f*g*Log[c*(d + e*x^2)^p])/x^2 + (g^2*Log[-((
e*x^2)/d)]*Log[c*(d + e*x^2)^p])/2 + (g^2*p*PolyLog[2, 1 + (e*x^2)/d])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)^2\log\left(c(d+ex)^p\right)}{x^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{f^2\log\left(c(d+ex)^p\right)}{x^3}+\frac{2fg\log\left(c(d+ex)^p\right)}{x^2}+\frac{g^2\log\left(c(d+ex)^p\right)}{x}\right)\,dx,x,x^2\right)
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{2} f^2 \text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^3}\,dx, x, x^2\right) \\
&\quad + (fg)\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^2}\,dx, x, x^2\right) \\
&\quad + \frac{1}{2} g^2 \text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^2\right) \\
&= -\frac{f^2 \log\left(c(d+ex^2)^p\right)}{4x^4} - \frac{fg \log\left(c(d+ex^2)^p\right)}{x^2} \\
&\quad + \frac{1}{2} g^2 \log\left(-\frac{ex^2}{d}\right) \log\left(c(d+ex^2)^p\right) + \frac{1}{4}(ef^2 p)\,\text{Subst}\left(\int \frac{1}{x^2(d+ex)}\,dx, x, x^2\right) \\
&\quad + (efgp)\text{Subst}\left(\int \frac{1}{x(d+ex)}\,dx, x, x^2\right) - \frac{1}{2}(eg^2 p)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^2\right) \\
&= -\frac{f^2 \log\left(c(d+ex^2)^p\right)}{4x^4} - \frac{fg \log\left(c(d+ex^2)^p\right)}{x^2} + \frac{1}{2} g^2 \log\left(-\frac{ex^2}{d}\right) \log\left(c(d+ex^2)^p\right) \\
&\quad + \frac{1}{2} g^2 p \text{Li}_2\left(1+\frac{ex^2}{d}\right) + \frac{1}{4}(ef^2 p)\,\text{Subst}\left(\int \left(\frac{1}{dx^2} - \frac{e}{d^2 x} + \frac{e^2}{d^2(d+ex)}\right)dx, x, x^2\right) \\
&\quad + \frac{(efgp)\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{d} - \frac{(e^2 fgp)\,\text{Subst}\left(\int \frac{1}{d+ex}\,dx, x, x^2\right)}{d} \\
&= -\frac{ef^2 p}{4dx^2} - \frac{e^2 f^2 p \log(x)}{2d^2} + \frac{2efgp \log(x)}{d} + \frac{e^2 f^2 p \log(d+ex^2)}{4d^2} \\
&\quad - \frac{efgp \log(d+ex^2)}{d} - \frac{f^2 \log\left(c(d+ex^2)^p\right)}{4x^4} - \frac{fg \log\left(c(d+ex^2)^p\right)}{x^2} \\
&\quad + \frac{1}{2} g^2 \log\left(-\frac{ex^2}{d}\right) \log\left(c(d+ex^2)^p\right) + \frac{1}{2} g^2 p \text{Li}_2\left(1+\frac{ex^2}{d}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.88

$$
\begin{aligned}
\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^5}\,dx = \frac{1}{4}\Bigg(& \frac{8efgp \log(x)}{d} - \frac{4efgp \log(d+ex^2)}{d} \\
& - \frac{ef^2 p(d+2ex^2 \log(x) - ex^2 \log(d+ex^2))}{d^2 x^2} \\
& - \frac{f^2 \log\left(c(d+ex^2)^p\right)}{x^4} - \frac{4fg \log\left(c(d+ex^2)^p\right)}{x^2} \\
& + 2g^2 \Bigg(\log\left(-\frac{ex^2}{d}\right) \log\left(c(d+ex^2)^p\right) \\
& + p\,\text{PolyLog}\left(2, 1+\frac{ex^2}{d}\right)\Bigg)\Bigg)
\end{aligned}
$$

[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^5,x]

[Out] ((8*e*f*g*p*Log[x])/d - (4*e*f*g*p*Log[d + e*x^2])/d - (e*f^2*p*(d + 2*e*x^2*Log[x] - e*x^2*Log[d + e*x^2]))/(d^2*x^2) - (f^2*Log[c*(d + e*x^2)^p])/x^4 - (4*f*g*Log[c*(d + e*x^2)^p])/x^2 + 2*g^2*(Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + p*PolyLog[2, 1 + (e*x^2)/d]))/4

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.21

method	result
parts	$\ln\left(c\left(e\,x^{2}+d\right)^{p}\right) g^{2} \ln\left(x\right)-\frac{f g \ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{x^{2}}-\frac{f^{2} \ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{4 x^{4}}-\frac{p e \left(4 g^{2} \left(\frac{\ln\left(x\right)\left(\ln\left(\frac{-e x+\sqrt{-d e}}{\sqrt{-d e}}\right)+\ln\left(\frac{e x+\sqrt{-d e}}{\sqrt{-d e}}\right)\right)}{2 e}\right.\right.}{}$
risch	$\ln\left(\left(e\,x^{2}+d\right)^{p}\right) g^{2} \ln\left(x\right)-\frac{\ln\left(\left(e\,x^{2}+d\right)^{p}\right) f g}{x^{2}}-\frac{\ln\left(\left(e\,x^{2}+d\right)^{p}\right) f^{2}}{4 x^{4}}-p\,g^{2} \ln\left(x\right) \ln\left(\frac{-e x+\sqrt{-d e}}{\sqrt{-d e}}\right)-p\,g^{2} \ln\left(x\right) \ln$

[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^5,x,method=_RETURNVERBOSE)

[Out] ln(c*(e*x^2+d)^p)*g^2*ln(x)-f*g*ln(c*(e*x^2+d)^p)/x^2-1/4*f^2*ln(c*(e*x^2+d)^p)/x^4-1/2*p*e*(4*g^2*(1/2*ln(x)*(ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+ln((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)))/e+1/2*(dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)))/e)-f*(-1/2*(4*d*g-e*f)/d^2*ln(e*x^2+d)-1/2*f/d/x^2+(4*d*g-e*f)/d^2*ln(x)))

Fricas [F]

$$\int \frac{\left(f+g x^{2}\right)^{2} \log\left(c\left(d+e x^{2}\right)^{p}\right)}{x^{5}}\,dx=\int \frac{\left(g x^{2}+f\right)^{2} \log\left(\left(e x^{2}+d\right)^{p} c\right)}{x^{5}}\,dx$$

[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^5,x, algorithm="fricas")

[Out] integral((g^2*x^4 + 2*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)/x^5, x)

Sympy [F]

$$\int \frac{\left(f+g x^{2}\right)^{2} \log\left(c\left(d+e x^{2}\right)^{p}\right)}{x^{5}}\,dx=\int \frac{\left(f+g x^{2}\right)^{2} \log\left(c\left(d+e x^{2}\right)^{p}\right)}{x^{5}}\,dx$$

[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**5,x)

[Out] Integral((f + g*x**2)**2*log(c*(d + e*x**2)**p)/x**5, x)

Maxima [F]

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^5}\,dx = \int \frac{(gx^2+f)^2 \log\left((ex^2+d)^p c\right)}{x^5}\,dx$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^5,x, algorithm="maxima")
[Out] integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)/x^5, x)
```

Giac [F]

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^5}\,dx = \int \frac{(gx^2+f)^2 \log\left((ex^2+d)^p c\right)}{x^5}\,dx$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^5,x, algorithm="giac")
[Out] integrate((g*x^2 + f)^2*log((e*x^2 + d)^p*c)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^5}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)\,\left(g\,x^2+f\right)^2}{x^5}\,dx$$

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^5,x)
[Out] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^5, x)
```

3.329 $\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x^7}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x^7}\,dx = -\frac{ef^2p}{12dx^4}+\frac{ef(ef-3dg)p}{6d^2x^2} + \frac{e(e^2f^2-3defg+3d^2g^2)\,p\log(x)}{3d^3} - \frac{(ef-dg)^3p\log\left(d+ex^2\right)}{6d^3f} - \frac{(f+gx^2)^3\log\left(c(d+ex^2)^p\right)}{6fx^6}$$

```
[Out] -1/12*e*f^2*p/d/x^4+1/6*e*f*(-3*d*g+e*f)*p/d^2/x^2+1/3*e*(3*d^2*g^2-3*d*e*f
*g+e^2*f^2)*p*ln(x)/d^3-1/6*(-d*g+e*f)^3*p*ln(e*x^2+d)/d^3/f-1/6*(g*x^2+f)^
3*ln(c*(e*x^2+d)^p)/f/x^6
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2525, 37, 2461, 12, 90}

$$\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x^7}\,dx = -\frac{(f+gx^2)^3\log\left(c(d+ex^2)^p\right)}{6fx^6} - \frac{p(ef-dg)^3\log\left(d+ex^2\right)}{6d^3f}+\frac{efp(ef-3dg)}{6d^2x^2} + \frac{ep\log(x)\left(3d^2g^2-3defg+e^2f^2\right)}{3d^3}-\frac{ef^2p}{12dx^4}$$

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^7,x]
```

```
[Out] -1/12*(e*f^2*p)/(d*x^4) + (e*f*(e*f - 3*d*g)*p)/(6*d^2*x^2) + (e*(e^2*f^2 -
 3*d*e*f*g + 3*d^2*g^2)*p*Log[x])/(3*d^3) - ((e*f - d*g)^3*p*Log[d + e*x^2]
)/(6*d^3*f) - ((f + g*x^2)^3*Log[c*(d + e*x^2)^p])/(6*f*x^6)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)^2\log\left(c(d+ex)^p\right)}{x^4}\,dx,x,x^2\right)$$

$$
\begin{aligned}
&= -\frac{(f+gx^2)^3\log\left(c(d+ex^2)^p\right)}{6fx^6} - \frac{1}{2}(ep)\text{Subst}\left(\int -\frac{(f+gx)^3}{3fx^3(d+ex)}\,dx, x, x^2\right) \\
&= -\frac{(f+gx^2)^3\log\left(c(d+ex^2)^p\right)}{6fx^6} + \frac{(ep)\text{Subst}\left(\int \frac{(f+gx)^3}{x^3(d+ex)}\,dx, x, x^2\right)}{6f} \\
&= -\frac{(f+gx^2)^3\log\left(c(d+ex^2)^p\right)}{6fx^6} \\
&\quad + \frac{(ep)\text{Subst}\left(\int \left(\frac{f^3}{dx^3} + \frac{f^2(-ef+3dg)}{d^2x^2} + \frac{f(e^2f^2-3defg+3d^2g^2)}{d^3x} + \frac{(-ef+dg)^3}{d^3(d+ex)}\right)\,dx, x, x^2\right)}{6f} \\
&= -\frac{ef^2p}{12dx^4} + \frac{ef(ef-3dg)p}{6d^2x^2} + \frac{e(e^2f^2-3defg+3d^2g^2)\,p\log(x)}{3d^3} \\
&\quad - \frac{(ef-dg)^3p\log(d+ex^2)}{6d^3f} - \frac{(f+gx^2)^3\log\left(c(d+ex^2)^p\right)}{6fx^6}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08

$$
\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x^7}\,dx = -\frac{defpx^2(-2efx^2+d(f+6gx^2)) - 4e(e^2f^2-3defg+3d^2g^2)\,px^6\log(x) + 2e(e^2f^2-3defg+3d^2g^2)\,px^6}{12d^3x^6}
$$

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^7,x]
```

```
[Out] -1/12*(d*e*f*p*x^2*(-2*e*f*x^2 + d*(f + 6*g*x^2)) - 4*e*(e^2*f^2 - 3*d*e*f*
g + 3*d^2*g^2)*p*x^6*Log[x] + 2*e*(e^2*f^2 - 3*d*e*f*g + 3*d^2*g^2)*p*x^6*L
og[d + e*x^2] + 2*d^3*(f^2 + 3*f*g*x^2 + 3*g^2*x^4)*Log[c*(d + e*x^2)^p])/(
d^3*x^6)
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.22

method	result
parts	$-\frac{\ln(c(e\,x^2+d)^p)g^2}{2x^2} - \frac{\ln(c(e\,x^2+d)^p)fg}{2x^4} - \frac{\ln(c(e\,x^2+d)^p)f^2}{6x^6} - \frac{pe\left(\frac{(-3g^2d^2+3defg-e^2f^2)\ln(x)}{d^3} + \frac{f^2}{4d\,x^4} + \frac{f(3dg-ef)}{2d^2x^2} + \frac{(3g}{}\right.}{3}$
parallelrisch	$\frac{12\ln(x)x^6d^2e^2g^2p^2 - 12\ln(x)x^6d\,e^3fg\,p^2 + 4\ln(x)x^6e^4f^2p^2 - 6x^6\ln(c(e\,x^2+d)^p)d^2e^2g^2p + 6x^6\ln(c(e\,x^2+d)^p)d\,e^3fgp - 2x^6\ln(c}{}$
risch	$-\frac{(3g^2x^4+3fg\,x^2+f^2)\ln((e\,x^2+d)^p)}{6x^6} + \frac{-3i\pi\,d^3fg\,x^2\text{csgn}(ic(e\,x^2+d)^p)^2\,\text{csgn}(ic) - i\pi\,d^3f^2\text{csgn}(ic(e\,x^2+d)^p)^2\,\text{csgn}(ic) + 3i\pi}{}$

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(c*(e*x^2+d)^p)*g^2/x^2-1/2*ln(c*(e*x^2+d)^p)*f*g/x^4-1/6*ln(c*(e*x^
2+d)^p)*f^2/x^6-1/3*p*e*(1/d^3*(-3*d^2*g^2+3*d*e*f*g-e^2*f^2)*ln(x)+1/4*f^2
/d/x^4+1/2*f*(3*d*g-e*f)/d^2/x^2+1/2*(3*d^2*g^2-3*d*e*f*g+e^2*f^2)/d^3*ln(e
*x^2+d))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.41

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^7}\,dx$$
$$= \frac{4\,(e^3f^2-3\,de^2fg+3\,d^2eg^2)px^6\log(x)-d^2ef^2px^2+2\,(de^2f^2-3\,d^2efg)px^4-2\,(3\,d^3g^2px^4+3\,d^3fgpx^2}{12\,d^3x^6}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(4*(e^3*f^2 - 3*d*e^2*f*g + 3*d^2*e*g^2)*p*x^6*log(x) - d^2*e*f^2*p*x^
2 + 2*(d*e^2*f^2 - 3*d^2*e*f*g)*p*x^4 - 2*(3*d^3*g^2*p*x^4 + 3*d^3*f*g*p*x^
2 + (e^3*f^2 - 3*d*e^2*f*g + 3*d^2*e*g^2)*p*x^6 + d^3*f^2*p)*log(e*x^2 + d)
 - 2*(3*d^3*g^2*x^4 + 3*d^3*f*g*x^2 + d^3*f^2)*log(c))/(d^3*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^7}\,dx = \text{Timed out}$$

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**7,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^7}\,dx =$$
$$-\frac{1}{12}\,ep\left(\frac{2\,(e^2f^2-3\,defg+3\,d^2g^2)\log(ex^2+d)}{d^3}-\frac{2\,(e^2f^2-3\,defg+3\,d^2g^2)\log(x^2)}{d^3}+\frac{df^2-2\,(ef^2-}{d^2x^4}\right.$$
$$-\frac{(3\,g^2x^4+3\,fgx^2+f^2)\log\left((ex^2+d)^pc\right)}{6\,x^6}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*e*p*(2*(e^2*f^2 - 3*d*e*f*g + 3*d^2*g^2)*log(e*x^2 + d)/d^3 - 2*(e^2*
f^2 - 3*d*e*f*g + 3*d^2*g^2)*log(x^2)/d^3 + (d*f^2 - 2*(e*f^2 - 3*d*f*g)*x^
2)/(d^2*x^4)) - 1/6*(3*g^2*x^4 + 3*f*g*x^2 + f^2)*log((e*x^2 + d)^p*c)/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(120) = 240$.

Time = 0.32 (sec) , antiderivative size = 464, normalized size of antiderivative = 3.57

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^7}\,dx = -\frac{\frac{2\left(e^4f^2p+3\,(ex^2+d)e^3fgp-3\,de^3fgp+3\,(ex^2+d)^2e^2g^2p-6\,(ex^2+d)de^2g^2p+3\,d^2e^2g^2p\right)\log(ex^2+d)}{(ex^2+d)^3-3\,(ex^2+d)^2d+3\,(ex^2+d)d^2-d^3} - \frac{2\,(ex^2+d)^2e^4f^2p-5\,(ex^2+d)de^4f^2p\cdots}{\cdots}}{\cdots}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^7,x, algorithm="giac")
```

```
[Out] -1/12*(2*(e^4*f^2*p + 3*(e*x^2 + d)*e^3*f*g*p - 3*d*e^3*f*g*p + 3*(e*x^2 +
d)^2*e^2*g^2*p - 6*(e*x^2 + d)*d*e^2*g^2*p + 3*d^2*e^2*g^2*p)*log(e*x^2 + d
)/((e*x^2 + d)^3 - 3*(e*x^2 + d)^2*d + 3*(e*x^2 + d)*d^2 - d^3) - (2*(e*x^2
 + d)^2*e^4*f^2*p - 5*(e*x^2 + d)*d*e^4*f^2*p + 3*d^2*e^4*f^2*p - 6*(e*x^2
+ d)^2*d*e^3*f*g*p + 12*(e*x^2 + d)*d^2*e^3*f*g*p - 6*d^3*e^3*f*g*p - 2*d^2
*e^4*f^2*log(c) - 6*(e*x^2 + d)*d^2*e^3*f*g*log(c) + 6*d^3*e^3*f*g*log(c) -
 6*(e*x^2 + d)^2*d^2*e^2*g^2*log(c) + 12*(e*x^2 + d)*d^3*e^2*g^2*log(c) - 6
*d^4*e^2*g^2*log(c))/((e*x^2 + d)^3*d^2 - 3*(e*x^2 + d)^2*d^3 + 3*(e*x^2 +
d)*d^4 - d^5) + 2*(e^4*f^2*p - 3*d*e^3*f*g*p + 3*d^2*e^2*g^2*p)*log(e*x^2 +
 d)/d^3 - 2*(e^4*f^2*p - 3*d*e^3*f*g*p + 3*d^2*e^2*g^2*p)*log(e*x^2)/d^3)/e
```

Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.16

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^7}\,dx = \frac{\ln(x)\,\left(3\,p\,d^2\,e\,g^2-3\,p\,d\,e^2\,f\,g+p\,e^3\,f^2\right)}{3\,d^3} - \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{f^2}{6}+\frac{f\,g\,x^2}{2}+\frac{g^2\,x^4}{2}\right)}{x^6} - \frac{\ln\left(e\,x^2+d\right)\,\left(3\,p\,d^2\,e\,g^2-3\,p\,d\,e^2\,f\,g+p\,e^3\,f^2\right)}{6\,d^3} - \frac{\frac{e\,f^2\,p}{4\,d}+\frac{e\,f\,p\,x^2\,\left(3\,d\,g-e\,f\right)}{2\,d^2}}{3\,x^4}$$

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^7,x)
```

```
[Out] (log(x)*(e^3*f^2*p + 3*d^2*e*g^2*p - 3*d*e^2*f*g*p))/(3*d^3) - (log(c*(d +
e*x^2)^p)*(f^2/6 + (g^2*x^4)/2 + (f*g*x^2)/2))/x^6 - (log(d + e*x^2)*(e^3*f
^2*p + 3*d^2*e*g^2*p - 3*d*e^2*f*g*p))/(6*d^3) - ((e*f^2*p)/(4*d) + (e*f*p*
x^2*(3*d*g - e*f))/(2*d^2))/(3*x^4)
```

3.330 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^9} dx$

Optimal result

Integrand size = 25, antiderivative size = 216

$$\begin{aligned}\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^9} dx =& -\frac{ef^2p}{24dx^6}+\frac{ef(3ef-8dg)p}{48d^2x^4} \\ & -\frac{e(3e^2f^2-8defg+6d^2g^2)\,p}{24d^3x^2} \\ & -\frac{e^2(3e^2f^2-8defg+6d^2g^2)\,p\log(x)}{12d^4} \\ & +\frac{e^2(3e^2f^2-8defg+6d^2g^2)\,p\log\left(d+ex^2\right)}{24d^4} \\ & -\frac{f^2\log\left(c(d+ex^2)^p\right)}{8x^8} \\ & -\frac{fg\log\left(c(d+ex^2)^p\right)}{3x^6}-\frac{g^2\log\left(c(d+ex^2)^p\right)}{4x^4}\end{aligned}$$

```
[Out] -1/24*e*f^2*p/d/x^6+1/48*e*f*(-8*d*g+3*e*f)*p/d^2/x^4-1/24*e*(6*d^2*g^2-8*d
*e*f*g+3*e^2*f^2)*p/d^3/x^2-1/12*e^2*(6*d^2*g^2-8*d*e*f*g+3*e^2*f^2)*p*ln(x
)/d^4+1/24*e^2*(6*d^2*g^2-8*d*e*f*g+3*e^2*f^2)*p*ln(e*x^2+d)/d^4-1/8*f^2*ln
(c*(e*x^2+d)^p)/x^8-1/3*f*g*ln(c*(e*x^2+d)^p)/x^6-1/4*g^2*ln(c*(e*x^2+d)^p)
/x^4
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2525, 45, 2461, 12, 907}

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^9}\,dx = -\frac{f^2\log\left(c(d+ex^2)^p\right)}{8x^8} - \frac{fg\log\left(c(d+ex^2)^p\right)}{3x^6} - \frac{g^2\log\left(c(d+ex^2)^p\right)}{4x^4} + \frac{efp(3ef-8dg)}{48d^2x^4} + \frac{e^2p(6d^2g^2-8defg+3e^2f^2)\log(d+ex^2)}{24d^4} - \frac{e^2p\log(x)\left(6d^2g^2-8defg+3e^2f^2\right)}{12d^4} - \frac{ep(6d^2g^2-8defg+3e^2f^2)}{24d^3x^2} - \frac{ef^2p}{24dx^6}$$

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^9,x]
```

```
[Out] -1/24*(e*f^2*p)/(d*x^6) + (e*f*(3*e*f - 8*d*g)*p)/(48*d^2*x^4) - (e*(3*e^2*
f^2 - 8*d*e*f*g + 6*d^2*g^2)*p)/(24*d^3*x^2) - (e^2*(3*e^2*f^2 - 8*d*e*f*g
+ 6*d^2*g^2)*p*Log[x])/(12*d^4) + (e^2*(3*e^2*f^2 - 8*d*e*f*g + 6*d^2*g^2)*
p*Log[d + e*x^2])/(24*d^4) - (f^2*Log[c*(d + e*x^2)^p])/(8*x^8) - (f*g*Log[
c*(d + e*x^2)^p])/(3*x^6) - (g^2*Log[c*(d + e*x^2)^p])/(4*x^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)^2\log\left(c(d+ex)^p\right)}{x^5}\,dx,x,x^2\right)\\
&= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{8x^8}-\frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{3x^6}-\frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{4x^4}\\
&\quad-\frac{1}{2}(ep)\text{Subst}\left(\int \frac{-3f^2-8fgx-6g^2x^2}{12x^4(d+ex)}\,dx,x,x^2\right)\\
&= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{8x^8}-\frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{3x^6}-\frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{4x^4}\\
&\quad-\frac{1}{24}(ep)\text{Subst}\left(\int \frac{-3f^2-8fgx-6g^2x^2}{x^4(d+ex)}\,dx,x,x^2\right)\\
&= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{8x^8}-\frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{3x^6}-\frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{4x^4}\\
&\quad-\frac{1}{24}(ep)\text{Subst}\left(\int \left(-\frac{3f^2}{dx^4}-\frac{f(-3ef+8dg)}{d^2x^3}+\frac{-3e^2f^2+8defg-6d^2g^2}{d^3x^2}\right.\right.\\
&\qquad\left.\left.+\frac{e(3e^2f^2-8defg+6d^2g^2)}{d^4x}-\frac{e^2(3e^2f^2-8defg+6d^2g^2)}{d^4(d+ex)}\right)\,dx,x,x^2\right)\\
&= -\frac{ef^2p}{24dx^6}+\frac{ef(3ef-8dg)p}{48d^2x^4}-\frac{e(3e^2f^2-8defg+6d^2g^2)\,p}{24d^3x^2}\\
&\quad-\frac{e^2(3e^2f^2-8defg+6d^2g^2)\,p\log(x)}{12d^4}+\frac{e^2(3e^2f^2-8defg+6d^2g^2)\,p\log\left(d+ex^2\right)}{24d^4}\\
&\quad-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{8x^8}-\frac{fg\log\left(c\left(d+ex^2\right)^p\right)}{3x^6}-\frac{g^2\log\left(c\left(d+ex^2\right)^p\right)}{4x^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.85

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^9}\,dx =$$
$$-\frac{depx^2(6e^2f^2x^4-defx^2(3f+16gx^2)+2d^2(f^2+4fgx^2+6g^2x^4))+4e^2(3e^2f^2-8defg+6d^2g^2)\,px^8\,\mathrm{lo}}{48d^4}$$

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^9,x]
```

```
[Out] -1/48*(d*e*p*x^2*(6*e^2*f^2*x^4 - d*e*f*x^2*(3*f + 16*g*x^2) + 2*d^2*(f^2 +
 4*f*g*x^2 + 6*g^2*x^4)) + 4*e^2*(3*e^2*f^2 - 8*d*e*f*g + 6*d^2*g^2)*p*x^8*
Log[x] - 2*e^2*(3*e^2*f^2 - 8*d*e*f*g + 6*d^2*g^2)*p*x^8*Log[d + e*x^2] + 2
*d^4*(3*f^2 + 8*f*g*x^2 + 6*g^2*x^4)*Log[c*(d + e*x^2)^p])/(d^4*x^8)
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.89

method	result
parts	$-\frac{g^2\ln\left(c\left(ex^2+d\right)^p\right)}{4x^4}-\frac{fg\ln\left(c\left(ex^2+d\right)^p\right)}{3x^6}-\frac{f^2\ln\left(c\left(ex^2+d\right)^p\right)}{8x^8}-\frac{pe\left(-\frac{-6g^2d^2+8defg-3e^2f^2}{2d^3x^2}+\frac{\left(6g^2d^2-8defg+3e^2f^2\right)e\,\mathrm{l}}{d^4}\right.}{}$
parallelrisch	$-\frac{24\ln(x)x^8d^2e^2g^2p^2-32\ln(x)x^8de^3fgp^2+12\ln(x)x^8e^4f^2p^2-12x^8\ln\left(c\left(ex^2+d\right)^p\right)d^2e^2g^2p+16x^8\ln\left(c\left(ex^2+d\right)^p\right)de^3fgp-}{}$
risch	$-\frac{\left(6g^2x^4+8fgx^2+3f^2\right)\ln\left(\left(ex^2+d\right)^p\right)}{24x^8}+\frac{-16\ln\left(-ex^2-d\right)de^3fgpx^8+32\ln(x)de^3fgpx^8-6\ln(c)d^4f^2+6i\pi d^4g^2x^4\,\mathrm{csgn}(i}{}$

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*g^2*ln(c*(e*x^2+d)^p)/x^4-1/3*f*g*ln(c*(e*x^2+d)^p)/x^6-1/8*f^2*ln(c*(
e*x^2+d)^p)/x^8-1/12*p*e*(-1/2*(-6*d^2*g^2+8*d*e*f*g-3*e^2*f^2)/d^3/x^2+(6*
d^2*g^2-8*d*e*f*g+3*e^2*f^2)/d^4*e*ln(x)+1/2*f^2/d/x^6+1/4*f*(8*d*g-3*e*f)/
d^2/x^4-1/2*e*(6*d^2*g^2-8*d*e*f*g+3*e^2*f^2)/d^4*ln(e*x^2+d))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^9}\,dx =$$
$$-\frac{4\,(3\,e^4f^2 - 8\,de^3fg + 6\,d^2e^2g^2)px^8\log(x) + 2\,d^3ef^2px^2 + 2\,(3\,de^3f^2 - 8\,d^2e^2fg + 6\,d^3eg^2)px^6 - (3\,d^2e^2f}{}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^9,x, algorithm="fricas")
```

```
[Out] -1/48*(4*(3*e^4*f^2 - 8*d*e^3*f*g + 6*d^2*e^2*g^2)*p*x^8*log(x) + 2*d^3*e*f
^2*p*x^2 + 2*(3*d*e^3*f^2 - 8*d^2*e^2*f*g + 6*d^3*e*g^2)*p*x^6 - (3*d^2*e^2
*f^2 - 8*d^3*e*f*g)*p*x^4 + 2*(6*d^4*g^2*p*x^4 - (3*e^4*f^2 - 8*d*e^3*f*g +
 6*d^2*e^2*g^2)*p*x^8 + 8*d^4*f*g*p*x^2 + 3*d^4*f^2*p)*log(e*x^2 + d) + 2*(
6*d^4*g^2*x^4 + 8*d^4*f*g*x^2 + 3*d^4*f^2)*log(c))/(d^4*x^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^9}\,dx = \text{Timed out}$$

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**9,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.85

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^9}\,dx$$
$$= \frac{1}{48}\,ep\left(\frac{2\,(3\,e^3f^2 - 8\,de^2fg + 6\,d^2eg^2)\log(ex^2+d)}{d^4} - \frac{2\,(3\,e^3f^2 - 8\,de^2fg + 6\,d^2eg^2)\log(x^2)}{d^4} - \frac{2\,(3\,e^2f^2 -}{}\right.$$
$$- \frac{(6\,g^2x^4 + 8\,fgx^2 + 3\,f^2)\log\left((ex^2+d)^pc\right)}{24\,x^8}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^9,x, algorithm="maxima")
```

```
[Out] 1/48*e*p*(2*(3*e^3*f^2 - 8*d*e^2*f*g + 6*d^2*e*g^2)*log(e*x^2 + d)/d^4 - 2*
(3*e^3*f^2 - 8*d*e^2*f*g + 6*d^2*e*g^2)*log(x^2)/d^4 - (2*(3*e^2*f^2 - 8*d*
e*f*g + 6*d^2*g^2)*x^4 + 2*d^2*f^2 - (3*d*e*f^2 - 8*d^2*f*g)*x^2)/(d^3*x^6)
) - 1/24*(6*g^2*x^4 + 8*f*g*x^2 + 3*f^2)*log((e*x^2 + d)^p*c)/x^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(200) = 400$.

Time = 0.32 (sec) , antiderivative size = 604, normalized size of antiderivative = 2.80

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^9}\,dx =$$
$$-\frac{\frac{2\left(3e^5f^2p+8(ex^2+d)e^4fgp-8de^4fgp+6(ex^2+d)^2e^3g^2p-12(ex^2+d)de^3g^2p+6d^2e^3g^2p\right)\log(ex^2+d)}{(ex^2+d)^4-4(ex^2+d)^3d+6(ex^2+d)^2d^2-4(ex^2+d)d^3+d^4}+\frac{6(ex^2+d)^3e^5f^2p-21(ex^2+d)^2}{}}{}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^9,x, algorithm="giac")
```

```
[Out] -1/48*(2*(3*e^5*f^2*p + 8*(e*x^2 + d)*e^4*f*g*p - 8*d*e^4*f*g*p + 6*(e*x^2
+ d)^2*e^3*g^2*p - 12*(e*x^2 + d)*d*e^3*g^2*p + 6*d^2*e^3*g^2*p)*log(e*x^2
+ d)/((e*x^2 + d)^4 - 4*(e*x^2 + d)^3*d + 6*(e*x^2 + d)^2*d^2 - 4*(e*x^2 +
d)*d^3 + d^4) + (6*(e*x^2 + d)^3*e^5*f^2*p - 21*(e*x^2 + d)^2*d*e^5*f^2*p +
 26*(e*x^2 + d)*d^2*e^5*f^2*p - 11*d^3*e^5*f^2*p - 16*(e*x^2 + d)^3*d*e^4*f
*g*p + 56*(e*x^2 + d)^2*d^2*e^4*f*g*p - 64*(e*x^2 + d)*d^3*e^4*f*g*p + 24*d
^4*e^4*f*g*p + 12*(e*x^2 + d)^3*d^2*e^3*g^2*p - 36*(e*x^2 + d)^2*d^3*e^3*g^
2*p + 36*(e*x^2 + d)*d^4*e^3*g^2*p - 12*d^5*e^3*g^2*p + 6*d^3*e^5*f^2*log(c
) + 16*(e*x^2 + d)*d^3*e^4*f*g*log(c) - 16*d^4*e^4*f*g*log(c) + 12*(e*x^2 +
 d)^2*d^3*e^3*g^2*log(c) - 24*(e*x^2 + d)*d^4*e^3*g^2*log(c) + 12*d^5*e^3*g
^2*log(c))/((e*x^2 + d)^4*d^3 - 4*(e*x^2 + d)^3*d^4 + 6*(e*x^2 + d)^2*d^5 -
 4*(e*x^2 + d)*d^6 + d^7) - 2*(3*e^5*f^2*p - 8*d*e^4*f*g*p + 6*d^2*e^3*g^2*
p)*log(e*x^2 + d)/d^4 + 2*(3*e^5*f^2*p - 8*d*e^4*f*g*p + 6*d^2*e^3*g^2*p)*l
og(e*x^2)/d^4)/e
```

Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.88

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^9}\,dx = \frac{\ln\left(e\,x^2+d\right)\,\left(6\,p\,d^2\,e^2\,g^2-8\,p\,d\,e^3\,f\,g+3\,p\,e^4\,f^2\right)}{24\,d^4} - \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{f^2}{8}+\frac{f\,g\,x^2}{3}+\frac{g^2\,x^4}{4}\right)}{x^8} - \frac{\frac{e\,f^2\,p}{2\,d}+\frac{e\,p\,x^4\,\left(6\,d^2\,g^2-8\,d\,e\,f\,g+3\,e^2\,f^2\right)}{2\,d^3}+\frac{e\,f\,p\,x^2\,\left(8\,d\,g-3\,e\,f\right)}{4\,d^2}}{12\,x^6} - \frac{\ln\left(x\right)\,\left(6\,p\,d^2\,e^2\,g^2-8\,p\,d\,e^3\,f\,g+3\,p\,e^4\,f^2\right)}{12\,d^4}$$

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^9,x)
```

```
[Out] (log(d + e*x^2)*(3*e^4*f^2*p + 6*d^2*e^2*g^2*p - 8*d*e^3*f*g*p))/(24*d^4) -
 (log(c*(d + e*x^2)^p)*(f^2/8 + (g^2*x^4)/4 + (f*g*x^2)/3))/x^8 - ((e*f^2*p
)/(2*d) + (e*p*x^4*(6*d^2*g^2 + 3*e^2*f^2 - 8*d*e*f*g))/(2*d^3) + (e*f*p*x^
2*(8*d*g - 3*e*f))/(4*d^2))/(12*x^6) - (log(x)*(3*e^4*f^2*p + 6*d^2*e^2*g^2
*p - 8*d*e^3*f*g*p))/(12*d^4)
```

3.331 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^{11}} dx$

Optimal result

Integrand size = 25, antiderivative size = 253

$$\begin{aligned}\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^{11}} dx = &-\frac{ef^2p}{40dx^8}+\frac{ef(2ef-5dg)p}{60d^2x^6} \\ &-\frac{e(6e^2f^2-15defg+10d^2g^2)\,p}{120d^3x^4} \\ &+\frac{e^2(6e^2f^2-15defg+10d^2g^2)\,p}{60d^4x^2} \\ &+\frac{e^3(6e^2f^2-15defg+10d^2g^2)\,p\log(x)}{30d^5} \\ &-\frac{e^3(6e^2f^2-15defg+10d^2g^2)\,p\log\left(d+ex^2\right)}{60d^5} \\ &-\frac{f^2\log\left(c(d+ex^2)^p\right)}{10x^{10}} \\ &-\frac{fg\log\left(c(d+ex^2)^p\right)}{4x^8}-\frac{g^2\log\left(c(d+ex^2)^p\right)}{6x^6}\end{aligned}$$

```
[Out] -1/40*e*f^2*p/d/x^8+1/60*e*f*(-5*d*g+2*e*f)*p/d^2/x^6-1/120*e*(10*d^2*g^2-1
5*d*e*f*g+6*e^2*f^2)*p/d^3/x^4+1/60*e^2*(10*d^2*g^2-15*d*e*f*g+6*e^2*f^2)*p
/d^4/x^2+1/30*e^3*(10*d^2*g^2-15*d*e*f*g+6*e^2*f^2)*p*ln(x)/d^5-1/60*e^3*(1
0*d^2*g^2-15*d*e*f*g+6*e^2*f^2)*p*ln(e*x^2+d)/d^5-1/10*f^2*ln(c*(e*x^2+d)^p
)/x^10-1/4*f*g*ln(c*(e*x^2+d)^p)/x^8-1/6*g^2*ln(c*(e*x^2+d)^p)/x^6
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2525, 45, 2461, 12, 907}

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^{11}}\,dx = -\frac{f^2\log\left(c(d+ex^2)^p\right)}{10x^{10}} - \frac{fg\log\left(c(d+ex^2)^p\right)}{4x^8} - \frac{g^2\log\left(c(d+ex^2)^p\right)}{6x^6} + \frac{efp(2ef-5dg)}{60d^2x^6} - \frac{e^3p(10d^2g^2-15defg+6e^2f^2)\log(d+ex^2)}{60d^5} + \frac{e^3p\log(x)\,(10d^2g^2-15defg+6e^2f^2)}{30d^5} + \frac{e^2p(10d^2g^2-15defg+6e^2f^2)}{60d^4x^2} - \frac{ep(10d^2g^2-15defg+6e^2f^2)}{120d^3x^4} - \frac{ef^2p}{40dx^8}$$

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^11,x]
```

```
[Out] -1/40*(e*f^2*p)/(d*x^8) + (e*f*(2*e*f - 5*d*g)*p)/(60*d^2*x^6) - (e*(6*e^2*
f^2 - 15*d*e*f*g + 10*d^2*g^2)*p)/(120*d^3*x^4) + (e^2*(6*e^2*f^2 - 15*d*e*
f*g + 10*d^2*g^2)*p)/(60*d^4*x^2) + (e^3*(6*e^2*f^2 - 15*d*e*f*g + 10*d^2*g
^2)*p*Log[x])/(30*d^5) - (e^3*(6*e^2*f^2 - 15*d*e*f*g + 10*d^2*g^2)*p*Log[d
 + e*x^2])/(60*d^5) - (f^2*Log[c*(d + e*x^2)^p])/(10*x^10) - (f*g*Log[c*(d
+ e*x^2)^p])/(4*x^8) - (g^2*Log[c*(d + e*x^2)^p])/(6*x^6)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
```

)

Rule 2461

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*(x_)^(m_.)*((f_) +
 (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(f + g*x^r)^q,
 x]}, Dist[a + b*Log[c*(d + e*x)^n], u, x] - Dist[b*e*n, Int[SimplifyIntegr
and[u/(d + e*x), x], x], x] /; InverseFunctionFreeQ[u, x]] /; FreeQ[{a, b,
c, d, e, f, g, m, n, q, r}, x] && IntegerQ[m] && IntegerQ[q] && IntegerQ[r]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(f+gx)^2\log\left(c(d+ex)^p\right)}{x^6}\,dx,x,x^2\right)\\
&= -\frac{f^2\log\left(c(d+ex^2)^p\right)}{10x^{10}}-\frac{fg\log\left(c(d+ex^2)^p\right)}{4x^8}-\frac{g^2\log\left(c(d+ex^2)^p\right)}{6x^6}\\
&\quad-\frac{1}{2}(ep)\text{Subst}\left(\int \frac{-6f^2-15fgx-10g^2x^2}{30x^5(d+ex)}\,dx,x,x^2\right)\\
&= -\frac{f^2\log\left(c(d+ex^2)^p\right)}{10x^{10}}-\frac{fg\log\left(c(d+ex^2)^p\right)}{4x^8}-\frac{g^2\log\left(c(d+ex^2)^p\right)}{6x^6}\\
&\quad-\frac{1}{60}(ep)\text{Subst}\left(\int \frac{-6f^2-15fgx-10g^2x^2}{x^5(d+ex)}\,dx,x,x^2\right)\\
&= -\frac{f^2\log\left(c(d+ex^2)^p\right)}{10x^{10}}-\frac{fg\log\left(c(d+ex^2)^p\right)}{4x^8}-\frac{g^2\log\left(c(d+ex^2)^p\right)}{6x^6}\\
&\quad-\frac{1}{60}(ep)\text{Subst}\left(\int \left(-\frac{6f^2}{dx^5}-\frac{3f(-2ef+5dg)}{d^2x^4}+\frac{-6e^2f^2+15defg-10d^2g^2}{d^3x^3}\right.\right.\\
&\qquad+\frac{e(6e^2f^2-15defg+10d^2g^2)}{d^4x^2}-\frac{e^2(6e^2f^2-15defg+10d^2g^2)}{d^5x}\\
&\qquad\left.\left.+\frac{e^3(6e^2f^2-15defg+10d^2g^2)}{d^5(d+ex)}\right)\,dx,x,x^2\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{ef^2p}{40dx^8} + \frac{ef(2ef-5dg)p}{60d^2x^6} - \frac{e(6e^2f^2-15defg+10d^2g^2)\,p}{120d^3x^4} \\
&\quad + \frac{e^2(6e^2f^2-15defg+10d^2g^2)\,p}{60d^4x^2} + \frac{e^3(6e^2f^2-15defg+10d^2g^2)\,p\log(x)}{30d^5} \\
&\quad - \frac{e^3(6e^2f^2-15defg+10d^2g^2)\,p\log(d+ex^2)}{60d^5} \\
&\quad - \frac{f^2\log\left(c(d+ex^2)^p\right)}{10x^{10}} - \frac{fg\log\left(c(d+ex^2)^p\right)}{4x^8} - \frac{g^2\log\left(c(d+ex^2)^p\right)}{6x^6}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.85

$$
\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x^{11}}\,dx =
$$
$$
-\frac{depx^2(-12e^3f^2x^6+6de^2fx^4(f+5gx^2)+d^3(3f^2+10fgx^2+10g^2x^4)-d^2ex^2(4f^2+15fgx^2+20g^2x^4)}{}
$$

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^11,x]
```

```
[Out] -1/120*(d*e*p*x^2*(-12*e^3*f^2*x^6 + 6*d*e^2*f*x^4*(f + 5*g*x^2) + d^3*(3*f
^2 + 10*f*g*x^2 + 10*g^2*x^4) - d^2*e*x^2*(4*f^2 + 15*f*g*x^2 + 20*g^2*x^4)
) - 4*e^3*(6*e^2*f^2 - 15*d*e*f*g + 10*d^2*g^2)*p*x^10*Log[x] + 2*e^3*(6*e^
2*f^2 - 15*d*e*f*g + 10*d^2*g^2)*p*x^10*Log[d + e*x^2] + 2*d^5*(6*f^2 + 15*
f*g*x^2 + 10*g^2*x^4)*Log[c*(d + e*x^2)^p])/(d^5*x^10)
```

Maple [A] (verified)

Time = 3.25 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.91

method	result
parts	$-\frac{g^2\ln\left(c(e\,x^2+d)^p\right)}{6x^6}-\frac{fg\ln\left(c(e\,x^2+d)^p\right)}{4x^8}-\frac{f^2\ln\left(c(e\,x^2+d)^p\right)}{10x^{10}}-\frac{pe\left(-\frac{-10g^2d^2+15defg-6e^2f^2}{4d^3x^4}-\frac{\left(10g^2d^2-15defg+6e^2f^2\right)}{2d^4x^2}\right.}{}$
parallelrisch	$\frac{40\ln(x)x^{10}d^2e^3g^2p^2-60\ln(x)x^{10}d\,e^4fg\,p^2+24\ln(x)x^{10}e^5f^2p^2-20x^{10}\ln\left(c(e\,x^2+d)^p\right)d^2e^3g^2p+30x^{10}\ln\left(c(e\,x^2+d)^p\right)d\,e^4fgp-}{}$
risch	$-\frac{\left(10g^2x^4+15fg\,x^2+6f^2\right)\ln\left((e\,x^2+d)^p\right)}{60x^{10}}-\frac{30\ln(c)d^5fg\,x^2-10i\pi\,d^5g^2x^4\,\mathrm{csgn}\left(i(e\,x^2+d)^p\right)\mathrm{csgn}\left(ic(e\,x^2+d)^p\right)\mathrm{csgn}(ic)-30\ln}{}$

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*g^2*ln(c*(e*x^2+d)^p)/x^6-1/4*f*g*ln(c*(e*x^2+d)^p)/x^8-1/10*f^2*ln(c*
(e*x^2+d)^p)/x^10-1/30*p*e*(-1/4*(-10*d^2*g^2+15*d*e*f*g-6*e^2*f^2)/d^3/x^4
-1/2*(10*d^2*g^2-15*d*e*f*g+6*e^2*f^2)/d^4*e/x^2+3/4*f^2/d/x^8+1/2*f*(5*d*g
-2*e*f)/d^2/x^6-(10*d^2*g^2-15*d*e*f*g+6*e^2*f^2)/d^5*e^2*ln(x)+1/2*e^2*(10
*d^2*g^2-15*d*e*f*g+6*e^2*f^2)/d^5*ln(e*x^2+d))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^{11}}\,dx = \frac{4\,(6\,e^5f^2-15\,de^4fg+10\,d^2e^3g^2)px^{10}\log(x)-3\,d^4efpx^2+2\,(6\,de^4f^2-15\,d^2e^3fg+10\,d^3e^2g^2)px^8-(6$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^11,x, algorithm="fricas")
```

```
[Out] 1/120*(4*(6*e^5*f^2 - 15*d*e^4*f*g + 10*d^2*e^3*g^2)*p*x^10*log(x) - 3*d^4*
e*f^2*p*x^2 + 2*(6*d*e^4*f^2 - 15*d^2*e^3*f*g + 10*d^3*e^2*g^2)*p*x^8 - (6*
d^2*e^3*f^2 - 15*d^3*e^2*f*g + 10*d^4*e*g^2)*p*x^6 + 2*(2*d^3*e^2*f^2 - 5*d
^4*e*f*g)*p*x^4 - 2*(10*d^5*g^2*p*x^4 + (6*e^5*f^2 - 15*d*e^4*f*g + 10*d^2*
e^3*g^2)*p*x^10 + 15*d^5*f*g*p*x^2 + 6*d^5*f^2*p)*log(e*x^2 + d) - 2*(10*d^
5*g^2*x^4 + 15*d^5*f*g*x^2 + 6*d^5*f^2)*log(c))/(d^5*x^10)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^{11}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**11,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.88

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^{11}}\,dx = -\frac{1}{120}\,ep\left(\frac{2\,(6\,e^4f^2-15\,de^3fg+10\,d^2e^2g^2)\log(ex^2+d)}{d^5}-\frac{2\,(6\,e^4f^2-15\,de^3fg+10\,d^2e^2g^2)\log(x^2)}{d^5}-\right.$$
$$-\frac{(10\,g^2x^4+15\,fgx^2+6\,f^2)\log\left((ex^2+d)^pc\right)}{60\,x^{10}}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^11,x, algorithm="maxima")
```

```
[Out] -1/120*e*p*(2*(6*e^4*f^2 - 15*d*e^3*f*g + 10*d^2*e^2*g^2)*log(e*x^2 + d)/d^
5 - 2*(6*e^4*f^2 - 15*d*e^3*f*g + 10*d^2*e^2*g^2)*log(x^2)/d^5 - (2*(6*e^3*
f^2 - 15*d*e^2*f*g + 10*d^2*e*g^2)*x^6 - 3*d^3*f^2 - (6*d*e^2*f^2 - 15*d^2*
e*f*g + 10*d^3*g^2)*x^4 + 2*(2*d^2*e*f^2 - 5*d^3*f*g)*x^2)/(d^4*x^8)) - 1/6
0*(10*g^2*x^4 + 15*f*g*x^2 + 6*f^2)*log((e*x^2 + d)^p*c)/x^10
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(235) = 470$.

Time = 0.34 (sec) , antiderivative size = 699, normalized size of antiderivative = 2.76

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^{11}}\,dx = -\frac{\frac{2\left(6e^6f^2p+15(ex^2+d)e^5fgp-15de^5fgp+10(ex^2+d)^2e^4g^2p-20(ex^2+d)de^4g^2p+10d^2e^4g^2p\right)\log(ex^2+d)}{(ex^2+d)^5-5(ex^2+d)^4d+10(ex^2+d)^3d^2-10(ex^2+d)^2d^3+5(ex^2+d)d^4-d^5} - \frac{12(ex^2+d)^4e^6f^2p-54(ex^2+}{}}{}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^11,x, algorithm="giac")
```

```
[Out] -1/120*(2*(6*e^6*f^2*p + 15*(e*x^2 + d)*e^5*f*g*p - 15*d*e^5*f*g*p + 10*(e*
x^2 + d)^2*e^4*g^2*p - 20*(e*x^2 + d)*d*e^4*g^2*p + 10*d^2*e^4*g^2*p)*log(e
*x^2 + d)/((e*x^2 + d)^5 - 5*(e*x^2 + d)^4*d + 10*(e*x^2 + d)^3*d^2 - 10*(e
*x^2 + d)^2*d^3 + 5*(e*x^2 + d)*d^4 - d^5) - (12*(e*x^2 + d)^4*e^6*f^2*p -
54*(e*x^2 + d)^3*d*e^6*f^2*p + 94*(e*x^2 + d)^2*d^2*e^6*f^2*p - 77*(e*x^2 +
 d)*d^3*e^6*f^2*p + 25*d^4*e^6*f^2*p - 30*(e*x^2 + d)^4*d*e^5*f*g*p + 135*(
e*x^2 + d)^3*d^2*e^5*f*g*p - 235*(e*x^2 + d)^2*d^3*e^5*f*g*p + 185*(e*x^2 +
 d)*d^4*e^5*f*g*p - 55*d^5*e^5*f*g*p + 20*(e*x^2 + d)^4*d^2*e^4*g^2*p - 90*
(e*x^2 + d)^3*d^3*e^4*g^2*p + 150*(e*x^2 + d)^2*d^4*e^4*g^2*p - 110*(e*x^2
+ d)*d^5*e^4*g^2*p + 30*d^6*e^4*g^2*p - 12*d^4*e^6*f^2*log(c) - 30*(e*x^2 +
 d)*d^4*e^5*f*g*log(c) + 30*d^5*e^5*f*g*log(c) - 20*(e*x^2 + d)^2*d^4*e^4*g
^2*log(c) + 40*(e*x^2 + d)*d^5*e^4*g^2*log(c) - 20*d^6*e^4*g^2*log(c))/((e*
x^2 + d)^5*d^4 - 5*(e*x^2 + d)^4*d^5 + 10*(e*x^2 + d)^3*d^6 - 10*(e*x^2 + d
)^2*d^7 + 5*(e*x^2 + d)*d^8 - d^9) + 2*(6*e^6*f^2*p - 15*d*e^5*f*g*p + 10*d
^2*e^4*g^2*p)*log(e*x^2 + d)/d^5 - 2*(6*e^6*f^2*p - 15*d*e^5*f*g*p + 10*d^2
*e^4*g^2*p)*log(e*x^2)/d^5)/e
```

Mupad [B] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.89

$$\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x^{11}}\,dx$$
$$=\frac{\ln(x)\,\left(10\,p\,d^2\,e^3\,g^2-15\,p\,d\,e^4\,f\,g+6\,p\,e^5\,f^2\right)}{30\,d^5}-\frac{\ln\left(c\,(e\,x^2+d)^p\right)\,\left(\frac{f^2}{10}+\frac{f\,g\,x^2}{4}+\frac{g^2\,x^4}{6}\right)}{x^{10}}$$
$$-\frac{\ln(e\,x^2+d)\,\left(10\,p\,d^2\,e^3\,g^2-15\,p\,d\,e^4\,f\,g+6\,p\,e^5\,f^2\right)}{60\,d^5}$$
$$-\frac{\frac{3\,e\,f^2\,p}{4\,d}-\frac{e^2\,p\,x^6\,\left(10\,d^2\,g^2-15\,d\,e\,f\,g+6\,e^2\,f^2\right)}{2\,d^4}+\frac{e\,p\,x^4\,\left(10\,d^2\,g^2-15\,d\,e\,f\,g+6\,e^2\,f^2\right)}{4\,d^3}+\frac{e\,f\,p\,x^2\,(5\,d\,g-2\,e\,f)}{2\,d^2}}{30\,x^8}$$

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^11,x)
```

```
[Out] (log(x)*(6*e^5*f^2*p + 10*d^2*e^3*g^2*p - 15*d*e^4*f*g*p))/(30*d^5) - (log(
c*(d + e*x^2)^p)*(f^2/10 + (g^2*x^4)/6 + (f*g*x^2)/4))/x^10 - (log(d + e*x^
2)*(6*e^5*f^2*p + 10*d^2*e^3*g^2*p - 15*d*e^4*f*g*p))/(60*d^5) - ((3*e*f^2*
p)/(4*d) - (e^2*p*x^6*(10*d^2*g^2 + 6*e^2*f^2 - 15*d*e*f*g))/(2*d^4) + (e*p
*x^4*(10*d^2*g^2 + 6*e^2*f^2 - 15*d*e*f*g))/(4*d^3) + (e*f*p*x^2*(5*d*g - 2
*e*f))/(2*d^2))/(30*x^8)
```

3.332 $\int x^2(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 278

$$\begin{aligned}
&\int x^2(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx \\
&= \frac{2df^2px}{3e}-\frac{4d^2fgpx}{5e^2}+\frac{2d^3g^2px}{7e^3}-\frac{2}{9}f^2px^3+\frac{4dfgpx^3}{15e}-\frac{2d^2g^2px^3}{21e^2}-\frac{4}{25}fgpx^5+\frac{2dg^2px^5}{35e} \\
&\quad-\frac{2}{49}g^2px^7-\frac{2d^{3/2}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{4d^{5/2}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}}-\frac{2d^{7/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}} \\
&\quad+\frac{1}{3}f^2x^3\log\left(c(d+ex^2)^p\right)+\frac{2}{5}fgx^5\log\left(c(d+ex^2)^p\right)+\frac{1}{7}g^2x^7\log\left(c(d+ex^2)^p\right)
\end{aligned}$$

```
[Out] 2/3*d*f^2*p*x/e-4/5*d^2*f*g*p*x/e^2+2/7*d^3*g^2*p*x/e^3-2/9*f^2*p*x^3+4/15*
d*f*g*p*x^3/e-2/21*d^2*g^2*p*x^3/e^2-4/25*f*g*p*x^5+2/35*d*g^2*p*x^5/e-2/49
*g^2*p*x^7-2/3*d^(3/2)*f^2*p*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)+4/5*d^(5/2)*
f*g*p*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)-2/7*d^(7/2)*g^2*p*arctan(x*e^(1/2)/
d^(1/2))/e^(7/2)+1/3*f^2*x^3*ln(c*(e*x^2+d)^p)+2/5*f*g*x^5*ln(c*(e*x^2+d)^p
)+1/7*g^2*x^7*ln(c*(e*x^2+d)^p)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2526, 2505, 308, 211}

$$\begin{aligned}
&\int x^2(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx \\
&= -\frac{2d^{3/2}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{4d^{5/2}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}}-\frac{2d^{7/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}} \\
&\quad+\frac{1}{3}f^2x^3\log\left(c(d+ex^2)^p\right)+\frac{2}{5}fgx^5\log\left(c(d+ex^2)^p\right)+\frac{1}{7}g^2x^7\log\left(c(d+ex^2)^p\right)+\frac{2d^3g^2px}{7e^3}-\frac{4d^2fgpx}{5e^2}-\frac{2d^2g^2p
\end{aligned}$$

```
[In] Int[x^2*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (2*d*f^2*p*x)/(3*e) - (4*d^2*f*g*p*x)/(5*e^2) + (2*d^3*g^2*p*x)/(7*e^3) - (
2*f^2*p*x^3)/9 + (4*d*f*g*p*x^3)/(15*e) - (2*d^2*g^2*p*x^3)/(21*e^2) - (4*f
*g*p*x^5)/25 + (2*d*g^2*p*x^5)/(35*e) - (2*g^2*p*x^7)/49 - (2*d^(3/2)*f^2*p
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^(3/2)) + (4*d^(5/2)*f*g*p*ArcTan[(Sqrt[e
]*x)/Sqrt[d]])/(5*e^(5/2)) - (2*d^(7/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/
(7*e^(7/2)) + (f^2*x^3*Log[c*(d + e*x^2)^p])/3 + (2*f*g*x^5*Log[c*(d + e*x^
2)^p])/5 + (g^2*x^7*Log[c*(d + e*x^2)^p])/7
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(f^2x^2\log\left(c\left(d+ex^2\right)^p\right)+2fgx^4\log\left(c\left(d+ex^2\right)^p\right)+g^2x^6\log\left(c\left(d+ex^2\right)^p\right)\right)\,dx\\
&= f^2\int x^2\log\left(c\left(d+ex^2\right)^p\right)\,dx+(2fg)\int x^4\log\left(c\left(d+ex^2\right)^p\right)\,dx\\
&\quad +g^2\int x^6\log\left(c\left(d+ex^2\right)^p\right)\,dx\\
&= \frac{1}{3}f^2x^3\log\left(c\left(d+ex^2\right)^p\right)+\frac{2}{5}fgx^5\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{7}g^2x^7\log\left(c\left(d+ex^2\right)^p\right)\\
&\quad -\frac{1}{3}\left(2ef^2p\right)\int\frac{x^4}{d+ex^2}\,dx-\frac{1}{5}(4efgp)\int\frac{x^6}{d+ex^2}\,dx-\frac{1}{7}\left(2eg^2p\right)\int\frac{x^8}{d+ex^2}\,dx
\end{aligned}
$$

$$= \frac{1}{3}f^2x^3\log\left(c(d+ex^2)^p\right) + \frac{2}{5}fgx^5\log\left(c(d+ex^2)^p\right) + \frac{1}{7}g^2x^7\log\left(c(d+ex^2)^p\right) - \frac{1}{3}(2ef^2p)\int\left(-\frac{d}{e^2}+\frac{x^2}{e}+\frac{d^2}{e^2(d+ex^2)}\right)dx - \frac{1}{5}(4efgp)\int\left(\frac{d^2}{e^3}-\frac{dx^2}{e^2}+\frac{x^4}{e}-\frac{d^3}{e^3(d+ex^2)}\right)dx - \frac{1}{7}(2eg^2p)\int\left(-\frac{d^3}{e^4}+\frac{d^2x^2}{e^3}-\frac{dx^4}{e^2}+\frac{x^6}{e}+\frac{d^4}{e^4(d+ex^2)}\right)dx$$

$$= \frac{2df^2px}{3e} - \frac{4d^2fgpx}{5e^2} + \frac{2d^3g^2px}{7e^3} - \frac{2}{9}f^2px^3 + \frac{4dfgpx^3}{15e} - \frac{2d^2g^2px^3}{21e^2} - \frac{4}{25}fgpx^5 + \frac{2dg^2px^5}{35e} - \frac{2}{49}g^2px^7 + \frac{1}{3}f^2x^3\log\left(c(d+ex^2)^p\right) + \frac{2}{5}fgx^5\log\left(c(d+ex^2)^p\right) + \frac{1}{7}g^2x^7\log\left(c(d+ex^2)^p\right) - \frac{(2d^2f^2p)\int\frac{1}{d+ex^2}dx}{3e} + \frac{(4d^3fgp)\int\frac{1}{d+ex^2}dx}{5e^2} - \frac{(2d^4g^2p)\int\frac{1}{d+ex^2}dx}{7e^3}$$

$$= \frac{2df^2px}{3e} - \frac{4d^2fgpx}{5e^2} + \frac{2d^3g^2px}{7e^3} - \frac{2}{9}f^2px^3 + \frac{4dfgpx^3}{15e} - \frac{2d^2g^2px^3}{21e^2} - \frac{4}{25}fgpx^5 + \frac{2dg^2px^5}{35e} - \frac{2}{49}g^2px^7 - \frac{2d^{3/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}} + \frac{4d^{5/2}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}} - \frac{2d^{7/2}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7e^{7/2}} + \frac{1}{3}f^2x^3\log\left(c(d+ex^2)^p\right) + \frac{2}{5}fgx^5\log\left(c(d+ex^2)^p\right) + \frac{1}{7}g^2x^7\log\left(c(d+ex^2)^p\right)$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.68

$$\int x^2(f+gx^2)^2\log\left(c(d+ex^2)^p\right)dx$$

$$= \frac{-210d^{3/2}(35e^2f^2-42defg+15d^2g^2)\,p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\sqrt{e}x\left(2p(1575d^3g^2-105d^2eg(42f+5gx^2)+105de\right.}{}$$

```
[In] Integrate[x^2*(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] (-210*d^(3/2)*(35*e^2*f^2 - 42*d*e*f*g + 15*d^2*g^2)*p*ArcTan[(Sqrt[e]*x)/S
qrt[d]] + Sqrt[e]*x*(2*p*(1575*d^3*g^2 - 105*d^2*e*g*(42*f + 5*g*x^2) + 105
*d*e^2*(35*f^2 + 14*f*g*x^2 + 3*g^2*x^4) - e^3*x^2*(1225*f^2 + 882*f*g*x^2
+ 225*g^2*x^4)) + 105*e^3*x^2*(35*f^2 + 42*f*g*x^2 + 15*g^2*x^4)*Log[c*(d +
 e*x^2)^p]))/(11025*e^(7/2))
```

Maple [A] (verified)

Time = 3.05 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.76

method	result
parts	$\frac{g^{2} x^{7} \ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{7}+\frac{2 f g\,x^{5} \ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{5}+\frac{f^{2} x^{3} \ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{3}-\frac{2 p e \left(-\frac{-\frac{15}{7} e^{3} g^{2} x^{7}+3 d\,e^{2} g^{2} x^{5}-\frac{42}{5} e^{3} f g\,x^{5}-5 d^{2} e\,g^{2} x^{3}+1$
risch	$-\frac{2 \sqrt{-d e}\,p\,d^{2} \ln\left(-\sqrt{-d e}\,x-d\right) f g}{5 e^{3}}+\frac{2 \sqrt{-d e}\,p\,d^{2} \ln\left(\sqrt{-d e}\,x-d\right) f g}{5 e^{3}}-\frac{i \pi\,g^{2} x^{7} \operatorname{csgn}\left(i c\left(e\,x^{2}+d\right)^{p}\right)^{3}}{14}-\frac{i \pi\,f^{2} x^{3} \operatorname{csgn}\left(i c\left(e\,x^{2}+d\right)^{p}\right)^{3}}{6}$

```
[In] int(x^2*(g*x^2+f)^2*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*g^2*x^7*ln(c*(e*x^2+d)^p)+2/5*f*g*x^5*ln(c*(e*x^2+d)^p)+1/3*f^2*x^3*ln(
c*(e*x^2+d)^p)-2/105*p*e*(-1/e^4*(-15/7*e^3*g^2*x^7+3*d*e^2*g^2*x^5-42/5*e^
3*f*g*x^5-5*d^2*e*g^2*x^3+14*d*f*g*x^3*e^2-35/3*x^3*e^3*f^2+15*x*d^3*g^2-42
*d^2*e*f*g*x+35*d*e^2*f^2*x)+d^2*(15*d^2*g^2-42*d*e*f*g+35*e^2*f^2)/e^4/(d*
e)^(1/2)*arctan(x*e/(d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.77

$$\int x^{2}\left(f+gx^{2}\right)^{2}\log\left(c\left(d+ex^{2}\right)^{p}\right)\,dx$$
$$=\left[-\frac{450\,e^{3}g^{2}px^{7}+126\,(14\,e^{3}fg-5\,de^{2}g^{2})px^{5}+70\,(35\,e^{3}f^{2}-42\,de^{2}fg+15\,d^{2}eg^{2})px^{3}-105\,(35\,de^{2}f^{2}-4}{}\right.$$
$$-\frac{450\,e^{3}g^{2}px^{7}+126\,(14\,e^{3}fg-5\,de^{2}g^{2})px^{5}+70\,(35\,e^{3}f^{2}-42\,de^{2}fg+15\,d^{2}eg^{2})px^{3}+210\,(35\,de^{2}f^{2}-4}{}$$

```
[In] integrate(x^2*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/11025*(450*e^3*g^2*p*x^7 + 126*(14*e^3*f*g - 5*d*e^2*g^2)*p*x^5 + 70*(3
5*e^3*f^2 - 42*d*e^2*f*g + 15*d^2*e*g^2)*p*x^3 - 105*(35*d*e^2*f^2 - 42*d^2
*e*f*g + 15*d^3*g^2)*p*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2
 + d)) - 210*(35*d*e^2*f^2 - 42*d^2*e*f*g + 15*d^3*g^2)*p*x - 105*(15*e^3*g
^2*p*x^7 + 42*e^3*f*g*p*x^5 + 35*e^3*f^2*p*x^3)*log(e*x^2 + d) - 105*(15*e^
3*g^2*x^7 + 42*e^3*f*g*x^5 + 35*e^3*f^2*x^3)*log(c))/e^3, -1/11025*(450*e^3
*g^2*p*x^7 + 126*(14*e^3*f*g - 5*d*e^2*g^2)*p*x^5 + 70*(35*e^3*f^2 - 42*d*e
```

```
^2*f*g + 15*d^2*e*g^2)*p*x^3 + 210*(35*d*e^2*f^2 - 42*d^2*e*f*g + 15*d^3*g^
2)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) - 210*(35*d*e^2*f^2 - 42*d^2*e*f*g +
 15*d^3*g^2)*p*x - 105*(15*e^3*g^2*p*x^7 + 42*e^3*f*g*p*x^5 + 35*e^3*f^2*p*
x^3)*log(e*x^2 + d) - 105*(15*e^3*g^2*x^7 + 42*e^3*f*g*x^5 + 35*e^3*f^2*x^3
)*log(c))/e^3]
```

Sympy [A] (verification not implemented)

Time = 117.51 (sec) , antiderivative size = 559, normalized size of antiderivative = 2.01

$$\int x^2 (f+gx^2)^2 \log\left(c(d+ex^2)^p\right)\,dx$$
$$= \begin{cases} \left(\frac{f^2x^3}{3}+\frac{2fgx^5}{5}+\frac{g^2x^7}{7}\right)\log\left(0^pc\right) \\ \left(\frac{f^2x^3}{3}+\frac{2fgx^5}{5}+\frac{g^2x^7}{7}\right)\log\left(cd^p\right) \\ -\frac{2f^2px^3}{9}+\frac{f^2x^3\log\left(c(ex^2)^p\right)}{3}-\frac{4fgpx^5}{25}+\frac{2fgx^5\log\left(c(ex^2)^p\right)}{5}-\frac{2g^2px^7}{49}+\frac{g^2x^7\log\left(c(ex^2)^p\right)}{7} \\ -\frac{2d^4g^2p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{7e^4\sqrt{-\frac{d}{e}}}+\frac{d^4g^2\log\left(c(d+ex^2)^p\right)}{7e^4\sqrt{-\frac{d}{e}}}+\frac{4d^3fgp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{5e^3\sqrt{-\frac{d}{e}}}-\frac{2d^3fg\log\left(c(d+ex^2)^p\right)}{5e^3\sqrt{-\frac{d}{e}}}+\frac{2d^3g^2px}{7e^3}-\frac{2d^2f^2p\log\left(x-\sqrt{\ldots}\right.}{3e^2\sqrt{-\frac{d}{e}}} \end{cases}$$

```
[In] integrate(x**2*(g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise((((f**2*x**3/3 + 2*f*g*x**5/5 + g**2*x**7/7)*log(0**p*c), Eq(d, 0)
 & Eq(e, 0)), ((f**2*x**3/3 + 2*f*g*x**5/5 + g**2*x**7/7)*log(c*d**p), Eq(e
, 0)), (-2*f**2*p*x**3/9 + f**2*x**3*log(c*(e*x**2)**p)/3 - 4*f*g*p*x**5/25
 + 2*f*g*x**5*log(c*(e*x**2)**p)/5 - 2*g**2*p*x**7/49 + g**2*x**7*log(c*(e*
x**2)**p)/7, Eq(d, 0)), (-2*d**4*g**2*p*log(x - sqrt(-d/e))/(7*e**4*sqrt(-d
/e)) + d**4*g**2*log(c*(d + e*x**2)**p)/(7*e**4*sqrt(-d/e)) + 4*d**3*f*g*p*
log(x - sqrt(-d/e))/(5*e**3*sqrt(-d/e)) - 2*d**3*f*g*log(c*(d + e*x**2)**p)
/(5*e**3*sqrt(-d/e)) + 2*d**3*g**2*p*x/(7*e**3) - 2*d**2*f**2*p*log(x - sqr
t(-d/e))/(3*e**2*sqrt(-d/e)) + d**2*f**2*log(c*(d + e*x**2)**p)/(3*e**2*sqr
t(-d/e)) - 4*d**2*f*g*p*x/(5*e**2) - 2*d**2*g**2*p*x**3/(21*e**2) + 2*d*f**
2*p*x/(3*e) + 4*d*f*g*p*x**3/(15*e) + 2*d*g**2*p*x**5/(35*e) - 2*f**2*p*x**
3/9 + f**2*x**3*log(c*(d + e*x**2)**p)/3 - 4*f*g*p*x**5/25 + 2*f*g*x**5*log
(c*(d + e*x**2)**p)/5 - 2*g**2*p*x**7/49 + g**2*x**7*log(c*(d + e*x**2)**p)
/7, True))
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 (f + gx^2)^2 \log \left(c(d + ex^2)^p\right) \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.77

$$\begin{aligned}
&\int x^2 (f + gx^2)^2 \log \left(c(d + ex^2)^p\right) \, dx \\
&= -\frac{1}{49} \left(2 g^2 p - 7 g^2 \log (c)\right) x^7 - \frac{2 \left(14 efgp - 5 dg^2 p - 35 efg \log (c)\right) x^5}{175 e} \\
&\quad - \frac{\left(70 e^2 f^2 p - 84 defgp + 30 d^2 g^2 p - 105 e^2 f^2 \log (c)\right) x^3}{315 e^2} \\
&\quad + \frac{1}{105} \left(15 g^2 p x^7 + 42 fgp x^5 + 35 f^2 p x^3\right) \log \left(ex^2 + d\right) \\
&\quad + \frac{2 \left(35 de^2 f^2 p - 42 d^2 efgp + 15 d^3 g^2 p\right) x}{105 e^3} \\
&\quad - \frac{2 \left(35 d^2 e^2 f^2 p - 42 d^3 efgp + 15 d^4 g^2 p\right) \arctan \left(\frac{ex}{\sqrt{de}}\right)}{105 \sqrt{de} e^3}
\end{aligned}$$

```
[In] integrate(x^2*(g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -1/49*(2*g^2*p - 7*g^2*log(c))*x^7 - 2/175*(14*e*f*g*p - 5*d*g^2*p - 35*e*f
*g*log(c))*x^5/e - 1/315*(70*e^2*f^2*p - 84*d*e*f*g*p + 30*d^2*g^2*p - 105*
e^2*f^2*log(c))*x^3/e^2 + 1/105*(15*g^2*p*x^7 + 42*f*g*p*x^5 + 35*f^2*p*x^3
)*log(e*x^2 + d) + 2/105*(35*d*e^2*f^2*p - 42*d^2*e*f*g*p + 15*d^3*g^2*p)*x
/e^3 - 2/105*(35*d^2*e^2*f^2*p - 42*d^3*e*f*g*p + 15*d^4*g^2*p)*arctan(e*x/
sqrt(d*e))/(sqrt(d*e)*e^3)
```

Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.85

$$\int x^2 (f+gx^2)^2 \log \left(c(d+ex^2)^p\right) \, dx$$
$$= \ln \left(c \left(e x^2+d\right)^p\right) \left(\frac{f^2 x^3}{3}+\frac{2 f g x^5}{5}+\frac{g^2 x^7}{7}\right)-x^3 \left(\frac{2 f^2 p}{9}-\frac{d \left(\frac{4 f g p}{5}-\frac{2 d g^2 p}{7 e}\right)}{3 e}\right)$$
$$-x^5 \left(\frac{4 f g p}{25}-\frac{2 d g^2 p}{35 e}\right)-\frac{2 g^2 p x^7}{49}+\frac{d x \left(\frac{2 f^2 p}{3}-\frac{d \left(\frac{4 f g p}{5}-\frac{2 d g^2 p}{7 e}\right)}{e}\right)}{e}$$
$$-\frac{2 d^{3/2} p \operatorname{atan}\left(\frac{d^{3/2} \sqrt{e} p x \left(15 d^2 g^2-42 d e f g+35 e^2 f^2\right)}{15 p d^4 g^2-42 p d^3 e f g+35 p d^2 e^2 f^2}\right) \left(15 d^2 g^2-42 d e f g+35 e^2 f^2\right)}{105 e^{7/2}}$$

```
[In] int(x^2*log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

```
[Out] log(c*(d + e*x^2)^p)*((f^2*x^3)/3 + (g^2*x^7)/7 + (2*f*g*x^5)/5) - x^3*((2*
f^2*p)/9 - (d*((4*f*g*p)/5 - (2*d*g^2*p)/(7*e)))/(3*e)) - x^5*((4*f*g*p)/25
 - (2*d*g^2*p)/(35*e)) - (2*g^2*p*x^7)/49 + (d*x*((2*f^2*p)/3 - (d*((4*f*g*
p)/5 - (2*d*g^2*p)/(7*e)))/e))/e - (2*d^(3/2)*p*atan((d^(3/2)*e^(1/2)*p*x*(
15*d^2*g^2 + 35*e^2*f^2 - 42*d*e*f*g))/(15*d^4*g^2*p + 35*d^2*e^2*f^2*p - 4
2*d^3*e*f*g*p))*(15*d^2*g^2 + 35*e^2*f^2 - 42*d*e*f*g))/(105*e^(7/2))
```

3.333 $\int (f+gx^2)^2 \log(c(d+ex^2)^p)\, dx$

Optimal result

Integrand size = 22, antiderivative size = 221

$$\begin{aligned}\int (f+gx^2)^2 \log\left(c(d+ex^2)^p\right) dx &= -2f^2px+\frac{4dfgpx}{3e}-\frac{2d^2g^2px}{5e^2}-\frac{4}{9}fgpx^3+\frac{2dg^2px^3}{15e}-\frac{2}{25}g^2px^5\\&\quad+\frac{2\sqrt{d}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{4d^{3/2}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2d^{5/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}}\\&\quad+f^2x\log\left(c(d+ex^2)^p\right)+\frac{2}{3}fgx^3\log\left(c(d+ex^2)^p\right)+\frac{1}{5}g^2x^5\log\left(c(d+ex^2)^p\right)\end{aligned}$$

```
[Out] -2*f^2*p*x+4/3*d*f*g*p*x/e-2/5*d^2*g^2*p*x/e^2-4/9*f*g*p*x^3+2/15*d*g^2*p*x
^3/e-2/25*g^2*p*x^5-4/3*d^(3/2)*f*g*p*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)+2/5
*d^(5/2)*g^2*p*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)+f^2*x*ln(c*(e*x^2+d)^p)+2/
3*f*g*x^3*ln(c*(e*x^2+d)^p)+1/5*g^2*x^5*ln(c*(e*x^2+d)^p)+2*f^2*p*arctan(x*
e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2521, 2498, 327, 211, 2505, 308}

$$\begin{aligned}&\int (f+gx^2)^2 \log\left(c(d+ex^2)^p\right) dx\\&= -\frac{4d^{3/2}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2d^{5/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}}+\frac{2\sqrt{d}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}\\&\quad+f^2x\log\left(c(d+ex^2)^p\right)+\frac{2}{3}fgx^3\log\left(c(d+ex^2)^p\right)+\frac{1}{5}g^2x^5\log\left(c(d+ex^2)^p\right)-\frac{2d^2g^2px}{5e^2}+\frac{4dfgpx}{3e}+\frac{2dg^2px^3}{15e}\end{aligned}$$

```
[In] Int[(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]
```

```
[Out] -2*f^2*p*x + (4*d*f*g*p*x)/(3*e) - (2*d^2*g^2*p*x)/(5*e^2) - (4*f*g*p*x^3)/
9 + (2*d*g^2*p*x^3)/(15*e) - (2*g^2*p*x^5)/25 + (2*Sqrt[d]*f^2*p*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/Sqrt[e] - (4*d^(3/2)*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/
(3*e^(3/2)) + (2*d^(5/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*e^(5/2)) + f
^2*x*Log[c*(d + e*x^2)^p] + (2*f*g*x^3*Log[c*(d + e*x^2)^p])/3 + (g^2*x^5*L
og[c*(d + e*x^2)^p])/5
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
```

```
0] && LtQ[r, 0]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(f^2 \log \left(c\left(d+ex^2\right)^p\right)+2fgx^2 \log \left(c\left(d+ex^2\right)^p\right)+g^2x^4 \log \left(c\left(d+ex^2\right)^p\right)\right)\,dx \\
&= f^2 \int \log \left(c\left(d+ex^2\right)^p\right)\,dx+(2fg)\int x^2 \log \left(c\left(d+ex^2\right)^p\right)\,dx+g^2 \int x^4 \log \left(c\left(d+ex^2\right)^p\right)\,dx \\
&= f^2x \log \left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3 \log \left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5 \log \left(c\left(d+ex^2\right)^p\right) \\
&\quad -\left(2ef^2p\right)\int \frac{x^2}{d+ex^2}\,dx-\frac{1}{3}(4efgp)\int \frac{x^4}{d+ex^2}\,dx-\frac{1}{5}\left(2eg^2p\right)\int \frac{x^6}{d+ex^2}\,dx \\
&= -2f^2px+f^2x \log \left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3 \log \left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5 \log \left(c\left(d+ex^2\right)^p\right) \\
&\quad +\left(2df^2p\right)\int \frac{1}{d+ex^2}\,dx-\frac{1}{3}(4efgp)\int \left(-\frac{d}{e^2}+\frac{x^2}{e}+\frac{d^2}{e^2\left(d+ex^2\right)}\right)dx \\
&\quad -\frac{1}{5}\left(2eg^2p\right)\int \left(\frac{d^2}{e^3}-\frac{dx^2}{e^2}+\frac{x^4}{e}-\frac{d^3}{e^3\left(d+ex^2\right)}\right)dx \\
&= -2f^2px+\frac{4dfgpx}{3e}-\frac{2d^2g^2px}{5e^2}-\frac{4}{9}fgpx^3+\frac{2dg^2px^3}{15e}-\frac{2}{25}g^2px^5 \\
&\quad +\frac{2\sqrt{d}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+f^2x \log \left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3 \log \left(c\left(d+ex^2\right)^p\right) \\
&\quad +\frac{1}{5}g^2x^5 \log \left(c\left(d+ex^2\right)^p\right)-\frac{\left(4d^2fgp\right)\int \frac{1}{d+ex^2}\,dx}{3e}+\frac{\left(2d^3g^2p\right)\int \frac{1}{d+ex^2}\,dx}{5e^2} \\
&= -2f^2px+\frac{4dfgpx}{3e}-\frac{2d^2g^2px}{5e^2}-\frac{4}{9}fgpx^3+\frac{2dg^2px^3}{15e}-\frac{2}{25}g^2px^5 \\
&\quad +\frac{2\sqrt{d}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{4d^{3/2}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{2d^{5/2}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5e^{5/2}} \\
&\quad +f^2x \log \left(c\left(d+ex^2\right)^p\right)+\frac{2}{3}fgx^3 \log \left(c\left(d+ex^2\right)^p\right)+\frac{1}{5}g^2x^5 \log \left(c\left(d+ex^2\right)^p\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.68

$$\int (f+gx^2)^2 \log \left(c(d+ex^2)^p\right)\,dx$$
$$=\frac{30\sqrt{d}(15e^2f^2-10defg+3d^2g^2)\,p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\sqrt{e}x\left(-2p(45d^2g^2-15deg(10f+gx^2)+e^2(225f^2+50\right.}{225e^{5/2}}$$

[In] Integrate[(f + g*x^2)^2*Log[c*(d + e*x^2)^p],x]

[Out] (30*Sqrt[d]*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[e]*x*(-2*p*(45*d^2*g^2 - 15*d*e*g*(10*f + g*x^2) + e^2*(225*f^2 + 50*f*g*x^2 + 9*g^2*x^4)) + 15*e^2*(15*f^2 + 10*f*g*x^2 + 3*g^2*x^4)*Log[c*(d + e*x^2)^p]))/(225*e^(5/2))

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.76

method	result
parts	$\frac{g^2x^5\ln\left(c(e\,x^2+d)^p\right)}{5}+\frac{2fg\,x^3\ln\left(c(e\,x^2+d)^p\right)}{3}+f^2x\ln\left(c(e\,x^2+d)^p\right)-\frac{2pe\left(\frac{\frac{3}{5}e^2g^2x^5-de\,g^2x^3+\frac{10}{3}e^2fg\,x^3+3d^2g^2x-10def}{e^3}\right.}{}$
risch	$\frac{\ln(c)g^2x^5}{5}+x\ln(c)\,f^2-\frac{i\pi fg\,x^3\,\mathrm{csgn}\left(i(e\,x^2+d)^p\right)\mathrm{csgn}\left(ic(e\,x^2+d)^p\right)\mathrm{csgn}(ic)}{3}+\frac{2\sqrt{-de}\,p\ln\left(\sqrt{-de}\,x+d\right)dfg}{3e^2}-\frac{2\sqrt{-de}\,p\ln}{}$

[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p),x,method=_RETURNVERBOSE)

[Out] 1/5*g^2*x^5*ln(c*(e*x^2+d)^p)+2/3*f*g*x^3*ln(c*(e*x^2+d)^p)+f^2*x*ln(c*(e*x^2+d)^p)-2/15*p*e*(1/e^3*(3/5*e^2*g^2*x^5-d*e*g^2*x^3+10/3*e^2*f*g*x^3+3*d^2*g^2*x-10*d*e*f*g*x+15*e^2*f^2*x)-d*(3*d^2*g^2-10*d*e*f*g+15*e^2*f^2)/e^3/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.83

$$\int \left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$=\left[-\frac{18\,e^2g^2px^5+10\,(10\,e^2fg-3\,deg^2)px^3-15\,(15\,e^2f^2-10\,defg+3\,d^2g^2)p\sqrt{-\frac{d}{e}}\log\left(\frac{ex^2+2\,ex\sqrt{-\frac{d}{e}}-d}{ex^2+d}\right)-\right.$$
$$-\frac{18\,e^2g^2px^5+10\,(10\,e^2fg-3\,deg^2)px^3-30\,(15\,e^2f^2-10\,defg+3\,d^2g^2)p\sqrt{\frac{d}{e}}\arctan\left(\frac{ex\sqrt{\frac{d}{e}}}{d}\right)+30\,(15$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="fricas")
```

```
[Out] [-1/225*(18*e^2*g^2*p*x^5 + 10*(10*e^2*f*g - 3*d*e*g^2)*p*x^3 - 15*(15*e^2*
f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) -
d)/(e*x^2 + d)) + 30*(15*e^2*f^2 - 10*d*e*f*g + 3*d^2*g^2)*p*x - 15*(3*e^2*
g^2*p*x^5 + 10*e^2*f*g*p*x^3 + 15*e^2*f^2*p*x)*log(e*x^2 + d) - 15*(3*e^2*g
^2*x^5 + 10*e^2*f*g*x^3 + 15*e^2*f^2*x)*log(c))/e^2, -1/225*(18*e^2*g^2*p*x
^5 + 10*(10*e^2*f*g - 3*d*e*g^2)*p*x^3 - 30*(15*e^2*f^2 - 10*d*e*f*g + 3*d^
2*g^2)*p*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 30*(15*e^2*f^2 - 10*d*e*f*g +
3*d^2*g^2)*p*x - 15*(3*e^2*g^2*p*x^5 + 10*e^2*f*g*p*x^3 + 15*e^2*f^2*p*x)*l
og(e*x^2 + d) - 15*(3*e^2*g^2*x^5 + 10*e^2*f*g*x^3 + 15*e^2*f^2*x)*log(c))/
e^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(231) = 462$.

Time = 31.58 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.16

$$\int \left(f+gx^2\right)^2 \log\left(c\left(d+ex^2\right)^p\right)\,dx$$
$$=\begin{cases}\left(f^2x+\frac{2fgx^3}{3}+\frac{g^2x^5}{5}\right)\log\left(0^pc\right)\\ \left(f^2x+\frac{2fgx^3}{3}+\frac{g^2x^5}{5}\right)\log\left(cd^p\right)\\ -2f^2px+f^2x\log\left(c\left(ex^2\right)^p\right)-\frac{4fgpx^3}{9}+\frac{2fgx^3\log\left(c\left(ex^2\right)^p\right)}{3}-\frac{2g^2px^5}{25}+\frac{g^2x^5\log\left(c\left(ex^2\right)^p\right)}{5}\\ \frac{2d^3g^2p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{5e^3\sqrt{-\frac{d}{e}}}-\frac{d^3g^2\log\left(c\left(d+ex^2\right)^p\right)}{5e^3\sqrt{-\frac{d}{e}}}-\frac{4d^2fgp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{3e^2\sqrt{-\frac{d}{e}}}+\frac{2d^2fg\log\left(c\left(d+ex^2\right)^p\right)}{3e^2\sqrt{-\frac{d}{e}}}-\frac{2d^2g^2px}{5e^2}+\frac{2df^2p\log\left(x-\sqrt{-\right.}}{e\sqrt{-\frac{d}{e}}}\end{cases}$$

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p),x)
```

```
[Out] Piecewise(((f**2*x + 2*f*g*x**3/3 + g**2*x**5/5)*log(0**p*c), Eq(d, 0) & Eq
(e, 0)), ((f**2*x + 2*f*g*x**3/3 + g**2*x**5/5)*log(c*d**p), Eq(e, 0)), (-2
*f**2*p*x + f**2*x*log(c*(e*x**2)**p) - 4*f*g*p*x**3/9 + 2*f*g*x**3*log(c*(
e*x**2)**p)/3 - 2*g**2*p*x**5/25 + g**2*x**5*log(c*(e*x**2)**p)/5, Eq(d, 0)
), (2*d**3*g**2*p*log(x - sqrt(-d/e))/(5*e**3*sqrt(-d/e)) - d**3*g**2*log(c
*(d + e*x**2)**p)/(5*e**3*sqrt(-d/e)) - 4*d**2*f*g*p*log(x - sqrt(-d/e))/(3
*e**2*sqrt(-d/e)) + 2*d**2*f*g*log(c*(d + e*x**2)**p)/(3*e**2*sqrt(-d/e)) -
 2*d**2*g**2*p*x/(5*e**2) + 2*d*f**2*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) -
 d*f**2*log(c*(d + e*x**2)**p)/(e*sqrt(-d/e)) + 4*d*f*g*p*x/(3*e) + 2*d*g**
2*p*x**3/(15*e) - 2*f**2*p*x + f**2*x*log(c*(d + e*x**2)**p) - 4*f*g*p*x**3
/9 + 2*f*g*x**3*log(c*(d + e*x**2)**p)/3 - 2*g**2*p*x**5/25 + g**2*x**5*log
(c*(d + e*x**2)**p)/5, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(f+gx^2\right)^2 \log \left(c\left(d+ex^2\right)^p\right) \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.79

$$\begin{aligned}\int \left(f+gx^2\right)^2 \log \left(c\left(d+ex^2\right)^p\right) \, dx &= -\frac{1}{25}\left(2\,g^2p-5\,g^2\log\left(c\right)\right)x^5 \\ &\quad -\frac{2\,(10\,efgp-3\,dg^2p-15\,efg\log\left(c\right))x^3}{45\,e} \\ &\quad +\frac{1}{15}\left(3\,g^2px^5+10\,fgpx^3+15\,f^2px\right)\log\left(ex^2+d\right) \\ &\quad -\frac{(30\,e^2f^2p-20\,defgp+6\,d^2g^2p-15\,e^2f^2\log\left(c\right))x}{15\,e^2} \\ &\quad +\frac{2\,(15\,de^2f^2p-10\,d^2efgp+3\,d^3g^2p)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{15\,\sqrt{de}e^2}\end{aligned}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p),x, algorithm="giac")
```

```
[Out] -1/25*(2*g^2*p - 5*g^2*log(c))*x^5 - 2/45*(10*e*f*g*p - 3*d*g^2*p - 15*e*f*
g*log(c))*x^3/e + 1/15*(3*g^2*p*x^5 + 10*f*g*p*x^3 + 15*f^2*p*x)*log(e*x^2
+ d) - 1/15*(30*e^2*f^2*p - 20*d*e*f*g*p + 6*d^2*g^2*p - 15*e^2*f^2*log(c))
*x/e^2 + 2/15*(15*d*e^2*f^2*p - 10*d^2*e*f*g*p + 3*d^3*g^2*p)*arctan(e*x/sq
rt(d*e))/(sqrt(d*e)*e^2)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.87

$$\int \left(f+gx^2\right)^2 \log \left(c\left(d+ex^2\right)^p\right) \, dx = \ln \left(c\left(e\,x^2+d\right)^p\right)\,\left(f^2\,x+\frac{2\,f\,g\,x^3}{3}+\frac{g^2\,x^5}{5}\right) - x\,\left(2\,f^2\,p-\frac{d\,\left(\frac{4\,f\,g\,p}{3}-\frac{2\,d\,g^2\,p}{5\,e}\right)}{e}\right)-x^3\,\left(\frac{4\,f\,g\,p}{9}-\frac{2\,d\,g^2\,p}{15\,e}\right)-\frac{2\,g^2\,p\,x^5}{25} + \frac{2\,\sqrt{d}\,p\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e}\,p\,x\,\left(3\,d^2\,g^2-10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{3\,p\,d^3\,g^2-10\,p\,d^2\,e\,f\,g+15\,p\,d\,e^2\,f^2}\right)\,\left(3\,d^2\,g^2-10\,d\,e\,f\,g+15\,e^2\,f^2\right)}{15\,e^{5/2}}$$

```
[In] int(log(c*(d + e*x^2)^p)*(f + g*x^2)^2,x)
```

```
[Out] log(c*(d + e*x^2)^p)*(f^2*x + (g^2*x^5)/5 + (2*f*g*x^3)/3) - x*(2*f^2*p - (
d*((4*f*g*p)/3 - (2*d*g^2*p)/(5*e)))/e) - x^3*((4*f*g*p)/9 - (2*d*g^2*p)/(1
5*e)) - (2*g^2*p*x^5)/25 + (2*d^(1/2)*p*atan((d^(1/2)*e^(1/2)*p*x*(3*d^2*g^
2 + 15*e^2*f^2 - 10*d*e*f*g))/(3*d^3*g^2*p + 15*d*e^2*f^2*p - 10*d^2*e*f*g*
p))*(3*d^2*g^2 + 15*e^2*f^2 - 10*d*e*f*g))/(15*e^(5/2))
```

3.334 $\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 178

$$
\begin{aligned}
\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x^2}\,dx &= -4fgpx+\frac{2dg^2px}{3e}-\frac{2}{9}g^2px^3 \\
&\quad+\frac{2\sqrt{e}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{4\sqrt{d}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} \\
&\quad-\frac{2d^{3/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}}-\frac{f^2\log\left(c(d+ex^2)^p\right)}{x} \\
&\quad+2fgx\log\left(c(d+ex^2)^p\right)+\frac{1}{3}g^2x^3\log\left(c(d+ex^2)^p\right)
\end{aligned}
$$

```
[Out] -4*f*g*p*x+2/3*d*g^2*p*x/e-2/9*g^2*p*x^3-2/3*d^(3/2)*g^2*p*arctan(x*e^(1/2)
/d^(1/2))/e^(3/2)-f^2*ln(c*(e*x^2+d)^p)/x+2*f*g*x*ln(c*(e*x^2+d)^p)+1/3*g^2
*x^3*ln(c*(e*x^2+d)^p)+4*f*g*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)+2*
f^2*p*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used

$= \{2526, 2498, 327, 211, 2505, 308\}$

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^2}\,dx = -\frac{2d^{3/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}} + \frac{2\sqrt{e}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{4\sqrt{d}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{f^2\log\left(c(d+ex^2)^p\right)}{x} + 2fgx\log\left(c(d+ex^2)^p\right) + \frac{1}{3}g^2x^3\log\left(c(d+ex^2)^p\right) + \frac{2dg^2px}{3e} - 4fgpx - \frac{2}{9}g^2px^3$$

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^2,x]
```

```
[Out] -4*f*g*p*x + (2*d*g^2*p*x)/(3*e) - (2*g^2*p*x^3)/9 + (2*Sqrt[e]*f^2*p*ArcTa
n[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (4*Sqrt[d]*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[
d]])/Sqrt[e] - (2*d^(3/2)*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*e^(3/2)) -
(f^2*Log[c*(d + e*x^2)^p])/x + 2*f*g*x*Log[c*(d + e*x^2)^p] + (g^2*x^3*Log[
c*(d + e*x^2)^p])/3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(2fg\log\left(c\left(d+ex^2\right)^p\right)+\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{x^2}+g^2x^2\log\left(c\left(d+ex^2\right)^p\right)\right)\,dx\\
&= f^2\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2}\,dx+(2fg)\int \log\left(c\left(d+ex^2\right)^p\right)\,dx+g^2\int x^2\log\left(c\left(d+ex^2\right)^p\right)\,dx\\
&= -\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{x}+2fgx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}g^2x^3\log\left(c\left(d+ex^2\right)^p\right)\\
&\quad+\left(2ef^2p\right)\int \frac{1}{d+ex^2}\,dx-(4efgp)\int \frac{x^2}{d+ex^2}\,dx-\frac{1}{3}\left(2eg^2p\right)\int \frac{x^4}{d+ex^2}\,dx\\
&= -4fgpx+\frac{2\sqrt{e}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{x}\\
&\quad+2fgx\log\left(c\left(d+ex^2\right)^p\right)+\frac{1}{3}g^2x^3\log\left(c\left(d+ex^2\right)^p\right)\\
&\quad+(4dfgp)\int \frac{1}{d+ex^2}\,dx-\frac{1}{3}\left(2eg^2p\right)\int \left(-\frac{d}{e^2}+\frac{x^2}{e}+\frac{d^2}{e^2\left(d+ex^2\right)}\right)\,dx\\
&= -4fgpx+\frac{2dg^2px}{3e}-\frac{2}{9}g^2px^3+\frac{2\sqrt{e}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}\\
&\quad+\frac{4\sqrt{d}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{x}+2fgx\log\left(c\left(d+ex^2\right)^p\right)\\
&\quad+\frac{1}{3}g^2x^3\log\left(c\left(d+ex^2\right)^p\right)-\frac{\left(2d^2g^2p\right)\int \frac{1}{d+ex^2}\,dx}{3e}
\end{aligned}
$$

$$
\begin{aligned}
&= -4fgpx + \frac{2dg^2px}{3e} - \frac{2}{9}g^2px^3 + \frac{2\sqrt{e}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} \\
&\quad + \frac{4\sqrt{d}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{2d^{3/2}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}} - \frac{f^2\log\left(c(d+ex^2)^p\right)}{x} \\
&\quad + 2fgx\log\left(c(d+ex^2)^p\right) + \frac{1}{3}g^2x^3\log\left(c(d+ex^2)^p\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.63

$$
\begin{aligned}
\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x^2}\,dx = \frac{1}{9}\Bigg(& -\frac{2gpx(18ef-3dg+egx^2)}{e} \\
& + \frac{6(3e^2f^2+6defg-d^2g^2)\,p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{3/2}} \\
& + \left(-\frac{9f^2}{x}+18fgx+3g^2x^3\right)\log\left(c(d+ex^2)^p\right)\Bigg)
\end{aligned}
$$

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^2,x]
```

```
[Out] ((-2*g*p*x*(18*e*f - 3*d*g + e*g*x^2))/e + (6*(3*e^2*f^2 + 6*d*e*f*g - d^2*
g^2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(3/2)) + ((-9*f^2)/x + 18*f*
g*x + 3*g^2*x^3)*Log[c*(d + e*x^2)^p])/9
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.71

method	result
parts	$\frac{g^2x^3\ln\left(c(e\,x^2+d)^p\right)}{3}+2fgx\ln\left(c(e\,x^2+d)^p\right)-\frac{f^2\ln\left(c(e\,x^2+d)^p\right)}{x}-\frac{2pe\left(-\frac{g\left(-\frac{1}{3}eg\,x^3+dgx-6efx\right)}{e^2}+\frac{\left(g^2d^2-6defg-3e^2\right.}{e^2}\right.}{3}$
risch	$-\frac{\left(-g^2x^4-6fg\,x^2+3f^2\right)\ln\left((e\,x^2+d)^p\right)}{3x}-\frac{-3ig^2\pi\,x^4\mathrm{csgn}\left(ic(e\,x^2+d)^p\right)^2\mathrm{csgn}(ic)d\,e^2-3ig^2\pi\,x^4\,\mathrm{csgn}\left(i(e\,x^2+d)^p\right)\mathrm{csgn}\left(ic(e\,x^2+d\right.}{}$

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*g^2*x^3*ln(c*(e*x^2+d)^p)+2*f*g*x*ln(c*(e*x^2+d)^p)-f^2*ln(c*(e*x^2+d)^
p)/x-2/3*p*e*(-g/e^2*(-1/3*e*g*x^3+d*g*x-6*e*f*x)+(d^2*g^2-6*d*e*f*g-3*e^2*
f^2)/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.06

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^2}\,dx$$
$$= \left[-\frac{2\,de^2g^2px^4 - 3\,(3\,e^2f^2 + 6\,defg - d^2g^2)\sqrt{-de}px\log\left(\frac{ex^2+2\,\sqrt{-de}x-d}{ex^2+d}\right) + 6\,(6\,de^2fg - d^2eg^2)px^2 - 3\,(de^2g}{9\,de^2x}\right.$$
$$-\frac{2\,de^2g^2px^4 - 6\,(3\,e^2f^2 + 6\,defg - d^2g^2)\sqrt{de}px\arctan\left(\frac{\sqrt{de}x}{d}\right) + 6\,(6\,de^2fg - d^2eg^2)px^2 - 3\,(de^2g^2px^4 +}{9\,de^2x}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/9*(2*d*e^2*g^2*p*x^4 - 3*(3*e^2*f^2 + 6*d*e*f*g - d^2*g^2)*sqrt(-d*e)*p
*x*log((e*x^2 + 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 6*(6*d*e^2*f*g - d^2*e*g
^2)*p*x^2 - 3*(d*e^2*g^2*p*x^4 + 6*d*e^2*f*g*p*x^2 - 3*d*e^2*f^2*p)*log(e*x
^2 + d) - 3*(d*e^2*g^2*x^4 + 6*d*e^2*f*g*x^2 - 3*d*e^2*f^2)*log(c))/(d*e^2*
x), -1/9*(2*d*e^2*g^2*p*x^4 - 6*(3*e^2*f^2 + 6*d*e*f*g - d^2*g^2)*sqrt(d*e)
*p*x*arctan(sqrt(d*e)*x/d) + 6*(6*d*e^2*f*g - d^2*e*g^2)*p*x^2 - 3*(d*e^2*g
^2*p*x^4 + 6*d*e^2*f*g*p*x^2 - 3*d*e^2*f^2*p)*log(e*x^2 + d) - 3*(d*e^2*g^2
*x^4 + 6*d*e^2*f*g*x^2 - 3*d*e^2*f^2)*log(c))/(d*e^2*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(182) = 364.

Time = 60.65 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.25

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^2}\,dx$$
$$= \begin{cases} \left(-\frac{f^2}{x} + 2fgx + \frac{g^2x^3}{3}\right)\log\left(0^pc\right) \\ \left(-\frac{f^2}{x} + 2fgx + \frac{g^2x^3}{3}\right)\log\left(cd^p\right) \\ -\frac{2f^2p}{x} - \frac{f^2\log\left(c(ex^2)^p\right)}{x} - 4fgpx + 2fgx\log\left(c(ex^2)^p\right) - \frac{2g^2px^3}{9} + \frac{g^2x^3\log\left(c(ex^2)^p\right)}{3} \\ -\frac{2d^2g^2p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{3e^2\sqrt{-\frac{d}{e}}} + \frac{d^2g^2\log\left(c(d+ex^2)^p\right)}{3e^2\sqrt{-\frac{d}{e}}} + \frac{4dfgp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{e\sqrt{-\frac{d}{e}}} - \frac{2dfg\log\left(c(d+ex^2)^p\right)}{e\sqrt{-\frac{d}{e}}} + \frac{2dg^2px}{3e} + \frac{2f^2p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{\sqrt{-\frac{d}{e}}} \end{cases}$$

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**2,x)
```

```
[Out] Piecewise(((-f**2/x + 2*f*g*x + g**2*x**3/3)*log(0**p*c), Eq(d, 0) & Eq(e,
0)), ((-f**2/x + 2*f*g*x + g**2*x**3/3)*log(c*d**p), Eq(e, 0)), (-2*f**2*p/
x - f**2*log(c*(e*x**2)**p)/x - 4*f*g*p*x + 2*f*g*x*log(c*(e*x**2)**p) - 2*
g**2*p*x**3/9 + g**2*x**3*log(c*(e*x**2)**p)/3, Eq(d, 0)), (-2*d**2*g**2*p*
log(x - sqrt(-d/e))/(3*e**2*sqrt(-d/e)) + d**2*g**2*log(c*(d + e*x**2)**p)/
(3*e**2*sqrt(-d/e)) + 4*d*f*g*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) - 2*d*f*
g*log(c*(d + e*x**2)**p)/(e*sqrt(-d/e)) + 2*d*g**2*p*x/(3*e) + 2*f**2*p*log
(x - sqrt(-d/e))/sqrt(-d/e) - f**2*log(c*(d + e*x**2)**p)/sqrt(-d/e) - f**2
*log(c*(d + e*x**2)**p)/x - 4*f*g*p*x + 2*f*g*x*log(c*(d + e*x**2)**p) - 2*
g**2*p*x**3/9 + g**2*x**3*log(c*(d + e*x**2)**p)/3, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.76

$$\begin{aligned}\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^2}\,dx = &-\frac{1}{9}\left(2\,g^2p - 3\,g^2\log\left(c\right)\right)x^3 \\ &+ \frac{1}{3}\left(g^2px^3 + 6\,fgpx - \frac{3\,f^2p}{x}\right)\log\left(ex^2+d\right) \\ &- \frac{f^2\log\left(c\right)}{x} - \frac{2\left(6\,efgp - dg^2p - 3\,efg\log\left(c\right)\right)x}{3\,e} \\ &+ \frac{2\left(3\,e^2f^2p + 6\,defgp - d^2g^2p\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{3\sqrt{de}e}\end{aligned}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*g^2*p - 3*g^2*log(c))*x^3 + 1/3*(g^2*p*x^3 + 6*f*g*p*x - 3*f^2*p/x)
*log(e*x^2 + d) - f^2*log(c)/x - 2/3*(6*e*f*g*p - d*g^2*p - 3*e*f*g*log(c))
*x/e + 2/3*(3*e^2*f^2*p + 6*d*e*f*g*p - d^2*g^2*p)*arctan(e*x/sqrt(d*e))/(s
qrt(d*e)*e)
```

Mupad [B] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.01

$$\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x^2}\,dx$$
$$=\frac{2\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,p\,x\,\left(-d^2\,g^2+6\,d\,e\,f\,g+3\,e^2\,f^2\right)}{\sqrt{d}\,\left(-p\,d^2\,g^2+6\,p\,d\,e\,f\,g+3\,p\,e^2\,f^2\right)}\right)\,\left(-d^2\,g^2+6\,d\,e\,f\,g+3\,e^2\,f^2\right)}{3\,\sqrt{d}\,e^{3/2}}$$
$$-x\,\left(4\,f\,g\,p-\frac{2\,d\,g^2\,p}{3\,e}\right)-\frac{2\,g^2\,p\,x^3}{9}$$
$$-\ln\left(c\,\left(e\,x^2+d\right)^p\right)\,\left(\frac{f^2+2\,f\,g\,x^2+g^2\,x^4}{x}-\frac{\frac{4\,g^2\,x^4}{3}+4\,f\,g\,x^2}{x}\right)$$

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^2,x)
```

```
[Out] (2*p*atan((e^(1/2)*p*x*(3*e^2*f^2 - d^2*g^2 + 6*d*e*f*g))/(d^(1/2)*(3*e^2*f
^2*p - d^2*g^2*p + 6*d*e*f*g*p)))*(3*e^2*f^2 - d^2*g^2 + 6*d*e*f*g))/(3*d^(
1/2)*e^(3/2)) - x*(4*f*g*p - (2*d*g^2*p)/(3*e)) - (2*g^2*p*x^3)/9 - log(c*(
d + e*x^2)^p)*((f^2 + g^2*x^4 + 2*f*g*x^2)/x - ((4*g^2*x^4)/3 + 4*f*g*x^2)/
x)
```

3.335 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^4} dx$

Optimal result

Integrand size = 25, antiderivative size = 169

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^4} dx = -\frac{2ef^2p}{3dx} - 2g^2px - \frac{2e^{3/2}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}} + \frac{4\sqrt{e}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{d}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{f^2\log\left(c(d+ex^2)^p\right)}{3x^3} - \frac{2fg\log\left(c(d+ex^2)^p\right)}{x} + g^2x\log\left(c(d+ex^2)^p\right)$$

```
[Out] -2/3*e*f^2*p/d/x-2*g^2*p*x-2/3*e^(3/2)*f^2*p*arctan(x*e^(1/2)/d^(1/2))/d^(3
/2)-1/3*f^2*ln(c*(e*x^2+d)^p)/x^3-2*f*g*ln(c*(e*x^2+d)^p)/x+g^2*x*ln(c*(e*x
^2+d)^p)+2*g^2*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)+4*f*g*p*arctan(x
*e^(1/2)/d^(1/2))*e^(1/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used

$= \{2526, 2498, 327, 211, 2505, 331\}$

$$\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x^4}\,dx = -\frac{2e^{3/2}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}} + \frac{4\sqrt{e}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{d}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{f^2\log\left(c(d+ex^2)^p\right)}{3x^3} - \frac{2fg\log\left(c(d+ex^2)^p\right)}{x} + g^2x\log\left(c\left(d+ex^2\right)^p\right) - \frac{2ef^2p}{3dx} - 2g^2px$$

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^4,x]
```

```
[Out] (-2*e*f^2*p)/(3*d*x) - 2*g^2*p*x - (2*e^(3/2)*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]])/(3*d^(3/2)) + (4*Sqrt[e]*f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] +
 (2*Sqrt[d]*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (f^2*Log[c*(d + e*
x^2)^p])/(3*x^3) - (2*f*g*Log[c*(d + e*x^2)^p])/x + g^2*x*Log[c*(d + e*x^2)
^p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(g^2 \log \left(c\left(d+ex^2\right)^p\right)+\frac{f^2 \log \left(c\left(d+ex^2\right)^p\right)}{x^4}+\frac{2fg \log \left(c\left(d+ex^2\right)^p\right)}{x^2}\right) dx \\
&= f^2 \int \frac{\log \left(c\left(d+ex^2\right)^p\right)}{x^4} dx+(2fg) \int \frac{\log \left(c\left(d+ex^2\right)^p\right)}{x^2} dx+g^2 \int \log \left(c\left(d+ex^2\right)^p\right) dx \\
&= -\frac{f^2 \log \left(c\left(d+ex^2\right)^p\right)}{3x^3}-\frac{2fg \log \left(c\left(d+ex^2\right)^p\right)}{x}+g^2 x \log \left(c\left(d+ex^2\right)^p\right) \\
&\quad +\frac{1}{3}\left(2ef^2p\right) \int \frac{1}{x^2\left(d+ex^2\right)} dx+(4efgp) \int \frac{1}{d+ex^2} dx-\left(2eg^2p\right) \int \frac{x^2}{d+ex^2} dx \\
&= -\frac{2ef^2p}{3dx}-2g^2px+\frac{4\sqrt{e}fgp \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{f^2 \log \left(c\left(d+ex^2\right)^p\right)}{3x^3} \\
&\quad -\frac{2fg \log \left(c\left(d+ex^2\right)^p\right)}{x}+g^2 x \log \left(c\left(d+ex^2\right)^p\right) \\
&\quad -\frac{\left(2e^2f^2p\right) \int \frac{1}{d+ex^2} dx}{3d}+\left(2dg^2p\right) \int \frac{1}{d+ex^2} dx \\
&= -\frac{2ef^2p}{3dx}-2g^2px-\frac{2e^{3/2}f^2p \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}+\frac{4\sqrt{e}fgp \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} \\
&\quad +\frac{2\sqrt{d}g^2p \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{f^2 \log \left(c\left(d+ex^2\right)^p\right)}{3x^3} \\
&\quad -\frac{2fg \log \left(c\left(d+ex^2\right)^p\right)}{x}+g^2 x \log \left(c\left(d+ex^2\right)^p\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.67

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^4}\,dx = -2g^2px + \frac{2g(2ef+dg)p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}} - \frac{2ef^2p\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{ex^2}{d}\right)}{3dx} - \frac{(f^2+6fgx^2-3g^2x^4)\log\left(c(d+ex^2)^p\right)}{3x^3}$$

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^4,x]
```

```
[Out] -2*g^2*p*x + (2*g*(2*e*f + d*g)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqr
t[e]) - (2*e*f^2*p*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) -
 ((f^2 + 6*f*g*x^2 - 3*g^2*x^4)*Log[c*(d + e*x^2)^p])/(3*x^3)
```

Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.74

method	result
parts	$g^2x\ln\left(c(e\,x^2+d)^p\right)-\frac{f^2\ln(c(e\,x^2+d)^p)}{3x^3}-\frac{2fg\ln(c(e\,x^2+d)^p)}{x}-\frac{2pe\,(\frac{3x\,g^2}{e}+\frac{(-3g^2d^2-6defg+e^2f^2)\arctan(\frac{xe}{\sqrt{de}})}{de\sqrt{de}}+\frac{f^2}{dx}}{3}$
risch	$-\frac{(-3g^2x^4+6fg\,x^2+f^2)\ln((e\,x^2+d)^p)}{3x^3}+\frac{6i\pi dfg\,x^2\,\mathrm{csgn}(i(e\,x^2+d)^p)\,\mathrm{csgn}(ic(e\,x^2+d)^p)\,\mathrm{csgn}(ic)-3i\pi d\,g^2x^4\,\mathrm{csgn}(i(e\,x^2+d)^p)\,\mathrm{csg}}{}$

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] g^2*x*ln(c*(e*x^2+d)^p)-1/3*f^2*ln(c*(e*x^2+d)^p)/x^3-2*f*g*ln(c*(e*x^2+d)^
p)/x-2/3*p*e*(3*x*g^2/e+1/d/e*(-3*d^2*g^2-6*d*e*f*g+e^2*f^2)/(d*e)^(1/2)*ar
ctan(x*e/(d*e)^(1/2))+f^2/d/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.07

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^4}\,dx$$
$$= \left[-\frac{6\,d^2eg^2px^4+2\,de^2f^2px^2-(e^2f^2-6\,defg-3\,d^2g^2)\sqrt{-de}px^3\log\left(\frac{ex^2-2\,\sqrt{-de}x-d}{ex^2+d}\right)-(3\,d^2eg^2px^4-6\,d}{3\,d^2ex^3}\right.$$
$$-\frac{6\,d^2eg^2px^4+2\,de^2f^2px^2+2\,(e^2f^2-6\,defg-3\,d^2g^2)\sqrt{de}px^3\arctan\left(\frac{\sqrt{de}x}{d}\right)-(3\,d^2eg^2px^4-6\,d^2efgp}{3\,d^2ex^3}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/3*(6*d^2*e*g^2*p*x^4 + 2*d*e^2*f^2*p*x^2 - (e^2*f^2 - 6*d*e*f*g - 3*d^2
*g^2)*sqrt(-d*e)*p*x^3*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - (3*d
^2*e*g^2*p*x^4 - 6*d^2*e*f*g*p*x^2 - d^2*e*f^2*p)*log(e*x^2 + d) - (3*d^2*e
*g^2*x^4 - 6*d^2*e*f*g*x^2 - d^2*e*f^2)*log(c))/(d^2*e*x^3), -1/3*(6*d^2*e*
g^2*p*x^4 + 2*d*e^2*f^2*p*x^2 + 2*(e^2*f^2 - 6*d*e*f*g - 3*d^2*g^2)*sqrt(d*
e)*p*x^3*arctan(sqrt(d*e)*x/d) - (3*d^2*e*g^2*p*x^4 - 6*d^2*e*f*g*p*x^2 - d
^2*e*f^2*p)*log(e*x^2 + d) - (3*d^2*e*g^2*x^4 - 6*d^2*e*f*g*x^2 - d^2*e*f^2
)*log(c))/(d^2*e*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(170) = 340$.

Time = 72.93 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.25

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^4}\,dx$$
$$= \begin{cases} \left(-\frac{f^2}{3x^3}-\frac{2fg}{x}+g^2x\right)\log\left(0^pc\right) \\ -\frac{2f^2p}{9x^3}-\frac{f^2\log\left(c\left(ex^2\right)^p\right)}{3x^3}-\frac{4fgp}{x}-\frac{2fg\log\left(c\left(ex^2\right)^p\right)}{x}-2g^2px+g^2x\log\left(c\left(ex^2\right)^p\right) \\ \left(-\frac{f^2}{3x^3}-\frac{2fg}{x}+g^2x\right)\log\left(cd^p\right) \\ \frac{2dg^2p\log\left(x-\sqrt{-\frac{d}{e}}\right)}{e\sqrt{-\frac{d}{e}}}-\frac{dg^2\log\left(c(d+ex^2)^p\right)}{e\sqrt{-\frac{d}{e}}}-\frac{f^2\log\left(c(d+ex^2)^p\right)}{3x^3}+\frac{4fgp\log\left(x-\sqrt{-\frac{d}{e}}\right)}{\sqrt{-\frac{d}{e}}}-\frac{2fg\log\left(c(d+ex^2)^p\right)}{\sqrt{-\frac{d}{e}}}-\frac{2fg\log\left(c(d+\right.}{x} \end{cases}$$

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**4,x)
```

```
[Out] Piecewise((((-f**2/(3*x**3) - 2*f*g/x + g**2*x)*log(0**p*c), Eq(d, 0) & Eq(e
, 0)), (-2*f**2*p/(9*x**3) - f**2*log(c*(e*x**2)**p)/(3*x**3) - 4*f*g*p/x -
 2*f*g*log(c*(e*x**2)**p)/x - 2*g**2*p*x + g**2*x*log(c*(e*x**2)**p), Eq(d,
 0)), ((-f**2/(3*x**3) - 2*f*g/x + g**2*x)*log(c*d**p), Eq(e, 0)), (2*d*g**
2*p*log(x - sqrt(-d/e))/(e*sqrt(-d/e)) - d*g**2*log(c*(d + e*x**2)**p)/(e*s
qrt(-d/e)) - f**2*log(c*(d + e*x**2)**p)/(3*x**3) + 4*f*g*p*log(x - sqrt(-d
/e))/sqrt(-d/e) - 2*f*g*log(c*(d + e*x**2)**p)/sqrt(-d/e) - 2*f*g*log(c*(d
+ e*x**2)**p)/x - 2*g**2*p*x + g**2*x*log(c*(d + e*x**2)**p) - 2*e*f**2*p*l
og(x - sqrt(-d/e))/(3*d*sqrt(-d/e)) - 2*e*f**2*p/(3*d*x) + e*f**2*log(c*(d
+ e*x**2)**p)/(3*d*sqrt(-d/e)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.80

$$\begin{aligned}\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x^4}\,dx =& -\left(2\,g^2p-g^2\log\left(c\right)\right)x \\ &+\frac{1}{3}\left(3\,g^2px-\frac{6\,fgpx^2+f^2p}{x^3}\right)\log\left(ex^2+d\right) \\ &-\frac{2\left(e^2f^2p-6\,defgp-3\,d^2g^2p\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{3\sqrt{de}d} \\ &-\frac{2\,ef^2px^2+6\,dfgx^2\log\left(c\right)+df^2\log\left(c\right)}{3\,dx^3}\end{aligned}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^4,x, algorithm="giac")
```

```
[Out] -(2*g^2*p - g^2*log(c))*x + 1/3*(3*g^2*p*x - (6*f*g*p*x^2 + f^2*p)/x^3)*log
(e*x^2 + d) - 2/3*(e^2*f^2*p - 6*d*e*f*g*p - 3*d^2*g^2*p)*arctan(e*x/sqrt(d
*e))/(sqrt(d*e)*d) - 1/3*(2*e*f^2*p*x^2 + 6*d*f*g*x^2*log(c) + d*f^2*log(c)
)/(d*x^3)
```

Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.64

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^4}\,dx = \ln\left(c\left(e\,x^2+d\right)^p\right)\,\left(\frac{8\,g^2\,x}{3}-\frac{\frac{f^2}{3}+2\,f\,g\,x^2+\frac{5\,g^2\,x^4}{3}}{x^3}\right) - 2\,g^2\,p\,x + \frac{2\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(3\,d^2\,g^2+6\,d\,e\,f\,g-e^2\,f^2\right)}{3\,d^{3/2}\,\sqrt{e}} - \frac{2\,e\,f^2\,p}{3\,d\,x}$$

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^4,x)
```

```
[Out] log(c*(d + e*x^2)^p)*((8*g^2*x)/3 - (f^2/3 + (5*g^2*x^4)/3 + 2*f*g*x^2)/x^3
) - 2*g^2*p*x + (2*p*atan((e^(1/2)*x)/d^(1/2))*(3*d^2*g^2 - e^2*f^2 + 6*d*e
*f*g))/(3*d^(3/2)*e^(1/2)) - (2*e*f^2*p)/(3*d*x)
```

3.336 $\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^6} dx$

Optimal result

Integrand size = 25, antiderivative size = 200

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^6} dx = -\frac{2ef^2p}{15dx^3}+\frac{2e^2f^2p}{5d^2x}-\frac{4efgp}{3dx} +\frac{2e^{5/2}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{4e^{3/2}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}} +\frac{2\sqrt{e}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{f^2\log\left(c(d+ex^2)^p\right)}{5x^5} -\frac{2fg\log\left(c(d+ex^2)^p\right)}{3x^3}-\frac{g^2\log\left(c(d+ex^2)^p\right)}{x}$$

```
[Out] -2/15*e*f^2*p/d/x^3+2/5*e^2*f^2*p/d^2/x-4/3*e*f*g*p/d/x+2/5*e^(5/2)*f^2*p*a
rctan(x*e^(1/2)/d^(1/2))/d^(5/2)-4/3*e^(3/2)*f*g*p*arctan(x*e^(1/2)/d^(1/2)
)/d^(3/2)-1/5*f^2*ln(c*(e*x^2+d)^p)/x^5-2/3*f*g*ln(c*(e*x^2+d)^p)/x^3-g^2*l
n(c*(e*x^2+d)^p)/x+2*g^2*p*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used

= {2526, 2505, 331, 211}

$$\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x^6}\,dx = \frac{2e^{5/2}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}} - \frac{4e^{3/2}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}} + \frac{2\sqrt{e}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{f^2\log\left(c(d+ex^2)^p\right)}{5x^5} - \frac{2fg\log\left(c(d+ex^2)^p\right)}{3x^3} - \frac{g^2\log\left(c(d+ex^2)^p\right)}{x} + \frac{2e^2f^2p}{5d^2x} - \frac{2ef^2p}{15dx^3} - \frac{4efgp}{3dx}$$

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^6,x]
```

```
[Out] (-2*e*f^2*p)/(15*d*x^3) + (2*e^2*f^2*p)/(5*d^2*x) - (4*e*f*g*p)/(3*d*x) + (
2*e^(5/2)*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*d^(5/2)) - (4*e^(3/2)*f*g*p
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*d^(3/2)) + (2*Sqrt[e]*g^2*p*ArcTan[(Sqrt[e
]*x)/Sqrt[d]])/Sqrt[d] - (f^2*Log[c*(d + e*x^2)^p])/(5*x^5) - (2*f*g*Log[c*
(d + e*x^2)^p])/(3*x^3) - (g^2*Log[c*(d + e*x^2)^p])/x
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
```

`& IntegerQ[s]`

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{f^2\log\left(c(d+ex^2)^p\right)}{x^6}+\frac{2fg\log\left(c(d+ex^2)^p\right)}{x^4}+\frac{g^2\log\left(c(d+ex^2)^p\right)}{x^2}\right)dx\\
&= f^2\int\frac{\log\left(c(d+ex^2)^p\right)}{x^6}\,dx+(2fg)\int\frac{\log\left(c(d+ex^2)^p\right)}{x^4}\,dx+g^2\int\frac{\log\left(c(d+ex^2)^p\right)}{x^2}\,dx\\
&= -\frac{f^2\log\left(c(d+ex^2)^p\right)}{5x^5}-\frac{2fg\log\left(c(d+ex^2)^p\right)}{3x^3}\\
&\quad-\frac{g^2\log\left(c(d+ex^2)^p\right)}{x}+\frac{1}{5}\left(2ef^2p\right)\int\frac{1}{x^4\left(d+ex^2\right)}\,dx\\
&\quad+\frac{1}{3}(4efgp)\int\frac{1}{x^2\left(d+ex^2\right)}\,dx+\left(2eg^2p\right)\int\frac{1}{d+ex^2}\,dx\\
&= -\frac{2ef^2p}{15dx^3}-\frac{4efgp}{3dx}+\frac{2\sqrt{e}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{f^2\log\left(c(d+ex^2)^p\right)}{5x^5}\\
&\quad-\frac{2fg\log\left(c(d+ex^2)^p\right)}{3x^3}-\frac{g^2\log\left(c(d+ex^2)^p\right)}{x}\\
&\quad-\frac{\left(2e^2f^2p\right)\int\frac{1}{x^2(d+ex^2)}\,dx}{5d}-\frac{\left(4e^2fgp\right)\int\frac{1}{d+ex^2}\,dx}{3d}\\
&= -\frac{2ef^2p}{15dx^3}+\frac{2e^2f^2p}{5d^2x}-\frac{4efgp}{3dx}-\frac{4e^{3/2}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}+\frac{2\sqrt{e}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}\\
&\quad-\frac{f^2\log\left(c(d+ex^2)^p\right)}{5x^5}-\frac{2fg\log\left(c(d+ex^2)^p\right)}{3x^3}-\frac{g^2\log\left(c(d+ex^2)^p\right)}{x}\\
&\quad+\frac{\left(2e^3f^2p\right)\int\frac{1}{d+ex^2}\,dx}{5d^2}\\
&= -\frac{2ef^2p}{15dx^3}+\frac{2e^2f^2p}{5d^2x}-\frac{4efgp}{3dx}+\frac{2e^{5/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}}\\
&\quad-\frac{4e^{3/2}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}+\frac{2\sqrt{e}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}}\\
&\quad-\frac{f^2\log\left(c(d+ex^2)^p\right)}{5x^5}-\frac{2fg\log\left(c(d+ex^2)^p\right)}{3x^3}-\frac{g^2\log\left(c(d+ex^2)^p\right)}{x}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.78

$$\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x^6}\,dx = \frac{2\sqrt{e}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{2ef^2p\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\frac{ex^2}{d}\right)}{15dx^3} - \frac{4efgp\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{ex^2}{d}\right)}{3dx} - \frac{f^2\log(c(d+ex^2)^p)}{5x^5} - \frac{2fg\log(c(d+ex^2)^p)}{3x^3} - \frac{g^2\log(c(d+ex^2)^p)}{x}$$

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^6,x]
```

```
[Out] (2*Sqrt[e]*g^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] - (2*e*f^2*p*Hypergeo
metric2F1[-3/2, 1, -1/2, -((e*x^2)/d)])/(15*d*x^3) - (4*e*f*g*p*Hypergeomet
ric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) - (f^2*Log[c*(d + e*x^2)^p])/(5
*x^5) - (2*f*g*Log[c*(d + e*x^2)^p])/(3*x^3) - (g^2*Log[c*(d + e*x^2)^p])/x
```

Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.67

method	result
parts	$-\frac{g^2\ln(c(ex^2+d)^p)}{x} - \frac{2fg\ln(c(ex^2+d)^p)}{3x^3} - \frac{f^2\ln(c(ex^2+d)^p)}{5x^5} - \frac{2pe\left(\frac{f^2}{dx^3}+\frac{f(10dg-3ef)}{d^2x}+\frac{(-15g^2d^2+10defg-3e^2f^2)\arctan(}{d^2\sqrt{de}}\right.}{15}$
risch	$-\frac{(15g^2x^4+10fgx^2+3f^2)\ln((ex^2+d)^p)}{15x^5} - \frac{-10i\pi d^3fgx^2\text{csgn}(ic(ex^2+d)^p)^3-15i\pi d^3g^2x^4\text{csgn}(ic(ex^2+d)^p)^3-3i\pi d^3f^2\,\text{csgn}(}{}$

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -g^2*ln(c*(e*x^2+d)^p)/x-2/3*f*g*ln(c*(e*x^2+d)^p)/x^3-1/5*f^2*ln(c*(e*x^2+
d)^p)/x^5-2/15*p*e*(f^2/d/x^3+f*(10*d*g-3*e*f)/d^2/x+1/d^2*(-15*d^2*g^2+10*
d*e*f*g-3*e^2*f^2)/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.76

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^6}\,dx$$
$$= \left[\frac{(3e^2f^2 - 10defg + 15d^2g^2)px^5\sqrt{-\frac{e}{d}}\log\left(\frac{ex^2+2dx\sqrt{-\frac{e}{d}}-d}{ex^2+d}\right) - 2def^2px^2 + 2(3e^2f^2 - 10defg)px^4 - (15}{15d^2x^5} \right.$$

[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^6,x, algorithm="fricas")

[Out] [1/15*((3*e^2*f^2 - 10*d*e*f*g + 15*d^2*g^2)*p*x^5*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)) - 2*d*e*f^2*p*x^2 + 2*(3*e^2*f^2 - 10*d*e*f*g)*p*x^4 - (15*d^2*g^2*p*x^4 + 10*d^2*f*g*p*x^2 + 3*d^2*f^2*p)*log(e*x^2 + d) - (15*d^2*g^2*x^4 + 10*d^2*f*g*x^2 + 3*d^2*f^2)*log(c))/(d^2*x^5), 1/15*(2*(3*e^2*f^2 - 10*d*e*f*g + 15*d^2*g^2)*p*x^5*sqrt(e/d)*arctan(x*sqrt(e/d)) - 2*d*e*f^2*p*x^2 + 2*(3*e^2*f^2 - 10*d*e*f*g)*p*x^4 - (15*d^2*g^2*p*x^4 + 10*d^2*f*g*p*x^2 + 3*d^2*f^2*p)*log(e*x^2 + d) - (15*d^2*g^2*x^4 + 10*d^2*f*g*x^2 + 3*d^2*f^2)*log(c))/(d^2*x^5)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1603 vs. $2(199) = 398$.

Time = 156.42 (sec) , antiderivative size = 1603, normalized size of antiderivative = 8.02

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^6}\,dx = \text{Too large to display}$$

[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**6,x)

[Out] Piecewise(((-f**2/(5*x**5) - 2*f*g/(3*x**3) - g**2/x)*log(0**p*c), Eq(d, 0) & Eq(e, 0)), ((-f**2/(5*x**5) - 2*f*g/(3*x**3) - g**2/x)*log(c*d**p), Eq(e, 0)), (-2*f**2*p/(25*x**5) - f**2*log(c*(e*x**2)**p)/(5*x**5) - 4*f*g*p/(9*x**3) - 2*f*g*log(c*(e*x**2)**p)/(3*x**3) - 2*g**2*p/x - g**2*log(c*(e*x**2)**p)/x, Eq(d, 0)), ((-f**2/(5*x**5) - 2*f*g/(3*x**3) - g**2/x)*log(0**p*c), Eq(d, -e*x**2)), (-3*d**3*f**2*sqrt(-d/e)*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e) + 15*d**2*e*x**7*sqrt(-d/e)) - 10*d**3*f*g*x**2*sqrt(-d/e)*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e) + 15*d**2*e*x**7*sqrt(-d/e)) + 30*d**3*g**2*p*x**5*log(x - sqrt(-d/e))/(15*d**3*x**5*sqrt(-d/e) + 15*d**2*e*x**7*sqrt(-d/e)) - 15*d**3*g**2*x**5*log(c*(d + e*x**2)**p)/(15*d**

```
3*x**5*sqrt(-d/e) + 15*d**2*e*x**7*sqrt(-d/e)) - 15*d**3*g**2*x**4*sqrt(-d/
e)*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e) + 15*d**2*e*x**7*sqrt(-d
/e)) - 2*d**2*f**2*p*x**2*sqrt(-d/e)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x
**7*sqrt(-d/e)) - 3*d**2*f**2*x**2*sqrt(-d/e)*log(c*(d + e*x**2)**p)/(15*d*
*3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 20*d**2*f*g*p*x**5*log(x
- sqrt(-d/e))/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 20*d*
*2*f*g*p*x**4*sqrt(-d/e)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/
e)) + 10*d**2*f*g*x**5*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e)/e +
15*d**2*x**7*sqrt(-d/e)) - 10*d**2*f*g*x**4*sqrt(-d/e)*log(c*(d + e*x**2)**
p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) + 30*d**2*g**2*p*x
**7*log(x - sqrt(-d/e))/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e
)) - 15*d**2*g**2*x**7*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e)/e +
15*d**2*x**7*sqrt(-d/e)) - 15*d**2*g**2*x**6*sqrt(-d/e)*log(c*(d + e*x**2)*
*p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) + 6*d*e*f**2*p*x*
*5*log(x - sqrt(-d/e))/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)
) + 4*d*e*f**2*p*x**4*sqrt(-d/e)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*
sqrt(-d/e)) - 3*d*e*f**2*x**5*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/
e)/e + 15*d**2*x**7*sqrt(-d/e)) - 20*d*e*f*g*p*x**7*log(x - sqrt(-d/e))/(15
*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 20*d*e*f*g*p*x**6*sqrt
(-d/e)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) + 10*d*e*f*g*x
**7*log(c*(d + e*x**2)**p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-
d/e)) + 6*e**2*f**2*p*x**7*log(x - sqrt(-d/e))/(15*d**3*x**5*sqrt(-d/e)/e +
 15*d**2*x**7*sqrt(-d/e)) + 6*e**2*f**2*p*x**6*sqrt(-d/e)/(15*d**3*x**5*sqr
t(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)) - 3*e**2*f**2*x**7*log(c*(d + e*x**2)*
*p)/(15*d**3*x**5*sqrt(-d/e)/e + 15*d**2*x**7*sqrt(-d/e)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f + gx^2)^2 \log \left(c(d + ex^2)^p\right)}{x^6} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.82

$$\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x^6}\,dx = \frac{2\,(3\,e^3f^2p-10\,de^2fgp+15\,d^2eg^2p)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{15\,\sqrt{de}d^2} - \frac{(15\,g^2px^4+10\,fgpx^2+3\,f^2p)\log(ex^2+d)}{15\,x^5} + \frac{6\,e^2f^2px^4-20\,defgpx^4-15\,d^2g^2x^4\log(c)-2\,def^2px^2-10\,d^2fgx^2\log(c)-3\,d^2f^2\log(c)}{15\,d^2x^5}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^6,x, algorithm="giac")
```

```
[Out] 2/15*(3*e^3*f^2*p - 10*d*e^2*f*g*p + 15*d^2*e*g^2*p)*arctan(e*x/sqrt(d*e))/
(sqrt(d*e)*d^2) - 1/15*(15*g^2*p*x^4 + 10*f*g*p*x^2 + 3*f^2*p)*log(e*x^2 +
d)/x^5 + 1/15*(6*e^2*f^2*p*x^4 - 20*d*e*f*g*p*x^4 - 15*d^2*g^2*x^4*log(c) -
 2*d*e*f^2*p*x^2 - 10*d^2*f*g*x^2*log(c) - 3*d^2*f^2*log(c))/(d^2*x^5)
```

Mupad [B] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.58

$$\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x^6}\,dx = \frac{2\,\sqrt{e}\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(15\,d^2\,g^2-10\,d\,e\,f\,g+3\,e^2\,f^2)}{15\,d^{5/2}} - \frac{\ln\left(c\,(e\,x^2+d)^p\right)\,\left(\frac{f^2}{5}+\frac{2\,f\,g\,x^2}{3}+g^2\,x^4\right)}{x^5} - \frac{\frac{2\,e\,f^2\,p}{d}+\frac{2\,e\,f\,p\,x^2\,(10\,d\,g-3\,e\,f)}{d^2}}{15\,x^3}$$

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^6,x)
```

```
[Out] (2*e^(1/2)*p*atan((e^(1/2)*x)/d^(1/2))*(15*d^2*g^2 + 3*e^2*f^2 - 10*d*e*f*g
))/(15*d^(5/2)) - (log(c*(d + e*x^2)^p)*(f^2/5 + g^2*x^4 + (2*f*g*x^2)/3))/
x^5 - ((2*e*f^2*p)/d + (2*e*f*p*x^2*(10*d*g - 3*e*f))/d^2)/(15*x^3)
```

3.337 $\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x^8}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 252

$$\int \frac{(f+gx^2)^2\log\left(c(d+ex^2)^p\right)}{x^8}\,dx = -\frac{2ef^2p}{35dx^5}+\frac{2e^2f^2p}{21d^2x^3}-\frac{4efgp}{15dx^3}-\frac{2e^3f^2p}{7d^3x}+\frac{4e^2fgp}{5d^2x}-\frac{2eg^2p}{3dx}-\frac{2e^{7/2}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7d^{7/2}}+\frac{4e^{5/2}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{2e^{3/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}-\frac{f^2\log\left(c(d+ex^2)^p\right)}{7x^7}-\frac{2fg\log\left(c(d+ex^2)^p\right)}{5x^5}-\frac{g^2\log\left(c(d+ex^2)^p\right)}{3x^3}$$

```
[Out] -2/35*e*f^2*p/d/x^5+2/21*e^2*f^2*p/d^2/x^3-4/15*e*f*g*p/d/x^3-2/7*e^3*f^2*p
/d^3/x+4/5*e^2*f*g*p/d^2/x-2/3*e*g^2*p/d/x-2/7*e^(7/2)*f^2*p*arctan(x*e^(1/
2)/d^(1/2))/d^(7/2)+4/5*e^(5/2)*f*g*p*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)-2/3
*e^(3/2)*g^2*p*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)-1/7*f^2*ln(c*(e*x^2+d)^p)/
x^7-2/5*f*g*ln(c*(e*x^2+d)^p)/x^5-1/3*g^2*ln(c*(e*x^2+d)^p)/x^3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used

$= \{2526, 2505, 331, 211\}$

$$\int \frac{(f+gx^2)^2 \log(c(d+ex^2)^p)}{x^8}\,dx = -\frac{2e^{7/2}f^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7d^{7/2}} + \frac{4e^{5/2}fgp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}} - \frac{2e^{3/2}g^2p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}} - \frac{f^2\log(c(d+ex^2)^p)}{7x^7} - \frac{2fg\log(c(d+ex^2)^p)}{5x^5} - \frac{g^2\log(c(d+ex^2)^p)}{3x^3} - \frac{2e^3f^2p}{7d^3x} + \frac{2e^2f^2p}{21d^2x^3} + \frac{4e^2fgp}{5d^2x} - \frac{2ef^2p}{35dx^5} - \frac{4efgp}{15dx^3} - \frac{2eg^2p}{3dx}$$

```
[In] Int[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^8,x]
```

```
[Out] (-2*e*f^2*p)/(35*d*x^5) + (2*e^2*f^2*p)/(21*d^2*x^3) - (4*e*f*g*p)/(15*d*x^
3) - (2*e^3*f^2*p)/(7*d^3*x) + (4*e^2*f*g*p)/(5*d^2*x) - (2*e*g^2*p)/(3*d*x
) - (2*e^(7/2)*f^2*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(7*d^(7/2)) + (4*e^(5/2)*
f*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(5*d^(5/2)) - (2*e^(3/2)*g^2*p*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(3*d^(3/2)) - (f^2*Log[c*(d + e*x^2)^p])/(7*x^7) - (2*f
*g*Log[c*(d + e*x^2)^p])/(5*x^5) - (g^2*Log[c*(d + e*x^2)^p])/(3*x^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
```

`& IntegerQ[s]`

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{f^2\log\left(c(d+ex^2)^p\right)}{x^8}+\frac{2fg\log\left(c(d+ex^2)^p\right)}{x^6}+\frac{g^2\log\left(c(d+ex^2)^p\right)}{x^4}\right)dx\\
&= f^2\int\frac{\log\left(c(d+ex^2)^p\right)}{x^8}\,dx+(2fg)\int\frac{\log\left(c(d+ex^2)^p\right)}{x^6}\,dx+g^2\int\frac{\log\left(c(d+ex^2)^p\right)}{x^4}\,dx\\
&= -\frac{f^2\log\left(c(d+ex^2)^p\right)}{7x^7}-\frac{2fg\log\left(c(d+ex^2)^p\right)}{5x^5}\\
&\quad-\frac{g^2\log\left(c(d+ex^2)^p\right)}{3x^3}+\frac{1}{7}\left(2ef^2p\right)\int\frac{1}{x^6\left(d+ex^2\right)}\,dx\\
&\quad+\frac{1}{5}(4efgp)\int\frac{1}{x^4\left(d+ex^2\right)}\,dx+\frac{1}{3}\left(2eg^2p\right)\int\frac{1}{x^2\left(d+ex^2\right)}\,dx\\
&= -\frac{2ef^2p}{35dx^5}-\frac{4efgp}{15dx^3}-\frac{2eg^2p}{3dx}-\frac{f^2\log\left(c(d+ex^2)^p\right)}{7x^7}-\frac{2fg\log\left(c(d+ex^2)^p\right)}{5x^5}\\
&\quad-\frac{g^2\log\left(c(d+ex^2)^p\right)}{3x^3}-\frac{\left(2e^2f^2p\right)\int\frac{1}{x^4(d+ex^2)}\,dx}{7d}\\
&\quad-\frac{\left(4e^2fgp\right)\int\frac{1}{x^2(d+ex^2)}\,dx}{5d}-\frac{\left(2e^2g^2p\right)\int\frac{1}{d+ex^2}\,dx}{3d}\\
&= -\frac{2ef^2p}{35dx^5}+\frac{2e^2f^2p}{21d^2x^3}-\frac{4efgp}{15dx^3}+\frac{4e^2fgp}{5d^2x}-\frac{2eg^2p}{3dx}-\frac{2e^{3/2}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}\\
&\quad-\frac{f^2\log\left(c(d+ex^2)^p\right)}{7x^7}-\frac{2fg\log\left(c(d+ex^2)^p\right)}{5x^5}-\frac{g^2\log\left(c(d+ex^2)^p\right)}{3x^3}\\
&\quad+\frac{\left(2e^3f^2p\right)\int\frac{1}{x^2(d+ex^2)}\,dx}{7d^2}+\frac{\left(4e^3fgp\right)\int\frac{1}{d+ex^2}\,dx}{5d^2}\\
&= -\frac{2ef^2p}{35dx^5}+\frac{2e^2f^2p}{21d^2x^3}-\frac{4efgp}{15dx^3}-\frac{2e^3f^2p}{7d^3x}+\frac{4e^2fgp}{5d^2x}-\frac{2eg^2p}{3dx}\\
&\quad+\frac{4e^{5/2}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{2e^{3/2}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}-\frac{f^2\log\left(c(d+ex^2)^p\right)}{7x^7}\\
&\quad-\frac{2fg\log\left(c(d+ex^2)^p\right)}{5x^5}-\frac{g^2\log\left(c(d+ex^2)^p\right)}{3x^3}-\frac{\left(2e^4f^2p\right)\int\frac{1}{d+ex^2}\,dx}{7d^3}\\
&= -\frac{2ef^2p}{35dx^5}+\frac{2e^2f^2p}{21d^2x^3}-\frac{4efgp}{15dx^3}-\frac{2e^3f^2p}{7d^3x}+\frac{4e^2fgp}{5d^2x}-\frac{2eg^2p}{3dx}\\
&\quad-\frac{2e^{7/2}f^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{7d^{7/2}}+\frac{4e^{5/2}fgp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{5d^{5/2}}-\frac{2e^{3/2}g^2p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}}\\
&\quad-\frac{f^2\log\left(c(d+ex^2)^p\right)}{7x^7}-\frac{2fg\log\left(c(d+ex^2)^p\right)}{5x^5}-\frac{g^2\log\left(c(d+ex^2)^p\right)}{3x^3}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.64

$$
\begin{aligned}
\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^8}\,dx = & -\frac{2ef^2p\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\frac{ex^2}{d}\right)}{35dx^5} \\
& -\frac{4efgp\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\frac{ex^2}{d}\right)}{15dx^3} \\
& -\frac{2eg^2p\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{ex^2}{d}\right)}{3dx} \\
& -\frac{f^2\log\left(c(d+ex^2)^p\right)}{7x^7} \\
& -\frac{2fg\log\left(c(d+ex^2)^p\right)}{5x^5}-\frac{g^2\log\left(c(d+ex^2)^p\right)}{3x^3}
\end{aligned}
$$

```
[In] Integrate[((f + g*x^2)^2*Log[c*(d + e*x^2)^p])/x^8,x]
```

```
[Out] (-2*e*f^2*p*Hypergeometric2F1[-5/2, 1, -3/2, -((e*x^2)/d)])/(35*d*x^5) - (4
*e*f*g*p*Hypergeometric2F1[-3/2, 1, -1/2, -((e*x^2)/d)])/(15*d*x^3) - (2*e*
g^2*p*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(3*d*x) - (f^2*Log[c*(
d + e*x^2)^p])/(7*x^7) - (2*f*g*Log[c*(d + e*x^2)^p])/(5*x^5) - (g^2*Log[c*
(d + e*x^2)^p])/(3*x^3)
```

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.66

method	result
parts	$-\frac{g^2\ln\left(c\left(ex^2+d\right)^p\right)}{3x^3}-\frac{2fg\ln\left(c\left(ex^2+d\right)^p\right)}{5x^5}-\frac{f^2\ln\left(c\left(ex^2+d\right)^p\right)}{7x^7}-\frac{2pe\left(-\frac{-35g^2d^2+42defg-15e^2f^2}{d^3x}+\frac{3f^2}{dx^5}+\frac{f(14dg-5ef)}{d^2x^3}+\frac{e(35g^2}{}\right.}{105}$
risch	$-\frac{\left(35g^2x^4+42fgx^2+15f^2\right)\ln\left(\left(ex^2+d\right)^p\right)}{105x^7}-\frac{30\ln(c)d^4f^2-70\sqrt{-de}\,pe\ln\left(-ex+\sqrt{-de}\right)g^2d^2x^7+70\sqrt{-de}\,pe\ln\left(-ex-\sqrt{-de}\right)g^2d^2x^7}{}$

```
[In] int((g*x^2+f)^2*ln(c*(e*x^2+d)^p)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*g^2*ln(c*(e*x^2+d)^p)/x^3-2/5*f*g*ln(c*(e*x^2+d)^p)/x^5-1/7*f^2*ln(c*(
e*x^2+d)^p)/x^7-2/105*p*e*(-1/d^3*(-35*d^2*g^2+42*d*e*f*g-15*e^2*f^2)/x+3*f
^2/d/x^5+f*(14*d*g-5*e*f)/d^2/x^3+e*(35*d^2*g^2-42*d*e*f*g+15*e^2*f^2)/d^3/
(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.70

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^8}\,dx$$

$$= \left[\frac{(15\,e^3f^2 - 42\,de^2fg + 35\,d^2eg^2)px^7\sqrt{-\frac{e}{d}}\log\left(\frac{ex^2-2\,dx\sqrt{-\frac{e}{d}}-d}{ex^2+d}\right) - 6\,d^2ef^2px^2 - 2\,(15\,e^3f^2 - 42\,de^2fg + 3\right.$$

$$-\frac{2\,(15\,e^3f^2 - 42\,de^2fg + 35\,d^2eg^2)px^7\sqrt{\frac{e}{d}}\arctan\left(x\sqrt{\frac{e}{d}}\right) + 6\,d^2ef^2px^2 + 2\,(15\,e^3f^2 - 42\,de^2fg + 35\,d^2e}{}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^8,x, algorithm="fricas")
```

```
[Out] [1/105*((15*e^3*f^2 - 42*d*e^2*f*g + 35*d^2*e*g^2)*p*x^7*sqrt(-e/d)*log((e*
x^2 - 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)) - 6*d^2*e*f^2*p*x^2 - 2*(15*e^3*f^
2 - 42*d*e^2*f*g + 35*d^2*e*g^2)*p*x^6 + 2*(5*d*e^2*f^2 - 14*d^2*e*f*g)*p*x
^4 - (35*d^3*g^2*p*x^4 + 42*d^3*f*g*p*x^2 + 15*d^3*f^2*p)*log(e*x^2 + d) -
(35*d^3*g^2*x^4 + 42*d^3*f*g*x^2 + 15*d^3*f^2)*log(c))/(d^3*x^7), -1/105*(2
*(15*e^3*f^2 - 42*d*e^2*f*g + 35*d^2*e*g^2)*p*x^7*sqrt(e/d)*arctan(x*sqrt(e
/d)) + 6*d^2*e*f^2*p*x^2 + 2*(15*e^3*f^2 - 42*d*e^2*f*g + 35*d^2*e*g^2)*p*x
^6 - 2*(5*d*e^2*f^2 - 14*d^2*e*f*g)*p*x^4 + (35*d^3*g^2*p*x^4 + 42*d^3*f*g*
p*x^2 + 15*d^3*f^2*p)*log(e*x^2 + d) + (35*d^3*g^2*x^4 + 42*d^3*f*g*x^2 + 1
5*d^3*f^2)*log(c))/(d^3*x^7)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx^2)^2 \log\left(c(d+ex^2)^p\right)}{x^8}\,dx = \text{Timed out}$$

```
[In] integrate((g*x**2+f)**2*ln(c*(e*x**2+d)**p)/x**8,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^8} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.82

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^8} \, dx = -\frac{2\,(15\,e^4 f^2 p - 42\,de^3 fgp + 35\,d^2 e^2 g^2 p) \arctan\left(\frac{ex}{\sqrt{de}}\right)}{105\,\sqrt{de}d^3} - \frac{(35\,g^2 p x^4 + 42\,fgpx^2 + 15\,f^2 p) \log\left(ex^2+d\right)}{105\,x^7} - \frac{30\,e^3 f^2 p x^6 - 84\,de^2 fgpx^6 + 70\,d^2 e g^2 p x^6 - 10\,de^2 f^2 p x^4 + 28\,d^2 efgpx^4 + 35\,d^3 g^2 x^4 \log\left(c\right) + 6\,d^2 e f^2 p x^2 +}{105\,d^3 x^7}$$

```
[In] integrate((g*x^2+f)^2*log(c*(e*x^2+d)^p)/x^8,x, algorithm="giac")
```

```
[Out] -2/105*(15*e^4*f^2*p - 42*d*e^3*f*g*p + 35*d^2*e^2*g^2*p)*arctan(e*x/sqrt(d
*e))/(sqrt(d*e)*d^3) - 1/105*(35*g^2*p*x^4 + 42*f*g*p*x^2 + 15*f^2*p)*log(e
*x^2 + d)/x^7 - 1/105*(30*e^3*f^2*p*x^6 - 84*d*e^2*f*g*p*x^6 + 70*d^2*e*g^2
*p*x^6 - 10*d*e^2*f^2*p*x^4 + 28*d^2*e*f*g*p*x^4 + 35*d^3*g^2*x^4*log(c) +
6*d^2*e*f^2*p*x^2 + 42*d^3*f*g*x^2*log(c) + 15*d^3*f^2*log(c))/(d^3*x^7)
```

Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.59

$$\int \frac{(f+gx^2)^2 \log \left(c(d+ex^2)^p\right)}{x^8} \, dx = -\frac{\frac{6\,e\,f^2\,p}{d} + \frac{2\,e\,p\,x^4\,\left(35\,d^2\,g^2 - 42\,d\,e\,f\,g + 15\,e^2\,f^2\right)}{d^3} + \frac{2\,e\,f\,p\,x^2\,\left(14\,d\,g - 5\,e\,f\right)}{d^2}}{105\,x^5} - \frac{\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\,\left(\frac{f^2}{7} + \frac{2\,f\,g\,x^2}{5} + \frac{g^2\,x^4}{3}\right)}{x^7} - \frac{2\,e^{3/2}\,p\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(35\,d^2\,g^2 - 42\,d\,e\,f\,g + 15\,e^2\,f^2\right)}{105\,d^{7/2}}$$

```
[In] int((log(c*(d + e*x^2)^p)*(f + g*x^2)^2)/x^8,x)
```

```
[Out] - ((6*e*f^2*p)/d + (2*e*p*x^4*(35*d^2*g^2 + 15*e^2*f^2 - 42*d*e*f*g))/d^3 +
 (2*e*f*p*x^2*(14*d*g - 5*e*f))/d^2)/(105*x^5) - (log(c*(d + e*x^2)^p)*(f^2
/7 + (g^2*x^4)/3 + (2*f*g*x^2)/5))/x^7 - (2*e^(3/2)*p*atan((e^(1/2)*x)/d^(1
/2))*(35*d^2*g^2 + 15*e^2*f^2 - 42*d*e*f*g))/(105*d^(7/2))
```

3.338 $\int \frac{x^5 \log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 188

$$\begin{aligned}\int \frac{x^5 \log\left(c(d+ex^2)^p\right)}{f+gx^2} \, dx = {} & \frac{fpx^2}{2g^2}+\frac{dpx^2}{4eg}-\frac{px^4}{8g}-\frac{d^2p\log\left(d+ex^2\right)}{4e^2g} \\ & +\frac{x^4\log\left(c(d+ex^2)^p\right)}{4g}-\frac{f(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2eg^2} \\ & +\frac{f^2\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^3} \\ & +\frac{f^2p\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2g^3}\end{aligned}$$

```
[Out] 1/2*f*p*x^2/g^2+1/4*d*p*x^2/e/g-1/8*p*x^4/g-1/4*d^2*p*ln(e*x^2+d)/e^2/g+1/4
*x^4*ln(c*(e*x^2+d)^p)/g-1/2*f*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e/g^2+1/2*f^2*ln
(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(-d*g+e*f))/g^3+1/2*f^2*p*polylog(2,-g*(e*x^
2+d)/(-d*g+e*f))/g^3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used

$= \{2525, 45, 2463, 2436, 2332, 2442, 2441, 2440, 2438\}$

$$\int \frac{x^5 \log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \frac{f^2 \log\left(c(d+ex^2)^p\right) \log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^3} - \frac{f(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2eg^2} + \frac{x^4\log\left(c(d+ex^2)^p\right)}{4g} - \frac{d^2p\log\left(d+ex^2\right)}{4e^2g} + \frac{f^2p\,\text{PolyLog}\left(2,-\frac{g(ex^2+d)}{ef-dg}\right)}{2g^3} + \frac{dpx^2}{4eg} + \frac{fpx^2}{2g^2} - \frac{px^4}{8g}$$

```
[In] Int[(x^5*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (f*p*x^2)/(2*g^2) + (d*p*x^2)/(4*e*g) - (p*x^4)/(8*g) - (d^2*p*Log[d + e*x^
2])/(4*e^2*g) + (x^4*Log[c*(d + e*x^2)^p])/(4*g) - (f*(d + e*x^2)*Log[c*(d
+ e*x^2)^p])/(2*e*g^2) + (f^2*Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f
 - d*g)])/(2*g^3) + (f^2*p*PolyLog[2, -((g*(d + e*x^2))/(e*f - d*g))])/(2*g
^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2 \log \left(c(d+ex)^p\right)}{f+gx}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{f \log \left(c(d+ex)^p\right)}{g^2}+\frac{x \log \left(c(d+ex)^p\right)}{g}+\frac{f^2 \log \left(c(d+ex)^p\right)}{g^2(f+gx)}\right)\,dx, x, x^2\right) \\
&= -\frac{f\text{Subst}\left(\int \log \left(c(d+ex)^p\right)\,dx, x, x^2\right)}{2g^2}+\frac{f^2\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx, x, x^2\right)}{2g^2} \\
&\quad +\frac{\text{Subst}\left(\int x \log \left(c(d+ex)^p\right)\,dx, x, x^2\right)}{2g}
\end{aligned}$$

$$= \frac{x^4 \log\left(c(d+ex^2)^p\right)}{4g} + \frac{f^2 \log\left(c(d+ex^2)^p\right) \log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^3} - \frac{f\text{Subst}\left(\int \log\left(cx^p\right)\, dx, x, d+ex^2\right)}{2eg^2} - \frac{(ef^2p)\,\text{Subst}\left(\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\, dx, x, x^2\right)}{2g^3} - \frac{(ep)\text{Subst}\left(\int \frac{x^2}{d+ex}\, dx, x, x^2\right)}{4g}$$

$$= \frac{fpx^2}{2g^2} + \frac{x^4 \log\left(c(d+ex^2)^p\right)}{4g} - \frac{f(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2eg^2} + \frac{f^2 \log\left(c(d+ex^2)^p\right) \log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^3} - \frac{(f^2p)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\, dx, x, d+ex^2\right)}{2g^3} - \frac{(ep)\text{Subst}\left(\int \left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)\, dx, x, x^2\right)}{4g}$$

$$= \frac{fpx^2}{2g^2} + \frac{dpx^2}{4eg} - \frac{px^4}{8g} - \frac{d^2p\log\left(d+ex^2\right)}{4e^2g} + \frac{x^4 \log\left(c(d+ex^2)^p\right)}{4g} - \frac{f(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2eg^2} + \frac{f^2 \log\left(c(d+ex^2)^p\right) \log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^3} + \frac{f^2p\text{Li}_2\left(-\frac{g(d+ex^2)}{ef-dg}\right)}{2g^3}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.76

$$\int \frac{x^5 \log\left(c(d+ex^2)^p\right)}{f+gx^2}\, dx = \frac{egpx^2(4ef+2dg-egx^2) - 2d^2g^2p\log\left(d+ex^2\right) + e\log\left(c(d+ex^2)^p\right)\left(2g(-2df-2efx^2+egx^4)+4ef^2\right.}{8e^2g^3}$$

```
[In] Integrate[(x^5*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (e*g*p*x^2*(4*e*f + 2*d*g - e*g*x^2) - 2*d^2*g^2*p*Log[d + e*x^2] + e*Log[c
*(d + e*x^2)^p]*(2*g*(-2*d*f - 2*e*f*x^2 + e*g*x^4) + 4*e*f^2*Log[(e*(f + g
*x^2))/(e*f - d*g)]) + 4*e^2*f^2*p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g
)])/(8*e^2*g^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.48 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.18

method	result
parts	$\frac{x^{4} \ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{4g}-\frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right) f\,x^{2}}{2g^{2}}+\frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right) f^{2} \ln\left(g\,x^{2}+f\right)}{2g^{3}}-pe\left(\frac{f^{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(e\_Z^{2}+d\right)}\left(\ln(x-\_\alpha) \ln\left(g\,x^{2}\right.\right.\right.}{}\right.$
risch	$\frac{\ln\left(\left(e\,x^{2}+d\right)^{p}\right) x^{4}}{4g}-\frac{\ln\left(\left(e\,x^{2}+d\right)^{p}\right) f\,x^{2}}{2g^{2}}+\frac{\ln\left(\left(e\,x^{2}+d\right)^{p}\right) f^{2} \ln\left(g\,x^{2}+f\right)}{2g^{3}}-\frac{p\,f^{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(e\_Z^{2}+d\right)}\left(\ln(x-\_\alpha) \ln\left(g\,x^{2}+f\right)-\ln\right.\right.}{}$

```
[In] int(x^5*ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*ln(c*(e*x^2+d)^p)/g-1/2*ln(c*(e*x^2+d)^p)/g^2*f*x^2+1/2*ln(c*(e*x^2
+d)^p)*f^2/g^3*ln(g*x^2+f)-p*e*(1/2*f^2/g^3/e*sum(ln(x-_alpha)*ln(g*x^2+f)-
ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha
)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_
Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g
+e*f,index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_
alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2
*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha
*e*g-d*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d))-1/2/g^2*(-1/2/e^2*(1/2*e*g*
x^4-d*g*x^2-2*f*e*x^2)-1/2*d*(d*g+2*e*f)/e^3*ln(e*x^2+d)))
```

Fricas [F]

$$\int \frac{x^5 \log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx=\int \frac{x^5 \log\left(\left(ex^2+d\right)^p c\right)}{gx^2+f}\,dx$$

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(x^5*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**5*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^5 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x^5 \log \left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
[Out] integrate(x^5*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Giac [F]

$$\int \frac{x^5 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x^5 \log \left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
[Out] integrate(x^5*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x^5\,\ln \left(c\,{\left(e\,x^2+d\right)}^p\right)}{g\,x^2+f}\,dx$$

```
[In] int((x^5*log(c*(d + e*x^2)^p))/(f + g*x^2),x)
[Out] int((x^5*log(c*(d + e*x^2)^p))/(f + g*x^2), x)
```

3.339 $\int \frac{x^3 \log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 112

$$\int \frac{x^3 \log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = -\frac{px^2}{2g} + \frac{(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2eg} - \frac{f\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^2} - \frac{fp\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2g^2}$$

```
[Out] -1/2*p*x^2/g+1/2*(e*x^2+d)*ln(c*(e*x^2+d)^p)/e/g-1/2*f*ln(c*(e*x^2+d)^p)*ln
(e*(g*x^2+f)/(-d*g+e*f))/g^2-1/2*f*p*polylog(2,-g*(e*x^2+d)/(-d*g+e*f))/g^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2525, 45, 2463, 2436, 2332, 2441, 2440, 2438}

$$\int \frac{x^3 \log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = -\frac{f\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^2} + \frac{(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2eg} - \frac{fp\,\mathrm{PolyLog}\left(2,-\frac{g(ex^2+d)}{ef-dg}\right)}{2g^2} - \frac{px^2}{2g}$$

```
[In] Int[(x^3*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] -1/2*(p*x^2)/g + ((d + e*x^2)*Log[c*(d + e*x^2)^p])/(2*e*g) - (f*Log[c*(d +
 e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/(2*g^2) - (f*p*PolyLog[2, -((g
*(d + e*x^2))/(e*f - d*g))])/(2*g^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x\log\left(c(d+ex)^p\right)}{f+gx}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{\log\left(c(d+ex)^p\right)}{g}-\frac{f\log\left(c(d+ex)^p\right)}{g(f+gx)}\right)\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \log\left(c(d+ex)^p\right)\,dx, x, x^2\right)}{2g}-\frac{f\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx, x, x^2\right)}{2g}\\
&= -\frac{f\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^2}+\frac{\text{Subst}\left(\int \log\left(cx^p\right)\,dx, x, d+ex^2\right)}{2eg}\\
&\quad+\frac{(efp)\text{Subst}\left(\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx, x, x^2\right)}{2g^2}\\
&= -\frac{px^2}{2g}+\frac{(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2eg}-\frac{f\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^2}\\
&\quad+\frac{(fp)\text{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex^2\right)}{2g^2}\\
&= -\frac{px^2}{2g}+\frac{(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2eg}-\frac{f\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^2}-\frac{fp\text{Li}_2\left(-\frac{g(d+ex^2)}{ef-dg}\right)}{2g^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81

$$\int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$
$$= -\frac{egpx^2 - \log\left(c(d+ex^2)^p\right)\left(dg+egx^2-ef\log\left(\frac{e(f+gx^2)}{ef-dg}\right)\right)+efp\,\mathrm{PolyLog}\left(2,\frac{g(d+ex^2)}{-ef+dg}\right)}{2eg^2}$$

```
[In] Integrate[(x^3*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] -1/2*(e*g*p*x^2 - Log[c*(d + e*x^2)^p]*(d*g + e*g*x^2 - e*f*Log[(e*(f + g*x
^2))/(e*f - d*g)]) + e*f*p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)])/(e*g
^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.50 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.20

method	result
parts	$\frac{\ln(c(e\,x^2+d)^p)x^2}{2g} - \frac{\ln(c(e\,x^2+d)^p)f\ln(g\,x^2+f)}{2g^2} - pe\left(\frac{x^2}{2ge} - \frac{d\ln(e\,x^2+d)}{2g\,e^2} - \frac{f\left(\sum_{\_\alpha=\mathrm{RootOf}(e\_Z^2+d)}\left(\ln(x-\_\alpha)\ln(g\,x\right.\right.}{}\right.$
risch	$\frac{\ln((e\,x^2+d)^p)x^2}{2g} - \frac{\ln((e\,x^2+d)^p)f\ln(g\,x^2+f)}{2g^2} - \frac{p\,x^2}{2g} + \frac{pd\ln(e\,x^2+d)}{2eg} + \frac{pf\left(\sum_{\_\alpha=\mathrm{RootOf}(e\_Z^2+d)}\left(\ln(x-\_\alpha)\ln(g\,x^2+f)\right.\right.}{}$

```
[In] int(x^3*ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(c*(e*x^2+d)^p)*x^2/g-1/2*ln(c*(e*x^2+d)^p)*f/g^2*ln(g*x^2+f)-p*e*(1/
2/g/e*x^2-1/2/g*d/e^2*ln(e*x^2+d)-1/2*f/g^2/e*sum(ln(x-_alpha)*ln(g*x^2+f)-
ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha
)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_
Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g
+e*f,index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_
alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2
*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha
*e*g-d*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d)))
```

Fricas [F]

$$\int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{f+gx^2} \, dx = \int \frac{x^3 \log \left((ex^2+d)^p c\right)}{gx^2+f} \, dx$$

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
[Out] integral(x^3*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F]

$$\int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{f+gx^2} \, dx = \int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{f+gx^2} \, dx$$

```
[In] integrate(x**3*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
[Out] Integral(x**3*log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

Maxima [F]

$$\int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{f+gx^2} \, dx = \int \frac{x^3 \log \left((ex^2+d)^p c\right)}{gx^2+f} \, dx$$

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
[Out] integrate(x^3*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Giac [F]

$$\int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{f+gx^2} \, dx = \int \frac{x^3 \log \left((ex^2+d)^p c\right)}{gx^2+f} \, dx$$

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
[Out] integrate(x^3*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x^3\,\ln\left(c\,(e\,x^2+d)^p\right)}{g\,x^2+f}\,dx$$

```
[In] int((x^3*log(c*(d + e*x^2)^p))/(f + g*x^2),x)
[Out] int((x^3*log(c*(d + e*x^2)^p))/(f + g*x^2), x)
```

3.340 $\int \frac{x \log\left(c(d+ex^2)^p\right)}{f+gx^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{x \log\left(c(d+ex^2)^p\right)}{f+gx^2} dx = \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g} + \frac{p\,\text{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2g}$$

```
[Out] 1/2*ln(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(-d*g+e*f))/g+1/2*p*polylog(2,-g*(e*x^
2+d)/(-d*g+e*f))/g
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2525, 2441, 2440, 2438}

$$\int \frac{x \log\left(c(d+ex^2)^p\right)}{f+gx^2} dx = \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g} + \frac{p\,\text{PolyLog}\left(2,-\frac{g(ex^2+d)}{ef-dg}\right)}{2g}$$

```
[In] Int[(x*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/(2*g) + (p*PolyLog[
2, -((g*(d + e*x^2))/(e*f - d*g))])/(2*g)
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{f+gx}\,dx, x, x^2\right)\\
&= \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g} - \frac{(ep)\text{Subst}\left(\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx, x, x^2\right)}{2g}\\
&= \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g} - \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex^2\right)}{2g}\\
&= \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g} + \frac{p\text{Li}_2\left(-\frac{g(d+ex^2)}{ef-dg}\right)}{2g}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.91

$$\int \frac{x \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \frac{\log \left(c(d+ex^2)^p\right) \log \left(\frac{e(f+gx^2)}{ef-dg}\right)+p\,\mathrm{PolyLog}\left(2,\frac{g(d+ex^2)}{-ef+dg}\right)}{2g}$$

```
[In] Integrate[(x*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)] + p*PolyLog[2, (g*(d
 + e*x^2))/(-(e*f) + d*g)])/(2*g)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.74 (sec) , antiderivative size = 301, normalized size of antiderivative = 4.30

method	result
parts	$\frac{\ln(c(e\,x^2+d)^p)\ln(g\,x^2+f)}{2g} - \frac{p\Big(\sum_{\_\alpha=\mathrm{RootOf}(e\_Z^2+d)}\Big(\ln(x-\_\alpha)\ln(g\,x^2+f)-\ln(x-\_\alpha)\Big(\ln\Big(\frac{\mathrm{RootOf}(e\_Z^2 g+2\_\alpha\_Z g e-dg+e}{\mathrm{RootOf}(e\_Z^2 g+2\_\alpha\_Z g e-c}}{}$
risch	$\frac{\ln((e\,x^2+d)^p)\ln(g\,x^2+f)}{2g} - \frac{p\Big(\sum_{\_\alpha=\mathrm{RootOf}(e\_Z^2+d)}\Big(\ln(x-\_\alpha)\ln(g\,x^2+f)-\ln(x-\_\alpha)\Big(\ln\Big(\frac{\mathrm{RootOf}(e\_Z^2 g+2\_\alpha\_Z g e-dg+e}{\mathrm{RootOf}(e\_Z^2 g+2\_\alpha\_Z g e-d}}{}$

```
[In] int(x*ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(c*(e*x^2+d)^p)/g*ln(g*x^2+f)-1/2/g*p*sum(ln(x-_alpha)*ln(g*x^2+f)-ln
(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/
RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*
_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e
*f,index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_al
pha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e
*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e
*g-d*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d))
```

Fricas [F]

$$\int \frac{x \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x \log \left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")

[Out] integral(x*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)

Sympy [F]

$$\int \frac{x \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

[In] integrate(x*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)

[Out] Integral(x*log(c*(d + e*x**2)**p)/(f + g*x**2), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(65) = 130$.

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.97

$$\int \frac{x \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \frac{ep\left(\frac{\log(ex^2+d)\log(gx^2+f)}{e} - \frac{\log(gx^2+f)\log\left(-\frac{egx^2+ef}{ef-dg}+1\right)+\mathrm{Li}_2\left(\frac{egx^2+ef}{ef-dg}\right)}{e}\right)}{2\,g} - \frac{p\log(ex^2+d)\log(gx^2+f)}{2\,g} + \frac{\log(gx^2+f)\log\left((ex^2+d)^p c\right)}{2\,g}$$

[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")

[Out] 1/2*e*p*(log(e*x^2 + d)*log(g*x^2 + f)/e - (log(g*x^2 + f)*log(-(e*g*x^2 + e*f)/(e*f - d*g) + 1) + dilog((e*g*x^2 + e*f)/(e*f - d*g)))/e)/g - 1/2*p*log(e*x^2 + d)*log(g*x^2 + f)/g + 1/2*log(g*x^2 + f)*log((e*x^2 + d)^p*c)/g

Giac [F]

$$\int \frac{x \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x \log \left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

```
[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
[Out] integrate(x*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x\ \ln\left(c\,{\left(e\,x^2+d\right)}^p\right)}{g\,x^2+f}\,dx$$

```
[In] int((x*log(c*(d + e*x^2)^p))/(f + g*x^2),x)
[Out] int((x*log(c*(d + e*x^2)^p))/(f + g*x^2), x)
```

3.341 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x\left(f+gx^2\right)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)}\,dx=\frac{\log \left(-\frac{ex^2}{d}\right)\log \left(c(d+ex^2)^p\right)}{2f}-\frac{\log \left(c(d+ex^2)^p\right)\log \left(\frac{e(f+gx^2)}{ef-dg}\right)}{2f}-\frac{p\,\text{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2f}+\frac{p\,\text{PolyLog}\left(2,1+\frac{ex^2}{d}\right)}{2f}$$

```
[Out] 1/2*ln(-e*x^2/d)*ln(c*(e*x^2+d)^p)/f-1/2*ln(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(
-d*g+e*f))/f-1/2*p*polylog(2,-g*(e*x^2+d)/(-d*g+e*f))/f+1/2*p*polylog(2,1+e
*x^2/d)/f
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {2525, 36, 29, 31, 2463, 2441, 2352, 2440, 2438}

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)}\,dx=-\frac{\log \left(c(d+ex^2)^p\right)\log \left(\frac{e(f+gx^2)}{ef-dg}\right)}{2f}+\frac{\log \left(-\frac{ex^2}{d}\right)\log \left(c(d+ex^2)^p\right)}{2f}-\frac{p\,\text{PolyLog}\left(2,-\frac{g(ex^2+d)}{ef-dg}\right)}{2f}+\frac{p\,\text{PolyLog}\left(2,\frac{ex^2}{d}+1\right)}{2f}$$

```
[In] Int[Log[c*(d + e*x^2)^p]/(x*(f + g*x^2)),x]
```

```
[Out] (Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/(2*f) - (Log[c*(d + e*x^2)^p]*Log[
(e*(f + g*x^2))/(e*f - d*g)])/(2*f) - (p*PolyLog[2, -((g*(d + e*x^2))/(e*f
- d*g))])/(2*f) + (p*PolyLog[2, 1 + (e*x^2)/d])/(2*f)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x(f+gx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{\log\left(c(d+ex)^p\right)}{fx} - \frac{g\log\left(c(d+ex)^p\right)}{f(f+gx)}\right)\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^2\right)}{2f} - \frac{g\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx, x, x^2\right)}{2f}\\
&= \frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{2f} - \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2f}\\
&\quad - \frac{(ep)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^2\right)}{2f} + \frac{(ep)\text{Subst}\left(\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx, x, x^2\right)}{2f}\\
&= \frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{2f} - \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2f}\\
&\quad + \frac{p\text{Li}_2\left(1+\frac{ex^2}{d}\right)}{2f} + \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex^2\right)}{2f}\\
&= \frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{2f} - \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2f}\\
&\quad - \frac{p\text{Li}_2\left(-\frac{g(d+ex^2)}{ef-dg}\right)}{2f} + \frac{p\text{Li}_2\left(1+\frac{ex^2}{d}\right)}{2f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.77

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x \left(f+gx^2\right)} \, dx = \frac{\log \left(c(d+ex^2)^p\right) \left(\log \left(-\frac{ex^2}{d}\right)-\log \left(\frac{e(f+gx^2)}{ef-dg}\right)\right)-p \operatorname{PolyLog}\left(2,\frac{g(d+ex^2)}{-ef+dg}\right)+p \operatorname{PolyLog}\left(2,1+\frac{ex^2}{d}\right)}{2f}$$

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x*(f + g*x^2)),x]
```

```
[Out] (Log[c*(d + e*x^2)^p]*(Log[-((e*x^2)/d)] - Log[(e*(f + g*x^2))/(e*f - d*g)]
) - p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)] + p*PolyLog[2, 1 + (e*x^2)
/d])/(2*f)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.23 (sec) , antiderivative size = 420, normalized size of antiderivative = 3.53

method	result
parts	$\frac{\ln\left(c\left(e\,x^2+d\right)^p\right)\ln(x)}{f}-\frac{\ln\left(c\left(e\,x^2+d\right)^p\right)\ln\left(g\,x^2+f\right)}{2f}-ep\left(\frac{\frac{\ln(x)\left(\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)+\ln\left(\frac{ex+\sqrt{-de}}{\sqrt{-de}}\right)\right)}{e}+\frac{\operatorname{dilog}\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)+\operatorname{dilog}\left(\frac{ex\cdots}{\cdots}\right)}{e}\cdots}{f}\cdots\right.$
risch	$\frac{\ln\left(\left(e\,x^2+d\right)^p\right)\ln(x)}{f}-\frac{\ln\left(\left(e\,x^2+d\right)^p\right)\ln\left(g\,x^2+f\right)}{2f}-\frac{p\ln(x)\ln\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)}{f}-\frac{p\ln(x)\ln\left(\frac{ex+\sqrt{-de}}{\sqrt{-de}}\right)}{f}-\frac{p\operatorname{dilog}\left(\frac{-ex+\sqrt{-de}}{\sqrt{-de}}\right)}{f}-\cdots$

```
[In] int(ln(c*(e*x^2+d)^p)/x/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] ln(c*(e*x^2+d)^p)/f*ln(x)-1/2*ln(c*(e*x^2+d)^p)/f*ln(g*x^2+f)-e*p*(2/f*(1/2
*ln(x)*(ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+ln((e*x+(-d*e)^(1/2))/(-d*e)^(
1/2)))/e+1/2*(dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+dilog((e*x+(-d*e)^(1/
2))/(-d*e)^(1/2)))/e)-1/2/f/e*sum(ln(x-_alpha)*ln(g*x^2+f)-ln(x-_alpha)*(ln
((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*
g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g
+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)))-
dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z
^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha
*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,ind
ex=2)),_alpha=RootOf(_Z^2*e+d)))
```

Fricas [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)}\,dx = \int \frac{\log \left((ex^2+d)^p c\right)}{(gx^2+f)x}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log((e*x^2 + d)^p*c)/(g*x^3 + f*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(e*x**2+d)**p)/x/(g*x**2+f),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.18

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)}\,dx = -\frac{1}{2}\,ep\left(\frac{2\,\log\left(\frac{ex^2}{d}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{ex^2}{d}\right)}{ef}-\frac{\log\left(gx^2+f\right)\log\left(-\frac{egx^2+ef}{ef-dg}+1\right)+\mathrm{Li}_2\left(\frac{egx^2+ef}{ef-dg}\right)}{ef}\right) - \frac{1}{2}\left(\frac{\log\left(gx^2+f\right)}{f}-\frac{\log\left(x^2\right)}{f}\right)\log\left(\left(ex^2+d\right)^p c\right)$$

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f),x, algorithm="maxima")
[Out] -1/2*e*p*((2*log(e*x^2/d + 1)*log(x) + dilog(-e*x^2/d))/(e*f) - (log(g*x^2
+ f)*log(-(e*g*x^2 + e*f)/(e*f - d*g) + 1) + dilog((e*g*x^2 + e*f)/(e*f - d
*g)))/(e*f)) - 1/2*(log(g*x^2 + f)/f - log(x^2)/f)*log((e*x^2 + d)^p*c)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x\,(f+gx^2)}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{(gx^2+f)x}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x\,(f+gx^2)}\,dx = \int \frac{\ln\left(c\,(e\,x^2+d)^p\right)}{x\,\left(g\,x^2+f\right)}\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)/(x*(f + g*x^2)),x)
[Out] int(log(c*(d + e*x^2)^p)/(x*(f + g*x^2)), x)
```

3.342 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^3\left(f+gx^2\right)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 176

$$\begin{aligned}\int \frac{\log\left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)}\,dx &= \frac{ep\log(x)}{df} - \frac{ep\log\left(d+ex^2\right)}{2df} - \frac{\log\left(c(d+ex^2)^p\right)}{2fx^2} \\ &\quad - \frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{2f^2} \\ &\quad + \frac{g\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2f^2} \\ &\quad + \frac{gp\operatorname{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2f^2} - \frac{gp\operatorname{PolyLog}\left(2,1+\frac{ex^2}{d}\right)}{2f^2}\end{aligned}$$

```
[Out] e*p*ln(x)/d/f-1/2*e*p*ln(e*x^2+d)/d/f-1/2*ln(c*(e*x^2+d)^p)/f/x^2-1/2*g*ln(
-e*x^2/d)*ln(c*(e*x^2+d)^p)/f^2+1/2*g*ln(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(-d*
g+e*f))/f^2+1/2*g*p*polylog(2,-g*(e*x^2+d)/(-d*g+e*f))/f^2-1/2*g*p*polylog(
2,1+e*x^2/d)/f^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules

used = {2525, 46, 2463, 2442, 36, 29, 31, 2441, 2352, 2440, 2438}

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)}\,dx = -\frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{2f^2} + \frac{g\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2f^2} - \frac{\log\left(c(d+ex^2)^p\right)}{2fx^2} + \frac{gp\,\mathrm{PolyLog}\left(2,-\frac{g(ex^2+d)}{ef-dg}\right)}{2f^2} - \frac{gp\,\mathrm{PolyLog}\left(2,\frac{ex^2}{d}+1\right)}{2f^2} - \frac{ep\log\left(d+ex^2\right)}{2df} + \frac{ep\log(x)}{df}$$

```
[In] Int[Log[c*(d + e*x^2)^p]/(x^3*(f + g*x^2)),x]
```

```
[Out] (e*p*Log[x])/(d*f) - (e*p*Log[d + e*x^2])/(2*d*f) - Log[c*(d + e*x^2)^p]/(2
*f*x^2) - (g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/(2*f^2) + (g*Log[c*(d
+ e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/(2*f^2) + (g*p*PolyLog[2, -((
g*(d + e*x^2))/(e*f - d*g))])/(2*f^2) - (g*p*PolyLog[2, 1 + (e*x^2)/d])/(2*
f^2)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x^2(f+gx)}\,dx, x, x^2\right)$$

$$=\frac{1}{2}\text{Subst}\left(\int\left(\frac{\log\left(c(d+ex)^p\right)}{fx^2}-\frac{g\log\left(c(d+ex)^p\right)}{f^2x}+\frac{g^2\log\left(c(d+ex)^p\right)}{f^2(f+gx)}\right)\,dx,x,x^2\right)$$

$$=\frac{\text{Subst}\left(\int\frac{\log(c(d+ex)^p)}{x^2}\,dx,x,x^2\right)}{2f}-\frac{g\text{Subst}\left(\int\frac{\log(c(d+ex)^p)}{x}\,dx,x,x^2\right)}{2f^2}+\frac{g^2\text{Subst}\left(\int\frac{\log(c(d+ex)^p)}{f+gx}\,dx,x,x^2\right)}{2f^2}$$

$$=-\frac{\log\left(c(d+ex^2)^p\right)}{2fx^2}-\frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{2f^2}+\frac{g\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2f^2}+\frac{(ep)\text{Subst}\left(\int\frac{1}{x(d+ex)}\,dx,x,x^2\right)}{2f}+\frac{(egp)\text{Subst}\left(\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx,x,x^2\right)}{2f^2}-\frac{(egp)\text{Subst}\left(\int\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx,x,x^2\right)}{2f^2}$$

$$=-\frac{\log\left(c(d+ex^2)^p\right)}{2fx^2}-\frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{2f^2}+\frac{g\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2f^2}-\frac{gp\text{Li}_2\left(1+\frac{ex^2}{d}\right)}{2f^2}+\frac{(ep)\text{Subst}\left(\int\frac{1}{x}\,dx,x,x^2\right)}{2df}-\frac{(e^2p)\,\text{Subst}\left(\int\frac{1}{d+ex}\,dx,x,x^2\right)}{2df}-\frac{(gp)\text{Subst}\left(\int\frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx,x,d+ex^2\right)}{2f^2}$$

$$=\frac{ep\log(x)}{df}-\frac{ep\log\left(d+ex^2\right)}{2df}-\frac{\log\left(c(d+ex^2)^p\right)}{2fx^2}-\frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{2f^2}+\frac{g\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2f^2}+\frac{gp\text{Li}_2\left(-\frac{g(d+ex^2)}{ef-dg}\right)}{2f^2}-\frac{gp\text{Li}_2\left(1+\frac{ex^2}{d}\right)}{2f^2}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.95

$$\int\frac{\log\left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)}\,dx=\frac{ep\log(x)}{df}-\frac{ep\log\left(d+ex^2\right)}{2df}-\frac{\log\left(c(d+ex^2)^p\right)}{2fx^2}-\frac{g\left(\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)+p\,\text{PolyLog}\left(2,\frac{d+ex^2}{d}\right)\right)}{2f^2}+\frac{g\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)+gp\,\text{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2f^2}$$

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x^3*(f + g*x^2)),x]
```

```
[Out] (e*p*Log[x])/(d*f) - (e*p*Log[d + e*x^2])/(2*d*f) - Log[c*(d + e*x^2)^p]/(2
*f*x^2) - (g*(Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + p*PolyLog[2, (d + e*
x^2)/d]))/(2*f^2) + (g*Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)
] + g*p*PolyLog[2, -((g*(d + e*x^2))/(e*f - d*g))])/(2*f^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.14 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.68

method	result
parts	$-\frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{2 f\,x^{2}}-\frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right) g \ln \left(x \right)}{f^{2}}+\frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right) g \ln \left(g\,x^{2}+f \right)}{2 f^{2}}-pe\left(\frac{g\left(\sum_{\_\alpha=\operatorname{RootOf}\left(e\_Z^{2}+d\right)}\left(\ln(x-\_\alpha)\ln(g\right.\right.}{}\right.$
risch	Expression too large to display

```
[In] int(ln(c*(e*x^2+d)^p)/x^3/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(c*(e*x^2+d)^p)/f/x^2-ln(c*(e*x^2+d)^p)/f^2*g*ln(x)+1/2*ln(c*(e*x^2+
d)^p)*g/f^2*ln(g*x^2+f)-p*e*(1/2*g/f^2/e*sum(ln(x-_alpha)*ln(g*x^2+f)-ln(x-
_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/Roo
tOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_al
pha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,
index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha
)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g+
2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-
d*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d))+1/2/f/d*ln(e*x^2+d)-1/f/d*ln(x)-
2*g/f^2*(1/2*ln(x)*(ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+ln((e*x+(-d*e)^(1/
2))/(-d*e)^(1/2)))/e+1/2*(dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+dilog((e*
x+(-d*e)^(1/2))/(-d*e)^(1/2)))/e))
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)}\,dx=\int \frac{\log\left((ex^2+d)^pc\right)}{(gx^2+f)x^3}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g*x^5 + f*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(e*x**2+d)**p)/x**3/(g*x**2+f),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.01

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)}\,dx = \frac{1}{2}\,ep\left(\frac{\left(2\,\log\left(\frac{ex^2}{d}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{ex^2}{d}\right)\right)g}{ef^2}-\frac{\left(\log\left(gx^2+f\right)\log\left(-\frac{egx^2+ef}{ef-dg}+1\right)+\mathrm{Li}_2\left(\frac{egx^2+ef}{ef-dg}\right)\right)g}{ef^2}-\right. + \frac{1}{2}\left(\frac{g\log\left(gx^2+f\right)}{f^2}-\frac{g\log\left(x^2\right)}{f^2}-\frac{1}{fx^2}\right)\log\left(\left(ex^2+d\right)^p c\right)$$

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f),x, algorithm="maxima")
[Out] 1/2*e*p*((2*log(e*x^2/d + 1)*log(x) + dilog(-e*x^2/d))*g/(e*f^2) - (log(g*x
^2 + f)*log(-(e*g*x^2 + e*f)/(e*f - d*g) + 1) + dilog((e*g*x^2 + e*f)/(e*f
- d*g)))*g/(e*f^2) - log(e*x^2 + d)/(d*f) + 2*log(x)/(d*f)) + 1/2*(g*log(g*
x^2 + f)/f^2 - g*log(x^2)/f^2 - 1/(f*x^2))*log((e*x^2 + d)^p*c)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)}\,dx = \int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{(gx^2+f)x^3}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^3\,(f+gx^2)}\,dx = \int \frac{\ln \left(c\,(e\,x^2+d)^p\right)}{x^3\,\left(g\,x^2+f\right)}\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)/(x^3*(f + g*x^2)),x)
[Out] int(log(c*(d + e*x^2)^p)/(x^3*(f + g*x^2)), x)
```

3.343 $\int \frac{x^4 \log\left(c(d+ex^2)^p\right)}{f+gx^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 667

$$\begin{aligned}
\int \frac{x^4 \log\left(c(d+ex^2)^p\right)}{f+gx^2} \, dx &= \frac{2fpx}{g^2} + \frac{2dpx}{3eg} - \frac{2px^3}{9g} - \frac{2\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2} \\
&\quad - \frac{2d^{3/2}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}g} + \frac{2f^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{5/2}} \\
&\quad - \frac{f^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{5/2}} \\
&\quad - \frac{f^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{5/2}} \\
&\quad - \frac{fx\log\left(c(d+ex^2)^p\right)}{g^2} + \frac{x^3\log\left(c(d+ex^2)^p\right)}{3g} \\
&\quad + \frac{f^{3/2}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{5/2}} \\
&\quad - \frac{if^{3/2}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{5/2}} \\
&\quad + \frac{if^{3/2}p\,\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}} \\
&\quad + \frac{if^{3/2}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}}
\end{aligned}$$

```
[Out] 2*f*p*x/g^2+2/3*d*p*x/e/g-2/9*p*x^3/g-2/3*d^(3/2)*p*arctan(x*e^(1/2)/d^(1/2
))/e^(3/2)/g-f*x*ln(c*(e*x^2+d)^p)/g^2+1/3*x^3*ln(c*(e*x^2+d)^p)/g+f^(3/2)*
```

```
arctan(x*g^(1/2)/f^(1/2))*ln(c*(e*x^2+d)^p)/g^(5/2)+2*f^(3/2)*p*arctan(x*g^
(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/g^(5/2)-f^(3/2)*p*arctan
(x*g^(1/2)/f^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/g^(5/2)-f^(3/2)*p*arcta
n(x*g^(1/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/g^(5/2)-I*f^(3/2)*p*pol
ylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/g^(5/2)+1/2*I*f^(3/2)*p*polylog(2
,1+2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2
)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/g^(5/2)+1/2*I*f^(3/2)*p*polylog(2,1-2*((-d)^
(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(
-d)^(1/2)*g^(1/2)))/g^(5/2)-2*f*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/g^2/e^(
1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {2526, 2498, 327, 211, 2505, 308, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$\int \frac{x^4\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \frac{f^{3/2}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{5/2}}-\frac{2d^{3/2}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}g}-\frac{f^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{g^{5/2}}-\frac{f^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{g^{5/2}}-\frac{2\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}+\frac{2f^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{5/2}}-\frac{fx\log\left(c(d+ex^2)^p\right)}{g^2}+\frac{x^3\log\left(c(d+ex^2)^p\right)}{3g}+\frac{if^{3/2}p\operatorname{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}+1\right)}{2g^{5/2}}+\frac{if^{3/2}p\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}}+\frac{2dpx}{3eg}-\frac{if^{3/2}p\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{5/2}}+\frac{2fpx}{g^2}-\frac{2px^3}{9g}$$

```
[In] Int[(x^4*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (2*f*p*x)/g^2 + (2*d*p*x)/(3*e*g) - (2*p*x^3)/(9*g) - (2*Sqrt[d]*f*p*ArcTan
[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*g^2) - (2*d^(3/2)*p*ArcTan[(Sqrt[e]*x)/Sqrt
```

```
[d]])/(3*e^(3/2)*g) + (2*f^(3/2)*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[
f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(5/2) - (f^(3/2)*p*ArcTan[(Sqrt[g]*x)/Sqrt[
f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - S
qrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) - (f^(3/2)*p*ArcTan[(Sq
rt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e
]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) - (f*x*Log
[c*(d + e*x^2)^p])/g^2 + (x^3*Log[c*(d + e*x^2)^p])/(3*g) + (f^(3/2)*ArcTan
[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/g^(5/2) - (I*f^(3/2)*p*PolyLog[
2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(5/2) + ((I/2)*f^(3/2)*p*Pol
yLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f]
- Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) + ((I/2)*f^(3/2)*p*P
olyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f
] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{f \log\left(c(d+ex^2)^p\right)}{g^2}+\frac{x^2\log\left(c(d+ex^2)^p\right)}{g}+\frac{f^2\log\left(c(d+ex^2)^p\right)}{g^2\left(f+gx^2\right)}\right)dx\\
&= -\frac{f\int \log\left(c(d+ex^2)^p\right)\,dx}{g^2}+\frac{f^2\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx}{g^2}+\frac{\int x^2\log\left(c(d+ex^2)^p\right)\,dx}{g}\\
&= -\frac{fx\log\left(c(d+ex^2)^p\right)}{g^2}+\frac{x^3\log\left(c(d+ex^2)^p\right)}{3g}+\frac{f^{3/2}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{5/2}}\\
&\quad+\frac{(2efp)\int\frac{x^2}{d+ex^2}\,dx}{g^2}-\frac{(2ef^2p)\int\frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}(d+ex^2)}\,dx}{g^2}-\frac{(2ep)\int\frac{x^4}{d+ex^2}\,dx}{3g}\\
&= \frac{2fpx}{g^2}-\frac{fx\log\left(c(d+ex^2)^p\right)}{g^2}+\frac{x^3\log\left(c(d+ex^2)^p\right)}{3g}+\frac{f^{3/2}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{5/2}}\\
&\quad-\frac{(2ef^{3/2}p)\int\frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{g^{5/2}}-\frac{(2dfp)\int\frac{1}{d+ex^2}\,dx}{g^2}-\frac{(2ep)\int\left(-\frac{d}{e^2}+\frac{x^2}{e}+\frac{d^2}{e^2(d+ex^2)}\right)dx}{3g}\\
&= \frac{2fpx}{g^2}+\frac{2dpx}{3eg}-\frac{2px^3}{9g}-\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}-\frac{fx\log\left(c(d+ex^2)^p\right)}{g^2}\\
&\quad+\frac{x^3\log\left(c(d+ex^2)^p\right)}{3g}+\frac{f^{3/2}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{5/2}}\\
&\quad-\frac{(2ef^{3/2}p)\int\left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}(\sqrt{-d}-\sqrt{e}x)}+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}(\sqrt{-d}+\sqrt{e}x)}\right)dx}{g^{5/2}}-\frac{(2d^2p)\int\frac{1}{d+ex^2}\,dx}{3eg}\\
&= \frac{2fpx}{g^2}+\frac{2dpx}{3eg}-\frac{2px^3}{9g}-\frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}-\frac{2d^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}g}\\
&\quad-\frac{fx\log\left(c(d+ex^2)^p\right)}{g^2}+\frac{x^3\log\left(c(d+ex^2)^p\right)}{3g}+\frac{f^{3/2}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{5/2}}\\
&\quad+\frac{\left(\sqrt{e}f^{3/2}p\right)\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{g^{5/2}}-\frac{\left(\sqrt{e}f^{3/2}p\right)\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{g^{5/2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2fpx}{g^2} + \frac{2dpx}{3eg} - \frac{2px^3}{9g} - \frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2} \\
&\quad - \frac{2d^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}g} + \frac{2f^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{5/2}} \\
&\quad - \frac{f^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{5/2}} \\
&\quad - \frac{f^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{5/2}} \\
&\quad - \frac{fx\log\left(c(d+ex^2)^p\right)}{g^2} + \frac{x^3\log\left(c(d+ex^2)^p\right)}{3g} \\
&\quad + \frac{f^{3/2}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{5/2}} - 2\frac{(fp)\int\frac{\log\left(\frac{2}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{1+\frac{gx^2}{f}}dx}{g^2} \\
&\quad + \frac{(fp)\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{f}\left(-i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{g^2} + \frac{(fp)\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{f}\left(i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{g^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2fpx}{g^2} + \frac{2dpx}{3eg} - \frac{2px^3}{9g} - \frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2} \\
&\quad - \frac{2d^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}g} + \frac{2f^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{5/2}} \\
&\quad - \frac{f^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{5/2}} \\
&\quad - \frac{f^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{5/2}} - \frac{fx\log\left(c(d+ex^2)^p\right)}{g^2} \\
&\quad + \frac{x^3\log\left(c(d+ex^2)^p\right)}{3g} + \frac{f^{3/2}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{5/2}} \\
&\quad + \frac{if^{3/2}p\text{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}} \\
&\quad + \frac{if^{3/2}p\text{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}} \\
&\quad - 2\frac{(if^{3/2}p)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}dx, x, \frac{1}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{g^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2fpx}{g^2} + \frac{2dpx}{3eg} - \frac{2px^3}{9g} - \frac{2\sqrt{d}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2} \\
&\quad - \frac{2d^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}g} + \frac{2f^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{5/2}} \\
&\quad - \frac{f^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{5/2}} \\
&\quad - \frac{f^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{5/2}} - \frac{fx\log\left(c(d+ex^2)^p\right)}{g^2} \\
&\quad + \frac{x^3\log\left(c(d+ex^2)^p\right)}{3g} + \frac{f^{3/2}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{5/2}} \\
&\quad - \frac{if^{3/2}p\mathrm{Li}_2\left(1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{5/2}} + \frac{if^{3/2}p\mathrm{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}} \\
&\quad + \frac{if^{3/2}p\mathrm{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 697, normalized size of antiderivative = 1.04

$$
\begin{aligned}
&\int \frac{x^4\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx \\
&= \frac{2fpx}{g^2} + \frac{2dpx}{3eg} - \frac{2px^3}{9g} - \frac{2\sqrt{d}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2} - \frac{2d^{3/2}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3e^{3/2}g} \\
&\quad - \frac{fx\log\left(c(d+ex^2)^p\right)}{g^2} + \frac{x^3\log\left(c(d+ex^2)^p\right)}{3g} + \frac{f^{3/2}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{5/2}} \\
&\quad - \frac{if^{3/2}p\left(\log\left(\frac{\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)+\log\left(\frac{\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{-i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)-\log\left(\frac{\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{-i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\right.}{}
\end{aligned}
$$

```
[In] Integrate[(x^4*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (2*f*p*x)/g^2 + (2*d*p*x)/(3*e*g) - (2*p*x^3)/(9*g) - (2*Sqrt[d]*f*p*ArcTan
[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*g^2) - (2*d^(3/2)*p*ArcTan[(Sqrt[e]*x)/Sqrt
[d]])/(3*e^(3/2)*g) - (f*x*Log[c*(d + e*x^2)^p])/g^2 + (x^3*Log[c*(d + e*x^
2)^p])/(3*g) + (f^(3/2)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/g
^(5/2) - ((I/2)*f^(3/2)*p*(Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*
Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + Log[(Sqrt[g]*
(Sqrt[-d] + Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 -
(I*Sqrt[g]*x)/Sqrt[f]] - Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]
```

```
*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - Log[(Sqrt[g]
*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I
*Sqrt[g]*x)/Sqrt[f]] + PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e
]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] + PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]
*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])] - PolyLog[2, (Sqrt[e]*(Sqrt[f]
 + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - PolyLog[2, (Sqrt
[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]))/g^(5
/2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.45 (sec) , antiderivative size = 616, normalized size of antiderivative = 0.92

method	result
risch	$\left(\ln\left(\left(e\,x^2+d\right)^p\right)-p\ln\left(e\,x^2+d\right)\right)\left(\frac{\frac{1}{3}g\,x^3-fx}{g^2}+\frac{f^2\arctan\left(\frac{gx}{\sqrt{fg}}\right)}{g^2\sqrt{fg}}\right)+\frac{p\,x^3\ln\left(e\,x^2+d\right)}{3g}-\frac{2p\,x^3}{9g}+\frac{2dpx}{3eg}-$

```
[In] int(x^4*ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] (ln((e*x^2+d)^p)-p*ln(e*x^2+d))*(1/g^2*(1/3*g*x^3-f*x)+f^2/g^2/(f*g)^(1/2)*
arctan(g*x/(f*g)^(1/2)))+1/3*p/g*x^3*ln(e*x^2+d)-2/9*p*x^3/g+2/3*d*p*x/e/g-
2/3*p/g*d^2/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-p*f/g^2*x*ln(e*x^2+d)+2*f
*p*x/g^2-2*p*f/g^2*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+p*Sum(1/2*(ln(x-_a
lpha)*ln(e*x^2+d)-2*e*(1/2*ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*
g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=
1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_
Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))/e+1/2*(dilog((RootOf(_Z^2*e*g+2*
_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*
g-e*f,index=1))+dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_
alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))/e))*f^2/g^3/_alph
a,_alpha=RootOf(_Z^2*g+f))+(1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)
^p)^2-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I*Pi
*csgn(I*c*(e*x^2+d)^p)^3+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+ln(c))*
(1/g^2*(1/3*g*x^3-f*x)+f^2/g^2/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2)))
```

Fricas [F]

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x^4 \log \left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")
```

```
[Out] integral(x^4*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x^4 \log \left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(x^4*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x^4\,\ln\left(c\,(e\,x^2+d)^p\right)}{g\,x^2+f}\,dx$$

```
[In] int((x^4*log(c*(d + e*x^2)^p))/(f + g*x^2),x)
[Out] int((x^4*log(c*(d + e*x^2)^p))/(f + g*x^2), x)
```

3.344 $\int \frac{x^2 \log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 585

$$\begin{aligned}\int \frac{x^2 \log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = & -\frac{2px}{g}+\frac{2\sqrt{d}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g} \\ & -\frac{2\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{3/2}} \\ & +\frac{\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{3/2}} \\ & +\frac{\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{3/2}} \\ & +\frac{x\log\left(c(d+ex^2)^p\right)}{g}-\frac{\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{3/2}} \\ & +\frac{i\sqrt{f}p\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{3/2}} \\ & -\frac{i\sqrt{f}p\,\mathrm{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{3/2}} \\ & -\frac{i\sqrt{f}p\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{3/2}}\end{aligned}$$

```
[Out] -2*p*x/g+x*ln(c*(e*x^2+d)^p)/g+2*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/g/e^(1
/2)-arctan(x*g^(1/2)/f^(1/2))*ln(c*(e*x^2+d)^p)*f^(1/2)/g^(3/2)-2*p*arctan(
x*g^(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))*f^(1/2)/g^(3/2)+p*ar
```

```
ctan(x*g^(1/2)/f^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/
2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))*f^(1/2)/g^(3/2)+p*a
rctan(x*g^(1/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/
2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))*f^(1/2)/g^(3/2)+I*p
*polylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))*f^(1/2)/g^(3/2)-1/2*I*p*polyl
og(2,1+2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^
(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))*f^(1/2)/g^(3/2)-1/2*I*p*polylog(2,1-2*((
-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/
2)+(-d)^(1/2)*g^(1/2)))*f^(1/2)/g^(3/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2526, 2498, 327, 211, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$\begin{aligned}\int \frac{x^2\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = &-\frac{\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{3/2}}\\
&+\frac{\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{g^{3/2}}\\
&+\frac{\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{g^{3/2}}\\
&+\frac{2\sqrt{d}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g}\\
&-\frac{2\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{3/2}}+\frac{x\log\left(c(d+ex^2)^p\right)}{g}\\
&-\frac{i\sqrt{f}p\,\text{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}+1\right)}{2g^{3/2}}\\
&-\frac{i\sqrt{f}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{3/2}}\\
&+\frac{i\sqrt{f}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{3/2}}-\frac{2px}{g}\end{aligned}$$

```
[In] Int[(x^2*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (-2*p*x)/g + (2*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*g) - (2*Sqr
t[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)]
)/g^(3/2) + (Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
```

```
I*Sqrt[g]*x))])/g^(3/2) + (Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqr
t[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g]
)*(Sqrt[f] - I*Sqrt[g]*x))])/g^(3/2) + (x*Log[c*(d + e*x^2)^p])/g - (Sqrt[f
]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/g^(3/2) + (I*Sqrt[f]*p*
PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(3/2) - ((I/2)*Sqrt[
f]*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*
Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(3/2) - ((I/2)*Sqr
t[f]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e
]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log\left(c(d+ex^2)^p\right)}{g} - \frac{f\log\left(c(d+ex^2)^p\right)}{g\left(f+gx^2\right)}\right)dx$$

$$= \frac{\int \log\left(c(d+ex^2)^p\right)dx}{g} - \frac{f\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}dx}{g}$$

$$= \frac{x \log \left(c(d+ex^2)^p\right)}{g} - \frac{\sqrt{f} \tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \log \left(c(d+ex^2)^p\right)}{g^{3/2}} - \frac{(2ep)\int \frac{x^2}{d+ex^2}\,dx}{g} + \frac{(2efp)\int \frac{x \tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}(d+ex^2)}\,dx}{g}$$

$$= -\frac{2px}{g} + \frac{x \log \left(c(d+ex^2)^p\right)}{g} - \frac{\sqrt{f} \tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \log \left(c(d+ex^2)^p\right)}{g^{3/2}} + \frac{\left(2e\sqrt{f}p\right)\int \frac{x \tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{g^{3/2}} + \frac{(2dp)\int \frac{1}{d+ex^2}\,dx}{g}$$

$$= -\frac{2px}{g} + \frac{2\sqrt{d}p \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g} + \frac{x \log \left(c(d+ex^2)^p\right)}{g} - \frac{\sqrt{f} \tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \log \left(c(d+ex^2)^p\right)}{g^{3/2}} + \frac{\left(2e\sqrt{f}p\right)\int \left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{g^{3/2}}$$

$$= -\frac{2px}{g} + \frac{2\sqrt{d}p \tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g} + \frac{x \log \left(c(d+ex^2)^p\right)}{g} - \frac{\sqrt{f} \tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \log \left(c(d+ex^2)^p\right)}{g^{3/2}} - \frac{\left(\sqrt{e}\sqrt{f}p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{g^{3/2}} + \frac{\left(\sqrt{e}\sqrt{f}p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{g^{3/2}}$$

$$= -\frac{2px}{g} + \frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g} - \frac{2\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{3/2}}$$
$$+ \frac{\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{3/2}}$$
$$+ \frac{\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{3/2}} + \frac{x\log\left(c(d+ex^2)^p\right)}{g}$$
$$- \frac{\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{3/2}} + 2\frac{p\int\frac{\log\left(\frac{2}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{1+\frac{gx^2}{f}}dx}{g}$$
$$- \frac{p\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{f}\left(-i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{g} - \frac{p\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{f}\left(i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{g}$$

$$= -\frac{2px}{g} + \frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g} - \frac{2\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{3/2}}$$
$$+ \frac{\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{3/2}}$$
$$+ \frac{\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{3/2}} + \frac{x\log\left(c(d+ex^2)^p\right)}{g}$$
$$- \frac{\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{3/2}} - \frac{i\sqrt{f}p\operatorname{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{3/2}}$$
$$- \frac{i\sqrt{f}p\operatorname{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{3/2}}$$
$$+ 2\frac{(i\sqrt{f}p)\operatorname{Subst}\left(\int\frac{\log(2x)}{1-2x}dx, x, \frac{1}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{g^{3/2}}$$

$$
\begin{aligned}
&= -\frac{2px}{g} + \frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g} - \frac{2\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{3/2}} \\
&+ \frac{\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{3/2}} \\
&+ \frac{\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{3/2}} \\
&+ \frac{x\log\left(c(d+ex^2)^p\right)}{g} - \frac{\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{3/2}} \\
&+ \frac{i\sqrt{f}p\operatorname{Li}_2\left(1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{3/2}} - \frac{i\sqrt{f}p\operatorname{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{3/2}} \\
&- \frac{i\sqrt{f}p\operatorname{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 619, normalized size of antiderivative = 1.06

$$
\int \frac{x^2\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx
= \frac{-2\sqrt{g}px + \frac{2\sqrt{d}\sqrt{g}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \sqrt{g}x\log\left(c(d+ex^2)^p\right) - \sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right) + \frac{1}{2}i\sqrt{f}p\Big(\log}{}
$$

```
[In] Integrate[(x^2*Log[c*(d + e*x^2)^p])/(f + g*x^2),x]
```

```
[Out] (-2*Sqrt[g]*p*x + (2*Sqrt[d]*Sqrt[g]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
 + Sqrt[g]*x*Log[c*(d + e*x^2)^p] - Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log
[c*(d + e*x^2)^p] + (I/2)*Sqrt[f]*p*(Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(
I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + Log
[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])
]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] - Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-
I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - Lo
g[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*
Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] + PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x
))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] + PolyLog[2, (Sqrt[e]*(Sqrt[f] -
 I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])] - PolyLog[2, (Sqrt[e
]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - PolyLo
g[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g
])]))/g^(3/2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.15 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.90

method	result
risch	$\left(\ln\left(\left(e\,x^{2}+d\right)^{p}\right)-p\ln\left(e\,x^{2}+d\right)\right)\left(\frac{x}{g}-\frac{f\arctan\left(\frac{gx}{\sqrt{fg}}\right)}{g\sqrt{fg}}\right)+\frac{px\ln\left(e\,x^{2}+d\right)}{g}-\frac{2px}{g}+\frac{2pd\arctan\left(\frac{xe}{\sqrt{de}}\right)}{g\sqrt{de}}+p$

[In] int(x^2*ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)

[Out] (ln((e*x^2+d)^p)-p*ln(e*x^2+d))*(x/g-f/g/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2)))+p*x/g*ln(e*x^2+d)-2*p*x/g+2*p/g*d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+p*Sum(-1/2*(ln(x-_alpha)*ln(e*x^2+d)-2*e*(1/2*ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))/e+1/2*(dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))/e))*f/g^2/_alpha,_alpha=RootOf(_Z^2*g+f))+(1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+ln(c))*(x/g-f/g/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2)))

Fricas [F]

$$\int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x^2 \log \left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")

[Out] integral(x^2*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)

Sympy [F]

$$\int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

```
[In] integrate(x**2*ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
```

```
[Out] Integral(x**2*log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x^2 \log \left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

```
[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
```

```
[Out] integrate(x^2*log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{x^2\,\ln \left(c\,{\left(e\,x^2+d\right)}^p\right)}{g\,x^2+f}\,dx$$

```
[In] int((x^2*log(c*(d + e*x^2)^p))/(f + g*x^2),x)
```

```
[Out] int((x^2*log(c*(d + e*x^2)^p))/(f + g*x^2), x)
```

3.345 $\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 533

$$\begin{aligned}\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx &= \frac{2p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} \\ &- \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} \\ &- \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} \\ &+ \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} \\ &+ \frac{ip\,\mathrm{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}} \\ &+ \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}}\end{aligned}$$

```
[Out] arctan(x*g^(1/2)/f^(1/2))*ln(c*(e*x^2+d)^p)/f^(1/2)/g^(1/2)+2*p*arctan(x*g^
(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)-p*arctan
(x*g^(1/2)/f^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)-p*arcta
n(x*g^(1/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I
*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)-I*p*pol
ylog(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2
,1+2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2
```

```
)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)+1/2*I*p*polylog(2,1-2*((-d)^
(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(
-d)^(1/2)*g^(1/2)))/f^(1/2)/g^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {211, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$\begin{aligned}\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = {}& \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\sqrt{g}} \\ & -\frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{\sqrt{f}\sqrt{g}} \\ & -\frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{\sqrt{f}\sqrt{g}} \\ & +\frac{2p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} \\ & +\frac{ip\operatorname{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}+1\right)}{2\sqrt{f}\sqrt{g}} \\ & +\frac{ip\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}} \\ & -\frac{ip\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}}\end{aligned}$$

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

```
[Out] (2*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/
(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))]])/(Sqrt[f]*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*S
qrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[
g])*(Sqrt[f] - I*Sqrt[g]*x))]])/(Sqrt[f]*Sqrt[g]) + (ArcTan[(Sqrt[g]*x)/Sqrt
[f]]*Log[c*(d + e*x^2)^p])/(Sqrt[f]*Sqrt[g]) - (I*p*PolyLog[2, 1 - (2*Sqrt[
f])/(Sqrt[f] - I*Sqrt[g]*x)]])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1 + (
2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sq
rt[g])*(Sqrt[f] - I*Sqrt[g]*x))]])/(Sqrt[f]*Sqrt[g]) + ((I/2)*p*PolyLog[2, 1
 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d
]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))]])/(Sqrt[f]*Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
```

```
/; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\sqrt{g}} - (2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}\,(d+ex^2)}\,dx \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{(2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{\sqrt{f}\sqrt{g}} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{(2ep)\int \left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}(\sqrt{-d}-\sqrt{e}x)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}(\sqrt{-d}+\sqrt{e}x)}\right)dx}{\sqrt{f}\sqrt{g}} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{(\sqrt{e}p)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{\sqrt{f}\sqrt{g}} - \frac{(\sqrt{e}p)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{\sqrt{f}\sqrt{g}} \\
&= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} \\
&\quad - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} \\
&\quad + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\sqrt{g}} - 2\frac{p\int \frac{\log\left(\frac{2}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{1+\frac{gx^2}{f}}\,dx}{f} \\
&\quad + \frac{p\int \frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{f}\left(-i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}\,dx}{f} + \frac{p\int \frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{f}\left(i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}\,dx}{f} \\
&= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} \\
&\quad - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}} \\
&\quad + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\sqrt{g}} + \frac{ip\text{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}} \\
&\quad + \frac{ip\text{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}} - 2\frac{(ip)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{\sqrt{f}\sqrt{g}}
\end{aligned}
$$

$$= \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}}$$
$$- \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}\sqrt{g}}$$
$$+ \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}\sqrt{g}} - \frac{ip\mathrm{Li}_2\left(1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}\sqrt{g}}$$
$$+ \frac{ip\mathrm{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}} + \frac{ip\mathrm{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}\sqrt{g}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.06

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx =$$
$$-\frac{i\left(p\log\left(\frac{\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)+p\log\left(\frac{\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{-i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\log\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)-p\log\left(\frac{\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{-i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\cdots\right.}{\cdots}$$

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2),x]
```

```
[Out] ((-1/2*I)*(p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt
[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + p*Log[(Sqrt[g]*(Sqrt[-d] +
Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x
)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] +
 Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - p*Log[(Sqrt[g]*(Sqrt[-
d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]
*x)/Sqrt[f]] + (2*I)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p] + p*P
olyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*S
qrt[g])] + p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f]
+ I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sq
rt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*S
qrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]))/(Sqrt[f]*Sqrt[g])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.86 (sec) , antiderivative size = 449, normalized size of antiderivative = 0.84

method	result
risch	$\frac{\left(\ln\left(\left(e\,x^{2}+d\right)^{p}\right)-p\ln\left(e\,x^{2}+d\right)\right)\arctan\left(\frac{g x}{\sqrt{f g}}\right)}{\sqrt{f g}}+\frac{p\left(\sum_{\_\alpha=\mathrm{RootOf}\left(g\_Z^{2}+f\right)}\frac{\ln\left(x-\_\alpha\right)\ln\left(e\,x^{2}+d\right)-\ln\left(x-\_\alpha\right)\left(\ln\left(\frac{\mathrm{RootOf}\left(e\_Z^{2}g+2\_\alpha\right.}{\mathrm{RootOf}\left(e\_Z^{2}g+\right.}\right.\right.}{}\right.}{}$

[In] int(ln(c*(e*x^2+d)^p)/(g*x^2+f),x,method=_RETURNVERBOSE)

[Out] (ln((e*x^2+d)^p)-p*ln(e*x^2+d))/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2))+1/2*p/g*sum(1/_alpha*(ln(x-_alpha)*ln(e*x^2+d)-ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2))),_alpha=RootOf(_Z^2*g+f))+(1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+ln(c))/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2))

Fricas [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx=\int \frac{\log\left((ex^2+d)^p c\right)}{gx^2+f}\,dx$$

[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="fricas")

[Out] integral(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)

Sympy [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx$$

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f),x)
[Out] Integral(log(c*(d + e*x**2)**p)/(f + g*x**2), x)
```

Maxima [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log \left((ex^2+d)^pc\right)}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="maxima")
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Giac [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\log \left((ex^2+d)^pc\right)}{gx^2+f}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{f+gx^2}\,dx = \int \frac{\ln \left(c\,{\left(e\,x^2+d\right)}^p\right)}{g\,x^2+f}\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)/(f + g*x^2),x)
[Out] int(log(c*(d + e*x^2)^p)/(f + g*x^2), x)
```

3.346 $\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2(f+gx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 581

$$
\begin{aligned}
\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2(f+gx^2)}\,dx &= \frac{2\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f} - \frac{2\sqrt{g}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}} \\
&+ \frac{\sqrt{g}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{f^{3/2}} \\
&+ \frac{\sqrt{g}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{f^{3/2}} \\
&- \frac{\log\left(c(d+ex^2)^p\right)}{fx} - \frac{\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{3/2}} \\
&+ \frac{i\sqrt{g}p\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}} \\
&- \frac{i\sqrt{g}p\,\mathrm{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}} \\
&- \frac{i\sqrt{g}p\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}}
\end{aligned}
$$

```
[Out] -ln(c*(e*x^2+d)^p)/f/x+2*p*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/f/d^(1/2)-arct
an(x*g^(1/2)/f^(1/2))*ln(c*(e*x^2+d)^p)*g^(1/2)/f^(3/2)-2*p*arctan(x*g^(1/2
)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))*g^(1/2)/f^(3/2)+p*arctan(x*g
^(1/2)/f^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g
^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))*g^(1/2)/f^(3/2)+p*arctan(x*
g^(1/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g
```

```
^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))*g^(1/2)/f^(3/2)+I*p*polylog
(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))*g^(1/2)/f^(3/2)-1/2*I*p*polylog(2,1+2
*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^
(1/2)-(-d)^(1/2)*g^(1/2)))*g^(1/2)/f^(3/2)-1/2*I*p*polylog(2,1-2*((-d)^(1/2
)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^
(1/2)*g^(1/2)))*g^(1/2)/f^(3/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2526, 2505, 211, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$\begin{aligned}\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2\left(f+gx^2\right)}\,dx = &-\frac{\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^{3/2}}\\ &+\frac{\sqrt{g}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{f^{3/2}}\\ &+\frac{\sqrt{g}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(\sqrt{f}-i\sqrt{g}x\right)\left(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f}\right)}\right)}{f^{3/2}}\\ &+\frac{2\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f}\\ &-\frac{2\sqrt{g}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}}-\frac{\log\left(c\left(d+ex^2\right)^p\right)}{fx}\\ &-\frac{i\sqrt{g}p\,\text{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}+1\right)}{2f^{3/2}}\\ &-\frac{i\sqrt{g}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2f^{3/2}}\\ &+\frac{i\sqrt{g}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}}\end{aligned}$$

```
[In] Int[Log[c*(d + e*x^2)^p]/(x^2*(f + g*x^2)),x]
```

```
[Out] (2*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*f) - (2*Sqrt[g]*p*ArcTan
[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(3/2) + (
Sqrt[g]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - S
qrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))
])/f^(3/2) + (Sqrt[g]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/f^(3/2) - Log[c*(d + e*x^2)^p]/(f*x) - (Sqrt[g]*ArcTan[(Sqr
```

```
t[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/f^(3/2) + (I*Sqrt[g]*p*PolyLog[2, 1
- (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(3/2) - ((I/2)*Sqrt[g]*p*PolyLog[
2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqr
t[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(3/2) - ((I/2)*Sqrt[g]*p*PolyLo
g[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + S
qrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
```

```
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\log\left(c(d+ex^2)^p\right)}{fx^2}-\frac{g\log\left(c(d+ex^2)^p\right)}{f\left(f+gx^2\right)}\right)dx\\
&= \frac{\int \frac{\log(c(d+ex^2)^p)}{x^2}\,dx}{f}-\frac{g\int \frac{\log(c(d+ex^2)^p)}{f+gx^2}\,dx}{f}\\
&= -\frac{\log\left(c(d+ex^2)^p\right)}{fx}-\frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{3/2}}\\
&\quad+\frac{(2ep)\int \frac{1}{d+ex^2}\,dx}{f}+\frac{(2egp)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}(d+ex^2)}\,dx}{f}\\
&= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f}-\frac{\log\left(c(d+ex^2)^p\right)}{fx}\\
&\quad-\frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{3/2}}+\frac{\left(2e\sqrt{g}p\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{f^{3/2}}
\end{aligned}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f} - \frac{\log\left(c(d+ex^2)^p\right)}{fx} - \frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{3/2}} + \frac{(2e\sqrt{g}p)\int\left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}(\sqrt{-d}-\sqrt{e}x)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}(\sqrt{-d}+\sqrt{e}x)}\right)dx}{f^{3/2}}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f} - \frac{\log\left(c(d+ex^2)^p\right)}{fx} - \frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{3/2}} - \frac{(\sqrt{e}\sqrt{g}p)\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}x}dx}{f^{3/2}} + \frac{(\sqrt{e}\sqrt{g}p)\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}+\sqrt{e}x}dx}{f^{3/2}}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f} - \frac{2\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}} + \frac{\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{f^{3/2}} + \frac{\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{f^{3/2}} - \frac{\log\left(c(d+ex^2)^p\right)}{fx} - \frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{3/2}} + 2\frac{(gp)\int\frac{\log\left(\frac{2}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{1+\frac{gx^2}{f}}dx}{f^2} - \frac{(gp)\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{f}\left(-i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{f^2} - \frac{(gp)\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{f}\left(i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{f^2}$$

$$
\begin{aligned}
&= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f} - \frac{2\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}} \\
&+ \frac{\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{f^{3/2}} \\
&+ \frac{\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{f^{3/2}} - \frac{\log\left(c(d+ex^2)^p\right)}{fx} \\
&- \frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{3/2}} - \frac{i\sqrt{g}p\text{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}} \\
&- \frac{i\sqrt{g}p\text{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}} \\
&+ 2\frac{(i\sqrt{g}p)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{f^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f} - \frac{2\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}} \\
&+ \frac{\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{f^{3/2}} \\
&+ \frac{\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{f^{3/2}} \\
&- \frac{\log\left(c(d+ex^2)^p\right)}{fx} - \frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{3/2}} \\
&+ \frac{i\sqrt{g}p\text{Li}_2\left(1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}} - \frac{i\sqrt{g}p\text{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}} \\
&- \frac{i\sqrt{g}p\text{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 613, normalized size of antiderivative = 1.06

$$
\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)}\,dx
$$

$$
= \frac{\frac{2\sqrt{e}\sqrt{f}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{f}\log(c(d+ex^2)^p)}{x} - \sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right) + \frac{1}{2}i\sqrt{g}p\left(\log\left(\frac{\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}}\right)\text{lo}\right.}{}
$$

[In] Integrate[Log[c*(d + e*x^2)^p]/(x^2*(f + g*x^2)),x]

[Out] ((2*Sqrt[e]*Sqrt[f]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] - (Sqrt[f]*Log[c*(d + e*x^2)^p])/x - Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p] + (I/2)*Sqrt[g]*p*(Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] - Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] + PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] + PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])] - PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]))/f^(3/2)

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.58 (sec) , antiderivative size = 526, normalized size of antiderivative = 0.91

method	result
risch	$\left(\ln\left(\left(e\,x^{2}+d\right)^{p}\right)-p\ln\left(e\,x^{2}+d\right)\right)\left(-\frac{1}{fx}-\frac{g\arctan\left(\frac{gx}{\sqrt{fg}}\right)}{f\sqrt{fg}}\right)-\frac{p\ln\left(e\,x^{2}+d\right)}{fx}+\frac{2pe\arctan\left(\frac{xe}{\sqrt{de}}\right)}{f\sqrt{de}}+p\left(\ldots\right.$ $\_\alpha$=R

[In] int(ln(c*(e*x^2+d)^p)/x^2/(g*x^2+f),x,method=_RETURNVERBOSE)

[Out] (ln((e*x^2+d)^p)-p*ln(e*x^2+d))*(-1/f/x-1/f*g/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2)))-p/f/x*ln(e*x^2+d)+2*p/f*e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+p*Sum(-1/2*(ln(x-_alpha)*ln(e*x^2+d)-2*e*(1/2*ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))/e+1/2*(dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)))/e))/f/_alpha,_alpha=RootOf(_Z^2*g+f))+(1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(

```
e*x^2+d)^p)^2-1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-
1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3+1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)
+ln(c))*(-1/f/x-1/f*g/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2)))
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{(gx^2+f)x^2}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log((e*x^2 + d)^p*c)/(g*x^4 + f*x^2), x)
```

Sympy [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)}\,dx = \int \frac{\log\left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)}\,dx$$

```
[In] integrate(ln(c*(e*x**2+d)**p)/x**2/(g*x**2+f),x)
[Out] Integral(log(c*(d + e*x**2)**p)/(x**2*(f + g*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)}\,dx=\int \frac{\log \left((ex^2+d)^pc\right)}{(gx^2+f)x^2}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)}\,dx=\int \frac{\ln \left(c\,{\left(e\,x^2+d\right)}^p\right)}{x^2\,\left(g\,x^2+f\right)}\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)/(x^2*(f + g*x^2)),x)
[Out] int(log(c*(d + e*x^2)^p)/(x^2*(f + g*x^2)), x)
```

3.347 $\int \frac{\log\left(c(d+ex^2)^p\right)}{x^4(f+gx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 651

$$
\begin{aligned}
\int \frac{\log\left(c(d+ex^2)^p\right)}{x^4(f+gx^2)}\,dx = & -\frac{2ep}{3dfx} - \frac{2e^{3/2}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}f} - \frac{2\sqrt{e}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} \\
& + \frac{2g^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} \\
& - \frac{g^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{f^{5/2}} \\
& - \frac{g^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{f^{5/2}} \\
& - \frac{\log\left(c(d+ex^2)^p\right)}{3fx^3} + \frac{g\log\left(c(d+ex^2)^p\right)}{f^2x} \\
& + \frac{g^{3/2}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{5/2}} \\
& - \frac{ig^{3/2}p\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} \\
& + \frac{ig^{3/2}p\,\mathrm{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}} \\
& + \frac{ig^{3/2}p\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}}
\end{aligned}
$$

```
[Out] -2/3*e*p/d/f/x-2/3*e^(3/2)*p*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)/f-1/3*ln(c*(
e*x^2+d)^p)/f/x^3+g*ln(c*(e*x^2+d)^p)/f^2/x+g^(3/2)*arctan(x*g^(1/2)/f^(1/2
```

```
))*ln(c*(e*x^2+d)^p)/f^(5/2)+2*g^(3/2)*p*arctan(x*g^(1/2)/f^(1/2))*ln(2*f^(
1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(5/2)-g^(3/2)*p*arctan(x*g^(1/2)/f^(1/2))*ln(
-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*
f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(5/2)-g^(3/2)*p*arctan(x*g^(1/2)/f^(1/2))*ln
(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*
f^(1/2)+(-d)^(1/2)*g^(1/2)))/f^(5/2)-I*g^(3/2)*p*polylog(2,1-2*f^(1/2)/(f^(
1/2)-I*x*g^(1/2)))/f^(5/2)+1/2*I*g^(3/2)*p*polylog(2,1+2*((-d)^(1/2)-x*e^(1
/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^
(1/2)))/f^(5/2)+1/2*I*g^(3/2)*p*polylog(2,1-2*((-d)^(1/2)+x*e^(1/2))*f^(1/2
)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/f^(
5/2)-2*g*p*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/f^2/d^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 651, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2526, 2505, 331, 211, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$\begin{aligned}\int \frac{\log\left(c(d+ex^2)^p\right)}{x^4(f+gx^2)}\,dx = {} & \frac{g^{3/2}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{5/2}}-\frac{2e^{3/2}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}f}\\
&-\frac{g^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{f^{5/2}}\\
&-\frac{g^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{f^{5/2}}\\
&-\frac{2\sqrt{e}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2}+\frac{2g^{3/2}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}}\\
&+\frac{g\log\left(c(d+ex^2)^p\right)}{f^2x}-\frac{\log\left(c(d+ex^2)^p\right)}{3fx^3}\\
&+\frac{ig^{3/2}p\operatorname{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}+1\right)}{2f^{5/2}}\\
&+\frac{ig^{3/2}p\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}}\\
&-\frac{2ep}{3dfx}-\frac{ig^{3/2}p\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}}\end{aligned}$$

```
[In] Int[Log[c*(d + e*x^2)^p]/(x^4*(f + g*x^2)),x]
```

```
[Out] (-2*e*p)/(3*d*f*x) - (2*e^(3/2)*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(3*d^(3/2)*f
) - (2*Sqrt[e]*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*f^2) + (2*g^(3/2)*
```

```
p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(
5/2) - (g^(3/2)*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt
[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqr
t[g]*x))])/f^(5/2) - (g^(3/2)*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*
Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sq
rt[f] - I*Sqrt[g]*x))])/f^(5/2) - Log[c*(d + e*x^2)^p]/(3*f*x^3) + (g*Log[c
*(d + e*x^2)^p])/(f^2*x) + (g^(3/2)*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d +
e*x^2)^p])/f^(5/2) - (I*g^(3/2)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*S
qrt[g]*x)])/f^(5/2) + ((I/2)*g^(3/2)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(S
qrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*
Sqrt[g]*x))])/f^(5/2) + ((I/2)*g^(3/2)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*
(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
I*Sqrt[g]*x))])/f^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log \left(c(d+ex^2)^p\right)}{fx^4} - \frac{g\log \left(c(d+ex^2)^p\right)}{f^2x^2} + \frac{g^2\log \left(c(d+ex^2)^p\right)}{f^2\left(f+gx^2\right)} \right) dx$$

$$= \frac{\int \frac{\log(c(d+ex^2)^p)}{x^4}\,dx}{f} - \frac{g\int \frac{\log(c(d+ex^2)^p)}{x^2}\,dx}{f^2} + \frac{g^2\int \frac{\log(c(d+ex^2)^p)}{f+gx^2}\,dx}{f^2}$$

$$= -\frac{\log\left(c(d+ex^2)^p\right)}{3fx^3} + \frac{g\log\left(c(d+ex^2)^p\right)}{f^2x} + \frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{5/2}} + \frac{(2ep)\int \frac{1}{x^2(d+ex^2)}\,dx}{3f} - \frac{(2egp)\int \frac{1}{d+ex^2}\,dx}{f^2} - \frac{(2eg^2p)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}(d+ex^2)}\,dx}{f^2}$$

$$= -\frac{2ep}{3dfx} - \frac{2\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\log\left(c(d+ex^2)^p\right)}{3fx^3} + \frac{g\log\left(c(d+ex^2)^p\right)}{f^2x} + \frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{5/2}} - \frac{(2e^2p)\int \frac{1}{d+ex^2}\,dx}{3df} - \frac{(2eg^{3/2}p)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{f^{5/2}}$$

$$= -\frac{2ep}{3dfx} - \frac{2e^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}f} - \frac{2\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\log\left(c(d+ex^2)^p\right)}{3fx^3} + \frac{g\log\left(c(d+ex^2)^p\right)}{f^2x} + \frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{5/2}} - \frac{(2eg^{3/2}p)\int \left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{f^{5/2}}$$

$$= -\frac{2ep}{3dfx} - \frac{2e^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}f} - \frac{2\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\log\left(c(d+ex^2)^p\right)}{3fx^3} + \frac{g\log\left(c(d+ex^2)^p\right)}{f^2x} + \frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{5/2}} + \frac{\left(\sqrt{e}g^{3/2}p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{f^{5/2}} - \frac{\left(\sqrt{e}g^{3/2}p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{f^{5/2}}$$

$$
\begin{aligned}
&= -\frac{2ep}{3dfx} - \frac{2e^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}f} - \frac{2\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} \\
&+ \frac{2g^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} \\
&- \frac{g^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{f^{5/2}} \\
&- \frac{g^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{f^{5/2}} - \frac{\log\left(c(d+ex^2)^p\right)}{3fx^3} \\
&+ \frac{g\log\left(c(d+ex^2)^p\right)}{f^2x} + \frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{5/2}} \\
&- 2\frac{(g^2p)\int\frac{\log\left(\frac{2}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{1+\frac{gx^2}{f}}dx}{f^3} + \frac{(g^2p)\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{f}\left(-i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{f^3} \\
&+ \frac{(g^2p)\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{f}\left(i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{f^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2ep}{3dfx} - \frac{2e^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}f} - \frac{2\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} \\
&+ \frac{2g^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} \\
&- \frac{g^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{f^{5/2}} \\
&- \frac{g^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{f^{5/2}} - \frac{\log\left(c(d+ex^2)^p\right)}{3fx^3} \\
&+ \frac{g\log\left(c(d+ex^2)^p\right)}{f^2x} + \frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{5/2}} \\
&+ \frac{ig^{3/2}p\text{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}} \\
&+ \frac{ig^{3/2}p\text{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}} \\
&- 2\frac{(ig^{3/2}p)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}dx,x,\frac{1}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{f^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2ep}{3dfx} - \frac{2e^{3/2}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{3d^{3/2}f} - \frac{2\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} \\
&+ \frac{2g^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} \\
&- \frac{g^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{f^{5/2}} \\
&- \frac{g^{3/2}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{f^{5/2}} - \frac{\log\left(c(d+ex^2)^p\right)}{3fx^3} \\
&+ \frac{g\log\left(c(d+ex^2)^p\right)}{f^2x} + \frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{5/2}} \\
&- \frac{ig^{3/2}p\operatorname{Li}_2\left(1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} + \frac{ig^{3/2}p\operatorname{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}} \\
&+ \frac{ig^{3/2}p\operatorname{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.32 (sec) , antiderivative size = 670, normalized size of antiderivative = 1.03

$$
\int \frac{\log\left(c(d+ex^2)^p\right)}{x^4(f+gx^2)}\,dx
$$

$$
= \frac{-\frac{12\sqrt{e}\sqrt{f}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} - \frac{4ef^{3/2}p\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{ex^2}{d}\right)}{dx} - \frac{2f^{3/2}\log\left(c(d+ex^2)^p\right)}{x^3} + \frac{6\sqrt{f}g\log\left(c(d+ex^2)^p\right)}{x} + 6g^{3/2}
$$

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x^4*(f + g*x^2)),x]
```

```
[Out] ((-12*Sqrt[e]*Sqrt[f]*g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] - (4*e*f^(3/
2)*p*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^2)/d)])/(d*x) - (2*f^(3/2)*Log[
c*(d + e*x^2)^p])/x^3 + (6*Sqrt[f]*g*Log[c*(d + e*x^2)^p])/x + 6*g^(3/2)*Ar
cTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p] - (3*I)*g^(3/2)*p*(Log[(Sqrt
[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 -
 (I*Sqrt[g]*x)/Sqrt[f]] + Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((-I)*Sqrt[e
]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] - Log[(Sqrt[g
]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1
+ (I*Sqrt[g]*x)/Sqrt[f]] - Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*
Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] + PolyLog[2, (S
qrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] + P
```

```
olyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*S
qrt[g])] - PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] -
I*Sqrt[-d]*Sqrt[g])] - PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e
]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])]))/(6*f^(5/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.60 (sec) , antiderivative size = 602, normalized size of antiderivative = 0.92

method	result
risch	$\left(\ln\left(\left(e\,x^2+d\right)^p\right)-p\ln\left(e\,x^2+d\right)\right)\left(-\frac{1}{3f\,x^3}+\frac{g}{f^2x}+\frac{g^2\arctan\left(\frac{gx}{\sqrt{fg}}\right)}{f^2\sqrt{fg}}\right)-\frac{p\ln\left(e\,x^2+d\right)}{3f\,x^3}-\frac{2p\,e^2\arctan\left(\frac{xe}{\sqrt{de}}\right)}{3fd\sqrt{de}}$

```
[In] int(ln(c*(e*x^2+d)^p)/x^4/(g*x^2+f),x,method=_RETURNVERBOSE)
```

```
[Out] (ln((e*x^2+d)^p)-p*ln(e*x^2+d))*(-1/3/f/x^3+1/f^2*g/x+g^2/f^2/(f*g)^(1/2)*a
rctan(g*x/(f*g)^(1/2)))-1/3*p/f/x^3*ln(e*x^2+d)-2/3*p/f*e^2/d/(d*e)^(1/2)*a
rctan(x*e/(d*e)^(1/2))-2/3*e*p/d/f/x+p*g/f^2/x*ln(e*x^2+d)-2*p*g/f^2*e/(d*e
)^(1/2)*arctan(x*e/(d*e)^(1/2))+p*Sum(1/2*(ln(x-_alpha)*ln(e*x^2+d)-2*e*(1/
2*ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1)-x+_alp
ha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+ln((RootOf(_Z^2*e*g+2
*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d
*g-e*f,index=2)))/e+1/2*(dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,ind
ex=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=1))+dilog((Ro
otOf(_Z^2*e*g+2*_Z*_alpha*e*g+d*g-e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*
_Z*_alpha*e*g+d*g-e*f,index=2)))/e))*g/f^2/_alpha,_alpha=RootOf(_Z^2*g+f))+
(1/2*I*Pi*csgn(I*(e*x^2+d)^p)*csgn(I*c*(e*x^2+d)^p)^2-1/2*I*Pi*csgn(I*(e*x^
2+d)^p)*csgn(I*c*(e*x^2+d)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(e*x^2+d)^p)^3+1/
2*I*Pi*csgn(I*c*(e*x^2+d)^p)^2*csgn(I*c)+ln(c))*(-1/3/f/x^3+1/f^2*g/x+g^2/f
^2/(f*g)^(1/2)*arctan(g*x/(f*g)^(1/2)))
```

Fricas [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^4 (f+gx^2)} \, dx = \int \frac{\log \left((ex^2+d)^p c\right)}{(gx^2+f)x^4} \, dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/x^4/(g*x^2+f),x, algorithm="fricas")
[Out] integral(log((e*x^2 + d)^p*c)/(g*x^6 + f*x^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^4 (f+gx^2)} \, dx = \text{Timed out}$$

```
[In] integrate(ln(c*(e*x**2+d)**p)/x**4/(g*x**2+f),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^4 (f+gx^2)} \, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(c*(e*x^2+d)^p)/x^4/(g*x^2+f),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^4 (f+gx^2)} \, dx = \int \frac{\log \left((ex^2+d)^p c\right)}{(gx^2+f)x^4} \, dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/x^4/(g*x^2+f),x, algorithm="giac")
[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^4\left(f+gx^2\right)}\,dx = \int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{x^4\,\left(g\,x^2+f\right)}\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)/(x^4*(f + g*x^2)),x)
[Out] int(log(c*(d + e*x^2)^p)/(x^4*(f + g*x^2)), x)
```

3.348 $\int \frac{x^5 \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 199

$$\begin{aligned}\int \frac{x^5 \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = & -\frac{px^2}{2g^2}+\frac{ef^2p\log(d+ex^2)}{2g^3(ef-dg)}+\frac{(d+ex^2)\log\left(c(d+ex^2)^p\right)}{2eg^2} \\ & -\frac{f^2\log\left(c(d+ex^2)^p\right)}{2g^3(f+gx^2)}-\frac{ef^2p\log(f+gx^2)}{2g^3(ef-dg)} \\ & -\frac{f\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{g^3} \\ & -\frac{fp\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{g^3}\end{aligned}$$

```
[Out] -1/2*p*x^2/g^2+1/2*e*f^2*p*ln(e*x^2+d)/g^3/(-d*g+e*f)+1/2*(e*x^2+d)*ln(c*(e
*x^2+d)^p)/e/g^2-1/2*f^2*ln(c*(e*x^2+d)^p)/g^3/(g*x^2+f)-1/2*e*f^2*p*ln(g*x
^2+f)/g^3/(-d*g+e*f)-f*ln(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(-d*g+e*f))/g^3-f*p
*polylog(2,-g*(e*x^2+d)/(-d*g+e*f))/g^3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules

used = {2525, 45, 2463, 2436, 2332, 2442, 36, 31, 2441, 2440, 2438}

$$\int \frac{x^5 \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2} \, dx = -\frac{f^2 \log \left(c(d+ex^2)^p\right)}{2g^3 \left(f+gx^2\right)} - \frac{f \log \left(c(d+ex^2)^p\right) \log \left(\frac{e(f+gx^2)}{ef-dg}\right)}{g^3} + \frac{(d+ex^2) \log \left(c(d+ex^2)^p\right)}{2eg^2} + \frac{ef^2 p \log \left(d+ex^2\right)}{2g^3(ef-dg)} - \frac{ef^2 p \log \left(f+gx^2\right)}{2g^3(ef-dg)} - \frac{fp \operatorname{PolyLog}\left(2, -\frac{g(ex^2+d)}{ef-dg}\right)}{g^3} - \frac{px^2}{2g^2}$$

```
[In] Int[(x^5*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] -1/2*(p*x^2)/g^2 + (e*f^2*p*Log[d + e*x^2])/(2*g^3*(e*f - d*g)) + ((d + e*x
^2)*Log[c*(d + e*x^2)^p])/(2*e*g^2) - (f^2*Log[c*(d + e*x^2)^p])/(2*g^3*(f
+ g*x^2)) - (e*f^2*p*Log[f + g*x^2])/(2*g^3*(e*f - d*g)) - (f*Log[c*(d + e*
x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/g^3 - (f*p*PolyLog[2, -((g*(d + e
*x^2))/(e*f - d*g))])/g^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^2 \log \left(c(d+ex)^p\right)}{(f+gx)^2}\,dx, x, x^2\right)$$

$$=\frac{1}{2}\text{Subst}\left(\int\left(\frac{\log\left(c(d+ex)^p\right)}{g^2}+\frac{f^2\log\left(c(d+ex)^p\right)}{g^2(f+gx)^2}-\frac{2f\log\left(c(d+ex)^p\right)}{g^2(f+gx)}\right)dx,x,x^2\right)$$

$$=\frac{\text{Subst}\left(\int\log\left(c(d+ex)^p\right)dx,x,x^2\right)}{2g^2}-\frac{f\text{Subst}\left(\int\frac{\log(c(d+ex)^p)}{f+gx}dx,x,x^2\right)}{g^2}+\frac{f^2\text{Subst}\left(\int\frac{\log(c(d+ex)^p)}{(f+gx)^2}dx,x,x^2\right)}{2g^2}$$

$$=-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{2g^3\left(f+gx^2\right)}-\frac{f\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{g^3}+\frac{\text{Subst}\left(\int\log\left(cx^p\right)dx,x,d+ex^2\right)}{2eg^2}+\frac{(efp)\text{Subst}\left(\int\frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}dx,x,x^2\right)}{g^3}+\frac{\left(ef^2p\right)\text{Subst}\left(\int\frac{1}{(d+ex)(f+gx)}dx,x,x^2\right)}{2g^3}$$

$$=-\frac{px^2}{2g^2}+\frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2eg^2}-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{2g^3\left(f+gx^2\right)}-\frac{f\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{g^3}+\frac{(fp)\text{Subst}\left(\int\frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}dx,x,d+ex^2\right)}{g^3}+\frac{\left(e^2f^2p\right)\text{Subst}\left(\int\frac{1}{d+ex}dx,x,x^2\right)}{2g^3(ef-dg)}-\frac{\left(ef^2p\right)\text{Subst}\left(\int\frac{1}{f+gx}dx,x,x^2\right)}{2g^2(ef-dg)}$$

$$=-\frac{px^2}{2g^2}+\frac{ef^2p\log\left(d+ex^2\right)}{2g^3(ef-dg)}+\frac{\left(d+ex^2\right)\log\left(c\left(d+ex^2\right)^p\right)}{2eg^2}-\frac{f^2\log\left(c\left(d+ex^2\right)^p\right)}{2g^3\left(f+gx^2\right)}-\frac{ef^2p\log\left(f+gx^2\right)}{2g^3(ef-dg)}-\frac{f\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e\left(f+gx^2\right)}{ef-dg}\right)}{g^3}-\frac{fp\text{Li}_2\left(-\frac{g\left(d+ex^2\right)}{ef-dg}\right)}{g^3}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.83

$$\int\frac{x^5\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}dx=$$

$$-\frac{px^2-\frac{(d+ex^2)\log(c(d+ex^2)^p)}{e}+\frac{f^2\log(c(d+ex^2)^p)}{g(f+gx^2)}+\frac{ef^2p(\log(d+ex^2)-\log(f+gx^2))}{g(-ef+dg)}+\frac{2f\left(\log(c(d+ex^2)^p)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)+p\,\text{P}\right.}{g}}{2g^2}$$

```
[In] Integrate[(x^5*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] -1/2*(p*x^2 - ((d + e*x^2)*Log[c*(d + e*x^2)^p])/e + (f^2*Log[c*(d + e*x^2)
^p])/(g*(f + g*x^2)) + (e*f^2*p*(Log[d + e*x^2] - Log[f + g*x^2]))/(g*(-(e*
f) + d*g)) + (2*f*(Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)] +
p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)]))/g)/g^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.09 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.22

method	result
parts	$\frac{\ln\left(c\left(e\,x^2+d\right)^p\right)x^2}{2g^2}-\frac{f^2\ln\left(c\left(e\,x^2+d\right)^p\right)}{2g^3\left(g\,x^2+f\right)}-\frac{\ln\left(c\left(e\,x^2+d\right)^p\right)f\ln\left(g\,x^2+f\right)}{g^3}-pe\left(-\frac{f\left(\sum_{\_\alpha=\mathrm{RootOf}\left(e\_Z^2+d\right)}\left(\ln(x-\_\alpha)\ln(g\,x\right.\right.}{}\right.$
risch	$\frac{\ln\left(\left(e\,x^2+d\right)^p\right)x^2}{2g^2}-\frac{\ln\left(\left(e\,x^2+d\right)^p\right)f^2}{2g^3\left(g\,x^2+f\right)}-\frac{\ln\left(\left(e\,x^2+d\right)^p\right)f\ln\left(g\,x^2+f\right)}{g^3}-\frac{p\,x^2}{2g^2}+\frac{p\ln\left(e\,x^2+d\right)d^2}{2eg\left(dg-ef\right)}-\frac{p\ln\left(e\,x^2+d\right)df}{2g^2\left(dg-ef\right)}-\frac{pe\ln(e\,x}{2g^3(d}$

```
[In] int(x^5*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(c*(e*x^2+d)^p)*x^2/g^2-1/2*f^2*ln(c*(e*x^2+d)^p)/g^3/(g*x^2+f)-ln(c*
(e*x^2+d)^p)*f/g^3*ln(g*x^2+f)-p*e*(-f/g^3/e*sum(ln(x-_alpha)*ln(g*x^2+f)-l
n(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)
/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z
*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+
e*f,index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_a
lpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*
e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*
e*g-d*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d))-1/g^3*(-1/2*g*x^2/e+1/2*(d^2
*g^2-d*e*f*g-e^2*f^2)/(d*g-e*f)/e^2*ln(e*x^2+d)+1/2*f^2/(d*g-e*f)*ln(g*x^2+
f)))
```

Fricas [F]

$$\int \frac{x^5 \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{x^5 \log \left((ex^2+d)^p c\right)}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral(x^5*log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5 \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**5*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^5 \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{x^5 \log \left((ex^2+d)^p c\right)}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
[Out] integrate(x^5*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

Giac [F]

$$\int \frac{x^5 \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{x^5 \log \left((ex^2+d)^p c\right)}{(gx^2+f)^2}\,dx$$

```
[In] integrate(x^5*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate(x^5*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \frac{x^5\,\ln\left(c\,(e\,x^2+d)^p\right)}{\left(g\,x^2+f\right)^2}\,dx$$

```
[In] int((x^5*log(c*(d + e*x^2)^p))/(f + g*x^2)^2,x)
[Out] int((x^5*log(c*(d + e*x^2)^p))/(f + g*x^2)^2, x)
```

3.349 $\int \frac{x^3 \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 155

$$\int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2} dx = -\frac{efp\log(d+ex^2)}{2g^2(ef-dg)} + \frac{f\log\left(c(d+ex^2)^p\right)}{2g^2(f+gx^2)} + \frac{efp\log(f+gx^2)}{2g^2(ef-dg)} + \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^2} + \frac{p\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2g^2}$$

```
[Out] -1/2*e*f*p*ln(e*x^2+d)/g^2/(-d*g+e*f)+1/2*f*ln(c*(e*x^2+d)^p)/g^2/(g*x^2+f)
+1/2*e*f*p*ln(g*x^2+f)/g^2/(-d*g+e*f)+1/2*ln(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/
(-d*g+e*f))/g^2+1/2*p*polylog(2,-g*(e*x^2+d)/(-d*g+e*f))/g^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {2525, 45, 2463, 2442, 36, 31, 2441, 2440, 2438}

$$\int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2} dx = \frac{f\log\left(c(d+ex^2)^p\right)}{2g^2(f+gx^2)} + \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^2} + \frac{p\,\mathrm{PolyLog}\left(2,-\frac{g(ex^2+d)}{ef-dg}\right)}{2g^2} - \frac{efp\log(d+ex^2)}{2g^2(ef-dg)} + \frac{efp\log(f+gx^2)}{2g^2(ef-dg)}$$

```
[In] Int[(x^3*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] -1/2*(e*f*p*Log[d + e*x^2])/(g^2*(e*f - d*g)) + (f*Log[c*(d + e*x^2)^p])/(2
*g^2*(f + g*x^2)) + (e*f*p*Log[f + g*x^2])/(2*g^2*(e*f - d*g)) + (Log[c*(d
+ e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/(2*g^2) + (p*PolyLog[2, -((g*
(d + e*x^2))/(e*f - d*g))])/(2*g^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x\log\left(c(d+ex)^p\right)}{(f+gx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{f\log\left(c(d+ex)^p\right)}{g(f+gx)^2}+\frac{\log\left(c(d+ex)^p\right)}{g(f+gx)}\right)\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx,x,x^2\right)}{2g}-\frac{f\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{(f+gx)^2}\,dx,x,x^2\right)}{2g}\\
&= \frac{f\log\left(c(d+ex^2)^p\right)}{2g^2\left(f+gx^2\right)}+\frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^2}\\
&\quad-\frac{(ep)\text{Subst}\left(\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx,x,x^2\right)}{2g^2}-\frac{(efp)\text{Subst}\left(\int \frac{1}{(d+ex)(f+gx)}\,dx,x,x^2\right)}{2g^2}\\
&= \frac{f\log\left(c(d+ex^2)^p\right)}{2g^2\left(f+gx^2\right)}+\frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^2}\\
&\quad-\frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx,x,d+ex^2\right)}{2g^2}\\
&\quad-\frac{\left(e^2fp\right)\text{Subst}\left(\int \frac{1}{d+ex}\,dx,x,x^2\right)}{2g^2(ef-dg)}+\frac{(efp)\text{Subst}\left(\int \frac{1}{f+gx}\,dx,x,x^2\right)}{2g(ef-dg)}
\end{aligned}$$

$$= -\frac{efp\log(d+ex^2)}{2g^2(ef-dg)} + \frac{f\log\left(c(d+ex^2)^p\right)}{2g^2(f+gx^2)} + \frac{efp\log(f+gx^2)}{2g^2(ef-dg)} + \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2g^2} + \frac{p\mathrm{Li}_2\left(-\frac{g(d+ex^2)}{ef-dg}\right)}{2g^2}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.85

$$\int \frac{x^3\log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \frac{\frac{efp\log(d+ex^2)}{-ef+dg} + \frac{f\log\left(c(d+ex^2)^p\right)}{f+gx^2} + \frac{efp\log(f+gx^2)}{ef-dg} + \log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right) + p\,\mathrm{PolyLog}\left(2,\frac{g(d+ex^2)}{-ef+dg}\right)}{2g^2}$$

```
[In] Integrate[(x^3*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] ((e*f*p*Log[d + e*x^2])/(-(e*f) + d*g) + (f*Log[c*(d + e*x^2)^p])/(f + g*x^
2) + (e*f*p*Log[f + g*x^2])/(e*f - d*g) + Log[c*(d + e*x^2)^p]*Log[(e*(f +
g*x^2))/(e*f - d*g)] + p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)])/(2*g^2
)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.08 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.46

method	result
parts	$\frac{f\ln\left(c\left(e\,x^2+d\right)^p\right)}{2g^2\left(g\,x^2+f\right)} + \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)\ln\left(g\,x^2+f\right)}{2g^2} - pe\left(\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(e\_Z^2+d\right)}\left(\ln(x-\_\alpha)\ln\left(g\,x^2+f\right)-\ln(x-\_\alpha)\left(\ln\left(\frac{\mathrm{RootOf}(}{\mathrm{Roo}}\right.\right.\right.}{}\right.$
risch	$\frac{\ln\left(\left(e\,x^2+d\right)^p\right)f}{2g^2\left(g\,x^2+f\right)} + \frac{\ln\left(\left(e\,x^2+d\right)^p\right)\ln\left(g\,x^2+f\right)}{2g^2} - \frac{p\left(\sum_{\_\alpha=\mathrm{RootOf}\left(e\_Z^2+d\right)}\left(\ln(x-\_\alpha)\ln\left(g\,x^2+f\right)-\ln(x-\_\alpha)\left(\ln\left(\frac{\mathrm{RootOf}\left(e\_\right.}{\mathrm{RootOf}\left(e\right.}\right.\right.\right.\right.}{}$

```
[In] int(x^3*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f*ln(c*(e*x^2+d)^p)/g^2/(g*x^2+f)+1/2*ln(c*(e*x^2+d)^p)/g^2*ln(g*x^2+f)
-p*e*(1/2/g^2/e*sum(ln(x-_alpha)*ln(g*x^2+f)-ln(x-_alpha)*(ln((RootOf(_Z^2*
e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*
e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-
```

```
x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)))-dilog((RootOf(
_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_a
lpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,i
ndex=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)),_alpha=
RootOf(_Z^2*e+d))+f/g^2*(-1/2/(d*g-e*f)*ln(e*x^2+d)+1/2/(d*g-e*f)*ln(g*x^2+
f)))
```

Fricas [F]

$$\int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \frac{x^3 \log \left((ex^2+d)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral(x^3*log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \frac{x^3 \log \left((ex^2+d)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
[Out] integrate(x^3*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

Giac [F]

$$\int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2} \, dx = \int \frac{x^3 \log \left((ex^2+d)^p c\right)}{(gx^2+f)^2} \, dx$$

```
[In] integrate(x^3*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate(x^3*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2} \, dx = \int \frac{x^3 \ln \left(c \, (e \, x^2+d)^p\right)}{(g \, x^2+f)^2} \, dx$$

```
[In] int((x^3*log(c*(d + e*x^2)^p))/(f + g*x^2)^2,x)
[Out] int((x^3*log(c*(d + e*x^2)^p))/(f + g*x^2)^2, x)
```

3.350 $\int \frac{x \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{x \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2} dx = \frac{ep\log(d+ex^2)}{2g(ef-dg)} - \frac{\log\left(c(d+ex^2)^p\right)}{2g(f+gx^2)} - \frac{ep\log(f+gx^2)}{2g(ef-dg)}$$

```
[Out] 1/2*e*p*ln(e*x^2+d)/g/(-d*g+e*f)-1/2*ln(c*(e*x^2+d)^p)/g/(g*x^2+f)-1/2*e*p*
ln(g*x^2+f)/g/(-d*g+e*f)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2525, 2442, 36, 31}

$$\int \frac{x \log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2} dx = -\frac{\log\left(c(d+ex^2)^p\right)}{2g(f+gx^2)} + \frac{ep\log(d+ex^2)}{2g(ef-dg)} - \frac{ep\log(f+gx^2)}{2g(ef-dg)}$$

```
[In] Int[(x*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] (e*p*Log[d + e*x^2])/(2*g*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(2*g*(f + g*x
^2)) - (e*p*Log[f + g*x^2])/(2*g*(e*f - d*g))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{(f+gx)^2}\,dx, x, x^2\right)\\
&= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2g\left(f+gx^2\right)} + \frac{(ep)\text{Subst}\left(\int \frac{1}{(d+ex)(f+gx)}\,dx, x, x^2\right)}{2g}\\
&= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2g\left(f+gx^2\right)} - \frac{(ep)\text{Subst}\left(\int \frac{1}{f+gx}\,dx, x, x^2\right)}{2(ef-dg)} + \frac{\left(e^2p\right)\text{Subst}\left(\int \frac{1}{d+ex}\,dx, x, x^2\right)}{2g(ef-dg)}\\
&= \frac{ep\log\left(d+ex^2\right)}{2g(ef-dg)} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{2g\left(f+gx^2\right)} - \frac{ep\log\left(f+gx^2\right)}{2g(ef-dg)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$
\int \frac{x\log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx = \frac{\frac{ep\log\left(d+ex^2\right)}{ef-dg} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f+gx^2} + \frac{ep\log\left(f+gx^2\right)}{-ef+dg}}{2g}
$$

```
[In] Integrate[(x*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] ((e*p*Log[d + e*x^2])/(e*f - d*g) - Log[c*(d + e*x^2)^p]/(f + g*x^2) + (e*p
*Log[f + g*x^2])/(-(e*f) + d*g))/(2*g)
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

method	result
parts	$-\frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{2g\left(g\,x^{2}+f\right)}+\frac{pe\left(-\frac{\ln\left(e\,x^{2}+d\right)}{2\left(dg-ef\right)}+\frac{\ln\left(g\,x^{2}+f\right)}{2dg-2ef}\right)}{g}$
parallelrisch	$-\frac{\ln\left(e\,x^{2}+d\right)x^{2}e^{2}gp-\ln\left(g\,x^{2}+f\right)x^{2}e^{2}gp+\ln\left(e\,x^{2}+d\right)e^{2}fp-\ln\left(g\,x^{2}+f\right)e^{2}fp+\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)deg-\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)e^{2}f}{2\left(dg-ef\right)\left(g\,x^{2}+f\right)eg}$
risch	$-\frac{\ln\left(\left(e\,x^{2}+d\right)^{p}\right)}{2g\left(g\,x^{2}+f\right)}-\frac{i\pi dg\ \mathrm{csgn}\left(i\left(e\,x^{2}+d\right)^{p}\right)\mathrm{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right)^{2}-i\pi dg\ \mathrm{csgn}\left(i\left(e\,x^{2}+d\right)^{p}\right)\mathrm{csgn}\left(ic\left(e\,x^{2}+d\right)^{p}\right)\mathrm{csgn}\left(ic\right)-i\pi\mathrm{csgn}(\cdots}{}$

[In] int(x*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x,method=_RETURNVERBOSE)

[Out] -1/2*ln(c*(e*x^2+d)^p)/g/(g*x^2+f)+p*e/g*(-1/2/(d*g-e*f)*ln(e*x^2+d)+1/2/(d*g-e*f)*ln(g*x^2+f))

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10

$$\int \frac{x\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \frac{(egpx^2+dgp)\log\left(ex^2+d\right)-(egpx^2+efp)\log\left(gx^2+f\right)-(ef-dg)\log\left(c\right)}{2\left(ef^2g-dfg^2+(efg^2-dg^3)x^2\right)}$$

[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")

[Out] 1/2*((e*g*p*x^2 + d*g*p)*log(e*x^2 + d) - (e*g*p*x^2 + e*f*p)*log(g*x^2 + f) - (e*f - d*g)*log(c))/(e*f^2*g - d*f*g^2 + (e*f*g^2 - d*g^3)*x^2)

Sympy [F(-1)]

Timed out.

$$\int \frac{x\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \text{Timed out}$$

[In] integrate(x*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int \frac{x \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \frac{ep\left(\frac{\log(ex^2+d)}{ef-dg} - \frac{\log(gx^2+f)}{ef-dg}\right)}{2\,g} - \frac{\log\left((ex^2+d)^p c\right)}{2\,(gx^2+f)g}$$

```
[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] 1/2*e*p*(log(e*x^2 + d)/(e*f - d*g) - log(g*x^2 + f)/(e*f - d*g))/g - 1/2*l
og((e*x^2 + d)^p*c)/((g*x^2 + f)*g)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.52

$$\int \frac{x \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = -\frac{ep\log(ex^2+d)}{2\,(efg+(ex^2+d)g^2-dg^2)} + \frac{ep\log(ex^2+d)}{2\,(efg-dg^2)} - \frac{ep\log(ef+(ex^2+d)g-dg)}{2\,(efg-dg^2)} - \frac{e\log(c)}{2\,(efg+(ex^2+d)g^2-dg^2)}$$

```
[In] integrate(x*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] -1/2*e*p*log(e*x^2 + d)/(e*f*g + (e*x^2 + d)*g^2 - d*g^2) + 1/2*e*p*log(e*x
^2 + d)/(e*f*g - d*g^2) - 1/2*e*p*log(e*f + (e*x^2 + d)*g - d*g)/(e*f*g - d
*g^2) - 1/2*e*log(c)/(e*f*g + (e*x^2 + d)*g^2 - d*g^2)
```

Mupad [B] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \frac{x \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = -\frac{\ln\left(c\,(e\,x^2+d)^p\right)}{2\,g\,\left(g\,x^2+f\right)} - \frac{e\,p\,\mathrm{atan}\left(\frac{x^2\,(d\,g\,1\mathrm{i}-e\,f\,1\mathrm{i})}{2\,d\,f+d\,g\,x^2+e\,f\,x^2}\right)\,1\mathrm{i}}{d\,g^2-e\,f\,g}$$

```
[In] int((x*log(c*(d + e*x^2)^p))/(f + g*x^2)^2,x)
```

```
[Out] - log(c*(d + e*x^2)^p)/(2*g*(f + g*x^2)) - (e*p*atan((x^2*(d*g*1i - e*f*1i)
)/(2*d*f + d*g*x^2 + e*f*x^2))*1i)/(d*g^2 - e*f*g)
```

3.351 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x\left(f+gx^2\right)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 201

$$\begin{aligned}\int \frac{\log\left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)^2}\,dx = & -\frac{ep\log\left(d+ex^2\right)}{2f(ef-dg)}+\frac{\log\left(c(d+ex^2)^p\right)}{2f\left(f+gx^2\right)} \\ & +\frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{2f^2}+\frac{ep\log\left(f+gx^2\right)}{2f(ef-dg)} \\ & -\frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2f^2} \\ & -\frac{p\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{2f^2}+\frac{p\,\mathrm{PolyLog}\left(2,1+\frac{ex^2}{d}\right)}{2f^2}\end{aligned}$$

```
[Out] -1/2*e*p*ln(e*x^2+d)/f/(-d*g+e*f)+1/2*ln(c*(e*x^2+d)^p)/f/(g*x^2+f)+1/2*ln(
-e*x^2/d)*ln(c*(e*x^2+d)^p)/f^2+1/2*e*p*ln(g*x^2+f)/f/(-d*g+e*f)-1/2*ln(c*(
e*x^2+d)^p)*ln(e*(g*x^2+f)/(-d*g+e*f))/f^2-1/2*p*polylog(2,-g*(e*x^2+d)/(-d
*g+e*f))/f^2+1/2*p*polylog(2,1+e*x^2/d)/f^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules

used = {2525, 46, 2463, 2441, 2352, 2442, 36, 31, 2440, 2438}

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)^2}\,dx = -\frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2f^2} + \frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{2f^2} + \frac{\log\left(c(d+ex^2)^p\right)}{2f\left(f+gx^2\right)} - \frac{p\,\mathrm{PolyLog}\left(2,-\frac{g(ex^2+d)}{ef-dg}\right)}{2f^2} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{ex^2}{d}+1\right)}{2f^2} - \frac{ep\log\left(d+ex^2\right)}{2f(ef-dg)} + \frac{ep\log\left(f+gx^2\right)}{2f(ef-dg)}$$

```
[In] Int[Log[c*(d + e*x^2)^p]/(x*(f + g*x^2)^2),x]
```

```
[Out] -1/2*(e*p*Log[d + e*x^2])/(f*(e*f - d*g)) + Log[c*(d + e*x^2)^p]/(2*f*(f +
g*x^2)) + (Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/(2*f^2) + (e*p*Log[f + g
*x^2])/(2*f*(e*f - d*g)) - (Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f -
 d*g)])/(2*f^2) - (p*PolyLog[2, -((g*(d + e*x^2))/(e*f - d*g))])/(2*f^2) +
(p*PolyLog[2, 1 + (e*x^2)/d])/(2*f^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x(f+gx)^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{\log\left(c(d+ex)^p\right)}{f^2x} - \frac{g\log\left(c(d+ex)^p\right)}{f(f+gx)^2} - \frac{g\log\left(c(d+ex)^p\right)}{f^2(f+gx)}\right)\,dx, x, x^2\right)
\end{aligned}$$

$$= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^2\right)}{2f^2} - \frac{g\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx, x, x^2\right)}{2f^2} - \frac{g\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{(f+gx)^2}\,dx, x, x^2\right)}{2f}$$

$$= \frac{\log\left(c(d+ex^2)^p\right)}{2f\left(f+gx^2\right)} + \frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{2f^2} - \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2f^2} - \frac{(ep)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^2\right)}{2f^2} + \frac{(ep)\text{Subst}\left(\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx, x, x^2\right)}{2f^2} - \frac{(ep)\text{Subst}\left(\int \frac{1}{(d+ex)(f+gx)}\,dx, x, x^2\right)}{2f}$$

$$= \frac{\log\left(c(d+ex^2)^p\right)}{2f\left(f+gx^2\right)} + \frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{2f^2} - \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2f^2} + \frac{p\text{Li}_2\left(1+\frac{ex^2}{d}\right)}{2f^2} + \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex^2\right)}{2f^2} - \frac{(e^2p)\,\text{Subst}\left(\int \frac{1}{d+ex}\,dx, x, x^2\right)}{2f(ef-dg)} + \frac{(egp)\text{Subst}\left(\int \frac{1}{f+gx}\,dx, x, x^2\right)}{2f(ef-dg)}$$

$$= -\frac{ep\log\left(d+ex^2\right)}{2f(ef-dg)} + \frac{\log\left(c(d+ex^2)^p\right)}{2f\left(f+gx^2\right)} + \frac{\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{2f^2} + \frac{ep\log\left(f+gx^2\right)}{2f(ef-dg)} - \frac{\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{2f^2} - \frac{p\text{Li}_2\left(-\frac{g(d+ex^2)}{ef-dg}\right)}{2f^2} + \frac{p\text{Li}_2\left(1+\frac{ex^2}{d}\right)}{2f^2}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.85

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)^2}\,dx = \frac{\frac{efp\log(d+ex^2)}{-ef+dg} + \frac{f\log(c(d+ex^2)^p)}{f+gx^2} + \log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right) + \frac{efp\log(f+gx^2)}{ef-dg} - \log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+g\cdots}{ef-\cdots}\right.}{2f^2}$$

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x*(f + g*x^2)^2),x]
```

```
[Out] ((e*f*p*Log[d + e*x^2])/(-(e*f) + d*g) + (f*Log[c*(d + e*x^2)^p])/(f + g*x^
2) + Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + (e*f*p*Log[f + g*x^2])/(e*f -
 d*g) - Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)] - p*PolyLog[2
, (g*(d + e*x^2))/(-(e*f) + d*g)] + p*PolyLog[2, 1 + (e*x^2)/d])/(2*f^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.56 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.45

method	result
parts	$\frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)\ln\left(x\right)}{f^{2}}+\frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)}{2f\left(g\,x^{2}+f\right)}-\frac{\ln\left(c\left(e\,x^{2}+d\right)^{p}\right)\ln\left(g\,x^{2}+f\right)}{2f^{2}}-pe\left(-\frac{\ln\left(e\,x^{2}+d\right)}{2f\left(dg-ef\right)}+\frac{\ln\left(g\,x^{2}+f\right)}{2f\left(dg-ef\right)}+\frac{\ln\left(x\right)\left(\ln\left(\frac{-ex+}{\sqrt{-}}\right.\right.}{}\right.$
risch	Expression too large to display

```
[In] int(ln(c*(e*x^2+d)^p)/x/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ln(c*(e*x^2+d)^p)/f^2*ln(x)+1/2*ln(c*(e*x^2+d)^p)/f/(g*x^2+f)-1/2*ln(c*(e*x
^2+d)^p)/f^2*ln(g*x^2+f)-p*e*(-1/2/f/(d*g-e*f)*ln(e*x^2+d)+1/2/f/(d*g-e*f)*
ln(g*x^2+f)+2/f^2*(1/2*ln(x)*(ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+ln((e*x+
(-d*e)^(1/2))/(-d*e)^(1/2)))/e+1/2*(dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))
+dilog((e*x+(-d*e)^(1/2))/(-d*e)^(1/2)))/e)-1/2/f^2/e*sum(ln(x-_alpha)*ln(g
*x^2+f)-ln(x-_alpha)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-
x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2
*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha
*e*g-d*g+e*f,index=2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,inde
x=1)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((Roo
tOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_
Z*_alpha*e*g-d*g+e*f,index=2)),_alpha=RootOf(_Z^2*e+d)))
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)^2}\,dx=\int \frac{\log\left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)^2 x}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g^2*x^5 + 2*f*g*x^3 + f^2*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c\left(d+e x^2\right)^p\right)}{x \left(f+g x^2\right)^2} \, dx=\text{Timed out}$$

```
[In] integrate(ln(c*(e*x**2+d)**p)/x/(g*x**2+f)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.98

$$\int \frac{\log \left(c\left(d+e x^2\right)^p\right)}{x \left(f+g x^2\right)^2} \, dx= -\frac{1}{2}\,ep\left(\frac{\log\left(ex^2+d\right)}{ef^2-dfg}-\frac{\log\left(gx^2+f\right)}{ef^2-dfg}+\frac{2\,\log\left(\frac{ex^2}{d}+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-\frac{ex^2}{d}\right)}{ef^2}-\frac{\log\left(gx^2+f\right)\log\left(-\frac{egx^2}{ef-\cdots}\right)}{e\cdots}\right) +\frac{1}{2}\left(\frac{1}{fgx^2+f^2}-\frac{\log\left(gx^2+f\right)}{f^2}+\frac{\log\left(x^2\right)}{f^2}\right)\log\left(\left(ex^2+d\right)^pc\right)$$

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f)^2,x, algorithm="maxima")
[Out] -1/2*e*p*(log(e*x^2 + d)/(e*f^2 - d*f*g) - log(g*x^2 + f)/(e*f^2 - d*f*g) +
 (2*log(e*x^2/d + 1)*log(x) + dilog(-e*x^2/d))/(e*f^2) - (log(g*x^2 + f)*lo
g(-(e*g*x^2 + e*f)/(e*f - d*g) + 1) + dilog((e*g*x^2 + e*f)/(e*f - d*g)))/(
e*f^2)) + 1/2*(1/(f*g*x^2 + f^2) - log(g*x^2 + f)/f^2 + log(x^2)/f^2)*log((
e*x^2 + d)^p*c)
```

Giac [F]

$$\int \frac{\log \left(c\left(d+e x^2\right)^p\right)}{x \left(f+g x^2\right)^2} \, dx=\int \frac{\log\left(\left(ex^2+d\right)^pc\right)}{\left(gx^2+f\right)^2x}\, dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/x/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x\left(f+gx^2\right)^2}\,dx=\int \frac{\ln \left(c\left(e\,x^2+d\right)^p\right)}{x\left(g\,x^2+f\right)^2}\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)/(x*(f + g*x^2)^2),x)
[Out] int(log(c*(d + e*x^2)^p)/(x*(f + g*x^2)^2), x)
```

3.352 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^3\left(f+gx^2\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 251

$$\begin{aligned}\int \frac{\log\left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)^2}\,dx={}&\frac{ep\log(x)}{df^2}-\frac{ep\log\left(d+ex^2\right)}{2df^2}+\frac{egp\log\left(d+ex^2\right)}{2f^2(ef-dg)}-\frac{\log\left(c(d+ex^2)^p\right)}{2f^2x^2}\\&-\frac{g\log\left(c(d+ex^2)^p\right)}{2f^2\left(f+gx^2\right)}-\frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c(d+ex^2)^p\right)}{f^3}\\&-\frac{egp\log\left(f+gx^2\right)}{2f^2(ef-dg)}+\frac{g\log\left(c(d+ex^2)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{f^3}\\&+\frac{gp\operatorname{PolyLog}\left(2,-\frac{g(d+ex^2)}{ef-dg}\right)}{f^3}-\frac{gp\operatorname{PolyLog}\left(2,1+\frac{ex^2}{d}\right)}{f^3}\end{aligned}$$

```
[Out] e*p*ln(x)/d/f^2-1/2*e*p*ln(e*x^2+d)/d/f^2+1/2*e*g*p*ln(e*x^2+d)/f^2/(-d*g+e
*f)-1/2*ln(c*(e*x^2+d)^p)/f^2/x^2-1/2*g*ln(c*(e*x^2+d)^p)/f^2/(g*x^2+f)-g*l
n(-e*x^2/d)*ln(c*(e*x^2+d)^p)/f^3-1/2*e*g*p*ln(g*x^2+f)/f^2/(-d*g+e*f)+g*ln
(c*(e*x^2+d)^p)*ln(e*(g*x^2+f)/(-d*g+e*f))/f^3+g*p*polylog(2,-g*(e*x^2+d)/(
-d*g+e*f))/f^3-g*p*polylog(2,1+e*x^2/d)/f^3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2525, 46, 2463, 2442, 36, 29, 31, 2441, 2352, 2440, 2438}

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^3 \left(f+gx^2\right)^2} \, dx = -\frac{g \log \left(-\frac{ex^2}{d}\right) \log \left(c(d+ex^2)^p\right)}{f^3} + \frac{g \log \left(c(d+ex^2)^p\right) \log \left(\frac{e(f+gx^2)}{ef-dg}\right)}{f^3} - \frac{g \log \left(c(d+ex^2)^p\right)}{2f^2 \left(f+gx^2\right)} - \frac{\log \left(c(d+ex^2)^p\right)}{2f^2x^2} + \frac{gp \operatorname{PolyLog}\left(2,-\frac{g(ex^2+d)}{ef-dg}\right)}{f^3} - \frac{gp \operatorname{PolyLog}\left(2,\frac{ex^2}{d}+1\right)}{f^3} + \frac{egp \log \left(d+ex^2\right)}{2f^2(ef-dg)} - \frac{egp \log \left(f+gx^2\right)}{2f^2(ef-dg)} - \frac{ep \log \left(d+ex^2\right)}{2df^2} + \frac{ep \log(x)}{df^2}$$

```
[In] Int[Log[c*(d + e*x^2)^p]/(x^3*(f + g*x^2)^2),x]
```

```
[Out] (e*p*Log[x])/(d*f^2) - (e*p*Log[d + e*x^2])/(2*d*f^2) + (e*g*p*Log[d + e*x^2])/(2*f^2*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(2*f^2*x^2) - (g*Log[c*(d + e*x^2)^p])/(2*f^2*(f + g*x^2)) - (g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p])/f^3 - (e*g*p*Log[f + g*x^2])/(2*f^2*(e*f - d*g)) + (g*Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d*g)])/f^3 + (g*p*PolyLog[2, -((g*(d + e*x^2))/(e*f - d*g))])/f^3 - (g*p*PolyLog[2, 1 + (e*x^2)/d])/f^3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x^2(f+gx)^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{\log\left(c(d+ex)^p\right)}{f^2x^2} - \frac{2g\log\left(c(d+ex)^p\right)}{f^3x} + \frac{g^2\log\left(c(d+ex)^p\right)}{f^2(f+gx)^2} + \frac{2g^2\log\left(c(d+ex)^p\right)}{f^3(f+gx)}\right)dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^2}\,dx, x, x^2\right)}{2f^2} - \frac{g\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^2\right)}{f^3} + \frac{g^2\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx, x, x^2\right)}{f^3} + \frac{g^2\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{(f+gx)^2}\,dx, x, x^2\right)}{2f^2}\\
&= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2f^2x^2} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{2f^2\left(f+gx^2\right)} - \frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^3} + \frac{g\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{f^3}\\
&\quad + \frac{(ep)\text{Subst}\left(\int \frac{1}{x(d+ex)}\,dx, x, x^2\right)}{2f^2} + \frac{(egp)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^2\right)}{f^3} - \frac{(egp)\text{Subst}\left(\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx, x, x^2\right)}{f^3} + \frac{(egp)\text{Subst}\left(\int \frac{1}{(d+ex)(f+gx)}\,dx, x, x^2\right)}{2f^2}\\
&= -\frac{\log\left(c\left(d+ex^2\right)^p\right)}{2f^2x^2} - \frac{g\log\left(c\left(d+ex^2\right)^p\right)}{2f^2\left(f+gx^2\right)} - \frac{g\log\left(-\frac{ex^2}{d}\right)\log\left(c\left(d+ex^2\right)^p\right)}{f^3}\\
&\quad + \frac{g\log\left(c\left(d+ex^2\right)^p\right)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{f^3} - \frac{gp\text{Li}_2\left(1+\frac{ex^2}{d}\right)}{f^3} + \frac{(ep)\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2df^2}\\
&\quad - \frac{\left(e^2p\right)\text{Subst}\left(\int \frac{1}{d+ex}\,dx, x, x^2\right)}{2df^2} - \frac{(gp)\text{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex^2\right)}{f^3}\\
&\quad + \frac{\left(e^2gp\right)\text{Subst}\left(\int \frac{1}{d+ex}\,dx, x, x^2\right)}{2f^2(ef-dg)} - \frac{\left(eg^2p\right)\text{Subst}\left(\int \frac{1}{f+gx}\,dx, x, x^2\right)}{2f^2(ef-dg)}
\end{aligned}
$$

$$= \frac{ep\log(x)}{df^2} - \frac{ep\log(d+ex^2)}{2df^2} + \frac{egp\log(d+ex^2)}{2f^2(ef-dg)} - \frac{\log(c(d+ex^2)^p)}{2f^2x^2} - \frac{g\log(c(d+ex^2)^p)}{2f^2(f+gx^2)} - \frac{g\log\left(-\frac{ex^2}{d}\right)\log(c(d+ex^2)^p)}{f^3} - \frac{egp\log(f+gx^2)}{2f^2(ef-dg)} + \frac{g\log(c(d+ex^2)^p)\log\left(\frac{e(f+gx^2)}{ef-dg}\right)}{f^3} + \frac{gp\mathrm{Li}_2\left(-\frac{g(d+ex^2)}{ef-dg}\right)}{f^3} - \frac{gp\mathrm{Li}_2\left(1+\frac{ex^2}{d}\right)}{f^3}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.89

$$\int \frac{\log(c(d+ex^2)^p)}{x^3(f+gx^2)^2}\,dx = \frac{\frac{2efp\log(x)}{d} - \frac{efp\log(d+ex^2)}{d} + \frac{efgp\log(d+ex^2)}{ef-dg} - \frac{f\log(c(d+ex^2)^p)}{x^2} - \frac{fg\log(c(d+ex^2)^p)}{f+gx^2} - 2g\log\left(-\frac{ex^2}{d}\right)\log(c(d+ex^2)$$

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x^3*(f + g*x^2)^2),x]
```

```
[Out] ((2*e*f*p*Log[x])/d - (e*f*p*Log[d + e*x^2])/d + (e*f*g*p*Log[d + e*x^2])/(
e*f - d*g) - (f*Log[c*(d + e*x^2)^p])/x^2 - (f*g*Log[c*(d + e*x^2)^p])/(f +
 g*x^2) - 2*g*Log[-((e*x^2)/d)]*Log[c*(d + e*x^2)^p] + (e*f*g*p*Log[f + g*x
^2])/(-(e*f) + d*g) + 2*g*Log[c*(d + e*x^2)^p]*Log[(e*(f + g*x^2))/(e*f - d
*g)] + 2*g*p*PolyLog[2, (g*(d + e*x^2))/(-(e*f) + d*g)] - 2*g*p*PolyLog[2,
1 + (e*x^2)/d])/(2*f^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.67 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.13

method	result
parts	$-\frac{\ln(c(e\,x^2+d)^p)}{2f^2x^2} - \frac{2\ln(c(e\,x^2+d)^p)g\ln(x)}{f^3} - \frac{g\ln(c(e\,x^2+d)^p)}{2f^2(g\,x^2+f)} + \frac{\ln(c(e\,x^2+d)^p)g\ln(g\,x^2+f)}{f^3} - pe\left(-\frac{4g\left(\frac{\ln(x)\left(\ln\left(\frac{-e}{}\right.\right.}{}\right.}{}\right.$
risch	Expression too large to display

```
[In] int(ln(c*(e*x^2+d)^p)/x^3/(g*x^2+f)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(c*(e*x^2+d)^p)/f^2/x^2-2*ln(c*(e*x^2+d)^p)/f^3*g*ln(x)-1/2*g*ln(c*(
e*x^2+d)^p)/f^2/(g*x^2+f)+ln(c*(e*x^2+d)^p)*g/f^3*ln(g*x^2+f)-p*e*(-4*g/f^3
*(1/2*ln(x)*(ln((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+ln((e*x+(-d*e)^(1/2))/(-d
*e)^(1/2)))/e+1/2*(dilog((-e*x+(-d*e)^(1/2))/(-d*e)^(1/2))+dilog((e*x+(-d*e
)^(1/2))/(-d*e)^(1/2)))/e)+g/f^3/e*sum(ln(x-_alpha)*ln(g*x^2+f)-ln(x-_alpha
)*(ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/RootOf(_Z
^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))+ln((RootOf(_Z^2*e*g+2*_Z*_alpha*e*
g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=
2)))-dilog((RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1)-x+_alpha)/Root
Of(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*f,index=1))-dilog((RootOf(_Z^2*e*g+2*_Z*_
alpha*e*g-d*g+e*f,index=2)-x+_alpha)/RootOf(_Z^2*e*g+2*_Z*_alpha*e*g-d*g+e*
f,index=2)),_alpha=RootOf(_Z^2*e+d))-1/f^2*(-1/2*(2*d*g-e*f)/d/(d*g-e*f)*ln
(e*x^2+d)+1/d*ln(x)+1/2*g/(d*g-e*f)*ln(g*x^2+f)))
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{(gx^2+f)^2 x^3}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral(log((e*x^2 + d)^p*c)/(g^2*x^7 + 2*f*g*x^5 + f^2*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(e*x**2+d)**p)/x**3/(g*x**2+f)**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.18

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)^2}\,dx =$$

$$-\frac{1}{2}\left(f\left(\frac{e\log\left(ex^2+d\right)}{def^3-d^2f^2g}-\frac{g\log\left(gx^2+f\right)}{ef^4-df^3g}-\frac{\log\left(x^2\right)}{df^3}\right)-2\,g\left(\frac{\log\left(ex^2+d\right)}{ef^3-df^2g}-\frac{\log\left(gx^2+f\right)}{ef^3-df^2g}\right)-\frac{2\left(2\log\right.}{}\right.$$

$$-\frac{1}{2}\left(\frac{2\,gx^2+f}{f^2gx^4+f^3x^2}-\frac{2\,g\log\left(gx^2+f\right)}{f^3}+\frac{2\,g\log\left(x^2\right)}{f^3}\right)\log\left(\left(ex^2+d\right)^p c\right)$$

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(f*(e*log(e*x^2 + d)/(d*e*f^3 - d^2*f^2*g) - g*log(g*x^2 + f)/(e*f^4 -
 d*f^3*g) - log(x^2)/(d*f^3)) - 2*g*(log(e*x^2 + d)/(e*f^3 - d*f^2*g) - log
(g*x^2 + f)/(e*f^3 - d*f^2*g)) - 2*(2*log(e*x^2/d + 1)*log(x) + dilog(-e*x^
2/d))*g/(e*f^3) + 2*(log(g*x^2 + f)*log(-(e*g*x^2 + e*f)/(e*f - d*g) + 1) +
 dilog((e*g*x^2 + e*f)/(e*f - d*g)))*g/(e*f^3))*e*p - 1/2*((2*g*x^2 + f)/(f
^2*g*x^4 + f^3*x^2) - 2*g*log(g*x^2 + f)/f^3 + 2*g*log(x^2)/f^3)*log((e*x^2
 + d)^p*c)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)^2}\,dx=\int \frac{\log\left((ex^2+d)^pc\right)}{(gx^2+f)^2x^3}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/x^3/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^3\left(f+gx^2\right)^2}\,dx=\int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{x^3\left(g\,x^2+f\right)^2}\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)/(x^3*(f + g*x^2)^2),x)
```

```
[Out] int(log(c*(d + e*x^2)^p)/(x^3*(f + g*x^2)^2), x)
```

3.353 $\int \frac{x^4 \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 802

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = -\frac{2px}{g^2} + \frac{2\sqrt{d}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2} + \frac{\sqrt{d}\sqrt{e}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g^2(ef-dg)} - \frac{e(-f)^{3/2}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{5/2}(ef-dg)} - \frac{3\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{5/2}} + \frac{3\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}} + \frac{3\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}} + \frac{e(-f)^{3/2}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2g^{5/2}(ef-dg)} + \frac{x\log\left(c(d+ex^2)^p\right)}{g^2} - \frac{f\log\left(c(d+ex^2)^p\right)}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{f\log\left(c(d+ex^2)^p\right)}{4g^{5/2}\left(\sqrt{-f}+\sqrt{g}x\right)} - \frac{3\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2g^{5/2}} + \frac{3i\sqrt{f}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2g^{5/2}} - \frac{3i\sqrt{f}p\,\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4g^{5/2}} - \frac{3i\sqrt{f}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4g^{5/2}}$$

```
[Out] -2*p*x/g^2+x*ln(c*(e*x^2+d)^p)/g^2-1/2*e*(-f)^(3/2)*p*ln((-f)^(1/2)-x*g^(1/
2))/g^(5/2)/(-d*g+e*f)+1/2*e*(-f)^(3/2)*p*ln((-f)^(1/2)+x*g^(1/2))/g^(5/2)/
(-d*g+e*f)+2*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/g^2/e^(1/2)+f*p*arctan(x*e
^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/g^2/(-d*g+e*f)-3/2*arctan(x*g^(1/2)/f^(1/2)
)*ln(c*(e*x^2+d)^p)*f^(1/2)/g^(5/2)-3*p*arctan(x*g^(1/2)/f^(1/2))*ln(2*f^(1
/2)/(f^(1/2)-I*x*g^(1/2)))*f^(1/2)/g^(5/2)+3/2*p*arctan(x*g^(1/2)/f^(1/2))*
ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/
2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))*f^(1/2)/g^(5/2)+3/2*p*arctan(x*g^(1/2)/f^(1
/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e
^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))*f^(1/2)/g^(5/2)+3/2*I*p*polylog(2,1-2*f
^(1/2)/(f^(1/2)-I*x*g^(1/2)))*f^(1/2)/g^(5/2)-3/4*I*p*polylog(2,1+2*((-d)^(
```

```
1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-
d)^(1/2)*g^(1/2)))*f^(1/2)/g^(5/2)-3/4*I*p*polylog(2,1-2*((-d)^(1/2)+x*e^(1
/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^
(1/2)))*f^(1/2)/g^(5/2)-1/4*f*ln(c*(e*x^2+d)^p)/g^(5/2)/((-f)^(1/2)-x*g^(1/
2))+1/4*f*ln(c*(e*x^2+d)^p)/g^(5/2)/((-f)^(1/2)+x*g^(1/2))
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 802, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {2526, 2498, 327, 211, 2521, 2513, 815, 649, 266, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$\begin{aligned}
\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2} \, dx = &-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)(-f)^{3/2}}{2g^{5/2}(ef-dg)}+\frac{ep\log\left(\sqrt{g}x+\sqrt{-f}\right)(-f)^{3/2}}{2g^{5/2}(ef-dg)} \\
&-\frac{2px}{g^2}+\frac{\sqrt{d}\sqrt{e}fp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g^2(ef-dg)}+\frac{2\sqrt{d}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2} \\
&-\frac{3\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{5/2}} \\
&+\frac{3\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}} \\
&+\frac{3\sqrt{f}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}} \\
&+\frac{x\log\left(c(ex^2+d)^p\right)}{g^2}-\frac{3\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(ex^2+d)^p\right)}{2g^{5/2}} \\
&-\frac{f\log\left(c(ex^2+d)^p\right)}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{f\log\left(c(ex^2+d)^p\right)}{4g^{5/2}\left(\sqrt{g}x+\sqrt{-f}\right)} \\
&+\frac{3i\sqrt{f}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2g^{5/2}} \\
&-\frac{3i\sqrt{f}p\,\text{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}+1\right)}{4g^{5/2}} \\
&-\frac{3i\sqrt{f}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4g^{5/2}}
\end{aligned}$$

```
[In] Int[(x^4*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] (-2*p*x)/g^2 + (2*Sqrt[d]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*g^2) + (S
qrt[d]*Sqrt[e]*f*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(g^2*(e*f - d*g)) - (e*(-f)
```

```
^(3/2)*p*Log[Sqrt[-f] - Sqrt[g]*x])/(2*g^(5/2)*(e*f - d*g)) - (3*Sqrt[f]*p*
ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(5/
2) + (3*Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt
[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqr
t[g]*x))])/(2*g^(5/2)) + (3*Sqrt[f]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sq
rt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g
])*(Sqrt[f] - I*Sqrt[g]*x))])/(2*g^(5/2)) + (e*(-f)^(3/2)*p*Log[Sqrt[-f] +
Sqrt[g]*x])/(2*g^(5/2)*(e*f - d*g)) + (x*Log[c*(d + e*x^2)^p])/g^2 - (f*Log
[c*(d + e*x^2)^p])/(4*g^(5/2)*(Sqrt[-f] - Sqrt[g]*x)) + (f*Log[c*(d + e*x^2
)^p])/(4*g^(5/2)*(Sqrt[-f] + Sqrt[g]*x)) - (3*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sq
rt[f]]*Log[c*(d + e*x^2)^p])/(2*g^(5/2)) + (((3*I)/2)*Sqrt[f]*p*PolyLog[2,
1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/g^(5/2) - (((3*I)/4)*Sqrt[f]*p*Po
lyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f]
 - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2) - (((3*I)/4)*Sqrt[f
]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*S
qrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/g^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
```

```
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\log\left(c(d+ex^2)^p\right)}{g^2}+\frac{f^2\log\left(c(d+ex^2)^p\right)}{g^2\left(f+gx^2\right)^2}-\frac{2f\log\left(c(d+ex^2)^p\right)}{g^2\left(f+gx^2\right)}\right)dx\\
&= \frac{\int \log\left(c(d+ex^2)^p\right)dx}{g^2}-\frac{(2f)\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}dx}{g^2}+\frac{f^2\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}dx}{g^2}\\
&= \frac{x\log\left(c(d+ex^2)^p\right)}{g^2}-\frac{2\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{5/2}}\\
&\quad+\frac{f^2\int \left(-\frac{g\log\left(c(d+ex^2)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}-gx\right)^2}-\frac{g\log\left(c(d+ex^2)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}+gx\right)^2}-\frac{g\log\left(c(d+ex^2)^p\right)}{2f\left(-fg-g^2x^2\right)}\right)dx}{g^2}\\
&\quad-\frac{(2ep)\int \frac{x^2}{d+ex^2}dx}{g^2}+\frac{(4efp)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}(d+ex^2)}dx}{g^2}
\end{aligned}
$$

$$=-\frac{2px}{g^2}+\frac{x\log\left(c(d+ex^2)^p\right)}{g^2}-\frac{2\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{g^{5/2}}-\frac{f\int\frac{\log(c(d+ex^2)^p)}{(\sqrt{-f}\sqrt{g}-gx)^2}\,dx}{4g}$$
$$-\frac{f\int\frac{\log(c(d+ex^2)^p)}{(\sqrt{-f}\sqrt{g}+gx)^2}\,dx}{4g}-\frac{f\int\frac{\log(c(d+ex^2)^p)}{-fg-g^2x^2}\,dx}{2g}+\frac{(4e\sqrt{f}p)\int\frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{g^{5/2}}+\frac{(2dp)\int\frac{1}{d+ex^2}\,dx}{g^2}$$

$$=-\frac{2px}{g^2}+\frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}+\frac{x\log\left(c(d+ex^2)^p\right)}{g^2}-\frac{f\log\left(c(d+ex^2)^p\right)}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}x\right)}$$
$$+\frac{f\log\left(c(d+ex^2)^p\right)}{4g^{5/2}\left(\sqrt{-f}+\sqrt{g}x\right)}-\frac{3\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2g^{5/2}}$$
$$+\frac{(4e\sqrt{f}p)\int\left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}(\sqrt{-d}-\sqrt{e}x)}+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}(\sqrt{-d}+\sqrt{e}x)}\right)dx}{g^{5/2}}$$
$$+\frac{(efp)\int\frac{x}{(\sqrt{-f}\sqrt{g}-gx)(d+ex^2)}\,dx}{2g^2}$$
$$-\frac{(efp)\int\frac{x}{(\sqrt{-f}\sqrt{g}+gx)(d+ex^2)}\,dx}{2g^2}-\frac{(efp)\int\frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}g^{3/2}(d+ex^2)}\,dx}{g}$$

$$=-\frac{2px}{g^2}+\frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2}+\frac{x\log\left(c(d+ex^2)^p\right)}{g^2}-\frac{f\log\left(c(d+ex^2)^p\right)}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}x\right)}$$
$$+\frac{f\log\left(c(d+ex^2)^p\right)}{4g^{5/2}\left(\sqrt{-f}+\sqrt{g}x\right)}-\frac{3\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2g^{5/2}}$$
$$-\frac{\left(2\sqrt{e}\sqrt{f}p\right)\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{g^{5/2}}+\frac{\left(2\sqrt{e}\sqrt{f}p\right)\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{g^{5/2}}$$
$$-\frac{\left(e\sqrt{f}p\right)\int\frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{g^{5/2}}-\frac{(efp)\int\left(\frac{\sqrt{-f}}{(ef-dg)(\sqrt{-f}+\sqrt{g}x)}-\frac{-d\sqrt{g}-e\sqrt{-f}x}{\sqrt{g}(-ef+dg)(d+ex^2)}\right)dx}{2g^2}$$
$$+\frac{(efp)\int\left(\frac{\sqrt{-f}}{(ef-dg)(-\sqrt{-f}+\sqrt{g}x)}-\frac{d\sqrt{g}-e\sqrt{-f}x}{\sqrt{g}(-ef+dg)(d+ex^2)}\right)dx}{2g^2}$$

$$
\begin{aligned}
&= -\frac{2px}{g^2} + \frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2} - \frac{e(-f)^{3/2}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{5/2}(ef-dg)} \\
&- \frac{4\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{5/2}} \\
&+ \frac{2\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{5/2}} \\
&+ \frac{2\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{5/2}} \\
&+ \frac{e(-f)^{3/2}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2g^{5/2}(ef-dg)} + \frac{x\log\left(c(d+ex^2)^p\right)}{g^2} - \frac{f\log\left(c(d+ex^2)^p\right)}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}x\right)} \\
&+ \frac{f\log\left(c(d+ex^2)^p\right)}{4g^{5/2}\left(\sqrt{-f}+\sqrt{g}x\right)} - \frac{3\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2g^{5/2}} \\
&- \frac{(e\sqrt{f}p)\int\left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}(\sqrt{-d}-\sqrt{e}x)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}(\sqrt{-d}+\sqrt{e}x)}\right)dx}{g^{5/2}} + 2\frac{(2p)\int\frac{\log\left(\frac{2}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{1+\frac{gx^2}{f}}dx}{g^2} \\
&- \frac{(2p)\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{f}\left(-i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{g^2} - \frac{(2p)\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{f}\left(i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{g^2} \\
&- \frac{(efp)\int\frac{-d\sqrt{g}-e\sqrt{-f}x}{d+ex^2}dx}{2g^{5/2}(ef-dg)} + \frac{(efp)\int\frac{d\sqrt{g}-e\sqrt{-f}x}{d+ex^2}dx}{2g^{5/2}(ef-dg)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2px}{g^2} + \frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2} - \frac{e(-f)^{3/2}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{5/2}(ef-dg)} \\
&- \frac{4\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{5/2}} \\
&+ \frac{2\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{5/2}} \\
&+ \frac{2\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{5/2}} \\
&+ \frac{e(-f)^{3/2}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2g^{5/2}(ef-dg)} + \frac{x\log\left(c(d+ex^2)^p\right)}{g^2} \\
&- \frac{f\log\left(c(d+ex^2)^p\right)}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{f\log\left(c(d+ex^2)^p\right)}{4g^{5/2}\left(\sqrt{-f}+\sqrt{g}x\right)} \\
&- \frac{3\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2g^{5/2}} - \frac{i\sqrt{f}p\mathrm{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{5/2}} \\
&- \frac{i\sqrt{f}p\mathrm{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{5/2}} \\
&+ 2\frac{\left(2i\sqrt{f}p\right)\mathrm{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{g^{5/2}} + \frac{\left(\sqrt{e}\sqrt{f}p\right)\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2g^{5/2}} \\
&- \frac{\left(\sqrt{e}\sqrt{f}p\right)\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{2g^{5/2}} + 2\frac{(defp)\int\frac{1}{d+ex^2}\,dx}{2g^2(ef-dg)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2px}{g^2} + \frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2} + \frac{\sqrt{d}\sqrt{e}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g^2(ef-dg)} \\
&- \frac{e(-f)^{3/2}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{5/2}(ef-dg)} - \frac{3\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{5/2}} \\
&+ \frac{3\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}} \\
&+ \frac{3\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}} \\
&+ \frac{e(-f)^{3/2}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2g^{5/2}(ef-dg)} + \frac{x\log\left(c(d+ex^2)^p\right)}{g^2} - \frac{f\log\left(c(d+ex^2)^p\right)}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}x\right)} \\
&+ \frac{f\log\left(c(d+ex^2)^p\right)}{4g^{5/2}\left(\sqrt{-f}+\sqrt{g}x\right)} - \frac{3\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2g^{5/2}} \\
&+ \frac{2i\sqrt{f}p\mathrm{Li}_2\left(1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{5/2}} - \frac{i\sqrt{f}p\mathrm{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{5/2}} \\
&- \frac{i\sqrt{f}p\mathrm{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{g^{5/2}} - 2\frac{p\int\frac{\log\left(\frac{2}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{1+\frac{gx^2}{f}}dx}{2g^2} \\
&+ \frac{p\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{f}\left(-i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{2g^2} + \frac{p\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{f}\left(i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{2g^2}
\end{aligned}
$$

$$= -\frac{2px}{g^2} + \frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2} + \frac{\sqrt{d}\sqrt{e}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g^2(ef-dg)}$$
$$- \frac{e(-f)^{3/2}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{5/2}(ef-dg)} - \frac{3\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{5/2}}$$
$$+ \frac{3\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}}$$
$$+ \frac{3\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}}$$
$$+ \frac{e(-f)^{3/2}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2g^{5/2}(ef-dg)} + \frac{x\log\left(c(d+ex^2)^p\right)}{g^2} - \frac{f\log\left(c(d+ex^2)^p\right)}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}x\right)}$$
$$+ \frac{f\log\left(c(d+ex^2)^p\right)}{4g^{5/2}\left(\sqrt{-f}+\sqrt{g}x\right)} - \frac{3\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2g^{5/2}}$$
$$+ \frac{2i\sqrt{f}p\text{Li}_2\left(1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{5/2}} - \frac{3i\sqrt{f}p\text{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4g^{5/2}}$$
$$- \frac{3i\sqrt{f}p\text{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4g^{5/2}}$$
$$- 2\frac{\left(i\sqrt{f}p\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{2g^{5/2}}$$

$$= -\frac{2px}{g^2} + \frac{2\sqrt{d}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}g^2} + \frac{\sqrt{d}\sqrt{e}fp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g^2(ef-dg)} - \frac{e(-f)^{3/2}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{5/2}(ef-dg)} - \frac{3\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{g^{5/2}} + \frac{3\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}} + \frac{3\sqrt{f}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2g^{5/2}} + \frac{e(-f)^{3/2}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2g^{5/2}(ef-dg)} + \frac{x\log\left(c(d+ex^2)^p\right)}{g^2} - \frac{f\log\left(c(d+ex^2)^p\right)}{4g^{5/2}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{f\log\left(c(d+ex^2)^p\right)}{4g^{5/2}\left(\sqrt{-f}+\sqrt{g}x\right)} - \frac{3\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2g^{5/2}} + \frac{3i\sqrt{f}p\,\mathrm{Li}_2\left(1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2g^{5/2}} - \frac{3i\sqrt{f}p\,\mathrm{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4g^{5/2}} - \frac{3i\sqrt{f}p\,\mathrm{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4g^{5/2}}$$

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 915, normalized size of antiderivative = 1.14

$$\int \frac{x^4\log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \frac{-8\sqrt{g}px + \frac{8\sqrt{d}\sqrt{g}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{2\sqrt{-d}\sqrt{e}f\sqrt{g}p\log(\sqrt{-d}-\sqrt{e}x)}{-ef+dg} + \frac{2\sqrt{-d}\sqrt{e}f\sqrt{g}p\log(\sqrt{-d}+\sqrt{e}x)}{ef-dg} + \frac{2e\sqrt{-f}fp\log(\sqrt{-f}-\sqrt{g}x)}{ef-dg}}{}$$

```
[In] Integrate[(x^4*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] (-8*Sqrt[g]*p*x + (8*Sqrt[d]*Sqrt[g]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
 + (2*Sqrt[-d]*Sqrt[e]*f*Sqrt[g]*p*Log[Sqrt[-d] - Sqrt[e]*x])/(-(e*f) + d*g
) + (2*Sqrt[-d]*Sqrt[e]*f*Sqrt[g]*p*Log[Sqrt[-d] + Sqrt[e]*x])/(e*f - d*g)
+ (2*e*Sqrt[-f]*f*p*Log[Sqrt[-f] - Sqrt[g]*x])/(e*f - d*g) + (2*e*(-f)^(3/2
)*p*Log[Sqrt[-f] + Sqrt[g]*x])/(e*f - d*g) + (3*I)*Sqrt[f]*p*Log[(Sqrt[g]*(
Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*S
qrt[g]*x)/Sqrt[f]] + (3*I)*Sqrt[f]*p*Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(
(-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] -
(3*I)*Sqrt[f]*p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f]
+ Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - (3*I)*Sqrt[f]*p*Log[(
```

```
Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log
[1 + (I*Sqrt[g]*x)/Sqrt[f]] + 4*Sqrt[g]*x*Log[c*(d + e*x^2)^p] - (f*Log[c*(
d + e*x^2)^p])/(Sqrt[-f] - Sqrt[g]*x) + (f*Log[c*(d + e*x^2)^p])/(Sqrt[-f]
+ Sqrt[g]*x) - 6*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p] +
 (3*I)*Sqrt[f]*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt
[f] - I*Sqrt[-d]*Sqrt[g])] + (3*I)*Sqrt[f]*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] -
 I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])] - (3*I)*Sqrt[f]*p*Po
lyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sq
rt[g])] - (3*I)*Sqrt[f]*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqr
t[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])])/(4*g^(5/2))
```

Maple [F]

$$\int \frac{x^4 \ln \left(c(e\,x^2+d)^p\right)}{\left(g\,x^2+f\right)^2}dx$$

```
[In] int(x^4*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
[Out] int(x^4*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

Fricas [F]

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \frac{x^4 \log \left((ex^2+d)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral(x^4*log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**4*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \frac{x^4 \log \left((ex^2+d)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

```
[In] integrate(x^4*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \frac{x^4\,\ln \left(c\,{\left(e\,x^2+d\right)}^p\right)}{{\left(g\,x^2+f\right)}^2}\,dx$$

```
[In] int((x^4*log(c*(d + e*x^2)^p))/(f + g*x^2)^2,x)
```

```
[Out] int((x^4*log(c*(d + e*x^2)^p))/(f + g*x^2)^2, x)
```

3.354 $\int \frac{x^2 \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 746

$$
\begin{aligned}
\int \frac{x^2 \log\left(c\left(d+ex^2\right)^p\right)}{\left(f+gx^2\right)^2}\,dx &= -\frac{\sqrt{d}\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g(ef-dg)} - \frac{e\sqrt{-f}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{3/2}(ef-dg)} \\
&+ \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}g^{3/2}} \\
&- \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}g^{3/2}} \\
&- \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}g^{3/2}} \\
&+ \frac{e\sqrt{-f}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2g^{3/2}(ef-dg)} + \frac{\log\left(c\left(d+ex^2\right)^p\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}x\right)} \\
&- \frac{\log\left(c\left(d+ex^2\right)^p\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2\sqrt{f}g^{3/2}} \\
&- \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2\sqrt{f}g^{3/2}} \\
&+ \frac{ip\,\mathrm{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4\sqrt{f}g^{3/2}} \\
&+ \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4\sqrt{f}g^{3/2}}
\end{aligned}
$$

```
[Out] -p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/g/(-d*g+e*f)-1/2*e*p*ln((-f)^(
1/2)-x*g^(1/2))*(-f)^(1/2)/g^(3/2)/(-d*g+e*f)+1/2*e*p*ln((-f)^(1/2)+x*g^(1/
2))*(-f)^(1/2)/g^(3/2)/(-d*g+e*f)+1/2*arctan(x*g^(1/2)/f^(1/2))*ln(c*(e*x^2
+d)^p)/g^(3/2)/f^(1/2)+p*arctan(x*g^(1/2)/f^(1/2))*ln(2*f^(1/2)/(f^(1/2)-I*
x*g^(1/2)))/g^(3/2)/f^(1/2)-1/2*p*arctan(x*g^(1/2)/f^(1/2))*ln(-2*((-d)^(1/
2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)
^(1/2)*g^(1/2)))/g^(3/2)/f^(1/2)-1/2*p*arctan(x*g^(1/2)/f^(1/2))*ln(2*((-d)
^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+
(-d)^(1/2)*g^(1/2)))/g^(3/2)/f^(1/2)-1/2*I*p*polylog(2,1-2*f^(1/2)/(f^(1/2)
-I*x*g^(1/2)))/g^(3/2)/f^(1/2)+1/4*I*p*polylog(2,1+2*((-d)^(1/2)-x*e^(1/2))
*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2
)))/g^(3/2)/f^(1/2)+1/4*I*p*polylog(2,1-2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^
(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/g^(3/2)
/f^(1/2)+1/4*ln(c*(e*x^2+d)^p)/g^(3/2)/((-f)^(1/2)-x*g^(1/2))-1/4*ln(c*(e*x
^2+d)^p)/g^(3/2)/((-f)^(1/2)+x*g^(1/2))
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules

used = {2526, 2521, 2513, 815, 649, 211, 266, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$\int \frac{x^2 \log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2\sqrt{f}g^{3/2}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{2\sqrt{f}g^{3/2}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{2\sqrt{f}g^{3/2}} - \frac{\sqrt{d}\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g(ef-dg)} + \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}g^{3/2}} + \frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{ip\,\mathrm{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}+1\right)}{4\sqrt{f}g^{3/2}} + \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4\sqrt{f}g^{3/2}} - \frac{e\sqrt{-f}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{3/2}(ef-dg)} + \frac{e\sqrt{-f}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2g^{3/2}(ef-dg)} - \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2\sqrt{f}g^{3/2}}$$

```
[In] Int[(x^2*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] -((Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(g*(e*f - d*g))) - (e*Sqr
t[-f]*p*Log[Sqrt[-f] - Sqrt[g]*x])/(2*g^(3/2)*(e*f - d*g)) + (p*ArcTan[(Sqr
t[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*g^(3/2)
) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt
[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/
(2*Sqrt[f]*g^(3/2)) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]
*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] -
 I*Sqrt[g]*x))])/(2*Sqrt[f]*g^(3/2)) + (e*Sqrt[-f]*p*Log[Sqrt[-f] + Sqrt[g]
*x])/(2*g^(3/2)*(e*f - d*g)) + Log[c*(d + e*x^2)^p]/(4*g^(3/2)*(Sqrt[-f] -
Sqrt[g]*x)) - Log[c*(d + e*x^2)^p]/(4*g^(3/2)*(Sqrt[-f] + Sqrt[g]*x)) + (Ar
cTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/(2*Sqrt[f]*g^(3/2)) - ((I/2
)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(Sqrt[f]*g^(3/2))
+ ((I/4)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sq
rt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*g^(3/
2)) + ((I/4)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((
```

```
I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(Sqrt[f]*g
^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\text{integral} = \int \left(-\frac{f \log \left(c(d+ex^2)^p\right)}{g\left(f+gx^2\right)^2} + \frac{\log \left(c(d+ex^2)^p\right)}{g\left(f+gx^2\right)} \right) dx$$

$$= \frac{\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx}{g} - \frac{f\int \frac{\log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx}{g}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}g^{3/2}} - \frac{f\int\left(-\frac{g\log\left(c(d+ex^2)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}-gx\right)^2} - \frac{g\log\left(c(d+ex^2)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}+gx\right)^2} - \frac{g\log\left(c(d+ex^2)^p\right)}{2f\left(-fg-g^2x^2\right)}\right)dx}{g} - \frac{(2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}(d+ex^2)}\,dx}{g}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{\sqrt{f}g^{3/2}} + \frac{1}{4}\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(\sqrt{-f}\sqrt{g}-gx\right)^2}\,dx + \frac{1}{4}\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(\sqrt{-f}\sqrt{g}+gx\right)^2}\,dx + \frac{1}{2}\int \frac{\log\left(c(d+ex^2)^p\right)}{-fg-g^2x^2}\,dx - \frac{(2ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{\sqrt{f}g^{3/2}}$$

$$= \frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2\sqrt{f}g^{3/2}} + (ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}g^{3/2}\left(d+ex^2\right)}\,dx - \frac{(2ep)\int\left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{\sqrt{f}g^{3/2}} - \frac{(ep)\int \frac{x}{(\sqrt{-f}\sqrt{g}-gx \cdots}}{2g}$$

$$= \frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2\sqrt{f}g^{3/2}} + \frac{\left(\sqrt{e}p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{\sqrt{f}g^{3/2}} - \frac{\left(\sqrt{e}p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{\sqrt{f}g^{3/2}} + \frac{(ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{\sqrt{f}g^{3/2}} + \frac{(ep)\int\left(\frac{\sqrt{-f}}{(ef-dg)\left(\sqrt{-f}+\sqrt{g}x\right)} - \frac{-d\sqrt{g}-e\sqrt{-f}x}{\sqrt{g}(-ef+dg)(d+ex^2)}\right)dx}{2g} - \frac{(ep)\int\left(\frac{\sqrt{-f}}{(ef-dg)\left(-\sqrt{-f}+\sqrt{g}x\right)} - \frac{d\sqrt{g}-e\sqrt{-f}x}{\sqrt{g}(-ef+dg)(d+ex^2)}\right)dx}{2g}$$

$$= -\frac{e\sqrt{-f}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{3/2}(ef-dg)} + \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}g^{3/2}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}g^{3/2}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}g^{3/2}} + \frac{e\sqrt{-f}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2g^{3/2}(ef-dg)} + \frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2\sqrt{f}g^{3/2}} + \frac{(ep)\int\left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}(\sqrt{-d}-\sqrt{e}x)}+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}(\sqrt{-d}+\sqrt{e}x)}\right)dx}{\sqrt{f}g^{3/2}} - 2\frac{p\int\frac{\log\left(\frac{2}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{1+\frac{gx^2}{f}}dx}{fg} + \frac{p\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{f}\left(-i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{fg} + \frac{p\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{f}\left(i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{fg} + \frac{(ep)\int\frac{-d\sqrt{g}-e\sqrt{-f}x}{d+ex^2}dx}{2g^{3/2}(ef-dg)} - \frac{(ep)\int\frac{d\sqrt{g}-e\sqrt{-f}x}{d+ex^2}dx}{2g^{3/2}(ef-dg)}$$

$$= -\frac{e\sqrt{-f}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{3/2}(ef-dg)} + \frac{2p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}g^{3/2}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}g^{3/2}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{\sqrt{f}g^{3/2}} + \frac{e\sqrt{-f}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2g^{3/2}(ef-dg)} + \frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2\sqrt{f}g^{3/2}} + \frac{ip\text{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}g^{3/2}} + \frac{ip\text{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}g^{3/2}} - 2\frac{(ip)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}dx, x, \frac{1}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{\sqrt{f}g^{3/2}} - \frac{(\sqrt{e}p)\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}x}dx}{2\sqrt{f}g^{3/2}} + \frac{(\sqrt{e}p)\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}+\sqrt{e}x}dx}{2\sqrt{f}g^{3/2}} - 2\frac{(dep)\int\frac{1}{d+ex^2}dx}{2g(ef-dg)}$$

$$= -\frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g(ef-dg)} - \frac{e\sqrt{-f}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{3/2}(ef-dg)} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}g^{3/2}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}g^{3/2}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}g^{3/2}} + \frac{e\sqrt{-f}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2g^{3/2}(ef-dg)} + \frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2\sqrt{f}g^{3/2}} - \frac{ip\text{Li}_2\left(1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}g^{3/2}} + \frac{ip\text{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}g^{3/2}} + \frac{ip\text{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}g^{3/2}} + 2\frac{p\int\frac{\log\left(\frac{2}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{1+\frac{gx^2}{f}}dx}{2fg} - \frac{p\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{f}\left(-i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{2fg} - \frac{p\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{f}\left(i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{2fg}$$

$$= -\frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g(ef-dg)} - \frac{e\sqrt{-f}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{3/2}(ef-dg)} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}g^{3/2}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}g^{3/2}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}g^{3/2}} + \frac{e\sqrt{-f}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2g^{3/2}(ef-dg)} + \frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2\sqrt{f}g^{3/2}} - \frac{ip\text{Li}_2\left(1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}g^{3/2}} + \frac{ip\text{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4\sqrt{f}g^{3/2}} + \frac{ip\text{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4\sqrt{f}g^{3/2}} + 2\frac{(ip)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}dx, x, \frac{1}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{2\sqrt{f}g^{3/2}}$$

$$
\begin{aligned}
&= -\frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{g(ef-dg)} - \frac{e\sqrt{-f}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2g^{3/2}(ef-dg)} \\
&\quad + \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{\sqrt{f}g^{3/2}} - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}g^{3/2}} \\
&\quad - \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2\sqrt{f}g^{3/2}} \\
&\quad + \frac{e\sqrt{-f}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2g^{3/2}(ef-dg)} + \frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\log\left(c(d+ex^2)^p\right)}{4g^{3/2}\left(\sqrt{-f}+\sqrt{g}x\right)} \\
&\quad + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2\sqrt{f}g^{3/2}} - \frac{ip\operatorname{Li}_2\left(1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2\sqrt{f}g^{3/2}} \\
&\quad + \frac{ip\operatorname{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4\sqrt{f}g^{3/2}} + \frac{ip\operatorname{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4\sqrt{f}g^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 850, normalized size of antiderivative = 1.14

$$
\int \frac{x^2\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx
$$

$$
= \frac{\frac{2\sqrt{-d}\sqrt{e}\sqrt{g}p\log(\sqrt{-d}-\sqrt{e}x)}{ef-dg} + \frac{2\sqrt{-d}\sqrt{e}\sqrt{g}p\log(\sqrt{-d}+\sqrt{e}x)}{-ef+dg} - \frac{2e\sqrt{-f}p\log(\sqrt{-f}-\sqrt{g}x)}{ef-dg} + \frac{2e\sqrt{-f}p\log(\sqrt{-f}+\sqrt{g}x)}{ef-dg} - \frac{ip\log\left(\frac{\sqrt{g}(\sqrt{-d}}{i\sqrt{e}\sqrt{f}+}\right.}{}}{}
$$

```
[In] Integrate[(x^2*Log[c*(d + e*x^2)^p])/(f + g*x^2)^2,x]
```

```
[Out] ((2*Sqrt[-d]*Sqrt[e]*Sqrt[g]*p*Log[Sqrt[-d] - Sqrt[e]*x])/(e*f - d*g) + (2*
Sqrt[-d]*Sqrt[e]*Sqrt[g]*p*Log[Sqrt[-d] + Sqrt[e]*x])/(-(e*f) + d*g) - (2*e
*Sqrt[-f]*p*Log[Sqrt[-f] - Sqrt[g]*x])/(e*f - d*g) + (2*e*Sqrt[-f]*p*Log[Sq
rt[-f] + Sqrt[g]*x])/(e*f - d*g) - (I*p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x)
)/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]])/S
qrt[f] - (I*p*Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] +
Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] + (I*p*Log[(Sqrt
[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[
1 + (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] + (I*p*Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]
*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]]
)/Sqrt[f] + Log[c*(d + e*x^2)^p]/(Sqrt[-f] - Sqrt[g]*x) - Log[c*(d + e*x^2)
^p]/(Sqrt[-f] + Sqrt[g]*x) + (2*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^
2)^p])/Sqrt[f] - (I*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]
*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])])/Sqrt[f] - (I*p*PolyLog[2, (Sqrt[e]*(Sqrt[f
] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])])/Sqrt[f] + (I*p*P
```

```
olyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*S
qrt[g])])/Sqrt[f] + (I*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt
[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])])/Sqrt[f])/(4*g^(3/2))
```

Maple [F]

$$\int \frac{x^2 \ln \left(c(e\,x^2+d)^p\right)}{\left(g\,x^2+f\right)^2}dx$$

```
[In] int(x^2*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
[Out] int(x^2*ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

Fricas [F]

$$\int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \frac{x^2 \log \left((ex^2+d)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

```
[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral(x^2*log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{x^2 \log \left(\left(ex^2+d\right)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

```
[In] integrate(x^2*log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate(x^2*log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \log \left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx = \int \frac{x^2\,\ln \left(c\,{\left(e\,x^2+d\right)}^p\right)}{{\left(g\,x^2+f\right)}^2}\,dx$$

```
[In] int((x^2*log(c*(d + e*x^2)^p))/(f + g*x^2)^2,x)
[Out] int((x^2*log(c*(d + e*x^2)^p))/(f + g*x^2)^2, x)
```

3.355 $\int \frac{\log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 751

$$
\begin{aligned}
\int \frac{\log\left(c(d+ex^2)^p\right)}{(f+gx^2)^2}\,dx &= \frac{\sqrt{d}\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)} - \frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} \\
&+ \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}} \\
&- \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}\sqrt{g}} \\
&- \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}\sqrt{g}} \\
&+ \frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} - \frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} \\
&+ \frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2f^{3/2}\sqrt{g}} \\
&- \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2f^{3/2}\sqrt{g}} \\
&+ \frac{ip\,\mathrm{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{3/2}\sqrt{g}} \\
&+ \frac{ip\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{3/2}\sqrt{g}}
\end{aligned}
$$

```
[Out] p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/f/(-d*g+e*f)+1/2*arctan(x*g^(1/
2)/f^(1/2))*ln(c*(e*x^2+d)^p)/f^(3/2)/g^(1/2)+p*arctan(x*g^(1/2)/f^(1/2))*l
n(2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*p*arctan(x*g^(1/2)/f
^(1/2))*ln(-2*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/
(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*p*arctan(x*g^(1
/2)/f^(1/2))*ln(2*((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/
2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*I*p*polylog
(2,1-2*f^(1/2)/(f^(1/2)-I*x*g^(1/2)))/f^(3/2)/g^(1/2)+1/4*I*p*polylog(2,1+2
*((-d)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^
(1/2)-(-d)^(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)+1/4*I*p*polylog(2,1-2*((-d)^(1/2
)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^
(1/2)*g^(1/2)))/f^(3/2)/g^(1/2)-1/2*e*p*ln((-f)^(1/2)-x*g^(1/2))/(-d*g+e*f)
/(-f)^(1/2)/g^(1/2)+1/2*e*p*ln((-f)^(1/2)+x*g^(1/2))/(-d*g+e*f)/(-f)^(1/2)/
g^(1/2)-1/4*ln(c*(e*x^2+d)^p)/f/g^(1/2)/((-f)^(1/2)-x*g^(1/2))+1/4*ln(c*(e*
x^2+d)^p)/f/g^(1/2)/((-f)^(1/2)+x*g^(1/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 751, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules

used = {2521, 2513, 815, 649, 211, 266, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$\int \frac{\log (c(d+ex^2)^p)}{(f+gx^2)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log (c(d+ex^2)^p)}{2f^{3/2}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(-\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{2f^{3/2}\sqrt{g}} - \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{f}-i\sqrt{g}x)(\sqrt{-d}\sqrt{g}+i\sqrt{e}\sqrt{f})}\right)}{2f^{3/2}\sqrt{g}} + \frac{\sqrt{d}\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)} + \frac{p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}} - \frac{\log (c(d+ex^2)^p)}{4f\sqrt{g}(\sqrt{-f}-\sqrt{g}x)} + \frac{\log (c(d+ex^2)^p)}{4f\sqrt{g}(\sqrt{-f}+\sqrt{g}x)} + \frac{ip\operatorname{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}+1\right)}{4f^{3/2}\sqrt{g}} + \frac{ip\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{3/2}\sqrt{g}} - \frac{ep\log (\sqrt{-f}-\sqrt{g}x)}{2\sqrt{-f}\sqrt{g}(ef-dg)} + \frac{ep\log (\sqrt{-f}+\sqrt{g}x)}{2\sqrt{-f}\sqrt{g}(ef-dg)} - \frac{ip\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2f^{3/2}\sqrt{g}}$$

```
[In] Int[Log[c*(d + e*x^2)^p]/(f + g*x^2)^2,x]
```

```
[Out] (Sqrt[d]*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(f*(e*f - d*g)) - (e*p*Log[
Sqrt[-f] - Sqrt[g]*x])/(2*Sqrt[-f]*Sqrt[g]*(e*f - d*g)) + (p*ArcTan[(Sqrt[g
]*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3/2)*Sqrt[g]) -
 (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]
*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(2*
f^(3/2)*Sqrt[g]) - (p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(S
qrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*
Sqrt[g]*x))])/(2*f^(3/2)*Sqrt[g]) + (e*p*Log[Sqrt[-f] + Sqrt[g]*x])/(2*Sqrt
[-f]*Sqrt[g]*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqrt[-f] - S
qrt[g]*x)) + Log[c*(d + e*x^2)^p]/(4*f*Sqrt[g]*(Sqrt[-f] + Sqrt[g]*x)) + (A
rcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/(2*f^(3/2)*Sqrt[g]) - ((I/
2)*p*PolyLog[2, 1 - (2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/(f^(3/2)*Sqrt[g])
 + ((I/4)*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*S
qrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(f^(3/2)*Sqrt
[g]) + ((I/4)*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(
```

```
(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(f^(3/2)*
Sqrt[g])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\text{integral} = \int \left(-\frac{g\log\left(c(d+ex^2)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}-gx\right)^2}-\frac{g\log\left(c(d+ex^2)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}+gx\right)^2}-\frac{g\log\left(c(d+ex^2)^p\right)}{2f\left(-fg-g^2x^2\right)}\right)dx$$

$$= -\frac{g\int\frac{\log(c(d+ex^2)^p)}{\left(\sqrt{-f}\sqrt{g}-gx\right)^2}\,dx}{4f}-\frac{g\int\frac{\log(c(d+ex^2)^p)}{\left(\sqrt{-f}\sqrt{g}+gx\right)^2}\,dx}{4f}-\frac{g\int\frac{\log(c(d+ex^2)^p)}{-fg-g^2x^2}\,dx}{2f}$$

$$= -\frac{\log \left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{\log \left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log \left(c(d+ex^2)^p\right)}{2f^{3/2}\sqrt{g}} + \frac{(ep)\int \frac{x}{(\sqrt{-f}\sqrt{g}-gx)(d+ex^2)}\,dx}{2f} - \frac{(ep)\int \frac{x}{(\sqrt{-f}\sqrt{g}+gx)(d+ex^2)}\,dx}{2f} - \frac{(egp)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}g^{3/2}(d+ex^2)}\,dx}{f}$$

$$= -\frac{\log \left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{\log \left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log \left(c(d+ex^2)^p\right)}{2f^{3/2}\sqrt{g}} - \frac{(ep)\int \left(\frac{\sqrt{-f}}{(ef-dg)(\sqrt{-f}+\sqrt{g}x)} - \frac{-d\sqrt{g}-e\sqrt{-f}x}{\sqrt{g}(-ef+dg)(d+ex^2)}\right)dx}{2f} + \frac{(ep)\int \left(\frac{\sqrt{-f}}{(ef-dg)(-\sqrt{-f}+\sqrt{g}x)} - \frac{d\sqrt{g}-e\sqrt{-f}x}{\sqrt{g}(-ef+dg)(d+ex^2)}\right)dx}{2f} - \frac{(ep)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{f^{3/2}\sqrt{g}}$$

$$= -\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} + \frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} - \frac{\log \left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{\log \left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log \left(c(d+ex^2)^p\right)}{2f^{3/2}\sqrt{g}} - \frac{(ep)\int \left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}(\sqrt{-d}-\sqrt{e}x)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}(\sqrt{-d}+\sqrt{e}x)}\right)dx}{f^{3/2}\sqrt{g}} - \frac{(ep)\int \frac{-d\sqrt{g}-e\sqrt{-f}x}{d+ex^2}\,dx}{2f\sqrt{g}(ef-dg)} + \frac{(ep)\int \frac{d\sqrt{g}-e\sqrt{-f}x}{d+ex^2}\,dx}{2f\sqrt{g}(ef-dg)}$$

$$= -\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} + \frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} - \frac{\log \left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{\log \left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log \left(c(d+ex^2)^p\right)}{2f^{3/2}\sqrt{g}} + \frac{\left(\sqrt{e}p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2f^{3/2}\sqrt{g}} - \frac{\left(\sqrt{e}p\right)\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{2f^{3/2}\sqrt{g}} + 2\frac{(dep)\int \frac{1}{d+ex^2}\,dx}{2f(ef-dg)}$$

$$=\frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)}-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}+\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}}$$
$$-\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}\sqrt{g}}$$
$$-\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}\sqrt{g}}+\frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}$$
$$-\frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}$$
$$+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2f^{3/2}\sqrt{g}}-2\frac{p\int\frac{\log\left(\frac{2}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{1+\frac{gx^2}{f}}dx}{2f^2}$$
$$+\frac{p\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{f}\left(-i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{2f^2}+\frac{p\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{f}\left(i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{2f^2}$$

$$=\frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)}-\frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}+\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}}$$
$$-\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}\sqrt{g}}$$
$$-\frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}\sqrt{g}}+\frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)}$$
$$-\frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}$$
$$+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2f^{3/2}\sqrt{g}}+\frac{ip\text{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{3/2}\sqrt{g}}$$
$$+\frac{ip\text{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{3/2}\sqrt{g}}-2\frac{(ip)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}dx,x,\frac{1}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{2f^{3/2}\sqrt{g}}$$

$$
\begin{aligned}
&= \frac{\sqrt{d}\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f(ef-dg)} - \frac{ep\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} + \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{3/2}\sqrt{g}} \\
&- \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}\sqrt{g}} \\
&- \frac{p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{3/2}\sqrt{g}} + \frac{ep\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2\sqrt{-f}\sqrt{g}(ef-dg)} \\
&- \frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{\log\left(c(d+ex^2)^p\right)}{4f\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)} \\
&+ \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2f^{3/2}\sqrt{g}} - \frac{ip\text{Li}_2\left(1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2f^{3/2}\sqrt{g}} \\
&+ \frac{ip\text{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{3/2}\sqrt{g}} + \frac{ip\text{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{3/2}\sqrt{g}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.17

$$
\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx
$$

$$
= \frac{\frac{2\sqrt{-d}\sqrt{e}\sqrt{f}p\log(\sqrt{-d}-\sqrt{e}x)}{-ef+dg} + \frac{2\sqrt{-d}\sqrt{e}\sqrt{f}p\log(\sqrt{-d}+\sqrt{e}x)}{ef-dg} + \frac{2e\sqrt{-f^2}p\log(\sqrt{-f}-\sqrt{g}x)}{\sqrt{g}(ef-dg)} + \frac{2e\sqrt{-f^2}p\log(\sqrt{-f}+\sqrt{g}x)}{\sqrt{g}(-ef+dg)} - \frac{ip\log\left(\frac{\sqrt{g}(\sqrt{}}{i\sqrt{e}\sqrt{}}\right.}{}}{}
$$

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(f + g*x^2)^2,x]
```

```
[Out] ((2*Sqrt[-d]*Sqrt[e]*Sqrt[f]*p*Log[Sqrt[-d] - Sqrt[e]*x])/(-(e*f) + d*g) +
(2*Sqrt[-d]*Sqrt[e]*Sqrt[f]*p*Log[Sqrt[-d] + Sqrt[e]*x])/(e*f - d*g) + (2*e
*Sqrt[-f^2]*p*Log[Sqrt[-f] - Sqrt[g]*x])/(Sqrt[g]*(e*f - d*g)) + (2*e*Sqrt[
-f^2]*p*Log[Sqrt[-f] + Sqrt[g]*x])/(Sqrt[g]*(-(e*f) + d*g)) - (I*p*Log[(Sqr
t[g]*(Sqrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1
- (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[g] - (I*p*Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x
))/((-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]
])/Sqrt[g] + (I*p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f
] + Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[g] + (I*p*Log[(
Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log
[1 + (I*Sqrt[g]*x)/Sqrt[f]])/Sqrt[g] + (Sqrt[f]*Log[c*(d + e*x^2)^p])/(-(Sq
rt[-f]*Sqrt[g]) + g*x) + (Sqrt[f]*Log[c*(d + e*x^2)^p])/(Sqrt[-f]*Sqrt[g] +
 g*x) + (2*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[c*(d + e*x^2)^p])/Sqrt[g] - (I*p
*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]
*Sqrt[g])])/Sqrt[g] - (I*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sq
```

```
rt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])])/Sqrt[g] + (I*p*PolyLog[2, (Sqrt[e]*(S
qrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])])/Sqrt[g] + (
I*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[
-d]*Sqrt[g])])/Sqrt[g])/(4*f^(3/2))
```

Maple [F]

$$\int \frac{\ln\left(c(e\,x^2+d)^p\right)}{\left(g\,x^2+f\right)^2}dx$$

```
[In] int(ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

```
[Out] int(ln(c*(e*x^2+d)^p)/(g*x^2+f)^2,x)
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^2 + d)^p*c)/(g^2*x^4 + 2*f*g*x^2 + f^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(e*x**2+d)**p)/(g*x**2+f)**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{\left(gx^2+f\right)^2}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/(g*x^2+f)^2,x, algorithm="giac")
[Out] integrate(log((e*x^2 + d)^p*c)/(g*x^2 + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx = \int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{\left(g\,x^2+f\right)^2}\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)/(f + g*x^2)^2,x)
[Out] int(log(c*(d + e*x^2)^p)/(f + g*x^2)^2, x)
```

3.356 $\int \frac{\log\left(c\left(d+ex^2\right)^p\right)}{x^2\left(f+gx^2\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 803

$$
\begin{aligned}
\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)^2}\,dx &= \frac{2\sqrt{e}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\sqrt{d}\sqrt{e}gp\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f^2(ef-dg)} \\
&- \frac{e\sqrt{g}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{3\sqrt{g}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} \\
&+ \frac{3\sqrt{g}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}} \\
&+ \frac{3\sqrt{g}p\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}} \\
&+ \frac{e\sqrt{g}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{\log\left(c(d+ex^2)^p\right)}{f^2x} \\
&+ \frac{\sqrt{g}\log\left(c(d+ex^2)^p\right)}{4f^2\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\sqrt{g}\log\left(c(d+ex^2)^p\right)}{4f^2\left(\sqrt{-f}+\sqrt{g}x\right)} \\
&- \frac{3\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2f^{5/2}} \\
&+ \frac{3i\sqrt{g}p\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2f^{5/2}} \\
&- \frac{3i\sqrt{g}p\operatorname{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{5/2}} \\
&- \frac{3i\sqrt{g}p\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{5/2}}
\end{aligned}
$$

```
[Out] -ln(c*(e*x^2+d)^p)/f^2/x+2*p*arctan(x*e^(1/2)/d^(1/2))*e^(1/2)/f^2/d^(1/2)-
g*p*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/f^2/(-d*g+e*f)-3/2*arctan(x*g
^(1/2)/f^(1/2))*ln(c*(e*x^2+d)^p)*g^(1/2)/f^(5/2)-1/2*e*p*ln((-f)^(1/2)-x*g
^(1/2))*g^(1/2)/(-f)^(3/2)/(-d*g+e*f)-3*p*arctan(x*g^(1/2)/f^(1/2))*ln(2*f^
(1/2)/(f^(1/2)-I*x*g^(1/2)))*g^(1/2)/f^(5/2)+1/2*e*p*ln((-f)^(1/2)+x*g^(1/2
))*g^(1/2)/(-f)^(3/2)/(-d*g+e*f)+3/2*p*arctan(x*g^(1/2)/f^(1/2))*ln(-2*((-d
)^(1/2)-x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)
-(-d)^(1/2)*g^(1/2)))*g^(1/2)/f^(5/2)+3/2*p*arctan(x*g^(1/2)/f^(1/2))*ln(2*
((-d)^(1/2)+x*e^(1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(
1/2)+(-d)^(1/2)*g^(1/2)))*g^(1/2)/f^(5/2)+3/2*I*p*polylog(2,1-2*f^(1/2)/(f^
(1/2)-I*x*g^(1/2)))*g^(1/2)/f^(5/2)-3/4*I*p*polylog(2,1+2*((-d)^(1/2)-x*e^(
1/2))*f^(1/2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)-(-d)^(1/2)*g
^(1/2)))*g^(1/2)/f^(5/2)-3/4*I*p*polylog(2,1-2*((-d)^(1/2)+x*e^(1/2))*f^(1/
2)*g^(1/2)/(f^(1/2)-I*x*g^(1/2))/(I*e^(1/2)*f^(1/2)+(-d)^(1/2)*g^(1/2)))*g^
(1/2)/f^(5/2)+1/4*ln(c*(e*x^2+d)^p)*g^(1/2)/f^2/((-f)^(1/2)-x*g^(1/2))-1/4*
ln(c*(e*x^2+d)^p)*g^(1/2)/f^2/((-f)^(1/2)+x*g^(1/2))
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 803, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules

used = {2526, 2505, 211, 2521, 2513, 815, 649, 266, 2520, 12, 5048, 4966, 2449, 2352, 2497}

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^2 \left(f+gx^2\right)^2} \, dx = -\frac{\sqrt{d}\sqrt{e}gp \arctan \left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f^2(ef-dg)} + \frac{2\sqrt{e}p \arctan \left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{e\sqrt{g}p \log \left(\sqrt{-f}-\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{3\sqrt{g}p \arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \log \left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} + \frac{3\sqrt{g}p \arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \log \left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}} + \frac{3\sqrt{g}p \arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \log \left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}} + \frac{e\sqrt{g}p \log \left(\sqrt{g}x+\sqrt{-f}\right)}{2(-f)^{3/2}(ef-dg)} - \frac{3\sqrt{g} \arctan \left(\frac{\sqrt{g}x}{\sqrt{f}}\right) \log \left(c(ex^2+d)^p\right)}{2f^{5/2}} - \frac{\log \left(c(ex^2+d)^p\right)}{f^2x} + \frac{\sqrt{g} \log \left(c(ex^2+d)^p\right)}{4f^2\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\sqrt{g} \log \left(c(ex^2+d)^p\right)}{4f^2\left(\sqrt{g}x+\sqrt{-f}\right)} + \frac{3i\sqrt{g}p \operatorname{PolyLog} \left(2, 1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2f^{5/2}} - \frac{3i\sqrt{g}p \operatorname{PolyLog} \left(2, \frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}+1\right)}{4f^{5/2}} - \frac{3i\sqrt{g}p \operatorname{PolyLog} \left(2, 1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{e}x+\sqrt{-d})}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{5/2}}$$

```
[In] Int[Log[c*(d + e*x^2)^p]/(x^2*(f + g*x^2)^2),x]
```

```
[Out] (2*Sqrt[e]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*f^2) - (Sqrt[d]*Sqrt[e]*
g*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(f^2*(e*f - d*g)) - (e*Sqrt[g]*p*Log[Sqrt[
-f] - Sqrt[g]*x])/(2*(-f)^(3/2)*(e*f - d*g)) - (3*Sqrt[g]*p*ArcTan[(Sqrt[g]
*x)/Sqrt[f]]*Log[(2*Sqrt[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(5/2) + (3*Sqrt[g]
*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(-2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*
x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/(2*f
^(5/2)) + (3*Sqrt[g]*p*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[(2*Sqrt[f]*Sqrt[g]*(
Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])*(Sqrt[f] - I
*Sqrt[g]*x))])/(2*f^(5/2)) + (e*Sqrt[g]*p*Log[Sqrt[-f] + Sqrt[g]*x])/(2*(-f
)^(3/2)*(e*f - d*g)) - Log[c*(d + e*x^2)^p]/(f^2*x) + (Sqrt[g]*Log[c*(d + e
*x^2)^p])/(4*f^2*(Sqrt[-f] - Sqrt[g]*x)) - (Sqrt[g]*Log[c*(d + e*x^2)^p])/(
4*f^2*(Sqrt[-f] + Sqrt[g]*x)) - (3*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]*Log[
c*(d + e*x^2)^p])/(2*f^(5/2)) + (((3*I)/2)*Sqrt[g]*p*PolyLog[2, 1 - (2*Sqrt
[f])/(Sqrt[f] - I*Sqrt[g]*x)])/f^(5/2) - (((3*I)/4)*Sqrt[g]*p*PolyLog[2, 1
+ (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] - Sqrt[-d]
*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(5/2) - (((3*I)/4)*Sqrt[g]*p*PolyLog
```

```
[2, 1 - (2*Sqrt[f]*Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((I*Sqrt[e]*Sqrt[f] + Sq
rt[-d]*Sqrt[g])*(Sqrt[f] - I*Sqrt[g]*x))])/f^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2513

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.) + (g_.
)*(x_))^(r_.), x_Symbol] :> Simp[(f + g*x)^(r + 1)*((a + b*Log[c*(d + e*x^n
)^p])/(g*(r + 1))), x] - Dist[b*e*n*(p/(g*(r + 1))), Int[x^(n - 1)*((f + g*
x)^(r + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, r}, x]
 && (IGtQ[r, 0] || RationalQ[n]) && NeQ[r, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
```

```
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{\log\left(c(d+ex^2)^p\right)}{f^2x^2} - \frac{g\log\left(c(d+ex^2)^p\right)}{f\left(f+gx^2\right)^2} - \frac{g\log\left(c(d+ex^2)^p\right)}{f^2\left(f+gx^2\right)}\right) dx$$

$$= \frac{\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2}\,dx}{f^2} - \frac{g\int \frac{\log\left(c(d+ex^2)^p\right)}{f+gx^2}\,dx}{f^2} - \frac{g\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(f+gx^2\right)^2}\,dx}{f}$$

$$= -\frac{\log\left(c(d+ex^2)^p\right)}{f^2x} - \frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{5/2}} - \frac{g\int \left(-\frac{g\log\left(c(d+ex^2)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}-gx\right)^2} - \frac{g\log\left(c(d+ex^2)^p\right)}{4f\left(\sqrt{-f}\sqrt{g}+gx\right)^2} - \frac{g\log\left(c(d+ex^2)^p\right)}{2f\left(-fg-g^2x^2\right)}\right) dx}{f} + \frac{(2ep)\int \frac{1}{d+ex^2}\,dx}{f^2} + \frac{(2egp)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}(d+ex^2)}\,dx}{f^2}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\log\left(c(d+ex^2)^p\right)}{f^2x} - \frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{f^{5/2}} + \frac{g^2\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(\sqrt{-f}\sqrt{g}-gx\right)^2}\,dx}{4f^2} + \frac{g^2\int \frac{\log\left(c(d+ex^2)^p\right)}{\left(\sqrt{-f}\sqrt{g}+gx\right)^2}\,dx}{4f^2} + \frac{g^2\int \frac{\log\left(c(d+ex^2)^p\right)}{-fg-g^2x^2}\,dx}{2f^2} + \frac{\left(2e\sqrt{g}p\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{f^{5/2}}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\log\left(c(d+ex^2)^p\right)}{f^2x} + \frac{\sqrt{g}\log\left(c(d+ex^2)^p\right)}{4f^2\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\sqrt{g}\log\left(c(d+ex^2)^p\right)}{4f^2\left(\sqrt{-f}+\sqrt{g}x\right)} - \frac{3\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2f^{5/2}} + \frac{\left(2e\sqrt{g}p\right)\int \left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right) dx}{f^{5/2}} - \frac{(egp)\int \frac{x}{\left(\sqrt{-f}\sqrt{g}-gx\right)(d+ex^2)}\,dx}{2f^2} + \frac{(egp)\int \frac{x}{\left(\sqrt{-f}\sqrt{g}+gx\right)(d+ex^2)}\,dx}{2f^2} + \frac{\left(eg^2p\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}g^{3/2}(d+ex^2)}\,dx}{f^2}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\log\left(c(d+ex^2)^p\right)}{f^2x} + \frac{\sqrt{g}\log\left(c(d+ex^2)^p\right)}{4f^2\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\sqrt{g}\log\left(c(d+ex^2)^p\right)}{4f^2\left(\sqrt{-f}+\sqrt{g}x\right)} - \frac{3\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2f^{5/2}} - \frac{\left(\sqrt{e}\sqrt{g}p\right)\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{f^{5/2}} + \frac{\left(\sqrt{e}\sqrt{g}p\right)\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{f^{5/2}} + \frac{\left(e\sqrt{g}p\right)\int\frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{d+ex^2}\,dx}{f^{5/2}} + \frac{(egp)\int\left(\frac{\sqrt{-f}}{(ef-dg)\left(\sqrt{-f}+\sqrt{g}x\right)} - \frac{-d\sqrt{g}-e\sqrt{-f}x}{\sqrt{g}(-ef+dg)(d+ex^2)}\right)dx}{2f^2} - \frac{(egp)\int\left(\frac{\sqrt{-f}}{(ef-dg)\left(-\sqrt{-f}+\sqrt{g}x\right)} - \frac{d\sqrt{g}-e\sqrt{-f}x}{\sqrt{g}(-ef+dg)(d+ex^2)}\right)dx}{2f^2}$$

$$= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{e\sqrt{g}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{2\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} + \frac{\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{f^{5/2}} + \frac{\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{f^{5/2}} + \frac{e\sqrt{g}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{\log\left(c(d+ex^2)^p\right)}{f^2x} + \frac{\sqrt{g}\log\left(c(d+ex^2)^p\right)}{4f^2\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\sqrt{g}\log\left(c(d+ex^2)^p\right)}{4f^2\left(\sqrt{-f}+\sqrt{g}x\right)} - \frac{3\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2f^{5/2}} + \frac{\left(e\sqrt{g}p\right)\int\left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx}{f^{5/2}} + 2\frac{(gp)\int\frac{\log\left(\frac{2}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{1+\frac{gx^2}{f}}\,dx}{f^3} - \frac{(gp)\int\frac{\log\left(\frac{2\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt{f}\left(-i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}\,dx}{f^3} - \frac{(gp)\int\frac{\log\left(\frac{2\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\sqrt{f}\left(i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}\,dx}{f^3} + \frac{\left(e\sqrt{g}p\right)\int\frac{-d\sqrt{g}-e\sqrt{-f}x}{d+ex^2}\,dx}{2f^2(ef-dg)} - \frac{\left(e\sqrt{g}p\right)\int\frac{d\sqrt{g}-e\sqrt{-f}x}{d+ex^2}\,dx}{2f^2(ef-dg)}$$

$$
\begin{aligned}
&= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{e\sqrt{g}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{2\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} \\
&+ \frac{\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{f^{5/2}} \\
&+ \frac{\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{f^{5/2}} + \frac{e\sqrt{g}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} \\
&- \frac{\log\left(c(d+ex^2)^p\right)}{f^2x} + \frac{\sqrt{g}\log\left(c(d+ex^2)^p\right)}{4f^2\left(\sqrt{-f}-\sqrt{g}x\right)} - \frac{\sqrt{g}\log\left(c(d+ex^2)^p\right)}{4f^2\left(\sqrt{-f}+\sqrt{g}x\right)} \\
&- \frac{3\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2f^{5/2}} - \frac{i\sqrt{g}p\text{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}} \\
&- \frac{i\sqrt{g}p\text{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}} \\
&+ 2\frac{(i\sqrt{g}p)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{f^{5/2}} - \frac{\left(\sqrt{e}\sqrt{g}p\right)\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}-\sqrt{e}x}\,dx}{2f^{5/2}} \\
&+ \frac{\left(\sqrt{e}\sqrt{g}p\right)\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{-d}+\sqrt{e}x}\,dx}{2f^{5/2}} - 2\frac{(degp)\int\frac{1}{d+ex^2}\,dx}{2f^2(ef-dg)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\sqrt{d}\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f^2(ef-dg)} \\
&- \frac{e\sqrt{g}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{3\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} \\
&+ \frac{3\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}} \\
&+ \frac{3\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}} \\
&+ \frac{e\sqrt{g}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{\log\left(c(d+ex^2)^p\right)}{f^2x} + \frac{\sqrt{g}\log\left(c(d+ex^2)^p\right)}{4f^2\left(\sqrt{-f}-\sqrt{g}x\right)} \\
&- \frac{\sqrt{g}\log\left(c(d+ex^2)^p\right)}{4f^2\left(\sqrt{-f}+\sqrt{g}x\right)} - \frac{3\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2f^{5/2}} \\
&+ \frac{i\sqrt{g}p\text{Li}_2\left(1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} - \frac{i\sqrt{g}p\text{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}} \\
&- \frac{i\sqrt{g}p\text{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}} + 2\frac{(gp)\int\frac{\log\left(\frac{2}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{1+\frac{gx^2}{f}}dx}{2f^3} \\
&- \frac{(gp)\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{\sqrt{f}\left(-i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{2f^3} - \frac{(gp)\int\frac{\log\left(\frac{2\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{\sqrt{f}\left(i\sqrt{e}+\frac{\sqrt{-d}\sqrt{g}}{\sqrt{f}}\right)\left(1-\frac{i\sqrt{g}x}{\sqrt{f}}\right)}\right)}{1+\frac{gx^2}{f}}dx}{2f^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\sqrt{d}\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f^2(ef-dg)} \\
&\quad - \frac{e\sqrt{g}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{3\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} \\
&\quad + \frac{3\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2f^{5/2}} \\
&\quad + \frac{3\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{2f^{5/2}} \\
&\quad + \frac{e\sqrt{g}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{\log\left(c\left(d+ex^2\right)^p\right)}{f^2x} + \frac{\sqrt{g}\log\left(c\left(d+ex^2\right)^p\right)}{4f^2\left(\sqrt{-f}-\sqrt{g}x\right)} \\
&\quad - \frac{\sqrt{g}\log\left(c\left(d+ex^2\right)^p\right)}{4f^2\left(\sqrt{-f}+\sqrt{g}x\right)} - \frac{3\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c\left(d+ex^2\right)^p\right)}{2f^{5/2}} \\
&\quad + \frac{i\sqrt{g}p\text{Li}_2\left(1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} - \frac{3i\sqrt{g}p\text{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{4f^{5/2}} \\
&\quad - \frac{3i\sqrt{g}p\text{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g}\right)\left(\sqrt{f}-i\sqrt{g}x\right)}\right)}{4f^{5/2}} \\
&\quad + 2\frac{\left(i\sqrt{g}p\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1-\frac{i\sqrt{g}x}{\sqrt{f}}}\right)}{2f^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\sqrt{e}p\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}f^2} - \frac{\sqrt{d}\sqrt{e}gp\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{f^2(ef-dg)} \\
&\quad - \frac{e\sqrt{g}p\log\left(\sqrt{-f}-\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{3\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{f^{5/2}} \\
&\quad + \frac{3\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}} \\
&\quad + \frac{3\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{2f^{5/2}} \\
&\quad + \frac{e\sqrt{g}p\log\left(\sqrt{-f}+\sqrt{g}x\right)}{2(-f)^{3/2}(ef-dg)} - \frac{\log\left(c(d+ex^2)^p\right)}{f^2x} + \frac{\sqrt{g}\log\left(c(d+ex^2)^p\right)}{4f^2\left(\sqrt{-f}-\sqrt{g}x\right)} \\
&\quad - \frac{\sqrt{g}\log\left(c(d+ex^2)^p\right)}{4f^2\left(\sqrt{-f}+\sqrt{g}x\right)} - \frac{3\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)\log\left(c(d+ex^2)^p\right)}{2f^{5/2}} \\
&\quad + \frac{3i\sqrt{g}p\mathrm{Li}_2\left(1-\frac{2\sqrt{f}}{\sqrt{f}-i\sqrt{g}x}\right)}{2f^{5/2}} - \frac{3i\sqrt{g}p\mathrm{Li}_2\left(1+\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}-\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}-\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{5/2}} \\
&\quad - \frac{3i\sqrt{g}p\mathrm{Li}_2\left(1-\frac{2\sqrt{f}\sqrt{g}(\sqrt{-d}+\sqrt{e}x)}{(i\sqrt{e}\sqrt{f}+\sqrt{-d}\sqrt{g})(\sqrt{f}-i\sqrt{g}x)}\right)}{4f^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 939, normalized size of antiderivative = 1.17

$$
\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)^2}\,dx
$$

$$
= \frac{\frac{8\sqrt{e}\sqrt{f}p\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{-d}\sqrt{e}\sqrt{f}gp\log(\sqrt{-d}-\sqrt{e}x)}{ef-dg} + \frac{2\sqrt{-d}\sqrt{e}\sqrt{f}gp\log(\sqrt{-d}+\sqrt{e}x)}{-ef+dg} - \frac{2e\sqrt{-f^2}\sqrt{g}p\log(\sqrt{-f}-\sqrt{g}x)}{ef-dg} + \frac{2e\sqrt{-f^2}}{}}{}
$$

```
[In] Integrate[Log[c*(d + e*x^2)^p]/(x^2*(f + g*x^2)^2),x]
```

```
[Out] ((8*Sqrt[e]*Sqrt[f]*p*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (2*Sqrt[-d]*Sq
rt[e]*Sqrt[f]*g*p*Log[Sqrt[-d] - Sqrt[e]*x])/(e*f - d*g) + (2*Sqrt[-d]*Sqrt
[e]*Sqrt[f]*g*p*Log[Sqrt[-d] + Sqrt[e]*x])/(-(e*f) + d*g) - (2*e*Sqrt[-f^2]
*Sqrt[g]*p*Log[Sqrt[-f] - Sqrt[g]*x])/(e*f - d*g) + (2*e*Sqrt[-f^2]*Sqrt[g]
*p*Log[Sqrt[-f] + Sqrt[g]*x])/(e*f - d*g) + (3*I)*Sqrt[g]*p*Log[(Sqrt[g]*(S
qrt[-d] - Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sq
rt[g]*x)/Sqrt[f]] + (3*I)*Sqrt[g]*p*Log[(Sqrt[g]*(Sqrt[-d] + Sqrt[e]*x))/((
-I)*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] - (
3*I)*Sqrt[g]*p*Log[(Sqrt[g]*(Sqrt[-d] - Sqrt[e]*x))/((-I)*Sqrt[e]*Sqrt[f] +
 Sqrt[-d]*Sqrt[g])]*Log[1 + (I*Sqrt[g]*x)/Sqrt[f]] - (3*I)*Sqrt[g]*p*Log[(S
```

```
qrt[g]*(Sqrt[-d] + Sqrt[e]*x))/(I*Sqrt[e]*Sqrt[f] + Sqrt[-d]*Sqrt[g])]*Log[
1 + (I*Sqrt[g]*x)/Sqrt[f]] - (4*Sqrt[f]*Log[c*(d + e*x^2)^p])/x + (Sqrt[f]*
Sqrt[g]*Log[c*(d + e*x^2)^p])/(Sqrt[-f] - Sqrt[g]*x) - (Sqrt[f]*Sqrt[g]*Log
[c*(d + e*x^2)^p])/(Sqrt[-f] + Sqrt[g]*x) - 6*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sq
rt[f]]*Log[c*(d + e*x^2)^p] + (3*I)*Sqrt[g]*p*PolyLog[2, (Sqrt[e]*(Sqrt[f]
- I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] + (3*I)*Sqrt[g]*p*P
olyLog[2, (Sqrt[e]*(Sqrt[f] - I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*S
qrt[g])] - (3*I)*Sqrt[g]*p*PolyLog[2, (Sqrt[e]*(Sqrt[f] + I*Sqrt[g]*x))/(Sq
rt[e]*Sqrt[f] - I*Sqrt[-d]*Sqrt[g])] - (3*I)*Sqrt[g]*p*PolyLog[2, (Sqrt[e]*
(Sqrt[f] + I*Sqrt[g]*x))/(Sqrt[e]*Sqrt[f] + I*Sqrt[-d]*Sqrt[g])])/(4*f^(5/2
))
```

Maple [F]

$$\int \frac{\ln \left(c(e\,x^{2}+d)^{p}\right)}{x^{2} \left(g\,x^{2}+f\right)^{2}}dx$$

```
[In] int(ln(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x)
[Out] int(ln(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x)
```

Fricas [F]

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^2 \left(f+gx^2\right)^2}\, dx = \int \frac{\log \left((ex^2+d)^p c\right)}{(gx^2+f)^2 x^2}\, dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x, algorithm="fricas")
[Out] integral(log((e*x^2 + d)^p*c)/(g^2*x^6 + 2*f*g*x^4 + f^2*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^2)^p\right)}{x^2 \left(f+gx^2\right)^2}\, dx = \text{Timed out}$$

```
[In] integrate(ln(c*(e*x**2+d)**p)/x**2/(g*x**2+f)**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)^2}\,dx = \int \frac{\log\left((ex^2+d)^p c\right)}{(gx^2+f)^2 x^2}\,dx$$

```
[In] integrate(log(c*(e*x^2+d)^p)/x^2/(g*x^2+f)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^2 + d)^p*c)/((g*x^2 + f)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^2)^p\right)}{x^2\left(f+gx^2\right)^2}\,dx = \int \frac{\ln\left(c\left(e\,x^2+d\right)^p\right)}{x^2\left(g\,x^2+f\right)^2}\,dx$$

```
[In] int(log(c*(d + e*x^2)^p)/(x^2*(f + g*x^2)^2),x)
```

```
[Out] int(log(c*(d + e*x^2)^p)/(x^2*(f + g*x^2)^2), x)
```

3.357 $\int \frac{\log\left(c(a+bx^2)^n\right)}{a+bx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 163

$$\int \frac{\log\left(c(a+bx^2)^n\right)}{a+bx^2}\,dx = \frac{in\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} + \frac{2n\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^n\right)}{\sqrt{a}\sqrt{b}} + \frac{in\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] I*n*arctan(x*b^(1/2)/a^(1/2))^2/a^(1/2)/b^(1/2)+arctan(x*b^(1/2)/a^(1/2))*l
n(c*(b*x^2+a)^n)/a^(1/2)/b^(1/2)+2*n*arctan(x*b^(1/2)/a^(1/2))*ln(2*a^(1/2)
/(a^(1/2)+I*x*b^(1/2)))/a^(1/2)/b^(1/2)+I*n*polylog(2,1-2*a^(1/2)/(a^(1/2)+
I*x*b^(1/2)))/a^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {211, 2520, 12, 5040, 4964, 2449, 2352}

$$\int \frac{\log\left(c(a+bx^2)^n\right)}{a+bx^2}\,dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c(a+bx^2)^n\right)}{\sqrt{a}\sqrt{b}} + \frac{in\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} + \frac{2n\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}} + \frac{in\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{a}}{i\sqrt{b}x+\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[In] Int[Log[c*(a + b*x^2)^n]/(a + b*x^2),x]
```

```
[Out] (I*n*ArcTan[(Sqrt[b]*x)/Sqrt[a]]^2)/(Sqrt[a]*Sqrt[b]) + (2*n*ArcTan[(Sqrt[b
]*x)/Sqrt[a]]*Log[(2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b]) +
 (ArcTan[(Sqrt[b]*x)/Sqrt[a]]*Log[c*(a + b*x^2)^n])/(Sqrt[a]*Sqrt[b]) + (I*
n*PolyLog[2, 1 - (2*Sqrt[a])/(Sqrt[a] + I*Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}} - (2bn)\int \frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\left(a+bx^2\right)}\,dx \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}} - \frac{\left(2\sqrt{b}n\right)\int \frac{x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a+bx^2}\,dx}{\sqrt{a}} \\
&= \frac{in\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}} + \frac{(2n)\int \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{i-\frac{\sqrt{b}x}{\sqrt{a}}}\,dx}{a} \\
&= \frac{in\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} + \frac{2n\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}} \\
&\quad + \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}} - \frac{(2n)\int \frac{\log\left(\frac{2}{1+\frac{i\sqrt{b}x}{\sqrt{a}}}\right)}{1+\frac{bx^2}{a}}\,dx}{a} \\
&= \frac{in\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} + \frac{2n\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}} \\
&\quad + \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}} + \frac{(2in)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+\frac{i\sqrt{b}x}{\sqrt{a}}}\right)}{\sqrt{a}\sqrt{b}} \\
&= \frac{in\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)^2}{\sqrt{a}\sqrt{b}} + \frac{2n\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}} \\
&\quad + \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\log\left(c\left(a+bx^2\right)^n\right)}{\sqrt{a}\sqrt{b}} + \frac{in\text{Li}_2\left(1-\frac{2\sqrt{a}}{\sqrt{a}+i\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.79

$$
\int \frac{\log\left(c\left(a+bx^2\right)^n\right)}{a+bx^2}\,dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(in\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+2n\log\left(\frac{2i}{i-\frac{\sqrt{b}x}{\sqrt{a}}}\right)+\log\left(c\left(a+bx^2\right)^n\right)\right)+in\,\text{PolyLog}\left(2,\frac{i\sqrt{a}+\sqrt{b}x}{-i\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{a}\sqrt{b}}
$$

```
[In] Integrate[Log[c*(a + b*x^2)^n]/(a + b*x^2),x]
```

```
[Out] (ArcTan[(Sqrt[b]*x)/Sqrt[a]]*(I*n*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 2*n*Log[(2*
I)/(I - (Sqrt[b]*x)/Sqrt[a])] + Log[c*(a + b*x^2)^n]) + I*n*PolyLog[2, (I*S
qrt[a] + Sqrt[b]*x)/((-I)*Sqrt[a] + Sqrt[b]*x)])/(Sqrt[a]*Sqrt[b])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.25 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.62

method	result
risch	$\frac{\left(\ln\left(\left(b\,x^2+a\right)^n\right)-n\ln\left(b\,x^2+a\right)\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}+\frac{n\left(\sum_{\_\alpha=\operatorname{RootOf}\left(b\_Z^2+a\right)}\frac{2\ln(x-\_\alpha)\ln\left(b\,x^2+a\right)-b\left(\frac{\ln(x-\_\alpha)^2}{\_\alpha b}+\frac{2\_\alpha\ln(x-\_\alpha)\ln(}{a}\right.}{\_\alpha}\right.}{4b}$

```
[In] int(ln(c*(b*x^2+a)^n)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] (ln((b*x^2+a)^n)-n*ln(b*x^2+a))/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+1/4*n/b
*sum(1/_alpha*(2*ln(x-_alpha)*ln(b*x^2+a)-b*(1/_alpha/b*ln(x-_alpha)^2+2*_a
lpha/a*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+2*_alpha/a*dilog(1/2*(x+_alph
a)/_alpha))),_alpha=RootOf(_Z^2*b+a))+(1/2*I*Pi*csgn(I*(b*x^2+a)^n)*csgn(I*
c*(b*x^2+a)^n)^2-1/2*I*Pi*csgn(I*(b*x^2+a)^n)*csgn(I*c*(b*x^2+a)^n)*csgn(I*
c)-1/2*I*Pi*csgn(I*c*(b*x^2+a)^n)^3+1/2*I*Pi*csgn(I*c*(b*x^2+a)^n)^2*csgn(I
*c)+ln(c))/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [F]

$$\int \frac{\log\left(c(a+bx^2)^n\right)}{a+bx^2}\,dx=\int \frac{\log\left((bx^2+a)^n c\right)}{bx^2+a}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^n)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)^n*c)/(b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{\log\left(c(a+bx^2)^n\right)}{a+bx^2}\,dx = \int \frac{\log\left(c(a+bx^2)^n\right)}{a+bx^2}\,dx$$

```
[In] integrate(ln(c*(b*x**2+a)**n)/(b*x**2+a),x)
[Out] Integral(log(c*(a + b*x**2)**n)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\log\left(c(a+bx^2)^n\right)}{a+bx^2}\,dx = \int \frac{\log\left((bx^2+a)^n c\right)}{bx^2+a}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^n)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(log((b*x^2 + a)^n*c)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\log\left(c(a+bx^2)^n\right)}{a+bx^2}\,dx = \int \frac{\log\left((bx^2+a)^n c\right)}{bx^2+a}\,dx$$

```
[In] integrate(log(c*(b*x^2+a)^n)/(b*x^2+a),x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)^n*c)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(a+bx^2)^n\right)}{a+bx^2}\,dx = \int \frac{\ln\left(c\,(b\,x^2+a)^n\right)}{b\,x^2+a}\,dx$$

```
[In] int(log(c*(a + b*x^2)^n)/(a + b*x^2),x)
[Out] int(log(c*(a + b*x^2)^n)/(a + b*x^2), x)
```

3.358 $\int \frac{\log(1-x^2)}{2-x^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 239

$$\int \frac{\log(1-x^2)}{2-x^2} \, dx = \sqrt{2}\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right) - \frac{\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{(2-\sqrt{2})(\sqrt{2}+x)}\right)}{\sqrt{2}} - \frac{\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{4(1+x)}{(2+\sqrt{2})(\sqrt{2}+x)}\right)}{\sqrt{2}} + \frac{\text{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log(1-x^2)}{\sqrt{2}} - \frac{\text{PolyLog}\left(2,1-\frac{2\sqrt{2}}{\sqrt{2}+x}\right)}{\sqrt{2}} + \frac{\text{PolyLog}\left(2,1+\frac{4(1-x)}{(2-\sqrt{2})(\sqrt{2}+x)}\right)}{2\sqrt{2}} + \frac{\text{PolyLog}\left(2,1-\frac{4(1+x)}{(2+\sqrt{2})(\sqrt{2}+x)}\right)}{2\sqrt{2}}$$

```
[Out] 1/2*arctanh(1/2*x*2^(1/2))*ln(-x^2+1)*2^(1/2)-1/2*arctanh(1/2*x*2^(1/2))*ln
(-4*(1-x)/(2-2^(1/2))/(x+2^(1/2)))*2^(1/2)-1/2*arctanh(1/2*x*2^(1/2))*ln(4*
(1+x)/(2+2^(1/2))/(x+2^(1/2)))*2^(1/2)+1/4*polylog(2,1+4*(1-x)/(2-2^(1/2))/
(x+2^(1/2)))*2^(1/2)-1/2*polylog(2,1-2*2^(1/2)/(x+2^(1/2)))*2^(1/2)+1/4*pol
ylog(2,1-4*(1+x)/(2+2^(1/2))/(x+2^(1/2)))*2^(1/2)+arctanh(1/2*x*2^(1/2))*ln
(2*2^(1/2)/(x+2^(1/2)))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {212, 2520, 12, 6139, 6057, 2449, 2352, 2497}

$$\int \frac{\log(1-x^2)}{2-x^2}\,dx = \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log(1-x^2)}{\sqrt{2}} + \sqrt{2}\operatorname{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{x+\sqrt{2}}\right) - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(x+\sqrt{2}\right)}\right)}{\sqrt{2}} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{4(x+1)}{\left(2+\sqrt{2}\right)\left(x+\sqrt{2}\right)}\right)}{\sqrt{2}} - \frac{\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{2}}{x+\sqrt{2}}\right)}{\sqrt{2}} + \frac{\operatorname{PolyLog}\left(2,\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(x+\sqrt{2}\right)}+1\right)}{2\sqrt{2}} + \frac{\operatorname{PolyLog}\left(2,1-\frac{4(x+1)}{\left(2+\sqrt{2}\right)\left(x+\sqrt{2}\right)}\right)}{2\sqrt{2}}$$

```
[In] Int[Log[1 - x^2]/(2 - x^2),x]
```

```
[Out] Sqrt[2]*ArcTanh[x/Sqrt[2]]*Log[(2*Sqrt[2])/(Sqrt[2] + x)] - (ArcTanh[x/Sqrt
[2]]*Log[(-4*(1 - x))/((2 - Sqrt[2])*(Sqrt[2] + x))])/Sqrt[2] - (ArcTanh[x/
Sqrt[2]]*Log[(4*(1 + x))/((2 + Sqrt[2])*(Sqrt[2] + x))])/Sqrt[2] + (ArcTanh
[x/Sqrt[2]]*Log[1 - x^2])/Sqrt[2] - PolyLog[2, 1 - (2*Sqrt[2])/(Sqrt[2] + x
)]/Sqrt[2] + PolyLog[2, 1 + (4*(1 - x))/((2 - Sqrt[2])*(Sqrt[2] + x))]/(2*S
qrt[2]) + PolyLog[2, 1 - (4*(1 + x))/((2 + Sqrt[2])*(Sqrt[2] + x))]/(2*Sqrt
[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(1-x^2\right)}{\sqrt{2}}+2\int\frac{x\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}\left(1-x^2\right)}\,dx\\
&= \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(1-x^2\right)}{\sqrt{2}}+\sqrt{2}\int\frac{x\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{1-x^2}\,dx\\
&= \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(1-x^2\right)}{\sqrt{2}}+\sqrt{2}\int\left(-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2(-1+x)}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2(1+x)}\right)\,dx
\end{aligned}
$$

$$= \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(1-x^2\right)}{\sqrt{2}} - \frac{\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{-1+x}\,dx}{\sqrt{2}} - \frac{\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{1+x}\,dx}{\sqrt{2}}$$

$$= \sqrt{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right) - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right)}{\sqrt{2}}$$
$$- \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{4(1+x)}{\left(2+\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right)}{\sqrt{2}}$$
$$+ \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(1-x^2\right)}{\sqrt{2}} - 2\left(\frac{1}{2}\int \frac{\log\left(\frac{2}{1+\frac{x}{\sqrt{2}}}\right)}{1-\frac{x^2}{2}}\,dx\right)$$
$$+ \frac{1}{2}\int \frac{\log\left(\frac{\sqrt{2}(-1+x)}{\left(1-\frac{1}{\sqrt{2}}\right)\left(1+\frac{x}{\sqrt{2}}\right)}\right)}{1-\frac{x^2}{2}}\,dx + \frac{1}{2}\int \frac{\log\left(\frac{\sqrt{2}(1+x)}{\left(1+\frac{1}{\sqrt{2}}\right)\left(1+\frac{x}{\sqrt{2}}\right)}\right)}{1-\frac{x^2}{2}}\,dx$$

$$= \sqrt{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right) - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right)}{\sqrt{2}}$$
$$- \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{4(1+x)}{\left(2+\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right)}{\sqrt{2}}$$
$$+ \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(1-x^2\right)}{\sqrt{2}} + \frac{\mathrm{Li}_2\left(1+\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right)}{2\sqrt{2}}$$
$$+ \frac{\mathrm{Li}_2\left(1-\frac{4(1+x)}{\left(2+\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right)}{2\sqrt{2}} - 2\frac{\mathrm{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+\frac{x}{\sqrt{2}}}\right)}{\sqrt{2}}$$

$$= \sqrt{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right) - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right)}{\sqrt{2}}$$
$$- \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{4(1+x)}{\left(2+\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right)}{\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(1-x^2\right)}{\sqrt{2}}$$
$$- \frac{\mathrm{Li}_2\left(1-\frac{2\sqrt{2}}{\sqrt{2}+x}\right)}{\sqrt{2}} + \frac{\mathrm{Li}_2\left(1+\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right)}{2\sqrt{2}} + \frac{\mathrm{Li}_2\left(1-\frac{4(1+x)}{\left(2+\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right)}{2\sqrt{2}}$$

Mathematica [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.67

$$\int \frac{\log(1-x^2)}{2-x^2}\,dx$$
$$= \frac{\log\left(-1+\sqrt{2}\right)\log(-1+x)-\log\left(1+\sqrt{2}\right)\log(-1+x)-\log\left(-1+\sqrt{2}\right)\log(1+x)+\log\left(1+\sqrt{2}\right)\log(}{}$$

```
[In] Integrate[Log[1 - x^2]/(2 - x^2),x]
```

```
[Out] (Log[-1 + Sqrt[2]]*Log[-1 + x] - Log[1 + Sqrt[2]]*Log[-1 + x] - Log[-1 + Sq
rt[2]]*Log[1 + x] + Log[1 + Sqrt[2]]*Log[1 + x] - Log[Sqrt[2] - x]*Log[1 -
x^2] + Log[Sqrt[2] + x]*Log[1 - x^2] + PolyLog[2, -((-1 + Sqrt[2])*(-1 + x)
)] - PolyLog[2, (1 + Sqrt[2])*(-1 + x)] - PolyLog[2, (-1 + Sqrt[2])*(1 + x)
] + PolyLog[2, -((1 + Sqrt[2])*(1 + x))])/(2*Sqrt[2])
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.81

method	result
default	$-\frac{\left(\ln\left(x-\sqrt{2}\right)\ln(-x^2+1)-\operatorname{dilog}\left(\frac{x+1}{1+\sqrt{2}}\right)-\ln\left(x-\sqrt{2}\right)\ln\left(\frac{x+1}{1+\sqrt{2}}\right)-\operatorname{dilog}\left(\frac{-1+x}{\sqrt{2}-1}\right)-\ln\left(x-\sqrt{2}\right)\ln\left(\frac{-1+x}{\sqrt{2}-1}\right)\right)\sqrt{2}}{4}+\frac{\left(\ln\left(x+\sqrt{2}\right)\ln(-\right.}{}$
risch	$-\frac{\sqrt{2}\,\ln(-x^2+1)\ln\left(x-\sqrt{2}\right)}{4}+\frac{\sqrt{2}\,\ln\left(x-\sqrt{2}\right)\ln\left(\frac{x+1}{1+\sqrt{2}}\right)}{4}+\frac{\sqrt{2}\,\ln\left(x-\sqrt{2}\right)\ln\left(\frac{-1+x}{\sqrt{2}-1}\right)}{4}+\frac{\sqrt{2}\,\operatorname{dilog}\left(\frac{x+1}{1+\sqrt{2}}\right)}{4}+\frac{\sqrt{2}\,\operatorname{dilog}\left(\frac{-1}{\sqrt{2}}\right.}{4}$
parts	$\frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2}\right)\ln(-x^2+1)\sqrt{2}}{2}+\sqrt{2}\left(-\frac{\operatorname{arctanh}\left(\frac{x\sqrt{2}}{2}\right)\ln(x^2-1)}{2}+\frac{\ln\left(\frac{x\sqrt{2}}{2}+1\right)\ln(x^2-1)}{4}-\frac{\ln\left(\frac{x\sqrt{2}}{2}+1\right)\ln\left(\frac{\sqrt{2}-x\sqrt{2}}{2+\sqrt{2}}\right)}{4}-\right.$

```
[In] int(ln(-x^2+1)/(-x^2+2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(ln(x-2^(1/2))*ln(-x^2+1)-dilog((x+1)/(1+2^(1/2)))-ln(x-2^(1/2))*ln((x
+1)/(1+2^(1/2)))-dilog((-1+x)/(2^(1/2)-1))-ln(x-2^(1/2))*ln((-1+x)/(2^(1/2)
-1)))*2^(1/2)+1/4*(ln(x+2^(1/2))*ln(-x^2+1)-dilog((x+1)/(1-2^(1/2)))-ln(x+2
^(1/2))*ln((x+1)/(1-2^(1/2)))-dilog((-1+x)/(-1-2^(1/2)))-ln(x+2^(1/2))*ln((
-1+x)/(-1-2^(1/2))))*2^(1/2)
```

Fricas [F]

$$\int \frac{\log(1-x^2)}{2-x^2}\,dx = \int -\frac{\log(-x^2+1)}{x^2-2}\,dx$$

[In] integrate(log(-x^2+1)/(-x^2+2),x, algorithm="fricas")

[Out] integral(-log(-x^2 + 1)/(x^2 - 2), x)

Sympy [F]

$$\int \frac{\log(1-x^2)}{2-x^2}\,dx = -\int \frac{\log(1-x^2)}{x^2-2}\,dx$$

[In] integrate(ln(-x**2+1)/(-x**2+2),x)

[Out] -Integral(log(1 - x**2)/(x**2 - 2), x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.87

$$\int \frac{\log(1-x^2)}{2-x^2}\,dx$$
$$= \frac{1}{4}\sqrt{2}\left(\left(\log\left(2x+2\sqrt{2}\right)-\log\left(2x-2\sqrt{2}\right)\right)\log(-x^2+1)-\log\left(x+\sqrt{2}\right)\log\left(-\frac{x+\sqrt{2}}{\sqrt{2}+1}+1\right)+\log\left(\right.\right.$$

[In] integrate(log(-x^2+1)/(-x^2+2),x, algorithm="maxima")

[Out] 1/4*sqrt(2)*((log(2*x + 2*sqrt(2)) - log(2*x - 2*sqrt(2)))*log(-x^2 + 1) - log(x + sqrt(2))*log(-(x + sqrt(2))/(sqrt(2) + 1) + 1) + log(x - sqrt(2))*log((x - sqrt(2))/(sqrt(2) + 1) + 1) - log(x + sqrt(2))*log(-(x + sqrt(2))/(sqrt(2) - 1) + 1) + log(x - sqrt(2))*log((x - sqrt(2))/(sqrt(2) - 1) + 1) - dilog((x + sqrt(2))/(sqrt(2) + 1)) + dilog(-(x - sqrt(2))/(sqrt(2) + 1)) - dilog((x + sqrt(2))/(sqrt(2) - 1)) + dilog(-(x - sqrt(2))/(sqrt(2) - 1)))

Giac [F]

$$\int \frac{\log (1-x^2)}{2-x^2} \, dx = \int -\frac{\log (-x^2+1)}{x^2-2} \, dx$$

```
[In] integrate(log(-x^2+1)/(-x^2+2),x, algorithm="giac")
[Out] integrate(-log(-x^2 + 1)/(x^2 - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log (1-x^2)}{2-x^2} \, dx = -\int \frac{\ln (1-x^2)}{x^2-2} \, dx$$

```
[In] int(-log(1 - x^2)/(x^2 - 2),x)
[Out] -int(log(1 - x^2)/(x^2 - 2), x)
```

3.359 $\int \frac{\log(d+ex^2)}{1-x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 217

$$\int \frac{\log(d+ex^2)}{1-x^2}\,dx = 2\text{arctanh}(x)\log\left(\frac{2}{1+x}\right) - \text{arctanh}(x)\log\left(\frac{2(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{-d}-\sqrt{e})(1+x)}\right) - \text{arctanh}(x)\log\left(\frac{2(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{-d}+\sqrt{e})(1+x)}\right) + \text{arctanh}(x)\log(d+ex^2) - \text{PolyLog}\left(2,1-\frac{2}{1+x}\right) + \frac{1}{2}\text{PolyLog}\left(2,1-\frac{2(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{-d}-\sqrt{e})(1+x)}\right) + \frac{1}{2}\text{PolyLog}\left(2,1-\frac{2(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{-d}+\sqrt{e})(1+x)}\right)$$

```
[Out] 2*arctanh(x)*ln(2/(1+x))+arctanh(x)*ln(e*x^2+d)-arctanh(x)*ln(2*((-d)^(1/2)
-x*e^(1/2))/(1+x)/((-d)^(1/2)-e^(1/2)))-arctanh(x)*ln(2*((-d)^(1/2)+x*e^(1/
2))/(1+x)/((-d)^(1/2)+e^(1/2)))-polylog(2,1-2/(1+x))+1/2*polylog(2,1-2*((-d
)^(1/2)-x*e^(1/2))/(1+x)/((-d)^(1/2)-e^(1/2)))+1/2*polylog(2,1-2*((-d)^(1/2
)+x*e^(1/2))/(1+x)/((-d)^(1/2)+e^(1/2)))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used

$= \{212, 2520, 6139, 6057, 2449, 2352, 2497\}$

$$\int \frac{\log(d+ex^2)}{1-x^2}\,dx = \operatorname{arctanh}(x)\log(d+ex^2) - \operatorname{arctanh}(x)\log\left(\frac{2(\sqrt{-d}-\sqrt{e}x)}{(x+1)(\sqrt{-d}-\sqrt{e})}\right) - \operatorname{arctanh}(x)\log\left(\frac{2(\sqrt{-d}+\sqrt{e}x)}{(x+1)(\sqrt{-d}+\sqrt{e})}\right) + 2\operatorname{arctanh}(x)\log\left(\frac{2}{x+1}\right) + \frac{1}{2}\operatorname{PolyLog}\left(2, 1-\frac{2(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{-d}-\sqrt{e})(x+1)}\right) + \frac{1}{2}\operatorname{PolyLog}\left(2, 1-\frac{2(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}+\sqrt{e})(x+1)}\right) - \operatorname{PolyLog}\left(2, 1-\frac{2}{x+1}\right)$$

```
[In] Int[Log[d + e*x^2]/(1 - x^2),x]
```

```
[Out] 2*ArcTanh[x]*Log[2/(1 + x)] - ArcTanh[x]*Log[(2*(Sqrt[-d] - Sqrt[e]*x))/((S
qrt[-d] - Sqrt[e])*(1 + x))] - ArcTanh[x]*Log[(2*(Sqrt[-d] + Sqrt[e]*x))/((
Sqrt[-d] + Sqrt[e])*(1 + x))] + ArcTanh[x]*Log[d + e*x^2] - PolyLog[2, 1 -
2/(1 + x)] + PolyLog[2, 1 - (2*(Sqrt[-d] - Sqrt[e]*x))/((Sqrt[-d] - Sqrt[e]
)*(1 + x))]/2 + PolyLog[2, 1 - (2*(Sqrt[-d] + Sqrt[e]*x))/((Sqrt[-d] + Sqrt
[e])*(1 + x))]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \tanh^{-1}(x)\log\left(d+ex^2\right)-(2e)\int\frac{x\tanh^{-1}(x)}{d+ex^2}\,dx\\
&= \tanh^{-1}(x)\log\left(d+ex^2\right)-(2e)\int\left(-\frac{\tanh^{-1}(x)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\tanh^{-1}(x)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx\\
&= \tanh^{-1}(x)\log\left(d+ex^2\right)+\sqrt{e}\int\frac{\tanh^{-1}(x)}{\sqrt{-d}-\sqrt{e}x}\,dx-\sqrt{e}\int\frac{\tanh^{-1}(x)}{\sqrt{-d}+\sqrt{e}x}\,dx\\
&= 2\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right)-\tanh^{-1}(x)\log\left(\frac{2(\sqrt{-d}-\sqrt{e}x)}{\left(\sqrt{-d}-\sqrt{e}\right)(1+x)}\right)\\
&\quad-\tanh^{-1}(x)\log\left(\frac{2(\sqrt{-d}+\sqrt{e}x)}{\left(\sqrt{-d}+\sqrt{e}\right)(1+x)}\right)+\tanh^{-1}(x)\log\left(d+ex^2\right)\\
&\quad-2\int\frac{\log\left(\frac{2}{1+x}\right)}{1-x^2}\,dx+\int\frac{\log\left(\frac{2(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{-d}-\sqrt{e})(1+x)}\right)}{1-x^2}\,dx+\int\frac{\log\left(\frac{2(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{-d}+\sqrt{e})(1+x)}\right)}{1-x^2}\,dx
\end{aligned}
$$

$$= 2\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right) - \tanh^{-1}(x)\log\left(\frac{2(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{-d}-\sqrt{e})(1+x)}\right) - \tanh^{-1}(x)\log\left(\frac{2(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{-d}+\sqrt{e})(1+x)}\right) + \tanh^{-1}(x)\log(d+ex^2) + \frac{1}{2}\mathrm{Li}_2\left(1-\frac{2(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{-d}-\sqrt{e})(1+x)}\right) + \frac{1}{2}\mathrm{Li}_2\left(1-\frac{2(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{-d}+\sqrt{e})(1+x)}\right) - 2\mathrm{Subst}\left(\int\frac{\log(2x)}{1-2x}dx, x, \frac{1}{1+x}\right)$$

$$= 2\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right) - \tanh^{-1}(x)\log\left(\frac{2(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{-d}-\sqrt{e})(1+x)}\right) - \tanh^{-1}(x)\log\left(\frac{2(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{-d}+\sqrt{e})(1+x)}\right) + \tanh^{-1}(x)\log(d+ex^2) - \mathrm{Li}_2\left(1-\frac{2}{1+x}\right) + \frac{1}{2}\mathrm{Li}_2\left(1-\frac{2(\sqrt{-d}-\sqrt{e}x)}{(\sqrt{-d}-\sqrt{e})(1+x)}\right) + \frac{1}{2}\mathrm{Li}_2\left(1-\frac{2(\sqrt{-d}+\sqrt{e}x)}{(\sqrt{-d}+\sqrt{e})(1+x)}\right)$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.54

$$\int\frac{\log(d+ex^2)}{1-x^2}dx = \frac{1}{2}\log(1-x)\log\left(\frac{\sqrt{-d}-\sqrt{e}x}{\sqrt{-d}-\sqrt{e}}\right) - \frac{1}{2}\log(1+x)\log\left(\frac{\sqrt{-d}-\sqrt{e}x}{\sqrt{-d}+\sqrt{e}}\right) - \frac{1}{2}\log(1+x)\log\left(\frac{\sqrt{-d}+\sqrt{e}x}{\sqrt{-d}-\sqrt{e}}\right) + \frac{1}{2}\log(1-x)\log\left(\frac{\sqrt{-d}+\sqrt{e}x}{\sqrt{-d}+\sqrt{e}}\right) - \frac{1}{2}\log(1-x)\log(d+ex^2) + \frac{1}{2}\log(1+x)\log(d+ex^2) + \frac{1}{2}\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}(1-x)}{\sqrt{-d}-\sqrt{e}}\right) + \frac{1}{2}\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(1-x)}{\sqrt{-d}+\sqrt{e}}\right) - \frac{1}{2}\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}(1+x)}{\sqrt{-d}-\sqrt{e}}\right) - \frac{1}{2}\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(1+x)}{\sqrt{-d}+\sqrt{e}}\right)$$

```
[In] Integrate[Log[d + e*x^2]/(1 - x^2),x]
```

```
[Out] (Log[1 - x]*Log[(Sqrt[-d] - Sqrt[e]*x)/(Sqrt[-d] - Sqrt[e])])/2 - (Log[1 +
x]*Log[(Sqrt[-d] - Sqrt[e]*x)/(Sqrt[-d] + Sqrt[e])])/2 - (Log[1 + x]*Log[(S
qrt[-d] + Sqrt[e]*x)/(Sqrt[-d] - Sqrt[e])])/2 + (Log[1 - x]*Log[(Sqrt[-d] +
 Sqrt[e]*x)/(Sqrt[-d] + Sqrt[e])])/2 - (Log[1 - x]*Log[d + e*x^2])/2 + (Log
[1 + x]*Log[d + e*x^2])/2 + PolyLog[2, -((Sqrt[e]*(1 - x))/(Sqrt[-d] - Sqrt
```

```
[e]))]/2 + PolyLog[2, (Sqrt[e]*(1 - x))/(Sqrt[-d] + Sqrt[e])]/2 - PolyLog[2
, -((Sqrt[e]*(1 + x))/(Sqrt[-d] - Sqrt[e]))]/2 - PolyLog[2, (Sqrt[e]*(1 + x
))/(Sqrt[-d] + Sqrt[e])]/2
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.30

method	result
risch	$-\frac{\ln(-1+x)\ln(e\,x^2+d)}{2}+\frac{\ln(-1+x)\ln\left(\frac{-e(-1+x)+\sqrt{-de}-e}{-e+\sqrt{-de}}\right)}{2}+\frac{\ln(-1+x)\ln\left(\frac{e(-1+x)+\sqrt{-de}+e}{e+\sqrt{-de}}\right)}{2}+\frac{\operatorname{dilog}\left(\frac{-e(-1+x)+\sqrt{-de}-e}{-e+\sqrt{-de}}\right)}{2}$
default	$-\frac{\ln(-1+x)\ln(e\,x^2+d)}{2}+e\left(\frac{\ln(-1+x)\left(\ln\left(\frac{-e(-1+x)+\sqrt{-de}-e}{-e+\sqrt{-de}}\right)+\ln\left(\frac{e(-1+x)+\sqrt{-de}+e}{e+\sqrt{-de}}\right)\right)}{2e}+\frac{\operatorname{dilog}\left(\frac{-e(-1+x)+\sqrt{-de}-e}{-e+\sqrt{-de}}\right)+\operatorname{dilog}(}{2e}\right.$
parts	$\operatorname{arctanh}(x)\ln(e\,x^2+d)-2e\left(\frac{\operatorname{arctanh}(x)\ln(e\,x^2+d)}{2e}-\frac{-\frac{\ln(-1+x)\ln(e\,x^2+d)}{2}+e\left(\frac{\ln(-1+x)\left(\ln\left(\frac{-e(-1+x)+\sqrt{-de}-e}{-e+\sqrt{-de}}\right)+\right.}{2e}\right.}{}\right.$

```
[In] int(ln(e*x^2+d)/(-x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(-1+x)*ln(e*x^2+d)+1/2*ln(-1+x)*ln((-e*(-1+x)+(-d*e)^(1/2)-e)/(-e+(-
d*e)^(1/2)))+1/2*ln(-1+x)*ln((e*(-1+x)+(-d*e)^(1/2)+e)/(e+(-d*e)^(1/2)))+1/
2*dilog((-e*(-1+x)+(-d*e)^(1/2)-e)/(-e+(-d*e)^(1/2)))+1/2*dilog((e*(-1+x)+(
-d*e)^(1/2)+e)/(e+(-d*e)^(1/2)))+1/2*ln(x+1)*ln(e*x^2+d)-1/2*ln(x+1)*ln((-e
*(x+1)+(-d*e)^(1/2)+e)/(e+(-d*e)^(1/2)))-1/2*ln(x+1)*ln((e*(x+1)+(-d*e)^(1/
2)-e)/(-e+(-d*e)^(1/2)))-1/2*dilog((-e*(x+1)+(-d*e)^(1/2)+e)/(e+(-d*e)^(1/2
)))-1/2*dilog((e*(x+1)+(-d*e)^(1/2)-e)/(-e+(-d*e)^(1/2)))
```

Fricas [F]

$$\int \frac{\log(d+ex^2)}{1-x^2}\,dx = \int -\frac{\log(ex^2+d)}{x^2-1}\,dx$$

```
[In] integrate(log(e*x^2+d)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-log(e*x^2 + d)/(x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\log(d+ex^2)}{1-x^2}\,dx = -\int \frac{\log(d+ex^2)}{x^2-1}\,dx$$

```
[In] integrate(ln(e*x**2+d)/(-x**2+1),x)
[Out] -Integral(log(d + e*x**2)/(x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\log(d+ex^2)}{1-x^2}\,dx = \int -\frac{\log(ex^2+d)}{x^2-1}\,dx$$

```
[In] integrate(log(e*x^2+d)/(-x^2+1),x, algorithm="maxima")
[Out] -integrate(log(e*x^2 + d)/(x^2 - 1), x)
```

Giac [F]

$$\int \frac{\log(d+ex^2)}{1-x^2}\,dx = \int -\frac{\log(ex^2+d)}{x^2-1}\,dx$$

```
[In] integrate(log(e*x^2+d)/(-x^2+1),x, algorithm="giac")
[Out] integrate(-log(e*x^2 + d)/(x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(d+ex^2)}{1-x^2}\,dx = -\int \frac{\ln(e\,x^2+d)}{x^2-1}\,dx$$

```
[In] int(-log(d + e*x^2)/(x^2 - 1),x)
[Out] -int(log(d + e*x^2)/(x^2 - 1), x)
```

3.360 $\int \frac{(f+gx^{3n})\log(c(d+ex^n)^p)}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 144

$$\int \frac{(f+gx^{3n})\log(c(d+ex^n)^p)}{x}\,dx = -\frac{d^2gpx^n}{3e^2n}+\frac{dgpx^{2n}}{6en}-\frac{gpx^{3n}}{9n}+\frac{d^3gp\log(d+ex^n)}{3e^3n}+\frac{gx^{3n}\log(c(d+ex^n)^p)}{3n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n}+\frac{fp\,\mathrm{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}$$

```
[Out] -1/3*d^2*g*p*x^n/e^2/n+1/6*d*g*p*x^(2*n)/e/n-1/9*g*p*x^(3*n)/n+1/3*d^3*g*p*
ln(d+e*x^n)/e^3/n+1/3*g*x^(3*n)*ln(c*(d+e*x^n)^p)/n+f*ln(-e*x^n/d)*ln(c*(d+
e*x^n)^p)/n+f*p*polylog(2,1+e*x^n/d)/n
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2525, 14, 2463, 2441, 2352, 2442, 45}

$$\int \frac{(f+gx^{3n})\log(c(d+ex^n)^p)}{x}\,dx = \frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n}+\frac{gx^{3n}\log(c(d+ex^n)^p)}{3n}+\frac{d^3gp\log(d+ex^n)}{3e^3n}-\frac{d^2gpx^n}{3e^2n}+\frac{fp\,\mathrm{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}+\frac{dgpx^{2n}}{6en}-\frac{gpx^{3n}}{9n}$$

```
[In] Int[((f + g*x^(3*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -1/3*(d^2*g*p*x^n)/(e^2*n) + (d*g*p*x^(2*n))/(6*e*n) - (g*p*x^(3*n))/(9*n)
+ (d^3*g*p*Log[d + e*x^n])/(3*e^3*n) + (g*x^(3*n)*Log[c*(d + e*x^n)^p])/(3*
n) + (f*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f*p*PolyLog[2, 1 + (e*
x^n)/d])/n
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(f+gx^3)\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{f\log(c(d+ex)^p)}{x}+gx^2\log\left(c(d+ex)^p\right)\right)\,dx,x,x^n\right)}{n}\\
&= \frac{f\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}+\frac{g\text{Subst}\left(\int x^2\log\left(c(d+ex)^p\right)\,dx,x,x^n\right)}{n}\\
&= \frac{gx^{3n}\log\left(c(d+ex^n)^p\right)}{3n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}\\
&\quad-\frac{(efp)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx,x,x^n\right)}{n}-\frac{(egp)\text{Subst}\left(\int \frac{x^3}{d+ex}\,dx,x,x^n\right)}{3n}\\
&= \frac{gx^{3n}\log\left(c(d+ex^n)^p\right)}{3n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}\\
&\quad+\frac{fp\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}-\frac{(egp)\text{Subst}\left(\int \left(\frac{d^2}{e^3}-\frac{dx}{e^2}+\frac{x^2}{e}-\frac{d^3}{e^3(d+ex)}\right)\,dx,x,x^n\right)}{3n}\\
&= -\frac{d^2gpx^n}{3e^2n}+\frac{dgpx^{2n}}{6en}-\frac{gpx^{3n}}{9n}+\frac{d^3gp\log\left(d+ex^n\right)}{3e^3n}+\frac{gx^{3n}\log\left(c(d+ex^n)^p\right)}{3n}\\
&\quad+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}+\frac{fp\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.82

$$\int \frac{(f+gx^{3n})\log\left(c(d+ex^n)^p\right)}{x}\,dx$$
$$= \frac{-\frac{gp\left(ex^n\left(6d^2-3dex^n+2e^2x^{2n}\right)-6d^3\log(d+ex^n)\right)}{e^3}+6gx^{3n}\log\left(c(d+ex^n)^p\right)+18f\left(\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)+p\,\text{Po}\right.}{18n}$$

```
[In] Integrate[((f + g*x^(3*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-((g*p*(e*x^n*(6*d^2 - 3*d*e*x^n + 2*e^2*x^(2*n)) - 6*d^3*Log[d + e*x^n]))
/e^3) + 6*g*x^(3*n)*Log[c*(d + e*x^n)^p] + 18*f*(Log[-((e*x^n)/d)]*Log[c*(d
 + e*x^n)^p] + p*PolyLog[2, 1 + (e*x^n)/d]))/(18*n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.65 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.85

method	result
risch	$\frac{(g\,x^{3n}+3f\ln(x)n)\ln((d+e\,x^n)^p)}{3n}+\left(\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\mathrm{csgn}(ic(d+e\,x^n)^p)^2}{2}-\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\ \mathrm{csgn}(ic(d+e\,x^n)^p)\ \mathrm{csgn}(ic}{2}\right.$

```
[In] int((f+g*x^(3*n))*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(g*(x^n)^3+3*f*ln(x)*n)/n*ln((d+e*x^n)^p)+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)
*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)
*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^
2*csgn(I*c)+ln(c))*(f*ln(x)+1/3*g/n*(x^n)^3)-1/9*p/n*g*(x^n)^3+1/6*p/e/n*g*
d*(x^n)^2-1/3*d^2*g*p*x^n/e^2/n+1/3*d^3*g*p*ln(d+e*x^n)/e^3/n-p/n*f*dilog((
d+e*x^n)/d)-p*f*ln(x)*ln((d+e*x^n)/d)
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03

$$\int \frac{(f+gx^{3n})\log\left(c(d+ex^n)^p\right)}{x}\,dx = -\frac{18\,e^3fnp\log\left(x\right)\log\left(\frac{ex^n+d}{d}\right)-18\,e^3fn\log\left(c\right)\log\left(x\right)-3\,de^2gpx^{2\,n}+6\,d^2egpx^n+18\,e^3fp\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+\right.}{18\,e^3n}$$

```
[In] integrate((f+g*x^(3*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/18*(18*e^3*f*n*p*log(x)*log((e*x^n + d)/d) - 18*e^3*f*n*log(c)*log(x) -
3*d*e^2*g*p*x^(2*n) + 6*d^2*e*g*p*x^n + 18*e^3*f*p*dilog(-(e*x^n + d)/d + 1
) + 2*(e^3*g*p - 3*e^3*g*log(c))*x^(3*n) - 6*(3*e^3*f*n*p*log(x) + e^3*g*p*
x^(3*n) + d^3*g*p)*log(e*x^n + d))/(e^3*n)
```

Sympy [F]

$$\int \frac{(f+gx^{3n})\log\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{(f+gx^{3n})\log\left(c(d+ex^n)^p\right)}{x}\,dx$$

```
[In] integrate((f+g*x**(3*n))*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral((f + g*x**(3*n))*log(c*(d + e*x**n)**p)/x, x)
```

Maxima [F]

$$\int \frac{(f + gx^{3n}) \log (c(d + ex^n)^p)}{x} \, dx = \int \frac{(gx^{3\,n} + f) \log ((ex^n + d)^p c)}{x} \, dx$$

```
[In] integrate((f+g*x^(3*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/18*(9*e^3*f*n^2*p*log(x)^2 - 3*d*e^2*g*p*x^(2*n) + 6*d^2*e*g*p*x^n + 2*(
e^3*g*p - 3*e^3*g*log(c))*x^(3*n) - 6*(3*e^3*f*n*log(x) + e^3*g*x^(3*n))*lo
g((e*x^n + d)^p) - 6*(d^3*g*n*p + 3*e^3*f*n*log(c))*log(x))/(e^3*n) + integ
rate(1/3*(3*d*e^3*f*n*p*log(x) - d^4*g*p)/(e^4*x*x^n + d*e^3*x), x)
```

Giac [F]

$$\int \frac{(f + gx^{3n}) \log (c(d + ex^n)^p)}{x} \, dx = \int \frac{(gx^{3\,n} + f) \log ((ex^n + d)^p c)}{x} \, dx$$

```
[In] integrate((f+g*x^(3*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^(3*n) + f)*log((e*x^n + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f + gx^{3n}) \log (c(d + ex^n)^p)}{x} \, dx = \int \frac{\ln \left(c \left(d + e\,x^n\right)^p\right) \, \left(f + g\,x^{3\,n}\right)}{x} \, dx$$

```
[In] int((log(c*(d + e*x^n)^p)*(f + g*x^(3*n)))/x,x)
```

```
[Out] int((log(c*(d + e*x^n)^p)*(f + g*x^(3*n)))/x, x)
```

3.361 $\int \frac{(f+gx^{2n})\log(c(d+ex^n)^p)}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \frac{(f+gx^{2n})\log(c(d+ex^n)^p)}{x}\,dx = \frac{dgpx^n}{2en} - \frac{gpx^{2n}}{4n} - \frac{d^2gp\log(d+ex^n)}{2e^2n} + \frac{gx^{2n}\log(c(d+ex^n)^p)}{2n} + \frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{fp\,\mathrm{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}$$

```
[Out] 1/2*d*g*p*x^n/e/n-1/4*g*p*x^(2*n)/n-1/2*d^2*g*p*ln(d+e*x^n)/e^2/n+1/2*g*x^(
2*n)*ln(c*(d+e*x^n)^p)/n+f*ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/n+f*p*polylog(2,1
+e*x^n/d)/n
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2525, 14, 2463, 2441, 2352, 2442, 45}

$$\int \frac{(f+gx^{2n})\log(c(d+ex^n)^p)}{x}\,dx = \frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{gx^{2n}\log(c(d+ex^n)^p)}{2n} - \frac{d^2gp\log(d+ex^n)}{2e^2n} + \frac{fp\,\mathrm{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n} + \frac{dgpx^n}{2en} - \frac{gpx^{2n}}{4n}$$

```
[In] Int[((f + g*x^(2*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (d*g*p*x^n)/(2*e*n) - (g*p*x^(2*n))/(4*n) - (d^2*g*p*Log[d + e*x^n])/(2*e^2
*n) + (g*x^(2*n)*Log[c*(d + e*x^n)^p])/(2*n) + (f*Log[-((e*x^n)/d)]*Log[c*(
d + e*x^n)^p])/n + (f*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
```

```
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(f+gx^2)\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{f\log(c(d+ex)^p)}{x} + gx\log\left(c(d+ex)^p\right)\right)\,dx, x, x^n\right)}{n} \\
&= \frac{f\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n} + \frac{g\text{Subst}\left(\int x\log\left(c(d+ex)^p\right)\,dx, x, x^n\right)}{n} \\
&= \frac{gx^{2n}\log\left(c(d+ex^n)^p\right)}{2n} + \frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n} \\
&\quad - \frac{(efp)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^n\right)}{n} - \frac{(egp)\text{Subst}\left(\int \frac{x^2}{d+ex}\,dx, x, x^n\right)}{2n} \\
&= \frac{gx^{2n}\log\left(c(d+ex^n)^p\right)}{2n} + \frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n} \\
&\quad + \frac{fp\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n} - \frac{(egp)\text{Subst}\left(\int \left(-\frac{d}{e^2} + \frac{x}{e} + \frac{d^2}{e^2(d+ex)}\right)\,dx, x, x^n\right)}{2n} \\
&= \frac{dgpx^n}{2en} - \frac{gpx^{2n}}{4n} - \frac{d^2gp\log\left(d+ex^n\right)}{2e^2n} + \frac{gx^{2n}\log\left(c(d+ex^n)^p\right)}{2n} \\
&\quad + \frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n} + \frac{fp\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.81

$$
\begin{aligned}
&\int \frac{(f+gx^{2n})\log\left(c(d+ex^n)^p\right)}{x}\,dx \\
&= \frac{-egpx^n(-2d+ex^n) - 2d^2gp\log\left(d+ex^n\right) + 2e^2\left(gx^{2n} + 2f\log\left(-\frac{ex^n}{d}\right)\right)\log\left(c(d+ex^n)^p\right) + 4e^2fp\,\text{PolyLo}}{4e^2n}
\end{aligned}
$$

```
[In] Integrate[((f + g*x^(2*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-(e*g*p*x^n*(-2*d + e*x^n)) - 2*d^2*g*p*Log[d + e*x^n] + 2*e^2*(g*x^(2*n)
+ 2*f*Log[-((e*x^n)/d)])*Log[c*(d + e*x^n)^p] + 4*e^2*f*p*PolyLog[2, 1 + (e
*x^n)/d])/(4*e^2*n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.67 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.01

method	result
risch	$\frac{(2f\ln(x)n+g\,x^{2n})\ln((d+e\,x^n)^p)}{2n}+\left(\frac{i\pi\,\mathrm{csgn}(i(d+e\,x^n)^p)\mathrm{csgn}(ic(d+e\,x^n)^p)^2}{2}-\frac{i\pi\,\mathrm{csgn}(i(d+e\,x^n)^p)\,\mathrm{csgn}(ic(d+e\,x^n)^p)\,\mathrm{csgn}(ic)}{2}\right.$

[In] int((f+g*x^(2*n))*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)

[Out] 1/2*(2*f*ln(x)*n+g*(x^n)^2)/n*ln((d+e*x^n)^p)+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))*(f*ln(x)+1/2*g*(x^n)^2/n)-1/4*p/n*g*(x^n)^2+1/2*d*g*p*x^n/e/n-1/2*d^2*g*p*ln(d+e*x^n)/e^2/n-p/n*f*dilog((d+e*x^n)/d)-p*f*ln(x)*ln((d+e*x^n)/d)

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx^{2n})\log\left(c(d+ex^n)^p\right)}{x}\,dx = -\frac{4\,e^2fnp\log(x)\log\left(\frac{ex^n+d}{d}\right)-4\,e^2fn\log(c)\log(x)-2\,degpx^n+4\,e^2fp\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)+(e^2gp-2\,e^2g\,\mathrm{l}}{4\,e^2n}$$

[In] integrate((f+g*x^(2*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")

[Out] -1/4*(4*e^2*f*n*p*log(x)*log((e*x^n + d)/d) - 4*e^2*f*n*log(c)*log(x) - 2*d*e*g*p*x^n + 4*e^2*f*p*dilog(-(e*x^n + d)/d + 1) + (e^2*g*p - 2*e^2*g*log(c))*x^(2*n) - 2*(2*e^2*f*n*p*log(x) + e^2*g*p*x^(2*n) - d^2*g*p)*log(e*x^n + d))/(e^2*n)

Sympy [F]

$$\int \frac{(f+gx^{2n})\log\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{(f+gx^{2n})\log\left(c(d+ex^n)^p\right)}{x}\,dx$$

[In] integrate((f+g*x**(2*n))*ln(c*(d+e*x**n)**p)/x,x)

[Out] Integral((f + g*x**(2*n))*log(c*(d + e*x**n)**p)/x, x)

Maxima [F]

$$\int \frac{(f+gx^{2n})\log\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{(gx^{2n}+f)\log\left((ex^n+d)^p c\right)}{x}\,dx$$

```
[In] integrate((f+g*x^(2*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/4*(2*e^2*f*n^2*p*log(x)^2 - 2*d*e*g*p*x^n + (e^2*g*p - 2*e^2*g*log(c))*x
^(2*n) - 2*(2*e^2*f*n*log(x) + e^2*g*x^(2*n))*log((e*x^n + d)^p) + 2*(d^2*g
*n*p - 2*e^2*f*n*log(c))*log(x))/(e^2*n) + integrate(1/2*(2*d*e^2*f*n*p*log
(x) + d^3*g*p)/(e^3*x*x^n + d*e^2*x), x)
```

Giac [F]

$$\int \frac{(f+gx^{2n})\log\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{(gx^{2n}+f)\log\left((ex^n+d)^p c\right)}{x}\,dx$$

```
[In] integrate((f+g*x^(2*n))*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((g*x^(2*n) + f)*log((e*x^n + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx^{2n})\log\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)\,\left(f+g\,x^{2\,n}\right)}{x}\,dx$$

```
[In] int((log(c*(d + e*x^n)^p)*(f + g*x^(2*n)))/x,x)
```

```
[Out] int((log(c*(d + e*x^n)^p)*(f + g*x^(2*n)))/x, x)
```

3.362 $\int \frac{(f+gx^n)\log(c(d+ex^n)^p)}{x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{(f+gx^n)\log(c(d+ex^n)^p)}{x}\,dx = -\frac{gpx^n}{n}+\frac{g(d+ex^n)\log(c(d+ex^n)^p)}{en} + \frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{fp\,\mathrm{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}$$

```
[Out] -g*p*x^n/n+g*(d+e*x^n)*ln(c*(d+e*x^n)^p)/e/n+f*ln(-e*x^n/d)*ln(c*(d+e*x^n)^
p)/n+f*p*polylog(2,1+e*x^n/d)/n
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2525, 45, 2463, 2436, 2332, 2441, 2352}

$$\int \frac{(f+gx^n)\log(c(d+ex^n)^p)}{x}\,dx = \frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{g(d+ex^n)\log(c(d+ex^n)^p)}{en} + \frac{fp\,\mathrm{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n} - \frac{gpx^n}{n}$$

```
[In] Int[((f + g*x^n)*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -((g*p*x^n)/n) + (g*(d + e*x^n)*Log[c*(d + e*x^n)^p])/(e*n) + (f*Log[-((e*x
^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(f+gx)\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(g\log\left(c(d+ex)^p\right)+\frac{f\log(c(d+ex)^p)}{x}\right)\,dx,x,x^n\right)}{n}\\
&= \frac{f\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}+\frac{g\text{Subst}\left(\int \log\left(c(d+ex)^p\right)\,dx,x,x^n\right)}{n}\\
&= \frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}+\frac{g\text{Subst}\left(\int \log\left(cx^p\right)\,dx,x,d+ex^n\right)}{en}\\
&\quad-\frac{(efp)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx,x,x^n\right)}{n}\\
&= -\frac{gpx^n}{n}+\frac{g(d+ex^n)\log\left(c(d+ex^n)^p\right)}{en}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}+\frac{fp\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.82

$$\int \frac{(f+gx^n)\log\left(c(d+ex^n)^p\right)}{x}\,dx
= \frac{-egpx^n+\left(dg+egx^n+ef\log\left(-\frac{ex^n}{d}\right)\right)\log\left(c(d+ex^n)^p\right)+efp\,\text{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{en}$$

```
[In] Integrate[((f + g*x^n)*Log[c*(d + e*x^n)^p])/x,x]
[Out] (-(e*g*p*x^n) + (d*g + e*g*x^n + e*f*Log[-((e*x^n)/d)])*Log[c*(d + e*x^n)^p
] + e*f*p*PolyLog[2, 1 + (e*x^n)/d])/(e*n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.16 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.69

method	result
risch	$\frac{(f\ln(x)n+g\,x^n)\ln((d+e\,x^n)^p)}{n}+\left(\frac{i\pi\ \text{csgn}(i(d+e\,x^n)^p)\text{csgn}(ic(d+e\,x^n)^p)^2}{2}-\frac{i\pi\ \text{csgn}(i(d+e\,x^n)^p)\ \text{csgn}(ic(d+e\,x^n)^p)\ \text{csgn}(ic)}{2}-\right.$

```
[In] int((f+g*x^n)*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

[Out] (f*ln(x)*n+g*x^n)/n*ln((d+e*x^n)^p)+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))*(f*ln(x)+g*x^n/n)-g*p*x^n/n+p/e/n*g*d*ln(d+e*x^n)-p/n*f*dilog((d+e*x^n)/d)-p*f*ln(x)*ln((d+e*x^n)/d)

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.20

$$\int \frac{(f+gx^n)\log\left(c(d+ex^n)^p\right)}{x}\,dx = -\frac{efnp\log(x)\log\left(\frac{ex^n+d}{d}\right)-efn\log(c)\log(x)+efp\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)+(egp-eg\log(c))x^n-(efnp\log}{en}$$

[In] integrate((f+g*x^n)*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")

[Out] -(e*f*n*p*log(x)*log((e*x^n + d)/d) - e*f*n*log(c)*log(x) + e*f*p*dilog(-(e*x^n + d)/d + 1) + (e*g*p - e*g*log(c))*x^n - (e*f*n*p*log(x) + e*g*p*x^n + d*g*p)*log(e*x^n + d))/(e*n)

Sympy [F]

$$\int \frac{(f+gx^n)\log\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{(f+gx^n)\log\left(c(d+ex^n)^p\right)}{x}\,dx$$

[In] integrate((f+g*x**n)*ln(c*(d+e*x**n)**p)/x,x)

[Out] Integral((f + g*x**n)*log(c*(d + e*x**n)**p)/x, x)

Maxima [F]

$$\int \frac{(f+gx^n)\log\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{(gx^n+f)\log\left((ex^n+d)^pc\right)}{x}\,dx$$

[In] integrate((f+g*x^n)*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")

[Out] -1/2*(e*f*n^2*p*log(x)^2 + 2*(e*g*p - e*g*log(c))*x^n - 2*(e*f*n*log(x) + e*g*x^n)*log((e*x^n + d)^p) - 2*(d*g*n*p + e*f*n*log(c))*log(x))/(e*n) + integrate((d*e*f*n*p*log(x) - d^2*g*p)/(e^2*x*x^n + d*e*x), x)

Giac [F]

$$\int \frac{(f+gx^n)\log\left(c(d+ex^n)^p\right)}{x}\,dx=\int \frac{(gx^n+f)\log\left((ex^n+d)^pc\right)}{x}\,dx$$

```
[In] integrate((f+g*x^n)*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
[Out] integrate((g*x^n + f)*log((e*x^n + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx^n)\log\left(c(d+ex^n)^p\right)}{x}\,dx=\int \frac{\ln\left(c\left(d+e\,x^n\right)^p\right)\,\left(f+g\,x^n\right)}{x}\,dx$$

```
[In] int((log(c*(d + e*x^n)^p)*(f + g*x^n))/x,x)
[Out] int((log(c*(d + e*x^n)^p)*(f + g*x^n))/x, x)
```

3.363 $\int \frac{(f+gx^{-n})\log(c(d+ex^n)^p)}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{(f+gx^{-n})\log(c(d+ex^n)^p)}{x}\,dx = \frac{egp\log(x)}{d}-\frac{egp\log(d+ex^n)}{dn}-\frac{gx^{-n}\log(c(d+ex^n)^p)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n}+\frac{fp\operatorname{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}$$

```
[Out] e*g*p*ln(x)/d-e*g*p*ln(d+e*x^n)/d/n-g*ln(c*(d+e*x^n)^p)/n/(x^n)+f*ln(-e*x^n
/d)*ln(c*(d+e*x^n)^p)/n+f*p*polylog(2,1+e*x^n/d)/n
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {2525, 14, 2463, 2442, 36, 29, 31, 2441, 2352}

$$\int \frac{(f+gx^{-n})\log(c(d+ex^n)^p)}{x}\,dx = \frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n}-\frac{gx^{-n}\log(c(d+ex^n)^p)}{n}+\frac{fp\operatorname{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}-\frac{egp\log(d+ex^n)}{dn}+\frac{egp\log(x)}{d}$$

```
[In] Int[((f + g/x^n)*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (e*g*p*Log[x])/d - (e*g*p*Log[d + e*x^n])/(d*n) - (g*Log[c*(d + e*x^n)^p])/
(n*x^n) + (f*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f*p*PolyLog[2, 1
+ (e*x^n)/d])/n
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\left(f+\frac{g}{x}\right)\log\left(c(d+ex)^p\right)}{x}\,dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{g\log\left(c(d+ex)^p\right)}{x^2}+\frac{f\log\left(c(d+ex)^p\right)}{x}\right)\,dx, x, x^n\right)}{n}\\
&= \frac{f\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x}\,dx, x, x^n\right)}{n}+\frac{g\text{Subst}\left(\int \frac{\log\left(c(d+ex)^p\right)}{x^2}\,dx, x, x^n\right)}{n}\\
&= -\frac{gx^{-n}\log\left(c(d+ex^n)^p\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}\\
&\quad -\frac{(efp)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^n\right)}{n}+\frac{(egp)\text{Subst}\left(\int \frac{1}{x(d+ex)}\,dx, x, x^n\right)}{n}\\
&= -\frac{gx^{-n}\log\left(c(d+ex^n)^p\right)}{n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}+\frac{fp\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}\\
&\quad +\frac{(egp)\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^n\right)}{dn}-\frac{\left(e^2gp\right)\text{Subst}\left(\int \frac{1}{d+ex}\,dx, x, x^n\right)}{dn}\\
&= \frac{egp\log(x)}{d}-\frac{egp\log\left(d+ex^n\right)}{dn}-\frac{gx^{-n}\log\left(c(d+ex^n)^p\right)}{n}\\
&\quad +\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}+\frac{fp\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int \frac{(f+gx^{-n})\log(c(d+ex^n)^p)}{x}\,dx$$
$$=\frac{egnp\log(x)-egp\log(d+ex^n)-dgx^{-n}\log(c(d+ex^n)^p)+df\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)+dfp\,\mathrm{PolyLog}}{dn}$$

```
[In] Integrate[((f + g/x^n)*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (e*g*n*p*Log[x] - e*g*p*Log[d + e*x^n] - (d*g*Log[c*(d + e*x^n)^p])/x^n + d
*f*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + d*f*p*PolyLog[2, 1 + (e*x^n)/d]
)/(d*n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.90 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.49

method	result
risch	$\frac{(f\ln(x)n\,x^n-g)x^{-n}\ln((d+e\,x^n)^p)}{n}+\left(\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\mathrm{csgn}(ic(d+e\,x^n)^p)^2}{2}-\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\ \mathrm{csgn}(ic(d+e\,x^n)^p)\ \mathrm{csgn}(ic)}{2}\right.$

```
[In] int((f+g/(x^n))*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (f*ln(x)*n*x^n-g)/n/(x^n)*ln((d+e*x^n)^p)+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csg
n(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csg
n(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*cs
gn(I*c)+ln(c))*(-1/n*g/(x^n)+1/n*f*ln(x^n))-p/n*f*dilog((d+e*x^n)/d)-p*f*ln
(x)*ln((d+e*x^n)/d)-e*g*p*ln(d+e*x^n)/d/n+p*e/n*g/d*ln(x^n)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.18

$$\int \frac{(f+gx^{-n})\log(c(d+ex^n)^p)}{x}\,dx=$$
$$-\frac{dfnpx^n\log(x)\log\left(\frac{ex^n+d}{d}\right)+dfpx^n\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)+dg\log(c)-(egnp+dfn\log(c))x^n\log(x)+(dgp}{dnx^n}$$

```
[In] integrate((f+g/(x^n))*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -(d*f*n*p*x^n*log(x)*log((e*x^n + d)/d) + d*f*p*x^n*dilog(-(e*x^n + d)/d +
1) + d*g*log(c) - (e*g*n*p + d*f*n*log(c))*x^n*log(x) + (d*g*p - (d*f*n*p*l
og(x) - e*g*p)*x^n)*log(e*x^n + d))/(d*n*x^n)
```

Sympy [F]

$$\int \frac{(f + gx^{-n}) \log \left(c(d + ex^n)^p\right)}{x} \, dx = \int \frac{x^{-n}(fx^n + g) \log \left(c(d + ex^n)^p\right)}{x} \, dx$$

```
[In] integrate((f+g/(x**n))*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral((f*x**n + g)*log(c*(d + e*x**n)**p)/(x*x**n), x)
```

Maxima [F]

$$\int \frac{(f + gx^{-n}) \log \left(c(d + ex^n)^p\right)}{x} \, dx = \int \frac{\left(f + \frac{g}{x^n}\right) \log \left((ex^n + d)^p c\right)}{x} \, dx$$

```
[In] integrate((f+g/(x^n))*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/2*((f*n^2*p*log(x)^2 - 2*f*n*log(c)*log(x))*x^n - 2*(f*n*x^n*log(x) - g)
*log((e*x^n + d)^p) + 2*g*log(c))/(n*x^n) + integrate((d*f*n*p*log(x) + e*g
*p)/(e*x*x^n + d*x), x)
```

Giac [F]

$$\int \frac{(f + gx^{-n}) \log \left(c(d + ex^n)^p\right)}{x} \, dx = \int \frac{\left(f + \frac{g}{x^n}\right) \log \left((ex^n + d)^p c\right)}{x} \, dx$$

```
[In] integrate((f+g/(x^n))*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((f + g/x^n)*log((e*x^n + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx^{-n})\log(c(d+ex^n)^p)}{x}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)\,\left(f+\frac{g}{x^n}\right)}{x}\,dx$$

```
[In] int((log(c*(d + e*x^n)^p)*(f + g/x^n))/x,x)
[Out] int((log(c*(d + e*x^n)^p)*(f + g/x^n))/x, x)
```

3.364 $\int \frac{(f+gx^{-2n})\log(c(d+ex^n)^p)}{x}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 126

$$\int \frac{(f+gx^{-2n})\log(c(d+ex^n)^p)}{x}\,dx = -\frac{egpx^{-n}}{2dn}-\frac{e^2gp\log(x)}{2d^2}+\frac{e^2gp\log(d+ex^n)}{2d^2n}-\frac{gx^{-2n}\log(c(d+ex^n)^p)}{2n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n}+\frac{fp\,\text{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}$$

```
[Out] -1/2*e*g*p/d/n/(x^n)-1/2*e^2*g*p*ln(x)/d^2+1/2*e^2*g*p*ln(d+e*x^n)/d^2/n-1/
2*g*ln(c*(d+e*x^n)^p)/n/(x^(2*n))+f*ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/n+f*p*po
lylog(2,1+e*x^n/d)/n
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2525, 14, 2463, 2442, 46, 2441, 2352}

$$\int \frac{(f+gx^{-2n})\log(c(d+ex^n)^p)}{x}\,dx = \frac{f\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n}-\frac{gx^{-2n}\log(c(d+ex^n)^p)}{2n}+\frac{e^2gp\log(d+ex^n)}{2d^2n}-\frac{e^2gp\log(x)}{2d^2}+\frac{fp\,\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n}-\frac{egpx^{-n}}{2dn}$$

```
[In] Int[((f + g/x^(2*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -1/2*(e*g*p)/(d*n*x^n) - (e^2*g*p*Log[x])/(2*d^2) + (e^2*g*p*Log[d + e*x^n]
)/(2*d^2*n) - (g*Log[c*(d + e*x^n)^p])/(2*n*x^(2*n)) + (f*Log[-((e*x^n)/d)]
*Log[c*(d + e*x^n)^p])/n + (f*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
```

```
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\left(f+\frac{g}{x^2}\right)\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{g\log(c(d+ex)^p)}{x^3}+\frac{f\log(c(d+ex)^p)}{x}\right)\,dx, x, x^n\right)}{n}\\
&= \frac{f\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}+\frac{g\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^3}\,dx, x, x^n\right)}{n}\\
&= -\frac{gx^{-2n}\log\left(c(d+ex^n)^p\right)}{2n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}\\
&\quad-\frac{(efp)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^n\right)}{n}+\frac{(egp)\text{Subst}\left(\int \frac{1}{x^2(d+ex)}\,dx, x, x^n\right)}{2n}\\
&= -\frac{gx^{-2n}\log\left(c(d+ex^n)^p\right)}{2n}+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}\\
&\quad+\frac{fp\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}+\frac{(egp)\text{Subst}\left(\int \left(\frac{1}{dx^2}-\frac{e}{d^2x}+\frac{e^2}{d^2(d+ex)}\right)\,dx, x, x^n\right)}{2n}\\
&= -\frac{egpx^{-n}}{2dn}-\frac{e^2gp\log(x)}{2d^2}+\frac{e^2gp\log\left(d+ex^n\right)}{2d^2n}-\frac{gx^{-2n}\log\left(c(d+ex^n)^p\right)}{2n}\\
&\quad+\frac{f\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}+\frac{fp\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int \frac{(f+gx^{-2n})\log\left(c(d+ex^n)^p\right)}{x}\,dx = -\frac{\frac{egpx^{-n}(d+enx^n\log(x)-ex^n\log(d+ex^n))}{d^2}+gx^{-2n}\log\left(c(d+ex^n)^p\right)-2f\left(\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)+p\,\text{PolyL}\right.}{2n}$$

```
[In] Integrate[((f + g/x^(2*n))*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -1/2*((e*g*p*(d + e*n*x^n*Log[x] - e*x^n*Log[d + e*x^n]))/(d^2*x^n) + (g*Lo
g[c*(d + e*x^n)^p])/x^(2*n) - 2*f*(Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] +
 p*PolyLog[2, 1 + (e*x^n)/d]))/n
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.64 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.12

method	result
risch	$\frac{(2f\ln(x)n\,x^{2n}-g)x^{-2n}\ln((d+e\,x^n)^p)}{2n}+\left(\frac{i\pi\,\operatorname{csgn}(i(d+e\,x^n)^p)\operatorname{csgn}(ic(d+e\,x^n)^p)^2}{2}-\frac{i\pi\,\operatorname{csgn}(i(d+e\,x^n)^p)\operatorname{csgn}(ic(d+e\,x^n)^p)\operatorname{csgn}}{2}\right.$

```
[In] int((f+g/(x^(2*n)))*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*f*ln(x)*n*(x^n)^2-g)/n/(x^n)^2*ln((d+e*x^n)^p)+(1/2*I*Pi*csgn(I*(d+e
*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e
*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*
x^n)^p)^2*csgn(I*c)+ln(c))*(1/n*f*ln(x^n)-1/2/n*g/(x^n)^2)+1/2*e^2*g*p*ln(d
+e*x^n)/d^2/n-1/2*e*g*p/d/n/(x^n)-1/2*p*e^2/n*g/d^2*ln(x^n)-p/n*f*dilog((d+
e*x^n)/d)-p*f*ln(x)*ln((d+e*x^n)/d)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.19

$$\int \frac{(f+gx^{-2n})\log(c(d+ex^n)^p)}{x}\,dx = -\frac{2\,d^2fnpx^{2\,n}\log(x)\log\left(\frac{ex^n+d}{d}\right)+2\,d^2fpx^{2\,n}\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)+degpx^n+d^2g\log(c)+(e^2gnp-2\,d^2fn\log}{2\,d^2nx^{2\,n}}$$

```
[In] integrate((f+g/(x^(2*n)))*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(2*d^2*f*n*p*x^(2*n)*log(x)*log((e*x^n + d)/d) + 2*d^2*f*p*x^(2*n)*dil
og(-(e*x^n + d)/d + 1) + d*e*g*p*x^n + d^2*g*log(c) + (e^2*g*n*p - 2*d^2*f*
n*log(c))*x^(2*n)*log(x) + (d^2*g*p - (2*d^2*f*n*p*log(x) + e^2*g*p)*x^(2*n
))*log(e*x^n + d))/(d^2*n*x^(2*n))
```

Sympy [F]

$$\int \frac{(f+gx^{-2n})\log(c(d+ex^n)^p)}{x}\,dx = \int \frac{x^{-2n}(fx^{2n}+g)\log(c(d+ex^n)^p)}{x}\,dx$$

```
[In] integrate((f+g/(x**(2*n)))*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral((f*x**(2*n) + g)*log(c*(d + e*x**n)**p)/(x*x**(2*n)), x)
```

Maxima [F]

$$\int \frac{(f + gx^{-2n}) \log (c(d + ex^n)^p)}{x} \, dx = \int \frac{\left(f + \frac{g}{x^{2n}}\right) \log \left((ex^n + d)^p c\right)}{x} \, dx$$

```
[In] integrate((f+g/(x^(2*n)))*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(e*g*p*x^n + d*g*log(c) + (d*f*n^2*p*log(x)^2 - 2*d*f*n*log(c)*log(x))
*x^(2*n) - (2*d*f*n*x^(2*n)*log(x) - d*g)*log((e*x^n + d)^p))/(d*n*x^(2*n))
 + integrate(1/2*(2*d^2*f*n*p*log(x) - e^2*g*p)/(d*e*x*x^n + d^2*x), x)
```

Giac [F]

$$\int \frac{(f + gx^{-2n}) \log (c(d + ex^n)^p)}{x} \, dx = \int \frac{\left(f + \frac{g}{x^{2n}}\right) \log \left((ex^n + d)^p c\right)}{x} \, dx$$

```
[In] integrate((f+g/(x^(2*n)))*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
```

```
[Out] integrate((f + g/x^(2*n))*log((e*x^n + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f + gx^{-2n}) \log (c(d + ex^n)^p)}{x} \, dx = \int \frac{\ln \left(c \left(d + e\, x^n\right)^p\right) \, \left(f + \frac{g}{x^{2\,n}}\right)}{x} \, dx$$

```
[In] int((log(c*(d + e*x^n)^p)*(f + g/x^(2*n)))/x,x)
```

```
[Out] int((log(c*(d + e*x^n)^p)*(f + g/x^(2*n)))/x, x)
```

3.365 $\int \frac{(f+gx^{3n})^2\log(c(d+ex^n)^p)}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 327

$$\int \frac{(f+gx^{3n})^2\log(c(d+ex^n)^p)}{x}\,dx = -\frac{2d^2fgpx^n}{3e^2n}+\frac{d^5g^2px^n}{6e^5n}+\frac{dfgpx^{2n}}{3en}-\frac{d^4g^2px^{2n}}{12e^4n} -\frac{2fgpx^{3n}}{9n}+\frac{d^3g^2px^{3n}}{18e^3n}-\frac{d^2g^2px^{4n}}{24e^2n}+\frac{dg^2px^{5n}}{30en} -\frac{g^2px^{6n}}{36n}+\frac{2d^3fgp\log(d+ex^n)}{3e^3n}-\frac{d^6g^2p\log(d+ex^n)}{6e^6n} +\frac{2fgx^{3n}\log(c(d+ex^n)^p)}{3n}+\frac{g^2x^{6n}\log(c(d+ex^n)^p)}{6n} +\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} +\frac{f^2p\,\mathrm{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}$$

```
[Out] -2/3*d^2*f*g*p*x^n/e^2/n+1/6*d^5*g^2*p*x^n/e^5/n+1/3*d*f*g*p*x^(2*n)/e/n-1/
12*d^4*g^2*p*x^(2*n)/e^4/n-2/9*f*g*p*x^(3*n)/n+1/18*d^3*g^2*p*x^(3*n)/e^3/n
-1/24*d^2*g^2*p*x^(4*n)/e^2/n+1/30*d*g^2*p*x^(5*n)/e/n-1/36*g^2*p*x^(6*n)/n
+2/3*d^3*f*g*p*ln(d+e*x^n)/e^3/n-1/6*d^6*g^2*p*ln(d+e*x^n)/e^6/n+2/3*f*g*x^
(3*n)*ln(c*(d+e*x^n)^p)/n+1/6*g^2*x^(6*n)*ln(c*(d+e*x^n)^p)/n+f^2*ln(-e*x^n
/d)*ln(c*(d+e*x^n)^p)/n+f^2*p*polylog(2,1+e*x^n/d)/n
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2525, 272, 45, 2463, 2441, 2352, 2442}

$$\int \frac{(f+gx^{3n})^2\log(c(d+ex^n)^p)}{x}\,dx = \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{2fgx^{3n}\log(c(d+ex^n)^p)}{3n} + \frac{g^2x^{6n}\log(c(d+ex^n)^p)}{6n} - \frac{d^6g^2p\log(d+ex^n)}{6e^6n} + \frac{d^5g^2px^n}{6e^5n} - \frac{d^4g^2px^{2n}}{12e^4n} + \frac{2d^3fgp\log(d+ex^n)}{3e^3n} + \frac{d^3g^2px^{3n}}{18e^3n} - \frac{2d^2fgpx^n}{3e^2n} - \frac{d^2g^2px^{4n}}{24e^2n} + \frac{f^2p\operatorname{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n} + \frac{dfgpx^{2n}}{3en} + \frac{dg^2px^{5n}}{30en} - \frac{2fgpx^{3n}}{9n} - \frac{g^2px^{6n}}{36n}$$

```
[In] Int[((f + g*x^(3*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-2*d^2*f*g*p*x^n)/(3*e^2*n) + (d^5*g^2*p*x^n)/(6*e^5*n) + (d*f*g*p*x^(2*n)
)/(3*e*n) - (d^4*g^2*p*x^(2*n))/(12*e^4*n) - (2*f*g*p*x^(3*n))/(9*n) + (d^3
*g^2*p*x^(3*n))/(18*e^3*n) - (d^2*g^2*p*x^(4*n))/(24*e^2*n) + (d*g^2*p*x^(5
*n))/(30*e*n) - (g^2*p*x^(6*n))/(36*n) + (2*d^3*f*g*p*Log[d + e*x^n])/(3*e^
3*n) - (d^6*g^2*p*Log[d + e*x^n])/(6*e^6*n) + (2*f*g*x^(3*n)*Log[c*(d + e*x
^n)^p])/(3*n) + (g^2*x^(6*n)*Log[c*(d + e*x^n)^p])/(6*n) + (f^2*Log[-((e*x^
n)/d)]*Log[c*(d + e*x^n)^p])/n + (f^2*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(f+gx^3)^2\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{f^2\log(c(d+ex)^p)}{x} + 2fgx^2\log\left(c(d+ex)^p\right) + g^2x^5\log\left(c(d+ex)^p\right)\right)\,dx, x, x^n\right)}{n}\\
&= \frac{f^2\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n} + \frac{(2fg)\text{Subst}\left(\int x^2\log\left(c(d+ex)^p\right)\,dx, x, x^n\right)}{n}\\
&\quad + \frac{g^2\text{Subst}\left(\int x^5\log\left(c(d+ex)^p\right)\,dx, x, x^n\right)}{n}
\end{aligned}
$$

$$= \frac{2fgx^{3n}\log\left(c(d+ex^n)^p\right)}{3n} + \frac{g^2x^{6n}\log\left(c(d+ex^n)^p\right)}{6n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n} - \frac{(ef^2p)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx,x,x^n\right)}{n} - \frac{(2efgp)\text{Subst}\left(\int \frac{x^3}{d+ex}\,dx,x,x^n\right)}{3n} - \frac{(eg^2p)\,\text{Subst}\left(\int \frac{x^6}{d+ex}\,dx,x,x^n\right)}{6n}$$

$$= \frac{2fgx^{3n}\log\left(c(d+ex^n)^p\right)}{3n} + \frac{g^2x^{6n}\log\left(c(d+ex^n)^p\right)}{6n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n} + \frac{f^2p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n} - \frac{(2efgp)\text{Subst}\left(\int \left(\frac{d^2}{e^3}-\frac{dx}{e^2}+\frac{x^2}{e}-\frac{d^3}{e^3(d+ex)}\right)\,dx,x,x^n\right)}{3n} - \frac{(eg^2p)\,\text{Subst}\left(\int \left(-\frac{d^5}{e^6}+\frac{d^4x}{e^5}-\frac{d^3x^2}{e^4}+\frac{d^2x^3}{e^3}-\frac{dx^4}{e^2}+\frac{x^5}{e}+\frac{d^6}{e^6(d+ex)}\right)\,dx,x,x^n\right)}{6n}$$

$$= -\frac{2d^2fgpx^n}{3e^2n} + \frac{d^5g^2px^n}{6e^5n} + \frac{dfgpx^{2n}}{3en} - \frac{d^4g^2px^{2n}}{12e^4n} - \frac{2fgpx^{3n}}{9n} + \frac{d^3g^2px^{3n}}{18e^3n} - \frac{d^2g^2px^{4n}}{24e^2n} + \frac{dg^2px^{5n}}{30en} - \frac{g^2px^{6n}}{36n} + \frac{2d^3fgp\log\left(d+ex^n\right)}{3e^3n} - \frac{d^6g^2p\log\left(d+ex^n\right)}{6e^6n} + \frac{2fgx^{3n}\log\left(c(d+ex^n)^p\right)}{3n} + \frac{g^2x^{6n}\log\left(c(d+ex^n)^p\right)}{6n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n} + \frac{f^2p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.64

$$\int \frac{\left(f+gx^{3n}\right)^2\log\left(c(d+ex^n)^p\right)}{x}\,dx$$
$$= \frac{-egpx^n(-60d^5g+30d^4egx^n-20d^3e^2gx^{2n}+10e^5x^{2n}(8f+gx^{3n})-12de^4x^n(10f+gx^{3n})+15d^2e^3(16f+\cdots}{}$$

```
[In] Integrate[((f + g*x^(3*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-(e*g*p*x^n*(-60*d^5*g + 30*d^4*e*g*x^n - 20*d^3*e^2*g*x^(2*n) + 10*e^5*x^
(2*n)*(8*f + g*x^(3*n)) - 12*d*e^4*x^n*(10*f + g*x^(3*n)) + 15*d^2*e^3*(16*
f + g*x^(3*n)))) - 60*d^3*g*(-4*e^3*f + d^3*g)*p*Log[d + e*x^n] + 60*e^6*(g
*x^(3*n)*(4*f + g*x^(3*n)) + 6*f^2*Log[-((e*x^n)/d)])*Log[c*(d + e*x^n)^p]
+ 360*e^6*f^2*p*PolyLog[2, 1 + (e*x^n)/d])/(360*e^6*n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.08 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.33

method	result
risch	$\frac{(g^2 x^{6n}+4fg\,x^{3n}+6f^2\ln(x)n)\ln((d+e\,x^n)^p)}{6n}+\frac{\left(\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\mathrm{csgn}(ic(d+e\,x^n)^p)^2}{2}-\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\ \mathrm{csgn}(ic(d+e\,x^n)^p)\ \mathrm{csgn}(ic)}{2}\right.}{}$

```
[In] int((f+g*x^(3*n))^2*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(g^2*(x^n)^6+4*f*g*(x^n)^3+6*f^2*ln(x)*n)/n*ln((d+e*x^n)^p)+(1/2*I*Pi*c
sgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csg
n(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn
(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))/n*(1/6*g^2*(x^n)^6+2/3*f*g*(x^n)^3+f^2
*ln(x^n))-1/36*p/n*g^2*(x^n)^6+1/30*p/e/n*g^2*d*(x^n)^5-1/24*p/e^2/n*g^2*d^
2*(x^n)^4+1/18*p/e^3/n*g^2*d^3*(x^n)^3-1/12*p/e^4/n*g^2*d^4*(x^n)^2+1/6*d^5
*g^2*p*x^n/e^5/n-1/6*d^6*g^2*p*ln(d+e*x^n)/e^6/n-p/n*f^2*dilog((d+e*x^n)/d)
-p*f^2*ln(x)*ln((d+e*x^n)/d)-2/9*p/n*f*g*(x^n)^3+1/3*p/e/n*f*g*d*(x^n)^2-2/
3*d^2*f*g*p*x^n/e^2/n+2/3*d^3*f*g*p*ln(d+e*x^n)/e^3/n
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.89

$$\int \frac{(f+gx^{3n})^2 \log(c(d+ex^n)^p)}{x}\,dx = -\frac{360\,e^6 f^2 np\log(x)\log\left(\frac{ex^n+d}{d}\right)-360\,e^6 f^2 n\log(c)\log(x)-12\,de^5 g^2 px^{5\,n}+15\,d^2e^4g^2px^{4\,n}+360\,e^6f^2p\mathrm{Li}_2}{}$$

```
[In] integrate((f+g*x^(3*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/360*(360*e^6*f^2*n*p*log(x)*log((e*x^n + d)/d) - 360*e^6*f^2*n*log(c)*lo
g(x) - 12*d*e^5*g^2*p*x^(5*n) + 15*d^2*e^4*g^2*p*x^(4*n) + 360*e^6*f^2*p*di
log(-(e*x^n + d)/d + 1) - 30*(4*d*e^5*f*g - d^4*e^2*g^2)*p*x^(2*n) + 60*(4*
d^2*e^4*f*g - d^5*e*g^2)*p*x^n + 10*(e^6*g^2*p - 6*e^6*g^2*log(c))*x^(6*n)
- 20*(12*e^6*f*g*log(c) - (4*e^6*f*g - d^3*e^3*g^2)*p)*x^(3*n) - 60*(6*e^6*
f^2*n*p*log(x) + e^6*g^2*p*x^(6*n) + 4*e^6*f*g*p*x^(3*n) + (4*d^3*e^3*f*g -
 d^6*g^2)*p)*log(e*x^n + d))/(e^6*n)
```

Sympy [F]

$$\int \frac{(f+gx^{3n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \int \frac{(f+gx^{3n})^2 \log(c(d+ex^n)^p)}{x}\,dx$$

[In] integrate((f+g*x**(3*n))**2*ln(c*(d+e*x**n)**p)/x,x)

[Out] Integral((f + g*x**(3*n))**2*log(c*(d + e*x**n)**p)/x, x)

Maxima [F]

$$\int \frac{(f+gx^{3n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \int \frac{(gx^{3n}+f)^2 \log((ex^n+d)^p c)}{x}\,dx$$

[In] integrate((f+g*x^(3*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")

[Out] -1/360*(180*e^6*f^2*n^2*p*log(x)^2 - 12*d*e^5*g^2*p*x^(5*n) + 15*d^2*e^4*g^2*p*x^(4*n) + 10*(e^6*g^2*p - 6*e^6*g^2*log(c))*x^(6*n) + 20*(4*e^6*f*g*p - d^3*e^3*g^2*p - 12*e^6*f*g*log(c))*x^(3*n) - 30*(4*d*e^5*f*g*p - d^4*e^2*g^2*p)*x^(2*n) + 60*(4*d^2*e^4*f*g*p - d^5*e*g^2*p)*x^n - 60*(6*e^6*f^2*n*log(x) + e^6*g^2*x^(6*n) + 4*e^6*f*g*x^(3*n))*log((e*x^n + d)^p) - 60*(4*d^3*e^3*f*g*n*p - d^6*g^2*n*p + 6*e^6*f^2*n*log(c))*log(x))/(e^6*n) + integrate(1/6*(6*d*e^6*f^2*n*p*log(x) - 4*d^4*e^3*f*g*p + d^7*g^2*p)/(e^7*x*x^n + d*e^6*x), x)

Giac [F]

$$\int \frac{(f+gx^{3n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \int \frac{(gx^{3n}+f)^2 \log((ex^n+d)^p c)}{x}\,dx$$

[In] integrate((f+g*x^(3*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")

[Out] integrate((g*x^(3*n) + f)^2*log((e*x^n + d)^p*c)/x, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx^{3n})^2 \log \left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{\ln \left(c\left(d+e\,x^n\right)^p\right)\,\left(f+g\,x^{3\,n}\right)^2}{x}\,dx$$

```
[In] int((log(c*(d + e*x^n)^p)*(f + g*x^(3*n))^2)/x,x)
[Out] int((log(c*(d + e*x^n)^p)*(f + g*x^(3*n))^2)/x, x)
```

3.366 $\int \frac{(f+gx^{2n})^2 \log(c(d+ex^n)^p)}{x} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 254

$$\int \frac{(f+gx^{2n})^2 \log(c(d+ex^n)^p)}{x} \, dx = \frac{dfgpx^n}{en} + \frac{d^3g^2px^n}{4e^3n} - \frac{fgpx^{2n}}{2n} - \frac{d^2g^2px^{2n}}{8e^2n} + \frac{dg^2px^{3n}}{12en} - \frac{g^2px^{4n}}{16n} - \frac{d^2fgp\log(d+ex^n)}{e^2n} - \frac{d^4g^2p\log(d+ex^n)}{4e^4n} + \frac{fgx^{2n}\log(c(d+ex^n)^p)}{n} + \frac{g^2x^{4n}\log(c(d+ex^n)^p)}{4n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{f^2p\,\mathrm{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}$$

```
[Out] d*f*g*p*x^n/e/n+1/4*d^3*g^2*p*x^n/e^3/n-1/2*f*g*p*x^(2*n)/n-1/8*d^2*g^2*p*x
^(2*n)/e^2/n+1/12*d*g^2*p*x^(3*n)/e/n-1/16*g^2*p*x^(4*n)/n-d^2*f*g*p*ln(d+e
*x^n)/e^2/n-1/4*d^4*g^2*p*ln(d+e*x^n)/e^4/n+f*g*x^(2*n)*ln(c*(d+e*x^n)^p)/n
+1/4*g^2*x^(4*n)*ln(c*(d+e*x^n)^p)/n+f^2*ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/n+f
^2*p*polylog(2,1+e*x^n/d)/n
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used

$= \{2525, 272, 45, 2463, 2441, 2352, 2442\}$

$$\int \frac{(f+gx^{2n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{fgx^{2n}\log(c(d+ex^n)^p)}{n} + \frac{g^2x^{4n}\log(c(d+ex^n)^p)}{4n} - \frac{d^4g^2p\log(d+ex^n)}{4e^4n} + \frac{d^3g^2px^n}{4e^3n} - \frac{d^2fgp\log(d+ex^n)}{e^2n} - \frac{d^2g^2px^{2n}}{8e^2n} + \frac{f^2p\,\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n} + \frac{dfgpx^n}{en} + \frac{dg^2px^{3n}}{12en} - \frac{fgpx^{2n}}{2n} - \frac{g^2px^{4n}}{16n}$$

```
[In] Int[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (d*f*g*p*x^n)/(e*n) + (d^3*g^2*p*x^n)/(4*e^3*n) - (f*g*p*x^(2*n))/(2*n) - (
d^2*g^2*p*x^(2*n))/(8*e^2*n) + (d*g^2*p*x^(3*n))/(12*e*n) - (g^2*p*x^(4*n))
/(16*n) - (d^2*f*g*p*Log[d + e*x^n])/(e^2*n) - (d^4*g^2*p*Log[d + e*x^n])/(
4*e^4*n) + (f*g*x^(2*n)*Log[c*(d + e*x^n)^p])/n + (g^2*x^(4*n)*Log[c*(d + e
*x^n)^p])/(4*n) + (f^2*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f^2*p*P
olyLog[2, 1 + (e*x^n)/d])/n
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(f+gx^2)^2\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{f^2\log(c(d+ex)^p)}{x}+2fgx\log\left(c(d+ex)^p\right)+g^2x^3\log\left(c(d+ex)^p\right)\right)\,dx,x,x^n\right)}{n}\\
&= \frac{f^2\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx,x,x^n\right)}{n}+\frac{(2fg)\text{Subst}\left(\int x\log\left(c(d+ex)^p\right)\,dx,x,x^n\right)}{n}\\
&\quad+\frac{g^2\text{Subst}\left(\int x^3\log\left(c(d+ex)^p\right)\,dx,x,x^n\right)}{n}\\
&= \frac{fgx^{2n}\log\left(c(d+ex^n)^p\right)}{n}+\frac{g^2x^{4n}\log\left(c(d+ex^n)^p\right)}{4n}\\
&\quad+\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}-\frac{(ef^2p)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx,x,x^n\right)}{n}\\
&\quad-\frac{(efgp)\text{Subst}\left(\int \frac{x^2}{d+ex}\,dx,x,x^n\right)}{n}-\frac{(eg^2p)\,\text{Subst}\left(\int \frac{x^4}{d+ex}\,dx,x,x^n\right)}{4n}
\end{aligned}
$$

$$=\frac{fgx^{2n}\log\left(c(d+ex^n)^p\right)}{n}+\frac{g^2x^{4n}\log\left(c(d+ex^n)^p\right)}{4n}+\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}+\frac{f^2p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}-\frac{(efgp)\text{Subst}\left(\int\left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)dx,x,x^n\right)}{n}-\frac{(eg^2p)\,\text{Subst}\left(\int\left(-\frac{d^3}{e^4}+\frac{d^2x}{e^3}-\frac{dx^2}{e^2}+\frac{x^3}{e}+\frac{d^4}{e^4(d+ex)}\right)dx,x,x^n\right)}{4n}$$

$$=\frac{dfgpx^n}{en}+\frac{d^3g^2px^n}{4e^3n}-\frac{fgpx^{2n}}{2n}-\frac{d^2g^2px^{2n}}{8e^2n}+\frac{dg^2px^{3n}}{12en}-\frac{g^2px^{4n}}{16n}-\frac{d^2fgp\log(d+ex^n)}{e^2n}-\frac{d^4g^2p\log(d+ex^n)}{4e^4n}+\frac{fgx^{2n}\log\left(c(d+ex^n)^p\right)}{n}+\frac{g^2x^{4n}\log\left(c(d+ex^n)^p\right)}{4n}+\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}+\frac{f^2p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.67

$$\int\frac{(f+gx^{2n})^2\log\left(c(d+ex^n)^p\right)}{x}\,dx=\frac{-egpx^n(-12d^3g+6d^2egx^n+3e^3x^n(8f+gx^{2n})-4de^2(12f+gx^{2n}))-12d^2g(4e^2f+d^2g)\,p\log(d+ex^n)}{48e^4n}$$

[In] Integrate[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p])/x,x]

[Out] (-(e*g*p*x^n*(-12*d^3*g + 6*d^2*e*g*x^n + 3*e^3*x^n*(8*f + g*x^(2*n)) - 4*d*e^2*(12*f + g*x^(2*n)))) - 12*d^2*g*(4*e^2*f + d^2*g)*p*Log[d + e*x^n] + 12*e^4*(g*x^(2*n)*(4*f + g*x^(2*n)) + 4*f^2*Log[-((e*x^n)/d)])*Log[c*(d + e*x^n)^p] + 48*e^4*f^2*p*PolyLog[2, 1 + (e*x^n)/d])/(48*e^4*n)

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.57 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.48

method	result
risch	$\frac{(g^2x^{4n}+4f^2\ln(x)n+4fg\,x^{2n})\ln((d+e\,x^n)^p)}{4n}+\frac{\left(\frac{i\pi\,\text{csgn}(i(d+e\,x^n)^p)\text{csgn}(ic(d+e\,x^n)^p)^2}{2}-\frac{i\pi\,\text{csgn}(i(d+e\,x^n)^p)\,\text{csgn}(ic(d+e\,x^n)^p)\,\text{csgn}(ic)}{2}\right.}{}$

[In] int((f+g*x^(2*n))^2*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)

[Out] 1/4*(g^2*(x^n)^4+4*f^2*ln(x)*n+4*f*g*(x^n)^2)/n*ln((d+e*x^n)^p)+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn

```
(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))/n*(1/4*g^2*(x^n)^4+f*g*(x^n)^2+f^2*ln(
x^n))-1/16*p/n*g^2*(x^n)^4+1/12*p/e/n*g^2*d*(x^n)^3-1/8*p/e^2/n*g^2*d^2*(x^
n)^2+1/4*d^3*g^2*p*x^n/e^3/n-1/4*d^4*g^2*p*ln(d+e*x^n)/e^4/n-p/n*f^2*dilog(
(d+e*x^n)/d)-p*f^2*ln(x)*ln((d+e*x^n)/d)-1/2*p/n*f*g*(x^n)^2+d*f*g*p*x^n/e/
n-d^2*f*g*p*ln(d+e*x^n)/e^2/n
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.95

$$\int \frac{(f+gx^{2n})^2 \log(c(d+ex^n)^p)}{x}\,dx = -\frac{48\,e^4 f^2 np \log(x) \log\left(\frac{ex^n+d}{d}\right) - 48\,e^4 f^2 n \log(c)\log(x) - 4\,de^3 g^2 p x^{3\,n} + 48\,e^4 f^2 p \mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right) - 12\,(\ldots}{}$$

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/48*(48*e^4*f^2*n*p*log(x)*log((e*x^n + d)/d) - 48*e^4*f^2*n*log(c)*log(x
) - 4*d*e^3*g^2*p*x^(3*n) + 48*e^4*f^2*p*dilog(-(e*x^n + d)/d + 1) - 12*(4*
d*e^3*f*g + d^3*e*g^2)*p*x^n + 3*(e^4*g^2*p - 4*e^4*g^2*log(c))*x^(4*n) - 6
*(8*e^4*f*g*log(c) - (4*e^4*f*g + d^2*e^2*g^2)*p)*x^(2*n) - 12*(4*e^4*f^2*n
*p*log(x) + e^4*g^2*p*x^(4*n) + 4*e^4*f*g*p*x^(2*n) - (4*d^2*e^2*f*g + d^4*
g^2)*p)*log(e*x^n + d))/(e^4*n)
```

Sympy [F]

$$\int \frac{(f+gx^{2n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \int \frac{(f+gx^{2n})^2 \log(c(d+ex^n)^p)}{x}\,dx$$

```
[In] integrate((f+g*x**(2*n))**2*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral((f + g*x**(2*n))**2*log(c*(d + e*x**n)**p)/x, x)
```

Maxima [F]

$$\int \frac{(f+gx^{2n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \int \frac{(gx^{2\,n}+f)^2 \log((ex^n+d)^p c)}{x}\,dx$$

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/48*(24*e^4*f^2*n^2*p*log(x)^2 - 4*d*e^3*g^2*p*x^(3*n) + 3*(e^4*g^2*p - 4
*e^4*g^2*log(c))*x^(4*n) + 6*(4*e^4*f*g*p + d^2*e^2*g^2*p - 8*e^4*f*g*log(c
))*x^(2*n) - 12*(4*d*e^3*f*g*p + d^3*e*g^2*p)*x^n - 12*(4*e^4*f^2*n*log(x)
+ e^4*g^2*x^(4*n) + 4*e^4*f*g*x^(2*n))*log((e*x^n + d)^p) + 12*(4*d^2*e^2*f
*g*n*p + d^4*g^2*n*p - 4*e^4*f^2*n*log(c))*log(x))/(e^4*n) + integrate(1/4*
(4*d*e^4*f^2*n*p*log(x) + 4*d^3*e^2*f*g*p + d^5*g^2*p)/(e^5*x*x^n + d*e^4*x
), x)
```

Giac [F]

$$\int \frac{(f+gx^{2n})^2 \log(c(d+ex^n)^p)}{x} \, dx = \int \frac{(gx^{2n}+f)^2 \log((ex^n+d)^p c)}{x} \, dx$$

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
[Out] integrate((g*x^(2*n) + f)^2*log((e*x^n + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx^{2n})^2 \log(c(d+ex^n)^p)}{x} \, dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)\,\left(f+g\,x^{2\,n}\right)^2}{x} \, dx$$

```
[In] int((log(c*(d + e*x^n)^p)*(f + g*x^(2*n))^2)/x,x)
[Out] int((log(c*(d + e*x^n)^p)*(f + g*x^(2*n))^2)/x, x)
```

3.367 $\int \frac{(f+gx^n)^2 \log(c(d+ex^n)^p)}{x} dx$

Optimal result

Integrand size = 25, antiderivative size = 176

$$\begin{aligned}\int \frac{(f+gx^n)^2 \log(c(d+ex^n)^p)}{x} dx &= -\frac{2fgpx^n}{n} + \frac{dg^2px^n}{2en} - \frac{g^2px^{2n}}{4n} \\ &\quad - \frac{d^2g^2p\log(d+ex^n)}{2e^2n} + \frac{g^2x^{2n}\log(c(d+ex^n)^p)}{2n} \\ &\quad + \frac{2fg(d+ex^n)\log(c(d+ex^n)^p)}{en} \\ &\quad + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} \\ &\quad + \frac{f^2p\,\mathrm{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}\end{aligned}$$

```
[Out] -2*f*g*p*x^n/n+1/2*d*g^2*p*x^n/e/n-1/4*g^2*p*x^(2*n)/n-1/2*d^2*g^2*p*ln(d+e
*x^n)/e^2/n+1/2*g^2*x^(2*n)*ln(c*(d+e*x^n)^p)/n+2*f*g*(d+e*x^n)*ln(c*(d+e*x
^n)^p)/e/n+f^2*ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/n+f^2*p*polylog(2,1+e*x^n/d)/
n
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used

$= \{2525, 45, 2463, 2436, 2332, 2441, 2352, 2442\}$

$$\int \frac{(f+gx^n)^2 \log(c(d+ex^n)^p)}{x}\,dx = \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{2fg(d+ex^n)\log(c(d+ex^n)^p)}{en} + \frac{g^2x^{2n}\log(c(d+ex^n)^p)}{2n} - \frac{d^2g^2p\log(d+ex^n)}{2e^2n} + \frac{f^2p\,\mathrm{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n} + \frac{dg^2px^n}{2en} - \frac{2fgpx^n}{n} - \frac{g^2px^{2n}}{4n}$$

```
[In] Int[((f + g*x^n)^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-2*f*g*p*x^n)/n + (d*g^2*p*x^n)/(2*e*n) - (g^2*p*x^(2*n))/(4*n) - (d^2*g^2
*p*Log[d + e*x^n])/(2*e^2*n) + (g^2*x^(2*n)*Log[c*(d + e*x^n)^p])/(2*n) + (
2*f*g*(d + e*x^n)*Log[c*(d + e*x^n)^p])/(e*n) + (f^2*Log[-((e*x^n)/d)]*Log[
c*(d + e*x^n)^p])/n + (f^2*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(f+gx)^2\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(2fg\log\left(c(d+ex)^p\right)+\frac{f^2\log(c(d+ex)^p)}{x}+g^2x\log\left(c(d+ex)^p\right)\right)\,dx, x, x^n\right)}{n}\\
&= \frac{f^2\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}+\frac{(2fg)\text{Subst}\left(\int \log\left(c(d+ex)^p\right)\,dx, x, x^n\right)}{n}\\
&\quad+\frac{g^2\text{Subst}\left(\int x\log\left(c(d+ex)^p\right)\,dx, x, x^n\right)}{n}\\
&= \frac{g^2x^{2n}\log\left(c(d+ex^n)^p\right)}{2n}+\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n}\\
&\quad+\frac{(2fg)\text{Subst}\left(\int \log\left(cx^p\right)\,dx, x, d+ex^n\right)}{en}\\
&\quad-\frac{\left(ef^2p\right)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^n\right)}{n}-\frac{\left(eg^2p\right)\text{Subst}\left(\int \frac{x^2}{d+ex}\,dx, x, x^n\right)}{2n}
\end{aligned}
$$

$$= -\frac{2fgpx^n}{n} + \frac{g^2x^{2n}\log\left(c(d+ex^n)^p\right)}{2n} + \frac{2fg(d+ex^n)\log\left(c(d+ex^n)^p\right)}{en} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n} + \frac{f^2p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n} - \frac{(eg^2p)\,\text{Subst}\left(\int\left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)dx, x, x^n\right)}{2n}$$

$$= -\frac{2fgpx^n}{n} + \frac{dg^2px^n}{2en} - \frac{g^2px^{2n}}{4n} - \frac{d^2g^2p\log(d+ex^n)}{2e^2n} + \frac{g^2x^{2n}\log\left(c(d+ex^n)^p\right)}{2n} + \frac{2fg(d+ex^n)\log\left(c(d+ex^n)^p\right)}{en} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{n} + \frac{f^2p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.70

$$\int \frac{(f+gx^n)^2\log\left(c(d+ex^n)^p\right)}{x}\,dx = \frac{-egpx^n(8ef-2dg+egx^n) - 2d^2g^2p\log(d+ex^n) + 2e\left(4dfg+egx^n(4f+gx^n)+2ef^2\log\left(-\frac{ex^n}{d}\right)\right)\log(c(\cdots}{4e^2n}$$

```
[In] Integrate[((f + g*x^n)^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] (-(e*g*p*x^n*(8*e*f - 2*d*g + e*g*x^n)) - 2*d^2*g^2*p*Log[d + e*x^n] + 2*e*
(4*d*f*g + e*g*x^n*(4*f + g*x^n) + 2*e*f^2*Log[-((e*x^n)/d)])*Log[c*(d + e*
x^n)^p] + 4*e^2*f^2*p*PolyLog[2, 1 + (e*x^n)/d])/(4*e^2*n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.21 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.78

method	result
risch	$\frac{(2f^2\ln(x)n+g^2x^{2n}+4fg\,x^n)\ln((d+e\,x^n)^p)}{2n} + \frac{\left(\frac{i\pi\,\text{csgn}(i(d+e\,x^n)^p)\text{csgn}(ic(d+e\,x^n)^p)^2}{2} - \frac{i\pi\,\text{csgn}(i(d+e\,x^n)^p)\,\text{csgn}(ic(d+e\,x^n)^p)\,\text{csgn}(ic)}{2}\cdots\right.}{\cdots}$

```
[In] int((f+g*x^n)^2*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*f^2*ln(x)*n+g^2*(x^n)^2+4*f*g*x^n)/n*ln((d+e*x^n)^p)+(1/2*I*Pi*csgn(
I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*
c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c
*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))/n*(1/2*g^2*(x^n)^2+2*f*g*x^n+f^2*ln(x^n))-
```

```
1/4*p/n*g^2*(x^n)^2+1/2*d*g^2*p*x^n/e/n-1/2*d^2*g^2*p*ln(d+e*x^n)/e^2/n-p/n
*f^2*dilog((d+e*x^n)/d)-p*f^2*ln(x)*ln((d+e*x^n)/d)-2*f*g*p*x^n/n+2*p/e/n*f
*g*d*ln(d+e*x^n)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.09

$$\int \frac{(f+gx^n)^2\log\left(c(d+ex^n)^p\right)}{x}\,dx = -\frac{4\,e^2f^2np\log(x)\log\left(\frac{ex^n+d}{d}\right)-4\,e^2f^2n\log(c)\log(x)+4\,e^2f^2p\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)+\left(e^2g^2p-2\,e^2g^2\log(c)\right.}{}$$

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")
```

```
[Out] -1/4*(4*e^2*f^2*n*p*log(x)*log((e*x^n + d)/d) - 4*e^2*f^2*n*log(c)*log(x) +
 4*e^2*f^2*p*dilog(-(e*x^n + d)/d + 1) + (e^2*g^2*p - 2*e^2*g^2*log(c))*x^(
2*n) - 2*(4*e^2*f*g*log(c) - (4*e^2*f*g - d*e*g^2)*p)*x^n - 2*(2*e^2*f^2*n*
p*log(x) + e^2*g^2*p*x^(2*n) + 4*e^2*f*g*p*x^n + (4*d*e*f*g - d^2*g^2)*p)*l
og(e*x^n + d))/(e^2*n)
```

Sympy [F]

$$\int \frac{(f+gx^n)^2\log\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{(f+gx^n)^2\log\left(c(d+ex^n)^p\right)}{x}\,dx$$

```
[In] integrate((f+g*x**n)**2*ln(c*(d+e*x**n)**p)/x,x)
```

```
[Out] Integral((f + g*x**n)**2*log(c*(d + e*x**n)**p)/x, x)
```

Maxima [F]

$$\int \frac{(f+gx^n)^2\log\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{(gx^n+f)^2\log\left((ex^n+d)^pc\right)}{x}\,dx$$

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
```

```
[Out] -1/4*(2*e^2*f^2*n^2*p*log(x)^2 + (e^2*g^2*p - 2*e^2*g^2*log(c))*x^(2*n) + 2
*(4*e^2*f*g*p - d*e*g^2*p - 4*e^2*f*g*log(c))*x^n - 2*(2*e^2*f^2*n*log(x) +
 e^2*g^2*x^(2*n) + 4*e^2*f*g*x^n)*log((e*x^n + d)^p) - 2*(4*d*e*f*g*n*p - d
^2*g^2*n*p + 2*e^2*f^2*n*log(c))*log(x))/(e^2*n) + integrate(1/2*(2*d*e^2*f
^2*n*p*log(x) - 4*d^2*e*f*g*p + d^3*g^2*p)/(e^3*x*x^n + d*e^2*x), x)
```

Giac [F]

$$\int \frac{(f + gx^n)^2 \log\left(c(d + ex^n)^p\right)}{x}\, dx = \int \frac{(gx^n + f)^2 \log\left((ex^n + d)^p c\right)}{x}\, dx$$

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
[Out] integrate((g*x^n + f)^2*log((e*x^n + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f + gx^n)^2 \log\left(c(d + ex^n)^p\right)}{x}\, dx = \int \frac{\ln\left(c\,(d + e\,x^n)^p\right)\,(f + g\,x^n)^2}{x}\, dx$$

```
[In] int((log(c*(d + e*x^n)^p)*(f + g*x^n)^2)/x,x)
[Out] int((log(c*(d + e*x^n)^p)*(f + g*x^n)^2)/x, x)
```

3.368 $\int \frac{(f+gx^{-n})^2 \log(c(d+ex^n)^p)}{x} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 193

$$\int \frac{(f+gx^{-n})^2 \log (c(d+ex^n)^p)}{x} \, dx = -\frac{eg^2px^{-n}}{2dn} + \frac{2efgp\log(x)}{d} - \frac{e^2g^2p\log(x)}{2d^2} - \frac{2efgp\log(d+ex^n)}{dn} + \frac{e^2g^2p\log(d+ex^n)}{2d^2n} - \frac{g^2x^{-2n}\log(c(d+ex^n)^p)}{2n} - \frac{2fgx^{-n}\log(c(d+ex^n)^p)}{n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{f^2p\operatorname{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}$$

```
[Out] -1/2*e*g^2*p/d/n/(x^n)+2*e*f*g*p*ln(x)/d-1/2*e^2*g^2*p*ln(x)/d^2-2*e*f*g*p*
ln(d+e*x^n)/d/n+1/2*e^2*g^2*p*ln(d+e*x^n)/d^2/n-1/2*g^2*ln(c*(d+e*x^n)^p)/n
/(x^(2*n))-2*f*g*ln(c*(d+e*x^n)^p)/n/(x^n)+f^2*ln(-e*x^n/d)*ln(c*(d+e*x^n)^
p)/n+f^2*p*polylog(2,1+e*x^n/d)/n
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules

used = {2525, 269, 45, 2463, 2442, 46, 36, 29, 31, 2441, 2352}

$$\int \frac{(f+gx^{-n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} - \frac{2fgx^{-n}\log(c(d+ex^n)^p)}{n} - \frac{g^2x^{-2n}\log(c(d+ex^n)^p)}{2n} + \frac{e^2g^2p\log(d+ex^n)}{2d^2n} - \frac{e^2g^2p\log(x)}{2d^2} + \frac{f^2p\,\mathrm{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n} - \frac{2efgp\log(d+ex^n)}{dn} + \frac{2efgp\log(x)}{d} - \frac{eg^2px^{-n}}{2dn}$$

```
[In] Int[((f + g/x^n)^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -1/2*(e*g^2*p)/(d*n*x^n) + (2*e*f*g*p*Log[x])/d - (e^2*g^2*p*Log[x])/(2*d^2
) - (2*e*f*g*p*Log[d + e*x^n])/(d*n) + (e^2*g^2*p*Log[d + e*x^n])/(2*d^2*n)
 - (g^2*Log[c*(d + e*x^n)^p])/(2*n*x^(2*n)) - (2*f*g*Log[c*(d + e*x^n)^p])/
(n*x^n) + (f^2*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n + (f^2*p*PolyLog[2
, 1 + (e*x^n)/d])/n
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(f+\frac{g}{x}\right)^2 \log(c(d+ex)^p)}{x} \, dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{g^2 \log(c(d+ex)^p)}{x^3} + \frac{2fg \log(c(d+ex)^p)}{x^2} + \frac{f^2 \log(c(d+ex)^p)}{x}\right) \, dx, x, x^n\right)}{n}$$

$$= \frac{f^2 \text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x} \, dx, x, x^n\right)}{n} + \frac{(2fg)\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^2} \, dx, x, x^n\right)}{n} + \frac{g^2 \text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^3} \, dx, x, x^n\right)}{n}$$

$$= -\frac{g^2 x^{-2n} \log(c(d+ex^n)^p)}{2n} - \frac{2fgx^{-n} \log(c(d+ex^n)^p)}{n} + \frac{f^2 \log\left(-\frac{ex^n}{d}\right) \log(c(d+ex^n)^p)}{n} - \frac{(ef^2p)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex} \, dx, x, x^n\right)}{n} + \frac{(2efgp)\text{Subst}\left(\int \frac{1}{x(d+ex)} \, dx, x, x^n\right)}{n} + \frac{(eg^2p)\,\text{Subst}\left(\int \frac{1}{x^2(d+ex)} \, dx, x, x^n\right)}{2n}$$

$$= -\frac{g^2 x^{-2n} \log(c(d+ex^n)^p)}{2n} - \frac{2fgx^{-n} \log(c(d+ex^n)^p)}{n} + \frac{f^2 \log\left(-\frac{ex^n}{d}\right) \log(c(d+ex^n)^p)}{n} + \frac{f^2 p \text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n} + \frac{(2efgp)\text{Subst}\left(\int \frac{1}{x} \, dx, x, x^n\right)}{dn} - \frac{(2e^2fgp)\,\text{Subst}\left(\int \frac{1}{d+ex} \, dx, x, x^n\right)}{dn} + \frac{(eg^2p)\,\text{Subst}\left(\int \left(\frac{1}{dx^2} - \frac{e}{d^2x} + \frac{e^2}{d^2(d+ex)}\right) \, dx, x, x^n\right)}{2n}$$

$$= -\frac{eg^2px^{-n}}{2dn} + \frac{2efgp\log(x)}{d} - \frac{e^2g^2p\log(x)}{2d^2} - \frac{2efgp\log(d+ex^n)}{dn} + \frac{e^2g^2p\log(d+ex^n)}{2d^2n} - \frac{g^2x^{-2n}\log(c(d+ex^n)^p)}{2n} - \frac{2fgx^{-n}\log(c(d+ex^n)^p)}{n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{f^2p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.83

$$\int \frac{(f+gx^{-n})^2 \log(c(d+ex^n)^p)}{x} \, dx =$$
$$-\frac{-4defgnp\log(x)+4defgp\log(d+ex^n)+eg^2p(dx^{-n}+en\log(x)-e\log(d+ex^n))+d^2g^2x^{-2n}\log(c\cdots}{2\cdots}$$

[In] Integrate[((f + g/x^n)^2*Log[c*(d + e*x^n)^p])/x,x]

[Out] -1/2*(-4*d*e*f*g*n*p*Log[x] + 4*d*e*f*g*p*Log[d + e*x^n] + e*g^2*p*(d/x^n + e*n*Log[x] - e*Log[d + e*x^n]) + (d^2*g^2*Log[c*(d + e*x^n)^p])/x^(2*n) + (4*d^2*f*g*Log[c*(d + e*x^n)^p])/x^n - 2*d^2*f^2*(Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*PolyLog[2, 1 + (e*x^n)/d]))/(d^2*n)

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.57 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.73

method	result
risch	$\frac{(2f^2\ln(x)n\,x^{2n}-4fg\,x^n-g^2)x^{-2n}\ln((d+e\,x^n)^p)}{2n}+\frac{\left(\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\mathrm{csgn}(ic(d+e\,x^n)^p)^2}{2}-\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\ \mathrm{csgn}(ic(d+e\,x^n)^p)\cdots}{2}\right.\cdots}{\cdots}$

[In] int((f+g/(x^n))^2*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)

[Out] 1/2*(2*f^2*ln(x)*n*(x^n)^2-4*f*g*x^n-g^2)/n/(x^n)^2*ln((d+e*x^n)^p)+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))/n*(-2*f*g/(x^n)+f^2*ln(x^n)-1/2*g^2/(x^n)^2)-2*e*f*g*p*ln(d+e*x^n)/d/n+2*p*e/n*f*g/d*ln(x^n)+1/2*e^2*g^2*p*ln(d+e*x^n)/d^2/n-1/2*e*g^2*p/d/n/(x^n)-1/2*p*e^2/n*g^2/d^2*ln(x^n)-p/n*f^2*dilog((d+e*x^n)/d)-p*f^2*ln(x)*ln((d+e*x^n)/d)

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.09

$$\int \frac{(f+gx^{-n})^2 \log(c(d+ex^n)^p)}{x}\,dx = -\frac{2\,d^2f^2npx^{2\,n}\log(x)\log\left(\frac{ex^n+d}{d}\right)+2\,d^2f^2px^{2\,n}\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)+d^2g^2\log(c)-(2\,d^2f^2n\log(c)+(4\,defg}{}$$

[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")

[Out] -1/2*(2*d^2*f^2*n*p*x^(2*n)*log(x)*log((e*x^n + d)/d) + 2*d^2*f^2*p*x^(2*n)*dilog(-(e*x^n + d)/d + 1) + d^2*g^2*log(c) - (2*d^2*f^2*n*log(c) + (4*d*e*f*g - e^2*g^2)*n*p)*x^(2*n)*log(x) + (d*e*g^2*p + 4*d^2*f*g*log(c))*x^n + (4*d^2*f*g*p*x^n + d^2*g^2*p - (2*d^2*f^2*n*p*log(x) - (4*d*e*f*g - e^2*g^2)*p)*x^(2*n))*log(e*x^n + d))/(d^2*n*x^(2*n))

Sympy [F]

$$\int \frac{(f+gx^{-n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \int \frac{x^{-2n}(fx^n+g)^2 \log(c(d+ex^n)^p)}{x}\,dx$$

[In] integrate((f+g/(x**n))**2*ln(c*(d+e*x**n)**p)/x,x)

[Out] Integral((f*x**n + g)**2*log(c*(d + e*x**n)**p)/(x*x**(2*n)), x)

Maxima [F]

$$\int \frac{(f+gx^{-n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \int \frac{\left(f+\frac{g}{x^n}\right)^2 \log((ex^n+d)^p c)}{x}\,dx$$

[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")

[Out] -1/2*(d*g^2*log(c) + (d*f^2*n^2*p*log(x)^2 - 2*d*f^2*n*log(c)*log(x))*x^(2*n) + (e*g^2*p + 4*d*f*g*log(c))*x^n - (2*d*f^2*n*x^(2*n)*log(x) - 4*d*f*g*x^n - d*g^2)*log((e*x^n + d)^p))/(d*n*x^(2*n)) + integrate(1/2*(2*d^2*f^2*n*p*log(x) + 4*d*e*f*g*p - e^2*g^2*p)/(d*e*x*x^n + d^2*x), x)

Giac [F]

$$\int \frac{(f+gx^{-n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \int \frac{\left(f+\frac{g}{x^n}\right)^2 \log\left(\left(ex^n+d\right)^p c\right)}{x}\,dx$$

```
[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
[Out] integrate((f + g/x^n)^2*log((e*x^n + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx^{-n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \int \frac{\ln\left(c\left(d+e\,x^n\right)^p\right)\,\left(f+\frac{g}{x^n}\right)^2}{x}\,dx$$

```
[In] int((log(c*(d + e*x^n)^p)*(f + g/x^n)^2)/x,x)
[Out] int((log(c*(d + e*x^n)^p)*(f + g/x^n)^2)/x, x)
```

3.369 $\int \frac{(f+gx^{-2n})^2\log(c(d+ex^n)^p)}{x}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 257

$$\begin{aligned}\int \frac{(f+gx^{-2n})^2\log(c(d+ex^n)^p)}{x}\,dx = &-\frac{eg^2px^{-3n}}{12dn}+\frac{e^2g^2px^{-2n}}{8d^2n}-\frac{efgpx^{-n}}{dn}-\frac{e^3g^2px^{-n}}{4d^3n}\\ &-\frac{e^2fgp\log(x)}{d^2}-\frac{e^4g^2p\log(x)}{4d^4}+\frac{e^2fgp\log(d+ex^n)}{d^2n}\\ &+\frac{e^4g^2p\log(d+ex^n)}{4d^4n}-\frac{g^2x^{-4n}\log(c(d+ex^n)^p)}{4n}\\ &-\frac{fgx^{-2n}\log(c(d+ex^n)^p)}{n}\\ &+\frac{f^2\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n}\\ &+\frac{f^2p\,\mathrm{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{n}\end{aligned}$$

```
[Out] -1/12*e*g^2*p/d/n/(x^(3*n))+1/8*e^2*g^2*p/d^2/n/(x^(2*n))-e*f*g*p/d/n/(x^n)
-1/4*e^3*g^2*p/d^3/n/(x^n)-e^2*f*g*p*ln(x)/d^2-1/4*e^4*g^2*p*ln(x)/d^4+e^2*
f*g*p*ln(d+e*x^n)/d^2/n+1/4*e^4*g^2*p*ln(d+e*x^n)/d^4/n-1/4*g^2*ln(c*(d+e*x
^n)^p)/n/(x^(4*n))-f*g*ln(c*(d+e*x^n)^p)/n/(x^(2*n))+f^2*ln(-e*x^n/d)*ln(c*
(d+e*x^n)^p)/n+f^2*p*polylog(2,1+e*x^n/d)/n
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2525, 269, 272, 45, 2463, 2442, 46, 2441, 2352}

$$\int \frac{(f+gx^{-2n})^2\log(c(d+ex^n)^p)}{x}\,dx = \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} - \frac{fgx^{-2n}\log(c(d+ex^n)^p)}{n} - \frac{g^2x^{-4n}\log(c(d+ex^n)^p)}{4n} + \frac{e^4g^2p\log(d+ex^n)}{4d^4n} - \frac{e^4g^2p\log(x)}{4d^4} - \frac{e^3g^2px^{-n}}{4d^3n} + \frac{e^2fgp\log(d+ex^n)}{d^2n} - \frac{e^2fgp\log(x)}{d^2} + \frac{e^2g^2px^{-2n}}{8d^2n} + \frac{f^2p\,\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{n} - \frac{efgpx^{-n}}{dn} - \frac{eg^2px^{-3n}}{12dn}$$

```
[In] Int[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

```
[Out] -1/12*(e*g^2*p)/(d*n*x^(3*n)) + (e^2*g^2*p)/(8*d^2*n*x^(2*n)) - (e*f*g*p)/(
d*n*x^n) - (e^3*g^2*p)/(4*d^3*n*x^n) - (e^2*f*g*p*Log[x])/d^2 - (e^4*g^2*p*
Log[x])/(4*d^4) + (e^2*f*g*p*Log[d + e*x^n])/(d^2*n) + (e^4*g^2*p*Log[d + e
*x^n])/(4*d^4*n) - (g^2*Log[c*(d + e*x^n)^p])/(4*n*x^(4*n)) - (f*g*Log[c*(d
 + e*x^n)^p])/(n*x^(2*n)) + (f^2*Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/n
+ (f^2*p*PolyLog[2, 1 + (e*x^n)/d])/n
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(f+\frac{g}{x^2}\right)^2 \log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{g^2\log(c(d+ex)^p)}{x^5} + \frac{2fg\log(c(d+ex)^p)}{x^3} + \frac{f^2\log(c(d+ex)^p)}{x}\right)dx, x, x^n\right)}{n}$$

$$= \frac{f^2\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}dx, x, x^n\right)}{n} + \frac{(2fg)\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^3}dx, x, x^n\right)}{n} + \frac{g^2\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x^5}dx, x, x^n\right)}{n}$$

$$= -\frac{g^2x^{-4n}\log(c(d+ex^n)^p)}{4n} - \frac{fgx^{-2n}\log(c(d+ex^n)^p)}{n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} - \frac{(ef^2p)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}dx, x, x^n\right)}{n} + \frac{(efgp)\text{Subst}\left(\int \frac{1}{x^2(d+ex)}dx, x, x^n\right)}{n} + \frac{(eg^2p)\,\text{Subst}\left(\int \frac{1}{x^4(d+ex)}dx, x, x^n\right)}{4n}$$

$$= -\frac{g^2x^{-4n}\log(c(d+ex^n)^p)}{4n} - \frac{fgx^{-2n}\log(c(d+ex^n)^p)}{n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{f^2p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n} + \frac{(efgp)\text{Subst}\left(\int \left(\frac{1}{dx^2} - \frac{e}{d^2x} + \frac{e^2}{d^2(d+ex)}\right)dx, x, x^n\right)}{n} + \frac{(eg^2p)\,\text{Subst}\left(\int \left(\frac{1}{dx^4} - \frac{e}{d^2x^3} + \frac{e^2}{d^3x^2} - \frac{e^3}{d^4x} + \frac{e^4}{d^4(d+ex)}\right)dx, x, x^n\right)}{4n}$$

$$= -\frac{eg^2px^{-3n}}{12dn} + \frac{e^2g^2px^{-2n}}{8d^2n} - \frac{efgpx^{-n}}{dn} - \frac{e^3g^2px^{-n}}{4d^3n} - \frac{e^2fgp\log(x)}{d^2} - \frac{e^4g^2p\log(x)}{4d^4} + \frac{e^2fgp\log(d+ex^n)}{d^2n} + \frac{e^4g^2p\log(d+ex^n)}{4d^4n} - \frac{g^2x^{-4n}\log(c(d+ex^n)^p)}{4n} - \frac{fgx^{-2n}\log(c(d+ex^n)^p)}{n} + \frac{f^2\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{n} + \frac{f^2p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{n}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.73

$$\int \frac{(f+gx^{-2n})^2\log(c(d+ex^n)^p)}{x}\,dx =$$

$$-\frac{\frac{24efgp(dx^{-n}+en\log(x)-e\log(d+ex^n))}{d^2} + \frac{eg^2p(dx^{-3n}(2d^2-3dex^n+6e^2x^{2n})+6e^3n\log(x)-6e^3\log(d+ex^n))}{d^4} + 6g^2x^{-4n}\log(c(d+\cdots}{24\cdots}$$

```
[In] Integrate[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p])/x,x]
```

[Out] -1/24*((24*e*f*g*p*(d/x^n + e*n*Log[x] - e*Log[d + e*x^n]))/d^2 + (e*g^2*p*((d*(2*d^2 - 3*d*e*x^n + 6*e^2*x^(2*n)))/x^(3*n) + 6*e^3*n*Log[x] - 6*e^3*Log[d + e*x^n]))/d^4 + (6*g^2*Log[c*(d + e*x^n)^p])/x^(4*n) + (24*f*g*Log[c*(d + e*x^n)^p])/x^(2*n) - 24*f^2*(Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + p*PolyLog[2, 1 + (e*x^n)/d]))/n

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.13 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.54

method	result
risch	$\frac{(4f^2\ln(x)n\,x^{4n}-4fg\,x^{2n}-g^2)x^{-4n}\ln((d+e\,x^n)^p)}{4n}+\frac{\left(\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\mathrm{csgn}(ic(d+e\,x^n)^p)^2}{2}-\frac{i\pi\ \mathrm{csgn}(i(d+e\,x^n)^p)\ \mathrm{csgn}(ic(d+e\,x^n)^p)\ \mathrm{c}}{2}\right.}{}$

[In] int((f+g/(x^(2*n)))^2*ln(c*(d+e*x^n)^p)/x,x,method=_RETURNVERBOSE)

[Out] 1/4*(4*f^2*ln(x)*n*(x^n)^4-4*f*g*(x^n)^2-g^2)/n/(x^n)^4*ln((d+e*x^n)^p)+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))/n*(f^2*ln(x^n)-f*g/(x^n)^2-1/4*g^2/(x^n)^4)+1/4*e^4*g^2*p*ln(d+e*x^n)/d^4/n-1/12*p*e/n*g^2/d/(x^n)^3-1/4*e^3*g^2*p/d^3/n/(x^n)+1/8*p*e^2/n*g^2/d^2/(x^n)^2-1/4*p*e^4/n*g^2/d^4*ln(x^n)+e^2*f*g*p*ln(d+e*x^n)/d^2/n-e*f*g*p/d/n/(x^n)-p*e^2/n*f*g/d^2*ln(x^n)-p/n*f^2*dilog((d+e*x^n)/d)-p*f^2*ln(x)*ln((d+e*x^n)/d)

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.03

$$\int \frac{(f+gx^{-2n})^2\log\left(c(d+ex^n)^p\right)}{x}\,dx = -\frac{24\,d^4f^2npx^{4\,n}\log\left(x\right)\log\left(\frac{ex^n+d}{d}\right)+24\,d^4f^2px^{4\,n}\mathrm{Li}_2\left(-\frac{ex^n+d}{d}+1\right)+2\,d^3eg^2px^n+6\,d^4g^2\log\left(c\right)+6\,(4\,d^3e\ldots}{}$$

[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="fricas")

[Out] -1/24*(24*d^4*f^2*n*p*x^(4*n)*log(x)*log((e*x^n + d)/d) + 24*d^4*f^2*p*x^(4*n)*dilog(-(e*x^n + d)/d + 1) + 2*d^3*e*g^2*p*x^n + 6*d^4*g^2*log(c) + 6*(4*d^3*e*f*g + d*e^3*g^2)*p*x^(3*n) - 6*(4*d^4*f^2*n*log(c) - (4*d^2*e^2*f*g + e^4*g^2)*n*p)*x^(4*n)*log(x) - 3*(d^2*e^2*g^2*p - 8*d^4*f*g*log(c))*x^(2*n) + 6*(4*d^4*f*g*p*x^(2*n) + d^4*g^2*p - (4*d^4*f^2*n*p*log(x) + (4*d^2*e^2*f*g + e^4*g^2)*p)*x^(4*n))*log(e*x^n + d))/(d^4*n*x^(4*n))

Sympy [F]

$$\int \frac{(f+gx^{-2n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \int \frac{x^{-4n}(fx^{2n}+g)^2 \log(c(d+ex^n)^p)}{x}\,dx$$

```
[In] integrate((f+g/(x**(2*n)))**2*ln(c*(d+e*x**n)**p)/x,x)
[Out] Integral((f*x**(2*n) + g)**2*log(c*(d + e*x**n)**p)/(x*x**(4*n)), x)
```

Maxima [F]

$$\int \frac{(f+gx^{-2n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \int \frac{\left(f+\frac{g}{x^{2n}}\right)^2 \log((ex^n+d)^p c)}{x}\,dx$$

```
[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="maxima")
[Out] -1/24*(2*d^2*e*g^2*p*x^n + 6*d^3*g^2*log(c) + 12*(d^3*f^2*n^2*p*log(x)^2 -
2*d^3*f^2*n*log(c)*log(x))*x^(4*n) + 6*(4*d^2*e*f*g*p + e^3*g^2*p)*x^(3*n)
- 3*(d*e^2*g^2*p - 8*d^3*f*g*log(c))*x^(2*n) - 6*(4*d^3*f^2*n*x^(4*n)*log(x
) - 4*d^3*f*g*x^(2*n) - d^3*g^2)*log((e*x^n + d)^p))/(d^3*n*x^(4*n)) + inte
grate(1/4*(4*d^4*f^2*n*p*log(x) - 4*d^2*e^2*f*g*p - e^4*g^2*p)/(d^3*e*x*x^n
 + d^4*x), x)
```

Giac [F]

$$\int \frac{(f+gx^{-2n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \int \frac{\left(f+\frac{g}{x^{2n}}\right)^2 \log((ex^n+d)^p c)}{x}\,dx$$

```
[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)/x,x, algorithm="giac")
[Out] integrate((f + g/x^(2*n))^2*log((e*x^n + d)^p*c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx^{-2n})^2 \log(c(d+ex^n)^p)}{x}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)\,\left(f+\frac{g}{x^{2\,n}}\right)^2}{x}\,dx$$

```
[In] int((log(c*(d + e*x^n)^p)*(f + g/x^(2*n))^2)/x,x)
[Out] int((log(c*(d + e*x^n)^p)*(f + g/x^(2*n))^2)/x, x)
```

3.370 $\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{2n})}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 266

$$\begin{aligned}\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{2n})}\,dx &= \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{fn} - \frac{\log(c(d+ex^n)^p)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x^n)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2fn} \\ &\quad - \frac{\log(c(d+ex^n)^p)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x^n)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2fn} \\ &\quad - \frac{p\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex^n)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2fn} \\ &\quad - \frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{g}(d+ex^n)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2fn} + \frac{p\,\text{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{fn}\end{aligned}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/f/n-1/2*ln(c*(d+e*x^n)^p)*ln(e*((-f)^(1/2)-x
^n*g^(1/2))/(e*(-f)^(1/2)+d*g^(1/2)))/f/n-1/2*ln(c*(d+e*x^n)^p)*ln(e*((-f)^
(1/2)+x^n*g^(1/2))/(e*(-f)^(1/2)-d*g^(1/2)))/f/n+p*polylog(2,1+e*x^n/d)/f/n
-1/2*p*polylog(2,-(d+e*x^n)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/f/n-1/2*p*pol
ylog(2,(d+e*x^n)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/f/n
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2525, 272, 36, 29, 31, 2463, 2441, 2352, 266, 2440, 2438}

$$\begin{aligned}\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{2n})}\,dx = &-\frac{\log(c(d+ex^n)^p)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x^n)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2fn}\\ &-\frac{\log(c(d+ex^n)^p)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x^n)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2fn}\\ &+\frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{fn}-\frac{p\,\text{PolyLog}\left(2,-\frac{\sqrt{g}(ex^n+d)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2fn}\\ &-\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{g}(ex^n+d)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2fn}+\frac{p\,\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{fn}\end{aligned}$$

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))),x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f*n) - (Log[c*(d + e*x^n)^p]*Log[
(e*(Sqrt[-f] - Sqrt[g]*x^n))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*f*n) - (Log[c*(d
 + e*x^n)^p]*Log[(e*(Sqrt[-f] + Sqrt[g]*x^n))/(e*Sqrt[-f] - d*Sqrt[g])])/(2
*f*n) - (p*PolyLog[2, -((Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] - d*Sqrt[g]))])/(
2*f*n) - (p*PolyLog[2, (Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] + d*Sqrt[g])])/(2*
f*n) + (p*PolyLog[2, 1 + (e*x^n)/d])/(f*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x(f+gx^2)}\,dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{\log(c(d+ex)^p)}{fx} - \frac{gx\log(c(d+ex)^p)}{f(f+gx^2)}\right)\,dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{fn} - \frac{g\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{f+gx^2}\,dx, x, x^n\right)}{fn}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{fn}\\
&\quad - \frac{g\text{Subst}\left(\int \left(-\frac{\log(c(d+ex)^p)}{2\sqrt{g}\left(\sqrt{-f}-\sqrt{g}x\right)} + \frac{\log(c(d+ex)^p)}{2\sqrt{g}\left(\sqrt{-f}+\sqrt{g}x\right)}\right)\,dx, x, x^n\right)}{fn}\\
&\quad - \frac{(ep)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^n\right)}{fn}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{fn} + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn}\\
&\quad + \frac{\sqrt{g}\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{-f}-\sqrt{g}x}\,dx, x, x^n\right)}{2fn} - \frac{\sqrt{g}\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{-f}+\sqrt{g}x}\,dx, x, x^n\right)}{2fn}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{fn} - \frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x^n\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2fn}\\
&\quad - \frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x^n\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2fn} + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn}\\
&\quad + \frac{(ep)\text{Subst}\left(\int \frac{\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}\,dx, x, x^n\right)}{2fn}\\
&\quad + \frac{(ep)\text{Subst}\left(\int \frac{\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}\,dx, x, x^n\right)}{2fn}
\end{aligned}
$$

$$=\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{fn}-\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x^n)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2fn}-\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x^n)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2fn}+\frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn}+\frac{p\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}dx,x,d+ex^n\right)}{2fn}+\frac{p\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}dx,x,d+ex^n\right)}{2fn}$$

$$=\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{fn}-\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x^n)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2fn}-\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x^n)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2fn}-\frac{p\text{Li}_2\left(-\frac{\sqrt{g}(d+ex^n)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2fn}-\frac{p\text{Li}_2\left(\frac{\sqrt{g}(d+ex^n)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2fn}+\frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn}$$

Mathematica [F]

$$\int\frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}dx=\int\frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}dx$$

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))),x]
[Out] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))), x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.08 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.80

method	result
risch	$\frac{\ln((d+e\,x^n)^p)\ln(x^n)}{nf}-\frac{\ln((d+e\,x^n)^p)\ln(f+g\,x^{2n})}{2nf}-\frac{p\,\text{dilog}\left(\frac{d+e\,x^n}{d}\right)}{nf}-\frac{p\ln(x^n)\ln\left(\frac{d+e\,x^n}{d}\right)}{nf}+\frac{p\ln(d+e\,x^n)\ln(f+g\,x^{2n})}{2nf}-$

```
[In] int(ln(c*(d+e*x^n)^p)/x/(f+g*x^(2*n)),x,method=_RETURNVERBOSE)
[Out] 1/n*ln((d+e*x^n)^p)/f*ln(x^n)-1/2/n*ln((d+e*x^n)^p)/f*ln(f+g*(x^n)^2)-1/n*p
/f*dilog((d+e*x^n)/d)-1/n*p/f*ln(x^n)*ln((d+e*x^n)/d)+1/2/n*p/f*ln(d+e*x^n)
```

```
*ln(f+g*(x^n)^2)-1/2/n*p/f*ln(d+e*x^n)*ln((e*(-f*g)^(1/2)-g*(d+e*x^n)+d*g)/
(e*(-f*g)^(1/2)+d*g))-1/2/n*p/f*ln(d+e*x^n)*ln((e*(-f*g)^(1/2)+g*(d+e*x^n)-
d*g)/(e*(-f*g)^(1/2)-d*g))-1/2/n*p/f*dilog((e*(-f*g)^(1/2)-g*(d+e*x^n)+d*g)
/(e*(-f*g)^(1/2)+d*g))-1/2/n*p/f*dilog((e*(-f*g)^(1/2)+g*(d+e*x^n)-d*g)/(e*
(-f*g)^(1/2)-d*g))+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/
2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*
c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))*(1/n/f*l
n(x^n)-1/2/n/f*ln(f+g*(x^n)^2))
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx = \int \frac{\log\left((ex^n+d)^pc\right)}{(gx^{2\,n}+f)x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n)),x, algorithm="fricas")
[Out] integral(log((e*x^n + d)^p*c)/(g*x*x^(2*n) + f*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g*x**(2*n)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx = \int \frac{\log\left((ex^n+d)^pc\right)}{(gx^{2\,n}+f)x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n)),x, algorithm="maxima")
[Out] integrate(log((e*x^n + d)^p*c)/((g*x^(2*n) + f)*x), x)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx = \int \frac{\log\left((ex^n+d)^pc\right)}{(gx^{2\,n}+f)x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((g*x^(2*n) + f)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx = \int \frac{\ln\left(c\left(d+e\,x^n\right)^p\right)}{x\,\left(f+g\,x^{2\,n}\right)}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^(2*n))),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^(2*n))), x)
```

3.371 $\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^n)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^n)}\,dx = \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{fn} - \frac{\log(c(d+ex^n)^p)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{fn} - \frac{p\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex^n)}{ef-dg}\right)}{fn} + \frac{p\,\mathrm{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{fn}$$

```
[Out] ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/f/n-ln(c*(d+e*x^n)^p)*ln(e*(f+g*x^n)/(-d*g+e
*f))/f/n-p*polylog(2,-g*(d+e*x^n)/(-d*g+e*f))/f/n+p*polylog(2,1+e*x^n/d)/f/
n
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {2525, 36, 29, 31, 2463, 2441, 2352, 2440, 2438}

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^n)}\,dx = -\frac{\log(c(d+ex^n)^p)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{fn} + \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{fn} - \frac{p\,\mathrm{PolyLog}\left(2,-\frac{g(ex^n+d)}{ef-dg}\right)}{fn} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{fn}$$

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g*x^n)),x]
```

```
[Out] (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f*n) - (Log[c*(d + e*x^n)^p]*Log[
(e*(f + g*x^n))/(e*f - d*g)])/(f*n) - (p*PolyLog[2, -((g*(d + e*x^n))/(e*f
- d*g))])/(f*n) + (p*PolyLog[2, 1 + (e*x^n)/d])/(f*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x(f+gx)}\,dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{\log(c(d+ex)^p)}{fx} - \frac{g\log(c(d+ex)^p)}{f(f+gx)}\right)\,dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{fn} - \frac{g\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx, x, x^n\right)}{fn}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{fn} - \frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{fn}\\
&\quad - \frac{(ep)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^n\right)}{fn} + \frac{(ep)\text{Subst}\left(\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx, x, x^n\right)}{fn}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{fn} - \frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{fn}\\
&\quad + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn} + \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex^n\right)}{fn}\\
&= \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{fn} - \frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{fn}\\
&\quad - \frac{p\text{Li}_2\left(-\frac{g(d+ex^n)}{ef-dg}\right)}{fn} + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{fn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.76

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x \left(f+gx^n\right)} \, dx = \frac{\log \left(c(d+ex^n)^p\right) \left(\log \left(-\frac{ex^n}{d}\right)-\log \left(\frac{e(f+gx^n)}{ef-dg}\right)\right)-p \operatorname{PolyLog}\left(2,\frac{g(d+ex^n)}{-ef+dg}\right)+p \operatorname{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{fn}$$

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^n)),x]
```

```
[Out] (Log[c*(d + e*x^n)^p]*(Log[-((e*x^n)/d)] - Log[(e*(f + g*x^n))/(e*f - d*g)]
) - p*PolyLog[2, (g*(d + e*x^n))/(-(e*f) + d*g)] + p*PolyLog[2, 1 + (e*x^n)
/d])/(f*n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.72 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.67

method	result
risch	$\frac{\ln((d+e\,x^n)^p)\ln(x^n)}{nf}-\frac{\ln((d+e\,x^n)^p)\ln(f+g\,x^n)}{nf}-\frac{p\,\mathrm{dilog}\left(\frac{d+e\,x^n}{d}\right)}{nf}-\frac{p\ln(x^n)\ln\left(\frac{d+e\,x^n}{d}\right)}{nf}+\frac{p\,\mathrm{dilog}\left(\frac{(f+g\,x^n)e+dg-ef}{dg-ef}\right)}{nf}+$

```
[In] int(ln(c*(d+e*x^n)^p)/x/(f+g*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*ln((d+e*x^n)^p)/f*ln(x^n)-1/n*ln((d+e*x^n)^p)/f*ln(f+g*x^n)-1/n*p/f*dil
og((d+e*x^n)/d)-1/n*p/f*ln(x^n)*ln((d+e*x^n)/d)+1/n*p/f*dilog(((f+g*x^n)*e+
d*g-e*f)/(d*g-e*f))+1/n*p/f*ln(f+g*x^n)*ln(((f+g*x^n)*e+d*g-e*f)/(d*g-e*f))
+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e
*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1
/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))*(1/n/f*ln(x^n)-1/n/f*ln(f+
g*x^n))
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{(gx^n+f)x}\,dx$$

[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n),x, algorithm="fricas")

[Out] integral(log((e*x^n + d)^p*c)/(g*x*x^n + f*x), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx = \text{Timed out}$$

[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g*x**n),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.27

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx = -enp\left(\frac{\log\left(x^n\right)\log\left(\frac{ex^n}{d}+1\right)+\operatorname{Li}_2\left(-\frac{ex^n}{d}\right)}{efn^2} - \frac{\log\left(gx^n+f\right)\log\left(-\frac{egx^n+ef}{ef-dg}+1\right)+\operatorname{Li}_2\left(\frac{egx^n+ef}{ef-dg}\right)}{efn^2}\right) - \left(\frac{\log\left(gx^n+f\right)}{fn} - \frac{\log\left(x^n\right)}{fn}\right)\log\left((ex^n+d)^p c\right)$$

[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n),x, algorithm="maxima")

[Out] -e*n*p*((log(x^n)*log(e*x^n/d + 1) + dilog(-e*x^n/d))/(e*f*n^2) - (log(g*x^
n + f)*log(-(e*g*x^n + e*f)/(e*f - d*g) + 1) + dilog((e*g*x^n + e*f)/(e*f -
 d*g)))/(e*f*n^2)) - (log(g*x^n + f)/(f*n) - log(x^n)/(f*n))*log((e*x^n + d
)^p*c)

Giac [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx = \int \frac{\log\left((ex^n+d)^pc\right)}{(gx^n+f)x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n),x, algorithm="giac")
[Out] integrate(log((e*x^n + d)^p*c)/((g*x^n + f)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)}{x\,\left(f+g\,x^n\right)}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^n)),x)
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^n)), x)
```

3.372 $\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-n})}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 70

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx = \frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e(g+fx^n)}{df-eg}\right)}{fn} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{f(d+ex^n)}{df-eg}\right)}{fn}$$

```
[Out] ln(c*(d+e*x^n)^p)*ln(-e*(g+f*x^n)/(d*f-e*g))/f/n+p*polylog(2,f*(d+e*x^n)/(d
*f-e*g))/f/n
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2525, 2459, 2441, 2440, 2438}

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx = \frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e(fx^n+g)}{df-eg}\right)}{fn} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{f(ex^n+d)}{df-eg}\right)}{fn}$$

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g/x^n)),x]
```

```
[Out] (Log[c*(d + e*x^n)^p]*Log[-((e*(g + f*x^n))/(d*f - e*g))])/(f*n) + (p*PolyL
og[2, (f*(d + e*x^n))/(d*f - e*g)])/(f*n)
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2459

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)/(x_))^(q_.)*(x_)^(m_.), x_Symbol] :> Int[(g + f*x)^q*(a + b*Log[c*(d + e*
x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x] && EqQ[m, q] &&
 IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\left(f+\frac{g}{x}\right)x}\,dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{g+fx}\,dx, x, x^n\right)}{n} \\
&= \frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e(g+fx^n)}{df-eg}\right)}{fn} - \frac{(ep)\text{Subst}\left(\int \frac{\log\left(\frac{e(g+fx)}{-df+eg}\right)}{d+ex}\,dx, x, x^n\right)}{fn} \\
&= \frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e(g+fx^n)}{df-eg}\right)}{fn} - \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{fx}{-df+eg}\right)}{x}\,dx, x, d+ex^n\right)}{fn} \\
&= \frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e(g+fx^n)}{df-eg}\right)}{fn} + \frac{p\text{Li}_2\left(\frac{f(d+ex^n)}{df-eg}\right)}{fn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.91

$$\int \frac{\log (c(d+ex^n)^p)}{x(f+gx^{-n})}\,dx = \frac{\log (c(d+ex^n)^p)\log \left(\frac{e(g+fx^n)}{-df+eg}\right)+p\,\text{PolyLog}\left(2,\frac{f(d+ex^n)}{df-eg}\right)}{fn}$$

[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^n)),x]

[Out] (Log[c*(d + e*x^n)^p]*Log[(e*(g + f*x^n))/(-(d*f) + e*g)] + p*PolyLog[2, (f*(d + e*x^n))/(d*f - e*g)])/(f*n)

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.04 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.47

method	result
risch	$\frac{\ln((d+e\,x^n)^p)\ln(g+f\,x^n)}{nf}-\frac{p\,\text{dilog}\left(\frac{(g+f\,x^n)e+df-eg}{df-eg}\right)}{nf}-\frac{p\ln(g+f\,x^n)\ln\left(\frac{(g+f\,x^n)e+df-eg}{df-eg}\right)}{nf}+$

[In] int(ln(c*(d+e*x^n)^p)/x/(f+g/(x^n)),x,method=_RETURNVERBOSE)

[Out] 1/n*ln((d+e*x^n)^p)*ln(g+f*x^n)/f-1/n/f*p*dilog(((g+f*x^n)*e+d*f-e*g)/(d*f-e*g))-1/n/f*p*ln(g+f*x^n)*ln(((g+f*x^n)*e+d*f-e*g)/(d*f-e*g))+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))/n*ln(g+f*x^n)/f

Fricas [F]

$$\int \frac{\log (c(d+ex^n)^p)}{x(f+gx^{-n})}\,dx = \int \frac{\log ((ex^n+d)^p c)}{(f+\frac{g}{x^n})x}\,dx$$

[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n)),x, algorithm="fricas")

[Out] integral(x^n*log((e*x^n + d)^p*c)/(f*x*x^n + g*x), x)

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g/(x**n)),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.60

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx = \left(\frac{\log\left(f+\frac{g}{x^n}\right)}{fn} - \frac{\log\left(\frac{1}{x^n}\right)}{fn}\right)\log\left((ex^n+d)^pc\right) - \frac{\left(\log\left(fx^n+g\right)\log\left(\frac{efx^n+eg}{df-eg}+1\right)+\mathrm{Li}_2\left(-\frac{efx^n+eg}{df-eg}\right)\right)p}{fn}$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n)),x, algorithm="maxima")
[Out] (log(f + g/x^n)/(f*n) - log(1/(x^n))/(f*n))*log((e*x^n + d)^p*c) - (log(f*x
^n + g)*log((e*f*x^n + e*g)/(d*f - e*g) + 1) + dilog(-(e*f*x^n + e*g)/(d*f
- e*g)))*p/(f*n)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx = \int \frac{\log\left((ex^n+d)^pc\right)}{\left(f+\frac{g}{x^n}\right)x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n)),x, algorithm="giac")
[Out] integrate(log((e*x^n + d)^p*c)/((f + g/x^n)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-n})}\,dx = \int \frac{\ln\left(c\left(d+e\,x^n\right)^p\right)}{x\,\left(f+\frac{g}{x^n}\right)}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^n)),x)
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^n)), x)
```

3.373 $\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-2n})}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 221

$$\begin{aligned}\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx &= \frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn}\\&\quad+\frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e\left(\sqrt{g}+\sqrt{-f}x^n\right)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn}\\&\quad+\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-f}(d+ex^n)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn}+\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-f}(d+ex^n)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn}\end{aligned}$$

```
[Out] 1/2*ln(c*(d+e*x^n)^p)*ln(-e*(x^n*(-f)^(1/2)+g^(1/2))/(d*(-f)^(1/2)-e*g^(1/2
)))/f/n+1/2*ln(c*(d+e*x^n)^p)*ln(e*(-x^n*(-f)^(1/2)+g^(1/2))/(d*(-f)^(1/2)+
e*g^(1/2)))/f/n+1/2*p*polylog(2,(d+e*x^n)*(-f)^(1/2)/(d*(-f)^(1/2)-e*g^(1/2
)))/f/n+1/2*p*polylog(2,(d+e*x^n)*(-f)^(1/2)/(d*(-f)^(1/2)+e*g^(1/2)))/f/n
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2525, 269, 266, 2463, 2441, 2440, 2438}

$$\begin{aligned}\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx &= \frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn}\\&\quad+\frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e\left(\sqrt{-f}x^n+\sqrt{g}\right)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn}\\&\quad+\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-f}(ex^n+d)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn}+\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-f}(ex^n+d)}{\sqrt{-f}d+e\sqrt{g}}\right)}{2fn}\end{aligned}$$

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))),x]
```

```
[Out] (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[g] - Sqrt[-f]*x^n))/(d*Sqrt[-f] + e*Sqrt
[g])])/(2*f*n) + (Log[c*(d + e*x^n)^p]*Log[-((e*(Sqrt[g] + Sqrt[-f]*x^n))/(
d*Sqrt[-f] - e*Sqrt[g]))])/(2*f*n) + (p*PolyLog[2, (Sqrt[-f]*(d + e*x^n))/(
d*Sqrt[-f] - e*Sqrt[g])])/(2*f*n) + (p*PolyLog[2, (Sqrt[-f]*(d + e*x^n))/(d
*Sqrt[-f] + e*Sqrt[g])])/(2*f*n)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
```

```
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\left(f+\frac{g}{x^2}\right)x}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{\sqrt{-f}\log(c(d+ex)^p)}{2f\left(\sqrt{g}-\sqrt{-f}x\right)}+\frac{\sqrt{-f}\log(c(d+ex)^p)}{2f\left(\sqrt{g}+\sqrt{-f}x\right)}\right)\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{g}-\sqrt{-f}x}\,dx,x,x^n\right)}{2\sqrt{-f}n}-\frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{g}+\sqrt{-f}x}\,dx,x,x^n\right)}{2\sqrt{-f}n}\\
&= \frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn}+\frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e\left(\sqrt{g}+\sqrt{-f}x^n\right)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn}\\
&\quad-\frac{(ep)\text{Subst}\left(\int \frac{\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}x\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{d+ex}\,dx,x,x^n\right)}{2fn}-\frac{(ep)\text{Subst}\left(\int \frac{\log\left(\frac{e\left(\sqrt{g}+\sqrt{-f}x\right)}{-d\sqrt{-f}+e\sqrt{g}}\right)}{d+ex}\,dx,x,x^n\right)}{2fn}\\
&= \frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn}+\frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e\left(\sqrt{g}+\sqrt{-f}x^n\right)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn}\\
&\quad-\frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-f}x}{-d\sqrt{-f}+e\sqrt{g}}\right)}{x}\,dx,x,d+ex^n\right)}{2fn}-\frac{p\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-f}x}{d\sqrt{-f}+e\sqrt{g}}\right)}{x}\,dx,x,d+ex^n\right)}{2fn}\\
&= \frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn}+\frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e\left(\sqrt{g}+\sqrt{-f}x^n\right)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn}\\
&\quad+\frac{p\text{Li}_2\left(\frac{\sqrt{-f}(d+ex^n)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2fn}+\frac{p\text{Li}_2\left(\frac{\sqrt{-f}(d+ex^n)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2fn}
\end{aligned}
$$

Mathematica [F]

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx = \int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx$$

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))),x]
[Out] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))), x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.89 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.80

method	result
risch	$\frac{\ln((d+e\,x^n)^p)\ln(g+f\,x^{2n})}{2nf}-\frac{p\ln(d+e\,x^n)\ln(g+f\,x^{2n})}{2nf}+\frac{p\ln(d+e\,x^n)\ln\left(\frac{e\sqrt{-fg}-f(d+e\,x^n)+df}{e\sqrt{-fg}+df}\right)}{2nf}+$

```
[In] int(ln(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n))),x,method=_RETURNVERBOSE)
[Out] 1/2/n*ln((d+e*x^n)^p)/f*ln(g+f*(x^n)^2)-1/2/n/f*p*ln(d+e*x^n)*ln(g+f*(x^n)^
2)+1/2/n/f*p*ln(d+e*x^n)*ln((e*(-f*g)^(1/2)-f*(d+e*x^n)+d*f)/(e*(-f*g)^(1/2
)+d*f))+1/2/n/f*p*ln(d+e*x^n)*ln((e*(-f*g)^(1/2)+f*(d+e*x^n)-d*f)/(e*(-f*g)
^(1/2)-d*f))+1/2/n/f*p*dilog((e*(-f*g)^(1/2)-f*(d+e*x^n)+d*f)/(e*(-f*g)^(1/
2)+d*f))+1/2/n/f*p*dilog((e*(-f*g)^(1/2)+f*(d+e*x^n)-d*f)/(e*(-f*g)^(1/2)-d
*f))+1/2*(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csg
n(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n
)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))/n/f*ln(g+f*(x^n)^2
)
```

Fricas [F]

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx = \int \frac{\log \left((ex^n+d)^p c\right)}{\left(f+\frac{g}{x^{2\,n}}\right)x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n))),x, algorithm="fricas")
[Out] integral(x^(2*n)*log((e*x^n + d)^p*c)/(f*x*x^(2*n) + g*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g/(x**(2*n))),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{\left(f+\frac{g}{x^{2n}}\right)x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n))),x, algorithm="maxima")
[Out] integrate(log((e*x^n + d)^p*c)/((f + g/x^(2*n))*x), x)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{\left(f+\frac{g}{x^{2n}}\right)x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n))),x, algorithm="giac")
[Out] integrate(log((e*x^n + d)^p*c)/((f + g/x^(2*n))*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)}{x\,\left(f+\frac{g}{x^{2\,n}}\right)}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^(2*n))),x)
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^(2*n))), x)
```

3.374 $\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{2n})^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 419

$$\begin{aligned}\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{2n})^2}\,dx = & -\frac{de\sqrt{g}p\arctan\left(\frac{\sqrt{g}x^n}{\sqrt{f}}\right)}{2f^{3/2}(e^2f+d^2g)\,n}-\frac{e^2p\log(d+ex^n)}{2f(e^2f+d^2g)\,n}\\ & +\frac{\log(c(d+ex^n)^p)}{2fn(f+gx^{2n})}+\frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{f^2n}\\ & -\frac{\log(c(d+ex^n)^p)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x^n)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2n}\\ & -\frac{\log(c(d+ex^n)^p)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x^n)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2n}\\ & +\frac{e^2p\log(f+gx^{2n})}{4f(e^2f+d^2g)\,n}-\frac{p\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(d+ex^n)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2n}\\ & -\frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(d+ex^n)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2n}+\frac{p\,\mathrm{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{f^2n}\end{aligned}$$

```
[Out] -1/2*e^2*p*ln(d+e*x^n)/f/(d^2*g+e^2*f)/n+1/2*ln(c*(d+e*x^n)^p)/f/n/(f+g*x^(
2*n))+ln(-e*x^n/d)*ln(c*(d+e*x^n)^p)/f^2/n+1/4*e^2*p*ln(f+g*x^(2*n))/f/(d^2
*g+e^2*f)/n-1/2*ln(c*(d+e*x^n)^p)*ln(e*((-f)^(1/2)-x^n*g^(1/2))/(e*(-f)^(1/
2)+d*g^(1/2)))/f^2/n-1/2*ln(c*(d+e*x^n)^p)*ln(e*((-f)^(1/2)+x^n*g^(1/2))/(e
*(-f)^(1/2)-d*g^(1/2)))/f^2/n+p*polylog(2,1+e*x^n/d)/f^2/n-1/2*p*polylog(2,
-(d+e*x^n)*g^(1/2)/(e*(-f)^(1/2)-d*g^(1/2)))/f^2/n-1/2*p*polylog(2,(d+e*x^n
)*g^(1/2)/(e*(-f)^(1/2)+d*g^(1/2)))/f^2/n-1/2*d*e*p*arctan(x^n*g^(1/2)/f^(1
/2))*g^(1/2)/f^(3/2)/(d^2*g+e^2*f)/n
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.519, Rules used = {2525, 272, 46, 2463, 2441, 2352, 2460, 720, 31, 649, 211, 266, 2440, 2438}

$$\int \frac{\log (c(d+ex^n)^p)}{x(f+gx^{2n})^2}\,dx = -\frac{de\sqrt{g}p\arctan\left(\frac{\sqrt{g}x^n}{\sqrt{f}}\right)}{2f^{3/2}n(d^2g+e^2f)} - \frac{\log(c(d+ex^n)^p)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x^n)}{d\sqrt{g}+e\sqrt{-f}}\right)}{2f^2n} - \frac{\log(c(d+ex^n)^p)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x^n)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2n} + \frac{\log\left(-\frac{ex^n}{d}\right)\log(c(d+ex^n)^p)}{f^2n} + \frac{\log(c(d+ex^n)^p)}{2fn(f+gx^{2n})} + \frac{e^2p\log(f+gx^{2n})}{4fn(d^2g+e^2f)} - \frac{e^2p\log(d+ex^n)}{2fn(d^2g+e^2f)} - \frac{p\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{g}(ex^n+d)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2n} - \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(ex^n+d)}{\sqrt{g}d+e\sqrt{-f}}\right)}{2f^2n} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{f^2n}$$

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))^2),x]
```

```
[Out] -1/2*(d*e*Sqrt[g]*p*ArcTan[(Sqrt[g]*x^n)/Sqrt[f]])/(f^(3/2)*(e^2*f + d^2*g)
*n) - (e^2*p*Log[d + e*x^n])/(2*f*(e^2*f + d^2*g)*n) + Log[c*(d + e*x^n)^p]
/(2*f*n*(f + g*x^(2*n))) + (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f^2*n)
 - (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[-f] - Sqrt[g]*x^n))/(e*Sqrt[-f] + d*S
qrt[g])])/(2*f^2*n) - (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[-f] + Sqrt[g]*x^n)
)/(e*Sqrt[-f] - d*Sqrt[g])])/(2*f^2*n) + (e^2*p*Log[f + g*x^(2*n)])/(4*f*(e
^2*f + d^2*g)*n) - (p*PolyLog[2, -((Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] - d*Sq
rt[g]))])/(2*f^2*n) - (p*PolyLog[2, (Sqrt[g]*(d + e*x^n))/(e*Sqrt[-f] + d*S
qrt[g])])/(2*f^2*n) + (p*PolyLog[2, 1 + (e*x^n)/d])/(f^2*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2460

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(x_)^(m_.)*(
(f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Simp[(f + g*x^r)^(q + 1)*((a
+ b*Log[c*(d + e*x)^n])^p/(g*r*(q + 1))), x] - Dist[b*e*n*(p/(g*r*(q + 1)))
, Int[(f + g*x^r)^(q + 1)*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x
], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q, r}, x] && EqQ[m, r - 1] && Ne
Q[q, -1] && IGtQ[p, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x(f+gx^2)^2}\,dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{\log(c(d+ex)^p)}{f^2x} - \frac{gx\log(c(d+ex)^p)}{f(f+gx^2)^2} - \frac{gx\log(c(d+ex)^p)}{f^2(f+gx^2)}\right)dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{f^2n} - \frac{g\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{f+gx^2}\,dx, x, x^n\right)}{f^2n} \\
&\quad - \frac{g\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{(f+gx^2)^2}\,dx, x, x^n\right)}{fn}
\end{aligned}
$$

$$= \frac{\log\left(c(d+ex^n)^p\right)}{2fn\left(f+gx^{2n}\right)} + \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{f^2n} - \frac{g\text{Subst}\left(\int\left(-\frac{\log(c(d+ex)^p)}{2\sqrt{g}(\sqrt{-f}-\sqrt{g}x)}+\frac{\log(c(d+ex)^p)}{2\sqrt{g}(\sqrt{-f}+\sqrt{g}x)}\right)dx,x,x^n\right)}{f^2n} - \frac{(ep)\text{Subst}\left(\int\frac{\log\left(-\frac{ex}{d}\right)}{d+ex}dx,x,x^n\right)}{f^2n} - \frac{(ep)\text{Subst}\left(\int\frac{1}{(d+ex)(f+gx^2)}dx,x,x^n\right)}{2fn}$$

$$= \frac{\log\left(c(d+ex^n)^p\right)}{2fn\left(f+gx^{2n}\right)} + \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{f^2n} + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{f^2n} + \frac{\sqrt{g}\text{Subst}\left(\int\frac{\log(c(d+ex)^p)}{\sqrt{-f}-\sqrt{g}x}dx,x,x^n\right)}{2f^2n} - \frac{\sqrt{g}\text{Subst}\left(\int\frac{\log(c(d+ex)^p)}{\sqrt{-f}+\sqrt{g}x}dx,x,x^n\right)}{2f^2n} - \frac{(ep)\text{Subst}\left(\int\frac{dg-egx}{f+gx^2}dx,x,x^n\right)}{2f\left(e^2f+d^2g\right)n} - \frac{\left(e^3p\right)\text{Subst}\left(\int\frac{1}{d+ex}dx,x,x^n\right)}{2f\left(e^2f+d^2g\right)n}$$

$$= -\frac{e^2p\log\left(d+ex^n\right)}{2f\left(e^2f+d^2g\right)n} + \frac{\log\left(c(d+ex^n)^p\right)}{2fn\left(f+gx^{2n}\right)} + \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{f^2n} - \frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x^n)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2n} - \frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x^n)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2n} + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{f^2n} + \frac{(ep)\text{Subst}\left(\int\frac{\log\left(\frac{e(\sqrt{-f}-\sqrt{g}x)}{e\sqrt{-f}+d\sqrt{g}}\right)}{d+ex}dx,x,x^n\right)}{2f^2n} + \frac{(ep)\text{Subst}\left(\int\frac{\log\left(\frac{e(\sqrt{-f}+\sqrt{g}x)}{e\sqrt{-f}-d\sqrt{g}}\right)}{d+ex}dx,x,x^n\right)}{2f^2n} - \frac{(degp)\text{Subst}\left(\int\frac{1}{f+gx^2}dx,x,x^n\right)}{2f\left(e^2f+d^2g\right)n} + \frac{\left(e^2gp\right)\text{Subst}\left(\int\frac{x}{f+gx^2}dx,x,x^n\right)}{2f\left(e^2f+d^2g\right)n}$$

$$=-\frac{de\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x^n}{\sqrt{f}}\right)}{2f^{3/2}\left(e^2f+d^2g\right)n}-\frac{e^2p\log\left(d+ex^n\right)}{2f\left(e^2f+d^2g\right)n}+\frac{\log\left(c(d+ex^n)^p\right)}{2fn\left(f+gx^{2n}\right)}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{f^2n}-\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x^n\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2n}-\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x^n\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2n}+\frac{e^2p\log\left(f+gx^{2n}\right)}{4f\left(e^2f+d^2g\right)n}+\frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{f^2n}+\frac{p\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{g}x}{e\sqrt{-f}-d\sqrt{g}}\right)}{x}\,dx,x,d+ex^n\right)}{2f^2n}+\frac{p\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{g}x}{e\sqrt{-f}+d\sqrt{g}}\right)}{x}\,dx,x,d+ex^n\right)}{2f^2n}$$

$$=-\frac{de\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{g}x^n}{\sqrt{f}}\right)}{2f^{3/2}\left(e^2f+d^2g\right)n}-\frac{e^2p\log\left(d+ex^n\right)}{2f\left(e^2f+d^2g\right)n}+\frac{\log\left(c(d+ex^n)^p\right)}{2fn\left(f+gx^{2n}\right)}+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{f^2n}-\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{-f}-\sqrt{g}x^n\right)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2n}-\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{-f}+\sqrt{g}x^n\right)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2n}+\frac{e^2p\log\left(f+gx^{2n}\right)}{4f\left(e^2f+d^2g\right)n}-\frac{p\text{Li}_2\left(-\frac{\sqrt{g}(d+ex^n)}{e\sqrt{-f}-d\sqrt{g}}\right)}{2f^2n}-\frac{p\text{Li}_2\left(\frac{\sqrt{g}(d+ex^n)}{e\sqrt{-f}+d\sqrt{g}}\right)}{2f^2n}+\frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{f^2n}$$

Mathematica [F]

$$\int\frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)^2}\,dx=\int\frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)^2}\,dx$$

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))^2),x]
[Out] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^(2*n))^2), x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 11.00 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.52

method	result
risch	$\frac{\ln((d+e\,x^n)^p)\ln(x^n)}{n\,f^2}+\frac{\ln((d+e\,x^n)^p)}{2nf(f+g\,x^{2n})}-\frac{\ln((d+e\,x^n)^p)\ln(f+g\,x^{2n})}{2n\,f^2}-\frac{e^2p\ln(d+e\,x^n)}{2f(d^2g+f\,e^2)n}+\frac{e^2p\ln(f+g\,x^{2n})}{4f(d^2g+f\,e^2)n}-\frac{pegd\arctan(}{2nf(d^2g+f\,e^2}$

```
[In] int(ln(c*(d+e*x^n)^p)/x/(f+g*x^(2*n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*ln((d+e*x^n)^p)/f^2*ln(x^n)+1/2/n*ln((d+e*x^n)^p)/f/(f+g*(x^n)^2)-1/2/n
*ln((d+e*x^n)^p)/f^2*ln(f+g*(x^n)^2)-1/2*e^2*p*ln(d+e*x^n)/f/(d^2*g+e^2*f)/
n+1/4/n*p*e^2/f/(d^2*g+e^2*f)*ln(f+g*(x^n)^2)-1/2/n*p*e/f/(d^2*g+e^2*f)*g*d
/(f*g)^(1/2)*arctan(x^n*g/(f*g)^(1/2))-1/n*p/f^2*dilog((d+e*x^n)/d)-1/n*p/f
^2*ln(x^n)*ln((d+e*x^n)/d)+1/2/n*p/f^2*ln(d+e*x^n)*ln(f+g*(x^n)^2)-1/2/n*p/
f^2*ln(d+e*x^n)*ln((e*(-f*g)^(1/2)-g*(d+e*x^n)+d*g)/(e*(-f*g)^(1/2)+d*g))-1
/2/n*p/f^2*ln(d+e*x^n)*ln((e*(-f*g)^(1/2)+g*(d+e*x^n)-d*g)/(e*(-f*g)^(1/2)-
d*g))-1/2/n*p/f^2*dilog((e*(-f*g)^(1/2)-g*(d+e*x^n)+d*g)/(e*(-f*g)^(1/2)+d*
g))-1/2/n*p/f^2*dilog((e*(-f*g)^(1/2)+g*(d+e*x^n)-d*g)/(e*(-f*g)^(1/2)-d*g)
)+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+
e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+
1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))*(1/n/f^2*ln(x^n)+1/2/n/f/
(f+g*(x^n)^2)-1/2/n/f^2*ln(f+g*(x^n)^2))
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)^2}\,dx=\int \frac{\log\left((ex^n+d)^pc\right)}{\left(gx^{2\,n}+f\right)^2x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)/(g^2*x*x^(4*n) + 2*f*g*x*x^(2*n) + f^2*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g*x**(2*n))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)^2}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{\left(gx^{2\,n}+f\right)^2 x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n))^2,x, algorithm="maxima")
[Out] integrate(log((e*x^n + d)^p*c)/((g*x^(2*n) + f)^2*x), x)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)^2}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)}{\left(gx^{2\,n}+f\right)^2 x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^(2*n))^2,x, algorithm="giac")
[Out] integrate(log((e*x^n + d)^p*c)/((g*x^(2*n) + f)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)^2}\,dx = \int \frac{\ln\left(c\,\left(d+e\,x^n\right)^p\right)}{x\,\left(f+g\,x^{2\,n}\right)^2}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^(2*n))^2),x)
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^(2*n))^2), x)
```

3.375 $\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^n)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 204

$$\begin{aligned}\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)^2}\,dx =& -\frac{ep\log\left(d+ex^n\right)}{f(ef-dg)n}+\frac{\log\left(c(d+ex^n)^p\right)}{fn\left(f+gx^n\right)} \\ &+\frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{f^2n}+\frac{ep\log\left(f+gx^n\right)}{f(ef-dg)n} \\ &-\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{f^2n} \\ &-\frac{p\,\mathrm{PolyLog}\left(2,-\frac{g(d+ex^n)}{ef-dg}\right)}{f^2n}+\frac{p\,\mathrm{PolyLog}\left(2,1+\frac{ex^n}{d}\right)}{f^2n}\end{aligned}$$

```
[Out] -e*p*ln(d+e*x^n)/f/(-d*g+e*f)/n+ln(c*(d+e*x^n)^p)/f/n/(f+g*x^n)+ln(-e*x^n/d
)*ln(c*(d+e*x^n)^p)/f^2/n+e*p*ln(f+g*x^n)/f/(-d*g+e*f)/n-ln(c*(d+e*x^n)^p)*
ln(e*(f+g*x^n)/(-d*g+e*f))/f^2/n-p*polylog(2,-g*(d+e*x^n)/(-d*g+e*f))/f^2/n
+p*polylog(2,1+e*x^n/d)/f^2/n
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules

used = {2525, 46, 2463, 2441, 2352, 2442, 36, 31, 2440, 2438}

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)^2}\,dx = -\frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{f^2n} + \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{f^2n} + \frac{\log\left(c(d+ex^n)^p\right)}{fn\left(f+gx^n\right)} - \frac{p\,\text{PolyLog}\left(2,-\frac{g(ex^n+d)}{ef-dg}\right)}{f^2n} + \frac{p\,\text{PolyLog}\left(2,\frac{ex^n}{d}+1\right)}{f^2n} - \frac{ep\log\left(d+ex^n\right)}{fn(ef-dg)} + \frac{ep\log\left(f+gx^n\right)}{fn(ef-dg)}$$

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g*x^n)^2),x]
```

```
[Out] -((e*p*Log[d + e*x^n])/(f*(e*f - d*g)*n)) + Log[c*(d + e*x^n)^p]/(f*n*(f +
g*x^n)) + (Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p])/(f^2*n) + (e*p*Log[f + g
*x^n])/(f*(e*f - d*g)*n) - (Log[c*(d + e*x^n)^p]*Log[(e*(f + g*x^n))/(e*f -
 d*g)])/(f^2*n) - (p*PolyLog[2, -((g*(d + e*x^n))/(e*f - d*g))])/(f^2*n) +
(p*PolyLog[2, 1 + (e*x^n)/d])/(f^2*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x(f+gx)^2}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{\log(c(d+ex)^p)}{f^2x} - \frac{g\log(c(d+ex)^p)}{f(f+gx)^2} - \frac{g\log(c(d+ex)^p)}{f^2(f+gx)}\right)dx, x, x^n\right)}{n}\end{aligned}$$

$$
\begin{aligned}
&= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{x}\,dx, x, x^n\right)}{f^2 n} - \frac{g\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{f+gx}\,dx, x, x^n\right)}{f^2 n} \\
&\quad - \frac{g\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{(f+gx)^2}\,dx, x, x^n\right)}{fn} \\
&= \frac{\log\left(c(d+ex^n)^p\right)}{fn\,(f+gx^n)} + \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{f^2 n} \\
&\quad - \frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{f^2 n} - \frac{(ep)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^n\right)}{f^2 n} \\
&\quad + \frac{(ep)\text{Subst}\left(\int \frac{\log\left(\frac{e(f+gx)}{ef-dg}\right)}{d+ex}\,dx, x, x^n\right)}{f^2 n} - \frac{(ep)\text{Subst}\left(\int \frac{1}{(d+ex)(f+gx)}\,dx, x, x^n\right)}{fn} \\
&= \frac{\log\left(c(d+ex^n)^p\right)}{fn\,(f+gx^n)} + \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{f^2 n} - \frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{f^2 n} \\
&\quad + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{f^2 n} + \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{gx}{ef-dg}\right)}{x}\,dx, x, d+ex^n\right)}{f^2 n} \\
&\quad - \frac{(e^2 p)\,\text{Subst}\left(\int \frac{1}{d+ex}\,dx, x, x^n\right)}{f(ef-dg)n} + \frac{(egp)\text{Subst}\left(\int \frac{1}{f+gx}\,dx, x, x^n\right)}{f(ef-dg)n} \\
&= -\frac{ep\log\left(d+ex^n\right)}{f(ef-dg)n} + \frac{\log\left(c(d+ex^n)^p\right)}{fn\,(f+gx^n)} \\
&\quad + \frac{\log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right)}{f^2 n} + \frac{ep\log\left(f+gx^n\right)}{f(ef-dg)n} \\
&\quad - \frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e(f+gx^n)}{ef-dg}\right)}{f^2 n} - \frac{p\text{Li}_2\left(-\frac{g(d+ex^n)}{ef-dg}\right)}{f^2 n} + \frac{p\text{Li}_2\left(1+\frac{ex^n}{d}\right)}{f^2 n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.84

$$
\int \frac{\log\left(c(d+ex^n)^p\right)}{x\,(f+gx^n)^2}\,dx
$$

$$
= \frac{-\frac{efp\log(d+ex^n)}{ef-dg} + \frac{f\log(c(d+ex^n)^p)}{f+gx^n} + \log\left(-\frac{ex^n}{d}\right)\log\left(c(d+ex^n)^p\right) + \frac{efp\log(f+gx^n)}{ef-dg} - \log\left(c(d+ex^n)^p\right)\log\left(\frac{e(f+ga}{ef-d}\right.}{f^2 n}
$$

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g*x^n)^2),x]
```

```
[Out] (-((e*f*p*Log[d + e*x^n])/(e*f - d*g)) + (f*Log[c*(d + e*x^n)^p])/(f + g*x^
n) + Log[-((e*x^n)/d)]*Log[c*(d + e*x^n)^p] + (e*f*p*Log[f + g*x^n])/(e*f -
```

```
d*g) - Log[c*(d + e*x^n)^p]*Log[(e*(f + g*x^n))/(e*f - d*g)] - p*PolyLog[2
, (g*(d + e*x^n))/(-(e*f) + d*g)] + p*PolyLog[2, 1 + (e*x^n)/d])/(f^2*n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.93 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.03

method	result
risch	$\frac{\ln((d+e\,x^{n})^{p})\ln(x^{n})}{n\,f^{2}}-\frac{\ln((d+e\,x^{n})^{p})\ln(f+g\,x^{n})}{n\,f^{2}}+\frac{\ln((d+e\,x^{n})^{p})}{nf(f+g\,x^{n})}+\frac{pe\ln(d+e\,x^{n})}{nf(dg-ef)}-\frac{pe\ln(f+g\,x^{n})}{nf(dg-ef)}-\frac{p\,\mathrm{dilog}\left(\frac{d+e\,x^{n}}{d}\right)}{n\,f^{2}}-$

```
[In] int(ln(c*(d+e*x^n)^p)/x/(f+g*x^n)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*ln((d+e*x^n)^p)/f^2*ln(x^n)-1/n*ln((d+e*x^n)^p)/f^2*ln(f+g*x^n)+1/n*ln(
(d+e*x^n)^p)/f/(f+g*x^n)+1/n*p*e/f/(d*g-e*f)*ln(d+e*x^n)-1/n*p*e/f/(d*g-e*f
)*ln(f+g*x^n)-1/n*p/f^2*dilog((d+e*x^n)/d)-1/n*p/f^2*ln(x^n)*ln((d+e*x^n)/d
)+1/n*p/f^2*dilog(((f+g*x^n)*e+d*g-e*f)/(d*g-e*f))+1/n*p/f^2*ln(f+g*x^n)*ln
(((f+g*x^n)*e+d*g-e*f)/(d*g-e*f))+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d
+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1
/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+
ln(c))/n*(ln(x^n)/f^2-ln(f+g*x^n)/f^2+1/f/(f+g*x^n))
```

Fricas [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)^2}\,dx=\int \frac{\log\left((ex^n+d)^pc\right)}{(gx^n+f)^2x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(log((e*x^n + d)^p*c)/(g^2*x*x^(2*n) + 2*f*g*x*x^n + f^2*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g*x**n)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.14

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)^2}\,dx =$$
$$-enp\left(\frac{\log\left(\frac{ex^n+d}{e}\right)}{ef^2n^2-dfgn^2}-\frac{\log\left(\frac{gx^n+f}{g}\right)}{ef^2n^2-dfgn^2}+\frac{\log\left(x^n\right)\log\left(\frac{ex^n}{d}+1\right)+\mathrm{Li}_2\left(-\frac{ex^n}{d}\right)}{ef^2n^2}-\frac{\log\left(gx^n+f\right)\log\left(-\frac{egx^n}{ef-}\right.}{ef}\right.$$
$$+\left(\frac{1}{fgnx^n+f^2n}-\frac{\log\left(gx^n+f\right)}{f^2n}+\frac{\log\left(x^n\right)}{f^2n}\right)\log\left(\left(ex^n+d\right)^pc\right)$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n)^2,x, algorithm="maxima")
```

```
[Out] -e*n*p*(log((e*x^n + d)/e)/(e*f^2*n^2 - d*f*g*n^2) - log((g*x^n + f)/g)/(e*
f^2*n^2 - d*f*g*n^2) + (log(x^n)*log(e*x^n/d + 1) + dilog(-e*x^n/d))/(e*f^2
*n^2) - (log(g*x^n + f)*log(-(e*g*x^n + e*f)/(e*f - d*g) + 1) + dilog((e*g*
x^n + e*f)/(e*f - d*g)))/(e*f^2*n^2)) + (1/(f*g*n*x^n + f^2*n) - log(g*x^n
+ f)/(f^2*n) + log(x^n)/(f^2*n))*log((e*x^n + d)^p*c)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)^2}\,dx = \int \frac{\log\left(\left(ex^n+d\right)^pc\right)}{\left(gx^n+f\right)^2x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((g*x^n + f)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)^2}\,dx = \int \frac{\ln\left(c\left(d+e\,x^n\right)^p\right)}{x\left(f+g\,x^n\right)^2}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^n)^2),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g*x^n)^2), x)
```

3.376 $\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-n})^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 156

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)^2}\,dx = \frac{egp\log\left(d+ex^n\right)}{f^2(df-eg)n}+\frac{g\log\left(c(d+ex^n)^p\right)}{f^2n\left(g+fx^n\right)}-\frac{egp\log\left(g+fx^n\right)}{f^2(df-eg)n}+\frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e(g+fx^n)}{df-eg}\right)}{f^2n}+\frac{p\operatorname{PolyLog}\left(2,\frac{f(d+ex^n)}{df-eg}\right)}{f^2n}$$

```
[Out] e*g*p*ln(d+e*x^n)/f^2/(d*f-e*g)/n+g*ln(c*(d+e*x^n)^p)/f^2/n/(g+f*x^n)-e*g*p
*ln(g+f*x^n)/f^2/(d*f-e*g)/n+ln(c*(d+e*x^n)^p)*ln(-e*(g+f*x^n)/(d*f-e*g))/f
^2/n+p*polylog(2,f*(d+e*x^n)/(d*f-e*g))/f^2/n
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {2525, 269, 45, 2463, 2442, 36, 31, 2441, 2440, 2438}

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)^2}\,dx = \frac{g\log\left(c(d+ex^n)^p\right)}{f^2n\left(fx^n+g\right)}+\frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e(fx^n+g)}{df-eg}\right)}{f^2n}+\frac{p\operatorname{PolyLog}\left(2,\frac{f(ex^n+d)}{df-eg}\right)}{f^2n}+\frac{egp\log\left(d+ex^n\right)}{f^2n(df-eg)}-\frac{egp\log\left(fx^n+g\right)}{f^2n(df-eg)}$$

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g/x^n)^2),x]
```

```
[Out] (e*g*p*Log[d + e*x^n])/(f^2*(d*f - e*g)*n) + (g*Log[c*(d + e*x^n)^p])/(f^2*
n*(g + f*x^n)) - (e*g*p*Log[g + f*x^n])/(f^2*(d*f - e*g)*n) + (Log[c*(d + e
```

```
*x^n)^p]*Log[-((e*(g + f*x^n))/(d*f - e*g))])/(f^2*n) + (p*PolyLog[2, (f*(d
 + e*x^n))/(d*f - e*g)])/(f^2*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\left(f+\frac{g}{x}\right)^2 x}\,dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{g\log(c(d+ex)^p)}{f(g+fx)^2}+\frac{\log(c(d+ex)^p)}{f(g+fx)}\right)\,dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{g+fx}\,dx, x, x^n\right)}{fn} - \frac{g\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{(g+fx)^2}\,dx, x, x^n\right)}{fn}\\
&= \frac{g\log\left(c(d+ex^n)^p\right)}{f^2n\left(g+fx^n\right)} + \frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e(g+fx^n)}{df-eg}\right)}{f^2n}\\
&\quad - \frac{(ep)\text{Subst}\left(\int \frac{\log\left(\frac{e(g+fx)}{-df+eg}\right)}{d+ex}\,dx, x, x^n\right)}{f^2n} - \frac{(egp)\text{Subst}\left(\int \frac{1}{(d+ex)(g+fx)}\,dx, x, x^n\right)}{f^2n}
\end{aligned}
$$

$$= \frac{g\log\left(c(d+ex^n)^p\right)}{f^2n\left(g+fx^n\right)} + \frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e(g+fx^n)}{df-eg}\right)}{f^2n} - \frac{p\text{Subst}\left(\int\frac{\log\left(1+\frac{fx}{-df+eg}\right)}{x}\,dx,x,d+ex^n\right)}{f^2n} + \frac{(e^2gp)\,\text{Subst}\left(\int\frac{1}{d+ex}\,dx,x,x^n\right)}{f^2(df-eg)n} - \frac{(egp)\text{Subst}\left(\int\frac{1}{g+fx}\,dx,x,x^n\right)}{f(df-eg)n}$$

$$= \frac{egp\log\left(d+ex^n\right)}{f^2(df-eg)n} + \frac{g\log\left(c(d+ex^n)^p\right)}{f^2n\left(g+fx^n\right)} - \frac{egp\log\left(g+fx^n\right)}{f^2(df-eg)n} + \frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e(g+fx^n)}{df-eg}\right)}{f^2n} + \frac{p\text{Li}_2\left(\frac{f(d+ex^n)}{df-eg}\right)}{f^2n}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 433 vs. $2(156) = 312$.

Time = 1.05 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.78

$$\int\frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)^2}\,dx$$

$$= \frac{gp\log\left(f-fx^{-n}\right)+fpx^n\log\left(f-fx^{-n}\right)-gnp\log(x)\log\left(f-fx^{-n}\right)-fnpx^n\log(x)\log\left(f-fx^{-n}\right)-p\,\mathrm{l}}{} - \frac{p\left(-\frac{df\log(e+dx^{-n})}{df-eg}+\frac{fx^n\log(e+dx^{-n})}{g+fx^n}+\log\left(-\frac{dx^{-n}}{e}\right)\log\left(e+dx^{-n}\right)+\frac{df\log(f+gx^{-n})}{df-eg}-\log\left(e+dx^{-n}\right)\log\left(\frac{d(}{}\right.\right.}{f^2n}$$

```
[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^n)^2),x]
```

```
[Out] (g*p*Log[f - f/x^n] + f*p*x^n*Log[f - f/x^n] - g*n*p*Log[x]*Log[f - f/x^n]
- f*n*p*x^n*Log[x]*Log[f - f/x^n] - p*Log[e + d/x^n]*(-(f*x^n) + (g + f*x^n
)*Log[f - f/x^n]) - f*x^n*Log[c*(d + e*x^n)^p] + g*Log[f - f/x^n]*Log[c*(d
+ e*x^n)^p] + f*x^n*Log[f - f/x^n]*Log[c*(d + e*x^n)^p] + g*n*p*Log[x]*Log[
1 + (f*x^n)/g] + f*n*p*x^n*Log[x]*Log[1 + (f*x^n)/g] + p*(g + f*x^n)*PolyLo
g[2, -((f*x^n)/g)])/(f^2*n*(g + f*x^n)) - (p*(-((d*f*Log[e + d/x^n])/(d*f -
 e*g)) + (f*x^n*Log[e + d/x^n])/(g + f*x^n) + Log[-(d/(e*x^n))]*Log[e + d/x
^n] + (d*f*Log[f + g/x^n])/(d*f - e*g) - Log[e + d/x^n]*Log[(d*(f + g/x^n))
/(d*f - e*g)] - PolyLog[2, -((g*(e + d/x^n))/(d*f - e*g))] + PolyLog[2, 1 +
 d/(e*x^n)]))/(f^2*n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.88 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.28

method	result
risch	$\frac{\ln((d+e\,x^n)^p)\ln(g+f\,x^n)}{n\,f^2}+\frac{\ln((d+e\,x^n)^p)g}{n\,f^2(g+f\,x^n)}-\frac{p\,\mathrm{dilog}\left(\frac{(g+f\,x^n)e+df-eg}{df-eg}\right)}{n\,f^2}-\frac{p\ln(g+f\,x^n)\ln\left(\frac{(g+f\,x^n)e+df-eg}{df-eg}\right)}{n\,f^2}+\frac{peg\ln((g+}{n\,f}$

[In] int(ln(c*(d+e*x^n)^p)/x/(f+g/(x^n))^2,x,method=_RETURNVERBOSE)

[Out] 1/n*ln((d+e*x^n)^p)/f^2*ln(g+f*x^n)+1/n*ln((d+e*x^n)^p)*g/f^2/(g+f*x^n)-1/n *p/f^2*dilog(((g+f*x^n)*e+d*f-e*g)/(d*f-e*g))-1/n*p/f^2*ln(g+f*x^n)*ln(((g+ f*x^n)*e+d*f-e*g)/(d*f-e*g))+1/n*p*e/f^2*g/(d*f-e*g)*ln((g+f*x^n)*e+d*f-e*g)-e*g*p*ln(g+f*x^n)/f^2/(d*f-e*g)/n+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c* (d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c) -1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))*(1/n/f^2*ln(g+f*x^n)+1/n*g/f^2/(g+f*x^n))

Fricas [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)^2}\,dx=\int \frac{\log\left((ex^n+d)^pc\right)}{\left(f+\frac{g}{x^n}\right)^2x}\,dx$$

[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n))^2,x, algorithm="fricas")

[Out] integral(log((e*x^n + d)^p*c)/(f^2*x + 2*f*g*x*x^n/x^(2*n) + g^2*x/x^(2*n)) , x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)^2}\,dx=\text{Timed out}$$

[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g/(x**n))**2,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.34

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)^2}\,dx$$
$$= enp\left(\frac{d\log\left(\frac{ex^n+d}{e}\right)}{def^2n^2-e^2fgn^2}-\frac{g\log\left(\frac{fx^n+g}{f}\right)}{df^3n^2-ef^2gn^2}-\frac{\log\left(fx^n+g\right)\log\left(\frac{efx^n+eg}{df-eg}+1\right)+\mathrm{Li}_2\left(-\frac{efx^n+eg}{df-eg}\right)}{ef^2n^2}\right)$$
$$-\left(\frac{1}{f^2n+\frac{fgn}{x^n}}-\frac{\log\left(f+\frac{g}{x^n}\right)}{f^2n}+\frac{\log\left(\frac{1}{x^n}\right)}{f^2n}\right)\log\left((ex^n+d)^pc\right)$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n))^2,x, algorithm="maxima")
```

```
[Out] e*n*p*(d*log((e*x^n + d)/e)/(d*e*f^2*n^2 - e^2*f*g*n^2) - g*log((f*x^n + g)
/f)/(d*f^3*n^2 - e*f^2*g*n^2) - (log(f*x^n + g)*log((e*f*x^n + e*g)/(d*f -
e*g) + 1) + dilog(-(e*f*x^n + e*g)/(d*f - e*g)))/(e*f^2*n^2)) - (1/(f^2*n +
 f*g*n/x^n) - log(f + g/x^n)/(f^2*n) + log(1/(x^n))/(f^2*n))*log((e*x^n + d
)^p*c)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)^2}\,dx=\int \frac{\log\left((ex^n+d)^pc\right)}{\left(f+\frac{g}{x^n}\right)^2x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^n))^2,x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)/((f + g/x^n)^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)^2}\,dx=\int \frac{\ln\left(c\,\left(d+e\,x^n\right)^p\right)}{x\,\left(f+\frac{g}{x^n}\right)^2}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^n)^2),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^n)^2), x)
```

3.377 $\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-2n})^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 377

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-2n})^2}\,dx = -\frac{de\sqrt{g}p\arctan\left(\frac{\sqrt{f}x^n}{\sqrt{g}}\right)}{2f^{3/2}(d^2f+e^2g)n} - \frac{e^2gp\log(d+ex^n)}{2f^2(d^2f+e^2g)n} + \frac{g\log(c(d+ex^n)^p)}{2f^2n(g+fx^{2n})} + \frac{\log(c(d+ex^n)^p)\log\left(\frac{e(\sqrt{g}-\sqrt{-f}x^n)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2f^2n} + \frac{\log(c(d+ex^n)^p)\log\left(-\frac{e(\sqrt{g}+\sqrt{-f}x^n)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2f^2n} + \frac{e^2gp\log(g+fx^{2n})}{4f^2(d^2f+e^2g)n} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-f}(d+ex^n)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2f^2n} + \frac{p\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-f}(d+ex^n)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2f^2n}$$

```
[Out] -1/2*e^2*g*p*ln(d+e*x^n)/f^2/(d^2*f+e^2*g)/n+1/2*g*ln(c*(d+e*x^n)^p)/f^2/n/
(g+f*x^(2*n))+1/4*e^2*g*p*ln(g+f*x^(2*n))/f^2/(d^2*f+e^2*g)/n+1/2*ln(c*(d+e
*x^n)^p)*ln(-e*(x^n*(-f)^(1/2)+g^(1/2))/(d*(-f)^(1/2)-e*g^(1/2)))/f^2/n+1/2
*ln(c*(d+e*x^n)^p)*ln(e*(-x^n*(-f)^(1/2)+g^(1/2))/(d*(-f)^(1/2)+e*g^(1/2)))
/f^2/n+1/2*p*polylog(2,(d+e*x^n)*(-f)^(1/2)/(d*(-f)^(1/2)-e*g^(1/2)))/f^2/n
+1/2*p*polylog(2,(d+e*x^n)*(-f)^(1/2)/(d*(-f)^(1/2)+e*g^(1/2)))/f^2/n-1/2*d
*e*p*arctan(x^n*f^(1/2)/g^(1/2))*g^(1/2)/f^(3/2)/(d^2*f+e^2*g)/n
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {2525, 269, 272, 45, 2463, 2460, 720, 31, 649, 211, 266, 2441, 2440, 2438}

$$\int \frac{\log(c(d+ex^n)^p)}{x(f+gx^{-2n})^2}\,dx = -\frac{de\sqrt{g}p\arctan\left(\frac{\sqrt{f}x^n}{\sqrt{g}}\right)}{2f^{3/2}n(d^2f+e^2g)}+\frac{\log(c(d+ex^n)^p)\log\left(\frac{e(\sqrt{g}-\sqrt{-f}x^n)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2f^2n}+\frac{\log(c(d+ex^n)^p)\log\left(-\frac{e(\sqrt{-f}x^n+\sqrt{g})}{d\sqrt{-f}-e\sqrt{g}}\right)}{2f^2n}+\frac{g\log(c(d+ex^n)^p)}{2f^2n(fx^{2n}+g)}+\frac{e^2gp\log(fx^{2n}+g)}{4f^2n(d^2f+e^2g)}-\frac{e^2gp\log(d+ex^n)}{2f^2n(d^2f+e^2g)}+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-f}(ex^n+d)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2f^2n}+\frac{p\,\text{PolyLog}\left(2,\frac{\sqrt{-f}(ex^n+d)}{\sqrt{-f}d+e\sqrt{g}}\right)}{2f^2n}$$

```
[In] Int[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))^2),x]
```

```
[Out] -1/2*(d*e*Sqrt[g]*p*ArcTan[(Sqrt[f]*x^n)/Sqrt[g]])/(f^(3/2)*(d^2*f + e^2*g)
*n) - (e^2*g*p*Log[d + e*x^n])/(2*f^2*(d^2*f + e^2*g)*n) + (g*Log[c*(d + e*
x^n)^p])/(2*f^2*n*(g + f*x^(2*n))) + (Log[c*(d + e*x^n)^p]*Log[(e*(Sqrt[g]
- Sqrt[-f]*x^n))/(d*Sqrt[-f] + e*Sqrt[g])]])/(2*f^2*n) + (Log[c*(d + e*x^n)^
p]*Log[-((e*(Sqrt[g] + Sqrt[-f]*x^n))/(d*Sqrt[-f] - e*Sqrt[g]))])/(2*f^2*n)
 + (e^2*g*p*Log[g + f*x^(2*n)])/(4*f^2*(d^2*f + e^2*g)*n) + (p*PolyLog[2, (
Sqrt[-f]*(d + e*x^n))/(d*Sqrt[-f] - e*Sqrt[g])]])/(2*f^2*n) + (p*PolyLog[2,
(Sqrt[-f]*(d + e*x^n))/(d*Sqrt[-f] + e*Sqrt[g])]])/(2*f^2*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2460

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(x_)^(m_.)*(
(f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Simp[(f + g*x^r)^(q + 1)*((a
+ b*Log[c*(d + e*x)^n])^p/(g*r*(q + 1))), x] - Dist[b*e*n*(p/(g*r*(q + 1)))
, Int[(f + g*x^r)^(q + 1)*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x
], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, q, r}, x] && EqQ[m, r - 1] && Ne
Q[q, -1] && IGtQ[p, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\left(f+\frac{g}{x^2}\right)^2 x}\,dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{gx\log(c(d+ex)^p)}{f(g+fx^2)^2}+\frac{x\log(c(d+ex)^p)}{f(g+fx^2)}\right)\,dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{g+fx^2}\,dx, x, x^n\right)}{fn}-\frac{g\text{Subst}\left(\int \frac{x\log(c(d+ex)^p)}{(g+fx^2)^2}\,dx, x, x^n\right)}{fn} \\
&= \frac{g\log\left(c(d+ex^n)^p\right)}{2f^2n\left(g+fx^{2n}\right)}+\frac{\text{Subst}\left(\int \left(-\frac{\sqrt{-f}\log(c(d+ex)^p)}{2f\left(\sqrt{g}-\sqrt{-f}x\right)}+\frac{\sqrt{-f}\log(c(d+ex)^p)}{2f\left(\sqrt{g}+\sqrt{-f}x\right)}\right)\,dx, x, x^n\right)}{fn} \\
&\quad -\frac{(egp)\text{Subst}\left(\int \frac{1}{(d+ex)(g+fx^2)}\,dx, x, x^n\right)}{2f^2n}
\end{aligned}
$$

$$= \frac{g\log\left(c(d+ex^n)^p\right)}{2f^2n\left(g+fx^{2n}\right)} - \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{g}-\sqrt{-f}x}\,dx, x, x^n\right)}{2(-f)^{3/2}n} + \frac{\text{Subst}\left(\int \frac{\log(c(d+ex)^p)}{\sqrt{g}+\sqrt{-f}x}\,dx, x, x^n\right)}{2(-f)^{3/2}n} - \frac{(egp)\text{Subst}\left(\int \frac{df-efx}{g+fx^2}\,dx, x, x^n\right)}{2f^2\left(d^2f+e^2g\right)n} - \frac{\left(e^3gp\right)\text{Subst}\left(\int \frac{1}{d+ex}\,dx, x, x^n\right)}{2f^2\left(d^2f+e^2g\right)n}$$

$$= -\frac{e^2gp\log\left(d+ex^n\right)}{2f^2\left(d^2f+e^2g\right)n} + \frac{g\log\left(c(d+ex^n)^p\right)}{2f^2n\left(g+fx^{2n}\right)} + \frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2f^2n} + \frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e\left(\sqrt{g}+\sqrt{-f}x^n\right)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2f^2n} - \frac{(ep)\text{Subst}\left(\int \frac{\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}x\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{d+ex}\,dx, x, x^n\right)}{2f^2n} - \frac{(ep)\text{Subst}\left(\int \frac{\log\left(\frac{e\left(\sqrt{g}+\sqrt{-f}x\right)}{-d\sqrt{-f}+e\sqrt{g}}\right)}{d+ex}\,dx, x, x^n\right)}{2f^2n} - \frac{(degp)\text{Subst}\left(\int \frac{1}{g+fx^2}\,dx, x, x^n\right)}{2f\left(d^2f+e^2g\right)n} + \frac{\left(e^2gp\right)\text{Subst}\left(\int \frac{x}{g+fx^2}\,dx, x, x^n\right)}{2f\left(d^2f+e^2g\right)n}$$

$$= -\frac{de\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{f}x^n}{\sqrt{g}}\right)}{2f^{3/2}\left(d^2f+e^2g\right)n} - \frac{e^2gp\log\left(d+ex^n\right)}{2f^2\left(d^2f+e^2g\right)n} + \frac{g\log\left(c(d+ex^n)^p\right)}{2f^2n\left(g+fx^{2n}\right)} + \frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2f^2n} + \frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e\left(\sqrt{g}+\sqrt{-f}x^n\right)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2f^2n} + \frac{e^2gp\log\left(g+fx^{2n}\right)}{4f^2\left(d^2f+e^2g\right)n} - \frac{p\text{Subst}\left(\int \frac{\log\left(1+\frac{\sqrt{-f}x}{-d\sqrt{-f}+e\sqrt{g}}\right)}{x}\,dx, x, d+ex^n\right)}{2f^2n} - \frac{p\text{Subst}\left(\int \frac{\log\left(1-\frac{\sqrt{-f}x}{d\sqrt{-f}+e\sqrt{g}}\right)}{x}\,dx, x, d+ex^n\right)}{2f^2n}$$

$$= -\frac{de\sqrt{g}p\tan^{-1}\left(\frac{\sqrt{f}x^n}{\sqrt{g}}\right)}{2f^{3/2}\left(d^2f+e^2g\right)n} - \frac{e^2gp\log\left(d+ex^n\right)}{2f^2\left(d^2f+e^2g\right)n} + \frac{g\log\left(c(d+ex^n)^p\right)}{2f^2n\left(g+fx^{2n}\right)} + \frac{\log\left(c(d+ex^n)^p\right)\log\left(\frac{e\left(\sqrt{g}-\sqrt{-f}x^n\right)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2f^2n} + \frac{\log\left(c(d+ex^n)^p\right)\log\left(-\frac{e\left(\sqrt{g}+\sqrt{-f}x^n\right)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2f^2n} + \frac{e^2gp\log\left(g+fx^{2n}\right)}{4f^2\left(d^2f+e^2g\right)n} + \frac{p\text{Li}_2\left(\frac{\sqrt{-f}(d+ex^n)}{d\sqrt{-f}-e\sqrt{g}}\right)}{2f^2n} + \frac{p\text{Li}_2\left(\frac{\sqrt{-f}(d+ex^n)}{d\sqrt{-f}+e\sqrt{g}}\right)}{2f^2n}$$

Mathematica [F]

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)^2}\,dx=\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)^2}\,dx$$

[In] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))^2),x]

[Out] Integrate[Log[c*(d + e*x^n)^p]/(x*(f + g/x^(2*n))^2), x]

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 59.38 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.49

method	result
risch	$\frac{\ln\left(\left(d+e\,x^{n}\right)^{p}\right)\ln\left(g+f\,x^{2n}\right)}{2n\,f^{2}}+\frac{\ln\left(\left(d+e\,x^{n}\right)^{p}\right)g}{2n\,f^{2}\left(g+f\,x^{2n}\right)}-\frac{p\ln\left(d+e\,x^{n}\right)\ln\left(g+f\,x^{2n}\right)}{2n\,f^{2}}+\frac{p\ln\left(d+e\,x^{n}\right)\ln\left(\frac{e\sqrt{-fg}-f\left(d+e\,x^{n}\right)+df}{e\sqrt{-fg}+df}\right)}{2n\,f^{2}}+\frac{p\ln(d+$

[In] int(ln(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n)))^2,x,method=_RETURNVERBOSE)

[Out] 1/2/n*ln((d+e*x^n)^p)/f^2*ln(g+f*(x^n)^2)+1/2/n*ln((d+e*x^n)^p)*g/f^2/(g+f*(x^n)^2)-1/2/n*p/f^2*ln(d+e*x^n)*ln(g+f*(x^n)^2)+1/2/n*p/f^2*ln(d+e*x^n)*ln((e*(-f*g)^(1/2)-f*(d+e*x^n)+d*f)/(e*(-f*g)^(1/2)+d*f))+1/2/n*p/f^2*ln(d+e*x^n)*ln((e*(-f*g)^(1/2)+f*(d+e*x^n)-d*f)/(e*(-f*g)^(1/2)-d*f))+1/2/n*p/f^2*dilog((e*(-f*g)^(1/2)-f*(d+e*x^n)+d*f)/(e*(-f*g)^(1/2)+d*f))+1/2/n*p/f^2*dilog((e*(-f*g)^(1/2)+f*(d+e*x^n)-d*f)/(e*(-f*g)^(1/2)-d*f))-1/2*e^2*g*p*ln(d+e*x^n)/f^2/(d^2*f+e^2*g)/n+1/4/n*p*e^2*g/f^2/(d^2*f+e^2*g)*ln(g+f*(x^n)^2)-1/2/n*p*e*g/f/(d^2*f+e^2*g)*d/(f*g)^(1/2)*arctan(x^n*f/(f*g)^(1/2))+(1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)^2-1/2*I*Pi*csgn(I*(d+e*x^n)^p)*csgn(I*c*(d+e*x^n)^p)*csgn(I*c)-1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^3+1/2*I*Pi*csgn(I*c*(d+e*x^n)^p)^2*csgn(I*c)+ln(c))*(1/2/n/f^2*ln(g+f*(x^n)^2)+1/2/n*g/f^2/(g+f*(x^n)^2))

Fricas [F]

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)^2}\,dx=\int \frac{\log \left(\left(ex^n+d\right)^p c\right)}{\left(f+\frac{g}{x^{2n}}\right)^2 x}\,dx$$

[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n)))^2,x, algorithm="fricas")

[Out] integral(log((e*x^n + d)^p*c)/(f^2*x + 2*f*g*x*x^(2*n)/x^(4*n) + g^2*x/x^(4*n)), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(d+e*x**n)**p)/x/(f+g/(x**(2*n)))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)^2}\,dx = \int \frac{\log \left((ex^n+d)^p c\right)}{\left(f+\frac{g}{x^{2n}}\right)^2 x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n)))^2,x, algorithm="maxima")
[Out] integrate(log((e*x^n + d)^p*c)/((f + g/x^(2*n))^2*x), x)
```

Giac [F]

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)^2}\,dx = \int \frac{\log \left((ex^n+d)^p c\right)}{\left(f+\frac{g}{x^{2n}}\right)^2 x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)/x/(f+g/(x^(2*n)))^2,x, algorithm="giac")
[Out] integrate(log((e*x^n + d)^p*c)/((f + g/x^(2*n))^2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)^2}\,dx = \int \frac{\ln \left(c\,{\left(d+e\,x^n\right)}^p\right)}{x\,{\left(f+\frac{g}{x^{2\,n}}\right)}^2}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^(2*n))^2),x)
[Out] int(log(c*(d + e*x^n)^p)/(x*(f + g/x^(2*n))^2), x)
```

3.378 $\int \frac{\log(c(d+ex^n))}{x(ce-(1-cd)x^{-n})}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 25

$$\int \frac{\log (c(d+ex^n))}{x\left(ce-(1-cd)x^{-n}\right)}\,dx=-\frac{\text{PolyLog}(2,1-c(d+ex^n))}{cen}$$

```
[Out] -polylog(2,1-c*(d+e*x^n))/c/e/n
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2525, 2459, 2440, 2438}

$$\int \frac{\log (c(d+ex^n))}{x\left(ce-(1-cd)x^{-n}\right)}\,dx=-\frac{\text{PolyLog}(2,1-c(ex^n+d))}{cen}$$

```
[In] Int[Log[c*(d + e*x^n)]/(x*(c*e - (1 - c*d)/x^n)),x]
[Out] -(PolyLog[2, 1 - c*(d + e*x^n)]/(c*e*n))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2459

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)/(x_))^(q_.)*(x_)^(m_.), x_Symbol] :> Int[(g + f*x)^q*(a + b*Log[c*(d + e*
x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x] && EqQ[m, q] &&
 IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{\left(ce+\frac{-1+cd}{x}\right)x}\,dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{-1+cd+cex}\,dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, -1+cd+cex^n\right)}{cen}\\
&= -\frac{\text{Li}_2(1-c(d+ex^n))}{cen}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{\log\left(c(d+ex^n)\right)}{x\left(ce-(1-cd)x^{-n}\right)}\,dx = -\frac{\text{PolyLog}\left(2, 1-cd-cex^n\right)}{cen}$$

```
[In] Integrate[Log[c*(d + e*x^n)]/(x*(c*e - (1 - c*d)/x^n)),x]
[Out] -(PolyLog[2, 1 - c*d - c*e*x^n]/(c*e*n))
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

method	result
derivativedivides	$-\frac{\operatorname{dilog}(ce\,x^{n}+cd)}{nce}$
default	$-\frac{\operatorname{dilog}(ce\,x^{n}+cd)}{nce}$
risch	$\frac{\ln(1-c(d+e\,x^{n}))\ln(d+e\,x^{n})}{nec}-\frac{\ln(1-c(d+e\,x^{n}))\ln(c(d+e\,x^{n}))}{nec}-\frac{\operatorname{dilog}(c(d+e\,x^{n}))}{nec}+\frac{\left(\frac{i\pi\ \operatorname{csgn}(i(d+e\,x^{n}))\operatorname{csgn}(ic(d+e}{2}\right.}{}$

[In] int(ln(c*(d+e*x^n))/x/(c*e+(c*d-1)/(x^n)),x,method=_RETURNVERBOSE)

[Out] -1/n/c/e*dilog(c*e*x^n+c*d)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(c(d+ex^{n})\right)}{x\left(ce-(1-cd)x^{-n}\right)}\,dx=-\frac{\operatorname{Li}_{2}(-cex^{n}-cd+1)}{cen}$$

[In] integrate(log(c*(d+e*x^n))/x/(c*e+(c*d-1)/(x^n)),x, algorithm="fricas")

[Out] -dilog(-c*e*x^n - c*d + 1)/(c*e*n)

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log\left(c(d+ex^{n})\right)}{x\left(ce-(1-cd)x^{-n}\right)}\,dx=\text{Exception raised: TypeError}$$

[In] integrate(ln(c*(d+e*x**n))/x/(c*e+(c*d-1)/(x**n)),x)

[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(24) = 48$.

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 4.24

$$\int \frac{\log\left(c(d+ex^n)\right)}{x\left(ce-(1-cd)x^{-n}\right)}\,dx = \left(\frac{\log\left(ce+\frac{cd-1}{x^n}\right)}{cen} - \frac{\log\left(\frac{1}{x^n}\right)}{cen}\right)\log\left((ex^n+d)c\right) - \frac{\log\left(cex^n+cd\right)\log\left(cex^n+cd-1\right)+\mathrm{Li}_2(-cex^n-cd+1)}{cen}$$

```
[In] integrate(log(c*(d+e*x^n))/x/(c*e+(c*d-1)/(x^n)),x, algorithm="maxima")
[Out] (log(c*e + (c*d - 1)/x^n)/(c*e*n) - log(1/(x^n))/(c*e*n))*log((e*x^n + d)*c
) - (log(c*e*x^n + c*d)*log(c*e*x^n + c*d - 1) + dilog(-c*e*x^n - c*d + 1))
/(c*e*n)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^n)\right)}{x\left(ce-(1-cd)x^{-n}\right)}\,dx = \int \frac{\log\left((ex^n+d)c\right)}{\left(ce+\frac{cd-1}{x^n}\right)x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n))/x/(c*e+(c*d-1)/(x^n)),x, algorithm="giac")
[Out] integrate(log((e*x^n + d)*c)/((c*e + (c*d - 1)/x^n)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^n)\right)}{x\left(ce-(1-cd)x^{-n}\right)}\,dx = \int \frac{\ln\left(c\left(d+e\,x^n\right)\right)}{x\,\left(c\,e+\frac{c\,d-1}{x^n}\right)}\,dx$$

```
[In] int(log(c*(d + e*x^n))/(x*(c*e + (c*d - 1)/x^n)),x)
[Out] int(log(c*(d + e*x^n))/(x*(c*e + (c*d - 1)/x^n)), x)
```

3.379 $\int \frac{x^{-1+n}\log(c(d+ex^n))}{-1+cd+cex^n}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 25

$$\int \frac{x^{-1+n}\log\left(c(d+ex^n)\right)}{-1+cd+cex^n}\,dx=-\frac{\text{PolyLog}\left(2,1-c(d+ex^n)\right)}{cen}$$

```
[Out] -polylog(2,1-c*(d+e*x^n))/c/e/n
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2525, 2440, 2438}

$$\int \frac{x^{-1+n}\log\left(c(d+ex^n)\right)}{-1+cd+cex^n}\,dx=-\frac{\text{PolyLog}\left(2,1-c(ex^n+d)\right)}{cen}$$

```
[In] Int[(x^(-1 + n)*Log[c*(d + e*x^n)])/(-1 + c*d + c*e*x^n),x]
```

```
[Out] -(PolyLog[2, 1 - c*(d + e*x^n)]/(c*e*n))
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{-1+cd+cex}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, -1+cd+cex^n\right)}{cen}\\ &= -\frac{\text{Li}_2(1-c(d+ex^n))}{cen}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{x^{-1+n}\log\left(c(d+ex^n)\right)}{-1+cd+cex^n}\,dx = -\frac{\text{PolyLog}\left(2, 1-cd-cex^n\right)}{cen}$$

```
[In] Integrate[(x^(-1 + n)*Log[c*(d + e*x^n)])/(-1 + c*d + c*e*x^n),x]
[Out] -(PolyLog[2, 1 - c*d - c*e*x^n]/(c*e*n))
```

Maple [A] (verified)

Time = 2.69 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\operatorname{dilog}(ce\,x^n+cd)}{nce}$
risch	$\frac{\ln(1-c(d+e\,x^n))\ln(d+e\,x^n)}{nec}-\frac{\ln(1-c(d+e\,x^n))\ln(c(d+e\,x^n))}{nec}-\frac{\operatorname{dilog}(c(d+e\,x^n))}{nec}+\frac{\left(\frac{i\pi\ \operatorname{csgn}(i(d+e\,x^n))\operatorname{csgn}(ic(d+e\,x^n))^2}{2}-i\pi \right.}{}$

```
[In] int(x^(n-1)*ln(c*(d+e*x^n))/(-1+c*d+c*e*x^n),x,method=_RETURNVERBOSE)
[Out] -1/n/c/e*dilog(c*e*x^n+c*d)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+n} \log \left(c(d+ex^n)\right)}{-1+cd+cex^n}\,dx = -\frac{\mathrm{Li}_2(-cex^n-cd+1)}{cen}$$

```
[In] integrate(x^(-1+n)*log(c*(d+e*x^n))/(-1+c*d+c*e*x^n),x, algorithm="fricas")
[Out] -dilog(-c*e*x^n - c*d + 1)/(c*e*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1+n} \log \left(c(d+ex^n)\right)}{-1+cd+cex^n}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x**(-1+n)*ln(c*(d+e*x**n))/(-1+c*d+c*e*x**n),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(24) = 48.

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 4.36

$$\begin{aligned}\int \frac{x^{-1+n} \log \left(c(d+ex^n)\right)}{-1+cd+cex^n}\,dx &= \frac{\log \left(cex^n+cd-1\right) \log \left((ex^n+d)c\right)}{cen} \\ &\quad - \frac{\log \left(cex^n+cd-1\right) \log \left(ex^n+d\right)}{cen} \\ &\quad + \frac{\log \left(-cex^n-cd+1\right) \log \left(ex^n+d\right)+\mathrm{Li}_2(cex^n+cd)}{cen}\end{aligned}$$

```
[In] integrate(x^(-1+n)*log(c*(d+e*x^n))/(-1+c*d+c*e*x^n),x, algorithm="maxima")
[Out] log(c*e*x^n + c*d - 1)*log((e*x^n + d)*c)/(c*e*n) - log(c*e*x^n + c*d - 1)*
log(e*x^n + d)/(c*e*n) + (log(-c*e*x^n - c*d + 1)*log(e*x^n + d) + dilog(c*
e*x^n + c*d))/(c*e*n)
```

Giac [F]

$$\int \frac{x^{-1+n} \log\left(c(d+ex^n)\right)}{-1+cd+cex^n}\,dx = \int \frac{x^{n-1} \log\left((ex^n+d)c\right)}{cex^n+cd-1}\,dx$$

```
[In] integrate(x^(-1+n)*log(c*(d+e*x^n))/(-1+c*d+c*e*x^n),x, algorithm="giac")
[Out] integrate(x^(n - 1)*log((e*x^n + d)*c)/(c*e*x^n + c*d - 1), x)
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^{-1+n} \log\left(c(d+ex^n)\right)}{-1+cd+cex^n}\,dx = -\frac{\mathrm{Li}_2\left(c\left(d+e\,x^n\right)\right)}{c\,e\,n}$$

```
[In] int((x^(n - 1)*log(c*(d + e*x^n)))/(c*d + c*e*x^n - 1),x)
[Out] -dilog(c*(d + e*x^n))/(c*e*n)
```

3.380 $\int \frac{\log(c(d+ex^{-n}))}{x(ce-(1-cd)x^n)} dx$

Optimal result

Integrand size = 33, antiderivative size = 26

$$\int \frac{\log (c(d+ex^{-n}))}{x (ce-(1-cd)x^n)} dx = \frac{\text{PolyLog}(2, 1-c(d+ex^{-n}))}{cen}$$

```
[Out] polylog(2,1-c*(d+e/(x^n)))/c/e/n
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2525, 2459, 2440, 2438}

$$\int \frac{\log (c(d+ex^{-n}))}{x (ce-(1-cd)x^n)} dx = \frac{\text{PolyLog}(2, 1-c(ex^{-n}+d))}{cen}$$

```
[In] Int[Log[c*(d + e/x^n)]/(x*(c*e - (1 - c*d)*x^n)),x]
```

```
[Out] PolyLog[2, 1 - c*(d + e/x^n)]/(c*e*n)
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2459

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)/(x_))^(q_.)*(x_)^(m_.), x_Symbol] :> Int[(g + f*x)^q*(a + b*Log[c*(d + e*
x)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x] && EqQ[m, q] &&
 IntegerQ[q]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{\left(ce+\frac{-1+cd}{x}\right)x}\,dx, x, x^{-n}\right)}{n}\\
&= -\frac{\text{Subst}\left(\int \frac{\log(c(d+ex))}{-1+cd+cex}\,dx, x, x^{-n}\right)}{n}\\
&= -\frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, -1+cd+cex^{-n}\right)}{cen}\\
&= \frac{\text{Li}_2(1-cd-cex^{-n})}{cen}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{\log \left(c(d+ex^{-n})\right)}{x\left(ce-(1-cd)x^n\right)}\,dx = \frac{\text{PolyLog}\left(2,-x^{-n}(ce-x^n+cdx^n)\right)}{cen}$$

```
[In] Integrate[Log[c*(d + e/x^n)]/(x*(c*e - (1 - c*d)*x^n)),x]
[Out] PolyLog[2, -((c*e - x^n + c*d*x^n)/x^n)]/(c*e*n)
```

Maple [A] (verified)

Time = 3.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\mathrm{dilog}\left(cd+ce\,x^{-n}\right)}{nce}$	24
default	$\frac{\mathrm{dilog}\left(cd+ce\,x^{-n}\right)}{nce}$	24
risch	Expression too large to display	1900

```
[In] int(ln(c*(d+e/(x^n)))/x/(c*e-(-c*d+1)*x^n),x,method=_RETURNVERBOSE)
[Out] 1/n/c/e*dilog(c*d+c*e/(x^n))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\log \left(c(d+ex^{-n})\right)}{x\left(ce-(1-cd)x^n\right)}\,dx=\frac{\mathrm{Li}_2\left(-\frac{cdx^n+ce}{x^n}+1\right)}{cen}$$

```
[In] integrate(log(c*(d+e/(x^n)))/x/(c*e-(-c*d+1)*x^n),x, algorithm="fricas")
[Out] dilog(-(c*d*x^n + c*e)/x^n + 1)/(c*e*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log \left(c(d+ex^{-n})\right)}{x\left(ce-(1-cd)x^n\right)}\,dx=\text{Timed out}$$

```
[In] integrate(ln(c*(d+e/(x**n)))/x/(c*e-(-c*d+1)*x**n),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log\left(c(d+ex^{-n})\right)}{x\left(ce-(1-cd)x^n\right)}\,dx = \int \frac{\log\left(c\left(d+\frac{e}{x^n}\right)\right)}{(ce+(cd-1)x^n)x}\,dx$$

```
[In] integrate(log(c*(d+e/(x^n)))/x/(c*e-(-c*d+1)*x^n),x, algorithm="maxima")
```

```
[Out] n*integrate(log(x)/(c*d*x*x^n + c*e*x), x) + (log(d*x^n + e)*log(x) + log(c
)*log(x) - log(x)*log(x^n))/(c*e) - log(c)*log((c*e + (c*d - 1)*x^n)/(c*d -
 1))/(c*e*n) - (log(d*x^n + e)*log((c*d*e + (c*d^2 - d)*x^n - e)/e + 1) + d
ilog(-(c*d*e + (c*d^2 - d)*x^n - e)/e))/(c*e*n) + (log(x^n)*log((c*d - 1)*x
^n/(c*e) + 1) + dilog(-(c*d - 1)*x^n/(c*e)))/(c*e*n)
```

Giac [F]

$$\int \frac{\log\left(c(d+ex^{-n})\right)}{x\left(ce-(1-cd)x^n\right)}\,dx = \int \frac{\log\left(c\left(d+\frac{e}{x^n}\right)\right)}{(ce+(cd-1)x^n)x}\,dx$$

```
[In] integrate(log(c*(d+e/(x^n)))/x/(c*e-(-c*d+1)*x^n),x, algorithm="giac")
```

```
[Out] integrate(log(c*(d + e/x^n))/((c*e + (c*d - 1)*x^n)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(c(d+ex^{-n})\right)}{x\left(ce-(1-cd)x^n\right)}\,dx = \int \frac{\ln\left(c\left(d+\frac{e}{x^n}\right)\right)}{x\,\left(c\,e+x^n\,\left(c\,d-1\right)\right)}\,dx$$

```
[In] int(log(c*(d + e/x^n))/(x*(c*e + x^n*(c*d - 1))),x)
```

```
[Out] int(log(c*(d + e/x^n))/(x*(c*e + x^n*(c*d - 1))), x)
```

3.381 $\int \frac{(f+gx^{2n})^2 \log^q(c(d+ex^n)^p)}{x} dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\begin{aligned}
&\int \frac{(f+gx^{2n})^2 \log^q(c(d+ex^n)^p)}{x} dx \\
&= \frac{4^{-1-q}g^2(d+ex^n)^4 (c(d+ex^n)^p)^{-4/p}\Gamma\left(1+q,-\frac{4\log(c(d+ex^n)^p)}{p}\right)\log^q(c(d+ex^n)^p)\left(-\frac{\log(c(d+ex^n)^p)}{p}\right)^{-q}}{e^4n} \\
&- \frac{3^{-q}dg^2(d+ex^n)^3 (c(d+ex^n)^p)^{-3/p}\Gamma\left(1+q,-\frac{3\log(c(d+ex^n)^p)}{p}\right)\log^q(c(d+ex^n)^p)\left(-\frac{\log(c(d+ex^n)^p)}{p}\right)^{-q}}{e^4n} \\
&+ \frac{2^{-q}fg(d+ex^n)^2 (c(d+ex^n)^p)^{-2/p}\Gamma\left(1+q,-\frac{2\log(c(d+ex^n)^p)}{p}\right)\log^q(c(d+ex^n)^p)\left(-\frac{\log(c(d+ex^n)^p)}{p}\right)^{-q}}{e^2n} \\
&+ \frac{3\,2^{-1-q}d^2g^2(d+ex^n)^2 (c(d+ex^n)^p)^{-2/p}\Gamma\left(1+q,-\frac{2\log(c(d+ex^n)^p)}{p}\right)\log^q(c(d+ex^n)^p)\left(-\frac{\log(c(d+ex^n)^p)}{p}\right)^{-q}}{e^4n} \\
&- \frac{2dfg(d+ex^n)(c(d+ex^n)^p)^{-1/p}\Gamma\left(1+q,-\frac{\log(c(d+ex^n)^p)}{p}\right)\log^q(c(d+ex^n)^p)\left(-\frac{\log(c(d+ex^n)^p)}{p}\right)^{-q}}{e^2n} \\
&- \frac{d^3g^2(d+ex^n)(c(d+ex^n)^p)^{-1/p}\Gamma\left(1+q,-\frac{\log(c(d+ex^n)^p)}{p}\right)\log^q(c(d+ex^n)^p)\left(-\frac{\log(c(d+ex^n)^p)}{p}\right)^{-q}}{e^4n} \\
&+ f^2\text{Int}\left(\frac{\log^q(c(d+ex^n)^p)}{x},x\right)
\end{aligned}$$

```
[Out] 4^(-1-q)*g^2*(d+e*x^n)^4*GAMMA(1+q,-4*ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p
)^q/e^4/n/((c*(d+e*x^n)^p)^(4/p))/((-ln(c*(d+e*x^n)^p)/p)^q)-d*g^2*(d+e*x^n
)^3*GAMMA(1+q,-3*ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p)^q/(3^q)/e^4/n/((c*(
d+e*x^n)^p)^(3/p))/((-ln(c*(d+e*x^n)^p)/p)^q)+f*g*(d+e*x^n)^2*GAMMA(1+q,-2*
ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p)^q/(2^q)/e^2/n/((c*(d+e*x^n)^p)^(2/p)
```

```
)/((-ln(c*(d+e*x^n)^p)/p)^q)+3*2^(-1-q)*d^2*g^2*(d+e*x^n)^2*GAMMA(1+q,-2*ln
(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p)^q/e^4/n/((c*(d+e*x^n)^p)^(2/p))/((-ln(
c*(d+e*x^n)^p)/p)^q)-2*d*f*g*(d+e*x^n)*GAMMA(1+q,-ln(c*(d+e*x^n)^p)/p)*ln(c
*(d+e*x^n)^p)^q/e^2/n/((c*(d+e*x^n)^p)^(1/p))/((-ln(c*(d+e*x^n)^p)/p)^q)-d^
3*g^2*(d+e*x^n)*GAMMA(1+q,-ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p)^q/e^4/n/(
(c*(d+e*x^n)^p)^(1/p))/((-ln(c*(d+e*x^n)^p)/p)^q)+f^2*Unintegrable(ln(c*(d+
e*x^n)^p)^q/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(f+gx^{2n})^2 \log^q(c(d+ex^n)^p)}{x}\,dx = \int \frac{(f+gx^{2n})^2 \log^q(c(d+ex^n)^p)}{x}\,dx$$

```
[In] Int[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x,x]
[Out] Defer[Int][((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{(f+gx^{2n})^2 \log^q(c(d+ex^n)^p)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx^{2n})^2 \log^q(c(d+ex^n)^p)}{x}\,dx = \int \frac{(f+gx^{2n})^2 \log^q(c(d+ex^n)^p)}{x}\,dx$$

```
[In] Integrate[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x,x]
[Out] Integrate[((f + g*x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\left(f+g\,x^{2n}\right)^{2} \ln \left(c(d+e\,x^{n})^{p}\right)^{q}}{x}dx$$

```
[In] int((f+g*x^(2*n))^2*ln(c*(d+e*x^n)^p)^q/x,x)
[Out] int((f+g*x^(2*n))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.45

$$\int \frac{\left(f+gx^{2n}\right)^{2} \log^{q}\left(c(d+ex^{n})^{p}\right)}{x}\,dx=\int \frac{\left(gx^{2\,n}+f\right)^{2} \log\left((ex^{n}+d)^{p}c\right)^{q}}{x}\,dx$$

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="fricas")
[Out] integral((g^2*x^(4*n) + 2*f*g*x^(2*n) + f^2)*log((e*x^n + d)^p*c)^q/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(f+gx^{2n}\right)^{2} \log^{q}\left(c(d+ex^{n})^{p}\right)}{x}\,dx=\text{Timed out}$$

```
[In] integrate((f+g*x**(2*n))**2*ln(c*(d+e*x**n)**p)**q/x,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(f+gx^{2n}\right)^{2} \log^{q}\left(c(d+ex^{n})^{p}\right)}{x}\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [N/A]

Not integrable

Time = 3.77 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx^{2n})^2\log^q\left(c(d+ex^n)^p\right)}{x}\,dx=\int \frac{(gx^{2\,n}+f)^2\log\left((ex^n+d)^pc\right)^q}{x}\,dx$$

```
[In] integrate((f+g*x^(2*n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="giac")
[Out] integrate((g*x^(2*n) + f)^2*log((e*x^n + d)^p*c)^q/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx^{2n})^2\log^q\left(c(d+ex^n)^p\right)}{x}\,dx=\int \frac{\ln\left(c\,(d+e\,x^n)^p\right)^q\,(f+g\,x^{2\,n})^2}{x}\,dx$$

```
[In] int((log(c*(d + e*x^n)^p)^q*(f + g*x^(2*n))^2)/x,x)
[Out] int((log(c*(d + e*x^n)^p)^q*(f + g*x^(2*n))^2)/x, x)
```

3.382 $\int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\begin{aligned}
&\int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x} \, dx \\
&= \frac{2^{-1-q}g^2(d+ex^n)^2 (c(d+ex^n)^p)^{-2/p} \Gamma\left(1+q, -\frac{2\log(c(d+ex^n)^p)}{p}\right) \log^q(c(d+ex^n)^p) \left(-\frac{\log(c(d+ex^n)^p)}{p}\right)^{-q}}{e^2n} \\
&\quad + \frac{2fg(d+ex^n) (c(d+ex^n)^p)^{-1/p} \Gamma\left(1+q, -\frac{\log(c(d+ex^n)^p)}{p}\right) \log^q(c(d+ex^n)^p) \left(-\frac{\log(c(d+ex^n)^p)}{p}\right)^{-q}}{en} \\
&\quad - \frac{dg^2(d+ex^n) (c(d+ex^n)^p)^{-1/p} \Gamma\left(1+q, -\frac{\log(c(d+ex^n)^p)}{p}\right) \log^q(c(d+ex^n)^p) \left(-\frac{\log(c(d+ex^n)^p)}{p}\right)^{-q}}{e^2n} \\
&\quad + f^2 \text{Int}\left(\frac{\log^q(c(d+ex^n)^p)}{x}, x\right)
\end{aligned}$$

```
[Out] 2^(-1-q)*g^2*(d+e*x^n)^2*GAMMA(1+q,-2*ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p
)^q/e^2/n/((c*(d+e*x^n)^p)^(2/p))/((-ln(c*(d+e*x^n)^p)/p)^q)+2*f*g*(d+e*x^n
)*GAMMA(1+q,-ln(c*(d+e*x^n)^p)/p)*ln(c*(d+e*x^n)^p)^q/e/n/((c*(d+e*x^n)^p)^
(1/p))/((-ln(c*(d+e*x^n)^p)/p)^q)-d*g^2*(d+e*x^n)*GAMMA(1+q,-ln(c*(d+e*x^n)
^p)/p)*ln(c*(d+e*x^n)^p)^q/e^2/n/((c*(d+e*x^n)^p)^(1/p))/((-ln(c*(d+e*x^n)^
p)/p)^q)+f^2*Unintegrable(ln(c*(d+e*x^n)^p)^q/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x}\,dx = \int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x}\,dx$$

```
[In] Int[((f + g*x^n)^2*Log[c*(d + e*x^n)^p]^q)/x,x]
[Out] Defer[Int][((f + g*x^n)^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x}\,dx = \int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x}\,dx$$

```
[In] Integrate[((f + g*x^n)^2*Log[c*(d + e*x^n)^p]^q)/x,x]
[Out] Integrate[((f + g*x^n)^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(f+g\,x^n)^2 \ln\left(c(d+e\,x^n)^p\right)^q}{x}dx$$

```
[In] int((f+g*x^n)^2*ln(c*(d+e*x^n)^p)^q/x,x)
[Out] int((f+g*x^n)^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x}\,dx = \int \frac{(gx^n+f)^2 \log\left(\left(ex^n+d\right)^p c\right)^q}{x}\,dx$$

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="fricas")
[Out] integral((g^2*x^(2*n) + 2*f*g*x^n + f^2)*log((e*x^n + d)^p*c)^q/x, x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((f+g*x**n)**2*ln(c*(d+e*x**n)**p)**q/x,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [N/A]

Not integrable

Time = 2.75 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x}\,dx = \int \frac{(gx^n+f)^2 \log\left(\left(ex^n+d\right)^p c\right)^q}{x}\,dx$$

```
[In] integrate((f+g*x^n)^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="giac")
[Out] integrate((g*x^n + f)^2*log((e*x^n + d)^p*c)^q/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx^n)^2 \log^q(c(d+ex^n)^p)}{x}\,dx = \int \frac{\ln\left(c\left(d+e\,x^n\right)^p\right)^q \left(f+g\,x^n\right)^2}{x}\,dx$$

```
[In] int((log(c*(d + e*x^n)^p)^q*(f + g*x^n)^2)/x,x)
[Out] int((log(c*(d + e*x^n)^p)^q*(f + g*x^n)^2)/x, x)
```

3.383 $\int \frac{(f+gx^{-n})^2 \log^q(c(d+ex^n)^p)}{x} dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(f+gx^{-n})^2 \log^q(c(d+ex^n)^p)}{x} dx = \text{Int}\left(\frac{(f+gx^{-n})^2 \log^q(c(d+ex^n)^p)}{x}, x\right)$$

```
[Out] Unintegrable((f+g/(x^n))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(f+gx^{-n})^2 \log^q(c(d+ex^n)^p)}{x} dx = \int \frac{(f+gx^{-n})^2 \log^q(c(d+ex^n)^p)}{x} dx$$

```
[In] Int[((f + g/x^n)^2*Log[c*(d + e*x^n)^p]^q)/x,x]
[Out] Defer[Int][((f + g/x^n)^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{(f+gx^{-n})^2 \log^q(c(d+ex^n)^p)}{x} dx$$

Mathematica [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx^{-n})^2 \log^q (c(d+ex^n)^p)}{x} \, dx = \int \frac{(f+gx^{-n})^2 \log^q (c(d+ex^n)^p)}{x} \, dx$$

```
[In] Integrate[((f + g/x^n)^2*Log[c*(d + e*x^n)^p]^q)/x,x]
[Out] Integrate[((f + g/x^n)^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(f+g\,x^{-n})^2 \ln (c(d+e\,x^n)^p)^q}{x}dx$$

```
[In] int((f+g/(x^n))^2*ln(c*(d+e*x^n)^p)^q/x,x)
[Out] int((f+g/(x^n))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62

$$\int \frac{(f+gx^{-n})^2 \log^q (c(d+ex^n)^p)}{x} \, dx = \int \frac{\left(f+\frac{g}{x^n}\right)^2 \log \left((ex^n+d)^p c\right)^q}{x} \, dx$$

```
[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="fricas")
[Out] integral((f^2*x^(2*n) + 2*f*g*x^n + g^2)*log((e*x^n + d)^p*c)^q/(x*x^(2*n))
, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx^{-n})^2 \log^q\left(c(d+ex^n)^p\right)}{x}\,dx = \text{Timed out}$$

```
[In] integrate((f+g/(x**n))**2*ln(c*(d+e*x**n)**p)**q/x,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx^{-n})^2 \log^q\left(c(d+ex^n)^p\right)}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [N/A]

Not integrable

Time = 2.50 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx^{-n})^2 \log^q\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{\left(f+\frac{g}{x^n}\right)^2 \log\left((ex^n+d)^p c\right)^q}{x}\,dx$$

```
[In] integrate((f+g/(x^n))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="giac")
[Out] integrate((f + g/x^n)^2*log((e*x^n + d)^p*c)^q/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx^{-n})^2 \log^q\left(c(d+ex^n)^p\right)}{x}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)^q\,\left(f+\frac{g}{x^n}\right)^2}{x}\,dx$$

```
[In] int((log(c*(d + e*x^n)^p)^q*(f + g/x^n)^2)/x,x)
[Out] int((log(c*(d + e*x^n)^p)^q*(f + g/x^n)^2)/x, x)
```

3.384 $\int \frac{(f+gx^{-2n})^2 \log^q(c(d+ex^n)^p)}{x} dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{(f+gx^{-2n})^2 \log^q(c(d+ex^n)^p)}{x} dx = \mathrm{Int}\left(\frac{(f+gx^{-2n})^2 \log^q(c(d+ex^n)^p)}{x}, x\right)$$

```
[Out] Unintegrable((f+g/(x^(2*n)))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(f+gx^{-2n})^2 \log^q(c(d+ex^n)^p)}{x} dx = \int \frac{(f+gx^{-2n})^2 \log^q(c(d+ex^n)^p)}{x} dx$$

```
[In] Int[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x,x]
[Out] Defer[Int][((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{(f+gx^{-2n})^2 \log^q(c(d+ex^n)^p)}{x} dx$$

Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+gx^{-2n})^2\log^q\left(c(d+ex^n)^p\right)}{x}\,dx=\int \frac{(f+gx^{-2n})^2\log^q\left(c(d+ex^n)^p\right)}{x}\,dx$$

```
[In] Integrate[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x,x]
[Out] Integrate[((f + g/x^(2*n))^2*Log[c*(d + e*x^n)^p]^q)/x, x]
```

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(f+g\,x^{-2n})^2\ln\left(c(d+e\,x^n)^p\right)^q}{x}dx$$

```
[In] int((f+g/(x^(2*n)))^2*ln(c*(d+e*x^n)^p)^q/x,x)
[Out] int((f+g/(x^(2*n)))^2*ln(c*(d+e*x^n)^p)^q/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \frac{(f+gx^{-2n})^2\log^q\left(c(d+ex^n)^p\right)}{x}\,dx=\int \frac{\left(f+\frac{g}{x^{2n}}\right)^2\log\left((ex^n+d)^pc\right)^q}{x}\,dx$$

```
[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="fricas")
[Out] integral((f^2*x^(4*n) + 2*f*g*x^(2*n) + g^2)*log((e*x^n + d)^p*c)^q/(x*x^(4
*n)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx^{-2n})^2 \log^q(c(d+ex^n)^p)}{x}\,dx = \text{Timed out}$$

```
[In] integrate((f+g/(x**(2*n)))**2*ln(c*(d+e*x**n)**p)**q/x,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx^{-2n})^2 \log^q(c(d+ex^n)^p)}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{(f+gx^{-2n})^2 \log^q(c(d+ex^n)^p)}{x}\,dx = \int \frac{\left(f+\frac{g}{x^{2n}}\right)^2 \log\left((ex^n+d)^p c\right)^q}{x}\,dx$$

```
[In] integrate((f+g/(x^(2*n)))^2*log(c*(d+e*x^n)^p)^q/x,x, algorithm="giac")
[Out] integrate((f + g/x^(2*n))^2*log((e*x^n + d)^p*c)^q/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{(f+gx^{-2n})^2 \log^q(c(d+ex^n)^p)}{x}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)^q \left(f+\frac{g}{x^{2\,n}}\right)^2}{x}\,dx$$

```
[In] int((log(c*(d + e*x^n)^p)^q*(f + g/x^(2*n))^2)/x,x)
[Out] int((log(c*(d + e*x^n)^p)^q*(f + g/x^(2*n))^2)/x, x)
```

3.385 $\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{2n})} dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{2n})}\,dx = \text{Int}\left(\frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{2n})},x\right)$$

```
[Out] Unintegrable(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{2n})}\,dx = \int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{2n})}\,dx$$

```
[In] Int[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^(2*n))),x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^(2*n))), x]
```

Rubi steps

$$\text{integral} = \int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{2n})}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx = \int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx$$

```
[In] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^(2*n))),x]
[Out] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^(2*n))), x]
```

Maple [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c(d+e\,x^n)^p\right)^q}{x\left(f+g\,x^{2n}\right)}dx$$

```
[In] int(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x)
[Out] int(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx = \int \frac{\log\left((ex^n+d)^pc\right)^q}{(gx^{2\,n}+f)x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x, algorithm="fricas")
[Out] integral(log((e*x^n + d)^p*c)^q/(g*x*x^(2*n) + f*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx = \text{Timed out}$$

```
[In] integrate(ln(c*(d+e*x**n)**p)**q/x/(f+g*x**(2*n)),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [N/A]

Not integrable

Time = 2.99 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)^q}{(gx^{2n}+f)x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)^q/((g*x^(2*n) + f)*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{2n}\right)}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)^q}{x\,\left(f+g\,x^{2\,n}\right)}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)^q/(x*(f + g*x^(2*n))),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)^q/(x*(f + g*x^(2*n))), x)
```

3.386 $\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^n)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx=\operatorname{Int}\left(\frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)},x\right)$$

```
[Out] Unintegrable(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx=\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx$$

```
[In] Int[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^n)),x]
[Out] Defer[Int][Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^n)), x]
```

Rubi steps

$$\text{integral}=\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx=\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx$$

```
[In] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^n)),x]
[Out] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g*x^n)), x]
```

Maple [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c(d+e\,x^n)^p\right)^q}{x\left(f+g\,x^n\right)}dx$$

```
[In] int(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x)
[Out] int(ln(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x)
```

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx=\int \frac{\log\left((ex^n+d)^pc\right)^q}{(gx^n+f)x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x, algorithm="fricas")
[Out] integral(log((e*x^n + d)^p*c)^q/(g*x*x^n + f*x), x)
```

Sympy [N/A]

Not integrable

Time = 9.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx=\int \frac{\log\left(c(d+ex^n)^p\right)^q}{x\left(f+gx^n\right)}\,dx$$

```
[In] integrate(ln(c*(d+e*x**n)**p)**q/x/(f+g*x**n),x)
[Out] Integral(log(c*(d + e*x**n)**p)**q/(x*(f + g*x**n)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [N/A]

Not integrable

Time = 2.47 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx = \int \frac{\log\left((ex^n+d)^pc\right)^q}{(gx^n+f)x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g*x^n),x, algorithm="giac")
[Out] integrate(log((e*x^n + d)^p*c)^q/((g*x^n + f)*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^n\right)}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)^q}{x\,\left(f+g\,x^n\right)}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)^q/(x*(f + g*x^n)),x)
[Out] int(log(c*(d + e*x^n)^p)^q/(x*(f + g*x^n)), x)
```

3.387 $\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{-n})} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)} \, dx = \text{Int}\left(\frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)},x\right)$$

```
[Out] Unintegrable(ln(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)} \, dx = \int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)} \, dx$$

```
[In] Int[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^n)),x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^n)), x]
```

Rubi steps

$$\text{integral} = \int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx=\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx$$

```
[In] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^n)),x]
[Out] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^n)), x]
```

Maple [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(c(d+e\,x^n)^p\right)^q}{x\left(f+g\,x^{-n}\right)}dx$$

```
[In] int(ln(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x)
[Out] int(ln(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x)
```

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx=\int \frac{\log\left((ex^n+d)^pc\right)^q}{\left(f+\frac{g}{x^n}\right)x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x, algorithm="fricas")
[Out] integral(x^n*log((e*x^n + d)^p*c)^q/(f*x*x^n + g*x), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-n}\right)}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(ln(c*(d+e*x**n)**p)**q/x/(f+g/(x**n)),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{-n})}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [N/A]

Not integrable

Time = 2.52 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{-n})}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)^q}{\left(f+\frac{g}{x^n}\right)x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^n)),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)^q/((f + g/x^n)*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{-n})}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)^q}{x\,\left(f+\frac{g}{x^n}\right)}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)^q/(x*(f + g/x^n)),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)^q/(x*(f + g/x^n)), x)
```

3.388 $\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{-2n})} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 29

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)} \, dx = \operatorname{Int}\left(\frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}, x\right)$$

```
[Out] Unintegrable(ln(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)} \, dx = \int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)} \, dx$$

```
[In] Int[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^(2*n))),x]
```

```
[Out] Defer[Int][Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^(2*n))), x]
```

Rubi steps

$$\text{integral} = \int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx=\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx$$

```
[In] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^(2*n))),x]
[Out] Integrate[Log[c*(d + e*x^n)^p]^q/(x*(f + g/x^(2*n))), x]
```

Maple [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\ln\left(c(d+e\,x^n)^p\right)^q}{x\left(f+g\,x^{-2n}\right)}dx$$

```
[In] int(ln(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x)
[Out] int(ln(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x)
```

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx=\int \frac{\log\left((ex^n+d)^pc\right)^q}{\left(f+\frac{g}{x^{2n}}\right)x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x, algorithm="fricas")
[Out] integral(x^(2*n)*log((e*x^n + d)^p*c)^q/(f*x*x^(2*n) + g*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^q\left(c(d+ex^n)^p\right)}{x\left(f+gx^{-2n}\right)}\,dx=\text{Timed out}$$

```
[In] integrate(ln(c*(d+e*x**n)**p)**q/x/(f+g/(x**(2*n))),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{-2n})}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [N/A]

Not integrable

Time = 3.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{-2n})}\,dx = \int \frac{\log\left((ex^n+d)^p c\right)^q}{\left(f+\frac{g}{x^{2n}}\right)x}\,dx$$

```
[In] integrate(log(c*(d+e*x^n)^p)^q/x/(f+g/(x^(2*n))),x, algorithm="giac")
```

```
[Out] integrate(log((e*x^n + d)^p*c)^q/((f + g/x^(2*n))*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{\log^q(c(d+ex^n)^p)}{x(f+gx^{-2n})}\,dx = \int \frac{\ln\left(c\,(d+e\,x^n)^p\right)^q}{x\,\left(f+\frac{g}{x^{2\,n}}\right)}\,dx$$

```
[In] int(log(c*(d + e*x^n)^p)^q/(x*(f + g/x^(2*n))),x)
```

```
[Out] int(log(c*(d + e*x^n)^p)^q/(x*(f + g/x^(2*n))), x)
```

3.389 $\int \frac{\log(x)\log(d+ex^m)}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 69

$$\int \frac{\log(x)\log(d+ex^m)}{x}\,dx = \frac{1}{2}\log^2(x)\log(d+ex^m) - \frac{1}{2}\log^2(x)\log\left(1+\frac{ex^m}{d}\right) - \frac{\log(x)\,\mathrm{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m} + \frac{\mathrm{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}$$

```
[Out] 1/2*ln(x)^2*ln(d+e*x^m)-1/2*ln(x)^2*ln(1+e*x^m/d)-ln(x)*polylog(2,-e*x^m/d)
/m+polylog(3,-e*x^m/d)/m^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2422, 2375, 2421, 6724}

$$\int \frac{\log(x)\log(d+ex^m)}{x}\,dx = \frac{\mathrm{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2} - \frac{\log(x)\,\mathrm{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m} + \frac{1}{2}\log^2(x)\log(d+ex^m) - \frac{1}{2}\log^2(x)\log\left(\frac{ex^m}{d}+1\right)$$

```
[In] Int[(Log[x]*Log[d + e*x^m])/x,x]
```

```
[Out] (Log[x]^2*Log[d + e*x^m])/2 - (Log[x]^2*Log[1 + (e*x^m)/d])/2 - (Log[x]*Pol
yLog[2, -((e*x^m)/d)])/m + PolyLog[3, -((e*x^m)/d)]/m^2
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
```

```
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2422

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_
.)]*(b_.))^(p_.))/(x_), x_Symbol] :> Simp[Log[d*(e + f*x^m)^r]*((a + b*Log[
c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[f*m*(r/(b*n*(p + 1))), Int[x^(m -
 1)*((a + b*Log[c*x^n])^(p + 1)/(e + f*x^m)), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\log^2(x)\log\left(d+ex^m\right)-\frac{1}{2}(em)\int\frac{x^{-1+m}\log^2(x)}{d+ex^m}\,dx\\
&= \frac{1}{2}\log^2(x)\log\left(d+ex^m\right)-\frac{1}{2}\log^2(x)\log\left(1+\frac{ex^m}{d}\right)+\int\frac{\log(x)\log\left(1+\frac{ex^m}{d}\right)}{x}\,dx\\
&= \frac{1}{2}\log^2(x)\log\left(d+ex^m\right)-\frac{1}{2}\log^2(x)\log\left(1+\frac{ex^m}{d}\right)-\frac{\log(x)\mathrm{Li}_2\left(-\frac{ex^m}{d}\right)}{m}+\frac{\int\frac{\mathrm{Li}_2\left(-\frac{ex^m}{d}\right)}{x}\,dx}{m}\\
&= \frac{1}{2}\log^2(x)\log\left(d+ex^m\right)-\frac{1}{2}\log^2(x)\log\left(1+\frac{ex^m}{d}\right)-\frac{\log(x)\mathrm{Li}_2\left(-\frac{ex^m}{d}\right)}{m}+\frac{\mathrm{Li}_3\left(-\frac{ex^m}{d}\right)}{m^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{\log(x)\log(d+ex^m)}{x}\,dx = -\frac{1}{6}\log^2(x)\left(m\log(x)+3\log\left(1+\frac{dx^{-m}}{e}\right)-3\log(d+ex^m)\right) + \frac{\log(x)\,\text{PolyLog}\left(2,-\frac{dx^{-m}}{e}\right)}{m} + \frac{\text{PolyLog}\left(3,-\frac{dx^{-m}}{e}\right)}{m^2}$$

```
[In] Integrate[(Log[x]*Log[d + e*x^m])/x,x]
[Out] -1/6*(Log[x]^2*(m*Log[x] + 3*Log[1 + d/(e*x^m)] - 3*Log[d + e*x^m])) + (Log
[x]*PolyLog[2, -(d/(e*x^m))])/m + PolyLog[3, -(d/(e*x^m))]/m^2
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

method	result	size
risch	$\frac{\ln(x)^2\ln(d+e\,x^m)}{2}-\frac{\ln(x)^2\ln\left(1+\frac{e\,x^m}{d}\right)}{2}-\frac{\ln(x)\,\mathrm{Li}_2\left(-\frac{e\,x^m}{d}\right)}{m}+\frac{\mathrm{Li}_3\left(-\frac{e\,x^m}{d}\right)}{m^2}$	66

```
[In] int(ln(x)*ln(d+e*x^m)/x,x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x)^2*ln(d+e*x^m)-1/2*ln(x)^2*ln(1+e*x^m/d)-ln(x)*polylog(2,-e*x^m/d)
/m+polylog(3,-e*x^m/d)/m^2
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \frac{\log(x)\log(d+ex^m)}{x}\,dx = \frac{m^2\log(ex^m+d)\log(x)^2 - m^2\log(x)^2\log\left(\frac{ex^m+d}{d}\right) - 2\,m\mathrm{Li}_2\left(-\frac{ex^m+d}{d}+1\right)\log(x) + 2\,\mathrm{polylog}\left(3,-\frac{ex^m}{d}\right)}{2\,m^2}$$

```
[In] integrate(log(x)*log(d+e*x^m)/x,x, algorithm="fricas")
[Out] 1/2*(m^2*log(e*x^m + d)*log(x)^2 - m^2*log(x)^2*log((e*x^m + d)/d) - 2*m*di
log(-(e*x^m + d)/d + 1)*log(x) + 2*polylog(3, -e*x^m/d))/m^2
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log(x)\log(d+ex^m)}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(ln(x)*ln(d+e*x**m)/x,x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{\log(x)\log(d+ex^m)}{x}\,dx = \int \frac{\log(ex^m+d)\log(x)}{x}\,dx$$

```
[In] integrate(log(x)*log(d+e*x^m)/x,x, algorithm="maxima")
```

```
[Out] -1/6*m*log(x)^3 + d*m*integrate(1/2*log(x)^2/(e*x*x^m + d*x), x) + 1/2*log(
e*x^m + d)*log(x)^2
```

Giac [F]

$$\int \frac{\log(x)\log(d+ex^m)}{x}\,dx = \int \frac{\log(ex^m+d)\log(x)}{x}\,dx$$

```
[In] integrate(log(x)*log(d+e*x^m)/x,x, algorithm="giac")
```

```
[Out] integrate(log(e*x^m + d)*log(x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(x)\log(d+ex^m)}{x}\,dx = \int \frac{\ln(d+e\,x^m)\,\ln(x)}{x}\,dx$$

```
[In] int((log(d + e*x^m)*log(x))/x,x)
```

```
[Out] int((log(d + e*x^m)*log(x))/x, x)
```

3.390 $\int \frac{\log\left(\frac{a+x}{x}\right)}{x} dx$

Optimal result

Integrand size = 12, antiderivative size = 8

$$\int \frac{\log\left(\frac{a+x}{x}\right)}{x} dx = \text{PolyLog}\left(2,-\frac{a}{x}\right)$$

```
[Out] polylog(2,-a/x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2497}

$$\int \frac{\log\left(\frac{a+x}{x}\right)}{x} dx = \text{PolyLog}\left(2,1-\frac{a+x}{x}\right)$$

```
[In] Int[Log[(a + x)/x]/x,x]
```

```
[Out] PolyLog[2, 1 - (a + x)/x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\text{integral} = \text{Li}_2\left(1-\frac{a+x}{x}\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 34 vs. $2(8) = 16$.

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 4.25

$$\int \frac{\log\left(\frac{a+x}{x}\right)}{x}\,dx = -\log\left(-\frac{a}{x}\right)\log\left(\frac{a+x}{x}\right) - \text{PolyLog}\left(2,-\frac{-a-x}{x}\right)$$

```
[In] Integrate[Log[(a + x)/x]/x,x]
[Out] -(Log[-(a/x)]*Log[(a + x)/x]) - PolyLog[2, -((-a - x)/x)]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$\operatorname{dilog}\left(1+\frac{a}{x}\right)$	9
default	$\operatorname{dilog}\left(1+\frac{a}{x}\right)$	9
risch	$\operatorname{dilog}\left(1+\frac{a}{x}\right)$	9
parts	$\ln\left(\frac{a+x}{x}\right)\ln\left(x\right)+\frac{\ln\left(x\right)^{2}}{2}-\operatorname{dilog}\left(\frac{a+x}{a}\right)-\ln\left(x\right)\ln\left(\frac{a+x}{a}\right)$	41

```
[In] int(ln((a+x)/x)/x,x,method=_RETURNVERBOSE)
[Out] dilog(1+a/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.38

$$\int \frac{\log\left(\frac{a+x}{x}\right)}{x}\,dx = \text{Li}_2\left(-\frac{a+x}{x}+1\right)$$

```
[In] integrate(log((a+x)/x)/x,x, algorithm="fricas")
[Out] dilog(-(a + x)/x + 1)
```

Sympy [F]

$$\int \frac{\log\left(\frac{a+x}{x}\right)}{x}\,dx = \int \frac{\log\left(\frac{a}{x}+1\right)}{x}\,dx$$

```
[In] integrate(ln((a+x)/x)/x,x)
```

```
[Out] Integral(log(a/x + 1)/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(7) = 14$.

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 7.38

$$\int \frac{\log\left(\frac{a+x}{x}\right)}{x}\,dx = -(\log(a+x)-\log(x))\log(x)+\log(a+x)\log(x)-\frac{1}{2}\log(x)^2 + \log(x)\log\left(\frac{a+x}{x}\right)-\log(x)\log\left(\frac{x}{a}+1\right)-\mathrm{Li}_2\left(-\frac{x}{a}\right)$$

```
[In] integrate(log((a+x)/x)/x,x, algorithm="maxima")
```

```
[Out] -(log(a + x) - log(x))*log(x) + log(a + x)*log(x) - 1/2*log(x)^2 + log(x)*l
og((a + x)/x) - log(x)*log(x/a + 1) - dilog(-x/a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(7) = 14$.

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 8.50

$$\int \frac{\log\left(\frac{a+x}{x}\right)}{x}\,dx = -\frac{a^3\left(\frac{1}{\frac{a+x}{x}-1}-\log\left(\frac{|a+x|}{|x|}\right)+\log\left(\left|\frac{a+x}{x}-1\right|\right)\right)+\frac{a^3\log\left(\frac{a+x}{x}\right)}{\left(\frac{a+x}{x}-1\right)^2}}{2\,a^2}$$

```
[In] integrate(log((a+x)/x)/x,x, algorithm="giac")
```

```
[Out] -1/2*(a^3*(1/((a + x)/x - 1) - log(abs(a + x)/abs(x)) + log(abs((a + x)/x -
 1))) + a^3*log((a + x)/x)/((a + x)/x - 1)^2)/a^2
```

Mupad [B] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(\frac{a+x}{x}\right)}{x}\,dx = \text{polylog}\left(2, -\frac{a}{x}\right)$$

```
[In] int(log((a + x)/x)/x,x)
[Out] polylog(2, -a/x)
```

3.391 $\int \frac{\log\left(\frac{a+x^2}{x^2}\right)}{x} dx$

Optimal result

Integrand size = 14, antiderivative size = 12

$$\int \frac{\log\left(\frac{a+x^2}{x^2}\right)}{x} dx = \frac{1}{2}\text{PolyLog}\left(2,-\frac{a}{x^2}\right)$$

```
[Out] 1/2*polylog(2,-a/x^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2511, 2438}

$$\int \frac{\log\left(\frac{a+x^2}{x^2}\right)}{x} dx = \frac{1}{2}\text{PolyLog}\left(2,-\frac{a}{x^2}\right)$$

```
[In] Int[Log[(a + x^2)/x^2]/x,x]
```

```
[Out] PolyLog[2, -(a/x^2)]/2
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2511

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
```

```
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{\log\left(1+\frac{a}{x^2}\right)}{x}\,dx \\ &= \frac{1}{2}\text{Li}_2\left(-\frac{a}{x^2}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(\frac{a+x^2}{x^2}\right)}{x}\,dx = \frac{1}{2}\,\text{PolyLog}\left(2,-\frac{a}{x^2}\right)$$

```
[In] Integrate[Log[(a + x^2)/x^2]/x,x]
[Out] PolyLog[2, -(a/x^2)]/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(10) = 20$.

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 6.33

method	result
risch	$-\ln\left(\frac{1}{x}\right)\ln\left(1+\frac{a}{x^2}\right)+\ln\left(\frac{1}{x}\right)\ln\left(1+\frac{\sqrt{-a}}{x}\right)+\ln\left(\frac{1}{x}\right)\ln\left(1-\frac{\sqrt{-a}}{x}\right)+\text{dilog}\left(1+\frac{\sqrt{-a}}{x}\right)+$
derivativedivides	$-\ln\left(\frac{1}{x}\right)\ln\left(1+\frac{a}{x^2}\right)+2a\left(\frac{\ln\left(\frac{1}{x}\right)\left(\ln\left(1+\frac{\sqrt{-a}}{x}\right)+\ln\left(1-\frac{\sqrt{-a}}{x}\right)\right)}{2a}+\frac{\text{dilog}\left(1+\frac{\sqrt{-a}}{x}\right)+\text{dilog}\left(1-\frac{\sqrt{-a}}{x}\right)}{2a}\right)$
default	$-\ln\left(\frac{1}{x}\right)\ln\left(1+\frac{a}{x^2}\right)+2a\left(\frac{\ln\left(\frac{1}{x}\right)\left(\ln\left(1+\frac{\sqrt{-a}}{x}\right)+\ln\left(1-\frac{\sqrt{-a}}{x}\right)\right)}{2a}+\frac{\text{dilog}\left(1+\frac{\sqrt{-a}}{x}\right)+\text{dilog}\left(1-\frac{\sqrt{-a}}{x}\right)}{2a}\right)$
parts	$\ln\left(\frac{x^2+a}{x^2}\right)\ln\left(x\right)+\ln\left(x\right)^2-\ln\left(x\right)\ln\left(\frac{\sqrt{-a}-x}{\sqrt{-a}}\right)-\ln\left(x\right)\ln\left(\frac{\sqrt{-a}+x}{\sqrt{-a}}\right)-\text{dilog}\left(\frac{\sqrt{-a}-x}{\sqrt{-a}}\right)-$

```
[In] int(ln((x^2+a)/x^2)/x,x,method=_RETURNVERBOSE)
[Out] -ln(1/x)*ln(1+1/x^2*a)+ln(1/x)*ln(1+1/x*(-a)^(1/2))+ln(1/x)*ln(1-1/x*(-a)^(
1/2))+dilog(1+1/x*(-a)^(1/2))+dilog(1-1/x*(-a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\log\left(\frac{a+x^2}{x^2}\right)}{x}\,dx = \frac{1}{2}\,\mathrm{Li}_2\left(-\frac{x^2+a}{x^2}+1\right)$$

```
[In] integrate(log((x^2+a)/x^2)/x,x, algorithm="fricas")
[Out] 1/2*dilog(-(x^2 + a)/x^2 + 1)
```

Sympy [F]

$$\int \frac{\log\left(\frac{a+x^2}{x^2}\right)}{x}\,dx = \int \frac{\log\left(\frac{a}{x^2}+1\right)}{x}\,dx$$

```
[In] integrate(ln((x**2+a)/x**2)/x,x)
[Out] Integral(log(a/x**2 + 1)/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(9) = 18$.

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 5.75

$$\int \frac{\log\left(\frac{a+x^2}{x^2}\right)}{x}\,dx = -\left(\log\left(x^2+a\right)-2\,\log\left(x\right)\right)\log\left(x\right)+\log\left(x^2+a\right)\log\left(x\right)-\log\left(x\right)^2 -\log\left(x\right)\log\left(\frac{x^2}{a}+1\right)+\log\left(x\right)\log\left(\frac{x^2+a}{x^2}\right)-\frac{1}{2}\,\mathrm{Li}_2\left(-\frac{x^2}{a}\right)$$

```
[In] integrate(log((x^2+a)/x^2)/x,x, algorithm="maxima")
[Out] -(log(x^2 + a) - 2*log(x))*log(x) + log(x^2 + a)*log(x) - log(x)^2 - log(x)
*log(x^2/a + 1) + log(x)*log((x^2 + a)/x^2) - 1/2*dilog(-x^2/a)
```

Giac [F]

$$\int \frac{\log\left(\frac{a+x^2}{x^2}\right)}{x}\,dx = \int \frac{\log\left(\frac{x^2+a}{x^2}\right)}{x}\,dx$$

```
[In] integrate(log((x^2+a)/x^2)/x,x, algorithm="giac")
[Out] integrate(log((x^2 + a)/x^2)/x, x)
```

Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\log\left(\frac{a+x^2}{x^2}\right)}{x}\,dx = \frac{\mathrm{polylog}\left(2,-\frac{a}{x^2}\right)}{2}$$

```
[In] int(log((a + x^2)/x^2)/x,x)
[Out] polylog(2, -a/x^2)/2
```

3.392 $\int \frac{\log(x^{-n}(a+x^n))}{x} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 14

$$\int \frac{\log(x^{-n}(a+x^n))}{x} \, dx = \frac{\text{PolyLog}(2, -ax^{-n})}{n}$$

```
[Out] polylog(2,-a/(x^n))/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2511, 2438}

$$\int \frac{\log(x^{-n}(a+x^n))}{x} \, dx = \frac{\text{PolyLog}(2, -ax^{-n})}{n}$$

```
[In] Int[Log[(a + x^n)/x^n]/x,x]
```

```
[Out] PolyLog[2, -(a/x^n)]/n
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2511

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\log(1+ax^{-n})}{x}\,dx\\ &= \frac{\text{Li}_2(-ax^{-n})}{n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\log(x^{-n}(a+x^n))}{x}\,dx = \frac{\text{PolyLog}(2,-ax^{-n})}{n}$$

```
[In] Integrate[Log[(a + x^n)/x^n]/x,x]
[Out] PolyLog[2, -(a/x^n)]/n
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{\operatorname{dilog}(1+a\,x^{-n})}{n}$
default	$\frac{\operatorname{dilog}(1+a\,x^{-n})}{n}$
risch	$-\ln(x)\ln(x^n)+\frac{n\ln(x)^2}{2}+\frac{i\pi\ln(x)\operatorname{csgn}(ix^{-n})\operatorname{csgn}(ix^{-n}(a+x^n))^2}{2}+\frac{i\pi\ln(x)\operatorname{csgn}(i(a+x^n))\operatorname{csgn}(ix^{-n}(a$

```
[In] int(ln((a+x^n)/(x^n))/x,x,method=_RETURNVERBOSE)
[Out] 1/n*dilog(1+a/(x^n))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(13) = 26.

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 4.36

$$\int \frac{\log(x^{-n}(a+x^n))}{x}\,dx = \frac{n^2\log(x)^2-2\,n\log(x)\log\left(\frac{a+x^n}{a}\right)+2\,n\log(x)\log\left(\frac{a+x^n}{x^n}\right)-2\,\text{Li}_2\left(-\frac{a+x^n}{a}+1\right)}{2\,n}$$

```
[In] integrate(log((a+x^n)/(x^n))/x,x, algorithm="fricas")
```

```
[Out] 1/2*(n^2*log(x)^2 - 2*n*log(x)*log((a + x^n)/a) + 2*n*log(x)*log((a + x^n)/
x^n) - 2*dilog(-(a + x^n)/a + 1))/n
```

Sympy [F]

$$\int \frac{\log\left(x^{-n}(a+x^n)\right)}{x}\,dx = \int \frac{\log\left(ax^{-n}+1\right)}{x}\,dx$$

```
[In] integrate(ln((a+x**n)/(x**n))/x,x)
[Out] Integral(log(a/x**n + 1)/x, x)
```

Maxima [F]

$$\int \frac{\log\left(x^{-n}(a+x^n)\right)}{x}\,dx = \int \frac{\log\left(\frac{a+x^n}{x^n}\right)}{x}\,dx$$

```
[In] integrate(log((a+x^n)/(x^n))/x,x, algorithm="maxima")
[Out] a*n*integrate(log(x)/(a*x + x*x^n), x) + log(a + x^n)*log(x) - log(x)*log(x
^n)
```

Giac [F]

$$\int \frac{\log\left(x^{-n}(a+x^n)\right)}{x}\,dx = \int \frac{\log\left(\frac{a+x^n}{x^n}\right)}{x}\,dx$$

```
[In] integrate(log((a+x^n)/(x^n))/x,x, algorithm="giac")
[Out] integrate(log((a + x^n)/x^n)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(x^{-n}(a+x^n)\right)}{x}\,dx = \int \frac{\ln\left(\frac{a+x^n}{x^n}\right)}{x}\,dx$$

```
[In] int(log((a + x^n)/x^n)/x,x)
[Out] int(log((a + x^n)/x^n)/x, x)
```

3.393 $\int \frac{\log\left(\frac{a+bx}{x}\right)}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 35

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{x}\,dx = -\log\left(b+\frac{a}{x}\right)\log\left(-\frac{a}{bx}\right) - \operatorname{PolyLog}\left(2,1+\frac{a}{bx}\right)$$

```
[Out] -ln(b+a/x)*ln(-a/b/x)-polylog(2,1+a/b/x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2511, 2504, 2441, 2352}

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{x}\,dx = -\operatorname{PolyLog}\left(2,\frac{a}{bx}+1\right) - \log\left(\frac{a}{x}+b\right)\log\left(-\frac{a}{bx}\right)$$

```
[In] Int[Log[(a + b*x)/x]/x,x]
[Out] -(Log[b + a/x]*Log[-(a/(b*x))]) - PolyLog[2, 1 + a/(b*x)]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2511

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\log\left(b+\frac{a}{x}\right)}{x}\,dx\\
&= -\text{Subst}\left(\int \frac{\log(b+ax)}{x}\,dx, x, \frac{1}{x}\right)\\
&= -\log\left(b+\frac{a}{x}\right)\log\left(-\frac{a}{bx}\right)+a\text{Subst}\left(\int \frac{\log\left(-\frac{ax}{b}\right)}{b+ax}\,dx, x, \frac{1}{x}\right)\\
&= -\log\left(b+\frac{a}{x}\right)\log\left(-\frac{a}{bx}\right)-\text{Li}_2\left(1+\frac{a}{bx}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{x}\,dx = -\log\left(b+\frac{a}{x}\right)\log\left(-\frac{a}{bx}\right)-\text{PolyLog}\left(2,\frac{b+\frac{a}{x}}{b}\right)$$

```
[In] Integrate[Log[(a + b*x)/x]/x,x]
[Out] -(Log[b + a/x]*Log[-(a/(b*x))]) - PolyLog[2, (b + a/x)/b]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$-\operatorname{dilog}\left(-\frac{a}{xb}\right)-\ln\left(b+\frac{a}{x}\right)\ln\left(-\frac{a}{xb}\right)$	34
default	$-\operatorname{dilog}\left(-\frac{a}{xb}\right)-\ln\left(b+\frac{a}{x}\right)\ln\left(-\frac{a}{xb}\right)$	34
risch	$-\operatorname{dilog}\left(-\frac{a}{xb}\right)-\ln\left(b+\frac{a}{x}\right)\ln\left(-\frac{a}{xb}\right)$	34
parts	$\ln\left(\frac{bx+a}{x}\right)\ln\left(x\right)+\frac{\ln(x)^2}{2}-b\left(\frac{\operatorname{dilog}\left(\frac{bx+a}{a}\right)}{b}+\frac{\ln(x)\ln\left(\frac{bx+a}{a}\right)}{b}\right)$	55

```
[In] int(ln((b*x+a)/x)/x,x,method=_RETURNVERBOSE)
[Out] -dilog(-1/x*a/b)-ln(b+a/x)*ln(-1/x*a/b)
```

Fricas [F]

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{x}\,dx=\int \frac{\log\left(\frac{bx+a}{x}\right)}{x}\,dx$$

```
[In] integrate(log((b*x+a)/x)/x,x, algorithm="fricas")
[Out] integral(log((b*x + a)/x)/x, x)
```

Sympy [F]

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{x}\,dx=\int \frac{\log\left(\frac{a}{x}+b\right)}{x}\,dx$$

```
[In] integrate(ln((b*x+a)/x)/x,x)
[Out] Integral(log(a/x + b)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.91

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{x}\,dx = -(\log(bx+a)-\log(x))\log(x)+\log(bx+a)\log(x) - \log\left(\frac{bx}{a}+1\right)\log(x)-\frac{1}{2}\log(x)^2+\log(x)\log\left(\frac{bx+a}{x}\right)-\mathrm{Li}_2\left(-\frac{bx}{a}\right)$$

```
[In] integrate(log((b*x+a)/x)/x,x, algorithm="maxima")
```

```
[Out] -(log(b*x + a) - log(x))*log(x) + log(b*x + a)*log(x) - log(b*x/a + 1)*log(
x) - 1/2*log(x)^2 + log(x)*log((b*x + a)/x) - dilog(-b*x/a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(34) = 68$.

Time = 0.40 (sec) , antiderivative size = 204, normalized size of antiderivative = 5.83

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{x}\,dx = \frac{a^3\left(\frac{\log\left(\frac{|bx+a|}{|x|}\right)}{b^2}-\frac{\log\left(\left|-b+\frac{bx+a}{x}\right|\right)}{b^2}+\frac{1}{\left(b-\frac{bx+a}{x}\right)b}\right)-\frac{a^3\log\left(-\left(a-\frac{b}{\frac{\left(a-\frac{b}{\frac{b}{a}-\frac{bx+a}{ax}}\right)\left(\frac{b}{a}-\frac{bx+a}{ax}\right)}{a}+\frac{b}{a}}\right)\left(\frac{\left(a-\frac{b}{\frac{b}{a}-\frac{bx+a}{ax}}\right)\left(\frac{b}{a}-\frac{bx+a}{ax}\right)}{a}+\frac{b}{a}\right)\right)}{\left(b-\frac{bx+a}{x}\right)^2}}{2a^2}$$

```
[In] integrate(log((b*x+a)/x)/x,x, algorithm="giac")
```

```
[Out] 1/2*(a^3*(log(abs(b*x + a)/abs(x))/b^2 - log(abs(-b + (b*x + a)/x))/b^2 + 1
/((b - (b*x + a)/x)*b)) - a^3*log(-(a - b/((a - b/(b/a - (b*x + a)/(a*x)))*
(b/a - (b*x + a)/(a*x))/a + b/a))*((a - b/(b/a - (b*x + a)/(a*x)))*(b/a - (
b*x + a)/(a*x))/a + b/a))/(b - (b*x + a)/x)^2)/a^2
```

Mupad [B] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{x}\,dx = -\mathrm{polylog}\left(2, \frac{a}{b\,x}+1\right) - \ln\left(\frac{a+b\,x}{x}\right)\,\ln\left(-\frac{a}{b\,x}\right)$$

```
[In] int(log((a + b*x)/x)/x,x)
[Out] - polylog(2, a/(b*x) + 1) - log((a + b*x)/x)*log(-a/(b*x))
```

3.394 $\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x} dx$

Optimal result

Integrand size = 16, antiderivative size = 39

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x} dx = -\frac{1}{2}\log\left(b+\frac{a}{x^2}\right)\log\left(-\frac{a}{bx^2}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,1+\frac{a}{bx^2}\right)$$

```
[Out] -1/2*ln(b+a/x^2)*ln(-a/b/x^2)-1/2*polylog(2,1+a/b/x^2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2511, 2504, 2441, 2352}

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x} dx = -\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{a}{bx^2}+1\right)-\frac{1}{2}\log\left(\frac{a}{x^2}+b\right)\log\left(-\frac{a}{bx^2}\right)$$

```
[In] Int[Log[(a + b*x^2)/x^2]/x,x]
```

```
[Out] -1/2*(Log[b + a/x^2]*Log[-(a/(b*x^2))]) - PolyLog[2, 1 + a/(b*x^2)]/2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2511

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\log\left(b+\frac{a}{x^2}\right)}{x}\,dx\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\log(b+ax)}{x}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\frac{1}{2}\log\left(b+\frac{a}{x^2}\right)\log\left(-\frac{a}{bx^2}\right)+\frac{1}{2}a\text{Subst}\left(\int \frac{\log\left(-\frac{ax}{b}\right)}{b+ax}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{1}{2}\log\left(b+\frac{a}{x^2}\right)\log\left(-\frac{a}{bx^2}\right)-\frac{1}{2}\text{Li}_2\left(1+\frac{a}{bx^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$
\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x}\,dx = -\frac{1}{2}\log\left(b+\frac{a}{x^2}\right)\log\left(-\frac{a}{bx^2}\right)-\frac{1}{2}\,\text{PolyLog}\left(2, \frac{b+\frac{a}{x^2}}{b}\right)
$$

```
[In] Integrate[Log[(a + b*x^2)/x^2]/x,x]
[Out] -1/2*(Log[b + a/x^2]*Log[-(a/(b*x^2))]) - PolyLog[2, (b + a/x^2)/b]/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(35) = 70$.

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.77

method	result
risch	$-\ln\left(\frac{1}{x}\right)\ln\left(b+\frac{a}{x^2}\right)+\ln\left(\frac{1}{x}\right)\ln\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)+\ln\left(\frac{1}{x}\right)\ln\left(\frac{\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)+\mathrm{dilog}\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)+$
parts	$\ln\left(\frac{b\,x^2+a}{x^2}\right)\ln(x)+\ln(x)^2-2b\left(\frac{\ln(x)\left(\ln\left(\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}\right)+\ln\left(\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}\right)\right)}{2b}+\frac{\mathrm{dilog}\left(\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}\right)+\mathrm{dilog}\left(\frac{bx+}{}\right.}{2b}\right.$
derivativedivides	$-\ln\left(\frac{1}{x}\right)\ln\left(b+\frac{a}{x^2}\right)+2a\left(\frac{\ln\left(\frac{1}{x}\right)\left(\ln\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)+\ln\left(\frac{\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)\right)}{2a}+\frac{\mathrm{dilog}\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)+\mathrm{dilog}\left(\frac{\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right.}{2a}\right.$
default	$-\ln\left(\frac{1}{x}\right)\ln\left(b+\frac{a}{x^2}\right)+2a\left(\frac{\ln\left(\frac{1}{x}\right)\left(\ln\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)+\ln\left(\frac{\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)\right)}{2a}+\frac{\mathrm{dilog}\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)+\mathrm{dilog}\left(\frac{\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right.}{2a}\right.$

```
[In] int(ln((b*x^2+a)/x^2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -ln(1/x)*ln(b+1/x^2*a)+ln(1/x)*ln((-a/x+(-a*b)^(1/2))/(-a*b)^(1/2))+ln(1/x)
*ln((a/x+(-a*b)^(1/2))/(-a*b)^(1/2))+dilog((-a/x+(-a*b)^(1/2))/(-a*b)^(1/2)
)+dilog((a/x+(-a*b)^(1/2))/(-a*b)^(1/2))
```

Fricas [F]

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x}\,dx = \int \frac{\log\left(\frac{bx^2+a}{x^2}\right)}{x}\,dx$$

```
[In] integrate(log((b*x^2+a)/x^2)/x,x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)/x^2)/x, x)
```

Sympy [F]

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x}\,dx = \int \frac{\log\left(\frac{a}{x^2}+b\right)}{x}\,dx$$

```
[In] integrate(ln((b*x**2+a)/x**2)/x,x)
```

```
[Out] Integral(log(a/x**2 + b)/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(34) = 68$.

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.97

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x}\,dx = -\left(\log\left(bx^2+a\right) - 2\,\log\left(x\right)\right)\log\left(x\right) + \log\left(bx^2+a\right)\log\left(x\right) - \log\left(\frac{bx^2}{a}+1\right)\log\left(x\right) - \log\left(x\right)^2 + \log\left(x\right)\log\left(\frac{bx^2+a}{x^2}\right) - \frac{1}{2}\,\mathrm{Li}_2\left(-\frac{bx^2}{a}\right)$$

```
[In] integrate(log((b*x^2+a)/x^2)/x,x, algorithm="maxima")
```

```
[Out] -(log(b*x^2 + a) - 2*log(x))*log(x) + log(b*x^2 + a)*log(x) - log(b*x^2/a +
 1)*log(x) - log(x)^2 + log(x)*log((b*x^2 + a)/x^2) - 1/2*dilog(-b*x^2/a)
```

Giac [F]

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x}\,dx = \int \frac{\log\left(\frac{bx^2+a}{x^2}\right)}{x}\,dx$$

```
[In] integrate(log((b*x^2+a)/x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^2 + a)/x^2)/x, x)
```

Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{x}\,dx = -\frac{\mathrm{Li}_2\left(-\frac{a}{b\,x^2}\right)}{2} - \frac{\ln\left(b+\frac{a}{x^2}\right)\,\ln\left(-\frac{a}{b\,x^2}\right)}{2}$$

```
[In] int(log((a + b*x^2)/x^2)/x,x)
```

```
[Out] - dilog(-a/(b*x^2))/2 - (log(b + a/x^2)*log(-a/(b*x^2)))/2
```

3.395 $\int \frac{\log(x^{-n}(a+bx^n))}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 47

$$\int \frac{\log(x^{-n}(a+bx^n))}{x}\,dx = -\frac{\log\left(-\frac{ax^{-n}}{b}\right)\log(b+ax^{-n})}{n} - \frac{\operatorname{PolyLog}\left(2, 1+\frac{ax^{-n}}{b}\right)}{n}$$

```
[Out] -ln(-a/b/(x^n))*ln(b+a/(x^n))/n-polylog(2,1+a/b/(x^n))/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2511, 2504, 2441, 2352}

$$\int \frac{\log(x^{-n}(a+bx^n))}{x}\,dx = -\frac{\operatorname{PolyLog}\left(2, \frac{ax^{-n}}{b}+1\right)}{n} - \frac{\log\left(-\frac{ax^{-n}}{b}\right)\log(ax^{-n}+b)}{n}$$

```
[In] Int[Log[(a + b*x^n)/x^n]/x,x]
[Out] -((Log[-(a/(b*x^n))]*Log[b + a/x^n])/n) - PolyLog[2, 1 + a/(b*x^n)]/n
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
```

```
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2511

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\log\left(b+ax^{-n}\right)}{x}\,dx\\
&= -\frac{\text{Subst}\left(\int \frac{\log(b+ax)}{x}\,dx, x, x^{-n}\right)}{n}\\
&= -\frac{\log\left(-\frac{ax^{-n}}{b}\right)\log\left(b+ax^{-n}\right)}{n}+\frac{a\text{Subst}\left(\int \frac{\log\left(-\frac{ax}{b}\right)}{b+ax}\,dx, x, x^{-n}\right)}{n}\\
&= -\frac{\log\left(-\frac{ax^{-n}}{b}\right)\log\left(b+ax^{-n}\right)}{n}-\frac{\text{Li}_2\left(1+\frac{ax^{-n}}{b}\right)}{n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{\log\left(x^{-n}(a+bx^n)\right)}{x}\,dx = -\frac{\log\left(-\frac{ax^{-n}}{b}\right)\log\left(b+ax^{-n}\right)+\text{PolyLog}\left(2,\frac{b+ax^{-n}}{b}\right)}{n}$$

```
[In] Integrate[Log[(a + b*x^n)/x^n]/x,x]
[Out] -((Log[-(a/(b*x^n))]*Log[b + a/x^n] + PolyLog[2, (b + a/x^n)/b])/n)
```

Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{-\operatorname{dilog}\left(-\frac{a\,x^{-n}}{b}\right)-\ln\left(b+a\,x^{-n}\right)\ln\left(-\frac{a\,x^{-n}}{b}\right)}{n}$
default	$\frac{-\operatorname{dilog}\left(-\frac{a\,x^{-n}}{b}\right)-\ln\left(b+a\,x^{-n}\right)\ln\left(-\frac{a\,x^{-n}}{b}\right)}{n}$
risch	$-\ln\left(x\right)\ln\left(x^{n}\right)+\frac{n\ln(x)^{2}}{2}+\frac{i\ln(x)\pi\ \operatorname{csgn}\left(ix^{-n}\right)\operatorname{csgn}\left(ix^{-n}(a+b\,x^{n})\right)^{2}}{2}+\frac{i\ln(x)\pi\ \operatorname{csgn}\left(i(a+b\,x^{n})\right)\operatorname{csgn}\left(ix^{-n}\right.}{2}$

```
[In] int(ln((a+b*x^n)/(x^n))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-dilog(-a/b/(x^n))-ln(b+a/(x^n))*ln(-a/b/(x^n)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int \frac{\log\left(x^{-n}(a+bx^n)\right)}{x}\,dx = \frac{n^2\log\left(x\right)^2-2\,n\log\left(x\right)\log\left(\frac{bx^n+a}{a}\right)+2\,n\log\left(x\right)\log\left(\frac{bx^n+a}{x^n}\right)-2\,\mathrm{Li}_2\left(-\frac{bx^n+a}{a}+1\right)}{2\,n}$$

```
[In] integrate(log((a+b*x^n)/(x^n))/x,x, algorithm="fricas")
```

```
[Out] 1/2*(n^2*log(x)^2 - 2*n*log(x)*log((b*x^n + a)/a) + 2*n*log(x)*log((b*x^n +
 a)/x^n) - 2*dilog(-(b*x^n + a)/a + 1))/n
```

Sympy [F]

$$\int \frac{\log\left(x^{-n}(a+bx^n)\right)}{x}\,dx = \int \frac{\log\left(ax^{-n}+b\right)}{x}\,dx$$

```
[In] integrate(ln((a+b*x**n)/(x**n))/x,x)
```

```
[Out] Integral(log(a/x**n + b)/x, x)
```

Maxima [F]

$$\int \frac{\log(x^{-n}(a+bx^n))}{x}\,dx = \int \frac{\log\left(\frac{bx^n+a}{x^n}\right)}{x}\,dx$$

```
[In] integrate(log((a+b*x^n)/(x^n))/x,x, algorithm="maxima")
```

```
[Out] a*n*integrate(log(x)/(b*x*x^n + a*x), x) + log(b*x^n + a)*log(x) - log(x)*l
og(x^n)
```

Giac [F]

$$\int \frac{\log(x^{-n}(a+bx^n))}{x}\,dx = \int \frac{\log\left(\frac{bx^n+a}{x^n}\right)}{x}\,dx$$

```
[In] integrate(log((a+b*x^n)/(x^n))/x,x, algorithm="giac")
```

```
[Out] integrate(log((b*x^n + a)/x^n)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(x^{-n}(a+bx^n))}{x}\,dx = \int \frac{\ln\left(\frac{a+b\,x^n}{x^n}\right)}{x}\,dx$$

```
[In] int(log((a + b*x^n)/x^n)/x,x)
```

```
[Out] int(log((a + b*x^n)/x^n)/x, x)
```

3.396 $\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx} dx$

Optimal result

Integrand size = 18, antiderivative size = 105

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx} dx = \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d} + \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{d} - \frac{\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{d} + \frac{\text{PolyLog}\left(2,1+\frac{dx}{c}\right)}{d}$$

```
[Out] ln(b+a/x)*ln(d*x+c)/d+ln(-d*x/c)*ln(d*x+c)/d-ln(-d*(b*x+a)/(-a*d+b*c))*ln(d
*x+c)/d-polylog(2,b*(d*x+c)/(-a*d+b*c))/d+polylog(2,1+d*x/c)/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2515, 2512, 266, 2463, 2441, 2352, 2440, 2438}

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx} dx = -\frac{\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{d} + \frac{\log\left(\frac{a}{x}+b\right)\log(c+dx)}{d} - \frac{\log(c+dx)\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{d} + \frac{\text{PolyLog}\left(2,\frac{dx}{c}+1\right)}{d} + \frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}$$

```
[In] Int[Log[(a + b*x)/x]/(c + d*x),x]
```

```
[Out] (Log[b + a/x]*Log[c + d*x])/d + (Log[-((d*x)/c)]*Log[c + d*x])/d - (Log[-((
d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/d - PolyLog[2, (b*(c + d*x))/(b*c
- a*d)]/d + PolyLog[2, 1 + (d*x)/c]/d
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2515

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*(u_)^(r_.), x_Symbol] :> In
t[ExpandToSum[u, x]^r*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a,
 b, c, p, q, r}, x] && LinearQ[u, x] && BinomialQ[v, x] &&  !(LinearMatchQ[
u, x] && BinomialMatchQ[v, x])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\log\left(b+\frac{a}{x}\right)}{c+dx}\,dx\\
&= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d}+\frac{a\int \frac{\log(c+dx)}{\left(b+\frac{a}{x}\right)x^2}\,dx}{d}\\
&= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d}+\frac{a\int \left(\frac{\log(c+dx)}{ax}-\frac{b\log(c+dx)}{a(a+bx)}\right)dx}{d}\\
&= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d}+\frac{\int \frac{\log(c+dx)}{x}\,dx}{d}-\frac{b\int \frac{\log(c+dx)}{a+bx}\,dx}{d}\\
&= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}\\
&\quad -\frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{d}-\int \frac{\log\left(-\frac{dx}{c}\right)}{c+dx}\,dx+\int \frac{\log\left(\frac{d(a+bx)}{-bc+ad}\right)}{c+dx}\,dx\\
&= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}-\frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{d}\\
&\quad +\frac{\operatorname{Li}_2\left(1+\frac{dx}{c}\right)}{d}+\frac{\operatorname{Subst}\left(\int \frac{\log\left(1+\frac{bx}{-bc+ad}\right)}{x}\,dx,x,c+dx\right)}{d}\\
&= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)}{d}+\frac{\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}\\
&\quad -\frac{\log\left(-\frac{d(a+bx)}{bc-ad}\right)\log(c+dx)}{d}-\frac{\operatorname{Li}_2\left(\frac{b(c+dx)}{bc-ad}\right)}{d}+\frac{\operatorname{Li}_2\left(1+\frac{dx}{c}\right)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx}\,dx$$
$$= \frac{\log\left(b+\frac{a}{x}\right)\log(c+dx)+\log(x)\log(c+dx)-\log\left(\frac{a}{b}+x\right)\log(c+dx)+\log\left(\frac{a}{b}+x\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)-\log(x}{d}$$

```
[In] Integrate[Log[(a + b*x)/x]/(c + d*x),x]
```

```
[Out] (Log[b + a/x]*Log[c + d*x] + Log[x]*Log[c + d*x] - Log[a/b + x]*Log[c + d*x
] + Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] - Log[x]*Log[1 + (d*x)/c] -
 PolyLog[2, -((d*x)/c)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/d
```

Maple [A] (verified)

Time = 3.77 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.09

method	result
risch	$\frac{\operatorname{dilog}\left(\frac{ad-cb+c\left(b+\frac{a}{x}\right)}{ad-cb}\right)}{d}+\frac{\ln\left(b+\frac{a}{x}\right)\ln\left(\frac{ad-cb+c\left(b+\frac{a}{x}\right)}{ad-cb}\right)}{d}-\frac{\ln\left(b+\frac{a}{x}\right)\ln\left(-\frac{a}{xb}\right)}{d}-\frac{\operatorname{dilog}\left(-\frac{a}{xb}\right)}{d}$
derivativedivides	$-a\left(-\frac{c\left(\frac{\operatorname{dilog}\left(\frac{ad-cb+c\left(b+\frac{a}{x}\right)}{ad-cb}\right)}{c}+\frac{\ln\left(b+\frac{a}{x}\right)\ln\left(\frac{ad-cb+c\left(b+\frac{a}{x}\right)}{ad-cb}\right)}{c}\right)}{da}+\frac{\operatorname{dilog}\left(-\frac{a}{xb}\right)+\ln\left(b+\frac{a}{x}\right)\ln\left(-\frac{a}{xb}\right)}{da}\right)$
default	$-a\left(-\frac{c\left(\frac{\operatorname{dilog}\left(\frac{ad-cb+c\left(b+\frac{a}{x}\right)}{ad-cb}\right)}{c}+\frac{\ln\left(b+\frac{a}{x}\right)\ln\left(\frac{ad-cb+c\left(b+\frac{a}{x}\right)}{ad-cb}\right)}{c}\right)}{da}+\frac{\operatorname{dilog}\left(-\frac{a}{xb}\right)+\ln\left(b+\frac{a}{x}\right)\ln\left(-\frac{a}{xb}\right)}{da}\right)$
parts	$\frac{\ln\left(\frac{bx+a}{x}\right)\ln(dx+c)}{d}-\frac{-d^2\left(\operatorname{dilog}\left(-\frac{xd}{c}\right)+\ln(dx+c)\ln\left(-\frac{xd}{c}\right)\right)+d^2b\left(\frac{\operatorname{dilog}\left(\frac{ad-cb+b(dx+c)}{ad-cb}\right)}{b}+\frac{\ln(dx+c)\ln\left(\frac{ad-cb+b(d}{ad-cb}\right.}{b}\right.}{d^3}$

```
[In] int(ln((b*x+a)/x)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*dilog((a*d-c*b+c*(b+a/x))/(a*d-b*c))+1/d*ln(b+a/x)*ln((a*d-c*b+c*(b+a/x
))/(a*d-b*c))-1/d*ln(b+a/x)*ln(-1/x*a/b)-1/d*dilog(-1/x*a/b)
```

Fricas [F]

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx}\,dx=\int \frac{\log\left(\frac{bx+a}{x}\right)}{dx+c}\,dx$$

```
[In] integrate(log((b*x+a)/x)/(d*x+c),x, algorithm="fricas")
[Out] integral(log((b*x + a)/x)/(d*x + c), x)
```

Sympy [F]

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx}\,dx=\int \frac{\log\left(\frac{a}{x}+b\right)}{c+dx}\,dx$$

```
[In] integrate(ln((b*x+a)/x)/(d*x+c),x)
[Out] Integral(log(a/x + b)/(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.18

$$\begin{aligned}\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx}\,dx=&-\frac{(\log(bx+a)-\log(x))\log(dx+c)}{d}\\&+\frac{\log(dx+c)\log\left(\frac{bx+a}{x}\right)}{d}-\frac{\log\left(\frac{dx}{c}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{dx}{c}\right)}{d}\\&+\frac{\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)}{d}\end{aligned}$$

```
[In] integrate(log((b*x+a)/x)/(d*x+c),x, algorithm="maxima")
[Out] -(log(b*x + a) - log(x))*log(d*x + c)/d + log(d*x + c)*log((b*x + a)/x)/d -
 (log(d*x/c + 1)*log(x) + dilog(-d*x/c))/d + (log(b*x + a)*log((b*d*x + a*d
)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/d
```

Giac [F]

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx}\,dx = \int \frac{\log\left(\frac{bx+a}{x}\right)}{dx+c}\,dx$$

```
[In] integrate(log((b*x+a)/x)/(d*x+c),x, algorithm="giac")
[Out] integrate(log((b*x + a)/x)/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a+bx}{x}\right)}{c+dx}\,dx = \int \frac{\ln\left(\frac{a+b\,x}{x}\right)}{c+d\,x}\,dx$$

```
[In] int(log((a + b*x)/x)/(c + d*x),x)
[Out] int(log((a + b*x)/x)/(c + d*x), x)
```

3.397 $\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{c+dx} dx$

Optimal result

Integrand size = 20, antiderivative size = 227

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{c+dx} dx = \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}c+\sqrt{-a}d}\right)\log(c+dx)}{d} - \frac{\log\left(-\frac{d\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}c-\sqrt{-a}d}\right)\log(c+dx)}{d} - \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}\right)}{d} - \frac{\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{-a}d}\right)}{d} + \frac{2\operatorname{PolyLog}\left(2,1+\frac{dx}{c}\right)}{d}$$

```
[Out] ln(b+a/x^2)*ln(d*x+c)/d+2*ln(-d*x/c)*ln(d*x+c)/d-ln(d*x+c)*ln(d*((-a)^(1/2)
-x*b^(1/2))/(d*(-a)^(1/2)+c*b^(1/2)))/d-ln(d*x+c)*ln(-d*((-a)^(1/2)+x*b^(1/
2))/(-d*(-a)^(1/2)+c*b^(1/2)))/d+2*polylog(2,1+d*x/c)/d-polylog(2,(d*x+c)*b
^(1/2)/(-d*(-a)^(1/2)+c*b^(1/2)))/d-polylog(2,(d*x+c)*b^(1/2)/(d*(-a)^(1/2)
+c*b^(1/2)))/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2515, 2512, 266, 2463, 2441, 2352, 2440, 2438}

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{c+dx}\,dx = -\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}\right)}{d}-\frac{\operatorname{PolyLog}\left(2,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{-a}d}\right)}{d}+\frac{\log\left(\frac{a}{x^2}+b\right)\log(c+dx)}{d}-\frac{\log(c+dx)\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{-a}d+\sqrt{b}c}\right)}{d}-\frac{\log(c+dx)\log\left(-\frac{d\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}c-\sqrt{-a}d}\right)}{d}+\frac{2\operatorname{PolyLog}\left(2,\frac{dx}{c}+1\right)}{d}+\frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d}$$

```
[In] Int[Log[(a + b*x^2)/x^2]/(c + d*x),x]
```

```
[Out] (Log[b + a/x^2]*Log[c + d*x])/d + (2*Log[-((d*x)/c)]*Log[c + d*x])/d - (Log
[(d*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*c + Sqrt[-a]*d)]*Log[c + d*x])/d - (Lo
g[-((d*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*c - Sqrt[-a]*d))]*Log[c + d*x])/d -
 PolyLog[2, (Sqrt[b]*(c + d*x))/(Sqrt[b]*c - Sqrt[-a]*d)]/d - PolyLog[2, (S
qrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[-a]*d)]/d + (2*PolyLog[2, 1 + (d*x)/c])
/d
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2463

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2512

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[f + g*x]*((a + b*Log[c*(d + e*x^n)^p])/g), x
] - Dist[b*e*n*(p/g), Int[x^(n - 1)*(Log[f + g*x]/(d + e*x^n)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

Rule 2515

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*(u_)^(r_.), x_Symbol] :> In
t[ExpandToSum[u, x]^r*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a,
 b, c, p, q, r}, x] && LinearQ[u, x] && BinomialQ[v, x] &&  !(LinearMatchQ[
u, x] && BinomialMatchQ[v, x])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\log\left(b+\frac{a}{x^2}\right)}{c+dx}\,dx \\
&= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d}+\frac{(2a)\int \frac{\log(c+dx)}{\left(b+\frac{a}{x^2}\right)x^3}\,dx}{d} \\
&= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d}+\frac{(2a)\int \left(\frac{\log(c+dx)}{ax}-\frac{bx\log(c+dx)}{a(a+bx^2)}\right)dx}{d} \\
&= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d}+\frac{2\int \frac{\log(c+dx)}{x}\,dx}{d}-\frac{(2b)\int \frac{x\log(c+dx)}{a+bx^2}\,dx}{d}
\end{aligned}
$$

$$= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - 2\int \frac{\log\left(-\frac{dx}{c}\right)}{c+dx}\,dx - \frac{(2b)\int\left(-\frac{\log(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\log(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{d}$$

$$= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} + \frac{2\text{Li}_2\left(1+\frac{dx}{c}\right)}{d} + \frac{\sqrt{b}\int\frac{\log(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{d} - \frac{\sqrt{b}\int\frac{\log(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{d}$$

$$= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}c+\sqrt{-a}d}\right)\log(c+dx)}{d} - \frac{\log\left(-\frac{d\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}c-\sqrt{-a}d}\right)\log(c+dx)}{d} + \frac{2\text{Li}_2\left(1+\frac{dx}{c}\right)}{d} + \int\frac{\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}c+\sqrt{-a}d}\right)}{c+dx}\,dx + \int\frac{\log\left(\frac{d\left(\sqrt{-a}+\sqrt{b}x\right)}{-\sqrt{b}c+\sqrt{-a}d}\right)}{c+dx}\,dx$$

$$= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}c+\sqrt{-a}d}\right)\log(c+dx)}{d} - \frac{\log\left(-\frac{d\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}c-\sqrt{-a}d}\right)\log(c+dx)}{d} + \frac{2\text{Li}_2\left(1+\frac{dx}{c}\right)}{d} + \frac{\text{Subst}\left(\int\frac{\log\left(1+\frac{\sqrt{b}x}{-\sqrt{b}c+\sqrt{-a}d}\right)}{x}\,dx, x, c+dx\right)}{d} + \frac{\text{Subst}\left(\int\frac{\log\left(1-\frac{\sqrt{b}x}{\sqrt{b}c+\sqrt{-a}d}\right)}{x}\,dx, x, c+dx\right)}{d}$$

$$= \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}c+\sqrt{-a}d}\right)\log(c+dx)}{d} - \frac{\log\left(-\frac{d\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}c-\sqrt{-a}d}\right)\log(c+dx)}{d} - \frac{\text{Li}_2\left(\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}\right)}{d} - \frac{\text{Li}_2\left(\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{-a}d}\right)}{d} + \frac{2\text{Li}_2\left(1+\frac{dx}{c}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{c+dx}\,dx = \frac{\log\left(b+\frac{a}{x^2}\right)\log(c+dx)}{d} + \frac{2\log\left(-\frac{dx}{c}\right)\log(c+dx)}{d} - \frac{\log\left(\frac{d\left(\sqrt{-a}-\sqrt{b}x\right)}{\sqrt{b}c+\sqrt{-a}d}\right)\log(c+dx)}{d} - \frac{\log\left(-\frac{d\left(\sqrt{-a}+\sqrt{b}x\right)}{\sqrt{b}c-\sqrt{-a}d}\right)\log(c+dx)}{d} + \frac{2\,\mathrm{PolyLog}\left(2,\frac{c+dx}{c}\right)}{d} - \frac{\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}\right)}{d} - \frac{\mathrm{PolyLog}\left(2,\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{-a}d}\right)}{d}$$

```
[In] Integrate[Log[(a + b*x^2)/x^2]/(c + d*x),x]
```

```
[Out] (Log[b + a/x^2]*Log[c + d*x])/d + (2*Log[-((d*x)/c)]*Log[c + d*x])/d - (Log
[(d*(Sqrt[-a] - Sqrt[b]*x))/(Sqrt[b]*c + Sqrt[-a]*d)]*Log[c + d*x])/d - (Lo
g[-((d*(Sqrt[-a] + Sqrt[b]*x))/(Sqrt[b]*c - Sqrt[-a]*d))]*Log[c + d*x])/d +
 (2*PolyLog[2, (c + d*x)/c])/d - PolyLog[2, (Sqrt[b]*(c + d*x))/(Sqrt[b]*c
- Sqrt[-a]*d)]/d - PolyLog[2, (Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[-a]*d)]
/d
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.01

method	result
parts	$\frac{\ln\left(\frac{b\,x^{2}+a}{x^{2}}\right)\ln(dx+c)}{d}-\frac{2\left(-b\,d^{3}\left(-\frac{\ln(dx+c)\left(\ln\left(\frac{d\sqrt{-ab}+cb-b(dx+c)}{d\sqrt{-ab}+cb}\right)+\ln\left(\frac{d\sqrt{-ab}-cb+b(dx+c)}{d\sqrt{-ab}-cb}\right)\right)}{2b}-\frac{\mathrm{dilog}\left(\frac{d\sqrt{-ab}+cb-}{d\sqrt{-ab}-}\right.}{}\right.\right.}{d^{4}}$
derivativedivides	$-\frac{\ln\left(\frac{1}{x}\right)\ln\left(b+\frac{a}{x^{2}}\right)-2a\left(\frac{\ln\left(\frac{1}{x}\right)\left(\ln\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)+\ln\left(\frac{\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)\right)}{2a}+\frac{\mathrm{dilog}\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)+\mathrm{dilog}\left(\frac{\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{2a}\right)}{d}+\frac{\left(\ln\left(\frac{c}{x}\right.\right.}{}$
default	$-\frac{\ln\left(\frac{1}{x}\right)\ln\left(b+\frac{a}{x^{2}}\right)-2a\left(\frac{\ln\left(\frac{1}{x}\right)\left(\ln\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)+\ln\left(\frac{\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)\right)}{2a}+\frac{\mathrm{dilog}\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)+\mathrm{dilog}\left(\frac{\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{2a}\right)}{d}+\frac{\left(\ln\left(\frac{c}{x}\right.\right.}{}$
risch	$\frac{\ln\left(\frac{1}{x}\right)\ln\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{d}+\frac{\ln\left(\frac{1}{x}\right)\ln\left(\frac{\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{d}-\frac{\ln\left(\frac{1}{x}\right)\ln\left(b+\frac{a}{x^{2}}\right)}{d}+\frac{\mathrm{dilog}\left(\frac{-\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{d}+\frac{\mathrm{dilog}\left(\frac{\frac{a}{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{d}-$

```
[In] int(ln((b*x^2+a)/x^2)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] ln((b*x^2+a)/x^2)/d*ln(d*x+c)-2/d^4*(-b*d^3*(-1/2*ln(d*x+c)*(ln((d*(-a*b)^(
1/2)+c*b-b*(d*x+c))/(d*(-a*b)^(1/2)+c*b))+ln((d*(-a*b)^(1/2)-c*b+b*(d*x+c))
/(d*(-a*b)^(1/2)-c*b)))/b-1/2*(dilog((d*(-a*b)^(1/2)+c*b-b*(d*x+c))/(d*(-a*
b)^(1/2)+c*b))+dilog((d*(-a*b)^(1/2)-c*b+b*(d*x+c))/(d*(-a*b)^(1/2)-c*b)))/
b)-d^3*(dilog(-x*d/c)+ln(d*x+c)*ln(-x*d/c)))
```

Fricas [F]

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{c+dx}\,dx=\int \frac{\log\left(\frac{bx^2+a}{x^2}\right)}{dx+c}\,dx$$

```
[In] integrate(log((b*x^2+a)/x^2)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(log((b*x^2 + a)/x^2)/(d*x + c), x)
```

Sympy [F]

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{c+dx}\,dx = \int \frac{\log\left(\frac{a}{x^2}+b\right)}{c+dx}\,dx$$

```
[In] integrate(ln((b*x**2+a)/x**2)/(d*x+c),x)
[Out] Integral(log(a/x**2 + b)/(c + d*x), x)
```

Maxima [F]

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{c+dx}\,dx = \int \frac{\log\left(\frac{bx^2+a}{x^2}\right)}{dx+c}\,dx$$

```
[In] integrate(log((b*x^2+a)/x^2)/(d*x+c),x, algorithm="maxima")
[Out] integrate(log((b*x^2 + a)/x^2)/(d*x + c), x)
```

Giac [F]

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{c+dx}\,dx = \int \frac{\log\left(\frac{bx^2+a}{x^2}\right)}{dx+c}\,dx$$

```
[In] integrate(log((b*x^2+a)/x^2)/(d*x+c),x, algorithm="giac")
[Out] integrate(log((b*x^2 + a)/x^2)/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a+bx^2}{x^2}\right)}{c+dx}\,dx = \int \frac{\ln\left(\frac{b\,x^2+a}{x^2}\right)}{c+d\,x}\,dx$$

```
[In] int(log((a + b*x^2)/x^2)/(c + d*x),x)
[Out] int(log((a + b*x^2)/x^2)/(c + d*x), x)
```

3.398 $\int \frac{\log(x^{-n}(a+bx^n))}{c+dx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\log(x^{-n}(a+bx^n))}{c+dx}\,dx = \text{Int}\left(\frac{\log(b+ax^{-n})}{c+dx},x\right)$$

```
[Out] Unintegrable(ln(b+a/(x^n))/(d*x+c),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\log(x^{-n}(a+bx^n))}{c+dx}\,dx = \int \frac{\log(x^{-n}(a+bx^n))}{c+dx}\,dx$$

```
[In] Int[Log[(a + b*x^n)/x^n]/(c + d*x),x]
[Out] Defer[Int][Log[b + a/x^n]/(c + d*x), x]
```

Rubi steps

$$\text{integral} = \int \frac{\log(b+ax^{-n})}{c+dx}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\log\left(x^{-n}(a+bx^n)\right)}{c+dx}\,dx=\int \frac{\log\left(x^{-n}(a+bx^n)\right)}{c+dx}\,dx$$

```
[In] Integrate[Log[(a + b*x^n)/x^n]/(c + d*x),x]
[Out] Integrate[Log[(a + b*x^n)/x^n]/(c + d*x), x]
```

Maple [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\ln\left(\left(a+b\,x^{n}\right)x^{-n}\right)}{dx+c}dx$$

```
[In] int(ln((a+b*x^n)/(x^n))/(d*x+c),x)
[Out] int(ln((a+b*x^n)/(x^n))/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\log\left(x^{-n}(a+bx^n)\right)}{c+dx}\,dx=\int \frac{\log\left(\frac{bx^n+a}{x^n}\right)}{dx+c}\,dx$$

```
[In] integrate(log((a+b*x^n)/(x^n))/(d*x+c),x, algorithm="fricas")
[Out] integral(log((b*x^n + a)/x^n)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 23.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \frac{\log\left(x^{-n}(a+bx^n)\right)}{c+dx}\,dx=\int \frac{\log\left(ax^{-n}+b\right)}{c+dx}\,dx$$

```
[In] integrate(ln((a+b*x**n)/(x**n))/(d*x+c),x)
[Out] Integral(log(a/x**n + b)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\log(x^{-n}(a+bx^n))}{c+dx}\,dx = \int \frac{\log\left(\frac{bx^n+a}{x^n}\right)}{dx+c}\,dx$$

```
[In] integrate(log((a+b*x^n)/(x^n))/(d*x+c),x, algorithm="maxima")
[Out] integrate(log((b*x^n + a)/x^n)/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\log(x^{-n}(a+bx^n))}{c+dx}\,dx = \int \frac{\log\left(\frac{bx^n+a}{x^n}\right)}{dx+c}\,dx$$

```
[In] integrate(log((a+b*x^n)/(x^n))/(d*x+c),x, algorithm="giac")
[Out] integrate(log((b*x^n + a)/x^n)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\log(x^{-n}(a+bx^n))}{c+dx}\,dx = \int \frac{\ln\left(\frac{a+b\,x^n}{x^n}\right)}{c+d\,x}\,dx$$

```
[In] int(log((a + b*x^n)/x^n)/(c + d*x),x)
[Out] int(log((a + b*x^n)/x^n)/(c + d*x), x)
```

3.399 $\int (fx)^q (a+b\log(c(d+ex^m)^n))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 92

$$\int (fx)^q (a+b\log(c(d+ex^m)^n))\,dx = -\frac{bemnx^{1+m}(fx)^q \text{Hypergeometric2F1}\left(1,\frac{1+m+q}{m},\frac{1+2m+q}{m},-\frac{ex^m}{d}\right)}{d(1+q)(1+m+q)} + \frac{(fx)^{1+q}(a+b\log(c(d+ex^m)^n))}{f(1+q)}$$

```
[Out] -b*e*m*n*x^(1+m)*(f*x)^q*hypergeom([1, (1+m+q)/m],[(1+2*m+q)/m],-e*x^m/d)/d
/(1+q)/(1+m+q)+(f*x)^(1+q)*(a+b*ln(c*(d+e*x^m)^n))/f/(1+q)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2505, 20, 371}

$$\int (fx)^q (a+b\log(c(d+ex^m)^n))\,dx = \frac{(fx)^{q+1}(a+b\log(c(d+ex^m)^n))}{f(q+1)} - \frac{bemnx^{m+1}(fx)^q \text{Hypergeometric2F1}\left(1,\frac{m+q+1}{m},\frac{2m+q+1}{m},-\frac{ex^m}{d}\right)}{d(q+1)(m+q+1)}$$

```
[In] Int[(f*x)^q*(a + b*Log[c*(d + e*x^m)^n]),x]
```

```
[Out] -((b*e*m*n*x^(1 + m)*(f*x)^q*Hypergeometric2F1[1, (1 + m + q)/m, (1 + 2*m +
 q)/m, -((e*x^m)/d)])/(d*(1 + q)*(1 + m + q))) + ((f*x)^(1 + q)*(a + b*Log[
c*(d + e*x^m)^n]))/(f*(1 + q))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(fx)^{1+q}\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{f(1+q)}-\frac{(bemn)\int\frac{x^{-1+m}(fx)^{1+q}}{d+ex^m}\,dx}{f(1+q)}\\ &= \frac{(fx)^{1+q}\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{f(1+q)}-\frac{\left(bemnx^{-q}(fx)^q\right)\int\frac{x^{m+q}}{d+ex^m}\,dx}{1+q}\\ &= -\frac{bemnx^{1+m}(fx)^q\,{}_2F_1\left(1,\frac{1+m+q}{m};\frac{1+2m+q}{m};-\frac{ex^m}{d}\right)}{d(1+q)(1+m+q)}+\frac{(fx)^{1+q}\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{f(1+q)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int(fx)^q\left(a+b\log\left(c(d+ex^m)^n\right)\right)\,dx$$
$$=\frac{x(fx)^q\left(-bemnx^m\,\text{Hypergeometric2F1}\left(1,\frac{1+m+q}{m},\frac{1+2m+q}{m},-\frac{ex^m}{d}\right)+d(1+m+q)\left(a+b\log\left(c(d+ex^m)^n\right.\right.\right.}{d(1+q)(1+m+q)}$$

```
[In] Integrate[(f*x)^q*(a + b*Log[c*(d + e*x^m)^n]),x]
```

```
[Out] (x*(f*x)^q*(-(b*e*m*n*x^m*Hypergeometric2F1[1, (1 + m + q)/m, (1 + 2*m + q)
/m, -((e*x^m)/d)]) + d*(1 + m + q)*(a + b*Log[c*(d + e*x^m)^n])))/(d*(1 + q
)*(1 + m + q))
```

Maple [F]

$$\int (fx)^q (a + b\ln (c(d + e\,x^m)^n))\,dx$$

```
[In] int((f*x)^q*(a+b*ln(c*(d+e*x^m)^n)),x)
[Out] int((f*x)^q*(a+b*ln(c*(d+e*x^m)^n)),x)
```

Fricas [F]

$$\int (fx)^q (a + b\log (c(d + ex^m)^n))\ dx = \int (b\log ((ex^m + d)^n c) + a)(fx)^q\ dx$$

```
[In] integrate((f*x)^q*(a+b*log(c*(d+e*x^m)^n)),x, algorithm="fricas")
[Out] integral((f*x)^q*b*log((e*x^m + d)^n*c) + (f*x)^q*a, x)
```

Sympy [F]

$$\int (fx)^q (a + b\log (c(d + ex^m)^n))\ dx = \int (fx)^q (a + b\log (c(d + ex^m)^n))\ dx$$

```
[In] integrate((f*x)**q*(a+b*ln(c*(d+e*x**m)**n)),x)
[Out] Integral((f*x)**q*(a + b*log(c*(d + e*x**m)**n)), x)
```

Maxima [F]

$$\int (fx)^q (a + b\log (c(d + ex^m)^n))\ dx = \int (b\log ((ex^m + d)^n c) + a)(fx)^q\ dx$$

```
[In] integrate((f*x)^q*(a+b*log(c*(d+e*x^m)^n)),x, algorithm="maxima")
[Out] (d^2*f^q*m^2*n*integrate(x^q/((m*(q + 1) - q^2 - 2*q - 1)*e^2*x^(2*m) + 2*(
m*(q + 1) - q^2 - 2*q - 1)*d*e*x^m + (m*(q + 1) - q^2 - 2*q - 1)*d^2), x) -
 (((m*(q + 1) - q^2 - 2*q - 1)*e*f^q*x*x^m + (m*(q + 1) - q^2 - 2*q - 1)*d*
f^q*x)*x^q*log((e*x^m + d)^n) + (((m*(q + 1) - q^2 - 2*q - 1)*e*f^q*log(c)
- (m^2*n - m*n*(q + 1))*e*f^q)*x*x^m - (d*f^q*m^2*n - (m*(q + 1) - q^2 - 2*
q - 1)*d*f^q*log(c))*x)*x^q)/((q^3 - (q^2 + 2*q + 1)*m + 3*q^2 + 3*q + 1)*e
*x^m + (q^3 - (q^2 + 2*q + 1)*m + 3*q^2 + 3*q + 1)*d))*b + (f*x)^(q + 1)*a/
(f*(q + 1))
```

Giac [F]

$$\int (fx)^q (a + b\log (c(d + ex^m)^n)) \, dx = \int (b\log ((ex^m + d)^n c) + a)(fx)^q \, dx$$

```
[In] integrate((f*x)^q*(a+b*log(c*(d+e*x^m)^n)),x, algorithm="giac")
[Out] integrate((b*log((e*x^m + d)^n*c) + a)*(f*x)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int (fx)^q (a + b\log (c(d + ex^m)^n)) \, dx = \int (f\,x)^q \left(a + b\,\ln \left(c\,{\left(d + e\,x^m\right)}^n\right)\right) \, dx$$

```
[In] int((f*x)^q*(a + b*log(c*(d + e*x^m)^n)),x)
[Out] int((f*x)^q*(a + b*log(c*(d + e*x^m)^n)), x)
```

3.400 $\int x^3 \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 166

$$\int x^3 \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right) \, dx = \frac{b d^7 n \sqrt{x}}{4 e^7}-\frac{b d^6 n x}{8 e^6}+\frac{b d^5 n x^{3/2}}{12 e^5}-\frac{b d^4 n x^2}{16 e^4}+\frac{b d^3 n x^{5/2}}{20 e^3}-\frac{b d^2 n x^3}{24 e^2}+\frac{b d n x^{7/2}}{28 e}-\frac{1}{32} b n x^4-\frac{b d^8 n \log \left(d+e \sqrt{x}\right)}{4 e^8}+\frac{1}{4} x^4 \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right)$$

```
[Out] -1/8*b*d^6*n*x/e^6+1/12*b*d^5*n*x^(3/2)/e^5-1/16*b*d^4*n*x^2/e^4+1/20*b*d^3
*n*x^(5/2)/e^3-1/24*b*d^2*n*x^3/e^2+1/28*b*d*n*x^(7/2)/e-1/32*b*n*x^4-1/4*b
*d^8*n*ln(d+e*x^(1/2))/e^8+1/4*x^4*(a+b*ln(c*(d+e*x^(1/2))^n))+1/4*b*d^7*n*
x^(1/2)/e^7
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 45}

$$\int x^3 \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right) \, dx = \frac{1}{4} x^4 \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right)-\frac{b d^8 n \log \left(d+e \sqrt{x}\right)}{4 e^8}+\frac{b d^7 n \sqrt{x}}{4 e^7}-\frac{b d^6 n x}{8 e^6}+\frac{b d^5 n x^{3/2}}{12 e^5}-\frac{b d^4 n x^2}{16 e^4}+\frac{b d^3 n x^{5/2}}{20 e^3}-\frac{b d^2 n x^3}{24 e^2}+\frac{b d n x^{7/2}}{28 e}-\frac{1}{32} b n x^4$$

```
[In] Int[x^3*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

```
[Out] (b*d^7*n*Sqrt[x])/(4*e^7) - (b*d^6*n*x)/(8*e^6) + (b*d^5*n*x^(3/2))/(12*e^5
) - (b*d^4*n*x^2)/(16*e^4) + (b*d^3*n*x^(5/2))/(20*e^3) - (b*d^2*n*x^3)/(24
*e^2) + (b*d*n*x^(7/2))/(28*e) - (b*n*x^4)/32 - (b*d^8*n*Log[d + e*Sqrt[x]]
)/(4*e^8) + (x^4*(a + b*Log[c*(d + e*Sqrt[x])^n]))/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^7(a+b\log\left(c(d+ex)^n\right))\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-\frac{1}{4}(ben)\text{Subst}\left(\int \frac{x^8}{d+ex}\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-\frac{1}{4}(ben)\text{Subst}\left(\int \left(-\frac{d^7}{e^8}+\frac{d^6x}{e^7}-\frac{d^5x^2}{e^6}+\frac{d^4x^3}{e^5}\right.\right.\\
&\qquad \left.\left.-\frac{d^3x^4}{e^4}+\frac{d^2x^5}{e^3}-\frac{dx^6}{e^2}+\frac{x^7}{e}+\frac{d^8}{e^8(d+ex)}\right)\,dx,x,\sqrt{x}\right)\\
&= \frac{bd^7n\sqrt{x}}{4e^7}-\frac{bd^6nx}{8e^6}+\frac{bd^5nx^{3/2}}{12e^5}-\frac{bd^4nx^2}{16e^4}+\frac{bd^3nx^{5/2}}{20e^3}-\frac{bd^2nx^3}{24e^2}+\frac{bdnx^{7/2}}{28e}\\
&\quad -\frac{1}{32}bnx^4-\frac{bd^8n\log\left(d+e\sqrt{x}\right)}{4e^8}+\frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.95

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{ax^4}{4}-\frac{1}{8}ben\left(-\frac{2d^7\sqrt{x}}{e^8}+\frac{d^6x}{e^7}-\frac{2d^5x^{3/2}}{3e^6}+\frac{d^4x^2}{2e^5}-\frac{2d^3x^{5/2}}{5e^4}+\frac{d^2x^3}{3e^3}-\frac{2dx^{7/2}}{7e^2}+\frac{x^4}{4e}+\frac{2d^8\log\left(d+e\sqrt{x}\right)}{e^9}\right)+\frac{1}{4}bx^4\log\left(c\left(d+e\sqrt{x}\right)^n\right)$$

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

```
[Out] (a*x^4)/4 - (b*e*n*((-2*d^7*Sqrt[x])/e^8 + (d^6*x)/e^7 - (2*d^5*x^(3/2))/(3
*e^6) + (d^4*x^2)/(2*e^5) - (2*d^3*x^(5/2))/(5*e^4) + (d^2*x^3)/(3*e^3) - (
2*d*x^(7/2))/(7*e^2) + x^4/(4*e) + (2*d^8*Log[d + e*Sqrt[x]])/e^9))/8 + (b*
x^4*Log[c*(d + e*Sqrt[x])^n])/4
```

Maple [F]

$$\int x^3\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)dx$$

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.89

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{840\,be^8x^4\log\left(c\right)-140\,bd^2e^6nx^3-210\,bd^4e^4nx^2-420\,bd^6e^2nx-105\,(be^8n-8\,ae^8)x^4+840\,(be^8nx^4-bd^8}{3360\,e^8}$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="fricas")
```

```
[Out] 1/3360*(840*b*e^8*x^4*log(c) - 140*b*d^2*e^6*n*x^3 - 210*b*d^4*e^4*n*x^2 -
420*b*d^6*e^2*n*x - 105*(b*e^8*n - 8*a*e^8)*x^4 + 840*(b*e^8*n*x^4 - b*d^8*
n)*log(e*sqrt(x) + d) + 8*(15*b*d*e^7*n*x^3 + 21*b*d^3*e^5*n*x^2 + 35*b*d^5
*e^3*n*x + 105*b*d^7*e*n)*sqrt(x))/e^8
```

Sympy [A] (verification not implemented)

Time = 8.47 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.93

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{ax^4}{4}+b\left(-\frac{en\left(\frac{2d^8\left(\begin{cases}\frac{\sqrt{x}}{d} & \text{for } e=0\\ \frac{\log\left(d+e\sqrt{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^8}-\frac{2d^7\sqrt{x}}{e^8}+\frac{d^6x}{e^7}-\frac{2d^5x^{\frac{3}{2}}}{3e^6}+\frac{d^4x^2}{2e^5}-\frac{2d^3x^{\frac{5}{2}}}{5e^4}+\frac{d^2x^3}{3e^3}-\frac{2dx^{\frac{7}{2}}}{7e^2}+\frac{x^4}{4e}\right)}{8}+\frac{x^4\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{4}\right)$$

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/2))**n)),x)
```

```
[Out] a*x**4/4 + b*(-e*n*(2*d**8*Piecewise((sqrt(x)/d, Eq(e, 0)), (log(d + e*sqrt
(x))/e, True))/e**8 - 2*d**7*sqrt(x)/e**8 + d**6*x/e**7 - 2*d**5*x**(3/2)/(
3*e**6) + d**4*x**2/(2*e**5) - 2*d**3*x**(5/2)/(5*e**4) + d**2*x**3/(3*e**3
) - 2*d*x**(7/2)/(7*e**2) + x**4/(4*e))/8 + x**4*log(c*(d + e*sqrt(x))**n)/
4)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.77

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{1}{4}\,bx^4\log\left(\left(e\sqrt{x}+d\right)^nc\right)+\frac{1}{4}\,ax^4-\frac{1}{3360}\,ben\left(\frac{840\,d^8\log\left(e\sqrt{x}+d\right)}{e^9}+\frac{105\,e^7x^4-120\,de^6x^{\frac{7}{2}}+140\,d^2e^5x^3-168\,d^3e^4x^{\frac{5}{2}}+210\,d^4e^3x^2-280\,d\cdots}{e^8}\right.$$

[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="maxima")

[Out] 1/4*b*x^4*log((e*sqrt(x) + d)^n*c) + 1/4*a*x^4 - 1/3360*b*e*n*(840*d^8*log(e*sqrt(x) + d)/e^9 + (105*e^7*x^4 - 120*d*e^6*x^(7/2) + 140*d^2*e^5*x^3 - 168*d^3*e^4*x^(5/2) + 210*d^4*e^3*x^2 - 280*d^5*e^2*x^(3/2) + 420*d^6*e*x - 840*d^7*sqrt(x))/e^8)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(134) = 268.

Time = 0.32 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.10

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{840\,bex^4\log\left(c\right)+840\,aex^4+\left(\frac{840\,\left(e\sqrt{x}+d\right)^8\log\left(e\sqrt{x}+d\right)}{e^7}-\frac{6720\,\left(e\sqrt{x}+d\right)^7d\log\left(e\sqrt{x}+d\right)}{e^7}+\frac{23520\,\left(e\sqrt{x}+d\right)^6d^2\log\left(e\sqrt{x}+d\right)}{e^7}-\cdots\right.}{\cdots}$$

[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="giac")

[Out] 1/3360*(840*b*e*x^4*log(c) + 840*a*e*x^4 + (840*(e*sqrt(x) + d)^8*log(e*sqrt(x) + d)/e^7 - 6720*(e*sqrt(x) + d)^7*d*log(e*sqrt(x) + d)/e^7 + 23520*(e*sqrt(x) + d)^6*d^2*log(e*sqrt(x) + d)/e^7 - 47040*(e*sqrt(x) + d)^5*d^3*log(e*sqrt(x) + d)/e^7 + 58800*(e*sqrt(x) + d)^4*d^4*log(e*sqrt(x) + d)/e^7 - 47040*(e*sqrt(x) + d)^3*d^5*log(e*sqrt(x) + d)/e^7 + 23520*(e*sqrt(x) + d)^2*d^6*log(e*sqrt(x) + d)/e^7 - 6720*(e*sqrt(x) + d)*d^7*log(e*sqrt(x) + d)/e^7 - 105*(e*sqrt(x) + d)^8/e^7 + 960*(e*sqrt(x) + d)^7*d/e^7 - 3920*(e*sqrt(x) + d)^6*d^2/e^7 + 9408*(e*sqrt(x) + d)^5*d^3/e^7 - 14700*(e*sqrt(x) + d)^4*d^4/e^7 + 15680*(e*sqrt(x) + d)^3*d^5/e^7 - 11760*(e*sqrt(x) + d)^2*d^6/e^7 + 6720*(e*sqrt(x) + d)*d^7/e^7)*b*n)/e

Mupad [B] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.83

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{a\,x^4}{4}-\frac{b\,n\,x^4}{32}+\frac{b\,x^4\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{4}+\frac{b\,d\,n\,x^{7/2}}{28\,e}-\frac{b\,d^6\,n\,x}{8\,e^6}-\frac{b\,d^8\,n\,\ln\left(d+e\,\sqrt{x}\right)}{4\,e^8}-\frac{b\,d^2\,n\,x^3}{24\,e^2}-\frac{b\,d^4\,n\,x^2}{16\,e^4}+\frac{b\,d^3\,n\,x^{5/2}}{20\,e^3}+\frac{b\,d^5\,n\,x^{3/2}}{12\,e^5}+\frac{b\,d^7\,n\,\sqrt{x}}{4\,e^7}$$

```
[In] int(x^3*(a + b*log(c*(d + e*x^(1/2))^n)),x)
```

```
[Out] (a*x^4)/4 - (b*n*x^4)/32 + (b*x^4*log(c*(d + e*x^(1/2))^n))/4 + (b*d*n*x^(7
/2))/(28*e) - (b*d^6*n*x)/(8*e^6) - (b*d^8*n*log(d + e*x^(1/2)))/(4*e^8) -
(b*d^2*n*x^3)/(24*e^2) - (b*d^4*n*x^2)/(16*e^4) + (b*d^3*n*x^(5/2))/(20*e^3
) + (b*d^5*n*x^(3/2))/(12*e^5) + (b*d^7*n*x^(1/2))/(4*e^7)
```

3.401 $\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 134

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{bd^5n\sqrt{x}}{3e^5}-\frac{bd^4nx}{6e^4}+\frac{bd^3nx^{3/2}}{9e^3}-\frac{bd^2nx^2}{12e^2}+\frac{bdnx^{5/2}}{15e}-\frac{1}{18}bnx^3-\frac{bd^6n\log\left(d+e\sqrt{x}\right)}{3e^6}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)$$

```
[Out] -1/6*b*d^4*n*x/e^4+1/9*b*d^3*n*x^(3/2)/e^3-1/12*b*d^2*n*x^2/e^2+1/15*b*d*n*
x^(5/2)/e-1/18*b*n*x^3-1/3*b*d^6*n*ln(d+e*x^(1/2))/e^6+1/3*x^3*(a+b*ln(c*(d
+e*x^(1/2))^n))+1/3*b*d^5*n*x^(1/2)/e^5
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 45}

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-\frac{bd^6n\log\left(d+e\sqrt{x}\right)}{3e^6}+\frac{bd^5n\sqrt{x}}{3e^5}-\frac{bd^4nx}{6e^4}+\frac{bd^3nx^{3/2}}{9e^3}-\frac{bd^2nx^2}{12e^2}+\frac{bdnx^{5/2}}{15e}-\frac{1}{18}bnx^3$$

```
[In] Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

```
[Out] (b*d^5*n*Sqrt[x])/(3*e^5) - (b*d^4*n*x)/(6*e^4) + (b*d^3*n*x^(3/2))/(9*e^3)
 - (b*d^2*n*x^2)/(12*e^2) + (b*d*n*x^(5/2))/(15*e) - (b*n*x^3)/18 - (b*d^6*
n*Log[d + e*Sqrt[x]])/(3*e^6) + (x^3*(a + b*Log[c*(d + e*Sqrt[x])^n]))/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^5(a+b\log\left(c(d+ex)^n\right))\,dx, x, \sqrt{x}\right)\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-\frac{1}{3}(ben)\text{Subst}\left(\int\frac{x^6}{d+ex}\,dx, x, \sqrt{x}\right)\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-\frac{1}{3}(ben)\text{Subst}\left(\int\left(-\frac{d^5}{e^6}+\frac{d^4x}{e^5}-\frac{d^3x^2}{e^4}+\frac{d^2x^3}{e^3}\right.\right.\\
&\qquad\left.\left.-\frac{dx^4}{e^2}+\frac{x^5}{e}+\frac{d^6}{e^6(d+ex)}\right)dx, x, \sqrt{x}\right)\\
&= \frac{bd^5n\sqrt{x}}{3e^5}-\frac{bd^4nx}{6e^4}+\frac{bd^3nx^{3/2}}{9e^3}-\frac{bd^2nx^2}{12e^2}+\frac{bdnx^{5/2}}{15e}-\frac{1}{18}bnx^3\\
&\quad-\frac{bd^6n\log\left(d+e\sqrt{x}\right)}{3e^6}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.96

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{ax^3}{3}-\frac{1}{6}ben\left(-\frac{2d^5\sqrt{x}}{e^6}+\frac{d^4x}{e^5}-\frac{2d^3x^{3/2}}{3e^4}+\frac{d^2x^2}{2e^3}-\frac{2dx^{5/2}}{5e^2}+\frac{x^3}{3e}+\frac{2d^6\log\left(d+e\sqrt{x}\right)}{e^7}\right)+\frac{1}{3}bx^3\log\left(c\left(d+e\sqrt{x}\right)^n\right)$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

```
[Out] (a*x^3)/3 - (b*e*n*((-2*d^5*Sqrt[x])/e^6 + (d^4*x)/e^5 - (2*d^3*x^(3/2))/(3
*e^4) + (d^2*x^2)/(2*e^3) - (2*d*x^(5/2))/(5*e^2) + x^3/(3*e) + (2*d^6*Log[
d + e*Sqrt[x]])/e^7))/6 + (b*x^3*Log[c*(d + e*Sqrt[x])^n])/3
```

Maple [F]

$$\int x^2\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)dx$$

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.91

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$$
$$=\frac{60\,be^6x^3\log\left(c\right)-15\,bd^2e^4nx^2-30\,bd^4e^2nx-10\,(be^6n-6\,ae^6)x^3+60\,(be^6nx^3-bd^6n)\log\left(e\sqrt{x}+d\right)+4\,(}{180\,e^6}$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="fricas")
```

```
[Out] 1/180*(60*b*e^6*x^3*log(c) - 15*b*d^2*e^4*n*x^2 - 30*b*d^4*e^2*n*x - 10*(b*
e^6*n - 6*a*e^6)*x^3 + 60*(b*e^6*n*x^3 - b*d^6*n)*log(e*sqrt(x) + d) + 4*(3
*b*d*e^5*n*x^2 + 5*b*d^3*e^3*n*x + 15*b*d^5*e*n)*sqrt(x))/e^6
```

Sympy [A] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.96

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$$
$$=\frac{ax^3}{3}+b\left(-\frac{en\left(\frac{2d^6\left(\begin{cases}\frac{\sqrt{x}}{d} & \text{for } e=0\\ \frac{\log\left(d+e\sqrt{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^6}-\frac{2d^5\sqrt{x}}{e^6}+\frac{d^4x}{e^5}-\frac{2d^3x^{\frac{3}{2}}}{3e^4}+\frac{d^2x^2}{2e^3}-\frac{2dx^{\frac{5}{2}}}{5e^2}+\frac{x^3}{3e}\right)}{6}+\frac{x^3\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3}\right)$$

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))**n)),x)
```

```
[Out] a*x**3/3 + b*(-e*n*(2*d**6*Piecewise((sqrt(x)/d, Eq(e, 0)), (log(d + e*sqrt
(x))/e, True))/e**6 - 2*d**5*sqrt(x)/e**6 + d**4*x/e**5 - 2*d**3*x**(3/2)/(
3*e**4) + d**2*x**2/(2*e**3) - 2*d*x**(5/2)/(5*e**2) + x**3/(3*e))/6 + x**3
*log(c*(d + e*sqrt(x))**n)/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.79

$$\int x^2 \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right) \, dx = \frac{1}{3} \, b x^3 \log \left(\left(e \sqrt{x}+d\right)^n c\right) + \frac{1}{3} \, a x^3 - \frac{1}{180} \, b e n \left(\frac{60 \, d^6 \log \left(e \sqrt{x}+d\right)}{e^7} + \frac{10 \, e^5 x^3 - 12 \, d e^4 x^{\frac{5}{2}} + 15 \, d^2 e^3 x^2 - 20 \, d^3 e^2 x^{\frac{3}{2}} + 30 \, d^4 e x - 60 \, d^5 \sqrt{x}}{e^6}\right)$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3*log((e*sqrt(x) + d)^n*c) + 1/3*a*x^3 - 1/180*b*e*n*(60*d^6*log(e*
sqrt(x) + d)/e^7 + (10*e^5*x^3 - 12*d*e^4*x^(5/2) + 15*d^2*e^3*x^2 - 20*d^3
*e^2*x^(3/2) + 30*d^4*e*x - 60*d^5*sqrt(x))/e^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(108) = 216.

Time = 0.32 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.97

$$\int x^2 \left(a+b \log \left(c \left(d+e \sqrt{x}\right)^n\right)\right) \, dx = \frac{60 \, b e x^3 \log \left(c\right) + 60 \, a e x^3 + \left(\frac{60 \, \left(e \sqrt{x}+d\right)^6 \log \left(e \sqrt{x}+d\right)}{e^5} - \frac{360 \, \left(e \sqrt{x}+d\right)^5 d \log \left(e \sqrt{x}+d\right)}{e^5} + \frac{900 \, \left(e \sqrt{x}+d\right)^4 d^2 \log \left(e \sqrt{x}+d\right)}{e^5} - \frac{1200 \, (e \cdots}{}\right.}{}$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="giac")
```

```
[Out] 1/180*(60*b*e*x^3*log(c) + 60*a*e*x^3 + (60*(e*sqrt(x) + d)^6*log(e*sqrt(x)
 + d)/e^5 - 360*(e*sqrt(x) + d)^5*d*log(e*sqrt(x) + d)/e^5 + 900*(e*sqrt(x)
 + d)^4*d^2*log(e*sqrt(x) + d)/e^5 - 1200*(e*sqrt(x) + d)^3*d^3*log(e*sqrt(
x) + d)/e^5 + 900*(e*sqrt(x) + d)^2*d^4*log(e*sqrt(x) + d)/e^5 - 360*(e*sqr
t(x) + d)*d^5*log(e*sqrt(x) + d)/e^5 - 10*(e*sqrt(x) + d)^6/e^5 + 72*(e*sqr
t(x) + d)^5*d/e^5 - 225*(e*sqrt(x) + d)^4*d^2/e^5 + 400*(e*sqrt(x) + d)^3*d
^3/e^5 - 450*(e*sqrt(x) + d)^2*d^4/e^5 + 360*(e*sqrt(x) + d)*d^5/e^5)*b*n)/
e
```

Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{a\,x^3}{3}-\frac{b\,n\,x^3}{18}+\frac{b\,x^3\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)}{3}+\frac{b\,d\,n\,x^{5/2}}{15\,e}-\frac{b\,d^4\,n\,x}{6\,e^4}-\frac{b\,d^6\,n\,\ln\left(d+e\,\sqrt{x}\right)}{3\,e^6}-\frac{b\,d^2\,n\,x^2}{12\,e^2}+\frac{b\,d^3\,n\,x^{3/2}}{9\,e^3}+\frac{b\,d^5\,n\,\sqrt{x}}{3\,e^5}$$

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/2))^n)),x)
```

```
[Out] (a*x^3)/3 - (b*n*x^3)/18 + (b*x^3*log(c*(d + e*x^(1/2))^n))/3 + (b*d*n*x^(5
/2))/(15*e) - (b*d^4*n*x)/(6*e^4) - (b*d^6*n*log(d + e*x^(1/2)))/(3*e^6) -
(b*d^2*n*x^2)/(12*e^2) + (b*d^3*n*x^(3/2))/(9*e^3) + (b*d^5*n*x^(1/2))/(3*e
^5)
```

3.402 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 102

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{bd^3n\sqrt{x}}{2e^3}-\frac{bd^2nx}{4e^2}+\frac{bdnx^{3/2}}{6e}-\frac{1}{8}bnx^2-\frac{bd^4n\log\left(d+e\sqrt{x}\right)}{2e^4}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)$$

```
[Out] -1/4*b*d^2*n*x/e^2+1/6*b*d*n*x^(3/2)/e-1/8*b*n*x^2-1/2*b*d^4*n*ln(d+e*x^(1/
2))/e^4+1/2*x^2*(a+b*ln(c*(d+e*x^(1/2))^n))+1/2*b*d^3*n*x^(1/2)/e^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2504, 2442, 45}

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-\frac{bd^4n\log\left(d+e\sqrt{x}\right)}{2e^4}+\frac{bd^3n\sqrt{x}}{2e^3}-\frac{bd^2nx}{4e^2}+\frac{bdnx^{3/2}}{6e}-\frac{1}{8}bnx^2$$

```
[In] Int[x*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

```
[Out] (b*d^3*n*Sqrt[x])/(2*e^3) - (b*d^2*n*x)/(4*e^2) + (b*d*n*x^(3/2))/(6*e) - (
b*n*x^2)/8 - (b*d^4*n*Log[d + e*Sqrt[x]])/(2*e^4) + (x^2*(a + b*Log[c*(d +
e*Sqrt[x])^n]))/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^3(a+b\log\left(c(d+ex)^n\right))\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-\frac{1}{2}(ben)\text{Subst}\left(\int\frac{x^4}{d+ex}\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\\
&\quad-\frac{1}{2}(ben)\text{Subst}\left(\int\left(-\frac{d^3}{e^4}+\frac{d^2x}{e^3}-\frac{dx^2}{e^2}+\frac{x^3}{e}+\frac{d^4}{e^4(d+ex)}\right)dx,x,\sqrt{x}\right)\\
&= \frac{bd^3n\sqrt{x}}{2e^3}-\frac{bd^2nx}{4e^2}+\frac{bdnx^{3/2}}{6e}-\frac{1}{8}bnx^2-\frac{bd^4n\log\left(d+e\sqrt{x}\right)}{2e^4}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.99

$$
\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{ax^2}{2}-\frac{1}{4}ben\left(-\frac{2d^3\sqrt{x}}{e^4}+\frac{d^2x}{e^3}-\frac{2dx^{3/2}}{3e^2}+\frac{x^2}{2e}+\frac{2d^4\log\left(d+e\sqrt{x}\right)}{e^5}\right)+\frac{1}{2}bx^2\log\left(c\left(d+e\sqrt{x}\right)^n\right)
$$

```
[In] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^n]),x]
```

```
[Out] (a*x^2)/2 - (b*e*n*((-2*d^3*Sqrt[x])/e^4 + (d^2*x)/e^3 - (2*d*x^(3/2))/(3*e
^2) + x^2/(2*e) + (2*d^4*Log[d + e*Sqrt[x]])/e^5))/4 + (b*x^2*Log[c*(d + e*
Sqrt[x])^n])/2
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)dx$$

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(1/2))^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.93

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)\,dx = \frac{12\,be^{4}x^{2}\log\left(c\right)-6\,bd^{2}e^{2}nx-3\,(be^{4}n-4\,ae^{4})x^{2}+12\,(be^{4}nx^{2}-bd^{4}n)\log\left(e\sqrt{x}+d\right)+4\,(bde^{3}nx+3\,bd^{3}en)\sqrt{x}}{24\,e^{4}}$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="fricas")
```

```
[Out] 1/24*(12*b*e^4*x^2*log(c) - 6*b*d^2*e^2*n*x - 3*(b*e^4*n - 4*a*e^4)*x^2 + 1
2*(b*e^4*n*x^2 - b*d^4*n)*log(e*sqrt(x) + d) + 4*(b*d*e^3*n*x + 3*b*d^3*e*n
)*sqrt(x))/e^4
```

Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.98

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$$
$$=\frac{ax^2}{2}+b\left(-\frac{en\left(\frac{2d^4\left(\begin{cases}\frac{\sqrt{x}}{d} & \text{for } e=0\\ \frac{\log\left(d+e\sqrt{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^4}-\frac{2d^3\sqrt{x}}{e^4}+\frac{d^2x}{e^3}-\frac{2dx^{\frac{3}{2}}}{3e^2}+\frac{x^2}{2e}\right)}{4}+\frac{x^2\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2}\right)$$

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/2))**n)),x)
```

```
[Out] a*x**2/2 + b*(-e*n*(2*d**4*Piecewise((sqrt(x)/d, Eq(e, 0)), (log(d + e*sqrt
(x))/e, True))/e**4 - 2*d**3*sqrt(x)/e**4 + d**2*x/e**3 - 2*d*x**(3/2)/(3*e
**2) + x**2/(2*e))/4 + x**2*log(c*(d + e*sqrt(x))**n)/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)\,dx$$
$$=-\frac{1}{24}\,ben\left(\frac{12\,d^{4}\log\left(e\sqrt{x}+d\right)}{e^{5}}+\frac{3\,e^{3}x^{2}-4\,de^{2}x^{\frac{3}{2}}+6\,d^{2}ex-12\,d^{3}\sqrt{x}}{e^{4}}\right)$$
$$+\frac{1}{2}\,bx^{2}\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)+\frac{1}{2}\,ax^{2}$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="maxima")
```

```
[Out] -1/24*b*e*n*(12*d^4*log(e*sqrt(x) + d)/e^5 + (3*e^3*x^2 - 4*d*e^2*x^(3/2) +
 6*d^2*e*x - 12*d^3*sqrt(x))/e^4) + 1/2*b*x^2*log((e*sqrt(x) + d)^n*c) + 1/
2*a*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(82) = 164.

Time = 0.33 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.76

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)\,dx$$
$$=\frac{12\,bex^{2}\log\left(c\right)+12\,aex^{2}+\left(\frac{12\left(e\sqrt{x}+d\right)^{4}\log\left(e\sqrt{x}+d\right)}{e^{3}}-\frac{48\left(e\sqrt{x}+d\right)^{3}d\log\left(e\sqrt{x}+d\right)}{e^{3}}+\frac{72\left(e\sqrt{x}+d\right)^{2}d^{2}\log\left(e\sqrt{x}+d\right)}{e^{3}}-\frac{48\left(e\sqrt{x}+\right.}{}\right.}{24\,e}$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n)),x, algorithm="giac")
```

```
[Out] 1/24*(12*b*e*x^2*log(c) + 12*a*e*x^2 + (12*(e*sqrt(x) + d)^4*log(e*sqrt(x)
+ d)/e^3 - 48*(e*sqrt(x) + d)^3*d*log(e*sqrt(x) + d)/e^3 + 72*(e*sqrt(x) +
d)^2*d^2*log(e*sqrt(x) + d)/e^3 - 48*(e*sqrt(x) + d)*d^3*log(e*sqrt(x) + d)
/e^3 - 3*(e*sqrt(x) + d)^4/e^3 + 16*(e*sqrt(x) + d)^3*d/e^3 - 36*(e*sqrt(x)
 + d)^2*d^2/e^3 + 48*(e*sqrt(x) + d)*d^3/e^3)*b*n)/e
```

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{a\,x^2}{2}-\frac{b\,n\,x^2}{8}+\frac{b\,x^2\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)}{2}-\frac{b\,d^2\,n\,x}{4\,e^2}+\frac{b\,d\,n\,x^{3/2}}{6\,e}-\frac{b\,d^4\,n\,\ln\left(d+e\,\sqrt{x}\right)}{2\,e^4}+\frac{b\,d^3\,n\,\sqrt{x}}{2\,e^3}$$

```
[In] int(x*(a + b*log(c*(d + e*x^(1/2))^n)),x)
```

```
[Out] (a*x^2)/2 - (b*n*x^2)/8 + (b*x^2*log(c*(d + e*x^(1/2))^n))/2 - (b*d^2*n*x)/
(4*e^2) + (b*d*n*x^(3/2))/(6*e) - (b*d^4*n*log(d + e*x^(1/2)))/(2*e^4) + (b
*d^3*n*x^(1/2))/(2*e^3)
```

3.403 $\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 60

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=\frac{bdn\sqrt{x}}{e}+ax-\frac{bnx}{2}-\frac{bd^2n\log\left(d+e\sqrt{x}\right)}{e^2}+bx\log\left(c\left(d+e\sqrt{x}\right)^n\right)$$

```
[Out] a*x-1/2*b*n*x-b*d^2*n*ln(d+e*x^(1/2))/e^2+b*x*ln(c*(d+e*x^(1/2))^n)+b*d*n*x
^(1/2)/e
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2498, 272, 45}

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=ax+bx\log\left(c\left(d+e\sqrt{x}\right)^n\right)-\frac{bd^2n\log\left(d+e\sqrt{x}\right)}{e^2}+\frac{bdn\sqrt{x}}{e}-\frac{bnx}{2}$$

```
[In] Int[a + b*Log[c*(d + e*Sqrt[x])^n],x]
```

```
[Out] (b*d*n*Sqrt[x])/e + a*x - (b*n*x)/2 - (b*d^2*n*Log[d + e*Sqrt[x]])/e^2 + b*
x*Log[c*(d + e*Sqrt[x])^n]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= ax + b\int \log\left(c\left(d+e\sqrt{x}\right)^n\right)\,dx\\
&= ax + bx\log\left(c\left(d+e\sqrt{x}\right)^n\right) - \frac{1}{2}(ben)\int \frac{\sqrt{x}}{d+e\sqrt{x}}\,dx\\
&= ax + bx\log\left(c\left(d+e\sqrt{x}\right)^n\right) - (ben)\text{Subst}\left(\int \frac{x^2}{d+ex}\,dx, x, \sqrt{x}\right)\\
&= ax + bx\log\left(c\left(d+e\sqrt{x}\right)^n\right) - (ben)\text{Subst}\left(\int \left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{bdn\sqrt{x}}{e} + ax - \frac{bnx}{2} - \frac{bd^2n\log\left(d+e\sqrt{x}\right)}{e^2} + bx\log\left(c\left(d+e\sqrt{x}\right)^n\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx = \frac{bdn\sqrt{x}}{e} + ax - \frac{bnx}{2} - \frac{bd^2n\log\left(d+e\sqrt{x}\right)}{e^2} + bx\log\left(c\left(d+e\sqrt{x}\right)^n\right)$$

```
[In] Integrate[a + b*Log[c*(d + e*Sqrt[x])^n],x]
[Out] (b*d*n*Sqrt[x])/e + a*x - (b*n*x)/2 - (b*d^2*n*Log[d + e*Sqrt[x]])/e^2 + b*
x*Log[c*(d + e*Sqrt[x])^n]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

method	result	size
default	$ax-\frac{bnx}{2}-\frac{b\,d^{2}n\ln\left(d+e\sqrt{x}\right)}{e^{2}}+bx\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)+\frac{bdn\sqrt{x}}{e}$	53
parts	$ax-\frac{bnx}{2}-\frac{b\,d^{2}n\ln\left(d+e\sqrt{x}\right)}{e^{2}}+bx\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)+\frac{bdn\sqrt{x}}{e}$	53
derivativedivides	$ax-\frac{bnx}{2}+bx\ln\left(c\,\mathrm{e}^{n\ln\left(d+e\sqrt{x}\right)}\right)-\frac{b\,d^{2}n\ln\left(d+e\sqrt{x}\right)}{e^{2}}+\frac{bdn\sqrt{x}}{e}$	55

```
[In] int(a+b*ln(c*(d+e*x^(1/2))^n),x,method=_RETURNVERBOSE)
```

```
[Out] a*x-1/2*b*n*x-b*d^2*n*ln(d+e*x^(1/2))/e^2+b*x*ln(c*(d+e*x^(1/2))^n)+b*d*n*x
^(1/2)/e
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)\,dx$$
$$=\frac{2\,be^{2}x\log\left(c\right)+2\,bden\sqrt{x}-\left(be^{2}n-2\,ae^{2}\right)x+2\,\left(be^{2}nx-bd^{2}n\right)\log\left(e\sqrt{x}+d\right)}{2\,e^{2}}$$

```
[In] integrate(a+b*log(c*(d+e*x^(1/2))^n),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*e^2*x*log(c) + 2*b*d*e*n*sqrt(x) - (b*e^2*n - 2*a*e^2)*x + 2*(b*e^
2*n*x - b*d^2*n)*log(e*sqrt(x) + d))/e^2
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=ax+b\left(-\frac{en\left(\frac{2d^2\left(\begin{cases}\frac{\sqrt{x}}{d} & \text{for } e=0\\ \frac{\log\left(d+e\sqrt{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^2}-\frac{2d\sqrt{x}}{e^2}+\frac{x}{e}\right)}{2}+x\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)$$

```
[In] integrate(a+b*ln(c*(d+e*x**(1/2))**n),x)
[Out] a*x + b*(-e*n*(2*d**2*Piecewise((sqrt(x)/d, Eq(e, 0)), (log(d + e*sqrt(x))/
e, True))/e**2 - 2*d*sqrt(x)/e**2 + x/e)/2 + x*log(c*(d + e*sqrt(x))**n))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx$$
$$=-\frac{1}{2}\left(en\left(\frac{2\,d^2\log\left(e\sqrt{x}+d\right)}{e^3}+\frac{ex-2\,d\sqrt{x}}{e^2}\right)-2\,x\log\left(\left(e\sqrt{x}+d\right)^nc\right)\right)b+ax$$

```
[In] integrate(a+b*log(c*(d+e*x^(1/2))^n),x, algorithm="maxima")
```

```
[Out] -1/2*(e*n*(2*d^2*log(e*sqrt(x) + d)/e^3 + (e*x - 2*d*sqrt(x))/e^2) - 2*x*lo
g((e*sqrt(x) + d)^n*c))*b + a*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.70

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=ax+\frac{b\left(\frac{\left(2\left(e\sqrt{x}+d\right)^2\log\left(e\sqrt{x}+d\right)-4\left(e\sqrt{x}+d\right)d\log\left(e\sqrt{x}+d\right)-\left(e\sqrt{x}+d\right)^2+4\left(e\sqrt{x}+d\right)d\right)n}{e}+\frac{2\left(\left(e\sqrt{x}+d\right)^2-2\left(e\sqrt{x}+d\right)d\right)\log\left(c\right)}{e}\right)}{2\,e}$$

```
[In] integrate(a+b*log(c*(d+e*x^(1/2))^n),x, algorithm="giac")
```

```
[Out] a*x + 1/2*b*((2*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d) - 4*(e*sqrt(x) + d)*d*
log(e*sqrt(x) + d) - (e*sqrt(x) + d)^2 + 4*(e*sqrt(x) + d)*d)*n/e + 2*((e*s
qrt(x) + d)^2 - 2*(e*sqrt(x) + d)*d)*log(c)/e)/e
```

Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\,dx=a\,x+b\,x\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)-\frac{b\,n\,\left(e^2\,x+2\,d^2\,\ln\left(d+e\,\sqrt{x}\right)-2\,d\,e\,\sqrt{x}\right)}{2\,e^2}$$

```
[In] int(a + b*log(c*(d + e*x^(1/2))^n),x)
```

```
[Out] a*x + b*x*log(c*(d + e*x^(1/2))^n) - (b*n*(e^2*x + 2*d^2*log(d + e*x^(1/2))
 - 2*d*e*x^(1/2)))/(2*e^2)
```

3.404 $\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 51

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}\,dx = 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right) + 2bn\,\mathrm{PolyLog}\left(2,1+\frac{e\sqrt{x}}{d}\right)$$

```
[Out] 2*ln(-e*x^(1/2)/d)*(a+b*ln(c*(d+e*x^(1/2))^n))+2*b*n*polylog(2,1+e*x^(1/2)/
d)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2441, 2352}

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}\,dx = 2\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right) + 2bn\,\mathrm{PolyLog}\left(2,\frac{\sqrt{x}e}{d}+1\right)$$

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])/x,x]
```

```
[Out] 2*(a + b*Log[c*(d + e*Sqrt[x])^n])*Log[-((e*Sqrt[x])/d)] + 2*b*n*PolyLog[2,
 1 + (e*Sqrt[x])/d]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx, x, \sqrt{x}\right)\\
&= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right)-(2ben)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, \sqrt{x}\right)\\
&= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right)+2bn\text{Li}_2\left(1+\frac{e\sqrt{x}}{d}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\begin{aligned}
\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}\,dx &= 2b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\log\left(-\frac{e\sqrt{x}}{d}\right)\\
&\quad+a\log(x)+2bn\,\text{PolyLog}\left(2,\frac{d+e\sqrt{x}}{d}\right)
\end{aligned}$$

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])/x,x]
```

```
[Out] 2*b*Log[c*(d + e*Sqrt[x])^n]*Log[-((e*Sqrt[x])/d)] + a*Log[x] + 2*b*n*PolyL
og[2, (d + e*Sqrt[x])/d]
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)}{x}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))/x,x)
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))/x,x)
```

Fricas [F]

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)}{x}\,dx = \int \frac{b\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x,x, algorithm="fricas")
[Out] integral((b*log((e*sqrt(x) + d)^n*c) + a)/x, x)
```

Sympy [F]

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)}{x}\,dx = \int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)}{x}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))/x,x)
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(44) = 88$.

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.12

$$\begin{aligned}&\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)}{x}\,dx\\&= -2\left(\log\left(\frac{e\sqrt{x}}{d}+1\right)\log\left(\sqrt{x}\right)+\mathrm{Li}_2\left(-\frac{e\sqrt{x}}{d}\right)\right)bn+\frac{2\left(ben\sqrt{x}\log\left(\sqrt{x}\right)-ben\sqrt{x}\right)}{d}\\&\quad+\frac{bd\log\left(\left(e\sqrt{x}+d\right)^{n}\right)\log\left(x\right)+\left(bd\log\left(c\right)+ad\right)\log\left(x\right)-\frac{benx\log(x)-2\,benx}{\sqrt{x}}}{d}\end{aligned}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x,x, algorithm="maxima")
[Out] -2*(log(e*sqrt(x)/d + 1)*log(sqrt(x)) + dilog(-e*sqrt(x)/d))*b*n + 2*(b*e*n
*sqrt(x)*log(sqrt(x)) - b*e*n*sqrt(x))/d + (b*d*log((e*sqrt(x) + d)^n)*log(
x) + (b*d*log(c) + a*d)*log(x) - (b*e*n*x*log(x) - 2*b*e*n*x)/sqrt(x))/d
```

Giac [F]

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)}{x}\,dx=\int \frac{b\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)}{x}\,dx=\int \frac{a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^{n}\right)}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))/x,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))/x, x)
```

3.405 $\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}\,dx=-\frac{ben}{d\sqrt{x}}+\frac{be^2n\log\left(d+e\sqrt{x}\right)}{d^2}-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}-\frac{be^2n\log(x)}{2d^2}$$

```
[Out] -1/2*b*e^2*n*ln(x)/d^2+b*e^2*n*ln(d+e*x^(1/2))/d^2+(-a-b*ln(c*(d+e*x^(1/2))
^n))/x-b*e*n/d/x^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 46}

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}\,dx=-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}+\frac{be^2n\log\left(d+e\sqrt{x}\right)}{d^2}-\frac{be^2n\log(x)}{2d^2}-\frac{ben}{d\sqrt{x}}$$

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^2,x]
```

```
[Out] -((b*e*n)/(d*Sqrt[x])) + (b*e^2*n*Log[d + e*Sqrt[x]])/d^2 - (a + b*Log[c*(d
 + e*Sqrt[x])^n])/x - (b*e^2*n*Log[x])/(2*d^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^3}\,dx, x, \sqrt{x}\right)\\
&= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}+(ben)\text{Subst}\left(\int \frac{1}{x^2(d+ex)}\,dx, x, \sqrt{x}\right)\\
&= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}+(ben)\text{Subst}\left(\int \left(\frac{1}{dx^2}-\frac{e}{d^2x}+\frac{e^2}{d^2(d+ex)}\right)\,dx, x, \sqrt{x}\right)\\
&= -\frac{ben}{d\sqrt{x}}+\frac{be^2n\log\left(d+e\sqrt{x}\right)}{d^2}-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x}-\frac{be^2n\log(x)}{2d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}\,dx = -\frac{a}{x}-\frac{b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x} + \frac{1}{2}ben\left(-\frac{2}{d\sqrt{x}}+\frac{2e\log\left(d+e\sqrt{x}\right)}{d^2}-\frac{e\log(x)}{d^2}\right)$$

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^2,x]
```

```
[Out] -(a/x) - (b*Log[c*(d + e*Sqrt[x])^n])/x + (b*e*n*(-2/(d*Sqrt[x]) + (2*e*Log
[d + e*Sqrt[x]])/d^2 - (e*Log[x])/d^2))/2
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)}{x^{2}}dx$$

[In] int((a+b*ln(c*(d+e*x^(1/2))^n))/x^2,x)

[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))/x^2,x)

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)}{x^{2}}\,dx = -\frac{be^{2}nx\log\left(\sqrt{x}\right)+bden\sqrt{x}+bd^{2}\log\left(c\right)+ad^{2}-\left(be^{2}nx-bd^{2}n\right)\log\left(e\sqrt{x}+d\right)}{d^{2}x}$$

[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^2,x, algorithm="fricas")

[Out] -(b*e^2*n*x*log(sqrt(x)) + b*d*e*n*sqrt(x) + b*d^2*log(c) + a*d^2 - (b*e^2*n*x - b*d^2*n)*log(e*sqrt(x) + d))/(d^2*x)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(66) = 132.

Time = 18.30 (sec) , antiderivative size = 442, normalized size of antiderivative = 6.31

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)}{x^{2}}\,dx = \begin{cases} -\frac{a+b\log\left(0^{n}c\right)}{x} \\ -\frac{a}{x}-\frac{bn}{2x}-\frac{b\log\left(c\left(e\sqrt{x}\right)^{n}\right)}{x} \\ -\frac{a+b\log\left(0^{n}c\right)}{x} \\ -\frac{2ad^{3}\sqrt{x}}{2d^{3}x^{\frac{3}{2}}+2d^{2}ex^{2}}-\frac{2ad^{2}ex}{2d^{3}x^{\frac{3}{2}}+2d^{2}ex^{2}}-\frac{2bd^{3}\sqrt{x}\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)}{2d^{3}x^{\frac{3}{2}}+2d^{2}ex^{2}}-\frac{2bd^{2}enx}{2d^{3}x^{\frac{3}{2}}+2d^{2}ex^{2}}-\frac{2bd^{2}ex\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)}{2d^{3}x^{\frac{3}{2}}+2d^{2}ex^{2}}-\frac{bde^{2}nx^{\frac{3}{2}}\log\left(x\right)}{2d^{3}x^{\frac{3}{2}}+2d^{2}ex^{2}}- \end{cases}$$

[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))/x**2,x)

[Out] Piecewise((-(a + b*log(0**n*c))/x, Eq(d, 0) & Eq(e, 0)), (-a/x - b*n/(2*x) - b*log(c*(e*sqrt(x))**n)/x, Eq(d, 0)), (-(a + b*log(0**n*c))/x, Eq(d, -e*sqrt(x))), (-2*a*d**3*sqrt(x)/(2*d**3*x**(3/2) + 2*d**2*e*x**2) - 2*a*d**2*e*x/(2*d**3*x**(3/2) + 2*d**2*e*x**2) - 2*b*d**3*sqrt(x)*log(c*(d + e*sqrt(x

```
))**n)/(2*d**3*x**(3/2) + 2*d**2*e*x**2) - 2*b*d**2*e*n*x/(2*d**3*x**(3/2)
+ 2*d**2*e*x**2) - 2*b*d**2*e*x*log(c*(d + e*sqrt(x))**n)/(2*d**3*x**(3/2)
+ 2*d**2*e*x**2) - b*d*e**2*n*x**(3/2)*log(x)/(2*d**3*x**(3/2) + 2*d**2*e*x
**2) - 2*b*d*e**2*n*x**(3/2)/(2*d**3*x**(3/2) + 2*d**2*e*x**2) + 2*b*d*e**2
*x**(3/2)*log(c*(d + e*sqrt(x))**n)/(2*d**3*x**(3/2) + 2*d**2*e*x**2) - b*e
**3*n*x**2*log(x)/(2*d**3*x**(3/2) + 2*d**2*e*x**2) + 2*b*e**3*x**2*log(c*(
d + e*sqrt(x))**n)/(2*d**3*x**(3/2) + 2*d**2*e*x**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}\,dx = \frac{1}{2}\,ben\left(\frac{2\,e\log\left(e\sqrt{x}+d\right)}{d^2}-\frac{e\log\left(x\right)}{d^2}-\frac{2}{d\sqrt{x}}\right) - \frac{b\log\left(\left(e\sqrt{x}+d\right)^n c\right)}{x}-\frac{a}{x}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*e*n*(2*e*log(e*sqrt(x) + d)/d^2 - e*log(x)/d^2 - 2/(d*sqrt(x))) - b*l
og((e*sqrt(x) + d)^n*c)/x - a/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(62) = 124$.

Time = 0.31 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.06

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}\,dx = -\frac{\frac{be^3n\log\left(e\sqrt{x}+d\right)}{\left(e\sqrt{x}+d\right)^2-2\left(e\sqrt{x}+d\right)d+d^2}-\frac{be^3n\log\left(e\sqrt{x}+d\right)}{d^2}+\frac{be^3n\log\left(e\sqrt{x}\right)}{d^2}+\frac{\left(e\sqrt{x}+d\right)be^3n-bde^3n+bde^3\log\left(c\right)+ade^3}{\left(e\sqrt{x}+d\right)^2d-2\left(e\sqrt{x}+d\right)d^2+d^3}}{e}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^2,x, algorithm="giac")
```

```
[Out] -(b*e^3*n*log(e*sqrt(x) + d)/((e*sqrt(x) + d)^2 - 2*(e*sqrt(x) + d)*d + d^2
) - b*e^3*n*log(e*sqrt(x) + d)/d^2 + b*e^3*n*log(e*sqrt(x))/d^2 + ((e*sqrt(
x) + d)*b*e^3*n - b*d*e^3*n + b*d*e^3*log(c) + a*d*e^3)/((e*sqrt(x) + d)^2*
d - 2*(e*sqrt(x) + d)*d^2 + d^3))/e
```

Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^2}\,dx=\frac{2\,b\,e^2\,n\,\mathrm{atanh}\left(\frac{2\,e\,\sqrt{x}}{d}+1\right)}{d^2}-\frac{b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)}{x}-\frac{b\,e\,n}{d\,\sqrt{x}}-\frac{a}{x}$$

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))/x^2,x)
[Out] (2*b*e^2*n*atanh((2*e*x^(1/2))/d + 1))/d^2 - (b*log(c*(d + e*x^(1/2))^n))/x
 - (b*e*n)/(d*x^(1/2)) - a/x
```

3.406 $\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 109

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx = -\frac{ben}{6dx^{3/2}}+\frac{be^2n}{4d^2x}-\frac{be^3n}{2d^3\sqrt{x}}+\frac{be^4n\log\left(d+e\sqrt{x}\right)}{2d^4} - \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2}-\frac{be^4n\log(x)}{4d^4}$$

```
[Out] -1/6*b*e*n/d/x^(3/2)+1/4*b*e^2*n/d^2/x-1/4*b*e^4*n*ln(x)/d^4+1/2*b*e^4*n*ln
(d+e*x^(1/2))/d^4+1/2*(-a-b*ln(c*(d+e*x^(1/2))^n))/x^2-1/2*b*e^3*n/d^3/x^(1
/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 46}

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx = -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2}+\frac{be^4n\log\left(d+e\sqrt{x}\right)}{2d^4} - \frac{be^4n\log(x)}{4d^4}-\frac{be^3n}{2d^3\sqrt{x}}+\frac{be^2n}{4d^2x}-\frac{ben}{6dx^{3/2}}$$

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^3,x]
```

```
[Out] -1/6*(b*e*n)/(d*x^(3/2)) + (b*e^2*n)/(4*d^2*x) - (b*e^3*n)/(2*d^3*Sqrt[x])
+ (b*e^4*n*Log[d + e*Sqrt[x]])/(2*d^4) - (a + b*Log[c*(d + e*Sqrt[x])^n])/(
2*x^2) - (b*e^4*n*Log[x])/(4*d^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^5}\,dx, x, \sqrt{x}\right)\\
&= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\text{Subst}\left(\int \frac{1}{x^4(d+ex)}\,dx, x, \sqrt{x}\right)\\
&= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2}\\
&\quad+\frac{1}{2}(ben)\text{Subst}\left(\int \left(\frac{1}{dx^4}-\frac{e}{d^2x^3}+\frac{e^2}{d^3x^2}-\frac{e^3}{d^4x}+\frac{e^4}{d^4(d+ex)}\right)\,dx, x, \sqrt{x}\right)\\
&= -\frac{ben}{6dx^{3/2}}+\frac{be^2n}{4d^2x}-\frac{be^3n}{2d^3\sqrt{x}}+\frac{be^4n\log\left(d+e\sqrt{x}\right)}{2d^4}-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2}-\frac{be^4n\log(x)}{4d^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.92

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx=-\frac{a}{2x^2}-\frac{b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{2x^2}+\frac{1}{4}ben\left(-\frac{2}{3dx^{3/2}}+\frac{e}{d^2x}-\frac{2e^2}{d^3\sqrt{x}}+\frac{2e^3\log\left(d+e\sqrt{x}\right)}{d^4}-\frac{e^3\log(x)}{d^4}\right)$$

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*Log[c*(d + e*Sqrt[x])^n])/(2*x^2) + (b*e*n*(-2/(3*d*x^(3/2)
) + e/(d^2*x) - (2*e^2)/(d^3*Sqrt[x]) + (2*e^3*Log[d + e*Sqrt[x]])/d^4 - (e
^3*Log[x])/d^4))/4
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx = -\frac{6\,be^4nx^2\log\left(\sqrt{x}\right)-3\,bd^2e^2nx+6\,bd^4\log\left(c\right)+6\,ad^4-6\left(be^4nx^2-bd^4n\right)\log\left(e\sqrt{x}+d\right)+2\left(3\,bde^3nx+\cdots\right.}{12\,d^4x^2}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^3,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b*e^4*n*x^2*log(sqrt(x)) - 3*b*d^2*e^2*n*x + 6*b*d^4*log(c) + 6*a*
d^4 - 6*(b*e^4*n*x^2 - b*d^4*n)*log(e*sqrt(x) + d) + 2*(3*b*d*e^3*n*x + b*d
^3*e*n)*sqrt(x))/(d^4*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))/x**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.77

$$\begin{aligned}&\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx\\&=\frac{1}{12}\,ben\left(\frac{6\,e^3\log\left(e\sqrt{x}+d\right)}{d^4}-\frac{3\,e^3\log\left(x\right)}{d^4}-\frac{6\,e^2x-3\,de\sqrt{x}+2\,d^2}{d^3x^{\frac{3}{2}}}\right)\\&\quad-\frac{b\log\left(\left(e\sqrt{x}+d\right)^nc\right)}{2\,x^2}-\frac{a}{2\,x^2}\end{aligned}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^3,x, algorithm="maxima")
[Out] 1/12*b*e*n*(6*e^3*log(e*sqrt(x) + d)/d^4 - 3*e^3*log(x)/d^4 - (6*e^2*x - 3*
d*e*sqrt(x) + 2*d^2)/(d^3*x^(3/2))) - 1/2*b*log((e*sqrt(x) + d)^n*c)/x^2 -
1/2*a/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(89) = 178.

Time = 0.31 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.28

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx = -\frac{\frac{6\,be^5n\log\left(e\sqrt{x}+d\right)}{\left(e\sqrt{x}+d\right)^4-4\left(e\sqrt{x}+d\right)^3d+6\left(e\sqrt{x}+d\right)^2d^2-4\left(e\sqrt{x}+d\right)d^3+d^4}-\frac{6\,be^5n\log\left(e\sqrt{x}+d\right)}{d^4}+\frac{6\,be^5n\log\left(e\sqrt{x}\right)}{d^4}+\frac{6\left(e\sqrt{x}+d\right)^3be^5n-21\left(e\sqrt{x}+\right.}{\left(e\sqrt{x}+d\right)^4d^3-}}{12\,e}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^3,x, algorithm="giac")
[Out] -1/12*(6*b*e^5*n*log(e*sqrt(x) + d)/((e*sqrt(x) + d)^4 - 4*(e*sqrt(x) + d)^
3*d + 6*(e*sqrt(x) + d)^2*d^2 - 4*(e*sqrt(x) + d)*d^3 + d^4) - 6*b*e^5*n*lo
g(e*sqrt(x) + d)/d^4 + 6*b*e^5*n*log(e*sqrt(x))/d^4 + (6*(e*sqrt(x) + d)^3*
```

```
b*e^5*n - 21*(e*sqrt(x) + d)^2*b*d*e^5*n + 26*(e*sqrt(x) + d)*b*d^2*e^5*n -
 11*b*d^3*e^5*n + 6*b*d^3*e^5*log(c) + 6*a*d^3*e^5)/((e*sqrt(x) + d)^4*d^3
- 4*(e*sqrt(x) + d)^3*d^4 + 6*(e*sqrt(x) + d)^2*d^5 - 4*(e*sqrt(x) + d)*d^6
 + d^7))/e
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.76

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^3}\,dx=\frac{b\,e^4\,n\,\mathrm{atanh}\left(\frac{2\,e\,\sqrt{x}}{d}+1\right)}{d^4}-\frac{\frac{b\,e\,n}{3\,d}+\frac{b\,e^3\,n\,x}{d^3}-\frac{b\,e^2\,n\,\sqrt{x}}{2\,d^2}}{2\,x^{3/2}}-\frac{b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))/x^3,x)
```

```
[Out] (b*e^4*n*atanh((2*e*x^(1/2))/d + 1))/d^4 - ((b*e*n)/(3*d) + (b*e^3*n*x)/d^3
 - (b*e^2*n*x^(1/2))/(2*d^2))/(2*x^(3/2)) - (b*log(c*(d + e*x^(1/2))^n))/(2
*x^2) - a/(2*x^2)
```

3.407 $\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 141

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx = -\frac{ben}{15dx^{5/2}}+\frac{be^2n}{12d^2x^2}-\frac{be^3n}{9d^3x^{3/2}}+\frac{be^4n}{6d^4x}-\frac{be^5n}{3d^5\sqrt{x}}+\frac{be^6n\log\left(d+e\sqrt{x}\right)}{3d^6}-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3x^3}-\frac{be^6n\log(x)}{6d^6}$$

```
[Out] -1/15*b*e*n/d/x^(5/2)+1/12*b*e^2*n/d^2/x^2-1/9*b*e^3*n/d^3/x^(3/2)+1/6*b*e^
4*n/d^4/x-1/6*b*e^6*n*ln(x)/d^6+1/3*b*e^6*n*ln(d+e*x^(1/2))/d^6+1/3*(-a-b*l
n(c*(d+e*x^(1/2))^n))/x^3-1/3*b*e^5*n/d^5/x^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 46}

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx = -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3x^3}+\frac{be^6n\log\left(d+e\sqrt{x}\right)}{3d^6}-\frac{be^6n\log(x)}{6d^6}-\frac{be^5n}{3d^5\sqrt{x}}+\frac{be^4n}{6d^4x}-\frac{be^3n}{9d^3x^{3/2}}+\frac{be^2n}{12d^2x^2}-\frac{ben}{15dx^{5/2}}$$

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^4,x]
```

```
[Out] -1/15*(b*e*n)/(d*x^(5/2)) + (b*e^2*n)/(12*d^2*x^2) - (b*e^3*n)/(9*d^3*x^(3/
2)) + (b*e^4*n)/(6*d^4*x) - (b*e^5*n)/(3*d^5*Sqrt[x]) + (b*e^6*n*Log[d + e*
Sqrt[x]])/(3*d^6) - (a + b*Log[c*(d + e*Sqrt[x])^n])/(3*x^3) - (b*e^6*n*Log
[x])/(6*d^6)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^7}\,dx,x,\sqrt{x}\right)\\
&= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3x^3}+\frac{1}{3}(ben)\text{Subst}\left(\int \frac{1}{x^6(d+ex)}\,dx,x,\sqrt{x}\right)\\
&= -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3x^3}+\frac{1}{3}(ben)\text{Subst}\left(\int \left(\frac{1}{dx^6}-\frac{e}{d^2x^5}+\frac{e^2}{d^3x^4}-\frac{e^3}{d^4x^3}\right.\right.\\
&\qquad\left.\left.+\frac{e^4}{d^5x^2}-\frac{e^5}{d^6x}+\frac{e^6}{d^6(d+ex)}\right)\,dx,x,\sqrt{x}\right)\\
&= -\frac{ben}{15dx^{5/2}}+\frac{be^2n}{12d^2x^2}-\frac{be^3n}{9d^3x^{3/2}}+\frac{be^4n}{6d^4x}-\frac{be^5n}{3d^5\sqrt{x}}\\
&\quad+\frac{be^6n\log\left(d+e\sqrt{x}\right)}{3d^6}-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{3x^3}-\frac{be^6n\log(x)}{6d^6}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91

$$\int \frac{a+b\log \left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx = -\frac{a}{3x^3}-\frac{b\log \left(c\left(d+e\sqrt{x}\right)^n\right)}{3x^3}+\frac{1}{6}ben\left(-\frac{2}{5dx^{5/2}}+\frac{e}{2d^2x^2}-\frac{2e^2}{3d^3x^{3/2}}+\frac{e^3}{d^4x}-\frac{2e^4}{d^5\sqrt{x}}+\frac{2e^5\log \left(d+e\sqrt{x}\right)}{d^6}-\frac{e^5\log(x)}{d^6}\right)$$

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (b*Log[c*(d + e*Sqrt[x])^n])/(3*x^3) + (b*e*n*(-2/(5*d*x^(5/2)
) + e/(2*d^2*x^2) - (2*e^2)/(3*d^3*x^(3/2)) + e^3/(d^4*x) - (2*e^4)/(d^5*Sq
rt[x]) + (2*e^5*Log[d + e*Sqrt[x]])/d^6 - (e^5*Log[x])/d^6))/6
```

Maple [F]

$$\int \frac{a+b\ln \left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.88

$$\int \frac{a+b\log \left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx = -\frac{60\,be^6nx^3\log\left(\sqrt{x}\right)-30\,bd^2e^4nx^2-15\,bd^4e^2nx+60\,bd^6\log\left(c\right)+60\,ad^6-60\,\left(be^6nx^3-bd^6n\right)\log\left(e\sqrt{x}\right.}{180\,d^6x^3}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^4,x, algorithm="fricas")
```

```
[Out] -1/180*(60*b*e^6*n*x^3*log(sqrt(x)) - 30*b*d^2*e^4*n*x^2 - 15*b*d^4*e^2*n*x
 + 60*b*d^6*log(c) + 60*a*d^6 - 60*(b*e^6*n*x^3 - b*d^6*n)*log(e*sqrt(x) +
d) + 4*(15*b*d*e^5*n*x^2 + 5*b*d^3*e^3*n*x + 3*b*d^5*e*n)*sqrt(x))/(d^6*x^3
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))/x**4,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.75

$$\begin{aligned}\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx &= \frac{1}{180}\,ben\left(\frac{60\,e^5\log\left(e\sqrt{x}+d\right)}{d^6}-\frac{30\,e^5\log\left(x\right)}{d^6}-\frac{60\,e^4x^2-30\,de^3x^{\frac{3}{2}}+20\,d^2e^2x-15\,d^3e\sqrt{x}+12\,d^4}{d^5x^{\frac{5}{2}}}\right)\\&\quad-\frac{b\log\left(\left(e\sqrt{x}+d\right)^nc\right)}{3\,x^3}-\frac{a}{3\,x^3}\end{aligned}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^4,x, algorithm="maxima")
[Out] 1/180*b*e*n*(60*e^5*log(e*sqrt(x) + d)/d^6 - 30*e^5*log(x)/d^6 - (60*e^4*x^
2 - 30*d*e^3*x^(3/2) + 20*d^2*e^2*x - 15*d^3*e*sqrt(x) + 12*d^4)/(d^5*x^(5/
2))) - 1/3*b*log((e*sqrt(x) + d)^n*c)/x^3 - 1/3*a/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(115) = 230.

Time = 0.41 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.43

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx = -\frac{\frac{60\,be^7n\log\left(e\sqrt{x}+d\right)}{\left(e\sqrt{x}+d\right)^6-6\left(e\sqrt{x}+d\right)^5d+15\left(e\sqrt{x}+d\right)^4d^2-20\left(e\sqrt{x}+d\right)^3d^3+15\left(e\sqrt{x}+d\right)^2d^4-6\left(e\sqrt{x}+d\right)d^5+d^6}-\frac{60\,be^7n\log\left(e\sqrt{x}+d\right)}{d^6}+\frac{60\,be^7n\log\left(e\sqrt{}\right.}{d^6}}{}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))/x^4,x, algorithm="giac")
[Out] -1/180*(60*b*e^7*n*log(e*sqrt(x) + d)/((e*sqrt(x) + d)^6 - 6*(e*sqrt(x) + d
)^5*d + 15*(e*sqrt(x) + d)^4*d^2 - 20*(e*sqrt(x) + d)^3*d^3 + 15*(e*sqrt(x)
 + d)^2*d^4 - 6*(e*sqrt(x) + d)*d^5 + d^6) - 60*b*e^7*n*log(e*sqrt(x) + d)/
```

```
d^6 + 60*b*e^7*n*log(e*sqrt(x))/d^6 + (60*(e*sqrt(x) + d)^5*b*e^7*n - 330*(
e*sqrt(x) + d)^4*b*d*e^7*n + 740*(e*sqrt(x) + d)^3*b*d^2*e^7*n - 855*(e*sqr
t(x) + d)^2*b*d^3*e^7*n + 522*(e*sqrt(x) + d)*b*d^4*e^7*n - 137*b*d^5*e^7*n
 + 60*b*d^5*e^7*log(c) + 60*a*d^5*e^7)/((e*sqrt(x) + d)^6*d^5 - 6*(e*sqrt(x
) + d)^5*d^6 + 15*(e*sqrt(x) + d)^4*d^7 - 20*(e*sqrt(x) + d)^3*d^8 + 15*(e*
sqrt(x) + d)^2*d^9 - 6*(e*sqrt(x) + d)*d^10 + d^11))/e
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.78

$$\int \frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{x^4}\,dx = \frac{2\,b\,e^6\,n\,\mathrm{atanh}\left(\frac{2\,e\,\sqrt{x}}{d}+1\right)}{3\,d^6} - \frac{\frac{b\,e\,n}{5\,d}+\frac{b\,e^3\,n\,x}{3\,d^3}-\frac{b\,e^2\,n\,\sqrt{x}}{4\,d^2}+\frac{b\,e^5\,n\,x^2}{d^5}-\frac{b\,e^4\,n\,x^{3/2}}{2\,d^4}}{3\,x^{5/2}} - \frac{b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)}{3\,x^3} - \frac{a}{3\,x^3}$$

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))/x^4,x)
```

```
[Out] (2*b*e^6*n*atanh((2*e*x^(1/2))/d + 1))/(3*d^6) - ((b*e*n)/(5*d) + (b*e^3*n*
x)/(3*d^3) - (b*e^2*n*x^(1/2))/(4*d^2) + (b*e^5*n*x^2)/d^5 - (b*e^4*n*x^(3/
2))/(2*d^4))/(3*x^(5/2)) - (b*log(c*(d + e*x^(1/2))^n))/(3*x^3) - a/(3*x^3)
```

3.408 $\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 480

$$\begin{aligned}\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx &= \frac{5b^2d^4n^2\left(d+e\sqrt{x}\right)^2}{2e^6}-\frac{40b^2d^3n^2\left(d+e\sqrt{x}\right)^3}{27e^6}\\
&+\frac{5b^2d^2n^2\left(d+e\sqrt{x}\right)^4}{8e^6}-\frac{4b^2dn^2\left(d+e\sqrt{x}\right)^5}{25e^6}\\
&+\frac{b^2n^2\left(d+e\sqrt{x}\right)^6}{54e^6}-\frac{4b^2d^5n^2\sqrt{x}}{e^5}\\
&+\frac{b^2d^6n^2\log^2\left(d+e\sqrt{x}\right)}{3e^6}\\
&+\frac{4bd^5n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^6}\\
&-\frac{5bd^4n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^6}\\
&+\frac{40bd^3n\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9e^6}\\
&-\frac{5bd^2n\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^6}\\
&+\frac{4bdn\left(d+e\sqrt{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{5e^6}\\
&-\frac{bn\left(d+e\sqrt{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9e^6}\\
&-\frac{2bd^6n\log\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3e^6}\\
&+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\end{aligned}$$

```
[Out] 1/3*b^2*d^6*n^2*ln(d+e*x^(1/2))^2/e^6-2/3*b*d^6*n*ln(d+e*x^(1/2))*(a+b*ln(c
*(d+e*x^(1/2))^n))/e^6+1/3*x^3*(a+b*ln(c*(d+e*x^(1/2))^n))^2-4*b^2*d^5*n^2*
x^(1/2)/e^5+4*b*d^5*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))/e^6+5/2*b^2
*d^4*n^2*(d+e*x^(1/2))^2/e^6-5*b*d^4*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(
1/2))^2/e^6-40/27*b^2*d^3*n^2*(d+e*x^(1/2))^3/e^6+40/9*b*d^3*n*(a+b*ln(c*(d
+e*x^(1/2))^n))*(d+e*x^(1/2))^3/e^6+5/8*b^2*d^2*n^2*(d+e*x^(1/2))^4/e^6-5/2
*b*d^2*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^4/e^6-4/25*b^2*d*n^2*(d+
e*x^(1/2))^5/e^6+4/5*b*d*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^5/e^6+
1/54*b^2*n^2*(d+e*x^(1/2))^6/e^6-1/9*b*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x
^(1/2))^6/e^6
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$\begin{aligned}\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx=&-\frac{2bd^6n\log\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3e^6}\\&+\frac{4bd^5n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^6}\\&-\frac{5bd^4n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^6}\\&+\frac{40bd^3n\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9e^6}\\&-\frac{5bd^2n\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^6}\\&+\frac{4bdn\left(d+e\sqrt{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{5e^6}\\&-\frac{bn\left(d+e\sqrt{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9e^6}\\&+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\\&+\frac{b^2d^6n^2\log^2\left(d+e\sqrt{x}\right)}{3e^6}\\&-\frac{4b^2d^5n^2\sqrt{x}}{e^5}+\frac{5b^2d^4n^2\left(d+e\sqrt{x}\right)^2}{2e^6}\\&-\frac{40b^2d^3n^2\left(d+e\sqrt{x}\right)^3}{27e^6}+\frac{5b^2d^2n^2\left(d+e\sqrt{x}\right)^4}{8e^6}\\&-\frac{4b^2dn^2\left(d+e\sqrt{x}\right)^5}{25e^6}+\frac{b^2n^2\left(d+e\sqrt{x}\right)^6}{54e^6}\end{aligned}$$

```
[In] Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

```
[Out] (5*b^2*d^4*n^2*(d + e*Sqrt[x])^2)/(2*e^6) - (40*b^2*d^3*n^2*(d + e*Sqrt[x])
^3)/(27*e^6) + (5*b^2*d^2*n^2*(d + e*Sqrt[x])^4)/(8*e^6) - (4*b^2*d*n^2*(d
+ e*Sqrt[x])^5)/(25*e^6) + (b^2*n^2*(d + e*Sqrt[x])^6)/(54*e^6) - (4*b^2*d^
5*n^2*Sqrt[x])/e^5 + (b^2*d^6*n^2*Log[d + e*Sqrt[x]]^2)/(3*e^6) + (4*b*d^5*
n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n]))/e^6 - (5*b*d^4*n*(d + e
*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/e^6 + (40*b*d^3*n*(d + e*Sqrt
[x])^3*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(9*e^6) - (5*b*d^2*n*(d + e*Sqrt[x
])^4*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*e^6) + (4*b*d*n*(d + e*Sqrt[x])^5
*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(5*e^6) - (b*n*(d + e*Sqrt[x])^6*(a + b*
Log[c*(d + e*Sqrt[x])^n]))/(9*e^6) - (2*b*d^6*n*Log[d + e*Sqrt[x]]*(a + b*L
og[c*(d + e*Sqrt[x])^n]))/(3*e^6) + (x^3*(a + b*Log[c*(d + e*Sqrt[x])^n])^2
)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
```

```
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^5(a+b\log\left(c(d+ex)^n\right))^2\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-\frac{1}{3}(2ben)\text{Subst}\left(\int\frac{x^6(a+b\log\left(c(d+ex)^n\right))}{d+ex}\,dx,x,\sqrt{x}\right)\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\\
&\quad-\frac{1}{3}(2bn)\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6\left(a+b\log\left(cx^n\right)\right)}{x}\,dx,x,d+e\sqrt{x}\right)
\end{aligned}
$$

$$= \frac{4bd^5n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^6} - \frac{5bd^4n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^6} + \frac{40bd^3n\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9e^6} - \frac{5bd^2n\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^6} + \frac{4bdn\left(d+e\sqrt{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{5e^6} - \frac{bn\left(d+e\sqrt{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9e^6} - \frac{2bd^6n\log\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3e^6} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 + \frac{1}{3}\left(2b^2n^2\right)\text{Subst}\left(\int\frac{x\left(-360d^5+450d^4x-400d^3x^2+225d^2x^3-72dx^4+10x^5\right)+60d^6\log(x)}{60e^6x}\,dx + e\sqrt{x}\right)$$

$$= \frac{4bd^5n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^6} - \frac{5bd^4n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^6} + \frac{40bd^3n\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9e^6} - \frac{5bd^2n\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^6} + \frac{4bdn\left(d+e\sqrt{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{5e^6} - \frac{bn\left(d+e\sqrt{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9e^6} - \frac{2bd^6n\log\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3e^6} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 + \frac{\left(b^2n^2\right)\text{Subst}\left(\int\frac{x\left(-360d^5+450d^4x-400d^3x^2+225d^2x^3-72dx^4+10x^5\right)+60d^6\log(x)}{x}\,dx, x, d+e\sqrt{x}\right)}{90e^6}$$

$$= \frac{4bd^5n(d+e\sqrt{x})\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)}{e^6} - \frac{5bd^4n(d+e\sqrt{x})^2\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)}{e^6} + \frac{40bd^3n(d+e\sqrt{x})^3\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)}{9e^6} - \frac{5bd^2n(d+e\sqrt{x})^4\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)}{2e^6} + \frac{4bdn(d+e\sqrt{x})^5\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)}{5e^6} - \frac{bn(d+e\sqrt{x})^6\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)}{9e^6} - \frac{2bd^6n\log\left(d+e\sqrt{x}\right)\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)}{3e^6} + \frac{1}{3}x^3\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^2 + \frac{(b^2n^2)\,\text{Subst}\left(\int\left(-360d^5+450d^4x-400d^3x^2+225d^2x^3-72dx^4+10x^5+\frac{60d^6\log(x)}{x}\right)dx, x, d+\right.}{90e^6}$$

$$= \frac{5b^2d^4n^2(d+e\sqrt{x})^2}{2e^6} - \frac{40b^2d^3n^2(d+e\sqrt{x})^3}{27e^6} + \frac{5b^2d^2n^2(d+e\sqrt{x})^4}{8e^6} - \frac{4b^2dn^2(d+e\sqrt{x})^5}{25e^6} + \frac{b^2n^2(d+e\sqrt{x})^6}{54e^6} - \frac{4b^2d^5n^2\sqrt{x}}{e^5} + \frac{4bd^5n(d+e\sqrt{x})\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)}{e^6} - \frac{5bd^4n(d+e\sqrt{x})^2\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)}{e^6} + \frac{40bd^3n(d+e\sqrt{x})^3\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)}{9e^6} - \frac{5bd^2n(d+e\sqrt{x})^4\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)}{2e^6} + \frac{4bdn(d+e\sqrt{x})^5\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)}{5e^6} - \frac{bn(d+e\sqrt{x})^6\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)}{9e^6} - \frac{2bd^6n\log\left(d+e\sqrt{x}\right)\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)}{3e^6} + \frac{1}{3}x^3\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^2 + \frac{(2b^2d^6n^2)\,\text{Subst}\left(\int\frac{\log(x)}{x}dx, x, d+e\sqrt{x}\right)}{3e^6}$$

$$
\begin{aligned}
&= \frac{5b^2d^4n^2\left(d+e\sqrt{x}\right)^2}{2e^6} - \frac{40b^2d^3n^2\left(d+e\sqrt{x}\right)^3}{27e^6} + \frac{5b^2d^2n^2\left(d+e\sqrt{x}\right)^4}{8e^6} \\
&- \frac{4b^2dn^2\left(d+e\sqrt{x}\right)^5}{25e^6} + \frac{b^2n^2\left(d+e\sqrt{x}\right)^6}{54e^6} - \frac{4b^2d^5n^2\sqrt{x}}{e^5} \\
&+ \frac{b^2d^6n^2\log^2\left(d+e\sqrt{x}\right)}{3e^6} + \frac{4bd^5n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^6} \\
&- \frac{5bd^4n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^6} \\
&+ \frac{40bd^3n\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9e^6} \\
&- \frac{5bd^2n\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^6} \\
&+ \frac{4bdn\left(d+e\sqrt{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{5e^6} \\
&- \frac{bn\left(d+e\sqrt{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9e^6} \\
&- \frac{2bd^6n\log\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3e^6} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.65

$$
\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx
= \frac{e\sqrt{x}\left(1800a^2e^5x^{5/2}+60abn\left(60d^5-30d^4e\sqrt{x}+20d^3e^2x-15d^2e^3x^{3/2}+12de^4x^2-10e^5x^{5/2}\right)+b^2n^2\left(-882\right.\right.}{}
$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

```
[Out] (e*Sqrt[x]*(1800*a^2*e^5*x^(5/2) + 60*a*b*n*(60*d^5 - 30*d^4*e*Sqrt[x] + 20
*d^3*e^2*x - 15*d^2*e^3*x^(3/2) + 12*d*e^4*x^2 - 10*e^5*x^(5/2)) + b^2*n^2*
(-8820*d^5 + 2610*d^4*e*Sqrt[x] - 1140*d^3*e^2*x + 555*d^2*e^3*x^(3/2) - 26
4*d*e^4*x^2 + 100*e^5*x^(5/2))) + 180*b*d^6*n*(-20*a + 49*b*n)*Log[d + e*Sq
rt[x]] - 60*b*e*Sqrt[x]*(-60*a*e^5*x^(5/2) + b*n*(-60*d^5 + 30*d^4*e*Sqrt[x
] - 20*d^3*e^2*x + 15*d^2*e^3*x^(3/2) - 12*d*e^4*x^2 + 10*e^5*x^(5/2)))*Log
[c*(d + e*Sqrt[x])^n] - 1800*b^2*(d^6 - e^6*x^3)*Log[c*(d + e*Sqrt[x])^n]^2
)/(5400*e^6)
```

Maple [F]

$$\int x^2 \left(a + b \ln \left(c \left(d + e\sqrt{x}\right)^n\right)\right)^2 dx$$

[In] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n))^2,x)

[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n))^2,x)

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.01

$$\int x^2 \left(a + b \log \left(c \left(d + e\sqrt{x}\right)^n\right)\right)^2 dx = \frac{1800\, b^2 e^6 x^3 \log (c)^2 + 100\, (b^2 e^6 n^2 - 6\, abe^6 n + 18\, a^2 e^6) x^3 + 15\, (37\, b^2 d^2 e^4 n^2 - 60\, abd^2 e^4 n) x^2 + 1800\, (b^2 e^6 n}{}$$

[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="fricas")

[Out] 1/5400*(1800*b^2*e^6*x^3*log(c)^2 + 100*(b^2*e^6*n^2 - 6*a*b*e^6*n + 18*a^2*e^6)*x^3 + 15*(37*b^2*d^2*e^4*n^2 - 60*a*b*d^2*e^4*n)*x^2 + 1800*(b^2*e^6*n^2*x^3 - b^2*d^6*n^2)*log(e*sqrt(x) + d)^2 + 90*(29*b^2*d^4*e^2*n^2 - 20*a*b*d^4*e^2*n)*x - 60*(15*b^2*d^2*e^4*n^2*x^2 + 30*b^2*d^4*e^2*n^2*x - 147*b^2*d^6*n^2 + 60*a*b*d^6*n + 10*(b^2*e^6*n^2 - 6*a*b*e^6*n)*x^3 - 60*(b^2*e^6*n*x^3 - b^2*d^6*n)*log(c) - 4*(3*b^2*d*e^5*n^2*x^2 + 5*b^2*d^3*e^3*n^2*x + 15*b^2*d^5*e*n^2)*sqrt(x))*log(e*sqrt(x) + d) - 300*(3*b^2*d^2*e^4*n*x^2 + 6*b^2*d^4*e^2*n*x + 2*(b^2*e^6*n - 6*a*b*e^6)*x^3)*log(c) - 12*(735*b^2*d^5*e*n^2 - 300*a*b*d^5*e*n + 2*(11*b^2*d*e^5*n^2 - 30*a*b*d*e^5*n)*x^2 + 5*(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)*x - 20*(3*b^2*d*e^5*n*x^2 + 5*b^2*d^3*e^3*n*x + 15*b^2*d^5*e*n)*log(c))*sqrt(x))/e^6

Sympy [F]

$$\int x^2 \left(a + b \log \left(c \left(d + e\sqrt{x}\right)^n\right)\right)^2\, dx = \int x^2 \left(a + b \log \left(c \left(d + e\sqrt{x}\right)^n\right)\right)^2\, dx$$

[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))**n))**2,x)

[Out] Integral(x**2*(a + b*log(c*(d + e*sqrt(x))**n))**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.68

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$
$$= \frac{1}{3}\,b^2x^3\log\left(\left(e\sqrt{x}+d\right)^nc\right)^2 + \frac{2}{3}\,abx^3\log\left(\left(e\sqrt{x}+d\right)^nc\right) + \frac{1}{3}\,a^2x^3 - \frac{1}{90}\,aben\left(\frac{60\,d^6\log\left(e\sqrt{x}+d\right)}{e^7} + \frac{10\,e^5x^3 - 12\,de^4x^{\frac{5}{2}} + 15\,d^2e^3x^2 - 20\,d^3e^2x^{\frac{3}{2}} + 30\,d^4ex - 60\,d^5\sqrt{x}}{e^6}\right) - \frac{1}{5400}\left(60\,en\left(\frac{60\,d^6\log\left(e\sqrt{x}+d\right)}{e^7} + \frac{10\,e^5x^3 - 12\,de^4x^{\frac{5}{2}} + 15\,d^2e^3x^2 - 20\,d^3e^2x^{\frac{3}{2}} + 30\,d^4ex - 60\,d^5\sqrt{x}}{e^6}\right.\right.$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*log((e*sqrt(x) + d)^n*c)^2 + 2/3*a*b*x^3*log((e*sqrt(x) + d)^n*
c) + 1/3*a^2*x^3 - 1/90*a*b*e*n*(60*d^6*log(e*sqrt(x) + d)/e^7 + (10*e^5*x^
3 - 12*d*e^4*x^(5/2) + 15*d^2*e^3*x^2 - 20*d^3*e^2*x^(3/2) + 30*d^4*e*x - 6
0*d^5*sqrt(x))/e^6) - 1/5400*(60*e*n*(60*d^6*log(e*sqrt(x) + d)/e^7 + (10*e
^5*x^3 - 12*d*e^4*x^(5/2) + 15*d^2*e^3*x^2 - 20*d^3*e^2*x^(3/2) + 30*d^4*e*
x - 60*d^5*sqrt(x))/e^6)*log((e*sqrt(x) + d)^n*c) - (100*e^6*x^3 - 264*d*e^
5*x^(5/2) + 555*d^2*e^4*x^2 + 1800*d^6*log(e*sqrt(x) + d)^2 - 1140*d^3*e^3*
x^(3/2) + 2610*d^4*e^2*x + 8820*d^6*log(e*sqrt(x) + d) - 8820*d^5*e*sqrt(x)
)*n^2/e^6)*b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 930 vs. 2(412) = 824.

Time = 0.32 (sec) , antiderivative size = 930, normalized size of antiderivative = 1.94

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="giac")
```

```
[Out] 1/5400*(1800*b^2*e*x^3*log(c)^2 + 3600*a*b*e*x^3*log(c) + 1800*a^2*e*x^3 +
(1800*(e*sqrt(x) + d)^6*log(e*sqrt(x) + d)^2/e^5 - 10800*(e*sqrt(x) + d)^5*
d*log(e*sqrt(x) + d)^2/e^5 + 27000*(e*sqrt(x) + d)^4*d^2*log(e*sqrt(x) + d)
^2/e^5 - 36000*(e*sqrt(x) + d)^3*d^3*log(e*sqrt(x) + d)^2/e^5 + 27000*(e*sq
rt(x) + d)^2*d^4*log(e*sqrt(x) + d)^2/e^5 - 10800*(e*sqrt(x) + d)*d^5*log(e
*sqrt(x) + d)^2/e^5 - 600*(e*sqrt(x) + d)^6*log(e*sqrt(x) + d)/e^5 + 4320*(
e*sqrt(x) + d)^5*d*log(e*sqrt(x) + d)/e^5 - 13500*(e*sqrt(x) + d)^4*d^2*log
```

```
(e*sqrt(x) + d)/e^5 + 24000*(e*sqrt(x) + d)^3*d^3*log(e*sqrt(x) + d)/e^5 -
27000*(e*sqrt(x) + d)^2*d^4*log(e*sqrt(x) + d)/e^5 + 21600*(e*sqrt(x) + d)*
d^5*log(e*sqrt(x) + d)/e^5 + 100*(e*sqrt(x) + d)^6/e^5 - 864*(e*sqrt(x) + d
)^5*d/e^5 + 3375*(e*sqrt(x) + d)^4*d^2/e^5 - 8000*(e*sqrt(x) + d)^3*d^3/e^5
 + 13500*(e*sqrt(x) + d)^2*d^4/e^5 - 21600*(e*sqrt(x) + d)*d^5/e^5)*b^2*n^2
 + 60*(60*(e*sqrt(x) + d)^6*log(e*sqrt(x) + d)/e^5 - 360*(e*sqrt(x) + d)^5*
d*log(e*sqrt(x) + d)/e^5 + 900*(e*sqrt(x) + d)^4*d^2*log(e*sqrt(x) + d)/e^5
 - 1200*(e*sqrt(x) + d)^3*d^3*log(e*sqrt(x) + d)/e^5 + 900*(e*sqrt(x) + d)^
2*d^4*log(e*sqrt(x) + d)/e^5 - 360*(e*sqrt(x) + d)*d^5*log(e*sqrt(x) + d)/e
^5 - 10*(e*sqrt(x) + d)^6/e^5 + 72*(e*sqrt(x) + d)^5*d/e^5 - 225*(e*sqrt(x)
 + d)^4*d^2/e^5 + 400*(e*sqrt(x) + d)^3*d^3/e^5 - 450*(e*sqrt(x) + d)^2*d^4
/e^5 + 360*(e*sqrt(x) + d)*d^5/e^5)*b^2*n*log(c) + 60*(60*(e*sqrt(x) + d)^6
*log(e*sqrt(x) + d)/e^5 - 360*(e*sqrt(x) + d)^5*d*log(e*sqrt(x) + d)/e^5 +
900*(e*sqrt(x) + d)^4*d^2*log(e*sqrt(x) + d)/e^5 - 1200*(e*sqrt(x) + d)^3*d
^3*log(e*sqrt(x) + d)/e^5 + 900*(e*sqrt(x) + d)^2*d^4*log(e*sqrt(x) + d)/e^
5 - 360*(e*sqrt(x) + d)*d^5*log(e*sqrt(x) + d)/e^5 - 10*(e*sqrt(x) + d)^6/e
^5 + 72*(e*sqrt(x) + d)^5*d/e^5 - 225*(e*sqrt(x) + d)^4*d^2/e^5 + 400*(e*sq
rt(x) + d)^3*d^3/e^5 - 450*(e*sqrt(x) + d)^2*d^4/e^5 + 360*(e*sqrt(x) + d)*
d^5/e^5)*a*b*n)/e
```

Mupad [B] (verification not implemented)

Time = 2.89 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.90

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx=\frac{a^2\,x^3}{3}+\frac{b^2\,x^3\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)^2}{3}+\frac{b^2\,n^2\,x^3}{54}+\frac{2\,a\,b\,x^3\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)}{3}-\frac{b^2\,d^6\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)^2}{3\,e^6}-\frac{a\,b\,n\,x^3}{9}-\frac{b^2\,n\,x^3\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)}{9}+\frac{49\,b^2\,d^6\,n^2\,\ln\left(d+e\,\sqrt{x}\right)}{30\,e^6}+\frac{37\,b^2\,d^2\,n^2\,x^2}{360\,e^2}-\frac{19\,b^2\,d^3\,n^2\,x^{3/2}}{90\,e^3}-\frac{49\,b^2\,d^5\,n^2\,\sqrt{x}}{30\,e^5}-\frac{11\,b^2\,d\,n^2\,x^{5/2}}{225\,e}+\frac{29\,b^2\,d^4\,n^2\,x}{60\,e^4}-\frac{b^2\,d^2\,n\,x^2\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)}{6\,e^2}+\frac{2\,b^2\,d^3\,n\,x^{3/2}\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)}{9\,e^3}+\frac{2\,b^2\,d^5\,n\,\sqrt{x}\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)}{3\,e^5}+\frac{2\,a\,b\,d\,n\,x^{5/2}}{15\,e}-\frac{a\,b\,d^4\,n\,x}{3\,e^4}-\frac{2\,a\,b\,d^6\,n\,\ln\left(d+e\,\sqrt{x}\right)}{3\,e^6}+\frac{2\,b^2\,d\,n\,x^{5/2}\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)}{15\,e}-\frac{b^2\,d^4\,n\,x\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)}{3\,e^4}-\frac{a\,b\,d^2\,n\,x^2}{6\,e^2}+\frac{2\,a\,b\,d^3\,n\,x^{3/2}}{9\,e^3}+\frac{2\,a\,b\,d^5\,n\,\sqrt{x}}{3\,e^5}$$

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/2))^n))^2,x)
[Out] (a^2*x^3)/3 + (b^2*x^3*log(c*(d + e*x^(1/2))^n)^2)/3 + (b^2*n^2*x^3)/54 + (
2*a*b*x^3*log(c*(d + e*x^(1/2))^n))/3 - (b^2*d^6*log(c*(d + e*x^(1/2))^n)^2
)/(3*e^6) - (a*b*n*x^3)/9 - (b^2*n*x^3*log(c*(d + e*x^(1/2))^n))/9 + (49*b^
2*d^6*n^2*log(d + e*x^(1/2)))/(30*e^6) + (37*b^2*d^2*n^2*x^2)/(360*e^2) - (
19*b^2*d^3*n^2*x^(3/2))/(90*e^3) - (49*b^2*d^5*n^2*x^(1/2))/(30*e^5) - (11*
b^2*d*n^2*x^(5/2))/(225*e) + (29*b^2*d^4*n^2*x)/(60*e^4) - (b^2*d^2*n*x^2*l
og(c*(d + e*x^(1/2))^n))/(6*e^2) + (2*b^2*d^3*n*x^(3/2)*log(c*(d + e*x^(1/2
))^n))/(9*e^3) + (2*b^2*d^5*n*x^(1/2)*log(c*(d + e*x^(1/2))^n))/(3*e^5) + (
2*a*b*d*n*x^(5/2))/(15*e) - (a*b*d^4*n*x)/(3*e^4) - (2*a*b*d^6*n*log(d + e*
x^(1/2)))/(3*e^6) + (2*b^2*d*n*x^(5/2)*log(c*(d + e*x^(1/2))^n))/(15*e) - (
b^2*d^4*n*x*log(c*(d + e*x^(1/2))^n))/(3*e^4) - (a*b*d^2*n*x^2)/(6*e^2) + (
2*a*b*d^3*n*x^(3/2))/(9*e^3) + (2*a*b*d^5*n*x^(1/2))/(3*e^5)
```

3.409 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 342

$$\begin{aligned}\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx &= \frac{3b^2d^2n^2\left(d+e\sqrt{x}\right)^2}{2e^4}-\frac{4b^2dn^2\left(d+e\sqrt{x}\right)^3}{9e^4}\\
&+\frac{b^2n^2\left(d+e\sqrt{x}\right)^4}{16e^4}-\frac{4b^2d^3n^2\sqrt{x}}{e^3}\\
&+\frac{b^2d^4n^2\log^2\left(d+e\sqrt{x}\right)}{2e^4}\\
&+\frac{4bd^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}\\
&-\frac{3bd^2n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}\\
&+\frac{4bdn\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3e^4}\\
&-\frac{bn\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{4e^4}\\
&-\frac{bd^4n\log\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}\\
&+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\end{aligned}$$

```
[Out] 1/2*b^2*d^4*n^2*ln(d+e*x^(1/2))^2/e^4-b*d^4*n*ln(d+e*x^(1/2))*(a+b*ln(c*(d+
e*x^(1/2))^n))/e^4+1/2*x^2*(a+b*ln(c*(d+e*x^(1/2))^n))^2-4*b^2*d^3*n^2*x^(1
/2)/e^3+4*b*d^3*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))/e^4+3/2*b^2*d^2
*n^2*(d+e*x^(1/2))^2/e^4-3*b*d^2*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2)
)^2/e^4-4/9*b^2*d*n^2*(d+e*x^(1/2))^3/e^4+4/3*b*d*n*(a+b*ln(c*(d+e*x^(1/2))
^n))*(d+e*x^(1/2))^3/e^4+1/16*b^2*n^2*(d+e*x^(1/2))^4/e^4-1/4*b*n*(a+b*ln(c
*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^4/e^4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx=-\frac{bd^4n\log\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}+\frac{4bd^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}-\frac{3bd^2n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}+\frac{4bdn\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3e^4}-\frac{bn\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{4e^4}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2+\frac{b^2d^4n^2\log^2\left(d+e\sqrt{x}\right)}{2e^4}-\frac{4b^2d^3n^2\sqrt{x}}{e^3}+\frac{3b^2d^2n^2\left(d+e\sqrt{x}\right)^2}{2e^4}-\frac{4b^2dn^2\left(d+e\sqrt{x}\right)^3}{9e^4}+\frac{b^2n^2\left(d+e\sqrt{x}\right)^4}{16e^4}$$

```
[In] Int[x*(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

```
[Out] (3*b^2*d^2*n^2*(d + e*Sqrt[x])^2)/(2*e^4) - (4*b^2*d*n^2*(d + e*Sqrt[x])^3)
/(9*e^4) + (b^2*n^2*(d + e*Sqrt[x])^4)/(16*e^4) - (4*b^2*d^3*n^2*Sqrt[x])/e
^3 + (b^2*d^4*n^2*Log[d + e*Sqrt[x]]^2)/(2*e^4) + (4*b*d^3*n*(d + e*Sqrt[x]
)*(a + b*Log[c*(d + e*Sqrt[x])^n]))/e^4 - (3*b*d^2*n*(d + e*Sqrt[x])^2*(a +
 b*Log[c*(d + e*Sqrt[x])^n]))/e^4 + (4*b*d*n*(d + e*Sqrt[x])^3*(a + b*Log[c
*(d + e*Sqrt[x])^n]))/(3*e^4) - (b*n*(d + e*Sqrt[x])^4*(a + b*Log[c*(d + e*
Sqrt[x])^n]))/(4*e^4) - (b*d^4*n*Log[d + e*Sqrt[x]]*(a + b*Log[c*(d + e*Sqr
t[x])^n]))/e^4 + (x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int x^3(a+b\log(c(d+ex)^n))^2\,dx, x, \sqrt{x}\right)$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-(ben)\text{Subst}\left(\int \frac{x^4(a+b\log(c(d+ex)^n))}{d+ex}\,dx, x, \sqrt{x}\right)$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-(bn)\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^4(a+b\log(cx^n))}{x}\,dx, x, d+e\sqrt{x}\right)$$

$$= \frac{4bd^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}-\frac{3bd^2n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}+\frac{4bdn\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3e^4}-\frac{bn\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{4e^4}-\frac{bd^4n\log\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2+\left(b^2n^2\right)\text{Subst}\left(\int \frac{x(-48d^3+36d^2x-16dx^2+3x^3)+12d^4\log(x)}{12e^4x}\,dx, x, d+e\sqrt{x}\right)$$

$$= \frac{4bd^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}-\frac{3bd^2n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}+\frac{4bdn\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3e^4}-\frac{bn\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{4e^4}-\frac{bd^4n\log\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2+\frac{\left(b^2n^2\right)\text{Subst}\left(\int \frac{x(-48d^3+36d^2x-16dx^2+3x^3)+12d^4\log(x)}{x}\,dx, x, d+e\sqrt{x}\right)}{12e^4}$$

$$= \frac{4bd^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}-\frac{3bd^2n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}+\frac{4bdn\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3e^4}-\frac{bn\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{4e^4}-\frac{bd^4n\log\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^4}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2+\frac{\left(b^2n^2\right)\text{Subst}\left(\int \left(-48d^3+36d^2x-16dx^2+3x^3+\frac{12d^4\log(x)}{x}\right)dx, x, d+e\sqrt{x}\right)}{12e^4}$$

$$= \frac{3b^2d^2n^2(d+e\sqrt{x})^2}{2e^4} - \frac{4b^2dn^2(d+e\sqrt{x})^3}{9e^4} + \frac{b^2n^2(d+e\sqrt{x})^4}{16e^4} - \frac{4b^2d^3n^2\sqrt{x}}{e^3} + \frac{4bd^3n(d+e\sqrt{x})\,(a+b\log(c(d+e\sqrt{x})^n))}{e^4} - \frac{3bd^2n(d+e\sqrt{x})^2\,(a+b\log(c(d+e\sqrt{x})^n))}{e^4} + \frac{4bdn(d+e\sqrt{x})^3\,(a+b\log(c(d+e\sqrt{x})^n))}{3e^4} - \frac{bn(d+e\sqrt{x})^4\,(a+b\log(c(d+e\sqrt{x})^n))}{4e^4} - \frac{bd^4n\log(d+e\sqrt{x})\,(a+b\log(c(d+e\sqrt{x})^n))}{e^4} + \frac{1}{2}x^2(a+b\log(c(d+e\sqrt{x})^n))^2 + \frac{(b^2d^4n^2)\,\mathrm{Subst}\left(\int \frac{\log(x)}{x}\,dx, x, d+e\sqrt{x}\right)}{e^4}$$

$$= \frac{3b^2d^2n^2(d+e\sqrt{x})^2}{2e^4} - \frac{4b^2dn^2(d+e\sqrt{x})^3}{9e^4} + \frac{b^2n^2(d+e\sqrt{x})^4}{16e^4} - \frac{4b^2d^3n^2\sqrt{x}}{e^3} + \frac{b^2d^4n^2\log^2(d+e\sqrt{x})}{2e^4} + \frac{4bd^3n(d+e\sqrt{x})\,(a+b\log(c(d+e\sqrt{x})^n))}{e^4} - \frac{3bd^2n(d+e\sqrt{x})^2\,(a+b\log(c(d+e\sqrt{x})^n))}{e^4} + \frac{4bdn(d+e\sqrt{x})^3\,(a+b\log(c(d+e\sqrt{x})^n))}{3e^4} - \frac{bn(d+e\sqrt{x})^4\,(a+b\log(c(d+e\sqrt{x})^n))}{4e^4} - \frac{bd^4n\log(d+e\sqrt{x})\,(a+b\log(c(d+e\sqrt{x})^n))}{e^4} + \frac{1}{2}x^2(a+b\log(c(d+e\sqrt{x})^n))^2$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.65

$$\int x(a+b\log(c(d+e\sqrt{x})^n))^2\,dx = \frac{e\sqrt{x}(72a^2e^3x^{3/2} + 12abn(12d^3 - 6d^2e\sqrt{x} + 4de^2x - 3e^3x^{3/2}) + b^2n^2(-300d^3 + 78d^2e\sqrt{x} - 28de^2x + 9e^3}{}$$

```
[In] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

```
[Out] (e*Sqrt[x]*(72*a^2*e^3*x^(3/2) + 12*a*b*n*(12*d^3 - 6*d^2*e*Sqrt[x] + 4*d*e
^2*x - 3*e^3*x^(3/2)) + b^2*n^2*(-300*d^3 + 78*d^2*e*Sqrt[x] - 28*d*e^2*x +
 9*e^3*x^(3/2))) - 12*b*(12*a*(d^4 - e^4*x^2) + b*n*(-25*d^4 - 12*d^3*e*Sqr
t[x] + 6*d^2*e^2*x - 4*d*e^3*x^(3/2) + 3*e^4*x^2))*Log[c*(d + e*Sqrt[x])^n]
 - 72*b^2*(d^4 - e^4*x^2)*Log[c*(d + e*Sqrt[x])^n]^2)/(144*e^4)
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}dx$$

[In] int(x*(a+b*ln(c*(d+e*x^(1/2))^n))^2,x)

[Out] int(x*(a+b*ln(c*(d+e*x^(1/2))^n))^2,x)

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.04

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}\,dx = \frac{72\,b^{2}e^{4}x^{2}\log\left(c\right)^{2}+9\,\left(b^{2}e^{4}n^{2}-4\,abe^{4}n+8\,a^{2}e^{4}\right)x^{2}+72\,\left(b^{2}e^{4}n^{2}x^{2}-b^{2}d^{4}n^{2}\right)\log\left(e\sqrt{x}+d\right)^{2}+6\,\left(13\,b^{2}d^{2}e^{2}\right.}{}$$

[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="fricas")

[Out] 1/144*(72*b^2*e^4*x^2*log(c)^2 + 9*(b^2*e^4*n^2 - 4*a*b*e^4*n + 8*a^2*e^4)*x^2 + 72*(b^2*e^4*n^2*x^2 - b^2*d^4*n^2)*log(e*sqrt(x) + d)^2 + 6*(13*b^2*d^2*e^2*n^2 - 12*a*b*d^2*e^2*n)*x - 12*(6*b^2*d^2*e^2*n^2*x - 25*b^2*d^4*n^2 + 12*a*b*d^4*n + 3*(b^2*e^4*n^2 - 4*a*b*e^4*n)*x^2 - 12*(b^2*e^4*n*x^2 - b^2*d^4*n)*log(c) - 4*(b^2*d*e^3*n^2*x + 3*b^2*d^3*e*n^2)*sqrt(x))*log(e*sqrt(x) + d) - 36*(2*b^2*d^2*e^2*n*x + (b^2*e^4*n - 4*a*b*e^4)*x^2)*log(c) - 4*(75*b^2*d^3*e*n^2 - 36*a*b*d^3*e*n + (7*b^2*d*e^3*n^2 - 12*a*b*d*e^3*n)*x - 12*(b^2*d*e^3*n*x + 3*b^2*d^3*e*n)*log(c))*sqrt(x))/e^4

Sympy [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}\,dx=\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}\,dx$$

[In] integrate(x*(a+b*ln(c*(d+e*x**(1/2))**n))**2,x)

[Out] Integral(x*(a + b*log(c*(d + e*sqrt(x))**n))**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.75

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx=\frac{1}{2}\,b^2x^2\log\left(\left(e\sqrt{x}+d\right)^nc\right)^2$$
$$-\frac{1}{12}\,aben\left(\frac{12\,d^4\log\left(e\sqrt{x}+d\right)}{e^5}+\frac{3\,e^3x^2-4\,de^2x^{\frac{3}{2}}+6\,d^2ex-12\,d^3\sqrt{x}}{e^4}\right)$$
$$+abx^2\log\left(\left(e\sqrt{x}+d\right)^nc\right)+\frac{1}{2}\,a^2x^2$$
$$-\frac{1}{144}\left(12\,en\left(\frac{12\,d^4\log\left(e\sqrt{x}+d\right)}{e^5}+\frac{3\,e^3x^2-4\,de^2x^{\frac{3}{2}}+6\,d^2ex-12\,d^3\sqrt{x}}{e^4}\right)\log\left(\left(e\sqrt{x}+d\right)^nc\right)-\frac{\left(9\,e^4\right.}{}\right.$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*log((e*sqrt(x) + d)^n*c)^2 - 1/12*a*b*e*n*(12*d^4*log(e*sqrt(x)
 + d)/e^5 + (3*e^3*x^2 - 4*d*e^2*x^(3/2) + 6*d^2*e*x - 12*d^3*sqrt(x))/e^4)
 + a*b*x^2*log((e*sqrt(x) + d)^n*c) + 1/2*a^2*x^2 - 1/144*(12*e*n*(12*d^4*l
og(e*sqrt(x) + d)/e^5 + (3*e^3*x^2 - 4*d*e^2*x^(3/2) + 6*d^2*e*x - 12*d^3*s
qrt(x))/e^4)*log((e*sqrt(x) + d)^n*c) - (9*e^4*x^2 + 72*d^4*log(e*sqrt(x) +
 d)^2 - 28*d*e^3*x^(3/2) + 78*d^2*e^2*x + 300*d^4*log(e*sqrt(x) + d) - 300*
d^3*e*sqrt(x))*n^2/e^4)*b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(296) = 592$.

Time = 0.31 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.82

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$
$$=\frac{72\,b^2ex^2\log\left(c\right)^2+144\,abex^2\log\left(c\right)+\left(\frac{72\left(e\sqrt{x}+d\right)^4\log\left(e\sqrt{x}+d\right)^2}{e^3}-\frac{288\left(e\sqrt{x}+d\right)^3d\log\left(e\sqrt{x}+d\right)^2}{e^3}+\frac{432\left(e\sqrt{x}+d\right)^2d^2\log\left(e\right.}{e^3}\right.}{}$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="giac")
```

```
[Out] 1/144*(72*b^2*e*x^2*log(c)^2 + 144*a*b*e*x^2*log(c) + (72*(e*sqrt(x) + d)^4
*log(e*sqrt(x) + d)^2/e^3 - 288*(e*sqrt(x) + d)^3*d*log(e*sqrt(x) + d)^2/e^
3 + 432*(e*sqrt(x) + d)^2*d^2*log(e*sqrt(x) + d)^2/e^3 - 288*(e*sqrt(x) + d
)*d^3*log(e*sqrt(x) + d)^2/e^3 - 36*(e*sqrt(x) + d)^4*log(e*sqrt(x) + d)/e^
3 + 192*(e*sqrt(x) + d)^3*d*log(e*sqrt(x) + d)/e^3 - 432*(e*sqrt(x) + d)^2*
d^2*log(e*sqrt(x) + d)/e^3 + 576*(e*sqrt(x) + d)*d^3*log(e*sqrt(x) + d)/e^3
```

```
 + 9*(e*sqrt(x) + d)^4/e^3 - 64*(e*sqrt(x) + d)^3*d/e^3 + 216*(e*sqrt(x) +
d)^2*d^2/e^3 - 576*(e*sqrt(x) + d)*d^3/e^3)*b^2*n^2 + 72*a^2*e*x^2 + 12*(12
*(e*sqrt(x) + d)^4*log(e*sqrt(x) + d)/e^3 - 48*(e*sqrt(x) + d)^3*d*log(e*sq
rt(x) + d)/e^3 + 72*(e*sqrt(x) + d)^2*d^2*log(e*sqrt(x) + d)/e^3 - 48*(e*sq
rt(x) + d)*d^3*log(e*sqrt(x) + d)/e^3 - 3*(e*sqrt(x) + d)^4/e^3 + 16*(e*sqr
t(x) + d)^3*d/e^3 - 36*(e*sqrt(x) + d)^2*d^2/e^3 + 48*(e*sqrt(x) + d)*d^3/e
^3)*b^2*n*log(c) + 12*(12*(e*sqrt(x) + d)^4*log(e*sqrt(x) + d)/e^3 - 48*(e*
sqrt(x) + d)^3*d*log(e*sqrt(x) + d)/e^3 + 72*(e*sqrt(x) + d)^2*d^2*log(e*sq
rt(x) + d)/e^3 - 48*(e*sqrt(x) + d)*d^3*log(e*sqrt(x) + d)/e^3 - 3*(e*sqrt(
x) + d)^4/e^3 + 16*(e*sqrt(x) + d)^3*d/e^3 - 36*(e*sqrt(x) + d)^2*d^2/e^3 +
 48*(e*sqrt(x) + d)*d^3/e^3)*a*b*n)/e
```

Mupad [B] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.23

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx = x\left(\frac{d\left(\frac{d\left(2a^2-abn+\frac{b^2n^2}{4}\right)}{e}-\frac{d\left(6a^2-b^2n^2\right)}{3e}\right)}{2e}+\frac{b^2d^2n^2}{4e^2}\right)$$
$$-x^{3/2}\left(\frac{d\left(2a^2-abn+\frac{b^2n^2}{4}\right)}{3e}-\frac{d\left(6a^2-b^2n^2\right)}{9e}\right)$$
$$+\ln\left(c\left(d+e\sqrt{x}\right)^n\right)^2\left(\frac{b^2x^2}{2}-\frac{b^2d^4}{2e^4}\right)+x^2\left(\frac{a^2}{2}-\frac{abn}{4}+\frac{b^2n^2}{16}\right)$$
$$-\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\left(x^{3/2}\left(\frac{bd\left(4a-bn\right)}{3e}-\frac{4abd}{3e}\right)-\frac{bx^2\left(4a-bn\right)}{4}+\frac{d^2\sqrt{x}\left(\frac{bd\left(4a-bn\right)}{e}-\frac{4abd}{e}\right)}{e^2}-dx\,(\right.$$
$$-\sqrt{x}\left(\frac{d\left(\frac{d\left(\frac{d\left(2a^2-abn+\frac{b^2n^2}{4}\right)}{e}-\frac{d\left(6a^2-b^2n^2\right)}{3e}\right)}{e}+\frac{b^2d^2n^2}{2e^2}\right)}{e}+\frac{b^2d^3n^2}{e^3}\right)$$
$$+\frac{\ln\left(d+e\sqrt{x}\right)\left(25b^2d^4n^2-12abd^4n\right)}{12e^4}$$

```
[In] int(x*(a + b*log(c*(d + e*x^(1/2))^n))^2,x)
```

```
[Out] x*((d*((d*(2*a^2 + (b^2*n^2)/4 - a*b*n))/e - (d*(6*a^2 - b^2*n^2))/(3*e)))/
(2*e) + (b^2*d^2*n^2)/(4*e^2)) - x^(3/2)*((d*(2*a^2 + (b^2*n^2)/4 - a*b*n))
```

```
/(3*e) - (d*(6*a^2 - b^2*n^2))/(9*e)) + log(c*(d + e*x^(1/2))^n)^2*((b^2*x^
2)/2 - (b^2*d^4)/(2*e^4)) + x^2*(a^2/2 + (b^2*n^2)/16 - (a*b*n)/4) - log(c*
(d + e*x^(1/2))^n)*(x^(3/2)*((b*d*(4*a - b*n))/(3*e) - (4*a*b*d)/(3*e)) - (
b*x^2*(4*a - b*n))/4 + (d^2*x^(1/2)*((b*d*(4*a - b*n))/e - (4*a*b*d)/e))/e^
2 - (d*x*((b*d*(4*a - b*n))/e - (4*a*b*d)/e))/(2*e)) - x^(1/2)*((d*((d*((d*
(2*a^2 + (b^2*n^2)/4 - a*b*n))/e - (d*(6*a^2 - b^2*n^2))/(3*e)))/e + (b^2*d
^2*n^2)/(2*e^2)))/e + (b^2*d^3*n^2)/e^3) + (log(d + e*x^(1/2))*(25*b^2*d^4*
n^2 - 12*a*b*d^4*n))/(12*e^4)
```

3.410 $\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 195

$$\begin{aligned}\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx =& \frac{b^2n^2\left(d+e\sqrt{x}\right)^2}{2e^2}+\frac{4abdn\sqrt{x}}{e}-\frac{4b^2dn^2\sqrt{x}}{e}\\&+\frac{4b^2dn\left(d+e\sqrt{x}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{e^2}\\&-\frac{bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^2}\\&-\frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2}\\&+\frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2}\end{aligned}$$

```
[Out] 4*a*b*d*n*x^(1/2)/e-4*b^2*d*n^2*x^(1/2)/e+4*b^2*d*n*ln(c*(d+e*x^(1/2))^n)*(
d+e*x^(1/2))/e^2-2*d*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))/e^2+1/2*b^
2*n^2*(d+e*x^(1/2))^2/e^2-b*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^2/e
^2+(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))^2/e^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used

$= \{2501, 2448, 2436, 2333, 2332, 2437, 2342, 2341\}$

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx = -\frac{bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^2} + \frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2} - \frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2} + \frac{4abdn\sqrt{x}}{e} + \frac{4b^2dn\left(d+e\sqrt{x}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{e^2} + \frac{b^2n^2\left(d+e\sqrt{x}\right)^2}{2e^2} - \frac{4b^2dn^2\sqrt{x}}{e}$$

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

```
[Out] (b^2*n^2*(d + e*Sqrt[x])^2)/(2*e^2) + (4*a*b*d*n*Sqrt[x])/e - (4*b^2*d*n^2*
Sqrt[x])/e + (4*b^2*d*n*(d + e*Sqrt[x])*Log[c*(d + e*Sqrt[x])^n])/e^2 - (b*
n*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/e^2 - (2*d*(d + e*Sqr
t[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/e^2 + ((d + e*Sqrt[x])^2*(a + b*L
og[c*(d + e*Sqrt[x])^n])^2)/e^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x(a+b\log(c(d+ex)^n))^2\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(-\frac{d(a+b\log(c(d+ex)^n))^2}{e}+\frac{(d+ex)\,(a+b\log(c(d+ex)^n))^2}{e}\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{2\text{Subst}\left(\int (d+ex)\,(a+b\log(c(d+ex)^n))^2\,dx, x, \sqrt{x}\right)}{e}\\
&\quad-\frac{(2d)\text{Subst}\left(\int (a+b\log(c(d+ex)^n))^2\,dx, x, \sqrt{x}\right)}{e}\\
&= \frac{2\text{Subst}\left(\int x(a+b\log(cx^n))^2\,dx, x, d+e\sqrt{x}\right)}{e^2}\\
&\quad-\frac{(2d)\text{Subst}\left(\int (a+b\log(cx^n))^2\,dx, x, d+e\sqrt{x}\right)}{e^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2} \\
&\quad+\frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2} \\
&\quad-\frac{(2bn)\mathrm{Subst}\left(\int x(a+b\log(cx^n))\,dx,x,d+e\sqrt{x}\right)}{e^2} \\
&\quad+\frac{(4bdn)\mathrm{Subst}\left(\int(a+b\log(cx^n))\,dx,x,d+e\sqrt{x}\right)}{e^2} \\
&= \frac{b^2n^2\left(d+e\sqrt{x}\right)^2}{2e^2}+\frac{4abdn\sqrt{x}}{e}-\frac{bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^2} \\
&\quad-\frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2} \\
&\quad+\frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2} \\
&\quad+\frac{(4b^2dn)\,\mathrm{Subst}\left(\int\log(cx^n)\,dx,x,d+e\sqrt{x}\right)}{e^2} \\
&= \frac{b^2n^2\left(d+e\sqrt{x}\right)^2}{2e^2}+\frac{4abdn\sqrt{x}}{e}-\frac{4b^2dn^2\sqrt{x}}{e}+\frac{4b^2dn\left(d+e\sqrt{x}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{e^2} \\
&\quad-\frac{bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{e^2} \\
&\quad-\frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2} \\
&\quad+\frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.77

$$
\int\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx
= \frac{-2abn\left(d-e\sqrt{x}\right)^2+b^2en^2\left(-6d+e\sqrt{x}\right)\sqrt{x}-2a^2(d^2-e^2x)+2b\left(d+e\sqrt{x}\right)\left(-2ad+3bdn+2ae\sqrt{x}-b\right.}{2e^2}
$$

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2,x]
```

```
[Out] (-2*a*b*n*(d - e*Sqrt[x])^2 + b^2*e*n^2*(-6*d + e*Sqrt[x])*Sqrt[x] - 2*a^2*
(d^2 - e^2*x) + 2*b*(d + e*Sqrt[x])*(-2*a*d + 3*b*d*n + 2*a*e*Sqrt[x] - b*e
*n*Sqrt[x])*Log[c*(d + e*Sqrt[x])^n] - 2*b^2*(d^2 - e^2*x)*Log[c*(d + e*Sqr
t[x])^n]^2)/(2*e^2)
```

Maple [F]

$$\int \left(a+b\ln \left(c{\left(d+e\sqrt{x}\right)}^{n}\right)\right)^{2}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.15

$$\int \left(a+b\log \left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 \, dx$$
$$= \frac{2\,b^{2}e^{2}x\log \left(c\right)^{2}+2\,(b^{2}e^{2}n^{2}x-b^{2}d^{2}n^{2})\log \left(e\sqrt{x}+d\right)^{2}-2\,(b^{2}e^{2}n-2\,abe^{2})x\log \left(c\right)+(b^{2}e^{2}n^{2}-2\,abe^{2}n+2}{}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*e^2*x*log(c)^2 + 2*(b^2*e^2*n^2*x - b^2*d^2*n^2)*log(e*sqrt(x) +
 d)^2 - 2*(b^2*e^2*n - 2*a*b*e^2)*x*log(c) + (b^2*e^2*n^2 - 2*a*b*e^2*n + 2
*a^2*e^2)*x + 2*(2*b^2*d*e*n^2*sqrt(x) + 3*b^2*d^2*n^2 - 2*a*b*d^2*n - (b^2
*e^2*n^2 - 2*a*b*e^2*n)*x + 2*(b^2*e^2*n*x - b^2*d^2*n)*log(c))*log(e*sqrt(
x) + d) - 2*(3*b^2*d*e*n^2 - 2*b^2*d*e*n*log(c) - 2*a*b*d*e*n)*sqrt(x))/e^2
```

Sympy [F]

$$\int \left(a+b\log \left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 \, dx = \int \left(a+b\log \left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 \, dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.92

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$
$$= -\left(en\left(\frac{2\,d^2\log\left(e\sqrt{x}+d\right)}{e^3}+\frac{ex-2\,d\sqrt{x}}{e^2}\right)-2\,x\log\left(\left(e\sqrt{x}+d\right)^nc\right)\right)ab$$
$$-\frac{1}{2}\left(2\,en\left(\frac{2\,d^2\log\left(e\sqrt{x}+d\right)}{e^3}+\frac{ex-2\,d\sqrt{x}}{e^2}\right)\log\left(\left(e\sqrt{x}+d\right)^nc\right)-2\,x\log\left(\left(e\sqrt{x}+d\right)^nc\right)^2-\frac{\left(2\,d^2\log\right.}{}\right.$$
$$+a^2x$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="maxima")
```

```
[Out] -(e*n*(2*d^2*log(e*sqrt(x) + d)/e^3 + (e*x - 2*d*sqrt(x))/e^2) - 2*x*log((e
*sqrt(x) + d)^n*c))*a*b - 1/2*(2*e*n*(2*d^2*log(e*sqrt(x) + d)/e^3 + (e*x -
 2*d*sqrt(x))/e^2)*log((e*sqrt(x) + d)^n*c) - 2*x*log((e*sqrt(x) + d)^n*c)^
2 - (2*d^2*log(e*sqrt(x) + d)^2 + e^2*x + 6*d^2*log(e*sqrt(x) + d) - 6*d*e*
sqrt(x))*n^2/e^2)*b^2 + a^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.74

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx$$
$$=\frac{\frac{\left(2\left(e\sqrt{x}+d\right)^2\log\left(e\sqrt{x}+d\right)^2-4\left(e\sqrt{x}+d\right)d\log\left(e\sqrt{x}+d\right)^2-2\left(e\sqrt{x}+d\right)^2\log\left(e\sqrt{x}+d\right)+8\left(e\sqrt{x}+d\right)d\log\left(e\sqrt{x}+d\right)+\left(e\sqrt{x}+d\right)^2-8\left(e\sqrt{x}+d\right)d\right)b^2n^2}{e}}{}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d)^2 - 4*(e*sqrt(x) + d)*d*log(e*
sqrt(x) + d)^2 - 2*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d) + 8*(e*sqrt(x) + d)
*d*log(e*sqrt(x) + d) + (e*sqrt(x) + d)^2 - 8*(e*sqrt(x) + d)*d)*b^2*n^2/e
+ 2*(2*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d) - 4*(e*sqrt(x) + d)*d*log(e*sqr
t(x) + d) - (e*sqrt(x) + d)^2 + 4*(e*sqrt(x) + d)*d)*b^2*n*log(c)/e + 2*((e
*sqrt(x) + d)^2 - 2*(e*sqrt(x) + d)*d)*b^2*log(c)^2/e + 2*(2*(e*sqrt(x) + d
)^2*log(e*sqrt(x) + d) - 4*(e*sqrt(x) + d)*d*log(e*sqrt(x) + d) - (e*sqrt(x
) + d)^2 + 4*(e*sqrt(x) + d)*d)*a*b*n/e + 4*((e*sqrt(x) + d)^2 - 2*(e*sqrt(
x) + d)*d)*a*b*log(c)/e + 2*((e*sqrt(x) + d)^2 - 2*(e*sqrt(x) + d)*d)*a^2/e
)/e
```

Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.95

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\,dx = x\left(a^2-a\,b\,n+\frac{b^2\,n^2}{2}\right) - \sqrt{x}\left(\frac{d\,(2\,a^2-2\,a\,b\,n+b^2\,n^2)}{e}-\frac{2\,d\,(a^2-b^2\,n^2)}{e}\right) + \ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)^2\left(b^2\,x-\frac{b^2\,d^2}{e^2}\right) - \ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)\left(\sqrt{x}\left(\frac{2\,b\,d\,(2\,a-b\,n)}{e}-\frac{4\,a\,b\,d}{e}\right) - b\,x\,(2\,a-b\,n)\right)+\frac{\ln\left(d+e\,\sqrt{x}\right)\,\left(3\,b^2\,d^2\,n^2-2\,a\,b\,d^2\,n\right)}{e^2}$$

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^2,x)
```

```
[Out] x*(a^2 + (b^2*n^2)/2 - a*b*n) - x^(1/2)*((d*(2*a^2 + b^2*n^2 - 2*a*b*n))/e
- (2*d*(a^2 - b^2*n^2))/e) + log(c*(d + e*x^(1/2))^n)^2*(b^2*x - (b^2*d^2)/
e^2) - log(c*(d + e*x^(1/2))^n)*(x^(1/2)*((2*b*d*(2*a - b*n))/e - (4*a*b*d)
/e) - b*x*(2*a - b*n)) + (log(d + e*x^(1/2))*(3*b^2*d^2*n^2 - 2*a*b*d^2*n))
/e^2
```

3.411 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x}\,dx &= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt{x}}{d}\right)\\ &\quad+4bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\operatorname{PolyLog}\left(2,1+\frac{e\sqrt{x}}{d}\right)\\ &\quad-4b^2n^2\operatorname{PolyLog}\left(3,1+\frac{e\sqrt{x}}{d}\right)\end{aligned}$$

```
[Out] 2*ln(-e*x^(1/2)/d)*(a+b*ln(c*(d+e*x^(1/2))^n))^2+4*b*n*(a+b*ln(c*(d+e*x^(1/
2))^n))*polylog(2,1+e*x^(1/2)/d)-4*b^2*n^2*polylog(3,1+e*x^(1/2)/d)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2504, 2443, 2481, 2421, 6724}

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x}\,dx &= 4bn\operatorname{PolyLog}\left(2,\frac{\sqrt{x}e}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\\ &\quad+2\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\\ &\quad-4b^2n^2\operatorname{PolyLog}\left(3,\frac{\sqrt{x}e}{d}+1\right)\end{aligned}$$

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x,x]
```

```
[Out] 2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2*Log[-((e*Sqrt[x])/d)] + 4*b*n*(a + b*L
og[c*(d + e*Sqrt[x])^n])*PolyLog[2, 1 + (e*Sqrt[x])/d] - 4*b^2*n^2*PolyLog[
3, 1 + (e*Sqrt[x])/d]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x}\,dx, x, \sqrt{x}\right)$$

$$
\begin{aligned}
&= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt{x}}{d}\right) \\
&\quad-(4ben)\text{Subst}\left(\int\frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex}dx,x,\sqrt{x}\right) \\
&= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt{x}}{d}\right) \\
&\quad-(4bn)\text{Subst}\left(\int\frac{\left(a+b\log\left(cx^n\right)\right)\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}dx,x,d+e\sqrt{x}\right) \\
&= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt{x}}{d}\right) \\
&\quad+4bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\text{Li}_2\left(1+\frac{e\sqrt{x}}{d}\right) \\
&\quad-\left(4b^2n^2\right)\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{x}{d}\right)}{x}dx,x,d+e\sqrt{x}\right) \\
&= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt{x}}{d}\right) \\
&\quad+4bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\text{Li}_2\left(1+\frac{e\sqrt{x}}{d}\right)-4b^2n^2\text{Li}_3\left(1+\frac{e\sqrt{x}}{d}\right)
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 195 vs. 2(93) = 186.

Time = 0.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.10

$$
\begin{aligned}
\int\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x}dx &= \left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\log(x) \\
&\quad+2bn\Big(a-bn\log\left(d+e\sqrt{x}\right) \\
&\quad+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\Big)\Bigg(\bigg(\log\left(d+e\sqrt{x}\right) \\
&\quad-\log\left(1+\frac{e\sqrt{x}}{d}\right)\bigg)\log(x)-2\,\text{PolyLog}\left(2,-\frac{e\sqrt{x}}{d}\right)\Bigg) \\
&\quad+2b^2n^2\Bigg(\log^2\left(d+e\sqrt{x}\right)\log\left(-\frac{e\sqrt{x}}{d}\right) \\
&\quad+2\log\left(d+e\sqrt{x}\right)\text{PolyLog}\left(2,1+\frac{e\sqrt{x}}{d}\right) \\
&\quad-2\,\text{PolyLog}\left(3,1+\frac{e\sqrt{x}}{d}\right)\Bigg)
\end{aligned}
$$

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])*((Log[d + e*Sqrt[
x]] - Log[1 + (e*Sqrt[x])/d])*Log[x] - 2*PolyLog[2, -((e*Sqrt[x])/d)]) + 2*
b^2*n^2*(Log[d + e*Sqrt[x]]^2*Log[-((e*Sqrt[x])/d)] + 2*Log[d + e*Sqrt[x]]*
PolyLog[2, 1 + (e*Sqrt[x])/d] - 2*PolyLog[3, 1 + (e*Sqrt[x])/d])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}}{x}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)+a\right)^{2}}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*sqrt(x) + d)^n*c)^2 + 2*a*b*log((e*sqrt(x) + d)^n*c) +
 a^2)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}}{x}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**2/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^nc\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x,x, algorithm="maxima")
```

```
[Out] b^2*log((e*sqrt(x) + d)^n)^2*log(x) + integrate(((b^2*e*log(c)^2 + 2*a*b*e*
log(c) + a^2*e)*x - (b^2*e*n*x*log(x) - 2*(b^2*e*log(c) + a*b*e)*x - 2*(b^2
*d*log(c) + a*b*d)*sqrt(x))*log((e*sqrt(x) + d)^n) + (b^2*d*log(c)^2 + 2*a*
b*d*log(c) + a^2*d)*sqrt(x))/(e*x^2 + d*x^(3/2)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^nc\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)\right)^2}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x, x)
```

3.412 $\int \frac{(a+b\log(c(d+e\sqrt{x})^n))^2}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 155

$$\int \frac{(a+b\log(c(d+e\sqrt{x})^n))^2}{x^2}\,dx = -\frac{2ben(d+e\sqrt{x})\,(a+b\log(c(d+e\sqrt{x})^n))}{d^2\sqrt{x}} - \frac{2be^2n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)(a+b\log(c(d+e\sqrt{x})^n))}{d^2} - \frac{(a+b\log(c(d+e\sqrt{x})^n))^2}{x} + \frac{b^2e^2n^2\log(x)}{d^2} + \frac{2b^2e^2n^2\operatorname{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)}{d^2}$$

```
[Out] b^2*e^2*n^2*ln(x)/d^2-(a+b*ln(c*(d+e*x^(1/2))^n))^2/x-2*b*e^2*n*(a+b*ln(c*(
d+e*x^(1/2))^n))*ln(1-d/(d+e*x^(1/2)))/d^2+2*b^2*e^2*n^2*polylog(2,d/(d+e*x
^(1/2)))/d^2-2*b*e*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))/d^2/x^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{2504, 2445, 2458, 2389, 2379, 2438, 2351, 31\}$

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}\,dx = -\frac{2be^2n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2} - \frac{2ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2\sqrt{x}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x} + \frac{2b^2e^2n^2\,\text{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)}{d^2} + \frac{b^2e^2n^2\log(x)}{d^2}$$

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^2,x]
```

```
[Out] (-2*b*e*n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(d^2*Sqrt[x]) -
 (2*b*e^2*n*Log[1 - d/(d + e*Sqrt[x])]*(a + b*Log[c*(d + e*Sqrt[x])^n]))/d^
2 - (a + b*Log[c*(d + e*Sqrt[x])^n])^2/x + (b^2*e^2*n^2*Log[x])/d^2 + (2*b^
2*e^2*n^2*PolyLog[2, d/(d + e*Sqrt[x])])/d^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3}\,dx, x, \sqrt{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x} + (2ben)\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^2(d+ex)}\,dx, x, \sqrt{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x} + (2bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x} + \frac{(2bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt{x}\right)}{d}\\
&\quad - \frac{(2ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx, x, d+e\sqrt{x}\right)}{d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2\sqrt{x}} \\
&\quad - \frac{2be^2n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2} \\
&\quad - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x} + \frac{(2b^2en^2)\operatorname{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x}{e}}\,dx,x,d+e\sqrt{x}\right)}{d^2} \\
&\quad + \frac{(2b^2e^2n^2)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{d}{x}\right)}{x}\,dx,x,d+e\sqrt{x}\right)}{d^2} \\
&= -\frac{2ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2\sqrt{x}} \\
&\quad - \frac{2be^2n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2} \\
&\quad - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x} + \frac{b^2e^2n^2\log(x)}{d^2} + \frac{2b^2e^2n^2\operatorname{Li}_2\left(\frac{d}{d+e\sqrt{x}}\right)}{d^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.22

$$
\begin{aligned}
\int\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}\,dx &= 2\Bigg(-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x} \\
&\quad + ben\Bigg(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{d\sqrt{x}} \\
&\quad + \frac{e\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2bd^2n} \\
&\quad - \frac{e\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right)}{d^2} \\
&\quad + \frac{bn\left(-\frac{e\log\left(d+e\sqrt{x}\right)}{d}+\frac{e\log(x)}{2d}\right)}{d} \\
&\quad - \frac{ben\operatorname{PolyLog}\left(2,\frac{d+e\sqrt{x}}{d}\right)}{d^2}\Bigg)\Bigg)
\end{aligned}
$$

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^2,x]
```

```
[Out] 2*(-1/2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x + b*e*n*(-((a + b*Log[c*(d + e
*Sqrt[x])^n])/(d*Sqrt[x])) + (e*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*b*d^
```

```
2*n) - (e*(a + b*Log[c*(d + e*Sqrt[x])^n])*Log[-((e*Sqrt[x])/d)])/d^2 + (b*
n*(-((e*Log[d + e*Sqrt[x]])/d) + (e*Log[x])/(2*d)))/d - (b*e*n*PolyLog[2, (
d + e*Sqrt[x])/d])/d^2))
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}}{x^{2}}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^2,x)
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}}{x^{2}}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)+a\right)^{2}}{x^{2}}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^2,x, algorithm="fricas")
[Out] integral((b^2*log((e*sqrt(x) + d)^n*c)^2 + 2*a*b*log((e*sqrt(x) + d)^n*c) +
 a^2)/x^2, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}}{x^{2}}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}}{x^{2}}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2/x**2,x)
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**2/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{2}}{x^{2}}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)+a\right)^{2}}{x^{2}}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^2,x, algorithm="maxima")
[Out] 2*(log(e*sqrt(x)/d + 1)*log(sqrt(x)) + dilog(-e*sqrt(x)/d))*b^2*e^2*n^2/d^2
 + 2*(a*b*e^2*n - (e^2*n^2 - e^2*n*log(c))*b^2)*log(e*sqrt(x) + d)/d^2 - 2*
```

```
(b^2*e^2*n*log(c) + a*b*e^2*n)*log(sqrt(x))/d^2 + integrate((b^2*e^4*n^2*x
+ b^2*d^2*e^2*n^2)/x, x)/d^4 + 1/3*(2*b^2*e^5*n^2*x^(3/2) - 6*b^2*d^2*e^3*n
^2*sqrt(x)*log(sqrt(x)) - 3*b^2*d*e^4*n^2*x + 12*b^2*d^2*e^3*n^2*sqrt(x))/d
^5 - 1/3*(3*b^2*d^3*e^2*n^2*x^(3/2)*log(e*sqrt(x) + d)^2 + 2*b^2*e^5*n^2*x^
3 - 3*b^2*d^2*e^3*n^2*x^2*log(x) + 12*b^2*d^2*e^3*n^2*x^2 + 3*b^2*d^5*sqrt(
x)*log((e*sqrt(x) + d)^n)^2 + 6*(b^2*d^4*e*n*log(c) + a*b*d^4*e*n)*x - 3*(2
*b^2*d^3*e^2*n*x^(3/2)*log(e*sqrt(x) + d) - 2*b^2*d^4*e*n*x - (b^2*d^3*e^2*
n*x*log(x) + 2*b^2*d^5*log(c) + 2*a*b*d^5)*sqrt(x))*log((e*sqrt(x) + d)^n))
/(d^5*x^(3/2))
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^2,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)\right)^2}{x^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^2, x)
```

3.413 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 293

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx &= -\frac{b^2e^2n^2}{6d^2x}+\frac{5b^2e^3n^2}{6d^3\sqrt{x}}-\frac{5b^2e^4n^2\log\left(d+e\sqrt{x}\right)}{6d^4}\\ &\quad-\frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3dx^{3/2}}\\ &\quad+\frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x}\\ &\quad-\frac{be^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4\sqrt{x}}\\ &\quad-\frac{be^4n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4}\\ &\quad-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x^2}\\ &\quad+\frac{11b^2e^4n^2\log(x)}{12d^4}+\frac{b^2e^4n^2\operatorname{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)}{d^4}\end{aligned}$$

```
[Out] -1/6*b^2*e^2*n^2/d^2/x+11/12*b^2*e^4*n^2*ln(x)/d^4-5/6*b^2*e^4*n^2*ln(d+e*x
^(1/2))/d^4-1/3*b*e*n*(a+b*ln(c*(d+e*x^(1/2))^n))/d/x^(3/2)+1/2*b*e^2*n*(a+
b*ln(c*(d+e*x^(1/2))^n))/d^2/x-1/2*(a+b*ln(c*(d+e*x^(1/2))^n))^2/x^2-b*e^4*
n*(a+b*ln(c*(d+e*x^(1/2))^n))*ln(1-d/(d+e*x^(1/2)))/d^4+b^2*e^4*n^2*polylog
(2,d/(d+e*x^(1/2)))/d^4+5/6*b^2*e^3*n^2/d^3/x^(1/2)-b*e^3*n*(a+b*ln(c*(d+e*
x^(1/2))^n))*(d+e*x^(1/2))/d^4/x^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx = -\frac{be^4n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4} - \frac{be^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4\sqrt{x}} + \frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3dx^{3/2}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x^2} + \frac{b^2e^4n^2\operatorname{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)}{d^4} - \frac{5b^2e^4n^2\log\left(d+e\sqrt{x}\right)}{6d^4} + \frac{11b^2e^4n^2\log(x)}{12d^4} + \frac{5b^2e^3n^2}{6d^3\sqrt{x}} - \frac{b^2e^2n^2}{6d^2x}$$

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^3,x]
```

```
[Out] -1/6*(b^2*e^2*n^2)/(d^2*x) + (5*b^2*e^3*n^2)/(6*d^3*Sqrt[x]) - (5*b^2*e^4*n
^2*Log[d + e*Sqrt[x]])/(6*d^4) - (b*e*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(
3*d*x^(3/2)) + (b*e^2*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*d^2*x) - (b*e^
3*n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(d^4*Sqrt[x]) - (b*e^
4*n*Log[1 - d/(d + e*Sqrt[x])]*(a + b*Log[c*(d + e*Sqrt[x])^n]))/d^4 - (a +
 b*Log[c*(d + e*Sqrt[x])^n])^2/(2*x^2) + (11*b^2*e^4*n^2*Log[x])/(12*d^4) +
 (b^2*e^4*n^2*PolyLog[2, d/(d + e*Sqrt[x])])/d^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
```

```
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^5}\,dx, x, \sqrt{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x^2} + (ben)\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^4(d+ex)}\,dx, x, \sqrt{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x^2} + (bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\sqrt{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x^2} + \frac{(bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\sqrt{x}\right)}{d}\\
&\quad - \frac{(ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt{x}\right)}{d}\\
&= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3dx^{3/2}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x^2}\\
&\quad - \frac{(ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt{x}\right)}{d^2}\\
&\quad + \frac{(be^2n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt{x}\right)}{d^2}\\
&\quad + \frac{(b^2en^2)\,\text{Subst}\left(\int \frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt{x}\right)}{3d}
\end{aligned}
$$

$$=-\frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3dx^{3/2}}+\frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x}-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x^2}+\frac{(be^2n)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+e\sqrt{x}\right)}{d^3}-\frac{(be^3n)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx,x,d+e\sqrt{x}\right)}{d^3}+\frac{(b^2en^2)\,\text{Subst}\left(\int\left(-\frac{e^3}{d(d-x)^3}-\frac{e^3}{d^2(d-x)^2}-\frac{e^3}{d^3(d-x)}-\frac{e^3}{d^3x}\right)\,dx,x,d+e\sqrt{x}\right)}{3d}-\frac{(b^2e^2n^2)\,\text{Subst}\left(\int\frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+e\sqrt{x}\right)}{2d^2}$$

$$=-\frac{b^2e^2n^2}{6d^2x}+\frac{b^2e^3n^2}{3d^3\sqrt{x}}-\frac{b^2e^4n^2\log\left(d+e\sqrt{x}\right)}{3d^4}-\frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3dx^{3/2}}+\frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x}-\frac{be^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4\sqrt{x}}-\frac{be^4n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4}-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x^2}+\frac{b^2e^4n^2\log(x)}{6d^4}-\frac{(b^2e^2n^2)\,\text{Subst}\left(\int\left(\frac{e^2}{d(d-x)^2}+\frac{e^2}{d^2(d-x)}+\frac{e^2}{d^2x}\right)\,dx,x,d+e\sqrt{x}\right)}{2d^2}+\frac{(b^2e^3n^2)\,\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x}{e}}\,dx,x,d+e\sqrt{x}\right)}{d^4}+\frac{(b^2e^4n^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{d}{x}\right)}{x}\,dx,x,d+e\sqrt{x}\right)}{d^4}$$

$$=-\frac{b^2e^2n^2}{6d^2x}+\frac{5b^2e^3n^2}{6d^3\sqrt{x}}-\frac{5b^2e^4n^2\log\left(d+e\sqrt{x}\right)}{6d^4}-\frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3dx^{3/2}}+\frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x}-\frac{be^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4\sqrt{x}}-\frac{be^4n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4}-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2x^2}+\frac{11b^2e^4n^2\log(x)}{12d^4}+\frac{b^2e^4n^2\text{Li}_2\left(\frac{d}{d+e\sqrt{x}}\right)}{d^4}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.20

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx =$$

$$-\frac{6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2+\frac{e\sqrt{x}\left(4bd^3n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)-6bd^2en\sqrt{x}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)+12bde^2nx\left(a+b\log\left(c\left(d+\right.\right.\right.\right.}{}}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^3,x]
```

```
[Out] -1/12*(6*(a + b*Log[c*(d + e*Sqrt[x])^n])^2 + (e*Sqrt[x]*(4*b*d^3*n*(a + b*
Log[c*(d + e*Sqrt[x])^n]) - 6*b*d^2*e*n*Sqrt[x]*(a + b*Log[c*(d + e*Sqrt[x]
)^n]) + 12*b*d*e^2*n*x*(a + b*Log[c*(d + e*Sqrt[x])^n]) - 6*e^3*x^(3/2)*(a
+ b*Log[c*(d + e*Sqrt[x])^n])^2 + 12*b*e^3*n*x^(3/2)*(a + b*Log[c*(d + e*Sq
rt[x])^n])*Log[-((e*Sqrt[x])/d)] + 6*b^2*e^3*n^2*x^(3/2)*(2*Log[d + e*Sqrt[
x]] - Log[x]) - 3*b^2*e^2*n^2*x*(2*d - 2*e*Sqrt[x]*Log[d + e*Sqrt[x]] + e*S
qrt[x]*Log[x]) + 2*b^2*e*n^2*Sqrt[x]*(d*(d - 2*e*Sqrt[x]) + 2*e^2*x*Log[d +
 e*Sqrt[x]] - e^2*x*Log[x]) + 12*b^2*e^3*n^2*x^(3/2)*PolyLog[2, 1 + (e*Sqrt
[x])/d]))/d^4)/x^2
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx = \int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^2}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*sqrt(x) + d)^n*c)^2 + 2*a*b*log((e*sqrt(x) + d)^n*c) +
 a^2)/x^3, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2/x**3,x)
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**2/x**3, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^2}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^3,x, algorithm="maxima")
[Out] -1/2*b^2*log((e*sqrt(x) + d)^n)^2/x^2 + integrate(1/2*(2*(b^2*e*log(c)^2 +
2*a*b*e*log(c) + a^2*e)*x + (b^2*e*n*x + 4*(b^2*e*log(c) + a*b*e)*x + 4*(b^
2*d*log(c) + a*b*d)*sqrt(x))*log((e*sqrt(x) + d)^n) + 2*(b^2*d*log(c)^2 + 2
*a*b*d*log(c) + a^2*d)*sqrt(x))/(e*x^4 + d*x^(7/2)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^2}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^3,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)\right)^2}{x^3}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^3,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^3, x)
```

3.414 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 408

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx &= -\frac{b^2e^2n^2}{30d^2x^2}+\frac{b^2e^3n^2}{10d^3x^{3/2}}-\frac{47b^2e^4n^2}{180d^4x} \\ &\quad+\frac{77b^2e^5n^2}{90d^5\sqrt{x}}-\frac{77b^2e^6n^2\log\left(d+e\sqrt{x}\right)}{90d^6} \\ &\quad-\frac{2ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{15dx^{5/2}} \\ &\quad+\frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{6d^2x^2} \\ &\quad-\frac{2be^3n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9d^3x^{3/2}} \\ &\quad+\frac{be^4n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3d^4x} \\ &\quad-\frac{2be^5n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3d^6\sqrt{x}} \\ &\quad-\frac{2be^6n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3d^6} \\ &\quad-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3}+\frac{137b^2e^6n^2\log(x)}{180d^6} \\ &\quad+\frac{2b^2e^6n^2\operatorname{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)}{3d^6}\end{aligned}$$

```
[Out] -1/30*b^2*e^2*n^2/d^2/x^2+1/10*b^2*e^3*n^2/d^3/x^(3/2)-47/180*b^2*e^4*n^2/d
^4/x+137/180*b^2*e^6*n^2*ln(x)/d^6-77/90*b^2*e^6*n^2*ln(d+e*x^(1/2))/d^6-2/
15*b*e*n*(a+b*ln(c*(d+e*x^(1/2))^n))/d/x^(5/2)+1/6*b*e^2*n*(a+b*ln(c*(d+e*x
```

```
^(1/2))^n))/d^2/x^2-2/9*b*e^3*n*(a+b*ln(c*(d+e*x^(1/2))^n))/d^3/x^(3/2)+1/3
*b*e^4*n*(a+b*ln(c*(d+e*x^(1/2))^n))/d^4/x-1/3*(a+b*ln(c*(d+e*x^(1/2))^n))^
2/x^3-2/3*b*e^6*n*(a+b*ln(c*(d+e*x^(1/2))^n))*ln(1-d/(d+e*x^(1/2)))/d^6+2/3
*b^2*e^6*n^2*polylog(2,d/(d+e*x^(1/2)))/d^6+77/90*b^2*e^5*n^2/d^5/x^(1/2)-2
/3*b*e^5*n*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))/d^6/x^(1/2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx = -\frac{2be^6n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3d^6} - \frac{2be^5n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3d^6\sqrt{x}} + \frac{be^4n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3d^4x} - \frac{2be^3n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9d^3x^{3/2}} + \frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{6d^2x^2} - \frac{2ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{15dx^{5/2}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3} + \frac{2b^2e^6n^2\operatorname{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)}{3d^6} - \frac{77b^2e^6n^2\log\left(d+e\sqrt{x}\right)}{90d^6} + \frac{137b^2e^6n^2\log(x)}{180d^6} + \frac{77b^2e^5n^2}{90d^5\sqrt{x}} - \frac{47b^2e^4n^2}{180d^4x} + \frac{b^2e^3n^2}{10d^3x^{3/2}} - \frac{b^2e^2n^2}{30d^2x^2}$$

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^4,x]
```

```
[Out] -1/30*(b^2*e^2*n^2)/(d^2*x^2) + (b^2*e^3*n^2)/(10*d^3*x^(3/2)) - (47*b^2*e^
4*n^2)/(180*d^4*x) + (77*b^2*e^5*n^2)/(90*d^5*Sqrt[x]) - (77*b^2*e^6*n^2*Lo
g[d + e*Sqrt[x]])/(90*d^6) - (2*b*e*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(15
*d*x^(5/2)) + (b*e^2*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(6*d^2*x^2) - (2*b
*e^3*n*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(9*d^3*x^(3/2)) + (b*e^4*n*(a + b*
Log[c*(d + e*Sqrt[x])^n]))/(3*d^4*x) - (2*b*e^5*n*(d + e*Sqrt[x])*(a + b*Lo
g[c*(d + e*Sqrt[x])^n]))/(3*d^6*Sqrt[x]) - (2*b*e^6*n*Log[1 - d/(d + e*Sqrt
```

```
[x])]*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(3*d^6) - (a + b*Log[c*(d + e*Sqrt[
x])^n])^2/(3*x^3) + (137*b^2*e^6*n^2*Log[x])/(180*d^6) + (2*b^2*e^6*n^2*Pol
yLog[2, d/(d + e*Sqrt[x])])/(3*d^6)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^7}\,dx, x, \sqrt{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3} + \frac{1}{3}(2ben)\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^6(d+ex)}\,dx, x, \sqrt{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3} + \frac{1}{3}(2bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+e\sqrt{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3} + \frac{(2bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+e\sqrt{x}\right)}{3d}\\
&\quad - \frac{(2ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+e\sqrt{x}\right)}{3d}
\end{aligned}
$$

$$= -\frac{2ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{15dx^{5/2}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3} - \frac{(2ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+e\sqrt{x}\right)}{3d^2} + \frac{(2be^2n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\sqrt{x}\right)}{3d^2} + \frac{(2b^2en^2)\,\text{Subst}\left(\int \frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+e\sqrt{x}\right)}{15d}$$

$$= -\frac{2ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{15dx^{5/2}} + \frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{6d^2x^2} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3} + \frac{(2be^2n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\sqrt{x}\right)}{3d^3} - \frac{(2be^3n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt{x}\right)}{3d^3} + \frac{(2b^2en^2)\,\text{Subst}\left(\int \left(-\frac{e^5}{d(d-x)^5} - \frac{e^5}{d^2(d-x)^4} - \frac{e^5}{d^3(d-x)^3} - \frac{e^5}{d^4(d-x)^2} - \frac{e^5}{d^5(d-x)} - \frac{e^5}{d^5x}\right)\,dx, x, d+e\sqrt{x}\right)}{15d} - \frac{(b^2e^2n^2)\,\text{Subst}\left(\int \frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\sqrt{x}\right)}{6d^2}$$

$$
\begin{aligned}
&= -\frac{b^2e^2n^2}{30d^2x^2} + \frac{2b^2e^3n^2}{45d^3x^{3/2}} - \frac{b^2e^4n^2}{15d^4x} + \frac{2b^2e^5n^2}{15d^5\sqrt{x}} - \frac{2b^2e^6n^2\log\left(d+e\sqrt{x}\right)}{15d^6} \\
&\quad - \frac{2ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{15dx^{5/2}} + \frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{6d^2x^2} \\
&\quad - \frac{2be^3n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9d^3x^{3/2}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3} \\
&\quad + \frac{b^2e^6n^2\log(x)}{15d^6} - \frac{(2be^3n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt{x}\right)}{3d^4} \\
&\quad + \frac{(2be^4n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt{x}\right)}{3d^4} \\
&\quad - \frac{(b^2e^2n^2)\,\text{Subst}\left(\int \left(\frac{e^4}{d(d-x)^4} + \frac{e^4}{d^2(d-x)^3} + \frac{e^4}{d^3(d-x)^2} + \frac{e^4}{d^4(d-x)} + \frac{e^4}{d^4x}\right)\,dx, x, d+e\sqrt{x}\right)}{6d^2} \\
&\quad + \frac{(2b^2e^3n^2)\,\text{Subst}\left(\int \frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt{x}\right)}{9d^3} \\
&= -\frac{b^2e^2n^2}{30d^2x^2} + \frac{b^2e^3n^2}{10d^3x^{3/2}} - \frac{3b^2e^4n^2}{20d^4x} + \frac{3b^2e^5n^2}{10d^5\sqrt{x}} - \frac{3b^2e^6n^2\log\left(d+e\sqrt{x}\right)}{10d^6} \\
&\quad - \frac{2ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{15dx^{5/2}} + \frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{6d^2x^2} \\
&\quad - \frac{2be^3n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9d^3x^{3/2}} + \frac{be^4n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3d^4x} \\
&\quad - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3} + \frac{3b^2e^6n^2\log(x)}{20d^6} \\
&\quad + \frac{(2be^4n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt{x}\right)}{3d^5} \\
&\quad - \frac{(2be^5n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx, x, d+e\sqrt{x}\right)}{3d^5} \\
&\quad + \frac{(2b^2e^3n^2)\,\text{Subst}\left(\int \left(-\frac{e^3}{d(d-x)^3} - \frac{e^3}{d^2(d-x)^2} - \frac{e^3}{d^3(d-x)} - \frac{e^3}{d^3x}\right)\,dx, x, d+e\sqrt{x}\right)}{9d^3} \\
&\quad - \frac{(b^2e^4n^2)\,\text{Subst}\left(\int \frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt{x}\right)}{3d^4}
\end{aligned}
$$

$$= -\frac{b^2e^2n^2}{30d^2x^2} + \frac{b^2e^3n^2}{10d^3x^{3/2}} - \frac{47b^2e^4n^2}{180d^4x} + \frac{47b^2e^5n^2}{90d^5\sqrt{x}} - \frac{47b^2e^6n^2\log\left(d+e\sqrt{x}\right)}{90d^6} - \frac{2ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{15dx^{5/2}} + \frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{6d^2x^2} - \frac{2be^3n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9d^3x^{3/2}} + \frac{be^4n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3d^4x} - \frac{2be^5n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3d^6\sqrt{x}} - \frac{2be^6n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3d^6} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3} + \frac{47b^2e^6n^2\log(x)}{180d^6} - \frac{(b^2e^4n^2)\,\mathrm{Subst}\left(\int\left(\frac{e^2}{d(d-x)^2}+\frac{e^2}{d^2(d-x)}+\frac{e^2}{d^2x}\right)dx, x, d+e\sqrt{x}\right)}{3d^4} + \frac{(2b^2e^5n^2)\,\mathrm{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x}{e}}\,dx, x, d+e\sqrt{x}\right)}{3d^6} + \frac{(2b^2e^6n^2)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{d}{x}\right)}{x}\,dx, x, d+e\sqrt{x}\right)}{3d^6}$$

$$= -\frac{b^2e^2n^2}{30d^2x^2} + \frac{b^2e^3n^2}{10d^3x^{3/2}} - \frac{47b^2e^4n^2}{180d^4x} + \frac{77b^2e^5n^2}{90d^5\sqrt{x}} - \frac{77b^2e^6n^2\log\left(d+e\sqrt{x}\right)}{90d^6} - \frac{2ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{15dx^{5/2}} + \frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{6d^2x^2} - \frac{2be^3n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9d^3x^{3/2}} + \frac{be^4n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3d^4x} - \frac{2be^5n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3d^6\sqrt{x}} - \frac{2be^6n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3d^6} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3} + \frac{137b^2e^6n^2\log(x)}{180d^6} + \frac{2b^2e^6n^2\mathrm{Li}_2\left(\frac{d}{d+e\sqrt{x}}\right)}{3d^6}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.07

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx=-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3x^3}$$
$$-\frac{be\Big(24ad^5n-30ad^4en\sqrt{x}+6bd^4en^2\sqrt{x}+40ad^3e^2nx-18bd^3e^2n^2x-60ad^2e^3nx^{3/2}+47bd^2e^3n^2x^{3/2}+1}{}$$

[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^4,x]

[Out] -1/3*(a + b*Log[c*(d + e*Sqrt[x])^n])^2/x^3 - (b*e*(24*a*d^5*n - 30*a*d^4*e*n*Sqrt[x] + 6*b*d^4*e*n^2*Sqrt[x] + 40*a*d^3*e^2*n*x - 18*b*d^3*e^2*n^2*x - 60*a*d^2*e^3*n*x^(3/2) + 47*b*d^2*e^3*n^2*x^(3/2) + 120*a*d*e^4*n*x^2 - 154*b*d*e^4*n^2*x^2 + 2*e^5*n*(-60*a + 137*b*n)*x^(5/2)*Log[d + e*Sqrt[x]] + 24*b*d^5*n*Log[c*(d + e*Sqrt[x])^n] - 30*b*d^4*e*n*Sqrt[x]*Log[c*(d + e*Sqrt[x])^n] + 40*b*d^3*e^2*n*x*Log[c*(d + e*Sqrt[x])^n] - 60*b*d^2*e^3*n*x^(3/2)*Log[c*(d + e*Sqrt[x])^n] + 120*b*d*e^4*n*x^2*Log[c*(d + e*Sqrt[x])^n] - 60*b*e^5*x^(5/2)*Log[c*(d + e*Sqrt[x])^n]^2 + 120*b*e^5*n*x^(5/2)*Log[c*(d + e*Sqrt[x])^n]*Log[-((e*Sqrt[x])/d)] + 60*a*e^5*n*x^(5/2)*Log[x] - 137*b*e^5*n^2*x^(5/2)*Log[x] + 120*b*e^5*n^2*x^(5/2)*PolyLog[2, 1 + (e*Sqrt[x])/d]))/(180*d^6*x^(5/2))

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}dx$$

[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^4,x)

[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^2/x^4,x)

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^nc\right)+a\right)^2}{x^4}\,dx$$

[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^4,x, algorithm="fricas")

[Out] integral((b^2*log((e*sqrt(x) + d)^n*c)^2 + 2*a*b*log((e*sqrt(x) + d)^n*c) + a^2)/x^4, x)

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**2/x**4,x)
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**2/x**4, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^2}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^4,x, algorithm="maxima")
[Out] -1/3*b^2*log((e*sqrt(x) + d)^n)^2/x^3 + integrate(1/3*(3*(b^2*e*log(c)^2 +
2*a*b*e*log(c) + a^2*e)*x + (b^2*e*n*x + 6*(b^2*e*log(c) + a*b*e)*x + 6*(b^
2*d*log(c) + a*b*d)*sqrt(x))*log((e*sqrt(x) + d)^n) + 3*(b^2*d*log(c)^2 + 2
*a*b*d*log(c) + a^2*d)*sqrt(x))/(e*x^5 + d*x^(9/2)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^2}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^2/x^4,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)\right)^2}{x^4}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^4,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))^2/x^4, x)
```

3.415 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 907

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx=-\frac{15b^3d^4n^3\left(d+e\sqrt{x}\right)^2}{4e^6}+\frac{40b^3d^3n^3\left(d+e\sqrt{x}\right)^3}{27e^6}-\frac{15b^3d^2n^3\left(d+e\sqrt{x}\right)^4}{32e^6}+\frac{12b^3dn^3\left(d+e\sqrt{x}\right)^5}{125e^6}-\frac{b^3n^3\left(d+e\sqrt{x}\right)^6}{108e^6}-\frac{12ab^2d^5n^2\sqrt{x}}{e^5}+\frac{12b^3d^5n^3\sqrt{x}}{e^5}-\frac{12b^3d^5n^2\left(d+e\sqrt{x}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{e^6}+\frac{15b^2d^4n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^6}-\frac{40b^2d^3n^2\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9e^6}+\frac{15b^2d^2n^2\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{8e^6}-\frac{12b^2dn^2\left(d+e\sqrt{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{25e^6}+\frac{b^2n^2\left(d+e\sqrt{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{18e^6}+\frac{6bd^5n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^6}-\frac{15bd^4n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2e^6}+\frac{20bd^3n\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3e^6}-\frac{15bd^2n\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{4e^6}+\frac{6bdn\left(d+e\sqrt{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{5e^6}-\frac{bn\left(d+e\sqrt{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{6e^6}-\frac{2d^5\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^6}+\frac{5d^4\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^6}-\frac{20d^3\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{3e^6}+\frac{5d^2\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^6}-\frac{2d\left(d+e\sqrt{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^6}+\frac{\left(d+e\sqrt{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2e^6}$$

```
[Out] 12*b^3*d^5*n^3*x^(1/2)/e^5-15/4*b^3*d^4*n^3*(d+e*x^(1/2))^2/e^6+40/27*b^3*d
^3*n^3*(d+e*x^(1/2))^3/e^6-15/32*b^3*d^2*n^3*(d+e*x^(1/2))^4/e^6+12/125*b^3
*d*n^3*(d+e*x^(1/2))^5/e^6+1/18*b^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^
(1/2))^6/e^6-1/6*b*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))^6/e^6-2*d^
5*(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x^(1/2))/e^6+5*d^4*(a+b*ln(c*(d+e*x^(1
/2))^n))^3*(d+e*x^(1/2))^2/e^6-20/3*d^3*(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*
x^(1/2))^3/e^6+5*d^2*(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x^(1/2))^4/e^6-2*d*
(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x^(1/2))^5/e^6-1/108*b^3*n^3*(d+e*x^(1/2
))^6/e^6+1/3*(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x^(1/2))^6/e^6-12*a*b^2*d^5
*n^2*x^(1/2)/e^5-12*b^3*d^5*n^2*ln(c*(d+e*x^(1/2))^n)*(d+e*x^(1/2))/e^6+6*b
*d^5*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))/e^6+15/2*b^2*d^4*n^2*(a+
b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^2/e^6-15/2*b*d^4*n*(a+b*ln(c*(d+e*x^
(1/2))^n))^2*(d+e*x^(1/2))^2/e^6-40/9*b^2*d^3*n^2*(a+b*ln(c*(d+e*x^(1/2))^n
))*(d+e*x^(1/2))^3/e^6+20/3*b*d^3*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1
/2))^3/e^6+15/8*b^2*d^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^4/e^6
-15/4*b*d^2*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))^4/e^6-12/25*b^2*d
*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^5/e^6+6/5*b*d*n*(a+b*ln(c*(d
+e*x^(1/2))^n))^2*(d+e*x^(1/2))^5/e^6
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 907, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341\}$

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx = -\frac{b^3n^3\left(d+e\sqrt{x}\right)^6}{108e^6} + \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\left(d+e\sqrt{x}\right)^6}{3e^6} - \frac{bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\left(d+e\sqrt{x}\right)^6}{6e^6} + \frac{b^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\left(d+e\sqrt{x}\right)^6}{18e^6} + \frac{12b^3dn^3\left(d+e\sqrt{x}\right)^5}{125e^6} - \frac{2d\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\left(d+e\sqrt{x}\right)^5}{e^6} + \frac{6bdn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\left(d+e\sqrt{x}\right)^5}{5e^6} - \frac{12b^2dn^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\left(d+e\sqrt{x}\right)^5}{25e^6} - \frac{15b^3d^2n^3\left(d+e\sqrt{x}\right)^4}{32e^6} + \frac{5d^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\left(d+e\sqrt{x}\right)^4}{e^6} - \frac{15bd^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\left(d+e\sqrt{x}\right)^4}{4e^6} + \frac{15b^2d^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\left(d+e\sqrt{x}\right)^4}{8e^6} + \frac{40b^3d^3n^3\left(d+e\sqrt{x}\right)^3}{27e^6} - \frac{20d^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\left(d+e\sqrt{x}\right)^3}{3e^6} + \frac{20bd^3n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\left(d+e\sqrt{x}\right)^3}{3e^6} - \frac{40b^2d^3n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\left(d+e\sqrt{x}\right)^3}{9e^6} - \frac{15b^3d^4n^3\left(d+e\sqrt{x}\right)^2}{4e^6} + \frac{5d^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\left(d+e\sqrt{x}\right)^2}{e^6} - \frac{15bd^4n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\left(d+e\sqrt{x}\right)^2}{2e^6} + \frac{15b^2d^4n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\left(d+e\sqrt{x}\right)^2}{2e^6} - \frac{2d^5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\left(d+e\sqrt{x}\right)}{e^6}$$

$$6bd^5n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\left(d+e\sqrt{x}\right)$$

```
[In] Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

```
[Out] (-15*b^3*d^4*n^3*(d + e*Sqrt[x])^2)/(4*e^6) + (40*b^3*d^3*n^3*(d + e*Sqrt[x
])^3)/(27*e^6) - (15*b^3*d^2*n^3*(d + e*Sqrt[x])^4)/(32*e^6) + (12*b^3*d*n^
3*(d + e*Sqrt[x])^5)/(125*e^6) - (b^3*n^3*(d + e*Sqrt[x])^6)/(108*e^6) - (1
2*a*b^2*d^5*n^2*Sqrt[x])/e^5 + (12*b^3*d^5*n^3*Sqrt[x])/e^5 - (12*b^3*d^5*n
^2*(d + e*Sqrt[x])*Log[c*(d + e*Sqrt[x])^n])/e^6 + (15*b^2*d^4*n^2*(d + e*S
qrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*e^6) - (40*b^2*d^3*n^2*(d +
e*Sqrt[x])^3*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(9*e^6) + (15*b^2*d^2*n^2*(d
 + e*Sqrt[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(8*e^6) - (12*b^2*d*n^2*(
d + e*Sqrt[x])^5*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(25*e^6) + (b^2*n^2*(d +
 e*Sqrt[x])^6*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(18*e^6) + (6*b*d^5*n*(d +
e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/e^6 - (15*b*d^4*n*(d + e*Sqr
t[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*e^6) + (20*b*d^3*n*(d + e*Sq
rt[x])^3*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(3*e^6) - (15*b*d^2*n*(d + e*S
qrt[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(4*e^6) + (6*b*d*n*(d + e*Sqr
t[x])^5*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(5*e^6) - (b*n*(d + e*Sqrt[x])^
6*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(6*e^6) - (2*d^5*(d + e*Sqrt[x])*(a +
 b*Log[c*(d + e*Sqrt[x])^n])^3)/e^6 + (5*d^4*(d + e*Sqrt[x])^2*(a + b*Log[c
*(d + e*Sqrt[x])^n])^3)/e^6 - (20*d^3*(d + e*Sqrt[x])^3*(a + b*Log[c*(d + e
*Sqrt[x])^n])^3)/(3*e^6) + (5*d^2*(d + e*Sqrt[x])^4*(a + b*Log[c*(d + e*Sqr
t[x])^n])^3)/e^6 - (2*d*(d + e*Sqrt[x])^5*(a + b*Log[c*(d + e*Sqrt[x])^n])^
3)/e^6 + ((d + e*Sqrt[x])^6*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/(3*e^6)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^5(a+b\log(c(d+ex)^n))^3\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int\left(-\frac{d^5(a+b\log(c(d+ex)^n))^3}{e^5}+\frac{5d^4(d+ex)(a+b\log(c(d+ex)^n))^3}{e^5}\right.\right.\\
&\qquad -\frac{10d^3(d+ex)^2(a+b\log(c(d+ex)^n))^3}{e^5}\\
&\qquad +\frac{10d^2(d+ex)^3(a+b\log(c(d+ex)^n))^3}{e^5}-\frac{5d(d+ex)^4(a+b\log(c(d+ex)^n))^3}{e^5}\\
&\qquad \left.\left.+\frac{(d+ex)^5(a+b\log(c(d+ex)^n))^3}{e^5}\right)\,dx, x, \sqrt{x}\right)
\end{aligned}
$$

$$= \frac{2\text{Subst}\left(\int (d+ex)^5 \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \, dx, x, \sqrt{x}\right)}{e^5} - \frac{(10d)\text{Subst}\left(\int (d+ex)^4 \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \, dx, x, \sqrt{x}\right)}{e^5} + \frac{(20d^2)\,\text{Subst}\left(\int (d+ex)^3 \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \, dx, x, \sqrt{x}\right)}{e^5} - \frac{(20d^3)\,\text{Subst}\left(\int (d+ex)^2 \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \, dx, x, \sqrt{x}\right)}{e^5} + \frac{(10d^4)\,\text{Subst}\left(\int (d+ex) \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \, dx, x, \sqrt{x}\right)}{e^5} - \frac{(2d^5)\,\text{Subst}\left(\int \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \, dx, x, \sqrt{x}\right)}{e^5}$$

$$= \frac{2\text{Subst}\left(\int x^5\left(a+b\log\left(cx^n\right)\right)^3 \, dx, x, d+e\sqrt{x}\right)}{e^6} - \frac{(10d)\text{Subst}\left(\int x^4\left(a+b\log\left(cx^n\right)\right)^3 \, dx, x, d+e\sqrt{x}\right)}{e^6} + \frac{(20d^2)\,\text{Subst}\left(\int x^3\left(a+b\log\left(cx^n\right)\right)^3 \, dx, x, d+e\sqrt{x}\right)}{e^6} - \frac{(20d^3)\,\text{Subst}\left(\int x^2\left(a+b\log\left(cx^n\right)\right)^3 \, dx, x, d+e\sqrt{x}\right)}{e^6} + \frac{(10d^4)\,\text{Subst}\left(\int x\left(a+b\log\left(cx^n\right)\right)^3 \, dx, x, d+e\sqrt{x}\right)}{e^6} - \frac{(2d^5)\,\text{Subst}\left(\int \left(a+b\log\left(cx^n\right)\right)^3 \, dx, x, d+e\sqrt{x}\right)}{e^6}$$

$$
\begin{aligned}
&= -\frac{2d^5\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^6} \\
&\quad+\frac{5d^4\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^6} \\
&\quad-\frac{20d^3\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{3e^6} \\
&\quad+\frac{5d^2\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^6} \\
&\quad-\frac{2d\left(d+e\sqrt{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^6} \\
&\quad+\frac{\left(d+e\sqrt{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{3e^6} \\
&\quad-\frac{(bn)\mathrm{Subst}\left(\int x^5(a+b\log(cx^n))^2\,dx,x,d+e\sqrt{x}\right)}{e^6} \\
&\quad+\frac{(6bdn)\mathrm{Subst}\left(\int x^4(a+b\log(cx^n))^2\,dx,x,d+e\sqrt{x}\right)}{e^6} \\
&\quad-\frac{(15bd^2n)\,\mathrm{Subst}\left(\int x^3(a+b\log(cx^n))^2\,dx,x,d+e\sqrt{x}\right)}{e^6} \\
&\quad+\frac{(20bd^3n)\,\mathrm{Subst}\left(\int x^2(a+b\log(cx^n))^2\,dx,x,d+e\sqrt{x}\right)}{e^6} \\
&\quad-\frac{(15bd^4n)\,\mathrm{Subst}\left(\int x(a+b\log(cx^n))^2\,dx,x,d+e\sqrt{x}\right)}{e^6} \\
&\quad+\frac{(6bd^5n)\,\mathrm{Subst}\left(\int (a+b\log(cx^n))^2\,dx,x,d+e\sqrt{x}\right)}{e^6}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{6bd^5n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^6} \\
&- \frac{15bd^4n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2e^6} \\
&+ \frac{20bd^3n\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3e^6} \\
&- \frac{15bd^2n\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{4e^6} \\
&+ \frac{6bdn\left(d+e\sqrt{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{5e^6} \\
&- \frac{bn\left(d+e\sqrt{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{6e^6} \\
&- \frac{2d^5\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^6} \\
&+ \frac{5d^4\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^6} \\
&- \frac{20d^3\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{3e^6} \\
&+ \frac{5d^2\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^6} \\
&- \frac{2d\left(d+e\sqrt{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^6} \\
&+ \frac{\left(d+e\sqrt{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{3e^6} \\
&+ \frac{(b^2n^2)\,\mathrm{Subst}\left(\int x^5(a+b\log\left(cx^n\right))\,dx, x, d+e\sqrt{x}\right)}{3e^6} \\
&- \frac{(12b^2dn^2)\,\mathrm{Subst}\left(\int x^4(a+b\log\left(cx^n\right))\,dx, x, d+e\sqrt{x}\right)}{5e^6} \\
&+ \frac{(15b^2d^2n^2)\,\mathrm{Subst}\left(\int x^3(a+b\log\left(cx^n\right))\,dx, x, d+e\sqrt{x}\right)}{2e^6} \\
&- \frac{(40b^2d^3n^2)\,\mathrm{Subst}\left(\int x^2(a+b\log\left(cx^n\right))\,dx, x, d+e\sqrt{x}\right)}{3e^6} \\
&+ \frac{(15b^2d^4n^2)\,\mathrm{Subst}\left(\int x(a+b\log\left(cx^n\right))\,dx, x, d+e\sqrt{x}\right)}{e^6} \\
&- \frac{(12b^2d^5n^2)\,\mathrm{Subst}\left(\int (a+b\log\left(cx^n\right))\,dx, x, d+e\sqrt{x}\right)}{e^6}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{15b^3d^4n^3\left(d+e\sqrt{x}\right)^2}{4e^6}+\frac{40b^3d^3n^3\left(d+e\sqrt{x}\right)^3}{27e^6}-\frac{15b^3d^2n^3\left(d+e\sqrt{x}\right)^4}{32e^6} \\
&+\frac{12b^3dn^3\left(d+e\sqrt{x}\right)^5}{125e^6}-\frac{b^3n^3\left(d+e\sqrt{x}\right)^6}{108e^6}-\frac{12ab^2d^5n^2\sqrt{x}}{e^5} \\
&+\frac{15b^2d^4n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^6} \\
&-\frac{40b^2d^3n^2\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9e^6} \\
&+\frac{15b^2d^2n^2\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{8e^6} \\
&-\frac{12b^2dn^2\left(d+e\sqrt{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{25e^6} \\
&+\frac{b^2n^2\left(d+e\sqrt{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{18e^6} \\
&+\frac{6bd^5n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^6} \\
&-\frac{15bd^4n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2e^6} \\
&+\frac{20bd^3n\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3e^6} \\
&-\frac{15bd^2n\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{4e^6} \\
&+\frac{6bdn\left(d+e\sqrt{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{5e^6} \\
&-\frac{bn\left(d+e\sqrt{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{6e^6} \\
&-\frac{2d^5\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^6} \\
&+\frac{5d^4\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^6} \\
&-\frac{20d^3\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{3e^6} \\
&+\frac{5d^2\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^6} \\
&-\frac{2d\left(d+e\sqrt{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^6} \\
&+\frac{\left(d+e\sqrt{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{3e^6} \\
&-\frac{\left(12b^3d^5n^2\right)\operatorname{Subst}\left(\int\log\left(cx^n\right)\,dx,x,d+e\sqrt{x}\right)}{e^6}
\end{aligned}
$$

$$= -\frac{15b^3d^4n^3\left(d+e\sqrt{x}\right)^2}{4e^6} + \frac{40b^3d^3n^3\left(d+e\sqrt{x}\right)^3}{27e^6} - \frac{15b^3d^2n^3\left(d+e\sqrt{x}\right)^4}{32e^6}$$
$$+ \frac{12b^3dn^3\left(d+e\sqrt{x}\right)^5}{125e^6} - \frac{b^3n^3\left(d+e\sqrt{x}\right)^6}{108e^6} - \frac{12ab^2d^5n^2\sqrt{x}}{e^5}$$
$$+ \frac{12b^3d^5n^3\sqrt{x}}{e^5} - \frac{12b^3d^5n^2\left(d+e\sqrt{x}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{e^6}$$
$$+ \frac{15b^2d^4n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^6}$$
$$- \frac{40b^2d^3n^2\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{9e^6}$$
$$+ \frac{15b^2d^2n^2\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{8e^6}$$
$$- \frac{12b^2dn^2\left(d+e\sqrt{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{25e^6}$$
$$+ \frac{b^2n^2\left(d+e\sqrt{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{18e^6}$$
$$+ \frac{6bd^5n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^6}$$
$$- \frac{15bd^4n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2e^6}$$
$$+ \frac{20bd^3n\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{3e^6}$$
$$- \frac{15bd^2n\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{4e^6}$$
$$+ \frac{6bdn\left(d+e\sqrt{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{5e^6}$$
$$- \frac{bn\left(d+e\sqrt{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{6e^6}$$
$$- \frac{2d^5\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^6}$$
$$+ \frac{5d^4\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^6}$$
$$- \frac{20d^3\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{3e^6}$$
$$+ \frac{5d^2\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^6}$$
$$- \frac{2d\left(d+e\sqrt{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^6}$$
$$+ \frac{\left(d+e\sqrt{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{3e^6}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 577, normalized size of antiderivative = 0.64

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3 dx$$
$$= \frac{b^3en^3\sqrt{x}\left(809340d^5-140070d^4e\sqrt{x}+41180d^3e^2x-13785d^2e^3x^{3/2}+4368de^4x^2-1000e^5x^{5/2}\right)+1800a^2}{}$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

```
[Out] (b^3*e*n^3*Sqrt[x]*(809340*d^5 - 140070*d^4*e*Sqrt[x] + 41180*d^3*e^2*x - 1
3785*d^2*e^3*x^(3/2) + 4368*d*e^4*x^2 - 1000*e^5*x^(5/2)) + 1800*a^2*b*n*(1
47*d^6 + 60*d^5*e*Sqrt[x] - 30*d^4*e^2*x + 20*d^3*e^3*x^(3/2) - 15*d^2*e^4*
x^2 + 12*d*e^5*x^(5/2) - 10*e^6*x^3) - 36000*a^3*(d^6 - e^6*x^3) + 60*a*b^2
*n^2*(8111*d^6 - 8820*d^5*e*Sqrt[x] + 2610*d^4*e^2*x - 1140*d^3*e^3*x^(3/2)
 + 555*d^2*e^4*x^2 - 264*d*e^5*x^(5/2) + 100*e^6*x^3) - 60*b*(b^2*n^2*(1348
9*d^6 + 8820*d^5*e*Sqrt[x] - 2610*d^4*e^2*x + 1140*d^3*e^3*x^(3/2) - 555*d^
2*e^4*x^2 + 264*d*e^5*x^(5/2) - 100*e^6*x^3) - 60*a*b*n*(147*d^6 + 60*d^5*e
*Sqrt[x] - 30*d^4*e^2*x + 20*d^3*e^3*x^(3/2) - 15*d^2*e^4*x^2 + 12*d*e^5*x^
(5/2) - 10*e^6*x^3) + 1800*a^2*(d^6 - e^6*x^3))*Log[c*(d + e*Sqrt[x])^n] -
1800*b^2*(60*a*(d^6 - e^6*x^3) + b*n*(-147*d^6 - 60*d^5*e*Sqrt[x] + 30*d^4*
e^2*x - 20*d^3*e^3*x^(3/2) + 15*d^2*e^4*x^2 - 12*d*e^5*x^(5/2) + 10*e^6*x^3
))*Log[c*(d + e*Sqrt[x])^n]^2 - 36000*b^3*(d^6 - e^6*x^3)*Log[c*(d + e*Sqrt
[x])^n]^3)/(108000*e^6)
```

Maple [F]

$$\int x^2 \left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3 dx$$

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n))^3,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^n))^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 1197, normalized size of antiderivative = 1.32

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3 dx = \text{Too large to display}$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/108000*(36000*b^3*e^6*x^3*log(c)^3 - 1000*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2
+ 18*a^2*b*e^6*n - 36*a^3*e^6)*x^3 + 36000*(b^3*e^6*n^3*x^3 - b^3*d^6*n^3)*
log(e*sqrt(x) + d)^3 - 15*(919*b^3*d^2*e^4*n^3 - 2220*a*b^2*d^2*e^4*n^2 + 1
800*a^2*b*d^2*e^4*n)*x^2 - 1800*(15*b^3*d^2*e^4*n^3*x^2 + 30*b^3*d^4*e^2*n^
3*x - 147*b^3*d^6*n^3 + 60*a*b^2*d^6*n^2 + 10*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^
2)*x^3 - 60*(b^3*e^6*n^2*x^3 - b^3*d^6*n^2)*log(c) - 4*(3*b^3*d*e^5*n^3*x^2
 + 5*b^3*d^3*e^3*n^3*x + 15*b^3*d^5*e*n^3)*sqrt(x))*log(e*sqrt(x) + d)^2 -
9000*(3*b^3*d^2*e^4*n*x^2 + 6*b^3*d^4*e^2*n*x + 2*(b^3*e^6*n - 6*a*b^2*e^6)
*x^3)*log(c)^2 - 30*(4669*b^3*d^4*e^2*n^3 - 5220*a*b^2*d^4*e^2*n^2 + 1800*a
^2*b*d^4*e^2*n)*x - 60*(13489*b^3*d^6*n^3 - 8820*a*b^2*d^6*n^2 + 1800*a^2*b
*d^6*n - 100*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2 + 18*a^2*b*e^6*n)*x^3 - 15*(37*
b^3*d^2*e^4*n^3 - 60*a*b^2*d^2*e^4*n^2)*x^2 - 1800*(b^3*e^6*n*x^3 - b^3*d^6
*n)*log(c)^2 - 90*(29*b^3*d^4*e^2*n^3 - 20*a*b^2*d^4*e^2*n^2)*x + 60*(15*b^
3*d^2*e^4*n^2*x^2 + 30*b^3*d^4*e^2*n^2*x - 147*b^3*d^6*n^2 + 60*a*b^2*d^6*n
 + 10*(b^3*e^6*n^2 - 6*a*b^2*e^6*n)*x^3)*log(c) + 12*(735*b^3*d^5*e*n^3 - 3
00*a*b^2*d^5*e*n^2 + 2*(11*b^3*d*e^5*n^3 - 30*a*b^2*d*e^5*n^2)*x^2 + 5*(19*
b^3*d^3*e^3*n^3 - 20*a*b^2*d^3*e^3*n^2)*x - 20*(3*b^3*d*e^5*n^2*x^2 + 5*b^3
*d^3*e^3*n^2*x + 15*b^3*d^5*e*n^2)*log(c))*sqrt(x))*log(e*sqrt(x) + d) + 30
0*(20*(b^3*e^6*n^2 - 6*a*b^2*e^6*n + 18*a^2*b*e^6)*x^3 + 3*(37*b^3*d^2*e^4*
n^2 - 60*a*b^2*d^2*e^4*n)*x^2 + 18*(29*b^3*d^4*e^2*n^2 - 20*a*b^2*d^4*e^2*n
)*x)*log(c) + 4*(202335*b^3*d^5*e*n^3 - 132300*a*b^2*d^5*e*n^2 + 27000*a^2*
b*d^5*e*n + 12*(91*b^3*d*e^5*n^3 - 330*a*b^2*d*e^5*n^2 + 450*a^2*b*d*e^5*n)
*x^2 + 1800*(3*b^3*d*e^5*n*x^2 + 5*b^3*d^3*e^3*n*x + 15*b^3*d^5*e*n)*log(c)
^2 + 5*(2059*b^3*d^3*e^3*n^3 - 3420*a*b^2*d^3*e^3*n^2 + 1800*a^2*b*d^3*e^3*
n)*x - 180*(735*b^3*d^5*e*n^2 - 300*a*b^2*d^5*e*n + 2*(11*b^3*d*e^5*n^2 - 3
0*a*b^2*d*e^5*n)*x^2 + 5*(19*b^3*d^3*e^3*n^2 - 20*a*b^2*d^3*e^3*n)*x)*log(c
))*sqrt(x))/e^6
```

Sympy [F]

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx=\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$$

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))**n))**3,x)
```

```
[Out] Integral(x**2*(a + b*log(c*(d + e*sqrt(x))**n))**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 666, normalized size of antiderivative = 0.73

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$$
$$=\frac{1}{3}\,b^3x^3\log\left(\left(e\sqrt{x}+d\right)^nc\right)^3+ab^2x^3\log\left(\left(e\sqrt{x}+d\right)^nc\right)^2+a^2bx^3\log\left(\left(e\sqrt{x}+d\right)^nc\right)+\frac{1}{3}\,a^3x^3$$
$$-\frac{1}{60}\,a^2ben\left(\frac{60\,d^6\log\left(e\sqrt{x}+d\right)}{e^7}+\frac{10\,e^5x^3-12\,de^4x^{\frac{5}{2}}+15\,d^2e^3x^2-20\,d^3e^2x^{\frac{3}{2}}+30\,d^4ex-60\,d^5\sqrt{x}}{e^6}\right)$$
$$-\frac{1}{1800}\left(60\,en\left(\frac{60\,d^6\log\left(e\sqrt{x}+d\right)}{e^7}+\frac{10\,e^5x^3-12\,de^4x^{\frac{5}{2}}+15\,d^2e^3x^2-20\,d^3e^2x^{\frac{3}{2}}+30\,d^4ex-60\,d^5\sqrt{x}}{e^6}\right.\right.$$
$$-\frac{1}{108000}\left(1800\,en\left(\frac{60\,d^6\log\left(e\sqrt{x}+d\right)}{e^7}+\frac{10\,e^5x^3-12\,de^4x^{\frac{5}{2}}+15\,d^2e^3x^2-20\,d^3e^2x^{\frac{3}{2}}+30\,d^4ex-60\,d\right.\right.$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3*log((e*sqrt(x) + d)^n*c)^3 + a*b^2*x^3*log((e*sqrt(x) + d)^n*c)
^2 + a^2*b*x^3*log((e*sqrt(x) + d)^n*c) + 1/3*a^3*x^3 - 1/60*a^2*b*e*n*(60*
d^6*log(e*sqrt(x) + d)/e^7 + (10*e^5*x^3 - 12*d*e^4*x^(5/2) + 15*d^2*e^3*x^
2 - 20*d^3*e^2*x^(3/2) + 30*d^4*e*x - 60*d^5*sqrt(x))/e^6) - 1/1800*(60*e*n
*(60*d^6*log(e*sqrt(x) + d)/e^7 + (10*e^5*x^3 - 12*d*e^4*x^(5/2) + 15*d^2*e
^3*x^2 - 20*d^3*e^2*x^(3/2) + 30*d^4*e*x - 60*d^5*sqrt(x))/e^6)*log((e*sqrt
(x) + d)^n*c) - (100*e^6*x^3 - 264*d*e^5*x^(5/2) + 555*d^2*e^4*x^2 + 1800*d
^6*log(e*sqrt(x) + d)^2 - 1140*d^3*e^3*x^(3/2) + 2610*d^4*e^2*x + 8820*d^6*
log(e*sqrt(x) + d) - 8820*d^5*e*sqrt(x))*n^2/e^6)*a*b^2 - 1/108000*(1800*e*
n*(60*d^6*log(e*sqrt(x) + d)/e^7 + (10*e^5*x^3 - 12*d*e^4*x^(5/2) + 15*d^2*
e^3*x^2 - 20*d^3*e^2*x^(3/2) + 30*d^4*e*x - 60*d^5*sqrt(x))/e^6)*log((e*sqr
t(x) + d)^n*c)^2 + e*n*((1000*e^6*x^3 + 36000*d^6*log(e*sqrt(x) + d)^3 - 43
68*d*e^5*x^(5/2) + 13785*d^2*e^4*x^2 + 264600*d^6*log(e*sqrt(x) + d)^2 - 41
180*d^3*e^3*x^(3/2) + 140070*d^4*e^2*x + 809340*d^6*log(e*sqrt(x) + d) - 80
9340*d^5*e*sqrt(x))*n^2/e^7 - 60*(100*e^6*x^3 - 264*d*e^5*x^(5/2) + 555*d^2
*e^4*x^2 + 1800*d^6*log(e*sqrt(x) + d)^2 - 1140*d^3*e^3*x^(3/2) + 2610*d^4*
e^2*x + 8820*d^6*log(e*sqrt(x) + d) - 8820*d^5*e*sqrt(x))*n*log((e*sqrt(x)
+ d)^n*c)/e^7))*b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2160 vs. $2(787) = 1574$.

Time = 0.35 (sec) , antiderivative size = 2160, normalized size of antiderivative = 2.38

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="giac")
```

```
[Out] 1/108000*(36000*b^3*e*x^3*log(c)^3 + 108000*a*b^2*e*x^3*log(c)^2 + 108000*a
^2*b*e*x^3*log(c) + (36000*(e*sqrt(x) + d)^6*log(e*sqrt(x) + d)^3/e^5 - 216
000*(e*sqrt(x) + d)^5*d*log(e*sqrt(x) + d)^3/e^5 + 540000*(e*sqrt(x) + d)^4
*d^2*log(e*sqrt(x) + d)^3/e^5 - 720000*(e*sqrt(x) + d)^3*d^3*log(e*sqrt(x)
+ d)^3/e^5 + 540000*(e*sqrt(x) + d)^2*d^4*log(e*sqrt(x) + d)^3/e^5 - 216000
*(e*sqrt(x) + d)*d^5*log(e*sqrt(x) + d)^3/e^5 - 18000*(e*sqrt(x) + d)^6*log
(e*sqrt(x) + d)^2/e^5 + 129600*(e*sqrt(x) + d)^5*d*log(e*sqrt(x) + d)^2/e^5
 - 405000*(e*sqrt(x) + d)^4*d^2*log(e*sqrt(x) + d)^2/e^5 + 720000*(e*sqrt(x
) + d)^3*d^3*log(e*sqrt(x) + d)^2/e^5 - 810000*(e*sqrt(x) + d)^2*d^4*log(e*
sqrt(x) + d)^2/e^5 + 648000*(e*sqrt(x) + d)*d^5*log(e*sqrt(x) + d)^2/e^5 +
6000*(e*sqrt(x) + d)^6*log(e*sqrt(x) + d)/e^5 - 51840*(e*sqrt(x) + d)^5*d*l
og(e*sqrt(x) + d)/e^5 + 202500*(e*sqrt(x) + d)^4*d^2*log(e*sqrt(x) + d)/e^5
 - 480000*(e*sqrt(x) + d)^3*d^3*log(e*sqrt(x) + d)/e^5 + 810000*(e*sqrt(x)
+ d)^2*d^4*log(e*sqrt(x) + d)/e^5 - 1296000*(e*sqrt(x) + d)*d^5*log(e*sqrt(
x) + d)/e^5 - 1000*(e*sqrt(x) + d)^6/e^5 + 10368*(e*sqrt(x) + d)^5*d/e^5 -
50625*(e*sqrt(x) + d)^4*d^2/e^5 + 160000*(e*sqrt(x) + d)^3*d^3/e^5 - 405000
*(e*sqrt(x) + d)^2*d^4/e^5 + 1296000*(e*sqrt(x) + d)*d^5/e^5)*b^3*n^3 + 360
00*a^3*e*x^3 + 60*(1800*(e*sqrt(x) + d)^6*log(e*sqrt(x) + d)^2/e^5 - 10800*
(e*sqrt(x) + d)^5*d*log(e*sqrt(x) + d)^2/e^5 + 27000*(e*sqrt(x) + d)^4*d^2*
log(e*sqrt(x) + d)^2/e^5 - 36000*(e*sqrt(x) + d)^3*d^3*log(e*sqrt(x) + d)^2
/e^5 + 27000*(e*sqrt(x) + d)^2*d^4*log(e*sqrt(x) + d)^2/e^5 - 10800*(e*sqrt
(x) + d)*d^5*log(e*sqrt(x) + d)^2/e^5 - 600*(e*sqrt(x) + d)^6*log(e*sqrt(x)
 + d)/e^5 + 4320*(e*sqrt(x) + d)^5*d*log(e*sqrt(x) + d)/e^5 - 13500*(e*sqrt
(x) + d)^4*d^2*log(e*sqrt(x) + d)/e^5 + 24000*(e*sqrt(x) + d)^3*d^3*log(e*s
qrt(x) + d)/e^5 - 27000*(e*sqrt(x) + d)^2*d^4*log(e*sqrt(x) + d)/e^5 + 2160
0*(e*sqrt(x) + d)*d^5*log(e*sqrt(x) + d)/e^5 + 100*(e*sqrt(x) + d)^6/e^5 -
864*(e*sqrt(x) + d)^5*d/e^5 + 3375*(e*sqrt(x) + d)^4*d^2/e^5 - 8000*(e*sqrt
(x) + d)^3*d^3/e^5 + 13500*(e*sqrt(x) + d)^2*d^4/e^5 - 21600*(e*sqrt(x) + d
)*d^5/e^5)*b^3*n^2*log(c) + 1800*(60*(e*sqrt(x) + d)^6*log(e*sqrt(x) + d)/e
^5 - 360*(e*sqrt(x) + d)^5*d*log(e*sqrt(x) + d)/e^5 + 900*(e*sqrt(x) + d)^4
*d^2*log(e*sqrt(x) + d)/e^5 - 1200*(e*sqrt(x) + d)^3*d^3*log(e*sqrt(x) + d)
/e^5 + 900*(e*sqrt(x) + d)^2*d^4*log(e*sqrt(x) + d)/e^5 - 360*(e*sqrt(x) +
d)*d^5*log(e*sqrt(x) + d)/e^5 - 10*(e*sqrt(x) + d)^6/e^5 + 72*(e*sqrt(x) +
d)^5*d/e^5 - 225*(e*sqrt(x) + d)^4*d^2/e^5 + 400*(e*sqrt(x) + d)^3*d^3/e^5
- 450*(e*sqrt(x) + d)^2*d^4/e^5 + 360*(e*sqrt(x) + d)*d^5/e^5)*b^3*n*log(c)
```

```
^2 + 60*(1800*(e*sqrt(x) + d)^6*log(e*sqrt(x) + d)^2/e^5 - 10800*(e*sqrt(x)
 + d)^5*d*log(e*sqrt(x) + d)^2/e^5 + 27000*(e*sqrt(x) + d)^4*d^2*log(e*sqrt
(x) + d)^2/e^5 - 36000*(e*sqrt(x) + d)^3*d^3*log(e*sqrt(x) + d)^2/e^5 + 270
00*(e*sqrt(x) + d)^2*d^4*log(e*sqrt(x) + d)^2/e^5 - 10800*(e*sqrt(x) + d)*d
^5*log(e*sqrt(x) + d)^2/e^5 - 600*(e*sqrt(x) + d)^6*log(e*sqrt(x) + d)/e^5
 + 4320*(e*sqrt(x) + d)^5*d*log(e*sqrt(x) + d)/e^5 - 13500*(e*sqrt(x) + d)^4
*d^2*log(e*sqrt(x) + d)/e^5 + 24000*(e*sqrt(x) + d)^3*d^3*log(e*sqrt(x) + d
)/e^5 - 27000*(e*sqrt(x) + d)^2*d^4*log(e*sqrt(x) + d)/e^5 + 21600*(e*sqrt(
x) + d)*d^5*log(e*sqrt(x) + d)/e^5 + 100*(e*sqrt(x) + d)^6/e^5 - 864*(e*sqr
t(x) + d)^5*d/e^5 + 3375*(e*sqrt(x) + d)^4*d^2/e^5 - 8000*(e*sqrt(x) + d)^3
*d^3/e^5 + 13500*(e*sqrt(x) + d)^2*d^4/e^5 - 21600*(e*sqrt(x) + d)*d^5/e^5)
*a*b^2*n^2 + 3600*(60*(e*sqrt(x) + d)^6*log(e*sqrt(x) + d)/e^5 - 360*(e*sqr
t(x) + d)^5*d*log(e*sqrt(x) + d)/e^5 + 900*(e*sqrt(x) + d)^4*d^2*log(e*sqrt
(x) + d)/e^5 - 1200*(e*sqrt(x) + d)^3*d^3*log(e*sqrt(x) + d)/e^5 + 900*(e*s
qrt(x) + d)^2*d^4*log(e*sqrt(x) + d)/e^5 - 360*(e*sqrt(x) + d)*d^5*log(e*sq
rt(x) + d)/e^5 - 10*(e*sqrt(x) + d)^6/e^5 + 72*(e*sqrt(x) + d)^5*d/e^5 - 22
5*(e*sqrt(x) + d)^4*d^2/e^5 + 400*(e*sqrt(x) + d)^3*d^3/e^5 - 450*(e*sqrt(x
) + d)^2*d^4/e^5 + 360*(e*sqrt(x) + d)*d^5/e^5)*a*b^2*n*log(c) + 1800*(60*(
e*sqrt(x) + d)^6*log(e*sqrt(x) + d)/e^5 - 360*(e*sqrt(x) + d)^5*d*log(e*sqr
t(x) + d)/e^5 + 900*(e*sqrt(x) + d)^4*d^2*log(e*sqrt(x) + d)/e^5 - 1200*(e*
sqrt(x) + d)^3*d^3*log(e*sqrt(x) + d)/e^5 + 900*(e*sqrt(x) + d)^2*d^4*log(e
*sqrt(x) + d)/e^5 - 360*(e*sqrt(x) + d)*d^5*log(e*sqrt(x) + d)/e^5 - 10*(e*
sqrt(x) + d)^6/e^5 + 72*(e*sqrt(x) + d)^5*d/e^5 - 225*(e*sqrt(x) + d)^4*d^2
/e^5 + 400*(e*sqrt(x) + d)^3*d^3/e^5 - 450*(e*sqrt(x) + d)^2*d^4/e^5 + 360*
(e*sqrt(x) + d)*d^5/e^5)*a^2*b*n)/e
```

Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 976, normalized size of antiderivative = 1.08

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx=\frac{a^3\,x^3}{3}+\frac{b^3\,x^3\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)^3}{3}-\frac{b^3\,n^3\,x^3}{108}+a\,b^2\,x^3\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)^2-\frac{b^3\,n\,x^3\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)^2}{6}+\frac{b^3\,n^2\,x^3\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{18}+\frac{a\,b^2\,n^2\,x^3}{18}-\frac{b^3\,d^6\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)^3}{3\,e^6}+a^2\,b\,x^3\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)-\frac{a^2\,b\,n\,x^3}{6}-\frac{a\,b^2\,n\,x^3\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{3}-\frac{13489\,b^3\,d^6\,n^3\,\ln\left(d+e\,\sqrt{x}\right)}{1800\,e^6}-\frac{919\,b^3\,d^2\,n^3\,x^2}{7200\,e^2}+\frac{2059\,b^3\,d^3\,n^3\,x^{3/2}}{5400\,e^3}+\frac{13489\,b^3\,d^5\,n^3\,\sqrt{x}}{1800\,e^5}-\frac{a\,b^2\,d^6\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)^2}{e^6}+\frac{49\,b^3\,d^6\,n\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)^2}{20\,e^6}+\frac{91\,b^3\,d\,n^3\,x^{5/2}}{2250\,e}-\frac{4669\,b^3\,d^4\,n^3\,x}{3600\,e^4}-\frac{a^2\,b\,d^6\,n\,\ln\left(d+e\,\sqrt{x}\right)}{e^6}+\frac{b^3\,d\,n\,x^{5/2}\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)^2}{5\,e}-\frac{11\,b^3\,d\,n^2\,x^{5/2}\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{75\,e}-\frac{b^3\,d^4\,n\,x\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)^2}{2\,e^4}+\frac{29\,b^3\,d^4\,n^2\,x\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{20\,e^4}-\frac{a^2\,b\,d^2\,n\,x^2}{4\,e^2}-\frac{11\,a\,b^2\,d\,n^2\,x^{5/2}}{75\,e}+\frac{29\,a\,b^2\,d^4\,n^2\,x}{20\,e^4}+\frac{a^2\,b\,d^3\,n\,x^{3/2}}{3\,e^3}+\frac{a^2\,b\,d^5\,n\,\sqrt{x}}{e^5}+\frac{49\,a\,b^2\,d^6\,n^2\,\ln\left(d+e\,\sqrt{x}\right)}{10\,e^6}-\frac{b^3\,d^2\,n\,x^2\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)^2}{4\,e^2}+\frac{37\,b^3\,d^2\,n^2\,x^2\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{120\,e^2}+\frac{b^3\,d^3\,n\,x^{3/2}\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)^2}{3\,e^3}-\frac{19\,b^3\,d^3\,n^2\,x^{3/2}\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)}{}$$

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/2))^n))^3,x)
```

```
[Out] (a^3*x^3)/3 + (b^3*x^3*log(c*(d + e*x^(1/2))^n)^3)/3 - (b^3*n^3*x^3)/108 +
a*b^2*x^3*log(c*(d + e*x^(1/2))^n)^2 - (b^3*n*x^3*log(c*(d + e*x^(1/2))^n)^
2)/6 + (b^3*n^2*x^3*log(c*(d + e*x^(1/2))^n))/18 + (a*b^2*n^2*x^3)/18 - (b^
3*d^6*log(c*(d + e*x^(1/2))^n)^3)/(3*e^6) + a^2*b*x^3*log(c*(d + e*x^(1/2))
^n) - (a^2*b*n*x^3)/6 - (a*b^2*n*x^3*log(c*(d + e*x^(1/2))^n))/3 - (13489*b
^3*d^6*n^3*log(d + e*x^(1/2)))/(1800*e^6) - (919*b^3*d^2*n^3*x^2)/(7200*e^2
) + (2059*b^3*d^3*n^3*x^(3/2))/(5400*e^3) + (13489*b^3*d^5*n^3*x^(1/2))/(18
00*e^5) - (a*b^2*d^6*log(c*(d + e*x^(1/2))^n)^2)/e^6 + (49*b^3*d^6*n*log(c*
(d + e*x^(1/2))^n)^2)/(20*e^6) + (91*b^3*d*n^3*x^(5/2))/(2250*e) - (4669*b^
3*d^4*n^3*x)/(3600*e^4) - (a^2*b*d^6*n*log(d + e*x^(1/2)))/e^6 + (b^3*d*n*x
^(5/2)*log(c*(d + e*x^(1/2))^n)^2)/(5*e) - (11*b^3*d*n^2*x^(5/2)*log(c*(d +
 e*x^(1/2))^n))/(75*e) - (b^3*d^4*n*x*log(c*(d + e*x^(1/2))^n)^2)/(2*e^4) +
 (29*b^3*d^4*n^2*x*log(c*(d + e*x^(1/2))^n))/(20*e^4) - (a^2*b*d^2*n*x^2)/(
4*e^2) - (11*a*b^2*d*n^2*x^(5/2))/(75*e) + (29*a*b^2*d^4*n^2*x)/(20*e^4) +
(a^2*b*d^3*n*x^(3/2))/(3*e^3) + (a^2*b*d^5*n*x^(1/2))/e^5 + (49*a*b^2*d^6*n
^2*log(d + e*x^(1/2)))/(10*e^6) - (b^3*d^2*n*x^2*log(c*(d + e*x^(1/2))^n)^2
)/(4*e^2) + (37*b^3*d^2*n^2*x^2*log(c*(d + e*x^(1/2))^n))/(120*e^2) + (b^3*
d^3*n*x^(3/2)*log(c*(d + e*x^(1/2))^n)^2)/(3*e^3) - (19*b^3*d^3*n^2*x^(3/2)
*log(c*(d + e*x^(1/2))^n))/(30*e^3) + (b^3*d^5*n*x^(1/2)*log(c*(d + e*x^(1/
2))^n)^2)/e^5 - (49*b^3*d^5*n^2*x^(1/2)*log(c*(d + e*x^(1/2))^n))/(10*e^5)
+ (37*a*b^2*d^2*n^2*x^2)/(120*e^2) - (19*a*b^2*d^3*n^2*x^(3/2))/(30*e^3) -
(49*a*b^2*d^5*n^2*x^(1/2))/(10*e^5) + (a^2*b*d*n*x^(5/2))/(5*e) - (a^2*b*d^
4*n*x)/(2*e^4) + (2*a*b^2*d*n*x^(5/2)*log(c*(d + e*x^(1/2))^n))/(5*e) - (a*
b^2*d^4*n*x*log(c*(d + e*x^(1/2))^n))/e^4 - (a*b^2*d^2*n*x^2*log(c*(d + e*x
^(1/2))^n))/(2*e^2) + (2*a*b^2*d^3*n*x^(3/2)*log(c*(d + e*x^(1/2))^n))/(3*e
^3) + (2*a*b^2*d^5*n*x^(1/2)*log(c*(d + e*x^(1/2))^n))/e^5
```

3.416 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 595

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx=-\frac{9b^3d^2n^3\left(d+e\sqrt{x}\right)^2}{4e^4}+\frac{4b^3dn^3\left(d+e\sqrt{x}\right)^3}{9e^4}-\frac{3b^3n^3\left(d+e\sqrt{x}\right)^4}{64e^4}-\frac{12ab^2d^3n^2\sqrt{x}}{e^3}+\frac{12b^3d^3n^3\sqrt{x}}{e^3}-\frac{12b^3d^3n^2\left(d+e\sqrt{x}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{e^4}+\frac{9b^2d^2n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^4}-\frac{4b^2dn^2\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3e^4}+\frac{3b^2n^2\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{16e^4}+\frac{6bd^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^4}-\frac{9bd^2n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2e^4}+\frac{2bdn\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^4}-\frac{3bn\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{8e^4}-\frac{2d^3\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^4}+\frac{3d^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^4}-\frac{2d\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^4}+\frac{\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2e^4}$$

```
[Out] -12*a*b^2*d^3*n^2*x^(1/2)/e^3+12*b^3*d^3*n^3*x^(1/2)/e^3-12*b^3*d^3*n^2*ln(
c*(d+e*x^(1/2))^n)*(d+e*x^(1/2))/e^4+6*b*d^3*n*(a+b*ln(c*(d+e*x^(1/2))^n))^
2*(d+e*x^(1/2))/e^4-2*d^3*(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x^(1/2))/e^4-9
/4*b^3*d^2*n^3*(d+e*x^(1/2))^2/e^4+9/2*b^2*d^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^
n))*(d+e*x^(1/2))^2/e^4-9/2*b*d^2*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1
/2))^2/e^4+3*d^2*(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x^(1/2))^2/e^4+4/9*b^3*
d*n^3*(d+e*x^(1/2))^3/e^4-4/3*b^2*d*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^
(1/2))^3/e^4+2*b*d*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))^3/e^4-2*d*
(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x^(1/2))^3/e^4-3/64*b^3*n^3*(d+e*x^(1/2)
```

```
)^4/e^4+3/16*b^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^4/e^4-3/8*b*
n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))^4/e^4+1/2*(a+b*ln(c*(d+e*x^(1
/2))^n))^3*(d+e*x^(1/2))^4/e^4
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\begin{aligned}\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx=&\frac{9b^2d^2n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^4}\\&+\frac{3b^2n^2\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{16e^4}\\&-\frac{4b^2dn^2\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3e^4}\\&-\frac{12ab^2d^3n^2\sqrt{x}}{e^3}\\&-\frac{2d^3\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^4}\\&+\frac{6bd^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^4}\\&+\frac{3d^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^4}\\&-\frac{9bd^2n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2e^4}\\&+\frac{\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2e^4}\\&-\frac{3bn\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{8e^4}\\&-\frac{2d\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^4}\\&+\frac{2bdn\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^4}\\&-\frac{12b^3d^3n^2\left(d+e\sqrt{x}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{e^4}\\&+\frac{12b^3d^3n^3\sqrt{x}}{e^3}-\frac{9b^3d^2n^3\left(d+e\sqrt{x}\right)^2}{4e^4}\\&-\frac{3b^3n^3\left(d+e\sqrt{x}\right)^4}{64e^4}+\frac{4b^3dn^3\left(d+e\sqrt{x}\right)^3}{9e^4}\end{aligned}$$

```
[In] Int[x*(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

```
[Out] (-9*b^3*d^2*n^3*(d + e*Sqrt[x])^2)/(4*e^4) + (4*b^3*d*n^3*(d + e*Sqrt[x])^3
)/(9*e^4) - (3*b^3*n^3*(d + e*Sqrt[x])^4)/(64*e^4) - (12*a*b^2*d^3*n^2*Sqrt
[x])/e^3 + (12*b^3*d^3*n^3*Sqrt[x])/e^3 - (12*b^3*d^3*n^2*(d + e*Sqrt[x])*L
og[c*(d + e*Sqrt[x])^n])/e^4 + (9*b^2*d^2*n^2*(d + e*Sqrt[x])^2*(a + b*Log[
c*(d + e*Sqrt[x])^n]))/(2*e^4) - (4*b^2*d*n^2*(d + e*Sqrt[x])^3*(a + b*Log[
c*(d + e*Sqrt[x])^n]))/(3*e^4) + (3*b^2*n^2*(d + e*Sqrt[x])^4*(a + b*Log[c*
(d + e*Sqrt[x])^n]))/(16*e^4) + (6*b*d^3*n*(d + e*Sqrt[x])*(a + b*Log[c*(d
+ e*Sqrt[x])^n])^2)/e^4 - (9*b*d^2*n*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*
Sqrt[x])^n])^2)/(2*e^4) + (2*b*d*n*(d + e*Sqrt[x])^3*(a + b*Log[c*(d + e*Sq
rt[x])^n])^2)/e^4 - (3*b*n*(d + e*Sqrt[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n
])^2)/(8*e^4) - (2*d^3*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/
e^4 + (3*d^2*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/e^4 - (2
*d*(d + e*Sqrt[x])^3*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/e^4 + ((d + e*Sqrt
[x])^4*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/(2*e^4)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^3(a+b\log\left(c(d+ex)^n\right))^3\,dx,x,\sqrt{x}\right)\\
&= 2\text{Subst}\left(\int\left(-\frac{d^3(a+b\log\left(c(d+ex)^n\right))^3}{e^3}+\frac{3d^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^3}\right.\right.\\
&\qquad -\frac{3d(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^3}\\
&\qquad \left.\left.+\frac{(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^3}\right)\,dx,x,\sqrt{x}\right)\\
&= \frac{2\text{Subst}\left(\int(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\sqrt{x}\right)}{e^3}\\
&\quad -\frac{(6d)\text{Subst}\left(\int(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\sqrt{x}\right)}{e^3}\\
&\quad +\frac{\left(6d^2\right)\text{Subst}\left(\int(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\sqrt{x}\right)}{e^3}\\
&\quad -\frac{\left(2d^3\right)\text{Subst}\left(\int\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\sqrt{x}\right)}{e^3}
\end{aligned}$$

$$= \frac{2\text{Subst}\left(\int x^3(a+b\log(cx^n))^3\,dx,x,d+e\sqrt{x}\right)}{e^4} - \frac{(6d)\text{Subst}\left(\int x^2(a+b\log(cx^n))^3\,dx,x,d+e\sqrt{x}\right)}{e^4} + \frac{(6d^2)\,\text{Subst}\left(\int x(a+b\log(cx^n))^3\,dx,x,d+e\sqrt{x}\right)}{e^4} - \frac{(2d^3)\,\text{Subst}\left(\int (a+b\log(cx^n))^3\,dx,x,d+e\sqrt{x}\right)}{e^4}$$

$$= -\frac{2d^3\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^4} + \frac{3d^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^4} - \frac{2d\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^4} + \frac{\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2e^4} - \frac{(3bn)\text{Subst}\left(\int x^3(a+b\log(cx^n))^2\,dx,x,d+e\sqrt{x}\right)}{2e^4} + \frac{(6bdn)\text{Subst}\left(\int x^2(a+b\log(cx^n))^2\,dx,x,d+e\sqrt{x}\right)}{e^4} - \frac{(9bd^2n)\,\text{Subst}\left(\int x(a+b\log(cx^n))^2\,dx,x,d+e\sqrt{x}\right)}{e^4} + \frac{(6bd^3n)\,\text{Subst}\left(\int (a+b\log(cx^n))^2\,dx,x,d+e\sqrt{x}\right)}{e^4}$$

$$
\begin{aligned}
&= \frac{6bd^3n(d+e\sqrt{x})\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^2}{e^4} \\
&\quad - \frac{9bd^2n(d+e\sqrt{x})^2\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^2}{2e^4} \\
&\quad + \frac{2bdn(d+e\sqrt{x})^3\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^2}{e^4} \\
&\quad - \frac{3bn(d+e\sqrt{x})^4\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^2}{8e^4} \\
&\quad - \frac{2d^3(d+e\sqrt{x})\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^3}{e^4} \\
&\quad + \frac{3d^2(d+e\sqrt{x})^2\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^3}{e^4} \\
&\quad - \frac{2d(d+e\sqrt{x})^3\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^3}{e^4} \\
&\quad + \frac{(d+e\sqrt{x})^4\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^3}{2e^4} \\
&\quad + \frac{(3b^2n^2)\,\mathrm{Subst}\left(\int x^3(a+b\log(cx^n))\,dx, x, d+e\sqrt{x}\right)}{4e^4} \\
&\quad - \frac{(4b^2dn^2)\,\mathrm{Subst}\left(\int x^2(a+b\log(cx^n))\,dx, x, d+e\sqrt{x}\right)}{e^4} \\
&\quad + \frac{(9b^2d^2n^2)\,\mathrm{Subst}\left(\int x(a+b\log(cx^n))\,dx, x, d+e\sqrt{x}\right)}{e^4} \\
&\quad - \frac{(12b^2d^3n^2)\,\mathrm{Subst}\left(\int (a+b\log(cx^n))\,dx, x, d+e\sqrt{x}\right)}{e^4}
\end{aligned}
$$

$$= -\frac{9b^3d^2n^3\left(d+e\sqrt{x}\right)^2}{4e^4} + \frac{4b^3dn^3\left(d+e\sqrt{x}\right)^3}{9e^4} - \frac{3b^3n^3\left(d+e\sqrt{x}\right)^4}{64e^4} - \frac{12ab^2d^3n^2\sqrt{x}}{e^3} + \frac{9b^2d^2n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^4} - \frac{4b^2dn^2\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3e^4} + \frac{3b^2n^2\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{16e^4} + \frac{6bd^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^4} - \frac{9bd^2n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2e^4} + \frac{2bdn\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^4} - \frac{3bn\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{8e^4} - \frac{2d^3\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^4} + \frac{3d^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^4} - \frac{2d\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^4} + \frac{\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2e^4} - \frac{\left(12b^3d^3n^2\right)\text{Subst}\left(\int\log\left(cx^n\right)\,dx,x,d+e\sqrt{x}\right)}{e^4}$$

$$= -\frac{9b^3d^2n^3\left(d+e\sqrt{x}\right)^2}{4e^4} + \frac{4b^3dn^3\left(d+e\sqrt{x}\right)^3}{9e^4} - \frac{3b^3n^3\left(d+e\sqrt{x}\right)^4}{64e^4}$$
$$- \frac{12ab^2d^3n^2\sqrt{x}}{e^3} + \frac{12b^3d^3n^3\sqrt{x}}{e^3} - \frac{12b^3d^3n^2\left(d+e\sqrt{x}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{e^4}$$
$$+ \frac{9b^2d^2n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^4}$$
$$- \frac{4b^2dn^2\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{3e^4}$$
$$+ \frac{3b^2n^2\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{16e^4}$$
$$+ \frac{6bd^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^4}$$
$$- \frac{9bd^2n\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2e^4}$$
$$+ \frac{2bdn\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^4}$$
$$- \frac{3bn\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{8e^4}$$
$$- \frac{2d^3\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^4}$$
$$+ \frac{3d^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^4}$$
$$- \frac{2d\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^4}$$
$$+ \frac{\left(d+e\sqrt{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2e^4}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 433, normalized size of antiderivative = 0.73

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$$
$$= \frac{b^3en^3\sqrt{x}\left(4980d^3-690d^2e\sqrt{x}+148de^2x-27e^3x^{3/2}\right)+72a^2bn\left(25d^4+12d^3e\sqrt{x}-6d^2e^2x+4de^3x^{3/2}-3e\right.}{}$$

```
[In] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

```
[Out] (b^3*e*n^3*Sqrt[x]*(4980*d^3 - 690*d^2*e*Sqrt[x] + 148*d*e^2*x - 27*e^3*x^(
3/2)) + 72*a^2*b*n*(25*d^4 + 12*d^3*e*Sqrt[x] - 6*d^2*e^2*x + 4*d*e^3*x^(3/
2) - 3*e^4*x^2) - 288*a^3*(d^4 - e^4*x^2) + 12*a*b^2*n^2*(161*d^4 - 300*d^3
*e*Sqrt[x] + 78*d^2*e^2*x - 28*d*e^3*x^(3/2) + 9*e^4*x^2) - 12*b*(b^2*n^2*(
```

```
415*d^4 + 300*d^3*e*Sqrt[x] - 78*d^2*e^2*x + 28*d*e^3*x^(3/2) - 9*e^4*x^2)
- 12*a*b*n*(25*d^4 + 12*d^3*e*Sqrt[x] - 6*d^2*e^2*x + 4*d*e^3*x^(3/2) - 3*e
^4*x^2) + 72*a^2*(d^4 - e^4*x^2))*Log[c*(d + e*Sqrt[x])^n] - 72*b^2*(12*a*(
d^4 - e^4*x^2) + b*n*(-25*d^4 - 12*d^3*e*Sqrt[x] + 6*d^2*e^2*x - 4*d*e^3*x^
(3/2) + 3*e^4*x^2))*Log[c*(d + e*Sqrt[x])^n]^2 - 288*b^3*(d^4 - e^4*x^2)*Lo
g[c*(d + e*Sqrt[x])^n]^3)/(576*e^4)
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}dx$$

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/2))^n))^3,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(1/2))^n))^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 861, normalized size of antiderivative = 1.45

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}\,dx = \frac{288\,b^{3}e^{4}x^{2}\log\left(c\right)^{3}+288\,\left(b^{3}e^{4}n^{3}x^{2}-b^{3}d^{4}n^{3}\right)\log\left(e\sqrt{x}+d\right)^{3}-9\,\left(3\,b^{3}e^{4}n^{3}-12\,ab^{2}e^{4}n^{2}+24\,a^{2}be^{4}n-32\right.}{}$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/576*(288*b^3*e^4*x^2*log(c)^3 + 288*(b^3*e^4*n^3*x^2 - b^3*d^4*n^3)*log(e
*sqrt(x) + d)^3 - 9*(3*b^3*e^4*n^3 - 12*a*b^2*e^4*n^2 + 24*a^2*b*e^4*n - 32
*a^3*e^4)*x^2 - 72*(6*b^3*d^2*e^2*n^3*x - 25*b^3*d^4*n^3 + 12*a*b^2*d^4*n^2
 + 3*(b^3*e^4*n^3 - 4*a*b^2*e^4*n^2)*x^2 - 12*(b^3*e^4*n^2*x^2 - b^3*d^4*n^
2)*log(c) - 4*(b^3*d*e^3*n^3*x + 3*b^3*d^3*e*n^3)*sqrt(x))*log(e*sqrt(x) +
d)^2 - 216*(2*b^3*d^2*e^2*n*x + (b^3*e^4*n - 4*a*b^2*e^4)*x^2)*log(c)^2 - 6
*(115*b^3*d^2*e^2*n^3 - 156*a*b^2*d^2*e^2*n^2 + 72*a^2*b*d^2*e^2*n)*x - 12*
(415*b^3*d^4*n^3 - 300*a*b^2*d^4*n^2 + 72*a^2*b*d^4*n - 9*(b^3*e^4*n^3 - 4*
a*b^2*e^4*n^2 + 8*a^2*b*e^4*n)*x^2 - 72*(b^3*e^4*n*x^2 - b^3*d^4*n)*log(c)^
2 - 6*(13*b^3*d^2*e^2*n^3 - 12*a*b^2*d^2*e^2*n^2)*x + 12*(6*b^3*d^2*e^2*n^2
*x - 25*b^3*d^4*n^2 + 12*a*b^2*d^4*n + 3*(b^3*e^4*n^2 - 4*a*b^2*e^4*n)*x^2)
*log(c) + 4*(75*b^3*d^3*e*n^3 - 36*a*b^2*d^3*e*n^2 + (7*b^3*d*e^3*n^3 - 12*
a*b^2*d*e^3*n^2)*x - 12*(b^3*d*e^3*n^2*x + 3*b^3*d^3*e*n^2)*log(c))*sqrt(x)
)*log(e*sqrt(x) + d) + 36*(3*(b^3*e^4*n^2 - 4*a*b^2*e^4*n + 8*a^2*b*e^4)*x^
2 + 2*(13*b^3*d^2*e^2*n^2 - 12*a*b^2*d^2*e^2*n)*x)*log(c) + 4*(1245*b^3*d^3
*e*n^3 - 900*a*b^2*d^3*e*n^2 + 216*a^2*b*d^3*e*n + 72*(b^3*d*e^3*n*x + 3*b^
3*d^3*e*n)*log(c)^2 + (37*b^3*d*e^3*n^3 - 84*a*b^2*d*e^3*n^2 + 72*a^2*b*d*e
^3*n)*x - 12*(75*b^3*d^3*e*n^2 - 36*a*b^2*d^3*e*n + (7*b^3*d*e^3*n^2 - 12*a
*b^2*d*e^3*n)*x)*log(c))*sqrt(x))/e^4
```

Sympy [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}\,dx=\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}\,dx$$

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/2))**n))**3,x)
[Out] Integral(x*(a + b*log(c*(d + e*sqrt(x))**n))**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.90

$$\begin{aligned}\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}\,dx&=\frac{1}{2}\,b^{3}x^{2}\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)^{3}+\frac{3}{2}\,ab^{2}x^{2}\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)^{2}\\&\quad-\frac{1}{8}\,a^{2}ben\left(\frac{12\,d^{4}\log\left(e\sqrt{x}+d\right)}{e^{5}}+\frac{3\,e^{3}x^{2}-4\,de^{2}x^{\frac{3}{2}}+6\,d^{2}ex-12\,d^{3}\sqrt{x}}{e^{4}}\right)\\&\quad+\frac{3}{2}\,a^{2}bx^{2}\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)+\frac{1}{2}\,a^{3}x^{2}\\&\quad-\frac{1}{48}\left(12\,en\left(\frac{12\,d^{4}\log\left(e\sqrt{x}+d\right)}{e^{5}}+\frac{3\,e^{3}x^{2}-4\,de^{2}x^{\frac{3}{2}}+6\,d^{2}ex-12\,d^{3}\sqrt{x}}{e^{4}}\right)\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)-\frac{\left(9\,e^{4}x^{2}\right.}{}\right.\\&\quad-\frac{1}{576}\left(72\,en\left(\frac{12\,d^{4}\log\left(e\sqrt{x}+d\right)}{e^{5}}+\frac{3\,e^{3}x^{2}-4\,de^{2}x^{\frac{3}{2}}+6\,d^{2}ex-12\,d^{3}\sqrt{x}}{e^{4}}\right)\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)^{2}+en\left(\right.\right.\end{aligned}$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="maxima")
[Out] 1/2*b^3*x^2*log((e*sqrt(x) + d)^n*c)^3 + 3/2*a*b^2*x^2*log((e*sqrt(x) + d)^
n*c)^2 - 1/8*a^2*b*e*n*(12*d^4*log(e*sqrt(x) + d)/e^5 + (3*e^3*x^2 - 4*d*e^
2*x^(3/2) + 6*d^2*e*x - 12*d^3*sqrt(x))/e^4) + 3/2*a^2*b*x^2*log((e*sqrt(x)
 + d)^n*c) + 1/2*a^3*x^2 - 1/48*(12*e*n*(12*d^4*log(e*sqrt(x) + d)/e^5 + (3
*e^3*x^2 - 4*d*e^2*x^(3/2) + 6*d^2*e*x - 12*d^3*sqrt(x))/e^4)*log((e*sqrt(x
) + d)^n*c) - (9*e^4*x^2 + 72*d^4*log(e*sqrt(x) + d)^2 - 28*d*e^3*x^(3/2) +
 78*d^2*e^2*x + 300*d^4*log(e*sqrt(x) + d) - 300*d^3*e*sqrt(x))*n^2/e^4)*a*
b^2 - 1/576*(72*e*n*(12*d^4*log(e*sqrt(x) + d)/e^5 + (3*e^3*x^2 - 4*d*e^2*x
^(3/2) + 6*d^2*e*x - 12*d^3*sqrt(x))/e^4)*log((e*sqrt(x) + d)^n*c)^2 + e*n*
((288*d^4*log(e*sqrt(x) + d)^3 + 27*e^4*x^2 + 1800*d^4*log(e*sqrt(x) + d)^2
 - 148*d*e^3*x^(3/2) + 690*d^2*e^2*x + 4980*d^4*log(e*sqrt(x) + d) - 4980*d
^3*e*sqrt(x))*n^2/e^5 - 12*(9*e^4*x^2 + 72*d^4*log(e*sqrt(x) + d)^2 - 28*d*
e^3*x^(3/2) + 78*d^2*e^2*x + 300*d^4*log(e*sqrt(x) + d) - 300*d^3*e*sqrt(x)
)*n*log((e*sqrt(x) + d)^n*c)/e^5))*b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1440 vs. 2(519) = 1038.

Time = 0.34 (sec) , antiderivative size = 1440, normalized size of antiderivative = 2.42

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="giac")
```

```
[Out] 1/576*(288*b^3*e*x^2*log(c)^3 + 864*a*b^2*e*x^2*log(c)^2 + (288*(e*sqrt(x)
+ d)^4*log(e*sqrt(x) + d)^3/e^3 - 1152*(e*sqrt(x) + d)^3*d*log(e*sqrt(x) +
d)^3/e^3 + 1728*(e*sqrt(x) + d)^2*d^2*log(e*sqrt(x) + d)^3/e^3 - 1152*(e*sq
rt(x) + d)*d^3*log(e*sqrt(x) + d)^3/e^3 - 216*(e*sqrt(x) + d)^4*log(e*sqrt(
x) + d)^2/e^3 + 1152*(e*sqrt(x) + d)^3*d*log(e*sqrt(x) + d)^2/e^3 - 2592*(e
*sqrt(x) + d)^2*d^2*log(e*sqrt(x) + d)^2/e^3 + 3456*(e*sqrt(x) + d)*d^3*log
(e*sqrt(x) + d)^2/e^3 + 108*(e*sqrt(x) + d)^4*log(e*sqrt(x) + d)/e^3 - 768*
(e*sqrt(x) + d)^3*d*log(e*sqrt(x) + d)/e^3 + 2592*(e*sqrt(x) + d)^2*d^2*log
(e*sqrt(x) + d)/e^3 - 6912*(e*sqrt(x) + d)*d^3*log(e*sqrt(x) + d)/e^3 - 27*
(e*sqrt(x) + d)^4/e^3 + 256*(e*sqrt(x) + d)^3*d/e^3 - 1296*(e*sqrt(x) + d)^
2*d^2/e^3 + 6912*(e*sqrt(x) + d)*d^3/e^3)*b^3*n^3 + 12*(72*(e*sqrt(x) + d)^
4*log(e*sqrt(x) + d)^2/e^3 - 288*(e*sqrt(x) + d)^3*d*log(e*sqrt(x) + d)^2/e
^3 + 432*(e*sqrt(x) + d)^2*d^2*log(e*sqrt(x) + d)^2/e^3 - 288*(e*sqrt(x) +
d)*d^3*log(e*sqrt(x) + d)^2/e^3 - 36*(e*sqrt(x) + d)^4*log(e*sqrt(x) + d)/e
^3 + 192*(e*sqrt(x) + d)^3*d*log(e*sqrt(x) + d)/e^3 - 432*(e*sqrt(x) + d)^2
*d^2*log(e*sqrt(x) + d)/e^3 + 576*(e*sqrt(x) + d)*d^3*log(e*sqrt(x) + d)/e^
3 + 9*(e*sqrt(x) + d)^4/e^3 - 64*(e*sqrt(x) + d)^3*d/e^3 + 216*(e*sqrt(x) +
 d)^2*d^2/e^3 - 576*(e*sqrt(x) + d)*d^3/e^3)*b^3*n^2*log(c) + 864*a^2*b*e*x
^2*log(c) + 72*(12*(e*sqrt(x) + d)^4*log(e*sqrt(x) + d)/e^3 - 48*(e*sqrt(x)
 + d)^3*d*log(e*sqrt(x) + d)/e^3 + 72*(e*sqrt(x) + d)^2*d^2*log(e*sqrt(x) +
 d)/e^3 - 48*(e*sqrt(x) + d)*d^3*log(e*sqrt(x) + d)/e^3 - 3*(e*sqrt(x) + d)
^4/e^3 + 16*(e*sqrt(x) + d)^3*d/e^3 - 36*(e*sqrt(x) + d)^2*d^2/e^3 + 48*(e*
sqrt(x) + d)*d^3/e^3)*b^3*n*log(c)^2 + 12*(72*(e*sqrt(x) + d)^4*log(e*sqrt(
x) + d)^2/e^3 - 288*(e*sqrt(x) + d)^3*d*log(e*sqrt(x) + d)^2/e^3 + 432*(e*s
qrt(x) + d)^2*d^2*log(e*sqrt(x) + d)^2/e^3 - 288*(e*sqrt(x) + d)*d^3*log(e*
sqrt(x) + d)^2/e^3 - 36*(e*sqrt(x) + d)^4*log(e*sqrt(x) + d)/e^3 + 192*(e*s
qrt(x) + d)^3*d*log(e*sqrt(x) + d)/e^3 - 432*(e*sqrt(x) + d)^2*d^2*log(e*sq
rt(x) + d)/e^3 + 576*(e*sqrt(x) + d)*d^3*log(e*sqrt(x) + d)/e^3 + 9*(e*sqrt
(x) + d)^4/e^3 - 64*(e*sqrt(x) + d)^3*d/e^3 + 216*(e*sqrt(x) + d)^2*d^2/e^3
 - 576*(e*sqrt(x) + d)*d^3/e^3)*a*b^2*n^2 + 288*a^3*e*x^2 + 144*(12*(e*sqrt
(x) + d)^4*log(e*sqrt(x) + d)/e^3 - 48*(e*sqrt(x) + d)^3*d*log(e*sqrt(x) +
d)/e^3 + 72*(e*sqrt(x) + d)^2*d^2*log(e*sqrt(x) + d)/e^3 - 48*(e*sqrt(x) +
d)*d^3*log(e*sqrt(x) + d)/e^3 - 3*(e*sqrt(x) + d)^4/e^3 + 16*(e*sqrt(x) + d
)^3*d/e^3 - 36*(e*sqrt(x) + d)^2*d^2/e^3 + 48*(e*sqrt(x) + d)*d^3/e^3)*a*b^
2*n*log(c) + 72*(12*(e*sqrt(x) + d)^4*log(e*sqrt(x) + d)/e^3 - 48*(e*sqrt(x
```

```
) + d)^3*d*log(e*sqrt(x) + d)/e^3 + 72*(e*sqrt(x) + d)^2*d^2*log(e*sqrt(x)
+ d)/e^3 - 48*(e*sqrt(x) + d)*d^3*log(e*sqrt(x) + d)/e^3 - 3*(e*sqrt(x) + d
)^4/e^3 + 16*(e*sqrt(x) + d)^3*d/e^3 - 36*(e*sqrt(x) + d)^2*d^2/e^3 + 48*(e
*sqrt(x) + d)*d^3/e^3)*a^2*b*n)/e
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 840, normalized size of antiderivative = 1.41

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx=\ln\left(c\left(d+e\,\sqrt{x}\right)^n\right)^3\left(\frac{b^3\,x^2}{2}-\frac{b^3\,d^4}{2\,e^4}\right)$$

$$-x^{3/2}\left(\frac{d\left(2\,a^3-\frac{3\,a^2\,b\,n}{2}+\frac{3\,a\,b^2\,n^2}{4}-\frac{3\,b^3\,n^3}{16}\right)}{3\,e}-\frac{d\,(24\,a^3-12\,a\,b^2\,n^2+7\,b^3\,n^3)}{36\,e}\right)$$

$$-\ln\left(c\left(d+e\,\sqrt{x}\right)^n\right)^2\left(\frac{x^{3/2}\left(\frac{b^2\,d\,(4\,a-b\,n)}{e}-\frac{4\,a\,b^2\,d}{e}\right)}{2}-\frac{3\,b^2\,x^2\,(4\,a-b\,n)}{8}\right.$$

$$+\frac{d\,(12\,a\,b^2\,d^3-25\,b^3\,d^3\,n)}{8\,e^4}+\frac{d^2\,\sqrt{x}\left(\frac{6\,b^2\,d\,(4\,a-b\,n)}{e}-\frac{24\,a\,b^2\,d}{e}\right)}{4\,e^2}$$

$$\left.-\frac{d\,x\left(\frac{6\,b^2\,d\,(4\,a-b\,n)}{e}-\frac{24\,a\,b^2\,d}{e}\right)}{8\,e}\right)$$

$$+x\left(\frac{d\left(\frac{d\left(2\,a^3-\frac{3\,a^2\,b\,n}{2}+\frac{3\,a\,b^2\,n^2}{4}-\frac{3\,b^3\,n^3}{16}\right)}{e}-\frac{d\,(24\,a^3-12\,a\,b^2\,n^2+7\,b^3\,n^3)}{12\,e}\right)}{2\,e}+\frac{b^2\,d^2\,n^2\,(12\,a-13\,b\,n)}{16\,e^2}\right)$$

$$-\sqrt{x}\left(\frac{d\left(\frac{d\left(\frac{d\left(2\,a^3-\frac{3\,a^2\,b\,n}{2}+\frac{3\,a\,b^2\,n^2}{4}-\frac{3\,b^3\,n^3}{16}\right)}{e}-\frac{d\,\left(24\,a^3-12\,a\,b^2\,n^2+7\,b^3\,n^3\right)}{12\,e}\right)}{e}+\frac{b^2\,d^2\,n^2\,(12\,a-13\,b\,n)}{8\,e^2}\right)}{e}+\frac{b^2\,d^3\,n^2\,(12\,a-25}{4\,e^3}\right.$$

$$+x^2\left(\frac{a^3}{2}-\frac{3\,a^2\,b\,n}{8}+\frac{3\,a\,b^2\,n^2}{16}-\frac{3\,b^3\,n^3}{64}\right)$$

$$+\frac{\ln\left(c\left(d+e\,\sqrt{x}\right)^n\right)\left(\frac{x^{3/2}\,\left(16\,b\,d\,e^3\,\left(6\,a^2-b^2\,n^2\right)-12\,b\,d\,e^3\,\left(8\,a^2-4\,a\,b\,n+b^2\,n^2\right)\right)}{12\,e^2}-\frac{x\left(\frac{d\left(16\,b\,d\,e^3\,\left(6\,a^2-b^2\,n^2\right)-12\,b\,d\,e^3\,\left(8\,a^2-4\,a\,b\,n\right.\right.}{e}\right.}{8\,e^2}\right.}{}$$

$$-\frac{\ln\left(d+e\,\sqrt{x}\right)\,\left(72\,a^2\,b\,d^4\,n-300\,a\,b^2\,d^4\,n^2+415\,b^3\,d^4\,n^3\right)}{48\,e^4}$$

```
[In] int(x*(a + b*log(c*(d + e*x^(1/2))^n))^3,x)
```

[Out] log(c*(d + e*x^(1/2))^n)^3*((b^3*x^2)/2 - (b^3*d^4)/(2*e^4)) - x^(3/2)*((d*(2*a^3 - (3*b^3*n^3)/16 + (3*a*b^2*n^2)/4 - (3*a^2*b*n)/2))/(3*e) - (d*(24*a^3 + 7*b^3*n^3 - 12*a*b^2*n^2))/(36*e)) - log(c*(d + e*x^(1/2))^n)^2*((x^(3/2)*((b^2*d*(4*a - b*n))/e - (4*a*b^2*d)/e))/2 - (3*b^2*x^2*(4*a - b*n))/8 + (d*(12*a*b^2*d^3 - 25*b^3*d^3*n))/(8*e^4) + (d^2*x^(1/2)*((6*b^2*d*(4*a - b*n))/e - (24*a*b^2*d)/e))/(4*e^2) - (d*x*((6*b^2*d*(4*a - b*n))/e - (24*a*b^2*d)/e))/(8*e)) + x*((d*((d*(2*a^3 - (3*b^3*n^3)/16 + (3*a*b^2*n^2)/4 - (3*a^2*b*n)/2))/e - (d*(24*a^3 + 7*b^3*n^3 - 12*a*b^2*n^2))/(12*e)))/(2*e) + (b^2*d^2*n^2*(12*a - 13*b*n))/(16*e^2)) - x^(1/2)*((d*((d*((d*(2*a^3 - (3*b^3*n^3)/16 + (3*a*b^2*n^2)/4 - (3*a^2*b*n)/2))/e - (d*(24*a^3 + 7*b^3*n^3 - 12*a*b^2*n^2))/(12*e)))/e + (b^2*d^2*n^2*(12*a - 13*b*n))/(8*e^2)))/e + (b^2*d^3*n^2*(12*a - 25*b*n))/(4*e^3)) + x^2*(a^3/2 - (3*b^3*n^3)/64 + (3*a*b^2*n^2)/16 - (3*a^2*b*n)/8) + (log(c*(d + e*x^(1/2))^n)*((x^(3/2)*(16*b*d*e^3*(6*a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2 + b^2*n^2 - 4*a*b*n)))/(12*e^2) - (x*((d*(16*b*d*e^3*(6*a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2 + b^2*n^2 - 4*a*b*n)))/e - 24*b^3*d^2*e^2*n^2))/(8*e^2) + (x^(1/2)*((d*((d*(16*b*d*e^3*(6*a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2 + b^2*n^2 - 4*a*b*n)))/e - 24*b^3*d^2*e^2*n^2))/e - 48*b^3*d^3*e*n^2))/(4*e^2) + (3*b*e^2*x^2*(8*a^2 + b^2*n^2 - 4*a*b*n))/4))/(4*e^2) - (log(d + e*x^(1/2))*(415*b^3*d^4*n^3 - 300*a*b^2*d^4*n^2 + 72*a^2*b*d^4*n))/(48*e^4)

3.417 $\int \left(a+b \log \left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 284

$$\begin{aligned}\int \left(a+b \log \left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3 \, dx = & -\frac{3b^3n^3\left(d+e\sqrt{x}\right)^2}{4e^2} - \frac{12ab^2dn^2\sqrt{x}}{e} + \frac{12b^3dn^3\sqrt{x}}{e} \\ & - \frac{12b^3dn^2\left(d+e\sqrt{x}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{e^2} \\ & + \frac{3b^2n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^2} \\ & + \frac{6bdn\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2} \\ & - \frac{3bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2e^2} \\ & - \frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^2} \\ & + \frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^2}\end{aligned}$$

```
[Out] -12*a*b^2*d*n^2*x^(1/2)/e+12*b^3*d*n^3*x^(1/2)/e-12*b^3*d*n^2*ln(c*(d+e*x^(
1/2))^n)*(d+e*x^(1/2))/e^2+6*b*d*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/
2))/e^2-2*d*(a+b*ln(c*(d+e*x^(1/2))^n))^3*(d+e*x^(1/2))/e^2-3/4*b^3*n^3*(d+
e*x^(1/2))^2/e^2+3/2*b^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))^2/e^
2-3/2*b*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))^2/e^2+(a+b*ln(c*(d+e*
x^(1/2))^n))^3*(d+e*x^(1/2))^2/e^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2501, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx=\frac{3b^2n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^2}-\frac{12ab^2dn^2\sqrt{x}}{e}-\frac{3bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2e^2}+\frac{6bdn\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2}+\frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^2}-\frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^2}-\frac{12b^3dn^2\left(d+e\sqrt{x}\right)\log\left(c\left(d+e\sqrt{x}\right)^n\right)}{e^2}-\frac{3b^3n^3\left(d+e\sqrt{x}\right)^2}{4e^2}+\frac{12b^3dn^3\sqrt{x}}{e}$$

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

```
[Out] (-3*b^3*n^3*(d + e*Sqrt[x])^2)/(4*e^2) - (12*a*b^2*d*n^2*Sqrt[x])/e + (12*b
^3*d*n^3*Sqrt[x])/e - (12*b^3*d*n^2*(d + e*Sqrt[x])*Log[c*(d + e*Sqrt[x])^n
])/e^2 + (3*b^2*n^2*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*
e^2) + (6*b*d*n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/e^2 - (
3*b*n*(d + e*Sqrt[x])^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*e^2) - (2*d*
(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/e^2 + ((d + e*Sqrt[x])^
2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3)/e^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int x(a+b\log\left(c(d+ex)^n\right))^3\,dx,x,\sqrt{x}\right)$$

$$=2\text{Subst}\left(\int\left(-\frac{d(a+b\log\left(c(d+ex)^n\right))^3}{e}+\frac{(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e}\right)\,dx,x,\sqrt{x}\right)$$

$$= \frac{2\text{Subst}\left(\int (d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\sqrt{x}\right)}{e} - \frac{(2d)\text{Subst}\left(\int \left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx,x,\sqrt{x}\right)}{e}$$

$$= \frac{2\text{Subst}\left(\int x\left(a+b\log\left(cx^n\right)\right)^3\,dx,x,d+e\sqrt{x}\right)}{e^2} - \frac{(2d)\text{Subst}\left(\int \left(a+b\log\left(cx^n\right)\right)^3\,dx,x,d+e\sqrt{x}\right)}{e^2}$$

$$= -\frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^2} + \frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^2} - \frac{(3bn)\text{Subst}\left(\int x\left(a+b\log\left(cx^n\right)\right)^2\,dx,x,d+e\sqrt{x}\right)}{e^2} + \frac{(6bdn)\text{Subst}\left(\int \left(a+b\log\left(cx^n\right)\right)^2\,dx,x,d+e\sqrt{x}\right)}{e^2}$$

$$= \frac{6bdn\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2} - \frac{3bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2e^2} - \frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^2} + \frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^2} + \frac{\left(3b^2n^2\right)\text{Subst}\left(\int x\left(a+b\log\left(cx^n\right)\right)\,dx,x,d+e\sqrt{x}\right)}{e^2} - \frac{\left(12b^2dn^2\right)\text{Subst}\left(\int \left(a+b\log\left(cx^n\right)\right)\,dx,x,d+e\sqrt{x}\right)}{e^2}$$

$$= -\frac{3b^3n^3\left(d+e\sqrt{x}\right)^2}{4e^2} - \frac{12ab^2dn^2\sqrt{x}}{e} + \frac{3b^2n^2\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2e^2} + \frac{6bdn\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{e^2} - \frac{3bn\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2e^2} - \frac{2d\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^2} + \frac{\left(d+e\sqrt{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{e^2} - \frac{\left(12b^3dn^2\right)\text{Subst}\left(\int \log\left(cx^n\right)\,dx,x,d+e\sqrt{x}\right)}{e^2}$$

$$
\begin{aligned}
&= -\frac{3b^3n^3(d+e\sqrt{x})^2}{4e^2} - \frac{12ab^2dn^2\sqrt{x}}{e} + \frac{12b^3dn^3\sqrt{x}}{e} \\
&\quad - \frac{12b^3dn^2(d+e\sqrt{x})\log\left(c(d+e\sqrt{x})^n\right)}{e^2} \\
&\quad + \frac{3b^2n^2(d+e\sqrt{x})^2\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)}{2e^2} \\
&\quad + \frac{6bdn(d+e\sqrt{x})\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^2}{e^2} \\
&\quad - \frac{3bn(d+e\sqrt{x})^2\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^2}{2e^2} \\
&\quad - \frac{2d(d+e\sqrt{x})\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^3}{e^2} \\
&\quad + \frac{(d+e\sqrt{x})^2\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^3}{e^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.85

$$
\int \left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^3 dx
$$

$$
= \frac{-8d(d+e\sqrt{x})\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^3 + 4(d+e\sqrt{x})^2\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^3 + 24bdn\Big((d+e\sqrt{}
$$

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^3,x]
```

```
[Out] (-8*d*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^3 + 4*(d + e*Sqrt[x]
)^2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3 + 24*b*d*n*((d + e*Sqrt[x])*(a + b*L
og[c*(d + e*Sqrt[x])^n])^2 - 2*b*n*(e*(a - b*n)*Sqrt[x] + b*(d + e*Sqrt[x])
*Log[c*(d + e*Sqrt[x])^n])) - 3*b*n*(2*(d + e*Sqrt[x])^2*(a + b*Log[c*(d +
e*Sqrt[x])^n])^2 + b*n*(b*e*n*(2*d*Sqrt[x] + e*x) - 2*(d + e*Sqrt[x])^2*(a
+ b*Log[c*(d + e*Sqrt[x])^n]))))/(4*e^2)
```

Maple [F]

$$
\int \left(a+b\ln\left(c(d+e\sqrt{x})^n\right)\right)^3 dx
$$

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. 2(248) = 496.

Time = 0.37 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.86

$$\int \left(a+b\log \left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$$
$$= \frac{4\,b^3e^2x\log \left(c\right)^3+4\,(b^3e^2n^3x-b^3d^2n^3)\log \left(e\sqrt{x}+d\right)^3-6\,(b^3e^2n-2\,ab^2e^2)x\log \left(c\right)^2+6\,\left(2\,b^3den^3\sqrt{x}+3\,b\right.}{}$$

[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="fricas")

[Out] 1/4*(4*b^3*e^2*x*log(c)^3 + 4*(b^3*e^2*n^3*x - b^3*d^2*n^3)*log(e*sqrt(x) + d)^3 - 6*(b^3*e^2*n - 2*a*b^2*e^2)*x*log(c)^2 + 6*(2*b^3*d*e*n^3*sqrt(x) + 3*b^3*d^2*n^3 - 2*a*b^2*d^2*n^2 - (b^3*e^2*n^3 - 2*a*b^2*e^2*n^2)*x + 2*(b^3*e^2*n^2*x - b^3*d^2*n^2)*log(c))*log(e*sqrt(x) + d)^2 + 6*(b^3*e^2*n^2 - 2*a*b^2*e^2*n + 2*a^2*b*e^2)*x*log(c) - (3*b^3*e^2*n^3 - 6*a*b^2*e^2*n^2 + 6*a^2*b*e^2*n - 4*a^3*e^2)*x - 6*(7*b^3*d^2*n^3 - 6*a*b^2*d^2*n^2 + 2*a^2*b*d^2*n - 2*(b^3*e^2*n*x - b^3*d^2*n)*log(c)^2 - (b^3*e^2*n^3 - 2*a*b^2*e^2*n^2 + 2*a^2*b*e^2*n)*x - 2*(3*b^3*d^2*n^2 - 2*a*b^2*d^2*n - (b^3*e^2*n^2 - 2*a*b^2*e^2*n)*x)*log(c) + 2*(3*b^3*d*e*n^3 - 2*b^3*d*e*n^2*log(c) - 2*a*b^2*d*e*n^2)*sqrt(x))*log(e*sqrt(x) + d) + 6*(7*b^3*d*e*n^3 + 2*b^3*d*e*n*log(c)^2 - 6*a*b^2*d*e*n^2 + 2*a^2*b*d*e*n - 2*(3*b^3*d*e*n^2 - 2*a*b^2*d*e*n)*log(c))*sqrt(x))/e^2

Sympy [F]

$$\int \left(a+b\log \left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx=\int \left(a+b\log \left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$$

[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**3,x)

[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**3, x)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.34

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$$
$$= -\frac{3}{2}\left(en\left(\frac{2\,d^2\log\left(e\sqrt{x}+d\right)}{e^3}+\frac{ex-2\,d\sqrt{x}}{e^2}\right)-2\,x\log\left(\left(e\sqrt{x}+d\right)^nc\right)\right)a^2b$$
$$-\frac{3}{2}\left(2\,en\left(\frac{2\,d^2\log\left(e\sqrt{x}+d\right)}{e^3}+\frac{ex-2\,d\sqrt{x}}{e^2}\right)\log\left(\left(e\sqrt{x}+d\right)^nc\right)-2\,x\log\left(\left(e\sqrt{x}+d\right)^nc\right)^2-\frac{\left(2\,d^2\log\right.}{}\right.$$
$$-\frac{1}{4}\left(6\,en\left(\frac{2\,d^2\log\left(e\sqrt{x}+d\right)}{e^3}+\frac{ex-2\,d\sqrt{x}}{e^2}\right)\log\left(\left(e\sqrt{x}+d\right)^nc\right)^2-4\,x\log\left(\left(e\sqrt{x}+d\right)^nc\right)^3+en\left(\frac{\left(4\,c\right.}{}\right.\right.$$
$$+a^3x$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="maxima")
```

```
[Out] -3/2*(e*n*(2*d^2*log(e*sqrt(x) + d)/e^3 + (e*x - 2*d*sqrt(x))/e^2) - 2*x*lo
g((e*sqrt(x) + d)^n*c))*a^2*b - 3/2*(2*e*n*(2*d^2*log(e*sqrt(x) + d)/e^3 +
(e*x - 2*d*sqrt(x))/e^2)*log((e*sqrt(x) + d)^n*c) - 2*x*log((e*sqrt(x) + d)
^n*c)^2 - (2*d^2*log(e*sqrt(x) + d)^2 + e^2*x + 6*d^2*log(e*sqrt(x) + d) -
6*d*e*sqrt(x))*n^2/e^2)*a*b^2 - 1/4*(6*e*n*(2*d^2*log(e*sqrt(x) + d)/e^3 +
(e*x - 2*d*sqrt(x))/e^2)*log((e*sqrt(x) + d)^n*c)^2 - 4*x*log((e*sqrt(x) +
d)^n*c)^3 + e*n*((4*d^2*log(e*sqrt(x) + d)^3 + 18*d^2*log(e*sqrt(x) + d)^2
+ 3*e^2*x + 42*d^2*log(e*sqrt(x) + d) - 42*d*e*sqrt(x))*n^2/e^3 - 6*(2*d^2*
log(e*sqrt(x) + d)^2 + e^2*x + 6*d^2*log(e*sqrt(x) + d) - 6*d*e*sqrt(x))*n*
log((e*sqrt(x) + d)^n*c)/e^3))*b^3 + a^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 714 vs. 2(248) = 496.

Time = 0.31 (sec) , antiderivative size = 714, normalized size of antiderivative = 2.51

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx$$
$$= \frac{\left(4\left(e\sqrt{x}+d\right)^2\log\left(e\sqrt{x}+d\right)^3-8\left(e\sqrt{x}+d\right)d\log\left(e\sqrt{x}+d\right)^3-6\left(e\sqrt{x}+d\right)^2\log\left(e\sqrt{x}+d\right)^2+24\left(e\sqrt{x}+d\right)d\log\left(e\sqrt{x}+d\right)^2+6\left(e\sqrt{x}+d\right)^2\log\left(e\sqrt{x}+d\right)-4\right.}{e}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3,x, algorithm="giac")
```

```
[Out] 1/4*((4*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d)^3 - 8*(e*sqrt(x) + d)*d*log(e*
sqrt(x) + d)^3 - 6*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d)^2 + 24*(e*sqrt(x) +
 d)*d*log(e*sqrt(x) + d)^2 + 6*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d) - 48*(e
```

```
*sqrt(x) + d)*d*log(e*sqrt(x) + d) - 3*(e*sqrt(x) + d)^2 + 48*(e*sqrt(x) +
d)*d)*b^3*n^3/e + 6*(2*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d)^2 - 4*(e*sqrt(x
) + d)*d*log(e*sqrt(x) + d)^2 - 2*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d) + 8*
(e*sqrt(x) + d)*d*log(e*sqrt(x) + d) + (e*sqrt(x) + d)^2 - 8*(e*sqrt(x) + d
)*d)*b^3*n^2*log(c)/e + 6*(2*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d) - 4*(e*sq
rt(x) + d)*d*log(e*sqrt(x) + d) - (e*sqrt(x) + d)^2 + 4*(e*sqrt(x) + d)*d)*
b^3*n*log(c)^2/e + 4*((e*sqrt(x) + d)^2 - 2*(e*sqrt(x) + d)*d)*b^3*log(c)^3
/e + 6*(2*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d)^2 - 4*(e*sqrt(x) + d)*d*log(
e*sqrt(x) + d)^2 - 2*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d) + 8*(e*sqrt(x) +
d)*d*log(e*sqrt(x) + d) + (e*sqrt(x) + d)^2 - 8*(e*sqrt(x) + d)*d)*a*b^2*n^
2/e + 12*(2*(e*sqrt(x) + d)^2*log(e*sqrt(x) + d) - 4*(e*sqrt(x) + d)*d*log(
e*sqrt(x) + d) - (e*sqrt(x) + d)^2 + 4*(e*sqrt(x) + d)*d)*a*b^2*n*log(c)/e
+ 12*((e*sqrt(x) + d)^2 - 2*(e*sqrt(x) + d)*d)*a*b^2*log(c)^2/e + 6*(2*(e*s
qrt(x) + d)^2*log(e*sqrt(x) + d) - 4*(e*sqrt(x) + d)*d*log(e*sqrt(x) + d) -
 (e*sqrt(x) + d)^2 + 4*(e*sqrt(x) + d)*d)*a^2*b*n/e + 12*((e*sqrt(x) + d)^2
 - 2*(e*sqrt(x) + d)*d)*a^2*b*log(c)/e + 4*((e*sqrt(x) + d)^2 - 2*(e*sqrt(x
) + d)*d)*a^3/e)/e
```

Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.23

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\,dx = x\left(a^3-\frac{3\,a^2\,b\,n}{2}+\frac{3\,a\,b^2\,n^2}{2}-\frac{3\,b^3\,n^3}{4}\right) - \sqrt{x}\left(\frac{d\left(2\,a^3-3\,a^2\,b\,n+3\,a\,b^2\,n^2-\frac{3\,b^3\,n^3}{2}\right)}{e} - \frac{d\,(2\,a^3-6\,a\,b^2\,n^2+9\,b^3\,n^3)}{e}\right) + \ln\left(c\left(d+e\sqrt{x}\right)^n\right)^3\left(b^3\,x-\frac{b^3\,d^2}{e^2}\right) - \ln\left(c\left(d+e\sqrt{x}\right)^n\right)\left(\sqrt{x}\left(\frac{3\,b\,d\,(2\,a^2-2\,a\,b\,n+b^2\,n^2)}{e} - \frac{6\,b\,d\,(a^2-b^2\,n^2)}{e}\right) - \frac{3\,b\,x\,(2\,a^2-2\,a\,b\,n+b^2\,n^2)}{2}\right) - \ln\left(c\left(d+e\sqrt{x}\right)^n\right)^2\left(\sqrt{x}\left(\frac{3\,b^2\,d\,(2\,a-b\,n)}{e} - \frac{6\,a\,b^2\,d}{e}\right) + \frac{3\,d\,(2\,a\,b^2\,d-3\,b^3\,d\,n)}{2\,e^2} - \frac{3\,b^2\,x\,(2\,a-b\,n)}{2}\right) - \frac{\ln\left(d+e\sqrt{x}\right)\,(6\,a^2\,b\,d^2\,n-18\,a\,b^2\,d^2\,n^2+21\,b^3\,d^2\,n^3)}{2\,e^2}$$

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^3,x)
```

```
[Out] x*(a^3 - (3*b^3*n^3)/4 + (3*a*b^2*n^2)/2 - (3*a^2*b*n)/2) - x^(1/2)*((d*(2*
a^3 - (3*b^3*n^3)/2 + 3*a*b^2*n^2 - 3*a^2*b*n))/e - (d*(2*a^3 + 9*b^3*n^3 -
 6*a*b^2*n^2))/e) + log(c*(d + e*x^(1/2))^n)^3*(b^3*x - (b^3*d^2)/e^2) - lo
g(c*(d + e*x^(1/2))^n)*(x^(1/2)*((3*b*d*(2*a^2 + b^2*n^2 - 2*a*b*n))/e - (6
*b*d*(a^2 - b^2*n^2))/e) - (3*b*x*(2*a^2 + b^2*n^2 - 2*a*b*n))/2) - log(c*(
d + e*x^(1/2))^n)^2*(x^(1/2)*((3*b^2*d*(2*a - b*n))/e - (6*a*b^2*d)/e) + (3
*d*(2*a*b^2*d - 3*b^3*d*n))/(2*e^2) - (3*b^2*x*(2*a - b*n))/2) - (log(d + e
*x^(1/2))*(21*b^3*d^2*n^3 - 18*a*b^2*d^2*n^2 + 6*a^2*b*d^2*n))/(2*e^2)
```

3.418 $\int \frac{(a+b\log(c(d+e\sqrt{x})^n))^3}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 135

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}\,dx = 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt{x}}{d}\right) + 6bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\operatorname{PolyLog}\left(2,1+\frac{e\sqrt{x}}{d}\right) - 12b^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\operatorname{PolyLog}\left(3,1+\frac{e\sqrt{x}}{d}\right) + 12b^3n^3\operatorname{PolyLog}\left(4,1+\frac{e\sqrt{x}}{d}\right)$$

```
[Out] 2*ln(-e*x^(1/2)/d)*(a+b*ln(c*(d+e*x^(1/2))^n))^3+6*b*n*(a+b*ln(c*(d+e*x^(1/
2))^n))^2*polylog(2,1+e*x^(1/2)/d)-12*b^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*p
olylog(3,1+e*x^(1/2)/d)+12*b^3*n^3*polylog(4,1+e*x^(1/2)/d)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

= {2504, 2443, 2481, 2421, 2430, 6724}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}\,dx = -12b^2n^2\,\text{PolyLog}\left(3,\frac{\sqrt{x}e}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right) + 6bn\,\text{PolyLog}\left(2,\frac{\sqrt{x}e}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2 + 2\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3 + 12b^3n^3\,\text{PolyLog}\left(4,\frac{\sqrt{x}e}{d}+1\right)$$

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x,x]
```

```
[Out] 2*(a + b*Log[c*(d + e*Sqrt[x])^n])^3*Log[-((e*Sqrt[x])/d)] + 6*b*n*(a + b*L
og[c*(d + e*Sqrt[x])^n])^2*PolyLog[2, 1 + (e*Sqrt[x])/d] - 12*b^2*n^2*(a +
b*Log[c*(d + e*Sqrt[x])^n])*PolyLog[3, 1 + (e*Sqrt[x])/d] + 12*b^3*n^3*Poly
Log[4, 1 + (e*Sqrt[x])/d]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
```

```
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x}\,dx, x, \sqrt{x}\right)\\
&= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt{x}}{d}\right)\\
&\quad-(6ben)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{d+ex}\,dx, x, \sqrt{x}\right)\\
&= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt{x}}{d}\right)\\
&\quad-(6bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx, x, d+e\sqrt{x}\right)\\
&= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt{x}}{d}\right)\\
&\quad+6bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\text{Li}_2\left(1+\frac{e\sqrt{x}}{d}\right)\\
&\quad-\left(12b^2n^2\right)\text{Subst}\left(\int \frac{(a+b\log(cx^n))\,\text{Li}_2\left(\frac{x}{d}\right)}{x}\,dx, x, d+e\sqrt{x}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt{x}}{d}\right) \\
&\quad + 6bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\operatorname{Li}_2\left(1+\frac{e\sqrt{x}}{d}\right) \\
&\quad - 12b^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\operatorname{Li}_3\left(1+\frac{e\sqrt{x}}{d}\right) \\
&\quad + \left(12b^3n^3\right)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(\frac{x}{d}\right)}{x}\,dx, x, d+e\sqrt{x}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= 2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt{x}}{d}\right) + 6bn\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\operatorname{Li}_2\left(1\right. \\
&\quad \left.+\frac{e\sqrt{x}}{d}\right) - 12b^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\operatorname{Li}_3\left(1+\frac{e\sqrt{x}}{d}\right) + 12b^3n^3\operatorname{Li}_4\left(1+\frac{e\sqrt{x}}{d}\right)
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 333 vs. 2(135) = 270.

Time = 0.11 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.47

$$
\begin{aligned}
\int\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}\,dx &= \left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3\log(x) \\
&\quad + 3bn\left(a-bn\log\left(d+e\sqrt{x}\right)\right. \\
&\qquad \left.+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2\left(\left(\log\left(d+e\sqrt{x}\right)\right.\right. \\
&\qquad \left.\left.-\log\left(1+\frac{e\sqrt{x}}{d}\right)\right)\log(x)-2\operatorname{PolyLog}\left(2,-\frac{e\sqrt{x}}{d}\right)\right) \\
&\quad + 6b^2n^2\left(a-bn\log\left(d+e\sqrt{x}\right)\right. \\
&\qquad \left.+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\left(\log^2\left(d+e\sqrt{x}\right)\log\left(-\frac{e\sqrt{x}}{d}\right)\right. \\
&\qquad +2\log\left(d+e\sqrt{x}\right)\operatorname{PolyLog}\left(2,1+\frac{e\sqrt{x}}{d}\right) \\
&\qquad \left.-2\operatorname{PolyLog}\left(3,1+\frac{e\sqrt{x}}{d}\right)\right) \\
&\quad + 2b^3n^3\left(\log^3\left(d+e\sqrt{x}\right)\log\left(-\frac{e\sqrt{x}}{d}\right)\right. \\
&\qquad +3\log^2\left(d+e\sqrt{x}\right)\operatorname{PolyLog}\left(2,1+\frac{e\sqrt{x}}{d}\right) \\
&\qquad -6\log\left(d+e\sqrt{x}\right)\operatorname{PolyLog}\left(3,1+\frac{e\sqrt{x}}{d}\right) \\
&\qquad \left.+6\operatorname{PolyLog}\left(4,1+\frac{e\sqrt{x}}{d}\right)\right)
\end{aligned}
$$

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x,x]
```

```
[Out] (a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^3*Log[x] + 3*b*n*
(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2*((Log[d + e*Sqr
t[x]] - Log[1 + (e*Sqrt[x])/d])*Log[x] - 2*PolyLog[2, -((e*Sqrt[x])/d)]) +
6*b^2*n^2*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])*(Log[d
+ e*Sqrt[x]]^2*Log[-((e*Sqrt[x])/d)] + 2*Log[d + e*Sqrt[x]]*PolyLog[2, 1 +
(e*Sqrt[x])/d] - 2*PolyLog[3, 1 + (e*Sqrt[x])/d]) + 2*b^3*n^3*(Log[d + e*Sq
rt[x]]^3*Log[-((e*Sqrt[x])/d)] + 3*Log[d + e*Sqrt[x]]^2*PolyLog[2, 1 + (e*S
qrt[x])/d] - 6*Log[d + e*Sqrt[x]]*PolyLog[3, 1 + (e*Sqrt[x])/d] + 6*PolyLog
[4, 1 + (e*Sqrt[x])/d])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}}{x}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}}{x}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)+a\right)^{3}}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*sqrt(x) + d)^n*c)^3 + 3*a*b^2*log((e*sqrt(x) + d)^n*c)
^2 + 3*a^2*b*log((e*sqrt(x) + d)^n*c) + a^3)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}}{x}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**3/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**3/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^nc\right)+a\right)^3}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x,x, algorithm="maxima")
```

```
[Out] b^3*log((e*sqrt(x) + d)^n)^3*log(x) + integrate(-1/2*(3*(b^3*e*n*x*log(x) -
 2*(b^3*e*log(c) + a*b^2*e)*x - 2*(b^3*d*log(c) + a*b^2*d)*sqrt(x))*log((e*
sqrt(x) + d)^n)^2 - 2*(b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(
c) + a^3*e)*x - 6*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b^3*d
*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*sqrt(x))*log((e*sqrt(x) + d)^n) - 2
*(b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*sqrt(x))/
(e*x^2 + d*x^(3/2)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^nc\right)+a\right)^3}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)\right)^3}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^3/x,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))^3/x, x)
```

3.419 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 263

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx = & -\frac{3ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2\sqrt{x}} \\ & -\frac{3be^2n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2} \\ & -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x} \\ & +\frac{6b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right)}{d^2} \\ & +\frac{6b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)}{d^2} \\ & +\frac{6b^3e^2n^3\operatorname{PolyLog}\left(2,1+\frac{e\sqrt{x}}{d}\right)}{d^2} \\ & +\frac{6b^3e^2n^3\operatorname{PolyLog}\left(3,\frac{d}{d+e\sqrt{x}}\right)}{d^2}\end{aligned}$$

```
[Out] 6*b^2*e^2*n^2*ln(-e*x^(1/2)/d)*(a+b*ln(c*(d+e*x^(1/2))^n))/d^2-(a+b*ln(c*(d
+e*x^(1/2))^n))^3/x-3*b*e^2*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*ln(1-d/(d+e*x^(
1/2)))/d^2+6*b^2*e^2*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*polylog(2,d/(d+e*x^(1/
2)))/d^2+6*b^3*e^2*n^3*polylog(2,1+e*x^(1/2)/d)/d^2+6*b^3*e^2*n^3*polylog(3
,d/(d+e*x^(1/2)))/d^2-3*b*e*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))/d
^2/x^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx = \frac{6b^2e^2n^2\operatorname{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2} + \frac{6b^2e^2n^2\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^2} - \frac{3be^2n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2} - \frac{3ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2\sqrt{x}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x} + \frac{6b^3e^2n^3\operatorname{PolyLog}\left(2,\frac{\sqrt{x}e}{d}+1\right)}{d^2} + \frac{6b^3e^2n^3\operatorname{PolyLog}\left(3,\frac{d}{d+e\sqrt{x}}\right)}{d^2}$$

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x^2,x]
```

```
[Out] (-3*b*e*n*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(d^2*Sqrt[x])
 - (3*b*e^2*n*Log[1 - d/(d + e*Sqrt[x])]*(a + b*Log[c*(d + e*Sqrt[x])^n])^2
)/d^2 - (a + b*Log[c*(d + e*Sqrt[x])^n])^3/x + (6*b^2*e^2*n^2*(a + b*Log[c*
(d + e*Sqrt[x])^n])*Log[-((e*Sqrt[x])/d)])/d^2 + (6*b^2*e^2*n^2*(a + b*Log[
c*(d + e*Sqrt[x])^n])*PolyLog[2, d/(d + e*Sqrt[x])])/d^2 + (6*b^3*e^2*n^3*P
olyLog[2, 1 + (e*Sqrt[x])/d])/d^2 + (6*b^3*e^2*n^3*PolyLog[3, d/(d + e*Sqrt
[x])])/d^2
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
```

```
, p}, x] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^3}\,dx, x, \sqrt{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}+(3ben)\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(d+ex)}\,dx, x, \sqrt{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}+(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}+\frac{(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt{x}\right)}{d}\\
&\quad-\frac{(3ben)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx, x, d+e\sqrt{x}\right)}{d}\\
&= -\frac{3ben\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2\sqrt{x}}\\
&\quad-\frac{3be^2n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{d^2}\\
&\quad-\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x}+\frac{(6b^2en^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{-\frac{d}{e}+\frac{x}{e}}\,dx, x, d+e\sqrt{x}\right)}{d^2}\\
&\quad+\frac{(6b^2e^2n^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{d}{x}\right)(a+b\log(cx^n))}{x}\,dx, x, d+e\sqrt{x}\right)}{d^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{3ben(d+e\sqrt{x})\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^2}{d^2\sqrt{x}} \\
&\quad -\frac{3be^2n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^2}{d^2} \\
&\quad -\frac{\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^3}{x}+\frac{6b^2e^2n^2\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right)}{d^2} \\
&\quad +\frac{6b^2e^2n^2\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)\operatorname{Li}_2\left(\frac{d}{d+e\sqrt{x}}\right)}{d^2} \\
&\quad -\frac{(6b^3e^2n^3)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{x}{d}\right)}{x}\,dx,x,d+e\sqrt{x}\right)}{d^2} \\
&\quad -\frac{(6b^3e^2n^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{d}{x}\right)}{x}\,dx,x,d+e\sqrt{x}\right)}{d^2} \\
&= -\frac{3ben(d+e\sqrt{x})\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^2}{d^2\sqrt{x}} \\
&\quad -\frac{3be^2n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^2}{d^2} \\
&\quad -\frac{\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^3}{x}+\frac{6b^2e^2n^2\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right)}{d^2} \\
&\quad +\frac{6b^2e^2n^2\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)\operatorname{Li}_2\left(\frac{d}{d+e\sqrt{x}}\right)}{d^2} \\
&\quad +\frac{6b^3e^2n^3\operatorname{Li}_2\left(1+\frac{e\sqrt{x}}{d}\right)}{d^2}+\frac{6b^3e^2n^3\operatorname{Li}_3\left(\frac{d}{d+e\sqrt{x}}\right)}{d^2}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 536 vs. $2(263) = 526$.

Time = 0.53 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.04

$$
\int\frac{\left(a+b\log\left(c(d+e\sqrt{x})^n\right)\right)^3}{x^2}\,dx
$$

$$
= \frac{-3bden\sqrt{x}\left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c(d+e\sqrt{x})^n\right)\right)^2-3bd^2n\log\left(d+e\sqrt{x}\right)\left(a-bn\log\left(d+e\sqrt{x}\right)+\right.}{}
$$

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x^2,x]
```

```
[Out] (-3*b*d*e*n*Sqrt[x]*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n
])^2 - 3*b*d^2*n*Log[d + e*Sqrt[x]]*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(
```

```
d + e*Sqrt[x])^n])^2 + 3*b*e^2*n*x*Log[d + e*Sqrt[x]]*(a - b*n*Log[d + e*Sq
rt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2 - d^2*(a - b*n*Log[d + e*Sqrt[x]] +
b*Log[c*(d + e*Sqrt[x])^n])^3 - (3*b*e^2*n*x*(a - b*n*Log[d + e*Sqrt[x]] +
b*Log[c*(d + e*Sqrt[x])^n])^2*Log[x])/2 + 3*b^2*n^2*(a - b*n*Log[d + e*Sqrt
[x]] + b*Log[c*(d + e*Sqrt[x])^n])*((d + e*Sqrt[x])*Log[d + e*Sqrt[x]]*(-2*
e*Sqrt[x] + (-d + e*Sqrt[x])*Log[d + e*Sqrt[x]]) - 2*e^2*x*(-1 + Log[d + e*
Sqrt[x]])*Log[-((e*Sqrt[x])/d)] - 2*e^2*x*PolyLog[2, 1 + (e*Sqrt[x])/d]) +
b^3*n^3*((d + e*Sqrt[x])*Log[d + e*Sqrt[x]]^2*(-3*e*Sqrt[x] + (-d + e*Sqrt[
x])*Log[d + e*Sqrt[x]]) - 3*e^2*x*(-2 + Log[d + e*Sqrt[x]])*Log[d + e*Sqrt[
x]]*Log[-((e*Sqrt[x])/d)] - 6*e^2*x*(-1 + Log[d + e*Sqrt[x]])*PolyLog[2, 1
+ (e*Sqrt[x])/d] + 6*e^2*x*PolyLog[3, 1 + (e*Sqrt[x])/d]))/(d^2*x)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}}{x^{2}}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}}{x^{2}}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)+a\right)^{3}}{x^{2}}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*sqrt(x) + d)^n*c)^3 + 3*a*b^2*log((e*sqrt(x) + d)^n*c)
^2 + 3*a^2*b*log((e*sqrt(x) + d)^n*c) + a^3)/x^2, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}}{x^{2}}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}}{x^{2}}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**3/x**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**3/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^nc\right)+a\right)^3}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^3*d^2*sqrt(x)*log((e*sqrt(x) + d)^n)^3 - 3*(2*b^3*e^2*n*x^(3/2)*l
og(e*sqrt(x) + d) - 2*b^3*d*e*n*x - (b^3*e^2*n*x*log(x) + 2*b^3*d^2*log(c)
+ 2*a*b^2*d^2)*sqrt(x))*log((e*sqrt(x) + d)^n)^2)/(d^2*x^(3/2)) - integrate
(-1/2*(2*(b^3*d^2*e*log(c)^3 + 3*a*b^2*d^2*e*log(c)^2 + 3*a^2*b*d^2*e*log(c
) + a^3*d^2*e)*x^(3/2) + 2*(b^3*d^3*log(c)^3 + 3*a*b^2*d^3*log(c)^2 + 3*a^2
*b*d^3*log(c) + a^3*d^3)*x - 3*(2*b^3*e^3*n^2*x^(5/2)*log(e*sqrt(x) + d) -
2*b^3*d*e^2*n^2*x^2 - 2*(b^3*d^2*e*log(c)^2 + 2*a*b^2*d^2*e*log(c) + a^2*b*
d^2*e)*x^(3/2) - 2*(b^3*d^3*log(c)^2 + 2*a*b^2*d^3*log(c) + a^2*b*d^3)*x -
(b^3*e^3*n^2*x^2*log(x) + 2*(b^3*d^2*e*n*log(c) + a*b^2*d^2*e*n)*x)*sqrt(x)
)*log((e*sqrt(x) + d)^n))/(d^2*e*x^(7/2) + d^3*x^3), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^nc\right)+a\right)^3}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^n\right)\right)^3}{x^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^3/x^2,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))^3/x^2, x)
```

3.420 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 573

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx = -\frac{b^3e^3n^3}{2d^3\sqrt{x}}+\frac{b^3e^4n^3\log\left(d+e\sqrt{x}\right)}{2d^4} - \frac{b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x} + \frac{5b^2e^3n^2\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^4\sqrt{x}} + \frac{5b^2e^4n^2\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^4} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2dx^{3/2}} + \frac{3be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{4d^2x} - \frac{3be^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2d^4\sqrt{x}} - \frac{3be^4n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2d^4} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x^2} + \frac{3b^2e^4n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right)}{d^4} - \frac{3b^3e^4n^3\log(x)}{2d^4} - \frac{5b^3e^4n^3\operatorname{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)}{2d^4} + \frac{3b^2e^4n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)}{d^4} + \frac{3b^3e^4n^3\operatorname{PolyLog}\left(2,1+\frac{e\sqrt{x}}{d}\right)}{d^4} + \frac{3b^3e^4n^3\operatorname{PolyLog}\left(3,\frac{d}{d+e\sqrt{x}}\right)}{d^4}$$

```
[Out] -3/2*b^3*e^4*n^3*ln(x)/d^4+1/2*b^3*e^4*n^3*ln(d+e*x^(1/2))/d^4-1/2*b^2*e^2*
n^2*(a+b*ln(c*(d+e*x^(1/2))^n))/d^2/x+3*b^2*e^4*n^2*ln(-e*x^(1/2)/d)*(a+b*l
n(c*(d+e*x^(1/2))^n))/d^4-1/2*b*e*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2/d/x^(3/2)
+3/4*b*e^2*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2/d^2/x-1/2*(a+b*ln(c*(d+e*x^(1/2)
)^n))^3/x^2+5/2*b^2*e^4*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*ln(1-d/(d+e*x^(1/2)
))/d^4-3/2*b*e^4*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*ln(1-d/(d+e*x^(1/2)))/d^4-
```

```
5/2*b^3*e^4*n^3*polylog(2,d/(d+e*x^(1/2)))/d^4+3*b^2*e^4*n^2*(a+b*ln(c*(d+e
*x^(1/2))^n))*polylog(2,d/(d+e*x^(1/2)))/d^4+3*b^3*e^4*n^3*polylog(2,1+e*x^
(1/2)/d)/d^4+3*b^3*e^4*n^3*polylog(3,d/(d+e*x^(1/2)))/d^4-1/2*b^3*e^3*n^3/d
^3/x^(1/2)+5/2*b^2*e^3*n^2*(a+b*ln(c*(d+e*x^(1/2))^n))*(d+e*x^(1/2))/d^4/x^
(1/2)-3/2*b*e^3*n*(a+b*ln(c*(d+e*x^(1/2))^n))^2*(d+e*x^(1/2))/d^4/x^(1/2)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules

used = {2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31, 46}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx = \frac{3b^2e^4n^2\,\mathrm{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4} + \frac{5b^2e^4n^2\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^4} + \frac{3b^2e^4n^2\log\left(-\frac{e\sqrt{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{d^4} + \frac{5b^2e^3n^2\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^4\sqrt{x}} - \frac{b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x} - \frac{3be^4n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2d^4} - \frac{3be^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2d^4\sqrt{x}} + \frac{3be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{4d^2x} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2dx^{3/2}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x^2} - \frac{5b^3e^4n^3\,\mathrm{PolyLog}\left(2,\frac{d}{d+e\sqrt{x}}\right)}{2d^4} + \frac{3b^3e^4n^3\,\mathrm{PolyLog}\left(2,\frac{\sqrt{x}e}{d}+1\right)}{d^4} + \frac{3b^3e^4n^3\,\mathrm{PolyLog}\left(3,\frac{d}{d+e\sqrt{x}}\right)}{d^4} + \frac{b^3e^4n^3\log\left(d+e\sqrt{x}\right)}{2d^4} - \frac{3b^3e^4n^3\log(x)}{2d^4} - \frac{b^3e^3n^3}{2d^3\sqrt{x}}$$

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x^3,x]
```

```
[Out] -1/2*(b^3*e^3*n^3)/(d^3*Sqrt[x]) + (b^3*e^4*n^3*Log[d + e*Sqrt[x]])/(2*d^4)
 - (b^2*e^2*n^2*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*d^2*x) + (5*b^2*e^3*n^
2*(d + e*Sqrt[x])*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*d^4*Sqrt[x]) + (5*b^
2*e^4*n^2*Log[1 - d/(d + e*Sqrt[x])]*(a + b*Log[c*(d + e*Sqrt[x])^n]))/(2*d
^4) - (b*e*n*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*d*x^(3/2)) + (3*b*e^2*n
*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(4*d^2*x) - (3*b*e^3*n*(d + e*Sqrt[x])
```

```
*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*d^4*Sqrt[x]) - (3*b*e^4*n*Log[1 - d
/(d + e*Sqrt[x])]*(a + b*Log[c*(d + e*Sqrt[x])^n])^2)/(2*d^4) - (a + b*Log[
c*(d + e*Sqrt[x])^n])^3/(2*x^2) + (3*b^2*e^4*n^2*(a + b*Log[c*(d + e*Sqrt[x
])^n])*Log[-((e*Sqrt[x])/d)])/d^4 - (3*b^3*e^4*n^3*Log[x])/(2*d^4) - (5*b^3
*e^4*n^3*PolyLog[2, d/(d + e*Sqrt[x])])/(2*d^4) + (3*b^2*e^4*n^2*(a + b*Log
[c*(d + e*Sqrt[x])^n])*PolyLog[2, d/(d + e*Sqrt[x])])/d^4 + (3*b^3*e^4*n^3*
PolyLog[2, 1 + (e*Sqrt[x])/d])/d^4 + (3*b^3*e^4*n^3*PolyLog[3, d/(d + e*Sqr
t[x])])/d^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
```

```
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
```

```
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^5}\,dx, x, \sqrt{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x^2}+\frac{1}{2}(3ben)\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4(d+ex)}\,dx, x, \sqrt{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x^2}+\frac{1}{2}(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\sqrt{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x^2}+\frac{(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\sqrt{x}\right)}{2d}\\
&\quad -\frac{(3ben)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt{x}\right)}{2d}
\end{aligned}
$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2dx^{3/2}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x^2} - \frac{(3ben)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt{x}\right)}{2d^2} + \frac{(3be^2n)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt{x}\right)}{2d^2} + \frac{(b^2en^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt{x}\right)}{d}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2dx^{3/2}} + \frac{3be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{4d^2x} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x^2} + \frac{(3be^2n)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt{x}\right)}{2d^3} - \frac{(3be^3n)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx, x, d+e\sqrt{x}\right)}{2d^3} + \frac{(b^2en^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt{x}\right)}{d^2} - \frac{(b^2e^2n^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt{x}\right)}{d^2} - \frac{(3b^2e^2n^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt{x}\right)}{2d^2}$$

$$= -\frac{b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2dx^{3/2}} + \frac{3be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{4d^2x} - \frac{3be^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2d^4\sqrt{x}} - \frac{3be^4n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2d^4} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x^2} - \frac{(b^2e^2n^2)\,\mathrm{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+e\sqrt{x}\right)}{d^3} - \frac{(3b^2e^2n^2)\,\mathrm{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+e\sqrt{x}\right)}{2d^3} + \frac{(3b^2e^3n^2)\,\mathrm{Subst}\left(\int\frac{a+b\log(cx^n)}{-\frac{d}{e}+\frac{x}{e}}\,dx,x,d+e\sqrt{x}\right)}{d^4} + \frac{(b^2e^3n^2)\,\mathrm{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx,x,d+e\sqrt{x}\right)}{d^3} + \frac{(3b^2e^3n^2)\,\mathrm{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx,x,d+e\sqrt{x}\right)}{2d^3} + \frac{(3b^2e^4n^2)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{d}{x}\right)(a+b\log(cx^n))}{x}\,dx,x,d+e\sqrt{x}\right)}{d^4} + \frac{(b^3e^2n^3)\,\mathrm{Subst}\left(\int\frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+e\sqrt{x}\right)}{2d^2}$$

$$
\begin{aligned}
&= -\frac{b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x} + \frac{5b^2e^3n^2\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^4\sqrt{x}} \\
&+ \frac{5b^2e^4n^2\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^4} \\
&- \frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2dx^{3/2}} + \frac{3be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{4d^2x} \\
&- \frac{3be^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2d^4\sqrt{x}} \\
&- \frac{3be^4n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2d^4} \\
&- \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x^2} + \frac{3b^2e^4n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right)}{d^4} \\
&+ \frac{3b^2e^4n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{d}{d+e\sqrt{x}}\right)}{d^4} \\
&+ \frac{\left(b^3e^2n^3\right)\operatorname{Subst}\left(\int\left(\frac{e^2}{d(d-x)^2}+\frac{e^2}{d^2(d-x)}+\frac{e^2}{d^2x}\right)dx, x, d+e\sqrt{x}\right)}{2d^2} \\
&- \frac{\left(b^3e^3n^3\right)\operatorname{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x}{e}}\,dx, x, d+e\sqrt{x}\right)}{d^4} \\
&- \frac{\left(3b^3e^3n^3\right)\operatorname{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x}{e}}\,dx, x, d+e\sqrt{x}\right)}{2d^4} \\
&- \frac{\left(b^3e^4n^3\right)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{d}{x}\right)}{x}\,dx, x, d+e\sqrt{x}\right)}{d^4} \\
&- \frac{\left(3b^3e^4n^3\right)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{d}{x}\right)}{x}\,dx, x, d+e\sqrt{x}\right)}{2d^4} \\
&- \frac{\left(3b^3e^4n^3\right)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{x}{d}\right)}{x}\,dx, x, d+e\sqrt{x}\right)}{d^4} \\
&- \frac{\left(3b^3e^4n^3\right)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{d}{x}\right)}{x}\,dx, x, d+e\sqrt{x}\right)}{d^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b^3e^3n^3}{2d^3\sqrt{x}} + \frac{b^3e^4n^3\log\left(d+e\sqrt{x}\right)}{2d^4} - \frac{b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^2x} \\
&\quad + \frac{5b^2e^3n^2\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^4\sqrt{x}} \\
&\quad + \frac{5b^2e^4n^2\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)}{2d^4} \\
&\quad - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2dx^{3/2}} + \frac{3be^2n\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{4d^2x} \\
&\quad - \frac{3be^3n\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2d^4\sqrt{x}} \\
&\quad - \frac{3be^4n\log\left(1-\frac{d}{d+e\sqrt{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2}{2d^4} - \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{2x^2} \\
&\quad + \frac{3b^2e^4n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt{x}}{d}\right)}{d^4} - \frac{3b^3e^4n^3\log(x)}{2d^4} \\
&\quad - \frac{5b^3e^4n^3\operatorname{Li}_2\left(\frac{d}{d+e\sqrt{x}}\right)}{2d^4} + \frac{3b^2e^4n^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{d}{d+e\sqrt{x}}\right)}{d^4} \\
&\quad + \frac{3b^3e^4n^3\operatorname{Li}_2\left(1+\frac{e\sqrt{x}}{d}\right)}{d^4} + \frac{3b^3e^4n^3\operatorname{Li}_3\left(\frac{d}{d+e\sqrt{x}}\right)}{d^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 841, normalized size of antiderivative = 1.47

$$
\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx =
$$

$$
-\frac{2bd^3en\sqrt{x}\left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^2-3bd^2e^2nx\left(a-bn\log\left(d+e\sqrt{x}\right)+b\log\left(c\left(\right.\right.\right.}{}
$$

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^n])^3/x^3,x]
```

```
[Out] -1/4*(2*b*d^3*e*n*Sqrt[x]*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt
[x])^n])^2 - 3*b*d^2*e^2*n*x*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*S
qrt[x])^n])^2 + 6*b*d*e^3*n*x^(3/2)*(a - b*n*Log[d + e*Sqrt[x]] + b*Log[c*(
d + e*Sqrt[x])^n])^2 + 6*b*d^4*n*Log[d + e*Sqrt[x]]*(a - b*n*Log[d + e*Sqrt
[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2 - 6*b*e^4*n*x^2*Log[d + e*Sqrt[x]]*(a
- b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2 + 2*d^4*(a - b*n*L
og[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^3 + 3*b*e^4*n*x^2*(a - b*n*
Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])^2*Log[x] - 2*b^2*n^2*(a -
b*n*Log[d + e*Sqrt[x]] + b*Log[c*(d + e*Sqrt[x])^n])*(-3*(d^4 - e^4*x^2)*Lo
g[d + e*Sqrt[x]]^2 + e^2*x*(-d^2 + 5*d*e*Sqrt[x] + 11*e^2*x*Log[-((e*Sqrt[x
```

```
])/d)]) - Log[d + e*Sqrt[x]]*(2*d^3*e*Sqrt[x] - 3*d^2*e^2*x + 6*d*e^3*x^(3/
2) + 11*e^4*x^2 + 6*e^4*x^2*Log[-((e*Sqrt[x])/d)]) - 6*e^4*x^2*PolyLog[2, 1
 + (e*Sqrt[x])/d]) + b^3*n^3*(d^2*e^2*x*(2 - 3*Log[d + e*Sqrt[x]])*Log[d +
e*Sqrt[x]] + 2*d^3*e*Sqrt[x]*Log[d + e*Sqrt[x]]^2 + 2*d^4*Log[d + e*Sqrt[x]
]^3 + 2*d*e^3*x^(3/2)*(1 - 5*Log[d + e*Sqrt[x]] + 3*Log[d + e*Sqrt[x]]^2) +
 12*e^4*x^2*(-Log[d + e*Sqrt[x]] + Log[-((e*Sqrt[x])/d)]) + 11*e^4*x^2*(Log
[d + e*Sqrt[x]]*(Log[d + e*Sqrt[x]] - 2*Log[-((e*Sqrt[x])/d)]) - 2*PolyLog[
2, 1 + (e*Sqrt[x])/d]) - 2*e^4*x^2*(Log[d + e*Sqrt[x]]^2*(Log[d + e*Sqrt[x]
] - 3*Log[-((e*Sqrt[x])/d)]) - 6*Log[d + e*Sqrt[x]]*PolyLog[2, 1 + (e*Sqrt[
x])/d] + 6*PolyLog[3, 1 + (e*Sqrt[x])/d])))/(d^4*x^2)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}}{x^{3}}dx$$

[In] int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x^3,x)

[Out] int((a+b*ln(c*(d+e*x^(1/2))^n))^3/x^3,x)

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}}{x^{3}}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^{n}c\right)+a\right)^{3}}{x^{3}}\,dx$$

[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^3,x, algorithm="fricas")

[Out] integral((b^3*log((e*sqrt(x) + d)^n*c)^3 + 3*a*b^2*log((e*sqrt(x) + d)^n*c)
^2 + 3*a^2*b*log((e*sqrt(x) + d)^n*c) + a^3)/x^3, x)

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}}{x^{3}}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{n}\right)\right)^{3}}{x^{3}}\,dx$$

[In] integrate((a+b*ln(c*(d+e*x**(1/2))**n))**3/x**3,x)

[Out] Integral((a + b*log(c*(d + e*sqrt(x))**n))**3/x**3, x)

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^3}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log((e*sqrt(x) + d)^n)^3/x^2 + integrate(1/4*(3*(b^3*e*n*x + 4*(b^
3*e*log(c) + a*b^2*e)*x + 4*(b^3*d*log(c) + a*b^2*d)*sqrt(x))*log((e*sqrt(x
) + d)^n)^2 + 4*(b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a
^3*e)*x + 12*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b^3*d*log(
c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*sqrt(x))*log((e*sqrt(x) + d)^n) + 4*(b^3
*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*sqrt(x))/(e*x^
4 + d*x^(7/2)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^n c\right)+a\right)^3}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^n))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*sqrt(x) + d)^n*c) + a)^3/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^n\right)\right)^3}{x^3}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/2))^n))^3/x^3,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/2))^n))^3/x^3, x)
```

3.421 $\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 171

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{be^7n\sqrt{x}}{4d^7}-\frac{be^6nx}{8d^6}+\frac{be^5nx^{3/2}}{12d^5}-\frac{be^4nx^2}{16d^4}+\frac{be^3nx^{5/2}}{20d^3}-\frac{be^2nx^3}{24d^2}+\frac{benx^{7/2}}{28d}-\frac{be^8n\log\left(d+\frac{e}{\sqrt{x}}\right)}{4d^8}+\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{be^8n\log(x)}{8d^8}$$

```
[Out] -1/8*b*e^6*n*x/d^6+1/12*b*e^5*n*x^(3/2)/d^5-1/16*b*e^4*n*x^2/d^4+1/20*b*e^3
*n*x^(5/2)/d^3-1/24*b*e^2*n*x^3/d^2+1/28*b*e*n*x^(7/2)/d-1/8*b*e^8*n*ln(x)/
d^8-1/4*b*e^8*n*ln(d+e/x^(1/2))/d^8+1/4*x^4*(a+b*ln(c*(d+e/x^(1/2))^n))+1/4
*b*e^7*n*x^(1/2)/d^7
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 46}

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{be^8n\log\left(d+\frac{e}{\sqrt{x}}\right)}{4d^8}-\frac{be^8n\log(x)}{8d^8}+\frac{be^7n\sqrt{x}}{4d^7}-\frac{be^6nx}{8d^6}+\frac{be^5nx^{3/2}}{12d^5}-\frac{be^4nx^2}{16d^4}+\frac{be^3nx^{5/2}}{20d^3}-\frac{be^2nx^3}{24d^2}+\frac{benx^{7/2}}{28d}$$

```
[In] Int[x^3*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

```
[Out] (b*e^7*n*Sqrt[x])/(4*d^7) - (b*e^6*n*x)/(8*d^6) + (b*e^5*n*x^(3/2))/(12*d^5
) - (b*e^4*n*x^2)/(16*d^4) + (b*e^3*n*x^(5/2))/(20*d^3) - (b*e^2*n*x^3)/(24
*d^2) + (b*e*n*x^(7/2))/(28*d) - (b*e^8*n*Log[d + e/Sqrt[x]])/(4*d^8) + (x^
4*(a + b*Log[c*(d + e/Sqrt[x])^n]))/4 - (b*e^8*n*Log[x])/(8*d^8)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(2\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^9}\,dx,x,\frac{1}{\sqrt{x}}\right)\right)\\
&= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{1}{4}(ben)\text{Subst}\left(\int \frac{1}{x^8(d+ex)}\,dx,x,\frac{1}{\sqrt{x}}\right)\\
&= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{1}{4}(ben)\text{Subst}\left(\int \left(\frac{1}{dx^8}-\frac{e}{d^2x^7}+\frac{e^2}{d^3x^6}-\frac{e^3}{d^4x^5}\right.\right.\\
&\quad \left.\left.+\frac{e^4}{d^5x^4}-\frac{e^5}{d^6x^3}+\frac{e^6}{d^7x^2}-\frac{e^7}{d^8x}+\frac{e^8}{d^8(d+ex)}\right)dx,x,\frac{1}{\sqrt{x}}\right)\\
&= \frac{be^7n\sqrt{x}}{4d^7}-\frac{be^6nx}{8d^6}+\frac{be^5nx^{3/2}}{12d^5}-\frac{be^4nx^2}{16d^4}+\frac{be^3nx^{5/2}}{20d^3}-\frac{be^2nx^3}{24d^2}+\frac{benx^{7/2}}{28d}\\
&\quad -\frac{be^8n\log\left(d+\frac{e}{\sqrt{x}}\right)}{4d^8}+\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{be^8n\log(x)}{8d^8}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.91

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{ax^4}{4}+\frac{1}{4}bx^4\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{8}ben\left(\frac{2e^6\sqrt{x}}{d^7}-\frac{e^5x}{d^6}+\frac{2e^4x^{3/2}}{3d^5}-\frac{e^3x^2}{2d^4}+\frac{2e^2x^{5/2}}{5d^3}-\frac{ex^3}{3d^2}+\frac{2x^{7/2}}{7d}-\frac{2e^7\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^8}-\frac{e^7\log(x)}{d^8}\right)$$

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

```
[Out] (a*x^4)/4 + (b*x^4*Log[c*(d + e/Sqrt[x])^n])/4 + (b*e*n*((2*e^6*Sqrt[x])/d^
7 - (e^5*x)/d^6 + (2*e^4*x^(3/2))/(3*d^5) - (e^3*x^2)/(2*d^4) + (2*e^2*x^(5
/2))/(5*d^3) - (e*x^3)/(3*d^2) + (2*x^(7/2))/(7*d) - (2*e^7*Log[d + e/Sqrt[
x]])/d^8 - (e^7*Log[x])/d^8))/8
```

Maple [F]

$$\int x^3\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx$$

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.05

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{420\,bd^8x^4\log\left(c\right)-70\,bd^6e^2nx^3+420\,ad^8x^4-105\,bd^4e^4nx^2-210\,bd^2e^6nx-420\,bd^8n\log\left(\sqrt{x}\right)+420\,(bd^8}{}$$

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="fricas")
```

```
[Out] 1/1680*(420*b*d^8*x^4*log(c) - 70*b*d^6*e^2*n*x^3 + 420*a*d^8*x^4 - 105*b*d
^4*e^4*n*x^2 - 210*b*d^2*e^6*n*x - 420*b*d^8*n*log(sqrt(x)) + 420*(b*d^8 -
b*e^8)*n*log(d*sqrt(x) + e) + 420*(b*d^8*n*x^4 - b*d^8*n)*log((d*x + e*sqrt
(x))/x) + 4*(15*b*d^7*e*n*x^3 + 21*b*d^5*e^3*n*x^2 + 35*b*d^3*e^5*n*x + 105
*b*d*e^7*n)*sqrt(x))/d^8
```

Sympy [A] (verification not implemented)

Time = 58.18 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.84

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{ax^4}{4}+b\left(\frac{en\left(\frac{2x^{\frac{7}{2}}}{7d}-\frac{ex^3}{3d^2}+\frac{2e^2x^{\frac{5}{2}}}{5d^3}-\frac{e^3x^2}{2d^4}+\frac{2e^4x^{\frac{3}{2}}}{3d^5}-\frac{e^5x}{d^6}-\frac{2e^7\left(\begin{cases}\frac{\sqrt{x}}{e}&\text{for }d=0\\\frac{\log\left(d\sqrt{x}+e\right)}{d}&\text{otherwise}\end{cases}\right)}{d^7}+\frac{2e^6\sqrt{x}}{d^7}\right)}{8}+\frac{x^4\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{4}\right)$$

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(1/2))**n)),x)
```

```
[Out] a*x**4/4 + b*(e*n*(2*x**(7/2)/(7*d) - e*x**3/(3*d**2) + 2*e**2*x**(5/2)/(5*
d**3) - e**3*x**2/(2*d**4) + 2*e**4*x**(3/2)/(3*d**5) - e**5*x/d**6 - 2*e**
7*Piecewise((sqrt(x)/e, Eq(d, 0)), (log(d*sqrt(x) + e)/d, True))/d**7 + 2*e
**6*sqrt(x)/d**7)/8 + x**4*log(c*(d + e/sqrt(x))**n)/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.69

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{1}{4}\,bx^4\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{4}\,ax^4 -\frac{1}{1680}\,ben\left(\frac{420\,e^7\log\left(d\sqrt{x}+e\right)}{d^8}-\frac{60\,d^6x^{\frac{7}{2}}-70\,d^5ex^3+84\,d^4e^2x^{\frac{5}{2}}-105\,d^3e^3x^2+140\,d^2e^4x^{\frac{3}{2}}-210\,de^5}{d^7}\right.$$

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4*log(c*(d + e/sqrt(x))^n) + 1/4*a*x^4 - 1/1680*b*e*n*(420*e^7*log(
d*sqrt(x) + e)/d^8 - (60*d^6*x^(7/2) - 70*d^5*e*x^3 + 84*d^4*e^2*x^(5/2) -
105*d^3*e^3*x^2 + 140*d^2*e^4*x^(3/2) - 210*d*e^5*x + 420*e^6*sqrt(x))/d^7)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.56

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{1}{4}\,bx^4\log\left(c\right)+\frac{1}{4}\,ax^4 -\frac{\left(e^9\left(\frac{420\,\log\left(\frac{|d\sqrt{x}+e|}{\sqrt{|x|}}\right)}{d^8}-\frac{420\,\log\left(\left|-d+\frac{d\sqrt{x}+e}{\sqrt{x}}\right|\right)}{d^8}+\frac{1089\,d^7-\frac{4683\,(d\sqrt{x}+e)d^6}{\sqrt{x}}+\frac{9639\,(d\sqrt{x}+e)^2d^5}{x}-\frac{11165\,(d\sqrt{x}+e)^3d^4}{x^{\frac{3}{2}}}+\frac{7490\,(d\sqrt{x}+e)^4d^3}{x^2}-}{\left(d-\frac{d\sqrt{x}+e}{\sqrt{x}}\right)^7d^8}\right.\right.}{1680\,e}$$

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4*log(c) + 1/4*a*x^4 - 1/1680*(e^9*(420*log(abs(d*sqrt(x) + e)/sqrt
(abs(x)))/d^8 - 420*log(abs(-d + (d*sqrt(x) + e)/sqrt(x)))/d^8 + (1089*d^7
- 4683*(d*sqrt(x) + e)*d^6/sqrt(x) + 9639*(d*sqrt(x) + e)^2*d^5/x - 11165*(
d*sqrt(x) + e)^3*d^4/x^(3/2) + 7490*(d*sqrt(x) + e)^4*d^3/x^2 - 2730*(d*sqr
t(x) + e)^5*d^2/x^(5/2) + 420*(d*sqrt(x) + e)^6*d/x^3)/((d - (d*sqrt(x) + e
)/sqrt(x))^7*d^8)) - 420*e^9*log(-(e - d/(d/e - (d*sqrt(x) + e)/(e*sqrt(x))
))*(d/e - (d*sqrt(x) + e)/(e*sqrt(x))))/(d - (d*sqrt(x) + e)/sqrt(x))^8)*b*
n/e
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.82

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$$
$$=\frac{\frac{b\,d\,e^7\,n\,\sqrt{x}}{4}-\frac{b\,d^2\,e^6\,n\,x}{8}+\frac{b\,d^7\,e\,n\,x^{7/2}}{28}-\frac{b\,d^4\,e^4\,n\,x^2}{16}-\frac{b\,d^6\,e^2\,n\,x^3}{24}+\frac{b\,d^3\,e^5\,n\,x^{3/2}}{12}+\frac{b\,d^5\,e^3\,n\,x^{5/2}}{20}+\frac{b\,e^8\,n\,\mathrm{atan}\left(\frac{d\,1\mathrm{i}+\frac{e\,2\mathrm{i}}{\sqrt{x}}}{d}\right)\,1\mathrm{i}}{2}}{d^8}$$
$$+\frac{a\,x^4}{4}+\frac{b\,x^4\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{4}$$

```
[In] int(x^3*(a + b*log(c*(d + e/x^(1/2))^n)),x)
```

```
[Out] ((b*e^8*n*atan((d*1i + (e*2i)/x^(1/2))/d)*1i)/2 - (b*d^2*e^6*n*x)/8 + (b*d*
e^7*n*x^(1/2))/4 + (b*d^7*e*n*x^(7/2))/28 - (b*d^4*e^4*n*x^2)/16 - (b*d^6*e
^2*n*x^3)/24 + (b*d^3*e^5*n*x^(3/2))/12 + (b*d^5*e^3*n*x^(5/2))/20)/d^8 + (
a*x^4)/4 + (b*x^4*log(c*(d + e/x^(1/2))^n))/4
```

3.422 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 139

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{be^5n\sqrt{x}}{3d^5}-\frac{be^4nx}{6d^4}+\frac{be^3nx^{3/2}}{9d^3}-\frac{be^2nx^2}{12d^2}+\frac{benx^{5/2}}{15d}-\frac{be^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3d^6}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{be^6n\log(x)}{6d^6}$$

```
[Out] -1/6*b*e^4*n*x/d^4+1/9*b*e^3*n*x^(3/2)/d^3-1/12*b*e^2*n*x^2/d^2+1/15*b*e*n*
x^(5/2)/d-1/6*b*e^6*n*ln(x)/d^6-1/3*b*e^6*n*ln(d+e/x^(1/2))/d^6+1/3*x^3*(a+
b*ln(c*(d+e/x^(1/2))^n))+1/3*b*e^5*n*x^(1/2)/d^5
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 46}

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{be^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3d^6}-\frac{be^6n\log(x)}{6d^6}+\frac{be^5n\sqrt{x}}{3d^5}-\frac{be^4nx}{6d^4}+\frac{be^3nx^{3/2}}{9d^3}-\frac{be^2nx^2}{12d^2}+\frac{benx^{5/2}}{15d}$$

```
[In] Int[x^2*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

```
[Out] (b*e^5*n*Sqrt[x])/(3*d^5) - (b*e^4*n*x)/(6*d^4) + (b*e^3*n*x^(3/2))/(9*d^3)
 - (b*e^2*n*x^2)/(12*d^2) + (b*e*n*x^(5/2))/(15*d) - (b*e^6*n*Log[d + e/Sqr
t[x]])/(3*d^6) + (x^3*(a + b*Log[c*(d + e/Sqrt[x])^n]))/3 - (b*e^6*n*Log[x]
)/(6*d^6)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(2\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^7}\,dx,x,\frac{1}{\sqrt{x}}\right)\right)\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{1}{3}(ben)\text{Subst}\left(\int \frac{1}{x^6(d+ex)}\,dx,x,\frac{1}{\sqrt{x}}\right)\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{1}{3}(ben)\text{Subst}\left(\int \left(\frac{1}{dx^6}-\frac{e}{d^2x^5}+\frac{e^2}{d^3x^4}-\frac{e^3}{d^4x^3}\right.\right.\\
&\qquad\left.\left.+\frac{e^4}{d^5x^2}-\frac{e^5}{d^6x}+\frac{e^6}{d^6(d+ex)}\right)\,dx,x,\frac{1}{\sqrt{x}}\right)\\
&= \frac{be^5n\sqrt{x}}{3d^5}-\frac{be^4nx}{6d^4}+\frac{be^3nx^{3/2}}{9d^3}-\frac{be^2nx^2}{12d^2}+\frac{benx^{5/2}}{15d}-\frac{be^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3d^6}\\
&\quad+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{be^6n\log(x)}{6d^6}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{ax^3}{3}+\frac{1}{3}bx^3\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{6}ben\left(\frac{2e^4\sqrt{x}}{d^5}-\frac{e^3x}{d^4}+\frac{2e^2x^{3/2}}{3d^3}-\frac{ex^2}{2d^2}+\frac{2x^{5/2}}{5d}-\frac{2e^5\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^6}-\frac{e^5\log(x)}{d^6}\right)$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

```
[Out] (a*x^3)/3 + (b*x^3*Log[c*(d + e/Sqrt[x])^n])/3 + (b*e*n*((2*e^4*Sqrt[x])/d^
5 - (e^3*x)/d^4 + (2*e^2*x^(3/2))/(3*d^3) - (e*x^2)/(2*d^2) + (2*x^(5/2))/(
5*d) - (2*e^5*Log[d + e/Sqrt[x]])/d^6 - (e^5*Log[x])/d^6))/6
```

Maple [F]

$$\int x^2\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx$$

```
[In] int(x^2*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.10

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{60\,bd^6x^3\log\left(c\right)-15\,bd^4e^2nx^2+60\,ad^6x^3-30\,bd^2e^4nx-60\,bd^6n\log\left(\sqrt{x}\right)+60\,(bd^6-be^6)n\log\left(d\sqrt{x}+e\right.}{180\,d^6}$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="fricas")
```

```
[Out] 1/180*(60*b*d^6*x^3*log(c) - 15*b*d^4*e^2*n*x^2 + 60*a*d^6*x^3 - 30*b*d^2*e
^4*n*x - 60*b*d^6*n*log(sqrt(x)) + 60*(b*d^6 - b*e^6)*n*log(d*sqrt(x) + e)
+ 60*(b*d^6*n*x^3 - b*d^6*n)*log((d*x + e*sqrt(x))/x) + 4*(3*b*d^5*e*n*x^2
+ 5*b*d^3*e^3*n*x + 15*b*d*e^5*n)*sqrt(x))/d^6
```

Sympy [A] (verification not implemented)

Time = 18.78 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$$
$$=\frac{ax^3}{3}+b\left(\frac{en\left(\frac{2x^{\frac{5}{2}}}{5d}-\frac{ex^2}{2d^2}+\frac{2e^2x^{\frac{3}{2}}}{3d^3}-\frac{e^3x}{d^4}-\frac{2e^5\left(\begin{cases}\frac{\sqrt{x}}{e} & \text{for } d=0\\ \frac{\log\left(d\sqrt{x}+e\right)}{d} & \text{otherwise}\end{cases}\right)}{d^5}+\frac{2e^4\sqrt{x}}{d^5}\right)}{6}+\frac{x^3\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3}\right)$$

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(1/2))**n)),x)
```

```
[Out] a*x**3/3 + b*(e*n*(2*x**(5/2)/(5*d) - e*x**2/(2*d**2) + 2*e**2*x**(3/2)/(3*
d**3) - e**3*x/d**4 - 2*e**5*Piecewise((sqrt(x)/e, Eq(d, 0)), (log(d*sqrt(x
) + e)/d, True))/d**5 + 2*e**4*sqrt(x)/d**5)/6 + x**3*log(c*(d + e/sqrt(x))
**n)/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.69

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{1}{3}\,bx^3\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{3}\,ax^3 -\frac{1}{180}\,ben\left(\frac{60\,e^5\log\left(d\sqrt{x}+e\right)}{d^6}-\frac{12\,d^4x^{\frac{5}{2}}-15\,d^3ex^2+20\,d^2e^2x^{\frac{3}{2}}-30\,de^3x+60\,e^4\sqrt{x}}{d^5}\right)$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3*log(c*(d + e/sqrt(x))^n) + 1/3*a*x^3 - 1/180*b*e*n*(60*e^5*log(d*
sqrt(x) + e)/d^6 - (12*d^4*x^(5/2) - 15*d^3*e*x^2 + 20*d^2*e^2*x^(3/2) - 30
*d*e^3*x + 60*e^4*sqrt(x))/d^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(113) = 226$.

Time = 0.36 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.67

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{1}{3}\,bx^3\log\left(c\right)+\frac{1}{3}\,ax^3 -\frac{\left(e^7\left(\frac{60\log\left(\frac{|d\sqrt{x}+e|}{\sqrt{|x|}}\right)}{d^6}-\frac{60\log\left(\left|-d+\frac{d\sqrt{x}+e}{\sqrt{x}}\right|\right)}{d^6}+\frac{137\,d^5-\frac{385\,(d\sqrt{x}+e)d^4}{\sqrt{x}}+\frac{470\,(d\sqrt{x}+e)^2d^3}{x}-\frac{270\,(d\sqrt{x}+e)^3d^2}{x^{\frac{3}{2}}}+\frac{60\,(d\sqrt{x}+e)^4d}{x^2}}{\left(d-\frac{d\sqrt{x}+e}{\sqrt{x}}\right)^5d^6}\right)-\frac{60\,e^{\cdots}}{\cdots}\right)}{180\,e}$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*log(c) + 1/3*a*x^3 - 1/180*(e^7*(60*log(abs(d*sqrt(x) + e)/sqrt(a
bs(x)))/d^6 - 60*log(abs(-d + (d*sqrt(x) + e)/sqrt(x)))/d^6 + (137*d^5 - 38
5*(d*sqrt(x) + e)*d^4/sqrt(x) + 470*(d*sqrt(x) + e)^2*d^3/x - 270*(d*sqrt(x
) + e)^3*d^2/x^(3/2) + 60*(d*sqrt(x) + e)^4*d/x^2)/((d - (d*sqrt(x) + e)/sq
rt(x))^5*d^6)) - 60*e^7*log(-(e - d/(d/e - (d*sqrt(x) + e)/(e*sqrt(x))))*(d
/e - (d*sqrt(x) + e)/(e*sqrt(x))))/(d - (d*sqrt(x) + e)/sqrt(x))^6)*b*n/e
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.76

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx = \frac{a\,x^3}{3} + \frac{b\left(60\,d^6\,x^3\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-120\,e^6\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,\sqrt{x}}+1\right)-15\,d^4\,e^2\,n\,x^2+20\,d^3\,e^3\,n\,x^{3/2}-30\,d^2\,e^4\,n\,x\right.}{180\,d^6}$$

```
[In] int(x^2*(a + b*log(c*(d + e/x^(1/2))^n)),x)
```

```
[Out] (a*x^3)/3 + (b*(60*d^6*x^3*log(c*(d + e/x^(1/2))^n) - 120*e^6*n*atanh((2*e)
/(d*x^(1/2)) + 1) - 15*d^4*e^2*n*x^2 + 20*d^3*e^3*n*x^(3/2) - 30*d^2*e^4*n*
x + 60*d*e^5*n*x^(1/2) + 12*d^5*e*n*x^(5/2)))/(180*d^6)
```

3.423 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 107

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{be^3n\sqrt{x}}{2d^3}-\frac{be^2nx}{4d^2}+\frac{benx^{3/2}}{6d}-\frac{be^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{be^4n\log(x)}{4d^4}$$

```
[Out] -1/4*b*e^2*n*x/d^2+1/6*b*e*n*x^(3/2)/d-1/4*b*e^4*n*ln(x)/d^4-1/2*b*e^4*n*ln
(d+e/x^(1/2))/d^4+1/2*x^2*(a+b*ln(c*(d+e/x^(1/2))^n))+1/2*b*e^3*n*x^(1/2)/d
^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2504, 2442, 46}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{be^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4}-\frac{be^4n\log(x)}{4d^4}+\frac{be^3n\sqrt{x}}{2d^3}-\frac{be^2nx}{4d^2}+\frac{benx^{3/2}}{6d}$$

```
[In] Int[x*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

```
[Out] (b*e^3*n*Sqrt[x])/(2*d^3) - (b*e^2*n*x)/(4*d^2) + (b*e*n*x^(3/2))/(6*d) - (
b*e^4*n*Log[d + e/Sqrt[x]])/(2*d^4) + (x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])
)/2 - (b*e^4*n*Log[x])/(4*d^4)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(2\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^5}\,dx, x, \frac{1}{\sqrt{x}}\right)\right) \\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{1}{2}(ben)\text{Subst}\left(\int \frac{1}{x^4(d+ex)}\,dx, x, \frac{1}{\sqrt{x}}\right) \\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right) \\
&\quad -\frac{1}{2}(ben)\text{Subst}\left(\int \left(\frac{1}{dx^4}-\frac{e}{d^2x^3}+\frac{e^2}{d^3x^2}-\frac{e^3}{d^4x}+\frac{e^4}{d^4(d+ex)}\right)\,dx, x, \frac{1}{\sqrt{x}}\right) \\
&= \frac{be^3n\sqrt{x}}{2d^3}-\frac{be^2nx}{4d^2}+\frac{benx^{3/2}}{6d}-\frac{be^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4} \\
&\quad +\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)-\frac{be^4n\log(x)}{4d^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{ax^2}{2}+\frac{1}{2}bx^2\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{4}ben\left(\frac{2e^2\sqrt{x}}{d^3}-\frac{ex}{d^2}+\frac{2x^{3/2}}{3d}-\frac{2e^3\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^4}-\frac{e^3\log(x)}{d^4}\right)$$

```
[In] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^n]),x]
```

```
[Out] (a*x^2)/2 + (b*x^2*Log[c*(d + e/Sqrt[x])^n])/2 + (b*e*n*((2*e^2*Sqrt[x])/d^
3 - (e*x)/d^2 + (2*x^(3/2))/(3*d) - (2*e^3*Log[d + e/Sqrt[x]])/d^4 - (e^3*L
og[x])/d^4))/4
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx$$

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(1/2))^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.18

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$$
$$=\frac{6\,bd^4x^2\log(c)-3\,bd^2e^2nx+6\,ad^4x^2-6\,bd^4n\log\left(\sqrt{x}\right)+6\,(bd^4-be^4)n\log\left(d\sqrt{x}+e\right)+6\,(bd^4nx^2-bd^4n}{12\,d^4}$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="fricas")
```

```
[Out] 1/12*(6*b*d^4*x^2*log(c) - 3*b*d^2*e^2*n*x + 6*a*d^4*x^2 - 6*b*d^4*n*log(sq
rt(x)) + 6*(b*d^4 - b*e^4)*n*log(d*sqrt(x) + e) + 6*(b*d^4*n*x^2 - b*d^4*n)
*log((d*x + e*sqrt(x))/x) + 2*(b*d^3*e*n*x + 3*b*d*e^3*n)*sqrt(x))/d^4
```

Sympy [A] (verification not implemented)

Time = 6.59 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.82

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$$
$$=\frac{ax^2}{2}+b\left(\frac{en\left(\frac{2x^{\frac{3}{2}}}{3d}-\frac{ex}{d^2}-\frac{2e^3\left(\begin{cases}\frac{\sqrt{x}}{e} & \text{for } d=0\\ \frac{\log\left(d\sqrt{x}+e\right)}{d} & \text{otherwise}\end{cases}\right)}{d^3}+\frac{2e^2\sqrt{x}}{d^3}\right)}{4}+\frac{x^2\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2}\right)$$

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/2))**n)),x)
```

```
[Out] a*x**2/2 + b*(e*n*(2*x**(3/2)/(3*d) - e*x/d**2 - 2*e**3*Piecewise((sqrt(x)/
e, Eq(d, 0)), (log(d*sqrt(x) + e)/d, True))/d**3 + 2*e**2*sqrt(x)/d**3)/4 +
 x**2*log(c*(d + e/sqrt(x))**n)/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.69

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$$
$$=-\frac{1}{12}\,ben\left(\frac{6\,e^3\log\left(d\sqrt{x}+e\right)}{d^4}-\frac{2\,d^2x^{\frac{3}{2}}-3\,dex+6\,e^2\sqrt{x}}{d^3}\right)+\frac{1}{2}\,bx^2\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{2}\,ax^2$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="maxima")
```

```
[Out] -1/12*b*e*n*(6*e^3*log(d*sqrt(x) + e)/d^4 - (2*d^2*x^(3/2) - 3*d*e*x + 6*e^
2*sqrt(x))/d^3) + 1/2*b*x^2*log(c*(d + e/sqrt(x))^n) + 1/2*a*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.76

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$$
$$=\frac{1}{2}\,bx^2\log\left(c\right)$$
$$+\frac{1}{12}\left(6\,x^2\log\left(d+\frac{e}{\sqrt{x}}\right)-e\left(\frac{6\,e^3\log\left(\left|d\sqrt{x}+e\right|\right)}{d^4}-\frac{2\,d^2x^{\frac{3}{2}}-3\,dex+6\,e^2\sqrt{x}}{d^3}\right)\right)bn$$
$$+\frac{1}{2}\,ax^2$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2*log(c) + 1/12*(6*x^2*log(d + e/sqrt(x)) - e*(6*e^3*log(abs(d*sqrt
(x) + e))/d^4 - (2*d^2*x^(3/2) - 3*d*e*x + 6*e^2*sqrt(x))/d^3))*b*n + 1/2*a
*x^2
```

Mupad [B] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{x^{3/2}\left(\frac{b\,e\,n}{3\,d}-\frac{b\,e^2\,n}{2\,d^2\,\sqrt{x}}+\frac{b\,e^3\,n}{d^3\,x}\right)}{2}+\frac{a\,x^2}{2}$$
$$+\frac{b\,x^2\,\ln\left(c\,{\left(d+\frac{e}{\sqrt{x}}\right)}^n\right)}{2}-\frac{b\,e^4\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,\sqrt{x}}+1\right)}{d^4}$$

```
[In] int(x*(a + b*log(c*(d + e/x^(1/2))^n)),x)
```

```
[Out] (x^(3/2)*((b*e*n)/(3*d) - (b*e^2*n)/(2*d^2*x^(1/2)) + (b*e^3*n)/(d^3*x)))/2
 + (a*x^2)/2 + (b*x^2*log(c*(d + e/x^(1/2))^n))/2 - (b*e^4*n*atanh((2*e)/(d
*x^(1/2)) + 1))/d^4
```

3.424 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 53

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=\frac{ben\sqrt{x}}{d}+ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{be^2n\log\left(e+d\sqrt{x}\right)}{d^2}$$

```
[Out] a*x+b*x*ln(c*(d+e/x^(1/2))^n)-b*e^2*n*ln(e+d*x^(1/2))/d^2+b*e*n*x^(1/2)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2498, 269, 196, 45}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{be^2n\log\left(d\sqrt{x}+e\right)}{d^2}+\frac{ben\sqrt{x}}{d}$$

```
[In] Int[a + b*Log[c*(d + e/Sqrt[x])^n],x]
```

```
[Out] (b*e*n*Sqrt[x])/d + a*x + b*x*Log[c*(d + e/Sqrt[x])^n] - (b*e^2*n*Log[e + d
*Sqrt[x]])/d^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= ax + b\int \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\,dx\\
&= ax + bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right) + \frac{1}{2}(ben)\int \frac{1}{\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}}\,dx\\
&= ax + bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right) + \frac{1}{2}(ben)\int \frac{1}{e+d\sqrt{x}}\,dx\\
&= ax + bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right) + (ben)\text{Subst}\left(\int \frac{x}{e+dx}\,dx, x, \sqrt{x}\right)\\
&= ax + bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right) + (ben)\text{Subst}\left(\int \left(\frac{1}{d}-\frac{e}{d(e+dx)}\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{ben\sqrt{x}}{d} + ax + bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right) - \frac{be^2n\log\left(e+d\sqrt{x}\right)}{d^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx=ax+bx\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+\frac{1}{2}ben\left(\frac{2\sqrt{x}}{d}-\frac{2e\log\left(d+\frac{e}{\sqrt{x}}\right)}{d^2}-\frac{e\log(x)}{d^2}\right)$$

```
[In] Integrate[a + b*Log[c*(d + e/Sqrt[x])^n],x]
```

```
[Out] a*x + b*x*Log[c*(d + e/Sqrt[x])^n] + (b*e*n*((2*Sqrt[x])/d - (2*e*Log[d + e
/Sqrt[x]])/d^2 - (e*Log[x])/d^2))/2
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.62

method	result	size
default	$ax+b\left(x\ln\left(c\left(\frac{e+d\sqrt{x}}{\sqrt{x}}\right)^n\right)+\frac{en\left(\frac{2\sqrt{x}}{d}-\frac{e\ln(e+d\sqrt{x})}{d^2}+\frac{e\ln(d\sqrt{x}-e)}{d^2}-\frac{e\ln\left(d^2x-e^2\right)}{d^2}\right)}{2}\right)$	86
parts	$ax+b\left(x\ln\left(c\left(\frac{e+d\sqrt{x}}{\sqrt{x}}\right)^n\right)+\frac{en\left(\frac{2\sqrt{x}}{d}-\frac{e\ln(e+d\sqrt{x})}{d^2}+\frac{e\ln(d\sqrt{x}-e)}{d^2}-\frac{e\ln\left(d^2x-e^2\right)}{d^2}\right)}{2}\right)$	86

```
[In] int(a+b*ln(c*(d+e/x^(1/2))^n),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*(x*ln(c*((e+d*x^(1/2))/x^(1/2))^n)+1/2*e*n*(2*x^(1/2)/d-1/d^2*e*ln(e+
d*x^(1/2))+1/d^2*e*ln(d*x^(1/2)-e)-e*ln(d^2*x-e^2)/d^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\,dx$$
$$=\frac{bd^2x\log\left(c\right)-bd^2n\log\left(\sqrt{x}\right)+bden\sqrt{x}+ad^2x+(bd^2-be^2)n\log\left(d\sqrt{x}+e\right)+(bd^2nx-bd^2n)\log\left(\frac{dx+e}{x}\right.}{d^2}$$

```
[In] integrate(a+b*log(c*(d+e/x^(1/2))^n),x, algorithm="fricas")
```

```
[Out] (b*d^2*x*log(c) - b*d^2*n*log(sqrt(x)) + b*d*e*n*sqrt(x) + a*d^2*x + (b*d^2
 - b*e^2)*n*log(d*sqrt(x) + e) + (b*d^2*n*x - b*d^2*n)*log((d*x + e*sqrt(x)
)/x))/d^2
```

Sympy [A] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)dx=ax+b\left(\frac{en\left(-\frac{2e\left(\begin{cases}\frac{\sqrt{x}}{e} & \text{for } d=0\\ \frac{\log\left(d\sqrt{x}+e\right)}{d} & \text{otherwise}\end{cases}\right)}{d}+\frac{2\sqrt{x}}{d}\right)}{2}+x\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)$$

```
[In] integrate(a+b*ln(c*(d+e/x**(1/2))**n),x)
```

```
[Out] a*x + b*(e*n*(-2*e*Piecewise((sqrt(x)/e, Eq(d, 0)), (log(d*sqrt(x) + e)/d,
True))/d + 2*sqrt(x)/d)/2 + x*log(c*(d + e/sqrt(x))**n))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)\,dx$$
$$=-\left(en\left(\frac{e\log\left(d\sqrt{x}+e\right)}{d^{2}}-\frac{\sqrt{x}}{d}\right)-x\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)b+ax$$

```
[In] integrate(a+b*log(c*(d+e/x^(1/2))^n),x, algorithm="maxima")
[Out] -(e*n*(e*log(d*sqrt(x) + e)/d^2 - sqrt(x)/d) - x*log(c*(d + e/sqrt(x))^n))*
b + a*x
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)\,dx$$
$$=-\left(\left(e\left(\frac{e\log\left(\left|d\sqrt{x}+e\right|\right)}{d^{2}}-\frac{\sqrt{x}}{d}\right)-x\log\left(d+\frac{e}{\sqrt{x}}\right)\right)n-x\log\left(c\right)\right)b+ax$$

```
[In] integrate(a+b*log(c*(d+e/x^(1/2))^n),x, algorithm="giac")
[Out] -((e*(e*log(abs(d*sqrt(x) + e))/d^2 - sqrt(x)/d) - x*log(d + e/sqrt(x)))*n
- x*log(c))*b + a*x
```

Mupad [B] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)\,dx=a\,x+b\,x\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)-\frac{b\,e\,n\,\left(e\,\ln\left(e+d\,\sqrt{x}\right)-d\,\sqrt{x}\right)}{d^{2}}$$

```
[In] int(a + b*log(c*(d + e/x^(1/2))^n),x)
[Out] a*x + b*x*log(c*(d + e/x^(1/2))^n) - (b*e*n*(e*log(e + d*x^(1/2)) - d*x^(1/
2)))/d^2
```

3.425 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 51

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx = -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt{x}}\right) - 2bn\,\mathrm{PolyLog}\left(2,1+\frac{e}{d\sqrt{x}}\right)$$

```
[Out] -2*(a+b*ln(c*(d+e/x^(1/2))^n))*ln(-e/d/x^(1/2))-2*b*n*polylog(2,1+e/d/x^(1/
2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2441, 2352}

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx = -2\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right) - 2bn\,\mathrm{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)$$

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])/x,x]
```

```
[Out] -2*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[x]))] - 2*b*n*PolyLog[2
, 1 + e/(d*Sqrt[x])]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(2\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx, x, \frac{1}{\sqrt{x}}\right)\right) \\
&= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt{x}}\right)+(2ben)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, \frac{1}{\sqrt{x}}\right) \\
&= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt{x}}\right) - 2bn\text{Li}_2\left(1+\frac{e}{d\sqrt{x}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$
\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx = -2b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\log\left(-\frac{e}{d\sqrt{x}}\right) + a\log(x) - 2bn\,\text{PolyLog}\left(2, \frac{d+\frac{e}{\sqrt{x}}}{d}\right)
$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])/x,x]
```

```
[Out] -2*b*Log[c*(d + e/Sqrt[x])^n]*Log[-(e/(d*Sqrt[x]))] + a*Log[x] - 2*b*n*Poly
Log[2, (d + e/Sqrt[x])/d]
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx$$

[In] int((a+b*ln(c*(d+e/x^(1/2))^n))/x,x)

[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))/x,x)

Fricas [F]

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx=\int \frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a}{x}\,dx$$

[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x,x, algorithm="fricas")

[Out] integral((b*log(c*((d*x + e*sqrt(x))/x)^n) + a)/x, x)

Sympy [F]

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx=\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx$$

[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))/x,x)

[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))/x, x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(44) = 88$.

Time = 0.48 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.43

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx=-2\left(\log\left(\frac{d\sqrt{x}}{e}+1\right)\log\left(\sqrt{x}\right)+\mathrm{Li}_2\left(-\frac{d\sqrt{x}}{e}\right)\right)bn$$
$$+\frac{ben\log(x)^2+4\,bdn\sqrt{x}\log(x)+4\,be\log\left(\left(d\sqrt{x}+e\right)^n\right)\log(x)-4\,be\log(x)\log\left(x^{\frac{1}{2}n}\right)-8\,bdn\sqrt{x}+4\,(b}{4\,e}$$

[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x,x, algorithm="maxima")

[Out] -2*(log(d*sqrt(x)/e + 1)*log(sqrt(x)) + dilog(-d*sqrt(x)/e))*b*n + 1/4*(b*e
*n*log(x)^2 + 4*b*d*n*sqrt(x)*log(x) + 4*b*e*log((d*sqrt(x) + e)^n)*log(x)
- 4*b*e*log(x)*log(x^(1/2*n)) - 8*b*d*n*sqrt(x) + 4*(b*e*log(c) + a*e)*log(
x) - 4*(b*d*n*x*log(x) - 2*b*d*n*x)/sqrt(x))/e

Giac [F]

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx = \int \frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx = \int \frac{a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))/x,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))^n))/x, x)
```

3.426 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 65

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}\,dx=\frac{bn}{2x}-\frac{bdn}{e\sqrt{x}}+\frac{bd^2n\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^2}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}$$

```
[Out] 1/2*b*n/x+b*d^2*n*ln(d+e/x^(1/2))/e^2+(-a-b*ln(c*(d+e/x^(1/2))^n))/x-b*d*n/
e/x^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 45}

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}\,dx=-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}+\frac{bd^2n\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^2}-\frac{bdn}{e\sqrt{x}}+\frac{bn}{2x}$$

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^2,x]
```

```
[Out] (b*n)/(2*x) - (b*d*n)/(e*Sqrt[x]) + (b*d^2*n*Log[d + e/Sqrt[x]])/e^2 - (a +
 b*Log[c*(d + e/Sqrt[x])^n])/x
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(2\text{Subst}\left(\int x(a+b\log(c(d+ex)^n))\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}+(ben)\text{Subst}\left(\int\frac{x^2}{d+ex}\,dx, x, \frac{1}{\sqrt{x}}\right)\\
&= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}+(ben)\text{Subst}\left(\int\left(-\frac{d}{e^2}+\frac{x}{e}+\frac{d^2}{e^2(d+ex)}\right)\,dx, x, \frac{1}{\sqrt{x}}\right)\\
&= \frac{bn}{2x}-\frac{bdn}{e\sqrt{x}}+\frac{bd^2n\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^2}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}\,dx=-\frac{a}{x}+\frac{bn}{2x}-\frac{bdn}{e\sqrt{x}}+\frac{bd^2n\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^2}-\frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^2,x]
```

```
[Out] -(a/x) + (b*n)/(2*x) - (b*d*n)/(e*Sqrt[x]) + (b*d^2*n*Log[d + e/Sqrt[x]])/e
^2 - (b*Log[c*(d + e/Sqrt[x])^n])/x
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$-\frac{a}{x}+\frac{bn}{2x}-\frac{b\ln\left(c\,{\mathrm e}^{n\ln\left(d+\frac{e}{\sqrt{x}}\right)}\right)}{x}+\frac{b\,d^{2} n\ln\left(d+\frac{e}{\sqrt{x}}\right)}{e^{2}}-\frac{bdn}{e\sqrt{x}}$	63
default	$-\frac{a}{x}+\frac{bn}{2x}-\frac{b\ln\left(c\,{\mathrm e}^{n\ln\left(d+\frac{e}{\sqrt{x}}\right)}\right)}{x}+\frac{b\,d^{2} n\ln\left(d+\frac{e}{\sqrt{x}}\right)}{e^{2}}-\frac{bdn}{e\sqrt{x}}$	63

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))/x^2,x,method=_RETURNVERBOSE)
[Out] -a/x+1/2*b*n/x-b/x*ln(c*exp(n*ln(d+e/x^(1/2))))+b*d^2*n*ln(d+e/x^(1/2))/e^2
-b*d*n/e/x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}\,dx = -\frac{2\,bden\sqrt{x}-be^2n+2\,be^2\log\left(c\right)+2\,ae^2-2\left(bd^2nx-be^2n\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right)}{2\,e^2x}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^2,x, algorithm="fricas")
[Out] -1/2*(2*b*d*e*n*sqrt(x) - b*e^2*n + 2*b*e^2*log(c) + 2*a*e^2 - 2*(b*d^2*n*x
 - b*e^2*n)*log((d*x + e*sqrt(x))/x))/(e^2*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(58) = 116$.

Time = 137.10 (sec) , antiderivative size = 391, normalized size of antiderivative = 6.02

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}\,dx = \begin{cases} -\frac{a+b\log\left(0^nc\right)}{x} \\ -\frac{a+b\log\left(cd^n\right)}{x} \\ -\frac{2ade^2x^3}{2de^2x^4+2e^3x^{\frac{7}{2}}}-\frac{2ae^3x^{\frac{5}{2}}}{2de^2x^4+2e^3x^{\frac{7}{2}}}+\frac{2bd^3x^4\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2de^2x^4+2e^3x^{\frac{7}{2}}}-\frac{2bd^2enx^{\frac{7}{2}}}{2de^2x^4+2e^3x^{\frac{7}{2}}}+\frac{2bd^2ex^{\frac{7}{2}}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2de^2x^4+2e^3x^{\frac{7}{2}}}-\frac{bde^2nx^3}{2de^2x^4+2e^3x^{\frac{7}{2}}}-\end{cases}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))/x**2,x)
```

```
[Out] Piecewise((-(a + b*log(0**n*c))/x, (Eq(d, 0) | Eq(d, -e/sqrt(x))) & (Eq(e,
0) | Eq(d, -e/sqrt(x)))), (-(a + b*log(c*d**n))/x, Eq(e, 0)), (-2*a*d*e**2*
x**3/(2*d*e**2*x**4 + 2*e**3*x**(7/2)) - 2*a*e**3*x**(5/2)/(2*d*e**2*x**4 +
 2*e**3*x**(7/2)) + 2*b*d**3*x**4*log(c*(d + e/sqrt(x))**n)/(2*d*e**2*x**4
+ 2*e**3*x**(7/2)) - 2*b*d**2*e*n*x**(7/2)/(2*d*e**2*x**4 + 2*e**3*x**(7/2)
) + 2*b*d**2*e*x**(7/2)*log(c*(d + e/sqrt(x))**n)/(2*d*e**2*x**4 + 2*e**3*x
**(7/2)) - b*d*e**2*n*x**3/(2*d*e**2*x**4 + 2*e**3*x**(7/2)) - 2*b*d*e**2*x
**3*log(c*(d + e/sqrt(x))**n)/(2*d*e**2*x**4 + 2*e**3*x**(7/2)) + b*e**3*n*
x**(5/2)/(2*d*e**2*x**4 + 2*e**3*x**(7/2)) - 2*b*e**3*x**(5/2)*log(c*(d + e
/sqrt(x))**n)/(2*d*e**2*x**4 + 2*e**3*x**(7/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}\,dx = \frac{1}{2}ben\left(\frac{2\,d^2\log\left(d\sqrt{x}+e\right)}{e^3} - \frac{d^2\log\left(x\right)}{e^3} - \frac{2\,d\sqrt{x}-e}{e^2x}\right) - \frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x} - \frac{a}{x}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*e*n*(2*d^2*log(d*sqrt(x) + e)/e^3 - d^2*log(x)/e^3 - (2*d*sqrt(x) - e
)/(e^2*x)) - b*log(c*(d + e/sqrt(x))^n)/x - a/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(57) = 114$.

Time = 0.31 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.78

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}\,dx = \frac{2\left(\frac{2\,(d\sqrt{x}+e)bdn}{e\sqrt{x}} - \frac{(d\sqrt{x}+e)^2bn}{ex}\right)\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right) + \frac{(bn-2\,b\log(c)-2\,a)(d\sqrt{x}+e)^2}{ex} - \frac{4\,(bdn-bd\log(c)-ad)(d\sqrt{x}+e)}{e\sqrt{x}}}{2\,e}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(2*(d*sqrt(x) + e)*b*d*n/(e*sqrt(x)) - (d*sqrt(x) + e)^2*b*n/(e*x))*
log((d*sqrt(x) + e)/sqrt(x)) + (b*n - 2*b*log(c) - 2*a)*(d*sqrt(x) + e)^2/(
e*x) - 4*(b*d*n - b*d*log(c) - a*d)*(d*sqrt(x) + e)/(e*sqrt(x)))/e
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^2}\,dx = \frac{b\,n}{2\,x}-\frac{a}{x}-\frac{b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}-\frac{b\,d\,n}{e\,\sqrt{x}}+\frac{b\,d^2\,n\,\ln\left(d+\frac{e}{\sqrt{x}}\right)}{e^2}$$

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))/x^2,x)
[Out] (b*n)/(2*x) - a/x - (b*log(c*(d + e/x^(1/2))^n))/x - (b*d*n)/(e*x^(1/2)) +
(b*d^2*n*log(d + e/x^(1/2)))/e^2
```

3.427 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx = \frac{bn}{8x^2}-\frac{bdn}{6ex^{3/2}}+\frac{bd^2n}{4e^2x}-\frac{bd^3n}{2e^3\sqrt{x}}+\frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2e^4}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2}$$

```
[Out] 1/8*b*n/x^2-1/6*b*d*n/e/x^(3/2)+1/4*b*d^2*n/e^2/x+1/2*b*d^4*n*ln(d+e/x^(1/2
))/e^4+1/2*(-a-b*ln(c*(d+e/x^(1/2))^n))/x^2-1/2*b*d^3*n/e^3/x^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 45}

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx = -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2}+\frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2e^4}-\frac{bd^3n}{2e^3\sqrt{x}}+\frac{bd^2n}{4e^2x}-\frac{bdn}{6ex^{3/2}}+\frac{bn}{8x^2}$$

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^3,x]
```

```
[Out] (b*n)/(8*x^2) - (b*d*n)/(6*e*x^(3/2)) + (b*d^2*n)/(4*e^2*x) - (b*d^3*n)/(2*
e^3*Sqrt[x]) + (b*d^4*n*Log[d + e/Sqrt[x]])/(2*e^4) - (a + b*Log[c*(d + e/S
qrt[x])^n])/(2*x^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(2\text{Subst}\left(\int x^3\left(a+b\log\left(c(d+ex)^n\right)\right)\,dx,x,\frac{1}{\sqrt{x}}\right)\right)\\
&= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\text{Subst}\left(\int\frac{x^4}{d+ex}\,dx,x,\frac{1}{\sqrt{x}}\right)\\
&= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2}\\
&\quad+\frac{1}{2}(ben)\text{Subst}\left(\int\left(-\frac{d^3}{e^4}+\frac{d^2x}{e^3}-\frac{dx^2}{e^2}+\frac{x^3}{e}+\frac{d^4}{e^4(d+ex)}\right)\,dx,x,\frac{1}{\sqrt{x}}\right)\\
&= \frac{bn}{8x^2}-\frac{bdn}{6ex^{3/2}}+\frac{bd^2n}{4e^2x}-\frac{bd^3n}{2e^3\sqrt{x}}+\frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)}{2e^4}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx = -\frac{a}{2x^2}-\frac{1}{4}ben\left(-\frac{1}{2ex^2}+\frac{2d}{3e^2x^{3/2}}-\frac{d^2}{e^3x}+\frac{2d^3}{e^4\sqrt{x}}-\frac{2d^4\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^5}\right)-\frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2x^2}$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*e*n*(-1/2*1/(e*x^2) + (2*d)/(3*e^2*x^(3/2)) - d^2/(e^3*x) +
 (2*d^3)/(e^4*Sqrt[x]) - (2*d^4*Log[d + e/Sqrt[x]])/e^5))/4 - (b*Log[c*(d +
 e/Sqrt[x])^n])/(2*x^2)
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.92

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx = \frac{6\,bd^2e^2nx+3\,be^4n-12\,be^4\log\left(c\right)-12\,ae^4+12\,(bd^4nx^2-be^4n)\log\left(\frac{dx+e\sqrt{x}}{x}\right)-4\,(3\,bd^3enx+bde^3n)\sqrt{x}}{24\,e^4x^2}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(6*b*d^2*e^2*n*x + 3*b*e^4*n - 12*b*e^4*log(c) - 12*a*e^4 + 12*(b*d^4*
n*x^2 - b*e^4*n)*log((d*x + e*sqrt(x))/x) - 4*(3*b*d^3*e*n*x + b*d*e^3*n)*s
qrt(x))/(e^4*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))/x**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.91

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx = \frac{1}{24}\,ben\left(\frac{12\,d^4\log\left(d\sqrt{x}+e\right)}{e^5} - \frac{6\,d^4\log\left(x\right)}{e^5} - \frac{12\,d^3x^{\frac{3}{2}} - 6\,d^2ex + 4\,de^2\sqrt{x} - 3\,e^3}{e^4x^2}\right) - \frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2\,x^2} - \frac{a}{2\,x^2}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^3,x, algorithm="maxima")
[Out] 1/24*b*e*n*(12*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*x^(3
/2) - 6*d^2*e*x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2)) - 1/2*b*log(c*(d + e/
sqrt(x))^n)/x^2 - 1/2*a/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(84) = 168$.

Time = 0.32 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.28

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx = \frac{12\left(\frac{4\,(d\sqrt{x}+e)bd^3n}{e^3\sqrt{x}} - \frac{6\,(d\sqrt{x}+e)^2bd^2n}{e^3x} + \frac{4\,(d\sqrt{x}+e)^3bdn}{e^3x^{\frac{3}{2}}} - \frac{(d\sqrt{x}+e)^4bn}{e^3x^2}\right)\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right) + \frac{3\,(bn-4\,b\log(c)-4\,a)(d\sqrt{x}+e)^4}{e^3x^2} - 16\,(}{24\,e}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/24*(12*(4*(d*sqrt(x) + e)*b*d^3*n/(e^3*sqrt(x)) - 6*(d*sqrt(x) + e)^2*b*d
^2*n/(e^3*x) + 4*(d*sqrt(x) + e)^3*b*d*n/(e^3*x^(3/2)) - (d*sqrt(x) + e)^4*
b*n/(e^3*x^2))*log((d*sqrt(x) + e)/sqrt(x)) + 3*(b*n - 4*b*log(c) - 4*a)*(d
*sqrt(x) + e)^4/(e^3*x^2) - 16*(b*d*n - 3*b*d*log(c) - 3*a*d)*(d*sqrt(x) +
e)^3/(e^3*x^(3/2)) + 36*(b*d^2*n - 2*b*d^2*log(c) - 2*a*d^2)*(d*sqrt(x) + e
)^2/(e^3*x) - 48*(b*d^3*n - b*d^3*log(c) - a*d^3)*(d*sqrt(x) + e)/(e^3*sqrt
(x)))/e
```

Mupad [B] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^3}\,dx=\frac{b\,n}{8\,x^2}-\frac{a}{2\,x^2}-\frac{b\,\ln\left(c\,{\left(d+\frac{e}{\sqrt{x}}\right)}^n\right)}{2\,x^2}-\frac{b\,d\,n}{6\,e\,x^{3/2}}+\frac{b\,d^4\,n\,\ln\left(d+\frac{e}{\sqrt{x}}\right)}{2\,e^4}+\frac{b\,d^2\,n}{4\,e^2\,x}-\frac{b\,d^3\,n}{2\,e^3\,\sqrt{x}}$$

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))/x^3,x)
```

```
[Out] (b*n)/(8*x^2) - a/(2*x^2) - (b*log(c*(d + e/x^(1/2))^n))/(2*x^2) - (b*d*n)/
(6*e*x^(3/2)) + (b*d^4*n*log(d + e/x^(1/2)))/(2*e^4) + (b*d^2*n)/(4*e^2*x)
- (b*d^3*n)/(2*e^3*x^(1/2))
```

3.428 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 136

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx=\frac{bn}{18x^3}-\frac{bdn}{15ex^{5/2}}+\frac{bd^2n}{12e^2x^2}-\frac{bd^3n}{9e^3x^{3/2}}+\frac{bd^4n}{6e^4x}-\frac{bd^5n}{3e^5\sqrt{x}}+\frac{bd^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3e^6}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3x^3}$$

```
[Out] 1/18*b*n/x^3-1/15*b*d*n/e/x^(5/2)+1/12*b*d^2*n/e^2/x^2-1/9*b*d^3*n/e^3/x^(3
/2)+1/6*b*d^4*n/e^4/x+1/3*b*d^6*n*ln(d+e/x^(1/2))/e^6+1/3*(-a-b*ln(c*(d+e/x
^(1/2))^n))/x^3-1/3*b*d^5*n/e^5/x^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 45}

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx=-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3x^3}+\frac{bd^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3e^6}-\frac{bd^5n}{3e^5\sqrt{x}}+\frac{bd^4n}{6e^4x}-\frac{bd^3n}{9e^3x^{3/2}}+\frac{bd^2n}{12e^2x^2}-\frac{bdn}{15ex^{5/2}}+\frac{bn}{18x^3}$$

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^4,x]
```

```
[Out] (b*n)/(18*x^3) - (b*d*n)/(15*e*x^(5/2)) + (b*d^2*n)/(12*e^2*x^2) - (b*d^3*n
)/(9*e^3*x^(3/2)) + (b*d^4*n)/(6*e^4*x) - (b*d^5*n)/(3*e^5*Sqrt[x]) + (b*d^
6*n*Log[d + e/Sqrt[x]])/(3*e^6) - (a + b*Log[c*(d + e/Sqrt[x])^n])/(3*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(2\text{Subst}\left(\int x^5(a+b\log\left(c(d+ex)^n\right))\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3x^3}+\frac{1}{3}(ben)\text{Subst}\left(\int\frac{x^6}{d+ex}\,dx, x, \frac{1}{\sqrt{x}}\right)\\
&= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3x^3}+\frac{1}{3}(ben)\text{Subst}\left(\int\left(-\frac{d^5}{e^6}+\frac{d^4x}{e^5}-\frac{d^3x^2}{e^4}+\frac{d^2x^3}{e^3}-\frac{dx^4}{e^2}\right.\right.\\
&\qquad\left.\left.+\frac{x^5}{e}+\frac{d^6}{e^6(d+ex)}\right)dx, x, \frac{1}{\sqrt{x}}\right)\\
&= \frac{bn}{18x^3}-\frac{bdn}{15ex^{5/2}}+\frac{bd^2n}{12e^2x^2}-\frac{bd^3n}{9e^3x^{3/2}}+\frac{bd^4n}{6e^4x}-\frac{bd^5n}{3e^5\sqrt{x}}\\
&\quad+\frac{bd^6n\log\left(d+\frac{e}{\sqrt{x}}\right)}{3e^6}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3x^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.97

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx=-\frac{a}{3x^3}-\frac{1}{6}ben\left(-\frac{1}{3ex^3}+\frac{2d}{5e^2x^{5/2}}-\frac{d^2}{2e^3x^2}+\frac{2d^3}{3e^4x^{3/2}}-\frac{d^4}{e^5x}+\frac{2d^5}{e^6\sqrt{x}}-\frac{2d^6\log\left(d+\frac{e}{\sqrt{x}}\right)}{e^7}\right)-\frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3x^3}$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (b*e*n*(-1/3*1/(e*x^3) + (2*d)/(5*e^2*x^(5/2)) - d^2/(2*e^3*x^
2) + (2*d^3)/(3*e^4*x^(3/2)) - d^4/(e^5*x) + (2*d^5)/(e^6*Sqrt[x]) - (2*d^6
*Log[d + e/Sqrt[x]])/e^7))/6 - (b*Log[c*(d + e/Sqrt[x])^n])/(3*x^3)
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx=\frac{30\,bd^4e^2nx^2+15\,bd^2e^4nx+10\,be^6n-60\,be^6\log(c)-60\,ae^6+60\,(bd^6nx^3-be^6n)\log\left(\frac{dx+e\sqrt{x}}{x}\right)-4\,(15\,bd^5}{180\,e^6x^3}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^4,x, algorithm="fricas")
```

```
[Out] 1/180*(30*b*d^4*e^2*n*x^2 + 15*b*d^2*e^4*n*x + 10*b*e^6*n - 60*b*e^6*log(c)
 - 60*a*e^6 + 60*(b*d^6*n*x^3 - b*e^6*n)*log((d*x + e*sqrt(x))/x) - 4*(15*b
*d^5*e*n*x^2 + 5*b*d^3*e^3*n*x + 3*b*d*e^5*n)*sqrt(x))/(e^6*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))/x**4,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.86

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx$$
$$= \frac{1}{180}ben\left(\frac{60\,d^6\log\left(d\sqrt{x}+e\right)}{e^7} - \frac{30\,d^6\log\left(x\right)}{e^7} - \frac{60\,d^5x^{\frac{5}{2}} - 30\,d^4ex^2 + 20\,d^3e^2x^{\frac{3}{2}} - 15\,d^2e^3x + 12\,de^4\sqrt{x} -}{e^6x^3}\right.$$
$$- \frac{b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3\,x^3} - \frac{a}{3\,x^3}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^4,x, algorithm="maxima")
```

```
[Out] 1/180*b*e*n*(60*d^6*log(d*sqrt(x) + e)/e^7 - 30*d^6*log(x)/e^7 - (60*d^5*x^
(5/2) - 30*d^4*e*x^2 + 20*d^3*e^2*x^(3/2) - 15*d^2*e^3*x + 12*d*e^4*sqrt(x)
 - 10*e^5)/(e^6*x^3)) - 1/3*b*log(c*(d + e/sqrt(x))^n)/x^3 - 1/3*a/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(110) = 220.

Time = 0.32 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.62

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx$$
$$= \frac{60\left(\frac{6\,(d\sqrt{x}+e)bd^5n}{e^5\sqrt{x}} - \frac{15\,(d\sqrt{x}+e)^2bd^4n}{e^5x} + \frac{20\,(d\sqrt{x}+e)^3bd^3n}{e^5x^{\frac{3}{2}}} - \frac{15\,(d\sqrt{x}+e)^4bd^2n}{e^5x^2} + \frac{6\,(d\sqrt{x}+e)^5bdn}{e^5x^{\frac{5}{2}}} - \frac{(d\sqrt{x}+e)^6bn}{e^5x^3}\right)\log\left(\frac{d\sqrt{x}-}{\sqrt{x}}\right.}{}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))/x^4,x, algorithm="giac")
```

```
[Out] 1/180*(60*(6*(d*sqrt(x) + e)*b*d^5*n/(e^5*sqrt(x)) - 15*(d*sqrt(x) + e)^2*b
*d^4*n/(e^5*x) + 20*(d*sqrt(x) + e)^3*b*d^3*n/(e^5*x^(3/2)) - 15*(d*sqrt(x)
 + e)^4*b*d^2*n/(e^5*x^2) + 6*(d*sqrt(x) + e)^5*b*d*n/(e^5*x^(5/2)) - (d*sq
rt(x) + e)^6*b*n/(e^5*x^3))*log((d*sqrt(x) + e)/sqrt(x)) + 10*(b*n - 6*b*lo
g(c) - 6*a)*(d*sqrt(x) + e)^6/(e^5*x^3) - 72*(b*d*n - 5*b*d*log(c) - 5*a*d)
*(d*sqrt(x) + e)^5/(e^5*x^(5/2)) + 225*(b*d^2*n - 4*b*d^2*log(c) - 4*a*d^2)
*(d*sqrt(x) + e)^4/(e^5*x^2) - 400*(b*d^3*n - 3*b*d^3*log(c) - 3*a*d^3)*(d*
sqrt(x) + e)^3/(e^5*x^(3/2)) + 450*(b*d^4*n - 2*b*d^4*log(c) - 2*a*d^4)*(d*
sqrt(x) + e)^2/(e^5*x) - 360*(b*d^5*n - b*d^5*log(c) - a*d^5)*(d*sqrt(x) +
e)/(e^5*sqrt(x)))/e
```

Mupad [B] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.83

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x^4}\,dx = \frac{b\,n}{18\,x^3}-\frac{a}{3\,x^3}-\frac{b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3\,x^3}-\frac{b\,d\,n}{15\,e\,x^{5/2}}+\frac{b\,d^6\,n\,\ln\left(d+\frac{e}{\sqrt{x}}\right)}{3\,e^6}+\frac{b\,d^2\,n}{12\,e^2\,x^2}+\frac{b\,d^4\,n}{6\,e^4\,x}-\frac{b\,d^3\,n}{9\,e^3\,x^{3/2}}-\frac{b\,d^5\,n}{3\,e^5\,\sqrt{x}}$$

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))/x^4,x)
```

```
[Out] (b*n)/(18*x^3) - a/(3*x^3) - (b*log(c*(d + e/x^(1/2))^n))/(3*x^3) - (b*d*n)
/(15*e*x^(5/2)) + (b*d^6*n*log(d + e/x^(1/2)))/(3*e^6) + (b*d^2*n)/(12*e^2*
x^2) + (b*d^4*n)/(6*e^4*x) - (b*d^3*n)/(9*e^3*x^(3/2)) - (b*d^5*n)/(3*e^5*x
^(1/2))
```

3.429 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 24, antiderivative size = 404

$$
\begin{aligned}
\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx
&= -\frac{77b^2e^5n^2\sqrt{x}}{90d^5}+\frac{47b^2e^4n^2x}{180d^4}-\frac{b^2e^3n^2x^{3/2}}{10d^3}+\frac{b^2e^2n^2x^2}{30d^2} \\
&\quad+\frac{77b^2e^6n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{90d^6}+\frac{2be^5n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d^6} \\
&\quad-\frac{be^4nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d^4}+\frac{2be^3nx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9d^3} \\
&\quad-\frac{be^2nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{6d^2}+\frac{2benx^{5/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{15d} \\
&\quad+\frac{2be^6n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d^6} \\
&\quad+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2+\frac{137b^2e^6n^2\log(x)}{180d^6}-\frac{2b^2e^6n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{3d^6}
\end{aligned}
$$

```
[Out] 47/180*b^2*e^4*n^2*x/d^4-1/10*b^2*e^3*n^2*x^(3/2)/d^3+1/30*b^2*e^2*n^2*x^2/
d^2+137/180*b^2*e^6*n^2*ln(x)/d^6+77/90*b^2*e^6*n^2*ln(d+e/x^(1/2))/d^6-1/3
*b*e^4*n*x*(a+b*ln(c*(d+e/x^(1/2))^n))/d^4+2/9*b*e^3*n*x^(3/2)*(a+b*ln(c*(d
+e/x^(1/2))^n))/d^3-1/6*b*e^2*n*x^2*(a+b*ln(c*(d+e/x^(1/2))^n))/d^2+2/15*b*
e*n*x^(5/2)*(a+b*ln(c*(d+e/x^(1/2))^n))/d+2/3*b*e^6*n*ln(1-d/(d+e/x^(1/2)))
*(a+b*ln(c*(d+e/x^(1/2))^n))/d^6+1/3*x^3*(a+b*ln(c*(d+e/x^(1/2))^n))^2-2/3*
b^2*e^6*n^2*polylog(2,d/(d+e/x^(1/2)))/d^6-77/90*b^2*e^5*n^2*x^(1/2)/d^5+2/
3*b*e^5*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))*x^(1/2)/d^6
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\begin{aligned}\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx=&\frac{2be^6n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d^6}\\&+\frac{2be^5n\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d^6}\\&-\frac{be^4nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d^4}\\&+\frac{2be^3nx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9d^3}\\&-\frac{be^2nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{6d^2}\\&+\frac{2benx^{5/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{15d}\\&+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\\&-\frac{2b^2e^6n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{3d^6}\\&+\frac{77b^2e^6n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{90d^6}\\&+\frac{137b^2e^6n^2\log(x)}{180d^6}-\frac{77b^2e^5n^2\sqrt{x}}{90d^5}\\&+\frac{47b^2e^4n^2x}{180d^4}-\frac{b^2e^3n^2x^{3/2}}{10d^3}+\frac{b^2e^2n^2x^2}{30d^2}\end{aligned}$$

```
[In] Int[x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

```
[Out] (-77*b^2*e^5*n^2*Sqrt[x])/(90*d^5) + (47*b^2*e^4*n^2*x)/(180*d^4) - (b^2*e^
3*n^2*x^(3/2))/(10*d^3) + (b^2*e^2*n^2*x^2)/(30*d^2) + (77*b^2*e^6*n^2*Log[
d + e/Sqrt[x]])/(90*d^6) + (2*b*e^5*n*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*
(d + e/Sqrt[x])^n]))/(3*d^6) - (b*e^4*n*x*(a + b*Log[c*(d + e/Sqrt[x])^n]))
/(3*d^4) + (2*b*e^3*n*x^(3/2)*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(9*d^3) - (
b*e^2*n*x^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(6*d^2) + (2*b*e*n*x^(5/2)*(a
 + b*Log[c*(d + e/Sqrt[x])^n]))/(15*d) + (2*b*e^6*n*Log[1 - d/(d + e/Sqrt[x
])]*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(3*d^6) + (x^3*(a + b*Log[c*(d + e/Sq
```

```
rt[x])^n])^2)/3 + (137*b^2*e^6*n^2*Log[x])/(180*d^6) - (2*b^2*e^6*n^2*PolyL
og[2, d/(d + e/Sqrt[x])])/(3*d^6)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(2\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^7}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 - \frac{1}{3}(2ben)\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^6(d+ex)}\,dx, x, \frac{1}{\sqrt{x}}\right)\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 - \frac{1}{3}(2bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+\frac{e}{\sqrt{x}}\right)\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 - \frac{(2bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{3d}\\
&\quad + \frac{(2ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{3d}
\end{aligned}
$$

$$=\frac{2benx^{5/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{15d}$$

$$+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2+\frac{(2ben)\mathrm{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{3d^2}$$

$$-\frac{(2be^2n)\,\mathrm{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{3d^2}$$

$$-\frac{(2b^2en^2)\,\mathrm{Subst}\left(\int\frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{15d}$$

$$=-\frac{be^2nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{6d^2}+\frac{2benx^{5/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{15d}$$

$$+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-\frac{(2be^2n)\,\mathrm{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{3d^3}$$

$$+\frac{(2be^3n)\,\mathrm{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{3d^3}$$

$$-\frac{(2b^2en^2)\,\mathrm{Subst}\left(\int\left(-\frac{e^5}{d(d-x)^5}-\frac{e^5}{d^2(d-x)^4}-\frac{e^5}{d^3(d-x)^3}-\frac{e^5}{d^4(d-x)^2}-\frac{e^5}{d^5(d-x)}-\frac{e^5}{d^5x}\right)\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{15d}$$

$$+\frac{(b^2e^2n^2)\,\mathrm{Subst}\left(\int\frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{6d^2}$$

$$=-\frac{2b^2e^5n^2\sqrt{x}}{15d^5}+\frac{b^2e^4n^2x}{15d^4}-\frac{2b^2e^3n^2x^{3/2}}{45d^3}+\frac{b^2e^2n^2x^2}{30d^2}$$

$$+\frac{2b^2e^6n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{15d^6}+\frac{2be^3nx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9d^3}$$

$$-\frac{be^2nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{6d^2}+\frac{2benx^{5/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{15d}$$

$$+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2+\frac{b^2e^6n^2\log(x)}{15d^6}+\frac{(2be^3n)\,\mathrm{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{3d^4}-$$

$$= -\frac{3b^2e^5n^2\sqrt{x}}{10d^5} + \frac{3b^2e^4n^2x}{20d^4} - \frac{b^2e^3n^2x^{3/2}}{10d^3} + \frac{b^2e^2n^2x^2}{30d^2} + \frac{3b^2e^6n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{10d^6} - \frac{be^4nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d^4} + \frac{2be^3nx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9d^3} - \frac{be^2nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{6d^2} + \frac{2benx^{5/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{15d} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 + \frac{3b^2e^6n^2\log(x)}{20d^6} - \frac{(2be^4n)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{3d^5} +$$

$$= -\frac{47b^2e^5n^2\sqrt{x}}{90d^5} + \frac{47b^2e^4n^2x}{180d^4} - \frac{b^2e^3n^2x^{3/2}}{10d^3} + \frac{b^2e^2n^2x^2}{30d^2} + \frac{47b^2e^6n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{90d^6} + \frac{2be^5n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d^6} - \frac{be^4nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d^4} + \frac{2be^3nx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9d^3} - \frac{be^2nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{6d^2} + \frac{2benx^{5/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{15d} + \frac{2be^6n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d^6} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 + \frac{47b^2e^6n^2\log(x)}{180d^6} + \frac{(b^2e^4n^2)\,\text{Subst}\left(\int\left(\frac{e^2}{d(d-x)^2}+\frac{e^2}{d^2(d-x)}+\frac{e^2}{d^2x}\right)\right.}{3d^4}$$

$$= -\frac{77b^2e^5n^2\sqrt{x}}{90d^5} + \frac{47b^2e^4n^2x}{180d^4} - \frac{b^2e^3n^2x^{3/2}}{10d^3} + \frac{b^2e^2n^2x^2}{30d^2} + \frac{77b^2e^6n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{90d^6} + \frac{2be^5n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d^6} - \frac{be^4nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d^4} + \frac{2be^3nx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9d^3} - \frac{be^2nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{6d^2} + \frac{2benx^{5/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{15d} + \frac{2be^6n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d^6} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 + \frac{137b^2e^6n^2\log(x)}{180d^6} - \frac{2b^2e^6n^2\text{Li}_2\left(\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{3d^6}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.07

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx=\frac{1}{3}\left(x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\right.$$
$$+\frac{ben\left(120ade^4\sqrt{x}-154bde^4n\sqrt{x}-60ad^2e^3x+47bd^2e^3nx+40ad^3e^2x^{3/2}-18bd^3e^2nx^{3/2}-30ad^4ex^2+\right.}{}$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

```
[Out] (x^3*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + (b*e*n*(120*a*d*e^4*Sqrt[x] - 154
*b*d*e^4*n*Sqrt[x] - 60*a*d^2*e^3*x + 47*b*d^2*e^3*n*x + 40*a*d^3*e^2*x^(3/
2) - 18*b*d^3*e^2*n*x^(3/2) - 30*a*d^4*e*x^2 + 6*b*d^4*e*n*x^2 + 24*a*d^5*x
^(5/2) + 214*b*e^5*n*Log[d + e/Sqrt[x]] + 120*b*d*e^4*Sqrt[x]*Log[c*(d + e/
Sqrt[x])^n] - 60*b*d^2*e^3*x*Log[c*(d + e/Sqrt[x])^n] + 40*b*d^3*e^2*x^(3/2
)*Log[c*(d + e/Sqrt[x])^n] - 30*b*d^4*e*x^2*Log[c*(d + e/Sqrt[x])^n] + 24*b
*d^5*x^(5/2)*Log[c*(d + e/Sqrt[x])^n] - 120*a*e^5*Log[e + d*Sqrt[x]] + 60*b
*e^5*n*Log[e + d*Sqrt[x]] - 120*b*e^5*Log[c*(d + e/Sqrt[x])^n]*Log[e + d*Sq
rt[x]] + 60*b*e^5*n*Log[e + d*Sqrt[x]]^2 - 120*b*e^5*n*Log[e + d*Sqrt[x]]*L
og[-((d*Sqrt[x])/e)] + 107*b*e^5*n*Log[x] - 120*b*e^5*n*PolyLog[2, 1 + (d*S
qrt[x])/e]))/(60*d^6))/3
```

Maple [F]

$$\int x^2\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx$$

```
[In] int(x^2*(a+b*ln(c*(d+e/x^(1/2))^n))^2,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e/x^(1/2))^n))^2,x)
```

Fricas [F]

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*log(c*((d*x + e*sqrt(x))/x)^n)^2 + 2*a*b*x^2*log(c*((d*x +
 e*sqrt(x))/x)^n) + a^2*x^2, x)
```

Sympy [F]

$$\int x^2 \left(a + b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx = \int x^2 \left(a + b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx$$

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(1/2))**n))**2,x)
[Out] Integral(x**2*(a + b*log(c*(d + e/sqrt(x))**n))**2, x)
```

Maxima [F]

$$\int x^2 \left(a + b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx = \int \left(b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right) + a\right)^2 x^2 dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="maxima")
[Out] 1/3*b^2*x^3*log((d*sqrt(x) + e)^n)^2 - integrate(-1/3*(3*(b^2*d*log(c)^2 +
2*a*b*d*log(c) + a^2*d)*x^3 + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x
^(5/2) + 3*(b^2*d*x^3 + b^2*e*x^(5/2))*log(x^(1/2*n))^2 - (b^2*d*n*x^3 - 6*
(b^2*d*log(c) + a*b*d)*x^3 - 6*(b^2*e*log(c) + a*b*e)*x^(5/2) + 6*(b^2*d*x^
3 + b^2*e*x^(5/2))*log(x^(1/2*n)))*log((d*sqrt(x) + e)^n) - 6*((b^2*d*log(c
) + a*b*d)*x^3 + (b^2*e*log(c) + a*b*e)*x^(5/2))*log(x^(1/2*n)))/(d*x + e*s
qrt(x)), x)
```

Giac [F]

$$\int x^2 \left(a + b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx = \int \left(b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^n\right) + a\right)^2 x^2 dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx=\int x^2\,\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{\sqrt{x}}\right)}^n\right)\right)^2\,dx$$

```
[In] int(x^2*(a + b*log(c*(d + e/x^(1/2))^n))^2,x)
[Out] int(x^2*(a + b*log(c*(d + e/x^(1/2))^n))^2, x)
```

3.430 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 22, antiderivative size = 288

$$\begin{aligned}\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx &= -\frac{5b^2e^3n^2\sqrt{x}}{6d^3}+\frac{b^2e^2n^2x}{6d^2}+\frac{5b^2e^4n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{6d^4}\\ &\quad+\frac{be^3n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4}\\ &\quad-\frac{be^2nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^2}\\ &\quad+\frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d}\\ &\quad+\frac{be^4n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4}\\ &\quad+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\\ &\quad+\frac{11b^2e^4n^2\log(x)}{12d^4}-\frac{b^2e^4n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^4}\end{aligned}$$

```
[Out] 1/6*b^2*e^2*n^2*x/d^2+11/12*b^2*e^4*n^2*ln(x)/d^4+5/6*b^2*e^4*n^2*ln(d+e/x^
(1/2))/d^4-1/2*b*e^2*n*x*(a+b*ln(c*(d+e/x^(1/2))^n))/d^2+1/3*b*e*n*x^(3/2)*
(a+b*ln(c*(d+e/x^(1/2))^n))/d+b*e^4*n*ln(1-d/(d+e/x^(1/2)))*(a+b*ln(c*(d+e/
x^(1/2))^n))/d^4+1/2*x^2*(a+b*ln(c*(d+e/x^(1/2))^n))^2-b^2*e^4*n^2*polylog(
2,d/(d+e/x^(1/2)))/d^4-5/6*b^2*e^3*n^2*x^(1/2)/d^3+b*e^3*n*(a+b*ln(c*(d+e/x
^(1/2))^n))*(d+e/x^(1/2))*x^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx=\frac{be^4n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4}+\frac{be^3n\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4}-\frac{be^2nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^2}+\frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-\frac{b^2e^4n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^4}+\frac{5b^2e^4n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{6d^4}+\frac{11b^2e^4n^2\log(x)}{12d^4}-\frac{5b^2e^3n^2\sqrt{x}}{6d^3}+\frac{b^2e^2n^2x}{6d^2}$$

```
[In] Int[x*(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

```
[Out] (-5*b^2*e^3*n^2*Sqrt[x])/(6*d^3) + (b^2*e^2*n^2*x)/(6*d^2) + (5*b^2*e^4*n^2
*Log[d + e/Sqrt[x]])/(6*d^4) + (b*e^3*n*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[
c*(d + e/Sqrt[x])^n]))/d^4 - (b*e^2*n*x*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(
2*d^2) + (b*e*n*x^(3/2)*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(3*d) + (b*e^4*n*
Log[1 - d/(d + e/Sqrt[x])]*(a + b*Log[c*(d + e/Sqrt[x])^n]))/d^4 + (x^2*(a
+ b*Log[c*(d + e/Sqrt[x])^n])^2)/2 + (11*b^2*e^4*n^2*Log[x])/(12*d^4) - (b^
2*e^4*n^2*PolyLog[2, d/(d + e/Sqrt[x])])/d^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(2\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^5}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-(ben)\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^4(d+ex)}\,dx, x, \frac{1}{\sqrt{x}}\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-(bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+\frac{e}{\sqrt{x}}\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-\frac{(bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{d}\\
&\quad+\frac{(ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{d}
\end{aligned}$$

$$=\frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d}$$

$$+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2+\frac{(ben)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{d^2}$$

$$-\frac{(be^2n)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{d^2}$$

$$-\frac{(b^2en^2)\,\text{Subst}\left(\int\frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{3d}$$

$$=-\frac{be^2nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^2}+\frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d}$$

$$+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-\frac{(be^2n)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{d^3}$$

$$+\frac{(be^3n)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{d^3}$$

$$-\frac{(b^2en^2)\,\text{Subst}\left(\int\left(-\frac{e^3}{d(d-x)^3}-\frac{e^3}{d^2(d-x)^2}-\frac{e^3}{d^3(d-x)}-\frac{e^3}{d^3x}\right)\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{3d}$$

$$+\frac{(b^2e^2n^2)\,\text{Subst}\left(\int\frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{2d^2}$$

$$
\begin{aligned}
&= -\frac{b^2e^3n^2\sqrt{x}}{3d^3} + \frac{b^2e^2n^2x}{6d^2} + \frac{b^2e^4n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{3d^4} \\
&\quad + \frac{be^3n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4} \\
&\quad - \frac{be^2nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^2} + \frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d} \\
&\quad + \frac{be^4n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4} \\
&\quad + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 + \frac{b^2e^4n^2\log(x)}{6d^4} \\
&\quad + \frac{(b^2e^2n^2)\,\mathrm{Subst}\left(\int\left(\frac{e^2}{d(d-x)^2}+\frac{e^2}{d^2(d-x)}+\frac{e^2}{d^2x}\right)dx, x, d+\frac{e}{\sqrt{x}}\right)}{2d^2} \\
&\quad - \frac{(b^2e^3n^2)\,\mathrm{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x}{e}}\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{d^4} \\
&\quad - \frac{(b^2e^4n^2)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{d}{x}\right)}{x}\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{d^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{5b^2e^3n^2\sqrt{x}}{6d^3} + \frac{b^2e^2n^2x}{6d^2} + \frac{5b^2e^4n^2\log\left(d+\frac{e}{\sqrt{x}}\right)}{6d^4} \\
&\quad + \frac{be^3n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4} \\
&\quad - \frac{be^2nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^2} + \frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3d} \\
&\quad + \frac{be^4n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4} \\
&\quad + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 + \frac{11b^2e^4n^2\log(x)}{12d^4} - \frac{b^2e^4n^2\mathrm{Li}_2\left(\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.11

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx=\frac{1}{6}\left(3x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\right.$$
$$+\frac{ben\left(6ade^2\sqrt{x}-5bde^2n\sqrt{x}-3ad^2ex+bd^2enx+2ad^3x^{3/2}+8be^3n\log\left(d+\frac{e}{\sqrt{x}}\right)+6bde^2\sqrt{x}\log\left(c\left(d+\right.\right.\right.}{}$$

[In] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]

[Out] (3*x^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + (b*e*n*(6*a*d*e^2*Sqrt[x] - 5*b*d*e^2*n*Sqrt[x] - 3*a*d^2*e*x + b*d^2*e*n*x + 2*a*d^3*x^(3/2) + 8*b*e^3*n*Log[d + e/Sqrt[x]] + 6*b*d*e^2*Sqrt[x]*Log[c*(d + e/Sqrt[x])^n] - 3*b*d^2*e*x*Log[c*(d + e/Sqrt[x])^n] + 2*b*d^3*x^(3/2)*Log[c*(d + e/Sqrt[x])^n] - 6*a*e^3*Log[e + d*Sqrt[x]] + 3*b*e^3*n*Log[e + d*Sqrt[x]] - 6*b*e^3*Log[c*(d + e/Sqrt[x])^n]*Log[e + d*Sqrt[x]] + 3*b*e^3*n*Log[e + d*Sqrt[x]]^2 - 6*b*e^3*n*Log[e + d*Sqrt[x]]*Log[-((d*Sqrt[x])/e)] + 4*b*e^3*n*Log[x] - 6*b*e^3*n*PolyLog[2, 1 + (d*Sqrt[x])/e]))/d^4)/6

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx$$

[In] int(x*(a+b*ln(c*(d+e/x^(1/2))^n))^2,x)

[Out] int(x*(a+b*ln(c*(d+e/x^(1/2))^n))^2,x)

Fricas [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2x\,dx$$

[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="fricas")

[Out] integral(b^2*x*log(c*((d*x + e*sqrt(x))/x)^n)^2 + 2*a*b*x*log(c*((d*x + e*sqrt(x))/x)^n) + a^2*x, x)

Sympy [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx$$

[In] integrate(x*(a+b*ln(c*(d+e/x**(1/2))**n))**2,x)

[Out] Integral(x*(a + b*log(c*(d + e/sqrt(x))**n))**2, x)

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2x\,dx$$

[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="maxima")

[Out] 1/2*b^2*x^2*log((d*sqrt(x) + e)^n)^2 - integrate(-1/2*(2*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x^2 + 2*(b^2*d*x^2 + b^2*e*x^(3/2))*log(x^(1/2*n))^2 + 2*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^(3/2) - (b^2*d*n*x^2 - 4*(b^2*d*log(c) + a*b*d)*x^2 - 4*(b^2*e*log(c) + a*b*e)*x^(3/2) + 4*(b^2*d*x^2 + b^2*e*x^(3/2))*log(x^(1/2*n)))*log((d*sqrt(x) + e)^n) - 4*((b^2*d*log(c) + a*b*d)*x^2 + (b^2*e*log(c) + a*b*e)*x^(3/2))*log(x^(1/2*n)))/(d*x + e*sqrt(x)), x)

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2x\,dx$$

[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="giac")

[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^2*x, x)

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx=\int x\,\left(a+b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx$$

```
[In] int(x*(a + b*log(c*(d + e/x^(1/2))^n))^2,x)
[Out] int(x*(a + b*log(c*(d + e/x^(1/2))^n))^2, x)
```

3.431 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 20, antiderivative size = 152

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx = \frac{2ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2} + \frac{2be^2n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 + \frac{b^2e^2n^2\log(x)}{d^2} - \frac{2b^2e^2n^2\,\mathrm{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^2}$$

```
[Out] b^2*e^2*n^2*ln(x)/d^2+2*b*e^2*n*ln(1-d/(d+e/x^(1/2)))*(a+b*ln(c*(d+e/x^(1/2
))^n))/d^2+x*(a+b*ln(c*(d+e/x^(1/2))^n))^2-2*b^2*e^2*n^2*polylog(2,d/(d+e/x
^(1/2)))/d^2+2*b*e*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))*x^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used

$= \{2501, 2504, 2445, 2458, 2389, 2379, 2438, 2351, 31\}$

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx = \frac{2be^2n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2} + \frac{2ben\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 - \frac{2b^2e^2n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^2} + \frac{b^2e^2n^2\log(x)}{d^2}$$

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

```
[Out] (2*b*e*n*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*(d + e/Sqrt[x])^n]))/d^2 + (2
*b*e^2*n*Log[1 - d/(d + e/Sqrt[x])]*(a + b*Log[c*(d + e/Sqrt[x])^n]))/d^2 +
 x*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + (b^2*e^2*n^2*Log[x])/d^2 - (2*b^2*e
^2*n^2*PolyLog[2, d/(d + e/Sqrt[x])])/d^2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^n\right)\right)^2\,dx, x, \sqrt{x}\right)\\
&= -\left(2\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^n\right)\right)^2}{x^3}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2-(2ben)\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^2(d+ex)}\,dx, x, \frac{1}{\sqrt{x}}\right)
\end{aligned}$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 - (2bn)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}dx, x, d+\frac{e}{\sqrt{x}}\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 - \frac{(2bn)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}dx, x, d+\frac{e}{\sqrt{x}}\right)}{d} + \frac{(2ben)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}dx, x, d+\frac{e}{\sqrt{x}}\right)}{d}$$

$$= \frac{2ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2} + \frac{2be^2n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 - \frac{(2b^2en^2)\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x}{e}}dx, x, d+\frac{e}{\sqrt{x}}\right)}{d^2} - \frac{(2b^2e^2n^2)\text{Subst}\left(\int\frac{\log\left(1-\frac{d}{x}\right)}{x}dx, x, d+\frac{e}{\sqrt{x}}\right)}{d^2}$$

$$= \frac{2ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2} + \frac{2be^2n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 + \frac{b^2e^2n^2\log(x)}{d^2} - \frac{2b^2e^2n^2\text{Li}_2\left(\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^2}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.13

$$\int\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 dx = x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 + \frac{ben\left(2ad\sqrt{x}+2ben\log\left(d+\frac{e}{\sqrt{x}}\right)+2bd\sqrt{x}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-2e\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(e+d\right.\right.}{d^2}$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2,x]
```

```
[Out] x*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + (b*e*n*(2*a*d*Sqrt[x] + 2*b*e*n*Log[
d + e/Sqrt[x]] + 2*b*d*Sqrt[x]*Log[c*(d + e/Sqrt[x])^n] - 2*e*(a + b*Log[c*
```

```
(d + e/Sqrt[x])^n])*Log[e + d*Sqrt[x]] + b*e*n*Log[x] + b*e*n*(Log[e + d*Sq
rt[x]]*(Log[e + d*Sqrt[x]] - 2*Log[-((d*Sqrt[x])/e)]) - 2*PolyLog[2, 1 + (d
*Sqrt[x])/e])))/d^2
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^2,x)
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^2,x)
```

Fricas [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}dx=\int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a\right)^{2}dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="fricas")
[Out] integral(b^2*log(c*((d*x + e*sqrt(x))/x)^n)^2 + 2*a*b*log(c*((d*x + e*sqrt(
x))/x)^n) + a^2, x)
```

Sympy [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}dx=\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}dx$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2,x)
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**2, x)
```

Maxima [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}dx=\int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a\right)^{2}dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="maxima")
[Out] -2*(e*n*(e*log(d*sqrt(x) + e)/d^2 - sqrt(x)/d) - x*log(c*(d + e/sqrt(x))^n)
)*a*b + (x*log((d*sqrt(x) + e)^n)^2 - integrate(-(d*x*log(c)^2 + e*sqrt(x)*
log(c)^2 + (d*x + e*sqrt(x))*log(x^(1/2*n))^2 - (d*n*x - 2*d*x*log(c) - 2*e
*sqrt(x)*log(c) + 2*(d*x + e*sqrt(x))*log(x^(1/2*n)))*log((d*sqrt(x) + e)^n
) - 2*(d*x*log(c) + e*sqrt(x)*log(c))*log(x^(1/2*n)))/(d*x + e*sqrt(x)), x)
)*b^2 + a^2*x
```

Giac [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx=\int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx=\int \left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^2,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))^n))^2, x)
```

3.432 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x}\,dx = -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt{x}}\right) - 4bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\operatorname{PolyLog}\left(2,1+\frac{e}{d\sqrt{x}}\right) + 4b^2n^2\operatorname{PolyLog}\left(3,1+\frac{e}{d\sqrt{x}}\right)$$

```
[Out] -2*(a+b*ln(c*(d+e/x^(1/2))^n))^2*ln(-e/d/x^(1/2))-4*b*n*(a+b*ln(c*(d+e/x^(1
/2))^n))*polylog(2,1+e/d/x^(1/2))+4*b^2*n^2*polylog(3,1+e/d/x^(1/2))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2504, 2443, 2481, 2421, 6724}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x}\,dx = -4bn\operatorname{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right) - 2\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 + 4b^2n^2\operatorname{PolyLog}\left(3,\frac{e}{d\sqrt{x}}+1\right)$$

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x,x]
```

```
[Out] -2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2*Log[-(e/(d*Sqrt[x]))] - 4*b*n*(a + b*
Log[c*(d + e/Sqrt[x])^n])*PolyLog[2, 1 + e/(d*Sqrt[x])] + 4*b^2*n^2*PolyLog
[3, 1 + e/(d*Sqrt[x])]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\left(2\text{Subst}\left(\int \frac{(a+b\log\left(c(d+ex)^n\right))^2}{x}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt{x}}\right) + (4ben)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{d+ex}dx, x, \frac{1}{\sqrt{x}}\right)$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt{x}}\right) + (4bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}dx, x, d+\frac{e}{\sqrt{x}}\right)$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt{x}}\right) - 4bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\text{Li}_2\left(1+\frac{e}{d\sqrt{x}}\right) + \left(4b^2n^2\right)\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{x}{d}\right)}{x}dx, x, d+\frac{e}{\sqrt{x}}\right)$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt{x}}\right) - 4bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\text{Li}_2\left(1+\frac{e}{d\sqrt{x}}\right) + 4b^2n^2\text{Li}_3\left(1+\frac{e}{d\sqrt{x}}\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 386 vs. $2(93) = 186$.

Time = 0.23 (sec) , antiderivative size = 386, normalized size of antiderivative = 4.15

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x}\,dx = \left(a-bn\log\left(d+\frac{e}{\sqrt{x}}\right)+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\log(x) + 2bn\left(a-bn\log\left(d+\frac{e}{\sqrt{x}}\right)+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\left(\left(\log\left(d+\frac{e}{\sqrt{x}}\right)-\log\left(1+\frac{e}{d\sqrt{x}}\right)\right)\log(x)+2\,\mathrm{PolyLog}\left(2,-\frac{e}{d\sqrt{x}}\right)\right) + \frac{1}{12}b^2n^2\left(24\log^2\left(\frac{e}{d}+\sqrt{x}\right)\log\left(-\frac{d\sqrt{x}}{e}\right)+12\log^2\left(d+\frac{e}{\sqrt{x}}\right)\log(x)-12\log^2\left(\frac{e}{d}+\sqrt{x}\right)\log(x)-24\log\left(d+\frac{e}{\sqrt{x}}\right)\log\left(1+\frac{d\sqrt{x}}{e}\right)\log(x)+24\log\left(\frac{e}{d}+\sqrt{x}\right)\log\left(1+\frac{d\sqrt{x}}{e}\right)\log(x)+6\log\left(d+\frac{e}{\sqrt{x}}\right)\log^2(x)-6\log\left(1+\frac{d\sqrt{x}}{e}\right)\log^2(x)+\log^3(x)+48\log\left(\frac{e}{d}+\sqrt{x}\right)\mathrm{PolyLog}\left(2,1+\frac{d\sqrt{x}}{e}\right)-48\left(\log\left(d+\frac{e}{\sqrt{x}}\right)-\log\left(\frac{e}{d}+\sqrt{x}\right)\right)\mathrm{PolyLog}\left(2,-\frac{d\sqrt{x}}{e}\right)-48\,\mathrm{PolyLog}\left(3,1+\frac{d\sqrt{x}}{e}\right)-48\,\mathrm{PolyLog}\left(3,-\frac{d\sqrt{x}}{e}\right)\right)$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e/Sqrt[x]] + b*Log[c*(d + e/Sqrt[x])^n])*((Log[d + e/Sqrt[
x]] - Log[1 + e/(d*Sqrt[x])])*Log[x] + 2*PolyLog[2, -(e/(d*Sqrt[x]))]) + (b
^2*n^2*(24*Log[e/d + Sqrt[x]]^2*Log[-((d*Sqrt[x])/e)] + 12*Log[d + e/Sqrt[x
]]^2*Log[x] - 12*Log[e/d + Sqrt[x]]^2*Log[x] - 24*Log[d + e/Sqrt[x]]*Log[1
+ (d*Sqrt[x])/e]*Log[x] + 24*Log[e/d + Sqrt[x]]*Log[1 + (d*Sqrt[x])/e]*Log[
x] + 6*Log[d + e/Sqrt[x]]*Log[x]^2 - 6*Log[1 + (d*Sqrt[x])/e]*Log[x]^2 + Lo
g[x]^3 + 48*Log[e/d + Sqrt[x]]*PolyLog[2, 1 + (d*Sqrt[x])/e] - 48*(Log[d +
e/Sqrt[x]] - Log[e/d + Sqrt[x]])*PolyLog[2, -((d*Sqrt[x])/e)] - 48*PolyLog[
3, 1 + (d*Sqrt[x])/e] - 48*PolyLog[3, -((d*Sqrt[x])/e)]))/12
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x}dx$$

[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x,x)

[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x,x)

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a\right)^{2}}{x}\,dx$$

[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x,x, algorithm="fricas")

[Out] integral((b^2*log(c*((d*x + e*sqrt(x))/x)^n)^2 + 2*a*b*log(c*((d*x + e*sqrt (x))/x)^n) + a^2)/x, x)

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x}\,dx$$

[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2/x,x)

[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**2/x, x)

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a\right)^{2}}{x}\,dx$$

[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x,x, algorithm="maxima")

[Out] b^2*log((d*sqrt(x) + e)^n)^2*log(x) - integrate(-((b^2*d*x + b^2*e*sqrt(x)) *log(x^(1/2*n))^2 + (b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x - (b^2*d*n* x*log(x) - 2*(b^2*d*log(c) + a*b*d)*x + 2*(b^2*d*x + b^2*e*sqrt(x))*log(x^(1/2*n)) - 2*(b^2*e*log(c) + a*b*e)*sqrt(x))*log((d*sqrt(x) + e)^n) - 2*((b^ 2*d*log(c) + a*b*d)*x + (b^2*e*log(c) + a*b*e)*sqrt(x))*log(x^(1/2*n)) + (b ^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*sqrt(x))/(d*x^2 + e*x^(3/2)), x)

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x}\,dx = \int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^2/x,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))^n))^2/x, x)
```

3.433 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 195

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx = -\frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^2}-\frac{4abdn}{e\sqrt{x}}+\frac{4b^2dn^2}{e\sqrt{x}} - \frac{4b^2dn\left(d+\frac{e}{\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{e^2} + \frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^2} + \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2} - \frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2}$$

```
[Out] -4*b^2*d*n*ln(c*(d+e/x^(1/2))^n)*(d+e/x^(1/2))/e^2+2*d*(a+b*ln(c*(d+e/x^(1/
2))^n))^2*(d+e/x^(1/2))/e^2-1/2*b^2*n^2*(d+e/x^(1/2))^2/e^2+b*n*(a+b*ln(c*(
d+e/x^(1/2))^n))*(d+e/x^(1/2))^2/e^2-(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(
1/2))^2/e^2-4*a*b*d*n/e/x^(1/2)+4*b^2*d*n^2/e/x^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx = \frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^2} - \frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2} + \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2} - \frac{4abdn}{e\sqrt{x}} - \frac{4b^2dn\left(d+\frac{e}{\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{e^2} - \frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^2} + \frac{4b^2dn^2}{e\sqrt{x}}$$

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^2,x]
```

```
[Out] -1/2*(b^2*n^2*(d + e/Sqrt[x])^2)/e^2 - (4*a*b*d*n)/(e*Sqrt[x]) + (4*b^2*d*n
^2)/(e*Sqrt[x]) - (4*b^2*d*n*(d + e/Sqrt[x])*Log[c*(d + e/Sqrt[x])^n])/e^2
+ (b*n*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/e^2 + (2*d*(d +
e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/e^2 - ((d + e/Sqrt[x])^2*(a
+ b*Log[c*(d + e/Sqrt[x])^n])^2)/e^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(2\text{Subst}\left(\int x(a+b\log(c(d+ex)^n))^2\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= -\left(2\text{Subst}\left(\int\left(-\frac{d(a+b\log(c(d+ex)^n))^2}{e}+\frac{(d+ex)(a+b\log(c(d+ex)^n))^2}{e}\right)\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= -\frac{2\text{Subst}\left(\int(d+ex)(a+b\log(c(d+ex)^n))^2\,dx, x, \frac{1}{\sqrt{x}}\right)}{e}\\
&\quad+\frac{(2d)\text{Subst}\left(\int(a+b\log(c(d+ex)^n))^2\,dx, x, \frac{1}{\sqrt{x}}\right)}{e}
\end{aligned}$$

$$= -\frac{2\text{Subst}\left(\int x(a+b\log(cx^n))^2\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^2} + \frac{(2d)\text{Subst}\left(\int (a+b\log(cx^n))^2\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^2}$$

$$= \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2} - \frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2} + \frac{(2bn)\text{Subst}\left(\int x(a+b\log(cx^n))\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^2} - \frac{(4bdn)\text{Subst}\left(\int (a+b\log(cx^n))\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^2}$$

$$= -\frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^2} - \frac{4abdn}{e\sqrt{x}} + \frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^2} + \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2} - \frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2} - \frac{(4b^2dn)\,\text{Subst}\left(\int \log(cx^n)\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^2}$$

$$= -\frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^2} - \frac{4abdn}{e\sqrt{x}} + \frac{4b^2dn^2}{e\sqrt{x}} - \frac{4b^2dn\left(d+\frac{e}{\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{e^2} + \frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^2} + \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2} - \frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.53

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx = -\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 + \frac{bn\left(4ade\sqrt{x}-4bden\sqrt{x}+bn\left(e(e-2d\sqrt{x})+2d^2x\log\left(d+\frac{e}{\sqrt{x}}\right)\right)+4bd(e+d\sqrt{x})\sqrt{x}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{}}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^2,x]
```

```
[Out] -1/2*(2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + (b*n*(4*a*d*e*Sqrt[x] - 4*b*d*
e*n*Sqrt[x] + b*n*(e*(e - 2*d*Sqrt[x]) + 2*d^2*x*Log[d + e/Sqrt[x]]) + 4*b*
d*(e + d*Sqrt[x])*Sqrt[x]*Log[c*(d + e/Sqrt[x])^n] - 2*e^2*(a + b*Log[c*(d
+ e/Sqrt[x])^n]) - 4*d^2*x*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[e + d*Sqrt[
x]] - 4*d^2*x*(a + b*Log[c*(d + e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[x]))] - 4*b*
d^2*n*x*PolyLog[2, 1 + e/(d*Sqrt[x])] + 2*b*d^2*n*x*(Log[e + d*Sqrt[x]]*(Lo
g[e + d*Sqrt[x]] - 2*Log[-((d*Sqrt[x])/e)]) - 2*PolyLog[2, 1 + (d*Sqrt[x])/
e])))/e^2)/x
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.21

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx = -\frac{b^2e^2n^2+2\,b^2e^2\log\left(c\right)^2-2\,abe^2n+2\,a^2e^2-2\,(b^2d^2n^2x-b^2e^2n^2)\log\left(\frac{dx+e\sqrt{x}}{x}\right)^2-2\,(b^2e^2n-2\,abe^2)\log}{}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*e^2*n^2 + 2*b^2*e^2*log(c)^2 - 2*a*b*e^2*n + 2*a^2*e^2 - 2*(b^2*d
^2*n^2*x - b^2*e^2*n^2)*log((d*x + e*sqrt(x))/x)^2 - 2*(b^2*e^2*n - 2*a*b*e
^2)*log(c) + 2*(2*b^2*d*e*n^2*sqrt(x) - b^2*e^2*n^2 + 2*a*b*e^2*n + (3*b^2*
d^2*n^2 - 2*a*b*d^2*n)*x - 2*(b^2*d^2*n*x - b^2*e^2*n)*log(c))*log((d*x + e
*sqrt(x))/x) - 2*(3*b^2*d*e*n^2 - 2*b^2*d*e*n*log(c) - 2*a*b*d*e*n)*sqrt(x)
)/(e^2*x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2/x**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**2/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.27

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx=aben\left(\frac{2\,d^2\log\left(d\sqrt{x}+e\right)}{e^3}-\frac{d^2\log\left(x\right)}{e^3}-\frac{2\,d\sqrt{x}-e}{e^2x}\right)$$
$$+\frac{1}{4}\left(4\,en\left(\frac{2\,d^2\log\left(d\sqrt{x}+e\right)}{e^3}-\frac{d^2\log\left(x\right)}{e^3}-\frac{2\,d\sqrt{x}-e}{e^2x}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{\left(4\,d^2x\log\left(d\sqrt{x}+e\right)^2\right.}{}\right.$$
$$-\frac{b^2\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2}{x}-\frac{2\,ab\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}-\frac{a^2}{x}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] a*b*e*n*(2*d^2*log(d*sqrt(x) + e)/e^3 - d^2*log(x)/e^3 - (2*d*sqrt(x) - e)/
(e^2*x)) + 1/4*(4*e*n*(2*d^2*log(d*sqrt(x) + e)/e^3 - d^2*log(x)/e^3 - (2*d
*sqrt(x) - e)/(e^2*x))*log(c*(d + e/sqrt(x))^n) - (4*d^2*x*log(d*sqrt(x) +
e)^2 + d^2*x*log(x)^2 - 6*d^2*x*log(x) - 12*d*e*sqrt(x) + 2*e^2 - 4*(d^2*x*
log(x) - 3*d^2*x)*log(d*sqrt(x) + e))*n^2/(e^2*x))*b^2 - b^2*log(c*(d + e/s
qrt(x))^n)^2/x - 2*a*b*log(c*(d + e/sqrt(x))^n)/x - a^2/x
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.43

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx$$
$$=\frac{2\left(\frac{2\,(d\sqrt{x}+e)b^2dn^2}{e\sqrt{x}}-\frac{(d\sqrt{x}+e)^2b^2n^2}{ex}\right)\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)^2+2\left(\frac{(b^2n^2-2\,b^2n\log(c)-2\,abn)\,(d\sqrt{x}+e)^2}{ex}-\frac{4\,(b^2dn^2-b^2dn\log(c)-abdn)\,(}{e\sqrt{x}}\right.}{}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(2*(d*sqrt(x) + e)*b^2*d*n^2/(e*sqrt(x)) - (d*sqrt(x) + e)^2*b^2*n^2
/(e*x))*log((d*sqrt(x) + e)/sqrt(x))^2 + 2*((b^2*n^2 - 2*b^2*n*log(c) - 2*a
*b*n)*(d*sqrt(x) + e)^2/(e*x) - 4*(b^2*d*n^2 - b^2*d*n*log(c) - a*b*d*n)*(d
*sqrt(x) + e)/(e*sqrt(x)))*log((d*sqrt(x) + e)/sqrt(x)) - (b^2*n^2 - 2*b^2*
n*log(c) + 2*b^2*log(c)^2 - 2*a*b*n + 4*a*b*log(c) + 2*a^2)*(d*sqrt(x) + e)
^2/(e*x) + 4*(2*b^2*d*n^2 - 2*b^2*d*n*log(c) + b^2*d*log(c)^2 - 2*a*b*d*n +
 2*a*b*d*log(c) + a^2*d)*(d*sqrt(x) + e)/(e*sqrt(x)))/e
```

Mupad [B] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^2}\,dx=\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\left(\frac{\frac{2\,b\,d\,(2\,a-b\,n)}{e}-\frac{4\,a\,b\,d}{e}}{\sqrt{x}}-\frac{b\,(2\,a-b\,n)}{x}\right)-\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2\left(\frac{b^2}{x}-\frac{b^2\,d^2}{e^2}\right)+\frac{\frac{d\,(2\,a^2-2\,a\,b\,n+b^2\,n^2)}{e}-\frac{2\,d\,(a^2-b^2\,n^2)}{e}}{\sqrt{x}}-\frac{a^2-a\,b\,n+\frac{b^2\,n^2}{2}}{x}-\frac{\ln\left(d+\frac{e}{\sqrt{x}}\right)\,(3\,b^2\,d^2\,n^2-2\,a\,b\,d^2\,n)}{e^2}$$

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^2/x^2,x)
```

```
[Out] log(c*(d + e/x^(1/2))^n)*(((2*b*d*(2*a - b*n))/e - (4*a*b*d)/e)/x^(1/2) - (
b*(2*a - b*n))/x) - log(c*(d + e/x^(1/2))^n)^2*(b^2/x - (b^2*d^2)/e^2) + ((
d*(2*a^2 + b^2*n^2 - 2*a*b*n))/e - (2*d*(a^2 - b^2*n^2))/e)/x^(1/2) - (a^2
+ (b^2*n^2)/2 - a*b*n)/x - (log(d + e/x^(1/2))*(3*b^2*d^2*n^2 - 2*a*b*d^2*n
))/e^2
```

3.434 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 341

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx = &-\frac{3b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^4}+\frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4}\\ &-\frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{16e^4}+\frac{4b^2d^3n^2}{e^3\sqrt{x}}-\frac{b^2d^4n^2\log^2\left(d+\frac{e}{\sqrt{x}}\right)}{2e^4}\\ &-\frac{4bd^3n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4}\\ &+\frac{3bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4}\\ &-\frac{4bdn\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^4}\\ &+\frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{4e^4}\\ &+\frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4}\\ &-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2x^2}\end{aligned}$$

```
[Out] -1/2*b^2*d^4*n^2*ln(d+e/x^(1/2))^2/e^4+b*d^4*n*ln(d+e/x^(1/2))*(a+b*ln(c*(d
+e/x^(1/2))^n))/e^4-1/2*(a+b*ln(c*(d+e/x^(1/2))^n))^2/x^2-4*b*d^3*n*(a+b*ln
(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))/e^4-3/2*b^2*d^2*n^2*(d+e/x^(1/2))^2/e^4+
```

```
3*b*d^2*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^2/e^4+4/9*b^2*d*n^2*(d+
e/x^(1/2))^3/e^4-4/3*b*d*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^3/e^4-
1/16*b^2*n^2*(d+e/x^(1/2))^4/e^4+1/4*b*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x
^(1/2))^4/e^4+4*b^2*d^3*n^2/e^3/x^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx = \frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4} - \frac{4bd^3n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4} + \frac{3bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4} - \frac{4bdn\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^4} + \frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{4e^4} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2x^2} - \frac{b^2d^4n^2\log^2\left(d+\frac{e}{\sqrt{x}}\right)}{2e^4} + \frac{4b^2d^3n^2}{e^3\sqrt{x}} - \frac{3b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^4} + \frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4} - \frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{16e^4}$$

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^3,x]
```

```
[Out] (-3*b^2*d^2*n^2*(d + e/Sqrt[x])^2)/(2*e^4) + (4*b^2*d*n^2*(d + e/Sqrt[x])^3
)/(9*e^4) - (b^2*n^2*(d + e/Sqrt[x])^4)/(16*e^4) + (4*b^2*d^3*n^2)/(e^3*Sqr
t[x]) - (b^2*d^4*n^2*Log[d + e/Sqrt[x]]^2)/(2*e^4) - (4*b*d^3*n*(d + e/Sqrt
[x])*(a + b*Log[c*(d + e/Sqrt[x])^n]))/e^4 + (3*b*d^2*n*(d + e/Sqrt[x])^2*(
a + b*Log[c*(d + e/Sqrt[x])^n]))/e^4 - (4*b*d*n*(d + e/Sqrt[x])^3*(a + b*Lo
g[c*(d + e/Sqrt[x])^n]))/(3*e^4) + (b*n*(d + e/Sqrt[x])^4*(a + b*Log[c*(d +
 e/Sqrt[x])^n]))/(4*e^4) + (b*d^4*n*Log[d + e/Sqrt[x]]*(a + b*Log[c*(d + e/
Sqrt[x])^n]))/e^4 - (a + b*Log[c*(d + e/Sqrt[x])^n])^2/(2*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(2\text{Subst}\left(\int x^3(a+b\log(c(d+ex)^n))^2\,dx,x,\frac{1}{\sqrt{x}}\right)\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2x^2}+(ben)\text{Subst}\left(\int\frac{x^4(a+b\log(c(d+ex)^n))}{d+ex}\,dx,x,\frac{1}{\sqrt{x}}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2x^2}+(bn)\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^4(a+b\log(cx^n))}{x}\,dx,x,d+\frac{e}{\sqrt{x}}\right)\\
&= -\frac{4bd^3n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4}+\frac{3bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4}\\
&\quad-\frac{4bdn\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^4}+\frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{4e^4}\\
&\quad+\frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2x^2}\\
&\quad-\left(b^2n^2\right)\text{Subst}\left(\int\frac{x(-48d^3+36d^2x-16dx^2+3x^3)+12d^4\log(x)}{12e^4x}\,dx,x,d+\frac{e}{\sqrt{x}}\right)
\end{aligned}$$

$$= -\frac{4bd^3n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4} + \frac{3bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4} - \frac{4bdn\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^4} + \frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{4e^4} + \frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2x^2} - \frac{(b^2n^2)\,\text{Subst}\left(\int\frac{x(-48d^3+36d^2x-16dx^2+3x^3)+12d^4\log(x)}{x}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{12e^4}$$

$$= -\frac{4bd^3n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4} + \frac{3bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4} - \frac{4bdn\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^4} + \frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{4e^4} + \frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2x^2} - \frac{(b^2n^2)\,\text{Subst}\left(\int\left(-48d^3+36d^2x-16dx^2+3x^3+\frac{12d^4\log(x)}{x}\right)dx,x,d+\frac{e}{\sqrt{x}}\right)}{12e^4}$$

$$
\begin{aligned}
&= -\frac{3b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^4} + \frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4} - \frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{16e^4} \\
&\quad + \frac{4b^2d^3n^2}{e^3\sqrt{x}} - \frac{4bd^3n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4} \\
&\quad + \frac{3bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4} \\
&\quad - \frac{4bdn\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^4} \\
&\quad + \frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{4e^4} \\
&\quad + \frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4} \\
&\quad - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2x^2} - \frac{(b^2d^4n^2)\,\mathrm{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^4} + \frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4} - \frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{16e^4} + \frac{4b^2d^3n^2}{e^3\sqrt{x}} \\
&\quad - \frac{b^2d^4n^2\log^2\left(d+\frac{e}{\sqrt{x}}\right)}{2e^4} - \frac{4bd^3n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4} \\
&\quad + \frac{3bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4} \\
&\quad - \frac{4bdn\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^4} \\
&\quad + \frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{4e^4} \\
&\quad + \frac{bd^4n\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^4} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2x^2}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.21 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.29

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx$$
$$= \frac{-72e^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2+bn\left(36ae^4-9be^4n-48ade^3\sqrt{x}+28bde^3n\sqrt{x}+72ad^2e^2x-78bd^2e^2n\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^3,x]
```

```
[Out] (-72*e^4*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + b*n*(36*a*e^4 - 9*b*e^4*n - 4
8*a*d*e^3*Sqrt[x] + 28*b*d*e^3*n*Sqrt[x] + 72*a*d^2*e^2*x - 78*b*d^2*e^2*n*
x - 144*a*d^3*e*x^(3/2) + 300*b*d^3*e*n*x^(3/2) - 300*b*d^4*n*x^2*Log[d + e
/Sqrt[x]] + 36*b*e^4*Log[c*(d + e/Sqrt[x])^n] - 48*b*d*e^3*Sqrt[x]*Log[c*(d
 + e/Sqrt[x])^n] + 72*b*d^2*e^2*x*Log[c*(d + e/Sqrt[x])^n] - 144*b*d^3*e*x^
(3/2)*Log[c*(d + e/Sqrt[x])^n] + 144*a*d^4*x^2*Log[e + d*Sqrt[x]] + 144*b*d
^4*x^2*Log[c*(d + e/Sqrt[x])^n]*Log[e + d*Sqrt[x]] - 72*b*d^4*n*x^2*Log[e +
 d*Sqrt[x]]^2 + 144*b*d^4*x^2*Log[c*(d + e/Sqrt[x])^n]*Log[-(e/(d*Sqrt[x]))
] + 144*b*d^4*n*x^2*Log[e + d*Sqrt[x]]*Log[-((d*Sqrt[x])/e)] - 72*a*d^4*x^2
*Log[x] + 144*b*d^4*n*x^2*PolyLog[2, 1 + e/(d*Sqrt[x])] + 144*b*d^4*n*x^2*P
olyLog[2, 1 + (d*Sqrt[x])/e]))/(144*e^4*x^2)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx =$$
$$-\frac{9\,b^2e^4n^2+72\,b^2e^4\log\left(c\right)^2-36\,abe^4n+72\,a^2e^4-72\,(b^2d^4n^2x^2-b^2e^4n^2)\log\left(\frac{dx+e\sqrt{x}}{x}\right)^2+6\,(13\,b^2d^2e^2n^2}{}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/144*(9*b^2*e^4*n^2 + 72*b^2*e^4*log(c)^2 - 36*a*b*e^4*n + 72*a^2*e^4 - 7
2*(b^2*d^4*n^2*x^2 - b^2*e^4*n^2)*log((d*x + e*sqrt(x))/x)^2 + 6*(13*b^2*d^
2*e^2*n^2 - 12*a*b*d^2*e^2*n)*x - 36*(2*b^2*d^2*e^2*n*x + b^2*e^4*n - 4*a*b
*e^4)*log(c) - 12*(6*b^2*d^2*e^2*n^2*x + 3*b^2*e^4*n^2 - 12*a*b*e^4*n - (25
*b^2*d^4*n^2 - 12*a*b*d^4*n)*x^2 + 12*(b^2*d^4*n*x^2 - b^2*e^4*n)*log(c) -
4*(3*b^2*d^3*e*n^2*x + b^2*d*e^3*n^2)*sqrt(x))*log((d*x + e*sqrt(x))/x) - 4
*(7*b^2*d*e^3*n^2 - 12*a*b*d*e^3*n + 3*(25*b^2*d^3*e*n^2 - 12*a*b*d^3*e*n)*
x - 12*(3*b^2*d^3*e*n*x + b^2*d*e^3*n)*log(c))*sqrt(x))/(e^4*x^2)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x^{3}}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x^{3}}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2/x**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**2/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.94

$$\begin{aligned}&\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x^{3}}\,dx\\&=\frac{1}{12}\,aben\left(\frac{12\,d^{4}\log\left(d\sqrt{x}+e\right)}{e^{5}}-\frac{6\,d^{4}\log\left(x\right)}{e^{5}}-\frac{12\,d^{3}x^{\frac{3}{2}}-6\,d^{2}ex+4\,de^{2}\sqrt{x}-3\,e^{3}}{e^{4}x^{2}}\right)\\&\quad+\frac{1}{144}\left(12\,en\left(\frac{12\,d^{4}\log\left(d\sqrt{x}+e\right)}{e^{5}}-\frac{6\,d^{4}\log\left(x\right)}{e^{5}}-\frac{12\,d^{3}x^{\frac{3}{2}}-6\,d^{2}ex+4\,de^{2}\sqrt{x}-3\,e^{3}}{e^{4}x^{2}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right.\right.\right.\\&\quad-\frac{b^{2}\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)^{2}}{2\,x^{2}}-\frac{ab\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)}{x^{2}}-\frac{a^{2}}{2\,x^{2}}\end{aligned}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*a*b*e*n*(12*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*x^
(3/2) - 6*d^2*e*x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2)) + 1/144*(12*e*n*(12
*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*x^(3/2) - 6*d^2*e*
x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2))*log(c*(d + e/sqrt(x))^n) - (72*d^4*
```

```
x^2*log(d*sqrt(x) + e)^2 + 18*d^4*x^2*log(x)^2 - 150*d^4*x^2*log(x) - 300*d
^3*e*x^(3/2) + 78*d^2*e^2*x - 28*d*e^3*sqrt(x) + 9*e^4 - 12*(6*d^4*x^2*log(
x) - 25*d^4*x^2)*log(d*sqrt(x) + e))*n^2/(e^4*x^2))*b^2 - 1/2*b^2*log(c*(d
+ e/sqrt(x))^n)^2/x^2 - a*b*log(c*(d + e/sqrt(x))^n)/x^2 - 1/2*a^2/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.70

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx$$

$$= \frac{72\left(\frac{4\,(d\sqrt{x}+e)b^2d^3n^2}{e^3\sqrt{x}} - \frac{6\,(d\sqrt{x}+e)^2b^2d^2n^2}{e^3x} + \frac{4\,(d\sqrt{x}+e)^3b^2dn^2}{e^3x^{\frac{3}{2}}} - \frac{(d\sqrt{x}+e)^4b^2n^2}{e^3x^2}\right)\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)^2 + 12\left(\frac{3\,(b^2n^2-4\,b^2n\log(c)-4}{e^3x^2}\right.}{}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^3,x, algorithm="giac")
```

```
[Out] 1/144*(72*(4*(d*sqrt(x) + e)*b^2*d^3*n^2/(e^3*sqrt(x)) - 6*(d*sqrt(x) + e)^
2*b^2*d^2*n^2/(e^3*x) + 4*(d*sqrt(x) + e)^3*b^2*d*n^2/(e^3*x^(3/2)) - (d*sq
rt(x) + e)^4*b^2*n^2/(e^3*x^2))*log((d*sqrt(x) + e)/sqrt(x))^2 + 12*(3*(b^2
*n^2 - 4*b^2*n*log(c) - 4*a*b*n)*(d*sqrt(x) + e)^4/(e^3*x^2) - 16*(b^2*d*n^
2 - 3*b^2*d*n*log(c) - 3*a*b*d*n)*(d*sqrt(x) + e)^3/(e^3*x^(3/2)) + 36*(b^2
*d^2*n^2 - 2*b^2*d^2*n*log(c) - 2*a*b*d^2*n)*(d*sqrt(x) + e)^2/(e^3*x) - 48
*(b^2*d^3*n^2 - b^2*d^3*n*log(c) - a*b*d^3*n)*(d*sqrt(x) + e)/(e^3*sqrt(x))
)*log((d*sqrt(x) + e)/sqrt(x)) - 9*(b^2*n^2 - 4*b^2*n*log(c) + 8*b^2*log(c)
^2 - 4*a*b*n + 16*a*b*log(c) + 8*a^2)*(d*sqrt(x) + e)^4/(e^3*x^2) + 32*(2*b
^2*d*n^2 - 6*b^2*d*n*log(c) + 9*b^2*d*log(c)^2 - 6*a*b*d*n + 18*a*b*d*log(c
) + 9*a^2*d)*(d*sqrt(x) + e)^3/(e^3*x^(3/2)) - 216*(b^2*d^2*n^2 - 2*b^2*d^2
*n*log(c) + 2*b^2*d^2*log(c)^2 - 2*a*b*d^2*n + 4*a*b*d^2*log(c) + 2*a^2*d^2
)*(d*sqrt(x) + e)^2/(e^3*x) + 288*(2*b^2*d^3*n^2 - 2*b^2*d^3*n*log(c) + b^2
*d^3*log(c)^2 - 2*a*b*d^3*n + 2*a*b*d^3*log(c) + a^2*d^3)*(d*sqrt(x) + e)/(
e^3*sqrt(x)))/e
```

Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.24

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^3}\,dx = \ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\left(\frac{\frac{b\,d\,(4\,a-b\,n)}{3\,e}-\frac{4\,a\,b\,d}{3\,e}}{x^{3/2}}-\frac{b\,(4\,a-b\,n)}{4\,x^2}-\frac{d\left(\frac{b\,d\,(4\,a-b\,n)}{e}-\frac{4\,a\,b\,d}{e}\right)}{2\,e\,x}+\frac{d^2\left(\frac{b\,d\,(4\,a-b\,n)}{e}-\frac{4\,a\,b\,d}{e}\right)}{e^2\,\sqrt{x}}\right)+\frac{\frac{d\left(2\,a^2-a\,b\,n+\frac{b^2\,n^2}{4}\right)}{3\,e}-\frac{d\,(6\,a^2-b^2\,n^2)}{9\,e}}{x^{3/2}}-\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2\left(\frac{b^2}{2\,x^2}-\frac{b^2\,d^4}{2\,e^4}\right)-\frac{\frac{a^2}{2}-\frac{a\,b\,n}{4}+\frac{b^2\,n^2}{16}}{x^2}-\frac{\frac{d\left(\frac{d\left(2\,a^2-a\,b\,n+\frac{b^2\,n^2}{4}\right)}{e}-\frac{d\,(6\,a^2-b^2\,n^2)}{3\,e}\right)}{2\,e}+\frac{b^2\,d^2\,n^2}{4\,e^2}}{x}+\frac{\frac{d\left(\frac{d\left(\frac{d\left(2\,a^2-a\,b\,n+\frac{b^2\,n^2}{4}\right)}{e}-\frac{d\,(6\,a^2-b^2\,n^2)}{3\,e}\right)}{e}+\frac{b^2\,d^2\,n^2}{2\,e^2}\right)}{e}+\frac{b^2\,d^3\,n^2}{e^3}}{\sqrt{x}}-\frac{\ln\left(d+\frac{e}{\sqrt{x}}\right)(25\,b^2\,d^4\,n^2-12\,a\,b\,d^4\,n)}{12\,e^4}$$

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^2/x^3,x)
```

```
[Out] log(c*(d + e/x^(1/2))^n)*(((b*d*(4*a - b*n))/(3*e) - (4*a*b*d)/(3*e))/x^(3/
2) - (b*(4*a - b*n))/(4*x^2) - (d*((b*d*(4*a - b*n))/e - (4*a*b*d)/e))/(2*e
*x) + (d^2*((b*d*(4*a - b*n))/e - (4*a*b*d)/e))/(e^2*x^(1/2))) + ((d*(2*a^2
 + (b^2*n^2)/4 - a*b*n))/(3*e) - (d*(6*a^2 - b^2*n^2))/(9*e))/x^(3/2) - log
(c*(d + e/x^(1/2))^n)^2*(b^2/(2*x^2) - (b^2*d^4)/(2*e^4)) - (a^2/2 + (b^2*n
^2)/16 - (a*b*n)/4)/x^2 - ((d*((d*(2*a^2 + (b^2*n^2)/4 - a*b*n))/e - (d*(6*
a^2 - b^2*n^2))/(3*e)))/(2*e) + (b^2*d^2*n^2)/(4*e^2))/x + ((d*((d*((d*(2*a
^2 + (b^2*n^2)/4 - a*b*n))/e - (d*(6*a^2 - b^2*n^2))/(3*e)))/e + (b^2*d^2*n
^2)/(2*e^2)))/e + (b^2*d^3*n^2)/e^3)/x^(1/2) - (log(d + e/x^(1/2))*(25*b^2*
d^4*n^2 - 12*a*b*d^4*n))/(12*e^4)
```

3.435 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 480

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx = -\frac{5b^2d^4n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^6}+\frac{40b^2d^3n^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6} - \frac{5b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{8e^6}+\frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^5}{25e^6} - \frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^6}{54e^6}+\frac{4b^2d^5n^2}{e^5\sqrt{x}}-\frac{b^2d^6n^2\log^2\left(d+\frac{e}{\sqrt{x}}\right)}{3e^6} - \frac{4bd^5n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^6} + \frac{5bd^4n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^6} - \frac{40bd^3n\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9e^6} + \frac{5bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^6} - \frac{4bdn\left(d+\frac{e}{\sqrt{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{5e^6} + \frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9e^6} + \frac{2bd^6n\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^6} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{3x^3}$$

```
[Out] -1/3*b^2*d^6*n^2*ln(d+e/x^(1/2))^2/e^6+2/3*b*d^6*n*ln(d+e/x^(1/2))*(a+b*ln(
c*(d+e/x^(1/2))^n))/e^6-1/3*(a+b*ln(c*(d+e/x^(1/2))^n))^2/x^3-4*b*d^5*n*(a+
b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))/e^6-5/2*b^2*d^4*n^2*(d+e/x^(1/2))^2/
e^6+5*b*d^4*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^2/e^6+40/27*b^2*d^3
*n^2*(d+e/x^(1/2))^3/e^6-40/9*b*d^3*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1
/2))^3/e^6-5/8*b^2*d^2*n^2*(d+e/x^(1/2))^4/e^6+5/2*b*d^2*n*(a+b*ln(c*(d+e/x
^(1/2))^n))*(d+e/x^(1/2))^4/e^6+4/25*b^2*d*n^2*(d+e/x^(1/2))^5/e^6-4/5*b*d*
n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^5/e^6-1/54*b^2*n^2*(d+e/x^(1/2)
)^6/e^6+1/9*b*n*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^6/e^6+4*b^2*d^5*n
^2/e^5/x^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx = \frac{2bd^6n\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^6} - \frac{4bd^5n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^6} + \frac{5bd^4n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^6} - \frac{40bd^3n\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9e^6} + \frac{5bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^6} - \frac{4bdn\left(d+\frac{e}{\sqrt{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{5e^6} + \frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9e^6} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{3x^3} - \frac{b^2d^6n^2\log^2\left(d+\frac{e}{\sqrt{x}}\right)}{3e^6} + \frac{4b^2d^5n^2}{e^5\sqrt{x}} - \frac{5b^2d^4n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^6} + \frac{40b^2d^3n^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6} - \frac{5b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{8e^6} + \frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^5}{25e^6} - \frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^6}{54e^6}$$

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^4,x]
```

```
[Out] (-5*b^2*d^4*n^2*(d + e/Sqrt[x])^2)/(2*e^6) + (40*b^2*d^3*n^2*(d + e/Sqrt[x]
)^3)/(27*e^6) - (5*b^2*d^2*n^2*(d + e/Sqrt[x])^4)/(8*e^6) + (4*b^2*d*n^2*(d
 + e/Sqrt[x])^5)/(25*e^6) - (b^2*n^2*(d + e/Sqrt[x])^6)/(54*e^6) + (4*b^2*d
^5*n^2)/(e^5*Sqrt[x]) - (b^2*d^6*n^2*Log[d + e/Sqrt[x]]^2)/(3*e^6) - (4*b*d
^5*n*(d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n]))/e^6 + (5*b*d^4*n*(d
+ e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/e^6 - (40*b*d^3*n*(d + e/S
```

```
qrt[x])^3*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(9*e^6) + (5*b*d^2*n*(d + e/Sqr
t[x])^4*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(2*e^6) - (4*b*d*n*(d + e/Sqrt[x]
)^5*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(5*e^6) + (b*n*(d + e/Sqrt[x])^6*(a +
 b*Log[c*(d + e/Sqrt[x])^n]))/(9*e^6) + (2*b*d^6*n*Log[d + e/Sqrt[x]]*(a +
b*Log[c*(d + e/Sqrt[x])^n]))/(3*e^6) - (a + b*Log[c*(d + e/Sqrt[x])^n])^2/(
3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(2\text{Subst}\left(\int x^5(a+b\log\left(c(d+ex)^n\right))^2\,dx,x,\frac{1}{\sqrt{x}}\right)\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{3x^3}+\frac{1}{3}(2ben)\text{Subst}\left(\int\frac{x^6(a+b\log\left(c(d+ex)^n\right))}{d+ex}\,dx,x,\frac{1}{\sqrt{x}}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{3x^3}+\frac{1}{3}(2bn)\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6\left(a+b\log\left(cx^n\right)\right)}{x}\,dx,x,d+\frac{e}{\sqrt{x}}\right)
\end{aligned}
$$

$$
\begin{aligned}
= &-\frac{4bd^5n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^6} \\
&+\frac{5bd^4n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^6} \\
&-\frac{40bd^3n\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9e^6} \\
&+\frac{5bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^6} \\
&-\frac{4bdn\left(d+\frac{e}{\sqrt{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{5e^6} \\
&+\frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9e^6} \\
&+\frac{2bd^6n\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^6}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{3x^3} \\
&-\frac{1}{3}\left(2b^2n^2\right)\mathrm{Subst}\left(\int\frac{x\left(-360d^5+450d^4x-400d^3x^2+225d^2x^3-72dx^4+10x^5\right)+60d^6\log(x)}{60e^6x}\,d\right. \\
&\qquad\left.+\frac{e}{\sqrt{x}}\right)
\end{aligned}
$$

$$
\begin{aligned}
= &-\frac{4bd^5n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^6} \\
&+\frac{5bd^4n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^6} \\
&-\frac{40bd^3n\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9e^6} \\
&+\frac{5bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^6} \\
&-\frac{4bdn\left(d+\frac{e}{\sqrt{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{5e^6} \\
&+\frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9e^6} \\
&+\frac{2bd^6n\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^6}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{3x^3} \\
&-\frac{\left(b^2n^2\right)\mathrm{Subst}\left(\int\frac{x\left(-360d^5+450d^4x-400d^3x^2+225d^2x^3-72dx^4+10x^5\right)+60d^6\log(x)}{x}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{90e^6}
\end{aligned}
$$

$$= -\frac{4bd^5n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^6} + \frac{5bd^4n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^6} - \frac{40bd^3n\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9e^6} + \frac{5bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^6} - \frac{4bdn\left(d+\frac{e}{\sqrt{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{5e^6} + \frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9e^6} + \frac{2bd^6n\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^6} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{3x^3} - \frac{(b^2n^2)\,\mathrm{Subst}\left(\int\left(-360d^5+450d^4x-400d^3x^2+225d^2x^3-72dx^4+10x^5+\frac{60d^6\log(x)}{x}\right)dx, x, d+\right.}{90e^6}$$

$$
\begin{aligned}
&= -\frac{5b^2d^4n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^6} + \frac{40b^2d^3n^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6} - \frac{5b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{8e^6} \\
&\quad + \frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^5}{25e^6} - \frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^6}{54e^6} + \frac{4b^2d^5n^2}{e^5\sqrt{x}} \\
&\quad - \frac{4bd^5n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^6} \\
&\quad + \frac{5bd^4n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^6} \\
&\quad - \frac{40bd^3n\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9e^6} \\
&\quad + \frac{5bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^6} \\
&\quad - \frac{4bdn\left(d+\frac{e}{\sqrt{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{5e^6} \\
&\quad + \frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9e^6} \\
&\quad + \frac{2bd^6n\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^6} \\
&\quad - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{3x^3} - \frac{(2b^2d^6n^2)\,\mathrm{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{3e^6}
\end{aligned}
$$

$$= -\frac{5b^2d^4n^2\left(d+\frac{e}{\sqrt{x}}\right)^2}{2e^6} + \frac{40b^2d^3n^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6} - \frac{5b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{8e^6} + \frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^5}{25e^6} - \frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^6}{54e^6} + \frac{4b^2d^5n^2}{e^5\sqrt{x}} - \frac{b^2d^6n^2\log^2\left(d+\frac{e}{\sqrt{x}}\right)}{3e^6} - \frac{4bd^5n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^6} + \frac{5bd^4n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{e^6} - \frac{40bd^3n\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9e^6} + \frac{5bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^6} - \frac{4bdn\left(d+\frac{e}{\sqrt{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{5e^6} + \frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9e^6} + \frac{2bd^6n\log\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^6} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{3x^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.37 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.14

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx = \frac{-1800\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 + \frac{bn\left(600ae^6-100be^6n-720ade^5\sqrt{x}+264bde^5n\sqrt{x}+900ad^2e^4x-555bd^2e^4nx-1200ad^3e^3x^{3/2}+1\cdots\right.}{}}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^2/x^4,x]
```

```
[Out] (-1800*(a + b*Log[c*(d + e/Sqrt[x])^n])^2 + (b*n*(600*a*e^6 - 100*b*e^6*n -
 720*a*d*e^5*Sqrt[x] + 264*b*d*e^5*n*Sqrt[x] + 900*a*d^2*e^4*x - 555*b*d^2*
e^4*n*x - 1200*a*d^3*e^3*x^(3/2) + 1140*b*d^3*e^3*n*x^(3/2) + 1800*a*d^4*e^
2*x^2 - 2610*b*d^4*e^2*n*x^2 - 3600*a*d^5*e*x^(5/2) + 8820*b*d^5*e*n*x^(5/2
) - 8820*b*d^6*n*x^3*Log[d + e/Sqrt[x]] + 600*b*e^6*Log[c*(d + e/Sqrt[x])^n
] - 720*b*d*e^5*Sqrt[x]*Log[c*(d + e/Sqrt[x])^n] + 900*b*d^2*e^4*x*Log[c*(d
```

```
 + e/Sqrt[x])^n] - 1200*b*d^3*e^3*x^(3/2)*Log[c*(d + e/Sqrt[x])^n] + 1800*b
*d^4*e^2*x^2*Log[c*(d + e/Sqrt[x])^n] - 3600*b*d^5*e*x^(5/2)*Log[c*(d + e/S
qrt[x])^n] + 3600*a*d^6*x^3*Log[e + d*Sqrt[x]] + 3600*b*d^6*x^3*Log[c*(d +
e/Sqrt[x])^n]*Log[e + d*Sqrt[x]] - 1800*b*d^6*n*x^3*Log[e + d*Sqrt[x]]^2 +
3600*b*d^6*x^3*Log[c*(d + e/Sqrt[x])^n]*Log[-(e/(d*Sqrt[x]))] + 3600*b*d^6*
n*x^3*Log[e + d*Sqrt[x]]*Log[-((d*Sqrt[x])/e)] - 1800*a*d^6*x^3*Log[x] + 36
00*b*d^6*n*x^3*PolyLog[2, 1 + e/(d*Sqrt[x])] + 3600*b*d^6*n*x^3*PolyLog[2,
1 + (d*Sqrt[x])/e]))/e^6)/(5400*x^3)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x^{4}}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^2/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{2}}{x^{4}}\,dx = -\frac{100\,b^{2}e^{6}n^{2}+1800\,b^{2}e^{6}\log\left(c\right)^{2}-600\,abe^{6}n+1800\,a^{2}e^{6}+90\,(29\,b^{2}d^{4}e^{2}n^{2}-20\,abd^{4}e^{2}n)x^{2}-1800\,(b^{2}d^{6}}{}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/5400*(100*b^2*e^6*n^2 + 1800*b^2*e^6*log(c)^2 - 600*a*b*e^6*n + 1800*a^2
*e^6 + 90*(29*b^2*d^4*e^2*n^2 - 20*a*b*d^4*e^2*n)*x^2 - 1800*(b^2*d^6*n^2*x
^3 - b^2*e^6*n^2)*log((d*x + e*sqrt(x))/x)^2 + 15*(37*b^2*d^2*e^4*n^2 - 60*
a*b*d^2*e^4*n)*x - 300*(6*b^2*d^4*e^2*n*x^2 + 3*b^2*d^2*e^4*n*x + 2*b^2*e^6
*n - 12*a*b*e^6)*log(c) - 60*(30*b^2*d^4*e^2*n^2*x^2 + 15*b^2*d^2*e^4*n^2*x
 + 10*b^2*e^6*n^2 - 60*a*b*e^6*n - 3*(49*b^2*d^6*n^2 - 20*a*b*d^6*n)*x^3 +
60*(b^2*d^6*n*x^3 - b^2*e^6*n)*log(c) - 4*(15*b^2*d^5*e*n^2*x^2 + 5*b^2*d^3
*e^3*n^2*x + 3*b^2*d*e^5*n^2)*sqrt(x))*log((d*x + e*sqrt(x))/x) - 12*(22*b^
2*d*e^5*n^2 - 60*a*b*d*e^5*n + 15*(49*b^2*d^5*e*n^2 - 20*a*b*d^5*e*n)*x^2 +
 5*(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)*x - 20*(15*b^2*d^5*e*n*x^2 + 5*b
^2*d^3*e^3*n*x + 3*b^2*d*e^5*n)*log(c))*sqrt(x))/(e^6*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**2/x**4,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx$$
$$= \frac{1}{90}aben\left(\frac{60\,d^6\log\left(d\sqrt{x}+e\right)}{e^7}-\frac{30\,d^6\log\left(x\right)}{e^7}-\frac{60\,d^5x^{\frac{5}{2}}-30\,d^4ex^2+20\,d^3e^2x^{\frac{3}{2}}-15\,d^2e^3x+12\,de^4\sqrt{x}-}{e^6x^3}\right.$$
$$+\frac{1}{5400}\left(60\,en\left(\frac{60\,d^6\log\left(d\sqrt{x}+e\right)}{e^7}-\frac{30\,d^6\log\left(x\right)}{e^7}-\frac{60\,d^5x^{\frac{5}{2}}-30\,d^4ex^2+20\,d^3e^2x^{\frac{3}{2}}-15\,d^2e^3x+12\,de}{e^6x^3}\right.\right.$$
$$-\frac{b^2\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2}{3\,x^3}-\frac{2\,ab\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3\,x^3}-\frac{a^2}{3\,x^3}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^4,x, algorithm="maxima")
[Out] 1/90*a*b*e*n*(60*d^6*log(d*sqrt(x) + e)/e^7 - 30*d^6*log(x)/e^7 - (60*d^5*x
^(5/2) - 30*d^4*e*x^2 + 20*d^3*e^2*x^(3/2) - 15*d^2*e^3*x + 12*d*e^4*sqrt(x
) - 10*e^5)/(e^6*x^3)) + 1/5400*(60*e*n*(60*d^6*log(d*sqrt(x) + e)/e^7 - 30
*d^6*log(x)/e^7 - (60*d^5*x^(5/2) - 30*d^4*e*x^2 + 20*d^3*e^2*x^(3/2) - 15*
d^2*e^3*x + 12*d*e^4*sqrt(x) - 10*e^5)/(e^6*x^3))*log(c*(d + e/sqrt(x))^n)
- (1800*d^6*x^3*log(d*sqrt(x) + e)^2 + 450*d^6*x^3*log(x)^2 - 4410*d^6*x^3*
log(x) - 8820*d^5*e*x^(5/2) + 2610*d^4*e^2*x^2 - 1140*d^3*e^3*x^(3/2) + 555
*d^2*e^4*x - 264*d*e^5*sqrt(x) + 100*e^6 - 180*(10*d^6*x^3*log(x) - 49*d^6*
x^3)*log(d*sqrt(x) + e))*n^2/(e^6*x^3))*b^2 - 1/3*b^2*log(c*(d + e/sqrt(x))
^n)^2/x^3 - 2/3*a*b*log(c*(d + e/sqrt(x))^n)/x^3 - 1/3*a^2/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 877 vs. $2(412) = 824$.

Time = 0.41 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.83

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^2/x^4,x, algorithm="giac")
```

```
[Out] 1/5400*(1800*(6*(d*sqrt(x) + e)*b^2*d^5*n^2/(e^5*sqrt(x)) - 15*(d*sqrt(x) +
 e)^2*b^2*d^4*n^2/(e^5*x) + 20*(d*sqrt(x) + e)^3*b^2*d^3*n^2/(e^5*x^(3/2))
- 15*(d*sqrt(x) + e)^4*b^2*d^2*n^2/(e^5*x^2) + 6*(d*sqrt(x) + e)^5*b^2*d*n^
2/(e^5*x^(5/2)) - (d*sqrt(x) + e)^6*b^2*n^2/(e^5*x^3))*log((d*sqrt(x) + e)/
sqrt(x))^2 + 60*(10*(b^2*n^2 - 6*b^2*n*log(c) - 6*a*b*n)*(d*sqrt(x) + e)^6/
(e^5*x^3) - 72*(b^2*d*n^2 - 5*b^2*d*n*log(c) - 5*a*b*d*n)*(d*sqrt(x) + e)^5
/(e^5*x^(5/2)) + 225*(b^2*d^2*n^2 - 4*b^2*d^2*n*log(c) - 4*a*b*d^2*n)*(d*sq
rt(x) + e)^4/(e^5*x^2) - 400*(b^2*d^3*n^2 - 3*b^2*d^3*n*log(c) - 3*a*b*d^3*
n)*(d*sqrt(x) + e)^3/(e^5*x^(3/2)) + 450*(b^2*d^4*n^2 - 2*b^2*d^4*n*log(c)
- 2*a*b*d^4*n)*(d*sqrt(x) + e)^2/(e^5*x) - 360*(b^2*d^5*n^2 - b^2*d^5*n*log
(c) - a*b*d^5*n)*(d*sqrt(x) + e)/(e^5*sqrt(x)))*log((d*sqrt(x) + e)/sqrt(x)
) - 100*(b^2*n^2 - 6*b^2*n*log(c) + 18*b^2*log(c)^2 - 6*a*b*n + 36*a*b*log(
c) + 18*a^2)*(d*sqrt(x) + e)^6/(e^5*x^3) + 432*(2*b^2*d*n^2 - 10*b^2*d*n*lo
g(c) + 25*b^2*d*log(c)^2 - 10*a*b*d*n + 50*a*b*d*log(c) + 25*a^2*d)*(d*sqrt
(x) + e)^5/(e^5*x^(5/2)) - 3375*(b^2*d^2*n^2 - 4*b^2*d^2*n*log(c) + 8*b^2*d
^2*log(c)^2 - 4*a*b*d^2*n + 16*a*b*d^2*log(c) + 8*a^2*d^2)*(d*sqrt(x) + e)^
4/(e^5*x^2) + 4000*(2*b^2*d^3*n^2 - 6*b^2*d^3*n*log(c) + 9*b^2*d^3*log(c)^2
 - 6*a*b*d^3*n + 18*a*b*d^3*log(c) + 9*a^2*d^3)*(d*sqrt(x) + e)^3/(e^5*x^(3
/2)) - 13500*(b^2*d^4*n^2 - 2*b^2*d^4*n*log(c) + 2*b^2*d^4*log(c)^2 - 2*a*b
*d^4*n + 4*a*b*d^4*log(c) + 2*a^2*d^4)*(d*sqrt(x) + e)^2/(e^5*x) + 10800*(2
*b^2*d^5*n^2 - 2*b^2*d^5*n*log(c) + b^2*d^5*log(c)^2 - 2*a*b*d^5*n + 2*a*b*
d^5*log(c) + a^2*d^5)*(d*sqrt(x) + e)/(e^5*sqrt(x)))/e
```

Mupad [B] (verification not implemented)

Time = 3.00 (sec) , antiderivative size = 440, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{x^4}\,dx = \frac{b^2\,d^6\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2}{3\,e^6}-\frac{b^2\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2}{3\,x^3}-\frac{b^2\,n^2}{54\,x^3}-\frac{2\,a\,b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3\,x^3}-\frac{a^2}{3\,x^3}+\frac{a\,b\,n}{9\,x^3}+\frac{b^2\,n\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{9\,x^3}-\frac{49\,b^2\,d^6\,n^2\,\ln\left(d+\frac{e}{\sqrt{x}}\right)}{30\,e^6}-\frac{37\,b^2\,d^2\,n^2}{360\,e^2\,x^2}-\frac{29\,b^2\,d^4\,n^2}{60\,e^4\,x}+\frac{19\,b^2\,d^3\,n^2}{90\,e^3\,x^{3/2}}+\frac{49\,b^2\,d^5\,n^2}{30\,e^5\,\sqrt{x}}+\frac{11\,b^2\,d\,n^2}{225\,e\,x^{5/2}}+\frac{b^2\,d^2\,n\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{6\,e^2\,x^2}+\frac{b^2\,d^4\,n\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3\,e^4\,x}-\frac{2\,b^2\,d^3\,n\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{9\,e^3\,x^{3/2}}-\frac{2\,b^2\,d^5\,n\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{3\,e^5\,\sqrt{x}}-\frac{2\,a\,b\,d\,n}{15\,e\,x^{5/2}}+\frac{2\,a\,b\,d^6\,n\,\ln\left(d+\frac{e}{\sqrt{x}}\right)}{3\,e^6}-\frac{2\,b^2\,d\,n\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{15\,e\,x^{5/2}}+\frac{a\,b\,d^2\,n}{6\,e^2\,x^2}+\frac{a\,b\,d^4\,n}{3\,e^4\,x}-\frac{2\,a\,b\,d^3\,n}{9\,e^3\,x^{3/2}}-\frac{2\,a\,b\,d^5\,n}{3\,e^5\,\sqrt{x}}$$

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^2/x^4,x)
```

```
[Out] (b^2*d^6*log(c*(d + e/x^(1/2))^n)^2)/(3*e^6) - (b^2*log(c*(d + e/x^(1/2))^n
)^2)/(3*x^3) - (b^2*n^2)/(54*x^3) - (2*a*b*log(c*(d + e/x^(1/2))^n))/(3*x^3
) - a^2/(3*x^3) + (a*b*n)/(9*x^3) + (b^2*n*log(c*(d + e/x^(1/2))^n))/(9*x^3
) - (49*b^2*d^6*n^2*log(d + e/x^(1/2)))/(30*e^6) - (37*b^2*d^2*n^2)/(360*e^
2*x^2) - (29*b^2*d^4*n^2)/(60*e^4*x) + (19*b^2*d^3*n^2)/(90*e^3*x^(3/2)) +
(49*b^2*d^5*n^2)/(30*e^5*x^(1/2)) + (11*b^2*d*n^2)/(225*e*x^(5/2)) + (b^2*d
^2*n*log(c*(d + e/x^(1/2))^n))/(6*e^2*x^2) + (b^2*d^4*n*log(c*(d + e/x^(1/2
))^n))/(3*e^4*x) - (2*b^2*d^3*n*log(c*(d + e/x^(1/2))^n))/(9*e^3*x^(3/2)) -
 (2*b^2*d^5*n*log(c*(d + e/x^(1/2))^n))/(3*e^5*x^(1/2)) - (2*a*b*d*n)/(15*e
*x^(5/2)) + (2*a*b*d^6*n*log(d + e/x^(1/2)))/(3*e^6) - (2*b^2*d*n*log(c*(d
+ e/x^(1/2))^n))/(15*e*x^(5/2)) + (a*b*d^2*n)/(6*e^2*x^2) + (a*b*d^4*n)/(3*
e^4*x) - (2*a*b*d^3*n)/(9*e^3*x^(3/2)) - (2*a*b*d^5*n)/(3*e^5*x^(1/2))
```

3.436 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 569

$$\begin{aligned}
&\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx\\
&=\frac{b^3e^3n^3\sqrt{x}}{2d^3}-\frac{b^3e^4n^3\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4}-\frac{5b^2e^3n^2\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^4}\\
&\quad+\frac{b^2e^2n^2x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^2}\\
&\quad-\frac{5b^2e^4n^2\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^4}\\
&\quad+\frac{3be^3n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d^4}\\
&\quad-\frac{3be^2nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{4d^2}+\frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d}\\
&\quad+\frac{3be^4n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d^4}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\\
&\quad-\frac{3b^2e^4n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt{x}}\right)}{d^4}-\frac{3b^3e^4n^3\log(x)}{2d^4}\\
&\quad+\frac{5b^3e^4n^3\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{2d^4}-\frac{3b^2e^4n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^4}\\
&\quad-\frac{3b^3e^4n^3\operatorname{PolyLog}\left(2,1+\frac{e}{d\sqrt{x}}\right)}{d^4}-\frac{3b^3e^4n^3\operatorname{PolyLog}\left(3,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^4}
\end{aligned}$$

```
[Out] -3/2*b^3*e^4*n^3*ln(x)/d^4-1/2*b^3*e^4*n^3*ln(d+e/x^(1/2))/d^4+1/2*b^2*e^2*
n^2*x*(a+b*ln(c*(d+e/x^(1/2))^n))/d^2-5/2*b^2*e^4*n^2*ln(1-d/(d+e/x^(1/2)))
*(a+b*ln(c*(d+e/x^(1/2))^n))/d^4-3/4*b*e^2*n*x*(a+b*ln(c*(d+e/x^(1/2))^n))^
2/d^2+1/2*b*e*n*x^(3/2)*(a+b*ln(c*(d+e/x^(1/2))^n))^2/d+3/2*b*e^4*n*ln(1-d/
(d+e/x^(1/2)))*(a+b*ln(c*(d+e/x^(1/2))^n))^2/d^4+1/2*x^2*(a+b*ln(c*(d+e/x^(
1/2))^n))^3-3*b^2*e^4*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*ln(-e/d/x^(1/2))/d^4+
5/2*b^3*e^4*n^3*polylog(2,d/(d+e/x^(1/2)))/d^4-3*b^2*e^4*n^2*(a+b*ln(c*(d+e
/x^(1/2))^n))*polylog(2,d/(d+e/x^(1/2)))/d^4-3*b^3*e^4*n^3*polylog(2,1+e/d/
x^(1/2))/d^4-3*b^3*e^4*n^3*polylog(3,d/(d+e/x^(1/2)))/d^4+1/2*b^3*e^3*n^3*x
^(1/2)/d^3-5/2*b^2*e^3*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))*x^(1/2
)/d^4+3/2*b*e^3*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(1/2))*x^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31, 46}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3dx$$
$$=-\frac{3b^2e^4n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4}-\frac{5b^2e^4n^2\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^4}-\frac{3b^2e^4n^2\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^4}-\frac{5b^2e^3n^2\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^4}+\frac{b^2e^2n^2x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^2}+\frac{3be^4n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d^4}+\frac{3be^3n\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d^4}-\frac{3be^2nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{4d^2}+\frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3+\frac{5b^3e^4n^3\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{2d^4}-\frac{3b^3e^4n^3\operatorname{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)}{d^4}-\frac{3b^3e^4n^3\operatorname{PolyLog}\left(3,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^4}-\frac{b^3e^4n^3\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4}-\frac{3b^3e^4n^3\log(x)}{2d^4}+\frac{b^3e^3n^3\sqrt{x}}{2d^3}$$

```
[In] Int[x*(a + b*Log[c*(d + e/Sqrt[x])^n])^3,x]
```

```
[Out] (b^3*e^3*n^3*Sqrt[x])/(2*d^3) - (b^3*e^4*n^3*Log[d + e/Sqrt[x]])/(2*d^4) -
(5*b^2*e^3*n^2*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(2
*d^4) + (b^2*e^2*n^2*x*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(2*d^2) - (5*b^2*e
^4*n^2*Log[1 - d/(d + e/Sqrt[x])]*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(2*d^4)
 + (3*b*e^3*n*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(
2*d^4) - (3*b*e^2*n*x*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(4*d^2) + (b*e*n*
x^(3/2)*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(2*d) + (3*b*e^4*n*Log[1 - d/(d
 + e/Sqrt[x])]*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(2*d^4) + (x^2*(a + b*Lo
g[c*(d + e/Sqrt[x])^n])^3)/2 - (3*b^2*e^4*n^2*(a + b*Log[c*(d + e/Sqrt[x])^
n])*Log[-(e/(d*Sqrt[x]))])/d^4 - (3*b^3*e^4*n^3*Log[x])/(2*d^4) + (5*b^3*e^
4*n^3*PolyLog[2, d/(d + e/Sqrt[x])])/(2*d^4) - (3*b^2*e^4*n^2*(a + b*Log[c*
(d + e/Sqrt[x])^n])*PolyLog[2, d/(d + e/Sqrt[x])])/d^4 - (3*b^3*e^4*n^3*Pol
yLog[2, 1 + e/(d*Sqrt[x])])/d^4 - (3*b^3*e^4*n^3*PolyLog[3, d/(d + e/Sqrt[x
])])/d^4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
```

```
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
```

```
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(2\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^5}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3-\frac{1}{2}(3ben)\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4(d+ex)}\,dx, x, \frac{1}{\sqrt{x}}\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3-\frac{1}{2}(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+\frac{e}{\sqrt{x}}\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3-\frac{(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{2d}\\
&\quad+\frac{(3ben)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{2d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d} \\
&\quad + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 + \frac{(3ben)\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}dx, x, d+\frac{e}{\sqrt{x}}\right)}{2d^2} \\
&\quad - \frac{(3be^2n)\,\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}dx, x, d+\frac{e}{\sqrt{x}}\right)}{2d^2} \\
&\quad - \frac{(b^2en^2)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}dx, x, d+\frac{e}{\sqrt{x}}\right)}{d} \\
&= -\frac{3be^2nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{4d^2} + \frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d} \\
&\quad + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 - \frac{(3be^2n)\,\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}dx, x, d+\frac{e}{\sqrt{x}}\right)}{2d^3} \\
&\quad + \frac{(3be^3n)\,\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}dx, x, d+\frac{e}{\sqrt{x}}\right)}{2d^3} \\
&\quad - \frac{(b^2en^2)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}dx, x, d+\frac{e}{\sqrt{x}}\right)}{d^2} \\
&\quad + \frac{(b^2e^2n^2)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}dx, x, d+\frac{e}{\sqrt{x}}\right)}{d^2} \\
&\quad + \frac{(3b^2e^2n^2)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}dx, x, d+\frac{e}{\sqrt{x}}\right)}{2d^2}
\end{aligned}
$$

$$= \frac{b^2e^2n^2x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^2} + \frac{3be^3n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d^4} - \frac{3be^2nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{4d^2} + \frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d} + \frac{3be^4n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d^4} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 + \frac{(b^2e^2n^2)\,\mathrm{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{d^3} + \frac{(3b^2e^2n^2)\,\mathrm{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{2d^3} - \frac{(3b^2e^3n^2)\,\mathrm{Subst}\left(\int\frac{a+b\log(cx^n)}{-\frac{d}{e}+\frac{x}{e}}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{d^4} - \frac{(b^2e^3n^2)\,\mathrm{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{d^3} - \frac{(3b^2e^3n^2)\,\mathrm{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{2d^3} - \frac{(3b^2e^4n^2)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{d}{x}\right)(a+b\log(cx^n))}{x}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{d^4} - \frac{(b^3e^2n^3)\,\mathrm{Subst}\left(\int\frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{2d^2}$$

$$= -\frac{5b^2e^3n^2\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^4} + \frac{b^2e^2n^2x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^2} - \frac{5b^2e^4n^2\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^4} + \frac{3be^3n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d^4} - \frac{3be^2nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{4d^2} + \frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d} + \frac{3be^4n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d^4} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 - \frac{3b^2e^4n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt{x}}\right)}{d^4} - \frac{3b^2e^4n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^4} - \frac{(b^3e^2n^3)\,\mathrm{Subst}\left(\int\left(\frac{e^2}{d(d-x)^2}+\frac{e^2}{d^2(d-x)}+\frac{e^2}{d^2x}\right)dx, x, d+\frac{e}{\sqrt{x}}\right)}{2d^2} + \frac{(b^3e^3n^3)\,\mathrm{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x}{e}}dx, x, d+\frac{e}{\sqrt{x}}\right)}{d^4} + \frac{(3b^3e^3n^3)\,\mathrm{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x}{e}}dx, x, d+\frac{e}{\sqrt{x}}\right)}{2d^4} + \frac{(b^3e^4n^3)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{d}{x}\right)}{x}dx, x, d+\frac{e}{\sqrt{x}}\right)}{d^4} + \frac{(3b^3e^4n^3)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{d}{x}\right)}{x}dx, x, d+\frac{e}{\sqrt{x}}\right)}{2d^4} + \frac{(3b^3e^4n^3)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{x}{d}\right)}{x}dx, x, d+\frac{e}{\sqrt{x}}\right)}{d^4} + \frac{(3b^3e^4n^3)\,\mathrm{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{d}{x}\right)}{x}dx, x, d+\frac{e}{\sqrt{x}}\right)}{d^4}$$

$$= \frac{b^3e^3n^3\sqrt{x}}{2d^3} - \frac{b^3e^4n^3\log\left(d+\frac{e}{\sqrt{x}}\right)}{2d^4} - \frac{5b^2e^3n^2\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^4}$$
$$+ \frac{b^2e^2n^2x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^2}$$
$$- \frac{5b^2e^4n^2\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2d^4}$$
$$+ \frac{3be^3n\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d^4}$$
$$- \frac{3be^2nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{4d^2} + \frac{benx^{3/2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d}$$
$$+ \frac{3be^4n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2d^4}$$
$$+ \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3$$
$$- \frac{3b^2e^4n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt{x}}\right)}{d^4} - \frac{3b^3e^4n^3\log(x)}{2d^4}$$
$$+ \frac{5b^3e^4n^3\text{Li}_2\left(\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{2d^4} - \frac{3b^2e^4n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\text{Li}_2\left(\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^4}$$
$$- \frac{3b^3e^4n^3\text{Li}_2\left(1+\frac{e}{d\sqrt{x}}\right)}{d^4} - \frac{3b^3e^4n^3\text{Li}_3\left(\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^4}$$

Mathematica [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx = \int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx$$

```
[In] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^n])^3,x]
[Out] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^n])^3, x]
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}dx$$

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/2))^n))^3,x)
[Out] int(x*(a+b*ln(c*(d+e/x^(1/2))^n))^3,x)
```

Fricas [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}\,dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a\right)^{3}x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="fricas")
[Out] integral(b^3*x*log(c*((d*x + e*sqrt(x))/x)^n)^3 + 3*a*b^2*x*log(c*((d*x + e
*sqrt(x))/x)^n)^2 + 3*a^2*b*x*log(c*((d*x + e*sqrt(x))/x)^n) + a^3*x, x)
```

Sympy [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}\,dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}\,dx$$

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/2))**n))**3,x)
[Out] Integral(x*(a + b*log(c*(d + e/sqrt(x))**n))**3, x)
```

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}\,dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)+a\right)^{3}x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="maxima")
[Out] 1/2*b^3*x^2*log((d*sqrt(x) + e)^n)^3 - integrate(1/4*(4*(b^3*d*x^2 + b^3*e*
x^(3/2))*log(x^(1/2*n))^3 - 4*(b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*
b*d*log(c) + a^3*d)*x^2 + 3*(b^3*d*n*x^2 - 4*(b^3*d*log(c) + a*b^2*d)*x^2 -
 4*(b^3*e*log(c) + a*b^2*e)*x^(3/2) + 4*(b^3*d*x^2 + b^3*e*x^(3/2))*log(x^(
1/2*n)))*log((d*sqrt(x) + e)^n)^2 - 12*((b^3*d*log(c) + a*b^2*d)*x^2 + (b^3
*e*log(c) + a*b^2*e)*x^(3/2))*log(x^(1/2*n))^2 - 4*(b^3*e*log(c)^3 + 3*a*b^
2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x^(3/2) - 12*((b^3*d*log(c)^2 + 2*
```

```
a*b^2*d*log(c) + a^2*b*d)*x^2 + (b^3*d*x^2 + b^3*e*x^(3/2))*log(x^(1/2*n))^
2 + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(3/2) - 2*((b^3*d*log(c
) + a*b^2*d)*x^2 + (b^3*e*log(c) + a*b^2*e)*x^(3/2))*log(x^(1/2*n)))*log((d
*sqrt(x) + e)^n) + 12*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^2 +
(b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(3/2))*log(x^(1/2*n)))/(d*x
 + e*sqrt(x)), x)
```

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^3x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^3*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx=\int x\,\left(a+b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx$$

```
[In] int(x*(a + b*log(c*(d + e/x^(1/2))^n))^3,x)
[Out] int(x*(a + b*log(c*(d + e/x^(1/2))^n))^3, x)
```

3.437 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 260

$$\begin{aligned}
&\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\,dx \\
&= \frac{3ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} \\
&\quad+\frac{3be^2n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} \\
&\quad+x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3-\frac{6b^2e^2n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt{x}}\right)}{d^2} \\
&\quad-\frac{6b^2e^2n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^2} \\
&\quad-\frac{6b^3e^2n^3\operatorname{PolyLog}\left(2,1+\frac{e}{d\sqrt{x}}\right)}{d^2}-\frac{6b^3e^2n^3\operatorname{PolyLog}\left(3,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^2}
\end{aligned}$$

```
[Out] 3*b*e^2*n*ln(1-d/(d+e/x^(1/2)))*(a+b*ln(c*(d+e/x^(1/2))^n))^2/d^2+x*(a+b*ln
(c*(d+e/x^(1/2))^n))^3-6*b^2*e^2*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*ln(-e/d/x^
(1/2))/d^2-6*b^2*e^2*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*polylog(2,d/(d+e/x^(1/
2)))/d^2-6*b^3*e^2*n^3*polylog(2,1+e/d/x^(1/2))/d^2-6*b^3*e^2*n^3*polylog(3
,d/(d+e/x^(1/2)))/d^2+3*b*e*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(1/2))*x
^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {2501, 2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438}

$$\begin{aligned}
&\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx \\
&= -\frac{6b^2e^2n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2} \\
&\quad -\frac{6b^2e^2n^2\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{d^2} \\
&\quad +\frac{3be^2n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} \\
&\quad +\frac{3ben\sqrt{x}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2}+x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 \\
&\quad -\frac{6b^3e^2n^3\operatorname{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)}{d^2}-\frac{6b^3e^2n^3\operatorname{PolyLog}\left(3,\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^2}
\end{aligned}$$

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3,x]
```

```
[Out] (3*b*e*n*(d + e/Sqrt[x])*Sqrt[x]*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/d^2 +
(3*b*e^2*n*Log[1 - d/(d + e/Sqrt[x])]*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/d
^2 + x*(a + b*Log[c*(d + e/Sqrt[x])^n])^3 - (6*b^2*e^2*n^2*(a + b*Log[c*(d
+ e/Sqrt[x])^n])*Log[-(e/(d*Sqrt[x]))])/d^2 - (6*b^2*e^2*n^2*(a + b*Log[c*(
d + e/Sqrt[x])^n])*PolyLog[2, d/(d + e/Sqrt[x])])/d^2 - (6*b^3*e^2*n^3*Poly
Log[2, 1 + e/(d*Sqrt[x])])/d^2 - (6*b^3*e^2*n^3*PolyLog[3, d/(d + e/Sqrt[x]
)])/d^2
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int x\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^n\right)\right)^3\,dx, x, \sqrt{x}\right)$$

$$= -\left(2\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^3}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3-(3ben)\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^2(d+ex)}\,dx, x, \frac{1}{\sqrt{x}}\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3-(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+\frac{e}{\sqrt{x}}\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3-\frac{(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{d} + \frac{(3ben)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{d}$$

$$= \frac{3ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} + \frac{3be^2n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 - \frac{(6b^2en^2)\,\mathrm{Subst}\left(\int\frac{a+b\log(cx^n)}{-\frac{d}{e}+\frac{x}{e}}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{d^2} - \frac{(6b^2e^2n^2)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{d}{x}\right)(a+b\log(cx^n))}{x}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{d^2}$$

$$= \frac{3ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} + \frac{3be^2n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 - \frac{6b^2e^2n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt{x}}\right)}{d^2} - \frac{6b^2e^2n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\mathrm{Li}_2\left(\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^2} + \frac{(6b^3e^2n^3)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{x}{d}\right)}{x}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{d^2} + \frac{(6b^3e^2n^3)\,\mathrm{Subst}\left(\int\frac{\mathrm{Li}_2\left(\frac{d}{x}\right)}{x}\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{d^2}$$

$$= \frac{3ben\left(d+\frac{e}{\sqrt{x}}\right)\sqrt{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} + \frac{3be^2n\log\left(1-\frac{d}{d+\frac{e}{\sqrt{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{d^2} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 - \frac{6b^2e^2n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt{x}}\right)}{d^2} - \frac{6b^2e^2n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\mathrm{Li}_2\left(\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^2} - \frac{6b^3e^2n^3\mathrm{Li}_2\left(1+\frac{e}{d\sqrt{x}}\right)}{d^2} - \frac{6b^3e^2n^3\mathrm{Li}_3\left(\frac{d}{d+\frac{e}{\sqrt{x}}}\right)}{d^2}$$

Mathematica [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx = \int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3,x]
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3, x]
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^3,x)
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^3,x)
```

Fricas [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx = \int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^3 dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="fricas")
[Out] integral(b^3*log(c*((d*x + e*sqrt(x))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*sqr
t(x))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*sqrt(x))/x)^n) + a^3, x)
```

Sympy [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx = \int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3,x)
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**3, x)
```

Maxima [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx = \int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^3 dx$$

[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="maxima")

[Out] b^3*x*log((d*sqrt(x) + e)^n)^3 - 3*(e*n*(e*log(d*sqrt(x) + e)/d^2 - sqrt(x)/d) - x*log(c*(d + e/sqrt(x))^n))*a^2*b + a^3*x - integrate(1/2*(2*(b^3*d*x + b^3*e*sqrt(x))*log(x^(1/2*n))^3 + 3*(b^3*d*n*x - 2*(b^3*d*log(c) + a*b^2*d)*x + 2*(b^3*d*x + b^3*e*sqrt(x))*log(x^(1/2*n)) - 2*(b^3*e*log(c) + a*b^2*e)*sqrt(x))*log((d*sqrt(x) + e)^n)^2 - 6*((b^3*d*log(c) + a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*sqrt(x))*log(x^(1/2*n))^2 - 2*(b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2)*x - 6*((b^3*d*x + b^3*e*sqrt(x))*log(x^(1/2*n))^2 + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c))*x - 2*((b^3*d*log(c) + a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*sqrt(x))*log(x^(1/2*n)) + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c))*sqrt(x))*log((d*sqrt(x) + e)^n) + 6*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c))*x + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c))*sqrt(x))*log(x^(1/2*n)) - 2*(b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2)*sqrt(x))/(d*x + e*sqrt(x)), x)

Giac [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx = \int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^3 dx$$

[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3,x, algorithm="giac")

[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx = \int \left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 dx$$

[In] int((a + b*log(c*(d + e/x^(1/2))^n))^3,x)

[Out] int((a + b*log(c*(d + e/x^(1/2))^n))^3, x)

3.438 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 135

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx = -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt{x}}\right)-6bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\operatorname{PolyLog}\left(2,1+\frac{e}{d\sqrt{x}}\right)+12b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\operatorname{PolyLog}\left(3,1+\frac{e}{d\sqrt{x}}\right)-12b^3n^3\operatorname{PolyLog}\left(4,1+\frac{e}{d\sqrt{x}}\right)$$

```
[Out] -2*(a+b*ln(c*(d+e/x^(1/2))^n))^3*ln(-e/d/x^(1/2))-6*b*n*(a+b*ln(c*(d+e/x^(1
/2))^n))^2*polylog(2,1+e/d/x^(1/2))+12*b^2*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*
polylog(3,1+e/d/x^(1/2))-12*b^3*n^3*polylog(4,1+e/d/x^(1/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

$= \{2504, 2443, 2481, 2421, 2430, 6724\}$

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx = 12b^2n^2\,\text{PolyLog}\left(3,\frac{e}{d\sqrt{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right) - 6bn\,\text{PolyLog}\left(2,\frac{e}{d\sqrt{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2 - 2\log\left(-\frac{e}{d\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3 - 12b^3n^3\,\text{PolyLog}\left(4,\frac{e}{d\sqrt{x}}+1\right)$$

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x,x]
```

```
[Out] -2*(a + b*Log[c*(d + e/Sqrt[x])^n])^3*Log[-(e/(d*Sqrt[x]))] - 6*b*n*(a + b*
Log[c*(d + e/Sqrt[x])^n])^2*PolyLog[2, 1 + e/(d*Sqrt[x])] + 12*b^2*n^2*(a +
 b*Log[c*(d + e/Sqrt[x])^n])*PolyLog[3, 1 + e/(d*Sqrt[x])] - 12*b^3*n^3*Pol
yLog[4, 1 + e/(d*Sqrt[x])]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(2\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x}\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt{x}}\right)\\
&\quad+(6ben)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{d+ex}\,dx, x, \frac{1}{\sqrt{x}}\right)\\
&= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt{x}}\right)\\
&\quad+(6bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx, x, d+\frac{e}{\sqrt{x}}\right)\\
&= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt{x}}\right)\\
&\quad-6bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\text{Li}_2\left(1+\frac{e}{d\sqrt{x}}\right)\\
&\quad+\left(12b^2n^2\right)\text{Subst}\left(\int \frac{(a+b\log(cx^n))\,\text{Li}_2\left(\frac{x}{d}\right)}{x}\,dx, x, d+\frac{e}{\sqrt{x}}\right)
\end{aligned}
$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt{x}}\right) - 6bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\text{Li}_2\left(1+\frac{e}{d\sqrt{x}}\right) + 12b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\text{Li}_3\left(1+\frac{e}{d\sqrt{x}}\right) - \left(12b^3n^3\right)\text{Subst}\left(\int\frac{\text{Li}_3\left(\frac{x}{d}\right)}{x}\,dx, x, d+\frac{e}{\sqrt{x}}\right)$$

$$= -2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt{x}}\right) - 6bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\text{Li}_2\left(1+\frac{e}{d\sqrt{x}}\right) + 12b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\text{Li}_3\left(1+\frac{e}{d\sqrt{x}}\right) - 12b^3n^3\text{Li}_4\left(1+\frac{e}{d\sqrt{x}}\right)$$

Mathematica [F]

$$\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx = \int\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x,x]
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x, x]
```

Maple [F]

$$\int\frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x,x)
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^3}{x}\,dx$$

[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x,x, algorithm="fricas")

[Out] integral((b^3*log(c*((d*x + e*sqrt(x))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*sqrt(x))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*sqrt(x))/x)^n) + a^3)/x, x)

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx$$

[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3/x,x)

[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**3/x, x)

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^3}{x}\,dx$$

[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x,x, algorithm="maxima")

[Out] b^3*log((d*sqrt(x) + e)^n)^3*log(x) - integrate(1/2*(2*(b^3*d*x + b^3*e*sqrt(x))*log(x^(1/2*n))^3 + 3*(b^3*d*n*x*log(x) - 2*(b^3*d*log(c) + a*b^2*d)*x + 2*(b^3*d*x + b^3*e*sqrt(x))*log(x^(1/2*n)) - 2*(b^3*e*log(c) + a*b^2*e)*sqrt(x))*log((d*sqrt(x) + e)^n)^2 - 6*((b^3*d*log(c) + a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*sqrt(x))*log(x^(1/2*n))^2 - 2*(b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x - 6*((b^3*d*x + b^3*e*sqrt(x))*log(x^(1/2*n))^2 + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x - 2*((b^3*d*log(c) + a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*sqrt(x))*log(x^(1/2*n)) + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*sqrt(x))*log((d*sqrt(x) + e)^n) + 6*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*sqrt(x))*log(x^(1/2*n)) - 2*(b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*sqrt(x))/(d*x^2 + e*x^(3/2)), x)

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)+a\right)^3}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))^n) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^3/x,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))^n))^3/x, x)
```

3.439 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 285

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx &= \frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^2}+\frac{12ab^2dn^2}{e\sqrt{x}}-\frac{12b^3dn^3}{e\sqrt{x}} \\ &+\frac{12b^3dn^2\left(d+\frac{e}{\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{e^2} \\ &-\frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^2} \\ &-\frac{6bdn\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2} \\ &+\frac{3bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2e^2} \\ &+\frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^2} \\ &-\frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^2}\end{aligned}$$

```
[Out] 12*b^3*d*n^2*ln(c*(d+e/x^(1/2))^n)*(d+e/x^(1/2))/e^2-6*b*d*n*(a+b*ln(c*(d+e
/x^(1/2))^n))^2*(d+e/x^(1/2))/e^2+2*d*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^
(1/2))/e^2+3/4*b^3*n^3*(d+e/x^(1/2))^2/e^2-3/2*b^2*n^2*(a+b*ln(c*(d+e/x^(1/
2))^n))*(d+e/x^(1/2))^2/e^2+3/2*b*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(1
/2))^2/e^2-(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/2))^2/e^2+12*a*b^2*d*n^2
/e/x^(1/2)-12*b^3*d*n^3/e/x^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx = -\frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^2} + \frac{12ab^2dn^2}{e\sqrt{x}} + \frac{3bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2e^2} - \frac{6bdn\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2} - \frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^2} + \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^2} + \frac{12b^3dn^2\left(d+\frac{e}{\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{e^2} + \frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^2} - \frac{12b^3dn^3}{e\sqrt{x}}$$

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^2,x]
```

```
[Out] (3*b^3*n^3*(d + e/Sqrt[x])^2)/(4*e^2) + (12*a*b^2*d*n^2)/(e*Sqrt[x]) - (12*
b^3*d*n^3)/(e*Sqrt[x]) + (12*b^3*d*n^2*(d + e/Sqrt[x])*Log[c*(d + e/Sqrt[x]
)^n])/e^2 - (3*b^2*n^2*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/
(2*e^2) - (6*b*d*n*(d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/e^2
+ (3*b*n*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(2*e^2) + (2
*d*(d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^2 - ((d + e/Sqrt[x
])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
```

```
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = -\left(2\text{Subst}\left(\int x(a+b\log(c(d+ex)^n))^3\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\text{Subst}\left(\int \left(-\frac{d(a+b\log(c(d+ex)^n))^3}{e}+\frac{(d+ex)(a+b\log(c(d+ex)^n))^3}{e}\right)dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\frac{2\text{Subst}\left(\int (d+ex)(a+b\log(c(d+ex)^n))^3\,dx, x, \frac{1}{\sqrt{x}}\right)}{e} + \frac{(2d)\text{Subst}\left(\int (a+b\log(c(d+ex)^n))^3\,dx, x, \frac{1}{\sqrt{x}}\right)}{e}$$

$$= -\frac{2\text{Subst}\left(\int x(a+b\log(cx^n))^3\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^2} + \frac{(2d)\text{Subst}\left(\int (a+b\log(cx^n))^3\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^2}$$

$$= \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^2} - \frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^2} + \frac{(3bn)\text{Subst}\left(\int x(a+b\log(cx^n))^2\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^2} - \frac{(6bdn)\text{Subst}\left(\int (a+b\log(cx^n))^2\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^2}$$

$$= -\frac{6bdn\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2} + \frac{3bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2e^2} + \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^2} - \frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^2} - \frac{(3b^2n^2)\,\text{Subst}\left(\int x(a+b\log(cx^n))\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^2} + \frac{(12b^2dn^2)\,\text{Subst}\left(\int (a+b\log(cx^n))\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^2}$$

$$= \frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^2} + \frac{12ab^2dn^2}{e\sqrt{x}} - \frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^2} - \frac{6bdn\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2} + \frac{3bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2e^2} + \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^2} - \frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^2} + \frac{(12b^3dn^2)\operatorname{Subst}\left(\int\log(cx^n)\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^2}$$

$$= \frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^2} + \frac{12ab^2dn^2}{e\sqrt{x}} - \frac{12b^3dn^3}{e\sqrt{x}} + \frac{12b^3dn^2\left(d+\frac{e}{\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{e^2} - \frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^2} - \frac{6bdn\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^2} + \frac{3bn\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2e^2} + \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^2} - \frac{\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^2}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.96

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx$$

$$= \frac{-4a^3e^2+6a^2be^2n-6ab^2e^2n^2+3b^3e^2n^3-12a^2bden\sqrt{x}+36ab^2den^2\sqrt{x}-42b^3den^3\sqrt{x}-8b^3d^2n^3x\log^3(}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^2,x]
```

```
[Out] (-4*a^3*e^2 + 6*a^2*b*e^2*n - 6*a*b^2*e^2*n^2 + 3*b^3*e^2*n^3 - 12*a^2*b*d*
e*n*Sqrt[x] + 36*a*b^2*d*e*n^2*Sqrt[x] - 42*b^3*d*e*n^3*Sqrt[x] - 8*b^3*d^2
*n^3*x*Log[d + e/Sqrt[x]]^3 - 4*b^3*e^2*Log[c*(d + e/Sqrt[x])^n]^3 + 12*a^2
*b*d^2*n*x*Log[e + d*Sqrt[x]] - 36*a*b^2*d^2*n^2*x*Log[e + d*Sqrt[x]] + 42*
b^3*d^2*n^3*x*Log[e + d*Sqrt[x]] + 6*b^2*d^2*n^2*x*Log[d + e/Sqrt[x]]*(-2*a
 + 3*b*n - 2*b*Log[c*(d + e/Sqrt[x])^n])*(2*Log[e + d*Sqrt[x]] - Log[x]) -
6*a^2*b*d^2*n*x*Log[x] + 18*a*b^2*d^2*n^2*x*Log[x] - 21*b^3*d^2*n^3*x*Log[x
] + 6*b^2*d^2*n^2*x*Log[d + e/Sqrt[x]]^2*(2*a - 3*b*n + 2*b*Log[c*(d + e/Sq
rt[x])^n] + 2*b*n*Log[e + d*Sqrt[x]] - b*n*Log[x]) + 6*b^2*Log[c*(d + e/Sqr
t[x])^n]^2*(e*(-2*a*e + b*n*(e - 2*d*Sqrt[x])) + 2*b*d^2*n*x*Log[e + d*Sqrt
[x]] - b*d^2*n*x*Log[x]) - 6*b*Log[c*(d + e/Sqrt[x])^n]*(e*(2*a^2*e + b^2*n
^2*(e - 6*d*Sqrt[x]) - 2*a*b*n*(e - 2*d*Sqrt[x])) + 2*b*d^2*n*(-2*a + 3*b*n
)*x*Log[e + d*Sqrt[x]] + b*d^2*n*(2*a - 3*b*n)*x*Log[x]))/(4*e^2*x)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}}{x^{2}}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. 2(249) = 498.

Time = 0.34 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.90

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^{n}\right)\right)^{3}}{x^{2}}\,dx = \frac{3\,b^{3}e^{2}n^{3}-4\,b^{3}e^{2}\log\left(c\right)^{3}-6\,ab^{2}e^{2}n^{2}+6\,a^{2}be^{2}n-4\,a^{3}e^{2}+4\left(b^{3}d^{2}n^{3}x-b^{3}e^{2}n^{3}\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right)^{3}+6\left(b^{3}e^{2}n-\right.}{}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(3*b^3*e^2*n^3 - 4*b^3*e^2*log(c)^3 - 6*a*b^2*e^2*n^2 + 6*a^2*b*e^2*n -
 4*a^3*e^2 + 4*(b^3*d^2*n^3*x - b^3*e^2*n^3)*log((d*x + e*sqrt(x))/x)^3 + 6
*(b^3*e^2*n - 2*a*b^2*e^2)*log(c)^2 - 6*(2*b^3*d*e*n^3*sqrt(x) - b^3*e^2*n^
3 + 2*a*b^2*e^2*n^2 + (3*b^3*d^2*n^3 - 2*a*b^2*d^2*n^2)*x - 2*(b^3*d^2*n^2*
x - b^3*e^2*n^2)*log(c))*log((d*x + e*sqrt(x))/x)^2 - 6*(b^3*e^2*n^2 - 2*a*
b^2*e^2*n + 2*a^2*b*e^2)*log(c) - 6*(b^3*e^2*n^3 - 2*a*b^2*e^2*n^2 + 2*a^2*
b*e^2*n - 2*(b^3*d^2*n*x - b^3*e^2*n)*log(c)^2 - (7*b^3*d^2*n^3 - 6*a*b^2*d
```

```
^2*n^2 + 2*a^2*b*d^2*n)*x - 2*(b^3*e^2*n^2 - 2*a*b^2*e^2*n - (3*b^3*d^2*n^2
 - 2*a*b^2*d^2*n)*x)*log(c) - 2*(3*b^3*d*e*n^3 - 2*b^3*d*e*n^2*log(c) - 2*a
*b^2*d*e*n^2)*sqrt(x))*log((d*x + e*sqrt(x))/x) - 6*(7*b^3*d*e*n^3 + 2*b^3*
d*e*n*log(c)^2 - 6*a*b^2*d*e*n^2 + 2*a^2*b*d*e*n - 2*(3*b^3*d*e*n^2 - 2*a*b
^2*d*e*n)*log(c))*sqrt(x))/(e^2*x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3/x**2,x)
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**3/x**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(249) = 498$.

Time = 0.23 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.99

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx$$
$$=\frac{3}{2}\,a^2ben\left(\frac{2\,d^2\log\left(d\sqrt{x}+e\right)}{e^3}-\frac{d^2\log\left(x\right)}{e^3}-\frac{2\,d\sqrt{x}-e}{e^2x}\right)-\frac{b^3\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^3}{x}$$
$$+\frac{3}{4}\left(4\,en\left(\frac{2\,d^2\log\left(d\sqrt{x}+e\right)}{e^3}-\frac{d^2\log\left(x\right)}{e^3}-\frac{2\,d\sqrt{x}-e}{e^2x}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)-\frac{\left(4\,d^2x\log\left(d\sqrt{x}+e\right)^2\right.}{}\right.$$
$$+\frac{1}{8}\left(12\,en\left(\frac{2\,d^2\log\left(d\sqrt{x}+e\right)}{e^3}-\frac{d^2\log\left(x\right)}{e^3}-\frac{2\,d\sqrt{x}-e}{e^2x}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2+en\left(\frac{\left(8\,d^2x\log\left(d\sqrt{\right.\right.}{}\right.\right.$$
$$-\frac{3\,ab^2\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2}{x}-\frac{3\,a^2b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{x}-\frac{a^3}{x}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^2,x, algorithm="maxima")
[Out] 3/2*a^2*b*e*n*(2*d^2*log(d*sqrt(x) + e)/e^3 - d^2*log(x)/e^3 - (2*d*sqrt(x)
 - e)/(e^2*x)) - b^3*log(c*(d + e/sqrt(x))^n)^3/x + 3/4*(4*e*n*(2*d^2*log(d
*sqrt(x) + e)/e^3 - d^2*log(x)/e^3 - (2*d*sqrt(x) - e)/(e^2*x))*log(c*(d +
e/sqrt(x))^n) - (4*d^2*x*log(d*sqrt(x) + e)^2 + d^2*x*log(x)^2 - 6*d^2*x*lo
```

```
g(x) - 12*d*e*sqrt(x) + 2*e^2 - 4*(d^2*x*log(x) - 3*d^2*x)*log(d*sqrt(x) +
e))*n^2/(e^2*x))*a*b^2 + 1/8*(12*e*n*(2*d^2*log(d*sqrt(x) + e)/e^3 - d^2*lo
g(x)/e^3 - (2*d*sqrt(x) - e)/(e^2*x))*log(c*(d + e/sqrt(x))^n)^2 + e*n*((8*
d^2*x*log(d*sqrt(x) + e)^3 - d^2*x*log(x)^3 + 9*d^2*x*log(x)^2 - 42*d^2*x*l
og(x) - 12*(d^2*x*log(x) - 3*d^2*x)*log(d*sqrt(x) + e)^2 - 84*d*e*sqrt(x) +
 6*e^2 + 6*(d^2*x*log(x)^2 - 6*d^2*x*log(x) + 14*d^2*x)*log(d*sqrt(x) + e))
*n^2/(e^3*x) - 6*(4*d^2*x*log(d*sqrt(x) + e)^2 + d^2*x*log(x)^2 - 6*d^2*x*l
og(x) - 12*d*e*sqrt(x) + 2*e^2 - 4*(d^2*x*log(x) - 3*d^2*x)*log(d*sqrt(x) +
 e))*n*log(c*(d + e/sqrt(x))^n)/(e^3*x)))*b^3 - 3*a*b^2*log(c*(d + e/sqrt(x
))^n)^2/x - 3*a^2*b*log(c*(d + e/sqrt(x))^n)/x - a^3/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(249) = 498$.

Time = 0.40 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.91

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx$$

$$= \frac{4\left(\frac{2\,(d\sqrt{x}+e)b^3dn^3}{e\sqrt{x}}-\frac{(d\sqrt{x}+e)^2b^3n^3}{ex}\right)\log\left(\frac{d\sqrt{x}+e}{\sqrt{x}}\right)^3+6\left(\frac{(b^3n^3-2\,b^3n^2\log(c)-2\,ab^2n^2)\,(d\sqrt{x}+e)^2}{ex}-\frac{4\,(b^3dn^3-b^3dn^2\log(c)-ab^2d}{e\sqrt{x}}\right.}{}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^2,x, algorithm="giac")
```

```
[Out] 1/4*(4*(2*(d*sqrt(x) + e)*b^3*d*n^3/(e*sqrt(x)) - (d*sqrt(x) + e)^2*b^3*n^3
/(e*x))*log((d*sqrt(x) + e)/sqrt(x))^3 + 6*((b^3*n^3 - 2*b^3*n^2*log(c) - 2
*a*b^2*n^2)*(d*sqrt(x) + e)^2/(e*x) - 4*(b^3*d*n^3 - b^3*d*n^2*log(c) - a*b
^2*d*n^2)*(d*sqrt(x) + e)/(e*sqrt(x)))*log((d*sqrt(x) + e)/sqrt(x))^2 - 6*(
(b^3*n^3 - 2*b^3*n^2*log(c) + 2*b^3*n*log(c)^2 - 2*a*b^2*n^2 + 4*a*b^2*n*lo
g(c) + 2*a^2*b*n)*(d*sqrt(x) + e)^2/(e*x) - 4*(2*b^3*d*n^3 - 2*b^3*d*n^2*lo
g(c) + b^3*d*n*log(c)^2 - 2*a*b^2*d*n^2 + 2*a*b^2*d*n*log(c) + a^2*b*d*n)*(
d*sqrt(x) + e)/(e*sqrt(x)))*log((d*sqrt(x) + e)/sqrt(x)) + (3*b^3*n^3 - 6*b
^3*n^2*log(c) + 6*b^3*n*log(c)^2 - 4*b^3*log(c)^3 - 6*a*b^2*n^2 + 12*a*b^2*
n*log(c) - 12*a*b^2*log(c)^2 + 6*a^2*b*n - 12*a^2*b*log(c) - 4*a^3)*(d*sqrt
(x) + e)^2/(e*x) - 8*(6*b^3*d*n^3 - 6*b^3*d*n^2*log(c) + 3*b^3*d*n*log(c)^2
 - b^3*d*log(c)^3 - 6*a*b^2*d*n^2 + 6*a*b^2*d*n*log(c) - 3*a*b^2*d*log(c)^2
 + 3*a^2*b*d*n - 3*a^2*b*d*log(c) - a^3*d)*(d*sqrt(x) + e)/(e*sqrt(x)))/e
```

Mupad [B] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.25

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^2}\,dx$$
$$= \frac{\frac{d\left(2\,a^3-3\,a^2\,b\,n+3\,a\,b^2\,n^2-\frac{3\,b^3\,n^3}{2}\right)}{e}-\frac{d\,(2\,a^3-6\,a\,b^2\,n^2+9\,b^3\,n^3)}{e}}{\sqrt{x}}-\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^3\left(\frac{b^3}{x}-\frac{b^3\,d^2}{e^2}\right)$$
$$+\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\left(\frac{\frac{3\,b\,d\,(2\,a^2-2\,a\,b\,n+b^2\,n^2)}{e}-\frac{6\,b\,d\,(a^2-b^2\,n^2)}{e}}{\sqrt{x}}-\frac{3\,b\,(2\,a^2-2\,a\,b\,n+b^2\,n^2)}{2\,x}\right)+\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2\left(\frac{\frac{3\,b^2\,d\,(2\,a-b\,n)}{e}-\frac{6\,a\,b^2\,d}{e}}{\sqrt{x}}-\frac{3\,b^2\,(2\,a-b\,n)}{2\,x}+\frac{3\,d\,(2\,a\,b^2\,d-3\,b^3\,d\,n)}{2\,e^2}\right)$$
$$-\frac{a^3-\frac{3\,a^2\,b\,n}{2}+\frac{3\,a\,b^2\,n^2}{2}-\frac{3\,b^3\,n^3}{4}}{x}+\frac{\ln\left(d+\frac{e}{\sqrt{x}}\right)\,(6\,a^2\,b\,d^2\,n-18\,a\,b^2\,d^2\,n^2+21\,b^3\,d^2\,n^3)}{2\,e^2}$$

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^3/x^2,x)
```

```
[Out] ((d*(2*a^3 - (3*b^3*n^3)/2 + 3*a*b^2*n^2 - 3*a^2*b*n))/e - (d*(2*a^3 + 9*b^
3*n^3 - 6*a*b^2*n^2))/e)/x^(1/2) - log(c*(d + e/x^(1/2))^n)^3*(b^3/x - (b^3
*d^2)/e^2) + log(c*(d + e/x^(1/2))^n)*(((3*b*d*(2*a^2 + b^2*n^2 - 2*a*b*n))
/e - (6*b*d*(a^2 - b^2*n^2))/e)/x^(1/2) - (3*b*(2*a^2 + b^2*n^2 - 2*a*b*n))
/(2*x)) + log(c*(d + e/x^(1/2))^n)^2*(((3*b^2*d*(2*a - b*n))/e - (6*a*b^2*d
)/e)/x^(1/2) - (3*b^2*(2*a - b*n))/(2*x) + (3*d*(2*a*b^2*d - 3*b^3*d*n))/(2
*e^2)) - (a^3 - (3*b^3*n^3)/4 + (3*a*b^2*n^2)/2 - (3*a^2*b*n)/2)/x + (log(d
 + e/x^(1/2))*(21*b^3*d^2*n^3 - 18*a*b^2*d^2*n^2 + 6*a^2*b*d^2*n))/(2*e^2)
```

3.440 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 595

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}\,dx = \frac{9b^3d^2n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^4} - \frac{4b^3dn^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4} + \frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^4}{64e^4} + \frac{12ab^2d^3n^2}{e^3\sqrt{x}} - \frac{12b^3d^3n^3}{e^3\sqrt{x}} + \frac{12b^3d^3n^2\left(d+\frac{e}{\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{e^4} - \frac{9b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^4} + \frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^4} - \frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{16e^4} - \frac{6bd^3n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^4} + \frac{9bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2e^4} - \frac{2bdn\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^4} + \frac{3bn\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{8e^4} + \frac{2d^3\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4} - \frac{3d^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4} + \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4} - \frac{\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{2e^4}$$

```
[Out] 12*b^3*d^3*n^2*ln(c*(d+e/x^(1/2))^n)*(d+e/x^(1/2))/e^4-6*b*d^3*n*(a+b*ln(c*
(d+e/x^(1/2))^n))^2*(d+e/x^(1/2))/e^4+2*d^3*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(
```

```
d+e/x^(1/2))/e^4+9/4*b^3*d^2*n^3*(d+e/x^(1/2))^2/e^4-9/2*b^2*d^2*n^2*(a+b*l
n(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^2/e^4+9/2*b*d^2*n*(a+b*ln(c*(d+e/x^(1/2
))^n))^2*(d+e/x^(1/2))^2/e^4-3*d^2*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/
2))^2/e^4-4/9*b^3*d*n^3*(d+e/x^(1/2))^3/e^4+4/3*b^2*d*n^2*(a+b*ln(c*(d+e/x^
(1/2))^n))*(d+e/x^(1/2))^3/e^4-2*b*d*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x
^(1/2))^3/e^4+2*d*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/2))^3/e^4+3/64*b^
3*n^3*(d+e/x^(1/2))^4/e^4-3/16*b^2*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(
1/2))^4/e^4+3/8*b*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(1/2))^4/e^4-1/2*(
a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/2))^4/e^4+12*a*b^2*d^3*n^2/e^3/x^(1/
2)-12*b^3*d^3*n^3/e^3/x^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341\}$

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}\,dx = -\frac{9b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^4} - \frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{16e^4} + \frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^4} + \frac{12ab^2d^3n^2}{e^3\sqrt{x}} + \frac{2d^3\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4} - \frac{6bd^3n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^4} - \frac{3d^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4} + \frac{9bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2e^4} - \frac{\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{2e^4} + \frac{3bn\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{8e^4} + \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4} - \frac{2bdn\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^4} + \frac{12b^3d^3n^2\left(d+\frac{e}{\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{e^4} - \frac{12b^3d^3n^3}{e^3\sqrt{x}} + \frac{9b^3d^2n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^4} + \frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^4}{64e^4} - \frac{4b^3dn^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4}$$

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^3,x]
```

```
[Out] (9*b^3*d^2*n^3*(d + e/Sqrt[x])^2)/(4*e^4) - (4*b^3*d*n^3*(d + e/Sqrt[x])^3)
/(9*e^4) + (3*b^3*n^3*(d + e/Sqrt[x])^4)/(64*e^4) + (12*a*b^2*d^3*n^2)/(e^3
*Sqrt[x]) - (12*b^3*d^3*n^3)/(e^3*Sqrt[x]) + (12*b^3*d^3*n^2*(d + e/Sqrt[x]
)*Log[c*(d + e/Sqrt[x])^n])/e^4 - (9*b^2*d^2*n^2*(d + e/Sqrt[x])^2*(a + b*L
og[c*(d + e/Sqrt[x])^n]))/(2*e^4) + (4*b^2*d*n^2*(d + e/Sqrt[x])^3*(a + b*L
og[c*(d + e/Sqrt[x])^n]))/(3*e^4) - (3*b^2*n^2*(d + e/Sqrt[x])^4*(a + b*Log
[c*(d + e/Sqrt[x])^n]))/(16*e^4) - (6*b*d^3*n*(d + e/Sqrt[x])*(a + b*Log[c*
(d + e/Sqrt[x])^n])^2)/e^4 + (9*b*d^2*n*(d + e/Sqrt[x])^2*(a + b*Log[c*(d +
 e/Sqrt[x])^n])^2)/(2*e^4) - (2*b*d*n*(d + e/Sqrt[x])^3*(a + b*Log[c*(d + e
/Sqrt[x])^n])^2)/e^4 + (3*b*n*(d + e/Sqrt[x])^4*(a + b*Log[c*(d + e/Sqrt[x]
)^n])^2)/(8*e^4) + (2*d^3*(d + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^
3)/e^4 - (3*d^2*(d + e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^4 +
 (2*d*(d + e/Sqrt[x])^3*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^4 - ((d + e/S
qrt[x])^4*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/(2*e^4)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(2\text{Subst}\left(\int x^3(a+b\log\left(c(d+ex)^n\right))^3\,dx, x, \frac{1}{\sqrt{x}}\right)\right)\\
&= -\left(2\text{Subst}\left(\int\left(-\frac{d^3(a+b\log\left(c(d+ex)^n\right))^3}{e^3}+\frac{3d^2(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^3}-\frac{3d(d+ex\right.\right.\right.\\
&= -\frac{2\text{Subst}\left(\int(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^3}\\
&\quad+\frac{(6d)\text{Subst}\left(\int(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^3}\\
&\quad-\frac{(6d^2)\,\text{Subst}\left(\int(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^3}\\
&\quad+\frac{(2d^3)\,\text{Subst}\left(\int\left(a+b\log\left(c(d+ex)^n\right)\right)^3\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2\text{Subst}\left(\int x^3(a+b\log(cx^n))^3\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^4}\\
&\quad+\frac{(6d)\text{Subst}\left(\int x^2(a+b\log(cx^n))^3\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^4}\\
&\quad-\frac{(6d^2)\,\text{Subst}\left(\int x(a+b\log(cx^n))^3\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^4}\\
&\quad+\frac{(2d^3)\,\text{Subst}\left(\int (a+b\log(cx^n))^3\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^4}\\
&= \frac{2d^3\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4}\\
&\quad-\frac{3d^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4}\\
&\quad+\frac{2d\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4}\\
&\quad-\frac{\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{2e^4}\\
&\quad+\frac{(3bn)\text{Subst}\left(\int x^3(a+b\log(cx^n))^2\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{2e^4}\\
&\quad-\frac{(6bdn)\text{Subst}\left(\int x^2(a+b\log(cx^n))^2\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^4}\\
&\quad+\frac{(9bd^2n)\,\text{Subst}\left(\int x(a+b\log(cx^n))^2\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^4}\\
&\quad-\frac{(6bd^3n)\,\text{Subst}\left(\int (a+b\log(cx^n))^2\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^4}
\end{aligned}
$$

$$= -\frac{6bd^3n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^4}$$
$$+\frac{9bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2e^4}$$
$$-\frac{2bdn\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^4}$$
$$+\frac{3bn\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{8e^4}$$
$$+\frac{2d^3\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4}$$
$$-\frac{3d^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4}$$
$$+\frac{2d\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4}$$
$$-\frac{\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{2e^4}$$
$$-\frac{(3b^2n^2)\,\mathrm{Subst}\left(\int x^3(a+b\log(cx^n))\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{4e^4}$$
$$+\frac{(4b^2dn^2)\,\mathrm{Subst}\left(\int x^2(a+b\log(cx^n))\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^4}$$
$$-\frac{(9b^2d^2n^2)\,\mathrm{Subst}\left(\int x(a+b\log(cx^n))\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^4}$$
$$+\frac{(12b^2d^3n^2)\,\mathrm{Subst}\left(\int (a+b\log(cx^n))\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^4}$$

$$
\begin{aligned}
&= \frac{9b^3d^2n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^4} - \frac{4b^3dn^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4} + \frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^4}{64e^4} \\
&\quad + \frac{12ab^2d^3n^2}{e^3\sqrt{x}} - \frac{9b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^4} \\
&\quad + \frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^4} \\
&\quad - \frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{16e^4} \\
&\quad - \frac{6bd^3n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^4} \\
&\quad + \frac{9bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2e^4} \\
&\quad - \frac{2bdn\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^4} \\
&\quad + \frac{3bn\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{8e^4} \\
&\quad + \frac{2d^3\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4} \\
&\quad - \frac{3d^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4} \\
&\quad + \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4} \\
&\quad - \frac{\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{2e^4} \\
&\quad + \frac{(12b^3d^3n^2)\operatorname{Subst}\left(\int \log\left(cx^n\right)\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^4}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{9b^3d^2n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^4} - \frac{4b^3dn^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^4} + \frac{3b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^4}{64e^4} \\
&\quad + \frac{12ab^2d^3n^2}{e^3\sqrt{x}} - \frac{12b^3d^3n^3}{e^3\sqrt{x}} + \frac{12b^3d^3n^2\left(d+\frac{e}{\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{e^4} \\
&\quad - \frac{9b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^4} \\
&\quad + \frac{4b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{3e^4} \\
&\quad - \frac{3b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{16e^4} \\
&\quad - \frac{6bd^3n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^4} \\
&\quad + \frac{9bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2e^4} \\
&\quad - \frac{2bdn\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^4} \\
&\quad + \frac{3bn\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{8e^4} \\
&\quad + \frac{2d^3\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4} \\
&\quad - \frac{3d^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4} \\
&\quad + \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^4} \\
&\quad - \frac{\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{2e^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 766, normalized size of antiderivative = 1.29

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}\,dx$$
$$= \frac{-288a^3e^4+216a^2be^4n-108ab^2e^4n^2+27b^3e^4n^3-288a^2bde^3n\sqrt{x}+336ab^2de^3n^2\sqrt{x}-148b^3de^3n^3\sqrt{x}+4}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^3,x]
```

```
[Out] (-288*a^3*e^4 + 216*a^2*b*e^4*n - 108*a*b^2*e^4*n^2 + 27*b^3*e^4*n^3 - 288*
a^2*b*d*e^3*n*Sqrt[x] + 336*a*b^2*d*e^3*n^2*Sqrt[x] - 148*b^3*d*e^3*n^3*Sqr
t[x] + 432*a^2*b*d^2*e^2*n*x - 936*a*b^2*d^2*e^2*n^2*x + 690*b^3*d^2*e^2*n^
3*x - 864*a^2*b*d^3*e*n*x^(3/2) + 3600*a*b^2*d^3*e*n^2*x^(3/2) - 4980*b^3*d
^3*e*n^3*x^(3/2) - 576*b^3*d^4*n^3*x^2*Log[d + e/Sqrt[x]]^3 - 288*b^3*e^4*L
og[c*(d + e/Sqrt[x])^n]^3 + 864*a^2*b*d^4*n*x^2*Log[e + d*Sqrt[x]] - 3600*a
*b^2*d^4*n^2*x^2*Log[e + d*Sqrt[x]] + 4980*b^3*d^4*n^3*x^2*Log[e + d*Sqrt[x
]] + 72*b^2*d^4*n^2*x^2*Log[d + e/Sqrt[x]]*(-12*a + 25*b*n - 12*b*Log[c*(d
+ e/Sqrt[x])^n])*(2*Log[e + d*Sqrt[x]] - Log[x]) - 432*a^2*b*d^4*n*x^2*Log[
x] + 1800*a*b^2*d^4*n^2*x^2*Log[x] - 2490*b^3*d^4*n^3*x^2*Log[x] + 72*b^2*d
^4*n^2*x^2*Log[d + e/Sqrt[x]]^2*(12*a - 25*b*n + 12*b*Log[c*(d + e/Sqrt[x])
^n] + 12*b*n*Log[e + d*Sqrt[x]] - 6*b*n*Log[x]) + 72*b^2*Log[c*(d + e/Sqrt[
x])^n]^2*(e*(-12*a*e^3 + 3*b*e^3*n - 4*b*d*e^2*n*Sqrt[x] + 6*b*d^2*e*n*x -
12*b*d^3*n*x^(3/2)) + 12*b*d^4*n*x^2*Log[e + d*Sqrt[x]] - 6*b*d^4*n*x^2*Log
[x]) - 12*b*Log[c*(d + e/Sqrt[x])^n]*(72*a^2*e^4 + b^2*e*n^2*(9*e^3 - 28*d*
e^2*Sqrt[x] + 78*d^2*e*x - 300*d^3*x^(3/2)) - 12*a*b*e*n*(3*e^3 - 4*d*e^2*S
qrt[x] + 6*d^2*e*x - 12*d^3*x^(3/2)) + 12*b*d^4*n*(-12*a + 25*b*n)*x^2*Log[
e + d*Sqrt[x]] + 6*b*d^4*n*(12*a - 25*b*n)*x^2*Log[x]))/(576*e^4*x^2)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 869, normalized size of antiderivative = 1.46

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}\,dx$$
$$=\frac{27\,b^3e^4n^3-288\,b^3e^4\log\left(c\right)^3-108\,ab^2e^4n^2+216\,a^2be^4n-288\,a^3e^4+288\,\left(b^3d^4n^3x^2-b^3e^4n^3\right)\log\left(\frac{dx+e\sqrt{x}}{x}\right.}{}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/576*(27*b^3*e^4*n^3 - 288*b^3*e^4*log(c)^3 - 108*a*b^2*e^4*n^2 + 216*a^2*
b*e^4*n - 288*a^3*e^4 + 288*(b^3*d^4*n^3*x^2 - b^3*e^4*n^3)*log((d*x + e*sq
rt(x))/x)^3 + 216*(2*b^3*d^2*e^2*n*x + b^3*e^4*n - 4*a*b^2*e^4)*log(c)^2 +
72*(6*b^3*d^2*e^2*n^3*x + 3*b^3*e^4*n^3 - 12*a*b^2*e^4*n^2 - (25*b^3*d^4*n^
3 - 12*a*b^2*d^4*n^2)*x^2 + 12*(b^3*d^4*n^2*x^2 - b^3*e^4*n^2)*log(c) - 4*(
3*b^3*d^3*e*n^3*x + b^3*d*e^3*n^3)*sqrt(x))*log((d*x + e*sqrt(x))/x)^2 + 6*
(115*b^3*d^2*e^2*n^3 - 156*a*b^2*d^2*e^2*n^2 + 72*a^2*b*d^2*e^2*n)*x - 36*(
3*b^3*e^4*n^2 - 12*a*b^2*e^4*n + 24*a^2*b*e^4 + 2*(13*b^3*d^2*e^2*n^2 - 12*
a*b^2*d^2*e^2*n)*x)*log(c) - 12*(9*b^3*e^4*n^3 - 36*a*b^2*e^4*n^2 + 72*a^2*
b*e^4*n - (415*b^3*d^4*n^3 - 300*a*b^2*d^4*n^2 + 72*a^2*b*d^4*n)*x^2 - 72*(
b^3*d^4*n*x^2 - b^3*e^4*n)*log(c)^2 + 6*(13*b^3*d^2*e^2*n^3 - 12*a*b^2*d^2*
e^2*n^2)*x - 12*(6*b^3*d^2*e^2*n^2*x + 3*b^3*e^4*n^2 - 12*a*b^2*e^4*n - (25
*b^3*d^4*n^2 - 12*a*b^2*d^4*n)*x^2)*log(c) - 4*(7*b^3*d*e^3*n^3 - 12*a*b^2*
d*e^3*n^2 + 3*(25*b^3*d^3*e*n^3 - 12*a*b^2*d^3*e*n^2)*x - 12*(3*b^3*d^3*e*n
^2*x + b^3*d*e^3*n^2)*log(c))*sqrt(x))*log((d*x + e*sqrt(x))/x) - 4*(37*b^3
*d*e^3*n^3 - 84*a*b^2*d*e^3*n^2 + 72*a^2*b*d*e^3*n + 72*(3*b^3*d^3*e*n*x +
b^3*d*e^3*n)*log(c)^2 + 3*(415*b^3*d^3*e*n^3 - 300*a*b^2*d^3*e*n^2 + 72*a^2
*b*d^3*e*n)*x - 12*(7*b^3*d*e^3*n^2 - 12*a*b^2*d*e^3*n + 3*(25*b^3*d^3*e*n^
2 - 12*a*b^2*d^3*e*n)*x)*log(c))*sqrt(x))/(e^4*x^2)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3/x**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e/sqrt(x))**n))**3/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 732, normalized size of antiderivative = 1.23

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}\,dx$$
$$= \frac{1}{8}a^2ben\left(\frac{12\,d^4\log\left(d\sqrt{x}+e\right)}{e^5}-\frac{6\,d^4\log\left(x\right)}{e^5}-\frac{12\,d^3x^{\frac{3}{2}}-6\,d^2ex+4\,de^2\sqrt{x}-3\,e^3}{e^4x^2}\right)$$
$$+\frac{1}{48}\left(12\,en\left(\frac{12\,d^4\log\left(d\sqrt{x}+e\right)}{e^5}-\frac{6\,d^4\log\left(x\right)}{e^5}-\frac{12\,d^3x^{\frac{3}{2}}-6\,d^2ex+4\,de^2\sqrt{x}-3\,e^3}{e^4x^2}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right.\right.$$
$$+\frac{1}{576}\left(72\,en\left(\frac{12\,d^4\log\left(d\sqrt{x}+e\right)}{e^5}-\frac{6\,d^4\log\left(x\right)}{e^5}-\frac{12\,d^3x^{\frac{3}{2}}-6\,d^2ex+4\,de^2\sqrt{x}-3\,e^3}{e^4x^2}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right.\right.\right.$$
$$-\frac{b^3\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^3}{2\,x^2}-\frac{3\,ab^2\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2}{2\,x^2}-\frac{3\,a^2b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{2\,x^2}-\frac{a^3}{2\,x^2}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*b*e*n*(12*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*x
^(3/2) - 6*d^2*e*x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2)) + 1/48*(12*e*n*(12
*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*x^(3/2) - 6*d^2*e*
x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2))*log(c*(d + e/sqrt(x))^n) - (72*d^4*
x^2*log(d*sqrt(x) + e)^2 + 18*d^4*x^2*log(x)^2 - 150*d^4*x^2*log(x) - 300*d
^3*e*x^(3/2) + 78*d^2*e^2*x - 28*d*e^3*sqrt(x) + 9*e^4 - 12*(6*d^4*x^2*log(
x) - 25*d^4*x^2)*log(d*sqrt(x) + e))*n^2/(e^4*x^2))*a*b^2 + 1/576*(72*e*n*(
12*d^4*log(d*sqrt(x) + e)/e^5 - 6*d^4*log(x)/e^5 - (12*d^3*x^(3/2) - 6*d^2*
e*x + 4*d*e^2*sqrt(x) - 3*e^3)/(e^4*x^2))*log(c*(d + e/sqrt(x))^n)^2 + e*n*
((288*d^4*x^2*log(d*sqrt(x) + e)^3 - 36*d^4*x^2*log(x)^3 + 450*d^4*x^2*log(
x)^2 - 2490*d^4*x^2*log(x) - 4980*d^3*e*x^(3/2) + 690*d^2*e^2*x - 148*d*e^3
*sqrt(x) + 27*e^4 - 72*(6*d^4*x^2*log(x) - 25*d^4*x^2)*log(d*sqrt(x) + e)^2
 + 12*(18*d^4*x^2*log(x)^2 - 150*d^4*x^2*log(x) + 415*d^4*x^2)*log(d*sqrt(x
) + e))*n^2/(e^5*x^2) - 12*(72*d^4*x^2*log(d*sqrt(x) + e)^2 + 18*d^4*x^2*lo
g(x)^2 - 150*d^4*x^2*log(x) - 300*d^3*e*x^(3/2) + 78*d^2*e^2*x - 28*d*e^3*s
qrt(x) + 9*e^4 - 12*(6*d^4*x^2*log(x) - 25*d^4*x^2)*log(d*sqrt(x) + e))*n*l
og(c*(d + e/sqrt(x))^n)/(e^5*x^2)))*b^3 - 1/2*b^3*log(c*(d + e/sqrt(x))^n)^
3/x^2 - 3/2*a*b^2*log(c*(d + e/sqrt(x))^n)^2/x^2 - 3/2*a^2*b*log(c*(d + e/s
qrt(x))^n)/x^2 - 1/2*a^3/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1147 vs. 2(519) = 1038.

Time = 0.40 (sec) , antiderivative size = 1147, normalized size of antiderivative = 1.93

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^3,x, algorithm="giac")
```

```
[Out] 1/576*(288*(4*(d*sqrt(x) + e)*b^3*d^3*n^3/(e^3*sqrt(x)) - 6*(d*sqrt(x) + e)
^2*b^3*d^2*n^3/(e^3*x) + 4*(d*sqrt(x) + e)^3*b^3*d*n^3/(e^3*x^(3/2)) - (d*s
qrt(x) + e)^4*b^3*n^3/(e^3*x^2))*log((d*sqrt(x) + e)/sqrt(x))^3 + 72*(3*(b^
3*n^3 - 4*b^3*n^2*log(c) - 4*a*b^2*n^2)*(d*sqrt(x) + e)^4/(e^3*x^2) - 16*(b
^3*d*n^3 - 3*b^3*d*n^2*log(c) - 3*a*b^2*d*n^2)*(d*sqrt(x) + e)^3/(e^3*x^(3/
2)) + 36*(b^3*d^2*n^3 - 2*b^3*d^2*n^2*log(c) - 2*a*b^2*d^2*n^2)*(d*sqrt(x)
+ e)^2/(e^3*x) - 48*(b^3*d^3*n^3 - b^3*d^3*n^2*log(c) - a*b^2*d^3*n^2)*(d*s
qrt(x) + e)/(e^3*sqrt(x)))*log((d*sqrt(x) + e)/sqrt(x))^2 - 12*(9*(b^3*n^3
- 4*b^3*n^2*log(c) + 8*b^3*n*log(c)^2 - 4*a*b^2*n^2 + 16*a*b^2*n*log(c) + 8
*a^2*b*n)*(d*sqrt(x) + e)^4/(e^3*x^2) - 32*(2*b^3*d*n^3 - 6*b^3*d*n^2*log(c
) + 9*b^3*d*n*log(c)^2 - 6*a*b^2*d*n^2 + 18*a*b^2*d*n*log(c) + 9*a^2*b*d*n)
*(d*sqrt(x) + e)^3/(e^3*x^(3/2)) + 216*(b^3*d^2*n^3 - 2*b^3*d^2*n^2*log(c)
+ 2*b^3*d^2*n*log(c)^2 - 2*a*b^2*d^2*n^2 + 4*a*b^2*d^2*n*log(c) + 2*a^2*b*d
^2*n)*(d*sqrt(x) + e)^2/(e^3*x) - 288*(2*b^3*d^3*n^3 - 2*b^3*d^3*n^2*log(c)
 + b^3*d^3*n*log(c)^2 - 2*a*b^2*d^3*n^2 + 2*a*b^2*d^3*n*log(c) + a^2*b*d^3*
n)*(d*sqrt(x) + e)/(e^3*sqrt(x)))*log((d*sqrt(x) + e)/sqrt(x)) + 9*(3*b^3*n
^3 - 12*b^3*n^2*log(c) + 24*b^3*n*log(c)^2 - 32*b^3*log(c)^3 - 12*a*b^2*n^2
 + 48*a*b^2*n*log(c) - 96*a*b^2*log(c)^2 + 24*a^2*b*n - 96*a^2*b*log(c) - 3
2*a^3)*(d*sqrt(x) + e)^4/(e^3*x^2) - 128*(2*b^3*d*n^3 - 6*b^3*d*n^2*log(c)
+ 9*b^3*d*n*log(c)^2 - 9*b^3*d*log(c)^3 - 6*a*b^2*d*n^2 + 18*a*b^2*d*n*log(
c) - 27*a*b^2*d*log(c)^2 + 9*a^2*b*d*n - 27*a^2*b*d*log(c) - 9*a^3*d)*(d*sq
rt(x) + e)^3/(e^3*x^(3/2)) + 432*(3*b^3*d^2*n^3 - 6*b^3*d^2*n^2*log(c) + 6*
b^3*d^2*n*log(c)^2 - 4*b^3*d^2*log(c)^3 - 6*a*b^2*d^2*n^2 + 12*a*b^2*d^2*n*
log(c) - 12*a*b^2*d^2*log(c)^2 + 6*a^2*b*d^2*n - 12*a^2*b*d^2*log(c) - 4*a^
3*d^2)*(d*sqrt(x) + e)^2/(e^3*x) - 1152*(6*b^3*d^3*n^3 - 6*b^3*d^3*n^2*log(
c) + 3*b^3*d^3*n*log(c)^2 - b^3*d^3*log(c)^3 - 6*a*b^2*d^3*n^2 + 6*a*b^2*d^
3*n*log(c) - 3*a*b^2*d^3*log(c)^2 + 3*a^2*b*d^3*n - 3*a^2*b*d^3*log(c) - a^
3*d^3)*(d*sqrt(x) + e)/(e^3*sqrt(x)))/e
```

Mupad [B] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 846, normalized size of antiderivative = 1.42

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^3}\,dx$$

$$= \frac{\frac{d\left(2a^3-\frac{3a^2bn}{2}+\frac{3ab^2n^2}{4}-\frac{3b^3n^3}{16}\right)}{3e}-\frac{d\left(24a^3-12ab^2n^2+7b^3n^3\right)}{36e}}{x^{3/2}}-\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^3\left(\frac{b^3}{2x^2}-\frac{b^3d^4}{2e^4}\right)$$

$$+\frac{\frac{d\left(\frac{d\left(\frac{d\left(2a^3-\frac{3a^2bn}{2}+\frac{3ab^2n^2}{4}-\frac{3b^3n^3}{16}\right)}{e}-\frac{d\left(24a^3-12ab^2n^2+7b^3n^3\right)}{12e}\right)}{e}+\frac{b^2d^2n^2(12a-13bn)}{8e^2}\right)}{e}+\frac{b^2d^3n^2(12a-25bn)}{4e^3}}{\sqrt{x}}$$

$$+\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)^2\left(\frac{\frac{b^2d(4a-bn)}{e}-\frac{4ab^2d}{e}}{2x^{3/2}}-\frac{3b^2(4a-bn)}{8x^2}+\frac{d\left(12ab^2d^3-25b^3d^3n\right)}{8e^4}\right.$$
$$\left.-\frac{d\left(\frac{6b^2d(4a-bn)}{e}-\frac{24ab^2d}{e}\right)}{8ex}+\frac{d^2\left(\frac{6b^2d(4a-bn)}{e}-\frac{24ab^2d}{e}\right)}{4e^2\sqrt{x}}\right)$$

$$-\frac{\frac{d\left(\frac{d\left(2a^3-\frac{3a^2bn}{2}+\frac{3ab^2n^2}{4}-\frac{3b^3n^3}{16}\right)}{e}-\frac{d\left(24a^3-12ab^2n^2+7b^3n^3\right)}{12e}\right)}{2e}+\frac{b^2d^2n^2(12a-13bn)}{16e^2}}{x}$$

$$-\frac{\frac{a^3}{2}-\frac{3a^2bn}{8}+\frac{3ab^2n^2}{16}-\frac{3b^3n^3}{64}}{x^2}$$

$$-\frac{\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\left(\frac{16bde^3\left(6a^2-b^2n^2\right)-12bde^3\left(8a^2-4abn+b^2n^2\right)}{12e^2x^{3/2}}+\frac{\frac{d\left(\frac{d\left(16bde^3\left(6a^2-b^2n^2\right)-12bde^3\left(8a^2-4abn+b^2n^2\right)\right)}{e}-\cdots\right)}{e}}{4e^2\sqrt{x}}\cdots\right)}{4e^2}$$

$$+\frac{\ln\left(d+\frac{e}{\sqrt{x}}\right)\left(72a^2bd^4n-300ab^2d^4n^2+415b^3d^4n^3\right)}{48e^4}$$

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^3/x^3,x)
```

```
[Out] ((d*(2*a^3 - (3*b^3*n^3)/16 + (3*a*b^2*n^2)/4 - (3*a^2*b*n)/2))/(3*e) - (d*
(24*a^3 + 7*b^3*n^3 - 12*a*b^2*n^2))/(36*e))/x^(3/2) - log(c*(d + e/x^(1/2)
)^n)^3*(b^3/(2*x^2) - (b^3*d^4)/(2*e^4)) + ((d*((d*((d*(2*a^3 - (3*b^3*n^3)
/16 + (3*a*b^2*n^2)/4 - (3*a^2*b*n)/2))/e - (d*(24*a^3 + 7*b^3*n^3 - 12*a*b
^2*n^2))/(12*e)))/e + (b^2*d^2*n^2*(12*a - 13*b*n))/(8*e^2)))/e + (b^2*d^3*
n^2*(12*a - 25*b*n))/(4*e^3))/x^(1/2) + log(c*(d + e/x^(1/2))^n)^2*(((b^2*d
*(4*a - b*n))/e - (4*a*b^2*d)/e)/(2*x^(3/2)) - (3*b^2*(4*a - b*n))/(8*x^2)
```

```
+ (d*(12*a*b^2*d^3 - 25*b^3*d^3*n))/(8*e^4) - (d*((6*b^2*d*(4*a - b*n))/e -
 (24*a*b^2*d)/e))/(8*e*x) + (d^2*((6*b^2*d*(4*a - b*n))/e - (24*a*b^2*d)/e)
)/(4*e^2*x^(1/2))) - ((d*((d*(2*a^3 - (3*b^3*n^3)/16 + (3*a*b^2*n^2)/4 - (3
*a^2*b*n)/2))/e - (d*(24*a^3 + 7*b^3*n^3 - 12*a*b^2*n^2))/(12*e)))/(2*e) +
(b^2*d^2*n^2*(12*a - 13*b*n))/(16*e^2))/x - (a^3/2 - (3*b^3*n^3)/64 + (3*a*
b^2*n^2)/16 - (3*a^2*b*n)/8)/x^2 - (log(c*(d + e/x^(1/2))^n)*((16*b*d*e^3*(
6*a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2 + b^2*n^2 - 4*a*b*n))/(12*e^2*x^(3/2))
 + ((d*((d*(16*b*d*e^3*(6*a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2 + b^2*n^2 - 4*
a*b*n)))/e - 24*b^3*d^2*e^2*n^2))/e - 48*b^3*d^3*e*n^2)/(4*e^2*x^(1/2)) - (
(d*(16*b*d*e^3*(6*a^2 - b^2*n^2) - 12*b*d*e^3*(8*a^2 + b^2*n^2 - 4*a*b*n)))
/e - 24*b^3*d^2*e^2*n^2)/(8*e^2*x) + (3*b*e^2*(8*a^2 + b^2*n^2 - 4*a*b*n))/
(4*x^2)))/(4*e^2) + (log(d + e/x^(1/2))*(415*b^3*d^4*n^3 - 300*a*b^2*d^4*n^
2 + 72*a^2*b*d^4*n))/(48*e^4)
```

3.441 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 907

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx = \frac{15b^3d^4n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^6}-\frac{40b^3d^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6}+\frac{15b^3d^2n^3\left(d+\frac{e}{\sqrt{x}}\right)^4}{32e^6}-\frac{12b^3dn^3\left(d+\frac{e}{\sqrt{x}}\right)^5}{125e^6}+\frac{b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^6}{108e^6}+\frac{12ab^2d^5n^2}{e^5\sqrt{x}}-\frac{12b^3d^5n^3}{e^5\sqrt{x}}+\frac{12b^3d^5n^2\left(d+\frac{e}{\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{e^6}-\frac{15b^2d^4n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^6}+\frac{40b^2d^3n^2\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9e^6}-\frac{15b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{8e^6}+\frac{12b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{25e^6}-\frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{18e^6}-\frac{6bd^5n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^6}+\frac{15bd^4n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2e^6}-\frac{20bd^3n\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{3e^6}+\frac{15bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{4e^6}-\frac{6bdn\left(d+\frac{e}{\sqrt{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{5e^6}+\frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{6e^6}+\frac{2d^5\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^6}-\frac{5d^4\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^6}$$

```
[Out] 2*d^5*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/2))/e^6-5*d^4*(a+b*ln(c*(d+e/
x^(1/2))^n))^3*(d+e/x^(1/2))^2/e^6+20/3*d^3*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(
d+e/x^(1/2))^3/e^6-5*d^2*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/2))^4/e^6+
2*d*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/2))^5/e^6+1/108*b^3*n^3*(d+e/x^
(1/2))^6/e^6-12/125*b^3*d*n^3*(d+e/x^(1/2))^5/e^6-1/18*b^2*n^2*(a+b*ln(c*(d
+e/x^(1/2))^n))*(d+e/x^(1/2))^6/e^6+1/6*b*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(
d+e/x^(1/2))^6/e^6-12*b^3*d^5*n^3/e^5/x^(1/2)+15/4*b^3*d^4*n^3*(d+e/x^(1/2)
)^2/e^6-40/27*b^3*d^3*n^3*(d+e/x^(1/2))^3/e^6+15/32*b^3*d^2*n^3*(d+e/x^(1/2
))^4/e^6-1/3*(a+b*ln(c*(d+e/x^(1/2))^n))^3*(d+e/x^(1/2))^6/e^6+12*b^3*d^5*n
^2*ln(c*(d+e/x^(1/2))^n)*(d+e/x^(1/2))/e^6-6*b*d^5*n*(a+b*ln(c*(d+e/x^(1/2)
)^n))^2*(d+e/x^(1/2))/e^6-15/2*b^2*d^4*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e
/x^(1/2))^2/e^6+15/2*b*d^4*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(1/2))^2/
e^6+40/9*b^2*d^3*n^2*(a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^3/e^6-20/3*b
*d^3*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(1/2))^3/e^6-15/8*b^2*d^2*n^2*(
a+b*ln(c*(d+e/x^(1/2))^n))*(d+e/x^(1/2))^4/e^6+15/4*b*d^2*n*(a+b*ln(c*(d+e/
x^(1/2))^n))^2*(d+e/x^(1/2))^4/e^6+12/25*b^2*d*n^2*(a+b*ln(c*(d+e/x^(1/2))^
n))*(d+e/x^(1/2))^5/e^6-6/5*b*d*n*(a+b*ln(c*(d+e/x^(1/2))^n))^2*(d+e/x^(1/2
))^5/e^6+12*a*b^2*d^5*n^2/e^5/x^(1/2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 907, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341\}$

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx = \frac{b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^6}{108e^6} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\left(d+\frac{e}{\sqrt{x}}\right)^6}{3e^6} + \frac{bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\left(d+\frac{e}{\sqrt{x}}\right)^6}{6e^6} - \frac{b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\left(d+\frac{e}{\sqrt{x}}\right)^6}{18e^6} - \frac{12b^3dn^3\left(d+\frac{e}{\sqrt{x}}\right)^5}{125e^6} + \frac{2d\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\left(d+\frac{e}{\sqrt{x}}\right)^5}{e^6} - \frac{6bdn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\left(d+\frac{e}{\sqrt{x}}\right)^5}{5e^6} + \frac{12b^2dn^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\left(d+\frac{e}{\sqrt{x}}\right)^5}{25e^6} + \frac{15b^3d^2n^3\left(d+\frac{e}{\sqrt{x}}\right)^4}{32e^6} - \frac{5d^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\left(d+\frac{e}{\sqrt{x}}\right)^4}{e^6} + \frac{15bd^2n\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\left(d+\frac{e}{\sqrt{x}}\right)^4}{4e^6} - \frac{15b^2d^2n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\left(d+\frac{e}{\sqrt{x}}\right)^4}{8e^6} - \frac{40b^3d^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6} + \frac{20d^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{3e^6} - \frac{20bd^3n\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2\left(d+\frac{e}{\sqrt{x}}\right)^3}{3e^6} + \frac{40b^2d^3n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)\left(d+\frac{e}{\sqrt{x}}\right)^3}{9e^6} + \frac{15b^3d^4n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^6} \; 5d^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3\left(d+\frac{e}{\sqrt{x}}\right)^2$$

[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^4,x]

[Out] (15*b^3*d^4*n^3*(d + e/Sqrt[x])^2)/(4*e^6) - (40*b^3*d^3*n^3*(d + e/Sqrt[x]
)^3)/(27*e^6) + (15*b^3*d^2*n^3*(d + e/Sqrt[x])^4)/(32*e^6) - (12*b^3*d*n^3
*(d + e/Sqrt[x])^5)/(125*e^6) + (b^3*n^3*(d + e/Sqrt[x])^6)/(108*e^6) + (12
*a*b^2*d^5*n^2)/(e^5*Sqrt[x]) - (12*b^3*d^5*n^3)/(e^5*Sqrt[x]) + (12*b^3*d^
5*n^2*(d + e/Sqrt[x])*Log[c*(d + e/Sqrt[x])^n])/e^6 - (15*b^2*d^4*n^2*(d +
e/Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(2*e^6) + (40*b^2*d^3*n^2*(d
 + e/Sqrt[x])^3*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(9*e^6) - (15*b^2*d^2*n^2
(d + e/Sqrt[x])^4(a + b*Log[c*(d + e/Sqrt[x])^n]))/(8*e^6) + (12*b^2*d*n^
2*(d + e/Sqrt[x])^5*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(25*e^6) - (b^2*n^2*(
d + e/Sqrt[x])^6*(a + b*Log[c*(d + e/Sqrt[x])^n]))/(18*e^6) - (6*b*d^5*n*(d
 + e/Sqrt[x])*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/e^6 + (15*b*d^4*n*(d + e/
Sqrt[x])^2*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(2*e^6) - (20*b*d^3*n*(d + e
/Sqrt[x])^3*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(3*e^6) + (15*b*d^2*n*(d +
e/Sqrt[x])^4*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(4*e^6) - (6*b*d*n*(d + e/
Sqrt[x])^5*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(5*e^6) + (b*n*(d + e/Sqrt[x
])^6*(a + b*Log[c*(d + e/Sqrt[x])^n])^2)/(6*e^6) + (2*d^5*(d + e/Sqrt[x])*(
a + b*Log[c*(d + e/Sqrt[x])^n])^3)/e^6 - (5*d^4*(d + e/Sqrt[x])^2*(a + b*Lo
g[c*(d + e/Sqrt[x])^n])^3)/e^6 + (20*d^3*(d + e/Sqrt[x])^3*(a + b*Log[c*(d
+ e/Sqrt[x])^n])^3)/(3*e^6) - (5*d^2*(d + e/Sqrt[x])^4*(a + b*Log[c*(d + e/
Sqrt[x])^n])^3)/e^6 + (2*d*(d + e/Sqrt[x])^5*(a + b*Log[c*(d + e/Sqrt[x])^n
])^3)/e^6 - ((d + e/Sqrt[x])^6*(a + b*Log[c*(d + e/Sqrt[x])^n])^3)/(3*e^6)

Rule 2332

Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]

Rule 2333

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]

Rule 2341

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rule 2342

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = -\left(2\text{Subst}\left(\int x^5(a+b\log(c(d+ex)^n))^3\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\text{Subst}\left(\int\left(-\frac{d^5(a+b\log(c(d+ex)^n))^3}{e^5}+\frac{5d^4(d+ex)(a+b\log(c(d+ex)^n))^3}{e^5}-\frac{10d^3(d+\right.\right.\right.$$

$$= -\frac{2\text{Subst}\left(\int (d+ex)^5 \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \, dx, x, \frac{1}{\sqrt{x}}\right)}{e^5} + \frac{(10d)\text{Subst}\left(\int (d+ex)^4 \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \, dx, x, \frac{1}{\sqrt{x}}\right)}{e^5} - \frac{(20d^2)\,\text{Subst}\left(\int (d+ex)^3 \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \, dx, x, \frac{1}{\sqrt{x}}\right)}{e^5} + \frac{(20d^3)\,\text{Subst}\left(\int (d+ex)^2 \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \, dx, x, \frac{1}{\sqrt{x}}\right)}{e^5} - \frac{(10d^4)\,\text{Subst}\left(\int (d+ex) \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \, dx, x, \frac{1}{\sqrt{x}}\right)}{e^5} + \frac{(2d^5)\,\text{Subst}\left(\int \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \, dx, x, \frac{1}{\sqrt{x}}\right)}{e^5}$$

$$= -\frac{2\text{Subst}\left(\int x^5 (a+b\log\left(cx^n\right))^3 \, dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6} + \frac{(10d)\text{Subst}\left(\int x^4 (a+b\log\left(cx^n\right))^3 \, dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6} - \frac{(20d^2)\,\text{Subst}\left(\int x^3 (a+b\log\left(cx^n\right))^3 \, dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6} + \frac{(20d^3)\,\text{Subst}\left(\int x^2 (a+b\log\left(cx^n\right))^3 \, dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6} - \frac{(10d^4)\,\text{Subst}\left(\int x (a+b\log\left(cx^n\right))^3 \, dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6} + \frac{(2d^5)\,\text{Subst}\left(\int (a+b\log\left(cx^n\right))^3 \, dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6}$$

$$
\begin{aligned}
&= \frac{2d^5\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^6} \\
&- \frac{5d^4\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^6} \\
&+ \frac{20d^3\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{3e^6} \\
&- \frac{5d^2\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^6} \\
&+ \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^6} \\
&- \frac{\left(d+\frac{e}{\sqrt{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{3e^6} \\
&+ \frac{(bn)\text{Subst}\left(\int x^5(a+b\log(cx^n))^2\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6} \\
&- \frac{(6bdn)\text{Subst}\left(\int x^4(a+b\log(cx^n))^2\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6} \\
&+ \frac{(15bd^2n)\,\text{Subst}\left(\int x^3(a+b\log(cx^n))^2\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6} \\
&- \frac{(20bd^3n)\,\text{Subst}\left(\int x^2(a+b\log(cx^n))^2\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6} \\
&+ \frac{(15bd^4n)\,\text{Subst}\left(\int x(a+b\log(cx^n))^2\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6} \\
&- \frac{(6bd^5n)\,\text{Subst}\left(\int (a+b\log(cx^n))^2\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6}
\end{aligned}
$$

$$= -\frac{6bd^5n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^6} + \frac{15bd^4n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2e^6} - \frac{20bd^3n\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{3e^6} + \frac{15bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{4e^6} - \frac{6bdn\left(d+\frac{e}{\sqrt{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{5e^6} + \frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{6e^6} + \frac{2d^5\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^6} - \frac{5d^4\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^6} + \frac{20d^3\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{3e^6} - \frac{5d^2\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^6} + \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^6} - \frac{\left(d+\frac{e}{\sqrt{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{3e^6} - \frac{(b^2n^2)\,\text{Subst}\left(\int x^5(a+b\log(cx^n))\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{3e^6} + \frac{(12b^2dn^2)\,\text{Subst}\left(\int x^4(a+b\log(cx^n))\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{5e^6} - \frac{(15b^2d^2n^2)\,\text{Subst}\left(\int x^3(a+b\log(cx^n))\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{2e^6} + \frac{(40b^2d^3n^2)\,\text{Subst}\left(\int x^2(a+b\log(cx^n))\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{3e^6} - \frac{(15b^2d^4n^2)\,\text{Subst}\left(\int x(a+b\log(cx^n))\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6} + \frac{(12b^2d^5n^2)\,\text{Subst}\left(\int (a+b\log(cx^n))\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6}$$

$$= \frac{15b^3d^4n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^6} - \frac{40b^3d^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6} + \frac{15b^3d^2n^3\left(d+\frac{e}{\sqrt{x}}\right)^4}{32e^6}$$
$$- \frac{12b^3dn^3\left(d+\frac{e}{\sqrt{x}}\right)^5}{125e^6} + \frac{b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^6}{108e^6} + \frac{12ab^2d^5n^2}{e^5\sqrt{x}}$$
$$- \frac{15b^2d^4n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^6}$$
$$+ \frac{40b^2d^3n^2\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9e^6}$$
$$- \frac{15b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{8e^6}$$
$$+ \frac{12b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{25e^6}$$
$$- \frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{18e^6}$$
$$- \frac{6bd^5n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^6}$$
$$+ \frac{15bd^4n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2e^6}$$
$$- \frac{20bd^3n\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{3e^6}$$
$$+ \frac{15bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{4e^6}$$
$$- \frac{6bdn\left(d+\frac{e}{\sqrt{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{5e^6}$$
$$+ \frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{6e^6}$$
$$+ \frac{2d^5\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^6}$$
$$- \frac{5d^4\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^6}$$
$$+ \frac{20d^3\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{3e^6}$$
$$- \frac{5d^2\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^6}$$
$$+ \frac{2d\left(d+\frac{e}{\sqrt{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^6}$$

$$= \frac{15b^3d^4n^3\left(d+\frac{e}{\sqrt{x}}\right)^2}{4e^6} - \frac{40b^3d^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^3}{27e^6} + \frac{15b^3d^2n^3\left(d+\frac{e}{\sqrt{x}}\right)^4}{32e^6}$$
$$- \frac{12b^3dn^3\left(d+\frac{e}{\sqrt{x}}\right)^5}{125e^6} + \frac{b^3n^3\left(d+\frac{e}{\sqrt{x}}\right)^6}{108e^6} + \frac{12ab^2d^5n^2}{e^5\sqrt{x}}$$
$$- \frac{12b^3d^5n^3}{e^5\sqrt{x}} + \frac{12b^3d^5n^2\left(d+\frac{e}{\sqrt{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)}{e^6}$$
$$- \frac{15b^2d^4n^2\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{2e^6}$$
$$+ \frac{40b^2d^3n^2\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{9e^6}$$
$$- \frac{15b^2d^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{8e^6}$$
$$+ \frac{12b^2dn^2\left(d+\frac{e}{\sqrt{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{25e^6}$$
$$- \frac{b^2n^2\left(d+\frac{e}{\sqrt{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)}{18e^6}$$
$$- \frac{6bd^5n\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{e^6}$$
$$+ \frac{15bd^4n\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{2e^6}$$
$$- \frac{20bd^3n\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{3e^6}$$
$$+ \frac{15bd^2n\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{4e^6}$$
$$- \frac{6bdn\left(d+\frac{e}{\sqrt{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{5e^6}$$
$$+ \frac{bn\left(d+\frac{e}{\sqrt{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^2}{6e^6}$$
$$+ \frac{2d^5\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^6}$$
$$- \frac{5d^4\left(d+\frac{e}{\sqrt{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^6}$$
$$+ \frac{20d^3\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{3e^6}$$
$$- \frac{5d^2\left(d+\frac{e}{\sqrt{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{e^6}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 950, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx$$
$$=\frac{-36000a^3e^6+18000a^2be^6n-6000ab^2e^6n^2+1000b^3e^6n^3-21600a^2bde^5n\sqrt{x}+15840ab^2de^5n^2\sqrt{x}-4368\cdots}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^n])^3/x^4,x]
```

```
[Out] (-36000*a^3*e^6 + 18000*a^2*b*e^6*n - 6000*a*b^2*e^6*n^2 + 1000*b^3*e^6*n^3
 - 21600*a^2*b*d*e^5*n*Sqrt[x] + 15840*a*b^2*d*e^5*n^2*Sqrt[x] - 4368*b^3*d
*e^5*n^3*Sqrt[x] + 27000*a^2*b*d^2*e^4*n*x - 33300*a*b^2*d^2*e^4*n^2*x + 13
785*b^3*d^2*e^4*n^3*x - 36000*a^2*b*d^3*e^3*n*x^(3/2) + 68400*a*b^2*d^3*e^3
*n^2*x^(3/2) - 41180*b^3*d^3*e^3*n^3*x^(3/2) + 54000*a^2*b*d^4*e^2*n*x^2 -
156600*a*b^2*d^4*e^2*n^2*x^2 + 140070*b^3*d^4*e^2*n^3*x^2 - 108000*a^2*b*d^
5*e*n*x^(5/2) + 529200*a*b^2*d^5*e*n^2*x^(5/2) - 809340*b^3*d^5*e*n^3*x^(5/
2) - 72000*b^3*d^6*n^3*x^3*Log[d + e/Sqrt[x]]^3 - 36000*b^3*e^6*Log[c*(d +
e/Sqrt[x])^n]^3 + 108000*a^2*b*d^6*n*x^3*Log[e + d*Sqrt[x]] - 529200*a*b^2*
d^6*n^2*x^3*Log[e + d*Sqrt[x]] + 809340*b^3*d^6*n^3*x^3*Log[e + d*Sqrt[x]]
+ 5400*b^2*d^6*n^2*x^3*Log[d + e/Sqrt[x]]*(-20*a + 49*b*n - 20*b*Log[c*(d +
 e/Sqrt[x])^n])*(2*Log[e + d*Sqrt[x]] - Log[x]) - 54000*a^2*b*d^6*n*x^3*Log
[x] + 264600*a*b^2*d^6*n^2*x^3*Log[x] - 404670*b^3*d^6*n^3*x^3*Log[x] + 540
0*b^2*d^6*n^2*x^3*Log[d + e/Sqrt[x]]^2*(20*a - 49*b*n + 20*b*Log[c*(d + e/S
qrt[x])^n] + 20*b*n*Log[e + d*Sqrt[x]] - 10*b*n*Log[x]) + 1800*b^2*Log[c*(d
 + e/Sqrt[x])^n]^2*(e*(-60*a*e^5 + 10*b*e^5*n - 12*b*d*e^4*n*Sqrt[x] + 15*b
*d^2*e^3*n*x - 20*b*d^3*e^2*n*x^(3/2) + 30*b*d^4*e*n*x^2 - 60*b*d^5*n*x^(5/
2)) + 60*b*d^6*n*x^3*Log[e + d*Sqrt[x]] - 30*b*d^6*n*x^3*Log[x]) - 60*b*Log
[c*(d + e/Sqrt[x])^n]*(1800*a^2*e^6 + b^2*e*n^2*(100*e^5 - 264*d*e^4*Sqrt[x
] + 555*d^2*e^3*x - 1140*d^3*e^2*x^(3/2) + 2610*d^4*e*x^2 - 8820*d^5*x^(5/2
)) - 60*a*b*e*n*(10*e^5 - 12*d*e^4*Sqrt[x] + 15*d^2*e^3*x - 20*d^3*e^2*x^(3
/2) + 30*d^4*e*x^2 - 60*d^5*x^(5/2)) + 180*b*d^6*n*(-20*a + 49*b*n)*x^3*Log
[e + d*Sqrt[x]] + 90*b*d^6*n*(20*a - 49*b*n)*x^3*Log[x]))/(108000*e^6*x^3)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/2))^n))^3/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 1203, normalized size of antiderivative = 1.33

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx=\text{Too large to display}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/108000*(1000*b^3*e^6*n^3 - 36000*b^3*e^6*log(c)^3 - 6000*a*b^2*e^6*n^2 +
18000*a^2*b*e^6*n - 36000*a^3*e^6 + 36000*(b^3*d^6*n^3*x^3 - b^3*e^6*n^3)*l
og((d*x + e*sqrt(x))/x)^3 + 30*(4669*b^3*d^4*e^2*n^3 - 5220*a*b^2*d^4*e^2*n
^2 + 1800*a^2*b*d^4*e^2*n)*x^2 + 9000*(6*b^3*d^4*e^2*n*x^2 + 3*b^3*d^2*e^4*
n*x + 2*b^3*e^6*n - 12*a*b^2*e^6)*log(c)^2 + 1800*(30*b^3*d^4*e^2*n^3*x^2 +
 15*b^3*d^2*e^4*n^3*x + 10*b^3*e^6*n^3 - 60*a*b^2*e^6*n^2 - 3*(49*b^3*d^6*n
^3 - 20*a*b^2*d^6*n^2)*x^3 + 60*(b^3*d^6*n^2*x^3 - b^3*e^6*n^2)*log(c) - 4*
(15*b^3*d^5*e*n^3*x^2 + 5*b^3*d^3*e^3*n^3*x + 3*b^3*d*e^5*n^3)*sqrt(x))*log
((d*x + e*sqrt(x))/x)^2 + 15*(919*b^3*d^2*e^4*n^3 - 2220*a*b^2*d^2*e^4*n^2
+ 1800*a^2*b*d^2*e^4*n)*x - 300*(20*b^3*e^6*n^2 - 120*a*b^2*e^6*n + 360*a^2
*b*e^6 + 18*(29*b^3*d^4*e^2*n^2 - 20*a*b^2*d^4*e^2*n)*x^2 + 3*(37*b^3*d^2*e
^4*n^2 - 60*a*b^2*d^2*e^4*n)*x)*log(c) - 60*(100*b^3*e^6*n^3 - 600*a*b^2*e^
6*n^2 + 1800*a^2*b*e^6*n - (13489*b^3*d^6*n^3 - 8820*a*b^2*d^6*n^2 + 1800*a
^2*b*d^6*n)*x^3 + 90*(29*b^3*d^4*e^2*n^3 - 20*a*b^2*d^4*e^2*n^2)*x^2 - 1800
*(b^3*d^6*n*x^3 - b^3*e^6*n)*log(c)^2 + 15*(37*b^3*d^2*e^4*n^3 - 60*a*b^2*d
^2*e^4*n^2)*x - 60*(30*b^3*d^4*e^2*n^2*x^2 + 15*b^3*d^2*e^4*n^2*x + 10*b^3*
e^6*n^2 - 60*a*b^2*e^6*n - 3*(49*b^3*d^6*n^2 - 20*a*b^2*d^6*n)*x^3)*log(c)
- 12*(22*b^3*d*e^5*n^3 - 60*a*b^2*d*e^5*n^2 + 15*(49*b^3*d^5*e*n^3 - 20*a*b
^2*d^5*e*n^2)*x^2 + 5*(19*b^3*d^3*e^3*n^3 - 20*a*b^2*d^3*e^3*n^2)*x - 20*(1
5*b^3*d^5*e*n^2*x^2 + 5*b^3*d^3*e^3*n^2*x + 3*b^3*d*e^5*n^2)*log(c))*sqrt(x
))*log((d*x + e*sqrt(x))/x) - 4*(1092*b^3*d*e^5*n^3 - 3960*a*b^2*d*e^5*n^2
+ 5400*a^2*b*d*e^5*n + 15*(13489*b^3*d^5*e*n^3 - 8820*a*b^2*d^5*e*n^2 + 180
0*a^2*b*d^5*e*n)*x^2 + 1800*(15*b^3*d^5*e*n*x^2 + 5*b^3*d^3*e^3*n*x + 3*b^3
*d*e^5*n)*log(c)^2 + 5*(2059*b^3*d^3*e^3*n^3 - 3420*a*b^2*d^3*e^3*n^2 + 180
0*a^2*b*d^3*e^3*n)*x - 180*(22*b^3*d*e^5*n^2 - 60*a*b^2*d*e^5*n + 15*(49*b^
3*d^5*e*n^2 - 20*a*b^2*d^5*e*n)*x^2 + 5*(19*b^3*d^3*e^3*n^2 - 20*a*b^2*d^3*
e^3*n)*x)*log(c))*sqrt(x))/(e^6*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**n))**3/x**4,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 864, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^4,x, algorithm="maxima")
[Out] 1/60*a^2*b*e*n*(60*d^6*log(d*sqrt(x) + e)/e^7 - 30*d^6*log(x)/e^7 - (60*d^5
*x^(5/2) - 30*d^4*e*x^2 + 20*d^3*e^2*x^(3/2) - 15*d^2*e^3*x + 12*d*e^4*sqrt
(x) - 10*e^5)/(e^6*x^3)) + 1/1800*(60*e*n*(60*d^6*log(d*sqrt(x) + e)/e^7 -
30*d^6*log(x)/e^7 - (60*d^5*x^(5/2) - 30*d^4*e*x^2 + 20*d^3*e^2*x^(3/2) - 1
5*d^2*e^3*x + 12*d*e^4*sqrt(x) - 10*e^5)/(e^6*x^3))*log(c*(d + e/sqrt(x))^n
) - (1800*d^6*x^3*log(d*sqrt(x) + e)^2 + 450*d^6*x^3*log(x)^2 - 4410*d^6*x^
3*log(x) - 8820*d^5*e*x^(5/2) + 2610*d^4*e^2*x^2 - 1140*d^3*e^3*x^(3/2) + 5
55*d^2*e^4*x - 264*d*e^5*sqrt(x) + 100*e^6 - 180*(10*d^6*x^3*log(x) - 49*d^
6*x^3)*log(d*sqrt(x) + e))*n^2/(e^6*x^3))*a*b^2 + 1/108000*(1800*e*n*(60*d^
6*log(d*sqrt(x) + e)/e^7 - 30*d^6*log(x)/e^7 - (60*d^5*x^(5/2) - 30*d^4*e*x
^2 + 20*d^3*e^2*x^(3/2) - 15*d^2*e^3*x + 12*d*e^4*sqrt(x) - 10*e^5)/(e^6*x^
3))*log(c*(d + e/sqrt(x))^n)^2 + e*n*((36000*d^6*x^3*log(d*sqrt(x) + e)^3 -
 4500*d^6*x^3*log(x)^3 + 66150*d^6*x^3*log(x)^2 - 404670*d^6*x^3*log(x) - 8
09340*d^5*e*x^(5/2) + 140070*d^4*e^2*x^2 - 41180*d^3*e^3*x^(3/2) + 13785*d^
2*e^4*x - 4368*d*e^5*sqrt(x) + 1000*e^6 - 5400*(10*d^6*x^3*log(x) - 49*d^6*
x^3)*log(d*sqrt(x) + e)^2 + 60*(450*d^6*x^3*log(x)^2 - 4410*d^6*x^3*log(x)
+ 13489*d^6*x^3)*log(d*sqrt(x) + e))*n^2/(e^7*x^3) - 60*(1800*d^6*x^3*log(d
*sqrt(x) + e)^2 + 450*d^6*x^3*log(x)^2 - 4410*d^6*x^3*log(x) - 8820*d^5*e*x
^(5/2) + 2610*d^4*e^2*x^2 - 1140*d^3*e^3*x^(3/2) + 555*d^2*e^4*x - 264*d*e^
5*sqrt(x) + 100*e^6 - 180*(10*d^6*x^3*log(x) - 49*d^6*x^3)*log(d*sqrt(x) +
e))*n*log(c*(d + e/sqrt(x))^n)/(e^7*x^3)))*b^3 - 1/3*b^3*log(c*(d + e/sqrt(
x))^n)^3/x^3 - a*b^2*log(c*(d + e/sqrt(x))^n)^2/x^3 - a^2*b*log(c*(d + e/sq
rt(x))^n)/x^3 - 1/3*a^3/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1747 vs. $2(787) = 1574$.

Time = 0.41 (sec) , antiderivative size = 1747, normalized size of antiderivative = 1.93

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^n))^3/x^4,x, algorithm="giac")
```

```
[Out] 1/108000*(36000*(6*(d*sqrt(x) + e)*b^3*d^5*n^3/(e^5*sqrt(x)) - 15*(d*sqrt(x
) + e)^2*b^3*d^4*n^3/(e^5*x) + 20*(d*sqrt(x) + e)^3*b^3*d^3*n^3/(e^5*x^(3/2
)) - 15*(d*sqrt(x) + e)^4*b^3*d^2*n^3/(e^5*x^2) + 6*(d*sqrt(x) + e)^5*b^3*d
*n^3/(e^5*x^(5/2)) - (d*sqrt(x) + e)^6*b^3*n^3/(e^5*x^3))*log((d*sqrt(x) +
e)/sqrt(x))^3 + 1800*(10*(b^3*n^3 - 6*b^3*n^2*log(c) - 6*a*b^2*n^2)*(d*sqrt
(x) + e)^6/(e^5*x^3) - 72*(b^3*d*n^3 - 5*b^3*d*n^2*log(c) - 5*a*b^2*d*n^2)*
(d*sqrt(x) + e)^5/(e^5*x^(5/2)) + 225*(b^3*d^2*n^3 - 4*b^3*d^2*n^2*log(c) -
 4*a*b^2*d^2*n^2)*(d*sqrt(x) + e)^4/(e^5*x^2) - 400*(b^3*d^3*n^3 - 3*b^3*d^
3*n^2*log(c) - 3*a*b^2*d^3*n^2)*(d*sqrt(x) + e)^3/(e^5*x^(3/2)) + 450*(b^3*
d^4*n^3 - 2*b^3*d^4*n^2*log(c) - 2*a*b^2*d^4*n^2)*(d*sqrt(x) + e)^2/(e^5*x)
 - 360*(b^3*d^5*n^3 - b^3*d^5*n^2*log(c) - a*b^2*d^5*n^2)*(d*sqrt(x) + e)/(
e^5*sqrt(x)))*log((d*sqrt(x) + e)/sqrt(x))^2 - 60*(100*(b^3*n^3 - 6*b^3*n^2
*log(c) + 18*b^3*n*log(c)^2 - 6*a*b^2*n^2 + 36*a*b^2*n*log(c) + 18*a^2*b*n)
*(d*sqrt(x) + e)^6/(e^5*x^3) - 432*(2*b^3*d*n^3 - 10*b^3*d*n^2*log(c) + 25*
b^3*d*n*log(c)^2 - 10*a*b^2*d*n^2 + 50*a*b^2*d*n*log(c) + 25*a^2*b*d*n)*(d*
sqrt(x) + e)^5/(e^5*x^(5/2)) + 3375*(b^3*d^2*n^3 - 4*b^3*d^2*n^2*log(c) + 8
*b^3*d^2*n*log(c)^2 - 4*a*b^2*d^2*n^2 + 16*a*b^2*d^2*n*log(c) + 8*a^2*b*d^2
*n)*(d*sqrt(x) + e)^4/(e^5*x^2) - 4000*(2*b^3*d^3*n^3 - 6*b^3*d^3*n^2*log(c
) + 9*b^3*d^3*n*log(c)^2 - 6*a*b^2*d^3*n^2 + 18*a*b^2*d^3*n*log(c) + 9*a^2*
b*d^3*n)*(d*sqrt(x) + e)^3/(e^5*x^(3/2)) + 13500*(b^3*d^4*n^3 - 2*b^3*d^4*n
^2*log(c) + 2*b^3*d^4*n*log(c)^2 - 2*a*b^2*d^4*n^2 + 4*a*b^2*d^4*n*log(c) +
 2*a^2*b*d^4*n)*(d*sqrt(x) + e)^2/(e^5*x) - 10800*(2*b^3*d^5*n^3 - 2*b^3*d^
5*n^2*log(c) + b^3*d^5*n*log(c)^2 - 2*a*b^2*d^5*n^2 + 2*a*b^2*d^5*n*log(c)
+ a^2*b*d^5*n)*(d*sqrt(x) + e)/(e^5*sqrt(x)))*log((d*sqrt(x) + e)/sqrt(x))
+ 1000*(b^3*n^3 - 6*b^3*n^2*log(c) + 18*b^3*n*log(c)^2 - 36*b^3*log(c)^3 -
6*a*b^2*n^2 + 36*a*b^2*n*log(c) - 108*a*b^2*log(c)^2 + 18*a^2*b*n - 108*a^2
*b*log(c) - 36*a^3)*(d*sqrt(x) + e)^6/(e^5*x^3) - 1728*(6*b^3*d*n^3 - 30*b^
3*d*n^2*log(c) + 75*b^3*d*n*log(c)^2 - 125*b^3*d*log(c)^3 - 30*a*b^2*d*n^2
+ 150*a*b^2*d*n*log(c) - 375*a*b^2*d*log(c)^2 + 75*a^2*b*d*n - 375*a^2*b*d*
log(c) - 125*a^3*d)*(d*sqrt(x) + e)^5/(e^5*x^(5/2)) + 16875*(3*b^3*d^2*n^3
- 12*b^3*d^2*n^2*log(c) + 24*b^3*d^2*n*log(c)^2 - 32*b^3*d^2*log(c)^3 - 12*
a*b^2*d^2*n^2 + 48*a*b^2*d^2*n*log(c) - 96*a*b^2*d^2*log(c)^2 + 24*a^2*b*d^
2*n - 96*a^2*b*d^2*log(c) - 32*a^3*d^2)*(d*sqrt(x) + e)^4/(e^5*x^2) - 80000
*(2*b^3*d^3*n^3 - 6*b^3*d^3*n^2*log(c) + 9*b^3*d^3*n*log(c)^2 - 9*b^3*d^3*l
```

```
og(c)^3 - 6*a*b^2*d^3*n^2 + 18*a*b^2*d^3*n*log(c) - 27*a*b^2*d^3*log(c)^2 +
 9*a^2*b*d^3*n - 27*a^2*b*d^3*log(c) - 9*a^3*d^3)*(d*sqrt(x) + e)^3/(e^5*x^
(3/2)) + 135000*(3*b^3*d^4*n^3 - 6*b^3*d^4*n^2*log(c) + 6*b^3*d^4*n*log(c)^
2 - 4*b^3*d^4*log(c)^3 - 6*a*b^2*d^4*n^2 + 12*a*b^2*d^4*n*log(c) - 12*a*b^2
*d^4*log(c)^2 + 6*a^2*b*d^4*n - 12*a^2*b*d^4*log(c) - 4*a^3*d^4)*(d*sqrt(x)
 + e)^2/(e^5*x) - 216000*(6*b^3*d^5*n^3 - 6*b^3*d^5*n^2*log(c) + 3*b^3*d^5*
n*log(c)^2 - b^3*d^5*log(c)^3 - 6*a*b^2*d^5*n^2 + 6*a*b^2*d^5*n*log(c) - 3*
a*b^2*d^5*log(c)^2 + 3*a^2*b*d^5*n - 3*a^2*b*d^5*log(c) - a^3*d^5)*(d*sqrt(
x) + e)/(e^5*sqrt(x)))/e
```

Mupad [B] (verification not implemented)

Time = 9.31 (sec) , antiderivative size = 989, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^n\right)\right)^3}{x^4}\,dx = \text{Too large to display}$$

```
[In] int((a + b*log(c*(d + e/x^(1/2))^n))^3/x^4,x)
```

```
[Out] (b^3*n^3)/(108*x^3) - (b^3*log(c*(d + e/x^(1/2))^n)^3)/(3*x^3) - a^3/(3*x^3
) - (a*b^2*log(c*(d + e/x^(1/2))^n)^2)/x^3 + (b^3*n*log(c*(d + e/x^(1/2))^n
)^2)/(6*x^3) - (b^3*n^2*log(c*(d + e/x^(1/2))^n))/(18*x^3) - (a*b^2*n^2)/(1
8*x^3) + (b^3*d^6*log(c*(d + e/x^(1/2))^n)^3)/(3*e^6) - (a^2*b*log(c*(d + e
/x^(1/2))^n))/x^3 + (a^2*b*n)/(6*x^3) + (a*b^2*n*log(c*(d + e/x^(1/2))^n))/
(3*x^3) + (13489*b^3*d^6*n^3*log(d + e/x^(1/2)))/(1800*e^6) + (919*b^3*d^2*
n^3)/(7200*e^2*x^2) + (4669*b^3*d^4*n^3)/(3600*e^4*x) - (2059*b^3*d^3*n^3)/
(5400*e^3*x^(3/2)) - (13489*b^3*d^5*n^3)/(1800*e^5*x^(1/2)) + (a*b^2*d^6*lo
g(c*(d + e/x^(1/2))^n)^2)/e^6 - (49*b^3*d^6*n*log(c*(d + e/x^(1/2))^n)^2)/(
20*e^6) - (91*b^3*d*n^3)/(2250*e*x^(5/2)) + (a^2*b*d^6*n*log(d + e/x^(1/2))
)/e^6 - (b^3*d*n*log(c*(d + e/x^(1/2))^n)^2)/(5*e*x^(5/2)) + (11*b^3*d*n^2*
log(c*(d + e/x^(1/2))^n))/(75*e*x^(5/2)) + (a^2*b*d^2*n)/(4*e^2*x^2) + (a^2
*b*d^4*n)/(2*e^4*x) + (11*a*b^2*d*n^2)/(75*e*x^(5/2)) - (a^2*b*d^3*n)/(3*e^
3*x^(3/2)) - (a^2*b*d^5*n)/(e^5*x^(1/2)) - (49*a*b^2*d^6*n^2*log(d + e/x^(1
/2)))/(10*e^6) + (b^3*d^2*n*log(c*(d + e/x^(1/2))^n)^2)/(4*e^2*x^2) - (37*b
^3*d^2*n^2*log(c*(d + e/x^(1/2))^n))/(120*e^2*x^2) + (b^3*d^4*n*log(c*(d +
e/x^(1/2))^n)^2)/(2*e^4*x) - (29*b^3*d^4*n^2*log(c*(d + e/x^(1/2))^n))/(20*
e^4*x) - (b^3*d^3*n*log(c*(d + e/x^(1/2))^n)^2)/(3*e^3*x^(3/2)) + (19*b^3*d
^3*n^2*log(c*(d + e/x^(1/2))^n))/(30*e^3*x^(3/2)) - (b^3*d^5*n*log(c*(d + e
/x^(1/2))^n)^2)/(e^5*x^(1/2)) + (49*b^3*d^5*n^2*log(c*(d + e/x^(1/2))^n))/(
10*e^5*x^(1/2)) - (37*a*b^2*d^2*n^2)/(120*e^2*x^2) - (29*a*b^2*d^4*n^2)/(20
*e^4*x) + (19*a*b^2*d^3*n^2)/(30*e^3*x^(3/2)) + (49*a*b^2*d^5*n^2)/(10*e^5*
x^(1/2)) - (a^2*b*d*n)/(5*e*x^(5/2)) - (2*a*b^2*d*n*log(c*(d + e/x^(1/2))^n
))/(5*e*x^(5/2)) + (a*b^2*d^2*n*log(c*(d + e/x^(1/2))^n))/(2*e^2*x^2) + (a*
b^2*d^4*n*log(c*(d + e/x^(1/2))^n))/(e^4*x) - (2*a*b^2*d^3*n*log(c*(d + e/x
```

```
^(1/2))^n))/(3*e^3*x^(3/2)) - (2*a*b^2*d^5*n*log(c*(d + e/x^(1/2))^n))/(e^5
*x^(1/2))
```

3.442 $\int x^3 \left(a+b \log \left(c \left(d+e \sqrt[3]{x}\right)^n\right)\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 234

$$\int x^3 \left(a+b \log \left(c \left(d+e \sqrt[3]{x}\right)^n\right)\right) \, dx = \frac{b d^{11} n \sqrt[3]{x}}{4 e^{11}}-\frac{b d^{10} n x^{2/3}}{8 e^{10}}+\frac{b d^9 n x}{12 e^9}-\frac{b d^8 n x^{4/3}}{16 e^8}+\frac{b d^7 n x^{5/3}}{20 e^7}-\frac{b d^6 n x^2}{24 e^6}+\frac{b d^5 n x^{7/3}}{28 e^5}-\frac{b d^4 n x^{8/3}}{32 e^4}+\frac{b d^3 n x^3}{36 e^3}-\frac{b d^2 n x^{10/3}}{40 e^2}+\frac{b d n x^{11/3}}{44 e}-\frac{1}{48} b n x^4-\frac{b d^{12} n \log \left(d+e \sqrt[3]{x}\right)}{4 e^{12}}+\frac{1}{4} x^4 \left(a+b \log \left(c \left(d+e \sqrt[3]{x}\right)^n\right)\right)$$

```
[Out] 1/4*b*d^11*n*x^(1/3)/e^11-1/8*b*d^10*n*x^(2/3)/e^10+1/12*b*d^9*n*x/e^9-1/16
*b*d^8*n*x^(4/3)/e^8+1/20*b*d^7*n*x^(5/3)/e^7-1/24*b*d^6*n*x^2/e^6+1/28*b*d
^5*n*x^(7/3)/e^5-1/32*b*d^4*n*x^(8/3)/e^4+1/36*b*d^3*n*x^3/e^3-1/40*b*d^2*n
*x^(10/3)/e^2+1/44*b*d*n*x^(11/3)/e-1/48*b*n*x^4-1/4*b*d^12*n*ln(d+e*x^(1/3
))/e^12+1/4*x^4*(a+b*ln(c*(d+e*x^(1/3))^n))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used

$= \{2504, 2442, 45\}$

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx = \frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right) - \frac{bd^{12}n\log\left(d+e\sqrt[3]{x}\right)}{4e^{12}} + \frac{bd^{11}n\sqrt[3]{x}}{4e^{11}} - \frac{bd^{10}nx^{2/3}}{8e^{10}} + \frac{bd^9nx}{12e^9} - \frac{bd^8nx^{4/3}}{16e^8} + \frac{bd^7nx^{5/3}}{20e^7} - \frac{bd^6nx^2}{24e^6} + \frac{bd^5nx^{7/3}}{28e^5} - \frac{bd^4nx^{8/3}}{32e^4} + \frac{bd^3nx^3}{36e^3} - \frac{bd^2nx^{10/3}}{40e^2} + \frac{bdnx^{11/3}}{44e} - \frac{1}{48}bnx^4$$

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

```
[Out] (b*d^11*n*x^(1/3))/(4*e^11) - (b*d^10*n*x^(2/3))/(8*e^10) + (b*d^9*n*x)/(12
*e^9) - (b*d^8*n*x^(4/3))/(16*e^8) + (b*d^7*n*x^(5/3))/(20*e^7) - (b*d^6*n*
x^2)/(24*e^6) + (b*d^5*n*x^(7/3))/(28*e^5) - (b*d^4*n*x^(8/3))/(32*e^4) + (
b*d^3*n*x^3)/(36*e^3) - (b*d^2*n*x^(10/3))/(40*e^2) + (b*d*n*x^(11/3))/(44*
e) - (b*n*x^4)/48 - (b*d^12*n*Log[d + e*x^(1/3)])/(4*e^12) + (x^4*(a + b*Lo
g[c*(d + e*x^(1/3))^n]))/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^{11}(a+b\log(c(d+ex)^n))\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{4}(ben)\text{Subst}\left(\int\frac{x^{12}}{d+ex}\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{4}(ben)\text{Subst}\left(\int\left(-\frac{d^{11}}{e^{12}}+\frac{d^{10}x}{e^{11}}-\frac{d^9x^2}{e^{10}}+\frac{d^8x^3}{e^9}-\frac{d^7x^4}{e^8}\right.\right.\\
&\quad\left.\left.+\frac{d^6x^5}{e^7}-\frac{d^5x^6}{e^6}+\frac{d^4x^7}{e^5}-\frac{d^3x^8}{e^4}+\frac{d^2x^9}{e^3}-\frac{dx^{10}}{e^2}+\frac{x^{11}}{e}+\frac{d^{12}}{e^{12}(d+ex)}\right)dx, x, \sqrt[3]{x}\right)\\
&= \frac{bd^{11}n\sqrt[3]{x}}{4e^{11}}-\frac{bd^{10}nx^{2/3}}{8e^{10}}+\frac{bd^9nx}{12e^9}-\frac{bd^8nx^{4/3}}{16e^8}+\frac{bd^7nx^{5/3}}{20e^7}-\frac{bd^6nx^2}{24e^6}\\
&\quad+\frac{bd^5nx^{7/3}}{28e^5}-\frac{bd^4nx^{8/3}}{32e^4}+\frac{bd^3nx^3}{36e^3}-\frac{bd^2nx^{10/3}}{40e^2}+\frac{bdnx^{11/3}}{44e}\\
&\quad-\frac{1}{48}bnx^4-\frac{bd^{12}n\log\left(d+e\sqrt[3]{x}\right)}{4e^{12}}+\frac{1}{4}x^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.93

$$\begin{aligned}
\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)dx &= \frac{ax^4}{4}-\frac{1}{12}ben\left(-\frac{3d^{11}\sqrt[3]{x}}{e^{12}}+\frac{3d^{10}x^{2/3}}{2e^{11}}-\frac{d^9x}{e^{10}}+\frac{3d^8x^{4/3}}{4e^9}\right.\\
&\quad-\frac{3d^7x^{5/3}}{5e^8}+\frac{d^6x^2}{2e^7}-\frac{3d^5x^{7/3}}{7e^6}+\frac{3d^4x^{8/3}}{8e^5}-\frac{d^3x^3}{3e^4}\\
&\quad+\frac{3d^2x^{10/3}}{10e^3}-\frac{3dx^{11/3}}{11e^2}+\frac{x^4}{4e}\\
&\quad\left.+\frac{3d^{12}\log\left(d+e\sqrt[3]{x}\right)}{e^{13}}\right)+\frac{1}{4}bx^4\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)
\end{aligned}$$

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
[Out] (a*x^4)/4 - (b*e*n*((-3*d^11*x^(1/3))/e^12 + (3*d^10*x^(2/3))/(2*e^11) - (d
^9*x)/e^10 + (3*d^8*x^(4/3))/(4*e^9) - (3*d^7*x^(5/3))/(5*e^8) + (d^6*x^2)/
(2*e^7) - (3*d^5*x^(7/3))/(7*e^6) + (3*d^4*x^(8/3))/(8*e^5) - (d^3*x^3)/(3*
e^4) + (3*d^2*x^(10/3))/(10*e^3) - (3*d*x^(11/3))/(11*e^2) + x^4/(4*e) + (3
*d^12*Log[d + e*x^(1/3)])/e^13))/12 + (b*x^4*Log[c*(d + e*x^(1/3))^n])/4
```

Maple [F]

$$\int x^3\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^n\right)\right)dx$$

[In] int(x^3*(a+b*ln(c*(d+e*x^(1/3))^n)),x)

[Out] int(x^3*(a+b*ln(c*(d+e*x^(1/3))^n)),x)

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.86

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$$
$$=\frac{27720\,be^{12}x^4\log\left(c\right)+3080\,bd^3e^9nx^3-4620\,bd^6e^6nx^2+9240\,bd^9e^3nx-2310\,(be^{12}n-12\,ae^{12})x^4+27720\,(\ldots}{\ldots}$$

[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="fricas")

[Out] 1/110880*(27720*b*e^12*x^4*log(c) + 3080*b*d^3*e^9*n*x^3 - 4620*b*d^6*e^6*n
*x^2 + 9240*b*d^9*e^3*n*x - 2310*(b*e^12*n - 12*a*e^12)*x^4 + 27720*(b*e^12
*n*x^4 - b*d^12*n)*log(e*x^(1/3) + d) + 63*(40*b*d*e^11*n*x^3 - 55*b*d^4*e^
8*n*x^2 + 88*b*d^7*e^5*n*x - 220*b*d^10*e^2*n)*x^(2/3) - 198*(14*b*d^2*e^10
*n*x^3 - 20*b*d^5*e^7*n*x^2 + 35*b*d^8*e^4*n*x - 140*b*d^11*e*n)*x^(1/3))/e
^12

Sympy [A] (verification not implemented)

Time = 45.06 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.92

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{ax^4}{4}$$

$$+b\left(-\frac{en\left(\frac{3d^{12}\left(\begin{cases}\frac{\sqrt[3]{x}}{d} & \text{for } e=0\\ \frac{\log\left(d+e\sqrt[3]{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^{12}}-\frac{3d^{11}\sqrt[3]{x}}{e^{12}}+\frac{3d^{10}x^{\frac{2}{3}}}{2e^{11}}-\frac{d^9x}{e^{10}}+\frac{3d^8x^{\frac{4}{3}}}{4e^9}-\frac{3d^7x^{\frac{5}{3}}}{5e^8}+\frac{d^6x^2}{2e^7}-\frac{3d^5x^{\frac{7}{3}}}{7e^6}+3\right.}{12}\right.$$

$$\left.+\frac{x^4\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{4}\right)$$

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/3))**n)),x)
```

```
[Out] a*x**4/4 + b*(-e*n*(3*d**12*Piecewise((x**(1/3)/d, Eq(e, 0)), (log(d + e*x*
*(1/3))/e, True))/e**12 - 3*d**11*x**(1/3)/e**12 + 3*d**10*x**(2/3)/(2*e**1
1) - d**9*x/e**10 + 3*d**8*x**(4/3)/(4*e**9) - 3*d**7*x**(5/3)/(5*e**8) + d
**6*x**2/(2*e**7) - 3*d**5*x**(7/3)/(7*e**6) + 3*d**4*x**(8/3)/(8*e**5) - d
**3*x**3/(3*e**4) + 3*d**2*x**(10/3)/(10*e**3) - 3*d*x**(11/3)/(11*e**2) +
x**4/(4*e))/12 + x**4*log(c*(d + e*x**(1/3))**n)/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.74

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{1}{4}\,bx^4\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+\frac{1}{4}\,ax^4$$
$$-\frac{1}{110880}\,ben\left(\frac{27720\,d^{12}\log\left(ex^{\frac{1}{3}}+d\right)}{e^{13}}+\frac{2310\,e^{11}x^4-2520\,de^{10}x^{\frac{11}{3}}+2772\,d^2e^9x^{\frac{10}{3}}-3080\,d^3e^8x^3+3465\cdots}{\cdots}\right.$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4*log((e*x^(1/3) + d)^n*c) + 1/4*a*x^4 - 1/110880*b*e*n*(27720*d^12
*log(e*x^(1/3) + d)/e^13 + (2310*e^11*x^4 - 2520*d*e^10*x^(11/3) + 2772*d^2
*e^9*x^(10/3) - 3080*d^3*e^8*x^3 + 3465*d^4*e^7*x^(8/3) - 3960*d^5*e^6*x^(7
/3) + 4620*d^6*e^5*x^2 - 5544*d^7*e^4*x^(5/3) + 6930*d^8*e^3*x^(4/3) - 9240
*d^9*e^2*x + 13860*d^10*e*x^(2/3) - 27720*d^11*x^(1/3))/e^12)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(186) = 372$.

Time = 0.30 (sec) , antiderivative size = 516, normalized size of antiderivative = 2.21

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$$
$$=\frac{27720\,bex^4\log(c)+27720\,aex^4+\left(\frac{27720\left(ex^{\frac{1}{3}}+d\right)^{12}\log\left(ex^{\frac{1}{3}}+d\right)}{e^{11}}-\frac{332640\left(ex^{\frac{1}{3}}+d\right)^{11}d\log\left(ex^{\frac{1}{3}}+d\right)}{e^{11}}+\frac{1829520\left(ex^{\frac{1}{3}}+d\right)^{10}\cdots}{e^{11}}\cdots\right.}{\cdots}$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="giac")
```

```
[Out] 1/110880*(27720*b*e*x^4*log(c) + 27720*a*e*x^4 + (27720*(e*x^(1/3) + d)^12*
log(e*x^(1/3) + d)/e^11 - 332640*(e*x^(1/3) + d)^11*d*log(e*x^(1/3) + d)/e^
11 + 1829520*(e*x^(1/3) + d)^10*d^2*log(e*x^(1/3) + d)/e^11 - 6098400*(e*x^
(1/3) + d)^9*d^3*log(e*x^(1/3) + d)/e^11 + 13721400*(e*x^(1/3) + d)^8*d^4*l
og(e*x^(1/3) + d)/e^11 - 21954240*(e*x^(1/3) + d)^7*d^5*log(e*x^(1/3) + d)/
e^11 + 25613280*(e*x^(1/3) + d)^6*d^6*log(e*x^(1/3) + d)/e^11 - 21954240*(e
*x^(1/3) + d)^5*d^7*log(e*x^(1/3) + d)/e^11 + 13721400*(e*x^(1/3) + d)^4*d^
8*log(e*x^(1/3) + d)/e^11 - 6098400*(e*x^(1/3) + d)^3*d^9*log(e*x^(1/3) + d
)/e^11 + 1829520*(e*x^(1/3) + d)^2*d^10*log(e*x^(1/3) + d)/e^11 - 332640*(e
*x^(1/3) + d)*d^11*log(e*x^(1/3) + d)/e^11 - 2310*(e*x^(1/3) + d)^12/e^11 +
 30240*(e*x^(1/3) + d)^11*d/e^11 - 182952*(e*x^(1/3) + d)^10*d^2/e^11 + 677
600*(e*x^(1/3) + d)^9*d^3/e^11 - 1715175*(e*x^(1/3) + d)^8*d^4/e^11 + 31363
```

```
20*(e*x^(1/3) + d)^7*d^5/e^11 - 4268880*(e*x^(1/3) + d)^6*d^6/e^11 + 439084
8*(e*x^(1/3) + d)^5*d^7/e^11 - 3430350*(e*x^(1/3) + d)^4*d^8/e^11 + 2032800
*(e*x^(1/3) + d)^3*d^9/e^11 - 914760*(e*x^(1/3) + d)^2*d^10/e^11 + 332640*(
e*x^(1/3) + d)*d^11/e^11)*b*n)/e
```

Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.81

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{a\,x^4}{4}-\frac{b\,n\,x^4}{48}+\frac{b\,x^4\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{4}+\frac{b\,d\,n\,x^{11/3}}{44\,e}+\frac{b\,d^9\,n\,x}{12\,e^9}-\frac{b\,d^{12}\,n\,\ln\left(d+e\,x^{1/3}\right)}{4\,e^{12}}+\frac{b\,d^3\,n\,x^3}{36\,e^3}-\frac{b\,d^6\,n\,x^2}{24\,e^6}-\frac{b\,d^2\,n\,x^{10/3}}{40\,e^2}-\frac{b\,d^4\,n\,x^{8/3}}{32\,e^4}+\frac{b\,d^5\,n\,x^{7/3}}{28\,e^5}+\frac{b\,d^7\,n\,x^{5/3}}{20\,e^7}-\frac{b\,d^8\,n\,x^{4/3}}{16\,e^8}-\frac{b\,d^{10}\,n\,x^{2/3}}{8\,e^{10}}+\frac{b\,d^{11}\,n\,x^{1/3}}{4\,e^{11}}$$

```
[In] int(x^3*(a + b*log(c*(d + e*x^(1/3))^n)),x)
```

```
[Out] (a*x^4)/4 - (b*n*x^4)/48 + (b*x^4*log(c*(d + e*x^(1/3))^n))/4 + (b*d*n*x^(1
1/3))/(44*e) + (b*d^9*n*x)/(12*e^9) - (b*d^12*n*log(d + e*x^(1/3)))/(4*e^12
) + (b*d^3*n*x^3)/(36*e^3) - (b*d^6*n*x^2)/(24*e^6) - (b*d^2*n*x^(10/3))/(4
0*e^2) - (b*d^4*n*x^(8/3))/(32*e^4) + (b*d^5*n*x^(7/3))/(28*e^5) + (b*d^7*n
*x^(5/3))/(20*e^7) - (b*d^8*n*x^(4/3))/(16*e^8) - (b*d^10*n*x^(2/3))/(8*e^1
0) + (b*d^11*n*x^(1/3))/(4*e^11)
```

3.443 $\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 185

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=-\frac{bd^8n\sqrt[3]{x}}{3e^8}+\frac{bd^7nx^{2/3}}{6e^7}-\frac{bd^6nx}{9e^6}+\frac{bd^5nx^{4/3}}{12e^5}-\frac{bd^4nx^{5/3}}{15e^4}+\frac{bd^3nx^2}{18e^3}-\frac{bd^2nx^{7/3}}{21e^2}+\frac{bdnx^{8/3}}{24e}-\frac{1}{27}bnx^3+\frac{bd^9n\log\left(d+e\sqrt[3]{x}\right)}{3e^9}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)$$

```
[Out] -1/3*b*d^8*n*x^(1/3)/e^8+1/6*b*d^7*n*x^(2/3)/e^7-1/9*b*d^6*n*x/e^6+1/12*b*d
^5*n*x^(4/3)/e^5-1/15*b*d^4*n*x^(5/3)/e^4+1/18*b*d^3*n*x^2/e^3-1/21*b*d^2*n
*x^(7/3)/e^2+1/24*b*d*n*x^(8/3)/e-1/27*b*n*x^3+1/3*b*d^9*n*ln(d+e*x^(1/3))/
e^9+1/3*x^3*(a+b*ln(c*(d+e*x^(1/3))^n))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 45}

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)+\frac{bd^9n\log\left(d+e\sqrt[3]{x}\right)}{3e^9}-\frac{bd^8n\sqrt[3]{x}}{3e^8}+\frac{bd^7nx^{2/3}}{6e^7}-\frac{bd^6nx}{9e^6}+\frac{bd^5nx^{4/3}}{12e^5}-\frac{bd^4nx^{5/3}}{15e^4}+\frac{bd^3nx^2}{18e^3}-\frac{bd^2nx^{7/3}}{21e^2}+\frac{bdnx^{8/3}}{24e}-\frac{1}{27}bnx^3$$

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

```
[Out] -1/3*(b*d^8*n*x^(1/3))/e^8 + (b*d^7*n*x^(2/3))/(6*e^7) - (b*d^6*n*x)/(9*e^6
) + (b*d^5*n*x^(4/3))/(12*e^5) - (b*d^4*n*x^(5/3))/(15*e^4) + (b*d^3*n*x^2)
/(18*e^3) - (b*d^2*n*x^(7/3))/(21*e^2) + (b*d*n*x^(8/3))/(24*e) - (b*n*x^3)
/27 + (b*d^9*n*Log[d + e*x^(1/3)])/(3*e^9) + (x^3*(a + b*Log[c*(d + e*x^(1/
3))^n]))/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^8(a+b\log(c(d+ex)^n))\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{3}(ben)\text{Subst}\left(\int\frac{x^9}{d+ex}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{3}(ben)\text{Subst}\left(\int\left(\frac{d^8}{e^9}-\frac{d^7x}{e^8}+\frac{d^6x^2}{e^7}-\frac{d^5x^3}{e^6}\right.\right.\\
&\qquad\left.\left.+\frac{d^4x^4}{e^5}-\frac{d^3x^5}{e^4}+\frac{d^2x^6}{e^3}-\frac{dx^7}{e^2}+\frac{x^8}{e}-\frac{d^9}{e^9(d+ex)}\right)dx,x,\sqrt[3]{x}\right)\\
&= -\frac{bd^8n\sqrt[3]{x}}{3e^8}+\frac{bd^7nx^{2/3}}{6e^7}-\frac{bd^6nx}{9e^6}+\frac{bd^5nx^{4/3}}{12e^5}-\frac{bd^4nx^{5/3}}{15e^4}+\frac{bd^3nx^2}{18e^3}-\frac{bd^2nx^{7/3}}{21e^2}\\
&\qquad+\frac{bdnx^{8/3}}{24e}-\frac{1}{27}bnx^3+\frac{bd^9n\log\left(d+e\sqrt[3]{x}\right)}{3e^9}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.94

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{ax^3}{3}-\frac{1}{9}ben\left(\frac{3d^8\sqrt[3]{x}}{e^9}-\frac{3d^7x^{2/3}}{2e^8}+\frac{d^6x}{e^7}-\frac{3d^5x^{4/3}}{4e^6}+\frac{3d^4x^{5/3}}{5e^5}-\frac{d^3x^2}{2e^4}+\frac{3d^2x^{7/3}}{7e^3}-\frac{3dx^{8/3}}{8e^2}+\frac{x^3}{3e}-\frac{3d^9\log\left(d+e\sqrt[3]{x}\right)}{e^{10}}\right)+\frac{1}{3}bx^3\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

```
[Out] (a*x^3)/3 - (b*e*n*((3*d^8*x^(1/3))/e^9 - (3*d^7*x^(2/3))/(2*e^8) + (d^6*x)
/e^7 - (3*d^5*x^(4/3))/(4*e^6) + (3*d^4*x^(5/3))/(5*e^5) - (d^3*x^2)/(2*e^4
) + (3*d^2*x^(7/3))/(7*e^3) - (3*d*x^(8/3))/(8*e^2) + x^3/(3*e) - (3*d^9*Lo
g[d + e*x^(1/3)])/e^10))/9 + (b*x^3*Log[c*(d + e*x^(1/3))^n])/3
```

Maple [F]

$$\int x^2\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^n\right)\right)dx$$

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.87

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{2520\,be^9x^3\log\left(c\right)+420\,bd^3e^6nx^2-840\,bd^6e^3nx-280\,(be^9n-9\,ae^9)x^3+2520\,(be^9nx^3+bd^9n)\log\left(ex^{\frac{1}{3}}+\right.}{7560\,e^9}$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/7560*(2520*b*e^9*x^3*log(c) + 420*b*d^3*e^6*n*x^2 - 840*b*d^6*e^3*n*x - 2
80*(b*e^9*n - 9*a*e^9)*x^3 + 2520*(b*e^9*n*x^3 + b*d^9*n)*log(e*x^(1/3) + d
) + 63*(5*b*d*e^8*n*x^2 - 8*b*d^4*e^5*n*x + 20*b*d^7*e^2*n)*x^(2/3) - 90*(4
*b*d^2*e^7*n*x^2 - 7*b*d^5*e^4*n*x + 28*b*d^8*e*n)*x^(1/3))/e^9
```

Sympy [A] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.94

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{ax^3}{3}+b\left(-\frac{en\left(-\frac{3d^9\left(\begin{cases}\frac{\sqrt[3]{x}}{d} & \text{for } e=0\\ \frac{\log\left(d+e\sqrt[3]{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^9}+\frac{3d^8\sqrt[3]{x}}{e^9}-\frac{3d^7x^{\frac{2}{3}}}{2e^8}+\frac{d^6x}{e^7}-\frac{3d^5x^{\frac{4}{3}}}{4e^6}+\frac{3d^4x^{\frac{5}{3}}}{5e^5}-\frac{d^3x^2}{2e^4}+\frac{3d^2x^{\frac{7}{3}}}{7e^3}-\cdots\right)}{9}+\frac{x^3\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3}\right)$$

[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))**n)),x)

[Out] a*x**3/3 + b*(-e*n*(-3*d**9*Piecewise((x**(1/3)/d, Eq(e, 0)), (log(d + e*x**(1/3))/e, True))/e**9 + 3*d**8*x**(1/3)/e**9 - 3*d**7*x**(2/3)/(2*e**8) + d**6*x/e**7 - 3*d**5*x**(4/3)/(4*e**6) + 3*d**4*x**(5/3)/(5*e**5) - d**3*x**2/(2*e**4) + 3*d**2*x**(7/3)/(7*e**3) - 3*d*x**(8/3)/(8*e**2) + x**3/(3*e))/9 + x**3*log(c*(d + e*x**(1/3))**n)/3)

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.76

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{1}{3}bx^3\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+\frac{1}{3}ax^3$$
$$+\frac{1}{7560}ben\left(\frac{2520\,d^9\log\left(ex^{\frac{1}{3}}+d\right)}{e^{10}}-\frac{280\,e^8x^3-315\,de^7x^{\frac{8}{3}}+360\,d^2e^6x^{\frac{7}{3}}-420\,d^3e^5x^2+504\,d^4e^4x^{\frac{5}{3}}-63\right.}{e^9}$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3*log((e*x^(1/3) + d)^n*c) + 1/3*a*x^3 + 1/7560*b*e*n*(2520*d^9*log
(e*x^(1/3) + d)/e^10 - (280*e^8*x^3 - 315*d*e^7*x^(8/3) + 360*d^2*e^6*x^(7/
3) - 420*d^3*e^5*x^2 + 504*d^4*e^4*x^(5/3) - 630*d^5*e^3*x^(4/3) + 840*d^6*
e^2*x - 1260*d^7*e*x^(2/3) + 2520*d^8*x^(1/3))/e^9)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. 2(147) = 294.

Time = 0.30 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.11

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$$
$$=\frac{2520\,bex^3\log(c)+2520\,aex^3+\left(\frac{2520\left(ex^{\frac{1}{3}}+d\right)^9\log\left(ex^{\frac{1}{3}}+d\right)}{e^8}-\frac{22680\left(ex^{\frac{1}{3}}+d\right)^8d\log\left(ex^{\frac{1}{3}}+d\right)}{e^8}+\frac{90720\left(ex^{\frac{1}{3}}+d\right)^7d^2\log\left(ex^{\frac{1}{3}}\right.}{e^8}\right.}{}$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="giac")
```

```
[Out] 1/7560*(2520*b*e*x^3*log(c) + 2520*a*e*x^3 + (2520*(e*x^(1/3) + d)^9*log(e*
x^(1/3) + d)/e^8 - 22680*(e*x^(1/3) + d)^8*d*log(e*x^(1/3) + d)/e^8 + 90720
*(e*x^(1/3) + d)^7*d^2*log(e*x^(1/3) + d)/e^8 - 211680*(e*x^(1/3) + d)^6*d^
3*log(e*x^(1/3) + d)/e^8 + 317520*(e*x^(1/3) + d)^5*d^4*log(e*x^(1/3) + d)/
e^8 - 317520*(e*x^(1/3) + d)^4*d^5*log(e*x^(1/3) + d)/e^8 + 211680*(e*x^(1/
3) + d)^3*d^6*log(e*x^(1/3) + d)/e^8 - 90720*(e*x^(1/3) + d)^2*d^7*log(e*x^
(1/3) + d)/e^8 + 22680*(e*x^(1/3) + d)*d^8*log(e*x^(1/3) + d)/e^8 - 280*(e*
x^(1/3) + d)^9/e^8 + 2835*(e*x^(1/3) + d)^8*d/e^8 - 12960*(e*x^(1/3) + d)^7
*d^2/e^8 + 35280*(e*x^(1/3) + d)^6*d^3/e^8 - 63504*(e*x^(1/3) + d)^5*d^4/e^
8 + 79380*(e*x^(1/3) + d)^4*d^5/e^8 - 70560*(e*x^(1/3) + d)^3*d^6/e^8 + 453
60*(e*x^(1/3) + d)^2*d^7/e^8 - 22680*(e*x^(1/3) + d)*d^8/e^8)*b*n)/e
```

Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.81

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{a\,x^3}{3}-\frac{b\,n\,x^3}{27}+\frac{b\,x^3\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{3}+\frac{b\,d\,n\,x^{8/3}}{24\,e}-\frac{b\,d^6\,n\,x}{9\,e^6}+\frac{b\,d^9\,n\,\ln\left(d+e\,x^{1/3}\right)}{3\,e^9}+\frac{b\,d^3\,n\,x^2}{18\,e^3}-\frac{b\,d^2\,n\,x^{7/3}}{21\,e^2}-\frac{b\,d^4\,n\,x^{5/3}}{15\,e^4}+\frac{b\,d^5\,n\,x^{4/3}}{12\,e^5}+\frac{b\,d^7\,n\,x^{2/3}}{6\,e^7}-\frac{b\,d^8\,n\,x^{1/3}}{3\,e^8}$$

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/3))^n)),x)
```

```
[Out] (a*x^3)/3 - (b*n*x^3)/27 + (b*x^3*log(c*(d + e*x^(1/3))^n))/3 + (b*d*n*x^(8
/3))/(24*e) - (b*d^6*n*x)/(9*e^6) + (b*d^9*n*log(d + e*x^(1/3)))/(3*e^9) +
(b*d^3*n*x^2)/(18*e^3) - (b*d^2*n*x^(7/3))/(21*e^2) - (b*d^4*n*x^(5/3))/(15
*e^4) + (b*d^5*n*x^(4/3))/(12*e^5) + (b*d^7*n*x^(2/3))/(6*e^7) - (b*d^8*n*x
^(1/3))/(3*e^8)
```

3.444 $\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 136

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{bd^5n\sqrt[3]{x}}{2e^5}-\frac{bd^4nx^{2/3}}{4e^4}+\frac{bd^3nx}{6e^3}-\frac{bd^2nx^{4/3}}{8e^2}+\frac{bdnx^{5/3}}{10e}-\frac{1}{12}bnx^2-\frac{bd^6n\log\left(d+e\sqrt[3]{x}\right)}{2e^6}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)$$

```
[Out] 1/2*b*d^5*n*x^(1/3)/e^5-1/4*b*d^4*n*x^(2/3)/e^4+1/6*b*d^3*n*x/e^3-1/8*b*d^2
*n*x^(4/3)/e^2+1/10*b*d*n*x^(5/3)/e-1/12*b*n*x^2-1/2*b*d^6*n*ln(d+e*x^(1/3)
)/e^6+1/2*x^2*(a+b*ln(c*(d+e*x^(1/3))^n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2504, 2442, 45}

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{bd^6n\log\left(d+e\sqrt[3]{x}\right)}{2e^6}+\frac{bd^5n\sqrt[3]{x}}{2e^5}-\frac{bd^4nx^{2/3}}{4e^4}+\frac{bd^3nx}{6e^3}-\frac{bd^2nx^{4/3}}{8e^2}+\frac{bdnx^{5/3}}{10e}-\frac{1}{12}bnx^2$$

```
[In] Int[x*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

```
[Out] (b*d^5*n*x^(1/3))/(2*e^5) - (b*d^4*n*x^(2/3))/(4*e^4) + (b*d^3*n*x)/(6*e^3)
 - (b*d^2*n*x^(4/3))/(8*e^2) + (b*d*n*x^(5/3))/(10*e) - (b*n*x^2)/12 - (b*d
^6*n*Log[d + e*x^(1/3)])/(2*e^6) + (x^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^5(a+b\log\left(c(d+ex)^n\right))\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{2}(ben)\text{Subst}\left(\int\frac{x^6}{d+ex}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)-\frac{1}{2}(ben)\text{Subst}\left(\int\left(-\frac{d^5}{e^6}+\frac{d^4x}{e^5}-\frac{d^3x^2}{e^4}+\frac{d^2x^3}{e^3}\right.\right.\\
&\qquad\left.\left.-\frac{dx^4}{e^2}+\frac{x^5}{e}+\frac{d^6}{e^6(d+ex)}\right)\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{bd^5n\sqrt[3]{x}}{2e^5}-\frac{bd^4nx^{2/3}}{4e^4}+\frac{bd^3nx}{6e^3}-\frac{bd^2nx^{4/3}}{8e^2}+\frac{bdnx^{5/3}}{10e}-\frac{1}{12}bnx^2\\
&\quad-\frac{bd^6n\log\left(d+e\sqrt[3]{x}\right)}{2e^6}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.97

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{ax^2}{2}-\frac{1}{6}ben\left(-\frac{3d^5\sqrt[3]{x}}{e^6}+\frac{3d^4x^{2/3}}{2e^5}-\frac{d^3x}{e^4}+\frac{3d^2x^{4/3}}{4e^3}-\frac{3dx^{5/3}}{5e^2}+\frac{x^2}{2e}+\frac{3d^6\log\left(d+e\sqrt[3]{x}\right)}{e^7}\right)+\frac{1}{2}bx^2\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)$$

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^n]),x]
```

```
[Out] (a*x^2)/2 - (b*e*n*((-3*d^5*x^(1/3))/e^6 + (3*d^4*x^(2/3))/(2*e^5) - (d^3*x
)/e^4 + (3*d^2*x^(4/3))/(4*e^3) - (3*d*x^(5/3))/(5*e^2) + x^2/(2*e) + (3*d^
6*Log[d + e*x^(1/3)])/e^7))/6 + (b*x^2*Log[c*(d + e*x^(1/3))^n])/2
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)dx$$

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(1/3))^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.90

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{60\,be^6x^2\log\left(c\right)+20\,bd^3e^3nx-10\,(be^6n-6\,ae^6)x^2+60\,(be^6nx^2-bd^6n)\log\left(ex^{\frac{1}{3}}+d\right)+6\,(2\,bde^5nx-5\,b}{120\,e^6}$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/120*(60*b*e^6*x^2*log(c) + 20*b*d^3*e^3*n*x - 10*(b*e^6*n - 6*a*e^6)*x^2
+ 60*(b*e^6*n*x^2 - b*d^6*n)*log(e*x^(1/3) + d) + 6*(2*b*d*e^5*n*x - 5*b*d^
4*e^2*n)*x^(2/3) - 15*(b*d^2*e^4*n*x - 4*b*d^5*e*n)*x^(1/3))/e^6
```

Sympy [A] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{ax^2}{2}+b\left(-\frac{en\left(\frac{3d^6\left(\begin{cases}\frac{\sqrt[3]{x}}{d} & \text{for } e=0\\ \frac{\log\left(d+e\sqrt[3]{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^6}-\frac{3d^5\sqrt[3]{x}}{e^6}+\frac{3d^4x^{\frac{2}{3}}}{2e^5}-\frac{d^3x}{e^4}+\frac{3d^2x^{\frac{4}{3}}}{4e^3}-\frac{3dx^{\frac{5}{3}}}{5e^2}+\frac{x^2}{2e}\right)}{6}+\frac{x^2\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2}\right)$$

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/3))**n)),x)
```

```
[Out] a*x**2/2 + b*(-e*n*(3*d**6*Piecewise((x**(1/3)/d, Eq(e, 0)), (log(d + e*x**
(1/3))/e, True))/e**6 - 3*d**5*x**(1/3)/e**6 + 3*d**4*x**(2/3)/(2*e**5) - d
**3*x/e**4 + 3*d**2*x**(4/3)/(4*e**3) - 3*d*x**(5/3)/(5*e**2) + x**2/(2*e))
/6 + x**2*log(c*(d + e*x**(1/3))**n)/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.78

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx = -\frac{1}{120}\,ben\left(\frac{60\,d^6\log\left(ex^{\frac{1}{3}}+d\right)}{e^7}+\frac{10\,e^5x^2-12\,de^4x^{\frac{5}{3}}+15\,d^2e^3x^{\frac{4}{3}}-20\,d^3e^2x+30\,d^4ex^{\frac{2}{3}}-60\,d^5x^{\frac{1}{3}}}{e^6}\right)+\frac{1}{2}\,bx^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+\frac{1}{2}\,ax^2$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="maxima")
```

```
[Out] -1/120*b*e*n*(60*d^6*log(e*x^(1/3) + d)/e^7 + (10*e^5*x^2 - 12*d*e^4*x^(5/3
) + 15*d^2*e^3*x^(4/3) - 20*d^3*e^2*x + 30*d^4*e*x^(2/3) - 60*d^5*x^(1/3))/
e^6) + 1/2*b*x^2*log((e*x^(1/3) + d)^n*c) + 1/2*a*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(108) = 216.

Time = 0.31 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.94

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx = \frac{60\,bex^2\log\left(c\right)+60\,aex^2+\left(\frac{60\left(ex^{\frac{1}{3}}+d\right)^6\log\left(ex^{\frac{1}{3}}+d\right)}{e^5}-\frac{360\left(ex^{\frac{1}{3}}+d\right)^5d\log\left(ex^{\frac{1}{3}}+d\right)}{e^5}+\frac{900\left(ex^{\frac{1}{3}}+d\right)^4d^2\log\left(ex^{\frac{1}{3}}+d\right)}{e^5}-\frac{1200}{}\right.}{}$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n)),x, algorithm="giac")
```

```
[Out] 1/120*(60*b*e*x^2*log(c) + 60*a*e*x^2 + (60*(e*x^(1/3) + d)^6*log(e*x^(1/3)
 + d)/e^5 - 360*(e*x^(1/3) + d)^5*d*log(e*x^(1/3) + d)/e^5 + 900*(e*x^(1/3)
 + d)^4*d^2*log(e*x^(1/3) + d)/e^5 - 1200*(e*x^(1/3) + d)^3*d^3*log(e*x^(1/
3) + d)/e^5 + 900*(e*x^(1/3) + d)^2*d^4*log(e*x^(1/3) + d)/e^5 - 360*(e*x^(
1/3) + d)*d^5*log(e*x^(1/3) + d)/e^5 - 10*(e*x^(1/3) + d)^6/e^5 + 72*(e*x^(
1/3) + d)^5*d/e^5 - 225*(e*x^(1/3) + d)^4*d^2/e^5 + 400*(e*x^(1/3) + d)^3*d
^3/e^5 - 450*(e*x^(1/3) + d)^2*d^4/e^5 + 360*(e*x^(1/3) + d)*d^5/e^5)*b*n)/
e
```

Mupad [B] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.82

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=\frac{a\,x^2}{2}-\frac{b\,n\,x^2}{12}+\frac{b\,x^2\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{2}+\frac{b\,d^3\,n\,x}{6\,e^3}+\frac{b\,d\,n\,x^{5/3}}{10\,e}-\frac{b\,d^6\,n\,\ln\left(d+e\,x^{1/3}\right)}{2\,e^6}-\frac{b\,d^2\,n\,x^{4/3}}{8\,e^2}-\frac{b\,d^4\,n\,x^{2/3}}{4\,e^4}+\frac{b\,d^5\,n\,x^{1/3}}{2\,e^5}$$

```
[In] int(x*(a + b*log(c*(d + e*x^(1/3))^n)),x)

[Out] (a*x^2)/2 - (b*n*x^2)/12 + (b*x^2*log(c*(d + e*x^(1/3))^n))/2 + (b*d^3*n*x)
/(6*e^3) + (b*d*n*x^(5/3))/(10*e) - (b*d^6*n*log(d + e*x^(1/3)))/(2*e^6) -
(b*d^2*n*x^(4/3))/(8*e^2) - (b*d^4*n*x^(2/3))/(4*e^4) + (b*d^5*n*x^(1/3))/(
2*e^5)
```

3.445 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 77

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=-\frac{bd^2n\sqrt[3]{x}}{e^2}+\frac{bdnx^{2/3}}{2e}+ax-\frac{bnx}{3}+\frac{bd^3n\log\left(d+e\sqrt[3]{x}\right)}{e^3}+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)$$

```
[Out] -b*d^2*n*x^(1/3)/e^2+1/2*b*d*n*x^(2/3)/e+a*x-1/3*b*n*x+b*d^3*n*ln(d+e*x^(1/
3))/e^3+b*x*ln(c*(d+e*x^(1/3))^n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2498, 272, 45}

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=ax+bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)+\frac{bd^3n\log\left(d+e\sqrt[3]{x}\right)}{e^3}-\frac{bd^2n\sqrt[3]{x}}{e^2}+\frac{bdnx^{2/3}}{2e}-\frac{bnx}{3}$$

```
[In] Int[a + b*Log[c*(d + e*x^(1/3))^n],x]
```

```
[Out] -((b*d^2*n*x^(1/3))/e^2) + (b*d*n*x^(2/3))/(2*e) + a*x - (b*n*x)/3 + (b*d^3
*n*Log[d + e*x^(1/3)])/e^3 + b*x*Log[c*(d + e*x^(1/3))^n]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= ax + b\int \log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\,dx\\
&= ax + bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right) - \frac{1}{3}(ben)\int \frac{\sqrt[3]{x}}{d+e\sqrt[3]{x}}\,dx\\
&= ax + bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right) - (ben)\text{Subst}\left(\int \frac{x^3}{d+ex}\,dx, x, \sqrt[3]{x}\right)\\
&= ax + bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right) - (ben)\text{Subst}\left(\int \left(\frac{d^2}{e^3} - \frac{dx}{e^2} + \frac{x^2}{e} - \frac{d^3}{e^3(d+ex)}\right)\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{bd^2n\sqrt[3]{x}}{e^2} + \frac{bdnx^{2/3}}{2e} + ax - \frac{bnx}{3} + \frac{bd^3n\log\left(d+e\sqrt[3]{x}\right)}{e^3} + bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx = -\frac{bd^2n\sqrt[3]{x}}{e^2} + \frac{bdnx^{2/3}}{2e} + ax - \frac{bnx}{3} + \frac{bd^3n\log\left(d+e\sqrt[3]{x}\right)}{e^3} + bx\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)$$

```
[In] Integrate[a + b*Log[c*(d + e*x^(1/3))^n],x]
```

```
[Out] -((b*d^2*n*x^(1/3))/e^2) + (b*d*n*x^(2/3))/(2*e) + a*x - (b*n*x)/3 + (b*d^3
*n*Log[d + e*x^(1/3)])/e^3 + b*x*Log[c*(d + e*x^(1/3))^n]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{b\,d^{2} n\,x^{\frac{1}{3}}}{e^{2}}+\frac{bdn\,x^{\frac{2}{3}}}{2e}+ax-\frac{bnx}{3}+\frac{b\,d^{3} n\ln\left(d+e\,x^{\frac{1}{3}}\right)}{e^{3}}+bx\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)$	66
parts	$-\frac{b\,d^{2} n\,x^{\frac{1}{3}}}{e^{2}}+\frac{bdn\,x^{\frac{2}{3}}}{2e}+ax-\frac{bnx}{3}+\frac{b\,d^{3} n\ln\left(d+e\,x^{\frac{1}{3}}\right)}{e^{3}}+bx\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)$	66

```
[In] int(a+b*ln(c*(d+e*x^(1/3))^n),x,method=_RETURNVERBOSE)
```

```
[Out] -b*d^2*n*x^(1/3)/e^2+1/2*b*d*n*x^(2/3)/e+a*x-1/3*b*n*x+b*d^3*n*ln(d+e*x^(1/
3))/e^3+b*x*ln(c*(d+e*x^(1/3))^n)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)\,dx$$
$$=\frac{6\,be^{3}x\log\left(c\right)+3\,bde^{2}nx^{\frac{2}{3}}-6\,bd^{2}enx^{\frac{1}{3}}-2\,(be^{3}n-3\,ae^{3})x+6\,(be^{3}nx+bd^{3}n)\log\left(ex^{\frac{1}{3}}+d\right)}{6\,e^{3}}$$

```
[In] integrate(a+b*log(c*(d+e*x^(1/3))^n),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b*e^3*x*log(c) + 3*b*d*e^2*n*x^(2/3) - 6*b*d^2*e*n*x^(1/3) - 2*(b*e^
3*n - 3*a*e^3)*x + 6*(b*e^3*n*x + b*d^3*n)*log(e*x^(1/3) + d))/e^3
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06

$$\int \left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^n\right)\right)\,dx$$
$$= ax + b\left(-\frac{en\left(-\frac{3d^3\left(\begin{cases}\frac{\sqrt[3]{x}}{d} & \text{for } e = 0\\ \frac{\log\left(d+e\sqrt[3]{x}\right)}{e} & \text{otherwise}\end{cases}\right)}{e^3} + \frac{3d^2\sqrt[3]{x}}{e^3} - \frac{3dx^{\frac{2}{3}}}{2e^2} + \frac{x}{e}\right)}{3} + x\log\left(c\left(d + e\sqrt[3]{x}\right)^n\right)\right)$$

```
[In] integrate(a+b*ln(c*(d+e*x**(1/3))**n),x)
```

```
[Out] a*x + b*(-e*n*(-3*d**3*Piecewise((x**(1/3)/d, Eq(e, 0)), (log(d + e*x**(1/3
))/e, True))/e**3 + 3*d**2*x**(1/3)/e**3 - 3*d*x**(2/3)/(2*e**2) + x/e)/3 +
 x*log(c*(d + e*x**(1/3))**n))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx$$
$$=\frac{1}{6}\left(en\left(\frac{6\,d^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^4}-\frac{2\,e^2x-3\,dex^{\frac{2}{3}}+6\,d^2x^{\frac{1}{3}}}{e^3}\right)+6\,x\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)\right)b$$
$$+ax$$

[In] integrate(a+b*log(c*(d+e*x^(1/3))^n),x, algorithm="maxima")

[Out] 1/6*(e*n*(6*d^3*log(e*x^(1/3) + d)/e^4 - (2*e^2*x - 3*d*e*x^(2/3) + 6*d^2*x^(1/3))/e^3) + 6*x*log((e*x^(1/3) + d)^n*c))*b + a*x

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(65) = 130.

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.71

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=ax$$
$$+\frac{\left(6\,ex\log\left(c\right)+\left(\frac{6\left(ex^{\frac{1}{3}}+d\right)^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^2}-\frac{18\left(ex^{\frac{1}{3}}+d\right)^2d\log\left(ex^{\frac{1}{3}}+d\right)}{e^2}+\frac{18\left(ex^{\frac{1}{3}}+d\right)d^2\log\left(ex^{\frac{1}{3}}+d\right)}{e^2}-\frac{2\left(ex^{\frac{1}{3}}+d\right)^3}{e^2}+\frac{9\left(e\right.}{}\right.\right.}{6\,e}$$

[In] integrate(a+b*log(c*(d+e*x^(1/3))^n),x, algorithm="giac")

[Out] a*x + 1/6*(6*e*x*log(c) + (6*(e*x^(1/3) + d)^3*log(e*x^(1/3) + d)/e^2 - 18*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) + d)/e^2 + 18*(e*x^(1/3) + d)*d^2*log(e*x^(1/3) + d)/e^2 - 2*(e*x^(1/3) + d)^3/e^2 + 9*(e*x^(1/3) + d)^2*d/e^2 - 18*(e*x^(1/3) + d)*d^2/e^2)*n)*b/e

Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\,dx=a\,x+b\,x\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)-\frac{b\,n\,x}{3}$$
$$+\frac{b\,d\,n\,x^{2/3}}{2\,e}+\frac{b\,d^3\,n\,\ln\left(d+e\,x^{1/3}\right)}{e^3}-\frac{b\,d^2\,n\,x^{1/3}}{e^2}$$

[In] int(a + b*log(c*(d + e*x^(1/3))^n),x)

[Out] a*x + b*x*log(c*(d + e*x^(1/3))^n) - (b*n*x)/3 + (b*d*n*x^(2/3))/(2*e) + (b*d^3*n*log(d + e*x^(1/3)))/e^3 - (b*d^2*n*x^(1/3))/e^2

3.446 $\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 51

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}\,dx = 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 3bn\,\text{PolyLog}\left(2,1+\frac{e\sqrt[3]{x}}{d}\right)$$

```
[Out] 3*(a+b*ln(c*(d+e*x^(1/3))^n))*ln(-e*x^(1/3)/d)+3*b*n*polylog(2,1+e*x^(1/3)/
d)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2504, 2441, 2352}

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}\,dx = 3\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right) + 3bn\,\text{PolyLog}\left(2,\frac{\sqrt[3]{x}e}{d}+1\right)$$

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])/x,x]
```

```
[Out] 3*(a + b*Log[c*(d + e*x^(1/3))^n])*Log[-((e*x^(1/3))/d)] + 3*b*n*PolyLog[2,
 1 + (e*x^(1/3))/d]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= 3\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right) - (3ben)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 3bn\text{Li}_2\left(1+\frac{e\sqrt[3]{x}}{d}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}\,dx = 3b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + a\log(x) + 3bn\,\text{PolyLog}\left(2, \frac{d+e\sqrt[3]{x}}{d}\right)$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])/x,x]
[Out] 3*b*Log[c*(d + e*x^(1/3))^n]*Log[-((e*x^(1/3))/d)] + a*Log[x] + 3*b*n*PolyL
og[2, (d + e*x^(1/3))/d]
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)}{x}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))/x,x)
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))/x,x)
```

Fricas [F]

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)}{x}\,dx = \int \frac{b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x,x, algorithm="fricas")
[Out] integral((b*log((e*x^(1/3) + d)^n*c) + a)/x, x)
```

Sympy [F]

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)}{x}\,dx = \int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)}{x}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))/x,x)
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(44) = 88$.

Time = 0.34 (sec) , antiderivative size = 166, normalized size of antiderivative = 3.25

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)}{x}\,dx = -3\left(\log\left(\frac{ex^{\frac{1}{3}}}{d}+1\right)\log\left(x^{\frac{1}{3}}\right)+\mathrm{Li}_2\left(-\frac{ex^{\frac{1}{3}}}{d}\right)\right)bn + \frac{4\,bd^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}\right)\log\left(x\right)+4\left(bd^2\log\left(c\right)+ad^2\right)\log\left(x\right)+\frac{2\,be^2nx\log(x)-3\,be^2nx}{x^{\frac{1}{3}}}-\frac{4\,(bdenx\log(x)-3\,bdenx)}{x^{\frac{2}{3}}}}{4\,d^2} + \frac{3\left(be^2nx^{\frac{2}{3}}-4\,bdenx^{\frac{1}{3}}-2\left(be^2nx^{\frac{2}{3}}-2\,bdenx^{\frac{1}{3}}\right)\log\left(x^{\frac{1}{3}}\right)\right)}{4\,d^2}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x,x, algorithm="maxima")
```

```
[Out] -3*(log(e*x^(1/3)/d + 1)*log(x^(1/3)) + dilog(-e*x^(1/3)/d))*b*n + 1/4*(4*b
*d^2*log((e*x^(1/3) + d)^n)*log(x) + 4*(b*d^2*log(c) + a*d^2)*log(x) + (2*b
*e^2*n*x*log(x) - 3*b*e^2*n*x)/x^(1/3) - 4*(b*d*e*n*x*log(x) - 3*b*d*e*n*x)
/x^(2/3))/d^2 + 3/4*(b*e^2*n*x^(2/3) - 4*b*d*e*n*x^(1/3) - 2*(b*e^2*n*x^(2/
3) - 2*b*d*e*n*x^(1/3))*log(x^(1/3)))/d^2
```

Giac [F]

$$\int \frac{a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^n\right)}{x}\,dx = \int \frac{b\log\left(\left(ex^{\frac{1}{3}} + d\right)^n c\right) + a}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x,x, algorithm="giac")
[Out] integrate((b*log((e*x^(1/3) + d)^n*c) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^n\right)}{x}\,dx = \int \frac{a + b\,\ln\left(c\left(d + e\,x^{1/3}\right)^n\right)}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))/x,x)
[Out] int((a + b*log(c*(d + e*x^(1/3))^n))/x, x)
```

3.447 $\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^2}\,dx = -\frac{ben}{2dx^{2/3}}+\frac{be^2n}{d^2\sqrt[3]{x}}-\frac{be^3n\log\left(d+e\sqrt[3]{x}\right)}{d^3}-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}+\frac{be^3n\log(x)}{3d^3}$$

```
[Out] -1/2*b*e*n/d/x^(2/3)+b*e^2*n/d^2/x^(1/3)-b*e^3*n*ln(d+e*x^(1/3))/d^3+(-a-b*
ln(c*(d+e*x^(1/3))^n))/x+1/3*b*e^3*n*ln(x)/d^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 46}

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^2}\,dx = -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x}-\frac{be^3n\log\left(d+e\sqrt[3]{x}\right)}{d^3}+\frac{be^3n\log(x)}{3d^3}+\frac{be^2n}{d^2\sqrt[3]{x}}-\frac{ben}{2dx^{2/3}}$$

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])/x^2,x]
```

```
[Out] -1/2*(b*e*n)/(d*x^(2/3)) + (b*e^2*n)/(d^2*x^(1/3)) - (b*e^3*n*Log[d + e*x^(
1/3)])/d^3 - (a + b*Log[c*(d + e*x^(1/3))^n])/x + (b*e^3*n*Log[x])/(3*d^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^4}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x} + (ben)\text{Subst}\left(\int \frac{1}{x^3(d+ex)}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x} + (ben)\text{Subst}\left(\int \left(\frac{1}{dx^3}-\frac{e}{d^2x^2}+\frac{e^2}{d^3x}-\frac{e^3}{d^3(d+ex)}\right)\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{ben}{2dx^{2/3}} + \frac{be^2n}{d^2\sqrt[3]{x}} - \frac{be^3n\log\left(d+e\sqrt[3]{x}\right)}{d^3} - \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x} + \frac{be^3n\log(x)}{3d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^2}\,dx = -\frac{a}{x} - \frac{b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x} + \frac{1}{3}ben\left(-\frac{3}{2dx^{2/3}} + \frac{3e}{d^2\sqrt[3]{x}} - \frac{3e^2\log\left(d+e\sqrt[3]{x}\right)}{d^3} + \frac{e^2\log(x)}{d^3}\right)$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])/x^2,x]
```

```
[Out] -(a/x) - (b*Log[c*(d + e*x^(1/3))^n])/x + (b*e*n*(-3/(2*d*x^(2/3)) + (3*e)/
(d^2*x^(1/3)) - (3*e^2*Log[d + e*x^(1/3)])/d^3 + (e^2*Log[x])/d^3))/3
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)}{x^{2}}dx$$

[In] int((a+b*ln(c*(d+e*x^(1/3))^n))/x^2,x)

[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))/x^2,x)

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)}{x^{2}}\,dx = \frac{2\,be^{3}nx\log\left(x^{\frac{1}{3}}\right)+2\,bde^{2}nx^{\frac{2}{3}}-bd^{2}enx^{\frac{1}{3}}-2\,bd^{3}\log\left(c\right)-2\,ad^{3}-2\,(be^{3}nx+bd^{3}n)\log\left(ex^{\frac{1}{3}}+d\right)}{2\,d^{3}x}$$

[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^2,x, algorithm="fricas")

[Out] 1/2*(2*b*e^3*n*x*log(x^(1/3)) + 2*b*d*e^2*n*x^(2/3) - b*d^2*e*n*x^(1/3) - 2*b*d^3*log(c) - 2*a*d^3 - 2*(b*e^3*n*x + b*d^3*n)*log(e*x^(1/3) + d))/(d^3*x)

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)}{x^{2}}\,dx = \text{Timed out}$$

[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))/x**2,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^2}\,dx$$
$$= -\frac{1}{6}\,ben\left(\frac{6\,e^2\log\left(ex^{\frac{1}{3}}+d\right)}{d^3}-\frac{2\,e^2\log\left(x\right)}{d^3}-\frac{3\left(2\,ex^{\frac{1}{3}}-d\right)}{d^2x^{\frac{2}{3}}}\right)$$
$$-\frac{b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)}{x}-\frac{a}{x}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^2,x, algorithm="maxima")
```

```
[Out] -1/6*b*e*n*(6*e^2*log(e*x^(1/3) + d)/d^3 - 2*e^2*log(x)/d^3 - 3*(2*e*x^(1/3
) - d)/(d^2*x^(2/3))) - b*log((e*x^(1/3) + d)^n*c)/x - a/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(75) = 150.

Time = 0.30 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.38

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^2}\,dx =$$
$$-\frac{\frac{2\,be^4n\log\left(ex^{\frac{1}{3}}+d\right)}{\left(ex^{\frac{1}{3}}+d\right)^3-3\left(ex^{\frac{1}{3}}+d\right)^2d+3\left(ex^{\frac{1}{3}}+d\right)d^2-d^3}+\frac{2\,be^4n\log\left(ex^{\frac{1}{3}}+d\right)}{d^3}-\frac{2\,be^4n\log\left(ex^{\frac{1}{3}}\right)}{d^3}-\frac{2\left(ex^{\frac{1}{3}}+d\right)^2be^4n-5\left(ex^{\frac{1}{3}}+d\right)bde^4n+3\,bd^2e^{4}}{\left(ex^{\frac{1}{3}}+d\right)^3d^2-3\left(ex^{\frac{1}{3}}+d\right)^2d^3+3}}{2\,e}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*e^4*n*log(e*x^(1/3) + d)/((e*x^(1/3) + d)^3 - 3*(e*x^(1/3) + d)^2
*d + 3*(e*x^(1/3) + d)*d^2 - d^3) + 2*b*e^4*n*log(e*x^(1/3) + d)/d^3 - 2*b*
e^4*n*log(e*x^(1/3))/d^3 - (2*(e*x^(1/3) + d)^2*b*e^4*n - 5*(e*x^(1/3) + d)
*b*d*e^4*n + 3*b*d^2*e^4*n - 2*b*d^2*e^4*log(c) - 2*a*d^2*e^4)/((e*x^(1/3)
+ d)^3*d^2 - 3*(e*x^(1/3) + d)^2*d^3 + 3*(e*x^(1/3) + d)*d^4 - d^5))/e
```

Mupad [B] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^2}\,dx = -\frac{\frac{b\,e\,n}{2\,d}-\frac{b\,e^2\,n\,x^{1/3}}{d^2}}{x^{2/3}}-\frac{a}{x}-\frac{b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)^n\right)}{x}-\frac{2\,b\,e^3\,n\,\mathrm{atanh}\left(\frac{2\,e\,x^{1/3}}{d}+1\right)}{d^3}$$

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))/x^2,x)
```

```
[Out] - ((b*e*n)/(2*d) - (b*e^2*n*x^(1/3))/d^2)/x^(2/3) - a/x - (b*log(c*(d + e*x
^(1/3))^n))/x - (2*b*e^3*n*atanh((2*e*x^(1/3))/d + 1))/d^3
```

3.448 $\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 143

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx = -\frac{ben}{10dx^{5/3}}+\frac{be^2n}{8d^2x^{4/3}}-\frac{be^3n}{6d^3x}+\frac{be^4n}{4d^4x^{2/3}} -\frac{be^5n}{2d^5\sqrt[3]{x}}+\frac{be^6n\log\left(d+e\sqrt[3]{x}\right)}{2d^6} -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2x^2}-\frac{be^6n\log(x)}{6d^6}$$

```
[Out] -1/10*b*e*n/d/x^(5/3)+1/8*b*e^2*n/d^2/x^(4/3)-1/6*b*e^3*n/d^3/x+1/4*b*e^4*n
/d^4/x^(2/3)-1/2*b*e^5*n/d^5/x^(1/3)+1/2*b*e^6*n*ln(d+e*x^(1/3))/d^6+1/2*(-
a-b*ln(c*(d+e*x^(1/3))^n))/x^2-1/6*b*e^6*n*ln(x)/d^6
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 46}

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx = -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2x^2}+\frac{be^6n\log\left(d+e\sqrt[3]{x}\right)}{2d^6} -\frac{be^6n\log(x)}{6d^6}-\frac{be^5n}{2d^5\sqrt[3]{x}}+\frac{be^4n}{4d^4x^{2/3}} -\frac{be^3n}{6d^3x}+\frac{be^2n}{8d^2x^{4/3}}-\frac{ben}{10dx^{5/3}}$$

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])/x^3,x]
```

```
[Out] -1/10*(b*e*n)/(d*x^(5/3)) + (b*e^2*n)/(8*d^2*x^(4/3)) - (b*e^3*n)/(6*d^3*x)
 + (b*e^4*n)/(4*d^4*x^(2/3)) - (b*e^5*n)/(2*d^5*x^(1/3)) + (b*e^6*n*Log[d +
 e*x^(1/3)])/(2*d^6) - (a + b*Log[c*(d + e*x^(1/3))^n])/(2*x^2) - (b*e^6*n*
Log[x])/(6*d^6)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^7}\,dx, x, \sqrt[3]{x}\right) \\
&= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\text{Subst}\left(\int \frac{1}{x^6(d+ex)}\,dx, x, \sqrt[3]{x}\right) \\
&= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\text{Subst}\left(\int \left(\frac{1}{dx^6}-\frac{e}{d^2x^5}+\frac{e^2}{d^3x^4}-\frac{e^3}{d^4x^3}\right.\right. \\
&\qquad \left.\left.+\frac{e^4}{d^5x^2}-\frac{e^5}{d^6x}+\frac{e^6}{d^6(d+ex)}\right)\,dx, x, \sqrt[3]{x}\right) \\
&= -\frac{ben}{10dx^{5/3}}+\frac{be^2n}{8d^2x^{4/3}}-\frac{be^3n}{6d^3x}+\frac{be^4n}{4d^4x^{2/3}}-\frac{be^5n}{2d^5\sqrt[3]{x}} \\
&\quad +\frac{be^6n\log\left(d+e\sqrt[3]{x}\right)}{2d^6}-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2x^2}-\frac{be^6n\log(x)}{6d^6}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.92

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx=-\frac{a}{2x^2}-\frac{b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{2x^2}+\frac{1}{6}ben\left(-\frac{3}{5dx^{5/3}}+\frac{3e}{4d^2x^{4/3}}-\frac{e^2}{d^3x}+\frac{3e^3}{2d^4x^{2/3}}-\frac{3e^4}{d^5\sqrt[3]{x}}+\frac{3e^5\log\left(d+e\sqrt[3]{x}\right)}{d^6}-\frac{e^5\log(x)}{d^6}\right)$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*Log[c*(d + e*x^(1/3))^n])/(2*x^2) + (b*e*n*(-3/(5*d*x^(5/3)
) + (3*e)/(4*d^2*x^(4/3)) - e^2/(d^3*x) + (3*e^3)/(2*d^4*x^(2/3)) - (3*e^4)
/(d^5*x^(1/3)) + (3*e^5*Log[d + e*x^(1/3)])/d^6 - (e^5*Log[x])/d^6))/6
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)}{x^{3}}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx=-\frac{60\,be^6nx^2\log\left(x^{\frac{1}{3}}\right)+20\,bd^3e^3nx+60\,bd^6\log\left(c\right)+60\,ad^6-60\,(be^6nx^2-bd^6n)\log\left(ex^{\frac{1}{3}}+d\right)+15\,(4\,bd}{120\,d^6x^2}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^3,x, algorithm="fricas")
```

```
[Out] -1/120*(60*b*e^6*n*x^2*log(x^(1/3)) + 20*b*d^3*e^3*n*x + 60*b*d^6*log(c) +
60*a*d^6 - 60*(b*e^6*n*x^2 - b*d^6*n)*log(e*x^(1/3) + d) + 15*(4*b*d*e^5*n*
x - b*d^4*e^2*n)*x^(2/3) - 6*(5*b*d^2*e^4*n*x - 2*b*d^5*e*n)*x^(1/3))/(d^6*
x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))/x**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.74

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx = \frac{1}{120}\,ben\left(\frac{60\,e^5\log\left(ex^{\frac{1}{3}}+d\right)}{d^6}-\frac{20\,e^5\log\left(x\right)}{d^6}-\frac{60\,e^4x^{\frac{4}{3}}-30\,de^3x+20\,d^2e^2x^{\frac{2}{3}}-15\,d^3ex^{\frac{1}{3}}+12\,d^4}{d^5x^{\frac{5}{3}}}\right) - \frac{b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^3,x, algorithm="maxima")
[Out] 1/120*b*e*n*(60*e^5*log(e*x^(1/3) + d)/d^6 - 20*e^5*log(x)/d^6 - (60*e^4*x^
(4/3) - 30*d*e^3*x + 20*d^2*e^2*x^(2/3) - 15*d^3*e*x^(1/3) + 12*d^4)/(d^5*x
^(5/3))) - 1/2*b*log((e*x^(1/3) + d)^n*c)/x^2 - 1/2*a/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(115) = 230.

Time = 0.33 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.40

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx = -\frac{\frac{60\,be^7n\log\left(ex^{\frac{1}{3}}+d\right)}{\left(ex^{\frac{1}{3}}+d\right)^6-6\left(ex^{\frac{1}{3}}+d\right)^5d+15\left(ex^{\frac{1}{3}}+d\right)^4d^2-20\left(ex^{\frac{1}{3}}+d\right)^3d^3+15\left(ex^{\frac{1}{3}}+d\right)^2d^4-6\left(ex^{\frac{1}{3}}+d\right)d^5+d^6}-\frac{60\,be^7n\log\left(ex^{\frac{1}{3}}+d\right)}{d^6}+\cdots}{\cdots}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^3,x, algorithm="giac")
```

```
[Out] -1/120*(60*b*e^7*n*log(e*x^(1/3) + d)/((e*x^(1/3) + d)^6 - 6*(e*x^(1/3) + d
)^5*d + 15*(e*x^(1/3) + d)^4*d^2 - 20*(e*x^(1/3) + d)^3*d^3 + 15*(e*x^(1/3)
 + d)^2*d^4 - 6*(e*x^(1/3) + d)*d^5 + d^6) - 60*b*e^7*n*log(e*x^(1/3) + d)/
d^6 + 60*b*e^7*n*log(e*x^(1/3))/d^6 + (60*(e*x^(1/3) + d)^5*b*e^7*n - 330*(
e*x^(1/3) + d)^4*b*d*e^7*n + 740*(e*x^(1/3) + d)^3*b*d^2*e^7*n - 855*(e*x^(
1/3) + d)^2*b*d^3*e^7*n + 522*(e*x^(1/3) + d)*b*d^4*e^7*n - 137*b*d^5*e^7*n
 + 60*b*d^5*e^7*log(c) + 60*a*d^5*e^7)/((e*x^(1/3) + d)^6*d^5 - 6*(e*x^(1/3
) + d)^5*d^6 + 15*(e*x^(1/3) + d)^4*d^7 - 20*(e*x^(1/3) + d)^3*d^8 + 15*(e*
x^(1/3) + d)^2*d^9 - 6*(e*x^(1/3) + d)*d^10 + d^11))/e
```

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.76

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^3}\,dx=\frac{b\,e^6\,n\,\mathrm{atanh}\left(\frac{2\,e\,x^{1/3}}{d}+1\right)}{d^6}-\frac{\frac{b\,e\,n}{5\,d}-\frac{b\,e^4\,n\,x}{2\,d^4}-\frac{b\,e^2\,n\,x^{1/3}}{4\,d^2}+\frac{b\,e^3\,n\,x^{2/3}}{3\,d^3}+\frac{b\,e^5\,n\,x^{4/3}}{d^5}}{2\,x^{5/3}}-\frac{b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))/x^3,x)
```

```
[Out] (b*e^6*n*atanh((2*e*x^(1/3))/d + 1))/d^6 - ((b*e*n)/(5*d) - (b*e^4*n*x)/(2*
d^4) - (b*e^2*n*x^(1/3))/(4*d^2) + (b*e^3*n*x^(2/3))/(3*d^3) + (b*e^5*n*x^(
4/3))/d^5)/(2*x^(5/3)) - (b*log(c*(d + e*x^(1/3))^n))/(2*x^2) - a/(2*x^2)
```

3.449 $\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 192

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx=-\frac{ben}{24dx^{8/3}}+\frac{be^2n}{21d^2x^{7/3}}-\frac{be^3n}{18d^3x^2}+\frac{be^4n}{15d^4x^{5/3}}-\frac{be^5n}{12d^5x^{4/3}}+\frac{be^6n}{9d^6x}-\frac{be^7n}{6d^7x^{2/3}}+\frac{be^8n}{3d^8\sqrt[3]{x}}-\frac{be^9n\log\left(d+e\sqrt[3]{x}\right)}{3d^9}-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x^3}+\frac{be^9n\log(x)}{9d^9}$$

```
[Out] -1/24*b*e*n/d/x^(8/3)+1/21*b*e^2*n/d^2/x^(7/3)-1/18*b*e^3*n/d^3/x^2+1/15*b*
e^4*n/d^4/x^(5/3)-1/12*b*e^5*n/d^5/x^(4/3)+1/9*b*e^6*n/d^6/x-1/6*b*e^7*n/d^
7/x^(2/3)+1/3*b*e^8*n/d^8/x^(1/3)-1/3*b*e^9*n*ln(d+e*x^(1/3))/d^9+1/3*(-a-b
*ln(c*(d+e*x^(1/3))^n))/x^3+1/9*b*e^9*n*ln(x)/d^9
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 46}

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx=-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x^3}-\frac{be^9n\log\left(d+e\sqrt[3]{x}\right)}{3d^9}+\frac{be^9n\log(x)}{9d^9}+\frac{be^8n}{3d^8\sqrt[3]{x}}-\frac{be^7n}{6d^7x^{2/3}}+\frac{be^6n}{9d^6x}-\frac{be^5n}{12d^5x^{4/3}}+\frac{be^4n}{15d^4x^{5/3}}-\frac{be^3n}{18d^3x^2}+\frac{be^2n}{21d^2x^{7/3}}-\frac{ben}{24dx^{8/3}}$$

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])/x^4,x]
```

```
[Out] -1/24*(b*e*n)/(d*x^(8/3)) + (b*e^2*n)/(21*d^2*x^(7/3)) - (b*e^3*n)/(18*d^3*
x^2) + (b*e^4*n)/(15*d^4*x^(5/3)) - (b*e^5*n)/(12*d^5*x^(4/3)) + (b*e^6*n)/
(9*d^6*x) - (b*e^7*n)/(6*d^7*x^(2/3)) + (b*e^8*n)/(3*d^8*x^(1/3)) - (b*e^9*
n*Log[d + e*x^(1/3)])/(3*d^9) - (a + b*Log[c*(d + e*x^(1/3))^n])/(3*x^3) +
(b*e^9*n*Log[x])/(9*d^9)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^{10}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x^3}+\frac{1}{3}(ben)\text{Subst}\left(\int \frac{1}{x^9(d+ex)}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x^3}+\frac{1}{3}(ben)\text{Subst}\left(\int \left(\frac{1}{dx^9}-\frac{e}{d^2x^8}+\frac{e^2}{d^3x^7}-\frac{e^3}{d^4x^6}\right.\right.\\
&\qquad \left.\left.+\frac{e^4}{d^5x^5}-\frac{e^5}{d^6x^4}+\frac{e^6}{d^7x^3}-\frac{e^7}{d^8x^2}+\frac{e^8}{d^9x}-\frac{e^9}{d^9(d+ex)}\right)\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{ben}{24dx^{8/3}}+\frac{be^2n}{21d^2x^{7/3}}-\frac{be^3n}{18d^3x^2}+\frac{be^4n}{15d^4x^{5/3}}-\frac{be^5n}{12d^5x^{4/3}}+\frac{be^6n}{9d^6x}-\frac{be^7n}{6d^7x^{2/3}}\\
&\qquad +\frac{be^8n}{3d^8\sqrt[3]{x}}-\frac{be^9n\log\left(d+e\sqrt[3]{x}\right)}{3d^9}-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x^3}+\frac{be^9n\log(x)}{9d^9}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.90

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx = -\frac{a}{3x^3}-\frac{b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{3x^3} +\frac{1}{9}ben\left(-\frac{3}{8dx^{8/3}}+\frac{3e}{7d^2x^{7/3}}-\frac{e^2}{2d^3x^2}+\frac{3e^3}{5d^4x^{5/3}}-\frac{3e^4}{4d^5x^{4/3}} +\frac{e^5}{d^6x}-\frac{3e^6}{2d^7x^{2/3}}+\frac{3e^7}{d^8\sqrt[3]{x}}-\frac{3e^8\log\left(d+e\sqrt[3]{x}\right)}{d^9}+\frac{e^8\log(x)}{d^9}\right)$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (b*Log[c*(d + e*x^(1/3))^n])/(3*x^3) + (b*e*n*(-3/(8*d*x^(8/3)
) + (3*e)/(7*d^2*x^(7/3)) - e^2/(2*d^3*x^2) + (3*e^3)/(5*d^4*x^(5/3)) - (3*
e^4)/(4*d^5*x^(4/3)) + e^5/(d^6*x) - (3*e^6)/(2*d^7*x^(2/3)) + (3*e^7)/(d^8
*x^(1/3)) - (3*e^8*Log[d + e*x^(1/3)])/d^9 + (e^8*Log[x])/d^9))/9
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)}{x^{4}}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.85

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx = \frac{840\,be^9nx^3\log\left(x^{\frac{1}{3}}\right)+280\,bd^3e^6nx^2-140\,bd^6e^3nx-840\,bd^9\log\left(c\right)-840\,ad^9-840\left(be^9nx^3+bd^9n\right)\log}{2520\,d^9}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^4,x, algorithm="fricas")
```

```
[Out] 1/2520*(840*b*e^9*n*x^3*log(x^(1/3)) + 280*b*d^3*e^6*n*x^2 - 140*b*d^6*e^3*
n*x - 840*b*d^9*log(c) - 840*a*d^9 - 840*(b*e^9*n*x^3 + b*d^9*n)*log(e*x^(1
/3) + d) + 30*(28*b*d*e^8*n*x^2 - 7*b*d^4*e^5*n*x + 4*b*d^7*e^2*n)*x^(2/3)
- 21*(20*b*d^2*e^7*n*x^2 - 8*b*d^5*e^4*n*x + 5*b*d^8*e*n)*x^(1/3))/(d^9*x^3
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))/x**4,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.72

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx =$$
$$-\frac{1}{2520}\,ben\left(\frac{840\,e^8\log\left(ex^{\frac{1}{3}}+d\right)}{d^9}-\frac{280\,e^8\log\left(x\right)}{d^9}-\frac{840\,e^7x^{\frac{7}{3}}-420\,de^6x^2+280\,d^2e^5x^{\frac{5}{3}}-210\,d^3e^4x^{\frac{4}{3}}+1}{d^8x^{\frac{8}{3}}}\right.$$
$$-\frac{b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)}{3\,x^3}-\frac{a}{3\,x^3}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^4,x, algorithm="maxima")
[Out] -1/2520*b*e*n*(840*e^8*log(e*x^(1/3) + d)/d^9 - 280*e^8*log(x)/d^9 - (840*e
^7*x^(7/3) - 420*d*e^6*x^2 + 280*d^2*e^5*x^(5/3) - 210*d^3*e^4*x^(4/3) + 16
8*d^4*e^3*x - 140*d^5*e^2*x^(2/3) + 120*d^6*e*x^(1/3) - 105*d^7)/(d^8*x^(8/
3))) - 1/3*b*log((e*x^(1/3) + d)^n*c)/x^3 - 1/3*a/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. 2(154) = 308.

Time = 0.31 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.55

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx =$$
$$-\frac{840\,be^{10}n\log\left(ex^{\frac{1}{3}}+d\right)}{\left(ex^{\frac{1}{3}}+d\right)^9-9\left(ex^{\frac{1}{3}}+d\right)^8d+36\left(ex^{\frac{1}{3}}+d\right)^7d^2-84\left(ex^{\frac{1}{3}}+d\right)^6d^3+126\left(ex^{\frac{1}{3}}+d\right)^5d^4-126\left(ex^{\frac{1}{3}}+d\right)^4d^5+84\left(ex^{\frac{1}{3}}+d\right)^3d^6-36\left(ex^{\frac{1}{3}}+d\right)^2d^7+9\left(\right.}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))/x^4,x, algorithm="giac")
```

```
[Out] -1/2520*(840*b*e^10*n*log(e*x^(1/3) + d)/((e*x^(1/3) + d)^9 - 9*(e*x^(1/3)
+ d)^8*d + 36*(e*x^(1/3) + d)^7*d^2 - 84*(e*x^(1/3) + d)^6*d^3 + 126*(e*x^(
1/3) + d)^5*d^4 - 126*(e*x^(1/3) + d)^4*d^5 + 84*(e*x^(1/3) + d)^3*d^6 - 36
*(e*x^(1/3) + d)^2*d^7 + 9*(e*x^(1/3) + d)*d^8 - d^9) + 840*b*e^10*n*log(e*
x^(1/3) + d)/d^9 - 840*b*e^10*n*log(e*x^(1/3))/d^9 - (840*(e*x^(1/3) + d)^8
*b*e^10*n - 7140*(e*x^(1/3) + d)^7*b*d*e^10*n + 26740*(e*x^(1/3) + d)^6*b*d
^2*e^10*n - 57750*(e*x^(1/3) + d)^5*b*d^3*e^10*n + 78918*(e*x^(1/3) + d)^4*
b*d^4*e^10*n - 70252*(e*x^(1/3) + d)^3*b*d^5*e^10*n + 40188*(e*x^(1/3) + d)
^2*b*d^6*e^10*n - 13827*(e*x^(1/3) + d)*b*d^7*e^10*n + 2283*b*d^8*e^10*n -
840*b*d^8*e^10*log(c) - 840*a*d^8*e^10)/((e*x^(1/3) + d)^9*d^8 - 9*(e*x^(1/
3) + d)^8*d^9 + 36*(e*x^(1/3) + d)^7*d^10 - 84*(e*x^(1/3) + d)^6*d^11 + 126
*(e*x^(1/3) + d)^5*d^12 - 126*(e*x^(1/3) + d)^4*d^13 + 84*(e*x^(1/3) + d)^3
*d^14 - 36*(e*x^(1/3) + d)^2*d^15 + 9*(e*x^(1/3) + d)*d^16 - d^17))/e
```

Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.80

$$\int \frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{x^4}\,dx = -\frac{\frac{a\,d^9}{3}+\frac{b\,d^9\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{3}+\frac{b\,d^6\,e^3\,n\,x}{18}+\frac{b\,d^8\,e\,n\,x^{1/3}}{24}-\frac{b\,d\,e^8\,n\,x^{8/3}}{3}-\frac{b\,d^3\,e^6\,n\,x^2}{9}-\frac{b\,d^7\,e^2\,n\,x^{2/3}}{21}-\frac{b\,d^5\,e^4\,n\,x^{4/3}}{15}+b\,d^4\,e\cdots}{d^9\,x^3} - \frac{2\,b\,e^9\,n\,\mathrm{atanh}\left(\frac{2\,e\,x^{1/3}}{d}+1\right)}{3\,d^9}$$

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))/x^4,x)
```

```
[Out] - ((a*d^9)/3 + (b*d^9*log(c*(d + e*x^(1/3))^n))/3 + (b*d^6*e^3*n*x)/18 + (b
*d^8*e*n*x^(1/3))/24 - (b*d*e^8*n*x^(8/3))/3 - (b*d^3*e^6*n*x^2)/9 - (b*d^7
*e^2*n*x^(2/3))/21 - (b*d^5*e^4*n*x^(4/3))/15 + (b*d^4*e^5*n*x^(5/3))/12 +
(b*d^2*e^7*n*x^(7/3))/6)/(d^9*x^3) - (2*b*e^9*n*atanh((2*e*x^(1/3))/d + 1))
/(3*d^9)
```

3.450 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 680

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx=-\frac{6b^2d^7n^2\left(d+e\sqrt[3]{x}\right)^2}{e^9}+\frac{56b^2d^6n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^9}-\frac{21b^2d^5n^2\left(d+e\sqrt[3]{x}\right)^4}{4e^9}+\frac{84b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^5}{25e^9}-\frac{14b^2d^3n^2\left(d+e\sqrt[3]{x}\right)^6}{9e^9}+\frac{24b^2d^2n^2\left(d+e\sqrt[3]{x}\right)^7}{49e^9}-\frac{3b^2dn^2\left(d+e\sqrt[3]{x}\right)^8}{32e^9}+\frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^9}{243e^9}+\frac{6b^2d^8n^2\sqrt[3]{x}}{e^8}-\frac{b^2d^9n^2\log^2\left(d+e\sqrt[3]{x}\right)}{3e^9}-\frac{6bd^8n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9}+\frac{12bd^7n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9}-\frac{56bd^6n\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9}+\frac{21bd^5n\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9}-\frac{84bd^4n\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5e^9}+\frac{28bd^3n\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9}-\frac{24bd^2n\left(d+e\sqrt[3]{x}\right)^7\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{7e^9}+\frac{3bdn\left(d+e\sqrt[3]{x}\right)^8\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4e^9}-\frac{2bn\left(d+e\sqrt[3]{x}\right)^9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{27e^9}+\frac{2bd^9n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2$$

```
[Out] -6*b^2*d^7*n^2*(d+e*x^(1/3))^2/e^9+56/9*b^2*d^6*n^2*(d+e*x^(1/3))^3/e^9-21/
4*b^2*d^5*n^2*(d+e*x^(1/3))^4/e^9+84/25*b^2*d^4*n^2*(d+e*x^(1/3))^5/e^9-14/
9*b^2*d^3*n^2*(d+e*x^(1/3))^6/e^9+24/49*b^2*d^2*n^2*(d+e*x^(1/3))^7/e^9-3/3
2*b^2*d*n^2*(d+e*x^(1/3))^8/e^9+2/243*b^2*n^2*(d+e*x^(1/3))^9/e^9+6*b^2*d^8
*n^2*x^(1/3)/e^8-1/3*b^2*d^9*n^2*ln(d+e*x^(1/3))^2/e^9-6*b*d^8*n*(d+e*x^(1/
```

```
3))*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+12*b*d^7*n*(d+e*x^(1/3))^2*(a+b*ln(c*(d
+e*x^(1/3))^n))/e^9-56/3*b*d^6*n*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n)
)/e^9+21*b*d^5*n*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9-84/5*b*d^4
*n*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+28/3*b*d^3*n*(d+e*x^(1/3
))^6*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9-24/7*b*d^2*n*(d+e*x^(1/3))^7*(a+b*ln(c
*(d+e*x^(1/3))^n))/e^9+3/4*b*d*n*(d+e*x^(1/3))^8*(a+b*ln(c*(d+e*x^(1/3))^n)
)/e^9-2/27*b*n*(d+e*x^(1/3))^9*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+2/3*b*d^9*n*
ln(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+1/3*x^3*(a+b*ln(c*(d+e*x^(1
/3))^n))^2
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{2504, 2445, 2458, 45, 2372, 12, 14, 2338\}$

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx = \frac{2bd^9n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9} - \frac{6bd^8n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9} + \frac{12bd^7n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9} - \frac{56bd^6n\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9} + \frac{21bd^5n\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9} - \frac{84bd^4n\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5e^9} + \frac{28bd^3n\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9} - \frac{24bd^2n\left(d+e\sqrt[3]{x}\right)^7\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{7e^9} + \frac{3bdn\left(d+e\sqrt[3]{x}\right)^8\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4e^9} - \frac{2bn\left(d+e\sqrt[3]{x}\right)^9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{27e^9} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 - \frac{b^2d^9n^2\log^2\left(d+e\sqrt[3]{x}\right)}{3e^9} + \frac{6b^2d^8n^2\sqrt[3]{x}}{e^8} - \frac{6b^2d^7n^2\left(d+e\sqrt[3]{x}\right)^2}{e^9} + \frac{56b^2d^6n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^9} - \frac{21b^2d^5n^2\left(d+e\sqrt[3]{x}\right)^4}{4e^9} + \frac{84b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^5}{25e^9} - \frac{14b^2d^3n^2\left(d+e\sqrt[3]{x}\right)^6}{9e^9} + \frac{24b^2d^2n^2\left(d+e\sqrt[3]{x}\right)^7}{49e^9} - \frac{3b^2dn^2\left(d+e\sqrt[3]{x}\right)^8}{32e^9} + \frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^9}{243e^9}$$

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

```
[Out] (-6*b^2*d^7*n^2*(d + e*x^(1/3))^2)/e^9 + (56*b^2*d^6*n^2*(d + e*x^(1/3))^3)
/(9*e^9) - (21*b^2*d^5*n^2*(d + e*x^(1/3))^4)/(4*e^9) + (84*b^2*d^4*n^2*(d
+ e*x^(1/3))^5)/(25*e^9) - (14*b^2*d^3*n^2*(d + e*x^(1/3))^6)/(9*e^9) + (24
*b^2*d^2*n^2*(d + e*x^(1/3))^7)/(49*e^9) - (3*b^2*d*n^2*(d + e*x^(1/3))^8)/
(32*e^9) + (2*b^2*n^2*(d + e*x^(1/3))^9)/(243*e^9) + (6*b^2*d^8*n^2*x^(1/3)
```

```
)/e^8 - (b^2*d^9*n^2*Log[d + e*x^(1/3)]^2)/(3*e^9) - (6*b*d^8*n*(d + e*x^(1
/3))*(a + b*Log[c*(d + e*x^(1/3))^n]))/e^9 + (12*b*d^7*n*(d + e*x^(1/3))^2*
(a + b*Log[c*(d + e*x^(1/3))^n]))/e^9 - (56*b*d^6*n*(d + e*x^(1/3))^3*(a +
b*Log[c*(d + e*x^(1/3))^n]))/(3*e^9) + (21*b*d^5*n*(d + e*x^(1/3))^4*(a + b
*Log[c*(d + e*x^(1/3))^n]))/e^9 - (84*b*d^4*n*(d + e*x^(1/3))^5*(a + b*Log[
c*(d + e*x^(1/3))^n]))/(5*e^9) + (28*b*d^3*n*(d + e*x^(1/3))^6*(a + b*Log[c
*(d + e*x^(1/3))^n]))/(3*e^9) - (24*b*d^2*n*(d + e*x^(1/3))^7*(a + b*Log[c*
(d + e*x^(1/3))^n]))/(7*e^9) + (3*b*d*n*(d + e*x^(1/3))^8*(a + b*Log[c*(d +
 e*x^(1/3))^n]))/(4*e^9) - (2*b*n*(d + e*x^(1/3))^9*(a + b*Log[c*(d + e*x^(
1/3))^n]))/(27*e^9) + (2*b*d^9*n*Log[d + e*x^(1/3)]*(a + b*Log[c*(d + e*x^(
1/3))^n]))/(3*e^9) + (x^3*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
```

```
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^8(a+b\log\left(c(d+ex)^n\right))^2\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-\frac{1}{3}(2ben)\text{Subst}\left(\int\frac{x^9(a+b\log\left(c(d+ex)^n\right))}{d+ex}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\\
&\quad-\frac{1}{3}(2bn)\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^9(a+b\log\left(cx^n\right))}{x}\,dx,x,d+e\sqrt[3]{x}\right)
\end{aligned}
$$

$$
\begin{aligned}
=&-\frac{6bd^8n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9} \\
&+\frac{12bd^7n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9} \\
&-\frac{56bd^6n\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9} \\
&+\frac{21bd^5n\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9} \\
&-\frac{84bd^4n\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5e^9} \\
&+\frac{28bd^3n\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9} \\
&-\frac{24bd^2n\left(d+e\sqrt[3]{x}\right)^7\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{7e^9} \\
&+\frac{3bdn\left(d+e\sqrt[3]{x}\right)^8\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4e^9} \\
&-\frac{2bn\left(d+e\sqrt[3]{x}\right)^9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{27e^9} \\
&+\frac{2bd^9n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 \\
&+\frac{1}{3}\left(2b^2n^2\right)\text{Subst}\left(\int\frac{22680d^8x-45360d^7x^2+70560d^6x^3-79380d^5x^4+63504d^4x^5-35280d^3x^6}{2520e^9x}\right. \\
&\left.+e\sqrt[3]{x}\right)
\end{aligned}
$$

$$= -\frac{6bd^8n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9}$$

$$+\frac{12bd^7n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9}$$

$$-\frac{56bd^6n\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9}$$

$$+\frac{21bd^5n\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9}$$

$$-\frac{84bd^4n\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5e^9}$$

$$+\frac{28bd^3n\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9}$$

$$-\frac{24bd^2n\left(d+e\sqrt[3]{x}\right)^7\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{7e^9}$$

$$+\frac{3bdn\left(d+e\sqrt[3]{x}\right)^8\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4e^9}$$

$$-\frac{2bn\left(d+e\sqrt[3]{x}\right)^9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{27e^9}$$

$$+\frac{2bd^9n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2$$

$$+\frac{(b^2n^2)\,\mathrm{Subst}\left(\int\frac{22680d^8x-45360d^7x^2+70560d^6x^3-79380d^5x^4+63504d^4x^5-35280d^3x^6+12960d^2x^7-2835dx^8+280x^9-252}{x}\right.}{3780e^9}$$

$$
\begin{aligned}
&= -\frac{6bd^8n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9} \\
&+ \frac{12bd^7n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9} \\
&- \frac{56bd^6n\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9} \\
&+ \frac{21bd^5n\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9} \\
&- \frac{84bd^4n\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5e^9} \\
&+ \frac{28bd^3n\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9} \\
&- \frac{24bd^2n\left(d+e\sqrt[3]{x}\right)^7\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{7e^9} \\
&+ \frac{3bdn\left(d+e\sqrt[3]{x}\right)^8\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4e^9} \\
&- \frac{2bn\left(d+e\sqrt[3]{x}\right)^9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{27e^9} \\
&+ \frac{2bd^9n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 \\
&+ \frac{(b^2n^2)\operatorname{Subst}\left(\int\left(22680d^8 - 45360d^7x + 70560d^6x^2 - 79380d^5x^3 + 63504d^4x^4 - 35280d^3x^5 + 12\right.\right.}{3780e^9}
\end{aligned}
$$

$$
\begin{aligned}
= & -\frac{6b^2d^7n^2\left(d+e\sqrt[3]{x}\right)^2}{e^9}+\frac{56b^2d^6n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^9}-\frac{21b^2d^5n^2\left(d+e\sqrt[3]{x}\right)^4}{4e^9} \\
& +\frac{84b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^5}{25e^9}-\frac{14b^2d^3n^2\left(d+e\sqrt[3]{x}\right)^6}{9e^9} \\
& +\frac{24b^2d^2n^2\left(d+e\sqrt[3]{x}\right)^7}{49e^9}-\frac{3b^2dn^2\left(d+e\sqrt[3]{x}\right)^8}{32e^9}+\frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^9}{243e^9} \\
& +\frac{6b^2d^8n^2\sqrt[3]{x}}{e^8}-\frac{6bd^8n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9} \\
& +\frac{12bd^7n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9} \\
& -\frac{56bd^6n\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9} \\
& +\frac{21bd^5n\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9} \\
& -\frac{84bd^4n\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5e^9} \\
& +\frac{28bd^3n\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9} \\
& -\frac{24bd^2n\left(d+e\sqrt[3]{x}\right)^7\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{7e^9} \\
& +\frac{3bdn\left(d+e\sqrt[3]{x}\right)^8\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4e^9} \\
& -\frac{2bn\left(d+e\sqrt[3]{x}\right)^9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{27e^9} \\
& +\frac{2bd^9n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9} \\
& +\frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-\frac{(2b^2d^9n^2)\,\mathrm{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,d+e\sqrt[3]{x}\right)}{3e^9}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{6b^2d^7n^2\left(d+e\sqrt[3]{x}\right)^2}{e^9} + \frac{56b^2d^6n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^9} - \frac{21b^2d^5n^2\left(d+e\sqrt[3]{x}\right)^4}{4e^9} \\
&+ \frac{84b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^5}{25e^9} - \frac{14b^2d^3n^2\left(d+e\sqrt[3]{x}\right)^6}{9e^9} + \frac{24b^2d^2n^2\left(d+e\sqrt[3]{x}\right)^7}{49e^9} \\
&- \frac{3b^2dn^2\left(d+e\sqrt[3]{x}\right)^8}{32e^9} + \frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^9}{243e^9} + \frac{6b^2d^8n^2\sqrt[3]{x}}{e^8} \\
&- \frac{b^2d^9n^2\log^2\left(d+e\sqrt[3]{x}\right)}{3e^9} - \frac{6bd^8n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9} \\
&+ \frac{12bd^7n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9} \\
&- \frac{56bd^6n\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9} \\
&+ \frac{21bd^5n\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9} \\
&- \frac{84bd^4n\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5e^9} \\
&+ \frac{28bd^3n\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9} \\
&- \frac{24bd^2n\left(d+e\sqrt[3]{x}\right)^7\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{7e^9} \\
&+ \frac{3bdn\left(d+e\sqrt[3]{x}\right)^8\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4e^9} \\
&- \frac{2bn\left(d+e\sqrt[3]{x}\right)^9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{27e^9} \\
&+ \frac{2bd^9n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 427, normalized size of antiderivative = 0.63

$$
\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx
= \frac{e\sqrt[3]{x}\left(3175200a^2e^8x^{8/3} - 2520abn\left(2520d^8 - 1260d^7e\sqrt[3]{x} + 840d^6e^2x^{2/3} - 630d^5e^3x + 504d^4e^4x^{4/3} - 420d^3e\right.\right.}{}
$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

```
[Out] (e*x^(1/3)*(3175200*a^2*e^8*x^(8/3) - 2520*a*b*n*(2520*d^8 - 1260*d^7*e*x^(
1/3) + 840*d^6*e^2*x^(2/3) - 630*d^5*e^3*x + 504*d^4*e^4*x^(4/3) - 420*d^3*
e^5*x^(5/3) + 360*d^2*e^6*x^2 - 315*d*e^7*x^(7/3) + 280*e^8*x^(8/3)) + b^2*
n^2*(17965080*d^8 - 5807340*d^7*e*x^(1/3) + 2813160*d^6*e^2*x^(2/3) - 15806
```

```
70*d^5*e^3*x + 947016*d^4*e^4*x^(4/3) - 577500*d^3*e^5*x^(5/3) + 343800*d^2
*e^6*x^2 - 187425*d*e^7*x^(7/3) + 78400*e^8*x^(8/3))) + 2520*b*d^9*n*(2520*
a - 7129*b*n)*Log[d + e*x^(1/3)] - 2520*b*e*x^(1/3)*(-2520*a*e^8*x^(8/3) +
b*n*(2520*d^8 - 1260*d^7*e*x^(1/3) + 840*d^6*e^2*x^(2/3) - 630*d^5*e^3*x +
504*d^4*e^4*x^(4/3) - 420*d^3*e^5*x^(5/3) + 360*d^2*e^6*x^2 - 315*d*e^7*x^(
7/3) + 280*e^8*x^(8/3)))*Log[c*(d + e*x^(1/3))^n] + 3175200*b^2*(d^9 + e^9*
x^3)*Log[c*(d + e*x^(1/3))^n]^2)/(9525600*e^9)
```

Maple [F]

$$\int x^2\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{2}dx$$

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n))^2,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n))^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 674, normalized size of antiderivative = 0.99

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$$
$$=\frac{3175200\,b^2e^9x^3\log\left(c\right)^2+39200\,(2\,b^2e^9n^2-18\,abe^9n+81\,a^2e^9)x^3-2100\,(275\,b^2d^3e^6n^2-504\,abd^3e^6n)x^2}{}$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/9525600*(3175200*b^2*e^9*x^3*log(c)^2 + 39200*(2*b^2*e^9*n^2 - 18*a*b*e^9
*n + 81*a^2*e^9)*x^3 - 2100*(275*b^2*d^3*e^6*n^2 - 504*a*b*d^3*e^6*n)*x^2 +
 3175200*(b^2*e^9*n^2*x^3 + b^2*d^9*n^2)*log(e*x^(1/3) + d)^2 + 840*(3349*b
^2*d^6*e^3*n^2 - 2520*a*b*d^6*e^3*n)*x + 2520*(420*b^2*d^3*e^6*n^2*x^2 - 84
0*b^2*d^6*e^3*n^2*x - 7129*b^2*d^9*n^2 + 2520*a*b*d^9*n - 280*(b^2*e^9*n^2
- 9*a*b*e^9*n)*x^3 + 2520*(b^2*e^9*n*x^3 + b^2*d^9*n)*log(c) + 63*(5*b^2*d*
e^8*n^2*x^2 - 8*b^2*d^4*e^5*n^2*x + 20*b^2*d^7*e^2*n^2)*x^(2/3) - 90*(4*b^2
*d^2*e^7*n^2*x^2 - 7*b^2*d^5*e^4*n^2*x + 28*b^2*d^8*e*n^2)*x^(1/3))*log(e*x
^(1/3) + d) + 352800*(3*b^2*d^3*e^6*n*x^2 - 6*b^2*d^6*e^3*n*x - 2*(b^2*e^9*
n - 9*a*b*e^9)*x^3)*log(c) - 63*(92180*b^2*d^7*e^2*n^2 - 50400*a*b*d^7*e^2*
n + 175*(17*b^2*d*e^8*n^2 - 72*a*b*d*e^8*n)*x^2 - 8*(1879*b^2*d^4*e^5*n^2 -
 2520*a*b*d^4*e^5*n)*x - 2520*(5*b^2*d*e^8*n*x^2 - 8*b^2*d^4*e^5*n*x + 20*b
^2*d^7*e^2*n)*log(c))*x^(2/3) + 90*(199612*b^2*d^8*e*n^2 - 70560*a*b*d^8*e*
n + 20*(191*b^2*d^2*e^7*n^2 - 504*a*b*d^2*e^7*n)*x^2 - 7*(2509*b^2*d^5*e^4*
n^2 - 2520*a*b*d^5*e^4*n)*x - 2520*(4*b^2*d^2*e^7*n*x^2 - 7*b^2*d^5*e^4*n*x
 + 28*b^2*d^8*e*n)*log(c))*x^(1/3))/e^9
```

Sympy [F]

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx=\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$$

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))**n))**2,x)
```

```
[Out] Integral(x**2*(a + b*log(c*(d + e*x**(1/3))**n))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 424, normalized size of antiderivative = 0.62

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$$
$$=\frac{1}{3}\,b^2x^3\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)^2+\frac{2}{3}\,abx^3\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+\frac{1}{3}\,a^2x^3$$
$$+\frac{1}{3780}\,aben\left(\frac{2520\,d^9\log\left(ex^{\frac{1}{3}}+d\right)}{e^{10}}-\frac{280\,e^8x^3-315\,de^7x^{\frac{8}{3}}+360\,d^2e^6x^{\frac{7}{3}}-420\,d^3e^5x^2+504\,d^4e^4x^{\frac{5}{3}}-6}{e^9}\right.$$
$$+\frac{1}{9525600}\left(2520\,en\left(\frac{2520\,d^9\log\left(ex^{\frac{1}{3}}+d\right)}{e^{10}}-\frac{280\,e^8x^3-315\,de^7x^{\frac{8}{3}}+360\,d^2e^6x^{\frac{7}{3}}-420\,d^3e^5x^2+504\,d^4e}{e^9}\right.\right.$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*log((e*x^(1/3) + d)^n*c)^2 + 2/3*a*b*x^3*log((e*x^(1/3) + d)^n*
c) + 1/3*a^2*x^3 + 1/3780*a*b*e*n*(2520*d^9*log(e*x^(1/3) + d)/e^10 - (280*
e^8*x^3 - 315*d*e^7*x^(8/3) + 360*d^2*e^6*x^(7/3) - 420*d^3*e^5*x^2 + 504*d
^4*e^4*x^(5/3) - 630*d^5*e^3*x^(4/3) + 840*d^6*e^2*x - 1260*d^7*e*x^(2/3) +
 2520*d^8*x^(1/3))/e^9) + 1/9525600*(2520*e*n*(2520*d^9*log(e*x^(1/3) + d)/
e^10 - (280*e^8*x^3 - 315*d*e^7*x^(8/3) + 360*d^2*e^6*x^(7/3) - 420*d^3*e^5
*x^2 + 504*d^4*e^4*x^(5/3) - 630*d^5*e^3*x^(4/3) + 840*d^6*e^2*x - 1260*d^7
*e*x^(2/3) + 2520*d^8*x^(1/3))/e^9)*log((e*x^(1/3) + d)^n*c) + (78400*e^9*x
^3 - 187425*d*e^8*x^(8/3) + 343800*d^2*e^7*x^(7/3) - 577500*d^3*e^6*x^2 - 3
175200*d^9*log(e*x^(1/3) + d)^2 + 947016*d^4*e^5*x^(5/3) - 1580670*d^5*e^4*
x^(4/3) + 2813160*d^6*e^3*x - 17965080*d^9*log(e*x^(1/3) + d) - 5807340*d^7
*e^2*x^(2/3) + 17965080*d^8*e*x^(1/3))*n^2/e^9)*b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1389 vs. $2(586) = 1172$.

Time = 0.32 (sec) , antiderivative size = 1389, normalized size of antiderivative = 2.04

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="giac")
```

```
[Out] 1/9525600*(3175200*b^2*e*x^3*log(c)^2 + 6350400*a*b*e*x^3*log(c) + 3175200*
a^2*e*x^3 + (3175200*(e*x^(1/3) + d)^9*log(e*x^(1/3) + d)^2/e^8 - 28576800*
(e*x^(1/3) + d)^8*d*log(e*x^(1/3) + d)^2/e^8 + 114307200*(e*x^(1/3) + d)^7*
d^2*log(e*x^(1/3) + d)^2/e^8 - 266716800*(e*x^(1/3) + d)^6*d^3*log(e*x^(1/3
) + d)^2/e^8 + 400075200*(e*x^(1/3) + d)^5*d^4*log(e*x^(1/3) + d)^2/e^8 - 4
00075200*(e*x^(1/3) + d)^4*d^5*log(e*x^(1/3) + d)^2/e^8 + 266716800*(e*x^(1
/3) + d)^3*d^6*log(e*x^(1/3) + d)^2/e^8 - 114307200*(e*x^(1/3) + d)^2*d^7*l
og(e*x^(1/3) + d)^2/e^8 + 28576800*(e*x^(1/3) + d)*d^8*log(e*x^(1/3) + d)^2
/e^8 - 705600*(e*x^(1/3) + d)^9*log(e*x^(1/3) + d)/e^8 + 7144200*(e*x^(1/3)
 + d)^8*d*log(e*x^(1/3) + d)/e^8 - 32659200*(e*x^(1/3) + d)^7*d^2*log(e*x^(
1/3) + d)/e^8 + 88905600*(e*x^(1/3) + d)^6*d^3*log(e*x^(1/3) + d)/e^8 - 160
030080*(e*x^(1/3) + d)^5*d^4*log(e*x^(1/3) + d)/e^8 + 200037600*(e*x^(1/3)
+ d)^4*d^5*log(e*x^(1/3) + d)/e^8 - 177811200*(e*x^(1/3) + d)^3*d^6*log(e*x
^(1/3) + d)/e^8 + 114307200*(e*x^(1/3) + d)^2*d^7*log(e*x^(1/3) + d)/e^8 -
57153600*(e*x^(1/3) + d)*d^8*log(e*x^(1/3) + d)/e^8 + 78400*(e*x^(1/3) + d)
^9/e^8 - 893025*(e*x^(1/3) + d)^8*d/e^8 + 4665600*(e*x^(1/3) + d)^7*d^2/e^8
 - 14817600*(e*x^(1/3) + d)^6*d^3/e^8 + 32006016*(e*x^(1/3) + d)^5*d^4/e^8
- 50009400*(e*x^(1/3) + d)^4*d^5/e^8 + 59270400*(e*x^(1/3) + d)^3*d^6/e^8 -
 57153600*(e*x^(1/3) + d)^2*d^7/e^8 + 57153600*(e*x^(1/3) + d)*d^8/e^8)*b^2
*n^2 + 2520*(2520*(e*x^(1/3) + d)^9*log(e*x^(1/3) + d)/e^8 - 22680*(e*x^(1/
3) + d)^8*d*log(e*x^(1/3) + d)/e^8 + 90720*(e*x^(1/3) + d)^7*d^2*log(e*x^(1
/3) + d)/e^8 - 211680*(e*x^(1/3) + d)^6*d^3*log(e*x^(1/3) + d)/e^8 + 317520
*(e*x^(1/3) + d)^5*d^4*log(e*x^(1/3) + d)/e^8 - 317520*(e*x^(1/3) + d)^4*d^
5*log(e*x^(1/3) + d)/e^8 + 211680*(e*x^(1/3) + d)^3*d^6*log(e*x^(1/3) + d)/
e^8 - 90720*(e*x^(1/3) + d)^2*d^7*log(e*x^(1/3) + d)/e^8 + 22680*(e*x^(1/3)
 + d)*d^8*log(e*x^(1/3) + d)/e^8 - 280*(e*x^(1/3) + d)^9/e^8 + 2835*(e*x^(1
/3) + d)^8*d/e^8 - 12960*(e*x^(1/3) + d)^7*d^2/e^8 + 35280*(e*x^(1/3) + d)^
6*d^3/e^8 - 63504*(e*x^(1/3) + d)^5*d^4/e^8 + 79380*(e*x^(1/3) + d)^4*d^5/e
^8 - 70560*(e*x^(1/3) + d)^3*d^6/e^8 + 45360*(e*x^(1/3) + d)^2*d^7/e^8 - 22
680*(e*x^(1/3) + d)*d^8/e^8)*b^2*n*log(c) + 2520*(2520*(e*x^(1/3) + d)^9*lo
g(e*x^(1/3) + d)/e^8 - 22680*(e*x^(1/3) + d)^8*d*log(e*x^(1/3) + d)/e^8 + 9
0720*(e*x^(1/3) + d)^7*d^2*log(e*x^(1/3) + d)/e^8 - 211680*(e*x^(1/3) + d)^
6*d^3*log(e*x^(1/3) + d)/e^8 + 317520*(e*x^(1/3) + d)^5*d^4*log(e*x^(1/3) +
 d)/e^8 - 317520*(e*x^(1/3) + d)^4*d^5*log(e*x^(1/3) + d)/e^8 + 211680*(e*x
^(1/3) + d)^3*d^6*log(e*x^(1/3) + d)/e^8 - 90720*(e*x^(1/3) + d)^2*d^7*log(
```

```
e*x^(1/3) + d)/e^8 + 22680*(e*x^(1/3) + d)*d^8*log(e*x^(1/3) + d)/e^8 - 280
*(e*x^(1/3) + d)^9/e^8 + 2835*(e*x^(1/3) + d)^8*d/e^8 - 12960*(e*x^(1/3) +
d)^7*d^2/e^8 + 35280*(e*x^(1/3) + d)^6*d^3/e^8 - 63504*(e*x^(1/3) + d)^5*d^
4/e^8 + 79380*(e*x^(1/3) + d)^4*d^5/e^8 - 70560*(e*x^(1/3) + d)^3*d^6/e^8 +
 45360*(e*x^(1/3) + d)^2*d^7/e^8 - 22680*(e*x^(1/3) + d)*d^8/e^8)*a*b*n)/e
```

Mupad [B] (verification not implemented)

Time = 6.03 (sec) , antiderivative size = 608, normalized size of antiderivative = 0.89

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx=\frac{a^2\,x^3}{3}+\frac{b^2\,x^3\ln\left(c\,\left(d+e\,x^{1/3}\right)^n\right)^2}{3}+\frac{2\,b^2\,n^2\,x^3}{243}+\frac{2\,a\,b\,x^3\,\ln\left(c\,\left(d+e\,x^{1/3}\right)^n\right)}{3}+\frac{b^2\,d^9\ln\left(c\,\left(d+e\,x^{1/3}\right)^n\right)^2}{3\,e^9}-\frac{2\,a\,b\,n\,x^3}{27}-\frac{2\,b^2\,n\,x^3\,\ln\left(c\,\left(d+e\,x^{1/3}\right)^n\right)}{27}-\frac{7129\,b^2\,d^9\,n^2\,\ln\left(d+e\,x^{1/3}\right)}{3780\,e^9}-\frac{275\,b^2\,d^3\,n^2\,x^2}{4536\,e^3}+\frac{191\,b^2\,d^2\,n^2\,x^{7/3}}{5292\,e^2}+\frac{1879\,b^2\,d^4\,n^2\,x^{5/3}}{18900\,e^4}-\frac{2509\,b^2\,d^5\,n^2\,x^{4/3}}{15120\,e^5}-\frac{4609\,b^2\,d^7\,n^2\,x^{2/3}}{7560\,e^7}+\frac{7129\,b^2\,d^8\,n^2\,x^{1/3}}{3780\,e^8}-\frac{17\,b^2\,d\,n^2\,x^{8/3}}{864\,e}+\frac{3349\,b^2\,d^6\,n^2\,x}{11340\,e^6}+\frac{b^2\,d^3\,n\,x^2\,\ln\left(c\,\left(d+e\,x^{1/3}\right)^n\right)}{9\,e^3}-\frac{2\,b^2\,d^2\,n\,x^{7/3}\,\ln\left(c\,\left(d+e\,x^{1/3}\right)^n\right)}{21\,e^2}-\frac{2\,b^2\,d^4\,n\,x^{5/3}\,\ln\left(c\,\left(d+e\,x^{1/3}\right)^n\right)}{15\,e^4}+\frac{b^2\,d^5\,n\,x^{4/3}\,\ln\left(c\,\left(d+e\,x^{1/3}\right)^n\right)}{6\,e^5}+\frac{b^2\,d^7\,n\,x^{2/3}\,\ln\left(c\,\left(d+e\,x^{1/3}\right)^n\right)}{3\,e^7}-\frac{2\,b^2\,d^8\,n\,x^{1/3}\,\ln\left(c\,\left(d+e\,x^{1/3}\right)^n\right)}{3\,e^8}+\frac{a\,b\,d\,n\,x^{8/3}}{12\,e}-\frac{2\,a\,b\,d^6\,n\,x}{9\,e^6}+\frac{2\,a\,b\,d^9\,n\,\ln\left(d+e\,x^{1/3}\right)}{3\,e^9}+\frac{b^2\,d\,n\,x^{8/3}\,\ln\left(c\,\left(d+e\,x^{1/3}\right)^n\right)}{12\,e}-\frac{2\,b^2\,d^6\,n\,x\,\ln\left(c\,\left(d+e\,x^{1/3}\right)^n\right)}{9\,e^6}+\frac{a\,b\,d^3\,n\,x^2}{9\,e^3}-\frac{2\,a\,b\,d^2\,n\,x^{7/3}}{21\,e^2}-\frac{2\,a\,b\,d^4\,n\,x^{5/3}}{15\,e^4}+\frac{a\,b\,d^5\,n\,x^{4/3}}{6\,e^5}+\frac{a\,b\,d^7\,n\,x^{2/3}}{3\,e^7}-\frac{2\,a\,b\,d^8\,n\,x^{1/3}}{3\,e^8}$$

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/3))^n))^2,x)
```

```
[Out] (a^2*x^3)/3 + (b^2*x^3*log(c*(d + e*x^(1/3))^n)^2)/3 + (2*b^2*n^2*x^3)/243
+ (2*a*b*x^3*log(c*(d + e*x^(1/3))^n))/3 + (b^2*d^9*log(c*(d + e*x^(1/3))^n
)^2)/(3*e^9) - (2*a*b*n*x^3)/27 - (2*b^2*n*x^3*log(c*(d + e*x^(1/3))^n))/27
 - (7129*b^2*d^9*n^2*log(d + e*x^(1/3)))/(3780*e^9) - (275*b^2*d^3*n^2*x^2)
/(4536*e^3) + (191*b^2*d^2*n^2*x^(7/3))/(5292*e^2) + (1879*b^2*d^4*n^2*x^(5
/3))/(18900*e^4) - (2509*b^2*d^5*n^2*x^(4/3))/(15120*e^5) - (4609*b^2*d^7*n
^2*x^(2/3))/(7560*e^7) + (7129*b^2*d^8*n^2*x^(1/3))/(3780*e^8) - (17*b^2*d*
n^2*x^(8/3))/(864*e) + (3349*b^2*d^6*n^2*x)/(11340*e^6) + (b^2*d^3*n*x^2*lo
g(c*(d + e*x^(1/3))^n))/(9*e^3) - (2*b^2*d^2*n*x^(7/3)*log(c*(d + e*x^(1/3)
)^n))/(21*e^2) - (2*b^2*d^4*n*x^(5/3)*log(c*(d + e*x^(1/3))^n))/(15*e^4) +
(b^2*d^5*n*x^(4/3)*log(c*(d + e*x^(1/3))^n))/(6*e^5) + (b^2*d^7*n*x^(2/3)*l
og(c*(d + e*x^(1/3))^n))/(3*e^7) - (2*b^2*d^8*n*x^(1/3)*log(c*(d + e*x^(1/3
))^n))/(3*e^8) + (a*b*d*n*x^(8/3))/(12*e) - (2*a*b*d^6*n*x)/(9*e^6) + (2*a*
b*d^9*n*log(d + e*x^(1/3)))/(3*e^9) + (b^2*d*n*x^(8/3)*log(c*(d + e*x^(1/3)
)^n))/(12*e) - (2*b^2*d^6*n*x*log(c*(d + e*x^(1/3))^n))/(9*e^6) + (a*b*d^3*
n*x^2)/(9*e^3) - (2*a*b*d^2*n*x^(7/3))/(21*e^2) - (2*a*b*d^4*n*x^(5/3))/(15
*e^4) + (a*b*d^5*n*x^(4/3))/(6*e^5) + (a*b*d^7*n*x^(2/3))/(3*e^7) - (2*a*b*
d^8*n*x^(1/3))/(3*e^8)
```

3.451 $\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 480

$$\begin{aligned}\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx &= \frac{15b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^2}{4e^6}-\frac{20b^2d^3n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^6}\\
&+\frac{15b^2d^2n^2\left(d+e\sqrt[3]{x}\right)^4}{16e^6}-\frac{6b^2dn^2\left(d+e\sqrt[3]{x}\right)^5}{25e^6}\\
&+\frac{b^2n^2\left(d+e\sqrt[3]{x}\right)^6}{36e^6}-\frac{6b^2d^5n^2\sqrt[3]{x}}{e^5}\\
&+\frac{b^2d^6n^2\log^2\left(d+e\sqrt[3]{x}\right)}{2e^6}\\
&+\frac{6bd^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^6}\\
&-\frac{15bd^4n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2e^6}\\
&+\frac{20bd^3n\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^6}\\
&-\frac{15bd^2n\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4e^6}\\
&+\frac{6bdn\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5e^6}\\
&-\frac{bn\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{6e^6}\\
&-\frac{bd^6n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^6}\\
&+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\end{aligned}$$

```
[Out] 15/4*b^2*d^4*n^2*(d+e*x^(1/3))^2/e^6-20/9*b^2*d^3*n^2*(d+e*x^(1/3))^3/e^6+1
5/16*b^2*d^2*n^2*(d+e*x^(1/3))^4/e^6-6/25*b^2*d*n^2*(d+e*x^(1/3))^5/e^6+1/3
6*b^2*n^2*(d+e*x^(1/3))^6/e^6-6*b^2*d^5*n^2*x^(1/3)/e^5+1/2*b^2*d^6*n^2*ln(
d+e*x^(1/3))^2/e^6+6*b*d^5*n*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6-
15/2*b*d^4*n*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6+20/3*b*d^3*n*(
d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6-15/4*b*d^2*n*(d+e*x^(1/3))^4
*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6+6/5*b*d*n*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x
^(1/3))^n))/e^6-1/6*b*n*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6-b*d
^6*n*ln(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6+1/2*x^2*(a+b*ln(c*(d+e
*x^(1/3))^n))^2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$\begin{aligned}\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx= & -\frac{bd^6n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^6}\\ & +\frac{6bd^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^6}\\ & -\frac{15bd^4n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2e^6}\\ & +\frac{20bd^3n\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^6}\\ & -\frac{15bd^2n\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4e^6}\\ & +\frac{6bdn\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5e^6}\\ & -\frac{bn\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{6e^6}\\ & +\frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\\ & +\frac{b^2d^6n^2\log^2\left(d+e\sqrt[3]{x}\right)}{2e^6}\\ & -\frac{6b^2d^5n^2\sqrt[3]{x}}{e^5}+\frac{15b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^2}{4e^6}\\ & -\frac{20b^2d^3n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^6}+\frac{15b^2d^2n^2\left(d+e\sqrt[3]{x}\right)^4}{16e^6}\\ & -\frac{6b^2dn^2\left(d+e\sqrt[3]{x}\right)^5}{25e^6}+\frac{b^2n^2\left(d+e\sqrt[3]{x}\right)^6}{36e^6}\end{aligned}$$

```
[In] Int[x*(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

```
[Out] (15*b^2*d^4*n^2*(d + e*x^(1/3))^2)/(4*e^6) - (20*b^2*d^3*n^2*(d + e*x^(1/3)
)^3)/(9*e^6) + (15*b^2*d^2*n^2*(d + e*x^(1/3))^4)/(16*e^6) - (6*b^2*d*n^2*(
d + e*x^(1/3))^5)/(25*e^6) + (b^2*n^2*(d + e*x^(1/3))^6)/(36*e^6) - (6*b^2*
d^5*n^2*x^(1/3))/e^5 + (b^2*d^6*n^2*Log[d + e*x^(1/3)]^2)/(2*e^6) + (6*b*d^
5*n*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n]))/e^6 - (15*b*d^4*n*(d
+ e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(2*e^6) + (20*b*d^3*n*(d +
 e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^6) - (15*b*d^2*n*(d +
e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n]))/(4*e^6) + (6*b*d*n*(d + e*x^
(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n]))/(5*e^6) - (b*n*(d + e*x^(1/3))^6
*(a + b*Log[c*(d + e*x^(1/3))^n]))/(6*e^6) - (b*d^6*n*Log[d + e*x^(1/3)]*(a
 + b*Log[c*(d + e*x^(1/3))^n]))/e^6 + (x^2*(a + b*Log[c*(d + e*x^(1/3))^n])
^2)/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
```

```
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^5(a+b\log\left(c(d+ex)^n\right))^2\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-(ben)\text{Subst}\left(\int \frac{x^6(a+b\log\left(c(d+ex)^n\right))}{d+ex}\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-(bn)\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6(a+b\log\left(cx^n\right))}{x}\,dx, x, d+e\sqrt[3]{x}\right)
\end{aligned}
$$

$$= \frac{6bd^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^6} - \frac{15bd^4n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2e^6} + \frac{20bd^3n\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^6} - \frac{15bd^2n\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4e^6} + \frac{6bdn\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5e^6} - \frac{bn\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{6e^6} - \frac{bd^6n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^6} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 + \left(b^2n^2\right)\operatorname{Subst}\left(\int \frac{x\left(-360d^5+450d^4x-400d^3x^2+225d^2x^3-72dx^4+10x^5\right)+60d^6\log(x)}{60e^6x}\,dx, + e\sqrt[3]{x}\right)$$

$$= \frac{6bd^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^6} - \frac{15bd^4n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2e^6} + \frac{20bd^3n\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^6} - \frac{15bd^2n\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4e^6} + \frac{6bdn\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5e^6} - \frac{bn\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{6e^6} - \frac{bd^6n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^6} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 + \frac{\left(b^2n^2\right)\operatorname{Subst}\left(\int \frac{x\left(-360d^5+450d^4x-400d^3x^2+225d^2x^3-72dx^4+10x^5\right)+60d^6\log(x)}{x}\,dx, x, d+e\sqrt[3]{x}\right)}{60e^6}$$

$$= \frac{6bd^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^6} - \frac{15bd^4n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2e^6} + \frac{20bd^3n\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^6} - \frac{15bd^2n\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4e^6} + \frac{6bdn\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5e^6} - \frac{bn\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{6e^6} - \frac{bd^6n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^6} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 + \frac{(b^2n^2)\,\mathrm{Subst}\left(\int\left(-360d^5+450d^4x-400d^3x^2+225d^2x^3-72dx^4+10x^5+\frac{60d^6\log(x)}{x}\right)dx, x, d+\right.}{60e^6}$$

$$= \frac{15b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^2}{4e^6} - \frac{20b^2d^3n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^6} + \frac{15b^2d^2n^2\left(d+e\sqrt[3]{x}\right)^4}{16e^6} - \frac{6b^2dn^2\left(d+e\sqrt[3]{x}\right)^5}{25e^6} + \frac{b^2n^2\left(d+e\sqrt[3]{x}\right)^6}{36e^6} - \frac{6b^2d^5n^2\sqrt[3]{x}}{e^5} + \frac{6bd^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^6} - \frac{15bd^4n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2e^6} + \frac{20bd^3n\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^6} - \frac{15bd^2n\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4e^6} + \frac{6bdn\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5e^6} - \frac{bn\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{6e^6} - \frac{bd^6n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^6} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 + \frac{(b^2d^6n^2)\,\mathrm{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, d+e\sqrt[3]{x}\right)}{e^6}$$

$$
\begin{aligned}
&= \frac{15b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^2}{4e^6} - \frac{20b^2d^3n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^6} + \frac{15b^2d^2n^2\left(d+e\sqrt[3]{x}\right)^4}{16e^6} \\
&\quad - \frac{6b^2dn^2\left(d+e\sqrt[3]{x}\right)^5}{25e^6} + \frac{b^2n^2\left(d+e\sqrt[3]{x}\right)^6}{36e^6} - \frac{6b^2d^5n^2\sqrt[3]{x}}{e^5} \\
&\quad + \frac{b^2d^6n^2\log^2\left(d+e\sqrt[3]{x}\right)}{2e^6} + \frac{6bd^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^6} \\
&\quad - \frac{15bd^4n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2e^6} \\
&\quad + \frac{20bd^3n\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^6} \\
&\quad - \frac{15bd^2n\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4e^6} \\
&\quad + \frac{6bdn\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5e^6} \\
&\quad - \frac{bn\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{6e^6} \\
&\quad - \frac{bd^6n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^6} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.66

$$
\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx
$$

$$
= \frac{e\sqrt[3]{x}\left(1800a^2e^5x^{5/3} + 60abn\left(60d^5 - 30d^4e\sqrt[3]{x} + 20d^3e^2x^{2/3} - 15d^2e^3x + 12de^4x^{4/3} - 10e^5x^{5/3}\right) + b^2n^2\left(-\right.\right.}{}
$$

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

```
[Out] (e*x^(1/3)*(1800*a^2*e^5*x^(5/3) + 60*a*b*n*(60*d^5 - 30*d^4*e*x^(1/3) + 20
*d^3*e^2*x^(2/3) - 15*d^2*e^3*x + 12*d*e^4*x^(4/3) - 10*e^5*x^(5/3)) + b^2*
n^2*(-8820*d^5 + 2610*d^4*e*x^(1/3) - 1140*d^3*e^2*x^(2/3) + 555*d^2*e^3*x
- 264*d*e^4*x^(4/3) + 100*e^5*x^(5/3))) + 180*b*d^6*n*(-20*a + 49*b*n)*Log[
d + e*x^(1/3)] - 60*b*e*x^(1/3)*(-60*a*e^5*x^(5/3) + b*n*(-60*d^5 + 30*d^4*
e*x^(1/3) - 20*d^3*e^2*x^(2/3) + 15*d^2*e^3*x - 12*d*e^4*x^(4/3) + 10*e^5*x
^(5/3)))*Log[c*(d + e*x^(1/3))^n] - 1800*b^2*(d^6 - e^6*x^2)*Log[c*(d + e*x
^(1/3))^n]^2)/(3600*e^6)
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{2}dx$$

[In] int(x*(a+b*ln(c*(d+e*x^(1/3))^n))^2,x)

[Out] int(x*(a+b*ln(c*(d+e*x^(1/3))^n))^2,x)

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.01

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{2}\,dx$$
$$=\frac{1800\,b^{2}e^{6}x^{2}\log\left(c\right)^{2}+100\,(b^{2}e^{6}n^{2}-6\,abe^{6}n+18\,a^{2}e^{6})x^{2}+1800\,(b^{2}e^{6}n^{2}x^{2}-b^{2}d^{6}n^{2})\log\left(ex^{\frac{1}{3}}+d\right)^{2}-60\,(}{}$$

[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="fricas")

[Out] 1/3600*(1800*b^2*e^6*x^2*log(c)^2 + 100*(b^2*e^6*n^2 - 6*a*b*e^6*n + 18*a^2*e^6)*x^2 + 1800*(b^2*e^6*n^2*x^2 - b^2*d^6*n^2)*log(e*x^(1/3) + d)^2 - 60*(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)*x + 60*(20*b^2*d^3*e^3*n^2*x + 147*b^2*d^6*n^2 - 60*a*b*d^6*n - 10*(b^2*e^6*n^2 - 6*a*b*e^6*n)*x^2 + 60*(b^2*e^6*n*x^2 - b^2*d^6*n)*log(c) + 6*(2*b^2*d*e^5*n^2*x - 5*b^2*d^4*e^2*n^2)*x^(2/3) - 15*(b^2*d^2*e^4*n^2*x - 4*b^2*d^5*e*n^2)*x^(1/3))*log(e*x^(1/3) + d) + 600*(2*b^2*d^3*e^3*n*x - (b^2*e^6*n - 6*a*b*e^6)*x^2)*log(c) + 6*(435*b^2*d^4*e^2*n^2 - 300*a*b*d^4*e^2*n - 4*(11*b^2*d*e^5*n^2 - 30*a*b*d*e^5*n)*x + 60*(2*b^2*d*e^5*n*x - 5*b^2*d^4*e^2*n)*log(c))*x^(2/3) - 15*(588*b^2*d^5*e*n^2 - 240*a*b*d^5*e*n - (37*b^2*d^2*e^4*n^2 - 60*a*b*d^2*e^4*n)*x + 60*(b^2*d^2*e^4*n*x - 4*b^2*d^5*e*n)*log(c))*x^(1/3))/e^6

Sympy [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{2}\,dx=\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{2}\,dx$$

[In] integrate(x*(a+b*ln(c*(d+e*x**(1/3))**n))**2,x)

[Out] Integral(x*(a + b*log(c*(d + e*x**(1/3))**n))**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.67

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx=\frac{1}{2}\,b^2x^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)^2$$
$$-\frac{1}{60}\,aben\left(\frac{60\,d^6\log\left(ex^{\frac{1}{3}}+d\right)}{e^7}+\frac{10\,e^5x^2-12\,de^4x^{\frac{5}{3}}+15\,d^2e^3x^{\frac{4}{3}}-20\,d^3e^2x+30\,d^4ex^{\frac{2}{3}}-60\,d^5x^{\frac{1}{3}}}{e^6}\right)$$
$$+abx^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+\frac{1}{2}\,a^2x^2$$
$$-\frac{1}{3600}\left(60\,en\left(\frac{60\,d^6\log\left(ex^{\frac{1}{3}}+d\right)}{e^7}+\frac{10\,e^5x^2-12\,de^4x^{\frac{5}{3}}+15\,d^2e^3x^{\frac{4}{3}}-20\,d^3e^2x+30\,d^4ex^{\frac{2}{3}}-60\,d^5x^{\frac{1}{3}}}{e^6}\right.\right.$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*log((e*x^(1/3) + d)^n*c)^2 - 1/60*a*b*e*n*(60*d^6*log(e*x^(1/3)
 + d)/e^7 + (10*e^5*x^2 - 12*d*e^4*x^(5/3) + 15*d^2*e^3*x^(4/3) - 20*d^3*e^
2*x + 30*d^4*e*x^(2/3) - 60*d^5*x^(1/3))/e^6) + a*b*x^2*log((e*x^(1/3) + d)
^n*c) + 1/2*a^2*x^2 - 1/3600*(60*e*n*(60*d^6*log(e*x^(1/3) + d)/e^7 + (10*e
^5*x^2 - 12*d*e^4*x^(5/3) + 15*d^2*e^3*x^(4/3) - 20*d^3*e^2*x + 30*d^4*e*x^
(2/3) - 60*d^5*x^(1/3))/e^6)*log((e*x^(1/3) + d)^n*c) - (100*e^6*x^2 + 1800
*d^6*log(e*x^(1/3) + d)^2 - 264*d*e^5*x^(5/3) + 555*d^2*e^4*x^(4/3) - 1140*
d^3*e^3*x + 8820*d^6*log(e*x^(1/3) + d) + 2610*d^4*e^2*x^(2/3) - 8820*d^5*e
*x^(1/3))*n^2/e^6)*b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 930 vs. $2(412) = 824$.

Time = 0.32 (sec) , antiderivative size = 930, normalized size of antiderivative = 1.94

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="giac")
```

```
[Out] 1/3600*(1800*b^2*e*x^2*log(c)^2 + 3600*a*b*e*x^2*log(c) + (1800*(e*x^(1/3)
+ d)^6*log(e*x^(1/3) + d)^2/e^5 - 10800*(e*x^(1/3) + d)^5*d*log(e*x^(1/3) +
 d)^2/e^5 + 27000*(e*x^(1/3) + d)^4*d^2*log(e*x^(1/3) + d)^2/e^5 - 36000*(e
*x^(1/3) + d)^3*d^3*log(e*x^(1/3) + d)^2/e^5 + 27000*(e*x^(1/3) + d)^2*d^4*
log(e*x^(1/3) + d)^2/e^5 - 10800*(e*x^(1/3) + d)*d^5*log(e*x^(1/3) + d)^2/e
```

```
^5 - 600*(e*x^(1/3) + d)^6*log(e*x^(1/3) + d)/e^5 + 4320*(e*x^(1/3) + d)^5*
d*log(e*x^(1/3) + d)/e^5 - 13500*(e*x^(1/3) + d)^4*d^2*log(e*x^(1/3) + d)/e
^5 + 24000*(e*x^(1/3) + d)^3*d^3*log(e*x^(1/3) + d)/e^5 - 27000*(e*x^(1/3)
+ d)^2*d^4*log(e*x^(1/3) + d)/e^5 + 21600*(e*x^(1/3) + d)*d^5*log(e*x^(1/3)
 + d)/e^5 + 100*(e*x^(1/3) + d)^6/e^5 - 864*(e*x^(1/3) + d)^5*d/e^5 + 3375*
(e*x^(1/3) + d)^4*d^2/e^5 - 8000*(e*x^(1/3) + d)^3*d^3/e^5 + 13500*(e*x^(1/
3) + d)^2*d^4/e^5 - 21600*(e*x^(1/3) + d)*d^5/e^5)*b^2*n^2 + 1800*a^2*e*x^2
 + 60*(60*(e*x^(1/3) + d)^6*log(e*x^(1/3) + d)/e^5 - 360*(e*x^(1/3) + d)^5*
d*log(e*x^(1/3) + d)/e^5 + 900*(e*x^(1/3) + d)^4*d^2*log(e*x^(1/3) + d)/e^5
 - 1200*(e*x^(1/3) + d)^3*d^3*log(e*x^(1/3) + d)/e^5 + 900*(e*x^(1/3) + d)^
2*d^4*log(e*x^(1/3) + d)/e^5 - 360*(e*x^(1/3) + d)*d^5*log(e*x^(1/3) + d)/e
^5 - 10*(e*x^(1/3) + d)^6/e^5 + 72*(e*x^(1/3) + d)^5*d/e^5 - 225*(e*x^(1/3)
 + d)^4*d^2/e^5 + 400*(e*x^(1/3) + d)^3*d^3/e^5 - 450*(e*x^(1/3) + d)^2*d^4
/e^5 + 360*(e*x^(1/3) + d)*d^5/e^5)*b^2*n*log(c) + 60*(60*(e*x^(1/3) + d)^6
*log(e*x^(1/3) + d)/e^5 - 360*(e*x^(1/3) + d)^5*d*log(e*x^(1/3) + d)/e^5 +
900*(e*x^(1/3) + d)^4*d^2*log(e*x^(1/3) + d)/e^5 - 1200*(e*x^(1/3) + d)^3*d
^3*log(e*x^(1/3) + d)/e^5 + 900*(e*x^(1/3) + d)^2*d^4*log(e*x^(1/3) + d)/e^
5 - 360*(e*x^(1/3) + d)*d^5*log(e*x^(1/3) + d)/e^5 - 10*(e*x^(1/3) + d)^6/e
^5 + 72*(e*x^(1/3) + d)^5*d/e^5 - 225*(e*x^(1/3) + d)^4*d^2/e^5 + 400*(e*x^
(1/3) + d)^3*d^3/e^5 - 450*(e*x^(1/3) + d)^2*d^4/e^5 + 360*(e*x^(1/3) + d)*
d^5/e^5)*a*b*n)/e
```

Mupad [B] (verification not implemented)

Time = 3.01 (sec) , antiderivative size = 431, normalized size of antiderivative = 0.90

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx=\frac{a^2\,x^2}{2}+\frac{b^2\,x^2\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)^2}{2}+\frac{b^2\,n^2\,x^2}{36}+a\,b\,x^2\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)-\frac{b^2\,d^6\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)^2}{2\,e^6}-\frac{a\,b\,n\,x^2}{6}-\frac{b^2\,n\,x^2\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{6}+\frac{49\,b^2\,d^6\,n^2\,\ln\left(d+e\,x^{1/3}\right)}{20\,e^6}+\frac{37\,b^2\,d^2\,n^2\,x^{4/3}}{240\,e^2}+\frac{29\,b^2\,d^4\,n^2\,x^{2/3}}{40\,e^4}-\frac{49\,b^2\,d^5\,n^2\,x^{1/3}}{20\,e^5}-\frac{19\,b^2\,d^3\,n^2\,x}{60\,e^3}-\frac{11\,b^2\,d\,n^2\,x^{5/3}}{150\,e}-\frac{b^2\,d^2\,n\,x^{4/3}\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{4\,e^2}-\frac{b^2\,d^4\,n\,x^{2/3}\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{2\,e^4}+\frac{b^2\,d^5\,n\,x^{1/3}\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{e^5}+\frac{a\,b\,d^3\,n\,x}{3\,e^3}+\frac{a\,b\,d\,n\,x^{5/3}}{5\,e}-\frac{a\,b\,d^6\,n\,\ln\left(d+e\,x^{1/3}\right)}{e^6}+\frac{b^2\,d^3\,n\,x\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{3\,e^3}+\frac{b^2\,d\,n\,x^{5/3}\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)}{5\,e}-\frac{a\,b\,d^2\,n\,x^{4/3}}{4\,e^2}-\frac{a\,b\,d^4\,n\,x^{2/3}}{2\,e^4}+\frac{a\,b\,d^5\,n\,x^{1/3}}{e^5}$$

```
[In] int(x*(a + b*log(c*(d + e*x^(1/3))^n))^2,x)
```

```
[Out] (a^2*x^2)/2 + (b^2*x^2*log(c*(d + e*x^(1/3))^n)^2)/2 + (b^2*n^2*x^2)/36 + a
*b*x^2*log(c*(d + e*x^(1/3))^n) - (b^2*d^6*log(c*(d + e*x^(1/3))^n)^2)/(2*e
^6) - (a*b*n*x^2)/6 - (b^2*n*x^2*log(c*(d + e*x^(1/3))^n))/6 + (49*b^2*d^6*
n^2*log(d + e*x^(1/3)))/(20*e^6) + (37*b^2*d^2*n^2*x^(4/3))/(240*e^2) + (29
*b^2*d^4*n^2*x^(2/3))/(40*e^4) - (49*b^2*d^5*n^2*x^(1/3))/(20*e^5) - (19*b^
2*d^3*n^2*x)/(60*e^3) - (11*b^2*d*n^2*x^(5/3))/(150*e) - (b^2*d^2*n*x^(4/3)
*log(c*(d + e*x^(1/3))^n))/(4*e^2) - (b^2*d^4*n*x^(2/3)*log(c*(d + e*x^(1/3
))^n))/(2*e^4) + (b^2*d^5*n*x^(1/3)*log(c*(d + e*x^(1/3))^n))/e^5 + (a*b*d^
3*n*x)/(3*e^3) + (a*b*d*n*x^(5/3))/(5*e) - (a*b*d^6*n*log(d + e*x^(1/3)))/e
^6 + (b^2*d^3*n*x*log(c*(d + e*x^(1/3))^n))/(3*e^3) + (b^2*d*n*x^(5/3)*log(
c*(d + e*x^(1/3))^n))/(5*e) - (a*b*d^2*n*x^(4/3))/(4*e^2) - (a*b*d^4*n*x^(2
/3))/(2*e^4) + (a*b*d^5*n*x^(1/3))/e^5
```

3.452 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 267

$$\begin{aligned}\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx=&-\frac{3b^2dn^2\left(d+e\sqrt[3]{x}\right)^2}{2e^3}+\frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^3}\\&+\frac{6b^2d^2n^2\sqrt[3]{x}}{e^2}-\frac{b^2d^3n^2\log^2\left(d+e\sqrt[3]{x}\right)}{e^3}\\&-\frac{6bd^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}\\&+\frac{3bdn\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}\\&-\frac{2bn\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^3}\\&+\frac{2bd^3n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}\\&+x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\end{aligned}$$

```
[Out] -3/2*b^2*d*n^2*(d+e*x^(1/3))^2/e^3+2/9*b^2*n^2*(d+e*x^(1/3))^3/e^3+6*b^2*d^
2*n^2*x^(1/3)/e^2-b^2*d^3*n^2*ln(d+e*x^(1/3))^2/e^3-6*b*d^2*n*(d+e*x^(1/3))
*(a+b*ln(c*(d+e*x^(1/3))^n))/e^3+3*b*d*n*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(
1/3))^n))/e^3-2/3*b*n*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))/e^3+2*b*d
^3*n*ln(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/e^3+x*(a+b*ln(c*(d+e*x^(1/
3))^n))^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2501, 2445, 2458, 45, 2372, 12, 14, 2338}

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx=\frac{2bd^3n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}-\frac{6bd^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}+\frac{3bdn\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}-\frac{2bn\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^3}+x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-\frac{b^2d^3n^2\log^2\left(d+e\sqrt[3]{x}\right)}{e^3}+\frac{6b^2d^2n^2\sqrt[3]{x}}{e^2}-\frac{3b^2dn^2\left(d+e\sqrt[3]{x}\right)^2}{2e^3}+\frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^3}$$

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

```
[Out] (-3*b^2*d*n^2*(d + e*x^(1/3))^2)/(2*e^3) + (2*b^2*n^2*(d + e*x^(1/3))^3)/(9
*e^3) + (6*b^2*d^2*n^2*x^(1/3))/e^2 - (b^2*d^3*n^2*Log[d + e*x^(1/3)]^2)/e^
3 - (6*b*d^2*n*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n]))/e^3 + (3*b
*d*n*(d + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/e^3 - (2*b*n*(d +
e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^3) + (2*b*d^3*n*Log[d +
 e*x^(1/3)]*(a + b*Log[c*(d + e*x^(1/3))^n]))/e^3 + x*(a + b*Log[c*(d + e*x
^(1/3))^n])^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int x^2(a+b\log(c(d+ex)^n))^2\,dx, x, \sqrt[3]{x}\right)$$

$$= x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-(2ben)\text{Subst}\left(\int \frac{x^3(a+b\log(c(d+ex)^n))}{d+ex}\,dx, x, \sqrt[3]{x}\right)$$

$$= x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-(2bn)\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^3\left(a+b\log\left(cx^n\right)\right)}{x}\,dx,x,d+e\sqrt[3]{x}\right)$$

$$= -\frac{6bd^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}+\frac{3bdn\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}-\frac{2bn\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^3}+\frac{2bd^3n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}+x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2+\left(2b^2n^2\right)\text{Subst}\left(\int\frac{18d^2x-9dx^2+2x^3-6d^3\log(x)}{6e^3x}\,dx,x,d+e\sqrt[3]{x}\right)$$

$$= -\frac{6bd^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}+\frac{3bdn\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}-\frac{2bn\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^3}+\frac{2bd^3n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}+x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2+\frac{\left(b^2n^2\right)\text{Subst}\left(\int\frac{18d^2x-9dx^2+2x^3-6d^3\log(x)}{x}\,dx,x,d+e\sqrt[3]{x}\right)}{3e^3}$$

$$= -\frac{6bd^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}+\frac{3bdn\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}-\frac{2bn\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^3}+\frac{2bd^3n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3}+x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2+\frac{\left(b^2n^2\right)\text{Subst}\left(\int\left(18d^2-9dx+2x^2-\frac{6d^3\log(x)}{x}\right)\,dx,x,d+e\sqrt[3]{x}\right)}{3e^3}$$

$$= -\frac{3b^2dn^2\left(d+e\sqrt[3]{x}\right)^2}{2e^3} + \frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^3} + \frac{6b^2d^2n^2\sqrt[3]{x}}{e^2} - \frac{6bd^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3} + \frac{3bdn\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3} - \frac{2bn\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^3} + \frac{2bd^3n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3} + x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 - \frac{(2b^2d^3n^2)\,\mathrm{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, d+e\sqrt[3]{x}\right)}{e^3}$$

$$= -\frac{3b^2dn^2\left(d+e\sqrt[3]{x}\right)^2}{2e^3} + \frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3}{9e^3} + \frac{6b^2d^2n^2\sqrt[3]{x}}{e^2} - \frac{b^2d^3n^2\log^2\left(d+e\sqrt[3]{x}\right)}{e^3} - \frac{6bd^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3} + \frac{3bdn\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3} - \frac{2bn\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^3} + \frac{2bd^3n\log\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^3} + x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.74

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$$
$$= \frac{b^2en^2\left(66d^2-15de\sqrt[3]{x}+4e^2x^{2/3}\right)\sqrt[3]{x}+6abn\left(7d^3-6d^2e\sqrt[3]{x}+3de^2x^{2/3}-2e^3x\right)+18a^2(d^3+e^3x)+6b\left(6a\right.}{1}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^2,x]
```

```
[Out] (b^2*e*n^2*(66*d^2 - 15*d*e*x^(1/3) + 4*e^2*x^(2/3))*x^(1/3) + 6*a*b*n*(7*d
^3 - 6*d^2*e*x^(1/3) + 3*d*e^2*x^(2/3) - 2*e^3*x) + 18*a^2*(d^3 + e^3*x) +
6*b*(6*a*(d^3 + e^3*x) - b*n*(11*d^3 + 6*d^2*e*x^(1/3) - 3*d*e^2*x^(2/3) +
2*e^3*x))*Log[c*(d + e*x^(1/3))^n] + 18*b^2*(d^3 + e^3*x)*Log[c*(d + e*x^(1
/3))^n]^2)/(18*e^3)
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{2}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^2,x)
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.07

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{2}\,dx$$
$$=\frac{18\,b^{2}e^{3}x\log\left(c\right)^{2}+18\,(b^{2}e^{3}n^{2}x+b^{2}d^{3}n^{2})\log\left(ex^{\frac{1}{3}}+d\right)^{2}-12\,(b^{2}e^{3}n-3\,abe^{3})x\log\left(c\right)+2\,(2\,b^{2}e^{3}n^{2}-6\,a}{}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="fricas")
[Out] 1/18*(18*b^2*e^3*x*log(c)^2 + 18*(b^2*e^3*n^2*x + b^2*d^3*n^2)*log(e*x^(1/3
) + d)^2 - 12*(b^2*e^3*n - 3*a*b*e^3)*x*log(c) + 2*(2*b^2*e^3*n^2 - 6*a*b*e
^3*n + 9*a^2*e^3)*x + 6*(3*b^2*d*e^2*n^2*x^(2/3) - 6*b^2*d^2*e*n^2*x^(1/3)
- 11*b^2*d^3*n^2 + 6*a*b*d^3*n - 2*(b^2*e^3*n^2 - 3*a*b*e^3*n)*x + 6*(b^2*e
^3*n*x + b^2*d^3*n)*log(c))*log(e*x^(1/3) + d) - 3*(5*b^2*d*e^2*n^2 - 6*b^2
*d*e^2*n*log(c) - 6*a*b*d*e^2*n)*x^(2/3) + 6*(11*b^2*d^2*e*n^2 - 6*b^2*d^2*
e*n*log(c) - 6*a*b*d^2*e*n)*x^(1/3))/e^3
```

Sympy [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{2}\,dx=\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{2}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**2,x)
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.81

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$$
$$=\frac{1}{3}\left(en\left(\frac{6\,d^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^4}-\frac{2\,e^2x-3\,dex^{\frac{2}{3}}+6\,d^2x^{\frac{1}{3}}}{e^3}\right)+6\,x\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)\right)ab$$
$$+\frac{1}{18}\left(6\,en\left(\frac{6\,d^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^4}-\frac{2\,e^2x-3\,dex^{\frac{2}{3}}+6\,d^2x^{\frac{1}{3}}}{e^3}\right)\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+18\,x\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right.\right.$$
$$+a^2x$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(e*n*(6*d^3*log(e*x^(1/3) + d)/e^4 - (2*e^2*x - 3*d*e*x^(2/3) + 6*d^2*x
^(1/3))/e^3) + 6*x*log((e*x^(1/3) + d)^n*c))*a*b + 1/18*(6*e*n*(6*d^3*log(e
*x^(1/3) + d)/e^4 - (2*e^2*x - 3*d*e*x^(2/3) + 6*d^2*x^(1/3))/e^3)*log((e*x
^(1/3) + d)^n*c) + 18*x*log((e*x^(1/3) + d)^n*c)^2 - (18*d^3*log(e*x^(1/3)
+ d)^2 - 4*e^3*x + 66*d^3*log(e*x^(1/3) + d) + 15*d*e^2*x^(2/3) - 66*d^2*e*
x^(1/3))*n^2/e^3)*b^2 + a^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.74

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx$$
$$=\frac{18\,b^2ex\log\left(c\right)^2+\left(\frac{18\left(ex^{\frac{1}{3}}+d\right)^3\log\left(ex^{\frac{1}{3}}+d\right)^2}{e^2}-\frac{54\left(ex^{\frac{1}{3}}+d\right)^2d\log\left(ex^{\frac{1}{3}}+d\right)^2}{e^2}+\frac{54\left(ex^{\frac{1}{3}}+d\right)d^2\log\left(ex^{\frac{1}{3}}+d\right)^2}{e^2}-\frac{12\left(ex^{\frac{1}{3}}+d\right)^3\log\cdots}{e^2}\right.}{}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2,x, algorithm="giac")
```

```
[Out] 1/18*(18*b^2*e*x*log(c)^2 + (18*(e*x^(1/3) + d)^3*log(e*x^(1/3) + d)^2/e^2
- 54*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) + d)^2/e^2 + 54*(e*x^(1/3) + d)*d^2*
log(e*x^(1/3) + d)^2/e^2 - 12*(e*x^(1/3) + d)^3*log(e*x^(1/3) + d)/e^2 + 54
*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) + d)/e^2 - 108*(e*x^(1/3) + d)*d^2*log(e
*x^(1/3) + d)/e^2 + 4*(e*x^(1/3) + d)^3/e^2 - 27*(e*x^(1/3) + d)^2*d/e^2 +
```

```
108*(e*x^(1/3) + d)*d^2/e^2)*b^2*n^2 + 6*(6*(e*x^(1/3) + d)^3*log(e*x^(1/3)
 + d)/e^2 - 18*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) + d)/e^2 + 18*(e*x^(1/3) +
 d)*d^2*log(e*x^(1/3) + d)/e^2 - 2*(e*x^(1/3) + d)^3/e^2 + 9*(e*x^(1/3) + d
)^2*d/e^2 - 18*(e*x^(1/3) + d)*d^2/e^2)*b^2*n*log(c) + 36*a*b*e*x*log(c) +
6*(6*(e*x^(1/3) + d)^3*log(e*x^(1/3) + d)/e^2 - 18*(e*x^(1/3) + d)^2*d*log(
e*x^(1/3) + d)/e^2 + 18*(e*x^(1/3) + d)*d^2*log(e*x^(1/3) + d)/e^2 - 2*(e*x
^(1/3) + d)^3/e^2 + 9*(e*x^(1/3) + d)^2*d/e^2 - 18*(e*x^(1/3) + d)*d^2/e^2)
*a*b*n + 18*a^2*e*x)/e
```

Mupad [B] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.09

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\,dx = \ln\left(c\left(d+e\,x^{1/3}\right)^n\right)\left(\frac{2\,b\,x\,(3\,a-b\,n)}{3} - x^{2/3}\left(\frac{b\,d\,(3\,a-b\,n)}{e}-\frac{3\,a\,b\,d}{e}\right)+\frac{d\,x^{1/3}\left(\frac{2\,b\,d\,(3\,a-b\,n)}{e}-\frac{6\,a\,b\,d}{e}\right)}{e}\right) - x^{2/3}\left(\frac{d\left(3\,a^2-2\,a\,b\,n+\frac{2\,b^2\,n^2}{3}\right)}{2\,e} - \frac{d\,(3\cdots}{}\right.$$

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))^2,x)
```

```
[Out] log(c*(d + e*x^(1/3))^n)*((2*b*x*(3*a - b*n))/3 - x^(2/3)*((b*d*(3*a - b*n)
)/e - (3*a*b*d)/e) + (d*x^(1/3)*((2*b*d*(3*a - b*n))/e - (6*a*b*d)/e))/e) -
 x^(2/3)*((d*(3*a^2 + (2*b^2*n^2)/3 - 2*a*b*n))/(2*e) - (d*(3*a^2 - b^2*n^2
))/(2*e)) + x^(1/3)*((d*((d*(3*a^2 + (2*b^2*n^2)/3 - 2*a*b*n))/e - (d*(3*a^
2 - b^2*n^2))/e))/e + (2*b^2*d^2*n^2)/e^2) + x*(a^2 + (2*b^2*n^2)/9 - (2*a*
b*n)/3) + log(c*(d + e*x^(1/3))^n)^2*(b^2*x + (b^2*d^3)/e^3) - (log(d + e*x
^(1/3))*(11*b^2*d^3*n^2 - 6*a*b*d^3*n))/(3*e^3)
```

3.453 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}\,dx = 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 6bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\operatorname{PolyLog}\left(2,1+\frac{e\sqrt[3]{x}}{d}\right) - 6b^2n^2\operatorname{PolyLog}\left(3,1+\frac{e\sqrt[3]{x}}{d}\right)$$

```
[Out] 3*(a+b*ln(c*(d+e*x^(1/3))^n))^2*ln(-e*x^(1/3)/d)+6*b*n*(a+b*ln(c*(d+e*x^(1/
3))^n))*polylog(2,1+e*x^(1/3)/d)-6*b^2*n^2*polylog(3,1+e*x^(1/3)/d)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2504, 2443, 2481, 2421, 6724}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}\,dx = 6bn\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{x}e}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right) + 3\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 - 6b^2n^2\operatorname{PolyLog}\left(3,\frac{\sqrt[3]{x}e}{d}+1\right)$$

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x,x]
```

```
[Out] 3*(a + b*Log[c*(d + e*x^(1/3))^n])^2*Log[-((e*x^(1/3))/d)] + 6*b*n*(a + b*L
og[c*(d + e*x^(1/3))^n])*PolyLog[2, 1 + (e*x^(1/3))/d] - 6*b^2*n^2*PolyLog[
3, 1 + (e*x^(1/3))/d]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x}\,dx, x, \sqrt[3]{x}\right)$$

$$
\begin{aligned}
&= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt[3]{x}}{d}\right) \\
&\quad -(6ben)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)\left(a+b\log\left(c(d+ex)^n\right)\right)}{d+ex}\,dx,x,\sqrt[3]{x}\right) \\
&= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt[3]{x}}{d}\right) \\
&\quad -(6bn)\text{Subst}\left(\int \frac{\left(a+b\log\left(cx^n\right)\right)\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx,x,d+e\sqrt[3]{x}\right) \\
&= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt[3]{x}}{d}\right) \\
&\quad +6bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\text{Li}_2\left(1+\frac{e\sqrt[3]{x}}{d}\right) \\
&\quad -\left(6b^2n^2\right)\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{x}{d}\right)}{x}\,dx,x,d+e\sqrt[3]{x}\right) \\
&= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\log\left(-\frac{e\sqrt[3]{x}}{d}\right) \\
&\quad +6bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\text{Li}_2\left(1+\frac{e\sqrt[3]{x}}{d}\right)-6b^2n^2\text{Li}_3\left(1+\frac{e\sqrt[3]{x}}{d}\right)
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 195 vs. $2(93) = 186$.

Time = 0.09 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.10

$$
\begin{aligned}
\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}\,dx &= \left(a-bn\log\left(d+e\sqrt[3]{x}\right)+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\log(x) \\
&\quad +2bn\left(a-bn\log\left(d+e\sqrt[3]{x}\right)\right. \\
&\quad \left.+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\left(\left(\log\left(d+e\sqrt[3]{x}\right)\right.\right. \\
&\quad \left.\left.-\log\left(1+\frac{e\sqrt[3]{x}}{d}\right)\right)\log(x)-3\,\text{PolyLog}\left(2,-\frac{e\sqrt[3]{x}}{d}\right)\right) \\
&\quad +3b^2n^2\left(\log^2\left(d+e\sqrt[3]{x}\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\right. \\
&\quad +2\log\left(d+e\sqrt[3]{x}\right)\text{PolyLog}\left(2,1+\frac{e\sqrt[3]{x}}{d}\right) \\
&\quad \left.-2\,\text{PolyLog}\left(3,1+\frac{e\sqrt[3]{x}}{d}\right)\right)
\end{aligned}
$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])*((Log[d + e*x^(1/
3)] - Log[1 + (e*x^(1/3))/d])*Log[x] - 3*PolyLog[2, -((e*x^(1/3))/d)]) + 3*
b^2*n^2*(Log[d + e*x^(1/3)]^2*Log[-((e*x^(1/3))/d)] + 2*Log[d + e*x^(1/3)]*
PolyLog[2, 1 + (e*x^(1/3))/d] - 2*PolyLog[3, 1 + (e*x^(1/3))/d])
```

Maple [F]

$$\int \frac{\left(a+b\ln \left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{2}}{x}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)+a\right)^{2}}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x^(1/3) + d)^n*c)^2 + 2*a*b*log((e*x^(1/3) + d)^n*c) +
 a^2)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{2}}{x}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**2/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**2/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x,x, algorithm="maxima")
```

```
[Out] b^2*log((e*x^(1/3) + d)^n)^2*log(x) + integrate(1/3*(3*(b^2*e*log(c)^2 + 2*
a*b*e*log(c) + a^2*e)*x - 2*(b^2*e*n*x*log(x) - 3*(b^2*e*log(c) + a*b*e)*x
- 3*(b^2*d*log(c) + a*b*d)*x^(2/3))*log((e*x^(1/3) + d)^n) + 3*(b^2*d*log(c
)^2 + 2*a*b*d*log(c) + a^2*d)*x^(2/3))/(e*x^2 + d*x^(5/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^n*c) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)\right)^2}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))^2/x,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/3))^n))^2/x, x)
```

3.454 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 231

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^2}\,dx &= -\frac{b^2e^2n^2}{d^2\sqrt[3]{x}}+\frac{b^2e^3n^2\log\left(d+e\sqrt[3]{x}\right)}{d^3}\\ &\quad-\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{dx^{2/3}}\\ &\quad+\frac{2be^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}}\\ &\quad+\frac{2be^3n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3}\\ &\quad-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}-\frac{b^2e^3n^2\log(x)}{d^3}\\ &\quad-\frac{2b^2e^3n^2\,\mathrm{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^3}\end{aligned}$$

```
[Out] -b^2*e^2*n^2/d^2/x^(1/3)+b^2*e^3*n^2*ln(d+e*x^(1/3))/d^3-b*e*n*(a+b*ln(c*(d
+e*x^(1/3))^n))/d/x^(2/3)+2*b*e^2*n*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n
))/d^3/x^(1/3)+2*b*e^3*n*ln(1-d/(d+e*x^(1/3)))*(a+b*ln(c*(d+e*x^(1/3))^n))/
d^3-(a+b*ln(c*(d+e*x^(1/3))^n))^2/x-b^2*e^3*n^2*ln(x)/d^3-2*b^2*e^3*n^2*pol
ylog(2,d/(d+e*x^(1/3)))/d^3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^2}\,dx = \frac{2be^3n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3} + \frac{2be^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{dx^{2/3}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x} - \frac{2b^2e^3n^2\operatorname{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^3} + \frac{b^2e^3n^2\log\left(d+e\sqrt[3]{x}\right)}{d^3} - \frac{b^2e^3n^2\log(x)}{d^3} - \frac{b^2e^2n^2}{d^2\sqrt[3]{x}}$$

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x^2,x]
```

```
[Out] -((b^2*e^2*n^2)/(d^2*x^(1/3))) + (b^2*e^3*n^2*Log[d + e*x^(1/3)])/d^3 - (b*
e*n*(a + b*Log[c*(d + e*x^(1/3))^n]))/(d*x^(2/3)) + (2*b*e^2*n*(d + e*x^(1/
3))*(a + b*Log[c*(d + e*x^(1/3))^n]))/(d^3*x^(1/3)) + (2*b*e^3*n*Log[1 - d/
(d + e*x^(1/3))]*(a + b*Log[c*(d + e*x^(1/3))^n]))/d^3 - (a + b*Log[c*(d +
e*x^(1/3))^n])^2/x - (b^2*e^3*n^2*Log[x])/d^3 - (2*b^2*e^3*n^2*PolyLog[2, d
/(d + e*x^(1/3))])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
```

```
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
```

```
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}+(2ben)\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^3(d+ex)}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}+(2bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt[3]{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}+\frac{(2bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt[3]{x}\right)}{d}\\
&\quad-\frac{(2ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt[3]{x}\right)}{d}\\
&= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{dx^{2/3}}-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}\\
&\quad-\frac{(2ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt[3]{x}\right)}{d^2}\\
&\quad+\frac{(2be^2n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx, x, d+e\sqrt[3]{x}\right)}{d^2}\\
&\quad+\frac{(b^2en^2)\,\text{Subst}\left(\int \frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt[3]{x}\right)}{d}
\end{aligned}
$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{dx^{2/3}} + \frac{2be^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}}$$
$$+ \frac{2be^3n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3}$$
$$- \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}$$
$$+ \frac{(b^2en^2)\,\mathrm{Subst}\left(\int\left(\frac{e^2}{d(d-x)^2}+\frac{e^2}{d^2(d-x)}+\frac{e^2}{d^2x}\right)dx, x, d+e\sqrt[3]{x}\right)}{d}$$
$$- \frac{(2b^2e^2n^2)\,\mathrm{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x}{e}}dx, x, d+e\sqrt[3]{x}\right)}{d^3}$$
$$- \frac{(2b^2e^3n^2)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{d}{x}\right)}{x}dx, x, d+e\sqrt[3]{x}\right)}{d^3}$$
$$= -\frac{b^2e^2n^2}{d^2\sqrt[3]{x}} + \frac{b^2e^3n^2\log\left(d+e\sqrt[3]{x}\right)}{d^3} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{dx^{2/3}}$$
$$+ \frac{2be^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}}$$
$$+ \frac{2be^3n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3}$$
$$- \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x} - \frac{b^2e^3n^2\log(x)}{d^3} - \frac{2b^2e^3n^2\operatorname{Li}_2\left(\frac{d}{d+e\sqrt[3]{x}}\right)}{d^3}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.18

$$\int\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^2}dx = -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x}$$
$$-\frac{e\left(3bd^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right) - 6bden\sqrt[3]{x}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right) + 3e^2x^{2/3}\left(a+b\log\left(c\left(d+e\right.\right.\right.\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x^2,x]
```

```
[Out] -((a + b*Log[c*(d + e*x^(1/3))^n])^2/x) - (e*(3*b*d^2*n*(a + b*Log[c*(d + e
*x^(1/3))^n]) - 6*b*d*e*n*x^(1/3)*(a + b*Log[c*(d + e*x^(1/3))^n]) + 3*e^2*
x^(2/3)*(a + b*Log[c*(d + e*x^(1/3))^n])^2 - 6*b*e^2*n*x^(2/3)*(a + b*Log[c
*(d + e*x^(1/3))^n])*Log[-((e*x^(1/3))/d)] - 2*b^2*e^2*n^2*x^(2/3)*(3*Log[d
 + e*x^(1/3)] - Log[x]) + b^2*e*n^2*x^(1/3)*(3*d - 3*e*x^(1/3)*Log[d + e*x^
```

```
(1/3)] + e*x^(1/3)*Log[x]) - 6*b^2*e^2*n^2*x^(2/3)*PolyLog[2, 1 + (e*x^(1/3
))/d]))/(3*d^3*x^(2/3))
```

Maple [F]

$$\int \frac{\left(a+b\ln \left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{2}}{x^{2}}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x^2,x)
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{2}}{x^{2}}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)+a\right)^{2}}{x^{2}}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^2,x, algorithm="fricas")
[Out] integral((b^2*log((e*x^(1/3) + d)^n*c)^2 + 2*a*b*log((e*x^(1/3) + d)^n*c) +
 a^2)/x^2, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{2}}{x^{2}}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{2}}{x^{2}}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**2/x**2,x)
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**2/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{2}}{x^{2}}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)+a\right)^{2}}{x^{2}}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^2,x, algorithm="maxima")
[Out] -2*(log(e*x^(1/3)/d + 1)*log(x^(1/3)) + dilog(-e*x^(1/3)/d))*b^2*e^3*n^2/d^
3 - (2*a*b*e^3*n - (3*e^3*n^2 - 2*e^3*n*log(c))*b^2)*log(e*x^(1/3) + d)/d^3
```

```
 + 2*(b^2*e^3*n*log(c) + a*b*e^3*n)*log(x^(1/3))/d^3 + integrate((b^2*e^6*n
^2*x - b^2*d^3*e^3*n^2)/x, x)/d^6 - 1/20*(12*b^2*e^8*n^2*x^(5/3) - 15*b^2*d
*e^7*n^2*x^(4/3) + 20*b^2*d^2*e^6*n^2*x - 40*b^2*d^3*e^5*n^2*x^(2/3) + 100*
b^2*d^4*e^4*n^2*x^(1/3) + 20*(b^2*d^3*e^5*n^2*x^(2/3) - 2*b^2*d^4*e^4*n^2*x
^(1/3))*log(x^(1/3)))/d^8 + 1/60*(60*b^2*d^5*e^3*n^2*x^(5/3)*log(e*x^(1/3)
+ d)^2 - 45*b^2*d*e^7*n^2*x^3 - 40*b^2*d^4*e^4*n^2*x^2*log(x) + 300*b^2*d^4
*e^4*n^2*x^2 - 60*b^2*d^8*x^(2/3)*log((e*x^(1/3) + d)^n)^2 - 60*(b^2*d^7*e*
n*log(c) + a*b*d^7*e*n)*x - 20*(6*b^2*d^5*e^3*n*x^(5/3)*log(e*x^(1/3) + d)
- 6*b^2*d^6*e^2*n*x^(4/3) + 3*b^2*d^7*e*n*x - 2*(b^2*d^5*e^3*n*x*log(x) - 3
*b^2*d^8*log(c) - 3*a*b*d^8)*x^(2/3))*log((e*x^(1/3) + d)^n) - 60*(b^2*d^8*
log(c)^2 + 2*a*b*d^8*log(c) + a^2*d^8)*x^(2/3) + 4*(9*b^2*e^8*n^2*x^3 + 5*b
^2*d^3*e^5*n^2*x^2*log(x) - 15*b^2*d^3*e^5*n^2*x^2 + 30*(b^2*d^6*e^2*n*log(
c) + a*b*d^6*e^2*n)*x)*x^(1/3) - 60*(b^2*d^3*e^5*n^2*x^3 + b^2*d^6*e^2*n^2*
x^2)/x^(2/3))/(d^8*x^(5/3))
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^2,x, algorithm="giac")
[Out] integrate((b*log((e*x^(1/3) + d)^n*c) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^n\right)\right)^2}{x^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))^2/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(1/3))^n))^2/x^2, x)
```

3.455 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 405

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx =& -\frac{b^2e^2n^2}{20d^2x^{4/3}}+\frac{3b^2e^3n^2}{20d^3x}-\frac{47b^2e^4n^2}{120d^4x^{2/3}} \\ &+\frac{77b^2e^5n^2}{60d^5\sqrt[3]{x}}-\frac{77b^2e^6n^2\log\left(d+e\sqrt[3]{x}\right)}{60d^6} \\ &-\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5dx^{5/3}} \\ &+\frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4d^2x^{4/3}} \\ &-\frac{be^3n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3d^3x} \\ &+\frac{be^4n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2d^4x^{2/3}} \\ &-\frac{be^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^6\sqrt[3]{x}} \\ &-\frac{be^6n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^6} \\ &-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2}+\frac{137b^2e^6n^2\log(x)}{180d^6} \\ &+\frac{b^2e^6n^2\operatorname{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^6}\end{aligned}$$

```
[Out] -1/20*b^2*e^2*n^2/d^2/x^(4/3)+3/20*b^2*e^3*n^2/d^3/x-47/120*b^2*e^4*n^2/d^4
/x^(2/3)+77/60*b^2*e^5*n^2/d^5/x^(1/3)-77/60*b^2*e^6*n^2*ln(d+e*x^(1/3))/d^
```

```
6-1/5*b*e*n*(a+b*ln(c*(d+e*x^(1/3))^n))/d/x^(5/3)+1/4*b*e^2*n*(a+b*ln(c*(d+
e*x^(1/3))^n))/d^2/x^(4/3)-1/3*b*e^3*n*(a+b*ln(c*(d+e*x^(1/3))^n))/d^3/x+1/
2*b*e^4*n*(a+b*ln(c*(d+e*x^(1/3))^n))/d^4/x^(2/3)-b*e^5*n*(d+e*x^(1/3))*(a+
b*ln(c*(d+e*x^(1/3))^n))/d^6/x^(1/3)-b*e^6*n*ln(1-d/(d+e*x^(1/3)))*(a+b*ln(
c*(d+e*x^(1/3))^n))/d^6-1/2*(a+b*ln(c*(d+e*x^(1/3))^n))^2/x^2+137/180*b^2*e
^6*n^2*ln(x)/d^6+b^2*e^6*n^2*polylog(2,d/(d+e*x^(1/3)))/d^6
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx = & -\frac{be^6n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^6} \\ & -\frac{be^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^6\sqrt[3]{x}} \\ & +\frac{be^4n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2d^4x^{2/3}} \\ & -\frac{be^3n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3d^3x} \\ & +\frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4d^2x^{4/3}} \\ & -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5dx^{5/3}} \\ & -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2} \\ & +\frac{b^2e^6n^2\operatorname{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^6} \\ & -\frac{77b^2e^6n^2\log\left(d+e\sqrt[3]{x}\right)}{60d^6}+\frac{137b^2e^6n^2\log(x)}{180d^6} \\ & +\frac{77b^2e^5n^2}{60d^5\sqrt[3]{x}}-\frac{47b^2e^4n^2}{120d^4x^{2/3}}+\frac{3b^2e^3n^2}{20d^3x}-\frac{b^2e^2n^2}{20d^2x^{4/3}}\end{aligned}$$

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x^3,x]
```

```
[Out] -1/20*(b^2*e^2*n^2)/(d^2*x^(4/3)) + (3*b^2*e^3*n^2)/(20*d^3*x) - (47*b^2*e^
4*n^2)/(120*d^4*x^(2/3)) + (77*b^2*e^5*n^2)/(60*d^5*x^(1/3)) - (77*b^2*e^6*
n^2*Log[d + e*x^(1/3)])/(60*d^6) - (b*e*n*(a + b*Log[c*(d + e*x^(1/3))^n]))
/(5*d*x^(5/3)) + (b*e^2*n*(a + b*Log[c*(d + e*x^(1/3))^n]))/(4*d^2*x^(4/3))
 - (b*e^3*n*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*d^3*x) + (b*e^4*n*(a + b*L
```

```
og[c*(d + e*x^(1/3))^n]))/(2*d^4*x^(2/3)) - (b*e^5*n*(d + e*x^(1/3))*(a + b
*Log[c*(d + e*x^(1/3))^n]))/(d^6*x^(1/3)) - (b*e^6*n*Log[1 - d/(d + e*x^(1/
3))]*(a + b*Log[c*(d + e*x^(1/3))^n]))/d^6 - (a + b*Log[c*(d + e*x^(1/3))^n
])^2/(2*x^2) + (137*b^2*e^6*n^2*Log[x])/(180*d^6) + (b^2*e^6*n^2*PolyLog[2,
 d/(d + e*x^(1/3))])/d^6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^7}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2}+(ben)\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^6(d+ex)}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2}+(bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+e\sqrt[3]{x}\right)
\end{aligned}$$

$$= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2} + \frac{(bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+e\sqrt[3]{x}\right)}{d} - \frac{(ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+e\sqrt[3]{x}\right)}{d}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5dx^{5/3}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2} - \frac{(ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+e\sqrt[3]{x}\right)}{d^2} + \frac{(be^2n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\sqrt[3]{x}\right)}{d^2} + \frac{(b^2en^2)\,\text{Subst}\left(\int \frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+e\sqrt[3]{x}\right)}{5d}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5dx^{5/3}} + \frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4d^2x^{4/3}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2} + \frac{(be^2n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\sqrt[3]{x}\right)}{d^3} - \frac{(be^3n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt[3]{x}\right)}{d^3} + \frac{(b^2en^2)\,\text{Subst}\left(\int \left(-\frac{e^5}{d(d-x)^5} - \frac{e^5}{d^2(d-x)^4} - \frac{e^5}{d^3(d-x)^3} - \frac{e^5}{d^4(d-x)^2} - \frac{e^5}{d^5(d-x)} - \frac{e^5}{d^5x}\right)\,dx, x, d+e\sqrt[3]{x}\right)}{5d} - \frac{(b^2e^2n^2)\,\text{Subst}\left(\int \frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\sqrt[3]{x}\right)}{4d^2}$$

$$= -\frac{b^2e^2n^2}{20d^2x^{4/3}} + \frac{b^2e^3n^2}{15d^3x} - \frac{b^2e^4n^2}{10d^4x^{2/3}} + \frac{b^2e^5n^2}{5d^5\sqrt[3]{x}} - \frac{b^2e^6n^2\log\left(d+e\sqrt[3]{x}\right)}{5d^6} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5dx^{5/3}} + \frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4d^2x^{4/3}} - \frac{be^3n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3d^3x} - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2} + \frac{b^2e^6n^2\log(x)}{15d^6} - \frac{(be^3n)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt[3]{x}\right)}{d^4} + \frac{(be^4n)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt[3]{x}\right)}{d^4} - \frac{(b^2e^2n^2)\,\text{Subst}\left(\int\left(\frac{e^4}{d(d-x)^4}+\frac{e^4}{d^2(d-x)^3}+\frac{e^4}{d^3(d-x)^2}+\frac{e^4}{d^4(d-x)}+\frac{e^4}{d^4x}\right)dx, x, d+e\sqrt[3]{x}\right)}{4d^2} + \frac{(b^2e^3n^2)\,\text{Subst}\left(\int\frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt[3]{x}\right)}{3d^3}$$

$$= -\frac{b^2e^2n^2}{20d^2x^{4/3}} + \frac{3b^2e^3n^2}{20d^3x} - \frac{9b^2e^4n^2}{40d^4x^{2/3}} + \frac{9b^2e^5n^2}{20d^5\sqrt[3]{x}} - \frac{9b^2e^6n^2\log\left(d+e\sqrt[3]{x}\right)}{20d^6} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5dx^{5/3}} + \frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4d^2x^{4/3}} - \frac{be^3n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3d^3x} + \frac{be^4n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2d^4x^{2/3}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2} + \frac{3b^2e^6n^2\log(x)}{20d^6} + \frac{(be^4n)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt[3]{x}\right)}{d^5} - \frac{(be^5n)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx, x, d+e\sqrt[3]{x}\right)}{d^5} + \frac{(b^2e^3n^2)\,\text{Subst}\left(\int\left(-\frac{e^3}{d(d-x)^3}-\frac{e^3}{d^2(d-x)^2}-\frac{e^3}{d^3(d-x)}-\frac{e^3}{d^3x}\right)dx, x, d+e\sqrt[3]{x}\right)}{3d^3} - \frac{(b^2e^4n^2)\,\text{Subst}\left(\int\frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt[3]{x}\right)}{2d^4}$$

$$
\begin{aligned}
&= -\frac{b^2e^2n^2}{20d^2x^{4/3}} + \frac{3b^2e^3n^2}{20d^3x} - \frac{47b^2e^4n^2}{120d^4x^{2/3}} + \frac{47b^2e^5n^2}{60d^5\sqrt[3]{x}} \\
&\quad - \frac{47b^2e^6n^2\log\left(d+e\sqrt[3]{x}\right)}{60d^6} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5dx^{5/3}} \\
&\quad + \frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4d^2x^{4/3}} - \frac{be^3n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3d^3x} \\
&\quad + \frac{be^4n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2d^4x^{2/3}} - \frac{be^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^6\sqrt[3]{x}} \\
&\quad - \frac{be^6n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^6} - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2} \\
&\quad + \frac{47b^2e^6n^2\log(x)}{180d^6} - \frac{(b^2e^4n^2)\,\mathrm{Subst}\left(\int\left(\frac{e^2}{d(d-x)^2}+\frac{e^2}{d^2(d-x)}+\frac{e^2}{d^2x}\right)dx, x, d+e\sqrt[3]{x}\right)}{2d^4} \\
&\quad + \frac{(b^2e^5n^2)\,\mathrm{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x}{e}}\,dx, x, d+e\sqrt[3]{x}\right)}{d^6} \\
&\quad + \frac{(b^2e^6n^2)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{d}{x}\right)}{x}\,dx, x, d+e\sqrt[3]{x}\right)}{d^6} \\
&= -\frac{b^2e^2n^2}{20d^2x^{4/3}} + \frac{3b^2e^3n^2}{20d^3x} - \frac{47b^2e^4n^2}{120d^4x^{2/3}} + \frac{77b^2e^5n^2}{60d^5\sqrt[3]{x}} \\
&\quad - \frac{77b^2e^6n^2\log\left(d+e\sqrt[3]{x}\right)}{60d^6} - \frac{ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{5dx^{5/3}} \\
&\quad + \frac{be^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4d^2x^{4/3}} - \frac{be^3n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3d^3x} \\
&\quad + \frac{be^4n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2d^4x^{2/3}} - \frac{be^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^6\sqrt[3]{x}} \\
&\quad - \frac{be^6n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^6} \\
&\quad - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2} + \frac{137b^2e^6n^2\log(x)}{180d^6} + \frac{b^2e^6n^2\mathrm{Li}_2\left(\frac{d}{d+e\sqrt[3]{x}}\right)}{d^6}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx = -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2x^2}$$
$$-\frac{be\left(72ad^5n-90ad^4en\sqrt[3]{x}+18bd^4en^2\sqrt[3]{x}+120ad^3e^2nx^{2/3}-54bd^3e^2n^2x^{2/3}-180ad^2e^3nx+141bd^2e^3n\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^2/x^3,x]
```

```
[Out] -1/2*(a + b*Log[c*(d + e*x^(1/3))^n])^2/x^2 - (b*e*(72*a*d^5*n - 90*a*d^4*e
*n*x^(1/3) + 18*b*d^4*e*n^2*x^(1/3) + 120*a*d^3*e^2*n*x^(2/3) - 54*b*d^3*e^
2*n^2*x^(2/3) - 180*a*d^2*e^3*n*x + 141*b*d^2*e^3*n^2*x + 360*a*d*e^4*n*x^(
4/3) - 462*b*d*e^4*n^2*x^(4/3) + 6*e^5*n*(-60*a + 137*b*n)*x^(5/3)*Log[d +
e*x^(1/3)] + 72*b*d^5*n*Log[c*(d + e*x^(1/3))^n] - 90*b*d^4*e*n*x^(1/3)*Log
[c*(d + e*x^(1/3))^n] + 120*b*d^3*e^2*n*x^(2/3)*Log[c*(d + e*x^(1/3))^n] -
180*b*d^2*e^3*n*x*Log[c*(d + e*x^(1/3))^n] + 360*b*d*e^4*n*x^(4/3)*Log[c*(d
 + e*x^(1/3))^n] - 180*b*e^5*x^(5/3)*Log[c*(d + e*x^(1/3))^n]^2 + 360*b*e^5
*n*x^(5/3)*Log[c*(d + e*x^(1/3))^n]*Log[-((e*x^(1/3))/d)] + 120*a*e^5*n*x^(
5/3)*Log[x] - 274*b*e^5*n^2*x^(5/3)*Log[x] + 360*b*e^5*n^2*x^(5/3)*PolyLog[
2, 1 + (e*x^(1/3))/d]))/(360*d^6*x^(5/3))
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{2}}{x^{3}}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^2/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+a\right)^2}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x^(1/3) + d)^n*c)^2 + 2*a*b*log((e*x^(1/3) + d)^n*c) +
 a^2)/x^3, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**2/x**3,x)
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**2/x**3, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)+a\right)^2}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^3,x, algorithm="maxima")
[Out] -1/2*b^2*log((e*x^(1/3) + d)^n)^2/x^2 + integrate(1/3*(3*(b^2*e*log(c)^2 +
2*a*b*e*log(c) + a^2*e)*x + (b^2*e*n*x + 6*(b^2*e*log(c) + a*b*e)*x + 6*(b^
2*d*log(c) + a*b*d)*x^(2/3))*log((e*x^(1/3) + d)^n) + 3*(b^2*d*log(c)^2 + 2
*a*b*d*log(c) + a^2*d)*x^(2/3))/(e*x^4 + d*x^(11/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)+a\right)^2}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^2/x^3,x, algorithm="giac")
[Out] integrate((b*log((e*x^(1/3) + d)^n*c) + a)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)\right)^2}{x^3}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))^2/x^3,x)
[Out] int((a + b*log(c*(d + e*x^(1/3))^n))^2/x^3, x)
```

3.456 $\int x^3 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3 dx$

Optimal result

Integrand size = 24, antiderivative size = 1835

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=-\frac{99b^3d^{10}n^3\left(d+e\sqrt[3]{x}\right)^2}{8e^{12}}+\frac{110b^3d^9n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^{12}}-\frac{1485b^3d^8n^3\left(d+e\sqrt[3]{x}\right)^4}{128e^{12}}+\frac{1188b^3d^7n^3\left(d+e\sqrt[3]{x}\right)^5}{125e^{12}}-\frac{77b^3d^6n^3\left(d+e\sqrt[3]{x}\right)^6}{12e^{12}}+\frac{1188b^3d^5n^3\left(d+e\sqrt[3]{x}\right)^7}{343e^{12}}-\frac{1485b^3d^4n^3\left(d+e\sqrt[3]{x}\right)^8}{1024e^{12}}+\frac{110b^3d^3n^3\left(d+e\sqrt[3]{x}\right)^9}{243e^{12}}-\frac{99b^3d^2n^3\left(d+e\sqrt[3]{x}\right)^{10}}{1000e^{12}}+\frac{18b^3dn^3\left(d+e\sqrt[3]{x}\right)^{11}}{1331e^{12}}-\frac{b^3n^3\left(d+e\sqrt[3]{x}\right)^{12}}{1152e^{12}}-\frac{18ab^2d^{11}n^2\sqrt[3]{x}}{e^{11}}+\frac{18b^3d^{11}n^3\sqrt[3]{x}}{e^{11}}-\frac{18b^3d^{11}n^2\left(d+e\sqrt[3]{x}\right)\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{e^{12}}+\frac{99b^2d^{10}n^2\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4e^{12}}-\frac{110b^2d^9n^2\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^{12}}+\frac{1485b^2d^8n^2\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{32e^{12}}-\frac{1188b^2d^7n^2\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{25e^{12}}+\frac{77b^2d^6n^2\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2e^{12}}-\frac{1188b^2d^5n^2\left(d+e\sqrt[3]{x}\right)^7\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{49e^{12}}+\frac{1485b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^8\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{128e^{12}}-\frac{110b^2d^3n^2\left(d+e\sqrt[3]{x}\right)^9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{27e^{12}}+\frac{99b^2d^2n^2\left(d+e\sqrt[3]{x}\right)^{10}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{100e^{12}}-\frac{18b^2dn^2\left(d+e\sqrt[3]{x}\right)^{11}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{121e^{12}}+\frac{b^2n^2\left(d+e\sqrt[3]{x}\right)^{12}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{96e^{12}}+\frac{9bd^{11}n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^{12}}-\frac{99bd^{10}n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{4e^{12}}+\frac{55bd^9n\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^{12}}$$

[Out] -1/1152*b^3*n^3*(d+e*x^(1/3))^12/e^12-3*d^11*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12+33/2*d^10*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12-55*d^9*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12+495/4*d^8*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12-198*d^7*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12+231*d^6*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12-198*d^5*(d+e*x^(1/3))^7*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12+495/4*d^4*(d+e*x^(1/3))^8*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12-55*d^3*(d+e*x^(1/3))^9*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12+33/2*d^2*(d+e*x^(1/3))^10*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12-3*d*(d+e*x^(1/3))^11*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12+1/4*(d+e*x^(1/3))^12*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^12-18*a*b^2*d^11*n^2*x^(1/3)/e^11-18*b^3*d^11*n^2*(d+e*x^(1/3))*ln(c*(d+e*x^(1/3))^n)/e^12+99/4*b^2*d^10*n^2*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12-110/3*b^2*d^9*n^2*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12+1485/32*b^2*d^8*n^2*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12-1188/25*b^2*d^7*n^2*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12+77/2*b^2*d^6*n^2*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12-1188/49*b^2*d^5*n^2*(d+e*x^(1/3))^7*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12+1485/128*b^2*d^4*n^2*(d+e*x^(1/3))^8*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12-110/27*b^2*d^3*n^2*(d+e*x^(1/3))^9*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12+99/100*b^2*d^2*n^2*(d+e*x^(1/3))^10*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12-18/121*b^2*d*n^2*(d+e*x^(1/3))^11*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12+9*b*d^11*n*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12-99/4*b*d^10*n*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12+55*b*d^9*n*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12-1485/16*b*d^8*n*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12+594/5*b*d^7*n*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12-231/2*b*d^6*n*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12+594/7*b*d^5*n*(d+e*x^(1/3))^7*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12-1485/32*b*d^4*n*(d+e*x^(1/3))^8*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12+55/3*b*d^3*n*(d+e*x^(1/3))^9*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12-99/20*b*d^2*n*(d+e*x^(1/3))^10*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12+9/11*b*d*n*(d+e*x^(1/3))^11*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12-99/8*b^3*d^10*n^3*(d+e*x^(1/3))^2/e^12+110/9*b^3*d^9*n^3*(d+e*x^(1/3))^3/e^12-1485/128*b^3*d^8*n^3*(d+e*x^(1/3))^4/e^12+1188/125*b^3*d^7*n^3*(d+e*x^(1/3))^5/e^12-77/12*b^3*d^6*n^3*(d+e*x^(1/3))^6/e^12+1188/343*b^3*d^5*n^3*(d+e*x^(1/3))^7/e^12-1485/1024*b^3*d^4*n^3*(d+e*x^(1/3))^8/e^12+110/243*b^3*d^3*n^3*(d+e*x^(1/3))^9/e^12-99/1000*b^3*d^2*n^3*(d+e*x^(1/3))^10/e^12+18/1331*b^3*d*n^3*(d+e*x^(1/3))^11/e^12+18*b^3*d^11*n^3*x^(1/3)/e^11+1/96*b^2*n^2*(d+e*x^(1/3))^12*(a+b*ln(c*(d+e*x^(1/3))^n))/e^12-1/16*b*n*(d+e*x^(1/3))^12*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^12

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 1835, normalized size of antiderivative = 1.00, number of steps used = 52, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341\}$

$$
\begin{aligned}
\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx = & -\frac{b^3n^3\left(d+e\sqrt[3]{x}\right)^{12}}{1152e^{12}} \\
& + \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^{12}}{4e^{12}} \\
& - \frac{bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(d+e\sqrt[3]{x}\right)^{12}}{16e^{12}} \\
& + \frac{b^2n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\left(d+e\sqrt[3]{x}\right)^{12}}{96e^{12}} \\
& + \frac{18b^3dn^3\left(d+e\sqrt[3]{x}\right)^{11}}{1331e^{12}} \\
& - \frac{3d\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^{11}}{e^{12}} \\
& + \frac{9bdn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(d+e\sqrt[3]{x}\right)^{11}}{11e^{12}} \\
& - \frac{18b^2dn^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\left(d+e\sqrt[3]{x}\right)^{11}}{121e^{12}} \\
& - \frac{99b^3d^2n^3\left(d+e\sqrt[3]{x}\right)^{10}}{1000e^{12}} \\
& + \frac{33d^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^{10}}{2e^{12}} \\
& - \frac{99bd^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(d+e\sqrt[3]{x}\right)^{10}}{20e^{12}} \\
& + \frac{99b^2d^2n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\left(d+e\sqrt[3]{x}\right)^{10}}{100e^{12}} \\
& + \frac{110b^3d^3n^3\left(d+e\sqrt[3]{x}\right)^{9}}{243e^{12}} \\
& - \frac{55d^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^{9}}{e^{12}} \\
& + \frac{55bd^3n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(d+e\sqrt[3]{x}\right)^{9}}{3e^{12}} \\
& - \frac{110b^2d^3n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\left(d+e\sqrt[3]{x}\right)^{9}}{27e^{12}} \\
& - \frac{1485b^3d^4n^3\left(d+e\sqrt[3]{x}\right)^{8}}{1024e^{12}} \\
& + \frac{495d^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^{8}}{4e^{12}} \\
& - \frac{1485bd^4n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(d+e\sqrt[3]{x}\right)^{8}}{32e^{12}} \\
& + \frac{1485b^2d^4n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\left(d+e\sqrt[3]{x}\right)^{8}}{128e^{12}} \\
& + \frac{1188b^3d^5n^3\left(d+e\sqrt[3]{x}\right)^{7}}{343e^{12}} \\
& 198d^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^{7}
\end{aligned}
$$

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]

[Out] (-99*b^3*d^10*n^3*(d + e*x^(1/3))^2)/(8*e^12) + (110*b^3*d^9*n^3*(d + e*x^(
1/3))^3)/(9*e^12) - (1485*b^3*d^8*n^3*(d + e*x^(1/3))^4)/(128*e^12) + (1188
*b^3*d^7*n^3*(d + e*x^(1/3))^5)/(125*e^12) - (77*b^3*d^6*n^3*(d + e*x^(1/3)
)^6)/(12*e^12) + (1188*b^3*d^5*n^3*(d + e*x^(1/3))^7)/(343*e^12) - (1485*b^
3*d^4*n^3*(d + e*x^(1/3))^8)/(1024*e^12) + (110*b^3*d^3*n^3*(d + e*x^(1/3))
^9)/(243*e^12) - (99*b^3*d^2*n^3*(d + e*x^(1/3))^10)/(1000*e^12) + (18*b^3*
d*n^3*(d + e*x^(1/3))^11)/(1331*e^12) - (b^3*n^3*(d + e*x^(1/3))^12)/(1152*
e^12) - (18*a*b^2*d^11*n^2*x^(1/3))/e^11 + (18*b^3*d^11*n^3*x^(1/3))/e^11 -
 (18*b^3*d^11*n^2*(d + e*x^(1/3))*Log[c*(d + e*x^(1/3))^n])/e^12 + (99*b^2*
d^10*n^2*(d + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(4*e^12) - (11
0*b^2*d^9*n^2*(d + e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^12)
+ (1485*b^2*d^8*n^2*(d + e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n]))/(32
*e^12) - (1188*b^2*d^7*n^2*(d + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n
]))/(25*e^12) + (77*b^2*d^6*n^2*(d + e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/
3))^n]))/(2*e^12) - (1188*b^2*d^5*n^2*(d + e*x^(1/3))^7*(a + b*Log[c*(d + e
*x^(1/3))^n]))/(49*e^12) + (1485*b^2*d^4*n^2*(d + e*x^(1/3))^8*(a + b*Log[c
*(d + e*x^(1/3))^n]))/(128*e^12) - (110*b^2*d^3*n^2*(d + e*x^(1/3))^9*(a +
b*Log[c*(d + e*x^(1/3))^n]))/(27*e^12) + (99*b^2*d^2*n^2*(d + e*x^(1/3))^10
*(a + b*Log[c*(d + e*x^(1/3))^n]))/(100*e^12) - (18*b^2*d*n^2*(d + e*x^(1/3
))^11*(a + b*Log[c*(d + e*x^(1/3))^n]))/(121*e^12) + (b^2*n^2*(d + e*x^(1/3
))^12*(a + b*Log[c*(d + e*x^(1/3))^n]))/(96*e^12) + (9*b*d^11*n*(d + e*x^(1
/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^12 - (99*b*d^10*n*(d + e*x^(1/3)
)^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(4*e^12) + (55*b*d^9*n*(d + e*x^(1/
3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^12 - (1485*b*d^8*n*(d + e*x^(1/
3))^4*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(16*e^12) + (594*b*d^7*n*(d + e*x
^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(5*e^12) - (231*b*d^6*n*(d +
e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*e^12) + (594*b*d^5*n*(d
 + e*x^(1/3))^7*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(7*e^12) - (1485*b*d^4*
n*(d + e*x^(1/3))^8*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(32*e^12) + (55*b*d
^3*n*(d + e*x^(1/3))^9*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(3*e^12) - (99*b
*d^2*n*(d + e*x^(1/3))^10*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(20*e^12) + (
9*b*d*n*(d + e*x^(1/3))^11*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(11*e^12) -
(b*n*(d + e*x^(1/3))^12*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(16*e^12) - (3*
d^11*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 + (33*d^10*(d
 + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/(2*e^12) - (55*d^9*(d +
 e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 + (495*d^8*(d + e*x^
(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/(4*e^12) - (198*d^7*(d + e*x^(
1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 + (231*d^6*(d + e*x^(1/3))
^6*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 - (198*d^5*(d + e*x^(1/3))^7*(a
 + b*Log[c*(d + e*x^(1/3))^n])^3)/e^12 + (495*d^4*(d + e*x^(1/3))^8*(a + b*
Log[c*(d + e*x^(1/3))^n])^3)/(4*e^12) - (55*d^3*(d + e*x^(1/3))^9*(a + b*Lo
g[c*(d + e*x^(1/3))^n])^3)/e^12 + (33*d^2*(d + e*x^(1/3))^10*(a + b*Log[c*(
d + e*x^(1/3))^n])^3)/(2*e^12) - (3*d*(d + e*x^(1/3))^11*(a + b*Log[c*(d +
```

```
e*x^(1/3))^n])^3)/e^12 + ((d + e*x^(1/3))^12*(a + b*Log[c*(d + e*x^(1/3))^n
])^3)/(4*e^12)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int x^{11}(a+b\log(c(d+ex)^n))^3\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\text{Subst}\left(\int \left(-\frac{d^{11}(a+b\log(c(d+ex)^n))^3}{e^{11}} + \frac{11d^{10}(d+ex)(a+b\log(c(d+ex)^n))^3}{e^{11}} - \frac{55d^9(d+ex)^2(a+b\log(c(d+ex)^n))^3}{e^{11}} + \frac{165d^8(d+ex)^3(a+b\log(c(d+ex)^n))^3}{e^{11}} - \frac{330d^7(d+ex)^4(a+b\log(c(d+ex)^n))^3}{e^{11}} + \frac{462d^6(d+ex)^5(a+b\log(c(d+ex)^n))^3}{e^{11}} - \frac{462d^5(d+ex)^6(a+b\log(c(d+ex)^n))^3}{e^{11}} + \frac{330d^4(d+ex)^7(a+b\log(c(d+ex)^n))^3}{e^{11}} - \frac{165d^3(d+ex)^8(a+b\log(c(d+ex)^n))^3}{e^{11}} + \frac{55d^2(d+ex)^9(a+b\log(c(d+ex)^n))^3}{e^{11}} - \frac{11d(d+ex)^{10}(a+b\log(c(d+ex)^n))^3}{e^{11}} + \frac{(d+ex)^{11}(a+b\log(c(d+ex)^n))^3}{e^{11}}\right) dx, x, \sqrt[3]{x}\right)$$

$$= \frac{3\text{Subst}\left(\int (d+ex)^{11} \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \,dx, x, \sqrt[3]{x}\right)}{e^{11}}$$
$$- \frac{(33d)\text{Subst}\left(\int (d+ex)^{10} \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \,dx, x, \sqrt[3]{x}\right)}{e^{11}}$$
$$+ \frac{(165d^2)\,\text{Subst}\left(\int (d+ex)^9 \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \,dx, x, \sqrt[3]{x}\right)}{e^{11}}$$
$$- \frac{(495d^3)\,\text{Subst}\left(\int (d+ex)^8 \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \,dx, x, \sqrt[3]{x}\right)}{e^{11}}$$
$$+ \frac{(990d^4)\,\text{Subst}\left(\int (d+ex)^7 \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \,dx, x, \sqrt[3]{x}\right)}{e^{11}}$$
$$- \frac{(1386d^5)\,\text{Subst}\left(\int (d+ex)^6 \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \,dx, x, \sqrt[3]{x}\right)}{e^{11}}$$
$$+ \frac{(1386d^6)\,\text{Subst}\left(\int (d+ex)^5 \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \,dx, x, \sqrt[3]{x}\right)}{e^{11}}$$
$$- \frac{(990d^7)\,\text{Subst}\left(\int (d+ex)^4 \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \,dx, x, \sqrt[3]{x}\right)}{e^{11}}$$
$$+ \frac{(495d^8)\,\text{Subst}\left(\int (d+ex)^3 \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \,dx, x, \sqrt[3]{x}\right)}{e^{11}}$$
$$- \frac{(165d^9)\,\text{Subst}\left(\int (d+ex)^2 \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \,dx, x, \sqrt[3]{x}\right)}{e^{11}}$$
$$+ \frac{(33d^{10})\,\text{Subst}\left(\int (d+ex) \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \,dx, x, \sqrt[3]{x}\right)}{e^{11}}$$
$$- \frac{(3d^{11})\,\text{Subst}\left(\int \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \,dx, x, \sqrt[3]{x}\right)}{e^{11}}$$

$$= \frac{3\text{Subst}\left(\int x^{11}(a+b\log(cx^n))^3\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} - \frac{(33d)\text{Subst}\left(\int x^{10}(a+b\log(cx^n))^3\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} + \frac{(165d^2)\,\text{Subst}\left(\int x^9(a+b\log(cx^n))^3\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} - \frac{(495d^3)\,\text{Subst}\left(\int x^8(a+b\log(cx^n))^3\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} + \frac{(990d^4)\,\text{Subst}\left(\int x^7(a+b\log(cx^n))^3\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} - \frac{(1386d^5)\,\text{Subst}\left(\int x^6(a+b\log(cx^n))^3\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} + \frac{(1386d^6)\,\text{Subst}\left(\int x^5(a+b\log(cx^n))^3\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} - \frac{(990d^7)\,\text{Subst}\left(\int x^4(a+b\log(cx^n))^3\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} + \frac{(495d^8)\,\text{Subst}\left(\int x^3(a+b\log(cx^n))^3\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} - \frac{(165d^9)\,\text{Subst}\left(\int x^2(a+b\log(cx^n))^3\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} + \frac{(33d^{10})\,\text{Subst}\left(\int x(a+b\log(cx^n))^3\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} - \frac{(3d^{11})\,\text{Subst}\left(\int (a+b\log(cx^n))^3\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}}$$

$= \text{Too large to display}$

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 1025, normalized size of antiderivative = 0.56

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx = \frac{e\sqrt[3]{x}\left(3550000608000a^3e^{11}x^{11/3}+b^3n^3\left(119225632485960d^{11}-26563616859780d^{10}e\sqrt[3]{x}+10242678720120\right.\right.}{}$$

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (e*x^(1/3)*(3550000608000*a^3*e^11*x^(11/3) + b^3*n^3*(119225632485960*d^11
 - 26563616859780*d^10*e*x^(1/3) + 10242678720120*d^9*e^2*x^(2/3) - 4836309
598890*d^8*e^3*x + 2516628075192*d^7*e^4*x^(4/3) - 1373077023780*d^6*e^5*x^
(5/3) + 761128152840*d^5*e^6*x^2 - 417533743935*d^4*e^7*x^(7/3) + 220161492
320*d^3*e^8*x^(8/3) - 106944990768*d^2*e^9*x^3 + 44119404000*d*e^10*x^(10/3
) - 12326391000*e^11*x^(11/3)) - 27720*a*b^2*n^2*(2384502120*d^11 - 8080518
```

```
60*d^10*e*x^(1/3) + 410634840*d^9*e^2*x^(2/3) - 243942930*d^8*e^3*x + 15673
4424*d^7*e^4*x^(4/3) - 104998740*d^6*e^5*x^(5/3) + 71703720*d^5*e^6*x^2 - 4
9019355*d^4*e^7*x^(7/3) + 32900560*d^3*e^8*x^(8/3) - 21072744*d^2*e^9*x^3 +
 12171600*d*e^10*x^(10/3) - 5336100*e^11*x^(11/3)) + 384199200*a^2*b*n*(277
20*d^11 - 13860*d^10*e*x^(1/3) + 9240*d^9*e^2*x^(2/3) - 6930*d^8*e^3*x + 55
44*d^7*e^4*x^(4/3) - 4620*d^6*e^5*x^(5/3) + 3960*d^5*e^6*x^2 - 3465*d^4*e^7
*x^(7/3) + 3080*d^3*e^8*x^(8/3) - 2772*d^2*e^9*x^3 + 2520*d*e^10*x^(10/3) -
 2310*e^11*x^(11/3))) - 27720*b*d^12*n*(384199200*a^2 - 2384502120*a*b*n +
4301068993*b^2*n^2)*Log[d + e*x^(1/3)] + 27720*b*e*x^(1/3)*(384199200*a^2*e
^11*x^(11/3) + 27720*a*b*n*(27720*d^11 - 13860*d^10*e*x^(1/3) + 9240*d^9*e^
2*x^(2/3) - 6930*d^8*e^3*x + 5544*d^7*e^4*x^(4/3) - 4620*d^6*e^5*x^(5/3) +
3960*d^5*e^6*x^2 - 3465*d^4*e^7*x^(7/3) + 3080*d^3*e^8*x^(8/3) - 2772*d^2*e
^9*x^3 + 2520*d*e^10*x^(10/3) - 2310*e^11*x^(11/3)) + b^2*n^2*(-2384502120*
d^11 + 808051860*d^10*e*x^(1/3) - 410634840*d^9*e^2*x^(2/3) + 243942930*d^8
*e^3*x - 156734424*d^7*e^4*x^(4/3) + 104998740*d^6*e^5*x^(5/3) - 71703720*d
^5*e^6*x^2 + 49019355*d^4*e^7*x^(7/3) - 32900560*d^3*e^8*x^(8/3) + 21072744
*d^2*e^9*x^3 - 12171600*d*e^10*x^(10/3) + 5336100*e^11*x^(11/3)))*Log[c*(d
+ e*x^(1/3))^n] + 384199200*b^2*(b*n*(86021*d^12 + 27720*d^11*e*x^(1/3) - 1
3860*d^10*e^2*x^(2/3) + 9240*d^9*e^3*x - 6930*d^8*e^4*x^(4/3) + 5544*d^7*e^
5*x^(5/3) - 4620*d^6*e^6*x^2 + 3960*d^5*e^7*x^(7/3) - 3465*d^4*e^8*x^(8/3)
+ 3080*d^3*e^9*x^3 - 2772*d^2*e^10*x^(10/3) + 2520*d*e^11*x^(11/3) - 2310*e
^12*x^4) - 27720*a*(d^12 - e^12*x^4))*Log[c*(d + e*x^(1/3))^n]^2 - 35500006
08000*b^3*(d^12 - e^12*x^4)*Log[c*(d + e*x^(1/3))^n]^3)/(14200002432000*e^1
2)
```

Maple [F]

$$\int x^{3}\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{3}dx$$

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
[Out] int(x^3*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 2183, normalized size of antiderivative = 1.19

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="fricas")
[Out] 1/14200002432000*(3550000608000*b^3*e^12*x^4*log(c)^3 - 12326391000*(b^3*e^
12*n^3 - 12*a*b^2*e^12*n^2 + 72*a^2*b*e^12*n - 288*a^3*e^12)*x^4 + 603680*(
```

364699*b^3*d^3*e^9*n^3 - 1510740*a*b^2*d^3*e^9*n^2 + 1960200*a^2*b*d^3*e^9*n)*x^3 + 3550000608000*(b^3*e^12*n^3*x^4 - b^3*d^12*n^3)*log(e*x^(1/3) + d)^3 - 4620*(297202819*b^3*d^6*e^6*n^3 - 629992440*a*b^2*d^6*e^6*n^2 + 384199200*a^2*b*d^6*e^6*n)*x^2 + 384199200*(3080*b^3*d^3*e^9*n^3*x^3 - 4620*b^3*d^6*e^6*n^3*x^2 + 9240*b^3*d^9*e^3*n^3*x + 86021*b^3*d^12*n^3 - 27720*a*b^2*d^12*n^2 - 2310*(b^3*e^12*n^3 - 12*a*b^2*e^12*n^2)*x^4 + 27720*(b^3*e^12*n^2*x^4 - b^3*d^12*n^2)*log(c) + 63*(40*b^3*d*e^11*n^3*x^3 - 55*b^3*d^4*e^8*n^3*x^2 + 88*b^3*d^7*e^5*n^3*x - 220*b^3*d^10*e^2*n^3)*x^(2/3) - 198*(14*b^3*d^2*e^10*n^3*x^3 - 20*b^3*d^5*e^7*n^3*x^2 + 35*b^3*d^8*e^4*n^3*x - 140*b^3*d^11*e*n^3)*x^(1/3))*log(e*x^(1/3) + d)^2 + 295833384000*(4*b^3*d^3*e^9*n*x^3 - 6*b^3*d^6*e^6*n*x^2 + 12*b^3*d^9*e^3*n*x - 3*(b^3*e^12*n - 12*a*b^2*e^12)*x^4)*log(c)^2 + 9240*(1108515013*b^3*d^9*e^3*n^3 - 1231904520*a*b^2*d^9*e^3*n^2 + 384199200*a^2*b*d^9*e^3*n)*x - 27720*(4301068993*b^3*d^12*n^3 - 2384502120*a*b^2*d^12*n^2 + 384199200*a^2*b*d^12*n - 5336100*(b^3*e^12*n^3 - 12*a*b^2*e^12*n^2 + 72*a^2*b*e^12*n)*x^4 + 43120*(763*b^3*d^3*e^9*n^3 - 1980*a*b^2*d^3*e^9*n^2)*x^3 - 4620*(22727*b^3*d^6*e^6*n^3 - 27720*a*b^2*d^6*e^6*n^2)*x^2 - 384199200*(b^3*e^12*n*x^4 - b^3*d^12*n)*log(c)^2 + 9240*(44441*b^3*d^9*e^3*n^3 - 27720*a*b^2*d^9*e^3*n^2)*x - 27720*(3080*b^3*d^3*e^9*n^2*x^3 - 4620*b^3*d^6*e^6*n^2*x^2 + 9240*b^3*d^9*e^3*n^2*x + 86021*b^3*d^12*n^2 - 27720*a*b^2*d^12*n - 2310*(b^3*e^12*n^2 - 12*a*b^2*e^12*n)*x^4)*log(c) - 63*(12826220*b^3*d^10*e^2*n^3 - 6098400*a*b^2*d^10*e^2*n^2 - 8400*(23*b^3*d*e^11*n^3 - 132*a*b^2*d*e^11*n^2)*x^3 + 385*(2021*b^3*d^4*e^8*n^3 - 3960*a*b^2*d^4*e^8*n^2)*x^2 - 88*(28271*b^3*d^7*e^5*n^3 - 27720*a*b^2*d^7*e^5*n^2)*x + 27720*(40*b^3*d*e^11*n^2*x^3 - 55*b^3*d^4*e^8*n^2*x^2 + 88*b^3*d^7*e^5*n^2*x - 220*b^3*d^10*e^2*n^2)*log(c))*x^(2/3) + 198*(12042940*b^3*d^11*e*n^3 - 3880800*a*b^2*d^11*e*n^2 - 588*(181*b^3*d^2*e^10*n^3 - 660*a*b^2*d^2*e^10*n^2)*x^3 + 20*(18107*b^3*d^5*e^7*n^3 - 27720*a*b^2*d^5*e^7*n^2)*x^2 - 35*(35201*b^3*d^8*e^4*n^3 - 27720*a*b^2*d^8*e^4*n^2)*x + 27720*(14*b^3*d^2*e^10*n^2*x^3 - 20*b^3*d^5*e^7*n^2*x^2 + 35*b^3*d^8*e^4*n^2*x - 140*b^3*d^11*e*n^2)*log(c))*x^(1/3))*log(e*x^(1/3) + d) + 42688800*(3465*(b^3*e^12*n^2 - 12*a*b^2*e^12*n + 72*a^2*b*e^12)*x^4 - 28*(763*b^3*d^3*e^9*n^2 - 1980*a*b^2*d^3*e^9*n)*x^3 + 3*(22727*b^3*d^6*e^6*n^2 - 27720*a*b^2*d^6*e^6*n)*x^2 - 6*(44441*b^3*d^9*e^3*n^2 - 27720*a*b^2*d^9*e^3*n)*x)*log(c) - 63*(421644712060*b^3*d^10*e^2*n^3 - 355542818400*a*b^2*d^10*e^2*n^2 + 84523824000*a^2*b*d^10*e^2*n - 1764000*(397*b^3*d*e^11*n^3 - 3036*a*b^2*d*e^11*n^2 + 8712*a^2*b*d*e^11*n)*x^3 + 2695*(2459191*b^3*d^4*e^8*n^3 - 8003160*a*b^2*d^4*e^8*n^2 + 7840800*a^2*b*d^4*e^8*n)*x^2 - 384199200*(40*b^3*d*e^11*n*x^3 - 55*b^3*d^4*e^8*n*x^2 + 88*b^3*d^7*e^5*n*x - 220*b^3*d^10*e^2*n)*log(c)^2 - 88*(453937243*b^3*d^7*e^5*n^3 - 783672120*a*b^2*d^7*e^5*n^2 + 384199200*a^2*b*d^7*e^5*n)*x - 27720*(12826220*b^3*d^10*e^2*n^2 - 6098400*a*b^2*d^10*e^2*n - 8400*(23*b^3*d*e^11*n^2 - 132*a*b^2*d*e^11*n)*x^3 + 385*(2021*b^3*d^4*e^8*n^2 - 3960*a*b^2*d^4*e^8*n)*x^2 - 88*(28271*b^3*d^7*e^5*n^2 - 27720*a*b^2*d^7*e^5*n)*x)*log(c))*x^(2/3) + 198*(602149659020*b^3*d^11*e*n^3 - 333830296800*a*b^2*d^11*e*n^2 + 53787888000*a^2*b*d^11*e*n - 24696*(21871*b^3*d^2*e^10*n^3 - 119460*a*b^2*d^2*e^10*n^2 + 217800*a^2*b*d^2*e^10*n)*x^3 + 20*(

```
192204079*b^3*d^5*e^7*n^3 - 501926040*a*b^2*d^5*e^7*n^2 + 384199200*a^2*b*d
^5*e^7*n)*x^2 - 384199200*(14*b^3*d^2*e^10*n*x^3 - 20*b^3*d^5*e^7*n*x^2 + 3
5*b^3*d^8*e^4*n*x - 140*b^3*d^11*e*n)*log(c)^2 - 35*(697880173*b^3*d^8*e^4*
n^3 - 975771720*a*b^2*d^8*e^4*n^2 + 384199200*a^2*b*d^8*e^4*n)*x - 27720*(1
2042940*b^3*d^11*e*n^2 - 3880800*a*b^2*d^11*e*n - 588*(181*b^3*d^2*e^10*n^2
 - 660*a*b^2*d^2*e^10*n)*x^3 + 20*(18107*b^3*d^5*e^7*n^2 - 27720*a*b^2*d^5*
e^7*n)*x^2 - 35*(35201*b^3*d^8*e^4*n^2 - 27720*a*b^2*d^8*e^4*n)*x)*log(c))*
x^(1/3))/e^12
```

Sympy [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\text{Timed out}$$

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/3))**n))**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 1064, normalized size of antiderivative = 0.58

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4*log((e*x^(1/3) + d)^n*c)^3 + 3/4*a*b^2*x^4*log((e*x^(1/3) + d)^
n*c)^2 + 3/4*a^2*b*x^4*log((e*x^(1/3) + d)^n*c) + 1/4*a^3*x^4 - 1/36960*a^2
*b*e*n*(27720*d^12*log(e*x^(1/3) + d)/e^13 + (2310*e^11*x^4 - 2520*d*e^10*x
^(11/3) + 2772*d^2*e^9*x^(10/3) - 3080*d^3*e^8*x^3 + 3465*d^4*e^7*x^(8/3) -
 3960*d^5*e^6*x^(7/3) + 4620*d^6*e^5*x^2 - 5544*d^7*e^4*x^(5/3) + 6930*d^8*
e^3*x^(4/3) - 9240*d^9*e^2*x + 13860*d^10*e*x^(2/3) - 27720*d^11*x^(1/3))/e
^12) - 1/512265600*(27720*e*n*(27720*d^12*log(e*x^(1/3) + d)/e^13 + (2310*e
^11*x^4 - 2520*d*e^10*x^(11/3) + 2772*d^2*e^9*x^(10/3) - 3080*d^3*e^8*x^3 +
 3465*d^4*e^7*x^(8/3) - 3960*d^5*e^6*x^(7/3) + 4620*d^6*e^5*x^2 - 5544*d^7*
e^4*x^(5/3) + 6930*d^8*e^3*x^(4/3) - 9240*d^9*e^2*x + 13860*d^10*e*x^(2/3)
- 27720*d^11*x^(1/3))/e^12)*log((e*x^(1/3) + d)^n*c) - (5336100*e^12*x^4 -
12171600*d*e^11*x^(11/3) + 21072744*d^2*e^10*x^(10/3) - 32900560*d^3*e^9*x^
3 + 49019355*d^4*e^8*x^(8/3) - 71703720*d^5*e^7*x^(7/3) + 104998740*d^6*e^6
*x^2 + 384199200*d^12*log(e*x^(1/3) + d)^2 - 156734424*d^7*e^5*x^(5/3) + 24
3942930*d^8*e^4*x^(4/3) - 410634840*d^9*e^3*x + 2384502120*d^12*log(e*x^(1/
3) + d) + 808051860*d^10*e^2*x^(2/3) - 2384502120*d^11*e*x^(1/3))*n^2/e^12)
```

```
*a*b^2 - 1/14200002432000*(384199200*e*n*(27720*d^12*log(e*x^(1/3) + d)/e^1
3 + (2310*e^11*x^4 - 2520*d*e^10*x^(11/3) + 2772*d^2*e^9*x^(10/3) - 3080*d^
3*e^8*x^3 + 3465*d^4*e^7*x^(8/3) - 3960*d^5*e^6*x^(7/3) + 4620*d^6*e^5*x^2
- 5544*d^7*e^4*x^(5/3) + 6930*d^8*e^3*x^(4/3) - 9240*d^9*e^2*x + 13860*d^10
*e*x^(2/3) - 27720*d^11*x^(1/3))/e^12)*log((e*x^(1/3) + d)^n*c)^2 + e*n*((1
2326391000*e^12*x^4 - 44119404000*d*e^11*x^(11/3) + 106944990768*d^2*e^10*x
^(10/3) - 220161492320*d^3*e^9*x^3 + 3550000608000*d^12*log(e*x^(1/3) + d)^
3 + 417533743935*d^4*e^8*x^(8/3) - 761128152840*d^5*e^7*x^(7/3) + 137307702
3780*d^6*e^6*x^2 + 33049199383200*d^12*log(e*x^(1/3) + d)^2 - 2516628075192
*d^7*e^5*x^(5/3) + 4836309598890*d^8*e^4*x^(4/3) - 10242678720120*d^9*e^3*x
 + 119225632485960*d^12*log(e*x^(1/3) + d) + 26563616859780*d^10*e^2*x^(2/3
) - 119225632485960*d^11*e*x^(1/3))*n^2/e^13 - 27720*(5336100*e^12*x^4 - 12
171600*d*e^11*x^(11/3) + 21072744*d^2*e^10*x^(10/3) - 32900560*d^3*e^9*x^3
+ 49019355*d^4*e^8*x^(8/3) - 71703720*d^5*e^7*x^(7/3) + 104998740*d^6*e^6*x
^2 + 384199200*d^12*log(e*x^(1/3) + d)^2 - 156734424*d^7*e^5*x^(5/3) + 2439
42930*d^8*e^4*x^(4/3) - 410634840*d^9*e^3*x + 2384502120*d^12*log(e*x^(1/3)
 + d) + 808051860*d^10*e^2*x^(2/3) - 2384502120*d^11*e*x^(1/3))*n*log((e*x^
(1/3) + d)^n*c)/e^13))*b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4320 vs. $2(1591) = 3182$.

Time = 0.56 (sec) , antiderivative size = 4320, normalized size of antiderivative = 2.35

$$\int x^3 \left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^n\right)\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="giac")
```

```
[Out] 1/14200002432000*(3550000608000*b^3*e*x^4*log(c)^3 + 10650001824000*a*b^2*e
*x^4*log(c)^2 + 10650001824000*a^2*b*e*x^4*log(c) + 3550000608000*a^3*e*x^4
 + (3550000608000*(e*x^(1/3) + d)^12*log(e*x^(1/3) + d)^3/e^11 - 4260000729
6000*(e*x^(1/3) + d)^11*d*log(e*x^(1/3) + d)^3/e^11 + 234300040128000*(e*x^
(1/3) + d)^10*d^2*log(e*x^(1/3) + d)^3/e^11 - 781000133760000*(e*x^(1/3) +
d)^9*d^3*log(e*x^(1/3) + d)^3/e^11 + 1757250300960000*(e*x^(1/3) + d)^8*d^4
*log(e*x^(1/3) + d)^3/e^11 - 2811600481536000*(e*x^(1/3) + d)^7*d^5*log(e*x
^(1/3) + d)^3/e^11 + 3280200561792000*(e*x^(1/3) + d)^6*d^6*log(e*x^(1/3) +
 d)^3/e^11 - 2811600481536000*(e*x^(1/3) + d)^5*d^7*log(e*x^(1/3) + d)^3/e^
11 + 1757250300960000*(e*x^(1/3) + d)^4*d^8*log(e*x^(1/3) + d)^3/e^11 - 781
000133760000*(e*x^(1/3) + d)^3*d^9*log(e*x^(1/3) + d)^3/e^11 + 234300040128
000*(e*x^(1/3) + d)^2*d^10*log(e*x^(1/3) + d)^3/e^11 - 42600007296000*(e*x^
(1/3) + d)*d^11*log(e*x^(1/3) + d)^3/e^11 - 887500152000*(e*x^(1/3) + d)^12
*log(e*x^(1/3) + d)^2/e^11 + 11618183808000*(e*x^(1/3) + d)^11*d*log(e*x^(1
/3) + d)^2/e^11 - 70290012038400*(e*x^(1/3) + d)^10*d^2*log(e*x^(1/3) + d)^
2/e^11 + 260333377920000*(e*x^(1/3) + d)^9*d^3*log(e*x^(1/3) + d)^2/e^11 -
```

658968862860000*(e*x^(1/3) + d)^8*d^4*log(e*x^(1/3) + d)^2/e^11 + 120497163 4944000*(e*x^(1/3) + d)^7*d^5*log(e*x^(1/3) + d)^2/e^11 - 1640100280896000* (e*x^(1/3) + d)^6*d^6*log(e*x^(1/3) + d)^2/e^11 + 1686960288921600*(e*x^(1/ 3) + d)^5*d^7*log(e*x^(1/3) + d)^2/e^11 - 1317937725720000*(e*x^(1/3) + d)^ 4*d^8*log(e*x^(1/3) + d)^2/e^11 + 781000133760000*(e*x^(1/3) + d)^3*d^9*log (e*x^(1/3) + d)^2/e^11 - 351450060192000*(e*x^(1/3) + d)^2*d^10*log(e*x^(1/ 3) + d)^2/e^11 + 127800021888000*(e*x^(1/3) + d)*d^11*log(e*x^(1/3) + d)^2/ e^11 + 147916692000*(e*x^(1/3) + d)^12*log(e*x^(1/3) + d)/e^11 - 2112397056 000*(e*x^(1/3) + d)^11*d*log(e*x^(1/3) + d)/e^11 + 14058002407680*(e*x^(1/3) + d)^10*d^2*log(e*x^(1/3) + d)/e^11 - 57851861760000*(e*x^(1/3) + d)^9*d^ 3*log(e*x^(1/3) + d)/e^11 + 164742215715000*(e*x^(1/3) + d)^8*d^4*log(e*x^(1/3) + d)/e^11 - 344277609984000*(e*x^(1/3) + d)^7*d^5*log(e*x^(1/3) + d)/e ^11 + 546700093632000*(e*x^(1/3) + d)^6*d^6*log(e*x^(1/3) + d)/e^11 - 67478 4115568640*(e*x^(1/3) + d)^5*d^7*log(e*x^(1/3) + d)/e^11 + 658968862860000* (e*x^(1/3) + d)^4*d^8*log(e*x^(1/3) + d)/e^11 - 520666755840000*(e*x^(1/3) + d)^3*d^9*log(e*x^(1/3) + d)/e^11 + 351450060192000*(e*x^(1/3) + d)^2*d^10 *log(e*x^(1/3) + d)/e^11 - 255600043776000*(e*x^(1/3) + d)*d^11*log(e*x^(1/ 3) + d)/e^11 - 12326391000*(e*x^(1/3) + d)^12/e^11 + 192036096000*(e*x^(1/3) + d)^11*d/e^11 - 1405800240768*(e*x^(1/3) + d)^10*d^2/e^11 + 642798464000 0*(e*x^(1/3) + d)^9*d^3/e^11 - 20592776964375*(e*x^(1/3) + d)^8*d^4/e^11 + 49182515712000*(e*x^(1/3) + d)^7*d^5/e^11 - 91116682272000*(e*x^(1/3) + d)^ 6*d^6/e^11 + 134956823113728*(e*x^(1/3) + d)^5*d^7/e^11 - 164742215715000*(e*x^(1/3) + d)^4*d^8/e^11 + 173555585280000*(e*x^(1/3) + d)^3*d^9/e^11 - 17 572503009600 0*(e*x^(1/3) + d)^2*d^10/e^11 + 255600043776000*(e*x^(1/3) + d) *d^11/e^11)*b^3*n^3 + 27720*(384199200*(e*x^(1/3) + d)^12*log(e*x^(1/3) + d)^2/e^11 - 4610390400*(e*x^(1/3) + d)^11*d*log(e*x^(1/3) + d)^2/e^11 + 2535 7147200*(e*x^(1/3) + d)^10*d^2*log(e*x^(1/3) + d)^2/e^11 - 84523824000*(e*x ^(1/3) + d)^9*d^3*log(e*x^(1/3) + d)^2/e^11 + 190178604000*(e*x^(1/3) + d)^ 8*d^4*log(e*x^(1/3) + d)^2/e^11 - 304285766400*(e*x^(1/3) + d)^7*d^5*log(e* x^(1/3) + d)^2/e^11 + 355000060800*(e*x^(1/3) + d)^6*d^6*log(e*x^(1/3) + d) ^2/e^11 - 304285766400*(e*x^(1/3) + d)^5*d^7*log(e*x^(1/3) + d)^2/e^11 + 19 0178604000*(e*x^(1/3) + d)^4*d^8*log(e*x^(1/3) + d)^2/e^11 - 84523824000*(e *x^(1/3) + d)^3*d^9*log(e*x^(1/3) + d)^2/e^11 + 25357147200*(e*x^(1/3) + d) ^2*d^10*log(e*x^(1/3) + d)^2/e^11 - 4610390400*(e*x^(1/3) + d)*d^11*log(e*x ^(1/3) + d)^2/e^11 - 64033200*(e*x^(1/3) + d)^12*log(e*x^(1/3) + d)/e^11 + 838252800*(e*x^(1/3) + d)^11*d*log(e*x^(1/3) + d)/e^11 - 5071429440*(e*x^(1 /3) + d)^10*d^2*log(e*x^(1/3) + d)/e^11 + 18783072000*(e*x^(1/3) + d)^9*d^3 *log(e*x^(1/3) + d)/e^11 - 47544651000*(e*x^(1/3) + d)^8*d^4*log(e*x^(1/3) + d)/e^11 + 86938790400*(e*x^(1/3) + d)^7*d^5*log(e*x^(1/3) + d)/e^11 - 118 333353600*(e*x^(1/3) + d)^6*d^6*log(e*x^(1/3) + d)/e^11 + 121714306560*(e*x ^(1/3) + d)^5*d^7*log(e*x^(1/3) + d)/e^11 - 95089302000*(e*x^(1/3) + d)^4*d ^8*log(e*x^(1/3) + d)/e^11 + 56349216000*(e*x^(1/3) + d)^3*d^9*log(e*x^(1/3) + d)/e^11 - 25357147200*(e*x^(1/3) + d)^2*d^10*log(e*x^(1/3) + d)/e^11 + 9220780800*(e*x^(1/3) + d)*d^11*log(e*x^(1/3) + d)/e^11 + 5336100*(e*x^(1/3) + d)^12/e^11 - 76204800*(e*x^(1/3) + d)^11*d/e^11 + 507142944*(e*x^(1/3)

+ d)^10*d^2/e^11 - 2087008000*(e*x^(1/3) + d)^9*d^3/e^11 + 5943081375*(e*x^(1/3) + d)^8*d^4/e^11 - 12419827200*(e*x^(1/3) + d)^7*d^5/e^11 + 19722225600*(e*x^(1/3) + d)^6*d^6/e^11 - 24342861312*(e*x^(1/3) + d)^5*d^7/e^11 + 23772325500*(e*x^(1/3) + d)^4*d^8/e^11 - 18783072000*(e*x^(1/3) + d)^3*d^9/e^11 + 12678573600*(e*x^(1/3) + d)^2*d^10/e^11 - 9220780800*(e*x^(1/3) + d)*d^11/e^11)*b^3*n^2*log(c) + 384199200*(27720*(e*x^(1/3) + d)^12*log(e*x^(1/3) + d)/e^11 - 332640*(e*x^(1/3) + d)^11*d*log(e*x^(1/3) + d)/e^11 + 1829520*(e*x^(1/3) + d)^10*d^2*log(e*x^(1/3) + d)/e^11 - 6098400*(e*x^(1/3) + d)^9*d^3*log(e*x^(1/3) + d)/e^11 + 13721400*(e*x^(1/3) + d)^8*d^4*log(e*x^(1/3) + d)/e^11 - 21954240*(e*x^(1/3) + d)^7*d^5*log(e*x^(1/3) + d)/e^11 + 25613280*(e*x^(1/3) + d)^6*d^6*log(e*x^(1/3) + d)/e^11 - 21954240*(e*x^(1/3) + d)^5*d^7*log(e*x^(1/3) + d)/e^11 + 13721400*(e*x^(1/3) + d)^4*d^8*log(e*x^(1/3) + d)/e^11 - 6098400*(e*x^(1/3) + d)^3*d^9*log(e*x^(1/3) + d)/e^11 + 1829520*(e*x^(1/3) + d)^2*d^10*log(e*x^(1/3) + d)/e^11 - 332640*(e*x^(1/3) + d)*d^11*log(e*x^(1/3) + d)/e^11 - 2310*(e*x^(1/3) + d)^12/e^11 + 30240*(e*x^(1/3) + d)^11*d/e^11 - 182952*(e*x^(1/3) + d)^10*d^2/e^11 + 677600*(e*x^(1/3) + d)^9*d^3/e^11 - 1715175*(e*x^(1/3) + d)^8*d^4/e^11 + 3136320*(e*x^(1/3) + d)^7*d^5/e^11 - 4268880*(e*x^(1/3) + d)^6*d^6/e^11 + 4390848*(e*x^(1/3) + d)^5*d^7/e^11 - 3430350*(e*x^(1/3) + d)^4*d^8/e^11 + 2032800*(e*x^(1/3) + d)^3*d^9/e^11 - 914760*(e*x^(1/3) + d)^2*d^10/e^11 + 332640*(e*x^(1/3) + d)*d^11/e^11)*b^3*n*log(c)^2 + 27720*(384199200*(e*x^(1/3) + d)^12*log(e*x^(1/3) + d)^2/e^11 - 4610390400*(e*x^(1/3) + d)^11*d*log(e*x^(1/3) + d)^2/e^11 + 25357147200*(e*x^(1/3) + d)^10*d^2*log(e*x^(1/3) + d)^2/e^11 - 84523824000*(e*x^(1/3) + d)^9*d^3*log(e*x^(1/3) + d)^2/e^11 + 190178604000*(e*x^(1/3) + d)^8*d^4*log(e*x^(1/3) + d)^2/e^11 - 304285766400*(e*x^(1/3) + d)^7*d^5*log(e*x^(1/3) + d)^2/e^11 + 355000060800*(e*x^(1/3) + d)^6*d^6*log(e*x^(1/3) + d)^2/e^11 - 304285766400*(e*x^(1/3) + d)^5*d^7*log(e*x^(1/3) + d)^2/e^11 + 190178604000*(e*x^(1/3) + d)^4*d^8*log(e*x^(1/3) + d)^2/e^11 - 84523824000*(e*x^(1/3) + d)^3*d^9*log(e*x^(1/3) + d)^2/e^11 + 25357147200*(e*x^(1/3) + d)^2*d^10*log(e*x^(1/3) + d)^2/e^11 - 4610390400*(e*x^(1/3) + d)*d^11*log(e*x^(1/3) + d)^2/e^11 - 64033200*(e*x^(1/3) + d)^12*log(e*x^(1/3) + d)/e^11 + 838252800*(e*x^(1/3) + d)^11*d*log(e*x^(1/3) + d)/e^11 - 5071429440*(e*x^(1/3) + d)^10*d^2*log(e*x^(1/3) + d)/e^11 + 18783072000*(e*x^(1/3) + d)^9*d^3*log(e*x^(1/3) + d)/e^11 - 47544651000*(e*x^(1/3) + d)^8*d^4*log(e*x^(1/3) + d)/e^11 + 86938790400*(e*x^(1/3) + d)^7*d^5*log(e*x^(1/3) + d)/e^11 - 118333353600*(e*x^(1/3) + d)^6*d^6*log(e*x^(1/3) + d)/e^11 + 121714306560*(e*x^(1/3) + d)^5*d^7*log(e*x^(1/3) + d)/e^11 - 95089302000*(e*x^(1/3) + d)^4*d^8*log(e*x^(1/3) + d)/e^11 + 56349216000*(e*x^(1/3) + d)^3*d^9*log(e*x^(1/3) + d)/e^11 - 25357147200*(e*x^(1/3) + d)^2*d^10*log(e*x^(1/3) + d)/e^11 + 9220780800*(e*x^(1/3) + d)*d^11*log(e*x^(1/3) + d)/e^11 + 5336100*(e*x^(1/3) + d)^12/e^11 - 76204800*(e*x^(1/3) + d)^11*d/e^11 + 507142944*(e*x^(1/3) + d)^10*d^2/e^11 - 2087008000*(e*x^(1/3) + d)^9*d^3/e^11 + 5943081375*(e*x^(1/3) + d)^8*d^4/e^11 - 12419827200*(e*x^(1/3) + d)^7*d^5/e^11 + 19722225600*(e*x^(1/3) + d)^6*d^6/e^11 - 24342861312*(e*x^(1/3) + d)^5*d^7/e^11 + 23772325500*(e*x^(1/3) + d)^4*d^8/e^11 - 18783072000*(e*x^(1/3) + d)^3

```
*d^9/e^11 + 12678573600*(e*x^(1/3) + d)^2*d^10/e^11 - 9220780800*(e*x^(1/3)
 + d)*d^11/e^11)*a*b^2*n^2 + 768398400*(27720*(e*x^(1/3) + d)^12*log(e*x^(1
/3) + d)/e^11 - 332640*(e*x^(1/3) + d)^11*d*log(e*x^(1/3) + d)/e^11 + 18295
20*(e*x^(1/3) + d)^10*d^2*log(e*x^(1/3) + d)/e^11 - 6098400*(e*x^(1/3) + d)
^9*d^3*log(e*x^(1/3) + d)/e^11 + 13721400*(e*x^(1/3) + d)^8*d^4*log(e*x^(1/
3) + d)/e^11 - 21954240*(e*x^(1/3) + d)^7*d^5*log(e*x^(1/3) + d)/e^11 + 256
13280*(e*x^(1/3) + d)^6*d^6*log(e*x^(1/3) + d)/e^11 - 21954240*(e*x^(1/3) +
 d)^5*d^7*log(e*x^(1/3) + d)/e^11 + 13721400*(e*x^(1/3) + d)^4*d^8*log(e*x^
(1/3) + d)/e^11 - 6098400*(e*x^(1/3) + d)^3*d^9*log(e*x^(1/3) + d)/e^11 + 1
829520*(e*x^(1/3) + d)^2*d^10*log(e*x^(1/3) + d)/e^11 - 332640*(e*x^(1/3) +
 d)*d^11*log(e*x^(1/3) + d)/e^11 - 2310*(e*x^(1/3) + d)^12/e^11 + 30240*(e*
x^(1/3) + d)^11*d/e^11 - 182952*(e*x^(1/3) + d)^10*d^2/e^11 + 677600*(e*x^(
1/3) + d)^9*d^3/e^11 - 1715175*(e*x^(1/3) + d)^8*d^4/e^11 + 3136320*(e*x^(1
/3) + d)^7*d^5/e^11 - 4268880*(e*x^(1/3) + d)^6*d^6/e^11 + 4390848*(e*x^(1/
3) + d)^5*d^7/e^11 - 3430350*(e*x^(1/3) + d)^4*d^8/e^11 + 2032800*(e*x^(1/3
) + d)^3*d^9/e^11 - 914760*(e*x^(1/3) + d)^2*d^10/e^11 + 332640*(e*x^(1/3)
+ d)*d^11/e^11)*a*b^2*n*log(c) + 384199200*(27720*(e*x^(1/3) + d)^12*log(e*
x^(1/3) + d)/e^11 - 332640*(e*x^(1/3) + d)^11*d*log(e*x^(1/3) + d)/e^11 + 1
829520*(e*x^(1/3) + d)^10*d^2*log(e*x^(1/3) + d)/e^11 - 6098400*(e*x^(1/3)
+ d)^9*d^3*log(e*x^(1/3) + d)/e^11 + 13721400*(e*x^(1/3) + d)^8*d^4*log(e*x
^(1/3) + d)/e^11 - 21954240*(e*x^(1/3) + d)^7*d^5*log(e*x^(1/3) + d)/e^11 +
 25613280*(e*x^(1/3) + d)^6*d^6*log(e*x^(1/3) + d)/e^11 - 21954240*(e*x^(1/
3) + d)^5*d^7*log(e*x^(1/3) + d)/e^11 + 13721400*(e*x^(1/3) + d)^4*d^8*log(
e*x^(1/3) + d)/e^11 - 6098400*(e*x^(1/3) + d)^3*d^9*log(e*x^(1/3) + d)/e^11
 + 1829520*(e*x^(1/3) + d)^2*d^10*log(e*x^(1/3) + d)/e^11 - 332640*(e*x^(1/
3) + d)*d^11*log(e*x^(1/3) + d)/e^11 - 2310*(e*x^(1/3) + d)^12/e^11 + 30240
*(e*x^(1/3) + d)^11*d/e^11 - 182952*(e*x^(1/3) + d)^10*d^2/e^11 + 677600*(e
*x^(1/3) + d)^9*d^3/e^11 - 1715175*(e*x^(1/3) + d)^8*d^4/e^11 + 3136320*(e*
x^(1/3) + d)^7*d^5/e^11 - 4268880*(e*x^(1/3) + d)^6*d^6/e^11 + 4390848*(e*x
^(1/3) + d)^5*d^7/e^11 - 3430350*(e*x^(1/3) + d)^4*d^8/e^11 + 2032800*(e*x^
(1/3) + d)^3*d^9/e^11 - 914760*(e*x^(1/3) + d)^2*d^10/e^11 + 332640*(e*x^(1
/3) + d)*d^11/e^11)*a^2*b*n)/e
```

Mupad [B] (verification not implemented)

Time = 9.93 (sec) , antiderivative size = 1802, normalized size of antiderivative = 0.98

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

```
[In] int(x^3*(a + b*log(c*(d + e*x^(1/3))^n))^3,x)
```

```
[Out] (a^3*x^4)/4 + (b^3*x^4*log(c*(d + e*x^(1/3))^n)^3)/4 - (b^3*n^3*x^4)/1152 +
 (3*a*b^2*x^4*log(c*(d + e*x^(1/3))^n)^2)/4 - (b^3*n*x^4*log(c*(d + e*x^(1/
3))^n)^2)/16 + (b^3*n^2*x^4*log(c*(d + e*x^(1/3))^n))/96 + (a*b^2*n^2*x^4)/
```

96 - (b^3*d^12*log(c*(d + e*x^(1/3))^n)^3)/(4*e^12) + (3*a^2*b*x^4*log(c*(d + e*x^(1/3))^n))/4 - (a^2*b*n*x^4)/16 - (a*b^2*n*x^4*log(c*(d + e*x^(1/3))^n))/8 - (4301068993*b^3*d^12*n^3*log(d + e*x^(1/3)))/(512265600*e^12) + (364699*b^3*d^3*n^3*x^3)/(23522400*e^3) - (297202819*b^3*d^6*n^3*x^2)/(3073593600*e^6) - (21871*b^3*d^2*n^3*x^(10/3))/(2904000*e^2) - (2459191*b^3*d^4*n^3*x^(8/3))/(83635200*e^4) + (192204079*b^3*d^5*n^3*x^(7/3))/(3585859200*e^5) + (453937243*b^3*d^7*n^3*x^(5/3))/(2561328000*e^7) - (697880173*b^3*d^8*n^3*x^(4/3))/(2049062400*e^8) - (1916566873*b^3*d^10*n^3*x^(2/3))/(1024531200*e^10) + (4301068993*b^3*d^11*n^3*x^(1/3))/(512265600*e^11) - (3*a*b^2*d^12*log(c*(d + e*x^(1/3))^n)^2)/(4*e^12) + (86021*b^3*d^12*n*log(c*(d + e*x^(1/3))^n)^2)/(36960*e^12) + (397*b^3*d*n^3*x^(11/3))/(127776*e) + (1108515013*b^3*d^9*n^3*x)/(1536796800*e^9) - (3*a^2*b*d^12*n*log(d + e*x^(1/3)))/(4*e^12) + (3*b^3*d*n*x^(11/3)*log(c*(d + e*x^(1/3))^n)^2)/(44*e) - (23*b^3*d*n^2*x^(11/3)*log(c*(d + e*x^(1/3))^n))/(968*e) + (b^3*d^9*n*x*log(c*(d + e*x^(1/3))^n)^2)/(4*e^9) - (44441*b^3*d^9*n^2*x*log(c*(d + e*x^(1/3))^n))/(55440*e^9) + (a^2*b*d^3*n*x^3)/(12*e^3) - (a^2*b*d^6*n*x^2)/(8*e^6) - (23*a*b^2*d*n^2*x^(11/3))/(968*e) - (3*a^2*b*d^2*n*x^(10/3))/(40*e^2) - (3*a^2*b*d^4*n*x^(8/3))/(32*e^4) - (44441*a*b^2*d^9*n^2*x)/(55440*e^9) + (3*a^2*b*d^5*n*x^(7/3))/(28*e^5) + (3*a^2*b*d^7*n*x^(5/3))/(20*e^7) - (3*a^2*b*d^8*n*x^(4/3))/(16*e^8) - (3*a^2*b*d^10*n*x^(2/3))/(8*e^10) + (3*a^2*b*d^11*n*x^(1/3))/(4*e^11) + (86021*a*b^2*d^12*n^2*log(d + e*x^(1/3)))/(18480*e^12) + (b^3*d^3*n*x^3*log(c*(d + e*x^(1/3))^n)^2)/(12*e^3) - (763*b^3*d^3*n^2*x^3*log(c*(d + e*x^(1/3))^n))/(11880*e^3) - (b^3*d^6*n*x^2*log(c*(d + e*x^(1/3))^n)^2)/(8*e^6) + (22727*b^3*d^6*n^2*x^2*log(c*(d + e*x^(1/3))^n))/(110880*e^6) - (3*b^3*d^2*n*x^(10/3)*log(c*(d + e*x^(1/3))^n)^2)/(40*e^2) + (181*b^3*d^2*n^2*x^(10/3)*log(c*(d + e*x^(1/3))^n))/(4400*e^2) - (3*b^3*d^4*n*x^(8/3)*log(c*(d + e*x^(1/3))^n)^2)/(32*e^4) + (2021*b^3*d^4*n^2*x^(8/3)*log(c*(d + e*x^(1/3))^n))/(21120*e^4) + (3*b^3*d^5*n*x^(7/3)*log(c*(d + e*x^(1/3))^n)^2)/(28*e^5) - (18107*b^3*d^5*n^2*x^(7/3)*log(c*(d + e*x^(1/3))^n))/(129360*e^5) + (3*b^3*d^7*n*x^(5/3)*log(c*(d + e*x^(1/3))^n)^2)/(20*e^7) - (28271*b^3*d^7*n^2*x^(5/3)*log(c*(d + e*x^(1/3))^n))/(92400*e^7) - (3*b^3*d^8*n*x^(4/3)*log(c*(d + e*x^(1/3))^n)^2)/(16*e^8) + (35201*b^3*d^8*n^2*x^(4/3)*log(c*(d + e*x^(1/3))^n))/(73920*e^8) - (3*b^3*d^10*n*x^(2/3)*log(c*(d + e*x^(1/3))^n)^2)/(8*e^10) + (58301*b^3*d^10*n^2*x^(2/3)*log(c*(d + e*x^(1/3))^n))/(36960*e^10) + (3*b^3*d^11*n*x^(1/3)*log(c*(d + e*x^(1/3))^n)^2)/(4*e^11) - (86021*b^3*d^11*n^2*x^(1/3)*log(c*(d + e*x^(1/3))^n))/(18480*e^11) - (763*a*b^2*d^3*n^2*x^3)/(11880*e^3) + (22727*a*b^2*d^6*n^2*x^2)/(110880*e^6) + (181*a*b^2*d^2*n^2*x^(10/3))/(4400*e^2) + (2021*a*b^2*d^4*n^2*x^(8/3))/(21120*e^4) - (18107*a*b^2*d^5*n^2*x^(7/3))/(129360*e^5) - (28271*a*b^2*d^7*n^2*x^(5/3))/(92400*e^7) + (35201*a*b^2*d^8*n^2*x^(4/3))/(73920*e^8) + (58301*a*b^2*d^10*n^2*x^(2/3))/(36960*e^10) - (86021*a*b^2*d^11*n^2*x^(1/3))/(18480*e^11) + (3*a^2*b*d*n*x^(11/3))/(44*e) + (a^2*b*d^9*n*x)/(4*e^9) + (3*a*b^2*d*n*x^(11/3)*log(c*(d + e*x^(1/3))^n))/(22*e) + (a*b^2*d^9*n*x*log(c*(d + e*x^(1/3))^n))/(2*e^9) + (a*b^2*d^3*n*x^3*log(c*(d + e*x^(1/3))^n))/(6*e^3) - (a*b^2*d^6*n*x^2*log(c*(d + e*x^(1/3))^n))/(4*e^6) - (3*a*b^2*d^2*n*

```
x^(10/3)*log(c*(d + e*x^(1/3))^n))/(20*e^2) - (3*a*b^2*d^4*n*x^(8/3)*log(c*
(d + e*x^(1/3))^n))/(16*e^4) + (3*a*b^2*d^5*n*x^(7/3)*log(c*(d + e*x^(1/3))
^n))/(14*e^5) + (3*a*b^2*d^7*n*x^(5/3)*log(c*(d + e*x^(1/3))^n))/(10*e^7) -
 (3*a*b^2*d^8*n*x^(4/3)*log(c*(d + e*x^(1/3))^n))/(8*e^8) - (3*a*b^2*d^10*n
*x^(2/3)*log(c*(d + e*x^(1/3))^n))/(4*e^10) + (3*a*b^2*d^11*n*x^(1/3)*log(c
*(d + e*x^(1/3))^n))/(2*e^11)
```

3.457 $\int x^2 \left(a+b \log \left(c \left(d+e \sqrt[3]{x}\right)^n\right)\right)^3 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 1357

$$
\begin{aligned}
\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx &= \frac{9b^3d^7n^3\left(d+e\sqrt[3]{x}\right)^2}{e^9}-\frac{56b^3d^6n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^9} \\
&+\frac{63b^3d^5n^3\left(d+e\sqrt[3]{x}\right)^4}{16e^9}-\frac{252b^3d^4n^3\left(d+e\sqrt[3]{x}\right)^5}{125e^9} \\
&+\frac{7b^3d^3n^3\left(d+e\sqrt[3]{x}\right)^6}{9e^9}-\frac{72b^3d^2n^3\left(d+e\sqrt[3]{x}\right)^7}{343e^9} \\
&+\frac{9b^3dn^3\left(d+e\sqrt[3]{x}\right)^8}{256e^9}-\frac{2b^3n^3\left(d+e\sqrt[3]{x}\right)^9}{729e^9} \\
&+\frac{18ab^2d^8n^2\sqrt[3]{x}}{e^8}-\frac{18b^3d^8n^3\sqrt[3]{x}}{e^8} \\
&+\frac{18b^3d^8n^2\left(d+e\sqrt[3]{x}\right)\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{e^9} \\
&-\frac{18b^2d^7n^2\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{e^9} \\
&+\frac{56b^2d^6n^2\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9} \\
&-\frac{63b^2d^5n^2\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4e^9} \\
&+\frac{252b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{25e^9} \\
&-\frac{14b^2d^3n^2\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^9} \\
&+\frac{72b^2d^2n^2\left(d+e\sqrt[3]{x}\right)^7\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{49e^9} \\
&-\frac{9b^2dn^2\left(d+e\sqrt[3]{x}\right)^8\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{32e^9} \\
&+\frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{81e^9} \\
&-\frac{9bd^8n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^9} \\
&+\frac{18bd^7n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^9} \\
&-\frac{28bd^6n\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^9} \\
&+\frac{63bd^5n\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2e^9} \\
&-\frac{126bd^4n\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{5e^9} \\
&+\frac{14bd^3n\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^9} \\
&-\frac{36bd^2n\left(d+e\sqrt[3]{x}\right)^7\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{7e^9}
\end{aligned}
$$

```
[Out] -2/729*b^3*n^3*(d+e*x^(1/3))^9/e^9+3*d^8*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/
3))^n))^3/e^9-12*d^7*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^9+28*d
^6*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^9-42*d^5*(d+e*x^(1/3))^4
*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^9+42*d^4*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^
(1/3))^n))^3/e^9-28*d^3*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^9+1
2*d^2*(d+e*x^(1/3))^7*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^9-3*d*(d+e*x^(1/3))^8
*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^9+1/3*(d+e*x^(1/3))^9*(a+b*ln(c*(d+e*x^(1/
3))^n))^3/e^9+18*a*b^2*d^8*n^2*x^(1/3)/e^8-14/3*b^2*d^3*n^2*(d+e*x^(1/3))^6
*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9+72/49*b^2*d^2*n^2*(d+e*x^(1/3))^7*(a+b*ln(
c*(d+e*x^(1/3))^n))/e^9-9/32*b^2*d*n^2*(d+e*x^(1/3))^8*(a+b*ln(c*(d+e*x^(1/
3))^n))/e^9-9*b*d^8*n*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^9+18*b*
d^7*n*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^9-28*b*d^6*n*(d+e*x^(
1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^9+63/2*b*d^5*n*(d+e*x^(1/3))^4*(a+b
*ln(c*(d+e*x^(1/3))^n))^2/e^9-126/5*b*d^4*n*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*
x^(1/3))^n))^2/e^9+14*b*d^3*n*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))^2
/e^9-36/7*b*d^2*n*(d+e*x^(1/3))^7*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^9+9/8*b*d
*n*(d+e*x^(1/3))^8*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^9+18*b^3*d^8*n^2*(d+e*x^
(1/3))*ln(c*(d+e*x^(1/3))^n)/e^9-18*b^2*d^7*n^2*(d+e*x^(1/3))^2*(a+b*ln(c*(
d+e*x^(1/3))^n))/e^9+56/3*b^2*d^6*n^2*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3
))^n))/e^9-63/4*b^2*d^5*n^2*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9
+252/25*b^2*d^4*n^2*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9-252/125
*b^3*d^4*n^3*(d+e*x^(1/3))^5/e^9+7/9*b^3*d^3*n^3*(d+e*x^(1/3))^6/e^9-72/343
*b^3*d^2*n^3*(d+e*x^(1/3))^7/e^9+9/256*b^3*d*n^3*(d+e*x^(1/3))^8/e^9-18*b^3
*d^8*n^3*x^(1/3)/e^8+9*b^3*d^7*n^3*(d+e*x^(1/3))^2/e^9-56/9*b^3*d^6*n^3*(d+
e*x^(1/3))^3/e^9+63/16*b^3*d^5*n^3*(d+e*x^(1/3))^4/e^9+2/81*b^2*n^2*(d+e*x^
(1/3))^9*(a+b*ln(c*(d+e*x^(1/3))^n))/e^9-1/9*b*n*(d+e*x^(1/3))^9*(a+b*ln(c*
(d+e*x^(1/3))^n))^2/e^9
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 1357, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341\}$

$$
\begin{aligned}
\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx = & -\frac{2b^3n^3\left(d+e\sqrt[3]{x}\right)^9}{729e^9} \\
& + \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^9}{3e^9} \\
& - \frac{bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(d+e\sqrt[3]{x}\right)^9}{9e^9} \\
& + \frac{2b^2n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\left(d+e\sqrt[3]{x}\right)^9}{81e^9} \\
& + \frac{9b^3dn^3\left(d+e\sqrt[3]{x}\right)^8}{256e^9} \\
& - \frac{3d\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^8}{e^9} \\
& + \frac{9bdn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(d+e\sqrt[3]{x}\right)^8}{8e^9} \\
& - \frac{9b^2dn^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\left(d+e\sqrt[3]{x}\right)^8}{32e^9} \\
& - \frac{72b^3d^2n^3\left(d+e\sqrt[3]{x}\right)^7}{343e^9} \\
& + \frac{12d^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^7}{e^9} \\
& - \frac{36bd^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(d+e\sqrt[3]{x}\right)^7}{7e^9} \\
& + \frac{72b^2d^2n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\left(d+e\sqrt[3]{x}\right)^7}{49e^9} \\
& + \frac{7b^3d^3n^3\left(d+e\sqrt[3]{x}\right)^6}{9e^9} \\
& - \frac{28d^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^6}{e^9} \\
& + \frac{14bd^3n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(d+e\sqrt[3]{x}\right)^6}{e^9} \\
& - \frac{14b^2d^3n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\left(d+e\sqrt[3]{x}\right)^6}{3e^9} \\
& - \frac{252b^3d^4n^3\left(d+e\sqrt[3]{x}\right)^5}{125e^9} \\
& + \frac{42d^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^5}{e^9} \\
& - \frac{126bd^4n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(d+e\sqrt[3]{x}\right)^5}{5e^9} \\
& + \frac{252b^2d^4n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\left(d+e\sqrt[3]{x}\right)^5}{25e^9} \\
& + \frac{63b^3d^5n^3\left(d+e\sqrt[3]{x}\right)^4}{16e^9} \\
& \quad 42d^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^4
\end{aligned}
$$

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (9*b^3*d^7*n^3*(d + e*x^(1/3))^2)/e^9 - (56*b^3*d^6*n^3*(d + e*x^(1/3))^3)/
(9*e^9) + (63*b^3*d^5*n^3*(d + e*x^(1/3))^4)/(16*e^9) - (252*b^3*d^4*n^3*(d
 + e*x^(1/3))^5)/(125*e^9) + (7*b^3*d^3*n^3*(d + e*x^(1/3))^6)/(9*e^9) - (7
2*b^3*d^2*n^3*(d + e*x^(1/3))^7)/(343*e^9) + (9*b^3*d*n^3*(d + e*x^(1/3))^8
)/(256*e^9) - (2*b^3*n^3*(d + e*x^(1/3))^9)/(729*e^9) + (18*a*b^2*d^8*n^2*x
^(1/3))/e^8 - (18*b^3*d^8*n^3*x^(1/3))/e^8 + (18*b^3*d^8*n^2*(d + e*x^(1/3)
)*Log[c*(d + e*x^(1/3))^n])/e^9 - (18*b^2*d^7*n^2*(d + e*x^(1/3))^2*(a + b*
Log[c*(d + e*x^(1/3))^n]))/e^9 + (56*b^2*d^6*n^2*(d + e*x^(1/3))^3*(a + b*L
og[c*(d + e*x^(1/3))^n]))/(3*e^9) - (63*b^2*d^5*n^2*(d + e*x^(1/3))^4*(a +
b*Log[c*(d + e*x^(1/3))^n]))/(4*e^9) + (252*b^2*d^4*n^2*(d + e*x^(1/3))^5*(
a + b*Log[c*(d + e*x^(1/3))^n]))/(25*e^9) - (14*b^2*d^3*n^2*(d + e*x^(1/3))
^6*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^9) + (72*b^2*d^2*n^2*(d + e*x^(1/
3))^7*(a + b*Log[c*(d + e*x^(1/3))^n]))/(49*e^9) - (9*b^2*d*n^2*(d + e*x^(1
/3))^8*(a + b*Log[c*(d + e*x^(1/3))^n]))/(32*e^9) + (2*b^2*n^2*(d + e*x^(1/
3))^9*(a + b*Log[c*(d + e*x^(1/3))^n]))/(81*e^9) - (9*b*d^8*n*(d + e*x^(1/3
))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^9 + (18*b*d^7*n*(d + e*x^(1/3))^2*
(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^9 - (28*b*d^6*n*(d + e*x^(1/3))^3*(a
+ b*Log[c*(d + e*x^(1/3))^n])^2)/e^9 + (63*b*d^5*n*(d + e*x^(1/3))^4*(a + b
*Log[c*(d + e*x^(1/3))^n])^2)/(2*e^9) - (126*b*d^4*n*(d + e*x^(1/3))^5*(a +
 b*Log[c*(d + e*x^(1/3))^n])^2)/(5*e^9) + (14*b*d^3*n*(d + e*x^(1/3))^6*(a
+ b*Log[c*(d + e*x^(1/3))^n])^2)/e^9 - (36*b*d^2*n*(d + e*x^(1/3))^7*(a + b
*Log[c*(d + e*x^(1/3))^n])^2)/(7*e^9) + (9*b*d*n*(d + e*x^(1/3))^8*(a + b*L
og[c*(d + e*x^(1/3))^n])^2)/(8*e^9) - (b*n*(d + e*x^(1/3))^9*(a + b*Log[c*(
d + e*x^(1/3))^n])^2)/(9*e^9) + (3*d^8*(d + e*x^(1/3))*(a + b*Log[c*(d + e*
x^(1/3))^n])^3)/e^9 - (12*d^7*(d + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3)
)^n])^3)/e^9 + (28*d^6*(d + e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3
)/e^9 - (42*d^5*(d + e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 +
 (42*d^4*(d + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 - (28*d^
3*(d + e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 + (12*d^2*(d +
e*x^(1/3))^7*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 - (3*d*(d + e*x^(1/3))
^8*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/e^9 + ((d + e*x^(1/3))^9*(a + b*Log[
c*(d + e*x^(1/3))^n])^3)/(3*e^9)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int x^8(a+b\log\left(c(d+ex)^n\right))^3\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\text{Subst}\left(\int \left(\frac{d^8(a+b\log(c(d+ex)^n))^3}{e^8} - \frac{8d^7(d+ex)(a+b\log(c(d+ex)^n))^3}{e^8} + \frac{28d^6(d+ex)^2(a+b\log(c(d+ex)^n))^3}{e^8} - \frac{56d^5(d+ex)^3(a+b\log(c(d+ex)^n))^3}{e^8} + \frac{70d^4(d+ex)^4(a+b\log(c(d+ex)^n))^3}{e^8} - \frac{56d^3(d+ex)^5(a+b\log(c(d+ex)^n))^3}{e^8} + \frac{28d^2(d+ex)^6(a+b\log(c(d+ex)^n))^3}{e^8} - \frac{8d(d+ex)^7(a+b\log(c(d+ex)^n))^3}{e^8} + \frac{(d+ex)^8(a+b\log(c(d+ex)^n))^3}{e^8}\right) dx, x, \sqrt[3]{x}\right)$$

$$= \frac{3\text{Subst}\left(\int (d+ex)^8(a+b\log(c(d+ex)^n))^3\, dx, x, \sqrt[3]{x}\right)}{e^8} - \frac{(24d)\text{Subst}\left(\int (d+ex)^7(a+b\log(c(d+ex)^n))^3\, dx, x, \sqrt[3]{x}\right)}{e^8} + \frac{(84d^2)\,\text{Subst}\left(\int (d+ex)^6(a+b\log(c(d+ex)^n))^3\, dx, x, \sqrt[3]{x}\right)}{e^8} - \frac{(168d^3)\,\text{Subst}\left(\int (d+ex)^5(a+b\log(c(d+ex)^n))^3\, dx, x, \sqrt[3]{x}\right)}{e^8} + \frac{(210d^4)\,\text{Subst}\left(\int (d+ex)^4(a+b\log(c(d+ex)^n))^3\, dx, x, \sqrt[3]{x}\right)}{e^8} - \frac{(168d^5)\,\text{Subst}\left(\int (d+ex)^3(a+b\log(c(d+ex)^n))^3\, dx, x, \sqrt[3]{x}\right)}{e^8} + \frac{(84d^6)\,\text{Subst}\left(\int (d+ex)^2(a+b\log(c(d+ex)^n))^3\, dx, x, \sqrt[3]{x}\right)}{e^8} - \frac{(24d^7)\,\text{Subst}\left(\int (d+ex)(a+b\log(c(d+ex)^n))^3\, dx, x, \sqrt[3]{x}\right)}{e^8} + \frac{(3d^8)\,\text{Subst}\left(\int (a+b\log(c(d+ex)^n))^3\, dx, x, \sqrt[3]{x}\right)}{e^8}$$

$$
\begin{aligned}
&= \frac{3\text{Subst}\left(\int x^8(a+b\log(cx^n))^3\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} \\
&\quad - \frac{(24d)\text{Subst}\left(\int x^7(a+b\log(cx^n))^3\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} \\
&\quad + \frac{(84d^2)\,\text{Subst}\left(\int x^6(a+b\log(cx^n))^3\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} \\
&\quad - \frac{(168d^3)\,\text{Subst}\left(\int x^5(a+b\log(cx^n))^3\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} \\
&\quad + \frac{(210d^4)\,\text{Subst}\left(\int x^4(a+b\log(cx^n))^3\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} \\
&\quad - \frac{(168d^5)\,\text{Subst}\left(\int x^3(a+b\log(cx^n))^3\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} \\
&\quad + \frac{(84d^6)\,\text{Subst}\left(\int x^2(a+b\log(cx^n))^3\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} \\
&\quad - \frac{(24d^7)\,\text{Subst}\left(\int x(a+b\log(cx^n))^3\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} \\
&\quad + \frac{(3d^8)\,\text{Subst}\left(\int (a+b\log(cx^n))^3\,dx, x, d+e\sqrt[3]{x}\right)}{e^9}
\end{aligned}
$$

$$= \frac{3d^8\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^9} - \frac{12d^7\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^9} + \frac{28d^6\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^9} - \frac{42d^5\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^9} + \frac{42d^4\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^9} - \frac{28d^3\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^9} + \frac{12d^2\left(d+e\sqrt[3]{x}\right)^7\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^9} - \frac{3d\left(d+e\sqrt[3]{x}\right)^8\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^9} + \frac{\left(d+e\sqrt[3]{x}\right)^9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{3e^9} - \frac{(bn)\text{Subst}\left(\int x^8(a+b\log(cx^n))^2\,dx,x,d+e\sqrt[3]{x}\right)}{e^9} + \frac{(9bdn)\text{Subst}\left(\int x^7(a+b\log(cx^n))^2\,dx,x,d+e\sqrt[3]{x}\right)}{e^9} - \frac{(36bd^2n)\,\text{Subst}\left(\int x^6(a+b\log(cx^n))^2\,dx,x,d+e\sqrt[3]{x}\right)}{e^9} + \frac{(84bd^3n)\,\text{Subst}\left(\int x^5(a+b\log(cx^n))^2\,dx,x,d+e\sqrt[3]{x}\right)}{e^9} - \frac{(126bd^4n)\,\text{Subst}\left(\int x^4(a+b\log(cx^n))^2\,dx,x,d+e\sqrt[3]{x}\right)}{e^9} + \frac{(126bd^5n)\,\text{Subst}\left(\int x^3(a+b\log(cx^n))^2\,dx,x,d+e\sqrt[3]{x}\right)}{e^9} - \frac{(84bd^6n)\,\text{Subst}\left(\int x^2(a+b\log(cx^n))^2\,dx,x,d+e\sqrt[3]{x}\right)}{e^9} + \frac{(36bd^7n)\,\text{Subst}\left(\int x(a+b\log(cx^n))^2\,dx,x,d+e\sqrt[3]{x}\right)}{e^9} - \frac{(9bd^8n)\,\text{Subst}\left(\int (a+b\log(cx^n))^2\,dx,x,d+e\sqrt[3]{x}\right)}{e^9}$$

$$= \text{Too large to display}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 808, normalized size of antiderivative = 0.60

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$$
$$=\frac{b^3en^3\sqrt[3]{x}\left(-76356985320d^8+15542491860d^7e\sqrt[3]{x}-5483495640d^6e^2x^{2/3}+2340330930d^5e^3x-10756070\right.}{}$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (b^3*e*n^3*x^(1/3)*(-76356985320*d^8 + 15542491860*d^7*e*x^(1/3) - 54834956
40*d^6*e^2*x^(2/3) + 2340330930*d^5*e^3*x - 1075607064*d^4*e^4*x^(4/3) + 49
8592500*d^3*e^5*x^(5/3) - 219465000*d^2*e^6*x^2 + 83734875*d*e^7*x^(7/3) -
21952000*e^8*x^(8/3)) - 2520*a*b^2*n^2*(26853209*d^9 - 17965080*d^8*e*x^(1/
3) + 5807340*d^7*e^2*x^(2/3) - 2813160*d^6*e^3*x + 1580670*d^5*e^4*x^(4/3)
- 947016*d^4*e^5*x^(5/3) + 577500*d^3*e^6*x^2 - 343800*d^2*e^7*x^(7/3) + 18
7425*d*e^8*x^(8/3) - 78400*e^9*x^3) + 2667168000*a^3*(d^9 + e^9*x^3) - 3175
200*a^2*b*n*(7129*d^9 + 2520*d^8*e*x^(1/3) - 1260*d^7*e^2*x^(2/3) + 840*d^6
*e^3*x - 630*d^5*e^4*x^(4/3) + 504*d^4*e^5*x^(5/3) - 420*d^3*e^6*x^2 + 360*
d^2*e^7*x^(7/3) - 315*d*e^8*x^(8/3) + 280*e^9*x^3) + 2520*b*(3175200*a^2*(d
^9 + e^9*x^3) - 2520*a*b*n*(7129*d^9 + 2520*d^8*e*x^(1/3) - 1260*d^7*e^2*x^
(2/3) + 840*d^6*e^3*x - 630*d^5*e^4*x^(4/3) + 504*d^4*e^5*x^(5/3) - 420*d^3
*e^6*x^2 + 360*d^2*e^7*x^(7/3) - 315*d*e^8*x^(8/3) + 280*e^9*x^3) + b^2*n^2
*(30300391*d^9 + 17965080*d^8*e*x^(1/3) - 5807340*d^7*e^2*x^(2/3) + 2813160
*d^6*e^3*x - 1580670*d^5*e^4*x^(4/3) + 947016*d^4*e^5*x^(5/3) - 577500*d^3*
e^6*x^2 + 343800*d^2*e^7*x^(7/3) - 187425*d*e^8*x^(8/3) + 78400*e^9*x^3))*L
og[c*(d + e*x^(1/3))^n] + 3175200*b^2*(2520*a*(d^9 + e^9*x^3) - b*n*(7129*d
^9 + 2520*d^8*e*x^(1/3) - 1260*d^7*e^2*x^(2/3) + 840*d^6*e^3*x - 630*d^5*e^
4*x^(4/3) + 504*d^4*e^5*x^(5/3) - 420*d^3*e^6*x^2 + 360*d^2*e^7*x^(7/3) - 3
15*d*e^8*x^(8/3) + 280*e^9*x^3))*Log[c*(d + e*x^(1/3))^n]^2 + 2667168000*b^
3*(d^9 + e^9*x^3)*Log[c*(d + e*x^(1/3))^n]^3)/(8001504000*e^9)
```

Maple [F]

$$\int x^2\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^n\right)\right)^3dx$$

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 1688, normalized size of antiderivative = 1.24

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/8001504000*(2667168000*b^3*e^9*x^3*log(c)^3 - 10976000*(2*b^3*e^9*n^3 - 1
8*a*b^2*e^9*n^2 + 81*a^2*b*e^9*n - 243*a^3*e^9)*x^3 + 2667168000*(b^3*e^9*n
^3*x^3 + b^3*d^9*n^3)*log(e*x^(1/3) + d)^3 + 10500*(47485*b^3*d^3*e^6*n^3 -
 138600*a*b^2*d^3*e^6*n^2 + 127008*a^2*b*d^3*e^6*n)*x^2 + 3175200*(420*b^3*
d^3*e^6*n^3*x^2 - 840*b^3*d^6*e^3*n^3*x - 7129*b^3*d^9*n^3 + 2520*a*b^2*d^9
*n^2 - 280*(b^3*e^9*n^3 - 9*a*b^2*e^9*n^2)*x^3 + 2520*(b^3*e^9*n^2*x^3 + b^
3*d^9*n^2)*log(c) + 63*(5*b^3*d*e^8*n^3*x^2 - 8*b^3*d^4*e^5*n^3*x + 20*b^3*
d^7*e^2*n^3)*x^(2/3) - 90*(4*b^3*d^2*e^7*n^3*x^2 - 7*b^3*d^5*e^4*n^3*x + 28
*b^3*d^8*e*n^3)*x^(1/3))*log(e*x^(1/3) + d)^2 + 444528000*(3*b^3*d^3*e^6*n*
x^2 - 6*b^3*d^6*e^3*n*x - 2*(b^3*e^9*n - 9*a*b^2*e^9)*x^3)*log(c)^2 - 840*(
6527971*b^3*d^6*e^3*n^3 - 8439480*a*b^2*d^6*e^3*n^2 + 3175200*a^2*b*d^6*e^3
*n)*x + 2520*(30300391*b^3*d^9*n^3 - 17965080*a*b^2*d^9*n^2 + 3175200*a^2*b
*d^9*n + 39200*(2*b^3*e^9*n^3 - 18*a*b^2*e^9*n^2 + 81*a^2*b*e^9*n)*x^3 - 21
00*(275*b^3*d^3*e^6*n^3 - 504*a*b^2*d^3*e^6*n^2)*x^2 + 3175200*(b^3*e^9*n*x
^3 + b^3*d^9*n)*log(c)^2 + 840*(3349*b^3*d^6*e^3*n^3 - 2520*a*b^2*d^6*e^3*n
^2)*x + 2520*(420*b^3*d^3*e^6*n^2*x^2 - 840*b^3*d^6*e^3*n^2*x - 7129*b^3*d^
9*n^2 + 2520*a*b^2*d^9*n - 280*(b^3*e^9*n^2 - 9*a*b^2*e^9*n)*x^3)*log(c) -
63*(92180*b^3*d^7*e^2*n^3 - 50400*a*b^2*d^7*e^2*n^2 + 175*(17*b^3*d*e^8*n^3
 - 72*a*b^2*d*e^8*n^2)*x^2 - 8*(1879*b^3*d^4*e^5*n^3 - 2520*a*b^2*d^4*e^5*n
^2)*x - 2520*(5*b^3*d*e^8*n^2*x^2 - 8*b^3*d^4*e^5*n^2*x + 20*b^3*d^7*e^2*n^
2)*log(c))*x^(2/3) + 90*(199612*b^3*d^8*e*n^3 - 70560*a*b^2*d^8*e*n^2 + 20*
(191*b^3*d^2*e^7*n^3 - 504*a*b^2*d^2*e^7*n^2)*x^2 - 7*(2509*b^3*d^5*e^4*n^3
 - 2520*a*b^2*d^5*e^4*n^2)*x - 2520*(4*b^3*d^2*e^7*n^2*x^2 - 7*b^3*d^5*e^4*
n^2*x + 28*b^3*d^8*e*n^2)*log(c))*x^(1/3))*log(e*x^(1/3) + d) + 352800*(280
*(2*b^3*e^9*n^2 - 18*a*b^2*e^9*n + 81*a^2*b*e^9)*x^3 - 15*(275*b^3*d^3*e^6*
n^2 - 504*a*b^2*d^3*e^6*n)*x^2 + 6*(3349*b^3*d^6*e^3*n^2 - 2520*a*b^2*d^6*e
^3*n)*x)*log(c) + 63*(246706220*b^3*d^7*e^2*n^3 - 232293600*a*b^2*d^7*e^2*n
^2 + 63504000*a^2*b*d^7*e^2*n + 6125*(217*b^3*d*e^8*n^3 - 1224*a*b^2*d*e^8*
n^2 + 2592*a^2*b*d*e^8*n)*x^2 + 3175200*(5*b^3*d*e^8*n*x^2 - 8*b^3*d^4*e^5*
n*x + 20*b^3*d^7*e^2*n)*log(c)^2 - 8*(2134141*b^3*d^4*e^5*n^3 - 4735080*a*b
^2*d^4*e^5*n^2 + 3175200*a^2*b*d^4*e^5*n)*x - 2520*(92180*b^3*d^7*e^2*n^2 -
 50400*a*b^2*d^7*e^2*n + 175*(17*b^3*d*e^8*n^2 - 72*a*b^2*d*e^8*n)*x^2 - 8*
(1879*b^3*d^4*e^5*n^2 - 2520*a*b^2*d^4*e^5*n)*x)*log(c))*x^(2/3) - 90*(8484
10948*b^3*d^8*e*n^3 - 503022240*a*b^2*d^8*e*n^2 + 88905600*a^2*b*d^8*e*n +
100*(24385*b^3*d^2*e^7*n^3 - 96264*a*b^2*d^2*e^7*n^2 + 127008*a^2*b*d^2*e^7
```

```
*n)*x^2 + 3175200*(4*b^3*d^2*e^7*n*x^2 - 7*b^3*d^5*e^4*n*x + 28*b^3*d^8*e*n
)*log(c)^2 - 7*(3714811*b^3*d^5*e^4*n^3 - 6322680*a*b^2*d^5*e^4*n^2 + 31752
00*a^2*b*d^5*e^4*n)*x - 2520*(199612*b^3*d^8*e*n^2 - 70560*a*b^2*d^8*e*n +
20*(191*b^3*d^2*e^7*n^2 - 504*a*b^2*d^2*e^7*n)*x^2 - 7*(2509*b^3*d^5*e^4*n^
2 - 2520*a*b^2*d^5*e^4*n)*x)*log(c))*x^(1/3))/e^9
```

Sympy [F]

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$$

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))**n))**3,x)
```

```
[Out] Integral(x**2*(a + b*log(c*(d + e*x**(1/3))**n))**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 867, normalized size of antiderivative = 0.64

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3*log((e*x^(1/3) + d)^n*c)^3 + a*b^2*x^3*log((e*x^(1/3) + d)^n*c)
^2 + a^2*b*x^3*log((e*x^(1/3) + d)^n*c) + 1/3*a^3*x^3 + 1/2520*a^2*b*e*n*(2
520*d^9*log(e*x^(1/3) + d)/e^10 - (280*e^8*x^3 - 315*d*e^7*x^(8/3) + 360*d^
2*e^6*x^(7/3) - 420*d^3*e^5*x^2 + 504*d^4*e^4*x^(5/3) - 630*d^5*e^3*x^(4/3)
 + 840*d^6*e^2*x - 1260*d^7*e*x^(2/3) + 2520*d^8*x^(1/3))/e^9) + 1/3175200*
(2520*e*n*(2520*d^9*log(e*x^(1/3) + d)/e^10 - (280*e^8*x^3 - 315*d*e^7*x^(8
/3) + 360*d^2*e^6*x^(7/3) - 420*d^3*e^5*x^2 + 504*d^4*e^4*x^(5/3) - 630*d^5
*e^3*x^(4/3) + 840*d^6*e^2*x - 1260*d^7*e*x^(2/3) + 2520*d^8*x^(1/3))/e^9)*
log((e*x^(1/3) + d)^n*c) + (78400*e^9*x^3 - 187425*d*e^8*x^(8/3) + 343800*d
^2*e^7*x^(7/3) - 577500*d^3*e^6*x^2 - 3175200*d^9*log(e*x^(1/3) + d)^2 + 94
7016*d^4*e^5*x^(5/3) - 1580670*d^5*e^4*x^(4/3) + 2813160*d^6*e^3*x - 179650
80*d^9*log(e*x^(1/3) + d) - 5807340*d^7*e^2*x^(2/3) + 17965080*d^8*e*x^(1/3
))*n^2/e^9)*a*b^2 + 1/8001504000*(3175200*e*n*(2520*d^9*log(e*x^(1/3) + d)/
e^10 - (280*e^8*x^3 - 315*d*e^7*x^(8/3) + 360*d^2*e^6*x^(7/3) - 420*d^3*e^5
*x^2 + 504*d^4*e^4*x^(5/3) - 630*d^5*e^3*x^(4/3) + 840*d^6*e^2*x - 1260*d^7
*e*x^(2/3) + 2520*d^8*x^(1/3))/e^9)*log((e*x^(1/3) + d)^n*c)^2 - e*n*((2195
2000*e^9*x^3 - 2667168000*d^9*log(e*x^(1/3) + d)^3 - 83734875*d*e^8*x^(8/3)
 + 219465000*d^2*e^7*x^(7/3) - 498592500*d^3*e^6*x^2 - 22636000800*d^9*log(
```

```
e*x^(1/3) + d)^2 + 1075607064*d^4*e^5*x^(5/3) - 2340330930*d^5*e^4*x^(4/3)
+ 5483495640*d^6*e^3*x - 76356985320*d^9*log(e*x^(1/3) + d) - 15542491860*d
^7*e^2*x^(2/3) + 76356985320*d^8*e*x^(1/3))*n^2/e^10 - 2520*(78400*e^9*x^3
- 187425*d*e^8*x^(8/3) + 343800*d^2*e^7*x^(7/3) - 577500*d^3*e^6*x^2 - 3175
200*d^9*log(e*x^(1/3) + d)^2 + 947016*d^4*e^5*x^(5/3) - 1580670*d^5*e^4*x^(
4/3) + 2813160*d^6*e^3*x - 17965080*d^9*log(e*x^(1/3) + d) - 5807340*d^7*e^
2*x^(2/3) + 17965080*d^8*e*x^(1/3))*n*log((e*x^(1/3) + d)^n*c)/e^10))*b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3240 vs. $2(1189) = 2378$.

Time = 0.35 (sec) , antiderivative size = 3240, normalized size of antiderivative = 2.39

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="giac")
```

```
[Out] 1/8001504000*(2667168000*b^3*e*x^3*log(c)^3 + 8001504000*a*b^2*e*x^3*log(c)
^2 + 8001504000*a^2*b*e*x^3*log(c) + (2667168000*(e*x^(1/3) + d)^9*log(e*x^
(1/3) + d)^3/e^8 - 24004512000*(e*x^(1/3) + d)^8*d*log(e*x^(1/3) + d)^3/e^8
 + 96018048000*(e*x^(1/3) + d)^7*d^2*log(e*x^(1/3) + d)^3/e^8 - 22404211200
0*(e*x^(1/3) + d)^6*d^3*log(e*x^(1/3) + d)^3/e^8 + 336063168000*(e*x^(1/3)
+ d)^5*d^4*log(e*x^(1/3) + d)^3/e^8 - 336063168000*(e*x^(1/3) + d)^4*d^5*lo
g(e*x^(1/3) + d)^3/e^8 + 224042112000*(e*x^(1/3) + d)^3*d^6*log(e*x^(1/3) +
 d)^3/e^8 - 96018048000*(e*x^(1/3) + d)^2*d^7*log(e*x^(1/3) + d)^3/e^8 + 24
004512000*(e*x^(1/3) + d)*d^8*log(e*x^(1/3) + d)^3/e^8 - 889056000*(e*x^(1/
3) + d)^9*log(e*x^(1/3) + d)^2/e^8 + 9001692000*(e*x^(1/3) + d)^8*d*log(e*x
^(1/3) + d)^2/e^8 - 41150592000*(e*x^(1/3) + d)^7*d^2*log(e*x^(1/3) + d)^2/
e^8 + 112021056000*(e*x^(1/3) + d)^6*d^3*log(e*x^(1/3) + d)^2/e^8 - 2016379
00800*(e*x^(1/3) + d)^5*d^4*log(e*x^(1/3) + d)^2/e^8 + 252047376000*(e*x^(1
/3) + d)^4*d^5*log(e*x^(1/3) + d)^2/e^8 - 224042112000*(e*x^(1/3) + d)^3*d^
6*log(e*x^(1/3) + d)^2/e^8 + 144027072000*(e*x^(1/3) + d)^2*d^7*log(e*x^(1/
3) + d)^2/e^8 - 72013536000*(e*x^(1/3) + d)*d^8*log(e*x^(1/3) + d)^2/e^8 +
197568000*(e*x^(1/3) + d)^9*log(e*x^(1/3) + d)/e^8 - 2250423000*(e*x^(1/3)
+ d)^8*d*log(e*x^(1/3) + d)/e^8 + 11757312000*(e*x^(1/3) + d)^7*d^2*log(e*x
^(1/3) + d)/e^8 - 37340352000*(e*x^(1/3) + d)^6*d^3*log(e*x^(1/3) + d)/e^8
+ 80655160320*(e*x^(1/3) + d)^5*d^4*log(e*x^(1/3) + d)/e^8 - 126023688000*(
e*x^(1/3) + d)^4*d^5*log(e*x^(1/3) + d)/e^8 + 149361408000*(e*x^(1/3) + d)^
3*d^6*log(e*x^(1/3) + d)/e^8 - 144027072000*(e*x^(1/3) + d)^2*d^7*log(e*x^(
1/3) + d)/e^8 + 144027072000*(e*x^(1/3) + d)*d^8*log(e*x^(1/3) + d)/e^8 - 2
1952000*(e*x^(1/3) + d)^9/e^8 + 281302875*(e*x^(1/3) + d)^8*d/e^8 - 1679616
000*(e*x^(1/3) + d)^7*d^2/e^8 + 6223392000*(e*x^(1/3) + d)^6*d^3/e^8 - 1613
1032064*(e*x^(1/3) + d)^5*d^4/e^8 + 31505922000*(e*x^(1/3) + d)^4*d^5/e^8 -
 49787136000*(e*x^(1/3) + d)^3*d^6/e^8 + 72013536000*(e*x^(1/3) + d)^2*d^7/
```

e^8 - 144027072000*(e*x^(1/3) + d)*d^8/e^8)*b^3*n^3 + 2667168000*a^3*e*x^3 + 2520*(3175200*(e*x^(1/3) + d)^9*log(e*x^(1/3) + d)^2/e^8 - 28576800*(e*x^(1/3) + d)^8*d*log(e*x^(1/3) + d)^2/e^8 + 114307200*(e*x^(1/3) + d)^7*d^2*log(e*x^(1/3) + d)^2/e^8 - 266716800*(e*x^(1/3) + d)^6*d^3*log(e*x^(1/3) + d)^2/e^8 + 400075200*(e*x^(1/3) + d)^5*d^4*log(e*x^(1/3) + d)^2/e^8 - 400075200*(e*x^(1/3) + d)^4*d^5*log(e*x^(1/3) + d)^2/e^8 + 266716800*(e*x^(1/3) + d)^3*d^6*log(e*x^(1/3) + d)^2/e^8 - 114307200*(e*x^(1/3) + d)^2*d^7*log(e*x^(1/3) + d)^2/e^8 + 28576800*(e*x^(1/3) + d)*d^8*log(e*x^(1/3) + d)^2/e^8 - 705600*(e*x^(1/3) + d)^9*log(e*x^(1/3) + d)/e^8 + 7144200*(e*x^(1/3) + d)^8*d*log(e*x^(1/3) + d)/e^8 - 32659200*(e*x^(1/3) + d)^7*d^2*log(e*x^(1/3) + d)/e^8 + 88905600*(e*x^(1/3) + d)^6*d^3*log(e*x^(1/3) + d)/e^8 - 160030080*(e*x^(1/3) + d)^5*d^4*log(e*x^(1/3) + d)/e^8 + 200037600*(e*x^(1/3) + d)^4*d^5*log(e*x^(1/3) + d)/e^8 - 177811200*(e*x^(1/3) + d)^3*d^6*log(e*x^(1/3) + d)/e^8 + 114307200*(e*x^(1/3) + d)^2*d^7*log(e*x^(1/3) + d)/e^8 - 57153600*(e*x^(1/3) + d)*d^8*log(e*x^(1/3) + d)/e^8 + 78400*(e*x^(1/3) + d)^9/e^8 - 893025*(e*x^(1/3) + d)^8*d/e^8 + 4665600*(e*x^(1/3) + d)^7*d^2/e^8 - 14817600*(e*x^(1/3) + d)^6*d^3/e^8 + 32006016*(e*x^(1/3) + d)^5*d^4/e^8 - 50009400*(e*x^(1/3) + d)^4*d^5/e^8 + 59270400*(e*x^(1/3) + d)^3*d^6/e^8 - 57153600*(e*x^(1/3) + d)^2*d^7/e^8 + 57153600*(e*x^(1/3) + d)*d^8/e^8)*b^3*n^2*log(c) + 3175200*(2520*(e*x^(1/3) + d)^9*log(e*x^(1/3) + d)/e^8 - 22680*(e*x^(1/3) + d)^8*d*log(e*x^(1/3) + d)/e^8 + 90720*(e*x^(1/3) + d)^7*d^2*log(e*x^(1/3) + d)/e^8 - 211680*(e*x^(1/3) + d)^6*d^3*log(e*x^(1/3) + d)/e^8 + 317520*(e*x^(1/3) + d)^5*d^4*log(e*x^(1/3) + d)/e^8 - 317520*(e*x^(1/3) + d)^4*d^5*log(e*x^(1/3) + d)/e^8 + 211680*(e*x^(1/3) + d)^3*d^6*log(e*x^(1/3) + d)/e^8 - 90720*(e*x^(1/3) + d)^2*d^7*log(e*x^(1/3) + d)/e^8 + 22680*(e*x^(1/3) + d)*d^8*log(e*x^(1/3) + d)/e^8 - 280*(e*x^(1/3) + d)^9/e^8 + 2835*(e*x^(1/3) + d)^8*d/e^8 - 12960*(e*x^(1/3) + d)^7*d^2/e^8 + 35280*(e*x^(1/3) + d)^6*d^3/e^8 - 63504*(e*x^(1/3) + d)^5*d^4/e^8 + 79380*(e*x^(1/3) + d)^4*d^5/e^8 - 70560*(e*x^(1/3) + d)^3*d^6/e^8 + 45360*(e*x^(1/3) + d)^2*d^7/e^8 - 22680*(e*x^(1/3) + d)*d^8/e^8)*b^3*n*log(c)^2 + 2520*(3175200*(e*x^(1/3) + d)^9*log(e*x^(1/3) + d)^2/e^8 - 28576800*(e*x^(1/3) + d)^8*d*log(e*x^(1/3) + d)^2/e^8 + 114307200*(e*x^(1/3) + d)^7*d^2*log(e*x^(1/3) + d)^2/e^8 - 266716800*(e*x^(1/3) + d)^6*d^3*log(e*x^(1/3) + d)^2/e^8 + 400075200*(e*x^(1/3) + d)^5*d^4*log(e*x^(1/3) + d)^2/e^8 - 400075200*(e*x^(1/3) + d)^4*d^5*log(e*x^(1/3) + d)^2/e^8 + 266716800*(e*x^(1/3) + d)^3*d^6*log(e*x^(1/3) + d)^2/e^8 - 114307200*(e*x^(1/3) + d)^2*d^7*log(e*x^(1/3) + d)^2/e^8 + 28576800*(e*x^(1/3) + d)*d^8*log(e*x^(1/3) + d)^2/e^8 - 705600*(e*x^(1/3) + d)^9*log(e*x^(1/3) + d)/e^8 + 7144200*(e*x^(1/3) + d)^8*d*log(e*x^(1/3) + d)/e^8 - 32659200*(e*x^(1/3) + d)^7*d^2*log(e*x^(1/3) + d)/e^8 + 88905600*(e*x^(1/3) + d)^6*d^3*log(e*x^(1/3) + d)/e^8 - 160030080*(e*x^(1/3) + d)^5*d^4*log(e*x^(1/3) + d)/e^8 + 200037600*(e*x^(1/3) + d)^4*d^5*log(e*x^(1/3) + d)/e^8 - 177811200*(e*x^(1/3) + d)^3*d^6*log(e*x^(1/3) + d)/e^8 + 114307200*(e*x^(1/3) + d)^2*d^7*log(e*x^(1/3) + d)/e^8 - 57153600*(e*x^(1/3) + d)*d^8*log(e*x^(1/3) + d)/e^8 + 78400*(e*x^(1/3) + d)^9/e^8 - 893025*(e*x^(1/3) + d)^8*d/e^8 + 4665600*(e*x^(1/3) + d)^7*d^2/e^8 - 14817600*(e*x^(1/3) + d)^6*d

```
^3/e^8 + 32006016*(e*x^(1/3) + d)^5*d^4/e^8 - 50009400*(e*x^(1/3) + d)^4*d^
5/e^8 + 59270400*(e*x^(1/3) + d)^3*d^6/e^8 - 57153600*(e*x^(1/3) + d)^2*d^7
/e^8 + 57153600*(e*x^(1/3) + d)*d^8/e^8)*a*b^2*n^2 + 6350400*(2520*(e*x^(1/
3) + d)^9*log(e*x^(1/3) + d)/e^8 - 22680*(e*x^(1/3) + d)^8*d*log(e*x^(1/3)
+ d)/e^8 + 90720*(e*x^(1/3) + d)^7*d^2*log(e*x^(1/3) + d)/e^8 - 211680*(e*x
^(1/3) + d)^6*d^3*log(e*x^(1/3) + d)/e^8 + 317520*(e*x^(1/3) + d)^5*d^4*log
(e*x^(1/3) + d)/e^8 - 317520*(e*x^(1/3) + d)^4*d^5*log(e*x^(1/3) + d)/e^8 +
 211680*(e*x^(1/3) + d)^3*d^6*log(e*x^(1/3) + d)/e^8 - 90720*(e*x^(1/3) + d
)^2*d^7*log(e*x^(1/3) + d)/e^8 + 22680*(e*x^(1/3) + d)*d^8*log(e*x^(1/3) +
d)/e^8 - 280*(e*x^(1/3) + d)^9/e^8 + 2835*(e*x^(1/3) + d)^8*d/e^8 - 12960*(
e*x^(1/3) + d)^7*d^2/e^8 + 35280*(e*x^(1/3) + d)^6*d^3/e^8 - 63504*(e*x^(1/
3) + d)^5*d^4/e^8 + 79380*(e*x^(1/3) + d)^4*d^5/e^8 - 70560*(e*x^(1/3) + d)
^3*d^6/e^8 + 45360*(e*x^(1/3) + d)^2*d^7/e^8 - 22680*(e*x^(1/3) + d)*d^8/e^
8)*a*b^2*n*log(c) + 3175200*(2520*(e*x^(1/3) + d)^9*log(e*x^(1/3) + d)/e^8
- 22680*(e*x^(1/3) + d)^8*d*log(e*x^(1/3) + d)/e^8 + 90720*(e*x^(1/3) + d)^
7*d^2*log(e*x^(1/3) + d)/e^8 - 211680*(e*x^(1/3) + d)^6*d^3*log(e*x^(1/3) +
 d)/e^8 + 317520*(e*x^(1/3) + d)^5*d^4*log(e*x^(1/3) + d)/e^8 - 317520*(e*x
^(1/3) + d)^4*d^5*log(e*x^(1/3) + d)/e^8 + 211680*(e*x^(1/3) + d)^3*d^6*log
(e*x^(1/3) + d)/e^8 - 90720*(e*x^(1/3) + d)^2*d^7*log(e*x^(1/3) + d)/e^8 +
22680*(e*x^(1/3) + d)*d^8*log(e*x^(1/3) + d)/e^8 - 280*(e*x^(1/3) + d)^9/e^
8 + 2835*(e*x^(1/3) + d)^8*d/e^8 - 12960*(e*x^(1/3) + d)^7*d^2/e^8 + 35280*
(e*x^(1/3) + d)^6*d^3/e^8 - 63504*(e*x^(1/3) + d)^5*d^4/e^8 + 79380*(e*x^(1
/3) + d)^4*d^5/e^8 - 70560*(e*x^(1/3) + d)^3*d^6/e^8 + 45360*(e*x^(1/3) + d
)^2*d^7/e^8 - 22680*(e*x^(1/3) + d)*d^8/e^8)*a^2*b*n)/e
```

Mupad [B] (verification not implemented)

Time = 9.64 (sec) , antiderivative size = 1386, normalized size of antiderivative = 1.02

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/3))^n))^3,x)
```

```
[Out] (a^3*x^3)/3 + (b^3*x^3*log(c*(d + e*x^(1/3))^n)^3)/3 - (2*b^3*n^3*x^3)/729
+ a*b^2*x^3*log(c*(d + e*x^(1/3))^n)^2 - (b^3*n*x^3*log(c*(d + e*x^(1/3))^n
)^2)/9 + (2*b^3*n^2*x^3*log(c*(d + e*x^(1/3))^n))/81 + (2*a*b^2*n^2*x^3)/81
 + (b^3*d^9*log(c*(d + e*x^(1/3))^n)^3)/(3*e^9) + a^2*b*x^3*log(c*(d + e*x^
(1/3))^n) - (a^2*b*n*x^3)/9 - (2*a*b^2*n*x^3*log(c*(d + e*x^(1/3))^n))/9 +
(30300391*b^3*d^9*n^3*log(d + e*x^(1/3)))/(3175200*e^9) + (47485*b^3*d^3*n^
3*x^2)/(762048*e^3) - (24385*b^3*d^2*n^3*x^(7/3))/(889056*e^2) - (2134141*b
^3*d^4*n^3*x^(5/3))/(15876000*e^4) + (3714811*b^3*d^5*n^3*x^(4/3))/(1270080
0*e^5) + (12335311*b^3*d^7*n^3*x^(2/3))/(6350400*e^7) - (30300391*b^3*d^8*n
^3*x^(1/3))/(3175200*e^8) + (a*b^2*d^9*log(c*(d + e*x^(1/3))^n)^2)/e^9 - (7
129*b^3*d^9*n*log(c*(d + e*x^(1/3))^n)^2)/(2520*e^9) + (217*b^3*d*n^3*x^(8/
```

3))/(20736*e) - (6527971*b^3*d^6*n^3*x)/(9525600*e^6) + (a^2*b*d^9*n*log(d + e*x^(1/3)))/e^9 + (b^3*d*n*x^(8/3)*log(c*(d + e*x^(1/3))^n)^2)/(8*e) - (17*b^3*d*n^2*x^(8/3)*log(c*(d + e*x^(1/3))^n))/(288*e) - (b^3*d^6*n*x*log(c*(d + e*x^(1/3))^n)^2)/(3*e^6) + (3349*b^3*d^6*n^2*x*log(c*(d + e*x^(1/3))^n))/(3780*e^6) + (a^2*b*d^3*n*x^2)/(6*e^3) - (17*a*b^2*d*n^2*x^(8/3))/(288*e) + (3349*a*b^2*d^6*n^2*x)/(3780*e^6) - (a^2*b*d^2*n*x^(7/3))/(7*e^2) - (a^2*b*d^4*n*x^(5/3))/(5*e^4) + (a^2*b*d^5*n*x^(4/3))/(4*e^5) + (a^2*b*d^7*n*x^(2/3))/(2*e^7) - (a^2*b*d^8*n*x^(1/3))/e^8 - (7129*a*b^2*d^9*n^2*log(d + e*x^(1/3)))/(1260*e^9) + (b^3*d^3*n*x^2*log(c*(d + e*x^(1/3))^n)^2)/(6*e^3) - (275*b^3*d^3*n^2*x^2*log(c*(d + e*x^(1/3))^n))/(1512*e^3) - (b^3*d^2*n*x^(7/3)*log(c*(d + e*x^(1/3))^n)^2)/(7*e^2) + (191*b^3*d^2*n^2*x^(7/3)*log(c*(d + e*x^(1/3))^n))/(1764*e^2) - (b^3*d^4*n*x^(5/3)*log(c*(d + e*x^(1/3))^n)^2)/(5*e^4) + (1879*b^3*d^4*n^2*x^(5/3)*log(c*(d + e*x^(1/3))^n))/(6300*e^4) + (b^3*d^5*n*x^(4/3)*log(c*(d + e*x^(1/3))^n)^2)/(4*e^5) - (2509*b^3*d^5*n^2*x^(4/3)*log(c*(d + e*x^(1/3))^n))/(5040*e^5) + (b^3*d^7*n*x^(2/3)*log(c*(d + e*x^(1/3))^n)^2)/(2*e^7) - (4609*b^3*d^7*n^2*x^(2/3)*log(c*(d + e*x^(1/3))^n))/(2520*e^7) - (b^3*d^8*n*x^(1/3)*log(c*(d + e*x^(1/3))^n)^2)/e^8 + (7129*b^3*d^8*n^2*x^(1/3)*log(c*(d + e*x^(1/3))^n))/(1260*e^8) - (275*a*b^2*d^3*n^2*x^2)/(1512*e^3) + (191*a*b^2*d^2*n^2*x^(7/3))/(1764*e^2) + (1879*a*b^2*d^4*n^2*x^(5/3))/(6300*e^4) - (2509*a*b^2*d^5*n^2*x^(4/3))/(5040*e^5) - (4609*a*b^2*d^7*n^2*x^(2/3))/(2520*e^7) + (7129*a*b^2*d^8*n^2*x^(1/3))/(1260*e^8) + (a^2*b*d*n*x^(8/3))/(8*e) - (a^2*b*d^6*n*x)/(3*e^6) + (a*b^2*d*n*x^(8/3)*log(c*(d + e*x^(1/3))^n))/(4*e) - (2*a*b^2*d^6*n*x*log(c*(d + e*x^(1/3))^n))/(3*e^6) + (a*b^2*d^3*n*x^2*log(c*(d + e*x^(1/3))^n))/(3*e^3) - (2*a*b^2*d^2*n*x^(7/3)*log(c*(d + e*x^(1/3))^n))/(7*e^2) - (2*a*b^2*d^4*n*x^(5/3)*log(c*(d + e*x^(1/3))^n))/(5*e^4) + (a*b^2*d^5*n*x^(4/3)*log(c*(d + e*x^(1/3))^n))/(2*e^5) + (a*b^2*d^7*n*x^(2/3)*log(c*(d + e*x^(1/3))^n))/e^7 - (2*a*b^2*d^8*n*x^(1/3)*log(c*(d + e*x^(1/3))^n))/e^8

3.458 $\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 907

$$
\begin{aligned}
\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx = & -\frac{45b^3d^4n^3\left(d+e\sqrt[3]{x}\right)^2}{8e^6}+\frac{20b^3d^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^6} \\
& -\frac{45b^3d^2n^3\left(d+e\sqrt[3]{x}\right)^4}{64e^6}+\frac{18b^3dn^3\left(d+e\sqrt[3]{x}\right)^5}{125e^6} \\
& -\frac{b^3n^3\left(d+e\sqrt[3]{x}\right)^6}{72e^6}-\frac{18ab^2d^5n^2\sqrt[3]{x}}{e^5}+\frac{18b^3d^5n^3\sqrt[3]{x}}{e^5} \\
& -\frac{18b^3d^5n^2\left(d+e\sqrt[3]{x}\right)\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{e^6} \\
& +\frac{45b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4e^6} \\
& -\frac{20b^2d^3n^2\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^6} \\
& +\frac{45b^2d^2n^2\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{16e^6} \\
& -\frac{18b^2dn^2\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{25e^6} \\
& +\frac{b^2n^2\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{12e^6} \\
& +\frac{9bd^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^6} \\
& -\frac{45bd^4n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{4e^6} \\
& +\frac{10bd^3n\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^6} \\
& -\frac{45bd^2n\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{8e^6} \\
& +\frac{9bdn\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{5e^6} \\
& -\frac{bn\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{4e^6} \\
& -\frac{3d^5\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^6} \\
& +\frac{15d^4\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2e^6} \\
& -\frac{10d^3\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^6} \\
& +\frac{15d^2\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2e^6} \\
& -\frac{3d\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^6} \\
& +\frac{\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2e^6}
\end{aligned}
$$

```
[Out] -1/72*b^3*n^3*(d+e*x^(1/3))^6/e^6-3*d^5*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3
))^n))^3/e^6+15/2*d^4*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^6-10*
d^3*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^6+15/2*d^2*(d+e*x^(1/3)
)^4*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^6-3*d*(d+e*x^(1/3))^5*(a+b*ln(c*(d+e*x^
(1/3))^n))^3/e^6+1/2*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^6-18*a
*b^2*d^5*n^2*x^(1/3)/e^5-18*b^3*d^5*n^2*(d+e*x^(1/3))*ln(c*(d+e*x^(1/3))^n)
/e^6+45/4*b^2*d^4*n^2*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6-20/3*
b^2*d^3*n^2*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6+45/16*b^2*d^2*n
^2*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6-18/25*b^2*d*n^2*(d+e*x^(
1/3))^5*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6+9*b*d^5*n*(d+e*x^(1/3))*(a+b*ln(c*(
d+e*x^(1/3))^n))^2/e^6-45/4*b*d^4*n*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))
^n))^2/e^6+10*b*d^3*n*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^6-45/
8*b*d^2*n*(d+e*x^(1/3))^4*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^6+9/5*b*d*n*(d+e*
x^(1/3))^5*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^6-45/8*b^3*d^4*n^3*(d+e*x^(1/3))
^2/e^6+20/9*b^3*d^3*n^3*(d+e*x^(1/3))^3/e^6-45/64*b^3*d^2*n^3*(d+e*x^(1/3))
^4/e^6+18/125*b^3*d*n^3*(d+e*x^(1/3))^5/e^6+18*b^3*d^5*n^3*x^(1/3)/e^5+1/12
*b^2*n^2*(d+e*x^(1/3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))/e^6-1/4*b*n*(d+e*x^(1/
3))^6*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^6
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 907, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used

$= \{2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341\}$

$$
\begin{aligned}
\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx = &-\frac{b^3n^3\left(d+e\sqrt[3]{x}\right)^6}{72e^6} \\
&+\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^6}{2e^6} \\
&-\frac{bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(d+e\sqrt[3]{x}\right)^6}{4e^6} \\
&+\frac{b^2n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\left(d+e\sqrt[3]{x}\right)^6}{12e^6} \\
&+\frac{18b^3dn^3\left(d+e\sqrt[3]{x}\right)^5}{125e^6} \\
&-\frac{3d\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^5}{e^6} \\
&+\frac{9bdn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(d+e\sqrt[3]{x}\right)^5}{5e^6} \\
&-\frac{18b^2dn^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\left(d+e\sqrt[3]{x}\right)^5}{25e^6} \\
&-\frac{45b^3d^2n^3\left(d+e\sqrt[3]{x}\right)^4}{64e^6} \\
&+\frac{15d^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^4}{2e^6} \\
&-\frac{45bd^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(d+e\sqrt[3]{x}\right)^4}{8e^6} \\
&+\frac{45b^2d^2n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\left(d+e\sqrt[3]{x}\right)^4}{16e^6} \\
&+\frac{20b^3d^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^6} \\
&-\frac{10d^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^3}{e^6} \\
&+\frac{10bd^3n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(d+e\sqrt[3]{x}\right)^3}{e^6} \\
&-\frac{20b^2d^3n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\left(d+e\sqrt[3]{x}\right)^3}{3e^6} \\
&-\frac{45b^3d^4n^3\left(d+e\sqrt[3]{x}\right)^2}{8e^6} \\
&+\frac{15d^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)^2}{2e^6} \\
&-\frac{45bd^4n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(d+e\sqrt[3]{x}\right)^2}{4e^6} \\
&+\frac{45b^2d^4n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\left(d+e\sqrt[3]{x}\right)^2}{4e^6} \\
&-\frac{3d^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\left(d+e\sqrt[3]{x}\right)}{e^6} \\
&\quad 9bd^5n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(d+e\sqrt[3]{x}\right)
\end{aligned}
$$

```
[In] Int[x*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (-45*b^3*d^4*n^3*(d + e*x^(1/3))^2)/(8*e^6) + (20*b^3*d^3*n^3*(d + e*x^(1/3
))^3)/(9*e^6) - (45*b^3*d^2*n^3*(d + e*x^(1/3))^4)/(64*e^6) + (18*b^3*d*n^3
*(d + e*x^(1/3))^5)/(125*e^6) - (b^3*n^3*(d + e*x^(1/3))^6)/(72*e^6) - (18*
a*b^2*d^5*n^2*x^(1/3))/e^5 + (18*b^3*d^5*n^3*x^(1/3))/e^5 - (18*b^3*d^5*n^2
*(d + e*x^(1/3))*Log[c*(d + e*x^(1/3))^n])/e^6 + (45*b^2*d^4*n^2*(d + e*x^(
1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(4*e^6) - (20*b^2*d^3*n^2*(d + e*
x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^6) + (45*b^2*d^2*n^2*(d +
 e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3))^n]))/(16*e^6) - (18*b^2*d*n^2*(d
 + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n]))/(25*e^6) + (b^2*n^2*(d +
e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/3))^n]))/(12*e^6) + (9*b*d^5*n*(d + e
*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^6 - (45*b*d^4*n*(d + e*x^(1
/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(4*e^6) + (10*b*d^3*n*(d + e*x^(
1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^6 - (45*b*d^2*n*(d + e*x^(1/3
))^4*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(8*e^6) + (9*b*d*n*(d + e*x^(1/3))
^5*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(5*e^6) - (b*n*(d + e*x^(1/3))^6*(a
+ b*Log[c*(d + e*x^(1/3))^n])^2)/(4*e^6) - (3*d^5*(d + e*x^(1/3))*(a + b*Lo
g[c*(d + e*x^(1/3))^n])^3)/e^6 + (15*d^4*(d + e*x^(1/3))^2*(a + b*Log[c*(d
+ e*x^(1/3))^n])^3)/(2*e^6) - (10*d^3*(d + e*x^(1/3))^3*(a + b*Log[c*(d + e
*x^(1/3))^n])^3)/e^6 + (15*d^2*(d + e*x^(1/3))^4*(a + b*Log[c*(d + e*x^(1/3
))^n])^3)/(2*e^6) - (3*d*(d + e*x^(1/3))^5*(a + b*Log[c*(d + e*x^(1/3))^n])
^3)/e^6 + ((d + e*x^(1/3))^6*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/(2*e^6)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^5(a+b\log\left(c(d+ex)^n\right))^3\,dx, x, \sqrt[3]{x}\right)\\
&= 3\text{Subst}\left(\int \left(-\frac{d^5(a+b\log\left(c(d+ex)^n\right))^3}{e^5}+\frac{5d^4(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^5}\right.\right.\\
&\quad -\frac{10d^3(d+ex)^2\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^5}\\
&\quad +\frac{10d^2(d+ex)^3\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^5}-\frac{5d(d+ex)^4\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^5}\\
&\quad \left.\left.+\frac{(d+ex)^5\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^5}\right)\,dx, x, \sqrt[3]{x}\right)
\end{aligned}$$

$$
\begin{aligned}
&= \frac{3\text{Subst}\left(\int (d+ex)^5 \left(a+b\log \left(c(d+ex)^n\right)\right)^3 \,dx, x, \sqrt[3]{x}\right)}{e^5} \\
&\quad - \frac{(15d)\text{Subst}\left(\int (d+ex)^4 \left(a+b\log \left(c(d+ex)^n\right)\right)^3 \,dx, x, \sqrt[3]{x}\right)}{e^5} \\
&\quad + \frac{(30d^2)\,\text{Subst}\left(\int (d+ex)^3 \left(a+b\log \left(c(d+ex)^n\right)\right)^3 \,dx, x, \sqrt[3]{x}\right)}{e^5} \\
&\quad - \frac{(30d^3)\,\text{Subst}\left(\int (d+ex)^2 \left(a+b\log \left(c(d+ex)^n\right)\right)^3 \,dx, x, \sqrt[3]{x}\right)}{e^5} \\
&\quad + \frac{(15d^4)\,\text{Subst}\left(\int (d+ex) \left(a+b\log \left(c(d+ex)^n\right)\right)^3 \,dx, x, \sqrt[3]{x}\right)}{e^5} \\
&\quad - \frac{(3d^5)\,\text{Subst}\left(\int \left(a+b\log \left(c(d+ex)^n\right)\right)^3 \,dx, x, \sqrt[3]{x}\right)}{e^5} \\
&= \frac{3\text{Subst}\left(\int x^5(a+b\log \left(cx^n\right))^3 \,dx, x, d+e\sqrt[3]{x}\right)}{e^6} \\
&\quad - \frac{(15d)\text{Subst}\left(\int x^4(a+b\log \left(cx^n\right))^3 \,dx, x, d+e\sqrt[3]{x}\right)}{e^6} \\
&\quad + \frac{(30d^2)\,\text{Subst}\left(\int x^3(a+b\log \left(cx^n\right))^3 \,dx, x, d+e\sqrt[3]{x}\right)}{e^6} \\
&\quad - \frac{(30d^3)\,\text{Subst}\left(\int x^2(a+b\log \left(cx^n\right))^3 \,dx, x, d+e\sqrt[3]{x}\right)}{e^6} \\
&\quad + \frac{(15d^4)\,\text{Subst}\left(\int x(a+b\log \left(cx^n\right))^3 \,dx, x, d+e\sqrt[3]{x}\right)}{e^6} \\
&\quad - \frac{(3d^5)\,\text{Subst}\left(\int (a+b\log \left(cx^n\right))^3 \,dx, x, d+e\sqrt[3]{x}\right)}{e^6}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3d^5\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^6} \\
&+ \frac{15d^4\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2e^6} \\
&- \frac{10d^3\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^6} \\
&+ \frac{15d^2\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2e^6} \\
&- \frac{3d\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^6} \\
&+ \frac{\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2e^6} \\
&- \frac{(3bn)\mathrm{Subst}\left(\int x^5(a+b\log(cx^n))^2\,dx, x, d+e\sqrt[3]{x}\right)}{2e^6} \\
&+ \frac{(9bdn)\mathrm{Subst}\left(\int x^4(a+b\log(cx^n))^2\,dx, x, d+e\sqrt[3]{x}\right)}{e^6} \\
&- \frac{(45bd^2n)\,\mathrm{Subst}\left(\int x^3(a+b\log(cx^n))^2\,dx, x, d+e\sqrt[3]{x}\right)}{2e^6} \\
&+ \frac{(30bd^3n)\,\mathrm{Subst}\left(\int x^2(a+b\log(cx^n))^2\,dx, x, d+e\sqrt[3]{x}\right)}{e^6} \\
&- \frac{(45bd^4n)\,\mathrm{Subst}\left(\int x(a+b\log(cx^n))^2\,dx, x, d+e\sqrt[3]{x}\right)}{2e^6} \\
&+ \frac{(9bd^5n)\,\mathrm{Subst}\left(\int (a+b\log(cx^n))^2\,dx, x, d+e\sqrt[3]{x}\right)}{e^6}
\end{aligned}
$$

$$= \frac{9bd^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^6} - \frac{45bd^4n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{4e^6} + \frac{10bd^3n\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^6} - \frac{45bd^2n\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{8e^6} + \frac{9bdn\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{5e^6} - \frac{bn\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{4e^6} - \frac{3d^5\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^6} + \frac{15d^4\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2e^6} - \frac{10d^3\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^6} + \frac{15d^2\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2e^6} - \frac{3d\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^6} + \frac{\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2e^6} + \frac{(b^2n^2)\,\mathrm{Subst}\left(\int x^5(a+b\log(cx^n))\,dx,x,d+e\sqrt[3]{x}\right)}{2e^6} - \frac{(18b^2dn^2)\,\mathrm{Subst}\left(\int x^4(a+b\log(cx^n))\,dx,x,d+e\sqrt[3]{x}\right)}{5e^6} + \frac{(45b^2d^2n^2)\,\mathrm{Subst}\left(\int x^3(a+b\log(cx^n))\,dx,x,d+e\sqrt[3]{x}\right)}{4e^6} - \frac{(20b^2d^3n^2)\,\mathrm{Subst}\left(\int x^2(a+b\log(cx^n))\,dx,x,d+e\sqrt[3]{x}\right)}{e^6} + \frac{(45b^2d^4n^2)\,\mathrm{Subst}\left(\int x(a+b\log(cx^n))\,dx,x,d+e\sqrt[3]{x}\right)}{2e^6} - \frac{(18b^2d^5n^2)\,\mathrm{Subst}\left(\int (a+b\log(cx^n))\,dx,x,d+e\sqrt[3]{x}\right)}{e^6}$$

$$
\begin{aligned}
&= -\frac{45b^3d^4n^3\left(d+e\sqrt[3]{x}\right)^2}{8e^6}+\frac{20b^3d^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^6}-\frac{45b^3d^2n^3\left(d+e\sqrt[3]{x}\right)^4}{64e^6}\\
&+\frac{18b^3dn^3\left(d+e\sqrt[3]{x}\right)^5}{125e^6}-\frac{b^3n^3\left(d+e\sqrt[3]{x}\right)^6}{72e^6}-\frac{18ab^2d^5n^2\sqrt[3]{x}}{e^5}\\
&+\frac{45b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4e^6}\\
&-\frac{20b^2d^3n^2\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^6}\\
&+\frac{45b^2d^2n^2\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{16e^6}\\
&-\frac{18b^2dn^2\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{25e^6}\\
&+\frac{b^2n^2\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{12e^6}\\
&+\frac{9bd^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^6}\\
&-\frac{45bd^4n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{4e^6}\\
&+\frac{10bd^3n\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^6}\\
&-\frac{45bd^2n\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{8e^6}\\
&+\frac{9bdn\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{5e^6}\\
&-\frac{bn\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{4e^6}\\
&-\frac{3d^5\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^6}\\
&+\frac{15d^4\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2e^6}\\
&-\frac{10d^3\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^6}\\
&+\frac{15d^2\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2e^6}\\
&-\frac{3d\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^6}\\
&+\frac{\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2e^6}\\
&-\frac{(18b^3d^5n^2)\,\mathrm{Subst}\left(\int\log\left(cx^n\right)\,dx,x,d+e\sqrt[3]{x}\right)}{e^6}
\end{aligned}
$$

$$= -\frac{45b^3d^4n^3\left(d+e\sqrt[3]{x}\right)^2}{8e^6}+\frac{20b^3d^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^6}-\frac{45b^3d^2n^3\left(d+e\sqrt[3]{x}\right)^4}{64e^6}$$
$$+\frac{18b^3dn^3\left(d+e\sqrt[3]{x}\right)^5}{125e^6}-\frac{b^3n^3\left(d+e\sqrt[3]{x}\right)^6}{72e^6}-\frac{18ab^2d^5n^2\sqrt[3]{x}}{e^5}$$
$$+\frac{18b^3d^5n^3\sqrt[3]{x}}{e^5}-\frac{18b^3d^5n^2\left(d+e\sqrt[3]{x}\right)\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{e^6}$$
$$+\frac{45b^2d^4n^2\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{4e^6}$$
$$-\frac{20b^2d^3n^2\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^6}$$
$$+\frac{45b^2d^2n^2\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{16e^6}$$
$$-\frac{18b^2dn^2\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{25e^6}$$
$$+\frac{b^2n^2\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{12e^6}$$
$$+\frac{9bd^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^6}$$
$$-\frac{45bd^4n\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{4e^6}$$
$$+\frac{10bd^3n\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^6}$$
$$-\frac{45bd^2n\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{8e^6}$$
$$+\frac{9bdn\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{5e^6}$$
$$-\frac{bn\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{4e^6}$$
$$-\frac{3d^5\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^6}$$
$$+\frac{15d^4\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2e^6}$$
$$-\frac{10d^3\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^6}$$
$$+\frac{15d^2\left(d+e\sqrt[3]{x}\right)^4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2e^6}$$
$$-\frac{3d\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^6}$$
$$+\frac{\left(d+e\sqrt[3]{x}\right)^6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2e^6}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 589, normalized size of antiderivative = 0.65

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$$
$$=\frac{b^3en^3\sqrt[3]{x}\left(809340d^5-140070d^4e\sqrt[3]{x}+41180d^3e^2x^{2/3}-13785d^2e^3x+4368de^4x^{4/3}-1000e^5x^{5/3}\right)+1800}{}$$

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (b^3*e*n^3*x^(1/3)*(809340*d^5 - 140070*d^4*e*x^(1/3) + 41180*d^3*e^2*x^(2/
3) - 13785*d^2*e^3*x + 4368*d*e^4*x^(4/3) - 1000*e^5*x^(5/3)) + 1800*a^2*b*
n*(147*d^6 + 60*d^5*e*x^(1/3) - 30*d^4*e^2*x^(2/3) + 20*d^3*e^3*x - 15*d^2*
e^4*x^(4/3) + 12*d*e^5*x^(5/3) - 10*e^6*x^2) - 36000*a^3*(d^6 - e^6*x^2) +
60*a*b^2*n^2*(8111*d^6 - 8820*d^5*e*x^(1/3) + 2610*d^4*e^2*x^(2/3) - 1140*d
^3*e^3*x + 555*d^2*e^4*x^(4/3) - 264*d*e^5*x^(5/3) + 100*e^6*x^2) - 60*b*(b
^2*n^2*(13489*d^6 + 8820*d^5*e*x^(1/3) - 2610*d^4*e^2*x^(2/3) + 1140*d^3*e^
3*x - 555*d^2*e^4*x^(4/3) + 264*d*e^5*x^(5/3) - 100*e^6*x^2) - 60*a*b*n*(14
7*d^6 + 60*d^5*e*x^(1/3) - 30*d^4*e^2*x^(2/3) + 20*d^3*e^3*x - 15*d^2*e^4*x
^(4/3) + 12*d*e^5*x^(5/3) - 10*e^6*x^2) + 1800*a^2*(d^6 - e^6*x^2))*Log[c*(
d + e*x^(1/3))^n] - 1800*b^2*(60*a*(d^6 - e^6*x^2) + b*n*(-147*d^6 - 60*d^5
*e*x^(1/3) + 30*d^4*e^2*x^(2/3) - 20*d^3*e^3*x + 15*d^2*e^4*x^(4/3) - 12*d*
e^5*x^(5/3) + 10*e^6*x^2))*Log[c*(d + e*x^(1/3))^n]^2 - 36000*b^3*(d^6 - e^
6*x^2)*Log[c*(d + e*x^(1/3))^n]^3)/(72000*e^6)
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{3}dx$$

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 1190, normalized size of antiderivative = 1.31

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/72000*(36000*b^3*e^6*x^2*log(c)^3 + 36000*(b^3*e^6*n^3*x^2 - b^3*d^6*n^3)
*log(e*x^(1/3) + d)^3 - 1000*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2 + 18*a^2*b*e^6*
n - 36*a^3*e^6)*x^2 + 1800*(20*b^3*d^3*e^3*n^3*x + 147*b^3*d^6*n^3 - 60*a*b
^2*d^6*n^2 - 10*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2)*x^2 + 60*(b^3*e^6*n^2*x^2 -
 b^3*d^6*n^2)*log(c) + 6*(2*b^3*d*e^5*n^3*x - 5*b^3*d^4*e^2*n^3)*x^(2/3) -
15*(b^3*d^2*e^4*n^3*x - 4*b^3*d^5*e*n^3)*x^(1/3))*log(e*x^(1/3) + d)^2 + 18
000*(2*b^3*d^3*e^3*n*x - (b^3*e^6*n - 6*a*b^2*e^6)*x^2)*log(c)^2 + 20*(2059
*b^3*d^3*e^3*n^3 - 3420*a*b^2*d^3*e^3*n^2 + 1800*a^2*b*d^3*e^3*n)*x - 60*(1
3489*b^3*d^6*n^3 - 8820*a*b^2*d^6*n^2 + 1800*a^2*b*d^6*n - 100*(b^3*e^6*n^3
 - 6*a*b^2*e^6*n^2 + 18*a^2*b*e^6*n)*x^2 - 1800*(b^3*e^6*n*x^2 - b^3*d^6*n)
*log(c)^2 + 60*(19*b^3*d^3*e^3*n^3 - 20*a*b^2*d^3*e^3*n^2)*x - 60*(20*b^3*d
^3*e^3*n^2*x + 147*b^3*d^6*n^2 - 60*a*b^2*d^6*n - 10*(b^3*e^6*n^2 - 6*a*b^2
*e^6*n)*x^2)*log(c) - 6*(435*b^3*d^4*e^2*n^3 - 300*a*b^2*d^4*e^2*n^2 - 4*(1
1*b^3*d*e^5*n^3 - 30*a*b^2*d*e^5*n^2)*x + 60*(2*b^3*d*e^5*n^2*x - 5*b^3*d^4
*e^2*n^2)*log(c))*x^(2/3) + 15*(588*b^3*d^5*e*n^3 - 240*a*b^2*d^5*e*n^2 - (
37*b^3*d^2*e^4*n^3 - 60*a*b^2*d^2*e^4*n^2)*x + 60*(b^3*d^2*e^4*n^2*x - 4*b^
3*d^5*e*n^2)*log(c))*x^(1/3))*log(e*x^(1/3) + d) + 1200*(5*(b^3*e^6*n^2 - 6
*a*b^2*e^6*n + 18*a^2*b*e^6)*x^2 - 3*(19*b^3*d^3*e^3*n^2 - 20*a*b^2*d^3*e^3
*n)*x)*log(c) - 6*(23345*b^3*d^4*e^2*n^3 - 26100*a*b^2*d^4*e^2*n^2 + 9000*a
^2*b*d^4*e^2*n - 1800*(2*b^3*d*e^5*n*x - 5*b^3*d^4*e^2*n)*log(c)^2 - 8*(91*
b^3*d*e^5*n^3 - 330*a*b^2*d*e^5*n^2 + 450*a^2*b*d*e^5*n)*x - 60*(435*b^3*d^
4*e^2*n^2 - 300*a*b^2*d^4*e^2*n - 4*(11*b^3*d*e^5*n^2 - 30*a*b^2*d*e^5*n)*x
)*log(c))*x^(2/3) + 15*(53956*b^3*d^5*e*n^3 - 35280*a*b^2*d^5*e*n^2 + 7200*
a^2*b*d^5*e*n - 1800*(b^3*d^2*e^4*n*x - 4*b^3*d^5*e*n)*log(c)^2 - (919*b^3*
d^2*e^4*n^3 - 2220*a*b^2*d^2*e^4*n^2 + 1800*a^2*b*d^2*e^4*n)*x - 60*(588*b^
3*d^5*e*n^2 - 240*a*b^2*d^5*e*n - (37*b^3*d^2*e^4*n^2 - 60*a*b^2*d^2*e^4*n)
*x)*log(c))*x^(1/3))/e^6
```

Sympy [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$$

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/3))**n))**3,x)
```

```
[Out] Integral(x*(a + b*log(c*(d + e*x**(1/3))**n))**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 668, normalized size of antiderivative = 0.74

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$$
$$=\frac{1}{2}\,b^3x^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)^3+\frac{3}{2}\,ab^2x^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)^2$$
$$-\frac{1}{40}\,a^2ben\left(\frac{60\,d^6\log\left(ex^{\frac{1}{3}}+d\right)}{e^7}+\frac{10\,e^5x^2-12\,de^4x^{\frac{5}{3}}+15\,d^2e^3x^{\frac{4}{3}}-20\,d^3e^2x+30\,d^4ex^{\frac{2}{3}}-60\,d^5x^{\frac{1}{3}}}{e^6}\right)$$
$$+\frac{3}{2}\,a^2bx^2\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+\frac{1}{2}\,a^3x^2$$
$$-\frac{1}{1200}\left(60\,en\left(\frac{60\,d^6\log\left(ex^{\frac{1}{3}}+d\right)}{e^7}+\frac{10\,e^5x^2-12\,de^4x^{\frac{5}{3}}+15\,d^2e^3x^{\frac{4}{3}}-20\,d^3e^2x+30\,d^4ex^{\frac{2}{3}}-60\,d^5x^{\frac{1}{3}}}{e^6}\right.\right.$$
$$-\frac{1}{72000}\left(1800\,en\left(\frac{60\,d^6\log\left(ex^{\frac{1}{3}}+d\right)}{e^7}+\frac{10\,e^5x^2-12\,de^4x^{\frac{5}{3}}+15\,d^2e^3x^{\frac{4}{3}}-20\,d^3e^2x+30\,d^4ex^{\frac{2}{3}}-60\,d}{e^6}\right.\right.$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2*log((e*x^(1/3) + d)^n*c)^3 + 3/2*a*b^2*x^2*log((e*x^(1/3) + d)^
n*c)^2 - 1/40*a^2*b*e*n*(60*d^6*log(e*x^(1/3) + d)/e^7 + (10*e^5*x^2 - 12*d
*e^4*x^(5/3) + 15*d^2*e^3*x^(4/3) - 20*d^3*e^2*x + 30*d^4*e*x^(2/3) - 60*d^
5*x^(1/3))/e^6) + 3/2*a^2*b*x^2*log((e*x^(1/3) + d)^n*c) + 1/2*a^3*x^2 - 1/
1200*(60*e*n*(60*d^6*log(e*x^(1/3) + d)/e^7 + (10*e^5*x^2 - 12*d*e^4*x^(5/3
) + 15*d^2*e^3*x^(4/3) - 20*d^3*e^2*x + 30*d^4*e*x^(2/3) - 60*d^5*x^(1/3))/
e^6)*log((e*x^(1/3) + d)^n*c) - (100*e^6*x^2 + 1800*d^6*log(e*x^(1/3) + d)^
2 - 264*d*e^5*x^(5/3) + 555*d^2*e^4*x^(4/3) - 1140*d^3*e^3*x + 8820*d^6*log
(e*x^(1/3) + d) + 2610*d^4*e^2*x^(2/3) - 8820*d^5*e*x^(1/3))*n^2/e^6)*a*b^2
 - 1/72000*(1800*e*n*(60*d^6*log(e*x^(1/3) + d)/e^7 + (10*e^5*x^2 - 12*d*e^
4*x^(5/3) + 15*d^2*e^3*x^(4/3) - 20*d^3*e^2*x + 30*d^4*e*x^(2/3) - 60*d^5*x
^(1/3))/e^6)*log((e*x^(1/3) + d)^n*c)^2 + e*n*((36000*d^6*log(e*x^(1/3) + d
)^3 + 1000*e^6*x^2 + 264600*d^6*log(e*x^(1/3) + d)^2 - 4368*d*e^5*x^(5/3) +
 13785*d^2*e^4*x^(4/3) - 41180*d^3*e^3*x + 809340*d^6*log(e*x^(1/3) + d) +
140070*d^4*e^2*x^(2/3) - 809340*d^5*e*x^(1/3))*n^2/e^7 - 60*(100*e^6*x^2 +
1800*d^6*log(e*x^(1/3) + d)^2 - 264*d*e^5*x^(5/3) + 555*d^2*e^4*x^(4/3) - 1
140*d^3*e^3*x + 8820*d^6*log(e*x^(1/3) + d) + 2610*d^4*e^2*x^(2/3) - 8820*d
^5*e*x^(1/3))*n*log((e*x^(1/3) + d)^n*c)/e^7))*b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2160 vs. $2(787) = 1574$.

Time = 0.33 (sec) , antiderivative size = 2160, normalized size of antiderivative = 2.38

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx = \text{Too large to display}$$

[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="giac")

[Out] 1/72000*(36000*b^3*e*x^2*log(c)^3 + 108000*a*b^2*e*x^2*log(c)^2 + (36000*(e*x^(1/3) + d)^6*log(e*x^(1/3) + d)^3/e^5 - 216000*(e*x^(1/3) + d)^5*d*log(e*x^(1/3) + d)^3/e^5 + 540000*(e*x^(1/3) + d)^4*d^2*log(e*x^(1/3) + d)^3/e^5 - 720000*(e*x^(1/3) + d)^3*d^3*log(e*x^(1/3) + d)^3/e^5 + 540000*(e*x^(1/3) + d)^2*d^4*log(e*x^(1/3) + d)^3/e^5 - 216000*(e*x^(1/3) + d)*d^5*log(e*x^(1/3) + d)^3/e^5 - 18000*(e*x^(1/3) + d)^6*log(e*x^(1/3) + d)^2/e^5 + 129600*(e*x^(1/3) + d)^5*d*log(e*x^(1/3) + d)^2/e^5 - 405000*(e*x^(1/3) + d)^4*d^2*log(e*x^(1/3) + d)^2/e^5 + 720000*(e*x^(1/3) + d)^3*d^3*log(e*x^(1/3) + d)^2/e^5 - 810000*(e*x^(1/3) + d)^2*d^4*log(e*x^(1/3) + d)^2/e^5 + 648000*(e*x^(1/3) + d)*d^5*log(e*x^(1/3) + d)^2/e^5 + 6000*(e*x^(1/3) + d)^6*log(e*x^(1/3) + d)/e^5 - 51840*(e*x^(1/3) + d)^5*d*log(e*x^(1/3) + d)/e^5 + 202500*(e*x^(1/3) + d)^4*d^2*log(e*x^(1/3) + d)/e^5 - 480000*(e*x^(1/3) + d)^3*d^3*log(e*x^(1/3) + d)/e^5 + 810000*(e*x^(1/3) + d)^2*d^4*log(e*x^(1/3) + d)/e^5 - 1296000*(e*x^(1/3) + d)*d^5*log(e*x^(1/3) + d)/e^5 - 1000*(e*x^(1/3) + d)^6/e^5 + 10368*(e*x^(1/3) + d)^5*d/e^5 - 50625*(e*x^(1/3) + d)^4*d^2/e^5 + 160000*(e*x^(1/3) + d)^3*d^3/e^5 - 405000*(e*x^(1/3) + d)^2*d^4/e^5 + 1296000*(e*x^(1/3) + d)*d^5/e^5)*b^3*n^3 + 60*(1800*(e*x^(1/3) + d)^6*log(e*x^(1/3) + d)^2/e^5 - 10800*(e*x^(1/3) + d)^5*d*log(e*x^(1/3) + d)^2/e^5 + 27000*(e*x^(1/3) + d)^4*d^2*log(e*x^(1/3) + d)^2/e^5 - 36000*(e*x^(1/3) + d)^3*d^3*log(e*x^(1/3) + d)^2/e^5 + 27000*(e*x^(1/3) + d)^2*d^4*log(e*x^(1/3) + d)^2/e^5 - 10800*(e*x^(1/3) + d)*d^5*log(e*x^(1/3) + d)^2/e^5 - 600*(e*x^(1/3) + d)^6*log(e*x^(1/3) + d)/e^5 + 4320*(e*x^(1/3) + d)^5*d*log(e*x^(1/3) + d)/e^5 - 13500*(e*x^(1/3) + d)^4*d^2*log(e*x^(1/3) + d)/e^5 + 24000*(e*x^(1/3) + d)^3*d^3*log(e*x^(1/3) + d)/e^5 - 27000*(e*x^(1/3) + d)^2*d^4*log(e*x^(1/3) + d)/e^5 + 21600*(e*x^(1/3) + d)*d^5*log(e*x^(1/3) + d)/e^5 + 100*(e*x^(1/3) + d)^6/e^5 - 864*(e*x^(1/3) + d)^5*d/e^5 + 3375*(e*x^(1/3) + d)^4*d^2/e^5 - 8000*(e*x^(1/3) + d)^3*d^3/e^5 + 13500*(e*x^(1/3) + d)^2*d^4/e^5 - 21600*(e*x^(1/3) + d)*d^5/e^5)*b^3*n^2*log(c) + 108000*a^2*b*e*x^2*log(c) + 1800*(60*(e*x^(1/3) + d)^6*log(e*x^(1/3) + d)/e^5 - 360*(e*x^(1/3) + d)^5*d*log(e*x^(1/3) + d)/e^5 + 900*(e*x^(1/3) + d)^4*d^2*log(e*x^(1/3) + d)/e^5 - 1200*(e*x^(1/3) + d)^3*d^3*log(e*x^(1/3) + d)/e^5 + 900*(e*x^(1/3) + d)^2*d^4*log(e*x^(1/3) + d)/e^5 - 360*(e*x^(1/3) + d)*d^5*log(e*x^(1/3) + d)/e^5 - 10*(e*x^(1/3) + d)^6/e^5 + 72*(e*x^(1/3) + d)^5*d/e^5 - 225*(e*x^(1/3) + d)^4*d^2/e^5 + 400*(e*x^(1/3) + d)^3*d^3/e^5 - 450*(e*x^(1/3) + d)^2*d^4/e^5 + 360*(e*x^(1/3) + d)*d^5/e^5)*b^3*n*log(c)^2 + 60*(1800*(e*x^

```
(1/3) + d)^6*log(e*x^(1/3) + d)^2/e^5 - 10800*(e*x^(1/3) + d)^5*d*log(e*x^(
1/3) + d)^2/e^5 + 27000*(e*x^(1/3) + d)^4*d^2*log(e*x^(1/3) + d)^2/e^5 - 36
000*(e*x^(1/3) + d)^3*d^3*log(e*x^(1/3) + d)^2/e^5 + 27000*(e*x^(1/3) + d)^
2*d^4*log(e*x^(1/3) + d)^2/e^5 - 10800*(e*x^(1/3) + d)*d^5*log(e*x^(1/3) +
d)^2/e^5 - 600*(e*x^(1/3) + d)^6*log(e*x^(1/3) + d)/e^5 + 4320*(e*x^(1/3) +
 d)^5*d*log(e*x^(1/3) + d)/e^5 - 13500*(e*x^(1/3) + d)^4*d^2*log(e*x^(1/3)
+ d)/e^5 + 24000*(e*x^(1/3) + d)^3*d^3*log(e*x^(1/3) + d)/e^5 - 27000*(e*x^
(1/3) + d)^2*d^4*log(e*x^(1/3) + d)/e^5 + 21600*(e*x^(1/3) + d)*d^5*log(e*x
^(1/3) + d)/e^5 + 100*(e*x^(1/3) + d)^6/e^5 - 864*(e*x^(1/3) + d)^5*d/e^5 +
 3375*(e*x^(1/3) + d)^4*d^2/e^5 - 8000*(e*x^(1/3) + d)^3*d^3/e^5 + 13500*(e
*x^(1/3) + d)^2*d^4/e^5 - 21600*(e*x^(1/3) + d)*d^5/e^5)*a*b^2*n^2 + 36000*
a^3*e*x^2 + 3600*(60*(e*x^(1/3) + d)^6*log(e*x^(1/3) + d)/e^5 - 360*(e*x^(1
/3) + d)^5*d*log(e*x^(1/3) + d)/e^5 + 900*(e*x^(1/3) + d)^4*d^2*log(e*x^(1/
3) + d)/e^5 - 1200*(e*x^(1/3) + d)^3*d^3*log(e*x^(1/3) + d)/e^5 + 900*(e*x^
(1/3) + d)^2*d^4*log(e*x^(1/3) + d)/e^5 - 360*(e*x^(1/3) + d)*d^5*log(e*x^(
1/3) + d)/e^5 - 10*(e*x^(1/3) + d)^6/e^5 + 72*(e*x^(1/3) + d)^5*d/e^5 - 225
*(e*x^(1/3) + d)^4*d^2/e^5 + 400*(e*x^(1/3) + d)^3*d^3/e^5 - 450*(e*x^(1/3)
 + d)^2*d^4/e^5 + 360*(e*x^(1/3) + d)*d^5/e^5)*a*b^2*n*log(c) + 1800*(60*(e
*x^(1/3) + d)^6*log(e*x^(1/3) + d)/e^5 - 360*(e*x^(1/3) + d)^5*d*log(e*x^(1
/3) + d)/e^5 + 900*(e*x^(1/3) + d)^4*d^2*log(e*x^(1/3) + d)/e^5 - 1200*(e*x
^(1/3) + d)^3*d^3*log(e*x^(1/3) + d)/e^5 + 900*(e*x^(1/3) + d)^2*d^4*log(e*
x^(1/3) + d)/e^5 - 360*(e*x^(1/3) + d)*d^5*log(e*x^(1/3) + d)/e^5 - 10*(e*x
^(1/3) + d)^6/e^5 + 72*(e*x^(1/3) + d)^5*d/e^5 - 225*(e*x^(1/3) + d)^4*d^2/
e^5 + 400*(e*x^(1/3) + d)^3*d^3/e^5 - 450*(e*x^(1/3) + d)^2*d^4/e^5 + 360*(
e*x^(1/3) + d)*d^5/e^5)*a^2*b*n)/e
```

Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 979, normalized size of antiderivative = 1.08

$$
\begin{aligned}
\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx &= \frac{a^3\,x^2}{2}+\frac{b^3\,x^2\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)^3}{2} \\
&\quad -\frac{b^3\,n^3\,x^2}{72}+\frac{3\,a\,b^2\,x^2\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)^2}{2} \\
&\quad -\frac{b^3\,n\,x^2\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)^2}{4} \\
&\quad +\frac{b^3\,n^2\,x^2\,\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)}{12} \\
&\quad +\frac{a\,b^2\,n^2\,x^2}{12}-\frac{b^3\,d^6\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)^3}{2\,e^6} \\
&\quad +\frac{3\,a^2\,b\,x^2\,\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)}{2}-\frac{a^2\,b\,n\,x^2}{4} \\
&\quad -\frac{a\,b^2\,n\,x^2\,\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)}{2} \\
&\quad -\frac{13489\,b^3\,d^6\,n^3\,\ln\left(d+e\,x^{1/3}\right)}{1200\,e^6}-\frac{919\,b^3\,d^2\,n^3\,x^{4/3}}{4800\,e^2} \\
&\quad -\frac{4669\,b^3\,d^4\,n^3\,x^{2/3}}{2400\,e^4}+\frac{13489\,b^3\,d^5\,n^3\,x^{1/3}}{1200\,e^5} \\
&\quad -\frac{3\,a\,b^2\,d^6\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)^2}{2\,e^6} \\
&\quad +\frac{147\,b^3\,d^6\,n\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)^2}{40\,e^6}+\frac{2059\,b^3\,d^3\,n^3\,x}{3600\,e^3} \\
&\quad +\frac{91\,b^3\,d\,n^3\,x^{5/3}}{1500\,e}-\frac{3\,a^2\,b\,d^6\,n\,\ln\left(d+e\,x^{1/3}\right)}{2\,e^6} \\
&\quad +\frac{b^3\,d^3\,n\,x\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)^2}{2\,e^3} \\
&\quad -\frac{19\,b^3\,d^3\,n^2\,x\,\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)}{20\,e^3} \\
&\quad +\frac{3\,b^3\,d\,n\,x^{5/3}\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)^2}{10\,e} \\
&\quad -\frac{11\,b^3\,d\,n^2\,x^{5/3}\,\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)}{50\,e}-\frac{19\,a\,b^2\,d^3\,n^2\,x}{20\,e^3} \\
&\quad -\frac{11\,a\,b^2\,d\,n^2\,x^{5/3}}{50\,e}-\frac{3\,a^2\,b\,d^2\,n\,x^{4/3}}{8\,e^2}-\frac{3\,a^2\,b\,d^4\,n\,x^{2/3}}{4\,e^4} \\
&\quad +\frac{3\,a^2\,b\,d^5\,n\,x^{1/3}}{2\,e^5}+\frac{147\,a\,b^2\,d^6\,n^2\,\ln\left(d+e\,x^{1/3}\right)}{20\,e^6} \\
&\quad -\frac{3\,b^3\,d^2\,n\,x^{4/3}\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)^2}{8\,e^2} \\
&\quad +\frac{37\,b^3\,d^2\,n^2\,x^{4/3}\,\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)}{80\,e^2} \\
&\quad -\frac{3\,b^3\,d^4\,n\,x^{2/3}\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)^2}{4\,e^4} \\
&\quad 87\,b^3\,d^4\,n^2\,x^{2/3}\,\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)
\end{aligned}
$$

```
[In] int(x*(a + b*log(c*(d + e*x^(1/3))^n))^3,x)

[Out] (a^3*x^2)/2 + (b^3*x^2*log(c*(d + e*x^(1/3))^n)^3)/2 - (b^3*n^3*x^2)/72 + (
3*a*b^2*x^2*log(c*(d + e*x^(1/3))^n)^2)/2 - (b^3*n*x^2*log(c*(d + e*x^(1/3)
)^n)^2)/4 + (b^3*n^2*x^2*log(c*(d + e*x^(1/3))^n))/12 + (a*b^2*n^2*x^2)/12
- (b^3*d^6*log(c*(d + e*x^(1/3))^n)^3)/(2*e^6) + (3*a^2*b*x^2*log(c*(d + e*
x^(1/3))^n))/2 - (a^2*b*n*x^2)/4 - (a*b^2*n*x^2*log(c*(d + e*x^(1/3))^n))/2
 - (13489*b^3*d^6*n^3*log(d + e*x^(1/3)))/(1200*e^6) - (919*b^3*d^2*n^3*x^(
4/3))/(4800*e^2) - (4669*b^3*d^4*n^3*x^(2/3))/(2400*e^4) + (13489*b^3*d^5*n
^3*x^(1/3))/(1200*e^5) - (3*a*b^2*d^6*log(c*(d + e*x^(1/3))^n)^2)/(2*e^6) +
 (147*b^3*d^6*n*log(c*(d + e*x^(1/3))^n)^2)/(40*e^6) + (2059*b^3*d^3*n^3*x)
/(3600*e^3) + (91*b^3*d*n^3*x^(5/3))/(1500*e) - (3*a^2*b*d^6*n*log(d + e*x^
(1/3)))/(2*e^6) + (b^3*d^3*n*x*log(c*(d + e*x^(1/3))^n)^2)/(2*e^3) - (19*b^
3*d^3*n^2*x*log(c*(d + e*x^(1/3))^n))/(20*e^3) + (3*b^3*d*n*x^(5/3)*log(c*(
d + e*x^(1/3))^n)^2)/(10*e) - (11*b^3*d*n^2*x^(5/3)*log(c*(d + e*x^(1/3))^n
))/(50*e) - (19*a*b^2*d^3*n^2*x)/(20*e^3) - (11*a*b^2*d*n^2*x^(5/3))/(50*e)
 - (3*a^2*b*d^2*n*x^(4/3))/(8*e^2) - (3*a^2*b*d^4*n*x^(2/3))/(4*e^4) + (3*a
^2*b*d^5*n*x^(1/3))/(2*e^5) + (147*a*b^2*d^6*n^2*log(d + e*x^(1/3)))/(20*e^
6) - (3*b^3*d^2*n*x^(4/3)*log(c*(d + e*x^(1/3))^n)^2)/(8*e^2) + (37*b^3*d^2
*n^2*x^(4/3)*log(c*(d + e*x^(1/3))^n))/(80*e^2) - (3*b^3*d^4*n*x^(2/3)*log(
c*(d + e*x^(1/3))^n)^2)/(4*e^4) + (87*b^3*d^4*n^2*x^(2/3)*log(c*(d + e*x^(1
/3))^n))/(40*e^4) + (3*b^3*d^5*n*x^(1/3)*log(c*(d + e*x^(1/3))^n)^2)/(2*e^5
) - (147*b^3*d^5*n^2*x^(1/3)*log(c*(d + e*x^(1/3))^n))/(20*e^5) + (37*a*b^2
*d^2*n^2*x^(4/3))/(80*e^2) + (87*a*b^2*d^4*n^2*x^(2/3))/(40*e^4) - (147*a*b
^2*d^5*n^2*x^(1/3))/(20*e^5) + (a^2*b*d^3*n*x)/(2*e^3) + (3*a^2*b*d*n*x^(5/
3))/(10*e) + (a*b^2*d^3*n*x*log(c*(d + e*x^(1/3))^n))/e^3 + (3*a*b^2*d*n*x^
(5/3)*log(c*(d + e*x^(1/3))^n))/(5*e) - (3*a*b^2*d^2*n*x^(4/3)*log(c*(d + e
*x^(1/3))^n))/(4*e^2) - (3*a*b^2*d^4*n*x^(2/3)*log(c*(d + e*x^(1/3))^n))/(2
*e^4) + (3*a*b^2*d^5*n*x^(1/3)*log(c*(d + e*x^(1/3))^n))/e^5
```

3.459 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 438

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\frac{9b^3dn^3\left(d+e\sqrt[3]{x}\right)^2}{4e^3}-\frac{2b^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^3}+\frac{18ab^2d^2n^2\sqrt[3]{x}}{e^2}-\frac{18b^3d^2n^3\sqrt[3]{x}}{e^2}+\frac{18b^3d^2n^2\left(d+e\sqrt[3]{x}\right)\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{e^3}-\frac{9b^2dn^2\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2e^3}+\frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^3}-\frac{9bd^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^3}+\frac{9bdn\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2e^3}-\frac{bn\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^3}+\frac{3d^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3}-\frac{3d\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3}+\frac{\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3}$$

```
[Out] 9/4*b^3*d*n^3*(d+e*x^(1/3))^2/e^3-2/9*b^3*n^3*(d+e*x^(1/3))^3/e^3+18*a*b^2*
d^2*n^2*x^(1/3)/e^2-18*b^3*d^2*n^3*x^(1/3)/e^2+18*b^3*d^2*n^2*(d+e*x^(1/3))
```

```
*ln(c*(d+e*x^(1/3))^n)/e^3-9/2*b^2*d*n^2*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(
1/3))^n))/e^3+2/3*b^2*n^2*(d+e*x^(1/3))^3*(a+b*ln(c*(d+e*x^(1/3))^n))/e^3-9
*b*d^2*n*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^3+9/2*b*d*n*(d+e*x^(
1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))^2/e^3-b*n*(d+e*x^(1/3))^3*(a+b*ln(c*(d+
e*x^(1/3))^n))^2/e^3+3*d^2*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^3-
3*d*(d+e*x^(1/3))^2*(a+b*ln(c*(d+e*x^(1/3))^n))^3/e^3+(d+e*x^(1/3))^3*(a+b*
ln(c*(d+e*x^(1/3))^n))^3/e^3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2501, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\begin{aligned}\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3 dx = {} & \frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^3} \\ & -\frac{9b^2dn^2\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2e^3} \\ & +\frac{18ab^2d^2n^2\sqrt[3]{x}}{e^2} \\ & -\frac{9bd^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^3} \\ & +\frac{3d^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3} \\ & -\frac{bn\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^3} \\ & +\frac{9bdn\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2e^3} \\ & +\frac{\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3} \\ & -\frac{3d\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3} \\ & +\frac{18b^3d^2n^2\left(d+e\sqrt[3]{x}\right)\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{e^3} \\ & -\frac{18b^3d^2n^3\sqrt[3]{x}}{e^2}-\frac{2b^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^3}+\frac{9b^3dn^3\left(d+e\sqrt[3]{x}\right)^2}{4e^3}\end{aligned}$$

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (9*b^3*d*n^3*(d + e*x^(1/3))^2)/(4*e^3) - (2*b^3*n^3*(d + e*x^(1/3))^3)/(9*
e^3) + (18*a*b^2*d^2*n^2*x^(1/3))/e^2 - (18*b^3*d^2*n^3*x^(1/3))/e^2 + (18*
b^3*d^2*n^2*(d + e*x^(1/3))*Log[c*(d + e*x^(1/3))^n])/e^3 - (9*b^2*d*n^2*(d
```

```
 + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(2*e^3) + (2*b^2*n^2*(d +
 e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n]))/(3*e^3) - (9*b*d^2*n*(d + e
*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/e^3 + (9*b*d*n*(d + e*x^(1/3)
)^2*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*e^3) - (b*n*(d + e*x^(1/3))^3*(a
 + b*Log[c*(d + e*x^(1/3))^n])^2)/e^3 + (3*d^2*(d + e*x^(1/3))*(a + b*Log[c
*(d + e*x^(1/3))^n])^3)/e^3 - (3*d*(d + e*x^(1/3))^2*(a + b*Log[c*(d + e*x^
(1/3))^n])^3)/e^3 + ((d + e*x^(1/3))^3*(a + b*Log[c*(d + e*x^(1/3))^n])^3)/
e^3
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^2(a+b\log(c(d+ex)^n))^3\,dx,x,\sqrt[3]{x}\right)\\
&= 3\text{Subst}\left(\int\left(\frac{d^2(a+b\log(c(d+ex)^n))^3}{e^2}-\frac{2d(d+ex)\,(a+b\log(c(d+ex)^n))^3}{e^2}\right.\right.\\
&\qquad\left.\left.+\frac{(d+ex)^2\,(a+b\log(c(d+ex)^n))^3}{e^2}\right)dx,x,\sqrt[3]{x}\right)\\
&= \frac{3\text{Subst}\left(\int(d+ex)^2\,(a+b\log(c(d+ex)^n))^3\,dx,x,\sqrt[3]{x}\right)}{e^2}\\
&\quad-\frac{(6d)\text{Subst}\left(\int(d+ex)\,(a+b\log(c(d+ex)^n))^3\,dx,x,\sqrt[3]{x}\right)}{e^2}\\
&\quad+\frac{(3d^2)\,\text{Subst}\left(\int(a+b\log(c(d+ex)^n))^3\,dx,x,\sqrt[3]{x}\right)}{e^2}\\
&= \frac{3\text{Subst}\left(\int x^2(a+b\log(cx^n))^3\,dx,x,d+e\sqrt[3]{x}\right)}{e^3}\\
&\quad-\frac{(6d)\text{Subst}\left(\int x(a+b\log(cx^n))^3\,dx,x,d+e\sqrt[3]{x}\right)}{e^3}\\
&\quad+\frac{(3d^2)\,\text{Subst}\left(\int(a+b\log(cx^n))^3\,dx,x,d+e\sqrt[3]{x}\right)}{e^3}
\end{aligned}
$$

$$= \frac{3d^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3} - \frac{3d\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3} + \frac{\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3} - \frac{(3bn)\text{Subst}\left(\int x^2(a+b\log(cx^n))^2\,dx,x,d+e\sqrt[3]{x}\right)}{e^3} + \frac{(9bdn)\text{Subst}\left(\int x(a+b\log(cx^n))^2\,dx,x,d+e\sqrt[3]{x}\right)}{e^3} - \frac{(9bd^2n)\,\text{Subst}\left(\int (a+b\log(cx^n))^2\,dx,x,d+e\sqrt[3]{x}\right)}{e^3}$$

$$= -\frac{9bd^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^3} + \frac{9bdn\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2e^3} - \frac{bn\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^3} + \frac{3d^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3} - \frac{3d\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3} + \frac{\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3} + \frac{(2b^2n^2)\,\text{Subst}\left(\int x^2(a+b\log(cx^n))\,dx,x,d+e\sqrt[3]{x}\right)}{e^3} - \frac{(9b^2dn^2)\,\text{Subst}\left(\int x(a+b\log(cx^n))\,dx,x,d+e\sqrt[3]{x}\right)}{e^3} + \frac{(18b^2d^2n^2)\,\text{Subst}\left(\int (a+b\log(cx^n))\,dx,x,d+e\sqrt[3]{x}\right)}{e^3}$$

$$
\begin{aligned}
&= \frac{9b^3dn^3\left(d+e\sqrt[3]{x}\right)^2}{4e^3} - \frac{2b^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^3} + \frac{18ab^2d^2n^2\sqrt[3]{x}}{e^2} \\
&\quad - \frac{9b^2dn^2\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2e^3} \\
&\quad + \frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^3} \\
&\quad - \frac{9bd^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^3} \\
&\quad + \frac{9bdn\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2e^3} \\
&\quad - \frac{bn\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^3} \\
&\quad + \frac{3d^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3} \\
&\quad - \frac{3d\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3} \\
&\quad + \frac{\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3} \\
&\quad + \frac{\left(18b^3d^2n^2\right)\text{Subst}\left(\int\log\left(cx^n\right)\,dx,x,d+e\sqrt[3]{x}\right)}{e^3} \\
&= \frac{9b^3dn^3\left(d+e\sqrt[3]{x}\right)^2}{4e^3} - \frac{2b^3n^3\left(d+e\sqrt[3]{x}\right)^3}{9e^3} + \frac{18ab^2d^2n^2\sqrt[3]{x}}{e^2} \\
&\quad - \frac{18b^3d^2n^3\sqrt[3]{x}}{e^2} + \frac{18b^3d^2n^2\left(d+e\sqrt[3]{x}\right)\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)}{e^3} \\
&\quad - \frac{9b^2dn^2\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{2e^3} \\
&\quad + \frac{2b^2n^2\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{3e^3} \\
&\quad - \frac{9bd^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^3} \\
&\quad + \frac{9bdn\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2e^3} \\
&\quad - \frac{bn\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{e^3} \\
&\quad + \frac{3d^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3} \\
&\quad - \frac{3d\left(d+e\sqrt[3]{x}\right)^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3} \\
&\quad + \frac{\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{e^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.83

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$$
$$=\frac{b^3en^3\left(-510d^2+57de\sqrt[3]{x}-8e^2x^{2/3}\right)\sqrt[3]{x}-6ab^2n^2\left(23d^3-66d^2e\sqrt[3]{x}+15de^2x^{2/3}-4e^3x\right)+36a^3(d^3+e^3x)}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^3,x]
```

```
[Out] (b^3*e*n^3*(-510*d^2 + 57*d*e*x^(1/3) - 8*e^2*x^(2/3))*x^(1/3) - 6*a*b^2*n^
2*(23*d^3 - 66*d^2*e*x^(1/3) + 15*d*e^2*x^(2/3) - 4*e^3*x) + 36*a^3*(d^3 +
e^3*x) - 18*a^2*b*n*(11*d^3 + 6*d^2*e*x^(1/3) - 3*d*e^2*x^(2/3) + 2*e^3*x)
+ 6*b*(18*a^2*(d^2 - d*e*x^(1/3) + e^2*x^(2/3)) - 6*a*b*n*(11*d^2 - 5*d*e*x
^(1/3) + 2*e^2*x^(2/3)) + b^2*n^2*(85*d^2 - 19*d*e*x^(1/3) + 4*e^2*x^(2/3))
)*(d + e*x^(1/3))*Log[c*(d + e*x^(1/3))^n] + 18*b^2*(6*a*(d^3 + e^3*x) - b*
n*(11*d^3 + 6*d^2*e*x^(1/3) - 3*d*e^2*x^(2/3) + 2*e^3*x))*Log[c*(d + e*x^(1
/3))^n]^2 + 36*b^3*(d^3 + e^3*x)*Log[c*(d + e*x^(1/3))^n]^3)/(36*e^3)
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{3}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 690, normalized size of antiderivative = 1.58

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$$
$$=\frac{36\,b^3e^3x\log\left(c\right)^3+36\,(b^3e^3n^3x+b^3d^3n^3)\log\left(ex^{\frac{1}{3}}+d\right)^3-36\,(b^3e^3n-3\,ab^2e^3)x\log\left(c\right)^2+18\left(3\,b^3de^2n^3x^{\frac{2}{3}}\right.}{}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/36*(36*b^3*e^3*x*log(c)^3 + 36*(b^3*e^3*n^3*x + b^3*d^3*n^3)*log(e*x^(1/3
) + d)^3 - 36*(b^3*e^3*n - 3*a*b^2*e^3)*x*log(c)^2 + 18*(3*b^3*d*e^2*n^3*x^
(2/3) - 6*b^3*d^2*e*n^3*x^(1/3) - 11*b^3*d^3*n^3 + 6*a*b^2*d^3*n^2 - 2*(b^3
*e^3*n^3 - 3*a*b^2*e^3*n^2)*x + 6*(b^3*e^3*n^2*x + b^3*d^3*n^2)*log(c))*log
```

```
(e*x^(1/3) + d)^2 + 12*(2*b^3*e^3*n^2 - 6*a*b^2*e^3*n + 9*a^2*b*e^3)*x*log(
c) - 4*(2*b^3*e^3*n^3 - 6*a*b^2*e^3*n^2 + 9*a^2*b*e^3*n - 9*a^3*e^3)*x + 6*
(85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*d^3*n + 18*(b^3*e^3*n*x + b^3
*d^3*n)*log(c)^2 + 2*(2*b^3*e^3*n^3 - 6*a*b^2*e^3*n^2 + 9*a^2*b*e^3*n)*x -
6*(11*b^3*d^3*n^2 - 6*a*b^2*d^3*n + 2*(b^3*e^3*n^2 - 3*a*b^2*e^3*n)*x)*log(
c) - 3*(5*b^3*d*e^2*n^3 - 6*b^3*d*e^2*n^2*log(c) - 6*a*b^2*d*e^2*n^2)*x^(2/
3) + 6*(11*b^3*d^2*e*n^3 - 6*b^3*d^2*e*n^2*log(c) - 6*a*b^2*d^2*e*n^2)*x^(1
/3))*log(e*x^(1/3) + d) + 3*(19*b^3*d*e^2*n^3 + 18*b^3*d*e^2*n*log(c)^2 - 3
0*a*b^2*d*e^2*n^2 + 18*a^2*b*d*e^2*n - 6*(5*b^3*d*e^2*n^2 - 6*a*b^2*d*e^2*n
)*log(c))*x^(2/3) - 6*(85*b^3*d^2*e*n^3 + 18*b^3*d^2*e*n*log(c)^2 - 66*a*b^
2*d^2*e*n^2 + 18*a^2*b*d^2*e*n - 6*(11*b^3*d^2*e*n^2 - 6*a*b^2*d^2*e*n)*log
(c))*x^(1/3))/e^3
```

Sympy [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**3,x)
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.04

$$\begin{aligned}
&\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx\\
&=\frac{1}{2}\left(en\left(\frac{6\,d^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^4}-\frac{2\,e^2x-3\,dex^{\frac{2}{3}}+6\,d^2x^{\frac{1}{3}}}{e^3}\right)+6\,x\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)\right)a^2b\\
&\quad+\frac{1}{6}\left(6\,en\left(\frac{6\,d^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^4}-\frac{2\,e^2x-3\,dex^{\frac{2}{3}}+6\,d^2x^{\frac{1}{3}}}{e^3}\right)\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+18\,x\log\left(\left(ex^{\frac{1}{3}}+d\right)^n\right.\right.\\
&\quad+\frac{1}{36}\left(18\,en\left(\frac{6\,d^3\log\left(ex^{\frac{1}{3}}+d\right)}{e^4}-\frac{2\,e^2x-3\,dex^{\frac{2}{3}}+6\,d^2x^{\frac{1}{3}}}{e^3}\right)\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)^2+36\,x\log\left(\left(ex^{\frac{1}{3}}+\right.\right.\right.\\
&\quad+a^3x
\end{aligned}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(e*n*(6*d^3*log(e*x^(1/3) + d)/e^4 - (2*e^2*x - 3*d*e*x^(2/3) + 6*d^2*x
^(1/3))/e^3) + 6*x*log((e*x^(1/3) + d)^n*c))*a^2*b + 1/6*(6*e*n*(6*d^3*log(
e*x^(1/3) + d)/e^4 - (2*e^2*x - 3*d*e*x^(2/3) + 6*d^2*x^(1/3))/e^3)*log((e*
x^(1/3) + d)^n*c) + 18*x*log((e*x^(1/3) + d)^n*c)^2 - (18*d^3*log(e*x^(1/3)
 + d)^2 - 4*e^3*x + 66*d^3*log(e*x^(1/3) + d) + 15*d*e^2*x^(2/3) - 66*d^2*e
*x^(1/3))*n^2/e^3)*a*b^2 + 1/36*(18*e*n*(6*d^3*log(e*x^(1/3) + d)/e^4 - (2*
e^2*x - 3*d*e*x^(2/3) + 6*d^2*x^(1/3))/e^3)*log((e*x^(1/3) + d)^n*c)^2 + 36
*x*log((e*x^(1/3) + d)^n*c)^3 + e*n*((36*d^3*log(e*x^(1/3) + d)^3 + 198*d^3
*log(e*x^(1/3) + d)^2 - 8*e^3*x + 510*d^3*log(e*x^(1/3) + d) + 57*d*e^2*x^(
2/3) - 510*d^2*e*x^(1/3))*n^2/e^4 - 6*(18*d^3*log(e*x^(1/3) + d)^2 - 4*e^3*
x + 66*d^3*log(e*x^(1/3) + d) + 15*d*e^2*x^(2/3) - 66*d^2*e*x^(1/3))*n*log(
(e*x^(1/3) + d)^n*c)/e^4))*b^3 + a^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1072 vs. $2(384) = 768$.

Time = 0.32 (sec) , antiderivative size = 1072, normalized size of antiderivative = 2.45

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3,x, algorithm="giac")
```

```
[Out] 1/36*(36*b^3*e*x*log(c)^3 + (36*(e*x^(1/3) + d)^3*log(e*x^(1/3) + d)^3/e^2
- 108*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) + d)^3/e^2 + 108*(e*x^(1/3) + d)*d^
2*log(e*x^(1/3) + d)^3/e^2 - 36*(e*x^(1/3) + d)^3*log(e*x^(1/3) + d)^2/e^2
+ 162*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) + d)^2/e^2 - 324*(e*x^(1/3) + d)*d^
2*log(e*x^(1/3) + d)^2/e^2 + 24*(e*x^(1/3) + d)^3*log(e*x^(1/3) + d)/e^2 -
162*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) + d)/e^2 + 648*(e*x^(1/3) + d)*d^2*lo
g(e*x^(1/3) + d)/e^2 - 8*(e*x^(1/3) + d)^3/e^2 + 81*(e*x^(1/3) + d)^2*d/e^2
 - 648*(e*x^(1/3) + d)*d^2/e^2)*b^3*n^3 + 6*(18*(e*x^(1/3) + d)^3*log(e*x^(
1/3) + d)^2/e^2 - 54*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) + d)^2/e^2 + 54*(e*x
^(1/3) + d)*d^2*log(e*x^(1/3) + d)^2/e^2 - 12*(e*x^(1/3) + d)^3*log(e*x^(1/
3) + d)/e^2 + 54*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) + d)/e^2 - 108*(e*x^(1/3
) + d)*d^2*log(e*x^(1/3) + d)/e^2 + 4*(e*x^(1/3) + d)^3/e^2 - 27*(e*x^(1/3)
 + d)^2*d/e^2 + 108*(e*x^(1/3) + d)*d^2/e^2)*b^3*n^2*log(c) + 18*(6*(e*x^(1
/3) + d)^3*log(e*x^(1/3) + d)/e^2 - 18*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) +
d)/e^2 + 18*(e*x^(1/3) + d)*d^2*log(e*x^(1/3) + d)/e^2 - 2*(e*x^(1/3) + d)^
3/e^2 + 9*(e*x^(1/3) + d)^2*d/e^2 - 18*(e*x^(1/3) + d)*d^2/e^2)*b^3*n*log(c
)^2 + 108*a*b^2*e*x*log(c)^2 + 6*(18*(e*x^(1/3) + d)^3*log(e*x^(1/3) + d)^2
/e^2 - 54*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) + d)^2/e^2 + 54*(e*x^(1/3) + d)
*d^2*log(e*x^(1/3) + d)^2/e^2 - 12*(e*x^(1/3) + d)^3*log(e*x^(1/3) + d)/e^2
 + 54*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) + d)/e^2 - 108*(e*x^(1/3) + d)*d^2*
log(e*x^(1/3) + d)/e^2 + 4*(e*x^(1/3) + d)^3/e^2 - 27*(e*x^(1/3) + d)^2*d/e
^2 + 108*(e*x^(1/3) + d)*d^2/e^2)*a*b^2*n^2 + 36*(6*(e*x^(1/3) + d)^3*log(e
```

```
*x^(1/3) + d)/e^2 - 18*(e*x^(1/3) + d)^2*d*log(e*x^(1/3) + d)/e^2 + 18*(e*x
^(1/3) + d)*d^2*log(e*x^(1/3) + d)/e^2 - 2*(e*x^(1/3) + d)^3/e^2 + 9*(e*x^(
1/3) + d)^2*d/e^2 - 18*(e*x^(1/3) + d)*d^2/e^2)*a*b^2*n*log(c) + 108*a^2*b*
e*x*log(c) + 18*(6*(e*x^(1/3) + d)^3*log(e*x^(1/3) + d)/e^2 - 18*(e*x^(1/3)
 + d)^2*d*log(e*x^(1/3) + d)/e^2 + 18*(e*x^(1/3) + d)*d^2*log(e*x^(1/3) + d
)/e^2 - 2*(e*x^(1/3) + d)^3/e^2 + 9*(e*x^(1/3) + d)^2*d/e^2 - 18*(e*x^(1/3)
 + d)*d^2/e^2)*a^2*b*n + 36*a^3*e*x)/e
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.27

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\,dx = x\left(a^3-a^2\,b\,n+\frac{2\,a\,b^2\,n^2}{3}-\frac{2\,b^3\,n^3}{9}\right)$$
$$-x^{2/3}\left(\frac{d\left(3\,a^3-3\,a^2\,b\,n+2\,a\,b^2\,n^2-\frac{2\,b^3\,n^3}{3}\right)}{2\,e}-\frac{d\,(6\,a^3-6\,a\,b^2\,n^2+5\,b^3\,n^3)}{4\,e}\right)$$
$$+\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)^3\left(b^3\,x+\frac{b^3\,d^3}{e^3}\right)$$

$$+\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)^2\left(\frac{d\,(6\,a\,b^2\,d^2-11\,b^3\,d^2\,n)}{2\,e^3}-x^{2/3}\left(\frac{3\,b^2\,d\,(3\,a-b\,n)}{2\,e}-\frac{9\,a\,b^2\,d}{2\,e}\right)+b^2\,x\,(3\,a-b\,n)+\frac{d}{\ }\right.$$

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))^3,x)
```

```
[Out] x*(a^3 - (2*b^3*n^3)/9 + (2*a*b^2*n^2)/3 - a^2*b*n) - x^(2/3)*((d*(3*a^3 -
(2*b^3*n^3)/3 + 2*a*b^2*n^2 - 3*a^2*b*n))/(2*e) - (d*(6*a^3 + 5*b^3*n^3 - 6
*a*b^2*n^2))/(4*e)) + log(c*(d + e*x^(1/3))^n)^3*(b^3*x + (b^3*d^3)/e^3) +
log(c*(d + e*x^(1/3))^n)^2*((d*(6*a*b^2*d^2 - 11*b^3*d^2*n))/(2*e^3) - x^(2
/3)*((3*b^2*d*(3*a - b*n))/(2*e) - (9*a*b^2*d)/(2*e)) + b^2*x*(3*a - b*n) +
 (d*x^(1/3)*((3*b^2*d*(3*a - b*n))/e - (9*a*b^2*d)/e))/e) + x^(1/3)*((d*((d
*(3*a^3 - (2*b^3*n^3)/3 + 2*a*b^2*n^2 - 3*a^2*b*n))/e - (d*(6*a^3 + 5*b^3*n
^3 - 6*a*b^2*n^2))/(2*e)))/e + (b^2*d^2*n^2*(6*a - 11*b*n))/e^2) + (log(d +
 e*x^(1/3))*(85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*d^3*n))/(6*e^3) +
 (log(c*(d + e*x^(1/3))^n)*((x^(1/3)*((d*(b*d*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*
n) - 3*b*d*e*(3*a^2 - b^2*n^2)))/e + 6*b^3*d^2*n^2))/e - (x^(2/3)*(b*d*e*(9
*a^2 + 2*b^2*n^2 - 6*a*b*n) - 3*b*d*e*(3*a^2 - b^2*n^2)))/(2*e) + (b*e*x*(9
*a^2 + 2*b^2*n^2 - 6*a*b*n))/3))/e
```

3.460 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 135

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\,dx = 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt[3]{x}}{d}\right) + 9bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\operatorname{PolyLog}\left(2,1+\frac{e\sqrt[3]{x}}{d}\right) - 18b^2n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\operatorname{PolyLog}\left(3,1+\frac{e\sqrt[3]{x}}{d}\right) + 18b^3n^3\operatorname{PolyLog}\left(4,1+\frac{e\sqrt[3]{x}}{d}\right)$$

```
[Out] 3*(a+b*ln(c*(d+e*x^(1/3))^n))^3*ln(-e*x^(1/3)/d)+9*b*n*(a+b*ln(c*(d+e*x^(1/
3))^n))^2*polylog(2,1+e*x^(1/3)/d)-18*b^2*n^2*(a+b*ln(c*(d+e*x^(1/3))^n))*p
olylog(3,1+e*x^(1/3)/d)+18*b^3*n^3*polylog(4,1+e*x^(1/3)/d)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{2504, 2443, 2481, 2421, 2430, 6724\}$

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\,dx = -18b^2n^2\,\text{PolyLog}\left(3,\frac{\sqrt[3]{x}e}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right) + 9bn\,\text{PolyLog}\left(2,\frac{\sqrt[3]{x}e}{d}+1\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2 + 3\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3 + 18b^3n^3\,\text{PolyLog}\left(4,\frac{\sqrt[3]{x}e}{d}+1\right)$$

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x,x]
```

```
[Out] 3*(a + b*Log[c*(d + e*x^(1/3))^n])^3*Log[-((e*x^(1/3))/d)] + 9*b*n*(a + b*L
og[c*(d + e*x^(1/3))^n])^2*PolyLog[2, 1 + (e*x^(1/3))/d] - 18*b^2*n^2*(a +
b*Log[c*(d + e*x^(1/3))^n])*PolyLog[3, 1 + (e*x^(1/3))/d] + 18*b^3*n^3*Poly
Log[4, 1 + (e*x^(1/3))/d]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
```

```
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x}\,dx, x, \sqrt[3]{x}\right)\\
&= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\\
&\quad-(9ben)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{d+ex}\,dx, x, \sqrt[3]{x}\right)\\
&= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\\
&\quad-(9bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx, x, d+e\sqrt[3]{x}\right)\\
&= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\\
&\quad+9bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\text{Li}_2\left(1+\frac{e\sqrt[3]{x}}{d}\right)\\
&\quad-\left(18b^2n^2\right)\text{Subst}\left(\int \frac{(a+b\log(cx^n))\,\text{Li}_2\left(\frac{x}{d}\right)}{x}\,dx, x, d+e\sqrt[3]{x}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\\
&\quad+9bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\operatorname{Li}_2\left(1+\frac{e\sqrt[3]{x}}{d}\right)\\
&\quad-18b^2n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\operatorname{Li}_3\left(1+\frac{e\sqrt[3]{x}}{d}\right)\\
&\quad+\left(18b^3n^3\right)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(\frac{x}{d}\right)}{x}\,dx,x,d+e\sqrt[3]{x}\right)
\end{aligned}
$$

$$
= 3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log\left(-\frac{e\sqrt[3]{x}}{d}\right)+9bn\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\operatorname{Li}_2\left(1+\frac{e\sqrt[3]{x}}{d}\right)-18b^2n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\operatorname{Li}_3\left(1+\frac{e\sqrt[3]{x}}{d}\right)+18b^3n^3\operatorname{Li}_4\left(1+\frac{e\sqrt[3]{x}}{d}\right)
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 333 vs. 2(135) = 270.

Time = 0.13 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.47

$$
\begin{aligned}
\int\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\,dx &= \left(a-bn\log\left(d+e\sqrt[3]{x}\right)+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3\log(x)\\
&\quad+3bn\left(a-bn\log\left(d+e\sqrt[3]{x}\right)+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2\left(\left(\log\left(d+e\sqrt[3]{x}\right)-\log\left(1+\frac{e\sqrt[3]{x}}{d}\right)\right)\log(x)-3\operatorname{PolyLog}\left(2,-\frac{e\sqrt[3]{x}}{d}\right)\right)\\
&\quad+9b^2n^2\left(a-bn\log\left(d+e\sqrt[3]{x}\right)+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\left(\log^2\left(d+e\sqrt[3]{x}\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right)+2\log\left(d+e\sqrt[3]{x}\right)\operatorname{PolyLog}\left(2,1+\frac{e\sqrt[3]{x}}{d}\right)-2\operatorname{PolyLog}\left(3,1+\frac{e\sqrt[3]{x}}{d}\right)\right)\\
&\quad+3b^3n^3\left(\log^3\left(d+e\sqrt[3]{x}\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right)+3\log^2\left(d+e\sqrt[3]{x}\right)\operatorname{PolyLog}\left(2,1+\frac{e\sqrt[3]{x}}{d}\right)-6\log\left(d+e\sqrt[3]{x}\right)\operatorname{PolyLog}\left(3,1+\frac{e\sqrt[3]{x}}{d}\right)+6\operatorname{PolyLog}\left(4,1+\frac{e\sqrt[3]{x}}{d}\right)\right)
\end{aligned}
$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x,x]
```

```
[Out] (a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^3*Log[x] + 3*b*n*
(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2*((Log[d + e*x^(
1/3)] - Log[1 + (e*x^(1/3))/d])*Log[x] - 3*PolyLog[2, -((e*x^(1/3))/d)]) +
9*b^2*n^2*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])*(Log[d
+ e*x^(1/3)]^2*Log[-((e*x^(1/3))/d)] + 2*Log[d + e*x^(1/3)]*PolyLog[2, 1 +
(e*x^(1/3))/d] - 2*PolyLog[3, 1 + (e*x^(1/3))/d]) + 3*b^3*n^3*(Log[d + e*x^
(1/3)]^3*Log[-((e*x^(1/3))/d)] + 3*Log[d + e*x^(1/3)]^2*PolyLog[2, 1 + (e*x
^(1/3))/d] - 6*Log[d + e*x^(1/3)]*PolyLog[3, 1 + (e*x^(1/3))/d] + 6*PolyLog
[4, 1 + (e*x^(1/3))/d])
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{3}}{x}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{3}}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)+a\right)^{3}}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x^(1/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(1/3) + d)^n*c)
^2 + 3*a^2*b*log((e*x^(1/3) + d)^n*c) + a^3)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{3}}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{3}}{x}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**3/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**3/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+a\right)^3}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x,x, algorithm="maxima")
```

```
[Out] b^3*log((e*x^(1/3) + d)^n)^3*log(x) + integrate(-((b^3*e*n*x*log(x) - 3*(b^
3*e*log(c) + a*b^2*e)*x - 3*(b^3*d*log(c) + a*b^2*d)*x^(2/3))*log((e*x^(1/3
) + d)^n)^2 - (b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3
*e)*x - 3*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b^3*d*log(c)^
2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^(2/3))*log((e*x^(1/3) + d)^n) - (b^3*d*lo
g(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^(2/3))/(e*x^2 + d
*x^(5/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+a\right)^3}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(1/3) + d)^n*c) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)^n\right)\right)^3}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))^3/x,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(1/3))^n))^3/x, x)
```

3.461 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 439

$$\begin{aligned}
&\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^2}\,dx\\
&=-\frac{3b^2e^2n^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}}\\
&\quad-\frac{3b^2e^3n^2\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3}\\
&\quad-\frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2dx^{2/3}}+\frac{3be^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{d^3\sqrt[3]{x}}\\
&\quad+\frac{3be^3n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{d^3}\\
&\quad-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}-\frac{6b^2e^3n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right)}{d^3}\\
&\quad+\frac{b^3e^3n^3\log(x)}{d^3}+\frac{3b^3e^3n^3\,\mathrm{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^3}\\
&\quad-\frac{6b^2e^3n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\mathrm{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^3}\\
&\quad-\frac{6b^3e^3n^3\,\mathrm{PolyLog}\left(2,1+\frac{e\sqrt[3]{x}}{d}\right)}{d^3}-\frac{6b^3e^3n^3\,\mathrm{PolyLog}\left(3,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^3}
\end{aligned}$$

```
[Out] -3*b^2*e^2*n^2*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/d^3/x^(1/3)-3*b^2*
e^3*n^2*ln(1-d/(d+e*x^(1/3)))*(a+b*ln(c*(d+e*x^(1/3))^n))/d^3-3/2*b*e*n*(a+
b*ln(c*(d+e*x^(1/3))^n))^2/d/x^(2/3)+3*b*e^2*n*(d+e*x^(1/3))*(a+b*ln(c*(d+e
*x^(1/3))^n))^2/d^3/x^(1/3)+3*b*e^3*n*ln(1-d/(d+e*x^(1/3)))*(a+b*ln(c*(d+e*
x^(1/3))^n))^2/d^3-(a+b*ln(c*(d+e*x^(1/3))^n))^3/x-6*b^2*e^3*n^2*(a+b*ln(c*
(d+e*x^(1/3))^n))*ln(-e*x^(1/3)/d)/d^3+b^3*e^3*n^3*ln(x)/d^3+3*b^3*e^3*n^3*
polylog(2,d/(d+e*x^(1/3)))/d^3-6*b^2*e^3*n^2*(a+b*ln(c*(d+e*x^(1/3))^n))*po
lylog(2,d/(d+e*x^(1/3)))/d^3-6*b^3*e^3*n^3*polylog(2,1+e*x^(1/3)/d)/d^3-6*b
^3*e^3*n^3*polylog(3,d/(d+e*x^(1/3)))/d^3
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^2}\,dx$$
$$= -\frac{6b^2e^3n^2\operatorname{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3} - \frac{3b^2e^3n^2\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3} - \frac{6b^2e^3n^2\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3} - \frac{3b^2e^2n^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}} + \frac{3be^3n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{d^3} + \frac{3be^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{d^3\sqrt[3]{x}} - \frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2dx^{2/3}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x} + \frac{3b^3e^3n^3\operatorname{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^3} - \frac{6b^3e^3n^3\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{x}e}{d}+1\right)}{d^3} - \frac{6b^3e^3n^3\operatorname{PolyLog}\left(3,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^3} + \frac{b^3e^3n^3\log(x)}{d^3}$$

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x^2,x]
```

```
[Out] (-3*b^2*e^2*n^2*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n]))/(d^3*x^(1
/3)) - (3*b^2*e^3*n^2*Log[1 - d/(d + e*x^(1/3))]*(a + b*Log[c*(d + e*x^(1/3
))^n]))/d^3 - (3*b*e*n*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(2*d*x^(2/3)) +
(3*b*e^2*n*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(d^3*x^(1/3)
) + (3*b*e^3*n*Log[1 - d/(d + e*x^(1/3))]*(a + b*Log[c*(d + e*x^(1/3))^n])^
2)/d^3 - (a + b*Log[c*(d + e*x^(1/3))^n])^3/x - (6*b^2*e^3*n^2*(a + b*Log[c
*(d + e*x^(1/3))^n])*Log[-((e*x^(1/3))/d)])/d^3 + (b^3*e^3*n^3*Log[x])/d^3
+ (3*b^3*e^3*n^3*PolyLog[2, d/(d + e*x^(1/3))])/d^3 - (6*b^2*e^3*n^2*(a + b
*Log[c*(d + e*x^(1/3))^n])*PolyLog[2, d/(d + e*x^(1/3))])/d^3 - (6*b^3*e^3*
n^3*PolyLog[2, 1 + (e*x^(1/3))/d])/d^3 - (6*b^3*e^3*n^3*PolyLog[3, d/(d + e
*x^(1/3))])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^4}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}+(3ben)\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(d+ex)}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}+(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt[3]{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}+\frac{(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt[3]{x}\right)}{d}\\
&\quad-\frac{(3ben)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt[3]{x}\right)}{d}\\
&= -\frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2dx^{2/3}}-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}\\
&\quad-\frac{(3ben)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt[3]{x}\right)}{d^2}\\
&\quad+\frac{(3be^2n)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx, x, d+e\sqrt[3]{x}\right)}{d^2}\\
&\quad+\frac{(3b^2en^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt[3]{x}\right)}{d}
\end{aligned}$$

$$= -\frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2dx^{2/3}} + \frac{3be^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{d^3\sqrt[3]{x}}$$
$$+ \frac{3be^3n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{d^3}$$
$$- \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x} + \frac{(3b^2en^2)\,\mathrm{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt[3]{x}\right)}{d^2}$$
$$- \frac{(6b^2e^2n^2)\,\mathrm{Subst}\left(\int \frac{a+b\log(cx^n)}{-\frac{d}{e}+\frac{x}{e}}\,dx, x, d+e\sqrt[3]{x}\right)}{d^3}$$
$$- \frac{(3b^2e^2n^2)\,\mathrm{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx, x, d+e\sqrt[3]{x}\right)}{d^2}$$
$$- \frac{(6b^2e^3n^2)\,\mathrm{Subst}\left(\int \frac{\log\left(1-\frac{d}{x}\right)(a+b\log(cx^n))}{x}\,dx, x, d+e\sqrt[3]{x}\right)}{d^3}$$

$$
\begin{aligned}
&= -\frac{3b^2e^2n^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}} \\
&\quad - \frac{3b^2e^3n^2\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3} \\
&\quad - \frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2dx^{2/3}} + \frac{3be^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{d^3\sqrt[3]{x}} \\
&\quad + \frac{3be^3n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{d^3} \\
&\quad - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x} - \frac{6b^2e^3n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right)}{d^3} \\
&\quad - \frac{6b^2e^3n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{d}{d+e\sqrt[3]{x}}\right)}{d^3} \\
&\quad + \frac{(3b^3e^2n^3)\,\mathrm{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x}{e}}\,dx,x,d+e\sqrt[3]{x}\right)}{d^3} \\
&\quad + \frac{(3b^3e^3n^3)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{d}{x}\right)}{x}\,dx,x,d+e\sqrt[3]{x}\right)}{d^3} \\
&\quad + \frac{(6b^3e^3n^3)\,\mathrm{Subst}\left(\int\frac{\log\left(1-\frac{x}{d}\right)}{x}\,dx,x,d+e\sqrt[3]{x}\right)}{d^3} \\
&\quad + \frac{(6b^3e^3n^3)\,\mathrm{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{d}{x}\right)}{x}\,dx,x,d+e\sqrt[3]{x}\right)}{d^3}
\end{aligned}
$$

$$= -\frac{3b^2e^2n^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3\sqrt[3]{x}}$$
$$-\frac{3b^2e^3n^2\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^3}$$
$$-\frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2dx^{2/3}}+\frac{3be^2n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{d^3\sqrt[3]{x}}$$
$$+\frac{3be^3n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{d^3}$$
$$-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x}-\frac{6b^2e^3n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right)}{d^3}$$
$$+\frac{b^3e^3n^3\log(x)}{d^3}+\frac{3b^3e^3n^3\operatorname{Li}_2\left(\frac{d}{d+e\sqrt[3]{x}}\right)}{d^3}$$
$$-\frac{6b^2e^3n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{d}{d+e\sqrt[3]{x}}\right)}{d^3}$$
$$-\frac{6b^3e^3n^3\operatorname{Li}_2\left(1+\frac{e\sqrt[3]{x}}{d}\right)}{d^3}-\frac{6b^3e^3n^3\operatorname{Li}_3\left(\frac{d}{d+e\sqrt[3]{x}}\right)}{d^3}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 733, normalized size of antiderivative = 1.67

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^2}\,dx$$
$$= \frac{-3bd^2en\sqrt[3]{x}\left(a-bn\log\left(d+e\sqrt[3]{x}\right)+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2+6bde^2nx^{2/3}\left(a-bn\log\left(d+e\sqrt[3]{x}\right)+b\log\left(\right.\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x^2,x]
```

```
[Out] (-3*b*d^2*e*n*x^(1/3)*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))
^n])^2 + 6*b*d*e^2*n*x^(2/3)*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x
^(1/3))^n])^2 - 6*b*d^3*n*Log[d + e*x^(1/3)]*(a - b*n*Log[d + e*x^(1/3)] +
b*Log[c*(d + e*x^(1/3))^n])^2 - 6*b*e^3*n*x*Log[d + e*x^(1/3)]*(a - b*n*Log
[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2 - 2*d^3*(a - b*n*Log[d + e*
x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^3 + 2*b*e^3*n*x*(a - b*n*Log[d + e*x
^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2*Log[x] - 6*b^2*n^2*(a - b*n*Log[d +
 e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])*(d*e^2*x^(2/3) + (d^3 + e^3*x)*Lo
g[d + e*x^(1/3)]^2 + 3*e^3*x*Log[-((e*x^(1/3))/d)] + Log[d + e*x^(1/3)]*(d^
```

```
2*e*x^(1/3) - 2*d*e^2*x^(2/3) - 3*e^3*x - 2*e^3*x*Log[-((e*x^(1/3))/d)]) -
2*e^3*x*PolyLog[2, 1 + (e*x^(1/3))/d]) + b^3*n^3*(-6*d*e^2*x^(2/3)*Log[d +
e*x^(1/3)] - 6*e^3*x*Log[d + e*x^(1/3)] - 3*d^2*e*x^(1/3)*Log[d + e*x^(1/3)
]^2 + 6*d*e^2*x^(2/3)*Log[d + e*x^(1/3)]^2 + 9*e^3*x*Log[d + e*x^(1/3)]^2 -
 2*d^3*Log[d + e*x^(1/3)]^3 - 2*e^3*x*Log[d + e*x^(1/3)]^3 + 6*e^3*x*Log[-(
(e*x^(1/3))/d)] - 18*e^3*x*Log[d + e*x^(1/3)]*Log[-((e*x^(1/3))/d)] + 6*e^3
*x*Log[d + e*x^(1/3)]^2*Log[-((e*x^(1/3))/d)] + 6*e^3*x*(-3 + 2*Log[d + e*x
^(1/3)])*PolyLog[2, 1 + (e*x^(1/3))/d] - 12*e^3*x*PolyLog[3, 1 + (e*x^(1/3)
)/d]))/(2*d^3*x)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{3}}{x^{2}}dx$$

[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x^2,x)

[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x^2,x)

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{3}}{x^{2}}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)+a\right)^{3}}{x^{2}}\,dx$$

[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^2,x, algorithm="fricas")

[Out] integral((b^3*log((e*x^(1/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(1/3) + d)^n*c)
^2 + 3*a^2*b*log((e*x^(1/3) + d)^n*c) + a^3)/x^2, x)

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{3}}{x^{2}}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{3}}{x^{2}}\,dx$$

[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**3/x**2,x)

[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**3/x**2, x)

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^2}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)+a\right)^3}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^2,x, algorithm="maxima")
[Out] -1/2*(2*b^3*d^3*x^(2/3)*log((e*x^(1/3) + d)^n)^3 + (6*b^3*e^3*n*x^(5/3)*log
(e*x^(1/3) + d) - 6*b^3*d*e^2*n*x^(4/3) + 3*b^3*d^2*e*n*x - 2*(b^3*e^3*n*x*
log(x) - 3*b^3*d^3*log(c) - 3*a*b^2*d^3)*x^(2/3))*log((e*x^(1/3) + d)^n)^2)
/(d^3*x^(5/3)) + integrate(1/3*(3*(b^3*d^3*e*log(c)^3 + 3*a*b^2*d^3*e*log(c
)^2 + 3*a^2*b*d^3*e*log(c) + a^3*d^3*e)*x^(5/3) + 3*(b^3*d^4*log(c)^3 + 3*a
*b^2*d^4*log(c)^2 + 3*a^2*b*d^4*log(c) + a^3*d^4)*x^(4/3) + (6*b^3*e^4*n^2*
x^(8/3)*log(e*x^(1/3) + d) - 6*b^3*d*e^3*n^2*x^(7/3) + 3*b^3*d^2*e^2*n^2*x^
2 + 9*(b^3*d^3*e*log(c)^2 + 2*a*b^2*d^3*e*log(c) + a^2*b*d^3*e)*x^(5/3) + 9
*(b^3*d^4*log(c)^2 + 2*a*b^2*d^4*log(c) + a^2*b*d^4)*x^(4/3) - 2*(b^3*e^4*n
^2*x^2*log(x) - 3*(b^3*d^3*e*n*log(c) + a*b^2*d^3*e*n)*x)*x^(2/3))*log((e*x
^(1/3) + d)^n))/(d^3*e*x^(11/3) + d^4*x^(10/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^2}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^n c\right)+a\right)^3}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^2,x, algorithm="giac")
[Out] integrate((b*log((e*x^(1/3) + d)^n*c) + a)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^2}\,dx = \int \frac{\left(a+b\,\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)\right)^3}{x^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/3))^n))^3/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(1/3))^n))^3/x^2, x)
```

3.462 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 765

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^3}\,dx$$
$$= -\frac{b^3e^3n^3}{20d^3x}+\frac{3b^3e^4n^3}{10d^4x^{2/3}}-\frac{71b^3e^5n^3}{40d^5\sqrt[3]{x}}+\frac{71b^3e^6n^3\log\left(d+e\sqrt[3]{x}\right)}{40d^6}$$
$$-\frac{3b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{20d^2x^{4/3}}+\frac{9b^2e^3n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{20d^3x}$$
$$-\frac{47b^2e^4n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{40d^4x^{2/3}}+\frac{77b^2e^5n^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{20d^6\sqrt[3]{x}}$$
$$+\frac{77b^2e^6n^2\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{20d^6}$$
$$-\frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{10dx^{5/3}}+\frac{3be^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{8d^2x^{4/3}}$$
$$-\frac{be^3n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2d^3x}+\frac{3be^4n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{4d^4x^{2/3}}$$
$$-\frac{3be^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2d^6\sqrt[3]{x}}$$
$$-\frac{3be^6n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2d^6}$$
$$-\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2x^2}+\frac{3b^2e^6n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\log\left(-\frac{e\sqrt[3]{x}}{d}\right)}{d^6}$$
$$-\frac{15b^3e^6n^3\log(x)}{8d^6}-\frac{77b^3e^6n^3\,\mathrm{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)}{20d^6}$$
$$+\frac{3b^2e^6n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)\mathrm{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^6}$$
$$+\frac{3b^3e^6n^3\,\mathrm{PolyLog}\left(2,1+\frac{e\sqrt[3]{x}}{d}\right)}{d^6}+\frac{3b^3e^6n^3\,\mathrm{PolyLog}\left(3,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^6}$$

```
[Out] -1/20*b^3*e^3*n^3/d^3/x+3/10*b^3*e^4*n^3/d^4/x^(2/3)-71/40*b^3*e^5*n^3/d^5/
x^(1/3)+71/40*b^3*e^6*n^3*ln(d+e*x^(1/3))/d^6-3/20*b^2*e^2*n^2*(a+b*ln(c*(d
+e*x^(1/3))^n))/d^2/x^(4/3)+9/20*b^2*e^3*n^2*(a+b*ln(c*(d+e*x^(1/3))^n))/d^
3/x-47/40*b^2*e^4*n^2*(a+b*ln(c*(d+e*x^(1/3))^n))/d^4/x^(2/3)+77/20*b^2*e^5
*n^2*(d+e*x^(1/3))*(a+b*ln(c*(d+e*x^(1/3))^n))/d^6/x^(1/3)+77/20*b^2*e^6*n^
2*ln(1-d/(d+e*x^(1/3)))*(a+b*ln(c*(d+e*x^(1/3))^n))/d^6-3/10*b*e*n*(a+b*ln(
```

```
c*(d+e*x^(1/3))^n))^2/d/x^(5/3)+3/8*b*e^2*n*(a+b*ln(c*(d+e*x^(1/3))^n))^2/d
^2/x^(4/3)-1/2*b*e^3*n*(a+b*ln(c*(d+e*x^(1/3))^n))^2/d^3/x+3/4*b*e^4*n*(a+b
*ln(c*(d+e*x^(1/3))^n))^2/d^4/x^(2/3)-3/2*b*e^5*n*(d+e*x^(1/3))*(a+b*ln(c*(
d+e*x^(1/3))^n))^2/d^6/x^(1/3)-3/2*b*e^6*n*ln(1-d/(d+e*x^(1/3)))*(a+b*ln(c*
(d+e*x^(1/3))^n))^2/d^6-1/2*(a+b*ln(c*(d+e*x^(1/3))^n))^3/x^2+3*b^2*e^6*n^2
*(a+b*ln(c*(d+e*x^(1/3))^n))*ln(-e*x^(1/3)/d)/d^6-15/8*b^3*e^6*n^3*ln(x)/d^
6-77/20*b^3*e^6*n^3*polylog(2,d/(d+e*x^(1/3)))/d^6+3*b^2*e^6*n^2*(a+b*ln(c*
(d+e*x^(1/3))^n))*polylog(2,d/(d+e*x^(1/3)))/d^6+3*b^3*e^6*n^3*polylog(2,1+
e*x^(1/3)/d)/d^6+3*b^3*e^6*n^3*polylog(3,d/(d+e*x^(1/3)))/d^6
```

Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 765, normalized size of antiderivative = 1.00, number of steps used = 62, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules

used = {2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31, 46}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^3}\,dx$$

$$= \frac{3b^2e^6n^2\operatorname{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^6} + \frac{77b^2e^6n^2\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{20d^6} + \frac{3b^2e^6n^2\log\left(-\frac{e\sqrt[3]{x}}{d}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{d^6} + \frac{77b^2e^5n^2\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{20d^6\sqrt[3]{x}} - \frac{47b^2e^4n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{40d^4x^{2/3}} + \frac{9b^2e^3n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{20d^3x} - \frac{3b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{20d^2x^{4/3}} - \frac{3be^6n\log\left(1-\frac{d}{d+e\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2d^6} - \frac{3be^5n\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2d^6\sqrt[3]{x}} + \frac{3be^4n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{4d^4x^{2/3}} - \frac{be^3n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2d^3x} + \frac{3be^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{8d^2x^{4/3}} - \frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{10dx^{5/3}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2x^2} - \frac{77b^3e^6n^3\operatorname{PolyLog}\left(2,\frac{d}{d+e\sqrt[3]{x}}\right)}{20d^6} + \frac{3b^3e^6n^3\operatorname{PolyLog}\left(2,\frac{\sqrt[3]{x}e}{d}+1\right)}{d^6} + \frac{3b^3e^6n^3\operatorname{PolyLog}\left(3,\frac{d}{d+e\sqrt[3]{x}}\right)}{d^6} + \frac{71b^3e^6n^3\log\left(d+e\sqrt[3]{x}\right)}{40d^6} - \frac{15b^3e^6n^3\log(x)}{8d^6}$$

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x^3,x]
```

```
[Out] -1/20*(b^3*e^3*n^3)/(d^3*x) + (3*b^3*e^4*n^3)/(10*d^4*x^(2/3)) - (71*b^3*e^
5*n^3)/(40*d^5*x^(1/3)) + (71*b^3*e^6*n^3*Log[d + e*x^(1/3)])/(40*d^6) - (3
*b^2*e^2*n^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(20*d^2*x^(4/3)) + (9*b^2*e^
3*n^2*(a + b*Log[c*(d + e*x^(1/3))^n]))/(20*d^3*x) - (47*b^2*e^4*n^2*(a + b
*Log[c*(d + e*x^(1/3))^n]))/(40*d^4*x^(2/3)) + (77*b^2*e^5*n^2*(d + e*x^(1/
3))*(a + b*Log[c*(d + e*x^(1/3))^n]))/(20*d^6*x^(1/3)) + (77*b^2*e^6*n^2*Lo
g[1 - d/(d + e*x^(1/3))]*(a + b*Log[c*(d + e*x^(1/3))^n]))/(20*d^6) - (3*b*
e*n*(a + b*Log[c*(d + e*x^(1/3))^n])^2)/(10*d*x^(5/3)) + (3*b*e^2*n*(a + b*
Log[c*(d + e*x^(1/3))^n])^2)/(8*d^2*x^(4/3)) - (b*e^3*n*(a + b*Log[c*(d + e
*x^(1/3))^n])^2)/(2*d^3*x) + (3*b*e^4*n*(a + b*Log[c*(d + e*x^(1/3))^n])^2)
/(4*d^4*x^(2/3)) - (3*b*e^5*n*(d + e*x^(1/3))*(a + b*Log[c*(d + e*x^(1/3))^
n])^2)/(2*d^6*x^(1/3)) - (3*b*e^6*n*Log[1 - d/(d + e*x^(1/3))]*(a + b*Log[c
*(d + e*x^(1/3))^n])^2)/(2*d^6) - (a + b*Log[c*(d + e*x^(1/3))^n])^3/(2*x^2
) + (3*b^2*e^6*n^2*(a + b*Log[c*(d + e*x^(1/3))^n])*Log[-((e*x^(1/3))/d)])/
d^6 - (15*b^3*e^6*n^3*Log[x])/(8*d^6) - (77*b^3*e^6*n^3*PolyLog[2, d/(d + e
*x^(1/3))])/(20*d^6) + (3*b^2*e^6*n^2*(a + b*Log[c*(d + e*x^(1/3))^n])*Poly
Log[2, d/(d + e*x^(1/3))])/d^6 + (3*b^3*e^6*n^3*PolyLog[2, 1 + (e*x^(1/3))/
d])/d^6 + (3*b^3*e^6*n^3*PolyLog[3, d/(d + e*x^(1/3))])/d^6
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 46</u>

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

<u>Rule 2351</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

<u>Rule 2354</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
```

```
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^7}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2x^2}+\frac{1}{2}(3ben)\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^6(d+ex)}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2x^2}+\frac{1}{2}(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+e\sqrt[3]{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2x^2}+\frac{(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+e\sqrt[3]{x}\right)}{2d}\\
&\quad-\frac{(3ben)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+e\sqrt[3]{x}\right)}{2d}
\end{aligned}$$

$$= -\frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{10dx^{5/3}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2x^2} - \frac{(3ben)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+e\sqrt[3]{x}\right)}{2d^2} + \frac{(3be^2n)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\sqrt[3]{x}\right)}{2d^2} + \frac{(3b^2en^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+e\sqrt[3]{x}\right)}{5d}$$

$$= -\frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{10dx^{5/3}} + \frac{3be^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{8d^2x^{4/3}} - \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2x^2} + \frac{(3be^2n)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\sqrt[3]{x}\right)}{2d^3} - \frac{(3be^3n)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt[3]{x}\right)}{2d^3} + \frac{(3b^2en^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+e\sqrt[3]{x}\right)}{5d^2} - \frac{(3b^2e^2n^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\sqrt[3]{x}\right)}{5d^2} - \frac{(3b^2e^2n^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\sqrt[3]{x}\right)}{4d^2}$$

$$
\begin{aligned}
&= -\frac{3b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{20d^2x^{4/3}} - \frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{10dx^{5/3}} \\
&+ \frac{3be^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{8d^2x^{4/3}} - \frac{be^3n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2d^3x} \\
&- \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2x^2} - \frac{(3be^3n)\,\mathrm{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt[3]{x}\right)}{2d^4} \\
&+ \frac{(3be^4n)\,\mathrm{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt[3]{x}\right)}{2d^4} \\
&- \frac{(3b^2e^2n^2)\,\mathrm{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\sqrt[3]{x}\right)}{5d^3} \\
&- \frac{(3b^2e^2n^2)\,\mathrm{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\sqrt[3]{x}\right)}{4d^3} \\
&+ \frac{(3b^2e^3n^2)\,\mathrm{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt[3]{x}\right)}{5d^3} \\
&+ \frac{(3b^2e^3n^2)\,\mathrm{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt[3]{x}\right)}{4d^3} \\
&+ \frac{(b^2e^3n^2)\,\mathrm{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt[3]{x}\right)}{d^3} \\
&+ \frac{(3b^3e^2n^3)\,\mathrm{Subst}\left(\int \frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+e\sqrt[3]{x}\right)}{20d^2}
\end{aligned}
$$

$$= -\frac{3b^2e^2n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{20d^2x^{4/3}} + \frac{9b^2e^3n^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)}{20d^3x}$$
$$- \frac{3ben\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{10dx^{5/3}} + \frac{3be^2n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{8d^2x^{4/3}}$$
$$- \frac{be^3n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{2d^3x} + \frac{3be^4n\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2}{4d^4x^{2/3}}$$
$$- \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{2x^2} + \frac{(3be^4n)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt[3]{x}\right)}{2d^5}$$
$$- \frac{(3be^5n)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx, x, d+e\sqrt[3]{x}\right)}{2d^5}$$
$$+ \frac{(3b^2e^3n^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt[3]{x}\right)}{5d^4}$$
$$+ \frac{(3b^2e^3n^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt[3]{x}\right)}{4d^4}$$
$$+ \frac{(b^2e^3n^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt[3]{x}\right)}{d^4}$$
$$- \frac{(3b^2e^4n^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt[3]{x}\right)}{5d^4}$$
$$- \frac{(3b^2e^4n^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt[3]{x}\right)}{4d^4}$$
$$- \frac{(b^2e^4n^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt[3]{x}\right)}{d^4}$$
$$- \frac{(3b^2e^4n^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+e\sqrt[3]{x}\right)}{2d^4}$$
$$+ \frac{(3b^3e^2n^3)\,\text{Subst}\left(\int \left(\frac{e^4}{d(d-x)^4}+\frac{e^4}{d^2(d-x)^3}+\frac{e^4}{d^3(d-x)^2}+\frac{e^4}{d^4(d-x)}+\frac{e^4}{d^4x}\right)dx, x, d+e\sqrt[3]{x}\right)}{20d^2}$$
$$- \frac{(b^3e^3n^3)\,\text{Subst}\left(\int \frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt[3]{x}\right)}{5d^3}$$
$$- \frac{(b^3e^3n^3)\,\text{Subst}\left(\int \frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+e\sqrt[3]{x}\right)}{4d^3}$$

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Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 1074, normalized size of antiderivative = 1.40

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^3}\,dx =$$

$$-\frac{12bd^5en\sqrt[3]{x}\left(a-bn\log\left(d+e\sqrt[3]{x}\right)+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^2-15bd^4e^2nx^{2/3}\left(a-bn\log\left(d+e\sqrt[3]{x}\right)+b\log\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^n])^3/x^3,x]
```

```
[Out] -1/40*(12*b*d^5*e*n*x^(1/3)*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^
(1/3))^n])^2 - 15*b*d^4*e^2*n*x^(2/3)*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c
*(d + e*x^(1/3))^n])^2 + 20*b*d^3*e^3*n*x*(a - b*n*Log[d + e*x^(1/3)] + b*L
og[c*(d + e*x^(1/3))^n])^2 - 30*b*d^2*e^4*n*x^(4/3)*(a - b*n*Log[d + e*x^(1
/3)] + b*Log[c*(d + e*x^(1/3))^n])^2 + 60*b*d*e^5*n*x^(5/3)*(a - b*n*Log[d
+ e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2 + 60*b*d^6*n*Log[d + e*x^(1/3)
]*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^2 - 60*b*e^6*n*
x^2*Log[d + e*x^(1/3)]*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3)
)^n])^2 + 20*d^6*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n])^
3 + 20*b*e^6*n*x^2*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^n]
)^2*Log[x] + b^2*n^2*(a - b*n*Log[d + e*x^(1/3)] + b*Log[c*(d + e*x^(1/3))^
n])*(6*d^4*e^2*x^(2/3) - 18*d^3*e^3*x + 47*d^2*e^4*x^(4/3) - 154*d*e^5*x^(5
/3) + 60*(d^6 - e^6*x^2)*Log[d + e*x^(1/3)]^2 - 274*e^6*x^2*Log[-((e*x^(1/3
))/d)] + 2*Log[d + e*x^(1/3)]*(12*d^5*e*x^(1/3) - 15*d^4*e^2*x^(2/3) + 20*d
^3*e^3*x - 30*d^2*e^4*x^(4/3) + 60*d*e^5*x^(5/3) + 137*e^6*x^2 + 60*e^6*x^2
*Log[-((e*x^(1/3))/d)]) + 120*e^6*x^2*PolyLog[2, 1 + (e*x^(1/3))/d]) + b^3*
n^3*(3*d^4*e^2*x^(2/3)*(2 - 5*Log[d + e*x^(1/3)])*Log[d + e*x^(1/3)] + 12*d
^5*e*x^(1/3)*Log[d + e*x^(1/3)]^2 + 20*d^6*Log[d + e*x^(1/3)]^3 + 2*d^3*e^3
*x*(1 - 9*Log[d + e*x^(1/3)] + 10*Log[d + e*x^(1/3)]^2) - d^2*e^4*x^(4/3)*(
12 - 47*Log[d + e*x^(1/3)] + 30*Log[d + e*x^(1/3)]^2) + d*e^5*x^(5/3)*(71 -
 154*Log[d + e*x^(1/3)] + 60*Log[d + e*x^(1/3)]^2) + 225*e^6*x^2*(-Log[d +
e*x^(1/3)] + Log[-((e*x^(1/3))/d)]) + 137*e^6*x^2*(Log[d + e*x^(1/3)]*(Log[
d + e*x^(1/3)] - 2*Log[-((e*x^(1/3))/d)]) - 2*PolyLog[2, 1 + (e*x^(1/3))/d]
) - 20*e^6*x^2*(Log[d + e*x^(1/3)]^2*(Log[d + e*x^(1/3)] - 3*Log[-((e*x^(1/
3))/d)]) - 6*Log[d + e*x^(1/3)]*PolyLog[2, 1 + (e*x^(1/3))/d] + 6*PolyLog[3
, 1 + (e*x^(1/3))/d])))/(d^6*x^2)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{n}\right)\right)^{3}}{x^{3}}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x^3,x)
[Out] int((a+b*ln(c*(d+e*x^(1/3))^n))^3/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{3}}{x^{3}}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)+a\right)^{3}}{x^{3}}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^3,x, algorithm="fricas")
[Out] integral((b^3*log((e*x^(1/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(1/3) + d)^n*c)
^2 + 3*a^2*b*log((e*x^(1/3) + d)^n*c) + a^3)/x^3, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{3}}{x^{3}}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{3}}{x^{3}}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**n))**3/x**3,x)
[Out] Integral((a + b*log(c*(d + e*x**(1/3))**n))**3/x**3, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{n}\right)\right)^{3}}{x^{3}}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{n}c\right)+a\right)^{3}}{x^{3}}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^3,x, algorithm="maxima")
[Out] -1/2*b^3*log((e*x^(1/3) + d)^n)^3/x^2 + integrate(1/2*((b^3*e*n*x + 6*(b^3*
e*log(c) + a*b^2*e)*x + 6*(b^3*d*log(c) + a*b^2*d)*x^(2/3))*log((e*x^(1/3)
+ d)^n)^2 + 2*(b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3
*e)*x + 6*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b^3*d*log(c)^
2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^(2/3))*log((e*x^(1/3) + d)^n) + 2*(b^3*d*
log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^(2/3))/(e*x^4 +
 d*x^(11/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^nc\right)+a\right)^3}{x^3}\,dx$$

[In] integrate((a+b*log(c*(d+e*x^(1/3))^n))^3/x^3,x, algorithm="giac")

[Out] integrate((b*log((e*x^(1/3) + d)^n*c) + a)^3/x^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^n\right)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+e\,x^{1/3}\right)^n\right)\right)^3}{x^3}\,dx$$

[In] int((a + b*log(c*(d + e*x^(1/3))^n))^3/x^3,x)

[Out] int((a + b*log(c*(d + e*x^(1/3))^n))^3/x^3, x)

3.463 $\int x^3 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 138

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{bd^5nx^{2/3}}{4e^5}-\frac{bd^4nx^{4/3}}{8e^4}+\frac{bd^3nx^2}{12e^3}-\frac{bd^2nx^{8/3}}{16e^2}+\frac{bdnx^{10/3}}{20e}-\frac{1}{24}bnx^4-\frac{bd^6n\log\left(d+ex^{2/3}\right)}{4e^6}+\frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)$$

```
[Out] 1/4*b*d^5*n*x^(2/3)/e^5-1/8*b*d^4*n*x^(4/3)/e^4+1/12*b*d^3*n*x^2/e^3-1/16*b
*d^2*n*x^(8/3)/e^2+1/20*b*d*n*x^(10/3)/e-1/24*b*n*x^4-1/4*b*d^6*n*ln(d+e*x^
(2/3))/e^6+1/4*x^4*(a+b*ln(c*(d+e*x^(2/3))^n))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 45}

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{bd^6n\log\left(d+ex^{2/3}\right)}{4e^6}+\frac{bd^5nx^{2/3}}{4e^5}-\frac{bd^4nx^{4/3}}{8e^4}+\frac{bd^3nx^2}{12e^3}-\frac{bd^2nx^{8/3}}{16e^2}+\frac{bdnx^{10/3}}{20e}-\frac{1}{24}bnx^4$$

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

```
[Out] (b*d^5*n*x^(2/3))/(4*e^5) - (b*d^4*n*x^(4/3))/(8*e^4) + (b*d^3*n*x^2)/(12*e
^3) - (b*d^2*n*x^(8/3))/(16*e^2) + (b*d*n*x^(10/3))/(20*e) - (b*n*x^4)/24 -
 (b*d^6*n*Log[d + e*x^(2/3)])/(4*e^6) + (x^4*(a + b*Log[c*(d + e*x^(2/3))^n
]))/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3}{2}\text{Subst}\left(\int x^5(a+b\log\left(c(d+ex)^n\right))\,dx, x, x^{2/3}\right)\\
&= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{4}(ben)\text{Subst}\left(\int\frac{x^6}{d+ex}\,dx, x, x^{2/3}\right)\\
&= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\\
&\quad-\frac{1}{4}(ben)\text{Subst}\left(\int\left(-\frac{d^5}{e^6}+\frac{d^4x}{e^5}-\frac{d^3x^2}{e^4}+\frac{d^2x^3}{e^3}-\frac{dx^4}{e^2}+\frac{x^5}{e}+\frac{d^6}{e^6(d+ex)}\right)\,dx, x, x^{2/3}\right)\\
&= \frac{bd^5nx^{2/3}}{4e^5}-\frac{bd^4nx^{4/3}}{8e^4}+\frac{bd^3nx^2}{12e^3}-\frac{bd^2nx^{8/3}}{16e^2}+\frac{bdnx^{10/3}}{20e}-\frac{1}{24}bnx^4\\
&\quad-\frac{bd^6n\log\left(d+ex^{2/3}\right)}{4e^6}+\frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.98

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{ax^4}{4}-\frac{1}{4}ben\left(-\frac{d^5x^{2/3}}{e^6}+\frac{d^4x^{4/3}}{2e^5}-\frac{d^3x^2}{3e^4}+\frac{d^2x^{8/3}}{4e^3}-\frac{dx^{10/3}}{5e^2}+\frac{x^4}{6e}+\frac{d^6\log\left(d+ex^{2/3}\right)}{e^7}\right)+\frac{1}{4}bx^4\log\left(c\left(d+ex^{2/3}\right)^n\right)$$

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

```
[Out] (a*x^4)/4 - (b*e*n*(-((d^5*x^(2/3))/e^6) + (d^4*x^(4/3))/(2*e^5) - (d^3*x^2
)/(3*e^4) + (d^2*x^(8/3))/(4*e^3) - (d*x^(10/3))/(5*e^2) + x^4/(6*e) + (d^6
*Log[d + e*x^(2/3)])/e^7))/4 + (b*x^4*Log[c*(d + e*x^(2/3))^n])/4
```

Maple [F]

$$\int x^3\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^n\right)\right)dx$$

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.93

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{60\,be^6x^4\log\left(c\right)+20\,bd^3e^3nx^2-10\,(be^6n-6\,ae^6)x^4+60\,(be^6nx^4-bd^6n)\log\left(\right.}{240\,e^6}$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/240*(60*b*e^6*x^4*log(c) + 20*b*d^3*e^3*n*x^2 - 10*(b*e^6*n - 6*a*e^6)*x^
4 + 60*(b*e^6*n*x^4 - b*d^6*n)*log(e*x^(2/3) + d) - 15*(b*d^2*e^4*n*x^2 - 4
*b*d^5*e*n)*x^(2/3) + 6*(2*b*d*e^5*n*x^3 - 5*b*d^4*e^2*n*x)*x^(1/3))/e^6
```

Sympy [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\text{Timed out}$$

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))**n)),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.78

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{1}{4}\,bx^4\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+\frac{1}{4}\,ax^4-\frac{1}{240}\,ben\left(\frac{60\,d^6\log\left(ex^{\frac{2}{3}}+d\right)}{e^7}+\frac{10\,e^5x^4-12\,de^4x^{\frac{10}{3}}+15\,d^2e^3x^{\frac{8}{3}}-20\,d^3e^2x^2+30\,d^4ex^{\frac{4}{3}}-60\,d^5x^{\frac{2}{3}}}{e^6}\right)$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="maxima")
[Out] 1/4*b*x^4*log((e*x^(2/3) + d)^n*c) + 1/4*a*x^4 - 1/240*b*e*n*(60*d^6*log(e*
x^(2/3) + d)/e^7 + (10*e^5*x^4 - 12*d*e^4*x^(10/3) + 15*d^2*e^3*x^(8/3) - 2
0*d^3*e^2*x^2 + 30*d^4*e*x^(4/3) - 60*d^5*x^(2/3))/e^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(110)=220$.

Time = 0.50 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.84

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{1}{4}\,bx^4\log\left(c\right)+\frac{1}{4}\,ax^4+\frac{1}{240}\,bn\left(\frac{60\left(ex^{\frac{2}{3}}+d\right)^6\log\left(ex^{\frac{2}{3}}+d\right)}{e^6}-\frac{360\left(ex^{\frac{2}{3}}+d\right)^5d\log\left(ex^{\frac{2}{3}}+d\right)}{e^6}+\frac{900\left(ex^{\frac{2}{3}}+d\right)^4d^2\log\left(ex^{\frac{2}{3}}+d\right)}{e^6}\right.$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="giac")
[Out] 1/4*b*x^4*log(c) + 1/4*a*x^4 + 1/240*b*n*(60*(e*x^(2/3) + d)^6*log(e*x^(2/3
) + d)/e^6 - 360*(e*x^(2/3) + d)^5*d*log(e*x^(2/3) + d)/e^6 + 900*(e*x^(2/3
) + d)^4*d^2*log(e*x^(2/3) + d)/e^6 - 1200*(e*x^(2/3) + d)^3*d^3*log(e*x^(2
/3) + d)/e^6 + 900*(e*x^(2/3) + d)^2*d^4*log(e*x^(2/3) + d)/e^6 - 10*(e*x^(
2/3) + d)^6/e^6 + 72*(e*x^(2/3) + d)^5*d/e^6 - 225*(e*x^(2/3) + d)^4*d^2/e^
6 + 400*(e*x^(2/3) + d)^3*d^3/e^6 - 450*(e*x^(2/3) + d)^2*d^4/e^6 - 360*((e
*x^(2/3) + d)*log(e*x^(2/3) + d) - e*x^(2/3) - d)*d^5/e^6)
```

Mupad [B] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82

$$\int x^3 \left(a + b\log\left(c\left(d + ex^{2/3}\right)^n\right)\right)\,dx = \frac{a\,x^4}{4} - \frac{b\,n\,x^4}{24} + \frac{b\,x^4\,\ln\left(c\,\left(d + e\,x^{2/3}\right)^n\right)}{4} + \frac{b\,d\,n\,x^{10/3}}{20\,e} - \frac{b\,d^6\,n\,\ln\left(d + e\,x^{2/3}\right)}{4\,e^6} + \frac{b\,d^3\,n\,x^2}{12\,e^3} - \frac{b\,d^2\,n\,x^{8/3}}{16\,e^2} - \frac{b\,d^4\,n\,x^{4/3}}{8\,e^4} + \frac{b\,d^5\,n\,x^{2/3}}{4\,e^5}$$

```
[In] int(x^3*(a + b*log(c*(d + e*x^(2/3))^n)),x)
```

```
[Out] (a*x^4)/4 - (b*n*x^4)/24 + (b*x^4*log(c*(d + e*x^(2/3))^n))/4 + (b*d*n*x^(1
0/3))/(20*e) - (b*d^6*n*log(d + e*x^(2/3)))/(4*e^6) + (b*d^3*n*x^2)/(12*e^3
) - (b*d^2*n*x^(8/3))/(16*e^2) - (b*d^4*n*x^(4/3))/(8*e^4) + (b*d^5*n*x^(2/
3))/(4*e^5)
```

3.464 $\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 130

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=-\frac{2bd^4n\sqrt[3]{x}}{3e^4}+\frac{2bd^3nx}{9e^3}-\frac{2bd^2nx^{5/3}}{15e^2}+\frac{2bdnx^{7/3}}{21e}-\frac{2}{27}bnx^3+\frac{2bd^{9/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{9/2}}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)$$

```
[Out] -2/3*b*d^4*n*x^(1/3)/e^4+2/9*b*d^3*n*x/e^3-2/15*b*d^2*n*x^(5/3)/e^2+2/21*b*
d*n*x^(7/3)/e-2/27*b*n*x^3+2/3*b*d^(9/2)*n*arctan(x^(1/3)*e^(1/2)/d^(1/2))/
e^(9/2)+1/3*x^3*(a+b*ln(c*(d+e*x^(2/3))^n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2505, 348, 308, 211}

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{2bd^{9/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{9/2}}-\frac{2bd^4n\sqrt[3]{x}}{3e^4}+\frac{2bd^3nx}{9e^3}-\frac{2bd^2nx^{5/3}}{15e^2}+\frac{2bdnx^{7/3}}{21e}-\frac{2}{27}bnx^3$$

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

```
[Out] (-2*b*d^4*n*x^(1/3))/(3*e^4) + (2*b*d^3*n*x)/(9*e^3) - (2*b*d^2*n*x^(5/3))/
(15*e^2) + (2*b*d*n*x^(7/3))/(21*e) - (2*b*n*x^3)/27 + (2*b*d^(9/2)*n*ArcTa
n[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(3*e^(9/2)) + (x^3*(a + b*Log[c*(d + e*x^(2/3
))^n]))/3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{9}(2ben)\int\frac{x^{8/3}}{d+ex^{2/3}}\,dx\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{3}(2ben)\text{Subst}\left(\int\frac{x^{10}}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\\
&\quad-\frac{1}{3}(2ben)\text{Subst}\left(\int\left(\frac{d^4}{e^5}-\frac{d^3x^2}{e^4}+\frac{d^2x^4}{e^3}-\frac{dx^6}{e^2}+\frac{x^8}{e}-\frac{d^5}{e^5\left(d+ex^2\right)}\right)\,dx,x,\sqrt[3]{x}\right)\\
&= -\frac{2bd^4n\sqrt[3]{x}}{3e^4}+\frac{2bd^3nx}{9e^3}-\frac{2bd^2nx^{5/3}}{15e^2}+\frac{2bdnx^{7/3}}{21e}-\frac{2}{27}bnx^3\\
&\quad+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{\left(2bd^5n\right)\text{Subst}\left(\int\frac{1}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{3e^4}\\
&= -\frac{2bd^4n\sqrt[3]{x}}{3e^4}+\frac{2bd^3nx}{9e^3}-\frac{2bd^2nx^{5/3}}{15e^2}+\frac{2bdnx^{7/3}}{21e}-\frac{2}{27}bnx^3\\
&\quad+\frac{2bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{9/2}}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{ax^3}{3}-\frac{2}{9}ben\left(\frac{3d^4\sqrt[3]{x}}{e^5}-\frac{d^3x}{e^4}+\frac{3d^2x^{5/3}}{5e^3}-\frac{3dx^{7/3}}{7e^2}+\frac{x^3}{3e}-\frac{3d^{9/2}\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{e^{11/2}}\right)+\frac{1}{3}bx^3\log\left(c\left(d+ex^{2/3}\right)^n\right)$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

```
[Out] (a*x^3)/3 - (2*b*e*n*((3*d^4*x^(1/3))/e^5 - (d^3*x)/e^4 + (3*d^2*x^(5/3))/(
5*e^3) - (3*d*x^(7/3))/(7*e^2) + x^3/(3*e) - (3*d^(9/2)*ArcTan[(Sqrt[e]*x^(
1/3))/Sqrt[d]])/e^(11/2)))/9 + (b*x^3*Log[c*(d + e*x^(2/3))^n])/3
```

Maple [F]

$$\int x^2\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^n\right)\right)dx$$

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.59

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\left[\frac{315\,be^4nx^3\log\left(ex^{\frac{2}{3}}+d\right)+315\,be^4x^3\log\left(c\right)-126\,bd^2e^2nx^{\frac{5}{3}}+315\,bd^4n\sqrt{-\frac{d}{e}}\cdots}{\cdots}\right.$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="fricas")
```

```
[Out] [1/945*(315*b*e^4*n*x^3*log(e*x^(2/3) + d) + 315*b*e^4*x^3*log(c) - 126*b*d
^2*e^2*n*x^(5/3) + 315*b*d^4*n*sqrt(-d/e)*log((e^3*x^2 - 2*d*e^2*x*sqrt(-d/
```

```
e) - d^3 + 2*(e^3*x*sqrt(-d/e) + d^2*e)*x^(2/3) - 2*(d*e^2*x - d^2*e*sqrt(-
d/e))*x^(1/3))/(e^3*x^2 + d^3)) + 210*b*d^3*e*n*x - 35*(2*b*e^4*n - 9*a*e^4
)*x^3 + 90*(b*d*e^3*n*x^2 - 7*b*d^4*n)*x^(1/3))/e^4, 1/945*(315*b*e^4*n*x^3
*log(e*x^(2/3) + d) + 315*b*e^4*x^3*log(c) - 126*b*d^2*e^2*n*x^(5/3) + 630*
b*d^4*n*sqrt(d/e)*arctan(e*x^(1/3)*sqrt(d/e)/d) + 210*b*d^3*e*n*x - 35*(2*b
*e^4*n - 9*a*e^4)*x^3 + 90*(b*d*e^3*n*x^2 - 7*b*d^4*n)*x^(1/3))/e^4]
```

Sympy [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))**n)),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.88

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{1}{3}\,bx^3\log\left(c\right)+\frac{1}{3}\,ax^3+\frac{1}{945}\left(315\,x^3\log\left(ex^{\frac{2}{3}}+d\right)+2\,e\left(\frac{315\,d^5\arctan\left(\frac{ex^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}e^5}-\frac{35\,e^8x^3-45\,de^7x^{\frac{7}{3}}+63\,d^2e^6x^{\frac{5}{3}}-105\,d^3e^5x+3}{e^9}\right.\right.$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="giac")
[Out] 1/3*b*x^3*log(c) + 1/3*a*x^3 + 1/945*(315*x^3*log(e*x^(2/3) + d) + 2*e*(315
*d^5*arctan(e*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*e^5) - (35*e^8*x^3 - 45*d*e^7*x
^(7/3) + 63*d^2*e^6*x^(5/3) - 105*d^3*e^5*x + 315*d^4*e^4*x^(1/3))/e^9))*b*
n
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\int x^2\,\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)\,dx$$

```
[In] int(x^2*(a + b*log(c*(d + e*x^(2/3))^n)),x)
[Out] int(x^2*(a + b*log(c*(d + e*x^(2/3))^n)), x)
```

3.465 $\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 89

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=-\frac{bd^2nx^{2/3}}{2e^2}+\frac{bdnx^{4/3}}{4e}-\frac{1}{6}bnx^2+\frac{bd^3n\log\left(d+ex^{2/3}\right)}{2e^3}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)$$

```
[Out] -1/2*b*d^2*n*x^(2/3)/e^2+1/4*b*d*n*x^(4/3)/e-1/6*b*n*x^2+1/2*b*d^3*n*ln(d+e
*x^(2/3))/e^3+1/2*x^2*(a+b*ln(c*(d+e*x^(2/3))^n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2504, 2442, 45}

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{bd^3n\log\left(d+ex^{2/3}\right)}{2e^3}-\frac{bd^2nx^{2/3}}{2e^2}+\frac{bdnx^{4/3}}{4e}-\frac{1}{6}bnx^2$$

```
[In] Int[x*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
```

```
[Out] -1/2*(b*d^2*n*x^(2/3))/e^2 + (b*d*n*x^(4/3))/(4*e) - (b*n*x^2)/6 + (b*d^3*n
*Log[d + e*x^(2/3)])/(2*e^3) + (x^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3}{2}\text{Subst}\left(\int x^2(a+b\log\left(c(d+ex)^n\right))\,dx, x, x^{2/3}\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{1}{2}(ben)\text{Subst}\left(\int\frac{x^3}{d+ex}\,dx, x, x^{2/3}\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\\
&\quad -\frac{1}{2}(ben)\text{Subst}\left(\int\left(\frac{d^2}{e^3}-\frac{dx}{e^2}+\frac{x^2}{e}-\frac{d^3}{e^3(d+ex)}\right)\,dx, x, x^{2/3}\right)\\
&= -\frac{bd^2nx^{2/3}}{2e^2}+\frac{bdnx^{4/3}}{4e}-\frac{1}{6}bnx^2+\frac{bd^3n\log\left(d+ex^{2/3}\right)}{2e^3}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\begin{aligned}
\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx &= -\frac{bd^2nx^{2/3}}{2e^2}+\frac{bdnx^{4/3}}{4e}+\frac{ax^2}{2}\\
&\quad -\frac{1}{6}bnx^2+\frac{bd^3n\log\left(d+ex^{2/3}\right)}{2e^3}+\frac{1}{2}bx^2\log\left(c\left(d+ex^{2/3}\right)^n\right)
\end{aligned}$$

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^n]),x]
[Out] -1/2*(b*d^2*n*x^(2/3))/e^2 + (b*d*n*x^(4/3))/(4*e) + (a*x^2)/2 - (b*n*x^2)/
6 + (b*d^3*n*Log[d + e*x^(2/3)])/(2*e^3) + (b*x^2*Log[c*(d + e*x^(2/3))^n])
/2
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)dx$$

[In] int(x*(a+b*ln(c*(d+e*x^(2/3))^n)),x)

[Out] int(x*(a+b*ln(c*(d+e*x^(2/3))^n)),x)

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

$$\int x\Big(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\Big)\,dx=\frac{6\,be^3x^2\log\left(c\right)+3\,bde^2nx^{\frac{4}{3}}-6\,bd^2enx^{\frac{2}{3}}-2\,(be^3n-3\,ae^3)x^2+6\,(be^3nx^2+bd}{12\,e^3}$$

[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="fricas")

[Out] 1/12*(6*b*e^3*x^2*log(c) + 3*b*d*e^2*n*x^(4/3) - 6*b*d^2*e*n*x^(2/3) - 2*(b*e^3*n - 3*a*e^3)*x^2 + 6*(b*e^3*n*x^2 + b*d^3*n)*log(e*x^(2/3) + d))/e^3

Sympy [A] (verification not implemented)

Time = 103.41 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{ax^2}{2}+b\left(-\frac{en\left(-\frac{3d^3\left(\begin{cases}\frac{x^{\frac{2}{3}}}{d} & \text{for } e=0\\ \frac{\log\left(d+ex^{\frac{2}{3}}\right)}{e} & \text{otherwise}\end{cases}\right)}{2e^3}+\frac{3d^2x^{\frac{2}{3}}}{2e^3}-\frac{3dx^{\frac{4}{3}}}{4e^2}+\frac{x^2}{2e}\right)}{3}+\frac{x^2\log\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)}{2}\right)$$

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(2/3))**n)),x)
```

```
[Out] a*x**2/2 + b*(-e*n*(-3*d**3*Piecewise((x**(2/3)/d, Eq(e, 0)), (log(d + e*x*
*(2/3))/e, True))/(2*e**3) + 3*d**2*x**(2/3)/(2*e**3) - 3*d*x**(4/3)/(4*e**
2) + x**2/(2*e))/3 + x**2*log(c*(d + e*x**(2/3))**n)/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{1}{12}\,ben\left(\frac{6\,d^3\log\left(ex^{\frac{2}{3}}+d\right)}{e^4}-\frac{2\,e^2x^2-3\,dex^{\frac{4}{3}}+6\,d^2x^{\frac{2}{3}}}{e^3}\right)+\frac{1}{2}\,bx^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+\frac{1}{2}\,ax^2$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/12*b*e*n*(6*d^3*log(e*x^(2/3) + d)/e^4 - (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d
^2*x^(2/3))/e^3) + 1/2*b*x^2*log((e*x^(2/3) + d)^n*c) + 1/2*a*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{1}{2}\,bx^2\log\left(c\right)+\frac{1}{12}\left(6\,x^2\log\left(ex^{\frac{2}{3}}+d\right)+e\left(\frac{6\,d^3\log\left(\left|ex^{\frac{2}{3}}+d\right|\right)}{e^4}-\frac{2\,e^2x^2-3\,dex^{\frac{4}{3}}+6\,d^2x^{\frac{2}{3}}}{e^3}\right)\right)bn+\frac{1}{2}\,ax^2$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2*log(c) + 1/12*(6*x^2*log(e*x^(2/3) + d) + e*(6*d^3*log(abs(e*x^(2
/3) + d))/e^4 - (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^3))*b*n + 1/2
*a*x^2
```

Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{a\,x^2}{2}-\frac{b\,n\,x^2}{6}+\frac{b\,x^2\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)}{2}+\frac{b\,d\,n\,x^{4/3}}{4\,e}+\frac{b\,d^3\,n\,\ln\left(d+e\,x^{2/3}\right)}{2\,e^3}-\frac{b\,d^2\,n\,x^{2/3}}{2\,e^2}$$

```
[In] int(x*(a + b*log(c*(d + e*x^(2/3))^n)),x)
```

```
[Out] (a*x^2)/2 - (b*n*x^2)/6 + (b*x^2*log(c*(d + e*x^(2/3))^n))/2 + (b*d*n*x^(4/
3))/(4*e) + (b*d^3*n*log(d + e*x^(2/3)))/(2*e^3) - (b*d^2*n*x^(2/3))/(2*e^2
)
```

3.466 $\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 72

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) \, dx = \frac{2bdn\sqrt[3]{x}}{e}+ax-\frac{2bnx}{3}-\frac{2bd^{3/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{e^{3/2}}+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)$$

```
[Out] 2*b*d*n*x^(1/3)/e+a*x-2/3*b*n*x-2*b*d^(3/2)*n*arctan(x^(1/3)*e^(1/2)/d^(1/2
))/e^(3/2)+b*x*ln(c*(d+e*x^(2/3))^n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2498, 348, 308, 211}

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) \, dx = ax-\frac{2bd^{3/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{e^{3/2}}+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)+\frac{2bdn\sqrt[3]{x}}{e}-\frac{2bnx}{3}$$

```
[In] Int[a + b*Log[c*(d + e*x^(2/3))^n],x]
```

```
[Out] (2*b*d*n*x^(1/3))/e + a*x - (2*b*n*x)/3 - (2*b*d^(3/2)*n*ArcTan[(Sqrt[e]*x^
(1/3))/Sqrt[d]])/e^(3/2) + b*x*Log[c*(d + e*x^(2/3))^n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= ax+b\int \log\left(c\left(d+ex^{2/3}\right)^n\right)\,dx\\
&= ax+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)-\frac{1}{3}(2ben)\int \frac{x^{2/3}}{d+ex^{2/3}}\,dx\\
&= ax+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)-(2ben)\text{Subst}\left(\int \frac{x^4}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)\\
&= ax+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)-(2ben)\text{Subst}\left(\int \left(-\frac{d}{e^2}+\frac{x^2}{e}+\frac{d^2}{e^2\left(d+ex^2\right)}\right)\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{2bdn\sqrt[3]{x}}{e}+ax-\frac{2bnx}{3}+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)-\frac{\left(2bd^2n\right)\text{Subst}\left(\int \frac{1}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{e}\\
&= \frac{2bdn\sqrt[3]{x}}{e}+ax-\frac{2bnx}{3}-\frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{e^{3/2}}+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\frac{2bdn\sqrt[3]{x}}{e}+ax-\frac{2bnx}{3}-\frac{2bd^{3/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{e^{3/2}}+bx\log\left(c\left(d+ex^{2/3}\right)^n\right)$$

```
[In] Integrate[a + b*Log[c*(d + e*x^(2/3))^n],x]
```

```
[Out] (2*b*d*n*x^(1/3))/e + a*x - (2*b*n*x)/3 - (2*b*d^(3/2)*n*ArcTan[(Sqrt[e]*x^
(1/3))/Sqrt[d]])/e^(3/2) + b*x*Log[c*(d + e*x^(2/3))^n]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

method	result	size
default	$ax+bx\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)-\frac{2bnx}{3}+\frac{2bdn\,x^{\frac{1}{3}}}{e}-\frac{2bn\,d^{2}\arctan\left(\frac{x^{\frac{1}{3}}e}{\sqrt{de}}\right)}{e\sqrt{de}}$	62
parts	$ax+bx\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)-\frac{2bnx}{3}+\frac{2bdn\,x^{\frac{1}{3}}}{e}-\frac{2bn\,d^{2}\arctan\left(\frac{x^{\frac{1}{3}}e}{\sqrt{de}}\right)}{e\sqrt{de}}$	62

```
[In] int(a+b*ln(c*(d+e*x^(2/3))^n),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*x*ln(c*(d+e*x^(2/3))^n)-2/3*b*n*x+2*b*d*n*x^(1/3)/e-2*b/e*n*d^2/(d*e)
^(1/2)*arctan(x^(1/3)*e/(d*e)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.21

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\left[\frac{3\,benx\log\left(ex^{\frac{2}{3}}+d\right)+3\,bdn\sqrt{-\frac{d}{e}}\log\left(\frac{e^{3}x^{2}+2\,de^{2}x\sqrt{-\frac{d}{e}}-d^{3}-2\left(e^{3}x\sqrt{-\frac{d}{e}}-d^{2}e\right)x^{\frac{2}{3}}-}{e^{3}x^{2}+d^{3}}\right.}{3\,e}\right.$$

[In] integrate(a+b*log(c*(d+e*x^(2/3))^n),x, algorithm="fricas")

[Out] [1/3*(3*b*e*n*x*log(e*x^(2/3) + d) + 3*b*d*n*sqrt(-d/e)*log((e^3*x^2 + 2*d*e^2*x*sqrt(-d/e) - d^3 - 2*(e^3*x*sqrt(-d/e) - d^2*e)*x^(2/3) - 2*(d*e^2*x + d^2*e*sqrt(-d/e))*x^(1/3))/(e^3*x^2 + d^3)) + 3*b*e*x*log(c) + 6*b*d*n*x^(1/3) - (2*b*e*n - 3*a*e)*x)/e, 1/3*(3*b*e*n*x*log(e*x^(2/3) + d) - 6*b*d*n*sqrt(d/e)*arctan(e*x^(1/3)*sqrt(d/e)/d) + 3*b*e*x*log(c) + 6*b*d*n*x^(1/3) - (2*b*e*n - 3*a*e)*x)/e]

Sympy [A] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=ax+b\left(-\frac{2en\left(\frac{3d^2\operatorname{atan}\left(\frac{\sqrt[3]{x}}{\sqrt{\frac{d}{e}}}\right)}{e^3\sqrt{\frac{d}{e}}}-\frac{3d\sqrt[3]{x}}{e^2}+\frac{x}{e}\right)}{3}+x\log\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)$$

[In] integrate(a+b*ln(c*(d+e*x**(2/3))**n),x)

[Out] a*x + b*(-2*e*n*(3*d**2*atan(x**(1/3)/sqrt(d/e))/(e**3*sqrt(d/e)) - 3*d*x**(1/3)/e**2 + x/e)/3 + x*log(c*(d + e*x**(2/3))**n))

Maxima [F(-2)]

Exception generated.

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx=\text{Exception raised: ValueError}$$

[In] integrate(a+b*log(c*(d+e*x^(2/3))^n),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx = -\frac{1}{3}\left(\left(\left(2\,e\left(\frac{3\,d^2\arctan\left(\frac{ex^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}e^2}+\frac{e^2x-3\,dex^{\frac{1}{3}}}{e^3}\right)-3\,x\log\left(ex^{\frac{2}{3}}+d\right)\right)n-3\,x\log\left(c\right)\right)b + ax$$

```
[In] integrate(a+b*log(c*(d+e*x^(2/3))^n),x, algorithm="giac")
```

```
[Out] -1/3*((2*e*(3*d^2*arctan(e*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*e^2) + (e^2*x - 3*
d*e*x^(1/3))/e^3) - 3*x*log(e*x^(2/3) + d))*n - 3*x*log(c))*b + a*x
```

Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\,dx = a\,x + b\,x\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right) - \frac{2\,b\,n\,x}{3} + \frac{2\,b\,d\,n\,x^{1/3}}{e} - \frac{2\,b\,d^{3/2}\,n\,\mathrm{atan}\left(\frac{\sqrt{e}\,x^{1/3}}{\sqrt{d}}\right)}{e^{3/2}}$$

```
[In] int(a + b*log(c*(d + e*x^(2/3))^n),x)
```

```
[Out] a*x + b*x*log(c*(d + e*x^(2/3))^n) - (2*b*n*x)/3 + (2*b*d*n*x^(1/3))/e - (2
*b*d^(3/2)*n*atan((e^(1/2)*x^(1/3))/d^(1/2)))/e^(3/2)
```

3.467 $\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}\,dx=\frac{3}{2}\Big(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\Big)\log\left(-\frac{ex^{2/3}}{d}\right)+\frac{3}{2}bn\,\text{PolyLog}\left(2,1+\frac{ex^{2/3}}{d}\right)$$

```
[Out] 3/2*(a+b*ln(c*(d+e*x^(2/3))^n))*ln(-e*x^(2/3)/d)+3/2*b*n*polylog(2,1+e*x^(2
/3)/d)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2441, 2352}

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}\,dx=\frac{3}{2}\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{3}{2}bn\,\text{PolyLog}\left(2,\frac{x^{2/3}e}{d}+1\right)$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])/x,x]
```

```
[Out] (3*(a + b*Log[c*(d + e*x^(2/3))^n])*Log[-((e*x^(2/3))/d)])/2 + (3*b*n*PolyL
og[2, 1 + (e*x^(2/3))/d])/2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3}{2}\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx, x, x^{2/3}\right)\\
&= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\log\left(-\frac{ex^{2/3}}{d}\right)-\frac{1}{2}(3ben)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^{2/3}\right)\\
&= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\log\left(-\frac{ex^{2/3}}{d}\right)+\frac{3}{2}bn\text{Li}_2\left(1+\frac{ex^{2/3}}{d}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}\,dx = a\log(x) + \frac{3}{2}b\left(\log\left(c\left(d+ex^{2/3}\right)^n\right)\log\left(-\frac{ex^{2/3}}{d}\right)+n\,\text{PolyLog}\left(2,\frac{d+ex^{2/3}}{d}\right)\right)$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])/x,x]
```

```
[Out] a*Log[x] + (3*b*(Log[c*(d + e*x^(2/3))^n]*Log[-((e*x^(2/3))/d)] + n*PolyLog
[2, (d + e*x^(2/3))/d]))/2
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)}{x}dx$$

[In] int((a+b*ln(c*(d+e*x^(2/3))^n))/x,x)

[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))/x,x)

Fricas [F]

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)}{x}\,dx=\int \frac{b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a}{x}\,dx$$

[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x,x, algorithm="fricas")

[Out] integral((b*log((e*x^(2/3) + d)^n*c) + a)/x, x)

Sympy [F]

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)}{x}\,dx=\int \frac{a+b\log\left(c\left(d+ex^{\frac{2}{3}}\right)^{n}\right)}{x}\,dx$$

[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))/x,x)

[Out] Integral((a + b*log(c*(d + e*x**(2/3))**n))/x, x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(44) = 88$.

Time = 0.36 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.07

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)}{x}\,dx=-\frac{3}{2}\left(2\log\left(\frac{ex^{\frac{2}{3}}}{d}+1\right)\log\left(x^{\frac{1}{3}}\right)+\mathrm{Li}_2\left(-\frac{ex^{\frac{2}{3}}}{d}\right)\right)bn+\frac{3\left(2\,benx^{\frac{2}{3}}\log\left(x^{\frac{1}{3}}\right)-benx^{\frac{2}{3}}\right)}{2\,d}+\frac{2\,bd\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}\right)\log\left(x\right)+2\left(bd\log\left(c\right)+ad\right)\log\left(x\right)-\frac{2\,benx\log(x)-3\,benx}{x^{\frac{1}{3}}}}{2\,d}$$

[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x,x, algorithm="maxima")

```
[Out] -3/2*(2*log(e*x^(2/3)/d + 1)*log(x^(1/3)) + dilog(-e*x^(2/3)/d))*b*n + 3/2*
(2*b*e*n*x^(2/3)*log(x^(1/3)) - b*e*n*x^(2/3))/d + 1/2*(2*b*d*log((e*x^(2/3
) + d)^n)*log(x) + 2*(b*d*log(c) + a*d)*log(x) - (2*b*e*n*x*log(x) - 3*b*e*
n*x)/x^(1/3))/d
```

Giac [F]

$$\int \frac{a + b\log\left(c\left(d + ex^{2/3}\right)^n\right)}{x}\,dx = \int \frac{b\log\left(\left(ex^{\frac{2}{3}} + d\right)^n c\right) + a}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c\left(d + ex^{2/3}\right)^n\right)}{x}\,dx = \int \frac{a + b\,\ln\left(c\,\left(d + e\,x^{2/3}\right)^n\right)}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))/x,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))/x, x)
```

3.468 $\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 68

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\,dx=-\frac{2ben}{d\sqrt[3]{x}}-\frac{2be^{3/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}$$

```
[Out] -2*b*e*n/d/x^(1/3)-2*b*e^(3/2)*n*arctan(x^(1/3)*e^(1/2)/d^(1/2))/d^(3/2)+(-
a-b*ln(c*(d+e*x^(2/3))^n))/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2505, 348, 331, 211}

$$\begin{aligned}\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\,dx &= -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}\\ &\quad -\frac{2be^{3/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2ben}{d\sqrt[3]{x}}\end{aligned}$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])/x^2,x]
```

```
[Out] (-2*b*e*n)/(d*x^(1/3)) - (2*b*e^(3/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/
d^(3/2) - (a + b*Log[c*(d + e*x^(2/3))^n])/x
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}+\frac{1}{3}(2ben)\int\frac{1}{\left(d+ex^{2/3}\right)x^{4/3}}\,dx\\
&= -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}+(2ben)\text{Subst}\left(\int\frac{1}{x^2\left(d+ex^2\right)}\,dx,x,\sqrt[3]{x}\right)\\
&= -\frac{2ben}{d\sqrt[3]{x}}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}-\frac{\left(2be^2n\right)\text{Subst}\left(\int\frac{1}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{d}\\
&= -\frac{2ben}{d\sqrt[3]{x}}-\frac{2be^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\,dx=-\frac{a}{x}-\frac{2ben\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{ex^{2/3}}{d}\right)}{d\sqrt[3]{x}}-\frac{b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])/x^2,x]
[Out] -(a/x) - (2*b*e*n*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^(2/3))/d)])/(d*x^(
1/3)) - (b*Log[c*(d + e*x^(2/3))^n])/x
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)}{x^{2}}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))/x^2,x)
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))/x^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 208, normalized size of antiderivative = 3.06

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\,dx=\left[\frac{benx\sqrt{-\frac{e}{d}}\log\left(\frac{e^3x^2+2\,d^2ex\sqrt{-\frac{e}{d}}-d^3-2\left(de^2x\sqrt{-\frac{e}{d}}-d^2e\right)x^{\frac{2}{3}}-2\left(de^2x+d^3\sqrt{-\frac{e}{d}}\right)x^{\frac{1}{3}}}{e^3x^2+d^3}\right)-}{dx}\right.$$
$$\left.-\frac{2\,benx\sqrt{\frac{e}{d}}\arctan\left(x^{\frac{1}{3}}\sqrt{\frac{e}{d}}\right)+bdn\log\left(ex^{\frac{2}{3}}+d\right)+2\,benx^{\frac{2}{3}}+bd\log\left(c\right)+ad}{dx}\right]$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^2,x, algorithm="fricas")
[Out] [(b*e*n*x*sqrt(-e/d)*log((e^3*x^2 + 2*d^2*e*x*sqrt(-e/d) - d^3 - 2*(d*e^2*x
*sqrt(-e/d) - d^2*e)*x^(2/3) - 2*(d*e^2*x + d^3*sqrt(-e/d))*x^(1/3))/(e^3*x
```

```
^2 + d^3)) - b*d*n*log(e*x^(2/3) + d) - 2*b*e*n*x^(2/3) - b*d*log(c) - a*d)
/(d*x), -(2*b*e*n*x*sqrt(e/d)*arctan(x^(1/3)*sqrt(e/d)) + b*d*n*log(e*x^(2/
3) + d) + 2*b*e*n*x^(2/3) + b*d*log(c) + a*d)/(d*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))/x**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\,dx = -\left(2\,e\left(\frac{e\arctan\left(\frac{ex^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}d}+\frac{1}{dx^{\frac{1}{3}}}\right)+\frac{\log\left(ex^{\frac{2}{3}}+d\right)}{x}\right)bn-\frac{b\log\left(c\right)}{x}-\frac{a}{x}$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^2,x, algorithm="giac")
```

```
[Out] -(2*e*(e*arctan(e*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*d) + 1/(d*x^(1/3))) + log(e
*x^(2/3) + d)/x)*b*n - b*log(c)/x - a/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log \left(c\left(d+ex^{2/3}\right)^n\right)}{x^2}\, dx = \int \frac{a+b\,\ln \left(c\,\left(d+e\,x^{2/3}\right)^n\right)}{x^2}\, dx$$

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))/x^2, x)
```

3.469 $\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^3}\,dx = -\frac{ben}{4dx^{4/3}}+\frac{be^2n}{2d^2x^{2/3}} - \frac{be^3n\log\left(d+ex^{2/3}\right)}{2d^3}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2}+\frac{be^3n\log(x)}{3d^3}$$

```
[Out] -1/4*b*e*n/d/x^(4/3)+1/2*b*e^2*n/d^2/x^(2/3)-1/2*b*e^3*n*ln(d+e*x^(2/3))/d^
3+1/2*(-a-b*ln(c*(d+e*x^(2/3))^n))/x^2+1/3*b*e^3*n*ln(x)/d^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 46}

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^3}\,dx = -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2} - \frac{be^3n\log\left(d+ex^{2/3}\right)}{2d^3}+\frac{be^3n\log(x)}{3d^3}+\frac{be^2n}{2d^2x^{2/3}}-\frac{ben}{4dx^{4/3}}$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])/x^3,x]
```

```
[Out] -1/4*(b*e*n)/(d*x^(4/3)) + (b*e^2*n)/(2*d^2*x^(2/3)) - (b*e^3*n*Log[d + e*x
^(2/3)])/(2*d^3) - (a + b*Log[c*(d + e*x^(2/3))^n])/(2*x^2) + (b*e^3*n*Log[
x])/(3*d^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3}{2}\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^4}\,dx, x, x^{2/3}\right)\\
&= -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\text{Subst}\left(\int \frac{1}{x^3(d+ex)}\,dx, x, x^{2/3}\right)\\
&= -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2}\\
&\quad+\frac{1}{2}(ben)\text{Subst}\left(\int \left(\frac{1}{dx^3}-\frac{e}{d^2x^2}+\frac{e^2}{d^3x}-\frac{e^3}{d^3(d+ex)}\right)dx, x, x^{2/3}\right)\\
&= -\frac{ben}{4dx^{4/3}}+\frac{be^2n}{2d^2x^{2/3}}-\frac{be^3n\log\left(d+ex^{2/3}\right)}{2d^3}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2}+\frac{be^3n\log(x)}{3d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\begin{aligned}
\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^3}\,dx &= -\frac{a}{2x^2}-\frac{b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{2x^2}\\
&\quad+\frac{1}{3}ben\left(-\frac{3}{4dx^{4/3}}+\frac{3e}{2d^2x^{2/3}}-\frac{3e^2\log\left(d+ex^{2/3}\right)}{2d^3}+\frac{e^2\log(x)}{d^3}\right)
\end{aligned}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*Log[c*(d + e*x^(2/3))^n])/(2*x^2) + (b*e*n*(-3/(4*d*x^(4/3)
) + (3*e)/(2*d^2*x^(2/3)) - (3*e^2*Log[d + e*x^(2/3)])/(2*d^3) + (e^2*Log[x
])/d^3))/3
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)}{x^{3}}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.90

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)}{x^{3}}\,dx = \frac{4\,be^{3}nx^{2}\log\left(x^{\frac{1}{3}}\right)+2\,bde^{2}nx^{\frac{4}{3}}-bd^{2}enx^{\frac{2}{3}}-2\,bd^{3}\log\left(c\right)-2\,ad^{3}-2\,(be^{3}n}{4\,d^{3}x^{2}}$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*e^3*n*x^2*log(x^(1/3)) + 2*b*d*e^2*n*x^(4/3) - b*d^2*e*n*x^(2/3) -
 2*b*d^3*log(c) - 2*a*d^3 - 2*(b*e^3*n*x^2 + b*d^3*n)*log(e*x^(2/3) + d))/(
d^3*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)}{x^{3}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))/x**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.82

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^3}\,dx = -\frac{1}{4}\,ben\left(\frac{2\,e^2\log\left(ex^{\frac{2}{3}}+d\right)}{d^3}-\frac{2\,e^2\log\left(x^{\frac{2}{3}}\right)}{d^3}-\frac{2\,ex^{\frac{2}{3}}-d}{d^2x^{\frac{4}{3}}}\right)-\frac{b\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*e*n*(2*e^2*log(e*x^(2/3) + d)/d^3 - 2*e^2*log(x^(2/3))/d^3 - (2*e*x^
(2/3) - d)/(d^2*x^(4/3))) - 1/2*b*log((e*x^(2/3) + d)^n*c)/x^2 - 1/2*a/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^3}\,dx = -\frac{\left(e^4\left(\frac{2\,\log\left(\left|ex^{\frac{2}{3}}+d\right|\right)}{d^3}-\frac{2\,\log\left(\left|ex^{\frac{2}{3}}\right|\right)}{d^3}-\frac{2\,\left(ex^{\frac{2}{3}}+d\right)d-3\,d^2}{d^3e^2x^{\frac{4}{3}}}\right)+\frac{2\,e\log\left(ex^{\frac{2}{3}}+d\right)}{x^2}\right)bn}{4\,e}-\frac{b\log\left(c\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(e^4*(2*log(abs(e*x^(2/3) + d))/d^3 - 2*log(abs(e*x^(2/3)))/d^3 - (2*(
e*x^(2/3) + d)*d - 3*d^2)/(d^3*e^2*x^(4/3))) + 2*e*log(e*x^(2/3) + d)/x^2)*
b*n/e - 1/2*b*log(c)/x^2 - 1/2*a/x^2
```

Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.79

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^3}\,dx = -\frac{\frac{b\,e\,n}{2\,d}-\frac{b\,e^2\,n\,x^{2/3}}{d^2}}{2\,x^{4/3}}-\frac{a}{2\,x^2}-\frac{b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)}{2\,x^2}-\frac{b\,e^3\,n\,\mathrm{atanh}\left(\frac{2\,e\,x^{2/3}}{d}+1\right)}{d^3}$$

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))/x^3,x)
```

```
[Out] - ((b*e*n)/(2*d) - (b*e^2*n*x^(2/3))/d^2)/(2*x^(4/3)) - a/(2*x^2) - (b*log(
c*(d + e*x^(2/3))^n))/(2*x^2) - (b*e^3*n*atanh((2*e*x^(2/3))/d + 1))/d^3
```

3.470 $\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^4}\,dx = -\frac{2ben}{21dx^{7/3}}+\frac{2be^2n}{15d^2x^{5/3}}-\frac{2be^3n}{9d^3x}+\frac{2be^4n}{3d^4\sqrt[3]{x}}+\frac{2be^{9/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3d^{9/2}}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}$$

```
[Out] -2/21*b*e*n/d/x^(7/3)+2/15*b*e^2*n/d^2/x^(5/3)-2/9*b*e^3*n/d^3/x+2/3*b*e^4*
n/d^4/x^(1/3)+2/3*b*e^(9/2)*n*arctan(x^(1/3)*e^(1/2)/d^(1/2))/d^(9/2)+1/3*(
-a-b*ln(c*(d+e*x^(2/3))^n))/x^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2505, 348, 331, 211}

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^4}\,dx = -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}+\frac{2be^{9/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3d^{9/2}}+\frac{2be^4n}{3d^4\sqrt[3]{x}}-\frac{2be^3n}{9d^3x}+\frac{2be^2n}{15d^2x^{5/3}}-\frac{2ben}{21dx^{7/3}}$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])/x^4,x]
```

```
[Out] (-2*b*e*n)/(21*d*x^(7/3)) + (2*b*e^2*n)/(15*d^2*x^(5/3)) - (2*b*e^3*n)/(9*d
^3*x) + (2*b*e^4*n)/(3*d^4*x^(1/3)) + (2*b*e^(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3
))/Sqrt[d]])/(3*d^(9/2)) - (a + b*Log[c*(d + e*x^(2/3))^n])/(3*x^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}+\frac{1}{9}(2ben)\int\frac{1}{\left(d+ex^{2/3}\right)x^{10/3}}\,dx\\
&= -\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}+\frac{1}{3}(2ben)\text{Subst}\left(\int\frac{1}{x^8\left(d+ex^2\right)}\,dx,x,\sqrt[3]{x}\right)\\
&= -\frac{2ben}{21dx^{7/3}}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}-\frac{(2be^2n)\,\text{Subst}\left(\int\frac{1}{x^6(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{3d}\\
&= -\frac{2ben}{21dx^{7/3}}+\frac{2be^2n}{15d^2x^{5/3}}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}+\frac{(2be^3n)\,\text{Subst}\left(\int\frac{1}{x^4(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{3d^2}\\
&= -\frac{2ben}{21dx^{7/3}}+\frac{2be^2n}{15d^2x^{5/3}}-\frac{2be^3n}{9d^3x}-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}\\
&\quad-\frac{(2be^4n)\,\text{Subst}\left(\int\frac{1}{x^2(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{3d^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2ben}{21dx^{7/3}} + \frac{2be^2n}{15d^2x^{5/3}} - \frac{2be^3n}{9d^3x} + \frac{2be^4n}{3d^4\sqrt[3]{x}} \\
&\quad - \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3} + \frac{(2be^5n)\,\text{Subst}\left(\int \frac{1}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)}{3d^4} \\
&= -\frac{2ben}{21dx^{7/3}} + \frac{2be^2n}{15d^2x^{5/3}} - \frac{2be^3n}{9d^3x} + \frac{2be^4n}{3d^4\sqrt[3]{x}} \\
&\quad + \frac{2be^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3d^{9/2}} - \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.53

$$
\begin{aligned}
&\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^4}\,dx = -\frac{a}{3x^3} \\
&\quad - \frac{2ben\,\text{Hypergeometric2F1}\left(-\frac{7}{2}, 1, -\frac{5}{2}, -\frac{ex^{2/3}}{d}\right)}{21dx^{7/3}} - \frac{b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3x^3}
\end{aligned}
$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])/x^4,x]
[Out] -1/3*a/x^3 - (2*b*e*n*Hypergeometric2F1[-7/2, 1, -5/2, -((e*x^(2/3))/d)])/(
21*d*x^(7/3)) - (b*Log[c*(d + e*x^(2/3))^n])/(3*x^3)
```

Maple [F]

$$
\int \frac{a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^n\right)}{x^4}dx
$$

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))/x^4,x)
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.54

$$
\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^4}\,dx = \left[\frac{105\,be^4nx^3\sqrt{-\frac{e}{d}}\log\left(\frac{e^3x^2-2\,d^2ex\sqrt{-\frac{e}{d}}-d^3+2\left(de^2x\sqrt{-\frac{e}{d}}+d^2e\right)x^{\frac{2}{3}}-2\left(de^2x-d^3\sqrt{-\frac{e}{d}}\right)x}{e^3x^2+d^3}\right.}{}\right.
$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^4,x, algorithm="fricas")
```

```
[Out] [1/315*(105*b*e^4*n*x^3*sqrt(-e/d)*log((e^3*x^2 - 2*d^2*e*x*sqrt(-e/d) - d^
3 + 2*(d*e^2*x*sqrt(-e/d) + d^2*e)*x^(2/3) - 2*(d*e^2*x - d^3*sqrt(-e/d))*x
^(1/3))/(e^3*x^2 + d^3)) - 70*b*d*e^3*n*x^2 + 42*b*d^2*e^2*n*x^(4/3) - 105*
b*d^4*n*log(e*x^(2/3) + d) - 105*b*d^4*log(c) - 105*a*d^4 + 30*(7*b*e^4*n*x
^2 - b*d^3*e*n)*x^(2/3))/(d^4*x^3), 1/315*(210*b*e^4*n*x^3*sqrt(e/d)*arctan
(x^(1/3)*sqrt(e/d)) - 70*b*d*e^3*n*x^2 + 42*b*d^2*e^2*n*x^(4/3) - 105*b*d^4
*n*log(e*x^(2/3) + d) - 105*b*d^4*log(c) - 105*a*d^4 + 30*(7*b*e^4*n*x^2 -
b*d^3*e*n)*x^(2/3))/(d^4*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))/x**4,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.81

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^4}\,dx = \frac{1}{315}\left(2\,e\left(\frac{105\,e^4\arctan\left(\frac{ex^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}d^4} + \frac{105\,e^3x^2 - 35\,de^2x^{\frac{4}{3}} + 21\,d^2ex^{\frac{2}{3}} - 15\,d^3}{d^4x^{\frac{7}{3}}}\right.\right.$$
$$- \frac{b\log\left(c\right)}{3\,x^3} - \frac{a}{3\,x^3}$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))/x^4,x, algorithm="giac")
```

```
[Out] 1/315*(2*e*(105*e^4*arctan(e*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*d^4) + (105*e^3*
x^2 - 35*d*e^2*x^(4/3) + 21*d^2*e*x^(2/3) - 15*d^3)/(d^4*x^(7/3))) - 105*lo
g(e*x^(2/3) + d)/x^3)*b*n - 1/3*b*log(c)/x^3 - 1/3*a/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)}{x^4}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)}{x^4}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))/x^4,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))/x^4, x)
```

3.471 $\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 482

$$\begin{aligned}\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=&\ \frac{15b^2d^4n^2\left(d+ex^{2/3}\right)^2}{8e^6}\\&-\frac{10b^2d^3n^2\left(d+ex^{2/3}\right)^3}{9e^6}+\frac{15b^2d^2n^2\left(d+ex^{2/3}\right)^4}{32e^6}\\&-\frac{3b^2dn^2\left(d+ex^{2/3}\right)^5}{25e^6}+\frac{b^2n^2\left(d+ex^{2/3}\right)^6}{72e^6}-\frac{3b^2d^5n^2x^{2/3}}{e^5}\\&+\frac{b^2d^6n^2\log^2\left(d+ex^{2/3}\right)}{4e^6}+\frac{3bd^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^6}\\&-\frac{15bd^4n\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4e^6}\\&+\frac{10bd^3n\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^6}\\&-\frac{15bd^2n\left(d+ex^{2/3}\right)^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{8e^6}\\&+\frac{3bdn\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5e^6}\\&-\frac{bn\left(d+ex^{2/3}\right)^6\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{12e^6}\\&-\frac{bd^6n\log\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2e^6}\\&+\frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\end{aligned}$$

```
[Out] 15/8*b^2*d^4*n^2*(d+e*x^(2/3))^2/e^6-10/9*b^2*d^3*n^2*(d+e*x^(2/3))^3/e^6+1
5/32*b^2*d^2*n^2*(d+e*x^(2/3))^4/e^6-3/25*b^2*d*n^2*(d+e*x^(2/3))^5/e^6+1/7
```

```
2*b^2*n^2*(d+e*x^(2/3))^6/e^6-3*b^2*d^5*n^2*x^(2/3)/e^5+1/4*b^2*d^6*n^2*ln(
d+e*x^(2/3))^2/e^6+3*b*d^5*n*(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6-
15/4*b*d^4*n*(d+e*x^(2/3))^2*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6+10/3*b*d^3*n*(
d+e*x^(2/3))^3*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6-15/8*b*d^2*n*(d+e*x^(2/3))^4
*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6+3/5*b*d*n*(d+e*x^(2/3))^5*(a+b*ln(c*(d+e*x
^(2/3))^n))/e^6-1/12*b*n*(d+e*x^(2/3))^6*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6-1/
2*b*d^6*n*ln(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6+1/4*x^4*(a+b*ln(c
*(d+e*x^(2/3))^n))^2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx = -\frac{bd^6n\log\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2e^6}$$
$$+\frac{3bd^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^6}$$
$$-\frac{15bd^4n\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4e^6}$$
$$+\frac{10bd^3n\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^6}$$
$$-\frac{15bd^2n\left(d+ex^{2/3}\right)^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{8e^6}$$
$$+\frac{3bdn\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5e^6}-\frac{bn\left(d+ex^{2/3}\right)^6\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{12e^6}$$
$$+\frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2+\frac{b^2d^6n^2\log^2\left(d+ex^{2/3}\right)}{4e^6}-\frac{3b^2d^5n^2x^{2/3}}{e^5}+\frac{15b^2d^4n^2\left(d+ex^{2/3}\right)^2}{8e^6}-\frac{10b^2d^3n^2(}{9}$$

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] (15*b^2*d^4*n^2*(d + e*x^(2/3))^2)/(8*e^6) - (10*b^2*d^3*n^2*(d + e*x^(2/3)
)^3)/(9*e^6) + (15*b^2*d^2*n^2*(d + e*x^(2/3))^4)/(32*e^6) - (3*b^2*d*n^2*(
d + e*x^(2/3))^5)/(25*e^6) + (b^2*n^2*(d + e*x^(2/3))^6)/(72*e^6) - (3*b^2*
d^5*n^2*x^(2/3))/e^5 + (b^2*d^6*n^2*Log[d + e*x^(2/3)]^2)/(4*e^6) + (3*b*d^
5*n*(d + e*x^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n]))/e^6 - (15*b*d^4*n*(d
+ e*x^(2/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(4*e^6) + (10*b*d^3*n*(d +
 e*x^(2/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*e^6) - (15*b*d^2*n*(d +
e*x^(2/3))^4*(a + b*Log[c*(d + e*x^(2/3))^n]))/(8*e^6) + (3*b*d*n*(d + e*x^
(2/3))^5*(a + b*Log[c*(d + e*x^(2/3))^n]))/(5*e^6) - (b*n*(d + e*x^(2/3))^6
*(a + b*Log[c*(d + e*x^(2/3))^n]))/(12*e^6) - (b*d^6*n*Log[d + e*x^(2/3)]*(
a + b*Log[c*(d + e*x^(2/3))^n]))/(2*e^6) + (x^4*(a + b*Log[c*(d + e*x^(2/3)
)^n])^2)/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
```

```
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3}{2}\text{Subst}\left(\int x^5(a+b\log(c(d+ex)^n))^2\,dx, x, x^{2/3}\right)\\
&= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{1}{2}(ben)\text{Subst}\left(\int \frac{x^6(a+b\log(c(d+ex)^n))}{d+ex}\,dx, x, x^{2/3}\right)\\
&= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\\
&\quad -\frac{1}{2}(bn)\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6(a+b\log(cx^n))}{x}\,dx, x, d+ex^{2/3}\right)\\
&= \frac{3bd^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^6}\\
&\quad -\frac{15bd^4n\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4e^6}\\
&\quad +\frac{10bd^3n\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^6}\\
&\quad -\frac{15bd^2n\left(d+ex^{2/3}\right)^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{8e^6}\\
&\quad +\frac{3bdn\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5e^6}\\
&\quad -\frac{bn\left(d+ex^{2/3}\right)^6\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{12e^6}\\
&\quad -\frac{bd^6n\log\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2e^6}\\
&\quad +\frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2+\frac{1}{2}\left(b^2n^2\right)\text{Subst}\left(\int \frac{x(-360d^5+450d^4x-400d^3x^2+225d^2x^3-}{60e^6x}\right.
\end{aligned}$$

$$= \frac{3bd^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^6} - \frac{15bd^4n\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4e^6} + \frac{10bd^3n\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^6} - \frac{15bd^2n\left(d+ex^{2/3}\right)^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{8e^6} + \frac{3bdn\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5e^6} - \frac{bn\left(d+ex^{2/3}\right)^6\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{12e^6} - \frac{bd^6n\log\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2e^6} + \frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 + \frac{(b^2n^2)\,\mathrm{Subst}\left(\int \frac{x(-360d^5+450d^4x-400d^3x^2+225d^2x^3-72dx^4+10x^5)+60d^6}{x}\right.}{120e^6}$$

$$= \frac{3bd^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^6} - \frac{15bd^4n\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4e^6} + \frac{10bd^3n\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^6} - \frac{15bd^2n\left(d+ex^{2/3}\right)^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{8e^6} + \frac{3bdn\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5e^6} - \frac{bn\left(d+ex^{2/3}\right)^6\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{12e^6} - \frac{bd^6n\log\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2e^6} + \frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 + \frac{(b^2n^2)\,\mathrm{Subst}\left(\int \left(-360d^5+450d^4x-400d^3x^2+225d^2x^3-7\right.\right.}{120e^6}$$

$$= \frac{15b^2d^4n^2\left(d+ex^{2/3}\right)^2}{8e^6} - \frac{10b^2d^3n^2\left(d+ex^{2/3}\right)^3}{9e^6} + \frac{15b^2d^2n^2\left(d+ex^{2/3}\right)^4}{32e^6} - \frac{3b^2dn^2\left(d+ex^{2/3}\right)^5}{25e^6} + \frac{b^2n^2\left(d+ex^{2/3}\right)^6}{72e^6} - \frac{3b^2d^5n^2x^{2/3}}{e^5} + \frac{3bd^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^6} - \frac{15bd^4n\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4e^6} + \frac{10bd^3n\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^6} - \frac{15bd^2n\left(d+ex^{2/3}\right)^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{8e^6} + \frac{3bdn\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5e^6} - \frac{bn\left(d+ex^{2/3}\right)^6\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{12e^6} - \frac{bd^6n\log\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2e^6} + \frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 + \frac{(b^2d^6n^2)\,\text{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, d+ex^{2/3}\right)}{2e^6}$$

$$= \frac{15b^2d^4n^2\left(d+ex^{2/3}\right)^2}{8e^6} - \frac{10b^2d^3n^2\left(d+ex^{2/3}\right)^3}{9e^6} + \frac{15b^2d^2n^2\left(d+ex^{2/3}\right)^4}{32e^6} - \frac{3b^2dn^2\left(d+ex^{2/3}\right)^5}{25e^6} + \frac{b^2n^2\left(d+ex^{2/3}\right)^6}{72e^6} - \frac{3b^2d^5n^2x^{2/3}}{e^5} + \frac{b^2d^6n^2\log^2\left(d+ex^{2/3}\right)}{4e^6} + \frac{3bd^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^6} - \frac{15bd^4n\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4e^6} + \frac{10bd^3n\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^6} - \frac{15bd^2n\left(d+ex^{2/3}\right)^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{8e^6} + \frac{3bdn\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5e^6} - \frac{bn\left(d+ex^{2/3}\right)^6\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{12e^6} - \frac{bd^6n\log\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2e^6} + \frac{1}{4}x^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.67

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\frac{ex^{2/3}\left(1800a^2e^5x^{10/3}+60abn\left(60d^5-30d^4ex^{2/3}+20d^3e^2x^{4/3}-15d^2e^3x^2+1\right.\right.}{}$$

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] (e*x^(2/3)*(1800*a^2*e^5*x^(10/3) + 60*a*b*n*(60*d^5 - 30*d^4*e*x^(2/3) + 2
0*d^3*e^2*x^(4/3) - 15*d^2*e^3*x^2 + 12*d*e^4*x^(8/3) - 10*e^5*x^(10/3)) +
b^2*n^2*(-8820*d^5 + 2610*d^4*e*x^(2/3) - 1140*d^3*e^2*x^(4/3) + 555*d^2*e^
3*x^2 - 264*d*e^4*x^(8/3) + 100*e^5*x^(10/3))) + 180*b*d^6*n*(-20*a + 49*b*
n)*Log[d + e*x^(2/3)] - 60*b*e*x^(2/3)*(-60*a*e^5*x^(10/3) + b*n*(-60*d^5 +
 30*d^4*e*x^(2/3) - 20*d^3*e^2*x^(4/3) + 15*d^2*e^3*x^2 - 12*d*e^4*x^(8/3)
+ 10*e^5*x^(10/3)))*Log[c*(d + e*x^(2/3))^n] - 1800*b^2*(d^6 - e^6*x^4)*Log
[c*(d + e*x^(2/3))^n]^2)/(7200*e^6)
```

Maple [F]

$$\int x^3\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^n\right)\right)^2dx$$

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.05

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\frac{1800\,b^2e^6x^4\log\left(c\right)^2+100\left(b^2e^6n^2-6\,abe^6n+18\,a^2e^6\right)x^4-60\left(19\,b^2d^3e^3n^2-\right.}{}$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] 1/7200*(1800*b^2*e^6*x^4*log(c)^2 + 100*(b^2*e^6*n^2 - 6*a*b*e^6*n + 18*a^2
*e^6)*x^4 - 60*(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)*x^2 + 1800*(b^2*e^6*
n^2*x^4 - b^2*d^6*n^2)*log(e*x^(2/3) + d)^2 + 60*(20*b^2*d^3*e^3*n^2*x^2 +
147*b^2*d^6*n^2 - 60*a*b*d^6*n - 10*(b^2*e^6*n^2 - 6*a*b*e^6*n)*x^4 + 60*(b
```

```
^2*e^6*n*x^4 - b^2*d^6*n)*log(c) - 15*(b^2*d^2*e^4*n^2*x^2 - 4*b^2*d^5*e*n^
2)*x^(2/3) + 6*(2*b^2*d*e^5*n^2*x^3 - 5*b^2*d^4*e^2*n^2*x)*x^(1/3))*log(e*x
^(2/3) + d) + 600*(2*b^2*d^3*e^3*n*x^2 - (b^2*e^6*n - 6*a*b*e^6)*x^4)*log(c
) - 15*(588*b^2*d^5*e*n^2 - 240*a*b*d^5*e*n - (37*b^2*d^2*e^4*n^2 - 60*a*b*
d^2*e^4*n)*x^2 + 60*(b^2*d^2*e^4*n*x^2 - 4*b^2*d^5*e*n)*log(c))*x^(2/3) - 6
*(4*(11*b^2*d*e^5*n^2 - 30*a*b*d*e^5*n)*x^3 - 15*(29*b^2*d^4*e^2*n^2 - 20*a
*b*d^4*e^2*n)*x - 60*(2*b^2*d*e^5*n*x^3 - 5*b^2*d^4*e^2*n*x)*log(c))*x^(1/3
))/e^6
```

Sympy [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))**n))**2,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.68

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\frac{1}{4}\,b^2x^4\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)^2$$
$$+\frac{1}{2}\,abx^4\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+\frac{1}{4}\,a^2x^4$$
$$-\frac{1}{120}\,aben\left(\frac{60\,d^6\log\left(ex^{\frac{2}{3}}+d\right)}{e^7}+\frac{10\,e^5x^4-12\,de^4x^{\frac{10}{3}}+15\,d^2e^3x^{\frac{8}{3}}-20\,d^3e^2x^2+30\,d^4ex^{\frac{4}{3}}-60\,d^5x^{\frac{2}{3}}}{e^6}\right)$$
$$-\frac{1}{7200}\left(60\,en\left(\frac{60\,d^6\log\left(ex^{\frac{2}{3}}+d\right)}{e^7}+\frac{10\,e^5x^4-12\,de^4x^{\frac{10}{3}}+15\,d^2e^3x^{\frac{8}{3}}-20\,d^3e^2x^2+30\,d^4ex^{\frac{4}{3}}-60\,d^5x^{\frac{2}{3}}}{e^6}\right)\right.$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="maxima")
[Out] 1/4*b^2*x^4*log((e*x^(2/3) + d)^n*c)^2 + 1/2*a*b*x^4*log((e*x^(2/3) + d)^n*
c) + 1/4*a^2*x^4 - 1/120*a*b*e*n*(60*d^6*log(e*x^(2/3) + d)/e^7 + (10*e^5*x
^4 - 12*d*e^4*x^(10/3) + 15*d^2*e^3*x^(8/3) - 20*d^3*e^2*x^2 + 30*d^4*e*x^(
4/3) - 60*d^5*x^(2/3))/e^6) - 1/7200*(60*e*n*(60*d^6*log(e*x^(2/3) + d)/e^7
 + (10*e^5*x^4 - 12*d*e^4*x^(10/3) + 15*d^2*e^3*x^(8/3) - 20*d^3*e^2*x^2 +
30*d^4*e*x^(4/3) - 60*d^5*x^(2/3))/e^6)*log((e*x^(2/3) + d)^n*c) - (100*e^6
```

```
*x^4 - 264*d*e^5*x^(10/3) + 555*d^2*e^4*x^(8/3) - 1140*d^3*e^3*x^2 + 1800*d
^6*log(e*x^(2/3) + d)^2 + 2610*d^4*e^2*x^(4/3) + 8820*d^6*log(e*x^(2/3) + d
) - 8820*d^5*e*x^(2/3))*n^2/e^6)*b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 905 vs. $2(412) = 824$.

Time = 0.61 (sec) , antiderivative size = 905, normalized size of antiderivative = 1.88

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] 1/4*b^2*x^4*log(c)^2 + 1/2*a*b*x^4*log(c) + 1/4*a^2*x^4 + 1/7200*(1800*(e*x
^(2/3) + d)^6*log(e*x^(2/3) + d)^2/e^6 - 10800*(e*x^(2/3) + d)^5*d*log(e*x^
(2/3) + d)^2/e^6 + 27000*(e*x^(2/3) + d)^4*d^2*log(e*x^(2/3) + d)^2/e^6 - 3
6000*(e*x^(2/3) + d)^3*d^3*log(e*x^(2/3) + d)^2/e^6 + 27000*(e*x^(2/3) + d)
^2*d^4*log(e*x^(2/3) + d)^2/e^6 - 600*(e*x^(2/3) + d)^6*log(e*x^(2/3) + d)/
e^6 + 4320*(e*x^(2/3) + d)^5*d*log(e*x^(2/3) + d)/e^6 - 13500*(e*x^(2/3) +
d)^4*d^2*log(e*x^(2/3) + d)/e^6 + 24000*(e*x^(2/3) + d)^3*d^3*log(e*x^(2/3)
 + d)/e^6 - 27000*(e*x^(2/3) + d)^2*d^4*log(e*x^(2/3) + d)/e^6 + 100*(e*x^(
2/3) + d)^6/e^6 - 864*(e*x^(2/3) + d)^5*d/e^6 + 3375*(e*x^(2/3) + d)^4*d^2/
e^6 - 8000*(e*x^(2/3) + d)^3*d^3/e^6 + 13500*(e*x^(2/3) + d)^2*d^4/e^6 - 10
800*((e*x^(2/3) + d)*log(e*x^(2/3) + d)^2 - 2*(e*x^(2/3) + d)*log(e*x^(2/3)
 + d) + 2*e*x^(2/3) + 2*d)*d^5/e^6)*b^2*n^2 + 1/120*b^2*n*(60*(e*x^(2/3) +
d)^6*log(e*x^(2/3) + d)/e^6 - 360*(e*x^(2/3) + d)^5*d*log(e*x^(2/3) + d)/e^
6 + 900*(e*x^(2/3) + d)^4*d^2*log(e*x^(2/3) + d)/e^6 - 1200*(e*x^(2/3) + d)
^3*d^3*log(e*x^(2/3) + d)/e^6 + 900*(e*x^(2/3) + d)^2*d^4*log(e*x^(2/3) + d
)/e^6 - 10*(e*x^(2/3) + d)^6/e^6 + 72*(e*x^(2/3) + d)^5*d/e^6 - 225*(e*x^(2
/3) + d)^4*d^2/e^6 + 400*(e*x^(2/3) + d)^3*d^3/e^6 - 450*(e*x^(2/3) + d)^2*
d^4/e^6 - 360*((e*x^(2/3) + d)*log(e*x^(2/3) + d) - e*x^(2/3) - d)*d^5/e^6)
*log(c) + 1/120*a*b*n*(60*(e*x^(2/3) + d)^6*log(e*x^(2/3) + d)/e^6 - 360*(e
*x^(2/3) + d)^5*d*log(e*x^(2/3) + d)/e^6 + 900*(e*x^(2/3) + d)^4*d^2*log(e*
x^(2/3) + d)/e^6 - 1200*(e*x^(2/3) + d)^3*d^3*log(e*x^(2/3) + d)/e^6 + 900*
(e*x^(2/3) + d)^2*d^4*log(e*x^(2/3) + d)/e^6 - 10*(e*x^(2/3) + d)^6/e^6 + 7
2*(e*x^(2/3) + d)^5*d/e^6 - 225*(e*x^(2/3) + d)^4*d^2/e^6 + 400*(e*x^(2/3)
+ d)^3*d^3/e^6 - 450*(e*x^(2/3) + d)^2*d^4/e^6 - 360*((e*x^(2/3) + d)*log(e
*x^(2/3) + d) - e*x^(2/3) - d)*d^5/e^6)
```

Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 440, normalized size of antiderivative = 0.91

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\frac{a^2\,x^4}{4}+\frac{b^2\,x^4\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)^2}{4}$$
$$+\frac{b^2\,n^2\,x^4}{72}+\frac{a\,b\,x^4\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)}{2}-\frac{b^2\,d^6\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)^2}{4\,e^6}-\frac{a\,b\,n\,x^4}{12}$$
$$-\frac{b^2\,n\,x^4\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)}{12}+\frac{49\,b^2\,d^6\,n^2\,\ln\left(d+e\,x^{2/3}\right)}{40\,e^6}-\frac{19\,b^2\,d^3\,n^2\,x^2}{120\,e^3}$$
$$+\frac{37\,b^2\,d^2\,n^2\,x^{8/3}}{480\,e^2}+\frac{29\,b^2\,d^4\,n^2\,x^{4/3}}{80\,e^4}-\frac{49\,b^2\,d^5\,n^2\,x^{2/3}}{40\,e^5}-\frac{11\,b^2\,d\,n^2\,x^{10/3}}{300\,e}$$
$$+\frac{b^2\,d^3\,n\,x^2\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)}{6\,e^3}-\frac{b^2\,d^2\,n\,x^{8/3}\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)}{8\,e^2}$$
$$-\frac{b^2\,d^4\,n\,x^{4/3}\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)}{4\,e^4}+\frac{b^2\,d^5\,n\,x^{2/3}\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)}{2\,e^5}$$
$$+\frac{a\,b\,d\,n\,x^{10/3}}{10\,e}-\frac{a\,b\,d^6\,n\,\ln\left(d+e\,x^{2/3}\right)}{2\,e^6}+\frac{b^2\,d\,n\,x^{10/3}\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)}{10\,e}$$
$$+\frac{a\,b\,d^3\,n\,x^2}{6\,e^3}-\frac{a\,b\,d^2\,n\,x^{8/3}}{8\,e^2}-\frac{a\,b\,d^4\,n\,x^{4/3}}{4\,e^4}+\frac{a\,b\,d^5\,n\,x^{2/3}}{2\,e^5}$$

```
[In] int(x^3*(a + b*log(c*(d + e*x^(2/3))^n))^2,x)
```

```
[Out] (a^2*x^4)/4 + (b^2*x^4*log(c*(d + e*x^(2/3))^n)^2)/4 + (b^2*n^2*x^4)/72 + (
a*b*x^4*log(c*(d + e*x^(2/3))^n))/2 - (b^2*d^6*log(c*(d + e*x^(2/3))^n)^2)/
(4*e^6) - (a*b*n*x^4)/12 - (b^2*n*x^4*log(c*(d + e*x^(2/3))^n))/12 + (49*b^
2*d^6*n^2*log(d + e*x^(2/3)))/(40*e^6) - (19*b^2*d^3*n^2*x^2)/(120*e^3) + (
37*b^2*d^2*n^2*x^(8/3))/(480*e^2) + (29*b^2*d^4*n^2*x^(4/3))/(80*e^4) - (49
*b^2*d^5*n^2*x^(2/3))/(40*e^5) - (11*b^2*d*n^2*x^(10/3))/(300*e) + (b^2*d^3
*n*x^2*log(c*(d + e*x^(2/3))^n))/(6*e^3) - (b^2*d^2*n*x^(8/3)*log(c*(d + e*
x^(2/3))^n))/(8*e^2) - (b^2*d^4*n*x^(4/3)*log(c*(d + e*x^(2/3))^n))/(4*e^4)
 + (b^2*d^5*n*x^(2/3)*log(c*(d + e*x^(2/3))^n))/(2*e^5) + (a*b*d*n*x^(10/3)
)/(10*e) - (a*b*d^6*n*log(d + e*x^(2/3)))/(2*e^6) + (b^2*d*n*x^(10/3)*log(c
*(d + e*x^(2/3))^n))/(10*e) + (a*b*d^3*n*x^2)/(6*e^3) - (a*b*d^2*n*x^(8/3))
/(8*e^2) - (a*b*d^4*n*x^(4/3))/(4*e^4) + (a*b*d^5*n*x^(2/3))/(2*e^5)
```

3.472 $\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 275

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=-\frac{3b^2dn^2\left(d+ex^{2/3}\right)^2}{4e^3}+\frac{b^2n^2\left(d+ex^{2/3}\right)^3}{9e^3}+\frac{3b^2d^2n^2x^{2/3}}{e^2}-\frac{b^2d^3n^2\log^2\left(d+ex^{2/3}\right)}{2e^3}-\frac{3bd^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^3}+\frac{3bdn\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2e^3}-\frac{bn\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^3}+\frac{bd^3n\log\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^3}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2$$

```
[Out] -3/4*b^2*d*n^2*(d+e*x^(2/3))^2/e^3+1/9*b^2*n^2*(d+e*x^(2/3))^3/e^3+3*b^2*d^
2*n^2*x^(2/3)/e^2-1/2*b^2*d^3*n^2*ln(d+e*x^(2/3))^2/e^3-3*b*d^2*n*(d+e*x^(2
/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^3+3/2*b*d*n*(d+e*x^(2/3))^2*(a+b*ln(c*(d
+e*x^(2/3))^n))/e^3-1/3*b*n*(d+e*x^(2/3))^3*(a+b*ln(c*(d+e*x^(2/3))^n))/e^3
+b*d^3*n*ln(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^3+1/2*x^2*(a+b*ln(c*
(d+e*x^(2/3))^n))^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\frac{bd^3n\log\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^3}$$
$$-\frac{3bd^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^3}$$
$$+\frac{3bdn\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2e^3}-\frac{bn\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^3}$$
$$+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{b^2d^3n^2\log^2\left(d+ex^{2/3}\right)}{2e^3}+\frac{3b^2d^2n^2x^{2/3}}{e^2}-\frac{3b^2dn^2\left(d+ex^{2/3}\right)^2}{4e^3}+\frac{b^2n^2(d+ex}{9e^3}$$

```
[In] Int[x*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] (-3*b^2*d*n^2*(d + e*x^(2/3))^2)/(4*e^3) + (b^2*n^2*(d + e*x^(2/3))^3)/(9*e
^3) + (3*b^2*d^2*n^2*x^(2/3))/e^2 - (b^2*d^3*n^2*Log[d + e*x^(2/3)]^2)/(2*e
^3) - (3*b*d^2*n*(d + e*x^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n]))/e^3 + (3
*b*d*n*(d + e*x^(2/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(2*e^3) - (b*n*(
d + e*x^(2/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*e^3) + (b*d^3*n*Log[d
 + e*x^(2/3)]*(a + b*Log[c*(d + e*x^(2/3))^n]))/e^3 + (x^2*(a + b*Log[c*(d
+ e*x^(2/3))^n])^2)/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3}{2}\text{Subst}\left(\int x^2(a+b\log\left(c(d+ex)^n\right))^2\,dx,x,x^{2/3}\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-(ben)\text{Subst}\left(\int\frac{x^3(a+b\log\left(c(d+ex)^n\right))}{d+ex}\,dx,x,x^{2/3}\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\\
&\quad-(bn)\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^3(a+b\log\left(cx^n\right))}{x}\,dx,x,d+ex^{2/3}\right)
\end{aligned}$$

$$= -\frac{3bd^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^3} + \frac{3bdn\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2e^3} - \frac{bn\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^3} + \frac{bd^3n\log\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^3} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 + \left(b^2n^2\right)\text{Subst}\left(\int\frac{18d^2x-9dx^2+2x^3-6d^3\log(x)}{6e^3x}\,dx,x,d+ex$$

$$= -\frac{3bd^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^3} + \frac{3bdn\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2e^3} - \frac{bn\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^3} + \frac{bd^3n\log\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^3} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 + \frac{\left(b^2n^2\right)\text{Subst}\left(\int\frac{18d^2x-9dx^2+2x^3-6d^3\log(x)}{x}\,dx,x,d+ex^{2/3}\right)}{6e^3}$$

$$= -\frac{3bd^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^3} + \frac{3bdn\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2e^3} - \frac{bn\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^3} + \frac{bd^3n\log\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^3} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 + \frac{\left(b^2n^2\right)\text{Subst}\left(\int\left(18d^2-9dx+2x^2-\frac{6d^3\log(x)}{x}\right)dx,x,d+ex^{2/3}\right)}{6e^3}$$

$$
\begin{aligned}
&= -\frac{3b^2dn^2\left(d+ex^{2/3}\right)^2}{4e^3} + \frac{b^2n^2\left(d+ex^{2/3}\right)^3}{9e^3} + \frac{3b^2d^2n^2x^{2/3}}{e^2} \\
&\quad - \frac{3bd^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^3} \\
&\quad + \frac{3bdn\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2e^3} \\
&\quad - \frac{bn\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^3} \\
&\quad + \frac{bd^3n\log\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^3} \\
&\quad + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 - \frac{(b^2d^3n^2)\,\mathrm{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, d+ex^{2/3}\right)}{e^3} \\
&= -\frac{3b^2dn^2\left(d+ex^{2/3}\right)^2}{4e^3} + \frac{b^2n^2\left(d+ex^{2/3}\right)^3}{9e^3} + \frac{3b^2d^2n^2x^{2/3}}{e^2} \\
&\quad - \frac{b^2d^3n^2\log^2\left(d+ex^{2/3}\right)}{2e^3} - \frac{3bd^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^3} \\
&\quad + \frac{3bdn\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2e^3} \\
&\quad - \frac{bn\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^3} \\
&\quad + \frac{bd^3n\log\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^3} \\
&\quad + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.87

$$
\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx = \frac{18a^2d^3 - 36abd^2enx^{2/3} + 66b^2d^2en^2x^{2/3} + 18abde^2nx^{4/3} - 15b^2de^2n^2x^{4/3} + 1
$$

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] (18*a^2*d^3 - 36*a*b*d^2*e*n*x^(2/3) + 66*b^2*d^2*e*n^2*x^(2/3) + 18*a*b*d*
e^2*n*x^(4/3) - 15*b^2*d*e^2*n^2*x^(4/3) + 18*a^2*e^3*x^2 - 12*a*b*e^3*n*x^
2 + 4*b^2*e^3*n^2*x^2 - 30*b^2*d^3*n^2*Log[d + e*x^(2/3)] + 6*b*(6*a*(d^3 +
 e^3*x^2) - b*n*(6*d^3 + 6*d^2*e*x^(2/3) - 3*d*e^2*x^(4/3) + 2*e^3*x^2))*Lo
g[c*(d + e*x^(2/3))^n] + 18*b^2*(d^3 + e^3*x^2)*Log[c*(d + e*x^(2/3))^n]^2)
/(36*e^3)
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}dx$$

[In] int(x*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)

[Out] int(x*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.11

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\right)^{2}dx = \frac{18\,b^{2}e^{3}x^{2}\log\left(c\right)^{2}-12\,(b^{2}e^{3}n-3\,abe^{3})x^{2}\log\left(c\right)+2\,(2\,b^{2}e^{3}n^{2}-6\,abe^{3}n+9\,a^{2}e}{}$$

[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="fricas")

[Out] 1/36*(18*b^2*e^3*x^2*log(c)^2 - 12*(b^2*e^3*n - 3*a*b*e^3)*x^2*log(c) + 2*(2*b^2*e^3*n^2 - 6*a*b*e^3*n + 9*a^2*e^3)*x^2 + 18*(b^2*e^3*n^2*x^2 + b^2*d^3*n^2)*log(e*x^(2/3) + d)^2 + 6*(3*b^2*d*e^2*n^2*x^(4/3) - 6*b^2*d^2*e*n^2*x^(2/3) - 11*b^2*d^3*n^2 + 6*a*b*d^3*n - 2*(b^2*e^3*n^2 - 3*a*b*e^3*n)*x^2 + 6*(b^2*e^3*n*x^2 + b^2*d^3*n)*log(c))*log(e*x^(2/3) + d) + 6*(11*b^2*d^2*e*n^2 - 6*b^2*d^2*e*n*log(c) - 6*a*b*d^2*e*n)*x^(2/3) + 3*(6*b^2*d*e^2*n*x*log(c) - (5*b^2*d*e^2*n^2 - 6*a*b*d*e^2*n)*x)*x^(1/3))/e^3

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\right)^{2}dx = \text{Timed out}$$

[In] integrate(x*(a+b*ln(c*(d+e*x**(2/3))**n))**2,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.84

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\frac{1}{2}\,b^2x^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)^2$$
$$+\frac{1}{6}\,aben\left(\frac{6\,d^3\log\left(ex^{\frac{2}{3}}+d\right)}{e^4}-\frac{2\,e^2x^2-3\,dex^{\frac{4}{3}}+6\,d^2x^{\frac{2}{3}}}{e^3}\right)$$
$$+abx^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+\frac{1}{2}\,a^2x^2$$
$$+\frac{1}{36}\left(6\,en\left(\frac{6\,d^3\log\left(ex^{\frac{2}{3}}+d\right)}{e^4}-\frac{2\,e^2x^2-3\,dex^{\frac{4}{3}}+6\,d^2x^{\frac{2}{3}}}{e^3}\right)\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+\frac{\left(4\,e^3x^2-18\,d^3\log\left(ex\right.\right.}{}\right.$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*log((e*x^(2/3) + d)^n*c)^2 + 1/6*a*b*e*n*(6*d^3*log(e*x^(2/3) +
 d)/e^4 - (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^3) + a*b*x^2*log((e
*x^(2/3) + d)^n*c) + 1/2*a^2*x^2 + 1/36*(6*e*n*(6*d^3*log(e*x^(2/3) + d)/e^
4 - (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^3)*log((e*x^(2/3) + d)^n*
c) + (4*e^3*x^2 - 18*d^3*log(e*x^(2/3) + d)^2 - 15*d*e^2*x^(4/3) - 66*d^3*l
og(e*x^(2/3) + d) + 66*d^2*e*x^(2/3))*n^2/e^3)*b^2
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.14

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\frac{1}{2}\,b^2x^2\log\left(c\right)^2$$
$$+\frac{1}{36}\left(18\,x^2\log\left(ex^{\frac{2}{3}}+d\right)^2-\left(6\left(\frac{2\left(ex^{\frac{2}{3}}+d\right)^3}{e^4}-\frac{9\left(ex^{\frac{2}{3}}+d\right)^2 d}{e^4}+\frac{18\left(ex^{\frac{2}{3}}+d\right)d^2}{e^4}\right)\log\left(ex^{\frac{2}{3}}+d\right)-\right.\right.$$
$$+\frac{1}{6}\left(6\,x^2\log\left(ex^{\frac{2}{3}}+d\right)+e\left(\frac{6\,d^3\log\left(\left|ex^{\frac{2}{3}}+d\right|\right)}{e^4}-\frac{2\,e^2x^2-3\,dex^{\frac{4}{3}}+6\,d^2x^{\frac{2}{3}}}{e^3}\right)\right)b^2n\log\left(c\right)$$
$$+abx^2\log\left(c\right)$$
$$+\frac{1}{6}\left(6\,x^2\log\left(ex^{\frac{2}{3}}+d\right)+e\left(\frac{6\,d^3\log\left(\left|ex^{\frac{2}{3}}+d\right|\right)}{e^4}-\frac{2\,e^2x^2-3\,dex^{\frac{4}{3}}+6\,d^2x^{\frac{2}{3}}}{e^3}\right)\right)abn$$
$$+\frac{1}{2}\,a^2x^2$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2*log(c)^2 + 1/36*(18*x^2*log(e*x^(2/3) + d)^2 - (6*(2*(e*x^(2/3)
 + d)^3/e^4 - 9*(e*x^(2/3) + d)^2*d/e^4 + 18*(e*x^(2/3) + d)*d^2/e^4)*log(e
*x^(2/3) + d) - 18*d^3*log(e*x^(2/3) + d)^2/e^4 - 4*(e*x^(2/3) + d)^3/e^4 +
 27*(e*x^(2/3) + d)^2*d/e^4 - 108*(e*x^(2/3) + d)*d^2/e^4)*e)*b^2*n^2 + 1/6
*(6*x^2*log(e*x^(2/3) + d) + e*(6*d^3*log(abs(e*x^(2/3) + d))/e^4 - (2*e^2*
x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^3))*b^2*n*log(c) + a*b*x^2*log(c) +
1/6*(6*x^2*log(e*x^(2/3) + d) + e*(6*d^3*log(abs(e*x^(2/3) + d))/e^4 - (2*e
^2*x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^3))*a*b*n + 1/2*a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.09

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx = \ln\left(c\left(d+e\,x^{2/3}\right)^n\right)^2\left(\frac{b^2\,x^2}{2}+\frac{b^2\,d^3}{2\,e^3}\right)$$

$$-x^{4/3}\left(\frac{d\left(\frac{3\,a^2}{2}-a\,b\,n+\frac{b^2\,n^2}{3}\right)}{2\,e}-\frac{d\,(3\,a^2-b^2\,n^2)}{4\,e}\right)+x^2\left(\frac{a^2}{2}-\frac{a\,b\,n}{3}+\frac{b^2\,n^2}{9}\right)+\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)\left(\frac{b\,x^2\,(3\,a}{3}\right.$$

```
[In] int(x*(a + b*log(c*(d + e*x^(2/3))^n))^2,x)
```

```
[Out] log(c*(d + e*x^(2/3))^n)^2*((b^2*x^2)/2 + (b^2*d^3)/(2*e^3)) - x^(4/3)*((d*
((3*a^2)/2 + (b^2*n^2)/3 - a*b*n))/(2*e) - (d*(3*a^2 - b^2*n^2))/(4*e)) + x
^2*(a^2/2 + (b^2*n^2)/9 - (a*b*n)/3) + log(c*(d + e*x^(2/3))^n)*((b*x^2*(3*
a - b*n))/3 - x^(4/3)*((b*d*(3*a - b*n))/(2*e) - (3*a*b*d)/(2*e)) + (d*x^(2
/3)*((b*d*(3*a - b*n))/e - (3*a*b*d)/e))/e) + x^(2/3)*((d*((d*((3*a^2)/2 +
(b^2*n^2)/3 - a*b*n))/e - (d*(3*a^2 - b^2*n^2))/(2*e)))/e + (b^2*d^2*n^2)/e
^2) - (log(d + e*x^(2/3))*(11*b^2*d^3*n^2 - 6*a*b*d^3*n))/(6*e^3)
```

3.473 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 95

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}\,dx = \frac{3}{2}\Big(a$$

$$+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\Big)^2\log\left(-\frac{ex^{2/3}}{d}\right)+3bn\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\operatorname{PolyLog}\left(2,1+\frac{ex^{2/3}}{d}\right)-3b^2n^2\operatorname{Poly}$$

```
[Out] 3/2*(a+b*ln(c*(d+e*x^(2/3))^n))^2*ln(-e*x^(2/3)/d)+3*b*n*(a+b*ln(c*(d+e*x^(
2/3))^n))*polylog(2,1+e*x^(2/3)/d)-3*b^2*n^2*polylog(3,1+e*x^(2/3)/d)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2504, 2443, 2481, 2421, 6724}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}\,dx = 3bn\operatorname{PolyLog}\left(2,\frac{x^{2/3}e}{d}\right.$$

$$\left.+1\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{3}{2}\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-3b^2n^2\operatorname{PolyLog}\left(3,\frac{x^{2/3}e}{d}+1\right.$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x,x]
```

```
[Out] (3*(a + b*Log[c*(d + e*x^(2/3))^n])^2*Log[-((e*x^(2/3))/d)])/2 + 3*b*n*(a +
 b*Log[c*(d + e*x^(2/3))^n])*PolyLog[2, 1 + (e*x^(2/3))/d] - 3*b^2*n^2*Poly
Log[3, 1 + (e*x^(2/3))/d]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x}\,dx, x, x^{2/3}\right)\\
&= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\log\left(-\frac{ex^{2/3}}{d}\right)\\
&\quad-(3ben)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{d+ex}\,dx, x, x^{2/3}\right)
\end{aligned}
$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\log\left(-\frac{ex^{2/3}}{d}\right) - (3bn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}dx, x, d+ex^{2/3}\right)$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\log\left(-\frac{ex^{2/3}}{d}\right) + 3bn\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\text{Li}_2\left(1+\frac{ex^{2/3}}{d}\right) - \left(3b^2n^2\right)\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{x}{d}\right)}{x}dx, x, d+ex^{2/3}\right)$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\log\left(-\frac{ex^{2/3}}{d}\right) + 3bn\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\text{Li}_2\left(1+\frac{ex^{2/3}}{d}\right) - 3b^2n^2\text{Li}_3\left(1+\frac{ex^{2/3}}{d}\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 199 vs. 2(95) = 190.

Time = 0.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.09

$$\int\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}dx = \Big(a - bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\Big)^2\log(x)+2bn\Big(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\Big)\Big(\Big(\text{lo}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])*((Log[d + e*x^(2/
3)] - Log[1 + (e*x^(2/3))/d])*Log[x] - (3*PolyLog[2, -((e*x^(2/3))/d)])/2)
+ (3*b^2*n^2*(Log[d + e*x^(2/3)]^2*Log[-((e*x^(2/3))/d)] + 2*Log[d + e*x^(2
/3)]*PolyLog[2, 1 + (e*x^(2/3))/d] - 2*PolyLog[3, 1 + (e*x^(2/3))/d]))/2
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}}{x}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x,x)
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{2}}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x,x, algorithm="fricas")
[Out] integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c) +
 a^2)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{\frac{2}{3}}\right)^{n}\right)\right)^{2}}{x}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x,x)
[Out] Integral((a + b*log(c*(d + e*x**(2/3))**n))**2/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{2}}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x,x, algorithm="maxima")
[Out] b^2*log((e*x^(2/3) + d)^n)^2*log(x) + integrate(1/3*(3*(b^2*e*log(c)^2 + 2*
a*b*e*log(c) + a^2*e)*x - 2*(2*b^2*e*n*x*log(x) - 3*(b^2*e*log(c) + a*b*e)*
x - 3*(b^2*d*log(c) + a*b*d)*x^(1/3))*log((e*x^(2/3) + d)^n) + 3*(b^2*d*log
(c)^2 + 2*a*b*d*log(c) + a^2*d)*x^(1/3))/(e*x^2 + d*x^(4/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)\right)^2}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x, x)
```

3.474 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 238

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}\,dx &= -\frac{b^2e^2n^2}{2d^2x^{2/3}}+\frac{b^2e^3n^2\log\left(d+ex^{2/3}\right)}{2d^3}\\ &-\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2dx^{4/3}}+\frac{be^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3x^{2/3}}\\ &+\frac{be^3n\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3}\\ &-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2x^2}-\frac{b^2e^3n^2\log(x)}{d^3}-\frac{b^2e^3n^2\operatorname{PolyLog}\left(2,\frac{d}{d+ex^{2/3}}\right)}{d^3}\end{aligned}$$

```
[Out] -1/2*b^2*e^2*n^2/d^2/x^(2/3)+1/2*b^2*e^3*n^2*ln(d+e*x^(2/3))/d^3-1/2*b*e*n*
(a+b*ln(c*(d+e*x^(2/3))^n))/d/x^(4/3)+b*e^2*n*(d+e*x^(2/3))*(a+b*ln(c*(d+e*
x^(2/3))^n))/d^3/x^(2/3)+b*e^3*n*ln(1-d/(d+e*x^(2/3)))*(a+b*ln(c*(d+e*x^(2/
3))^n))/d^3-1/2*(a+b*ln(c*(d+e*x^(2/3))^n))^2/x^2-b^2*e^3*n^2*ln(x)/d^3-b^2
*e^3*n^2*polylog(2,d/(d+e*x^(2/3)))/d^3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules

used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}\,dx = \frac{be^3n\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3} + \frac{be^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3x^{2/3}} - \frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2dx^{4/3}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2x^2} - \frac{b^2e^3n^2\operatorname{PolyLog}\left(2,\frac{d}{d+ex^{2/3}}\right)}{d^3} + \frac{b^2e^3n^2\log\left(d+ex^{2/3}\right)}{2d^3} - \frac{b^2e^3n^2\log(x)}{d^3} - \frac{b^2e^2n^2}{2d^2x^{2/3}}$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^3,x]
```

```
[Out] -1/2*(b^2*e^2*n^2)/(d^2*x^(2/3)) + (b^2*e^3*n^2*Log[d + e*x^(2/3)])/(2*d^3)
 - (b*e*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(2*d*x^(4/3)) + (b*e^2*n*(d + e
*x^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n]))/(d^3*x^(2/3)) + (b*e^3*n*Log[1
- d/(d + e*x^(2/3))]*(a + b*Log[c*(d + e*x^(2/3))^n]))/d^3 - (a + b*Log[c*(
d + e*x^(2/3))^n])^2/(2*x^2) - (b^2*e^3*n^2*Log[x])/d^3 - (b^2*e^3*n^2*Poly
Log[2, d/(d + e*x^(2/3))])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
```

```
NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4}\,dx, x, x^{2/3}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2x^2} + (ben)\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^3(d+ex)}\,dx, x, x^{2/3}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2x^2} + (bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+ex^{2/3}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2x^2} + \frac{(bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+ex^{2/3}\right)}{d}\\
&\quad - \frac{(ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+ex^{2/3}\right)}{d}\\
&= -\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2dx^{4/3}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2x^2}\\
&\quad - \frac{(ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+ex^{2/3}\right)}{d^2}\\
&\quad + \frac{\left(be^2n\right)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx, x, d+ex^{2/3}\right)}{d^2}\\
&\quad + \frac{\left(b^2en^2\right)\text{Subst}\left(\int \frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+ex^{2/3}\right)}{2d}\\
&= -\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2dx^{4/3}} + \frac{be^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3x^{2/3}}\\
&\quad + \frac{be^3n\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2x^2}\\
&\quad + \frac{\left(b^2en^2\right)\text{Subst}\left(\int \left(\frac{e^2}{d(d-x)^2}+\frac{e^2}{d^2(d-x)}+\frac{e^2}{d^2x}\right)dx, x, d+ex^{2/3}\right)}{2d}\\
&\quad - \frac{\left(b^2e^2n^2\right)\text{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x}{e}}\,dx, x, d+ex^{2/3}\right)}{d^3}\\
&\quad - \frac{\left(b^2e^3n^2\right)\text{Subst}\left(\int \frac{\log\left(1-\frac{d}{x}\right)}{x}\,dx, x, d+ex^{2/3}\right)}{d^3}
\end{aligned}
$$

$$=-\frac{b^2e^2n^2}{2d^2x^{2/3}}+\frac{b^2e^3n^2\log\left(d+ex^{2/3}\right)}{2d^3}-\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2dx^{4/3}}$$
$$+\frac{be^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3x^{2/3}}$$
$$+\frac{be^3n\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3}$$
$$-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2x^2}-\frac{b^2e^3n^2\log(x)}{d^3}-\frac{b^2e^3n^2\operatorname{Li}_2\left(\frac{d}{d+ex^{2/3}}\right)}{d^3}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.11

$$\int\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}\,dx=$$
$$-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2+\frac{ex^{2/3}\left(3bd^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-6bdenx^{2/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+3e^2x^{4/3}\left(a+b\log\left(c(d-\right.\right.\right.}{}}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^3,x]
```

```
[Out] -1/6*(3*(a + b*Log[c*(d + e*x^(2/3))^n])^2 + (e*x^(2/3)*(3*b*d^2*n*(a + b*L
og[c*(d + e*x^(2/3))^n]) - 6*b*d*e*n*x^(2/3)*(a + b*Log[c*(d + e*x^(2/3))^n
]) + 3*e^2*x^(4/3)*(a + b*Log[c*(d + e*x^(2/3))^n])^2 - 2*b^2*e^2*n^2*x^(4/
3)*(3*Log[d + e*x^(2/3)] - 2*Log[x]) + b^2*e*n^2*x^(2/3)*(3*d - 3*e*x^(2/3)
*Log[d + e*x^(2/3)] + 2*e*x^(2/3)*Log[x]) - 6*b*e^2*n*x^(4/3)*((a + b*Log[c
*(d + e*x^(2/3))^n])*Log[-((e*x^(2/3))/d)] + b*n*PolyLog[2, 1 + (e*x^(2/3))
/d])))/d^3)/x^2
```

Maple [F]

$$\int\frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^n\right)\right)^2}{x^3}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^3,x, algorithm="fricas")
[Out] integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c) +
 a^2)/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^3,x, algorithm="maxima")
[Out] -1/2*b^2*log((e*x^(2/3) + d)^n)^2/x^2 + integrate(1/3*(3*(b^2*e*log(c)^2 +
2*a*b*e*log(c) + a^2*e)*x + 2*(b^2*e*n*x + 3*(b^2*e*log(c) + a*b*e)*x + 3*(
b^2*d*log(c) + a*b*d)*x^(1/3))*log((e*x^(2/3) + d)^n) + 3*(b^2*d*log(c)^2 +
 2*a*b*d*log(c) + a^2*d)*x^(1/3))/(e*x^4 + d*x^(10/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^3,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)\right)^2}{x^3}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^3,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^3, x)
```

3.475 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 412

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^5}\,dx &= -\frac{b^2e^2n^2}{40d^2x^{8/3}}+\frac{3b^2e^3n^2}{40d^3x^2}-\frac{47b^2e^4n^2}{240d^4x^{4/3}} \\ &+\frac{77b^2e^5n^2}{120d^5x^{2/3}}-\frac{77b^2e^6n^2\log\left(d+ex^{2/3}\right)}{120d^6}-\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{10dx^{10/3}} \\ &+\frac{be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{8d^2x^{8/3}}-\frac{be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{6d^3x^2} \\ &+\frac{be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4d^4x^{4/3}}-\frac{be^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^6x^{2/3}} \\ &-\frac{be^6n\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^6} \\ &-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4}+\frac{137b^2e^6n^2\log(x)}{180d^6}+\frac{b^2e^6n^2\operatorname{PolyLog}\left(2,\frac{d}{d+ex^{2/3}}\right)}{2d^6}\end{aligned}$$

```
[Out] -1/40*b^2*e^2*n^2/d^2/x^(8/3)+3/40*b^2*e^3*n^2/d^3/x^2-47/240*b^2*e^4*n^2/d
^4/x^(4/3)+77/120*b^2*e^5*n^2/d^5/x^(2/3)-77/120*b^2*e^6*n^2*ln(d+e*x^(2/3)
)/d^6-1/10*b*e*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d/x^(10/3)+1/8*b*e^2*n*(a+b*ln
(c*(d+e*x^(2/3))^n))/d^2/x^(8/3)-1/6*b*e^3*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^
3/x^2+1/4*b*e^4*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^4/x^(4/3)-1/2*b*e^5*n*(d+e*
x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^6/x^(2/3)-1/2*b*e^6*n*ln(1-d/(d+e*x^
(2/3)))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^6-1/4*(a+b*ln(c*(d+e*x^(2/3))^n))^2/x
^4+137/180*b^2*e^6*n^2*ln(x)/d^6+1/2*b^2*e^6*n^2*polylog(2,d/(d+e*x^(2/3)))
/d^6
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^5}\,dx = -\frac{be^6n\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^6} - \frac{be^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^6x^{2/3}} + \frac{be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4d^4x^{4/3}} - \frac{be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{6d^3x^2} + \frac{be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{8d^2x^{8/3}} - \frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{10dx^{10/3}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4} + \frac{b^2e^6n^2\,\mathrm{PolyLog}\left(2,\frac{d}{d+ex^{2/3}}\right)}{2d^6} - \frac{77b^2e^6n^2\log\left(d+ex^{2/3}\right)}{120d^6} + \frac{137b^2e^6n^2\log(x)}{180d^6} + \frac{77b^2e^5n^2}{120d^5x^{2/3}} - \frac{47b^2e^4n^2}{240d^4x^{4/3}} + \frac{3b^2e^3n^2}{40d^3x^2} - \frac{b^2e^2n^2}{40d^2x^{8/3}}$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^5,x]
```

```
[Out] -1/40*(b^2*e^2*n^2)/(d^2*x^(8/3)) + (3*b^2*e^3*n^2)/(40*d^3*x^2) - (47*b^2*
e^4*n^2)/(240*d^4*x^(4/3)) + (77*b^2*e^5*n^2)/(120*d^5*x^(2/3)) - (77*b^2*e
^6*n^2*Log[d + e*x^(2/3)])/(120*d^6) - (b*e*n*(a + b*Log[c*(d + e*x^(2/3))^
n]))/(10*d*x^(10/3)) + (b*e^2*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(8*d^2*x^
(8/3)) - (b*e^3*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(6*d^3*x^2) + (b*e^4*n*
(a + b*Log[c*(d + e*x^(2/3))^n]))/(4*d^4*x^(4/3)) - (b*e^5*n*(d + e*x^(2/3)
)*(a + b*Log[c*(d + e*x^(2/3))^n]))/(2*d^6*x^(2/3)) - (b*e^6*n*Log[1 - d/(d
 + e*x^(2/3))]*(a + b*Log[c*(d + e*x^(2/3))^n]))/(2*d^6) - (a + b*Log[c*(d
+ e*x^(2/3))^n])^2/(4*x^4) + (137*b^2*e^6*n^2*Log[x])/(180*d^6) + (b^2*e^6*
n^2*PolyLog[2, d/(d + e*x^(2/3))])/(2*d^6)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^7}\,dx, x, x^{2/3}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4}+\frac{1}{2}(ben)\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^6(d+ex)}\,dx, x, x^{2/3}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4}+\frac{1}{2}(bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+ex^{2/3}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4}+\frac{(bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+ex^{2/3}\right)}{2d}\\
&\quad-\frac{(ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+ex^{2/3}\right)}{2d}
\end{aligned}
$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{10dx^{10/3}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4} - \frac{(ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+ex^{2/3}\right)}{2d^2} + \frac{(be^2n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+ex^{2/3}\right)}{2d^2} + \frac{(b^2en^2)\,\text{Subst}\left(\int \frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+ex^{2/3}\right)}{10d}$$

$$= -\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{10dx^{10/3}} + \frac{be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{8d^2x^{8/3}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4} + \frac{(be^2n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+ex^{2/3}\right)}{2d^3} - \frac{(be^3n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+ex^{2/3}\right)}{2d^3} + \frac{(b^2en^2)\,\text{Subst}\left(\int \left(-\frac{e^5}{d(d-x)^5} - \frac{e^5}{d^2(d-x)^4} - \frac{e^5}{d^3(d-x)^3} - \frac{e^5}{d^4(d-x)^2} - \frac{e^5}{d^5(d-x)} - \frac{e^5}{d^5x}\right)\,dx, x, d+ex^{2/3}\right)}{10d} - \frac{(b^2e^2n^2)\,\text{Subst}\left(\int \frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+ex^{2/3}\right)}{8d^2}$$

$$= -\frac{b^2e^2n^2}{40d^2x^{8/3}} + \frac{b^2e^3n^2}{30d^3x^2} - \frac{b^2e^4n^2}{20d^4x^{4/3}} + \frac{b^2e^5n^2}{10d^5x^{2/3}} - \frac{b^2e^6n^2\log\left(d+ex^{2/3}\right)}{10d^6} - \frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{10dx^{10/3}} + \frac{be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{8d^2x^{8/3}} - \frac{be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{6d^3x^2} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4} + \frac{b^2e^6n^2\log(x)}{15d^6} - \frac{(be^3n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+ex^{2/3}\right)}{2d^4} + \frac{(be^4n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+ex^{2/3}\right)}{2d^4} - \frac{(b^2e^2n^2)\,\text{Subst}\left(\int \left(\frac{e^4}{d(d-x)^4} + \frac{e^4}{d^2(d-x)^3} + \frac{e^4}{d^3(d-x)^2} + \frac{e^4}{d^4(d-x)} + \frac{e^4}{d^4x}\right)\,dx, x, d+ex^{2/3}\right)}{8d^2} + \frac{(b^2e^3n^2)\,\text{Subst}\left(\int \frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+ex^{2/3}\right)}{6d^3}$$

$$= -\frac{b^2e^2n^2}{40d^2x^{8/3}} + \frac{3b^2e^3n^2}{40d^3x^2} - \frac{9b^2e^4n^2}{80d^4x^{4/3}} + \frac{9b^2e^5n^2}{40d^5x^{2/3}} - \frac{9b^2e^6n^2\log\left(d+ex^{2/3}\right)}{40d^6} - \frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{10dx^{10/3}} + \frac{be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{8d^2x^{8/3}} - \frac{be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{6d^3x^2} + \frac{be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4d^4x^{4/3}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4} + \frac{3b^2e^6n^2\log(x)}{20d^6} + \frac{(be^4n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+ex^{2/3}\right)}{2d^5} - \frac{(be^5n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx, x, d+ex^{2/3}\right)}{2d^5} + \frac{(b^2e^3n^2)\,\text{Subst}\left(\int \left(-\frac{e^3}{d(d-x)^3} - \frac{e^3}{d^2(d-x)^2} - \frac{e^3}{d^3(d-x)} - \frac{e^3}{d^3x}\right)\,dx, x, d+ex^{2/3}\right)}{6d^3} - \frac{(b^2e^4n^2)\,\text{Subst}\left(\int \frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+ex^{2/3}\right)}{4d^4}$$

$$
\begin{aligned}
&= -\frac{b^2e^2n^2}{40d^2x^{8/3}}+\frac{3b^2e^3n^2}{40d^3x^2}-\frac{47b^2e^4n^2}{240d^4x^{4/3}}+\frac{47b^2e^5n^2}{120d^5x^{2/3}} \\
&\quad -\frac{47b^2e^6n^2\log\left(d+ex^{2/3}\right)}{120d^6}-\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{10dx^{10/3}} \\
&\quad +\frac{be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{8d^2x^{8/3}}-\frac{be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{6d^3x^2} \\
&\quad +\frac{be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4d^4x^{4/3}}-\frac{be^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^6x^{2/3}} \\
&\quad -\frac{be^6n\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^6}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4} \\
&\quad +\frac{47b^2e^6n^2\log(x)}{180d^6}-\frac{(b^2e^4n^2)\,\text{Subst}\left(\int\left(\frac{e^2}{d(d-x)^2}+\frac{e^2}{d^2(d-x)}+\frac{e^2}{d^2x}\right)dx,x,d+ex^{2/3}\right)}{4d^4} \\
&\quad +\frac{(b^2e^5n^2)\,\text{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x}{e}}dx,x,d+ex^{2/3}\right)}{2d^6} \\
&\quad +\frac{(b^2e^6n^2)\,\text{Subst}\left(\int\frac{\log\left(1-\frac{d}{x}\right)}{x}dx,x,d+ex^{2/3}\right)}{2d^6} \\
&= -\frac{b^2e^2n^2}{40d^2x^{8/3}}+\frac{3b^2e^3n^2}{40d^3x^2}-\frac{47b^2e^4n^2}{240d^4x^{4/3}}+\frac{77b^2e^5n^2}{120d^5x^{2/3}} \\
&\quad -\frac{77b^2e^6n^2\log\left(d+ex^{2/3}\right)}{120d^6}-\frac{ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{10dx^{10/3}} \\
&\quad +\frac{be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{8d^2x^{8/3}}-\frac{be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{6d^3x^2} \\
&\quad +\frac{be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4d^4x^{4/3}}-\frac{be^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^6x^{2/3}} \\
&\quad -\frac{be^6n\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^6} \\
&\quad -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4}+\frac{137b^2e^6n^2\log(x)}{180d^6}+\frac{b^2e^6n^2\text{Li}_2\left(\frac{d}{d+ex^{2/3}}\right)}{2d^6}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.09

$$
\int\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^5}dx=-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4x^4}
$$

$$
-\frac{be\left(72ad^5n-90ad^4enx^{2/3}+18bd^4en^2x^{2/3}+120ad^3e^2nx^{4/3}-54bd^3e^2n^2x^{4/3}-180ad^2e^3nx^2+141bd^2e^3n^2\right.}{}
$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^5,x]
```

```
[Out] -1/4*(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^4 - (b*e*(72*a*d^5*n - 90*a*d^4*e
*n*x^(2/3) + 18*b*d^4*e*n^2*x^(2/3) + 120*a*d^3*e^2*n*x^(4/3) - 54*b*d^3*e^
2*n^2*x^(4/3) - 180*a*d^2*e^3*n*x^2 + 141*b*d^2*e^3*n^2*x^2 + 360*a*d*e^4*n
*x^(8/3) - 462*b*d*e^4*n^2*x^(8/3) + 6*e^5*n*(-60*a + 137*b*n)*x^(10/3)*Log
[d + e*x^(2/3)] + 72*b*d^5*n*Log[c*(d + e*x^(2/3))^n] - 90*b*d^4*e*n*x^(2/3
)*Log[c*(d + e*x^(2/3))^n] + 120*b*d^3*e^2*n*x^(4/3)*Log[c*(d + e*x^(2/3))^
n] - 180*b*d^2*e^3*n*x^2*Log[c*(d + e*x^(2/3))^n] + 360*b*d*e^4*n*x^(8/3)*L
og[c*(d + e*x^(2/3))^n] - 180*b*e^5*x^(10/3)*Log[c*(d + e*x^(2/3))^n]^2 + 3
60*b*e^5*n*x^(10/3)*Log[c*(d + e*x^(2/3))^n]*Log[-((e*x^(2/3))/d)] + 240*a*
e^5*n*x^(10/3)*Log[x] - 548*b*e^5*n^2*x^(10/3)*Log[x] + 360*b*e^5*n^2*x^(10
/3)*PolyLog[2, 1 + (e*x^(2/3))/d]))/(720*d^6*x^(10/3))
```

Maple [F]

$$\int \frac{\left(a+b\ln \left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}}{x^{5}}d x$$

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^5,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^5,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\right)^{2}}{x^{5}}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{2}}{x^{5}}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^5,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c) +
 a^2)/x^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\right)^{2}}{x^{5}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**5,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^5}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2}{x^5}\,dx$$

[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^5,x, algorithm="maxima")

[Out] -1/4*b^2*log((e*x^(2/3) + d)^n)^2/x^4 + integrate(1/3*(3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x + (b^2*e*n*x + 6*(b^2*e*log(c) + a*b*e)*x + 6*(b^2*d*log(c) + a*b*d)*x^(1/3))*log((e*x^(2/3) + d)^n) + 3*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x^(1/3))/(e*x^6 + d*x^(16/3)), x)

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^5}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2}{x^5}\,dx$$

[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^5,x, algorithm="giac")

[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x^5, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^5}\,dx = \int \frac{\left(a+b\,\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)\right)^2}{x^5}\,dx$$

[In] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^5,x)

[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^5, x)

3.476 $\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 547

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=-\frac{4abd^4n\sqrt[3]{x}}{3e^4}+\frac{4504b^2d^4n^2\sqrt[3]{x}}{945e^4}-\frac{1984b^2d^3n^2x}{2835e^3}+\frac{1144b^2d^2n^2x^{5/3}}{4725e^2}-\frac{128b^2dn^2x^{7/3}}{1323e}+\frac{8}{243}b^2n^2x^3-\frac{4504b^2d^{9/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{945e^{9/2}}+\frac{4ib^2d^{9/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{3e^{9/2}}+\frac{8b^2d^{9/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3e^{9/2}}-\frac{4b^2d^4n\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3e^4}+\frac{4bd^3nx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9e^3}-\frac{4bd^2nx^{5/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15e^2}+\frac{4bdnx^{7/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{21e}-\frac{4}{27}bnx^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{4bd^{9/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^{9/2}}+\frac{1}{3}x^3\left(a+b\log\left(c\right.\right.$$

```
[Out] -4/3*a*b*d^4*n*x^(1/3)/e^4+4504/945*b^2*d^4*n^2*x^(1/3)/e^4-1984/2835*b^2*d
^3*n^2*x/e^3+1144/4725*b^2*d^2*n^2*x^(5/3)/e^2-128/1323*b^2*d*n^2*x^(7/3)/e
+8/243*b^2*n^2*x^3-4504/945*b^2*d^(9/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))
/e^(9/2)+4/3*I*b^2*d^(9/2)*n^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(
1/2)))/e^(9/2)-4/3*b^2*d^4*n*x^(1/3)*ln(c*(d+e*x^(2/3))^n)/e^4+4/9*b*d^3*n*
x*(a+b*ln(c*(d+e*x^(2/3))^n))/e^3-4/15*b*d^2*n*x^(5/3)*(a+b*ln(c*(d+e*x^(2/
3))^n))/e^2+4/21*b*d*n*x^(7/3)*(a+b*ln(c*(d+e*x^(2/3))^n))/e-4/27*b*n*x^3*(
a+b*ln(c*(d+e*x^(2/3))^n))+4/3*b*d^(9/2)*n*arctan(x^(1/3)*e^(1/2)/d^(1/2))*
(a+b*ln(c*(d+e*x^(2/3))^n))/e^(9/2)+1/3*x^3*(a+b*ln(c*(d+e*x^(2/3))^n))^2+8
/3*b^2*d^(9/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*
```

```
x^(1/3)*e^(1/2)))/e^(9/2)+4/3*I*b^2*d^(9/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1
/2))^2/e^(9/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {2508, 2507, 2526, 2498, 327, 211, 2505, 308, 2520, 12, 5040, 4964, 2449, 2352}

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\frac{4bd^{9/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^{9/2}}+\frac{4bd^3nx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9e^3}-\frac{4bd^2nx^{5/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15e^2}+\frac{4bdnx^{7/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{21e}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\frac{4}{27}bnx^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{4abd^4n\sqrt[3]{x}}{3e^4}+\frac{4ib^2d^{9/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{9/2}}$$

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] (-4*a*b*d^4*n*x^(1/3))/(3*e^4) + (4504*b^2*d^4*n^2*x^(1/3))/(945*e^4) - (19
84*b^2*d^3*n^2*x)/(2835*e^3) + (1144*b^2*d^2*n^2*x^(5/3))/(4725*e^2) - (128
*b^2*d*n^2*x^(7/3))/(1323*e) + (8*b^2*n^2*x^3)/243 - (4504*b^2*d^(9/2)*n^2*
ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(945*e^(9/2)) + (((4*I)/3)*b^2*d^(9/2)*n
^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/e^(9/2) + (8*b^2*d^(9/2)*n^2*ArcTan
[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])
/(3*e^(9/2)) - (4*b^2*d^4*n*x^(1/3)*Log[c*(d + e*x^(2/3))^n])/(3*e^4) + (4*
b*d^3*n*x*(a + b*Log[c*(d + e*x^(2/3))^n]))/(9*e^3) - (4*b*d^2*n*x^(5/3)*(a
 + b*Log[c*(d + e*x^(2/3))^n]))/(15*e^2) + (4*b*d*n*x^(7/3)*(a + b*Log[c*(d
 + e*x^(2/3))^n]))/(21*e) - (4*b*n*x^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/27
 + (4*b*d^(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(
2/3))^n]))/(3*e^(9/2)) + (x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/3 + (((4*
I)/3)*b^2*d^(9/2)*n^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/
3))])/e^(9/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2508

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\\
&\quad -\frac{1}{3}(4ben)\text{Subst}\left(\int \frac{x^{10}\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)
\end{aligned}$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2$$
$$-\frac{1}{3}(4ben)\text{Subst}\left(\int\left(\frac{d^4\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)}{e^5}-\frac{d^3x^2\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)}{e^4}+\frac{d^2x^4\left(a+b\log\right.}{e\right.}\right.\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2$$
$$-\frac{1}{3}(4bn)\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)\,dx,x,\sqrt[3]{x}\right)$$
$$-\frac{\left(4bd^4n\right)\text{Subst}\left(\int\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)\,dx,x,\sqrt[3]{x}\right)}{3e^4}$$
$$+\frac{\left(4bd^5n\right)\text{Subst}\left(\int\frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{3e^4}$$
$$+\frac{\left(4bd^3n\right)\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)\,dx,x,\sqrt[3]{x}\right)}{3e^3}$$
$$-\frac{\left(4bd^2n\right)\text{Subst}\left(\int x^4\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)\,dx,x,\sqrt[3]{x}\right)}{3e^2}$$
$$+\frac{(4bdn)\text{Subst}\left(\int x^6\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)\,dx,x,\sqrt[3]{x}\right)}{3e}$$

$$= -\frac{4abd^4n\sqrt[3]{x}}{3e^4}+\frac{4bd^3nx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9e^3}$$
$$-\frac{4bd^2nx^{5/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15e^2}+\frac{4bdnx^{7/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{21e}$$
$$-\frac{4}{27}bnx^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{4bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^{9/2}}+\frac{1}{3}x^3\left(a\right.$$

$$= -\frac{4abd^4n\sqrt[3]{x}}{3e^4}-\frac{4b^2d^4n\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3e^4}+\frac{4bd^3nx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9e^3}$$
$$-\frac{4bd^2nx^{5/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15e^2}+\frac{4bdnx^{7/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{21e}$$
$$-\frac{4}{27}bnx^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{4bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^{9/2}}+\frac{1}{3}x^3\left(a\right.$$

$$= -\frac{4abd^4n\sqrt[3]{x}}{3e^4} + \frac{4504b^2d^4n^2\sqrt[3]{x}}{945e^4} - \frac{1984b^2d^3n^2x}{2835e^3} + \frac{1144b^2d^2n^2x^{5/3}}{4725e^2} - \frac{128b^2dn^2x^{7/3}}{1323e} + \frac{8}{243}b^2n^2x^3 + \frac{4ib^2d^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{3e^{9/2}} - \frac{4b^2d^4n\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3e^4} + \frac{4bd^3nx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9e^3} - \frac{4bd^2nx^{5/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15e^2} + \frac{4bdnx^{7/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{21e} - \frac{4}{27}bnx^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) + \frac{4bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^{9/2}} + \frac{1}{3}x^3\Big($$

$$= -\frac{4abd^4n\sqrt[3]{x}}{3e^4} + \frac{4504b^2d^4n^2\sqrt[3]{x}}{945e^4} - \frac{1984b^2d^3n^2x}{2835e^3} + \frac{1144b^2d^2n^2x^{5/3}}{4725e^2} - \frac{128b^2dn^2x^{7/3}}{1323e} + \frac{8}{243}b^2n^2x^3 - \frac{4504b^2d^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{945e^{9/2}} + \frac{4ib^2d^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{3e^{9/2}} + \frac{8b^2d^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3e^{9/2}} - \frac{4b^2d^4n\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3e^4} + \frac{4bd^3nx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9e^3} - \frac{4bd^2nx^{5/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15e^2} + \frac{4bdnx^{7/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{21e}$$

$$-\frac{4}{27}bnx^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) + \frac{4bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^{9/2}} + \frac{1}{3}x^3\Big($$

$$= -\frac{4abd^4n\sqrt[3]{x}}{3e^4} + \frac{4504b^2d^4n^2\sqrt[3]{x}}{945e^4} - \frac{1984b^2d^3n^2x}{2835e^3} + \frac{1144b^2d^2n^2x^{5/3}}{4725e^2} - \frac{128b^2dn^2x^{7/3}}{1323e} + \frac{8}{243}b^2n^2x^3 - \frac{4504b^2d^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{945e^{9/2}} + \frac{4ib^2d^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{3e^{9/2}} + \frac{8b^2d^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3e^{9/2}} - \frac{4b^2d^4n\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3e^4} + \frac{4bd^3nx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9e^3} - \frac{4bd^2nx^{5/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15e^2} + \frac{4bdnx^{7/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{21e} - \frac{4}{27}bnx^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) + \frac{4bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^{9/2}} + \frac{1}{3}x^3\left(a\right.$$

$$= -\frac{4abd^4n\sqrt[3]{x}}{3e^4} + \frac{4504b^2d^4n^2\sqrt[3]{x}}{945e^4} - \frac{1984b^2d^3n^2x}{2835e^3} + \frac{1144b^2d^2n^2x^{5/3}}{4725e^2} - \frac{128b^2dn^2x^{7/3}}{1323e} + \frac{8}{243}b^2n^2x^3 - \frac{4504b^2d^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{945e^{9/2}} + \frac{4ib^2d^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{3e^{9/2}} + \frac{8b^2d^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3e^{9/2}} - \frac{4b^2d^4n\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{3e^4} + \frac{4bd^3nx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9e^3} - \frac{4bd^2nx^{5/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15e^2} + \frac{4bdnx^{7/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{21e} - \frac{4}{27}bnx^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) + \frac{4bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^{9/2}} + \frac{1}{3}x^3\left(a\right.$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.80

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx = \frac{396900ib^2d^{9/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2 + 1260bd^{9/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(315a-112\right.}{}$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] ((396900*I)*b^2*d^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2 + 1260*b*d^
(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(315*a - 1126*b*n + 630*b*n*Log[(
2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))] + 315*b*Log[c*(d + e*x^(2/3))^n])
 + Sqrt[e]*x^(1/3)*(99225*a^2*e^4*x^(8/3) - 1260*a*b*n*(315*d^4 - 105*d^3*e
*x^(2/3) + 63*d^2*e^2*x^(4/3) - 45*d*e^3*x^2 + 35*e^4*x^(8/3)) + 8*b^2*n^2*
(177345*d^4 - 26040*d^3*e*x^(2/3) + 9009*d^2*e^2*x^(4/3) - 3600*d*e^3*x^2 +
 1225*e^4*x^(8/3)) - 630*b*(-315*a*e^4*x^(8/3) + 2*b*n*(315*d^4 - 105*d^3*e
*x^(2/3) + 63*d^2*e^2*x^(4/3) - 45*d*e^3*x^2 + 35*e^4*x^(8/3)))*Log[c*(d +
e*x^(2/3))^n] + 99225*b^2*e^4*x^(8/3)*Log[c*(d + e*x^(2/3))^n]^2) + (396900
*I)*b^2*d^(9/2)*n^2*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x^(1/3))/((-I)*Sqrt[d]
+ Sqrt[e]*x^(1/3))])/(297675*e^(9/2))
```

Maple [F]

$$\int x^2 \left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}dx$$

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

Fricas [F]

$$\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2 x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*x^2*log((e*x^(2/3) + d)
^n*c) + a^2*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))**n))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx = \int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2 x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx = \int x^2\,\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)\right)^2\,dx$$

```
[In] int(x^2*(a + b*log(c*(d + e*x^(2/3))^n))^2,x)
```

```
[Out] int(x^2*(a + b*log(c*(d + e*x^(2/3))^n))^2, x)
```

3.477 $\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 364

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\frac{4abdn\sqrt[3]{x}}{e}-\frac{32b^2dn^2\sqrt[3]{x}}{3e}+\frac{8}{9}b^2n^2x+\frac{32b^2d^{3/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}}-\frac{4ib^2d^{3/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{e^{3/2}}-\frac{8b^2d^{3/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{e^{3/2}}+\frac{4b^2dn\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e}-\frac{4}{3}bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{4bd^{3/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{3/2}}+x\left(a+b\log\left(c\left(d-\right.\right.\right.$$

```
[Out] 4*a*b*d*n*x^(1/3)/e-32/3*b^2*d*n^2*x^(1/3)/e+8/9*b^2*n^2*x+32/3*b^2*d^(3/2)
*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))/e^(3/2)-4*I*b^2*d^(3/2)*n^2*arctan(x^(
1/3)*e^(1/2)/d^(1/2))^2/e^(3/2)+4*b^2*d*n*x^(1/3)*ln(c*(d+e*x^(2/3))^n)/e-4
/3*b*n*x*(a+b*ln(c*(d+e*x^(2/3))^n))-4*b*d^(3/2)*n*arctan(x^(1/3)*e^(1/2)/d
^(1/2))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^(3/2)+x*(a+b*ln(c*(d+e*x^(2/3))^n))^2
-8*b^2*d^(3/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*
x^(1/3)*e^(1/2)))/e^(3/2)-4*I*b^2*d^(3/2)*n^2*polylog(2,1-2*d^(1/2)/(d^(1/2
)+I*x^(1/3)*e^(1/2)))/e^(3/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {2501, 2507, 2526, 2498, 327, 211, 2505, 308, 2520, 12, 5040, 4964, 2449, 2352}

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=-\frac{4bd^{3/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{3/2}}$$
$$-\frac{4}{3}bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2+\frac{4abdn\sqrt[3]{x}}{e}-\frac{4ib^2d^{3/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{e^{3/2}}$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] (4*a*b*d*n*x^(1/3))/e - (32*b^2*d*n^2*x^(1/3))/(3*e) + (8*b^2*n^2*x)/9 + (3
2*b^2*d^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(3*e^(3/2)) - ((4*I)*b
^2*d^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/e^(3/2) - (8*b^2*d^(3/2
)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e
]*x^(1/3))])/e^(3/2) + (4*b^2*d*n*x^(1/3)*Log[c*(d + e*x^(2/3))^n])/e - (4*
b*n*x*(a + b*Log[c*(d + e*x^(2/3))^n]))/3 - (4*b*d^(3/2)*n*ArcTan[(Sqrt[e]*
x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/e^(3/2) + x*(a + b*Log[
c*(d + e*x^(2/3))^n])^2 - ((4*I)*b^2*d^(3/2)*n^2*PolyLog[2, 1 - (2*Sqrt[d])
/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/e^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
```

```
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2\,dx,x,\sqrt[3]{x}\right)\\
&= x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-(4ben)\text{Subst}\left(\int \frac{x^4\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)\\
&= x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\\
&\quad -(4ben)\text{Subst}\left(\int\left(-\frac{d\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)}{e^2}+\frac{x^2\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)}{e}+\frac{d^2\left(a+b\log\left(c\left(d\right.\right.\right.}{e^2\left(d+ex\right.}\right.\right.\\
&= x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\\
&\quad -(4bn)\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)\,dx,x,\sqrt[3]{x}\right)\\
&\quad +\frac{(4bdn)\text{Subst}\left(\int\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)\,dx,x,\sqrt[3]{x}\right)}{e}\\
&\quad -\frac{\left(4bd^2n\right)\text{Subst}\left(\int\frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{e}
\end{aligned}
$$

$$=\frac{4abdn\sqrt[3]{x}}{e}-\frac{4}{3}bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)$$
$$-\frac{4bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{3/2}}$$
$$+x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2+\frac{(4b^2dn)\,\text{Subst}\left(\int\log\left(c\left(d+ex^2\right)^n\right)\,dx,x,\sqrt[3]{x}\right)}{e}+\left(8b^2d^2n^2\right)\text{Subst}$$

$$=\frac{4abdn\sqrt[3]{x}}{e}+\frac{4b^2dn\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e}-\frac{4}{3}bnx\left(a\right.$$
$$\left.+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{4bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{3/2}}$$
$$+x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2-\left(8b^2dn^2\right)\text{Subst}\left(\int\frac{x^2}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)+\frac{\left(8b^2d^{3/2}n^2\right)\text{Subst}\left(\int\right.}{\sqrt{}}$$

$$=\frac{4abdn\sqrt[3]{x}}{e}-\frac{32b^2dn^2\sqrt[3]{x}}{3e}+\frac{8}{9}b^2n^2x$$
$$-\frac{4ib^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{e^{3/2}}+\frac{4b^2dn\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e}$$
$$-\frac{4}{3}bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{4bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{3/2}}+x\left(a+b\right.$$

$$=\frac{4abdn\sqrt[3]{x}}{e}-\frac{32b^2dn^2\sqrt[3]{x}}{3e}+\frac{8}{9}b^2n^2x$$
$$+\frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}}-\frac{4ib^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{e^{3/2}}$$
$$-\frac{8b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{e^{3/2}}+\frac{4b^2dn\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e}$$

$$-\frac{4}{3}bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{4bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{3/2}}+x\left(a+b\right.$$

$$
\begin{aligned}
&= \frac{4abdn\sqrt[3]{x}}{e} - \frac{32b^2dn^2\sqrt[3]{x}}{3e} + \frac{8}{9}b^2n^2x \\
&+ \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}} - \frac{4ib^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{e^{3/2}} \\
&- \frac{8b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{e^{3/2}} + \frac{4b^2dn\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e} \\
&- \frac{4}{3}bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) - \frac{4bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{3/2}} + x\Big(a+b\,\mathrm{l}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4abdn\sqrt[3]{x}}{e} - \frac{32b^2dn^2\sqrt[3]{x}}{3e} + \frac{8}{9}b^2n^2x \\
&+ \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}} - \frac{4ib^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{e^{3/2}} \\
&- \frac{8b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{e^{3/2}} + \frac{4b^2dn\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e} \\
&- \frac{4}{3}bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) - \frac{4bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{3/2}} + x\Big(a+b\,\mathrm{l}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.88

$$
\int \left(a + b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 dx = \frac{-36ib^2d^{3/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2 - 12bd^{3/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\Big(3a-8bn+6bn}{}
$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2,x]
```

```
[Out] ((-36*I)*b^2*d^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2 - 12*b*d^(3/2)
*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(3*a - 8*b*n + 6*b*n*Log[(2*Sqrt[d])/(
Sqrt[d] + I*Sqrt[e]*x^(1/3))] + 3*b*Log[c*(d + e*x^(2/3))^n]) + Sqrt[e]*x^(
1/3)*(12*a*b*n*(3*d - e*x^(2/3)) + 8*b^2*n^2*(-12*d + e*x^(2/3)) + 9*a^2*e*
x^(2/3) + 6*b*(6*b*d*n + 3*a*e*x^(2/3) - 2*b*e*n*x^(2/3))*Log[c*(d + e*x^(2
/3))^n] + 9*b^2*e*x^(2/3)*Log[c*(d + e*x^(2/3))^n]^2) - (36*I)*b^2*d^(3/2)*
n^2*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x^(1/3))/((-I)*Sqrt[d] + Sqrt[e]*x^(1/3
))])/(9*e^(3/2))
```

Maple [F]

$$\int \left(a+b\ln \left(c \left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^2,x)
```

Fricas [F]

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx = \int \left(b\log \left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="fricas")
[Out] integral(b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c) +
a^2, x)
```

Sympy [F]

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx = \int \left(a+b\log \left(c\left(d+ex^{\frac{2}{3}}\right)^{n}\right)\right)^{2}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2,x)
[Out] Integral((a + b*log(c*(d + e*x**(2/3))**n))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+a\right)^2\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\,dx=\int \left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)\right)^2\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^2,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^2, x)
```

3.478 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 298

$$\begin{aligned}
\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^2}\,dx &= \frac{8b^2e^{3/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} \\
&- \frac{4ib^2e^{3/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} - \frac{8b^2e^{3/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}} \\
&- \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d\sqrt[3]{x}} \\
&- \frac{4be^{3/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}} \\
&- \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x} - \frac{4ib^2e^{3/2}n^2\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}}
\end{aligned}$$

```
[Out] 8*b^2*e^(3/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))/d^(3/2)-4*I*b^2*e^(3/2)*n
^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))^2/d^(3/2)-4*b*e*n*(a+b*ln(c*(d+e*x^(2/3)
)^n))/d/x^(1/3)-4*b*e^(3/2)*n*arctan(x^(1/3)*e^(1/2)/d^(1/2))*(a+b*ln(c*(d+
e*x^(2/3))^n))/d^(3/2)-(a+b*ln(c*(d+e*x^(2/3))^n))^2/x-8*b^2*e^(3/2)*n^2*ar
ctan(x^(1/3)*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/d^(
3/2)-4*I*b^2*e^(3/2)*n^2*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))
/d^(3/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {2508, 2507, 2526, 2505, 211, 2520, 12, 5040, 4964, 2449, 2352}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^2}\,dx = -\frac{4be^{3/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d\sqrt[3]{x}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x} - \frac{4ib^2e^{3/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} + \frac{8b^2e^{3/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{8b^2e^{3/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}} - \frac{4ib^2e^{3/2}n^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}}$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^2,x]
```

```
[Out] (8*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/d^(3/2) - ((4*I)*b^2*
e^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/d^(3/2) - (8*b^2*e^(3/2)*n
^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x
^(1/3))])/d^(3/2) - (4*b*e*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(d*x^(1/3))
- (4*b*e^(3/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2
/3))^n]))/d^(3/2) - (a + b*Log[c*(d + e*x^(2/3))^n])^2/x - ((4*I)*b^2*e^(3/
2)*n^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/d^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2508

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^4}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}+(4ben)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^2\left(d+ex^2\right)}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}\\
&\quad+(4ben)\text{Subst}\left(\int \left(\frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{dx^2}-\frac{e\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)}{d\left(d+ex^2\right)}\right)dx, x, \sqrt[3]{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}+\frac{(4ben)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^2}\,dx, x, \sqrt[3]{x}\right)}{d}\\
&\quad-\frac{\left(4be^2n\right)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)}{d}\\
&= -\frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d\sqrt[3]{x}}\\
&\quad-\frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}}\\
&\quad-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x}+\frac{\left(8b^2e^2n^2\right)\text{Subst}\left(\int \frac{1}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)}{d}\\
&\quad+\frac{\left(8b^2e^3n^2\right)\text{Subst}\left(\int \frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}\left(d+ex^2\right)}\,dx, x, \sqrt[3]{x}\right)}{d}
\end{aligned}$$

$$= \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d\sqrt[3]{x}} - \frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x} + \frac{(8b^2e^{5/2}n^2)\,\mathrm{Subst}\left(\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d+ex^2}dx, x, \sqrt[3]{x}\right)}{d^{3/2}}$$

$$= \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d\sqrt[3]{x}} - \frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x} - \frac{(8b^2e^2n^2)\,\mathrm{Subst}\left(\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{i-\frac{\sqrt{e}x}{\sqrt{d}}}dx, x, \sqrt[3]{x}\right)}{d^2}$$

$$= \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} - \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d\sqrt[3]{x}} - \frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x} + \frac{(8b^2e^2n^2)\,\mathrm{Subst}\left(\int\frac{\log\left(\frac{2}{1+\frac{i\sqrt{e}x}{\sqrt{d}}}\right)}{1+\frac{ex^2}{d}}dx, x, \sqrt[3]{x}\right)}{d^2}$$

$$
\begin{aligned}
&= \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} \\
&\quad - \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d\sqrt[3]{x}} \\
&\quad - \frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}} \\
&\quad - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x} - \frac{(8ib^2e^{3/2}n^2)\,\mathrm{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+\frac{i\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}}\right)}{d^{3/2}} \\
&= \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} \\
&\quad - \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d\sqrt[3]{x}} \\
&\quad - \frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}} \\
&\quad - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x} - \frac{4ib^2e^{3/2}n^2\mathrm{Li}_2\left(1-\frac{2}{1+\frac{i\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}}\right)}{d^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.83

$$
\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^2}\,dx = \frac{-4ib^2e^{3/2}n^2x\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2 - 4be^{3/2}nx\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a-2bn+\right.}{}
$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^2,x]
```

```
[Out] ((-4*I)*b^2*e^(3/2)*n^2*x*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2 - 4*b*e^(3/2)
*n*x*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - 2*b*n + 2*b*n*Log[(2*Sqrt[d])/(
Sqrt[d] + I*Sqrt[e]*x^(1/3))] + b*Log[c*(d + e*x^(2/3))^n]) - Sqrt[d]*(a +
b*Log[c*(d + e*x^(2/3))^n])*(a*d + 4*b*e*n*x^(2/3) + b*d*Log[c*(d + e*x^(2/
3))^n]) - (4*I)*b^2*e^(3/2)*n^2*x*PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x^(1/3))/
((-I)*Sqrt[d] + Sqrt[e]*x^(1/3))])/(d^(3/2)*x)
```

Maple [F]

$$\int \frac{\left(a+b\ln \left(c{\left(d+e\,x^{\frac{2}{3}}\right)}^{n}\right)\right)^{2}}{x^{2}}d x$$

[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^2,x)

[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^2,x)

Fricas [F]

$$\int \frac{(a+b\log{(c(d+ex^{2/3})^n)})^2}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{2}}{x^{2}}\,dx$$

[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^2,x, algorithm="fricas")

[Out] integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c) + a^2)/x^2, x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log{(c(d+ex^{2/3})^n)})^2}{x^2}\,dx=\text{Timed out}$$

[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**2,x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log{(c(d+ex^{2/3})^n)})^2}{x^2}\,dx=\text{Exception raised: ValueError}$$

[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^2,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)\right)^2}{x^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^2, x)
```

3.479 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 476

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^4}\,dx = -\frac{8b^2e^2n^2}{105d^2x^{5/3}} + \frac{32b^2e^3n^2}{105d^3x} - \frac{568b^2e^4n^2}{315d^4\sqrt[3]{x}} - \frac{1408b^2e^{9/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{315d^{9/2}} + \frac{4ib^2e^{9/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{3d^{9/2}} + \frac{8b^2e^{9/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3d^{9/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{21dx^{7/3}} + \frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^2x^{5/3}} - \frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9d^3x} + \frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^4\sqrt[3]{x}} + \frac{4be^{9/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^{9/2}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3} + \frac{4ib^2e^{9/2}n^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3d^{9/2}}$$

```
[Out] -8/105*b^2*e^2*n^2/d^2/x^(5/3)+32/105*b^2*e^3*n^2/d^3/x-568/315*b^2*e^4*n^2
/d^4/x^(1/3)-1408/315*b^2*e^(9/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))/d^(9/
2)+4/3*I*b^2*e^(9/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))^2/d^(9/2)-4/21*b*e
*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d/x^(7/3)+4/15*b*e^2*n*(a+b*ln(c*(d+e*x^(2/3
))^n))/d^2/x^(5/3)-4/9*b*e^3*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^3/x+4/3*b*e^4*
n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^4/x^(1/3)+4/3*b*e^(9/2)*n*arctan(x^(1/3)*e^
```

```
(1/2)/d^(1/2))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^(9/2)-1/3*(a+b*ln(c*(d+e*x^(2/
3))^n))^2/x^3+8/3*b^2*e^(9/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))*ln(2*d^(1
/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/d^(9/2)+4/3*I*b^2*e^(9/2)*n^2*polylog(2,1-
2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/d^(9/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2508, 2507, 2526, 2505, 331, 211, 2520, 12, 5040, 4964, 2449, 2352}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^4}\,dx = \frac{4be^{9/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^{9/2}} + \frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^4\sqrt[3]{x}} - \frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9d^3x} + \frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^2x^{5/3}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{21dx^{7/3}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3} + \frac{4ib^2e^{9/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{3d^{9/2}} - \frac{1408b^2e^{9/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{315d^{9/2}} + \frac{8b^2e^{9/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3d^{9/2}} + \frac{4ib^2e^{9/2}n^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3d^{9/2}} - \frac{568b^2e^4n^2}{315d^4\sqrt[3]{x}} + \frac{32b^2e^3n^2}{105d^3x} - \frac{8b^2e^2n^2}{105d^2x^{5/3}}$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^4,x]
```

```
[Out] (-8*b^2*e^2*n^2)/(105*d^2*x^(5/3)) + (32*b^2*e^3*n^2)/(105*d^3*x) - (568*b^
2*e^4*n^2)/(315*d^4*x^(1/3)) - (1408*b^2*e^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3
))/Sqrt[d]])/(315*d^(9/2)) + (((4*I)/3)*b^2*e^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(
1/3))/Sqrt[d]]^2)/d^(9/2) + (8*b^2*e^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqr
t[d]]*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/(3*d^(9/2)) - (4*b*e*
n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(21*d*x^(7/3)) + (4*b*e^2*n*(a + b*Log[
c*(d + e*x^(2/3))^n]))/(15*d^2*x^(5/3)) - (4*b*e^3*n*(a + b*Log[c*(d + e*x^
(2/3))^n]))/(9*d^3*x) + (4*b*e^4*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*d^4
*x^(1/3)) + (4*b*e^(9/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(
d + e*x^(2/3))^n]))/(3*d^(9/2)) - (a + b*Log[c*(d + e*x^(2/3))^n])^2/(3*x^3
) + (((4*I)/3)*b^2*e^(9/2)*n^2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt
[e]*x^(1/3))])/d^(9/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2508

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
```

```
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^{10}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3} + \frac{1}{3}(4ben)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^8\left(d+ex^2\right)}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3}$$

$$+\frac{1}{3}(4ben)\text{Subst}\left(\int \left(\frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{dx^8} - \frac{e\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)}{d^2x^6} + \frac{e^2\left(a+b\log\left(c\left(d+ex\right.\right.\right.}{d^3x^4}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3} + \frac{(4ben)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^8}\,dx, x, \sqrt[3]{x}\right)}{3d}$$
$$- \frac{(4be^2n)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^6}\,dx, x, \sqrt[3]{x}\right)}{3d^2}$$
$$+ \frac{(4be^3n)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^4}\,dx, x, \sqrt[3]{x}\right)}{3d^3}$$
$$- \frac{(4be^4n)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^2}\,dx, x, \sqrt[3]{x}\right)}{3d^4}$$
$$+ \frac{(4be^5n)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)}{3d^4}$$
$$= -\frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{21dx^{7/3}} + \frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^2x^{5/3}}$$
$$- \frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9d^3x} + \frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^4\sqrt[3]{x}}$$
$$+ \frac{4be^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^{9/2}}$$
$$- \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3} + \frac{(8b^2e^2n^2)\,\text{Subst}\left(\int \frac{1}{x^6(d+ex^2)}\,dx, x, \sqrt[3]{x}\right)}{21d}$$
$$- \frac{(8b^2e^3n^2)\,\text{Subst}\left(\int \frac{1}{x^4(d+ex^2)}\,dx, x, \sqrt[3]{x}\right)}{15d^2} + \frac{(8b^2e^4n^2)\,\text{Subst}\left(\int \frac{1}{x^2(d+ex^2)}\,dx, x, \sqrt[3]{x}\right)}{9d^3}$$
$$- \frac{(8b^2e^5n^2)\,\text{Subst}\left(\int \frac{1}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)}{3d^4} - \frac{(8b^2e^6n^2)\,\text{Subst}\left(\int \frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}(d+ex^2)}\,dx, x, \sqrt[3]{x}\right)}{3d^4}$$

$$= -\frac{8b^2e^2n^2}{105d^2x^{5/3}} + \frac{8b^2e^3n^2}{45d^3x} - \frac{8b^2e^4n^2}{9d^4\sqrt[3]{x}} - \frac{8b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3d^{9/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{21dx^{7/3}} + \frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^2x^{5/3}} - \frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9d^3x} + \frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^4\sqrt[3]{x}} + \frac{4be^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^{9/2}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3} - \frac{(8b^2e^3n^2)\,\mathrm{Subst}\left(\int\frac{1}{x^4(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{21d^2} + \frac{(8b^2e^4n^2)\,\mathrm{Subst}\left(\int\frac{1}{x^2(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{15d^3} - \frac{(8b^2e^5n^2)\,\mathrm{Subst}\left(\int\frac{1}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{9d^4} - \frac{(8b^2e^{11/2}n^2)\,\mathrm{Subst}\left(\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{3d^{9/2}}$$

$$= -\frac{8b^2e^2n^2}{105d^2x^{5/3}} + \frac{32b^2e^3n^2}{105d^3x} - \frac{64b^2e^4n^2}{45d^4\sqrt[3]{x}} - \frac{32b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{9d^{9/2}} + \frac{4ib^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{3d^{9/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{21dx^{7/3}} + \frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^2x^{5/3}} - \frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9d^3x} + \frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^4\sqrt[3]{x}} + \frac{4be^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^{9/2}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3} + \frac{(8b^2e^4n^2)\,\mathrm{Subst}\left(\int\frac{1}{x^2(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{21d^3} + \frac{(8b^2e^5n^2)\,\mathrm{Subst}\left(\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{i-\frac{\sqrt{e}x}{\sqrt{d}}}\,dx,x,\sqrt[3]{x}\right)}{3d^5} - \frac{(8b^2e^5n^2)\,\mathrm{Subst}\left(\int\frac{1}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{15d^4}$$

$$= -\frac{8b^2e^2n^2}{105d^2x^{5/3}} + \frac{32b^2e^3n^2}{105d^3x} - \frac{568b^2e^4n^2}{315d^4\sqrt[3]{x}} - \frac{184b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{45d^{9/2}} + \frac{4ib^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{3d^{9/2}} + \frac{8b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3d^{9/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{21dx^{7/3}} + \frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^2x^{5/3}} - \frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9d^3x} + \frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^4\sqrt[3]{x}} + \frac{4be^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^{9/2}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3} - \frac{(8b^2e^5n^2)\operatorname{Subst}\left(\int\frac{\log\left(\frac{2}{1+\frac{i\sqrt{e}x}{\sqrt{d}}}\right)}{1+\frac{ex^2}{d}}\,dx,x,\sqrt[3]{x}\right)}{3d^5} - \frac{(8b^2e^5n^2)\operatorname{Subst}\left(\int\frac{1}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{21d^4}$$

$$= -\frac{8b^2e^2n^2}{105d^2x^{5/3}} + \frac{32b^2e^3n^2}{105d^3x} - \frac{568b^2e^4n^2}{315d^4\sqrt[3]{x}} - \frac{1408b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{315d^{9/2}} + \frac{4ib^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{3d^{9/2}} + \frac{8b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3d^{9/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{21dx^{7/3}} + \frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^2x^{5/3}} - \frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9d^3x} + \frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^4\sqrt[3]{x}} + \frac{4be^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^{9/2}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3} + \frac{\left(8ib^2e^{9/2}n^2\right)\operatorname{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+\frac{i\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}}\right)}{3d^{9/2}}$$

$$= -\frac{8b^2e^2n^2}{105d^2x^{5/3}} + \frac{32b^2e^3n^2}{105d^3x} - \frac{568b^2e^4n^2}{315d^4\sqrt[3]{x}} - \frac{1408b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{315d^{9/2}} + \frac{4ib^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{3d^{9/2}} + \frac{8b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{3d^{9/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{21dx^{7/3}} + \frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^2x^{5/3}} - \frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{9d^3x} + \frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^4\sqrt[3]{x}} + \frac{4be^{9/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3d^{9/2}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3} + \frac{4ib^2e^{9/2}n^2\text{Li}_2\left(1-\frac{2}{1+\frac{i\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}}\right)}{3d^{9/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.39 (sec) , antiderivative size = 473, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^4}\,dx = -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3x^3} + \frac{4}{3}ben\left(-\frac{2be^{7/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{9/2}} - \frac{2ben\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\frac{ex^{2/3}}{d}\right)}{35d^2x^{5/3}} + \frac{2be^2n\,\text{Hypergeometric}}{15}\right.$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^4,x]
```

```
[Out] -1/3*(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^3 + (4*b*e*n*((-2*b*e^(7/2)*n*Arc
Tan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/d^(9/2) - (2*b*e*n*Hypergeometric2F1[-5/2,
1, -3/2, -((e*x^(2/3))/d)])/(35*d^2*x^(5/3)) + (2*b*e^2*n*Hypergeometric2F1
[-3/2, 1, -1/2, -((e*x^(2/3))/d)])/(15*d^3*x) - (2*b*e^3*n*Hypergeometric2F
1[-1/2, 1, 1/2, -((e*x^(2/3))/d)])/(3*d^4*x^(1/3)) - (a + b*Log[c*(d + e*x^
(2/3))^n])/(7*d*x^(7/3)) + (e*(a + b*Log[c*(d + e*x^(2/3))^n]))/(5*d^2*x^(5
/3)) - (e^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*d^3*x) + (e^3*(a + b*Log[c
*(d + e*x^(2/3))^n]))/(d^4*x^(1/3)) + (e^(7/2)*ArcTan[(Sqrt[e]*x^(1/3))/Sqr
t[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/d^(9/2) + (I*b*e^(7/2)*n*(ArcTan[(S
qrt[e]*x^(1/3))/Sqrt[d]]*(ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]] - (2*I)*Log[(2*
Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))]) + PolyLog[2, (I*Sqrt[d] + Sqrt[e]*
x^(1/3))/((-I)*Sqrt[d] + Sqrt[e]*x^(1/3))]))/d^(9/2)))/3
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}}{x^{4}}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^4,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\right)^{2}}{x^{4}}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{2}}{x^{4}}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c) +
 a^2)/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\right)^{2}}{x^{4}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**4,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\right)^{2}}{x^{4}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^4}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^4,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^4}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)\right)^2}{x^4}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^4,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^4, x)
```

3.480 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 640

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^6}\,dx &= -\frac{8b^2e^2n^2}{715d^2x^{11/3}}+\frac{64b^2e^3n^2}{2145d^3x^3}-\frac{2872b^2e^4n^2}{45045d^4x^{7/3}} \\ &+\frac{1216b^2e^5n^2}{9009d^5x^{5/3}}-\frac{224072b^2e^6n^2}{675675d^6x}+\frac{344192b^2e^7n^2}{225225d^7\sqrt[3]{x}}+\frac{704552b^2e^{15/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{225225d^{15/2}} \\ &-\frac{4ib^2e^{15/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{5d^{15/2}}-\frac{8b^2e^{15/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{5d^{15/2}} \\ &-\frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{65dx^{13/3}}+\frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{55d^2x^{11/3}} \\ &-\frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{45d^3x^3}+\frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^4x^{7/3}} \\ &-\frac{4be^5n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{25d^5x^{5/3}}+\frac{4be^6n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^6x} \\ &-\frac{4be^7n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^7\sqrt[3]{x}}-\frac{4be^{15/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^{15/2}} \\ &-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5}-\frac{4ib^2e^{15/2}n^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{5d^{15/2}}\end{aligned}$$

```
[Out] -8/715*b^2*e^2*n^2/d^2/x^(11/3)+64/2145*b^2*e^3*n^2/d^3/x^3-2872/45045*b^2*
e^4*n^2/d^4/x^(7/3)+1216/9009*b^2*e^5*n^2/d^5/x^(5/3)-224072/675675*b^2*e^6
*n^2/d^6/x+344192/225225*b^2*e^7*n^2/d^7/x^(1/3)+704552/225225*b^2*e^(15/2)
*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))/d^(15/2)-4/5*I*b^2*e^(15/2)*n^2*arctan
```

```
(x^(1/3)*e^(1/2)/d^(1/2))^2/d^(15/2)-4/65*b*e*n*(a+b*ln(c*(d+e*x^(2/3))^n))
/d/x^(13/3)+4/55*b*e^2*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^2/x^(11/3)-4/45*b*e^
3*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^3/x^3+4/35*b*e^4*n*(a+b*ln(c*(d+e*x^(2/3)
)^n))/d^4/x^(7/3)-4/25*b*e^5*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^5/x^(5/3)+4/15
*b*e^6*n*(a+b*ln(c*(d+e*x^(2/3))^n))/d^6/x-4/5*b*e^7*n*(a+b*ln(c*(d+e*x^(2/
3))^n))/d^7/x^(1/3)-4/5*b*e^(15/2)*n*arctan(x^(1/3)*e^(1/2)/d^(1/2))*(a+b*l
n(c*(d+e*x^(2/3))^n))/d^(15/2)-1/5*(a+b*ln(c*(d+e*x^(2/3))^n))^2/x^5-8/5*b^
2*e^(15/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x^(1
/3)*e^(1/2)))/d^(15/2)-4/5*I*b^2*e^(15/2)*n^2*polylog(2,1-2*d^(1/2)/(d^(1/2
)+I*x^(1/3)*e^(1/2)))/d^(15/2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.00, number of steps used = 45, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2508, 2507, 2526, 2505, 331, 211, 2520, 12, 5040, 4964, 2449, 2352}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^6}\,dx =$$
$$-\frac{4be^{15/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^{15/2}}$$
$$-\frac{4be^7n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^7\sqrt[3]{x}}+\frac{4be^6n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^6x}$$
$$-\frac{4be^5n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{25d^5x^{5/3}}+\frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^4x^{7/3}}$$
$$-\frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{45d^3x^3}+\frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{55d^2x^{11/3}}$$
$$-\frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{65dx^{13/3}}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5}$$
$$-\frac{4ib^2e^{15/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{5d^{15/2}}+\frac{704552b^2e^{15/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{225225d^{15/2}}$$
$$-\frac{8b^2e^{15/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{5d^{15/2}}$$
$$-\frac{4ib^2e^{15/2}n^2\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{5d^{15/2}}+\frac{344192b^2e^7n^2}{225225d^7\sqrt[3]{x}}$$
$$-\frac{224072b^2e^6n^2}{675675d^6x}+\frac{1216b^2e^5n^2}{9009d^5x^{5/3}}-\frac{2872b^2e^4n^2}{45045d^4x^{7/3}}+\frac{64b^2e^3n^2}{2145d^3x^3}-\frac{8b^2e^2n^2}{715d^2x^{11/3}}$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^6,x]
```

```
[Out] (-8*b^2*e^2*n^2)/(715*d^2*x^(11/3)) + (64*b^2*e^3*n^2)/(2145*d^3*x^3) - (28
72*b^2*e^4*n^2)/(45045*d^4*x^(7/3)) + (1216*b^2*e^5*n^2)/(9009*d^5*x^(5/3))
 - (224072*b^2*e^6*n^2)/(675675*d^6*x) + (344192*b^2*e^7*n^2)/(225225*d^7*x
^(1/3)) + (704552*b^2*e^(15/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(2252
25*d^(15/2)) - (((4*I)/5)*b^2*e^(15/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]
]^2)/d^(15/2) - (8*b^2*e^(15/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(
2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/(5*d^(15/2)) - (4*b*e*n*(a + b*L
og[c*(d + e*x^(2/3))^n]))/(65*d*x^(13/3)) + (4*b*e^2*n*(a + b*Log[c*(d + e*
x^(2/3))^n]))/(55*d^2*x^(11/3)) - (4*b*e^3*n*(a + b*Log[c*(d + e*x^(2/3))^n
]))/(45*d^3*x^3) + (4*b*e^4*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(35*d^4*x^(
7/3)) - (4*b*e^5*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(25*d^5*x^(5/3)) + (4*
b*e^6*n*(a + b*Log[c*(d + e*x^(2/3))^n]))/(15*d^6*x) - (4*b*e^7*n*(a + b*Lo
g[c*(d + e*x^(2/3))^n]))/(5*d^7*x^(1/3)) - (4*b*e^(15/2)*n*ArcTan[(Sqrt[e]*
x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/(5*d^(15/2)) - (a + b*L
og[c*(d + e*x^(2/3))^n])^2/(5*x^5) - (((4*I)/5)*b^2*e^(15/2)*n^2*PolyLog[2,
 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/d^(15/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2508

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
```

```
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^{16}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5} + \frac{1}{5}(4ben)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^{14}\left(d+ex^2\right)}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5} + \frac{1}{5}(4ben)\text{Subst}\left(\int \left(\frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{dx^{14}} - \frac{e\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)}{d^2x^{12}} + \frac{e^2\left(a+b\log\left(c\left(d+e\right.\right.\right.}{d^3x^{10}}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5} + \frac{(4ben)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^{14}}\,dx, x, \sqrt[3]{x}\right)}{5d} - \frac{(4be^2n)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^{12}}\,dx, x, \sqrt[3]{x}\right)}{5d^2} + \frac{(4be^3n)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^{10}}\,dx, x, \sqrt[3]{x}\right)}{5d^3} - \frac{(4be^4n)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^{8}}\,dx, x, \sqrt[3]{x}\right)}{5d^4} + \frac{(4be^5n)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^{6}}\,dx, x, \sqrt[3]{x}\right)}{5d^5} - \frac{(4be^6n)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^{4}}\,dx, x, \sqrt[3]{x}\right)}{5d^6} + \frac{(4be^7n)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^{2}}\,dx, x, \sqrt[3]{x}\right)}{5d^7} - \frac{(4be^8n)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)}{5d^7}$$

$$= -\frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{65dx^{13/3}} + \frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{55d^2x^{11/3}} - \frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{45d^3x^3} + \frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^4x^{7/3}} - \frac{4be^5n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{25d^5x^{5/3}} + \frac{4be^6n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^6x} - \frac{4be^7n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^7\sqrt[3]{x}} - \frac{4be^{15/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^{15/2}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5} + \frac{(8b^2e^2n^2)\operatorname{Subst}\left(\int\frac{1}{x^{12}(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{65d} - \frac{(8b^2e^3n^2)\operatorname{Subst}\left(\int\frac{1}{x^{10}(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{55d^2} + \frac{(8b^2e^4n^2)\operatorname{Subst}\left(\int\frac{1}{x^{8}(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{45d^3} - \frac{(8b^2e^5n^2)\operatorname{Subst}\left(\int\frac{1}{x^{6}(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{35d^4} + \frac{(8b^2e^6n^2)\operatorname{Subst}\left(\int\frac{1}{x^{4}(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{25d^5} - \frac{(8b^2e^7n^2)\operatorname{Subst}\left(\int\frac{1}{x^{2}(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{15d^6} + \frac{(8b^2e^8n^2)\operatorname{Subst}\left(\int\frac{1}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{5d^7} + \frac{(8b^2e^9n^2)\operatorname{Subst}\left(\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{5d^7}$$

$$
\begin{aligned}
&= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{8b^2e^3n^2}{495d^3x^3} - \frac{8b^2e^4n^2}{315d^4x^{7/3}} + \frac{8b^2e^5n^2}{175d^5x^{5/3}} - \frac{8b^2e^6n^2}{75d^6x} + \frac{8b^2e^7n^2}{15d^7\sqrt[3]{x}} \\
&\quad + \frac{8b^2e^{15/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{5d^{15/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{65dx^{13/3}} \\
&\quad + \frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{55d^2x^{11/3}} - \frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{45d^3x^3} \\
&\quad + \frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^4x^{7/3}} - \frac{4be^5n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{25d^5x^{5/3}} \\
&\quad + \frac{4be^6n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^6x} - \frac{4be^7n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^7\sqrt[3]{x}} \\
&\quad - \frac{4be^{15/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^{15/2}} \\
&\quad - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5} - \frac{\left(8b^2e^3n^2\right)\text{Subst}\left(\int \frac{1}{x^{10}(d+ex^2)}\,dx, x, \sqrt[3]{x}\right)}{65d^2} \\
&\quad + \frac{\left(8b^2e^4n^2\right)\text{Subst}\left(\int \frac{1}{x^8(d+ex^2)}\,dx, x, \sqrt[3]{x}\right)}{55d^3} \\
&\quad - \frac{\left(8b^2e^5n^2\right)\text{Subst}\left(\int \frac{1}{x^6(d+ex^2)}\,dx, x, \sqrt[3]{x}\right)}{45d^4} + \frac{\left(8b^2e^6n^2\right)\text{Subst}\left(\int \frac{1}{x^4(d+ex^2)}\,dx, x, \sqrt[3]{x}\right)}{35d^5} \\
&\quad - \frac{\left(8b^2e^7n^2\right)\text{Subst}\left(\int \frac{1}{x^2(d+ex^2)}\,dx, x, \sqrt[3]{x}\right)}{25d^6} + \frac{\left(8b^2e^8n^2\right)\text{Subst}\left(\int \frac{1}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)}{15d^7} \\
&\quad + \frac{\left(8b^2e^{17/2}n^2\right)\text{Subst}\left(\int \frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)}{5d^{15/2}}
\end{aligned}
$$

$$= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{64b^2e^3n^2}{2145d^3x^3} - \frac{32b^2e^4n^2}{693d^4x^{7/3}} + \frac{128b^2e^5n^2}{1575d^5x^{5/3}} - \frac{32b^2e^6n^2}{175d^6x} + \frac{64b^2e^7n^2}{75d^7\sqrt[3]{x}} + \frac{32b^2e^{15/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{15d^{15/2}} - \frac{4ib^2e^{15/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{5d^{15/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{65dx^{13/3}} + \frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{55d^2x^{11/3}} - \frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{45d^3x^3} + \frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^4x^{7/3}} - \frac{4be^5n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{25d^5x^{5/3}} + \frac{4be^6n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^6x} - \frac{4be^7n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^7\sqrt[3]{x}} - \frac{4be^{15/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^{15/2}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5} + \frac{(8b^2e^4n^2)\,\text{Subst}\left(\int\frac{1}{x^8(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{65d^3} - \frac{(8b^2e^5n^2)\,\text{Subst}\left(\int\frac{1}{x^6(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{55d^4} + \frac{(8b^2e^6n^2)\,\text{Subst}\left(\int\frac{1}{x^4(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{45d^5} - \frac{(8b^2e^7n^2)\,\text{Subst}\left(\int\frac{1}{x^2(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{35d^6} - \frac{(8b^2e^8n^2)\,\text{Subst}\left(\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{i-\frac{\sqrt{e}x}{\sqrt{d}}}\,dx,x,\sqrt[3]{x}\right)}{5d^8} + \frac{(8b^2e^8n^2)\,\text{Subst}\left(\int\frac{1}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{25d^7}$$

$$= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{64b^2e^3n^2}{2145d^3x^3} - \frac{2872b^2e^4n^2}{45045d^4x^{7/3}} + \frac{1912b^2e^5n^2}{17325d^5x^{5/3}} - \frac{1144b^2e^6n^2}{4725d^6x} + \frac{568b^2e^7n^2}{525d^7\sqrt[3]{x}} + \frac{184b^2e^{15/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{75d^{15/2}} - \frac{4ib^2e^{15/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{5d^{15/2}} - \frac{8b^2e^{15/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{5d^{15/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{65dx^{13/3}} + \frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{55d^2x^{11/3}} - \frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{45d^3x^3} + \frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^4x^{7/3}} - \frac{4be^5n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{25d^5x^{5/3}} + \frac{4be^6n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^6x} - \frac{4be^7n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^7\sqrt[3]{x}} - \frac{4be^{15/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^{15/2}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5} - \frac{(8b^2e^5n^2)\,\text{Subst}\left(\int\frac{1}{x^6(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{65d^4} + \frac{(8b^2e^6n^2)\,\text{Subst}\left(\int\frac{1}{x^4(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{55d^5} - \frac{(8b^2e^7n^2)\,\text{Subst}\left(\int\frac{1}{x^2(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{45d^6} + \frac{(8b^2e^8n^2)\,\text{Subst}\left(\int\frac{\log\left(\frac{2}{1+\frac{i\sqrt{e}x}{\sqrt{d}}}\right)}{1+\frac{ex^2}{d}}\,dx,x,\sqrt[3]{x}\right)}{5d^8} + \frac{(8b^2e^8n^2)\,\text{Subst}\left(\int\frac{1}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{35d^7}$$

$$= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{64b^2e^3n^2}{2145d^3x^3} - \frac{2872b^2e^4n^2}{45045d^4x^{7/3}} + \frac{1216b^2e^5n^2}{9009d^5x^{5/3}} - \frac{15104b^2e^6n^2}{51975d^6x} + \frac{1984b^2e^7n^2}{1575d^7\sqrt[3]{x}} + \frac{1408b^2e^{15/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{525d^{15/2}} - \frac{4ib^2e^{15/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{5d^{15/2}} - \frac{8b^2e^{15/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{5d^{15/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{65dx^{13/3}} + \frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{55d^2x^{11/3}} - \frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{45d^3x^3} + \frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^4x^{7/3}} - \frac{4be^5n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{25d^5x^{5/3}} + \frac{4be^6n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^6x} - \frac{4be^7n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^7\sqrt[3]{x}} - \frac{4be^{15/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^{15/2}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5} + \frac{(8b^2e^6n^2)\operatorname{Subst}\left(\int\frac{1}{x^4(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{65d^5} - \frac{(8b^2e^7n^2)\operatorname{Subst}\left(\int\frac{1}{x^2(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{55d^6} - \frac{(8ib^2e^{15/2}n^2)\operatorname{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+\frac{i\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}}\right)}{5d^{15/2}} + \frac{(8b^2e^8n^2)\operatorname{Subst}\left(\int\frac{1}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{45d^7}$$

$$
\begin{aligned}
&= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{64b^2e^3n^2}{2145d^3x^3} - \frac{2872b^2e^4n^2}{45045d^4x^{7/3}} + \frac{1216b^2e^5n^2}{9009d^5x^{5/3}} - \frac{224072b^2e^6n^2}{675675d^6x} \\
&+ \frac{24344b^2e^7n^2}{17325d^7\sqrt[3]{x}} + \frac{4504b^2e^{15/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{1575d^{15/2}} - \frac{4ib^2e^{15/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{5d^{15/2}} \\
&- \frac{8b^2e^{15/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{5d^{15/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{65dx^{13/3}} \\
&+ \frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{55d^2x^{11/3}} - \frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{45d^3x^3} \\
&+ \frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^4x^{7/3}} - \frac{4be^5n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{25d^5x^{5/3}} \\
&+ \frac{4be^6n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^6x} - \frac{4be^7n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^7\sqrt[3]{x}} \\
&- \frac{4be^{15/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^{15/2}} \\
&- \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5} - \frac{4ib^2e^{15/2}n^2\operatorname{Li}_2\left(1-\frac{2}{1+\frac{i\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}}\right)}{5d^{15/2}} \\
&- \frac{(8b^2e^7n^2)\operatorname{Subst}\left(\int\frac{1}{x^2(d+ex^2)}\,dx, x, \sqrt[3]{x}\right)}{65d^6} + \frac{(8b^2e^8n^2)\operatorname{Subst}\left(\int\frac{1}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)}{55d^7}
\end{aligned}
$$

$$= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{64b^2e^3n^2}{2145d^3x^3} - \frac{2872b^2e^4n^2}{45045d^4x^{7/3}} + \frac{1216b^2e^5n^2}{9009d^5x^{5/3}} - \frac{224072b^2e^6n^2}{675675d^6x} + \frac{344192b^2e^7n^2}{225225d^7\sqrt[3]{x}} + \frac{52064b^2e^{15/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{17325d^{15/2}} - \frac{4ib^2e^{15/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{5d^{15/2}} - \frac{8b^2e^{15/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{5d^{15/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{65dx^{13/3}} + \frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{55d^2x^{11/3}} - \frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{45d^3x^3} + \frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^4x^{7/3}} - \frac{4be^5n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{25d^5x^{5/3}} + \frac{4be^6n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^6x} - \frac{4be^7n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^7\sqrt[3]{x}} - \frac{4be^{15/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^{15/2}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5} - \frac{4ib^2e^{15/2}n^2\operatorname{Li}_2\left(1-\frac{2}{1+\frac{i\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}}\right)}{5d^{15/2}} + \frac{(8b^2e^8n^2)\operatorname{Subst}\left(\int\frac{1}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{65d^7}$$

$$
\begin{aligned}
&= -\frac{8b^2e^2n^2}{715d^2x^{11/3}} + \frac{64b^2e^3n^2}{2145d^3x^3} - \frac{2872b^2e^4n^2}{45045d^4x^{7/3}} + \frac{1216b^2e^5n^2}{9009d^5x^{5/3}} - \frac{224072b^2e^6n^2}{675675d^6x} \\
&+ \frac{344192b^2e^7n^2}{225225d^7\sqrt[3]{x}} + \frac{704552b^2e^{15/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{225225d^{15/2}} - \frac{4ib^2e^{15/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{5d^{15/2}} \\
&- \frac{8b^2e^{15/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{5d^{15/2}} - \frac{4ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{65dx^{13/3}} \\
&+ \frac{4be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{55d^2x^{11/3}} - \frac{4be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{45d^3x^3} \\
&+ \frac{4be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^4x^{7/3}} - \frac{4be^5n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{25d^5x^{5/3}} \\
&+ \frac{4be^6n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{15d^6x} - \frac{4be^7n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^7\sqrt[3]{x}} \\
&- \frac{4be^{15/2}n\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{5d^{15/2}} \\
&- \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5} - \frac{4ib^2e^{15/2}n^2\operatorname{Li}_2\left(1-\frac{2}{1+\frac{i\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}}\right)}{5d^{15/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.76 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.06

$$
\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^6}\,dx = -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5x^5}
$$

$$
+\frac{4}{5}ben\left(\frac{2be^{13/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{d^{15/2}} - \frac{2ben\,\text{Hypergeometric2F1}\left(-\frac{11}{2},1,-\frac{9}{2},-\frac{ex^{2/3}}{d}\right)}{143d^2x^{11/3}} + \frac{2be^2n\,\text{Hypergeometri}}{9}\right.
$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^6,x]
```

```
[Out] -1/5*(a + b*Log[c*(d + e*x^(2/3))^n])^2/x^5 + (4*b*e*n*((2*b*e^(13/2)*n*Arc
Tan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/d^(15/2) - (2*b*e*n*Hypergeometric2F1[-11/2
, 1, -9/2, -((e*x^(2/3))/d)])/(143*d^2*x^(11/3)) + (2*b*e^2*n*Hypergeometri
c2F1[-9/2, 1, -7/2, -((e*x^(2/3))/d)])/(99*d^3*x^3) - (2*b*e^3*n*Hypergeome
tric2F1[-7/2, 1, -5/2, -((e*x^(2/3))/d)])/(63*d^4*x^(7/3)) + (2*b*e^4*n*Hyp
ergeometric2F1[-5/2, 1, -3/2, -((e*x^(2/3))/d)])/(35*d^5*x^(5/3)) - (2*b*e^
5*n*Hypergeometric2F1[-3/2, 1, -1/2, -((e*x^(2/3))/d)])/(15*d^6*x) + (2*b*e
```

```
^6*n*Hypergeometric2F1[-1/2, 1, 1/2, -((e*x^(2/3))/d)])/(3*d^7*x^(1/3)) - (
a + b*Log[c*(d + e*x^(2/3))^n])/(13*d*x^(13/3)) + (e*(a + b*Log[c*(d + e*x^
(2/3))^n]))/(11*d^2*x^(11/3)) - (e^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(9*d
^3*x^3) + (e^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(7*d^4*x^(7/3)) - (e^4*(a
+ b*Log[c*(d + e*x^(2/3))^n]))/(5*d^5*x^(5/3)) + (e^5*(a + b*Log[c*(d + e*x
^(2/3))^n]))/(3*d^6*x) - (e^6*(a + b*Log[c*(d + e*x^(2/3))^n]))/(d^7*x^(1/3
)) - (e^(13/2)*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3
))^n]))/d^(15/2) - (I*b*e^(13/2)*n*(ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(ArcT
an[(Sqrt[e]*x^(1/3))/Sqrt[d]] - (2*I)*Log[(2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*
x^(1/3))]) + PolyLog[2, (I*Sqrt[d] + Sqrt[e]*x^(1/3))/((-I)*Sqrt[d] + Sqrt[
e]*x^(1/3))]))/d^(15/2)))/5
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{2}}{x^{6}}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^6,x)
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^2/x^6,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\right)^{2}}{x^{6}}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{2}}{x^{6}}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^6,x, algorithm="fricas")
[Out] integral((b^2*log((e*x^(2/3) + d)^n*c)^2 + 2*a*b*log((e*x^(2/3) + d)^n*c) +
 a^2)/x^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\right)^{2}}{x^{6}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**2/x**6,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^6}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^6}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^2}{x^6}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^2/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^2/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^6}\,dx = \int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)\right)^2}{x^6}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^6,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^2/x^6, x)
```

3.481 $\int x^3 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 913

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=-\frac{45b^3d^4n^3\left(d+ex^{2/3}\right)^2}{16e^6}$$
$$+\frac{10b^3d^3n^3\left(d+ex^{2/3}\right)^3}{9e^6}-\frac{45b^3d^2n^3\left(d+ex^{2/3}\right)^4}{128e^6}$$
$$+\frac{9b^3dn^3\left(d+ex^{2/3}\right)^5}{125e^6}-\frac{b^3n^3\left(d+ex^{2/3}\right)^6}{144e^6}-\frac{9ab^2d^5n^2x^{2/3}}{e^5}$$
$$+\frac{9b^3d^5n^3x^{2/3}}{e^5}-\frac{9b^3d^5n^2\left(d+ex^{2/3}\right)\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e^6}$$
$$+\frac{45b^2d^4n^2\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{8e^6}$$
$$-\frac{10b^2d^3n^2\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^6}$$
$$+\frac{45b^2d^2n^2\left(d+ex^{2/3}\right)^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{32e^6}$$
$$-\frac{9b^2dn^2\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{25e^6}$$
$$+\frac{b^2n^2\left(d+ex^{2/3}\right)^6\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{24e^6}$$
$$+\frac{9bd^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^6}$$
$$-\frac{45bd^4n\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{8e^6}$$
$$+\frac{5bd^3n\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e^6}$$
$$-\frac{45bd^2n\left(d+ex^{2/3}\right)^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{16e^6}$$
$$+\frac{9bdn\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{10e^6}$$
$$-\frac{bn\left(d+ex^{2/3}\right)^6\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{8e^6}$$
$$-\frac{3d^5\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^6}$$
$$+\frac{15d^4\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{4e^6}$$
$$-\frac{5d^3\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{e^6}$$
$$+\frac{15d^2\left(d+ex^{2/3}\right)^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{4e^6}$$
$$-\frac{3d\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^6}$$
$$+\frac{\left(d+ex^{2/3}\right)^6\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{}$$

```
[Out] -45/16*b^3*d^4*n^3*(d+e*x^(2/3))^2/e^6-1/144*b^3*n^3*(d+e*x^(2/3))^6/e^6-3/
2*d^5*(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))^3/e^6+15/4*d^4*(d+e*x^(2/3)
)^2*(a+b*ln(c*(d+e*x^(2/3))^n))^3/e^6-5*d^3*(d+e*x^(2/3))^3*(a+b*ln(c*(d+e*
x^(2/3))^n))^3/e^6+15/4*d^2*(d+e*x^(2/3))^4*(a+b*ln(c*(d+e*x^(2/3))^n))^3/e
^6-3/2*d*(d+e*x^(2/3))^5*(a+b*ln(c*(d+e*x^(2/3))^n))^3/e^6+1/4*(d+e*x^(2/3)
)^6*(a+b*ln(c*(d+e*x^(2/3))^n))^3/e^6+10/9*b^3*d^3*n^3*(d+e*x^(2/3))^3/e^6-
45/128*b^3*d^2*n^3*(d+e*x^(2/3))^4/e^6+9/125*b^3*d*n^3*(d+e*x^(2/3))^5/e^6+
9*b^3*d^5*n^3*x^(2/3)/e^5+1/24*b^2*n^2*(d+e*x^(2/3))^6*(a+b*ln(c*(d+e*x^(2/
3))^n))/e^6-1/8*b*n*(d+e*x^(2/3))^6*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^6-9*a*b
^2*d^5*n^2*x^(2/3)/e^5-9*b^3*d^5*n^2*(d+e*x^(2/3))*ln(c*(d+e*x^(2/3))^n)/e^
6+45/8*b^2*d^4*n^2*(d+e*x^(2/3))^2*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6-10/3*b^2
*d^3*n^2*(d+e*x^(2/3))^3*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6+45/32*b^2*d^2*n^2*
(d+e*x^(2/3))^4*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6-9/25*b^2*d*n^2*(d+e*x^(2/3)
)^5*(a+b*ln(c*(d+e*x^(2/3))^n))/e^6+9/2*b*d^5*n*(d+e*x^(2/3))*(a+b*ln(c*(d+
e*x^(2/3))^n))^2/e^6-45/8*b*d^4*n*(d+e*x^(2/3))^2*(a+b*ln(c*(d+e*x^(2/3))^n
))^2/e^6+5*b*d^3*n*(d+e*x^(2/3))^3*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^6-45/16*
b*d^2*n*(d+e*x^(2/3))^4*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^6+9/10*b*d*n*(d+e*x
^(2/3))^5*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^6
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 913, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341\}$

$$\int x^3 \left(a + b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 dx = -\frac{b^3n^3\left(d+ex^{2/3}\right)^6}{144e^6} + \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\left(d+ex^{2/3}\right)^6}{4e^6} - \frac{bn\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\left(d+ex^{2/3}\right)^6}{8e^6} + \frac{b^2n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\left(d+ex^{2/3}\right)^6}{24e^6} + \frac{9b^3dn^3\left(d+ex^{2/3}\right)^5}{125e^6} - \frac{3d\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\left(d+ex^{2/3}\right)^5}{2e^6} + \frac{9bdn\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\left(d+ex^{2/3}\right)^5}{10e^6} - \frac{9b^2dn^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\left(d+ex^{2/3}\right)^5}{25e^6} - \frac{45b^3d^2n^3\left(d+ex^{2/3}\right)^4}{128e^6} + \frac{15d^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\left(d+ex^{2/3}\right)^4}{4e^6} - \frac{45bd^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\left(d+ex^{2/3}\right)^4}{16e^6} + \frac{45b^2d^2n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\left(d+ex^{2/3}\right)^4}{32e^6} + \frac{10b^3d^3n^3\left(d+ex^{2/3}\right)^3}{9e^6} - \frac{5d^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\left(d+ex^{2/3}\right)^3}{e^6} + \frac{5bd^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\left(d+ex^{2/3}\right)^3}{e^6} - \frac{10b^2d^3n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\left(d+ex^{2/3}\right)^3}{3e^6} - \frac{45b^3d^4n^3\left(d+ex^{2/3}\right)^2}{16e^6} + \frac{15d^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\left(d+ex^{2/3}\right)^2}{4e^6} - \frac{45bd^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\left(d+ex^{2/3}\right)^2}{8e^6} + \frac{45b^2d^4n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\left(d+ex^{2/3}\right)^2}{8e^6} - \frac{3d^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\left(d+ex^{2/3}\right)}{2e^6} + \frac{9bd^5n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\left(d+ex^{2/3}\right)}{2e^6} - \frac{9b^3d^5n^2\log\left(c\left(d+ex^{2/3}\right)^n\right)\left(d+ex^{2/3}\right)}{e^6} + \frac{9b^3d^5n^3x^{2/3}}{e^5} - \frac{9ab^2d^5n^2x^{2/3}}{e^5}$$

[In] Int[x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]

[Out] (-45*b^3*d^4*n^3*(d + e*x^(2/3))^2)/(16*e^6) + (10*b^3*d^3*n^3*(d + e*x^(2/3))^3)/(9*e^6) - (45*b^3*d^2*n^3*(d + e*x^(2/3))^4)/(128*e^6) + (9*b^3*d*n^3*(d + e*x^(2/3))^5)/(125*e^6) - (b^3*n^3*(d + e*x^(2/3))^6)/(144*e^6) - (9*a*b^2*d^5*n^2*x^(2/3))/e^5 + (9*b^3*d^5*n^3*x^(2/3))/e^5 - (9*b^3*d^5*n^2*(d + e*x^(2/3))*Log[c*(d + e*x^(2/3))^n])/e^6 + (45*b^2*d^4*n^2*(d + e*x^(2/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(8*e^6) - (10*b^2*d^3*n^2*(d + e*x^(2/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*e^6) + (45*b^2*d^2*n^2*(d + e*x^(2/3))^4*(a + b*Log[c*(d + e*x^(2/3))^n]))/(32*e^6) - (9*b^2*d*n^2*(d + e*x^(2/3))^5*(a + b*Log[c*(d + e*x^(2/3))^n]))/(25*e^6) + (b^2*n^2*(d + e*x^(2/3))^6*(a + b*Log[c*(d + e*x^(2/3))^n]))/(24*e^6) + (9*b*d^5*n*(d + e*x^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(2*e^6) - (45*b*d^4*n*(d + e*x^(2/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(8*e^6) + (5*b*d^3*n*(d + e*x^(2/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/e^6 - (45*b*d^2*n*(d + e*x^(2/3))^4*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(16*e^6) + (9*b*d*n*(d + e*x^(2/3))^5*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(10*e^6) - (b*n*(d + e*x^(2/3))^6*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(8*e^6) - (3*d^5*(d + e*x^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n])^3)/(2*e^6) + (15*d^4*(d + e*x^(2/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n])^3)/(4*e^6) - (5*d^3*(d + e*x^(2/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n])^3)/e^6 + (15*d^2*(d + e*x^(2/3))^4*(a + b*Log[c*(d + e*x^(2/3))^n])^3)/(4*e^6) - (3*d*(d + e*x^(2/3))^5*(a + b*Log[c*(d + e*x^(2/3))^n])^3)/(2*e^6) + ((d + e*x^(2/3))^6*(a + b*Log[c*(d + e*x^(2/3))^n])^3)/(4*e^6)

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
```

```
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3}{2}\text{Subst}\left(\int x^5(a+b\log(c(d+ex)^n))^3\,dx, x, x^{2/3}\right)\\
&= \frac{3}{2}\text{Subst}\left(\int \left(-\frac{d^5(a+b\log(c(d+ex)^n))^3}{e^5}+\frac{5d^4(d+ex)(a+b\log(c(d+ex)^n))^3}{e^5}\right.\right.\\
&\quad -\frac{10d^3(d+ex)^2(a+b\log(c(d+ex)^n))^3}{e^5}\\
&\quad +\frac{10d^2(d+ex)^3(a+b\log(c(d+ex)^n))^3}{e^5}-\frac{5d(d+ex)^4(a+b\log(c(d+ex)^n))^3}{e^5}\\
&\quad \left.\left.+\frac{(d+ex)^5(a+b\log(c(d+ex)^n))^3}{e^5}\right)\,dx, x, x^{2/3}\right)
\end{aligned}
$$

$$= \frac{3\text{Subst}\left(\int (d+ex)^5 \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \, dx, x, x^{2/3}\right)}{2e^5} - \frac{(15d)\text{Subst}\left(\int (d+ex)^4 \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \, dx, x, x^{2/3}\right)}{2e^5} + \frac{(15d^2)\,\text{Subst}\left(\int (d+ex)^3 \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \, dx, x, x^{2/3}\right)}{e^5} - \frac{(15d^3)\,\text{Subst}\left(\int (d+ex)^2 \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \, dx, x, x^{2/3}\right)}{e^5} + \frac{(15d^4)\,\text{Subst}\left(\int (d+ex) \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \, dx, x, x^{2/3}\right)}{2e^5} - \frac{(3d^5)\,\text{Subst}\left(\int \left(a+b\log\left(c(d+ex)^n\right)\right)^3 \, dx, x, x^{2/3}\right)}{2e^5}$$

$$= \frac{3\text{Subst}\left(\int x^5 (a+b\log(cx^n))^3 \, dx, x, d+ex^{2/3}\right)}{2e^6} - \frac{(15d)\text{Subst}\left(\int x^4 (a+b\log(cx^n))^3 \, dx, x, d+ex^{2/3}\right)}{2e^6} + \frac{(15d^2)\,\text{Subst}\left(\int x^3 (a+b\log(cx^n))^3 \, dx, x, d+ex^{2/3}\right)}{e^6} - \frac{(15d^3)\,\text{Subst}\left(\int x^2 (a+b\log(cx^n))^3 \, dx, x, d+ex^{2/3}\right)}{e^6} + \frac{(15d^4)\,\text{Subst}\left(\int x (a+b\log(cx^n))^3 \, dx, x, d+ex^{2/3}\right)}{2e^6} - \frac{(3d^5)\,\text{Subst}\left(\int (a+b\log(cx^n))^3 \, dx, x, d+ex^{2/3}\right)}{2e^6}$$

$$
\begin{aligned}
&= -\frac{3d^5\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^6} \\
&\quad+\frac{15d^4\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{4e^6} \\
&\quad-\frac{5d^3\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{e^6} \\
&\quad+\frac{15d^2\left(d+ex^{2/3}\right)^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{4e^6} \\
&\quad-\frac{3d\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^6} \\
&\quad+\frac{\left(d+ex^{2/3}\right)^6\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{4e^6} \\
&\quad-\frac{(3bn)\text{Subst}\left(\int x^5(a+b\log(cx^n))^2\,dx,x,d+ex^{2/3}\right)}{4e^6} \\
&\quad+\frac{(9bdn)\text{Subst}\left(\int x^4(a+b\log(cx^n))^2\,dx,x,d+ex^{2/3}\right)}{2e^6} \\
&\quad-\frac{(45bd^2n)\,\text{Subst}\left(\int x^3(a+b\log(cx^n))^2\,dx,x,d+ex^{2/3}\right)}{4e^6} \\
&\quad+\frac{(15bd^3n)\,\text{Subst}\left(\int x^2(a+b\log(cx^n))^2\,dx,x,d+ex^{2/3}\right)}{e^6} \\
&\quad-\frac{(45bd^4n)\,\text{Subst}\left(\int x(a+b\log(cx^n))^2\,dx,x,d+ex^{2/3}\right)}{4e^6} \\
&\quad+\frac{(9bd^5n)\,\text{Subst}\left(\int (a+b\log(cx^n))^2\,dx,x,d+ex^{2/3}\right)}{2e^6}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{9bd^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^6} \\
&\quad - \frac{45bd^4n\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{8e^6} \\
&\quad + \frac{5bd^3n\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e^6} \\
&\quad - \frac{45bd^2n\left(d+ex^{2/3}\right)^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{16e^6} \\
&\quad + \frac{9bdn\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{10e^6} \\
&\quad - \frac{bn\left(d+ex^{2/3}\right)^6\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{8e^6} \\
&\quad - \frac{3d^5\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^6} \\
&\quad + \frac{15d^4\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{4e^6} \\
&\quad - \frac{5d^3\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{e^6} \\
&\quad + \frac{15d^2\left(d+ex^{2/3}\right)^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{4e^6} \\
&\quad - \frac{3d\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^6} \\
&\quad + \frac{\left(d+ex^{2/3}\right)^6\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{4e^6} \\
&\quad + \frac{(b^2n^2)\,\mathrm{Subst}\left(\int x^5(a+b\log(cx^n))\,dx, x, d+ex^{2/3}\right)}{4e^6} \\
&\quad - \frac{(9b^2dn^2)\,\mathrm{Subst}\left(\int x^4(a+b\log(cx^n))\,dx, x, d+ex^{2/3}\right)}{5e^6} \\
&\quad + \frac{(45b^2d^2n^2)\,\mathrm{Subst}\left(\int x^3(a+b\log(cx^n))\,dx, x, d+ex^{2/3}\right)}{8e^6} \\
&\quad - \frac{(10b^2d^3n^2)\,\mathrm{Subst}\left(\int x^2(a+b\log(cx^n))\,dx, x, d+ex^{2/3}\right)}{e^6} \\
&\quad + \frac{(45b^2d^4n^2)\,\mathrm{Subst}\left(\int x(a+b\log(cx^n))\,dx, x, d+ex^{2/3}\right)}{4e^6} \\
&\quad - \frac{(9b^2d^5n^2)\,\mathrm{Subst}\left(\int (a+b\log(cx^n))\,dx, x, d+ex^{2/3}\right)}{e^6}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{45b^3d^4n^3\left(d+ex^{2/3}\right)^2}{16e^6}+\frac{10b^3d^3n^3\left(d+ex^{2/3}\right)^3}{9e^6}-\frac{45b^3d^2n^3\left(d+ex^{2/3}\right)^4}{128e^6} \\
&+\frac{9b^3dn^3\left(d+ex^{2/3}\right)^5}{125e^6}-\frac{b^3n^3\left(d+ex^{2/3}\right)^6}{144e^6}-\frac{9ab^2d^5n^2x^{2/3}}{e^5} \\
&+\frac{45b^2d^4n^2\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{8e^6} \\
&-\frac{10b^2d^3n^2\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^6} \\
&+\frac{45b^2d^2n^2\left(d+ex^{2/3}\right)^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{32e^6} \\
&-\frac{9b^2dn^2\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{25e^6} \\
&+\frac{b^2n^2\left(d+ex^{2/3}\right)^6\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{24e^6} \\
&+\frac{9bd^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^6} \\
&-\frac{45bd^4n\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{8e^6} \\
&+\frac{5bd^3n\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e^6} \\
&-\frac{45bd^2n\left(d+ex^{2/3}\right)^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{16e^6} \\
&+\frac{9bdn\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{10e^6} \\
&-\frac{bn\left(d+ex^{2/3}\right)^6\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{8e^6} \\
&-\frac{3d^5\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^6} \\
&+\frac{15d^4\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{4e^6} \\
&-\frac{5d^3\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{e^6} \\
&+\frac{15d^2\left(d+ex^{2/3}\right)^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{4e^6} \\
&-\frac{3d\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^6} \\
&+\frac{\left(d+ex^{2/3}\right)^6\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{4e^6} \\
&-\frac{\left(9b^3d^5n^2\right)\operatorname{Subst}\left(\int\log\left(cx^n\right)\,dx,x,d+ex^{2/3}\right)}{e^6}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{45b^3d^4n^3\left(d+ex^{2/3}\right)^2}{16e^6} + \frac{10b^3d^3n^3\left(d+ex^{2/3}\right)^3}{9e^6} - \frac{45b^3d^2n^3\left(d+ex^{2/3}\right)^4}{128e^6} \\
&+ \frac{9b^3dn^3\left(d+ex^{2/3}\right)^5}{125e^6} - \frac{b^3n^3\left(d+ex^{2/3}\right)^6}{144e^6} - \frac{9ab^2d^5n^2x^{2/3}}{e^5} \\
&+ \frac{9b^3d^5n^3x^{2/3}}{e^5} - \frac{9b^3d^5n^2\left(d+ex^{2/3}\right)\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e^6} \\
&+ \frac{45b^2d^4n^2\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{8e^6} \\
&- \frac{10b^2d^3n^2\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^6} \\
&+ \frac{45b^2d^2n^2\left(d+ex^{2/3}\right)^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{32e^6} \\
&- \frac{9b^2dn^2\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{25e^6} \\
&+ \frac{b^2n^2\left(d+ex^{2/3}\right)^6\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{24e^6} \\
&+ \frac{9bd^5n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^6} \\
&- \frac{45bd^4n\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{8e^6} \\
&+ \frac{5bd^3n\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e^6} \\
&- \frac{45bd^2n\left(d+ex^{2/3}\right)^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{16e^6} \\
&+ \frac{9bdn\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{10e^6} \\
&- \frac{bn\left(d+ex^{2/3}\right)^6\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{8e^6} \\
&- \frac{3d^5\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^6} \\
&+ \frac{15d^4\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{4e^6} \\
&- \frac{5d^3\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{e^6} \\
&+ \frac{15d^2\left(d+ex^{2/3}\right)^4\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{4e^6} \\
&- \frac{3d\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^6} \\
&+ \frac{\left(d+ex^{2/3}\right)^6\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{4e^6}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 593, normalized size of antiderivative = 0.65

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx = \frac{ex^{2/3}\left(36000a^3e^5x^{10/3}+b^3n^3\left(809340d^5-140070d^4ex^{2/3}+41180d^3e^2x^{4/3}-1\right.\right.}{}$$

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (e*x^(2/3)*(36000*a^3*e^5*x^(10/3) + b^3*n^3*(809340*d^5 - 140070*d^4*e*x^(
2/3) + 41180*d^3*e^2*x^(4/3) - 13785*d^2*e^3*x^2 + 4368*d*e^4*x^(8/3) - 100
0*e^5*x^(10/3)) - 60*a*b^2*n^2*(8820*d^5 - 2610*d^4*e*x^(2/3) + 1140*d^3*e^
2*x^(4/3) - 555*d^2*e^3*x^2 + 264*d*e^4*x^(8/3) - 100*e^5*x^(10/3)) + 1800*
a^2*b*n*(60*d^5 - 30*d^4*e*x^(2/3) + 20*d^3*e^2*x^(4/3) - 15*d^2*e^3*x^2 +
12*d*e^4*x^(8/3) - 10*e^5*x^(10/3))) - 60*b*d^6*n*(1800*a^2 - 8820*a*b*n +
13489*b^2*n^2)*Log[d + e*x^(2/3)] + 60*b*e*x^(2/3)*(1800*a^2*e^5*x^(10/3) +
 60*a*b*n*(60*d^5 - 30*d^4*e*x^(2/3) + 20*d^3*e^2*x^(4/3) - 15*d^2*e^3*x^2
+ 12*d*e^4*x^(8/3) - 10*e^5*x^(10/3)) + b^2*n^2*(-8820*d^5 + 2610*d^4*e*x^(
2/3) - 1140*d^3*e^2*x^(4/3) + 555*d^2*e^3*x^2 - 264*d*e^4*x^(8/3) + 100*e^5
*x^(10/3)))*Log[c*(d + e*x^(2/3))^n] + 1800*b^2*(b*n*(147*d^6 + 60*d^5*e*x^
(2/3) - 30*d^4*e^2*x^(4/3) + 20*d^3*e^3*x^2 - 15*d^2*e^4*x^(8/3) + 12*d*e^5
*x^(10/3) - 10*e^6*x^4) - 60*a*(d^6 - e^6*x^4))*Log[c*(d + e*x^(2/3))^n]^2
- 36000*b^3*(d^6 - e^6*x^4)*Log[c*(d + e*x^(2/3))^n]^3)/(144000*e^6)
```

Maple [F]

$$\int x^3\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^n\right)\right)^3dx$$

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 1241, normalized size of antiderivative = 1.36

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/144000*(36000*b^3*e^6*x^4*log(c)^3 - 1000*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2
+ 18*a^2*b*e^6*n - 36*a^3*e^6)*x^4 + 36000*(b^3*e^6*n^3*x^4 - b^3*d^6*n^3)*
log(e*x^(2/3) + d)^3 + 20*(2059*b^3*d^3*e^3*n^3 - 3420*a*b^2*d^3*e^3*n^2 +
1800*a^2*b*d^3*e^3*n)*x^2 + 1800*(20*b^3*d^3*e^3*n^3*x^2 + 147*b^3*d^6*n^3
- 60*a*b^2*d^6*n^2 - 10*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2)*x^4 + 60*(b^3*e^6*n
^2*x^4 - b^3*d^6*n^2)*log(c) - 15*(b^3*d^2*e^4*n^3*x^2 - 4*b^3*d^5*e*n^3)*x
^(2/3) + 6*(2*b^3*d*e^5*n^3*x^3 - 5*b^3*d^4*e^2*n^3*x)*x^(1/3))*log(e*x^(2/
3) + d)^2 + 18000*(2*b^3*d^3*e^3*n*x^2 - (b^3*e^6*n - 6*a*b^2*e^6)*x^4)*log
(c)^2 - 60*(13489*b^3*d^6*n^3 - 8820*a*b^2*d^6*n^2 + 1800*a^2*b*d^6*n - 100
*(b^3*e^6*n^3 - 6*a*b^2*e^6*n^2 + 18*a^2*b*e^6*n)*x^4 + 60*(19*b^3*d^3*e^3*
n^3 - 20*a*b^2*d^3*e^3*n^2)*x^2 - 1800*(b^3*e^6*n*x^4 - b^3*d^6*n)*log(c)^2
 - 60*(20*b^3*d^3*e^3*n^2*x^2 + 147*b^3*d^6*n^2 - 60*a*b^2*d^6*n - 10*(b^3*
e^6*n^2 - 6*a*b^2*e^6*n)*x^4)*log(c) + 15*(588*b^3*d^5*e*n^3 - 240*a*b^2*d^
5*e*n^2 - (37*b^3*d^2*e^4*n^3 - 60*a*b^2*d^2*e^4*n^2)*x^2 + 60*(b^3*d^2*e^4
*n^2*x^2 - 4*b^3*d^5*e*n^2)*log(c))*x^(2/3) + 6*(4*(11*b^3*d*e^5*n^3 - 30*a
*b^2*d*e^5*n^2)*x^3 - 15*(29*b^3*d^4*e^2*n^3 - 20*a*b^2*d^4*e^2*n^2)*x - 60
*(2*b^3*d*e^5*n^2*x^3 - 5*b^3*d^4*e^2*n^2*x)*log(c))*x^(1/3))*log(e*x^(2/3)
 + d) + 1200*(5*(b^3*e^6*n^2 - 6*a*b^2*e^6*n + 18*a^2*b*e^6)*x^4 - 3*(19*b^
3*d^3*e^3*n^2 - 20*a*b^2*d^3*e^3*n)*x^2)*log(c) + 15*(53956*b^3*d^5*e*n^3 -
 35280*a*b^2*d^5*e*n^2 + 7200*a^2*b*d^5*e*n - (919*b^3*d^2*e^4*n^3 - 2220*a
*b^2*d^2*e^4*n^2 + 1800*a^2*b*d^2*e^4*n)*x^2 - 1800*(b^3*d^2*e^4*n*x^2 - 4*
b^3*d^5*e*n)*log(c)^2 - 60*(588*b^3*d^5*e*n^2 - 240*a*b^2*d^5*e*n - (37*b^3
*d^2*e^4*n^2 - 60*a*b^2*d^2*e^4*n)*x^2)*log(c))*x^(2/3) + 6*(8*(91*b^3*d*e^
5*n^3 - 330*a*b^2*d*e^5*n^2 + 450*a^2*b*d*e^5*n)*x^3 + 1800*(2*b^3*d*e^5*n*
x^3 - 5*b^3*d^4*e^2*n*x)*log(c)^2 - 5*(4669*b^3*d^4*e^2*n^3 - 5220*a*b^2*d^
4*e^2*n^2 + 1800*a^2*b*d^4*e^2*n)*x - 60*(4*(11*b^3*d*e^5*n^2 - 30*a*b^2*d*
e^5*n)*x^3 - 15*(29*b^3*d^4*e^2*n^2 - 20*a*b^2*d^4*e^2*n)*x)*log(c))*x^(1/3
))/e^6
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(a + b \log \left(c \left(d + e x^{2/3}\right)^n\right)\right)^3 \, dx = \text{Timed out}$$

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))**n))**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 680, normalized size of antiderivative = 0.74

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\frac{1}{4}\,b^3x^4\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)^3$$
$$+\frac{3}{4}\,ab^2x^4\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)^2+\frac{3}{4}\,a^2bx^4\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+\frac{1}{4}\,a^3x^4$$
$$-\frac{1}{80}\,a^2ben\left(\frac{60\,d^6\log\left(ex^{\frac{2}{3}}+d\right)}{e^7}+\frac{10\,e^5x^4-12\,de^4x^{\frac{10}{3}}+15\,d^2e^3x^{\frac{8}{3}}-20\,d^3e^2x^2+30\,d^4ex^{\frac{4}{3}}-60\,d^5x^{\frac{2}{3}}}{e^6}\right)$$
$$-\frac{1}{2400}\left(60\,en\left(\frac{60\,d^6\log\left(ex^{\frac{2}{3}}+d\right)}{e^7}+\frac{10\,e^5x^4-12\,de^4x^{\frac{10}{3}}+15\,d^2e^3x^{\frac{8}{3}}-20\,d^3e^2x^2+30\,d^4ex^{\frac{4}{3}}-60\,d^5x^{\frac{2}{3}}}{e^6}\right)\right.$$
$$-\frac{1}{144000}\left(1800\,en\left(\frac{60\,d^6\log\left(ex^{\frac{2}{3}}+d\right)}{e^7}+\frac{10\,e^5x^4-12\,de^4x^{\frac{10}{3}}+15\,d^2e^3x^{\frac{8}{3}}-20\,d^3e^2x^2+30\,d^4ex^{\frac{4}{3}}-60\,d\right.}{e^6}\right.$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4*log((e*x^(2/3) + d)^n*c)^3 + 3/4*a*b^2*x^4*log((e*x^(2/3) + d)^
n*c)^2 + 3/4*a^2*b*x^4*log((e*x^(2/3) + d)^n*c) + 1/4*a^3*x^4 - 1/80*a^2*b*
e*n*(60*d^6*log(e*x^(2/3) + d)/e^7 + (10*e^5*x^4 - 12*d*e^4*x^(10/3) + 15*d
^2*e^3*x^(8/3) - 20*d^3*e^2*x^2 + 30*d^4*e*x^(4/3) - 60*d^5*x^(2/3))/e^6) -
 1/2400*(60*e*n*(60*d^6*log(e*x^(2/3) + d)/e^7 + (10*e^5*x^4 - 12*d*e^4*x^(
10/3) + 15*d^2*e^3*x^(8/3) - 20*d^3*e^2*x^2 + 30*d^4*e*x^(4/3) - 60*d^5*x^(
2/3))/e^6)*log((e*x^(2/3) + d)^n*c) - (100*e^6*x^4 - 264*d*e^5*x^(10/3) + 5
55*d^2*e^4*x^(8/3) - 1140*d^3*e^3*x^2 + 1800*d^6*log(e*x^(2/3) + d)^2 + 261
0*d^4*e^2*x^(4/3) + 8820*d^6*log(e*x^(2/3) + d) - 8820*d^5*e*x^(2/3))*n^2/e
^6)*a*b^2 - 1/144000*(1800*e*n*(60*d^6*log(e*x^(2/3) + d)/e^7 + (10*e^5*x^4
 - 12*d*e^4*x^(10/3) + 15*d^2*e^3*x^(8/3) - 20*d^3*e^2*x^2 + 30*d^4*e*x^(4/
3) - 60*d^5*x^(2/3))/e^6)*log((e*x^(2/3) + d)^n*c)^2 + e*n*((1000*e^6*x^4 -
 4368*d*e^5*x^(10/3) + 36000*d^6*log(e*x^(2/3) + d)^3 + 13785*d^2*e^4*x^(8/
3) - 41180*d^3*e^3*x^2 + 264600*d^6*log(e*x^(2/3) + d)^2 + 140070*d^4*e^2*x
^(4/3) + 809340*d^6*log(e*x^(2/3) + d) - 809340*d^5*e*x^(2/3))*n^2/e^7 - 60
*(100*e^6*x^4 - 264*d*e^5*x^(10/3) + 555*d^2*e^4*x^(8/3) - 1140*d^3*e^3*x^2
 + 1800*d^6*log(e*x^(2/3) + d)^2 + 2610*d^4*e^2*x^(4/3) + 8820*d^6*log(e*x^
(2/3) + d) - 8820*d^5*e*x^(2/3))*n*log((e*x^(2/3) + d)^n*c)/e^7))*b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2104 vs. $2(787) = 1574$.

Time = 0.82 (sec) , antiderivative size = 2104, normalized size of antiderivative = 2.30

$$\int x^3 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 \, dx = \text{Too large to display}$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4*log(c)^3 + 3/4*a*b^2*x^4*log(c)^2 + 3/4*a^2*b*x^4*log(c) + 1/14
4000*(36000*(e*x^(2/3) + d)^6*log(e*x^(2/3) + d)^3/e^6 - 216000*(e*x^(2/3)
+ d)^5*d*log(e*x^(2/3) + d)^3/e^6 + 540000*(e*x^(2/3) + d)^4*d^2*log(e*x^(2
/3) + d)^3/e^6 - 720000*(e*x^(2/3) + d)^3*d^3*log(e*x^(2/3) + d)^3/e^6 + 54
0000*(e*x^(2/3) + d)^2*d^4*log(e*x^(2/3) + d)^3/e^6 - 18000*(e*x^(2/3) + d)
^6*log(e*x^(2/3) + d)^2/e^6 + 129600*(e*x^(2/3) + d)^5*d*log(e*x^(2/3) + d)
^2/e^6 - 405000*(e*x^(2/3) + d)^4*d^2*log(e*x^(2/3) + d)^2/e^6 + 720000*(e*
x^(2/3) + d)^3*d^3*log(e*x^(2/3) + d)^2/e^6 - 810000*(e*x^(2/3) + d)^2*d^4*
log(e*x^(2/3) + d)^2/e^6 + 6000*(e*x^(2/3) + d)^6*log(e*x^(2/3) + d)/e^6 -
51840*(e*x^(2/3) + d)^5*d*log(e*x^(2/3) + d)/e^6 + 202500*(e*x^(2/3) + d)^4
*d^2*log(e*x^(2/3) + d)/e^6 - 480000*(e*x^(2/3) + d)^3*d^3*log(e*x^(2/3) +
d)/e^6 + 810000*(e*x^(2/3) + d)^2*d^4*log(e*x^(2/3) + d)/e^6 - 1000*(e*x^(2
/3) + d)^6/e^6 + 10368*(e*x^(2/3) + d)^5*d/e^6 - 50625*(e*x^(2/3) + d)^4*d^
2/e^6 + 160000*(e*x^(2/3) + d)^3*d^3/e^6 - 405000*(e*x^(2/3) + d)^2*d^4/e^6
 - 216000*((e*x^(2/3) + d)*log(e*x^(2/3) + d)^3 - 3*(e*x^(2/3) + d)*log(e*x
^(2/3) + d)^2 + 6*(e*x^(2/3) + d)*log(e*x^(2/3) + d) - 6*e*x^(2/3) - 6*d)*d
^5/e^6)*b^3*n^3 + 1/4*a^3*x^4 + 1/2400*(1800*(e*x^(2/3) + d)^6*log(e*x^(2/3
) + d)^2/e^6 - 10800*(e*x^(2/3) + d)^5*d*log(e*x^(2/3) + d)^2/e^6 + 27000*(
e*x^(2/3) + d)^4*d^2*log(e*x^(2/3) + d)^2/e^6 - 36000*(e*x^(2/3) + d)^3*d^3
*log(e*x^(2/3) + d)^2/e^6 + 27000*(e*x^(2/3) + d)^2*d^4*log(e*x^(2/3) + d)^
2/e^6 - 600*(e*x^(2/3) + d)^6*log(e*x^(2/3) + d)/e^6 + 4320*(e*x^(2/3) + d)
^5*d*log(e*x^(2/3) + d)/e^6 - 13500*(e*x^(2/3) + d)^4*d^2*log(e*x^(2/3) + d
)/e^6 + 24000*(e*x^(2/3) + d)^3*d^3*log(e*x^(2/3) + d)/e^6 - 27000*(e*x^(2/
3) + d)^2*d^4*log(e*x^(2/3) + d)/e^6 + 100*(e*x^(2/3) + d)^6/e^6 - 864*(e*x
^(2/3) + d)^5*d/e^6 + 3375*(e*x^(2/3) + d)^4*d^2/e^6 - 8000*(e*x^(2/3) + d)
^3*d^3/e^6 + 13500*(e*x^(2/3) + d)^2*d^4/e^6 - 10800*((e*x^(2/3) + d)*log(e
*x^(2/3) + d)^2 - 2*(e*x^(2/3) + d)*log(e*x^(2/3) + d) + 2*e*x^(2/3) + 2*d)
*d^5/e^6)*b^3*n^2*log(c) + 1/80*b^3*n*(60*(e*x^(2/3) + d)^6*log(e*x^(2/3) +
 d)/e^6 - 360*(e*x^(2/3) + d)^5*d*log(e*x^(2/3) + d)/e^6 + 900*(e*x^(2/3) +
 d)^4*d^2*log(e*x^(2/3) + d)/e^6 - 1200*(e*x^(2/3) + d)^3*d^3*log(e*x^(2/3)
 + d)/e^6 + 900*(e*x^(2/3) + d)^2*d^4*log(e*x^(2/3) + d)/e^6 - 10*(e*x^(2/3
) + d)^6/e^6 + 72*(e*x^(2/3) + d)^5*d/e^6 - 225*(e*x^(2/3) + d)^4*d^2/e^6 +
 400*(e*x^(2/3) + d)^3*d^3/e^6 - 450*(e*x^(2/3) + d)^2*d^4/e^6 - 360*((e*x^
(2/3) + d)*log(e*x^(2/3) + d) - e*x^(2/3) - d)*d^5/e^6)*log(c)^2 + 1/2400*(
1800*(e*x^(2/3) + d)^6*log(e*x^(2/3) + d)^2/e^6 - 10800*(e*x^(2/3) + d)^5*d
```

```
*log(e*x^(2/3) + d)^2/e^6 + 27000*(e*x^(2/3) + d)^4*d^2*log(e*x^(2/3) + d)^
2/e^6 - 36000*(e*x^(2/3) + d)^3*d^3*log(e*x^(2/3) + d)^2/e^6 + 27000*(e*x^(
2/3) + d)^2*d^4*log(e*x^(2/3) + d)^2/e^6 - 600*(e*x^(2/3) + d)^6*log(e*x^(2
/3) + d)/e^6 + 4320*(e*x^(2/3) + d)^5*d*log(e*x^(2/3) + d)/e^6 - 13500*(e*x
^(2/3) + d)^4*d^2*log(e*x^(2/3) + d)/e^6 + 24000*(e*x^(2/3) + d)^3*d^3*log(
e*x^(2/3) + d)/e^6 - 27000*(e*x^(2/3) + d)^2*d^4*log(e*x^(2/3) + d)/e^6 + 1
00*(e*x^(2/3) + d)^6/e^6 - 864*(e*x^(2/3) + d)^5*d/e^6 + 3375*(e*x^(2/3) +
d)^4*d^2/e^6 - 8000*(e*x^(2/3) + d)^3*d^3/e^6 + 13500*(e*x^(2/3) + d)^2*d^4
/e^6 - 10800*((e*x^(2/3) + d)*log(e*x^(2/3) + d)^2 - 2*(e*x^(2/3) + d)*log(
e*x^(2/3) + d) + 2*e*x^(2/3) + 2*d)*d^5/e^6)*a*b^2*n^2 + 1/40*a*b^2*n*(60*(
e*x^(2/3) + d)^6*log(e*x^(2/3) + d)/e^6 - 360*(e*x^(2/3) + d)^5*d*log(e*x^(
2/3) + d)/e^6 + 900*(e*x^(2/3) + d)^4*d^2*log(e*x^(2/3) + d)/e^6 - 1200*(e*
x^(2/3) + d)^3*d^3*log(e*x^(2/3) + d)/e^6 + 900*(e*x^(2/3) + d)^2*d^4*log(e
*x^(2/3) + d)/e^6 - 10*(e*x^(2/3) + d)^6/e^6 + 72*(e*x^(2/3) + d)^5*d/e^6 -
 225*(e*x^(2/3) + d)^4*d^2/e^6 + 400*(e*x^(2/3) + d)^3*d^3/e^6 - 450*(e*x^(
2/3) + d)^2*d^4/e^6 - 360*((e*x^(2/3) + d)*log(e*x^(2/3) + d) - e*x^(2/3) -
 d)*d^5/e^6)*log(c) + 1/80*a^2*b*n*(60*(e*x^(2/3) + d)^6*log(e*x^(2/3) + d)
/e^6 - 360*(e*x^(2/3) + d)^5*d*log(e*x^(2/3) + d)/e^6 + 900*(e*x^(2/3) + d)
^4*d^2*log(e*x^(2/3) + d)/e^6 - 1200*(e*x^(2/3) + d)^3*d^3*log(e*x^(2/3) +
d)/e^6 + 900*(e*x^(2/3) + d)^2*d^4*log(e*x^(2/3) + d)/e^6 - 10*(e*x^(2/3) +
 d)^6/e^6 + 72*(e*x^(2/3) + d)^5*d/e^6 - 225*(e*x^(2/3) + d)^4*d^2/e^6 + 40
0*(e*x^(2/3) + d)^3*d^3/e^6 - 450*(e*x^(2/3) + d)^2*d^4/e^6 - 360*((e*x^(2/
3) + d)*log(e*x^(2/3) + d) - e*x^(2/3) - d)*d^5/e^6)
```

Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 992, normalized size of antiderivative = 1.09

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\text{Too large to display}$$

```
[In] int(x^3*(a + b*log(c*(d + e*x^(2/3))^n))^3,x)
```

```
[Out] (a^3*x^4)/4 + (b^3*x^4*log(c*(d + e*x^(2/3))^n)^3)/4 - (b^3*n^3*x^4)/144 +
(3*a*b^2*x^4*log(c*(d + e*x^(2/3))^n)^2)/4 - (b^3*n*x^4*log(c*(d + e*x^(2/3
))^n)^2)/8 + (b^3*n^2*x^4*log(c*(d + e*x^(2/3))^n))/24 + (a*b^2*n^2*x^4)/24
 - (b^3*d^6*log(c*(d + e*x^(2/3))^n)^3)/(4*e^6) + (3*a^2*b*x^4*log(c*(d + e
*x^(2/3))^n))/4 - (a^2*b*n*x^4)/8 - (a*b^2*n*x^4*log(c*(d + e*x^(2/3))^n))/
4 - (13489*b^3*d^6*n^3*log(d + e*x^(2/3)))/(2400*e^6) + (2059*b^3*d^3*n^3*x
^2)/(7200*e^3) - (919*b^3*d^2*n^3*x^(8/3))/(9600*e^2) - (4669*b^3*d^4*n^3*x
^(4/3))/(4800*e^4) + (13489*b^3*d^5*n^3*x^(2/3))/(2400*e^5) - (3*a*b^2*d^6*
log(c*(d + e*x^(2/3))^n)^2)/(4*e^6) + (147*b^3*d^6*n*log(c*(d + e*x^(2/3))^
n)^2)/(80*e^6) + (91*b^3*d*n^3*x^(10/3))/(3000*e) - (3*a^2*b*d^6*n*log(d +
e*x^(2/3)))/(4*e^6) + (3*b^3*d*n*x^(10/3)*log(c*(d + e*x^(2/3))^n)^2)/(20*e
) - (11*b^3*d*n^2*x^(10/3)*log(c*(d + e*x^(2/3))^n))/(100*e) + (a^2*b*d^3*n
```

```
*x^2)/(4*e^3) - (3*a^2*b*d^2*n*x^(8/3))/(16*e^2) - (3*a^2*b*d^4*n*x^(4/3))/
(8*e^4) + (3*a^2*b*d^5*n*x^(2/3))/(4*e^5) - (11*a*b^2*d*n^2*x^(10/3))/(100*
e) + (147*a*b^2*d^6*n^2*log(d + e*x^(2/3)))/(40*e^6) + (b^3*d^3*n*x^2*log(c
*(d + e*x^(2/3))^n)^2)/(4*e^3) - (19*b^3*d^3*n^2*x^2*log(c*(d + e*x^(2/3))^
n))/(40*e^3) - (3*b^3*d^2*n*x^(8/3)*log(c*(d + e*x^(2/3))^n)^2)/(16*e^2) +
(37*b^3*d^2*n^2*x^(8/3)*log(c*(d + e*x^(2/3))^n))/(160*e^2) - (3*b^3*d^4*n*
x^(4/3)*log(c*(d + e*x^(2/3))^n)^2)/(8*e^4) + (87*b^3*d^4*n^2*x^(4/3)*log(c
*(d + e*x^(2/3))^n))/(80*e^4) + (3*b^3*d^5*n*x^(2/3)*log(c*(d + e*x^(2/3))^
n)^2)/(4*e^5) - (147*b^3*d^5*n^2*x^(2/3)*log(c*(d + e*x^(2/3))^n))/(40*e^5)
 - (19*a*b^2*d^3*n^2*x^2)/(40*e^3) + (37*a*b^2*d^2*n^2*x^(8/3))/(160*e^2) +
 (87*a*b^2*d^4*n^2*x^(4/3))/(80*e^4) - (147*a*b^2*d^5*n^2*x^(2/3))/(40*e^5)
 + (3*a^2*b*d*n*x^(10/3))/(20*e) + (3*a*b^2*d*n*x^(10/3)*log(c*(d + e*x^(2/
3))^n))/(10*e) + (a*b^2*d^3*n*x^2*log(c*(d + e*x^(2/3))^n))/(2*e^3) - (3*a*
b^2*d^2*n*x^(8/3)*log(c*(d + e*x^(2/3))^n))/(8*e^2) - (3*a*b^2*d^4*n*x^(4/3
)*log(c*(d + e*x^(2/3))^n))/(4*e^4) + (3*a*b^2*d^5*n*x^(2/3)*log(c*(d + e*x
^(2/3))^n))/(2*e^5)
```

3.482 $\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 449

$$\begin{aligned}\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx &= \frac{9b^3dn^3\left(d+ex^{2/3}\right)^2}{8e^3}-\frac{b^3n^3\left(d+ex^{2/3}\right)^3}{9e^3}\\
&+\frac{9ab^2d^2n^2x^{2/3}}{e^2}-\frac{9b^3d^2n^3x^{2/3}}{e^2}+\frac{9b^3d^2n^2\left(d+ex^{2/3}\right)\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e^3}\\
&-\frac{9b^2dn^2\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4e^3}\\
&+\frac{b^2n^2\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^3}\\
&-\frac{9bd^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^3}\\
&+\frac{9bdn\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4e^3}\\
&-\frac{bn\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^3}\\
&+\frac{3d^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3}\\
&-\frac{3d\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3}\\
&+\frac{\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3}\end{aligned}$$

```
[Out] 9/8*b^3*d*n^3*(d+e*x^(2/3))^2/e^3-1/9*b^3*n^3*(d+e*x^(2/3))^3/e^3+9*a*b^2*d
^2*n^2*x^(2/3)/e^2-9*b^3*d^2*n^3*x^(2/3)/e^2+9*b^3*d^2*n^2*(d+e*x^(2/3))*ln
(c*(d+e*x^(2/3))^n)/e^3-9/4*b^2*d*n^2*(d+e*x^(2/3))^2*(a+b*ln(c*(d+e*x^(2/3
))^n))/e^3+1/3*b^2*n^2*(d+e*x^(2/3))^3*(a+b*ln(c*(d+e*x^(2/3))^n))/e^3-9/2*
```

```
b*d^2*n*(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^3+9/4*b*d*n*(d+e*x^(2
/3))^2*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^3-1/2*b*n*(d+e*x^(2/3))^3*(a+b*ln(c*
(d+e*x^(2/3))^n))^2/e^3+3/2*d^2*(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))^3
/e^3-3/2*d*(d+e*x^(2/3))^2*(a+b*ln(c*(d+e*x^(2/3))^n))^3/e^3+1/2*(d+e*x^(2/
3))^3*(a+b*ln(c*(d+e*x^(2/3))^n))^3/e^3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\begin{aligned}\int x\Big(a &+ b\log\Big(c\big(d+ex^{2/3}\big)^n\Big)\Big)^3\,dx = \frac{b^2n^2\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^3}\\
&-\frac{9b^2dn^2\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4e^3}\\
&+\frac{9ab^2d^2n^2x^{2/3}}{e^2}-\frac{9bd^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^3}\\
&+\frac{3d^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3}\\
&-\frac{bn\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^3}\\
&+\frac{9bdn\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4e^3}\\
&+\frac{\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3}\\
&-\frac{3d\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3}\\
&+\frac{9b^3d^2n^2\left(d+ex^{2/3}\right)\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e^3}-\frac{9b^3d^2n^3x^{2/3}}{e^2}\\
&-\frac{b^3n^3\left(d+ex^{2/3}\right)^3}{9e^3}+\frac{9b^3dn^3\left(d+ex^{2/3}\right)^2}{8e^3}\end{aligned}$$

```
[In] Int[x*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (9*b^3*d*n^3*(d + e*x^(2/3))^2)/(8*e^3) - (b^3*n^3*(d + e*x^(2/3))^3)/(9*e^
3) + (9*a*b^2*d^2*n^2*x^(2/3))/e^2 - (9*b^3*d^2*n^3*x^(2/3))/e^2 + (9*b^3*d
^2*n^2*(d + e*x^(2/3))*Log[c*(d + e*x^(2/3))^n])/e^3 - (9*b^2*d*n^2*(d + e*
x^(2/3))^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(4*e^3) + (b^2*n^2*(d + e*x^(2
/3))^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/(3*e^3) - (9*b*d^2*n*(d + e*x^(2/3
))*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(2*e^3) + (9*b*d*n*(d + e*x^(2/3))^2
```

```
*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(4*e^3) - (b*n*(d + e*x^(2/3))^3*(a +
b*Log[c*(d + e*x^(2/3))^n])^2)/(2*e^3) + (3*d^2*(d + e*x^(2/3))*(a + b*Log[
c*(d + e*x^(2/3))^n])^3)/(2*e^3) - (3*d*(d + e*x^(2/3))^2*(a + b*Log[c*(d +
 e*x^(2/3))^n])^3)/(2*e^3) + ((d + e*x^(2/3))^3*(a + b*Log[c*(d + e*x^(2/3)
)^n])^3)/(2*e^3)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
```

```
+ e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3}{2}\text{Subst}\left(\int x^2(a+b\log(c(d+ex)^n))^3\,dx,x,x^{2/3}\right)\\
&= \frac{3}{2}\text{Subst}\left(\int \left(\frac{d^2(a+b\log(c(d+ex)^n))^3}{e^2}-\frac{2d(d+ex)(a+b\log(c(d+ex)^n))^3}{e^2}\right.\right.\\
&\qquad\left.\left.+\frac{(d+ex)^2(a+b\log(c(d+ex)^n))^3}{e^2}\right)dx,x,x^{2/3}\right)\\
&= \frac{3\text{Subst}\left(\int (d+ex)^2(a+b\log(c(d+ex)^n))^3\,dx,x,x^{2/3}\right)}{2e^2}\\
&\quad-\frac{(3d)\text{Subst}\left(\int (d+ex)(a+b\log(c(d+ex)^n))^3\,dx,x,x^{2/3}\right)}{e^2}\\
&\quad+\frac{(3d^2)\,\text{Subst}\left(\int (a+b\log(c(d+ex)^n))^3\,dx,x,x^{2/3}\right)}{2e^2}\\
&= \frac{3\text{Subst}\left(\int x^2(a+b\log(cx^n))^3\,dx,x,d+ex^{2/3}\right)}{2e^3}\\
&\quad-\frac{(3d)\text{Subst}\left(\int x(a+b\log(cx^n))^3\,dx,x,d+ex^{2/3}\right)}{e^3}\\
&\quad+\frac{(3d^2)\,\text{Subst}\left(\int (a+b\log(cx^n))^3\,dx,x,d+ex^{2/3}\right)}{2e^3}
\end{aligned}
$$

$$= \frac{3d^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3} - \frac{3d\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3} + \frac{\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3} - \frac{(3bn)\text{Subst}\left(\int x^2(a+b\log(cx^n))^2\,dx,x,d+ex^{2/3}\right)}{2e^3} + \frac{(9bdn)\text{Subst}\left(\int x(a+b\log(cx^n))^2\,dx,x,d+ex^{2/3}\right)}{2e^3} - \frac{(9bd^2n)\,\text{Subst}\left(\int(a+b\log(cx^n))^2\,dx,x,d+ex^{2/3}\right)}{2e^3}$$

$$= -\frac{9bd^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^3} + \frac{9bdn\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4e^3} - \frac{bn\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^3} + \frac{3d^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3} - \frac{3d\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3} + \frac{\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3} + \frac{(b^2n^2)\,\text{Subst}\left(\int x^2(a+b\log(cx^n))\,dx,x,d+ex^{2/3}\right)}{e^3} - \frac{(9b^2dn^2)\,\text{Subst}\left(\int x(a+b\log(cx^n))\,dx,x,d+ex^{2/3}\right)}{2e^3} + \frac{(9b^2d^2n^2)\,\text{Subst}\left(\int(a+b\log(cx^n))\,dx,x,d+ex^{2/3}\right)}{e^3}$$

$$
\begin{aligned}
&= \frac{9b^3dn^3\left(d+ex^{2/3}\right)^2}{8e^3} - \frac{b^3n^3\left(d+ex^{2/3}\right)^3}{9e^3} + \frac{9ab^2d^2n^2x^{2/3}}{e^2} \\
&\quad - \frac{9b^2dn^2\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4e^3} \\
&\quad + \frac{b^2n^2\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^3} \\
&\quad - \frac{9bd^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^3} \\
&\quad + \frac{9bdn\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4e^3} \\
&\quad - \frac{bn\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^3} \\
&\quad + \frac{3d^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3} \\
&\quad - \frac{3d\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3} \\
&\quad + \frac{\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3} \\
&\quad + \frac{(9b^3d^2n^2)\,\mathrm{Subst}\left(\int\log\left(cx^n\right)\,dx,x,d+ex^{2/3}\right)}{e^3} \\
&= \frac{9b^3dn^3\left(d+ex^{2/3}\right)^2}{8e^3} - \frac{b^3n^3\left(d+ex^{2/3}\right)^3}{9e^3} + \frac{9ab^2d^2n^2x^{2/3}}{e^2} \\
&\quad - \frac{9b^3d^2n^3x^{2/3}}{e^2} + \frac{9b^3d^2n^2\left(d+ex^{2/3}\right)\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e^3} \\
&\quad - \frac{9b^2dn^2\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{4e^3} \\
&\quad + \frac{b^2n^2\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{3e^3} \\
&\quad - \frac{9bd^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^3} \\
&\quad + \frac{9bdn\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4e^3} \\
&\quad - \frac{bn\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2e^3} \\
&\quad + \frac{3d^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3} \\
&\quad - \frac{3d\left(d+ex^{2/3}\right)^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3} \\
&\quad + \frac{\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2e^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.95

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx = \frac{36a^3d^3-198a^2bd^3n-108a^2bd^2enx^{2/3}+396ab^2d^2en^2x^{2/3}-510b^3d^2en^3x^{2/3}}{}$$

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (36*a^3*d^3 - 198*a^2*b*d^3*n - 108*a^2*b*d^2*e*n*x^(2/3) + 396*a*b^2*d^2*e
*n^2*x^(2/3) - 510*b^3*d^2*e*n^3*x^(2/3) + 54*a^2*b*d*e^2*n*x^(4/3) - 90*a*
b^2*d*e^2*n^2*x^(4/3) + 57*b^3*d*e^2*n^3*x^(4/3) + 36*a^3*e^3*x^2 - 36*a^2*
b*e^3*n*x^2 + 24*a*b^2*e^3*n^2*x^2 - 8*b^3*e^3*n^3*x^2 + 114*b^3*d^3*n^3*Lo
g[d + e*x^(2/3)] + 6*b*(18*a^2*(d^3 + e^3*x^2) - 6*a*b*n*(11*d^3 + 6*d^2*e*
x^(2/3) - 3*d*e^2*x^(4/3) + 2*e^3*x^2) + b^2*n^2*(66*d^3 + 66*d^2*e*x^(2/3)
 - 15*d*e^2*x^(4/3) + 4*e^3*x^2))*Log[c*(d + e*x^(2/3))^n] + 18*b^2*(6*a*(d
^3 + e^3*x^2) - b*n*(11*d^3 + 6*d^2*e*x^(2/3) - 3*d*e^2*x^(4/3) + 2*e^3*x^2
))*Log[c*(d + e*x^(2/3))^n]^2 + 36*b^3*(d^3 + e^3*x^2)*Log[c*(d + e*x^(2/3)
)^n]^3)/(72*e^3)
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{3}dx$$

```
[In] int(x*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.60

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx = \frac{36\,b^3e^3x^2\log\left(c\right)^3-36\,(b^3e^3n-3\,ab^2e^3)x^2\log\left(c\right)^2+36\,(b^3e^3n^3x^2+b^3d^3n^3)\,\mathrm{lo}}{}$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] 1/72*(36*b^3*e^3*x^2*log(c)^3 - 36*(b^3*e^3*n - 3*a*b^2*e^3)*x^2*log(c)^2 +
 36*(b^3*e^3*n^3*x^2 + b^3*d^3*n^3)*log(e*x^(2/3) + d)^3 + 12*(2*b^3*e^3*n^
```

```
2 - 6*a*b^2*e^3*n + 9*a^2*b*e^3)*x^2*log(c) - 4*(2*b^3*e^3*n^3 - 6*a*b^2*e^
3*n^2 + 9*a^2*b*e^3*n - 9*a^3*e^3)*x^2 + 18*(3*b^3*d*e^2*n^3*x^(4/3) - 6*b^
3*d^2*e*n^3*x^(2/3) - 11*b^3*d^3*n^3 + 6*a*b^2*d^3*n^2 - 2*(b^3*e^3*n^3 - 3
*a*b^2*e^3*n^2)*x^2 + 6*(b^3*e^3*n^2*x^2 + b^3*d^3*n^2)*log(c))*log(e*x^(2/
3) + d)^2 + 6*(85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*d^3*n + 2*(2*b^
3*e^3*n^3 - 6*a*b^2*e^3*n^2 + 9*a^2*b*e^3*n)*x^2 + 18*(b^3*e^3*n*x^2 + b^3*
d^3*n)*log(c)^2 - 6*(11*b^3*d^3*n^2 - 6*a*b^2*d^3*n + 2*(b^3*e^3*n^2 - 3*a*
b^2*e^3*n)*x^2)*log(c) + 6*(11*b^3*d^2*e*n^3 - 6*b^3*d^2*e*n^2*log(c) - 6*a
*b^2*d^2*e*n^2)*x^(2/3) + 3*(6*b^3*d*e^2*n^2*x*log(c) - (5*b^3*d*e^2*n^3 -
6*a*b^2*d*e^2*n^2)*x)*x^(1/3))*log(e*x^(2/3) + d) - 6*(85*b^3*d^2*e*n^3 + 1
8*b^3*d^2*e*n*log(c)^2 - 66*a*b^2*d^2*e*n^2 + 18*a^2*b*d^2*e*n - 6*(11*b^3*
d^2*e*n^2 - 6*a*b^2*d^2*e*n)*log(c))*x^(2/3) + 3*(18*b^3*d*e^2*n*x*log(c)^2
 - 6*(5*b^3*d*e^2*n^2 - 6*a*b^2*d*e^2*n)*x*log(c) + (19*b^3*d*e^2*n^3 - 30*
a*b^2*d*e^2*n^2 + 18*a^2*b*d*e^2*n)*x)*x^(1/3))/e^3
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\text{Timed out}$$

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(2/3))**n))**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.08

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\frac{1}{2}\,b^3x^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)^3$$
$$+\frac{3}{2}\,ab^2x^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)^2+\frac{1}{4}\,a^2ben\left(\frac{6\,d^3\log\left(ex^{\frac{2}{3}}+d\right)}{e^4}-\frac{2\,e^2x^2-3\,dex^{\frac{4}{3}}+6\,d^2x^{\frac{2}{3}}}{e^3}\right)$$
$$+\frac{3}{2}\,a^2bx^2\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+\frac{1}{2}\,a^3x^2$$
$$+\frac{1}{12}\left(6\,en\left(\frac{6\,d^3\log\left(ex^{\frac{2}{3}}+d\right)}{e^4}-\frac{2\,e^2x^2-3\,dex^{\frac{4}{3}}+6\,d^2x^{\frac{2}{3}}}{e^3}\right)\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+\frac{\left(4\,e^3x^2-18\,d^3\log\left(ex^{\frac{2}{3}}\right.\right.}{}\right.$$
$$+\frac{1}{72}\left(18\,en\left(\frac{6\,d^3\log\left(ex^{\frac{2}{3}}+d\right)}{e^4}-\frac{2\,e^2x^2-3\,dex^{\frac{4}{3}}+6\,d^2x^{\frac{2}{3}}}{e^3}\right)\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)^2+en\left(\frac{\left(36\,d^3\log\left(ex^{\frac{2}{3}}\right.\right.}{}\right.\right.$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2*log((e*x^(2/3) + d)^n*c)^3 + 3/2*a*b^2*x^2*log((e*x^(2/3) + d)^
n*c)^2 + 1/4*a^2*b*e*n*(6*d^3*log(e*x^(2/3) + d)/e^4 - (2*e^2*x^2 - 3*d*e*x
^(4/3) + 6*d^2*x^(2/3))/e^3) + 3/2*a^2*b*x^2*log((e*x^(2/3) + d)^n*c) + 1/2
*a^3*x^2 + 1/12*(6*e*n*(6*d^3*log(e*x^(2/3) + d)/e^4 - (2*e^2*x^2 - 3*d*e*x
^(4/3) + 6*d^2*x^(2/3))/e^3)*log((e*x^(2/3) + d)^n*c) + (4*e^3*x^2 - 18*d^3
*log(e*x^(2/3) + d)^2 - 15*d*e^2*x^(4/3) - 66*d^3*log(e*x^(2/3) + d) + 66*d
^2*e*x^(2/3))*n^2/e^3)*a*b^2 + 1/72*(18*e*n*(6*d^3*log(e*x^(2/3) + d)/e^4 -
 (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^3)*log((e*x^(2/3) + d)^n*c)^
2 + e*n*((36*d^3*log(e*x^(2/3) + d)^3 - 8*e^3*x^2 + 198*d^3*log(e*x^(2/3) +
 d)^2 + 57*d*e^2*x^(4/3) + 510*d^3*log(e*x^(2/3) + d) - 510*d^2*e*x^(2/3))*
n^2/e^4 + 6*(4*e^3*x^2 - 18*d^3*log(e*x^(2/3) + d)^2 - 15*d*e^2*x^(4/3) - 6
6*d^3*log(e*x^(2/3) + d) + 66*d^2*e*x^(2/3))*n*log((e*x^(2/3) + d)^n*c)/e^4
))*b^3
```

Giac [A] (verification not implemented)

none

Time = 0.79 (sec) , antiderivative size = 755, normalized size of antiderivative = 1.68

$$\int x\Big(a+b\log\Big(c\big(d+ex^{2/3}\big)^n\Big)\Big)^3\,dx=\text{Too large to display}$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2*log(c)^3 + 1/72*(36*x^2*log(e*x^(2/3) + d)^3 - (18*(2*(e*x^(2/3
) + d)^3/e^4 - 9*(e*x^(2/3) + d)^2*d/e^4 + 18*(e*x^(2/3) + d)*d^2/e^4)*log(
e*x^(2/3) + d)^2 - 6*(4*(e*x^(2/3) + d)^3/e^4 - 27*(e*x^(2/3) + d)^2*d/e^4
+ 108*(e*x^(2/3) + d)*d^2/e^4)*log(e*x^(2/3) + d) - 36*d^3*log(e*x^(2/3) +
d)^3/e^4 + 8*(e*x^(2/3) + d)^3/e^4 - 81*(e*x^(2/3) + d)^2*d/e^4 + 648*(e*x^
(2/3) + d)*d^2/e^4)*e)*b^3*n^3 + 1/12*(18*x^2*log(e*x^(2/3) + d)^2 - (6*(2*
(e*x^(2/3) + d)^3/e^4 - 9*(e*x^(2/3) + d)^2*d/e^4 + 18*(e*x^(2/3) + d)*d^2/
e^4)*log(e*x^(2/3) + d) - 18*d^3*log(e*x^(2/3) + d)^2/e^4 - 4*(e*x^(2/3) +
d)^3/e^4 + 27*(e*x^(2/3) + d)^2*d/e^4 - 108*(e*x^(2/3) + d)*d^2/e^4)*e)*b^3
*n^2*log(c) + 1/4*(6*x^2*log(e*x^(2/3) + d) + e*(6*d^3*log(abs(e*x^(2/3) +
d))/e^4 - (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^3))*b^3*n*log(c)^2
+ 3/2*a*b^2*x^2*log(c)^2 + 1/12*(18*x^2*log(e*x^(2/3) + d)^2 - (6*(2*(e*x^(
2/3) + d)^3/e^4 - 9*(e*x^(2/3) + d)^2*d/e^4 + 18*(e*x^(2/3) + d)*d^2/e^4)*l
og(e*x^(2/3) + d) - 18*d^3*log(e*x^(2/3) + d)^2/e^4 - 4*(e*x^(2/3) + d)^3/e
^4 + 27*(e*x^(2/3) + d)^2*d/e^4 - 108*(e*x^(2/3) + d)*d^2/e^4)*e)*a*b^2*n^2
 + 1/2*(6*x^2*log(e*x^(2/3) + d) + e*(6*d^3*log(abs(e*x^(2/3) + d))/e^4 - (
2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^3))*a*b^2*n*log(c) + 3/2*a^2*b
*x^2*log(c) + 1/4*(6*x^2*log(e*x^(2/3) + d) + e*(6*d^3*log(abs(e*x^(2/3) +
d))/e^4 - (2*e^2*x^2 - 3*d*e*x^(4/3) + 6*d^2*x^(2/3))/e^3))*a^2*b*n + 1/2*a
^3*x^2
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.28

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)^3\left(\frac{b^3\,x^2}{2}+\frac{b^3\,d^3}{2\,e^3}\right)$$

$$-x^{4/3}\left(\frac{d\left(\frac{3\,a^3}{2}-\frac{3\,a^2\,b\,n}{2}+a\,b^2\,n^2-\frac{b^3\,n^3}{3}\right)}{2\,e}-\frac{d\left(6\,a^3-6\,a\,b^2\,n^2+5\,b^3\,n^3\right)}{8\,e}\right)+\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)^2\left(\frac{b^2\,x^2\,(3\,a}{2}\right.$$

```
[In] int(x*(a + b*log(c*(d + e*x^(2/3))^n))^3,x)
```

```
[Out] log(c*(d + e*x^(2/3))^n)^3*((b^3*x^2)/2 + (b^3*d^3)/(2*e^3)) - x^(4/3)*((d*
((3*a^3)/2 - (b^3*n^3)/3 + a*b^2*n^2 - (3*a^2*b*n)/2))/(2*e) - (d*(6*a^3 +
5*b^3*n^3 - 6*a*b^2*n^2))/(8*e)) + log(c*(d + e*x^(2/3))^n)^2*((b^2*x^2*(3*
a - b*n))/2 - (x^(4/3)*((3*b^2*d*(3*a - b*n))/(2*e) - (9*a*b^2*d)/(2*e)))/2
 + (d*(6*a*b^2*d^2 - 11*b^3*d^2*n))/(4*e^3) + (d*x^(2/3)*((6*b^2*d*(3*a - b
*n))/e - (18*a*b^2*d)/e))/(4*e)) + x^(2/3)*((d*((d*((3*a^3)/2 - (b^3*n^3)/3
 + a*b^2*n^2 - (3*a^2*b*n)/2))/e - (d*(6*a^3 + 5*b^3*n^3 - 6*a*b^2*n^2))/(4
*e)))/e + (b^2*d^2*n^2*(6*a - 11*b*n))/(2*e^2)) + x^2*(a^3/2 - (b^3*n^3)/9
+ (a*b^2*n^2)/3 - (a^2*b*n)/2) + (log(c*(d + e*x^(2/3))^n)*((x^(2/3)*((d*(2
*b*d*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*n) - 6*b*d*e*(3*a^2 - b^2*n^2)))/e + 12*b
^3*d^2*n^2))/(2*e) - (x^(4/3)*(2*b*d*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*n) - 6*b*
d*e*(3*a^2 - b^2*n^2)))/(4*e) + (b*e*x^2*(9*a^2 + 2*b^2*n^2 - 6*a*b*n))/3))
/(2*e) + (log(d + e*x^(2/3))*(85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*
d^3*n))/(12*e^3)
```

3.483 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 139

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\,dx = \frac{3}{2}\Big(a +b\log\left(c\left(d+ex^{2/3}\right)^n\right)\Big)^3\log\left(-\frac{ex^{2/3}}{d}\right)+\frac{9}{2}bn\Big(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\Big)^2\operatorname{PolyLog}\left(2,1+\frac{ex^{2/3}}{d}\right)-9b^2n^2\Big(a$$

```
[Out] 3/2*(a+b*ln(c*(d+e*x^(2/3))^n))^3*ln(-e*x^(2/3)/d)+9/2*b*n*(a+b*ln(c*(d+e*x
^(2/3))^n))^2*polylog(2,1+e*x^(2/3)/d)-9*b^2*n^2*(a+b*ln(c*(d+e*x^(2/3))^n)
)*polylog(3,1+e*x^(2/3)/d)+9*b^3*n^3*polylog(4,1+e*x^(2/3)/d)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2504, 2443, 2481, 2421, 2430, 6724}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\,dx = -9b^2n^2\operatorname{PolyLog}\left(3,\frac{x^{2/3}e}{d} +1\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{9}{2}bn\operatorname{PolyLog}\left(2,\frac{x^{2/3}e}{d}+1\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2+\frac{3}{2}\log\left(-\frac{ex^{2/3}}{d}\right.$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x,x]
```

```
[Out] (3*(a + b*Log[c*(d + e*x^(2/3))^n])^3*Log[-((e*x^(2/3))/d)])/2 + (9*b*n*(a
+ b*Log[c*(d + e*x^(2/3))^n])^2*PolyLog[2, 1 + (e*x^(2/3))/d])/2 - 9*b^2*n^
2*(a + b*Log[c*(d + e*x^(2/3))^n])*PolyLog[3, 1 + (e*x^(2/3))/d] + 9*b^3*n^
3*PolyLog[4, 1 + (e*x^(2/3))/d]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x}\,dx, x, x^{2/3}\right)$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\log\left(-\frac{ex^{2/3}}{d}\right) - \frac{1}{2}(9ben)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{d+ex}\,dx, x, x^{2/3}\right)$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\log\left(-\frac{ex^{2/3}}{d}\right) - \frac{1}{2}(9bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx, x, d+ex^{2/3}\right)$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\log\left(-\frac{ex^{2/3}}{d}\right) + \frac{9}{2}bn\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\text{Li}_2\left(1+\frac{ex^{2/3}}{d}\right) - \left(9b^2n^2\right)\text{Subst}\left(\int \frac{(a+b\log(cx^n))\text{Li}_2\left(\frac{x}{d}\right)}{x}\,dx$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\log\left(-\frac{ex^{2/3}}{d}\right) + \frac{9}{2}bn\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\text{Li}_2\left(1+\frac{ex^{2/3}}{d}\right) - 9b^2n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\text{Li}_3\left(1+\frac{ex^{2/3}}{d}\right.$$

$$= \frac{3}{2}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\log\left(-\frac{ex^{2/3}}{d}\right) + \frac{9}{2}bn\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\text{Li}_2\left(1+\frac{ex^{2/3}}{d}\right) - 9b^2n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\text{Li}_3\left(1+\frac{ex^{2/3}}{d}\right.$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 339 vs. 2(139) = 278.

Time = 0.14 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.44

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\,dx = \left(a - bn\log\left(d+ex^{2/3}\right) + b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\log(x) + 3bn\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2\left(\left(\right.\right.$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x,x]
```

```
[Out] (a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^3*Log[x] + 3*b*n*
(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2*((Log[d + e*x^(
2/3)] - Log[1 + (e*x^(2/3))/d])*Log[x] - (3*PolyLog[2, -((e*x^(2/3))/d)])/2
) + (9*b^2*n^2*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])*(L
og[d + e*x^(2/3)]^2*Log[-((e*x^(2/3))/d)] + 2*Log[d + e*x^(2/3)]*PolyLog[2,
 1 + (e*x^(2/3))/d] - 2*PolyLog[3, 1 + (e*x^(2/3))/d]))/2 + (3*b^3*n^3*(Log
[d + e*x^(2/3)]^3*Log[-((e*x^(2/3))/d)] + 3*Log[d + e*x^(2/3)]^2*PolyLog[2,
 1 + (e*x^(2/3))/d] - 6*Log[d + e*x^(2/3)]*PolyLog[3, 1 + (e*x^(2/3))/d] +
6*PolyLog[4, 1 + (e*x^(2/3))/d]))/2
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{3}}{x}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c(d+ex^{2/3})^{n}\right)\right)^{3}}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{3}}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(2/3) + d)^n*c)
^2 + 3*a^2*b*log((e*x^(2/3) + d)^n*c) + a^3)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c(d+ex^{2/3})^{n}\right)\right)^{3}}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{\frac{2}{3}}\right)^{n}\right)\right)^{3}}{x}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(2/3))**n))**3/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^3}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x,x, algorithm="maxima")
```

```
[Out] b^3*log((e*x^(2/3) + d)^n)^3*log(x) + integrate(-((2*b^3*e*n*x*log(x) - 3*(
b^3*e*log(c) + a*b^2*e)*x - 3*(b^3*d*log(c) + a*b^2*d)*x^(1/3))*log((e*x^(2
/3) + d)^n)^2 - (b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a
^3*e)*x - 3*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b^3*d*log(c
)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^(1/3))*log((e*x^(2/3) + d)^n) - (b^3*d*
log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^(1/3))/(e*x^2 +
 d*x^(4/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^3}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\,dx = \int \frac{\left(a+b\,\ln\left(c\left(d+e\,x^{2/3}\right)^n\right)\right)^3}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x, x)
```

3.484 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 451

$$\begin{aligned}
\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}\,dx =
&-\frac{3b^2e^2n^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3x^{2/3}}\\
&-\frac{3b^2e^3n^2\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3}\\
&-\frac{3ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4dx^{4/3}}\\
&+\frac{3be^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2d^3x^{2/3}}\\
&+\frac{3be^3n\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2d^3}\\
&-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2x^2}\\
&-\frac{3b^2e^3n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\log\left(-\frac{ex^{2/3}}{d}\right)}{d^3}\\
&+\frac{b^3e^3n^3\log(x)}{d^3}+\frac{3b^3e^3n^3\,\mathrm{PolyLog}\left(2,\frac{d}{d+ex^{2/3}}\right)}{2d^3}\\
&-\frac{3b^2e^3n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\mathrm{PolyLog}\left(2,\frac{d}{d+ex^{2/3}}\right)}{d^3}\\
&-\frac{3b^3e^3n^3\,\mathrm{PolyLog}\left(2,1+\frac{ex^{2/3}}{d}\right)}{d^3}-\frac{3b^3e^3n^3\,\mathrm{PolyLog}\left(3,\frac{d}{d+ex^{2/3}}\right)}{d^3}
\end{aligned}$$

```
[Out] -3/2*b^2*e^2*n^2*(d+e*x^(2/3))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^3/x^(2/3)-3/2*
b^2*e^3*n^2*ln(1-d/(d+e*x^(2/3)))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^3-3/4*b*e*n
*(a+b*ln(c*(d+e*x^(2/3))^n))^2/d/x^(4/3)+3/2*b*e^2*n*(d+e*x^(2/3))*(a+b*ln(
c*(d+e*x^(2/3))^n))^2/d^3/x^(2/3)+3/2*b*e^3*n*ln(1-d/(d+e*x^(2/3)))*(a+b*ln
(c*(d+e*x^(2/3))^n))^2/d^3-1/2*(a+b*ln(c*(d+e*x^(2/3))^n))^3/x^2-3*b^2*e^3*
n^2*(a+b*ln(c*(d+e*x^(2/3))^n))*ln(-e*x^(2/3)/d)/d^3+b^3*e^3*n^3*ln(x)/d^3+
3/2*b^3*e^3*n^3*polylog(2,d/(d+e*x^(2/3)))/d^3-3*b^2*e^3*n^2*(a+b*ln(c*(d+e
*x^(2/3))^n))*polylog(2,d/(d+e*x^(2/3)))/d^3-3*b^3*e^3*n^3*polylog(2,1+e*x^
(2/3)/d)/d^3-3*b^3*e^3*n^3*polylog(3,d/(d+e*x^(2/3)))/d^3
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}\,dx = -\frac{3b^2e^3n^2\,\text{PolyLog}\left(2,\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3} - \frac{3b^2e^3n^2\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3} - \frac{3b^2e^3n^2\log\left(-\frac{ex^{2/3}}{d}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^3} - \frac{3b^2e^2n^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3x^{2/3}} + \frac{3be^3n\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2d^3} + \frac{3be^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2d^3x^{2/3}} - \frac{3ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4dx^{4/3}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2x^2} + \frac{3b^3e^3n^3\,\text{PolyLog}\left(2,\frac{d}{d+ex^{2/3}}\right)}{2d^3} - \frac{3b^3e^3n^3\,\text{PolyLog}\left(2,\frac{x^{2/3}e}{d}+1\right)}{d^3} - \frac{3b^3e^3n^3\,\text{PolyLog}\left(3,\frac{d}{d+ex^{2/3}}\right)}{d^3} + \frac{b^3e^3n^3\log(x)}{d^3}$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^3,x]
```

```
[Out] (-3*b^2*e^2*n^2*(d + e*x^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n]))/(2*d^3*x^
(2/3)) - (3*b^2*e^3*n^2*Log[1 - d/(d + e*x^(2/3))]*(a + b*Log[c*(d + e*x^(2
```

```
/3))^n]))/(2*d^3) - (3*b*e*n*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(4*d*x^(4/
3)) + (3*b*e^2*n*(d + e*x^(2/3))*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(2*d^3
*x^(2/3)) + (3*b*e^3*n*Log[1 - d/(d + e*x^(2/3))]*(a + b*Log[c*(d + e*x^(2/
3))^n])^2)/(2*d^3) - (a + b*Log[c*(d + e*x^(2/3))^n])^3/(2*x^2) - (3*b^2*e^
3*n^2*(a + b*Log[c*(d + e*x^(2/3))^n])*Log[-((e*x^(2/3))/d)])/d^3 + (b^3*e^
3*n^3*Log[x])/d^3 + (3*b^3*e^3*n^3*PolyLog[2, d/(d + e*x^(2/3))])/(2*d^3) -
 (3*b^2*e^3*n^2*(a + b*Log[c*(d + e*x^(2/3))^n])*PolyLog[2, d/(d + e*x^(2/3
))])/d^3 - (3*b^3*e^3*n^3*PolyLog[2, 1 + (e*x^(2/3))/d])/d^3 - (3*b^3*e^3*n
^3*PolyLog[3, d/(d + e*x^(2/3))])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^4}\,dx, x, x^{2/3}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2x^2}+\frac{1}{2}(3ben)\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^3(d+ex)}\,dx, x, x^{2/3}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2x^2}+\frac{1}{2}(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+ex^{2/3}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2x^2}+\frac{(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+ex^{2/3}\right)}{2d}\\
&\quad -\frac{(3ben)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+ex^{2/3}\right)}{2d}\\
&= -\frac{3ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4dx^{4/3}}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2x^2}\\
&\quad -\frac{(3ben)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+ex^{2/3}\right)}{2d^2}\\
&\quad +\frac{\left(3be^2n\right)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx, x, d+ex^{2/3}\right)}{2d^2}\\
&\quad +\frac{\left(3b^2en^2\right)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+ex^{2/3}\right)}{2d}
\end{aligned}$$

$$= -\frac{3ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4dx^{4/3}} + \frac{3be^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2d^3x^{2/3}}$$
$$+ \frac{3be^3n\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2d^3}$$
$$- \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2x^2} + \frac{(3b^2en^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+ex^{2/3}\right)}{2d^2}$$
$$- \frac{(3b^2e^2n^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{-\frac{d}{e}+\frac{x}{e}}\,dx, x, d+ex^{2/3}\right)}{d^3}$$
$$- \frac{(3b^2e^2n^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx, x, d+ex^{2/3}\right)}{2d^2}$$
$$- \frac{(3b^2e^3n^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{d}{x}\right)(a+b\log(cx^n))}{x}\,dx, x, d+ex^{2/3}\right)}{d^3}$$

$$
\begin{aligned}
&= -\frac{3b^2e^2n^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3x^{2/3}} \\
&\quad - \frac{3b^2e^3n^2\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3} \\
&\quad - \frac{3ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4dx^{4/3}} \\
&\quad + \frac{3be^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2d^3x^{2/3}} \\
&\quad + \frac{3be^3n\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2d^3} \\
&\quad - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2x^2} - \frac{3b^2e^3n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\log\left(-\frac{ex^{2/3}}{d}\right)}{d^3} \\
&\quad - \frac{3b^2e^3n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{d}{d+ex^{2/3}}\right)}{d^3} \\
&\quad + \frac{(3b^3e^2n^3)\operatorname{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x}{e}}\,dx,x,d+ex^{2/3}\right)}{2d^3} \\
&\quad + \frac{(3b^3e^3n^3)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{d}{x}\right)}{x}\,dx,x,d+ex^{2/3}\right)}{2d^3} \\
&\quad + \frac{(3b^3e^3n^3)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{x}{d}\right)}{x}\,dx,x,d+ex^{2/3}\right)}{d^3} \\
&\quad + \frac{(3b^3e^3n^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{d}{x}\right)}{x}\,dx,x,d+ex^{2/3}\right)}{d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3b^2e^2n^2\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3x^{2/3}} \\
&- \frac{3b^2e^3n^2\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{2d^3} \\
&- \frac{3ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{4dx^{4/3}} \\
&+ \frac{3be^2n\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2d^3x^{2/3}} \\
&+ \frac{3be^3n\log\left(1-\frac{d}{d+ex^{2/3}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{2d^3} \\
&- \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{2x^2} - \frac{3b^2e^3n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\log\left(-\frac{ex^{2/3}}{d}\right)}{d^3} \\
&+ \frac{b^3e^3n^3\log(x)}{d^3} + \frac{3b^3e^3n^3\operatorname{Li}_2\left(\frac{d}{d+ex^{2/3}}\right)}{2d^3} \\
&- \frac{3b^2e^3n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{d}{d+ex^{2/3}}\right)}{d^3} \\
&- \frac{3b^3e^3n^3\operatorname{Li}_2\left(1+\frac{ex^{2/3}}{d}\right)}{d^3} - \frac{3b^3e^3n^3\operatorname{Li}_3\left(\frac{d}{d+ex^{2/3}}\right)}{d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.69

$$
\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}\,dx = \frac{-3bd^2enx^{2/3}\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2+6bde^2n
$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^3,x]
```

```
[Out] (-3*b*d^2*e*n*x^(2/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))
^n])^2 + 6*b*d*e^2*n*x^(4/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x
^(2/3))^n])^2 - 6*b*d^3*n*Log[d + e*x^(2/3)]*(a - b*n*Log[d + e*x^(2/3)] +
b*Log[c*(d + e*x^(2/3))^n])^2 - 6*b*e^3*n*x^2*Log[d + e*x^(2/3)]*(a - b*n*L
og[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 - 2*d^3*(a - b*n*Log[d +
e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^3 + 4*b*e^3*n*x^2*(a - b*n*Log[d +
 e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2*Log[x] - 6*b^2*n^2*(a - b*n*Log
[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])*((d^3 + e^3*x^2)*Log[d + e*x^
(2/3)]^2 + e^2*x^(4/3)*(d + 3*e*x^(2/3)*Log[-((e*x^(2/3))/d)]) + Log[d + e*
x^(2/3)]*(d^2*e*x^(2/3) - 2*d*e^2*x^(4/3) - 3*e^3*x^2 - 2*e^3*x^2*Log[-((e*
x^(2/3))/d)]) - 2*e^3*x^2*PolyLog[2, 1 + (e*x^(2/3))/d]) + b^3*n^3*(-6*d*e^
2*x^(4/3)*Log[d + e*x^(2/3)] - 6*e^3*x^2*Log[d + e*x^(2/3)] - 3*d^2*e*x^(2/
```

```
3)*Log[d + e*x^(2/3)]^2 + 6*d*e^2*x^(4/3)*Log[d + e*x^(2/3)]^2 + 9*e^3*x^2*
Log[d + e*x^(2/3)]^2 - 2*d^3*Log[d + e*x^(2/3)]^3 - 2*e^3*x^2*Log[d + e*x^(
2/3)]^3 + 6*e^3*x^2*Log[-((e*x^(2/3))/d)] - 18*e^3*x^2*Log[d + e*x^(2/3)]*L
og[-((e*x^(2/3))/d)] + 6*e^3*x^2*Log[d + e*x^(2/3)]^2*Log[-((e*x^(2/3))/d)]
 + 6*e^3*x^2*(-3 + 2*Log[d + e*x^(2/3)])*PolyLog[2, 1 + (e*x^(2/3))/d] - 12
*e^3*x^2*PolyLog[3, 1 + (e*x^(2/3))/d]))/(4*d^3*x^2)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{3}}{x^{3}}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\right)^{3}}{x^{3}}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{3}}{x^{3}}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(2/3) + d)^n*c)
^2 + 3*a^2*b*log((e*x^(2/3) + d)^n*c) + a^3)/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^{n}\right)\right)^{3}}{x^{3}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3/x**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^3}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log((e*x^(2/3) + d)^n)^3/x^2 + integrate(((b^3*e*n*x + 3*(b^3*e*lo
g(c) + a*b^2*e)*x + 3*(b^3*d*log(c) + a*b^2*d)*x^(1/3))*log((e*x^(2/3) + d)
^n)^2 + (b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x
+ 3*((b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x + (b^3*d*log(c)^2 + 2*
a*b^2*d*log(c) + a^2*b*d)*x^(1/3))*log((e*x^(2/3) + d)^n) + (b^3*d*log(c)^3
 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^(1/3))/(e*x^4 + d*x^(10
/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^3}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}\,dx = \int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)\right)^3}{x^3}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^3,x)
```

```
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^3, x)
```

3.485 $\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\frac{4504ab^2d^4n^2\sqrt[3]{x}}{315e^4}$$
$$-\frac{3475504b^3d^4n^3\sqrt[3]{x}}{99225e^4}+\frac{637984b^3d^3n^3x}{297675e^3}-\frac{221344b^3d^2n^3x^{5/3}}{496125e^2}$$
$$+\frac{3088b^3dn^3x^{7/3}}{27783e}-\frac{16}{729}b^3n^3x^3+\frac{3475504b^3d^{9/2}n^3\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{99225e^{9/2}}$$
$$-\frac{4504ib^3d^{9/2}n^3\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{315e^{9/2}}-\frac{9008b^3d^{9/2}n^3\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{315e^{9/2}}$$
$$+\frac{4504b^3d^4n^2\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{315e^4}-\frac{1984b^2d^3n^2x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{945e^3}$$
$$+\frac{1144b^2d^2n^2x^{5/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{1575e^2}-\frac{128b^2dn^2x^{7/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{441e}$$
$$+\frac{8}{81}b^2n^2x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)-\frac{4504b^2d^{9/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{315e^{9/2}}-\frac{2bd^4n\sqrt[3]{x}}{}$$

```
[Out] 4504/315*a*b^2*d^4*n^2*x^(1/3)/e^4-3475504/99225*b^3*d^4*n^3*x^(1/3)/e^4+63
7984/297675*b^3*d^3*n^3*x/e^3-221344/496125*b^3*d^2*n^3*x^(5/3)/e^2+3088/27
783*b^3*d*n^3*x^(7/3)/e-16/729*b^3*n^3*x^3+3475504/99225*b^3*d^(9/2)*n^3*ar
ctan(x^(1/3)*e^(1/2)/d^(1/2))/e^(9/2)-4504/315*I*b^3*d^(9/2)*n^3*polylog(2,
1-2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/e^(9/2)+4504/315*b^3*d^4*n^2*x^(1/
3)*ln(c*(d+e*x^(2/3))^n)/e^4-1984/945*b^2*d^3*n^2*x*(a+b*ln(c*(d+e*x^(2/3))
^n))/e^3+1144/1575*b^2*d^2*n^2*x^(5/3)*(a+b*ln(c*(d+e*x^(2/3))^n))/e^2-128/
441*b^2*d*n^2*x^(7/3)*(a+b*ln(c*(d+e*x^(2/3))^n))/e+8/81*b^2*n^2*x^3*(a+b*l
```

```
n(c*(d+e*x^(2/3))^n))-4504/315*b^2*d^(9/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/
2))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^(9/2)-2*b*d^4*n*x^(1/3)*(a+b*ln(c*(d+e*x^
(2/3))^n))^2/e^4+2/3*b*d^3*n*x*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^3-2/5*b*d^2*
n*x^(5/3)*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e^2+2/7*b*d*n*x^(7/3)*(a+b*ln(c*(d+
e*x^(2/3))^n))^2/e-2/9*b*n*x^3*(a+b*ln(c*(d+e*x^(2/3))^n))^2+1/3*x^3*(a+b*l
n(c*(d+e*x^(2/3))^n))^3-9008/315*b^3*d^(9/2)*n^3*arctan(x^(1/3)*e^(1/2)/d^(
1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/e^(9/2)-4504/315*I*b^3*d^(9
/2)*n^3*arctan(x^(1/3)*e^(1/2)/d^(1/2))^2/e^(9/2)+2/3*b*d^5*n*Unintegrable(
(a+b*ln(c*(d+e*x^(2/3))^n))^2/(d+e*x^(2/3))/x^(2/3),x)/e^4
```

Rubi [N/A]

Not integrable

Time = 1.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3dx=\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3dx$$

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (4504*a*b^2*d^4*n^2*x^(1/3))/(315*e^4) - (3475504*b^3*d^4*n^3*x^(1/3))/(992
25*e^4) + (637984*b^3*d^3*n^3*x)/(297675*e^3) - (221344*b^3*d^2*n^3*x^(5/3)
)/(496125*e^2) + (3088*b^3*d*n^3*x^(7/3))/(27783*e) - (16*b^3*n^3*x^3)/729
+ (3475504*b^3*d^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(99225*e^(9/2
)) - (((4504*I)/315)*b^3*d^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/e
^(9/2) - (9008*b^3*d^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqr
t[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/(315*e^(9/2)) + (4504*b^3*d^4*n^2*x^(
1/3)*Log[c*(d + e*x^(2/3))^n])/(315*e^4) - (1984*b^2*d^3*n^2*x*(a + b*Log[c
*(d + e*x^(2/3))^n]))/(945*e^3) + (1144*b^2*d^2*n^2*x^(5/3)*(a + b*Log[c*(d
 + e*x^(2/3))^n]))/(1575*e^2) - (128*b^2*d*n^2*x^(7/3)*(a + b*Log[c*(d + e*
x^(2/3))^n]))/(441*e) + (8*b^2*n^2*x^3*(a + b*Log[c*(d + e*x^(2/3))^n]))/81
 - (4504*b^2*d^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d
+ e*x^(2/3))^n]))/(315*e^(9/2)) - (2*b*d^4*n*x^(1/3)*(a + b*Log[c*(d + e*x^
(2/3))^n])^2)/e^4 + (2*b*d^3*n*x*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(3*e^3
) - (2*b*d^2*n*x^(5/3)*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(5*e^2) + (2*b*d
*n*x^(7/3)*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(7*e) - (2*b*n*x^3*(a + b*Lo
g[c*(d + e*x^(2/3))^n])^2)/9 + (x^3*(a + b*Log[c*(d + e*x^(2/3))^n])^3)/3 -
 (((4504*I)/315)*b^3*d^(9/2)*n^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sq
rt[e]*x^(1/3))])/e^(9/2) + (2*b*d^5*n*Defer[Subst][Defer[Int][(a + b*Log[c*
(d + e*x^2)^n])^2/(d + e*x^2), x], x, x^(1/3)])/e^4
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^3dx,x,\sqrt[3]{x}\right)$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3-(2ben)\text{Subst}\left(\int\frac{x^{10}\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}dx,x,\sqrt[3]{x}\right)$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 -(2ben)\text{Subst}\left(\int\left(\frac{d^4\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{e^5}-\frac{d^3x^2\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{e^4}+\frac{d^2x^4\left(a+b\log\right.}{}\right.\right.$$

$$=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 -(2bn)\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2dx,x,\sqrt[3]{x}\right) -\frac{\left(2bd^4n\right)\text{Subst}\left(\int\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2dx,x,\sqrt[3]{x}\right)}{e^4} +\frac{\left(2bd^5n\right)\text{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}dx,x,\sqrt[3]{x}\right)}{e^4} +\frac{\left(2bd^3n\right)\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2dx,x,\sqrt[3]{x}\right)}{e^3} -\frac{\left(2bd^2n\right)\text{Subst}\left(\int x^4\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2dx,x,\sqrt[3]{x}\right)}{e^2} +\frac{(2bdn)\text{Subst}\left(\int x^6\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2dx,x,\sqrt[3]{x}\right)}{e}$$

$$=-\frac{2bd^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e^4}+\frac{2bd^3nx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3e^3} -\frac{2bd^2nx^{5/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5e^2}+\frac{2bdnx^{7/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{7e} -\frac{2}{9}bnx^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3+\frac{\left(2bd^5n\right)\text{Subst}\left(\int\frac{\left(a+b\log\left(c\left(\right.\right.\right.}{d+e}\right.}{e^4}$$

$$=-\frac{2bd^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e^4}+\frac{2bd^3nx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3e^3} -\frac{2bd^2nx^{5/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5e^2}+\frac{2bdnx^{7/3}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{7e} -\frac{2}{9}bnx^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3+\frac{\left(2bd^5n\right)\text{Subst}\left(\int\frac{\left(a+b\log\left(c\left(\right.\right.\right.}{d+e}\right.}{e^4}$$

$=$ Too large to display

Mathematica [A] (verified)

Time = 7.80 (sec) , antiderivative size = 1552, normalized size of antiderivative = 64.67

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=$$
$$-\frac{2bd^4n\sqrt[3]{x}\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e^4}$$
$$+\frac{2bd^3nx\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3e^3}$$
$$-\frac{2bd^2nx^{5/3}\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5e^2}$$
$$+\frac{2bdnx^{7/3}\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{7e}$$
$$+\frac{2bd^{9/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e^{9/2}}$$

$$+bnx^3\log\left(d+ex^{2/3}\right)\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2+\frac{1}{9}x^3\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d\right.\right.\right.$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (-2*b*d^4*n*x^(1/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n
])^2)/e^4 + (2*b*d^3*n*x*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/
3))^n])^2)/(3*e^3) - (2*b*d^2*n*x^(5/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log
[c*(d + e*x^(2/3))^n])^2)/(5*e^2) + (2*b*d*n*x^(7/3)*(a - b*n*Log[d + e*x^(
2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2)/(7*e) + (2*b*d^(9/2)*n*ArcTan[(Sqrt[
e]*x^(1/3))/Sqrt[d]]*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^
n])^2)/e^(9/2) + b*n*x^3*Log[d + e*x^(2/3)]*(a - b*n*Log[d + e*x^(2/3)] + b
*Log[c*(d + e*x^(2/3))^n])^2 + (x^3*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(
d + e*x^(2/3))^n])^2*(3*a - 2*b*n - 3*b*n*Log[d + e*x^(2/3)] + 3*b*Log[c*(d
 + e*x^(2/3))^n]))/9 - (b^3*n^3*(1094783760*d^(9/2)*Sqrt[d + e*x^(2/3)]*Sqr
t[(e*x^(2/3))/(d + e*x^(2/3))]*ArcSin[Sqrt[d]/Sqrt[d + e*x^(2/3)]] - e*x^(2
/3)*(-16*(68423985*d^4 - 4186770*d^3*e*x^(2/3) + 871542*d^2*e^2*x^(4/3) - 2
17125*d*e^3*x^2 + 42875*e^4*x^(8/3)) + 2520*(177345*d^4 - 26040*d^3*e*x^(2/
3) + 9009*d^2*e^2*x^(4/3) - 3600*d*e^3*x^2 + 1225*e^4*x^(8/3))*Log[d + e*x^
(2/3)] - 198450*(315*d^4 - 105*d^3*e*x^(2/3) + 63*d^2*e^2*x^(4/3) - 45*d*e^
3*x^2 + 35*e^4*x^(8/3))*Log[d + e*x^(2/3)]^2 + 10418625*e^4*x^(8/3)*Log[d +
 e*x^(2/3)]^3) + 62511750*d^(9/2)*Sqrt[(e*x^(2/3))/(d + e*x^(2/3))]*(8*Sqrt
[d]*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, d/(d + e*x^(2/
3))] + Log[d + e*x^(2/3)]*(4*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/
```

```
2, 3/2}, d/(d + e*x^(2/3))] + Sqrt[d + e*x^(2/3)]*ArcSin[Sqrt[d]/Sqrt[d + e
*x^(2/3)]]*Log[d + e*x^(2/3)])) + 111727350*(-d)^(9/2)*(4*Sqrt[e*x^(2/3)]*A
rcTanh[Sqrt[e*x^(2/3)]/Sqrt[-d]]*(Log[d + e*x^(2/3)] - Log[1 + (e*x^(2/3))/
d]) - Sqrt[-d]*Sqrt[-((e*x^(2/3))/d)]*(2*Log[(1 + Sqrt[-((e*x^(2/3))/d)])/2
]^2 - 4*Log[(1 + Sqrt[-((e*x^(2/3))/d)])/2]*Log[1 + (e*x^(2/3))/d] + Log[1
+ (e*x^(2/3))/d]^2 - 4*PolyLog[2, 1/2 - Sqrt[-((e*x^(2/3))/d)]/2]))))/(3125
5875*e^5*x^(1/3)) + (b^2*n^2*x^(1/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*
(d + e*x^(2/3))^n])*((1418760*d^(9/2)*ArcSin[Sqrt[d]/Sqrt[d + e*x^(2/3)]])/
(Sqrt[d + e*x^(2/3)]*Sqrt[(e*x^(2/3))/(d + e*x^(2/3))]) + 1225*(d + e*x^(2/
3))^4*(8 - 36*Log[d + e*x^(2/3)] + 81*Log[d + e*x^(2/3)]^2) - 100*d*(d + e*
x^(2/3))^3*(680 - 2331*Log[d + e*x^(2/3)] + 3969*Log[d + e*x^(2/3)]^2) + d^
4*(1737752 - 709380*Log[d + e*x^(2/3)] + 99225*Log[d + e*x^(2/3)]^2) - 4*d^
3*(d + e*x^(2/3))*(119516 - 159390*Log[d + e*x^(2/3)] + 99225*Log[d + e*x^(
2/3)]^2) + 6*d^2*(d + e*x^(2/3))^2*(36212 - 85680*Log[d + e*x^(2/3)] + 9922
5*Log[d + e*x^(2/3)]^2) + (396900*(-d)^(9/2)*ArcTanh[Sqrt[e*x^(2/3)]/Sqrt[-
d]]*(Log[d + e*x^(2/3)] - Log[1 + (e*x^(2/3))/d]))/Sqrt[e*x^(2/3)] - (99225
*d^4*(2*Log[(1 + Sqrt[-((e*x^(2/3))/d)])/2]^2 - 4*Log[(1 + Sqrt[-((e*x^(2/3
))/d)])/2]*Log[1 + (e*x^(2/3))/d] + Log[1 + (e*x^(2/3))/d]^2 - 4*PolyLog[2,
 1/2 - Sqrt[-((e*x^(2/3))/d)]/2]))/Sqrt[-((e*x^(2/3))/d)]))/(99225*e^4)
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^{2}\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{3}dx$$

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
[Out] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.12

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+a\right)^3x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="fricas")
[Out] integral(b^3*x^2*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*x^2*log((e*x^(2/3) +
d)^n*c)^2 + 3*a^2*b*x^2*log((e*x^(2/3) + d)^n*c) + a^3*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))**n))**3,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+a\right)^3x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\int x^2\,\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)^3\,dx$$

```
[In] int(x^2*(a + b*log(c*(d + e*x^(2/3))^n))^3,x)
[Out] int(x^2*(a + b*log(c*(d + e*x^(2/3))^n))^3, x)
```

3.486 $\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx = -\frac{32ab^2dn^2\sqrt[3]{x}}{e}+\frac{208b^3dn^3\sqrt[3]{x}}{3e}-\frac{16}{9}b^3n^3x-\frac{208b^3d^{3/2}n^3\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{32ib^3d^{3/2}n^3\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{e^{3/2}}+\frac{64b^3d^{3/2}n^3\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{e^{3/2}}-\frac{32b^3dn^2\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e}+\frac{8}{3}b^2n^2x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{32b^2d^{3/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{3/2}}+\frac{6bdn\sqrt[3]{x}(a+$$

```
[Out] -32*a*b^2*d*n^2*x^(1/3)/e+208/3*b^3*d*n^3*x^(1/3)/e-16/9*b^3*n^3*x-208/3*b^
3*d^(3/2)*n^3*arctan(x^(1/3)*e^(1/2)/d^(1/2))/e^(3/2)+32*I*b^3*d^(3/2)*n^3*
arctan(x^(1/3)*e^(1/2)/d^(1/2))^2/e^(3/2)-32*b^3*d*n^2*x^(1/3)*ln(c*(d+e*x^
(2/3))^n)/e+8/3*b^2*n^2*x*(a+b*ln(c*(d+e*x^(2/3))^n))+32*b^2*d^(3/2)*n^2*ar
ctan(x^(1/3)*e^(1/2)/d^(1/2))*(a+b*ln(c*(d+e*x^(2/3))^n))/e^(3/2)+6*b*d*n*x
^(1/3)*(a+b*ln(c*(d+e*x^(2/3))^n))^2/e-2*b*n*x*(a+b*ln(c*(d+e*x^(2/3))^n))^
2+x*(a+b*ln(c*(d+e*x^(2/3))^n))^3+64*b^3*d^(3/2)*n^3*arctan(x^(1/3)*e^(1/2)
/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/e^(3/2)+32*I*b^3*d^(3/2
)*n^3*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/e^(3/2)-2*b*d^2*n*
Unintegrable((a+b*ln(c*(d+e*x^(2/3))^n))^2/(d+e*x^(2/3))/x^(2/3),x)/e
```

Rubi [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (-32*a*b^2*d*n^2*x^(1/3))/e + (208*b^3*d*n^3*x^(1/3))/(3*e) - (16*b^3*n^3*x
)/9 - (208*b^3*d^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(3*e^(3/2)) +
 ((32*I)*b^3*d^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/e^(3/2) + (64
*b^3*d^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d]
 + I*Sqrt[e]*x^(1/3))])/e^(3/2) - (32*b^3*d*n^2*x^(1/3)*Log[c*(d + e*x^(2/3
))^n])/e + (8*b^2*n^2*x*(a + b*Log[c*(d + e*x^(2/3))^n]))/3 + (32*b^2*d^(3/
2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/
e^(3/2) + (6*b*d*n*x^(1/3)*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/e - 2*b*n*x*
(a + b*Log[c*(d + e*x^(2/3))^n])^2 + x*(a + b*Log[c*(d + e*x^(2/3))^n])^3 +
 ((32*I)*b^3*d^(3/2)*n^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^
(1/3))])/e^(3/2) - (6*b*d^2*n*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x
^2)^n])^2/(d + e*x^2), x], x, x^(1/3)])/e
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^3\,dx,x,\sqrt[3]{x}\right)$$

$$=x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3-(6ben)\text{Subst}\left(\int \frac{x^4\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)$$

$$=x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3-(6ben)\text{Subst}\left(\int\left(-\frac{d\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{e^2}+\frac{x^2\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{e}+\frac{d^2\left(a+b\log\left(\right.\right.}{e^2\left(d\right.}\right.\right.$$

$$=x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3-(6bn)\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2\,dx,x,\sqrt[3]{x}\right)+\frac{(6bdn)\text{Subst}\left(\int\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2\,dx,x,\sqrt[3]{x}\right)}{e}-\frac{\left(6bd^2n\right)\text{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{e}$$

$$= \frac{6bdn\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e} - 2bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 + x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 - \frac{(6bd^2n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)}{e} - \left(24b^2dn^2\right)\,\text{Subst}$$

$$= \frac{6bdn\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e} - 2bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 + x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 - \frac{(6bd^2n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)}{e} - \left(24b^2dn^2\right)\,\text{Subst}$$

$$= \frac{6bdn\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e} - 2bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 + x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 - \frac{(6bd^2n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)}{e} + \left(8b^2n^2\right)\,\text{Subst}\Bigg($$

$$= -\frac{32ab^2dn^2\sqrt[3]{x}}{e} + \frac{8}{3}b^2n^2x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) + \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{3/2}} + \frac{6bdn\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e} - 2bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 + x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 - \frac{(6bd^2n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}\right.}{e}$$

$$= -\frac{32ab^2dn^2\sqrt[3]{x}}{e} - \frac{32b^3dn^2\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e} + \frac{8}{3}b^2n^2x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right) + \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{3/2}} + \frac{6bdn\sqrt[3]{x}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e} - 2bnx\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2 + x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 - \frac{(6bd^2n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}\right.}{e}$$

$$=-\frac{32ab^2dn^2\sqrt[3]{x}}{e}+\frac{208b^3dn^3\sqrt[3]{x}}{3e}-\frac{16}{9}b^3n^3x$$
$$+\frac{32ib^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{e^{3/2}}-\frac{32b^3dn^2\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e}$$
$$+\frac{8}{3}b^2n^2x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{3/2}}+\frac{6b}{}$$

$$=-\frac{32ab^2dn^2\sqrt[3]{x}}{e}+\frac{208b^3dn^3\sqrt[3]{x}}{3e}-\frac{16}{9}b^3n^3x$$
$$-\frac{208b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{32ib^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{e^{3/2}}$$
$$+\frac{64b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{e^{3/2}}-\frac{32b^3dn^2\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e}$$

$$+\frac{8}{3}b^2n^2x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{3/2}}+\frac{6b}{}$$

$$=-\frac{32ab^2dn^2\sqrt[3]{x}}{e}+\frac{208b^3dn^3\sqrt[3]{x}}{3e}-\frac{16}{9}b^3n^3x$$
$$-\frac{208b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}}+\frac{32ib^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{e^{3/2}}$$
$$+\frac{64b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{e^{3/2}}-\frac{32b^3dn^2\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e}$$

$$+\frac{8}{3}b^2n^2x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{3/2}}+\frac{6b}{}$$

$$= -\frac{32ab^2dn^2\sqrt[3]{x}}{e} + \frac{208b^3dn^3\sqrt[3]{x}}{3e} - \frac{16}{9}b^3n^3x - \frac{208b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{3e^{3/2}} + \frac{32ib^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{e^{3/2}} + \frac{64b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{e^{3/2}} - \frac{32b^3dn^2\sqrt[3]{x}\log\left(c\left(d+ex^{2/3}\right)^n\right)}{e}$$

$$+\frac{8}{3}b^2n^2x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)+\frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{e^{3/2}}+\frac{6bd}{}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1299 vs. 2(486) = 972.

Time = 5.89 (sec) , antiderivative size = 1299, normalized size of antiderivative = 64.95

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx = \frac{6bdn\sqrt[3]{x}\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e} - \frac{6bd^{3/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{e^{3/2}}$$

$$+3bnx\log\left(d+ex^{2/3}\right)\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2+x\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+e\right.\right.\right.$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3,x]
```

```
[Out] (6*b*d*n*x^(1/3)*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^
2)/e - (6*b*d^(3/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - b*n*Log[d + e*
x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2)/e^(3/2) + 3*b*n*x*Log[d + e*x^(2/
3)]*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2 + x*(a - b*
n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2*(a - 2*b*n - b*n*Log[d
 + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n]) + (b^2*n^2*x^(1/3)*(a - b*n*Log
[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])*((-96*d^(3/2)*ArcSin[Sqrt[d]/
Sqrt[d + e*x^(2/3)]])/(Sqrt[d + e*x^(2/3)]*Sqrt[(e*x^(2/3))/(d + e*x^(2/3))
]) - d*(104 - 48*Log[d + e*x^(2/3)] + 9*Log[d + e*x^(2/3)]^2) + (d + e*x^(2
/3))*(8 - 12*Log[d + e*x^(2/3)] + 9*Log[d + e*x^(2/3)]^2) + (36*(-d)^(3/2)*
ArcTanh[Sqrt[e*x^(2/3)]/Sqrt[-d]]*(Log[d + e*x^(2/3)] - Log[1 + (e*x^(2/3))
/d]))/Sqrt[e*x^(2/3)] + (9*d*(2*Log[(1 + Sqrt[-((e*x^(2/3))/d)])/2]^2 - 4*L
og[(1 + Sqrt[-((e*x^(2/3))/d)])/2]*Log[1 + (e*x^(2/3))/d] + Log[1 + (e*x^(2
```

```
/3))/d]^2 - 4*PolyLog[2, 1/2 - Sqrt[-((e*x^(2/3))/d)]/2]))/Sqrt[-((e*x^(2/3
))/d)]))/(3*e) + (b^3*n^3*(624*d*e*x^(2/3) - 16*e^2*x^(4/3) + 624*d^(3/2)*S
qrt[d + e*x^(2/3)]*Sqrt[(e*x^(2/3))/(d + e*x^(2/3))]*ArcSin[Sqrt[d]/Sqrt[d
+ e*x^(2/3)]] + 432*d^2*Sqrt[(e*x^(2/3))/(d + e*x^(2/3))]*HypergeometricPFQ
[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, d/(d + e*x^(2/3))] + 144*d^2*Sqrt[-
((e*x^(2/3))/d)]*Log[(1 + Sqrt[-((e*x^(2/3))/d)])/2]^2 - 288*d*e*x^(2/3)*Lo
g[d + e*x^(2/3)] + 24*e^2*x^(4/3)*Log[d + e*x^(2/3)] + 288*Sqrt[-d]*d*Sqrt[
e*x^(2/3)]*ArcTanh[Sqrt[e*x^(2/3)]/Sqrt[-d]]*Log[d + e*x^(2/3)] + 216*d^2*S
qrt[(e*x^(2/3))/(d + e*x^(2/3))]*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2, 3
/2}, d/(d + e*x^(2/3))]*Log[d + e*x^(2/3)] + 54*d*e*x^(2/3)*Log[d + e*x^(2/
3)]^2 - 18*e^2*x^(4/3)*Log[d + e*x^(2/3)]^2 + 54*d^(3/2)*Sqrt[d + e*x^(2/3)
]*Sqrt[(e*x^(2/3))/(d + e*x^(2/3))]*ArcSin[Sqrt[d]/Sqrt[d + e*x^(2/3)]]*Log
[d + e*x^(2/3)]^2 + 9*e^2*x^(4/3)*Log[d + e*x^(2/3)]^3 + 288*(-d)^(3/2)*Sqr
t[e*x^(2/3)]*ArcTanh[Sqrt[e*x^(2/3)]/Sqrt[-d]]*Log[1 + (e*x^(2/3))/d] - 288
*d^2*Sqrt[-((e*x^(2/3))/d)]*Log[(1 + Sqrt[-((e*x^(2/3))/d)])/2]*Log[1 + (e*
x^(2/3))/d] + 72*d^2*Sqrt[-((e*x^(2/3))/d)]*Log[1 + (e*x^(2/3))/d]^2 - 288*
d^2*Sqrt[-((e*x^(2/3))/d)]*PolyLog[2, 1/2 - Sqrt[-((e*x^(2/3))/d)]/2]))/(9*
e^2*x^(1/3))
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(a+b\ln \left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{3}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 3.10

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\int \left(b\log \left(\left(ex^{\frac{2}{3}}+d\right)^{n}c\right)+a\right)^{3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(2/3) + d)^n*c)^
2 + 3*a^2*b*log((e*x^(2/3) + d)^n*c) + a^3, x)
```

Sympy [N/A]

Not integrable

Time = 61.66 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 \,dx=\int \left(a+b\log\left(c\left(d+ex^{\frac{2}{3}}\right)^n\right)\right)^3 \,dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**(2/3))**n))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 \,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3 \,dx=\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^nc\right)+a\right)^3 \,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3\,dx=\int \left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)\right)^3\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^3,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^3, x)
```

3.487 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$
\begin{aligned}
\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx &= \frac{24ib^3e^{3/2}n^3\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} \\
&+ \frac{48b^3e^{3/2}n^3\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}} \\
&+ \frac{24b^2e^{3/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}} \\
&- \frac{6ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d\sqrt[3]{x}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x} \\
&+ \frac{24ib^3e^{3/2}n^3\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}} - \frac{2be^2n\,\mathrm{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{\left(d+ex^{2/3}\right)x^{2/3}},x\right)}{d}
\end{aligned}
$$

```
[Out] 24*I*b^3*e^(3/2)*n^3*arctan(x^(1/3)*e^(1/2)/d^(1/2))^2/d^(3/2)+24*b^2*e^(3/
2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^(3/2)-
6*b*e*n*(a+b*ln(c*(d+e*x^(2/3))^n))^2/d/x^(1/3)-(a+b*ln(c*(d+e*x^(2/3))^n))
^3/x+48*b^3*e^(3/2)*n^3*arctan(x^(1/3)*e^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/
2)+I*x^(1/3)*e^(1/2)))/d^(3/2)+24*I*b^3*e^(3/2)*n^3*polylog(2,1-2*d^(1/2)/(
d^(1/2)+I*x^(1/3)*e^(1/2)))/d^(3/2)-2*b*e^2*n*Unintegrable((a+b*ln(c*(d+e*x
^(2/3))^n))^2/(d+e*x^(2/3))/x^(2/3),x)/d
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^2,x]
```

```
[Out] ((24*I)*b^3*e^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/d^(3/2) + (48*
b^3*e^(3/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d]
+ I*Sqrt[e]*x^(1/3))])/d^(3/2) + (24*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3
))/Sqrt[d]]*(a + b*Log[c*(d + e*x^(2/3))^n]))/d^(3/2) - (6*b*e*n*(a + b*Log
[c*(d + e*x^(2/3))^n])^2)/(d*x^(1/3)) - (a + b*Log[c*(d + e*x^(2/3))^n])^3/
x + ((24*I)*b^3*e^(3/2)*n^3*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]
*x^(1/3))])/d^(3/2) - (6*b*e^2*n*Defer[Subst][Defer[Int][(a + b*Log[c*(d +
e*x^2)^n])^2/(d + e*x^2), x], x, x^(1/3)])/d
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^3}{x^4}\,dx,x,\sqrt[3]{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}+(6ben)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^2\left(d+ex^2\right)}\,dx,x,\sqrt[3]{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\\
&\quad+(6ben)\text{Subst}\left(\int \left(\frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{dx^2}-\frac{e\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d\left(d+ex^2\right)}\right)dx,x,\sqrt[3]{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}+\frac{(6ben)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^2}\,dx,x,\sqrt[3]{x}\right)}{d}\\
&\quad-\frac{\left(6be^2n\right)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{d}\\
&= -\frac{6ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d\sqrt[3]{x}}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}\\
&\quad-\frac{\left(6be^2n\right)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{d}\\
&\quad+\frac{\left(24b^2e^2n^2\right)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{d}
\end{aligned}$$

$$=\frac{24b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}}-\frac{6ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d\sqrt[3]{x}}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}-\frac{(6be^2n)\operatorname{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}dx,x,\sqrt[3]{x}\right)}{d}-\frac{(48b^3e^3n^3)\operatorname{Subst}\left(\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}(d+ex^2)}dx,x,\sqrt[3]{x}\right)}{d}$$

$$=\frac{24b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}}-\frac{6ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d\sqrt[3]{x}}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}-\frac{(6be^2n)\operatorname{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}dx,x,\sqrt[3]{x}\right)}{d}-\frac{(48b^3e^{5/2}n^3)\operatorname{Subst}\left(\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d+ex^2}dx,x,\sqrt[3]{x}\right)}{d^{3/2}}$$

$$=\frac{24ib^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}}+\frac{24b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}}-\frac{6ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d\sqrt[3]{x}}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x}-\frac{(6be^2n)\operatorname{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}dx,x,\sqrt[3]{x}\right)}{d}+\frac{(48b^3e^2n^3)\operatorname{Subst}\left(\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{i-\frac{\sqrt{e}x}{\sqrt{d}}}dx,x,\sqrt[3]{x}\right)}{d^2}$$

$$= \frac{24ib^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} + \frac{48b^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}} + \frac{24b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}} - \frac{6ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d\sqrt[3]{x}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x} - \frac{(6be^2n)\,\text{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)}{d} - \frac{(48b^3e^2n^3)\,\text{Subst}\left(\int\frac{\log\left(\frac{2}{1+\frac{i\sqrt{e}x}{\sqrt{d}}}\right)}{1+\frac{ex^2}{d}}\,dx, x, \sqrt[3]{x}\right)}{d^2}$$

$$= \frac{24ib^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} + \frac{48b^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}} + \frac{24b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{3/2}} - \frac{6ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d\sqrt[3]{x}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x} - \frac{(6be^2n)\,\text{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)}{d} + \frac{(48ib^3e^{3/2}n^3)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+\frac{i\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}}\right)}{d^{3/2}}$$

$$= \frac{24ib^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{3/2}} + \frac{48b^3e^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{3/2}}$$
$$+ \frac{24b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c(d+ex^{2/3})^n\right)\right)}{d^{3/2}}$$
$$- \frac{6ben\left(a+b\log\left(c(d+ex^{2/3})^n\right)\right)^2}{d\sqrt[3]{x}}$$
$$- \frac{\left(a+b\log\left(c(d+ex^{2/3})^n\right)\right)^3}{x} + \frac{24ib^3e^{3/2}n^3\mathrm{Li}_2\left(1-\frac{2}{1+\frac{i\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}}\right)}{d^{3/2}}$$
$$- \frac{(6be^2n)\,\mathrm{Subst}\left(\int\frac{(a+b\log(c(d+ex^2)^n))^2}{d+ex^2}dx, x, \sqrt[3]{x}\right)}{d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1158 vs. $2(319) = 638$.

Time = 6.63 (sec) , antiderivative size = 1158, normalized size of antiderivative = 48.25

$$\int\frac{\left(a+b\log\left(c(d+ex^{2/3})^n\right)\right)^3}{x^2}dx = -\frac{6ben\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c(d+ex^{2/3})^n\right)\right)^2}{d\sqrt[3]{x}}$$
$$- \frac{6be^{3/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c(d+ex^{2/3})^n\right)\right)^2}{d^{3/2}}$$
$$- \frac{3bn\log\left(d+ex^{2/3}\right)\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c(d+ex^{2/3})^n\right)\right)^2}{x}$$
$$- \frac{\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c(d+ex^{2/3})^n\right)\right)^3}{x}$$
$$+ \frac{3b^2en^2\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c(d+ex^{2/3})^n\right)\right)\left(-\frac{16\sqrt{d+ex^{2/3}}\sqrt{\frac{ex^{2/3}}{d+ex^{2/3}}}\arcsin\left(\frac{\sqrt{d}}{\sqrt{d+ex^{2/3}}}\right)}{d^{3/2}} - \frac{8\log(d+ex^{2/3})}{d}\right.}{}$$
$$+ \frac{b^3n^3\left(48\sqrt{-d^2}e\sqrt{\frac{ex^{2/3}}{d+ex^{2/3}}}x^{2/3}\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2},\frac{3}{2};\frac{d}{d+ex^{2/3}}\right) - 12d\sqrt{-d^2}\left(-\frac{ex^{2/3}}{d}\right)^{3/2}\log^2\left(\frac{1}{2}\left(1+\sqrt{-\frac{ex^{2/3}}{d}}\right.\right.\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^2,x]
```

```
[Out] (-6*b*e*n*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2)/(d*x
^(1/3)) - (6*b*e^(3/2)*n*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - b*n*Log[d +
```

```
e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2)/d^(3/2) - (3*b*n*Log[d + e*x^(
2/3)]*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2)/x - (a -
 b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^3/x + (3*b^2*e*n^2*(a
 - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])*((-16*Sqrt[d + e*x^
(2/3)]*Sqrt[(e*x^(2/3))/(d + e*x^(2/3))]*ArcSin[Sqrt[d]/Sqrt[d + e*x^(2/3)]
])/d^(3/2) - (8*Log[d + e*x^(2/3)])/d - (2*Log[d + e*x^(2/3)]^2)/(e*x^(2/3)
) - (8*Sqrt[e*x^(2/3)]*ArcTanh[Sqrt[e*x^(2/3)]/Sqrt[-d]]*(Log[d + e*x^(2/3)
] - Log[1 + (e*x^(2/3))/d]))/(-d)^(3/2) - (2*Sqrt[-((e*x^(2/3))/d)]*(2*Log[
(1 + Sqrt[-((e*x^(2/3))/d)])/2]^2 - 4*Log[(1 + Sqrt[-((e*x^(2/3))/d)])/2]*L
og[1 + (e*x^(2/3))/d] + Log[1 + (e*x^(2/3))/d]^2 - 4*PolyLog[2, 1/2 - Sqrt[
-((e*x^(2/3))/d)]/2]))/d))/(2*x^(1/3)) + (b^3*n^3*(48*Sqrt[-d^2]*e*Sqrt[(e*
x^(2/3))/(d + e*x^(2/3))]*x^(2/3)*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {
3/2, 3/2, 3/2}, d/(d + e*x^(2/3))] - 12*d*Sqrt[-d^2]*(-((e*x^(2/3))/d))^(3/
2)*Log[(1 + Sqrt[-((e*x^(2/3))/d)])/2]^2 - 24*Sqrt[d]*(e*x^(2/3))^(3/2)*Arc
Tanh[Sqrt[e*x^(2/3)]/Sqrt[-d]]*Log[d + e*x^(2/3)] + 24*Sqrt[-d^2]*e*Sqrt[(e
*x^(2/3))/(d + e*x^(2/3))]*x^(2/3)*HypergeometricPFQ[{1/2, 1/2, 1/2}, {3/2,
 3/2}, d/(d + e*x^(2/3))]*Log[d + e*x^(2/3)] - 6*Sqrt[-d^2]*e*x^(2/3)*Log[d
 + e*x^(2/3)]^2 + 6*Sqrt[-d]*(d + e*x^(2/3))^(3/2)*((e*x^(2/3))/(d + e*x^(2
/3)))^(3/2)*ArcSin[Sqrt[d]/Sqrt[d + e*x^(2/3)]]*Log[d + e*x^(2/3)]^2 + (d^(
5/2)*Log[d + e*x^(2/3)]^3)/Sqrt[-d] + 24*Sqrt[d]*(e*x^(2/3))^(3/2)*ArcTanh[
Sqrt[e*x^(2/3)]/Sqrt[-d]]*Log[1 + (e*x^(2/3))/d] + 24*d*Sqrt[-d^2]*(-((e*x^
(2/3))/d))^(3/2)*Log[(1 + Sqrt[-((e*x^(2/3))/d)])/2]*Log[1 + (e*x^(2/3))/d]
 - 6*d*Sqrt[-d^2]*(-((e*x^(2/3))/d))^(3/2)*Log[1 + (e*x^(2/3))/d]^2 + 24*d*
Sqrt[-d^2]*(-((e*x^(2/3))/d))^(3/2)*PolyLog[2, 1/2 - Sqrt[-((e*x^(2/3))/d)]
/2]))/(Sqrt[-d]*d^(3/2)*x)
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{3}}{x^{2}}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^3}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(2/3) + d)^n*c)
^2 + 3*a^2*b*log((e*x^(2/3) + d)^n*c) + a^3)/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3/x**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^3}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^2,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^2}\,dx = \int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^n\right)\right)^3}{x^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^2, x)
```

3.488 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx = -\frac{16b^3e^3n^3}{105d^3x}+\frac{16b^3e^4n^3}{7d^4\sqrt[3]{x}}+\frac{1376b^3e^{9/2}n^3\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}}-\frac{1408ib^3e^{9/2}n^3\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{105d^{9/2}}-\frac{2816b^3e^{9/2}n^3\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{105d^{9/2}}-\frac{8b^2e^2n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^2x^{5/3}}+\frac{32b^2e^3n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^3x}-\frac{568b^2e^4n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{105d^4\sqrt[3]{x}}-\frac{1408b^2e^{9/2}n^2\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{105d^{9/2}}-\frac{2ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{7dx^{7/3}}+\frac{2be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5d^2x^{5/3}}-\frac{2be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3d^3x}+\frac{2be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d^4\sqrt[3]{x}}-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^3}-\frac{1408ib^3e^{9/2}n^3\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{105d^{9/2}}+\frac{2be^5n\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{\left(d+ex^{2/3}\right)x^{2/3}},x\right)}{3d^4}$$

```
[Out] -16/105*b^3*e^3*n^3/d^3/x+16/7*b^3*e^4*n^3/d^4/x^(1/3)+1376/105*b^3*e^(9/2)
*n^3*arctan(x^(1/3)*e^(1/2)/d^(1/2))/d^(9/2)-1408/105*I*b^3*e^(9/2)*n^3*arc
tan(x^(1/3)*e^(1/2)/d^(1/2))^2/d^(9/2)-8/35*b^2*e^2*n^2*(a+b*ln(c*(d+e*x^(2
/3))^n))/d^2/x^(5/3)+32/35*b^2*e^3*n^2*(a+b*ln(c*(d+e*x^(2/3))^n))/d^3/x-56
8/105*b^2*e^4*n^2*(a+b*ln(c*(d+e*x^(2/3))^n))/d^4/x^(1/3)-1408/105*b^2*e^(9
/2)*n^2*arctan(x^(1/3)*e^(1/2)/d^(1/2))*(a+b*ln(c*(d+e*x^(2/3))^n))/d^(9/2)
-2/7*b*e*n*(a+b*ln(c*(d+e*x^(2/3))^n))^2/d/x^(7/3)+2/5*b*e^2*n*(a+b*ln(c*(d
+e*x^(2/3))^n))^2/d^2/x^(5/3)-2/3*b*e^3*n*(a+b*ln(c*(d+e*x^(2/3))^n))^2/d^3
/x+2*b*e^4*n*(a+b*ln(c*(d+e*x^(2/3))^n))^2/d^4/x^(1/3)-1/3*(a+b*ln(c*(d+e*x
^(2/3))^n))^3/x^3-2816/105*b^3*e^(9/2)*n^3*arctan(x^(1/3)*e^(1/2)/d^(1/2))*
ln(2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/d^(9/2)-1408/105*I*b^3*e^(9/2)*n^
3*polylog(2,1-2*d^(1/2)/(d^(1/2)+I*x^(1/3)*e^(1/2)))/d^(9/2)+2/3*b*e^5*n*Un
integrable((a+b*ln(c*(d+e*x^(2/3))^n))^2/(d+e*x^(2/3))/x^(2/3),x)/d^4
```

Rubi [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^4,x]
```

```
[Out] (-16*b^3*e^3*n^3)/(105*d^3*x) + (16*b^3*e^4*n^3)/(7*d^4*x^(1/3)) + (1376*b^
3*e^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]])/(105*d^(9/2)) - (((1408*I)
/105)*b^3*e^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]^2)/d^(9/2) - (2816*
b^3*e^(9/2)*n^3*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d]
+ I*Sqrt[e]*x^(1/3))])/(105*d^(9/2)) - (8*b^2*e^2*n^2*(a + b*Log[c*(d + e*x
^(2/3))^n]))/(35*d^2*x^(5/3)) + (32*b^2*e^3*n^2*(a + b*Log[c*(d + e*x^(2/3)
)^n]))/(35*d^3*x) - (568*b^2*e^4*n^2*(a + b*Log[c*(d + e*x^(2/3))^n]))/(105
*d^4*x^(1/3)) - (1408*b^2*e^(9/2)*n^2*ArcTan[(Sqrt[e]*x^(1/3))/Sqrt[d]]*(a
+ b*Log[c*(d + e*x^(2/3))^n]))/(105*d^(9/2)) - (2*b*e*n*(a + b*Log[c*(d + e
*x^(2/3))^n])^2)/(7*d*x^(7/3)) + (2*b*e^2*n*(a + b*Log[c*(d + e*x^(2/3))^n]
)^2)/(5*d^2*x^(5/3)) - (2*b*e^3*n*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(3*d^
3*x) + (2*b*e^4*n*(a + b*Log[c*(d + e*x^(2/3))^n])^2)/(d^4*x^(1/3)) - (a +
b*Log[c*(d + e*x^(2/3))^n])^3/(3*x^3) - (((1408*I)/105)*b^3*e^(9/2)*n^3*Pol
yLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] + I*Sqrt[e]*x^(1/3))])/d^(9/2) + (2*b*e^5*
n*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)^n])^2/(d + e*x^2), x], x
, x^(1/3)])/d^4
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^3}{x^{10}}\,dx,x,\sqrt[3]{x}\right)$$

$$=-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^3}+(2ben)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^8\left(d+ex^2\right)}\,dx,x,\sqrt[3]{x}\right)$$

$$=-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^3}$$

$$+(2ben)\text{Subst}\left(\int \left(\frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{dx^8}-\frac{e\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d^2x^6}+\frac{e^2\left(a+b\log\left(c\left(d+\right.\right.\right.}{d^3x^4}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^3} + \frac{(2ben)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^8}\,dx, x, \sqrt[3]{x}\right)}{d}$$
$$- \frac{\left(2be^2n\right)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^6}\,dx, x, \sqrt[3]{x}\right)}{d^2}$$
$$+ \frac{\left(2be^3n\right)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^4}\,dx, x, \sqrt[3]{x}\right)}{d^3}$$
$$- \frac{\left(2be^4n\right)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{x^2}\,dx, x, \sqrt[3]{x}\right)}{d^4}$$
$$+ \frac{\left(2be^5n\right)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)}{d^4}$$

$$= -\frac{2ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{7dx^{7/3}} + \frac{2be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5d^2x^{5/3}}$$
$$- \frac{2be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3d^3x} + \frac{2be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d^4\sqrt[3]{x}}$$
$$- \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^3} + \frac{\left(2be^5n\right)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)}{d^4}$$
$$+ \frac{\left(8b^2e^2n^2\right)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^6\left(d+ex^2\right)}\,dx, x, \sqrt[3]{x}\right)}{7d}$$
$$- \frac{\left(8b^2e^3n^2\right)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^4\left(d+ex^2\right)}\,dx, x, \sqrt[3]{x}\right)}{5d^2}$$
$$+ \frac{\left(8b^2e^4n^2\right)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^2\left(d+ex^2\right)}\,dx, x, \sqrt[3]{x}\right)}{3d^3}$$
$$- \frac{\left(8b^2e^5n^2\right)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)}{d^4}$$

$$= -\frac{8b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{9/2}}$$

$$-\frac{2ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{7dx^{7/3}}+\frac{2be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5d^2x^{5/3}}$$

$$-\frac{2be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3d^3x}+\frac{2be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d^4\sqrt[3]{x}}$$

$$-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^3}+\frac{(2be^5n)\,\mathrm{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{d^4}$$

$$+\frac{(8b^2e^2n^2)\,\mathrm{Subst}\left(\int\left(\frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{dx^6}-\frac{e\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)}{d^2x^4}+\frac{e^2\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)}{d^3x^2}-\frac{e^3(a+b\log(c(d+ex^2}{d^3(d+ex^2)}\right.\right.}{7d}$$

$$-\frac{(8b^2e^3n^2)\,\mathrm{Subst}\left(\int\left(\frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{dx^4}-\frac{e\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)}{d^2x^2}+\frac{e^2\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)}{d^2(d+ex^2)}\right)dx,x,\sqrt[3]{x}\right)}{5d^2}$$

$$+\frac{(8b^2e^4n^2)\,\mathrm{Subst}\left(\int\left(\frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{dx^2}-\frac{e\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)}{d(d+ex^2)}\right)dx,x,\sqrt[3]{x}\right)}{3d^3}$$

$$+\frac{(16b^3e^6n^3)\,\mathrm{Subst}\left(\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{d^4}$$

$$= -\frac{8b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{d^{9/2}}$$
$$-\frac{2ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{7dx^{7/3}}+\frac{2be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5d^2x^{5/3}}$$
$$-\frac{2be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3d^3x}+\frac{2be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d^4\sqrt[3]{x}}$$
$$-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^3}+\frac{(2be^5n)\,\mathrm{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{d^4}$$
$$+\frac{(8b^2e^2n^2)\,\mathrm{Subst}\left(\int\frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^6}\,dx,x,\sqrt[3]{x}\right)}{7d^2}$$
$$-\frac{(8b^2e^3n^2)\,\mathrm{Subst}\left(\int\frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^4}\,dx,x,\sqrt[3]{x}\right)}{7d^3}$$
$$-\frac{(8b^2e^3n^2)\,\mathrm{Subst}\left(\int\frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^4}\,dx,x,\sqrt[3]{x}\right)}{5d^3}$$
$$+\frac{(8b^2e^4n^2)\,\mathrm{Subst}\left(\int\frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^2}\,dx,x,\sqrt[3]{x}\right)}{7d^4}$$
$$+\frac{(8b^2e^4n^2)\,\mathrm{Subst}\left(\int\frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^2}\,dx,x,\sqrt[3]{x}\right)}{5d^4}$$
$$+\frac{(8b^2e^4n^2)\,\mathrm{Subst}\left(\int\frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{x^2}\,dx,x,\sqrt[3]{x}\right)}{3d^4}$$
$$-\frac{(8b^2e^5n^2)\,\mathrm{Subst}\left(\int\frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{7d^4}$$
$$-\frac{(8b^2e^5n^2)\,\mathrm{Subst}\left(\int\frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{5d^4}$$
$$-\frac{(8b^2e^5n^2)\,\mathrm{Subst}\left(\int\frac{a+b\log\left(c\left(d+ex^2\right)^n\right)}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{3d^4}$$
$$+\frac{(16b^3e^{11/2}n^3)\,\mathrm{Subst}\left(\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{d^{9/2}}$$

$$= -\frac{8ib^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{9/2}} - \frac{8b^2e^2n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^2x^{5/3}} + \frac{32b^2e^3n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^3x} - \frac{568b^2e^4n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{105d^4\sqrt[3]{x}} - \frac{1408b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{105d^{9/2}} - \frac{2ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{7dx^{7/3}} + \frac{2be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5d^2x^{5/3}} - \frac{2be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3d^3x} + \frac{2be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d^4\sqrt[3]{x}} - \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^3} + \frac{(2be^5n)\,\mathrm{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{d^4} + \frac{(16b^3e^3n^3)\,\mathrm{Subst}\left(\int\frac{1}{x^4(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{35d^2} - \frac{(16b^3e^4n^3)\,\mathrm{Subst}\left(\int\frac{1}{x^2(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{21d^3} - \frac{(16b^3e^4n^3)\,\mathrm{Subst}\left(\int\frac{1}{x^2(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{15d^3} - \frac{(16b^3e^5n^3)\,\mathrm{Subst}\left(\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{i-\frac{\sqrt{e}x}{\sqrt{d}}}\,dx,x,\sqrt[3]{x}\right)}{d^5} + \frac{(16b^3e^5n^3)\,\mathrm{Subst}\left(\int\frac{1}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{7d^4} + \frac{(16b^3e^5n^3)\,\mathrm{Subst}\left(\int\frac{1}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{5d^4} + \frac{(16b^3e^5n^3)\,\mathrm{Subst}\left(\int\frac{1}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{3d^4} + \frac{(16b^3e^6n^3)\,\mathrm{Subst}\left(\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{7d^4} + \frac{(16b^3e^6n^3)\,\mathrm{Subst}\left(\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{5d^4} + \frac{(16b^3e^6n^3)\,\mathrm{Subst}\left(\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{3d^4}$$

$$= -\frac{16b^3e^3n^3}{105d^3x} + \frac{64b^3e^4n^3}{35d^4\sqrt[3]{x}} + \frac{1136b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}}$$
$$-\frac{8ib^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{d^{9/2}} - \frac{16b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{9/2}}$$
$$-\frac{8b^2e^2n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^2x^{5/3}} + \frac{32b^2e^3n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^3x}$$
$$-\frac{568b^2e^4n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{105d^4\sqrt[3]{x}}$$
$$-\frac{1408b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{105d^{9/2}}$$
$$-\frac{2ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{7dx^{7/3}} + \frac{2be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5d^2x^{5/3}}$$
$$-\frac{2be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3d^3x} + \frac{2be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d^4\sqrt[3]{x}}$$
$$-\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^3} + \frac{(2be^5n)\,\mathrm{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{d^4}$$
$$-\frac{(16b^3e^4n^3)\,\mathrm{Subst}\left(\int\frac{1}{x^2(d+ex^2)}\,dx,x,\sqrt[3]{x}\right)}{35d^3}$$
$$+\frac{(16b^3e^5n^3)\,\mathrm{Subst}\left(\int\frac{\log\left(\frac{2}{1+\frac{i\sqrt{e}x}{\sqrt{d}}}\right)}{1+\frac{ex^2}{d}}\,dx,x,\sqrt[3]{x}\right)}{d^5}$$
$$+\frac{(16b^3e^5n^3)\,\mathrm{Subst}\left(\int\frac{1}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{21d^4} + \frac{(16b^3e^5n^3)\,\mathrm{Subst}\left(\int\frac{1}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{15d^4}$$
$$+\frac{\left(16b^3e^{11/2}n^3\right)\mathrm{Subst}\left(\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{7d^{9/2}}$$
$$+\frac{\left(16b^3e^{11/2}n^3\right)\mathrm{Subst}\left(\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{5d^{9/2}}$$
$$+\frac{\left(16b^3e^{11/2}n^3\right)\mathrm{Subst}\left(\int\frac{x\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d+ex^2}\,dx,x,\sqrt[3]{x}\right)}{3d^{9/2}}$$

$$= -\frac{16b^3e^3n^3}{105d^3x} + \frac{16b^3e^4n^3}{7d^4\sqrt[3]{x}} + \frac{1328b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}}$$

$$- \frac{1408ib^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{105d^{9/2}} - \frac{16b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{d^{9/2}}$$

$$- \frac{8b^2e^2n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^2x^{5/3}} + \frac{32b^2e^3n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^3x}$$

$$- \frac{568b^2e^4n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{105d^4\sqrt[3]{x}}$$

$$- \frac{1408b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{105d^{9/2}}$$

$$- \frac{2ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{7dx^{7/3}} + \frac{2be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5d^2x^{5/3}}$$

$$- \frac{2be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3d^3x} + \frac{2be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d^4\sqrt[3]{x}}$$

$$- \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^3} + \frac{\left(2be^5n\right)\text{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)}{d^4}$$

$$- \frac{\left(16ib^3e^{9/2}n^3\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+\frac{i\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}}\right)}{d^{9/2}}$$

$$- \frac{\left(16b^3e^5n^3\right)\text{Subst}\left(\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{i-\frac{\sqrt{e}x}{\sqrt{d}}}\,dx, x, \sqrt[3]{x}\right)}{7d^5}$$

$$- \frac{\left(16b^3e^5n^3\right)\text{Subst}\left(\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{i-\frac{\sqrt{e}x}{\sqrt{d}}}\,dx, x, \sqrt[3]{x}\right)}{5d^5}$$

$$- \frac{\left(16b^3e^5n^3\right)\text{Subst}\left(\int\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{i-\frac{\sqrt{e}x}{\sqrt{d}}}\,dx, x, \sqrt[3]{x}\right)}{3d^5}$$

$$+ \frac{\left(16b^3e^5n^3\right)\text{Subst}\left(\int\frac{1}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)}{35d^4}$$

$$= -\frac{16b^3e^3n^3}{105d^3x} + \frac{16b^3e^4n^3}{7d^4\sqrt[3]{x}} + \frac{1376b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}}$$

$$- \frac{1408ib^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{105d^{9/2}}$$

$$- \frac{2816b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{105d^{9/2}}$$

$$- \frac{8b^2e^2n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^2x^{5/3}} + \frac{32b^2e^3n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^3x}$$

$$- \frac{568b^2e^4n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{105d^4\sqrt[3]{x}}$$

$$- \frac{1408b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{105d^{9/2}}$$

$$- \frac{2ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{7dx^{7/3}} + \frac{2be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5d^2x^{5/3}}$$

$$- \frac{2be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3d^3x} + \frac{2be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d^4\sqrt[3]{x}}$$

$$- \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^3} - \frac{8ib^3e^{9/2}n^3\text{Li}_2\left(1-\frac{2}{1+\frac{i\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}}\right)}{d^{9/2}}$$

$$+ \frac{(2be^5n)\,\text{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)}{d^4}$$

$$+ \frac{(16b^3e^5n^3)\,\text{Subst}\left(\int\frac{\log\left(\frac{2}{1+\frac{i\sqrt{e}x}{\sqrt{d}}}\right)}{1+\frac{ex^2}{d}}\,dx, x, \sqrt[3]{x}\right)}{7d^5}$$

$$+ \frac{(16b^3e^5n^3)\,\text{Subst}\left(\int\frac{\log\left(\frac{2}{1+\frac{i\sqrt{e}x}{\sqrt{d}}}\right)}{1+\frac{ex^2}{d}}\,dx, x, \sqrt[3]{x}\right)}{5d^5}$$

$$+ \frac{(16b^3e^5n^3)\,\text{Subst}\left(\int\frac{\log\left(\frac{2}{1+\frac{i\sqrt{e}x}{\sqrt{d}}}\right)}{1+\frac{ex^2}{d}}\,dx, x, \sqrt[3]{x}\right)}{3d^5}$$

$$= -\frac{16b^3e^3n^3}{105d^3x} + \frac{16b^3e^4n^3}{7d^4\sqrt[3]{x}} + \frac{1376b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}}$$
$$- \frac{1408ib^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{105d^{9/2}}$$
$$- \frac{2816b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{105d^{9/2}}$$
$$- \frac{8b^2e^2n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^2x^{5/3}} + \frac{32b^2e^3n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^3x}$$
$$- \frac{568b^2e^4n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{105d^4\sqrt[3]{x}}$$
$$- \frac{1408b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{105d^{9/2}}$$
$$- \frac{2ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{7dx^{7/3}} + \frac{2be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5d^2x^{5/3}}$$
$$- \frac{2be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3d^3x} + \frac{2be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d^4\sqrt[3]{x}}$$
$$- \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^3} - \frac{8ib^3e^{9/2}n^3\operatorname{Li}_2\left(1-\frac{2}{1+\frac{i\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}}\right)}{d^{9/2}}$$
$$+ \frac{(2be^5n)\operatorname{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)}{d^4}$$
$$- \frac{\left(16ib^3e^{9/2}n^3\right)\operatorname{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+\frac{i\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}}\right)}{7d^{9/2}}$$
$$- \frac{\left(16ib^3e^{9/2}n^3\right)\operatorname{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+\frac{i\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}}\right)}{5d^{9/2}}$$
$$- \frac{\left(16ib^3e^{9/2}n^3\right)\operatorname{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+\frac{i\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}}\right)}{3d^{9/2}}$$

$$= -\frac{16b^3e^3n^3}{105d^3x} + \frac{16b^3e^4n^3}{7d^4\sqrt[3]{x}} + \frac{1376b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)}{105d^{9/2}}$$
$$- \frac{1408ib^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)^2}{105d^{9/2}}$$
$$- \frac{2816b^3e^{9/2}n^3\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}+i\sqrt{e}\sqrt[3]{x}}\right)}{105d^{9/2}}$$
$$- \frac{8b^2e^2n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^2x^{5/3}} + \frac{32b^2e^3n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{35d^3x}$$
$$- \frac{568b^2e^4n^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{105d^4\sqrt[3]{x}}$$
$$- \frac{1408b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)}{105d^{9/2}}$$
$$- \frac{2ben\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{7dx^{7/3}} + \frac{2be^2n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{5d^2x^{5/3}}$$
$$- \frac{2be^3n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{3d^3x} + \frac{2be^4n\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d^4\sqrt[3]{x}}$$
$$- \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{3x^3} - \frac{1408ib^3e^{9/2}n^3\text{Li}_2\left(1-\frac{2}{1+\frac{i\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}}\right)}{105d^{9/2}}$$
$$+ \frac{(2be^5n)\,\text{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+ex^2\right)^n\right)\right)^2}{d+ex^2}\,dx, x, \sqrt[3]{x}\right)}{d^4}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1385 vs. 2(632) = 1264.

Time = 7.58 (sec) , antiderivative size = 1385, normalized size of antiderivative = 57.71

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx=\frac{1}{210}\left(-\frac{60ben\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{dx^{7/3}}\right.$$

$$+\frac{84be^2n\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d^2x^{5/3}}$$

$$-\frac{140be^3n\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d^3x}$$

$$+\frac{420be^4n\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d^4\sqrt[3]{x}}$$

$$+\frac{420be^{9/2}n\arctan\left(\frac{\sqrt{e}\sqrt[3]{x}}{\sqrt{d}}\right)\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{d^{9/2}}$$

$$-\frac{210bn\log\left(d+ex^{2/3}\right)\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^2}{x^3}$$

$$-\frac{70\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^3}$$

$$-\frac{2b^3n^3\left(1376e^3\left(d+ex^{2/3}\right)^{3/2}\left(\frac{ex^{2/3}}{d+ex^{2/3}}\right)^{3/2}x^2\arcsin\left(\frac{\sqrt{d}}{\sqrt{d+ex^{2/3}}}\right)+\sqrt{d}\left(16e^3\left(d-15ex^{2/3}\right)x^2+8\left(3d^2e^2x^{4/3}-\right.\right.\right.}{}$$

$$+b^2e^5n^2\sqrt[3]{x}\left(a-bn\log\left(d+ex^{2/3}\right)+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)\left(\frac{2816\arcsin\left(\frac{\sqrt{d}}{\sqrt{d+ex^{2/3}}}\right)}{d^{9/2}\sqrt{d+ex^{2/3}}\sqrt{\frac{ex^{2/3}}{d+ex^{2/3}}}}-\frac{120\log\left(d+ex^{2/3}\right)}{de^4x^{8/3}}-\right.$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^n])^3/x^4,x]
```

```
[Out] ((-60*b*e*n*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])^2)/(d
*x^(7/3)) + (84*b*e^2*n*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2/3
))^n])^2)/(d^2*x^(5/3)) - (140*b*e^3*n*(a - b*n*Log[d + e*x^(2/3)] + b*Log[
```

```
c*(d + e*x^(2/3))^n])^2)/(d^3*x) + (420*b*e^4*n*(a - b*n*Log[d + e*x^(2/3)]
 + b*Log[c*(d + e*x^(2/3))^n])^2)/(d^4*x^(1/3)) + (420*b*e^(9/2)*n*ArcTan[(
Sqrt[e]*x^(1/3))/Sqrt[d]]*(a - b*n*Log[d + e*x^(2/3)] + b*Log[c*(d + e*x^(2
/3))^n])^2)/d^(9/2) - (210*b*n*Log[d + e*x^(2/3)]*(a - b*n*Log[d + e*x^(2/3
)] + b*Log[c*(d + e*x^(2/3))^n])^2)/x^3 - (70*(a - b*n*Log[d + e*x^(2/3)] +
 b*Log[c*(d + e*x^(2/3))^n])^3)/x^3 - (2*b^3*n^3*(1376*e^3*(d + e*x^(2/3))^
(3/2)*((e*x^(2/3))/(d + e*x^(2/3)))^(3/2)*x^2*ArcSin[Sqrt[d]/Sqrt[d + e*x^(
2/3)]] + Sqrt[d]*(16*e^3*(d - 15*e*x^(2/3))*x^2 + 8*(3*d^2*e^2*x^(4/3) - 12
*d*e^3*x^2 + 71*e^4*x^(8/3))*Log[d + e*x^(2/3)] + (30*d^3*e*x^(2/3) - 42*d^
2*e^2*x^(4/3) + 70*d*e^3*x^2 - 210*e^4*x^(8/3))*Log[d + e*x^(2/3)]^2 + 35*d
^4*Log[d + e*x^(2/3)]^3) + 210*e^4*Sqrt[(e*x^(2/3))/(d + e*x^(2/3))]*x^(8/3
)*(8*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2, 1/2, 1/2}, {3/2, 3/2, 3/2}, d/(d
+ e*x^(2/3))] + Log[d + e*x^(2/3)]*(4*Sqrt[d]*HypergeometricPFQ[{1/2, 1/2,
1/2}, {3/2, 3/2}, d/(d + e*x^(2/3))] + Sqrt[d + e*x^(2/3)]*ArcSin[Sqrt[d]/S
qrt[d + e*x^(2/3)]]*Log[d + e*x^(2/3)])) + (352*d^(3/2)*e^4*x^(8/3)*(4*Sqrt
[e*x^(2/3)]*ArcTanh[Sqrt[e*x^(2/3)]/Sqrt[-d]]*(Log[d + e*x^(2/3)] - Log[1 +
 (e*x^(2/3))/d]) - Sqrt[-d]*Sqrt[-((e*x^(2/3))/d)]*(2*Log[(1 + Sqrt[-((e*x^
(2/3))/d)])/2]^2 - 4*Log[(1 + Sqrt[-((e*x^(2/3))/d)])/2]*Log[1 + (e*x^(2/3)
)/d] + Log[1 + (e*x^(2/3))/d]^2 - 4*PolyLog[2, 1/2 - Sqrt[-((e*x^(2/3))/d)]
/2])))/(-d)^(3/2)))/(d^(9/2)*x^3) + b^2*e^5*n^2*x^(1/3)*(a - b*n*Log[d + e*
x^(2/3)] + b*Log[c*(d + e*x^(2/3))^n])*((2816*ArcSin[Sqrt[d]/Sqrt[d + e*x^(
2/3)]])/(d^(9/2)*Sqrt[d + e*x^(2/3)]*Sqrt[(e*x^(2/3))/(d + e*x^(2/3))]) - (
120*Log[d + e*x^(2/3)])/(d*e^4*x^(8/3)) - (210*Log[d + e*x^(2/3)]^2)/(e^5*x
^(10/3)) + (24*(-2 + 7*Log[d + e*x^(2/3)]))/(d^2*e^3*x^2) - (8*(-24 + 35*Lo
g[d + e*x^(2/3)]))/(d^3*e^2*x^(4/3)) + (8*(-142 + 105*Log[d + e*x^(2/3)]))/
(d^4*e*x^(2/3)) - (840*ArcTanh[Sqrt[e*x^(2/3)]/Sqrt[-d]]*(Log[d + e*x^(2/3)
] - Log[1 + (e*x^(2/3))/d]))/((-d)^(9/2)*Sqrt[e*x^(2/3)]) - (210*(2*Log[(1
+ Sqrt[-((e*x^(2/3))/d)])/2]^2 - 4*Log[(1 + Sqrt[-((e*x^(2/3))/d)])/2]*Log[
1 + (e*x^(2/3))/d] + Log[1 + (e*x^(2/3))/d]^2 - 4*PolyLog[2, 1/2 - Sqrt[-((
e*x^(2/3))/d)]/2]))/(d^5*Sqrt[-((e*x^(2/3))/d)])))/210
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{n}\right)\right)^{3}}{x^{4}}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e*x^(2/3))^n))^3/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^3}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*log((e*x^(2/3) + d)^n*c)^3 + 3*a*b^2*log((e*x^(2/3) + d)^n*c)
^2 + 3*a^2*b*log((e*x^(2/3) + d)^n*c) + a^3)/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**n))**3/x**4,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^n c\right)+a\right)^3}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^n))^3/x^4,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)^n*c) + a)^3/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^n\right)\right)^3}{x^4}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^n\right)\right)^3}{x^4}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^4,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^n))^3/x^4, x)
```

3.489 $\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 239

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx=\frac{be^{11}n\sqrt[3]{x}}{4d^{11}}-\frac{be^{10}nx^{2/3}}{8d^{10}}+\frac{be^9nx}{12d^9}-\frac{be^8nx^{4/3}}{16d^8}+\frac{be^7nx^{5/3}}{20d^7}-\frac{be^6nx^2}{24d^6}+\frac{be^5nx^{7/3}}{28d^5}-\frac{be^4nx^{8/3}}{32d^4}+\frac{be^3nx^3}{36d^3}-\frac{be^2nx^{10/3}}{40d^2}+\frac{benx^{11/3}}{44d}-\frac{be^{12}n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{4d^{12}}+\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{be^{12}n\log(x)}{12d^{12}}$$

```
[Out] 1/4*b*e^11*n*x^(1/3)/d^11-1/8*b*e^10*n*x^(2/3)/d^10+1/12*b*e^9*n*x/d^9-1/16
*b*e^8*n*x^(4/3)/d^8+1/20*b*e^7*n*x^(5/3)/d^7-1/24*b*e^6*n*x^2/d^6+1/28*b*e
^5*n*x^(7/3)/d^5-1/32*b*e^4*n*x^(8/3)/d^4+1/36*b*e^3*n*x^3/d^3-1/40*b*e^2*n
*x^(10/3)/d^2+1/44*b*e*n*x^(11/3)/d-1/4*b*e^12*n*ln(d+e/x^(1/3))/d^12+1/4*x
^4*(a+b*ln(c*(d+e/x^(1/3))^n))-1/12*b*e^12*n*ln(x)/d^12
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used

$= \{2504, 2442, 46\}$

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx = \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right) - \frac{be^{12}n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{4d^{12}} - \frac{be^{12}n\log(x)}{12d^{12}} + \frac{be^{11}n\sqrt[3]{x}}{4d^{11}} - \frac{be^{10}nx^{2/3}}{8d^{10}} + \frac{be^9nx}{12d^9} - \frac{be^8nx^{4/3}}{16d^8} + \frac{be^7nx^{5/3}}{20d^7} - \frac{be^6nx^2}{24d^6} + \frac{be^5nx^{7/3}}{28d^5} - \frac{be^4nx^{8/3}}{32d^4} + \frac{be^3nx^3}{36d^3} - \frac{be^2nx^{10/3}}{40d^2} + \frac{benx^{11/3}}{44d}$$

```
[In] Int[x^3*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

```
[Out] (b*e^11*n*x^(1/3))/(4*d^11) - (b*e^10*n*x^(2/3))/(8*d^10) + (b*e^9*n*x)/(12
*d^9) - (b*e^8*n*x^(4/3))/(16*d^8) + (b*e^7*n*x^(5/3))/(20*d^7) - (b*e^6*n*
x^2)/(24*d^6) + (b*e^5*n*x^(7/3))/(28*d^5) - (b*e^4*n*x^(8/3))/(32*d^4) + (
b*e^3*n*x^3)/(36*d^3) - (b*e^2*n*x^(10/3))/(40*d^2) + (b*e*n*x^(11/3))/(44*
d) - (b*e^12*n*Log[d + e/x^(1/3)])/(4*d^12) + (x^4*(a + b*Log[c*(d + e/x^(1
/3))^n]))/4 - (b*e^12*n*Log[x])/(12*d^12)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = -\left(3\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^{13}}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$
$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right) - \frac{1}{4}(ben)\text{Subst}\left(\int \frac{1}{x^{12}(d+ex)}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)$$
$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right) - \frac{1}{4}(ben)\text{Subst}\left(\int \left(\frac{1}{dx^{12}} - \frac{e}{d^2x^{11}} + \frac{e^2}{d^3x^{10}} - \frac{e^3}{d^4x^9} + \frac{e^4}{d^5x^8} - \frac{e^5}{d^6x^7} + \frac{e^6}{d^7x^6} - \frac{e^7}{d^8x^5} + \frac{e^8}{d^9x^4} - \frac{e^9}{d^{10}x^3} + \frac{e^{10}}{d^{11}x^2} - \frac{e^{11}}{d^{12}x} + \frac{e^{12}}{d^{12}(d+ex)}\right)dx, x, \frac{1}{\sqrt[3]{x}}\right)$$
$$= \frac{be^{11}n\sqrt[3]{x}}{4d^{11}} - \frac{be^{10}nx^{2/3}}{8d^{10}} + \frac{be^9nx}{12d^9} - \frac{be^8nx^{4/3}}{16d^8} + \frac{be^7nx^{5/3}}{20d^7} - \frac{be^6nx^2}{24d^6} + \frac{be^5nx^{7/3}}{28d^5} - \frac{be^4nx^{8/3}}{32d^4} + \frac{be^3nx^3}{36d^3} - \frac{be^2nx^{10/3}}{40d^2} + \frac{benx^{11/3}}{44d} - \frac{be^{12}n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{4d^{12}} + \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right) - \frac{be^{12}n\log(x)}{12d^{12}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.90

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)dx = \frac{ax^4}{4} + \frac{1}{4}bx^4\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right) + \frac{1}{12}ben\left(\frac{3e^{10}\sqrt[3]{x}}{d^{11}} - \frac{3e^9x^{2/3}}{2d^{10}} + \frac{e^8x}{d^9} - \frac{3e^7x^{4/3}}{4d^8} + \frac{3e^6x^{5/3}}{5d^7} - \frac{e^5x^2}{2d^6} + \frac{3e^4x^{7/3}}{7d^5} - \frac{3e^3x^{8/3}}{8d^4} + \frac{e^2x^3}{3d^3} - \frac{3ex^{10/3}}{10d^2} + \frac{3x^{11/3}}{11d} - \frac{3e^{11}\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^{12}} - \frac{e^{11}\log(x)}{d^{12}}\right)$$

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

```
[Out] (a*x^4)/4 + (b*x^4*Log[c*(d + e/x^(1/3))^n])/4 + (b*e*n*((3*e^10*x^(1/3))/d
^11 - (3*e^9*x^(2/3))/(2*d^10) + (e^8*x)/d^9 - (3*e^7*x^(4/3))/(4*d^8) + (3
*e^6*x^(5/3))/(5*d^7) - (e^5*x^2)/(2*d^6) + (3*e^4*x^(7/3))/(7*d^5) - (3*e^
3*x^(8/3))/(8*d^4) + (e^2*x^3)/(3*d^3) - (3*e*x^(10/3))/(10*d^2) + (3*x^(11
/3))/(11*d) - (3*e^11*Log[d + e/x^(1/3)])/d^12 - (e^11*Log[x])/d^12))/12
```

Maple [F]

$$\int x^3\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)\right)dx$$

[In] int(x^3*(a+b*ln(c*(d+e/x^(1/3))^n)),x)

[Out] int(x^3*(a+b*ln(c*(d+e/x^(1/3))^n)),x)

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.97

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)dx$$
$$=\frac{27720\,bd^{12}x^4\log(c)+3080\,bd^9e^3nx^3+27720\,ad^{12}x^4-4620\,bd^6e^6nx^2+9240\,bd^3e^9nx-27720\,bd^{12}n\log\Big(}{}$$

[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="fricas")

[Out] 1/110880*(27720*b*d^12*x^4*log(c) + 3080*b*d^9*e^3*n*x^3 + 27720*a*d^12*x^4 - 4620*b*d^6*e^6*n*x^2 + 9240*b*d^3*e^9*n*x - 27720*b*d^12*n*log(x^(1/3)) + 27720*(b*d^12 - b*e^12)*n*log(d*x^(1/3) + e) + 27720*(b*d^12*n*x^4 - b*d^12*n)*log((d*x + e*x^(2/3))/x) + 63*(40*b*d^11*e*n*x^3 - 55*b*d^8*e^4*n*x^2 + 88*b*d^5*e^7*n*x - 220*b*d^2*e^10*n)*x^(2/3) - 198*(14*b*d^10*e^2*n*x^3 - 20*b*d^7*e^5*n*x^2 + 35*b*d^4*e^8*n*x - 140*b*d*e^11*n)*x^(1/3))/d^12

Sympy [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)dx=\text{Timed out}$$

[In] integrate(x**3*(a+b*ln(c*(d+e/x**(1/3))**n)),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.68

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx=\frac{1}{4}\,bx^4\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+\frac{1}{4}\,ax^4$$
$$-\frac{1}{110880}\,ben\left(\frac{27720\,e^{11}\log\left(dx^{\frac{1}{3}}+e\right)}{d^{12}}-\frac{2520\,d^{10}x^{\frac{11}{3}}-2772\,d^9ex^{\frac{10}{3}}+3080\,d^8e^2x^3-3465\,d^7e^3x^{\frac{8}{3}}+3960\right.$$

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4*log(c*(d + e/x^(1/3))^n) + 1/4*a*x^4 - 1/110880*b*e*n*(27720*e^11
*log(d*x^(1/3) + e)/d^12 - (2520*d^10*x^(11/3) - 2772*d^9*e*x^(10/3) + 3080
*d^8*e^2*x^3 - 3465*d^7*e^3*x^(8/3) + 3960*d^6*e^4*x^(7/3) - 4620*d^5*e^5*x
^2 + 5544*d^4*e^6*x^(5/3) - 6930*d^3*e^7*x^(4/3) + 9240*d^2*e^8*x - 13860*d
*e^9*x^(2/3) + 27720*e^10*x^(1/3))/d^11)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.71

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx=\frac{1}{4}\,bx^4\log\left(c\right)+\frac{1}{4}\,ax^4$$
$$+\frac{1}{110880}\left(27720\,x^4\log\left(d+\frac{e}{x^{\frac{1}{3}}}\right)-e\left(\frac{27720\,e^{11}\log\left(\left|dx^{\frac{1}{3}}+e\right|\right)}{d^{12}}-\frac{2520\,d^{10}x^{\frac{11}{3}}-2772\,d^9ex^{\frac{10}{3}}+3080\,d^8\right.\right.$$

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4*log(c) + 1/4*a*x^4 + 1/110880*(27720*x^4*log(d + e/x^(1/3)) - e*(
27720*e^11*log(abs(d*x^(1/3) + e))/d^12 - (2520*d^10*x^(11/3) - 2772*d^9*e*
x^(10/3) + 3080*d^8*e^2*x^3 - 3465*d^7*e^3*x^(8/3) + 3960*d^6*e^4*x^(7/3) -
 4620*d^5*e^5*x^2 + 5544*d^4*e^6*x^(5/3) - 6930*d^3*e^7*x^(4/3) + 9240*d^2*
e^8*x - 13860*d*e^9*x^(2/3) + 27720*e^10*x^(1/3))/d^11))*b*n
```

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.80

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$$
$$=\frac{\frac{a\,d^{12}\,x^4}{4}-\frac{b\,e^{12}\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,x^{1/3}}+1\right)}{2}+\frac{b\,d^{12}\,x^4\,\ln\left(c\,\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{4}+\frac{b\,d^3\,e^9\,n\,x}{12}+\frac{b\,d\,e^{11}\,n\,x^{1/3}}{4}+\frac{b\,d^{11}\,e\,n\,x^{11/3}}{44}-\frac{b\,d^6\,e^6\,n\,x^2}{24}+\frac{b\,d^9}{}}{d^{12}}$$

```
[In] int(x^3*(a + b*log(c*(d + e/x^(1/3))^n)),x)
```

```
[Out] ((a*d^12*x^4)/4 - (b*e^12*n*atanh((2*e)/(d*x^(1/3)) + 1))/2 + (b*d^12*x^4*l
og(c*(d + e/x^(1/3))^n))/4 + (b*d^3*e^9*n*x)/12 + (b*d*e^11*n*x^(1/3))/4 +
(b*d^11*e*n*x^(11/3))/44 - (b*d^6*e^6*n*x^2)/24 + (b*d^9*e^3*n*x^3)/36 - (b
*d^2*e^10*n*x^(2/3))/8 - (b*d^4*e^8*n*x^(4/3))/16 + (b*d^5*e^7*n*x^(5/3))/2
0 + (b*d^7*e^5*n*x^(7/3))/28 - (b*d^8*e^4*n*x^(8/3))/32 - (b*d^10*e^2*n*x^(
10/3))/40)/d^12
```

3.490 $\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 190

$$\begin{aligned}\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx &= -\frac{be^8n\sqrt[3]{x}}{3d^8}+\frac{be^7nx^{2/3}}{6d^7}-\frac{be^6nx}{9d^6}+\frac{be^5nx^{4/3}}{12d^5}\\ &\quad-\frac{be^4nx^{5/3}}{15d^4}+\frac{be^3nx^2}{18d^3}-\frac{be^2nx^{7/3}}{21d^2}\\ &\quad+\frac{benx^{8/3}}{24d}+\frac{be^9n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{3d^9}\\ &\quad+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)+\frac{be^9n\log(x)}{9d^9}\end{aligned}$$

```
[Out] -1/3*b*e^8*n*x^(1/3)/d^8+1/6*b*e^7*n*x^(2/3)/d^7-1/9*b*e^6*n*x/d^6+1/12*b*e
^5*n*x^(4/3)/d^5-1/15*b*e^4*n*x^(5/3)/d^4+1/18*b*e^3*n*x^2/d^3-1/21*b*e^2*n
*x^(7/3)/d^2+1/24*b*e*n*x^(8/3)/d+1/3*b*e^9*n*ln(d+e/x^(1/3))/d^9+1/3*x^3*(
a+b*ln(c*(d+e/x^(1/3))^n))+1/9*b*e^9*n*ln(x)/d^9
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used

$= \{2504, 2442, 46\}$

$$\int x^2 \left(a + b\log\left(c\left(d + \frac{e}{\sqrt[3]{x}}\right)^n\right)\right) dx = \frac{1}{3}x^3\left(a + b\log\left(c\left(d + \frac{e}{\sqrt[3]{x}}\right)^n\right)\right) + \frac{be^9 n\log\left(d + \frac{e}{\sqrt[3]{x}}\right)}{3d^9} + \frac{be^9 n\log(x)}{9d^9} - \frac{be^8 n\sqrt[3]{x}}{3d^8} + \frac{be^7 nx^{2/3}}{6d^7} - \frac{be^6 nx}{9d^6} + \frac{be^5 nx^{4/3}}{12d^5} - \frac{be^4 nx^{5/3}}{15d^4} + \frac{be^3 nx^2}{18d^3} - \frac{be^2 nx^{7/3}}{21d^2} + \frac{benx^{8/3}}{24d}$$

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

```
[Out] -1/3*(b*e^8*n*x^(1/3))/d^8 + (b*e^7*n*x^(2/3))/(6*d^7) - (b*e^6*n*x)/(9*d^6
) + (b*e^5*n*x^(4/3))/(12*d^5) - (b*e^4*n*x^(5/3))/(15*d^4) + (b*e^3*n*x^2)
/(18*d^3) - (b*e^2*n*x^(7/3))/(21*d^2) + (b*e*n*x^(8/3))/(24*d) + (b*e^9*n*
Log[d + e/x^(1/3)])/(3*d^9) + (x^3*(a + b*Log[c*(d + e/x^(1/3))^n]))/3 + (b
*e^9*n*Log[x])/(9*d^9)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(3\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^{10}}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{1}{3}(ben)\text{Subst}\left(\int \frac{1}{x^9(d+ex)}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{1}{3}(ben)\text{Subst}\left(\int \left(\frac{1}{dx^9}-\frac{e}{d^2x^8}+\frac{e^2}{d^3x^7}-\frac{e^3}{d^4x^6}\right.\right.\\
&\qquad \left.\left.+\frac{e^4}{d^5x^5}-\frac{e^5}{d^6x^4}+\frac{e^6}{d^7x^3}-\frac{e^7}{d^8x^2}+\frac{e^8}{d^9x}-\frac{e^9}{d^9(d+ex)}\right)\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\\
&= -\frac{be^8n\sqrt[3]{x}}{3d^8}+\frac{be^7nx^{2/3}}{6d^7}-\frac{be^6nx}{9d^6}+\frac{be^5nx^{4/3}}{12d^5}-\frac{be^4nx^{5/3}}{15d^4}+\frac{be^3nx^2}{18d^3}-\frac{be^2nx^{7/3}}{21d^2}\\
&\quad +\frac{benx^{8/3}}{24d}+\frac{be^9n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{3d^9}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)+\frac{be^9n\log(x)}{9d^9}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.90

$$
\begin{aligned}
\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)dx &= \frac{ax^3}{3}+\frac{1}{3}bx^3\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\\
&\quad +\frac{1}{9}ben\left(-\frac{3e^7\sqrt[3]{x}}{d^8}+\frac{3e^6x^{2/3}}{2d^7}-\frac{e^5x}{d^6}+\frac{3e^4x^{4/3}}{4d^5}\right.\\
&\quad -\frac{3e^3x^{5/3}}{5d^4}+\frac{e^2x^2}{2d^3}-\frac{3ex^{7/3}}{7d^2}+\frac{3x^{8/3}}{8d}\\
&\quad \left.+\frac{3e^8\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^9}+\frac{e^8\log(x)}{d^9}\right)
\end{aligned}
$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

```
[Out] (a*x^3)/3 + (b*x^3*Log[c*(d + e/x^(1/3))^n])/3 + (b*e*n*((-3*e^7*x^(1/3))/d
^8 + (3*e^6*x^(2/3))/(2*d^7) - (e^5*x)/d^6 + (3*e^4*x^(4/3))/(4*d^5) - (3*e
^3*x^(5/3))/(5*d^4) + (e^2*x^2)/(2*d^3) - (3*e*x^(7/3))/(7*d^2) + (3*x^(8/3
))/(8*d) + (3*e^8*Log[d + e/x^(1/3)])/d^9 + (e^8*Log[x])/d^9))/9
```

Maple [F]

$$\int x^{2}\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)dx$$

[In] int(x^2*(a+b*ln(c*(d+e/x^(1/3))^n)),x)

[Out] int(x^2*(a+b*ln(c*(d+e/x^(1/3))^n)),x)

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.01

$$\int x^{2}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)\,dx$$
$$=\frac{840\,bd^{9}x^{3}\log\left(c\right)+140\,bd^{6}e^{3}nx^{2}+840\,ad^{9}x^{3}-280\,bd^{3}e^{6}nx-840\,bd^{9}n\log\left(x^{\frac{1}{3}}\right)+840\,(bd^{9}+be^{9})n\log\left(\right.}{}$$

[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="fricas")

[Out] 1/2520*(840*b*d^9*x^3*log(c) + 140*b*d^6*e^3*n*x^2 + 840*a*d^9*x^3 - 280*b*d^3*e^6*n*x - 840*b*d^9*n*log(x^(1/3)) + 840*(b*d^9 + b*e^9)*n*log(d*x^(1/3) + e) + 840*(b*d^9*n*x^3 - b*d^9*n)*log((d*x + e*x^(2/3))/x) + 21*(5*b*d^8*e*n*x^2 - 8*b*d^5*e^4*n*x + 20*b*d^2*e^7*n)*x^(2/3) - 30*(4*b*d^7*e^2*n*x^2 - 7*b*d^4*e^5*n*x + 28*b*d*e^8*n)*x^(1/3))/d^9

Sympy [A] (verification not implemented)

Time = 57.67 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.85

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx=\frac{ax^3}{3}+b\left(\frac{en\left(\frac{3x^{\frac{8}{3}}}{8d}-\frac{3ex^{\frac{7}{3}}}{7d^2}+\frac{e^2x^2}{2d^3}-\frac{3e^3x^{\frac{5}{3}}}{5d^4}+\frac{3e^4x^{\frac{4}{3}}}{4d^5}-\frac{e^5x}{d^6}+\frac{3e^6x^{\frac{2}{3}}}{2d^7}+\frac{3e^8\left(\begin{cases}\frac{\sqrt[3]{x}}{e} & \text{for } d=0\\ \frac{\log\left(d\sqrt[3]{x}+e\right)}{d} & \text{otherwise}\end{cases}\right)}{d^8}-\frac{3e^7\sqrt[3]{x}}{d^8}\right)}{9}+\frac{x^3\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3}\right)$$

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(1/3))**n)),x)
```

```
[Out] a*x**3/3 + b*(e*n*(3*x**(8/3)/(8*d) - 3*e*x**(7/3)/(7*d**2) + e**2*x**2/(2*
d**3) - 3*e**3*x**(5/3)/(5*d**4) + 3*e**4*x**(4/3)/(4*d**5) - e**5*x/d**6 +
 3*e**6*x**(2/3)/(2*d**7) + 3*e**8*Piecewise((x**(1/3)/e, Eq(d, 0)), (log(d
*x**(1/3) + e)/d, True))/d**8 - 3*e**7*x**(1/3)/d**8)/9 + x**3*log(c*(d + e
/x**(1/3))**n)/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.67

$$\int x^2 \left(a + b\log\left(c\left(d + \frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx = \frac{1}{3}\,bx^3\log\left(c\left(d + \frac{e}{x^{\frac{1}{3}}}\right)^n\right) + \frac{1}{3}\,ax^3$$
$$+ \frac{1}{2520}\,ben\left(\frac{840\,e^8\log\left(dx^{\frac{1}{3}} + e\right)}{d^9} + \frac{105\,d^7x^{\frac{8}{3}} - 120\,d^6ex^{\frac{7}{3}} + 140\,d^5e^2x^2 - 168\,d^4e^3x^{\frac{5}{3}} + 210\,d^3e^4x^{\frac{4}{3}} - 28}{d^8}\right.$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3*log(c*(d + e/x^(1/3))^n) + 1/3*a*x^3 + 1/2520*b*e*n*(840*e^8*log(
d*x^(1/3) + e)/d^9 + (105*d^7*x^(8/3) - 120*d^6*e*x^(7/3) + 140*d^5*e^2*x^2
 - 168*d^4*e^3*x^(5/3) + 210*d^3*e^4*x^(4/3) - 280*d^2*e^5*x + 420*d*e^6*x^
(2/3) - 840*e^7*x^(1/3))/d^8)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.71

$$\int x^2 \left(a + b\log\left(c\left(d + \frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx = \frac{1}{3}\,bx^3\log\left(c\right) + \frac{1}{3}\,ax^3$$
$$+ \frac{1}{2520}\left(840\,x^3\log\left(d + \frac{e}{x^{\frac{1}{3}}}\right) + e\left(\frac{840\,e^8\log\left(\left|dx^{\frac{1}{3}} + e\right|\right)}{d^9} + \frac{105\,d^7x^{\frac{8}{3}} - 120\,d^6ex^{\frac{7}{3}} + 140\,d^5e^2x^2 - 168\,d}{}\right.\right.$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*log(c) + 1/3*a*x^3 + 1/2520*(840*x^3*log(d + e/x^(1/3)) + e*(840*
e^8*log(abs(d*x^(1/3) + e))/d^9 + (105*d^7*x^(8/3) - 120*d^6*e*x^(7/3) + 14
0*d^5*e^2*x^2 - 168*d^4*e^3*x^(5/3) + 210*d^3*e^4*x^(4/3) - 280*d^2*e^5*x +
 420*d*e^6*x^(2/3) - 840*e^7*x^(1/3))/d^8))*b*n
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.81

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$$
$$=\frac{840\,a\,d^9\,x^3+1680\,b\,e^9\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,x^{1/3}}+1\right)+840\,b\,d^9\,x^3\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)-280\,b\,d^3\,e^6\,n\,x-840\,b\,d\,e^8\,n\,x^{1/3}$$

```
[In] int(x^2*(a + b*log(c*(d + e/x^(1/3))^n)),x)
```

```
[Out] (840*a*d^9*x^3 + 1680*b*e^9*n*atanh((2*e)/(d*x^(1/3)) + 1) + 840*b*d^9*x^3*
log(c*(d + e/x^(1/3))^n) - 280*b*d^3*e^6*n*x - 840*b*d*e^8*n*x^(1/3) + 105*
b*d^8*e*n*x^(8/3) + 140*b*d^6*e^3*n*x^2 + 420*b*d^2*e^7*n*x^(2/3) + 210*b*d
^4*e^5*n*x^(4/3) - 168*b*d^5*e^4*n*x^(5/3) - 120*b*d^7*e^2*n*x^(7/3))/(2520
*d^9)
```

3.491 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 141

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx=\frac{be^5n\sqrt[3]{x}}{2d^5}-\frac{be^4nx^{2/3}}{4d^4}+\frac{be^3nx}{6d^3}-\frac{be^2nx^{4/3}}{8d^2}+\frac{benx^{5/3}}{10d}-\frac{be^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2d^6}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{be^6n\log(x)}{6d^6}$$

```
[Out] 1/2*b*e^5*n*x^(1/3)/d^5-1/4*b*e^4*n*x^(2/3)/d^4+1/6*b*e^3*n*x/d^3-1/8*b*e^2
*n*x^(4/3)/d^2+1/10*b*e*n*x^(5/3)/d-1/2*b*e^6*n*ln(d+e/x^(1/3))/d^6+1/2*x^2
*(a+b*ln(c*(d+e/x^(1/3))^n))-1/6*b*e^6*n*ln(x)/d^6
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2504, 2442, 46}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{be^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2d^6}-\frac{be^6n\log(x)}{6d^6}+\frac{be^5n\sqrt[3]{x}}{2d^5}-\frac{be^4nx^{2/3}}{4d^4}+\frac{be^3nx}{6d^3}-\frac{be^2nx^{4/3}}{8d^2}+\frac{benx^{5/3}}{10d}$$

```
[In] Int[x*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
```

```
[Out] (b*e^5*n*x^(1/3))/(2*d^5) - (b*e^4*n*x^(2/3))/(4*d^4) + (b*e^3*n*x)/(6*d^3)
 - (b*e^2*n*x^(4/3))/(8*d^2) + (b*e*n*x^(5/3))/(10*d) - (b*e^6*n*Log[d + e/
x^(1/3)])/(2*d^6) + (x^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/2 - (b*e^6*n*Log
[x])/(6*d^6)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(3\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^7}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{1}{2}(ben)\text{Subst}\left(\int \frac{1}{x^6(d+ex)}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{1}{2}(ben)\text{Subst}\left(\int \left(\frac{1}{dx^6}-\frac{e}{d^2x^5}+\frac{e^2}{d^3x^4}-\frac{e^3}{d^4x^3}\right.\right.\\
&\qquad \left.\left.+\frac{e^4}{d^5x^2}-\frac{e^5}{d^6x}+\frac{e^6}{d^6(d+ex)}\right)dx, x, \frac{1}{\sqrt[3]{x}}\right)\\
&= \frac{be^5n\sqrt[3]{x}}{2d^5}-\frac{be^4nx^{2/3}}{4d^4}+\frac{be^3nx}{6d^3}-\frac{be^2nx^{4/3}}{8d^2}+\frac{benx^{5/3}}{10d}\\
&\quad -\frac{be^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2d^6}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-\frac{be^6n\log(x)}{6d^6}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx=\frac{ax^2}{2}+\frac{1}{2}bx^2\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+\frac{1}{6}ben\left(\frac{3e^4\sqrt[3]{x}}{d^5}-\frac{3e^3x^{2/3}}{2d^4}+\frac{e^2x}{d^3}-\frac{3ex^{4/3}}{4d^2}+\frac{3x^{5/3}}{5d}-\frac{3e^5\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^6}-\frac{e^5\log(x)}{d^6}\right)$$

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^n]),x]
[Out] (a*x^2)/2 + (b*x^2*Log[c*(d + e/x^(1/3))^n])/2 + (b*e*n*((3*e^4*x^(1/3))/d^
5 - (3*e^3*x^(2/3))/(2*d^4) + (e^2*x)/d^3 - (3*e*x^(4/3))/(4*d^2) + (3*x^(5
/3))/(5*d) - (3*e^5*Log[d + e/x^(1/3)])/d^6 - (e^5*Log[x])/d^6))/6
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)\right)dx$$

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/3))^n)),x)
[Out] int(x*(a+b*ln(c*(d+e/x^(1/3))^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.09

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx=\frac{60\,bd^6x^2\log(c)+20\,bd^3e^3nx+60\,ad^6x^2-60\,bd^6n\log\left(x^{\frac{1}{3}}\right)+60\,(bd^6-be^6)n\log\left(dx^{\frac{1}{3}}+e\right)+60\,(bd^6nx}{120\,d^6}$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="fricas")
[Out] 1/120*(60*b*d^6*x^2*log(c) + 20*b*d^3*e^3*n*x + 60*a*d^6*x^2 - 60*b*d^6*n*l
og(x^(1/3)) + 60*(b*d^6 - b*e^6)*n*log(d*x^(1/3) + e) + 60*(b*d^6*n*x^2 - b
*d^6*n)*log((d*x + e*x^(2/3))/x) + 6*(2*b*d^5*e*n*x - 5*b*d^2*e^4*n)*x^(2/3
) - 15*(b*d^4*e^2*n*x - 4*b*d*e^5*n)*x^(1/3))/d^6
```

Sympy [A] (verification not implemented)

Time = 11.43 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.84

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$$
$$=\frac{ax^2}{2}+b\left(\frac{en\left(\frac{3x^{\frac{5}{3}}}{5d}-\frac{3ex^{\frac{4}{3}}}{4d^2}+\frac{e^2x}{d^3}-\frac{3e^3x^{\frac{2}{3}}}{2d^4}-\frac{3e^5\left(\begin{cases}\frac{\sqrt[3]{x}}{e} & \text{for } d=0\\ \frac{\log\left(d\sqrt[3]{x}+e\right)}{d} & \text{otherwise}\end{cases}\right)}{d^5}+\frac{3e^4\sqrt[3]{x}}{d^5}\right)}{6}+\frac{x^2\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2}\right)$$

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/3))**n)),x)
```

```
[Out] a*x**2/2 + b*(e*n*(3*x**(5/3)/(5*d) - 3*e*x**(4/3)/(4*d**2) + e**2*x/d**3 -
 3*e**3*x**(2/3)/(2*d**4) - 3*e**5*Piecewise((x**(1/3)/e, Eq(d, 0)), (log(d
*x**(1/3) + e)/d, True))/d**5 + 3*e**4*x**(1/3)/d**5)/6 + x**2*log(c*(d + e
/x**(1/3))**n)/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.68

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx = -\frac{1}{120}\,ben\left(\frac{60\,e^5\log\left(dx^{\frac{1}{3}}+e\right)}{d^6}-\frac{12\,d^4x^{\frac{5}{3}}-15\,d^3ex^{\frac{4}{3}}+20\,d^2e^2x-30\,de^3x^{\frac{2}{3}}+60\,e^4x^{\frac{1}{3}}}{d^5}\right)+\frac{1}{2}\,bx^2\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+\frac{1}{2}\,ax^2$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="maxima")
```

```
[Out] -1/120*b*e*n*(60*e^5*log(d*x^(1/3) + e)/d^6 - (12*d^4*x^(5/3) - 15*d^3*e*x^
(4/3) + 20*d^2*e^2*x - 30*d*e^3*x^(2/3) + 60*e^4*x^(1/3))/d^5) + 1/2*b*x^2*
log(c*(d + e/x^(1/3))^n) + 1/2*a*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.73

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx = \frac{1}{2}\,bx^2\log\left(c\right)+\frac{1}{120}\left(60\,x^2\log\left(d+\frac{e}{x^{\frac{1}{3}}}\right)-e\left(\frac{60\,e^5\log\left(\left|dx^{\frac{1}{3}}+e\right|\right)}{d^6}-\frac{12\,d^4x^{\frac{5}{3}}-15\,d^3ex^{\frac{4}{3}}+20\,d^2e^2x-30\,de^3x^{\frac{2}{3}}+6}{d^5}\right.\right.+\frac{1}{2}\,ax^2$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2*log(c) + 1/120*(60*x^2*log(d + e/x^(1/3)) - e*(60*e^5*log(abs(d*x
^(1/3) + e))/d^6 - (12*d^4*x^(5/3) - 15*d^3*e*x^(4/3) + 20*d^2*e^2*x - 30*d
*e^3*x^(2/3) + 60*e^4*x^(1/3))/d^5))*b*n + 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.79

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx=\frac{x^{5/3}\left(\frac{b\,e\,n}{5\,d}-\frac{b\,e^2\,n}{4\,d^2\,x^{1/3}}-\frac{b\,e^4\,n}{2\,d^4\,x}+\frac{b\,e^3\,n}{3\,d^3\,x^{2/3}}+\frac{b\,e^5\,n}{d^5\,x^{4/3}}\right)}{2}+\frac{a\,x^2}{2}+\frac{b\,x^2\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{2}-\frac{b\,e^6\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,x^{1/3}}+1\right)}{d^6}$$

```
[In] int(x*(a + b*log(c*(d + e/x^(1/3))^n)),x)
```

```
[Out] (x^(5/3)*((b*e*n)/(5*d) - (b*e^2*n)/(4*d^2*x^(1/3)) - (b*e^4*n)/(2*d^4*x) +
 (b*e^3*n)/(3*d^3*x^(2/3)) + (b*e^5*n)/(d^5*x^(4/3))))/2 + (a*x^2)/2 + (b*x
^2*log(c*(d + e/x^(1/3))^n))/2 - (b*e^6*n*atanh((2*e)/(d*x^(1/3)) + 1))/d^6
```

3.492 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right) dx$

Optimal result

Integrand size = 18, antiderivative size = 70

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right) dx = -\frac{be^2n\sqrt[3]{x}}{d^2}+\frac{benx^{2/3}}{2d}+ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+\frac{be^3n\log\left(e+d\sqrt[3]{x}\right)}{d^3}$$

[Out] -b*e^2*n*x^(1/3)/d^2+1/2*b*e*n*x^(2/3)/d+a*x+b*x*ln(c*(d+e/x^(1/3))^n)+b*e^3*n*ln(e+d*x^(1/3))/d^3

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2498, 269, 196, 45}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right) dx = ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+\frac{be^3n\log\left(d\sqrt[3]{x}+e\right)}{d^3}-\frac{be^2n\sqrt[3]{x}}{d^2}+\frac{benx^{2/3}}{2d}$$

[In] Int[a + b*Log[c*(d + e/x^(1/3))^n],x]

[Out] -((b*e^2*n*x^(1/3))/d^2) + (b*e*n*x^(2/3))/(2*d) + a*x + b*x*Log[c*(d + e/x^(1/3))^n] + (b*e^3*n*Log[e + d*x^(1/3)])/d^3

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= ax + b\int \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)dx\\
&= ax + bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right) + \frac{1}{3}(ben)\int \frac{1}{\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}}\,dx\\
&= ax + bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right) + \frac{1}{3}(ben)\int \frac{1}{e+d\sqrt[3]{x}}\,dx\\
&= ax + bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right) + (ben)\text{Subst}\left(\int \frac{x^2}{e+dx}\,dx, x, \sqrt[3]{x}\right)\\
&= ax + bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right) + (ben)\text{Subst}\left(\int \left(-\frac{e}{d^2}+\frac{x}{d}+\frac{e^2}{d^2(e+dx)}\right)dx, x, \sqrt[3]{x}\right)\\
&= -\frac{be^2n\sqrt[3]{x}}{d^2} + \frac{benx^{2/3}}{2d} + ax + bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right) + \frac{be^3n\log\left(e+d\sqrt[3]{x}\right)}{d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx=ax+bx\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)+\frac{1}{3}ben\left(-\frac{3e\sqrt[3]{x}}{d^2}+\frac{3x^{2/3}}{2d}+\frac{3e^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^3}+\frac{e^2\log(x)}{d^3}\right)$$

```
[In] Integrate[a + b*Log[c*(d + e/x^(1/3))^n],x]
```

```
[Out] a*x + b*x*Log[c*(d + e/x^(1/3))^n] + (b*e*n*((-3*e*x^(1/3))/d^2 + (3*x^(2/3
))/(2*d) + (3*e^2*Log[d + e/x^(1/3)])/d^3 + (e^2*Log[x])/d^3))/3
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.54

method	result	size
default	$ax+b\left(x\ln\left(c\left(\frac{e+d\,x^{\frac{1}{3}}}{x^{\frac{1}{3}}}\right)^n\right)+\frac{en\left(\frac{e^2\ln\left(d^3x+e^3\right)}{d^3}+\frac{3x^{\frac{2}{3}}}{2d}+\frac{2e^2\ln\left(e+d\,x^{\frac{1}{3}}\right)}{d^3}-\frac{e^2\ln\left(d^2x^{\frac{2}{3}}-ed\,x^{\frac{1}{3}}+e^2\right)}{d^3}-\frac{3e\,x^{\frac{1}{3}}}{d^2}\right)}{3}\right)$	108
parts	$ax+b\left(x\ln\left(c\left(\frac{e+d\,x^{\frac{1}{3}}}{x^{\frac{1}{3}}}\right)^n\right)+\frac{en\left(\frac{e^2\ln\left(d^3x+e^3\right)}{d^3}+\frac{3x^{\frac{2}{3}}}{2d}+\frac{2e^2\ln\left(e+d\,x^{\frac{1}{3}}\right)}{d^3}-\frac{e^2\ln\left(d^2x^{\frac{2}{3}}-ed\,x^{\frac{1}{3}}+e^2\right)}{d^3}-\frac{3e\,x^{\frac{1}{3}}}{d^2}\right)}{3}\right)$	108

```
[In] int(a+b*ln(c*(d+e/x^(1/3))^n),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*(x*ln(c*((e+d*x^(1/3))/x^(1/3))^n)+1/3*e*n*(e^2*ln(d^3*x+e^3)/d^3+3/2
/d*x^(2/3)+2/d^3*e^2*ln(e+d*x^(1/3))-1/d^3*e^2*ln(d^2*x^(2/3)-e*d*x^(1/3)+e
^2)-3/d^2*e*x^(1/3)))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.53

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$$
$$=\frac{2\,bd^3x\log\left(c\right)-2\,bd^3n\log\left(x^{\frac{1}{3}}\right)+bd^2enx^{\frac{2}{3}}-2\,bde^2nx^{\frac{1}{3}}+2\,ad^3x+2\,(bd^3+be^3)n\log\left(dx^{\frac{1}{3}}+e\right)+2\,(bd^3n}{2\,d^3}$$

```
[In] integrate(a+b*log(c*(d+e/x^(1/3))^n),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d^3*x*log(c) - 2*b*d^3*n*log(x^(1/3)) + b*d^2*e*n*x^(2/3) - 2*b*d*
e^2*n*x^(1/3) + 2*a*d^3*x + 2*(b*d^3 + b*e^3)*n*log(d*x^(1/3) + e) + 2*(b*d
^3*n*x - b*d^3*n)*log((d*x + e*x^(2/3))/x))/d^3
```

Sympy [A] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx$$
$$=ax+b\left(\frac{en\left(\frac{3x^{\frac{2}{3}}}{2d}+\frac{3e^2\left(\begin{cases}\frac{\sqrt[3]{x}}{e} & \text{for } d=0\\ \frac{\log\left(d\sqrt[3]{x}+e\right)}{d} & \text{otherwise}\end{cases}\right)}{d^2}-\frac{3e\sqrt[3]{x}}{d^2}\right)}{3}+x\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)$$

```
[In] integrate(a+b*ln(c*(d+e/x**(1/3))**n),x)
```

```
[Out] a*x + b*(e*n*(3*x**(2/3)/(2*d) + 3*e**2*Piecewise((x**(1/3)/e, Eq(d, 0)), (
log(d*x**(1/3) + e)/d, True))/d**2 - 3*e*x**(1/3)/d**2)/3 + x*log(c*(d + e/
x**(1/3))**n))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx = \frac{1}{2}\left(en\left(\frac{2\,e^2\log\left(dx^{\frac{1}{3}}+e\right)}{d^3}+\frac{dx^{\frac{2}{3}}-2\,ex^{\frac{1}{3}}}{d^2}\right)+2\,x\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)\right)b+ax$$

```
[In] integrate(a+b*log(c*(d+e/x^(1/3))^n),x, algorithm="maxima")
```

```
[Out] 1/2*(e*n*(2*e^2*log(d*x^(1/3) + e)/d^3 + (d*x^(2/3) - 2*e*x^(1/3))/d^2) + 2
*x*log(c*(d + e/x^(1/3))^n))*b + a*x
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx = \frac{1}{2}\left(\left(\left(e\left(\frac{2\,e^2\log\left(\left|dx^{\frac{1}{3}}+e\right|\right)}{d^3}+\frac{dx^{\frac{2}{3}}-2\,ex^{\frac{1}{3}}}{d^2}\right)+2\,x\log\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)n+2\,x\log\left(c\right)\right)b + ax$$

```
[In] integrate(a+b*log(c*(d+e/x^(1/3))^n),x, algorithm="giac")
```

```
[Out] 1/2*((e*(2*e^2*log(abs(d*x^(1/3) + e))/d^3 + (d*x^(2/3) - 2*e*x^(1/3))/d^2)
 + 2*x*log(d + e/x^(1/3)))*n + 2*x*log(c))*b + a*x
```

Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\,dx = a\,x+b\,x\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)+\frac{b\,\left(2\,e^3\,n\,\ln\left(e+d\,x^{1/3}\right)-2\,d\,e^2\,n\,x^{1/3}+d^2\,e\,n\,x^{2/3}\right)}{2\,d^3}$$

```
[In] int(a + b*log(c*(d + e/x^(1/3))^n),x)
```

```
[Out] a*x + b*x*log(c*(d + e/x^(1/3))^n) + (b*(2*e^3*n*log(e + d*x^(1/3)) - 2*d*e
^2*n*x^(1/3) + d^2*e*n*x^(2/3)))/(2*d^3)
```

3.493 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 51

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}\,dx=-3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt[3]{x}}\right)-3bn\,\text{PolyLog}\left(2,1+\frac{e}{d\sqrt[3]{x}}\right)$$

```
[Out] -3*(a+b*ln(c*(d+e/x^(1/3))^n))*ln(-e/d/x^(1/3))-3*b*n*polylog(2,1+e/d/x^(1/
3))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2441, 2352}

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}\,dx=-3\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-3bn\,\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)$$

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])/x,x]
```

```
[Out] -3*(a + b*Log[c*(d + e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))] - 3*b*n*PolyLog[2
, 1 + e/(d*x^(1/3))]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(3\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)\\
&= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt[3]{x}}\right)+(3ben)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\\
&= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt[3]{x}}\right)-3bn\text{Li}_2\left(1+\frac{e}{d\sqrt[3]{x}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}\,dx = -3b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\log\left(-\frac{e}{d\sqrt[3]{x}}\right) + a\log(x) - 3bn\,\text{PolyLog}\left(2, \frac{d+\frac{e}{\sqrt[3]{x}}}{d}\right)$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])/x,x]
```

```
[Out] -3*b*Log[c*(d + e/x^(1/3))^n]*Log[-(e/(d*x^(1/3)))] + a*Log[x] - 3*b*n*Poly
Log[2, (d + e/x^(1/3))/d]
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)}{x}dx$$

[In] int((a+b*ln(c*(d+e/x^(1/3))^n))/x,x)

[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))/x,x)

Fricas [F]

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)}{x}\,dx=\int \frac{b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a}{x}\,dx$$

[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x,x, algorithm="fricas")

[Out] integral((b*log(c*((d*x + e*x^(2/3))/x)^n) + a)/x, x)

Sympy [F]

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)}{x}\,dx=\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)}{x}\,dx$$

[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))/x,x)

[Out] Integral((a + b*log(c*(d + e/x**(1/3))**n))/x, x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(44) = 88$.

Time = 0.46 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.63

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)}{x}\,dx=-3\left(\log\left(\frac{dx^{\frac{1}{3}}}{e}+1\right)\log\left(x^{\frac{1}{3}}\right)+\mathrm{Li}_2\left(-\frac{dx^{\frac{1}{3}}}{e}\right)\right)bn$$
$$+\frac{2\,be^{2}n\log(x)^{2}+12\,be^{2}\log\left(\left(dx^{\frac{1}{3}}+e\right)^{n}\right)\log(x)-12\,be^{2}\log(x)\log\left(x^{\frac{1}{3}n}\right)+9\,bd^{2}nx^{\frac{2}{3}}-36\,bdenx^{\frac{1}{3}}-6}{12}$$

[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x,x, algorithm="maxima")

[Out] -3*(log(d*x^(1/3)/e + 1)*log(x^(1/3)) + dilog(-d*x^(1/3)/e))*b*n + 1/12*(2*b*e^2*n*log(x)^2 + 12*b*e^2*log((d*x^(1/3) + e)^n)*log(x) - 12*b*e^2*log(x)*log(x^(1/3*n)) + 9*b*d^2*n*x^(2/3) - 36*b*d*e*n*x^(1/3) - 6*(b*d^2*n*x^(2/3) - 2*b*d*e*n*x^(1/3))*log(x) + 12*(b*e^2*log(c) + a*e^2)*log(x) + 3*(2*b*d^2*n*x*log(x) - 3*b*d^2*n*x)/x^(1/3) - 12*(b*d*e*n*x*log(x) - 3*b*d*e*n*x)/x^(2/3))/e^2

Giac [F]

$$\int \frac{a + b\log\left(c\left(d + \frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}\,dx = \int \frac{b\log\left(c\left(d + \frac{e}{x^{\frac{1}{3}}}\right)^n\right) + a}{x}\,dx$$

[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x,x, algorithm="giac")

[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)/x, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c\left(d + \frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}\,dx = \int \frac{a + b\,\ln\left(c\left(d + \frac{e}{x^{1/3}}\right)^n\right)}{x}\,dx$$

[In] int((a + b*log(c*(d + e/x^(1/3))^n))/x,x)

[Out] int((a + b*log(c*(d + e/x^(1/3))^n))/x, x)

3.494 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^2}\,dx=\frac{bn}{3x}-\frac{bdn}{2ex^{2/3}}+\frac{bd^2n}{e^2\sqrt[3]{x}}-\frac{bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}$$

```
[Out] 1/3*b*n/x-1/2*b*d*n/e/x^(2/3)+b*d^2*n/e^2/x^(1/3)-b*d^3*n*ln(d+e/x^(1/3))/e
^3+(-a-b*ln(c*(d+e/x^(1/3))^n))/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 45}

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^2}\,dx=-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}-\frac{bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}+\frac{bd^2n}{e^2\sqrt[3]{x}}-\frac{bdn}{2ex^{2/3}}+\frac{bn}{3x}$$

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])/x^2,x]
```

```
[Out] (b*n)/(3*x) - (b*d*n)/(2*e*x^(2/3)) + (b*d^2*n)/(e^2*x^(1/3)) - (b*d^3*n*Lo
g[d + e/x^(1/3)])/e^3 - (a + b*Log[c*(d + e/x^(1/3))^n])/x
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(3\text{Subst}\left(\int x^2(a+b\log\left(c(d+ex)^n\right))\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)\\
&= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x} + (ben)\text{Subst}\left(\int \frac{x^3}{d+ex}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\\
&= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x} + (ben)\text{Subst}\left(\int \left(\frac{d^2}{e^3}-\frac{dx}{e^2}+\frac{x^2}{e}-\frac{d^3}{e^3(d+ex)}\right)\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\\
&= \frac{bn}{3x}-\frac{bdn}{2ex^{2/3}}+\frac{bd^2n}{e^2\sqrt[3]{x}}-\frac{bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^2}\,dx=-\frac{a}{x}+\frac{bn}{3x}-\frac{bdn}{2ex^{2/3}}+\frac{bd^2n}{e^2\sqrt[3]{x}}-\frac{bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}-\frac{b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x}$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])/x^2,x]
```

```
[Out] -(a/x) + (b*n)/(3*x) - (b*d*n)/(2*e*x^(2/3)) + (b*d^2*n)/(e^2*x^(1/3)) - (b
*d^3*n*Log[d + e/x^(1/3)])/e^3 - (b*Log[c*(d + e/x^(1/3))^n])/x
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)}{x^2}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))/x^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.30

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^2}\,dx=\frac{6\,bd^2enx^{\frac{2}{3}}-3\,bde^2nx^{\frac{1}{3}}+2\,be^3n-6\,ae^3-2\,(be^3n-3\,ae^3)x+6\,(be^3x-be^3)\log\left(c\right)-6\,(bd^3nx+be^3n)\log}{6\,e^3x}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(6*b*d^2*e*n*x^(2/3) - 3*b*d*e^2*n*x^(1/3) + 2*b*e^3*n - 6*a*e^3 - 2*(b
*e^3*n - 3*a*e^3)*x + 6*(b*e^3*x - b*e^3)*log(c) - 6*(b*d^3*n*x + b*e^3*n)*
log((d*x + e*x^(2/3))/x))/(e^3*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))/x**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\begin{aligned}
&\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^2}\,dx \\
&= -\frac{1}{6}\,ben\left(\frac{6\,d^3\log\left(dx^{\frac{1}{3}}+e\right)}{e^4}-\frac{2\,d^3\log\left(x\right)}{e^4}-\frac{6\,d^2x^{\frac{2}{3}}-3\,dex^{\frac{1}{3}}+2\,e^2}{e^3x}\right) \\
&\quad -\frac{b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)}{x}-\frac{a}{x}
\end{aligned}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^2,x, algorithm="maxima")
```

```
[Out] -1/6*b*e*n*(6*d^3*log(d*x^(1/3) + e)/e^4 - 2*d^3*log(x)/e^4 - (6*d^2*x^(2/3
) - 3*d*e*x^(1/3) + 2*e^2)/(e^3*x)) - b*log(c*(d + e/x^(1/3))^n)/x - a/x
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.17

$$\begin{aligned}
&\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^2}\,dx = \\
&-\frac{1}{6}\left(e\left(\frac{6\,d^3\log\left(\left|dx^{\frac{1}{3}}+e\right|\right)}{e^4}-\frac{2\,d^3\log\left(|x|\right)}{e^4}-\frac{6\,d^2ex^{\frac{2}{3}}-3\,de^2x^{\frac{1}{3}}+2\,e^3}{e^4x}\right)+\frac{6\,\log\left(d+\frac{e}{x^{\frac{1}{3}}}\right)}{x}\right)bn \\
&-\frac{b\log\left(c\right)}{x}-\frac{a}{x}
\end{aligned}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^2,x, algorithm="giac")
```

```
[Out] -1/6*(e*(6*d^3*log(abs(d*x^(1/3) + e))/e^4 - 2*d^3*log(abs(x))/e^4 - (6*d^2
*e*x^(2/3) - 3*d*e^2*x^(1/3) + 2*e^3)/(e^4*x)) + 6*log(d + e/x^(1/3))/x)*b*
n - b*log(c)/x - a/x
```

Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.89

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^2}\,dx = \frac{b\,n}{3\,x}-\frac{a}{x}-\frac{b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)}{x}-\frac{b\,d\,n}{2\,e\,x^{2/3}} -\frac{b\,d^3\,n\,\ln\left(d+\frac{e}{x^{1/3}}\right)}{e^3}+\frac{b\,d^2\,n}{e^2\,x^{1/3}}$$

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))/x^2,x)
```

```
[Out] (b*n)/(3*x) - a/x - (b*log(c*(d + e/x^(1/3))^n))/x - (b*d*n)/(2*e*x^(2/3))
- (b*d^3*n*log(d + e/x^(1/3)))/e^3 + (b*d^2*n)/(e^2*x^(1/3))
```

3.495 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 138

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx=\frac{bn}{12x^2}-\frac{bdn}{10ex^{5/3}}+\frac{bd^2n}{8e^2x^{4/3}}-\frac{bd^3n}{6e^3x}+\frac{bd^4n}{4e^4x^{2/3}}-\frac{bd^5n}{2e^5\sqrt[3]{x}}+\frac{bd^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2e^6}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2x^2}$$

```
[Out] 1/12*b*n/x^2-1/10*b*d*n/e/x^(5/3)+1/8*b*d^2*n/e^2/x^(4/3)-1/6*b*d^3*n/e^3/x
+1/4*b*d^4*n/e^4/x^(2/3)-1/2*b*d^5*n/e^5/x^(1/3)+1/2*b*d^6*n*ln(d+e/x^(1/3)
)/e^6+1/2*(-a-b*ln(c*(d+e/x^(1/3))^n))/x^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 45}

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx=-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2x^2}+\frac{bd^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2e^6}-\frac{bd^5n}{2e^5\sqrt[3]{x}}+\frac{bd^4n}{4e^4x^{2/3}}-\frac{bd^3n}{6e^3x}+\frac{bd^2n}{8e^2x^{4/3}}-\frac{bdn}{10ex^{5/3}}+\frac{bn}{12x^2}$$

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])/x^3,x]
```

```
[Out] (b*n)/(12*x^2) - (b*d*n)/(10*e*x^(5/3)) + (b*d^2*n)/(8*e^2*x^(4/3)) - (b*d^
3*n)/(6*e^3*x) + (b*d^4*n)/(4*e^4*x^(2/3)) - (b*d^5*n)/(2*e^5*x^(1/3)) + (b
*d^6*n*Log[d + e/x^(1/3)])/(2*e^6) - (a + b*Log[c*(d + e/x^(1/3))^n])/(2*x^
2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(3\text{Subst}\left(\int x^5(a+b\log\left(c(d+ex)^n\right))\,dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)\\
&= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\text{Subst}\left(\int\frac{x^6}{d+ex}\,dx,x,\frac{1}{\sqrt[3]{x}}\right)\\
&= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\text{Subst}\left(\int\left(-\frac{d^5}{e^6}+\frac{d^4x}{e^5}-\frac{d^3x^2}{e^4}+\frac{d^2x^3}{e^3}\right.\right.\\
&\qquad\left.\left.-\frac{dx^4}{e^2}+\frac{x^5}{e}+\frac{d^6}{e^6(d+ex)}\right)\,dx,x,\frac{1}{\sqrt[3]{x}}\right)\\
&= \frac{bn}{12x^2}-\frac{bdn}{10ex^{5/3}}+\frac{bd^2n}{8e^2x^{4/3}}-\frac{bd^3n}{6e^3x}+\frac{bd^4n}{4e^4x^{2/3}}-\frac{bd^5n}{2e^5\sqrt[3]{x}}\\
&\quad+\frac{bd^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2e^6}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx=-\frac{a}{2x^2}-\frac{1}{6}ben\left(-\frac{1}{2ex^2}+\frac{3d}{5e^2x^{5/3}}-\frac{3d^2}{4e^3x^{4/3}}+\frac{d^3}{e^4x}-\frac{3d^4}{2e^5x^{2/3}}+\frac{3d^5}{e^6\sqrt[3]{x}}-\frac{3d^6\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^7}\right)-\frac{b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{2x^2}$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*e*n*(-1/2*1/(e*x^2) + (3*d)/(5*e^2*x^(5/3)) - (3*d^2)/(4*e^
3*x^(4/3)) + d^3/(e^4*x) - (3*d^4)/(2*e^5*x^(2/3)) + (3*d^5)/(e^6*x^(1/3))
- (3*d^6*Log[d + e/x^(1/3)])/e^7))/6 - (b*Log[c*(d + e/x^(1/3))^n])/(2*x^2)
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)}{x^3}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.20

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx = -\frac{20\,bd^3e^3nx-10\,be^6n+60\,ae^6-10\,(6\,ae^6+(2\,bd^3e^3-be^6)n)x^2-60\,(be^6x^2-be^6)\log\left(c\right)-60\,(bd^6nx^2}{120\,e^6x^2}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^3,x, algorithm="fricas")
```

```
[Out] -1/120*(20*b*d^3*e^3*n*x - 10*b*e^6*n + 60*a*e^6 - 10*(6*a*e^6 + (2*b*d^3*e
^3 - b*e^6)*n)*x^2 - 60*(b*e^6*x^2 - b*e^6)*log(c) - 60*(b*d^6*n*x^2 - b*e^
6*n)*log((d*x + e*x^(2/3))/x) + 15*(4*b*d^5*e*n*x - b*d^2*e^4*n)*x^(2/3) -
6*(5*b*d^4*e^2*n*x - 2*b*d*e^5*n)*x^(1/3))/(e^6*x^2)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))/x**3,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx$$

$$= \frac{1}{120}\,ben\left(\frac{60\,d^6\log\left(dx^{\frac{1}{3}}+e\right)}{e^7} - \frac{20\,d^6\log\left(x\right)}{e^7} - \frac{60\,d^5x^{\frac{5}{3}} - 30\,d^4ex^{\frac{4}{3}} + 20\,d^3e^2x - 15\,d^2e^3x^{\frac{2}{3}} + 12\,de^4x^{\frac{1}{3}} - \cdots}{e^6x^2}\right.$$

$$- \frac{b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)}{2\,x^2} - \frac{a}{2\,x^2}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^3,x, algorithm="maxima")
[Out] 1/120*b*e*n*(60*d^6*log(d*x^(1/3) + e)/e^7 - 20*d^6*log(x)/e^7 - (60*d^5*x^
(5/3) - 30*d^4*e*x^(4/3) + 20*d^3*e^2*x - 15*d^2*e^3*x^(2/3) + 12*d*e^4*x^(
1/3) - 10*e^5)/(e^6*x^2)) - 1/2*b*log(c*(d + e/x^(1/3))^n)/x^2 - 1/2*a/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx$$

$$= \frac{1}{120}\left(e\left(\frac{60\,d^6\log\left(\left|dx^{\frac{1}{3}}+e\right|\right)}{e^7} - \frac{20\,d^6\log\left(|x|\right)}{e^7} - \frac{60\,d^5ex^{\frac{5}{3}} - 30\,d^4e^2x^{\frac{4}{3}} + 20\,d^3e^3x - 15\,d^2e^4x^{\frac{2}{3}} + 12\,de^5x\cdots}{e^7x^2}\right.\right.$$

$$- \frac{b\log\left(c\right)}{2\,x^2} - \frac{a}{2\,x^2}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/120*(e*(60*d^6*log(abs(d*x^(1/3) + e))/e^7 - 20*d^6*log(abs(x))/e^7 - (60
*d^5*e*x^(5/3) - 30*d^4*e^2*x^(4/3) + 20*d^3*e^3*x - 15*d^2*e^4*x^(2/3) + 1
2*d*e^5*x^(1/3) - 10*e^6)/(e^7*x^2)) - 60*log(d + e/x^(1/3))/x^2)*b*n - 1/2
*b*log(c)/x^2 - 1/2*a/x^2
```

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^3}\,dx = \frac{b\,n}{12\,x^2}-\frac{a}{2\,x^2}-\frac{b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)}{2\,x^2}-\frac{b\,d\,n}{10\,e\,x^{5/3}}+\frac{b\,d^6\,n\,\ln\left(d+\frac{e}{x^{1/3}}\right)}{2\,e^6}-\frac{b\,d^3\,n}{6\,e^3\,x}+\frac{b\,d^2\,n}{8\,e^2\,x^{4/3}}+\frac{b\,d^4\,n}{4\,e^4\,x^{2/3}}-\frac{b\,d^5\,n}{2\,e^5\,x^{1/3}}$$

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))/x^3,x)
```

```
[Out] (b*n)/(12*x^2) - a/(2*x^2) - (b*log(c*(d + e/x^(1/3))^n))/(2*x^2) - (b*d*n)
/(10*e*x^(5/3)) + (b*d^6*n*log(d + e/x^(1/3)))/(2*e^6) - (b*d^3*n)/(6*e^3*x
) + (b*d^2*n)/(8*e^2*x^(4/3)) + (b*d^4*n)/(4*e^4*x^(2/3)) - (b*d^5*n)/(2*e^
5*x^(1/3))
```

3.496 $\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 187

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx = \frac{bn}{27x^3}-\frac{bdn}{24ex^{8/3}}+\frac{bd^2n}{21e^2x^{7/3}}-\frac{bd^3n}{18e^3x^2}+\frac{bd^4n}{15e^4x^{5/3}}-\frac{bd^5n}{12e^5x^{4/3}}+\frac{bd^6n}{9e^6x}-\frac{bd^7n}{6e^7x^{2/3}}+\frac{bd^8n}{3e^8\sqrt[3]{x}}-\frac{bd^9n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{3e^9}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x^3}$$

```
[Out] 1/27*b*n/x^3-1/24*b*d*n/e/x^(8/3)+1/21*b*d^2*n/e^2/x^(7/3)-1/18*b*d^3*n/e^3
/x^2+1/15*b*d^4*n/e^4/x^(5/3)-1/12*b*d^5*n/e^5/x^(4/3)+1/9*b*d^6*n/e^6/x-1/
6*b*d^7*n/e^7/x^(2/3)+1/3*b*d^8*n/e^8/x^(1/3)-1/3*b*d^9*n*ln(d+e/x^(1/3))/e
^9+1/3*(-a-b*ln(c*(d+e/x^(1/3))^n))/x^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used

$= \{2504, 2442, 45\}$

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx = -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x^3} - \frac{bd^9n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{3e^9} + \frac{bd^8n}{3e^8\sqrt[3]{x}} - \frac{bd^7n}{6e^7x^{2/3}} + \frac{bd^6n}{9e^6x} - \frac{bd^5n}{12e^5x^{4/3}} + \frac{bd^4n}{15e^4x^{5/3}} - \frac{bd^3n}{18e^3x^2} + \frac{bd^2n}{21e^2x^{7/3}} - \frac{bdn}{24ex^{8/3}} + \frac{bn}{27x^3}$$

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])/x^4,x]
```

```
[Out] (b*n)/(27*x^3) - (b*d*n)/(24*e*x^(8/3)) + (b*d^2*n)/(21*e^2*x^(7/3)) - (b*d
^3*n)/(18*e^3*x^2) + (b*d^4*n)/(15*e^4*x^(5/3)) - (b*d^5*n)/(12*e^5*x^(4/3)
) + (b*d^6*n)/(9*e^6*x) - (b*d^7*n)/(6*e^7*x^(2/3)) + (b*d^8*n)/(3*e^8*x^(1
/3)) - (b*d^9*n*Log[d + e/x^(1/3)])/(3*e^9) - (a + b*Log[c*(d + e/x^(1/3))^
n])/(3*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = -\left(3\text{Subst}\left(\int x^8(a+b\log\left(c(d+ex)^n\right))\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$
\begin{aligned}
&= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x^3}+\frac{1}{3}(ben)\text{Subst}\left(\int \frac{x^9}{d+ex}\,dx,x,\frac{1}{\sqrt[3]{x}}\right)\\
&= -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x^3}+\frac{1}{3}(ben)\text{Subst}\left(\int \left(\frac{d^8}{e^9}-\frac{d^7x}{e^8}+\frac{d^6x^2}{e^7}-\frac{d^5x^3}{e^6}\right.\right.\\
&\qquad \left.\left.+\frac{d^4x^4}{e^5}-\frac{d^3x^5}{e^4}+\frac{d^2x^6}{e^3}-\frac{dx^7}{e^2}+\frac{x^8}{e}-\frac{d^9}{e^9(d+ex)}\right)\,dx,x,\frac{1}{\sqrt[3]{x}}\right)\\
&= \frac{bn}{27x^3}-\frac{bdn}{24ex^{8/3}}+\frac{bd^2n}{21e^2x^{7/3}}-\frac{bd^3n}{18e^3x^2}+\frac{bd^4n}{15e^4x^{5/3}}-\frac{bd^5n}{12e^5x^{4/3}}+\frac{bd^6n}{9e^6x}\\
&\quad -\frac{bd^7n}{6e^7x^{2/3}}+\frac{bd^8n}{3e^8\sqrt[3]{x}}-\frac{bd^9n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{3e^9}-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.95

$$
\begin{aligned}
\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx &= -\frac{a}{3x^3}-\frac{1}{9}ben\left(-\frac{1}{3ex^3}+\frac{3d}{8e^2x^{8/3}}-\frac{3d^2}{7e^3x^{7/3}}+\frac{d^3}{2e^4x^2}\right.\\
&\quad -\frac{3d^4}{5e^5x^{5/3}}+\frac{3d^5}{4e^6x^{4/3}}-\frac{d^6}{e^7x}+\frac{3d^7}{2e^8x^{2/3}}-\frac{3d^8}{e^9\sqrt[3]{x}}\\
&\quad \left.+\frac{3d^9\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^{10}}\right)-\frac{b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{3x^3}
\end{aligned}
$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])/x^4,x]
[Out] -1/3*a/x^3 - (b*e*n*(-1/3*1/(e*x^3) + (3*d)/(8*e^2*x^(8/3)) - (3*d^2)/(7*e^
3*x^(7/3)) + d^3/(2*e^4*x^2) - (3*d^4)/(5*e^5*x^(5/3)) + (3*d^5)/(4*e^6*x^(
4/3)) - d^6/(e^7*x) + (3*d^7)/(2*e^8*x^(2/3)) - (3*d^8)/(e^9*x^(1/3)) + (3*
d^9*Log[d + e/x^(1/3)])/e^10))/9 - (b*Log[c*(d + e/x^(1/3))^n])/(3*x^3)
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)}{x^4}dx$$

[In] int((a+b*ln(c*(d+e/x^(1/3))^n))/x^4,x)

[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))/x^4,x)

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.14

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx = \frac{840\,bd^6e^3nx^2 - 420\,bd^3e^6nx + 280\,be^9n - 2520\,ae^9 + 140\,(18\,ae^9 - (6\,bd^6e^3 - 3\,bd^3e^6 + 2\,be^9)n)x^3 + 2520}{}$$

[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^4,x, algorithm="fricas")

[Out] 1/7560*(840*b*d^6*e^3*n*x^2 - 420*b*d^3*e^6*n*x + 280*b*e^9*n - 2520*a*e^9 + 140*(18*a*e^9 - (6*b*d^6*e^3 - 3*b*d^3*e^6 + 2*b*e^9)*n)*x^3 + 2520*(b*e^9*x^3 - b*e^9)*log(c) - 2520*(b*d^9*n*x^3 + b*e^9*n)*log((d*x + e*x^(2/3))/x) + 90*(28*b*d^8*e*n*x^2 - 7*b*d^5*e^4*n*x + 4*b*d^2*e^7*n)*x^(2/3) - 63*(20*b*d^7*e^2*n*x^2 - 8*b*d^4*e^5*n*x + 5*b*d*e^8*n)*x^(1/3))/(e^9*x^3)

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx = \text{Timed out}$$

[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))/x**4,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.80

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx = -\frac{1}{7560}\,ben\left(\frac{2520\,d^9\log\left(dx^{\frac{1}{3}}+e\right)}{e^{10}} - \frac{840\,d^9\log\left(x\right)}{e^{10}} - \frac{2520\,d^8x^{\frac{8}{3}} - 1260\,d^7ex^{\frac{7}{3}} + 840\,d^6e^2x^2 - 630\,d^5e^3x^{\frac{5}{3}}}{} \right. - \frac{b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)}{3\,x^3} - \frac{a}{3\,x^3}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^4,x, algorithm="maxima")
```

```
[Out] -1/7560*b*e*n*(2520*d^9*log(d*x^(1/3) + e)/e^10 - 840*d^9*log(x)/e^10 - (25
20*d^8*x^(8/3) - 1260*d^7*e*x^(7/3) + 840*d^6*e^2*x^2 - 630*d^5*e^3*x^(5/3)
 + 504*d^4*e^4*x^(4/3) - 420*d^3*e^5*x + 360*d^2*e^6*x^(2/3) - 315*d*e^7*x^
(1/3) + 280*e^8)/(e^9*x^3)) - 1/3*b*log(c*(d + e/x^(1/3))^n)/x^3 - 1/3*a/x^
3
```

Giac [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.86

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx = -\frac{1}{7560}\left(e\left(\frac{2520\,d^9\log\left(\left|dx^{\frac{1}{3}}+e\right|\right)}{e^{10}} - \frac{840\,d^9\log\left(|x|\right)}{e^{10}} - \frac{2520\,d^8ex^{\frac{8}{3}} - 1260\,d^7e^2x^{\frac{7}{3}} + 840\,d^6e^3x^2 - 630\,d^5}{}\right.\right. - \frac{b\log\left(c\right)}{3\,x^3} - \frac{a}{3\,x^3}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))/x^4,x, algorithm="giac")
```

```
[Out] -1/7560*(e*(2520*d^9*log(abs(d*x^(1/3) + e))/e^10 - 840*d^9*log(abs(x))/e^1
0 - (2520*d^8*e*x^(8/3) - 1260*d^7*e^2*x^(7/3) + 840*d^6*e^3*x^2 - 630*d^5*
e^4*x^(5/3) + 504*d^4*e^5*x^(4/3) - 420*d^3*e^6*x + 360*d^2*e^7*x^(2/3) - 3
15*d*e^8*x^(1/3) + 280*e^9)/(e^10*x^3)) + 2520*log(d + e/x^(1/3))/x^3)*b*n
 - 1/3*b*log(c)/x^3 - 1/3*a/x^3
```

Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.81

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{x^4}\,dx = \frac{b\,n}{27\,x^3}-\frac{a}{3\,x^3}-\frac{b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{3\,x^3}-\frac{b\,d\,n}{24\,e\,x^{8/3}}-\frac{b\,d^9\,n\,\ln\left(d+\frac{e}{x^{1/3}}\right)}{3\,e^9}-\frac{b\,d^3\,n}{18\,e^3\,x^2}+\frac{b\,d^6\,n}{9\,e^6\,x}+\frac{b\,d^2\,n}{21\,e^2\,x^{7/3}}+\frac{b\,d^4\,n}{15\,e^4\,x^{5/3}}-\frac{b\,d^5\,n}{12\,e^5\,x^{4/3}}-\frac{b\,d^7\,n}{6\,e^7\,x^{2/3}}+\frac{b\,d^8\,n}{3\,e^8\,x^{1/3}}$$

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))/x^4,x)
```

```
[Out] (b*n)/(27*x^3) - a/(3*x^3) - (b*log(c*(d + e/x^(1/3))^n))/(3*x^3) - (b*d*n)
/(24*e*x^(8/3)) - (b*d^9*n*log(d + e/x^(1/3)))/(3*e^9) - (b*d^3*n)/(18*e^3*
x^2) + (b*d^6*n)/(9*e^6*x) + (b*d^2*n)/(21*e^2*x^(7/3)) + (b*d^4*n)/(15*e^4
*x^(5/3)) - (b*d^5*n)/(12*e^5*x^(4/3)) - (b*d^7*n)/(6*e^7*x^(2/3)) + (b*d^8
*n)/(3*e^8*x^(1/3))
```

3.497 $\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 572

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx$$
$$= \frac{481b^2e^8n^2\sqrt[3]{x}}{420d^8}-\frac{341b^2e^7n^2x^{2/3}}{840d^7}+\frac{743b^2e^6n^2x}{3780d^6}-\frac{533b^2e^5n^2x^{4/3}}{5040d^5}$$
$$+\frac{73b^2e^4n^2x^{5/3}}{1260d^4}-\frac{5b^2e^3n^2x^2}{168d^3}+\frac{b^2e^2n^2x^{7/3}}{84d^2}-\frac{481b^2e^9n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{420d^9}$$
$$-\frac{2be^8n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^9}$$
$$+\frac{be^7nx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^7}$$
$$-\frac{2be^6nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{9d^6}+\frac{be^5nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{6d^5}$$
$$-\frac{2be^4nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{15d^4}+\frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{9d^3}$$
$$-\frac{2be^2nx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{21d^2}+\frac{benx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{12d}$$
$$-\frac{2be^9n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^9}$$
$$+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-\frac{761b^2e^9n^2\log(x)}{1260d^9}+\frac{2b^2e^9n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{3d^9}$$

```
[Out] 481/420*b^2*e^8*n^2*x^(1/3)/d^8-341/840*b^2*e^7*n^2*x^(2/3)/d^7+743/3780*b^
2*e^6*n^2*x/d^6-533/5040*b^2*e^5*n^2*x^(4/3)/d^5+73/1260*b^2*e^4*n^2*x^(5/3
)/d^4-5/168*b^2*e^3*n^2*x^2/d^3+1/84*b^2*e^2*n^2*x^(7/3)/d^2-481/420*b^2*e^
9*n^2*ln(d+e/x^(1/3))/d^9-2/3*b*e^8*n*(d+e/x^(1/3))*x^(1/3)*(a+b*ln(c*(d+e/
x^(1/3))^n))/d^9+1/3*b*e^7*n*x^(2/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^7-2/9*b*
e^6*n*x*(a+b*ln(c*(d+e/x^(1/3))^n))/d^6+1/6*b*e^5*n*x^(4/3)*(a+b*ln(c*(d+e/
x^(1/3))^n))/d^5-2/15*b*e^4*n*x^(5/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^4+1/9*b
*e^3*n*x^2*(a+b*ln(c*(d+e/x^(1/3))^n))/d^3-2/21*b*e^2*n*x^(7/3)*(a+b*ln(c*(
d+e/x^(1/3))^n))/d^2+1/12*b*e*n*x^(8/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d-2/3*b
*e^9*n*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(c*(d+e/x^(1/3))^n))/d^9+1/3*x^3*(a+b*l
```

```
n(c*(d+e/x^(1/3))^n))^2-761/1260*b^2*e^9*n^2*ln(x)/d^9+2/3*b^2*e^9*n^2*poly
log(2,d/(d+e/x^(1/3)))/d^9
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 572, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2dx$$
$$= -\frac{2be^9n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^9} - \frac{2be^8n\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^9} + \frac{be^7nx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^7} - \frac{2be^6nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{9d^6} + \frac{be^5nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{6d^5} - \frac{2be^4nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{15d^4} + \frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{9d^3} - \frac{2be^2nx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{21d^2} + \frac{benx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{12d} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 + \frac{2b^2e^9n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{3d^9} - \frac{481b^2e^9n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{420d^9} - \frac{761b^2e^9n}{126}$$

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] (481*b^2*e^8*n^2*x^(1/3))/(420*d^8) - (341*b^2*e^7*n^2*x^(2/3))/(840*d^7) +
 (743*b^2*e^6*n^2*x)/(3780*d^6) - (533*b^2*e^5*n^2*x^(4/3))/(5040*d^5) + (7
3*b^2*e^4*n^2*x^(5/3))/(1260*d^4) - (5*b^2*e^3*n^2*x^2)/(168*d^3) + (b^2*e^
2*n^2*x^(7/3))/(84*d^2) - (481*b^2*e^9*n^2*Log[d + e/x^(1/3)])/(420*d^9) -
(2*b*e^8*n*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*d^9
) + (b*e^7*n*x^(2/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*d^7) - (2*b*e^6*n
*x*(a + b*Log[c*(d + e/x^(1/3))^n]))/(9*d^6) + (b*e^5*n*x^(4/3)*(a + b*Log[
c*(d + e/x^(1/3))^n]))/(6*d^5) - (2*b*e^4*n*x^(5/3)*(a + b*Log[c*(d + e/x^(
```

```
1/3))^n]))/(15*d^4) + (b*e^3*n*x^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/(9*d^3
) - (2*b*e^2*n*x^(7/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(21*d^2) + (b*e*n*
x^(8/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(12*d) - (2*b*e^9*n*Log[1 - d/(d
+ e/x^(1/3))]*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*d^9) + (x^3*(a + b*Log[c
*(d + e/x^(1/3))^n])^2)/3 - (761*b^2*e^9*n^2*Log[x])/(1260*d^9) + (2*b^2*e^
9*n^2*PolyLog[2, d/(d + e/x^(1/3))])/(3*d^9)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
```

```
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(3\text{Subst}\left(\int \frac{(a+b\log (c(d+ex)^n))^2}{x^{10}}\,dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-\frac{1}{3}(2ben)\text{Subst}\left(\int \frac{a+b\log (c(d+ex)^n)}{x^9(d+ex)}\,dx,x,\frac{1}{\sqrt[3]{x}}\right)\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-\frac{1}{3}(2bn)\text{Subst}\left(\int \frac{a+b\log (cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^9}\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)
\end{aligned}$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{(2bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^9}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{3d} + \frac{(2ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^8}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{3d}$$

$$= \frac{benx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{12d} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 + \frac{(2ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^8}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{3d^2} - \frac{(2be^2n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^7}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{3d^2} - \frac{(b^2en^2)\,\text{Subst}\left(\int \frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^8}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{12d}$$

$$= -\frac{2be^2nx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{21d^2} + \frac{benx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{12d} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{(2be^2n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^7}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{3d^3} + \frac{(2be^3n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{3d^3} - \frac{(b^2en^2)\,\text{Subst}\left(\int \left(\frac{e^8}{d(d-x)^8} + \frac{e^8}{d^2(d-x)^7} + \frac{e^8}{d^3(d-x)^6} + \frac{e^8}{d^4(d-x)^5} + \frac{e^8}{d^5(d-x)^4} + \frac{e^8}{d^6(d-x)^3} + \frac{e^8}{d^7(d-x)^2} + \frac{e^8}{d^8(d-\right.\right.}{12d} + \frac{(2b^2e^2n^2)\,\text{Subst}\left(\int \frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^7}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{21d^2}$$

$$= \frac{b^2e^8n^2\sqrt[3]{x}}{12d^8} - \frac{b^2e^7n^2x^{2/3}}{24d^7} + \frac{b^2e^6n^2x}{36d^6} - \frac{b^2e^5n^2x^{4/3}}{48d^5} + \frac{b^2e^4n^2x^{5/3}}{60d^4} - \frac{b^2e^3n^2x^2}{72d^3} + \frac{b^2e^2n^2x^{7/3}}{84d^2} - \frac{b^2e^9n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{12d^9} + \frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{9d^3} - \frac{2be^2nx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{21d^2} + \frac{benx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{12d} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{b^2e^9n^2\log(x)}{36d^9} + \frac{(2be^3n)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{3d^4} -$$

$$= \frac{5b^2e^8n^2\sqrt[3]{x}}{28d^8} - \frac{5b^2e^7n^2x^{2/3}}{56d^7} + \frac{5b^2e^6n^2x}{84d^6} - \frac{5b^2e^5n^2x^{4/3}}{112d^5} + \frac{b^2e^4n^2x^{5/3}}{28d^4} - \frac{5b^2e^3n^2x^2}{168d^3} + \frac{b^2e^2n^2x^{7/3}}{84d^2} - \frac{5b^2e^9n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{28d^9} - \frac{2be^4nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{15d^4} + \frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{9d^3} - \frac{2be^2nx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{21d^2} + \frac{benx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{12d} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{5b^2e^9n^2\log(x)}{84d^9} - \frac{(2be^4n)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{3d^5}$$

$$= \frac{73b^2e^8n^2\sqrt[3]{x}}{252d^8} - \frac{73b^2e^7n^2x^{2/3}}{504d^7} + \frac{73b^2e^6n^2x}{756d^6} - \frac{73b^2e^5n^2x^{4/3}}{1008d^5} + \frac{73b^2e^4n^2x^{5/3}}{1260d^4} - \frac{5b^2e^3n^2x^2}{168d^3} + \frac{b^2e^2n^2x^{7/3}}{84d^2} - \frac{73b^2e^9n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{252d^9} + \frac{be^5nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{6d^5} - \frac{2be^4nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{15d^4} + \frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{9d^3} - \frac{2be^2nx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{21d^2} + \frac{benx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{12d} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{73b^2e^9n^2\log(x)}{756d^9} + \frac{(2be^5n)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{3d^6}$$

$$
\begin{aligned}
&= \frac{533b^2e^8n^2\sqrt[3]{x}}{1260d^8} - \frac{533b^2e^7n^2x^{2/3}}{2520d^7} + \frac{533b^2e^6n^2x}{3780d^6} - \frac{533b^2e^5n^2x^{4/3}}{5040d^5} \\
&\quad + \frac{73b^2e^4n^2x^{5/3}}{1260d^4} - \frac{5b^2e^3n^2x^2}{168d^3} + \frac{b^2e^2n^2x^{7/3}}{84d^2} - \frac{533b^2e^9n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{1260d^9} \\
&\quad - \frac{2be^6nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{9d^6} + \frac{be^5nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{6d^5} \\
&\quad - \frac{2be^4nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{15d^4} + \frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{9d^3} \\
&\quad - \frac{2be^2nx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{21d^2} + \frac{benx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{12d} \\
&\quad + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{533b^2e^9n^2\log(x)}{3780d^9} - \frac{(2be^6n)\,\mathrm{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx,x,d+\frac{e}{\sqrt[3]{}}\right.}{3d^7}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{743b^2e^8n^2\sqrt[3]{x}}{1260d^8} - \frac{743b^2e^7n^2x^{2/3}}{2520d^7} + \frac{743b^2e^6n^2x}{3780d^6} - \frac{533b^2e^5n^2x^{4/3}}{5040d^5} + \frac{73b^2e^4n^2x^{5/3}}{1260d^4} \\
&\quad - \frac{5b^2e^3n^2x^2}{168d^3} + \frac{b^2e^2n^2x^{7/3}}{84d^2} - \frac{743b^2e^9n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{1260d^9} + \frac{be^7nx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^7} \\
&\quad - \frac{2be^6nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{9d^6} + \frac{be^5nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{6d^5} \\
&\quad - \frac{2be^4nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{15d^4} + \frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{9d^3} \\
&\quad - \frac{2be^2nx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{21d^2} + \frac{benx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{12d} \\
&\quad + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{743b^2e^9n^2\log(x)}{3780d^9} + \frac{(2be^7n)\,\mathrm{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+\frac{e}{\sqrt[3]{}}\right.}{3d^8}
\end{aligned}
$$

$$= \frac{341b^2e^8n^2\sqrt[3]{x}}{420d^8} - \frac{341b^2e^7n^2x^{2/3}}{840d^7} + \frac{743b^2e^6n^2x}{3780d^6} - \frac{533b^2e^5n^2x^{4/3}}{5040d^5}$$
$$+ \frac{73b^2e^4n^2x^{5/3}}{1260d^4} - \frac{5b^2e^3n^2x^2}{168d^3} + \frac{b^2e^2n^2x^{7/3}}{84d^2} - \frac{341b^2e^9n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{420d^9}$$
$$- \frac{2be^8n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^9}$$
$$+ \frac{be^7nx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^7}$$
$$- \frac{2be^6nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{9d^6} + \frac{be^5nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{6d^5}$$
$$- \frac{2be^4nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{15d^4} + \frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{9d^3}$$
$$- \frac{2be^2nx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{21d^2} + \frac{benx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{12d}$$
$$- \frac{2be^9n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^9}$$
$$+ \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{341b^2e^9n^2\log(x)}{1260d^9} - \frac{(b^2e^7n^2)\,\mathrm{Subst}\left(\int\left(\frac{e^2}{d(d-x)^2}+\frac{e^2}{d^2(d-x)}+\frac{e^2}{d^2x}\right.\right.}{3d^7}$$

$$
\begin{aligned}
&= \frac{481b^2e^8n^2\sqrt[3]{x}}{420d^8} - \frac{341b^2e^7n^2x^{2/3}}{840d^7} + \frac{743b^2e^6n^2x}{3780d^6} - \frac{533b^2e^5n^2x^{4/3}}{5040d^5} \\
&+ \frac{73b^2e^4n^2x^{5/3}}{1260d^4} - \frac{5b^2e^3n^2x^2}{168d^3} + \frac{b^2e^2n^2x^{7/3}}{84d^2} - \frac{481b^2e^9n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{420d^9} \\
&- \frac{2be^8n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^9} \\
&+ \frac{be^7nx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^7} \\
&- \frac{2be^6nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{9d^6} + \frac{be^5nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{6d^5} \\
&- \frac{2be^4nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{15d^4} + \frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{9d^3} \\
&- \frac{2be^2nx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{21d^2} + \frac{benx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{12d} \\
&- \frac{2be^9n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^9} \\
&+ \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{761b^2e^9n^2\log(x)}{1260d^9} + \frac{2b^2e^9n^2\operatorname{Li}_2\left(\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{3d^9}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.05

$$
\begin{aligned}
\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx &= \frac{1}{3}\left(x^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\right. \\
&+ \frac{ben\left(-10080ade^7\sqrt[3]{x}+17316bde^7n\sqrt[3]{x}+5040ad^2e^6x^{2/3}-6138bd^2e^6nx^{2/3}-3360ad^3e^5x+2972bd^3e^5n\right.}{}
\end{aligned}
$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] (x^3*(a + b*Log[c*(d + e/x^(1/3))^n])^2 + (b*e*n*(-10080*a*d*e^7*x^(1/3) +
17316*b*d*e^7*n*x^(1/3) + 5040*a*d^2*e^6*x^(2/3) - 6138*b*d^2*e^6*n*x^(2/3)
 - 3360*a*d^3*e^5*x + 2972*b*d^3*e^5*n*x + 2520*a*d^4*e^4*x^(4/3) - 1599*b*
d^4*e^4*n*x^(4/3) - 2016*a*d^5*e^3*x^(5/3) + 876*b*d^5*e^3*n*x^(5/3) + 1680
*a*d^6*e^2*x^2 - 450*b*d^6*e^2*n*x^2 - 1440*a*d^7*e*x^(7/3) + 180*b*d^7*e*n
*x^(7/3) + 1260*a*d^8*x^(8/3) - 22356*b*e^8*n*Log[d + e/x^(1/3)] - 10080*b*
d*e^7*x^(1/3)*Log[c*(d + e/x^(1/3))^n] + 5040*b*d^2*e^6*x^(2/3)*Log[c*(d +
e/x^(1/3))^n] - 3360*b*d^3*e^5*x*Log[c*(d + e/x^(1/3))^n] + 2520*b*d^4*e^4*
x^(4/3)*Log[c*(d + e/x^(1/3))^n] - 2016*b*d^5*e^3*x^(5/3)*Log[c*(d + e/x^(1
/3))^n] + 1680*b*d^6*e^2*x^2*Log[c*(d + e/x^(1/3))^n] - 1440*b*d^7*e*x^(7/3
)*Log[c*(d + e/x^(1/3))^n] + 1260*b*d^8*x^(8/3)*Log[c*(d + e/x^(1/3))^n] +
10080*a*e^8*Log[e + d*x^(1/3)] - 5040*b*e^8*n*Log[e + d*x^(1/3)] + 10080*b*
e^8*Log[c*(d + e/x^(1/3))^n]*Log[e + d*x^(1/3)] - 5040*b*e^8*n*Log[e + d*x^
(1/3)]^2 + 10080*b*e^8*n*Log[e + d*x^(1/3)]*Log[-((d*x^(1/3))/e)] - 7452*b*
e^8*n*Log[x] + 10080*b*e^8*n*PolyLog[2, 1 + (d*x^(1/3))/e]))/(5040*d^9))/3
```

Maple [F]

$$\int x^2 \left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)\right)^2 dx$$

```
[In] int(x^2*(a+b*ln(c*(d+e/x^(1/3))^n))^2,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e/x^(1/3))^n))^2,x)
```

Fricas [F]

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx = \int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2 x^2\, dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*log(c*((d*x + e*x^(2/3))/x)^n)^2 + 2*a*b*x^2*log(c*((d*x +
 e*x^(2/3))/x)^n) + a^2*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2dx=\text{Timed out}$$

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(1/3))**n))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="maxima")
[Out] 1/3*b^2*x^3*log((d*x^(1/3) + e)^n)^2 - integrate(-1/9*(9*(b^2*d*log(c)^2 +
2*a*b*d*log(c) + a^2*d)*x^3 + 9*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x
^(8/3) + 9*(b^2*d*x^3 + b^2*e*x^(8/3))*log(x^(1/3*n))^2 - 2*(b^2*d*n*x^3 -
9*(b^2*d*log(c) + a*b*d)*x^3 - 9*(b^2*e*log(c) + a*b*e)*x^(8/3) + 9*(b^2*d*
x^3 + b^2*e*x^(8/3))*log(x^(1/3*n)))*log((d*x^(1/3) + e)^n) - 18*((b^2*d*lo
g(c) + a*b*d)*x^3 + (b^2*e*log(c) + a*b*e)*x^(8/3))*log(x^(1/3*n)))/(d*x +
e*x^(2/3)), x)
```

Giac [F]

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a + b \log \left(c \left(d + \frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx = \int x^2 \left(a + b \ln \left(c \left(d + \frac{e}{x^{1/3}}\right)^n\right)\right)^2 dx$$

```
[In] int(x^2*(a + b*log(c*(d + e/x^(1/3))^n))^2,x)
[Out] int(x^2*(a + b*log(c*(d + e/x^(1/3))^n))^2, x)
```

3.498 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 400

$$\begin{aligned}
&\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx \\
&= -\frac{77b^2e^5n^2\sqrt[3]{x}}{60d^5}+\frac{47b^2e^4n^2x^{2/3}}{120d^4}-\frac{3b^2e^3n^2x}{20d^3}+\frac{b^2e^2n^2x^{4/3}}{20d^2} \\
&\quad+\frac{77b^2e^6n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{60d^6}+\frac{be^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^6} \\
&\quad-\frac{be^4nx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2d^4}+\frac{be^3nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^3} \\
&\quad-\frac{be^2nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4d^2}+\frac{benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{5d} \\
&\quad+\frac{be^6n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^6} \\
&\quad+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2+\frac{137b^2e^6n^2\log(x)}{180d^6}-\frac{b^2e^6n^2\,\mathrm{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^6}
\end{aligned}$$

```
[Out] -77/60*b^2*e^5*n^2*x^(1/3)/d^5+47/120*b^2*e^4*n^2*x^(2/3)/d^4-3/20*b^2*e^3*
n^2*x/d^3+1/20*b^2*e^2*n^2*x^(4/3)/d^2+77/60*b^2*e^6*n^2*ln(d+e/x^(1/3))/d^
6+b*e^5*n*(d+e/x^(1/3))*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^6-1/2*b*e^4*n
```

```
*x^(2/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^4+1/3*b*e^3*n*x*(a+b*ln(c*(d+e/x^(1/
3))^n))/d^3-1/4*b*e^2*n*x^(4/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^2+1/5*b*e*n*x
^(5/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d+b*e^6*n*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(
c*(d+e/x^(1/3))^n))/d^6+1/2*x^2*(a+b*ln(c*(d+e/x^(1/3))^n))^2+137/180*b^2*e
^6*n^2*ln(x)/d^6-b^2*e^6*n^2*polylog(2,d/(d+e/x^(1/3)))/d^6
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx$$
$$=\frac{be^6n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^6}$$
$$+\frac{be^5n\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^6}$$
$$-\frac{be^4nx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2d^4}+\frac{be^3nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^3}$$
$$-\frac{be^2nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4d^2}+\frac{benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{5d}$$
$$+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-\frac{b^2e^6n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^6}+\frac{77b^2e^6n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{60d^6}+\frac{137b^2e^6n^2\,\mathrm{l}}{180d^6}$$

```
[In] Int[x*(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] (-77*b^2*e^5*n^2*x^(1/3))/(60*d^5) + (47*b^2*e^4*n^2*x^(2/3))/(120*d^4) - (
3*b^2*e^3*n^2*x)/(20*d^3) + (b^2*e^2*n^2*x^(4/3))/(20*d^2) + (77*b^2*e^6*n^
2*Log[d + e/x^(1/3)])/(60*d^6) + (b*e^5*n*(d + e/x^(1/3))*x^(1/3)*(a + b*Lo
g[c*(d + e/x^(1/3))^n]))/d^6 - (b*e^4*n*x^(2/3)*(a + b*Log[c*(d + e/x^(1/3)
)^n]))/(2*d^4) + (b*e^3*n*x*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*d^3) - (b*
e^2*n*x^(4/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(4*d^2) + (b*e*n*x^(5/3)*(a
 + b*Log[c*(d + e/x^(1/3))^n]))/(5*d) + (b*e^6*n*Log[1 - d/(d + e/x^(1/3))]
*(a + b*Log[c*(d + e/x^(1/3))^n]))/d^6 + (x^2*(a + b*Log[c*(d + e/x^(1/3))^
n])^2)/2 + (137*b^2*e^6*n^2*Log[x])/(180*d^6) - (b^2*e^6*n^2*PolyLog[2, d/(
d + e/x^(1/3))])/d^6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = -\left(3\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^7}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-(ben)\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^6(d+ex)}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-(bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-\frac{(bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d}+\frac{(ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d}$$

$$= \frac{benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{5d} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 + \frac{(ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d^2} - \frac{(be^2n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d^2} - \frac{(b^2en^2)\,\text{Subst}\left(\int \frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{5d}$$

$$= -\frac{be^2nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4d^2} + \frac{benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{5d} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{(be^2n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d^3} + \frac{(be^3n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d^3} - \frac{(b^2en^2)\,\text{Subst}\left(\int \left(-\frac{e^5}{d(d-x)^5}-\frac{e^5}{d^2(d-x)^4}-\frac{e^5}{d^3(d-x)^3}-\frac{e^5}{d^4(d-x)^2}-\frac{e^5}{d^5(d-x)}-\frac{e^5}{d^5x}\right)\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{5d} + \frac{(b^2e^2n^2)\,\text{Subst}\left(\int \frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{4d^2}$$

$$= -\frac{b^2e^5n^2\sqrt[3]{x}}{5d^5} + \frac{b^2e^4n^2x^{2/3}}{10d^4} - \frac{b^2e^3n^2x}{15d^3} + \frac{b^2e^2n^2x^{4/3}}{20d^2} + \frac{b^2e^6n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{5d^6} + \frac{be^3nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^3} - \frac{be^2nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4d^2} + \frac{benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{5d} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 + \frac{b^2e^6n^2\log(x)}{15d^6} + \frac{(be^3n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d^4} -$$

$$= -\frac{9b^2e^5n^2\sqrt[3]{x}}{20d^5} + \frac{9b^2e^4n^2x^{2/3}}{40d^4} - \frac{3b^2e^3n^2x}{20d^3} + \frac{b^2e^2n^2x^{4/3}}{20d^2} + \frac{9b^2e^6n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{20d^6} - \frac{be^4nx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2d^4} + \frac{be^3nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^3} - \frac{be^2nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4d^2} + \frac{benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{5d} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 + \frac{3b^2e^6n^2\log(x)}{20d^6} - \frac{(be^4n)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d^5} +$$

$$= -\frac{47b^2e^5n^2\sqrt[3]{x}}{60d^5} + \frac{47b^2e^4n^2x^{2/3}}{120d^4} - \frac{3b^2e^3n^2x}{20d^3} + \frac{b^2e^2n^2x^{4/3}}{20d^2} + \frac{47b^2e^6n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{60d^6} + \frac{be^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^6} - \frac{be^4nx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2d^4} + \frac{be^3nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^3} - \frac{be^2nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4d^2} + \frac{benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{5d} + \frac{be^6n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^6} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 + \frac{47b^2e^6n^2\log(x)}{180d^6} + \frac{(b^2e^4n^2)\,\text{Subst}\left(\int\left(\frac{e^2}{d(d-x)^2}+\frac{e^2}{d^2(d-x)}+\frac{e^2}{d^2x}\right)\right.}{2d^4}$$

$$= -\frac{77b^2e^5n^2\sqrt[3]{x}}{60d^5} + \frac{47b^2e^4n^2x^{2/3}}{120d^4} - \frac{3b^2e^3n^2x}{20d^3} + \frac{b^2e^2n^2x^{4/3}}{20d^2} + \frac{77b^2e^6n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{60d^6} + \frac{be^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^6} - \frac{be^4nx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2d^4} + \frac{be^3nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3d^3} - \frac{be^2nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4d^2} + \frac{benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{5d} + \frac{be^6n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^6} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 + \frac{137b^2e^6n^2\log(x)}{180d^6} - \frac{b^2e^6n^2\operatorname{Li}_2\left(\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^6}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.09

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx = \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 + \frac{ben\left(360ade^4\sqrt[3]{x} - 462bde^4n\sqrt[3]{x} - 180ad^2e^3x^{2/3} + 141bd^2e^3nx^{2/3} + 120ad^3e^2x - 54bd^3e^2nx - 90ad^4ex\right.}{}$$

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] (x^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/2 + (b*e*n*(360*a*d*e^4*x^(1/3) -
462*b*d*e^4*n*x^(1/3) - 180*a*d^2*e^3*x^(2/3) + 141*b*d^2*e^3*n*x^(2/3) + 1
20*a*d^3*e^2*x - 54*b*d^3*e^2*n*x - 90*a*d^4*e*x^(4/3) + 18*b*d^4*e*n*x^(4/
3) + 72*a*d^5*x^(5/3) + 642*b*e^5*n*Log[d + e/x^(1/3)] + 360*b*d*e^4*x^(1/3
)*Log[c*(d + e/x^(1/3))^n] - 180*b*d^2*e^3*x^(2/3)*Log[c*(d + e/x^(1/3))^n]
 + 120*b*d^3*e^2*x*Log[c*(d + e/x^(1/3))^n] - 90*b*d^4*e*x^(4/3)*Log[c*(d +
 e/x^(1/3))^n] + 72*b*d^5*x^(5/3)*Log[c*(d + e/x^(1/3))^n] - 360*a*e^5*Log[
e + d*x^(1/3)] + 180*b*e^5*n*Log[e + d*x^(1/3)] - 360*b*e^5*Log[c*(d + e/x^
(1/3))^n]*Log[e + d*x^(1/3)] + 180*b*e^5*n*Log[e + d*x^(1/3)]^2 - 360*b*e^5
*n*Log[e + d*x^(1/3)]*Log[-((d*x^(1/3))/e)] + 214*b*e^5*n*Log[x] - 360*b*e^
5*n*PolyLog[2, 1 + (d*x^(1/3))/e]))/(360*d^6)
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{2}dx$$

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/3))^n))^2,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(1/3))^n))^2,x)
```

Fricas [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{2}x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x*log(c*((d*x + e*x^(2/3))/x)^n)^2 + 2*a*b*x*log(c*((d*x + e*x
^(2/3))/x)^n) + a^2*x, x)
```

Sympy [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}dx$$

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/3))**n))**2,x)
```

```
[Out] Integral(x*(a + b*log(c*(d + e/x**(1/3))**n))**2, x)
```

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{2}x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*log((d*x^(1/3) + e)^n)^2 - integrate(-1/3*(3*(b^2*d*log(c)^2 +
2*a*b*d*log(c) + a^2*d)*x^2 + 3*(b^2*d*x^2 + b^2*e*x^(5/3))*log(x^(1/3*n))^
2 + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^(5/3) - (b^2*d*n*x^2 - 6*
(b^2*d*log(c) + a*b*d)*x^2 - 6*(b^2*e*log(c) + a*b*e)*x^(5/3) + 6*(b^2*d*x^
2 + b^2*e*x^(5/3))*log(x^(1/3*n)))*log((d*x^(1/3) + e)^n) - 6*((b^2*d*log(c
) + a*b*d)*x^2 + (b^2*e*log(c) + a*b*e)*x^(5/3))*log(x^(1/3*n)))/(d*x + e*x
^(2/3)), x)
```

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2x\,dx$$

[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="giac")

[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^2*x, x)

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx=\int x\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)\right)^2\,dx$$

[In] int(x*(a + b*log(c*(d + e/x^(1/3))^n))^2,x)

[Out] int(x*(a + b*log(c*(d + e/x^(1/3))^n))^2, x)

3.499 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 20, antiderivative size = 227

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx = \frac{b^2e^2n^2\sqrt[3]{x}}{d^2} - \frac{b^2e^3n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^3} - \frac{2be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} + \frac{benx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d} - \frac{2be^3n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{b^2e^3n^2\log(x)}{d^3} + \frac{2b^2e^3n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^3}$$

```
[Out] b^2*e^2*n^2*x^(1/3)/d^2-b^2*e^3*n^2*ln(d+e/x^(1/3))/d^3-2*b*e^2*n*(d+e/x^(1
/3))*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^3+b*e*n*x^(2/3)*(a+b*ln(c*(d+e/x
^(1/3))^n))/d-2*b*e^3*n*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(c*(d+e/x^(1/3))^n))/d
^3+x*(a+b*ln(c*(d+e/x^(1/3))^n))^2-b^2*e^3*n^2*ln(x)/d^3+2*b^2*e^3*n^2*poly
log(2,d/(d+e/x^(1/3)))/d^3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {2501, 2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx$$
$$= -\frac{2be^3n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}$$
$$-\frac{2be^2n\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}$$
$$+\frac{benx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d}+x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2$$
$$+\frac{2b^2e^3n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^3}-\frac{b^2e^3n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^3}-\frac{b^2e^3n^2\log(x)}{d^3}+\frac{b^2e^2n^2\sqrt[3]{x}}{d^2}$$

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] (b^2*e^2*n^2*x^(1/3))/d^2 - (b^2*e^3*n^2*Log[d + e/x^(1/3)])/d^3 - (2*b*e^2
*n*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/d^3 + (b*e*n*x
^(2/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/d - (2*b*e^3*n*Log[1 - d/(d + e/x^
(1/3))]*(a + b*Log[c*(d + e/x^(1/3))^n]))/d^3 + x*(a + b*Log[c*(d + e/x^(1/
3))^n])^2 - (b^2*e^3*n^2*Log[x])/d^3 + (2*b^2*e^3*n^2*PolyLog[2, d/(d + e/x
^(1/3))])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
```

```
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
```

```
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^n\right)\right)^2\,dx,x,\sqrt[3]{x}\right)\\
&= -\left(3\text{Subst}\left(\int\frac{(a+b\log(c(d+ex)^n))^2}{x^4}\,dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)\\
&= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-(2ben)\text{Subst}\left(\int\frac{a+b\log(c(d+ex)^n)}{x^3(d+ex)}\,dx,x,\frac{1}{\sqrt[3]{x}}\right)\\
&= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-(2bn)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)\\
&= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-\frac{(2bn)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{d}\\
&\quad+\frac{(2ben)\text{Subst}\left(\int\frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{d}
\end{aligned}
$$

$$= \frac{benx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 + \frac{(2ben)\mathrm{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d^2} - \frac{(2be^2n)\,\mathrm{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d^2} - \frac{(b^2en^2)\,\mathrm{Subst}\left(\int \frac{1}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d}$$

$$= -\frac{2be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} + \frac{benx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d} - \frac{2be^3n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{(b^2en^2)\,\mathrm{Subst}\left(\int \left(\frac{e^2}{d(d-x)^2}+\frac{e^2}{d^2(d-x)}+\frac{e^2}{d^2x}\right)\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d} + \frac{(2b^2e^2n^2)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{d}{e}+\frac{x}{e}}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d^3} + \frac{(2b^2e^3n^2)\,\mathrm{Subst}\left(\int \frac{\log\left(1-\frac{d}{x}\right)}{x}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d^3}$$

$$= \frac{b^2e^2n^2\sqrt[3]{x}}{d^2} - \frac{b^2e^3n^2\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{d^3} - \frac{2be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} + \frac{benx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d} - \frac{2be^3n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{b^2e^3n^2\log(x)}{d^3} + \frac{2b^2e^3n^2\operatorname{Li}_2\left(\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^3}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.06

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 dx = x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 - \frac{ben\left(6ade\sqrt[3]{x}+6be^2n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)+6bde\sqrt[3]{x}\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)-3d^2x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right.\right.\right.\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^2,x]
```

```
[Out] x*(a + b*Log[c*(d + e/x^(1/3))^n])^2 - (b*e*n*(6*a*d*e*x^(1/3) + 6*b*e^2*n*
Log[d + e/x^(1/3)] + 6*b*d*e*x^(1/3)*Log[c*(d + e/x^(1/3))^n] - 3*d^2*x^(2/
3)*(a + b*Log[c*(d + e/x^(1/3))^n]) - 6*e^2*(a + b*Log[c*(d + e/x^(1/3))^n]
)*Log[e + d*x^(1/3)] + 3*b*e*n*(-(d*x^(1/3)) + e*Log[e + d*x^(1/3)]) + 2*b*
e^2*n*Log[x] + 3*b*e^2*n*(Log[e + d*x^(1/3)]*(Log[e + d*x^(1/3)] - 2*Log[-(
(d*x^(1/3))/e)]) - 2*PolyLog[2, 1 + (d*x^(1/3))/e])))/(3*d^3)
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{2}dx$$

[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^2,x)

[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^2,x)

Fricas [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}\,dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{2}\,dx$$

[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="fricas")

[Out] integral(b^2*log(c*((d*x + e*x^(2/3))/x)^n)^2 + 2*a*b*log(c*((d*x + e*x^(2/3))/x)^n) + a^2, x)

Sympy [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}\,dx=\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}\,dx$$

[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**2,x)

[Out] Integral((a + b*log(c*(d + e/x**(1/3))**n))**2, x)

Maxima [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}\,dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{2}\,dx$$

[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="maxima")

[Out] (e*n*(2*e^2*log(d*x^(1/3) + e)/d^3 + (d*x^(2/3) - 2*e*x^(1/3))/d^2) + 2*x*log(c*(d + e/x^(1/3))^n))*a*b + (x*log((d*x^(1/3) + e)^n)^2 - integrate(-1/3*(3*d*x*log(c)^2 + 3*e*x^(2/3)*log(c)^2 + 3*(d*x + e*x^(2/3))*log(x^(1/3*n))^2 - 2*(d*n*x - 3*d*x*log(c) - 3*e*x^(2/3)*log(c) + 3*(d*x + e*x^(2/3))*log(x^(1/3*n)))*log((d*x^(1/3) + e)^n) - 6*(d*x*log(c) + e*x^(2/3)*log(c))*log(x^(1/3*n)))/(d*x + e*x^(2/3)), x))*b^2 + a^2*x

Giac [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2\,dx$$

[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2,x, algorithm="giac")

[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\,dx=\int \left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)\right)^2\,dx$$

[In] int((a + b*log(c*(d + e/x^(1/3))^n))^2,x)

[Out] int((a + b*log(c*(d + e/x^(1/3))^n))^2, x)

3.500 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}\,dx = -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right) - 6bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\operatorname{PolyLog}\left(2,1+\frac{e}{d\sqrt[3]{x}}\right) + 6b^2n^2\operatorname{PolyLog}\left(3,1+\frac{e}{d\sqrt[3]{x}}\right)$$

```
[Out] -3*(a+b*ln(c*(d+e/x^(1/3))^n))^2*ln(-e/d/x^(1/3))-6*b*n*(a+b*ln(c*(d+e/x^(1
/3))^n))*polylog(2,1+e/d/x^(1/3))+6*b^2*n^2*polylog(3,1+e/d/x^(1/3))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2504, 2443, 2481, 2421, 6724}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}\,dx = -6bn\,\text{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right) - 3\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 + 6b^2n^2\,\text{PolyLog}\left(3,\frac{e}{d\sqrt[3]{x}}+1\right)$$

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x,x]
```

```
[Out] -3*(a + b*Log[c*(d + e/x^(1/3))^n])^2*Log[-(e/(d*x^(1/3)))] - 6*b*n*(a + b*
Log[c*(d + e/x^(1/3))^n])*PolyLog[2, 1 + e/(d*x^(1/3))] + 6*b^2*n^2*PolyLog
[3, 1 + e/(d*x^(1/3))]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(3\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)\\
&= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\\
&\quad+(6ben)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{d+ex}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\\
&= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\\
&\quad+(6bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)\\
&= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\\
&\quad-6bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\text{Li}_2\left(1+\frac{e}{d\sqrt[3]{x}}\right)\\
&\quad+\left(6b^2n^2\right)\text{Subst}\left(\int \frac{\text{Li}_2\left(\frac{x}{d}\right)}{x}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)\\
&= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\\
&\quad-6bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\text{Li}_2\left(1+\frac{e}{d\sqrt[3]{x}}\right)+6b^2n^2\text{Li}_3\left(1+\frac{e}{d\sqrt[3]{x}}\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 389 vs. 2(93) = 186.

Time = 0.15 (sec) , antiderivative size = 389, normalized size of antiderivative = 4.18

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}\,dx = \left(a-bn\log\left(d+\frac{e}{\sqrt[3]{x}}\right)+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\log(x)+2bn\left(a-bn\log\left(d+\frac{e}{\sqrt[3]{x}}\right)+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\left(\left(\log\left(d+\frac{e}{\sqrt[3]{x}}\right)-\log\left(1+\frac{e}{d\sqrt[3]{x}}\right)\right)\log(x)+3\,\text{PolyLog}\left(2,-\frac{e}{d\sqrt[3]{x}}\right)\right)+3b^2n^2\left(2\log\left(\frac{e}{d}+\sqrt[3]{x}\right)\text{PolyLog}\left(2,1+\frac{d\sqrt[3]{x}}{e}\right)-2\left(\log\left(d+\frac{e}{\sqrt[3]{x}}\right)-\log\left(\frac{e}{d}+\sqrt[3]{x}\right)\right)\text{PolyLog}\left(2,-\frac{d\sqrt[3]{x}}{e}\right)+\frac{1}{81}\left(81\log^2\left(\frac{e}{d}+\sqrt[3]{x}\right)\log\left(-\frac{d\sqrt[3]{x}}{e}\right)+27\log^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\log(x)-27\log^2\left(\frac{e}{d}+\sqrt[3]{x}\right)\log(x)-54\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\log\left(1+\frac{d\sqrt[3]{x}}{e}\right)\log(x)+54\log\left(\frac{e}{d}+\sqrt[3]{x}\right)\log\left(1+\frac{d\sqrt[3]{x}}{e}\right)\log(x)+9\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\log^2(x)-9\log\left(1+\frac{d\sqrt[3]{x}}{e}\right)\log^2(x)+\log^3(x)-162\,\text{PolyLog}\left(3,1+\frac{d\sqrt[3]{x}}{e}\right)-162\,\text{PolyLog}\left(3,-\frac{d\sqrt[3]{x}}{e}\right)\right)\right)$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e/x^(1/3)] + b*Log[c*(d + e/x^(1/3))^n])*((Log[d + e/x^(1/
3)] - Log[1 + e/(d*x^(1/3))])*Log[x] + 3*PolyLog[2, -(e/(d*x^(1/3)))]) + 3*
b^2*n^2*(2*Log[e/d + x^(1/3)]*PolyLog[2, 1 + (d*x^(1/3))/e] - 2*(Log[d + e/
x^(1/3)] - Log[e/d + x^(1/3)])*PolyLog[2, -((d*x^(1/3))/e)] + (81*Log[e/d +
 x^(1/3)]^2*Log[-((d*x^(1/3))/e)] + 27*Log[d + e/x^(1/3)]^2*Log[x] - 27*Log
[e/d + x^(1/3)]^2*Log[x] - 54*Log[d + e/x^(1/3)]*Log[1 + (d*x^(1/3))/e]*Log
[x] + 54*Log[e/d + x^(1/3)]*Log[1 + (d*x^(1/3))/e]*Log[x] + 9*Log[d + e/x^(
1/3)]*Log[x]^2 - 9*Log[1 + (d*x^(1/3))/e]*Log[x]^2 + Log[x]^3 - 162*PolyLog
[3, 1 + (d*x^(1/3))/e] - 162*PolyLog[3, -((d*x^(1/3))/e)])/81)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{2}}{x}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{2}}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*log(c*((d*x + e*x^(2/3))/x)^n)^2 + 2*a*b*log(c*((d*x + e*x^(2
/3))/x)^n) + a^2)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}}{x}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**2/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e/x**(1/3))**n))**2/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x,x, algorithm="maxima")
```

```
[Out] b^2*log((d*x^(1/3) + e)^n)^2*log(x) - integrate(-1/3*(3*(b^2*d*x + b^2*e*x^
(2/3))*log(x^(1/3*n))^2 + 3*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x - 2
*(b^2*d*n*x*log(x) - 3*(b^2*d*log(c) + a*b*d)*x + 3*(b^2*d*x + b^2*e*x^(2/3
))*log(x^(1/3*n)) - 3*(b^2*e*log(c) + a*b*e)*x^(2/3))*log((d*x^(1/3) + e)^n
) - 6*((b^2*d*log(c) + a*b*d)*x + (b^2*e*log(c) + a*b*e)*x^(2/3))*log(x^(1/
3*n)) + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^(2/3))/(d*x^2 + e*x^(
5/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}\,dx = \int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)\right)^2}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))^2/x,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(1/3))^n))^2/x, x)
```

3.501 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 269

$$\begin{aligned}
&\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx \\
&= \frac{3b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{2e^3} - \frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3} - \frac{6b^2d^2n^2}{e^2\sqrt[3]{x}} \\
&\quad + \frac{b^2d^3n^2\log^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} + \frac{6bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3} \\
&\quad - \frac{3bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3} \\
&\quad + \frac{2bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^3} \\
&\quad - \frac{2bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}
\end{aligned}$$

```
[Out] 3/2*b^2*d*n^2*(d+e/x^(1/3))^2/e^3-2/9*b^2*n^2*(d+e/x^(1/3))^3/e^3-6*b^2*d^2
*n^2/e^2/x^(1/3)+b^2*d^3*n^2*ln(d+e/x^(1/3))^2/e^3+6*b*d^2*n*(d+e/x^(1/3))*
(a+b*ln(c*(d+e/x^(1/3))^n))/e^3-3*b*d*n*(d+e/x^(1/3))^2*(a+b*ln(c*(d+e/x^(1
/3))^n))/e^3+2/3*b*n*(d+e/x^(1/3))^3*(a+b*ln(c*(d+e/x^(1/3))^n))/e^3-2*b*d^
```

```
3*n*ln(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))/e^3-(a+b*ln(c*(d+e/x^(1/3))
^n))^2/x
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx$$
$$= -\frac{2bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3}$$
$$+\frac{6bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3}$$
$$-\frac{3bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3}$$
$$+\frac{2bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^3}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}$$
$$+\frac{b^2d^3n^2\log^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}-\frac{6b^2d^2n^2}{e^2\sqrt[3]{x}}+\frac{3b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{2e^3}-\frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3}$$

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x^2,x]
```

```
[Out] (3*b^2*d*n^2*(d + e/x^(1/3))^2)/(2*e^3) - (2*b^2*n^2*(d + e/x^(1/3))^3)/(9*
e^3) - (6*b^2*d^2*n^2)/(e^2*x^(1/3)) + (b^2*d^3*n^2*Log[d + e/x^(1/3)]^2)/e
^3 + (6*b*d^2*n*(d + e/x^(1/3))*(a + b*Log[c*(d + e/x^(1/3))^n]))/e^3 - (3*
b*d*n*(d + e/x^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/e^3 + (2*b*n*(d +
 e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*e^3) - (2*b*d^3*n*Log[d
+ e/x^(1/3)]*(a + b*Log[c*(d + e/x^(1/3))^n]))/e^3 - (a + b*Log[c*(d + e/x^
(1/3))^n])^2/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = -\left(3\text{Subst}\left(\int x^2(a+b\log(c(d+ex)^n))^2\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x} + (2ben)\text{Subst}\left(\int \frac{x^3(a+b\log(c(d+ex)^n))}{d+ex}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x} + (2bn)\text{Subst}\left(\int \frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^3(a+b\log(cx^n))}{x}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)$$

$$= \frac{6bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3} - \frac{3bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3} + \frac{2bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^3} - \frac{2bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x} - \left(2b^2n^2\right)\text{Subst}\left(\int \frac{18d^2x-9dx^2+2x^3-6d^3\log(x)}{6e^3x}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)$$

$$= \frac{6bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3} - \frac{3bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3} + \frac{2bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^3} - \frac{2bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x} - \frac{(b^2n^2)\operatorname{Subst}\left(\int\frac{18d^2x-9dx^2+2x^3-6d^3\log(x)}{x}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{3e^3}$$

$$= \frac{6bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3} - \frac{3bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3} + \frac{2bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^3} - \frac{2bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x} - \frac{(b^2n^2)\operatorname{Subst}\left(\int\left(18d^2-9dx+2x^2-\frac{6d^3\log(x)}{x}\right)dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{3e^3}$$

$$= \frac{3b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{2e^3} - \frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3} - \frac{6b^2d^2n^2}{e^2\sqrt[3]{x}} + \frac{6bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3} - \frac{3bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3} + \frac{2bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^3} - \frac{2bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x} + \frac{(2b^2d^3n^2)\,\mathrm{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}$$

$$= \frac{3b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{2e^3} - \frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3} - \frac{6b^2d^2n^2}{e^2\sqrt[3]{x}} + \frac{b^2d^3n^2\log^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} + \frac{6bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3} - \frac{3bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3} + \frac{2bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^3} - \frac{2bd^3n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^3} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.39

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx$$
$$=\frac{-18\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2+\frac{bn\left(-2ben\left(2e^2-3de\sqrt[3]{x}+6d^2x^{2/3}\right)+9bden\left(e-2d\sqrt[3]{x}\right)\sqrt[3]{x}+36ad^2ex^{2/3}-36bd^2enx^{2/3}+\cdots\right.}{}}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x^2,x]
```

```
[Out] (-18*(a + b*Log[c*(d + e/x^(1/3))^n])^2 + (b*n*(-2*b*e*n*(2*e^2 - 3*d*e*x^(
1/3) + 6*d^2*x^(2/3)) + 9*b*d*e*n*(e - 2*d*x^(1/3))*x^(1/3) + 36*a*d^2*e*x^
(2/3) - 36*b*d^2*e*n*x^(2/3) + 30*b*d^3*n*x*Log[d + e/x^(1/3)] + 36*b*d^2*(
e + d*x^(1/3))*x^(2/3)*Log[c*(d + e/x^(1/3))^n] + 12*e^3*(a + b*Log[c*(d +
e/x^(1/3))^n]) - 18*d*e^2*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]) - 36*d^3
*x*(a + b*Log[c*(d + e/x^(1/3))^n])*Log[e + d*x^(1/3)] - 36*d^3*x*(a + b*Lo
g[c*(d + e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))] + 18*b*d^3*n*x*Log[e + d*x^(1
/3)]*(Log[e + d*x^(1/3)] - 2*Log[-((d*x^(1/3))/e)]) - 36*b*d^3*n*x*PolyLog[
2, 1 + e/(d*x^(1/3))] - 36*b*d^3*n*x*PolyLog[2, 1 + (d*x^(1/3))/e]))/e^3)/(
18*x)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)\right)^2}{x^2}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.33

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx =$$
$$-\frac{4b^2e^3n^2-12abe^3n+18a^2e^3-18\left(b^2e^3x-b^2e^3\right)\log\left(c\right)^2+18\left(b^2d^3n^2x+b^2e^3n^2\right)\log\left(\frac{dx+ex^{\frac{2}{3}}}{x}\right)^2-2\left(2b\cdots\right.}{}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/18*(4*b^2*e^3*n^2 - 12*a*b*e^3*n + 18*a^2*e^3 - 18*(b^2*e^3*x - b^2*e^3)
*log(c)^2 + 18*(b^2*d^3*n^2*x + b^2*e^3*n^2)*log((d*x + e*x^(2/3))/x)^2 - 2
*(2*b^2*e^3*n^2 - 6*a*b*e^3*n + 9*a^2*e^3)*x - 12*(b^2*e^3*n - 3*a*b*e^3 -
(b^2*e^3*n - 3*a*b*e^3)*x)*log(c) - 6*(6*b^2*d^2*e*n^2*x^(2/3) - 3*b^2*d*e^
2*n^2*x^(1/3) + 2*b^2*e^3*n^2 - 6*a*b*e^3*n + (11*b^2*d^3*n^2 - 6*a*b*d^3*n
)*x - 6*(b^2*d^3*n*x + b^2*e^3*n)*log(c))*log((d*x + e*x^(2/3))/x) + 6*(11*
b^2*d^2*e*n^2 - 6*b^2*d^2*e*n*log(c) - 6*a*b*d^2*e*n)*x^(2/3) - 3*(5*b^2*d*
e^2*n^2 - 6*b^2*d*e^2*n*log(c) - 6*a*b*d*e^2*n)*x^(1/3))/(e^3*x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}}{x^{2}}\,dx = \int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}}{x^{2}}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**2/x**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e/x**(1/3))**n))**2/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}}{x^{2}}\,dx$$
$$= -\frac{1}{3}\,aben\left(\frac{6\,d^{3}\log\left(dx^{\frac{1}{3}}+e\right)}{e^{4}} - \frac{2\,d^{3}\log\left(x\right)}{e^{4}} - \frac{6\,d^{2}x^{\frac{2}{3}} - 3\,dex^{\frac{1}{3}} + 2\,e^{2}}{e^{3}x}\right)$$
$$- \frac{1}{18}\left(6\,en\left(\frac{6\,d^{3}\log\left(dx^{\frac{1}{3}}+e\right)}{e^{4}} - \frac{2\,d^{3}\log\left(x\right)}{e^{4}} - \frac{6\,d^{2}x^{\frac{2}{3}} - 3\,dex^{\frac{1}{3}} + 2\,e^{2}}{e^{3}x}\right)\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right) - \frac{\left(18\,d^{3}\right.}{}\right.$$
$$- \frac{b^{2}\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)^{2}}{x} - \frac{2\,ab\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)}{x} - \frac{a^{2}}{x}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/3*a*b*e*n*(6*d^3*log(d*x^(1/3) + e)/e^4 - 2*d^3*log(x)/e^4 - (6*d^2*x^(2
/3) - 3*d*e*x^(1/3) + 2*e^2)/(e^3*x)) - 1/18*(6*e*n*(6*d^3*log(d*x^(1/3) +
```

```
e)/e^4 - 2*d^3*log(x)/e^4 - (6*d^2*x^(2/3) - 3*d*e*x^(1/3) + 2*e^2)/(e^3*x)
)*log(c*(d + e/x^(1/3))^n) - (18*d^3*x*log(d*x^(1/3) + e)^2 + 2*d^3*x*log(x
)^2 - 22*d^3*x*log(x) - 66*d^2*e*x^(2/3) + 15*d*e^2*x^(1/3) - 4*e^3 - 6*(2*
d^3*x*log(x) - 11*d^3*x)*log(d*x^(1/3) + e))*n^2/(e^3*x))*b^2 - b^2*log(c*(
d + e/x^(1/3))^n)^2/x - 2*a*b*log(c*(d + e/x^(1/3))^n)/x - a^2/x
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.59

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx = -\frac{18\left(\frac{3\left(dx^{\frac{1}{3}}+e\right)b^2d^2n^2}{e^2x^{\frac{1}{3}}}-\frac{3\left(dx^{\frac{1}{3}}+e\right)^2b^2dn^2}{e^2x^{\frac{2}{3}}}+\frac{\left(dx^{\frac{1}{3}}+e\right)^3b^2n^2}{e^2x}\right)\log\left(\frac{dx^{\frac{1}{3}}+e}{x^{\frac{1}{3}}}\right)^2-6\left(\frac{2\left(b^2n^2-3b^2n\log(c)-3abn\right)\left(dx^{\frac{1}{3}}+e\right)^3}{e^2x}-\right.}{}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^2,x, algorithm="giac")
```

```
[Out] -1/18*(18*(3*(d*x^(1/3) + e)*b^2*d^2*n^2/(e^2*x^(1/3)) - 3*(d*x^(1/3) + e)^
2*b^2*d*n^2/(e^2*x^(2/3)) + (d*x^(1/3) + e)^3*b^2*n^2/(e^2*x))*log((d*x^(1/
3) + e)/x^(1/3))^2 - 6*(2*(b^2*n^2 - 3*b^2*n*log(c) - 3*a*b*n)*(d*x^(1/3) +
 e)^3/(e^2*x) - 9*(b^2*d*n^2 - 2*b^2*d*n*log(c) - 2*a*b*d*n)*(d*x^(1/3) + e
)^2/(e^2*x^(2/3)) + 18*(b^2*d^2*n^2 - b^2*d^2*n*log(c) - a*b*d^2*n)*(d*x^(1
/3) + e)/(e^2*x^(1/3)))*log((d*x^(1/3) + e)/x^(1/3)) + 2*(2*b^2*n^2 - 6*b^2
*n*log(c) + 9*b^2*log(c)^2 - 6*a*b*n + 18*a*b*log(c) + 9*a^2)*(d*x^(1/3) +
e)^3/(e^2*x) - 27*(b^2*d*n^2 - 2*b^2*d*n*log(c) + 2*b^2*d*log(c)^2 - 2*a*b*
d*n + 4*a*b*d*log(c) + 2*a^2*d)*(d*x^(1/3) + e)^2/(e^2*x^(2/3)) + 54*(2*b^2
*d^2*n^2 - 2*b^2*d^2*n*log(c) + b^2*d^2*log(c)^2 - 2*a*b*d^2*n + 2*a*b*d^2*
log(c) + a^2*d^2)*(d*x^(1/3) + e)/(e^2*x^(1/3)))/e
```

Mupad [B] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.11

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^2}\,dx$$
$$=\frac{\frac{d\left(3a^2-2abn+\frac{2b^2n^2}{3}\right)}{2e}-\frac{d\,(3a^2-b^2n^2)}{2e}}{x^{2/3}}-\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)^2\left(\frac{b^2}{x}+\frac{b^2d^3}{e^3}\right)$$
$$-\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)\left(\frac{2b\,(3a-bn)}{3x}-\frac{\frac{bd\,(3a-bn)}{e}-\frac{3abd}{e}}{x^{2/3}}+\frac{d\left(\frac{2bd\,(3a-bn)}{e}-\frac{6abd}{e}\right)}{e\,x^{1/3}}\right)-\ldots$$

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))^2/x^2,x)
```

```
[Out] ((d*(3*a^2 + (2*b^2*n^2)/3 - 2*a*b*n))/(2*e) - (d*(3*a^2 - b^2*n^2))/(2*e))
/x^(2/3) - log(c*(d + e/x^(1/3))^n)^2*(b^2/x + (b^2*d^3)/e^3) - log(c*(d +
e/x^(1/3))^n)*((2*b*(3*a - b*n))/(3*x) - ((b*d*(3*a - b*n))/e - (3*a*b*d)/e
)/x^(2/3) + (d*((2*b*d*(3*a - b*n))/e - (6*a*b*d)/e))/(e*x^(1/3))) - ((d*((
d*(3*a^2 + (2*b^2*n^2)/3 - 2*a*b*n))/e - (d*(3*a^2 - b^2*n^2))/e))/e + (2*b
^2*d^2*n^2)/e^2)/x^(1/3) - (a^2 + (2*b^2*n^2)/9 - (2*a*b*n)/3)/x + (log(d +
 e/x^(1/3))*(11*b^2*d^3*n^2 - 6*a*b*d^3*n))/(3*e^3)
```

3.502 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 479

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^3}\,dx = -\frac{15b^2d^4n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^6}+\frac{20b^2d^3n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6} - \frac{15b^2d^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{16e^6}+\frac{6b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^5}{25e^6} - \frac{b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{36e^6}+\frac{6b^2d^5n^2}{e^5\sqrt[3]{x}} - \frac{b^2d^6n^2\log^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2e^6} - \frac{6bd^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^6} + \frac{15bd^4n\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2e^6} - \frac{20bd^3n\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^6} + \frac{15bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4e^6} - \frac{6bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{5e^6} + \frac{bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{6e^6} + \frac{bd^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^6} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2x^2}$$

```
[Out] -15/4*b^2*d^4*n^2*(d+e/x^(1/3))^2/e^6+20/9*b^2*d^3*n^2*(d+e/x^(1/3))^3/e^6-
15/16*b^2*d^2*n^2*(d+e/x^(1/3))^4/e^6+6/25*b^2*d*n^2*(d+e/x^(1/3))^5/e^6-1/
```

```
36*b^2*n^2*(d+e/x^(1/3))^6/e^6+6*b^2*d^5*n^2/e^5/x^(1/3)-1/2*b^2*d^6*n^2*ln
(d+e/x^(1/3))^2/e^6-6*b*d^5*n*(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6
+15/2*b*d^4*n*(d+e/x^(1/3))^2*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6-20/3*b*d^3*n*
(d+e/x^(1/3))^3*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6+15/4*b*d^2*n*(d+e/x^(1/3))^
4*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6-6/5*b*d*n*(d+e/x^(1/3))^5*(a+b*ln(c*(d+e/
x^(1/3))^n))/e^6+1/6*b*n*(d+e/x^(1/3))^6*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6+b*
d^6*n*ln(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6-1/2*(a+b*ln(c*(d+e/x^
(1/3))^n))^2/x^2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{2504, 2445, 2458, 45, 2372, 12, 14, 2338\}$

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^3}\,dx = \frac{bd^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^6} - \frac{6bd^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^6} + \frac{15bd^4n\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2e^6} - \frac{20bd^3n\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^6} + \frac{15bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4e^6} - \frac{6bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{5e^6} + \frac{bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{6e^6} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2x^2} - \frac{b^2d^6n^2\log^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2e^6} + \frac{6b^2d^5n^2}{e^5\sqrt[3]{x}} - \frac{15b^2d^4n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^6} + \frac{20b^2d^3n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6} - \frac{15b^2d^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{16e^6} + \frac{6b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^5}{25e^6} - \frac{b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{36e^6}$$

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x^3,x]
[Out] (-15*b^2*d^4*n^2*(d + e/x^(1/3))^2)/(4*e^6) + (20*b^2*d^3*n^2*(d + e/x^(1/3
))^3)/(9*e^6) - (15*b^2*d^2*n^2*(d + e/x^(1/3))^4)/(16*e^6) + (6*b^2*d*n^2*
```

```
(d + e/x^(1/3))^5)/(25*e^6) - (b^2*n^2*(d + e/x^(1/3))^6)/(36*e^6) + (6*b^2
*d^5*n^2)/(e^5*x^(1/3)) - (b^2*d^6*n^2*Log[d + e/x^(1/3)]^2)/(2*e^6) - (6*b
*d^5*n*(d + e/x^(1/3))*(a + b*Log[c*(d + e/x^(1/3))^n]))/e^6 + (15*b*d^4*n*
(d + e/x^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/(2*e^6) - (20*b*d^3*n*(
d + e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*e^6) + (15*b*d^2*n*(d
 + e/x^(1/3))^4*(a + b*Log[c*(d + e/x^(1/3))^n]))/(4*e^6) - (6*b*d*n*(d + e
/x^(1/3))^5*(a + b*Log[c*(d + e/x^(1/3))^n]))/(5*e^6) + (b*n*(d + e/x^(1/3)
)^6*(a + b*Log[c*(d + e/x^(1/3))^n]))/(6*e^6) + (b*d^6*n*Log[d + e/x^(1/3)]
*(a + b*Log[c*(d + e/x^(1/3))^n]))/e^6 - (a + b*Log[c*(d + e/x^(1/3))^n])^2
/(2*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
```

```
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(3\text{Subst}\left(\int x^5(a+b\log\left(c(d+ex)^n\right))^2\,dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2x^2}+(ben)\text{Subst}\left(\int\frac{x^6(a+b\log\left(c(d+ex)^n\right))}{d+ex}\,dx,x,\frac{1}{\sqrt[3]{x}}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2x^2}\\
&\quad+(bn)\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6(a+b\log\left(cx^n\right))}{x}\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)
\end{aligned}$$

$$= -\frac{6bd^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^6}$$
$$+\frac{15bd^4n\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2e^6}$$
$$-\frac{20bd^3n\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^6}$$
$$+\frac{15bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4e^6}$$
$$-\frac{6bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{5e^6}$$
$$+\frac{bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{6e^6}$$
$$+\frac{bd^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^6}$$
$$-\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2x^2}$$
$$-\left(b^2n^2\right)\text{Subst}\left(\int\frac{x(-360d^5+450d^4x-400d^3x^2+225d^2x^3-72dx^4+10x^5)+60d^6\log(x)}{60e^6x}\,dx,x,\right.$$
$$\left.+\frac{e}{\sqrt[3]{x}}\right)$$

$$= -\frac{6bd^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^6} + \frac{15bd^4n\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2e^6} - \frac{20bd^3n\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^6} + \frac{15bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4e^6} - \frac{6bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{5e^6} + \frac{bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{6e^6} + \frac{bd^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^6} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2x^2} - \frac{(b^2n^2)\,\mathrm{Subst}\left(\int \frac{x(-360d^5+450d^4x-400d^3x^2+225d^2x^3-72dx^4+10x^5)+60d^6\log(x)}{x}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{60e^6}$$

$$= -\frac{6bd^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^6} + \frac{15bd^4n\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2e^6} - \frac{20bd^3n\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^6} + \frac{15bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4e^6} - \frac{6bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{5e^6} + \frac{bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{6e^6} + \frac{bd^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^6} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2x^2} - \frac{(b^2n^2)\,\mathrm{Subst}\left(\int\left(-360d^5+450d^4x-400d^3x^2+225d^2x^3-72dx^4+10x^5+\frac{60d^6\log(x)}{x}\right)dx, x, d+\right.}{60e^6}$$

$$= -\frac{15b^2d^4n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^6} + \frac{20b^2d^3n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6}$$
$$- \frac{15b^2d^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{16e^6} + \frac{6b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^5}{25e^6} - \frac{b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{36e^6}$$
$$+ \frac{6b^2d^5n^2}{e^5\sqrt[3]{x}} - \frac{6bd^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^6}$$
$$+ \frac{15bd^4n\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2e^6}$$
$$- \frac{20bd^3n\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^6}$$
$$+ \frac{15bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4e^6}$$
$$- \frac{6bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{5e^6}$$
$$+ \frac{bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{6e^6}$$
$$+ \frac{bd^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^6}$$
$$- \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2x^2} - \frac{(b^2d^6n^2)\,\mathrm{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^6}$$

$$= -\frac{15b^2d^4n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^6} + \frac{20b^2d^3n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6} - \frac{15b^2d^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{16e^6}$$
$$+ \frac{6b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^5}{25e^6} - \frac{b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{36e^6} + \frac{6b^2d^5n^2}{e^5\sqrt[3]{x}}$$
$$- \frac{b^2d^6n^2\log^2\left(d+\frac{e}{\sqrt[3]{x}}\right)}{2e^6} - \frac{6bd^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^6}$$
$$+ \frac{15bd^4n\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2e^6}$$
$$- \frac{20bd^3n\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^6}$$
$$+ \frac{15bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4e^6}$$
$$- \frac{6bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{5e^6}$$
$$+ \frac{bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{6e^6}$$
$$+ \frac{bd^6n\log\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{e^6}$$
$$- \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2x^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.37 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.15

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^3}\,dx$$

$$= \frac{-1800\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2 + \frac{bn\left(600ae^6-100be^6n-720ade^5\sqrt[3]{x}+264bde^5n\sqrt[3]{x}+900ad^2e^4x^{2/3}-555bd^2e^4nx^{2/3}-120\cdots\right.}{\cdots}}{\cdots}$$

[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^2/x^3,x]

[Out] (-1800*(a + b*Log[c*(d + e/x^(1/3))^n])^2 + (b*n*(600*a*e^6 - 100*b*e^6*n - 720*a*d*e^5*x^(1/3) + 264*b*d*e^5*n*x^(1/3) + 900*a*d^2*e^4*x^(2/3) - 555*b*d^2*e^4*n*x^(2/3) - 1200*a*d^3*e^3*x + 1140*b*d^3*e^3*n*x + 1800*a*d^4*e^2*x^(4/3) - 2610*b*d^4*e^2*n*x^(4/3) - 3600*a*d^5*e*x^(5/3) + 8820*b*d^5*e*n*x^(5/3) - 8820*b*d^6*n*x^2*Log[d + e/x^(1/3)] + 600*b*e^6*Log[c*(d + e/x^(1/3))^n] - 720*b*d*e^5*x^(1/3)*Log[c*(d + e/x^(1/3))^n] + 900*b*d^2*e^4*x^(2/3)*Log[c*(d + e/x^(1/3))^n] - 1200*b*d^3*e^3*x*Log[c*(d + e/x^(1/3))^n] + 1800*b*d^4*e^2*x^(4/3)*Log[c*(d + e/x^(1/3))^n] - 3600*b*d^5*e*x^(5/3)*Log[c*(d + e/x^(1/3))^n] + 3600*a*d^6*x^2*Log[e + d*x^(1/3)] + 3600*b*d^6*x^2*Log[c*(d + e/x^(1/3))^n]*Log[e + d*x^(1/3)] - 1800*b*d^6*n*x^2*Log[e + d*x^(1/3)]^2 + 3600*b*d^6*x^2*Log[c*(d + e/x^(1/3))^n]*Log[-(e/(d*x^(1/3)))] + 3600*b*d^6*n*x^2*Log[e + d*x^(1/3)]*Log[-((d*x^(1/3))/e)] - 1200*a*d^6*x^2*Log[x] + 3600*b*d^6*n*x^2*PolyLog[2, 1 + e/(d*x^(1/3))] + 3600*b*d^6*n*x^2*PolyLog[2, 1 + (d*x^(1/3))/e]))/e^6)/(3600*x^2)

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{2}}{x^{3}}dx$$

[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x^3,x)

[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^2/x^3,x)

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.25

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}}{x^{3}}\,dx = -\frac{100\,b^2e^6n^2-600\,abe^6n+1800\,a^2e^6-20\,(90\,a^2e^6-(57\,b^2d^3e^3-5\,b^2e^6)n^2+30\,(2\,abd^3e^3-abe^6)n)x^2-}{}$$

[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^3,x, algorithm="fricas")

[Out] -1/3600*(100*b^2*e^6*n^2 - 600*a*b*e^6*n + 1800*a^2*e^6 - 20*(90*a^2*e^6 - (57*b^2*d^3*e^3 - 5*b^2*e^6)*n^2 + 30*(2*a*b*d^3*e^3 - a*b*e^6)*n)*x^2 - 1800*(b^2*e^6*x^2 - b^2*e^6)*log(c)^2 - 1800*(b^2*d^6*n^2*x^2 - b^2*e^6*n^2)*log((d*x + e*x^(2/3))/x)^2 - 60*(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)*x + 600*(2*b^2*d^3*e^3*n*x - b^2*e^6*n + 6*a*b*e^6 - (6*a*b*e^6 + (2*b^2*d^3*e

```
^3 - b^2*e^6)*n)*x^2)*log(c) + 60*(20*b^2*d^3*e^3*n^2*x - 10*b^2*e^6*n^2 +
60*a*b*e^6*n + 3*(49*b^2*d^6*n^2 - 20*a*b*d^6*n)*x^2 - 60*(b^2*d^6*n*x^2 -
b^2*e^6*n)*log(c) + 15*(4*b^2*d^5*e*n^2*x - b^2*d^2*e^4*n^2)*x^(2/3) - 6*(5
*b^2*d^4*e^2*n^2*x - 2*b^2*d*e^5*n^2)*x^(1/3))*log((d*x + e*x^(2/3))/x) + 1
5*(37*b^2*d^2*e^4*n^2 - 60*a*b*d^2*e^4*n - 12*(49*b^2*d^5*e*n^2 - 20*a*b*d^
5*e*n)*x + 60*(4*b^2*d^5*e*n*x - b^2*d^2*e^4*n)*log(c))*x^(2/3) - 6*(44*b^2
*d*e^5*n^2 - 120*a*b*d*e^5*n - 15*(29*b^2*d^4*e^2*n^2 - 20*a*b*d^4*e^2*n)*x
 + 60*(5*b^2*d^4*e^2*n*x - 2*b^2*d*e^5*n)*log(c))*x^(1/3))/(e^6*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}}{x^{3}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**2/x**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{2}}{x^{3}}\,dx$$
$$=\frac{1}{60}aben\left(\frac{60\,d^{6}\log\left(dx^{\frac{1}{3}}+e\right)}{e^{7}}-\frac{20\,d^{6}\log\left(x\right)}{e^{7}}-\frac{60\,d^{5}x^{\frac{5}{3}}-30\,d^{4}ex^{\frac{4}{3}}+20\,d^{3}e^{2}x-15\,d^{2}e^{3}x^{\frac{2}{3}}+12\,de^{4}x^{\frac{1}{3}}-\cdots}{e^{6}x^{2}}\right.$$
$$+\frac{1}{3600}\left(60\,en\left(\frac{60\,d^{6}\log\left(dx^{\frac{1}{3}}+e\right)}{e^{7}}-\frac{20\,d^{6}\log\left(x\right)}{e^{7}}-\frac{60\,d^{5}x^{\frac{5}{3}}-30\,d^{4}ex^{\frac{4}{3}}+20\,d^{3}e^{2}x-15\,d^{2}e^{3}x^{\frac{2}{3}}+12\,de\cdots}{e^{6}x^{2}}\right.\right.$$
$$-\frac{b^{2}\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)^{2}}{2\,x^{2}}-\frac{ab\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)}{x^{2}}-\frac{a^{2}}{2\,x^{2}}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^3,x, algorithm="maxima")
[Out] 1/60*a*b*e*n*(60*d^6*log(d*x^(1/3) + e)/e^7 - 20*d^6*log(x)/e^7 - (60*d^5*x
^(5/3) - 30*d^4*e*x^(4/3) + 20*d^3*e^2*x - 15*d^2*e^3*x^(2/3) + 12*d*e^4*x^
(1/3) - 10*e^5)/(e^6*x^2)) + 1/3600*(60*e*n*(60*d^6*log(d*x^(1/3) + e)/e^7
```

```
- 20*d^6*log(x)/e^7 - (60*d^5*x^(5/3) - 30*d^4*e*x^(4/3) + 20*d^3*e^2*x - 1
5*d^2*e^3*x^(2/3) + 12*d*e^4*x^(1/3) - 10*e^5)/(e^6*x^2))*log(c*(d + e/x^(1
/3))^n) - (1800*d^6*x^2*log(d*x^(1/3) + e)^2 + 200*d^6*x^2*log(x)^2 - 2940*
d^6*x^2*log(x) - 8820*d^5*e*x^(5/3) + 2610*d^4*e^2*x^(4/3) - 1140*d^3*e^3*x
 + 555*d^2*e^4*x^(2/3) - 264*d*e^5*x^(1/3) + 100*e^6 - 60*(20*d^6*x^2*log(x
) - 147*d^6*x^2)*log(d*x^(1/3) + e))*n^2/(e^6*x^2))*b^2 - 1/2*b^2*log(c*(d
+ e/x^(1/3))^n)^2/x^2 - a*b*log(c*(d + e/x^(1/3))^n)/x^2 - 1/2*a^2/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 877 vs. $2(411) = 822$.

Time = 0.37 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.83

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^2/x^3,x, algorithm="giac")
```

```
[Out] 1/3600*(1800*(6*(d*x^(1/3) + e)*b^2*d^5*n^2/(e^5*x^(1/3)) - 15*(d*x^(1/3) +
 e)^2*b^2*d^4*n^2/(e^5*x^(2/3)) + 20*(d*x^(1/3) + e)^3*b^2*d^3*n^2/(e^5*x)
- 15*(d*x^(1/3) + e)^4*b^2*d^2*n^2/(e^5*x^(4/3)) + 6*(d*x^(1/3) + e)^5*b^2*
d*n^2/(e^5*x^(5/3)) - (d*x^(1/3) + e)^6*b^2*n^2/(e^5*x^2))*log((d*x^(1/3) +
 e)/x^(1/3))^2 + 60*(10*(b^2*n^2 - 6*b^2*n*log(c) - 6*a*b*n)*(d*x^(1/3) + e
)^6/(e^5*x^2) - 72*(b^2*d*n^2 - 5*b^2*d*n*log(c) - 5*a*b*d*n)*(d*x^(1/3) +
e)^5/(e^5*x^(5/3)) + 225*(b^2*d^2*n^2 - 4*b^2*d^2*n*log(c) - 4*a*b*d^2*n)*(
d*x^(1/3) + e)^4/(e^5*x^(4/3)) - 400*(b^2*d^3*n^2 - 3*b^2*d^3*n*log(c) - 3*
a*b*d^3*n)*(d*x^(1/3) + e)^3/(e^5*x) + 450*(b^2*d^4*n^2 - 2*b^2*d^4*n*log(c
) - 2*a*b*d^4*n)*(d*x^(1/3) + e)^2/(e^5*x^(2/3)) - 360*(b^2*d^5*n^2 - b^2*d
^5*n*log(c) - a*b*d^5*n)*(d*x^(1/3) + e)/(e^5*x^(1/3)))*log((d*x^(1/3) + e)
/x^(1/3)) - 100*(b^2*n^2 - 6*b^2*n*log(c) + 18*b^2*log(c)^2 - 6*a*b*n + 36*
a*b*log(c) + 18*a^2)*(d*x^(1/3) + e)^6/(e^5*x^2) + 432*(2*b^2*d*n^2 - 10*b^
2*d*n*log(c) + 25*b^2*d*log(c)^2 - 10*a*b*d*n + 50*a*b*d*log(c) + 25*a^2*d)
*(d*x^(1/3) + e)^5/(e^5*x^(5/3)) - 3375*(b^2*d^2*n^2 - 4*b^2*d^2*n*log(c) +
 8*b^2*d^2*log(c)^2 - 4*a*b*d^2*n + 16*a*b*d^2*log(c) + 8*a^2*d^2)*(d*x^(1/
3) + e)^4/(e^5*x^(4/3)) + 4000*(2*b^2*d^3*n^2 - 6*b^2*d^3*n*log(c) + 9*b^2*
d^3*log(c)^2 - 6*a*b*d^3*n + 18*a*b*d^3*log(c) + 9*a^2*d^3)*(d*x^(1/3) + e)
^3/(e^5*x) - 13500*(b^2*d^4*n^2 - 2*b^2*d^4*n*log(c) + 2*b^2*d^4*log(c)^2 -
 2*a*b*d^4*n + 4*a*b*d^4*log(c) + 2*a^2*d^4)*(d*x^(1/3) + e)^2/(e^5*x^(2/3)
) + 10800*(2*b^2*d^5*n^2 - 2*b^2*d^5*n*log(c) + b^2*d^5*log(c)^2 - 2*a*b*d^
5*n + 2*a*b*d^5*log(c) + a^2*d^5)*(d*x^(1/3) + e)/(e^5*x^(1/3)))/e
```

Mupad [B] (verification not implemented)

Time = 3.00 (sec) , antiderivative size = 439, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{x^3}\,dx = \frac{b^2\,d^6\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)^2}{2\,e^6}-\frac{b^2\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)^2}{2\,x^2}-\frac{b^2\,n^2}{36\,x^2}-\frac{a\,b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{x^2}-\frac{a^2}{2\,x^2}+\frac{a\,b\,n}{6\,x^2}+\frac{b^2\,n\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{6\,x^2}-\frac{49\,b^2\,d^6\,n^2\,\ln\left(d+\frac{e}{x^{1/3}}\right)}{20\,e^6}+\frac{19\,b^2\,d^3\,n^2}{60\,e^3\,x}-\frac{37\,b^2\,d^2\,n^2}{240\,e^2\,x^{4/3}}-\frac{29\,b^2\,d^4\,n^2}{40\,e^4\,x^{2/3}}+\frac{49\,b^2\,d^5\,n^2}{20\,e^5\,x^{1/3}}+\frac{11\,b^2\,d\,n^2}{150\,e\,x^{5/3}}-\frac{b^2\,d^3\,n\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{3\,e^3\,x}+\frac{b^2\,d^2\,n\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{4\,e^2\,x^{4/3}}+\frac{b^2\,d^4\,n\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{2\,e^4\,x^{2/3}}-\frac{b^2\,d^5\,n\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{e^5\,x^{1/3}}-\frac{a\,b\,d\,n}{5\,e\,x^{5/3}}+\frac{a\,b\,d^6\,n\,\ln\left(d+\frac{e}{x^{1/3}}\right)}{e^6}-\frac{b^2\,d\,n\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)}{5\,e\,x^{5/3}}-\frac{a\,b\,d^3\,n}{3\,e^3\,x}+\frac{a\,b\,d^2\,n}{4\,e^2\,x^{4/3}}+\frac{a\,b\,d^4\,n}{2\,e^4\,x^{2/3}}-\frac{a\,b\,d^5\,n}{e^5\,x^{1/3}}$$

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))^2/x^3,x)
```

```
[Out] (b^2*d^6*log(c*(d + e/x^(1/3))^n)^2)/(2*e^6) - (b^2*log(c*(d + e/x^(1/3))^n
)^2)/(2*x^2) - (b^2*n^2)/(36*x^2) - (a*b*log(c*(d + e/x^(1/3))^n))/x^2 - a^
2/(2*x^2) + (a*b*n)/(6*x^2) + (b^2*n*log(c*(d + e/x^(1/3))^n))/(6*x^2) - (4
9*b^2*d^6*n^2*log(d + e/x^(1/3)))/(20*e^6) + (19*b^2*d^3*n^2)/(60*e^3*x) -
(37*b^2*d^2*n^2)/(240*e^2*x^(4/3)) - (29*b^2*d^4*n^2)/(40*e^4*x^(2/3)) + (4
9*b^2*d^5*n^2)/(20*e^5*x^(1/3)) + (11*b^2*d*n^2)/(150*e*x^(5/3)) - (b^2*d^3
*n*log(c*(d + e/x^(1/3))^n))/(3*e^3*x) + (b^2*d^2*n*log(c*(d + e/x^(1/3))^n
))/(4*e^2*x^(4/3)) + (b^2*d^4*n*log(c*(d + e/x^(1/3))^n))/(2*e^4*x^(2/3)) -
 (b^2*d^5*n*log(c*(d + e/x^(1/3))^n))/(e^5*x^(1/3)) - (a*b*d*n)/(5*e*x^(5/3
)) + (a*b*d^6*n*log(d + e/x^(1/3)))/e^6 - (b^2*d*n*log(c*(d + e/x^(1/3))^n)
)/(5*e*x^(5/3)) - (a*b*d^3*n)/(3*e^3*x) + (a*b*d^2*n)/(4*e^2*x^(4/3)) + (a*
b*d^4*n)/(2*e^4*x^(2/3)) - (a*b*d^5*n)/(e^5*x^(1/3))
```

3.503 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 759

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\,dx=\frac{71b^3e^5n^3\sqrt[3]{x}}{40d^5}-\frac{3b^3e^4n^3x^{2/3}}{10d^4}+\frac{b^3e^3n^3x}{20d^3}-\frac{71b^3e^6n^3\log\left(d+\frac{e}{\sqrt[3]{x}}\right)}{40d^6}-\frac{77b^2e^5n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{20d^6}+\frac{47b^2e^4n^2x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{40d^4}-\frac{9b^2e^3n^2x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{20d^3}+\frac{3b^2e^2n^2x^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{20d^2}-\frac{77b^2e^6n^2\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{20d^6}+\frac{3be^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2d^6}-\frac{3be^4nx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{4d^4}+\frac{be^3nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2d^3}-\frac{3be^2nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{8d^2}+\frac{3benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{10d}+\frac{3be^6n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2d^6}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3-\frac{3b^2e^6n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt[3]{x}}\right)}{d^6}-\frac{15b^3e^6n^3\log(x}{8d^6}$$

```
[Out] 71/40*b^3*e^5*n^3*x^(1/3)/d^5-3/10*b^3*e^4*n^3*x^(2/3)/d^4+1/20*b^3*e^3*n^3
*x/d^3-71/40*b^3*e^6*n^3*ln(d+e/x^(1/3))/d^6-77/20*b^2*e^5*n^2*(d+e/x^(1/3)
)*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^6+47/40*b^2*e^4*n^2*x^(2/3)*(a+b*ln
(c*(d+e/x^(1/3))^n))/d^4-9/20*b^2*e^3*n^2*x*(a+b*ln(c*(d+e/x^(1/3))^n))/d^3
+3/20*b^2*e^2*n^2*x^(4/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^2-77/20*b^2*e^6*n^2
*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(c*(d+e/x^(1/3))^n))/d^6+3/2*b*e^5*n*(d+e/x^(
1/3))*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))^2/d^6-3/4*b*e^4*n*x^(2/3)*(a+b*ln
(c*(d+e/x^(1/3))^n))^2/d^4+1/2*b*e^3*n*x*(a+b*ln(c*(d+e/x^(1/3))^n))^2/d^3-
```

```
3/8*b*e^2*n*x^(4/3)*(a+b*ln(c*(d+e/x^(1/3))^n))^2/d^2+3/10*b*e*n*x^(5/3)*(a
+b*ln(c*(d+e/x^(1/3))^n))^2/d+3/2*b*e^6*n*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(c*(
d+e/x^(1/3))^n))^2/d^6+1/2*x^2*(a+b*ln(c*(d+e/x^(1/3))^n))^3-3*b^2*e^6*n^2*
(a+b*ln(c*(d+e/x^(1/3))^n))*ln(-e/d/x^(1/3))/d^6-15/8*b^3*e^6*n^3*ln(x)/d^6
+77/20*b^3*e^6*n^3*polylog(2,d/(d+e/x^(1/3)))/d^6-3*b^2*e^6*n^2*(a+b*ln(c*(
d+e/x^(1/3))^n))*polylog(2,d/(d+e/x^(1/3)))/d^6-3*b^3*e^6*n^3*polylog(2,1+e
/d/x^(1/3))/d^6-3*b^3*e^6*n^3*polylog(3,d/(d+e/x^(1/3)))/d^6
```

Rubi [A] (verified)

Time = 1.80 (sec) , antiderivative size = 759, normalized size of antiderivative = 1.00, number of steps used = 62, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules

used = {2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31, 46}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3dx$$

$$=-\frac{3b^2e^6n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^6}$$
$$-\frac{77b^2e^6n^2\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{20d^6}$$
$$-\frac{3b^2e^6n^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^6}$$
$$-\frac{77b^2e^5n^2\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{20d^6}$$
$$+\frac{47b^2e^4n^2x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{40d^4}$$
$$-\frac{9b^2e^3n^2x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{20d^3}+\frac{3b^2e^2n^2x^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{20d^2}$$
$$+\frac{3be^6n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2d^6}$$
$$+\frac{3be^5n\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2d^6}$$
$$-\frac{3be^4nx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{4d^4}+\frac{be^3nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2d^3}$$
$$-\frac{3be^2nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{8d^2}+\frac{3benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{10d}$$
$$+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3+\frac{77b^3e^6n^3\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{20d^6}-\frac{3b^3e^6n^3\operatorname{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)}{d^6}-\frac{3}{}$$

```
[In] Int[x*(a + b*Log[c*(d + e/x^(1/3))^n])^3,x]
[Out] (71*b^3*e^5*n^3*x^(1/3))/(40*d^5) - (3*b^3*e^4*n^3*x^(2/3))/(10*d^4) + (b^3
*e^3*n^3*x)/(20*d^3) - (71*b^3*e^6*n^3*Log[d + e/x^(1/3)])/(40*d^6) - (77*b
```

```
^2*e^5*n^2*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(20*d^
6) + (47*b^2*e^4*n^2*x^(2/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(40*d^4) - (
9*b^2*e^3*n^2*x*(a + b*Log[c*(d + e/x^(1/3))^n]))/(20*d^3) + (3*b^2*e^2*n^2
*x^(4/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/(20*d^2) - (77*b^2*e^6*n^2*Log[1
 - d/(d + e/x^(1/3))]*(a + b*Log[c*(d + e/x^(1/3))^n]))/(20*d^6) + (3*b*e^5
*n*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(2*d^6) - (3
*b*e^4*n*x^(2/3)*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(4*d^4) + (b*e^3*n*x*(
a + b*Log[c*(d + e/x^(1/3))^n])^2)/(2*d^3) - (3*b*e^2*n*x^(4/3)*(a + b*Log[
c*(d + e/x^(1/3))^n])^2)/(8*d^2) + (3*b*e*n*x^(5/3)*(a + b*Log[c*(d + e/x^(
1/3))^n])^2)/(10*d) + (3*b*e^6*n*Log[1 - d/(d + e/x^(1/3))]*(a + b*Log[c*(d
 + e/x^(1/3))^n])^2)/(2*d^6) + (x^2*(a + b*Log[c*(d + e/x^(1/3))^n])^3)/2 -
 (3*b^2*e^6*n^2*(a + b*Log[c*(d + e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))])/d^6
 - (15*b^3*e^6*n^3*Log[x])/(8*d^6) + (77*b^3*e^6*n^3*PolyLog[2, d/(d + e/x^
(1/3))])/(20*d^6) - (3*b^2*e^6*n^2*(a + b*Log[c*(d + e/x^(1/3))^n])*PolyLog
[2, d/(d + e/x^(1/3))])/d^6 - (3*b^3*e^6*n^3*PolyLog[2, 1 + e/(d*x^(1/3))])
/d^6 - (3*b^3*e^6*n^3*PolyLog[3, d/(d + e/x^(1/3))])/d^6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
```

```
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
```

```
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(3\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^7}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3-\frac{1}{2}(3ben)\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^6(d+ex)}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3-\frac{1}{2}(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3-\frac{(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{2d}\\
&\quad+\frac{(3ben)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{2d}
\end{aligned}
$$

$$= \frac{3benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{10d} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 + \frac{(3ben)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{2d^2} - \frac{(3be^2n)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{2d^2} - \frac{(3b^2en^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{5d}$$

$$= -\frac{3be^2nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{8d^2} + \frac{3benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{10d} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 - \frac{(3be^2n)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{2d^3} + \frac{(3be^3n)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{2d^3} - \frac{(3b^2en^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{5d^2} + \frac{(3b^2e^2n^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{5d^2} + \frac{(3b^2e^2n^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{4d^2}$$

$$= \frac{3b^2e^2n^2x^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{20d^2} + \frac{be^3nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2d^3} - \frac{3be^2nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{8d^2} + \frac{3benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{10d} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 + \frac{(3be^3n)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{2d^4} - \frac{(3be^4n)\,\text{Sul}}{}$$

$$= -\frac{9b^2e^3n^2x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{20d^3} + \frac{3b^2e^2n^2x^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{20d^2} - \frac{3be^4nx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{4d^4} + \frac{be^3nx\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2d^3} - \frac{3be^2nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{8d^2} + \frac{3benx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{10d} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 - \frac{(3be^4n)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{2d^5} + \frac{(3be^5n)\,\text{Sul}}{}$$

$$= \text{Too large to display}$$

Mathematica [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 dx = \int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 dx$$

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^n])^3,x]
[Out] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^n])^3, x]
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{3}dx$$

[In] int(x*(a+b*ln(c*(d+e/x^(1/3))^n))^3,x)

[Out] int(x*(a+b*ln(c*(d+e/x^(1/3))^n))^3,x)

Fricas [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{3}x\,dx$$

[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="fricas")

[Out] integral(b^3*x*log(c*((d*x + e*x^(2/3))/x)^n)^3 + 3*a*b^2*x*log(c*((d*x + e*x^(2/3))/x)^n)^2 + 3*a^2*b*x*log(c*((d*x + e*x^(2/3))/x)^n) + a^3*x, x)

Sympy [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}dx$$

[In] integrate(x*(a+b*ln(c*(d+e/x**(1/3))**n))**3,x)

[Out] Integral(x*(a + b*log(c*(d + e/x**(1/3))**n))**3, x)

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{3}x\,dx$$

[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="maxima")

[Out] 1/2*b^3*x^2*log((d*x^(1/3) + e)^n)^3 - integrate(1/2*(2*(b^3*d*x^2 + b^3*e*x^(5/3))*log(x^(1/3*n))^3 - 2*(b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^2 + (b^3*d*n*x^2 - 6*(b^3*d*log(c) + a*b^2*d)*x^2 - 6*(b^3*e*log(c) + a*b^2*e)*x^(5/3) + 6*(b^3*d*x^2 + b^3*e*x^(5/3))*log(x^(1/3*n)))*log((d*x^(1/3) + e)^n)^2 - 6*((b^3*d*log(c) + a*b^2*d)*x^2 + (b^3*e*log(c) + a*b^2*e)*x^(5/3))*log(x^(1/3*n))^2 - 2*(b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x^(5/3) - 6*((b^3*d*log(c)^2 + 2*a*b^

```
2*d*log(c) + a^2*b*d)*x^2 + (b^3*d*x^2 + b^3*e*x^(5/3))*log(x^(1/3*n))^2 +
(b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(5/3) - 2*((b^3*d*log(c) +
a*b^2*d)*x^2 + (b^3*e*log(c) + a*b^2*e)*x^(5/3))*log(x^(1/3*n)))*log((d*x^(
1/3) + e)^n) + 6*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^2 + (b^3*
e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(5/3))*log(x^(1/3*n)))/(d*x + e*
x^(2/3)), x)
```

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^3x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^3*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\,dx=\int x\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)\right)^3\,dx$$

```
[In] int(x*(a + b*log(c*(d + e/x^(1/3))^n))^3,x)
[Out] int(x*(a + b*log(c*(d + e/x^(1/3))^n))^3, x)
```

3.504 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 dx$

Optimal result

Integrand size = 20, antiderivative size = 436

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 dx$$
$$=\frac{3b^2e^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}$$
$$+\frac{3b^2e^3n^2\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}$$
$$-\frac{3be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{d^3}$$
$$+\frac{3benx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2d}$$
$$-\frac{3be^3n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{d^3}$$
$$+x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3+\frac{6b^2e^3n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt[3]{x}}\right)}{d^3}$$
$$+\frac{b^3e^3n^3\log(x)}{d^3}-\frac{3b^3e^3n^3\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^3}$$
$$+\frac{6b^2e^3n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^3}$$
$$+\frac{6b^3e^3n^3\operatorname{PolyLog}\left(2,1+\frac{e}{d\sqrt[3]{x}}\right)}{d^3}+\frac{6b^3e^3n^3\operatorname{PolyLog}\left(3,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^3}$$

```
[Out] 3*b^2*e^2*n^2*(d+e/x^(1/3))*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))/d^3+3*b^2*e
^3*n^2*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(c*(d+e/x^(1/3))^n))/d^3-3*b*e^2*n*(d+e
/x^(1/3))*x^(1/3)*(a+b*ln(c*(d+e/x^(1/3))^n))^2/d^3+3/2*b*e*n*x^(2/3)*(a+b*
ln(c*(d+e/x^(1/3))^n))^2/d-3*b*e^3*n*ln(1-d/(d+e/x^(1/3)))*(a+b*ln(c*(d+e/x
^(1/3))^n))^2/d^3+x*(a+b*ln(c*(d+e/x^(1/3))^n))^3+6*b^2*e^3*n^2*(a+b*ln(c*(
d+e/x^(1/3))^n))*ln(-e/d/x^(1/3))/d^3+b^3*e^3*n^3*ln(x)/d^3-3*b^3*e^3*n^3*p
olylog(2,d/(d+e/x^(1/3)))/d^3+6*b^2*e^3*n^2*(a+b*ln(c*(d+e/x^(1/3))^n))*pol
```

```
ylog(2,d/(d+e/x^(1/3)))/d^3+6*b^3*e^3*n^3*polylog(2,1+e/d/x^(1/3))/d^3+6*b^
3*e^3*n^3*polylog(3,d/(d+e/x^(1/3)))/d^3
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {2501, 2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 dx$$

$$= \frac{6b^2e^3n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} + \frac{3b^2e^3n^2\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} + \frac{6b^2e^3n^2\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} + \frac{3b^2e^2n^2\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} - \frac{3be^3n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{d^3} - \frac{3be^2n\sqrt[3]{x}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{d^3} + \frac{3benx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2d} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 - \frac{3b^3e^3n^3\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^3} + \frac{6b^3e^3n^3\operatorname{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)}{d^3} + \frac{6b^3e^3n^3\operatorname{PolyLog}\left(3,\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^3} + \frac{b^3e^3n^3\log(x)}{d^3}$$

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^3,x]
```

```
[Out] (3*b^2*e^2*n^2*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^n]))/d^
3 + (3*b^2*e^3*n^2*Log[1 - d/(d + e/x^(1/3))]*(a + b*Log[c*(d + e/x^(1/3))^
n]))/d^3 - (3*b*e^2*n*(d + e/x^(1/3))*x^(1/3)*(a + b*Log[c*(d + e/x^(1/3))^
n])^2)/d^3 + (3*b*e*n*x^(2/3)*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(2*d) - (
3*b*e^3*n*Log[1 - d/(d + e/x^(1/3))]*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/d^
3 + x*(a + b*Log[c*(d + e/x^(1/3))^n])^3 + (6*b^2*e^3*n^2*(a + b*Log[c*(d +
 e/x^(1/3))^n])*Log[-(e/(d*x^(1/3)))])/d^3 + (b^3*e^3*n^3*Log[x])/d^3 - (3*
b^3*e^3*n^3*PolyLog[2, d/(d + e/x^(1/3))])/d^3 + (6*b^2*e^3*n^2*(a + b*Log[
c*(d + e/x^(1/3))^n])*PolyLog[2, d/(d + e/x^(1/3))])/d^3 + (6*b^3*e^3*n^3*P
olyLog[2, 1 + e/(d*x^(1/3))])/d^3 + (6*b^3*e^3*n^3*PolyLog[3, d/(d + e/x^(1
/3))])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^n\right)\right)^3\,dx,x,\sqrt[3]{x}\right)\\
&= -\left(3\text{Subst}\left(\int\frac{(a+b\log(c(d+ex)^n))^3}{x^4}\,dx,x,\frac{1}{\sqrt[3]{x}}\right)\right)\\
&= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3-(3ben)\text{Subst}\left(\int\frac{(a+b\log(c(d+ex)^n))^2}{x^3(d+ex)}\,dx,x,\frac{1}{\sqrt[3]{x}}\right)\\
&= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3-(3bn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)\\
&= x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3-\frac{(3bn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{d}\\
&\quad+\frac{(3ben)\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{d}
\end{aligned}$$

$$= \frac{3benx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2d} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 + \frac{(3ben)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d^2} - \frac{(3be^2n)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d^2} - \frac{(3b^2en^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d}$$

$$= -\frac{3be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{d^3} + \frac{3benx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2d} - \frac{3be^3n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{d^3} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 - \frac{(3b^2en^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d^2} + \frac{(6b^2e^2n^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{-\frac{d}{e}+\frac{x}{e}}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d^3} + \frac{(3b^2e^2n^2)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d^2} + \frac{(6b^2e^3n^2)\,\text{Subst}\left(\int \frac{\log\left(1-\frac{d}{x}\right)(a+b\log(cx^n))}{x}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{d^3}$$

$$= \frac{3b^2e^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}$$
$$+ \frac{3b^2e^3n^2\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3}$$
$$- \frac{3be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{d^3}$$
$$+ \frac{3benx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2d}$$
$$- \frac{3be^3n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{d^3}$$
$$+ x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3$$
$$+ \frac{6b^2e^3n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt[3]{x}}\right)}{d^3}$$
$$+ \frac{6b^2e^3n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\operatorname{Li}_2\left(\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^3}$$
$$- \frac{(3b^3e^2n^3)\operatorname{Subst}\left(\int\frac{1}{-\frac{d}{e}+\frac{x}{e}}\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{d^3}$$
$$- \frac{(3b^3e^3n^3)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{d}{x}\right)}{x}\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{d^3}$$
$$- \frac{(6b^3e^3n^3)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{x}{d}\right)}{x}\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{d^3}$$
$$- \frac{(6b^3e^3n^3)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_2\left(\frac{d}{x}\right)}{x}\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{d^3}$$

$$= \frac{3b^2e^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} + \frac{3b^2e^3n^2\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{d^3} - \frac{3be^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{d^3} + \frac{3benx^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2d} - \frac{3be^3n\log\left(1-\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{d^3} + x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 + \frac{6b^2e^3n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\log\left(-\frac{e}{d\sqrt[3]{x}}\right)}{d^3} + \frac{b^3e^3n^3\log(x)}{d^3} - \frac{3b^3e^3n^3\mathrm{Li}_2\left(\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^3} + \frac{6b^2e^3n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\mathrm{Li}_2\left(\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^3} + \frac{6b^3e^3n^3\mathrm{Li}_2\left(1+\frac{e}{d\sqrt[3]{x}}\right)}{d^3} + \frac{6b^3e^3n^3\mathrm{Li}_3\left(\frac{d}{d+\frac{e}{\sqrt[3]{x}}}\right)}{d^3}$$

Mathematica [F]

$$\int\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\,dx=\int\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3, x]
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{3}dx$$

[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^3,x)

[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^3,x)

Fricas [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{3}dx$$

[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="fricas")

[Out] integral(b^3*log(c*((d*x + e*x^(2/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x^(2/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(2/3))/x)^n) + a^3, x)

Sympy [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}dx=\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}dx$$

[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**3,x)

[Out] Integral((a + b*log(c*(d + e/x**(1/3))**n))**3, x)

Maxima [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{3}dx$$

[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="maxima")

[Out] b^3*x*log((d*x^(1/3) + e)^n)^3 + 3/2*(e*n*(2*e^2*log(d*x^(1/3) + e)/d^3 + (d*x^(2/3) - 2*e*x^(1/3))/d^2) + 2*x*log(c*(d + e/x^(1/3))^n))*a^2*b + a^3*x - integrate(((b^3*d*x + b^3*e*x^(2/3))*log(x^(1/3*n))^3 + (b^3*d*n*x - 3*(b^3*d*log(c) + a*b^2*d)*x + 3*(b^3*d*x + b^3*e*x^(2/3))*log(x^(1/3*n)) - 3*(b^3*e*log(c) + a*b^2*e)*x^(2/3))*log((d*x^(1/3) + e)^n)^2 - 3*((b^3*d*log(c) + a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*x^(2/3))*log(x^(1/3*n))^2 - (b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2)*x - 3*((b^3*d*x + b^3*e*x^(2/3))*log(x^(1

```
/3*n))^2 + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c))*x - 2*((b^3*d*log(c) + a*b^2
*d)*x + (b^3*e*log(c) + a*b^2*e)*x^(2/3))*log(x^(1/3*n)) + (b^3*e*log(c)^2
+ 2*a*b^2*e*log(c))*x^(2/3))*log((d*x^(1/3) + e)^n) + 3*((b^3*d*log(c)^2 +
2*a*b^2*d*log(c))*x + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c))*x^(2/3))*log(x^(1
/3*n)) - (b^3*e*log(c)^3 + 3*a*b^2*e*log(c)^2)*x^(2/3))/(d*x + e*x^(2/3)),
x)
```

Giac [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 dx = \int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^3 dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3 dx = \int \left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)\right)^3 dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))^3,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))^n))^3, x)
```

3.505 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 135

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx = -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt[3]{x}}\right) - 9bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\operatorname{PolyLog}\left(2,1+\frac{e}{d\sqrt[3]{x}}\right) + 18b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\operatorname{PolyLog}\left(3,1+\frac{e}{d\sqrt[3]{x}}\right) - 18b^3n^3\operatorname{PolyLog}\left(4,1+\frac{e}{d\sqrt[3]{x}}\right)$$

```
[Out] -3*(a+b*ln(c*(d+e/x^(1/3))^n))^3*ln(-e/d/x^(1/3))-9*b*n*(a+b*ln(c*(d+e/x^(1
/3))^n))^2*polylog(2,1+e/d/x^(1/3))+18*b^2*n^2*(a+b*ln(c*(d+e/x^(1/3))^n))*
polylog(3,1+e/d/x^(1/3))-18*b^3*n^3*polylog(4,1+e/d/x^(1/3))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2504, 2443, 2481, 2421, 2430, 6724}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx = 18b^2n^2\operatorname{PolyLog}\left(3,\frac{e}{d\sqrt[3]{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)-9bn\operatorname{PolyLog}\left(2,\frac{e}{d\sqrt[3]{x}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2-3\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3-18b^3n^3\operatorname{PolyLog}\left(4,\frac{e}{d\sqrt[3]{x}}+1\right)$$

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x,x]
```

```
[Out] -3*(a + b*Log[c*(d + e/x^(1/3))^n])^3*Log[-(e/(d*x^(1/3)))] - 9*b*n*(a + b*
Log[c*(d + e/x^(1/3))^n])^2*PolyLog[2, 1 + e/(d*x^(1/3))] + 18*b^2*n^2*(a +
 b*Log[c*(d + e/x^(1/3))^n])*PolyLog[3, 1 + e/(d*x^(1/3))] - 18*b^3*n^3*Pol
yLog[4, 1 + e/(d*x^(1/3))]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
```

```
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(3\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)\\
&= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\\
&\quad+(9ben)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{d+ex}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\\
&= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt[3]{x}}\right)\\
&\quad+(9bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)
\end{aligned}
$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt[3]{x}}\right)$$
$$-9bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\operatorname{Li}_2\left(1+\frac{e}{d\sqrt[3]{x}}\right)$$
$$+\left(18b^2n^2\right)\operatorname{Subst}\left(\int\frac{(a+b\log(cx^n))\operatorname{Li}_2\left(\frac{x}{d}\right)}{x}\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt[3]{x}}\right)$$
$$-9bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\operatorname{Li}_2\left(1+\frac{e}{d\sqrt[3]{x}}\right)$$
$$+18b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\operatorname{Li}_3\left(1+\frac{e}{d\sqrt[3]{x}}\right)$$
$$-\left(18b^3n^3\right)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(\frac{x}{d}\right)}{x}\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)$$

$$= -3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\log\left(-\frac{e}{d\sqrt[3]{x}}\right)$$
$$-9bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\operatorname{Li}_2\left(1+\frac{e}{d\sqrt[3]{x}}\right)$$
$$+18b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\operatorname{Li}_3\left(1+\frac{e}{d\sqrt[3]{x}}\right)-18b^3n^3\operatorname{Li}_4\left(1+\frac{e}{d\sqrt[3]{x}}\right)$$

Mathematica [F]

$$\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx=\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x, x]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{3}}{x}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x,x)
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}}{x}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{3}}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x,x, algorithm="fricas")
[Out] integral((b^3*log(c*((d*x + e*x^(2/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x^
(2/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(2/3))/x)^n) + a^3)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}}{x}\,dx = \int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}}{x}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**3/x,x)
[Out] Integral((a + b*log(c*(d + e/x**(1/3))**n))**3/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}}{x}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)+a\right)^{3}}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x,x, algorithm="maxima")
[Out] b^3*log((d*x^(1/3) + e)^n)^3*log(x) - integrate(((b^3*d*x + b^3*e*x^(2/3))*
log(x^(1/3*n))^3 + (b^3*d*n*x*log(x) - 3*(b^3*d*log(c) + a*b^2*d)*x + 3*(b^
3*d*x + b^3*e*x^(2/3))*log(x^(1/3*n)) - 3*(b^3*e*log(c) + a*b^2*e)*x^(2/3))
```

```
*log((d*x^(1/3) + e)^n)^2 - 3*((b^3*d*log(c) + a*b^2*d)*x + (b^3*e*log(c) +
 a*b^2*e)*x^(2/3))*log(x^(1/3*n))^2 - (b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2
+ 3*a^2*b*d*log(c) + a^3*d)*x - 3*((b^3*d*x + b^3*e*x^(2/3))*log(x^(1/3*n))
^2 + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x - 2*((b^3*d*log(c) + a
*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*x^(2/3))*log(x^(1/3*n)) + (b^3*e*log(c
)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(2/3))*log((d*x^(1/3) + e)^n) + 3*((b^3
*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x + (b^3*e*log(c)^2 + 2*a*b^2*e*l
og(c) + a^2*b*e)*x^(2/3))*log(x^(1/3*n)) - (b^3*e*log(c)^3 + 3*a*b^2*e*log(
c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x^(2/3))/(d*x^2 + e*x^(5/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)+a\right)^3}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))^n) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^n\right)\right)^3}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))^3/x,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))^n))^3/x, x)
```

3.506 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 438

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}\,dx=-\frac{9b^3dn^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^3}+\frac{2b^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3}-\frac{18ab^2d^2n^2}{e^2\sqrt[3]{x}}+\frac{18b^3d^2n^3}{e^2\sqrt[3]{x}}-\frac{18b^3d^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{e^3}+\frac{9b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2e^3}-\frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^3}+\frac{9bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^3}-\frac{9bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2e^3}+\frac{bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^3}-\frac{3d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3}+\frac{3d\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3}-\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3}$$

```
[Out] -9/4*b^3*d*n^3*(d+e/x^(1/3))^2/e^3+2/9*b^3*n^3*(d+e/x^(1/3))^3/e^3-18*a*b^2
*d^2*n^2/e^2/x^(1/3)+18*b^3*d^2*n^3/e^2/x^(1/3)-18*b^3*d^2*n^2*(d+e/x^(1/3)
)*ln(c*(d+e/x^(1/3))^n)/e^3+9/2*b^2*d*n^2*(d+e/x^(1/3))^2*(a+b*ln(c*(d+e/x^
(1/3))^n))/e^3-2/3*b^2*n^2*(d+e/x^(1/3))^3*(a+b*ln(c*(d+e/x^(1/3))^n))/e^3+
9*b*d^2*n*(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))^2/e^3-9/2*b*d*n*(d+e/x^
(1/3))^2*(a+b*ln(c*(d+e/x^(1/3))^n))^2/e^3+b*n*(d+e/x^(1/3))^3*(a+b*ln(c*(d
```

```
+e/x^(1/3))^n))^2/e^3-3*d^2*(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^3
+3*d*(d+e/x^(1/3))^2*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^3-(d+e/x^(1/3))^3*(a+b
*ln(c*(d+e/x^(1/3))^n))^3/e^3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341\}$

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}\,dx = -\frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^3} + \frac{9b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2e^3} - \frac{18ab^2d^2n^2}{e^2\sqrt[3]{x}} + \frac{9bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^3} - \frac{3d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3} + \frac{bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^3} - \frac{9bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2e^3} - \frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3} + \frac{3d\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3} - \frac{18b^3d^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{e^3} + \frac{18b^3d^2n^3}{e^2\sqrt[3]{x}} + \frac{2b^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3} - \frac{9b^3dn^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^3}$$

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x^2,x]
```

```
[Out] (-9*b^3*d*n^3*(d + e/x^(1/3))^2)/(4*e^3) + (2*b^3*n^3*(d + e/x^(1/3))^3)/(9
*e^3) - (18*a*b^2*d^2*n^2)/(e^2*x^(1/3)) + (18*b^3*d^2*n^3)/(e^2*x^(1/3)) -
```

```
 (18*b^3*d^2*n^2*(d + e/x^(1/3))*Log[c*(d + e/x^(1/3))^n])/e^3 + (9*b^2*d*n
^2*(d + e/x^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/(2*e^3) - (2*b^2*n^2
*(d + e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*e^3) + (9*b*d^2*n*(
d + e/x^(1/3))*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/e^3 - (9*b*d*n*(d + e/x^
(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(2*e^3) + (b*n*(d + e/x^(1/3))
^3*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/e^3 - (3*d^2*(d + e/x^(1/3))*(a + b*
Log[c*(d + e/x^(1/3))^n])^3)/e^3 + (3*d*(d + e/x^(1/3))^2*(a + b*Log[c*(d +
 e/x^(1/3))^n])^3)/e^3 - ((d + e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n]
)^3)/e^3
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = -\left(3\text{Subst}\left(\int x^2(a+b\log(c(d+ex)^n))^3\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\text{Subst}\left(\int \left(\frac{d^2(a+b\log(c(d+ex)^n))^3}{e^2} - \frac{2d(d+ex)(a+b\log(c(d+ex)^n))^3}{e^2} + \frac{(d+ex)^2(a+\right.\right.\right.$$

$$= -\frac{3\text{Subst}\left(\int (d+ex)^2(a+b\log(c(d+ex)^n))^3\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^2} + \frac{(6d)\text{Subst}\left(\int (d+ex)(a+b\log(c(d+ex)^n))^3\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^2} - \frac{(3d^2)\,\text{Subst}\left(\int (a+b\log(c(d+ex)^n))^3\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^2}$$

$$= -\frac{3\text{Subst}\left(\int x^2(a+b\log(cx^n))^3\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} + \frac{(6d)\text{Subst}\left(\int x(a+b\log(cx^n))^3\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} - \frac{(3d^2)\,\text{Subst}\left(\int (a+b\log(cx^n))^3\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}$$

$$=-\frac{3d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3}+\frac{3d\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3}-\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3}+\frac{(3bn)\text{Subst}\left(\int x^2(a+b\log(cx^n))^2\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}-\frac{(9bdn)\text{Subst}\left(\int x(a+b\log(cx^n))^2\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}+\frac{(9bd^2n)\,\text{Subst}\left(\int(a+b\log(cx^n))^2\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}$$

$$= \frac{9bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^3}$$
$$- \frac{9bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2e^3}$$
$$+ \frac{bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^3}$$
$$- \frac{3d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3}$$
$$+ \frac{3d\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3}$$
$$- \frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3}$$
$$- \frac{(2b^2n^2)\,\mathrm{Subst}\left(\int x^2(a+b\log(cx^n))\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}$$
$$+ \frac{(9b^2dn^2)\,\mathrm{Subst}\left(\int x(a+b\log(cx^n))\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}$$
$$- \frac{(18b^2d^2n^2)\,\mathrm{Subst}\left(\int (a+b\log(cx^n))\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}$$

$$= -\frac{9b^3dn^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^3} + \frac{2b^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3} - \frac{18ab^2d^2n^2}{e^2\sqrt[3]{x}}$$
$$+ \frac{9b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2e^3}$$
$$- \frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^3}$$
$$+ \frac{9bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^3}$$
$$- \frac{9bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2e^3}$$
$$+ \frac{bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^3}$$
$$- \frac{3d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3}$$
$$+ \frac{3d\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3}$$
$$- \frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3}$$
$$- \frac{(18b^3d^2n^2)\,\text{Subst}\left(\int\log\left(cx^n\right)\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}$$

$$= -\frac{9b^3dn^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^3} + \frac{2b^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^3} - \frac{18ab^2d^2n^2}{e^2\sqrt[3]{x}} + \frac{18b^3d^2n^3}{e^2\sqrt[3]{x}} - \frac{18b^3d^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{e^3} + \frac{9b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{2e^3} - \frac{2b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^3} + \frac{9bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^3} - \frac{9bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{2e^3} + \frac{bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^3} - \frac{3d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3} + \frac{3d\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3} - \frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^3}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 666, normalized size of antiderivative = 1.52

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}\,dx$$

$$= \frac{-36a^3e^3+36a^2be^3n-24ab^2e^3n^2+8b^3e^3n^3-54a^2bde^2n\sqrt[3]{x}+90ab^2de^2n^2\sqrt[3]{x}-57b^3de^2n^3\sqrt[3]{x}+108a^2bd^2e}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x^2,x]
```

```
[Out] (-36*a^3*e^3 + 36*a^2*b*e^3*n - 24*a*b^2*e^3*n^2 + 8*b^3*e^3*n^3 - 54*a^2*b
*d*e^2*n*x^(1/3) + 90*a*b^2*d*e^2*n^2*x^(1/3) - 57*b^3*d*e^2*n^3*x^(1/3) +
108*a^2*b*d^2*e*n*x^(2/3) - 396*a*b^2*d^2*e*n^2*x^(2/3) + 510*b^3*d^2*e*n^3
*x^(2/3) + 72*b^3*d^3*n^3*x*Log[d + e/x^(1/3)]^3 - 36*b^3*e^3*Log[c*(d + e/
x^(1/3))^n]^3 - 108*a^2*b*d^3*n*x*Log[e + d*x^(1/3)] + 396*a*b^2*d^3*n^2*x*
Log[e + d*x^(1/3)] - 510*b^3*d^3*n^3*x*Log[e + d*x^(1/3)] + 12*b^2*d^3*n^2*
x*Log[d + e/x^(1/3)]*(6*a - 11*b*n + 6*b*Log[c*(d + e/x^(1/3))^n])*(3*Log[e
 + d*x^(1/3)] - Log[x]) + 36*a^2*b*d^3*n*x*Log[x] - 132*a*b^2*d^3*n^2*x*Log
[x] + 170*b^3*d^3*n^3*x*Log[x] - 18*b^2*d^3*n^2*x*Log[d + e/x^(1/3)]^2*(6*a
 - 11*b*n + 6*b*Log[c*(d + e/x^(1/3))^n] + 6*b*n*Log[e + d*x^(1/3)] - 2*b*n
*Log[x]) + 18*b^2*Log[c*(d + e/x^(1/3))^n]^2*(e*(-6*a*e^2 + 2*b*e^2*n - 3*b
*d*e*n*x^(1/3) + 6*b*d^2*n*x^(2/3)) - 6*b*d^3*n*x*Log[e + d*x^(1/3)] + 2*b*
d^3*n*x*Log[x]) - 6*b*Log[c*(d + e/x^(1/3))^n]*(18*a^2*e^3 - 6*a*b*e*n*(2*e
^2 - 3*d*e*x^(1/3) + 6*d^2*x^(2/3)) + b^2*e*n^2*(4*e^2 - 15*d*e*x^(1/3) + 6
6*d^2*x^(2/3)) + 6*b*d^3*n*(6*a - 11*b*n)*x*Log[e + d*x^(1/3)] + 2*b*d^3*n*
(-6*a + 11*b*n)*x*Log[x]))/(36*e^3*x)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{3}}{x^{2}}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(384) = 768$.

Time = 0.38 (sec) , antiderivative size = 814, normalized size of antiderivative = 1.86

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}}{x^{2}}\,dx$$
$$= \frac{8\,b^3e^3n^3 - 24\,ab^2e^3n^2 + 36\,a^2be^3n - 36\,a^3e^3 + 36\,(b^3e^3x - b^3e^3)\log(c)^3 - 36\,(b^3d^3n^3x + b^3e^3n^3)\log\left(\frac{dx+e}{x}\right.}{}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/36*(8*b^3*e^3*n^3 - 24*a*b^2*e^3*n^2 + 36*a^2*b*e^3*n - 36*a^3*e^3 + 36*(
b^3*e^3*x - b^3*e^3)*log(c)^3 - 36*(b^3*d^3*n^3*x + b^3*e^3*n^3)*log((d*x +
 e*x^(2/3))/x)^3 + 36*(b^3*e^3*n - 3*a*b^2*e^3 - (b^3*e^3*n - 3*a*b^2*e^3)*
```

```
x)*log(c)^2 + 18*(6*b^3*d^2*e*n^3*x^(2/3) - 3*b^3*d*e^2*n^3*x^(1/3) + 2*b^3
*e^3*n^3 - 6*a*b^2*e^3*n^2 + (11*b^3*d^3*n^3 - 6*a*b^2*d^3*n^2)*x - 6*(b^3*
d^3*n^2*x + b^3*e^3*n^2)*log(c))*log((d*x + e*x^(2/3))/x)^2 - 4*(2*b^3*e^3*
n^3 - 6*a*b^2*e^3*n^2 + 9*a^2*b*e^3*n - 9*a^3*e^3)*x - 12*(2*b^3*e^3*n^2 -
6*a*b^2*e^3*n + 9*a^2*b*e^3 - (2*b^3*e^3*n^2 - 6*a*b^2*e^3*n + 9*a^2*b*e^3)
*x)*log(c) - 6*(4*b^3*e^3*n^3 - 12*a*b^2*e^3*n^2 + 18*a^2*b*e^3*n + 18*(b^3
*d^3*n*x + b^3*e^3*n)*log(c)^2 + (85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^2 + 18*a^
2*b*d^3*n)*x - 6*(2*b^3*e^3*n^2 - 6*a*b^2*e^3*n + (11*b^3*d^3*n^2 - 6*a*b^2
*d^3*n)*x)*log(c) + 6*(11*b^3*d^2*e*n^3 - 6*b^3*d^2*e*n^2*log(c) - 6*a*b^2*
d^2*e*n^2)*x^(2/3) - 3*(5*b^3*d*e^2*n^3 - 6*b^3*d*e^2*n^2*log(c) - 6*a*b^2*
d*e^2*n^2)*x^(1/3))*log((d*x + e*x^(2/3))/x) + 6*(85*b^3*d^2*e*n^3 + 18*b^3
*d^2*e*n*log(c)^2 - 66*a*b^2*d^2*e*n^2 + 18*a^2*b*d^2*e*n - 6*(11*b^3*d^2*e
*n^2 - 6*a*b^2*d^2*e*n)*log(c))*x^(2/3) - 3*(19*b^3*d*e^2*n^3 + 18*b^3*d*e^
2*n*log(c)^2 - 30*a*b^2*d*e^2*n^2 + 18*a^2*b*d*e^2*n - 6*(5*b^3*d*e^2*n^2 -
 6*a*b^2*d*e^2*n)*log(c))*x^(1/3))/(e^3*x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**3/x**2,x)
```

```
[Out] Integral((a + b*log(c*(d + e/x**(1/3))**n))**3/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.46

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}\,dx$$
$$= -\frac{1}{2}\,a^2ben\left(\frac{6\,d^3\log\left(dx^{\frac{1}{3}}+e\right)}{e^4}-\frac{2\,d^3\log\left(x\right)}{e^4}-\frac{6\,d^2x^{\frac{2}{3}}-3\,dex^{\frac{1}{3}}+2\,e^2}{e^3x}\right)$$
$$-\frac{b^3\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)^3}{x}$$
$$-\frac{1}{6}\left(6\,en\left(\frac{6\,d^3\log\left(dx^{\frac{1}{3}}+e\right)}{e^4}-\frac{2\,d^3\log\left(x\right)}{e^4}-\frac{6\,d^2x^{\frac{2}{3}}-3\,dex^{\frac{1}{3}}+2\,e^2}{e^3x}\right)\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)-\frac{\left(18\,d^3x\right.}{}\right.$$
$$-\frac{1}{108}\left(54\,en\left(\frac{6\,d^3\log\left(dx^{\frac{1}{3}}+e\right)}{e^4}-\frac{2\,d^3\log\left(x\right)}{e^4}-\frac{6\,d^2x^{\frac{2}{3}}-3\,dex^{\frac{1}{3}}+2\,e^2}{e^3x}\right)\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)^2+en\left(\right.\right.$$
$$-\frac{3\,ab^2\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)^2}{x}-\frac{3\,a^2b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^n\right)}{x}-\frac{a^3}{x}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*b*e*n*(6*d^3*log(d*x^(1/3) + e)/e^4 - 2*d^3*log(x)/e^4 - (6*d^2*x^
(2/3) - 3*d*e*x^(1/3) + 2*e^2)/(e^3*x)) - b^3*log(c*(d + e/x^(1/3))^n)^3/x
- 1/6*(6*e*n*(6*d^3*log(d*x^(1/3) + e)/e^4 - 2*d^3*log(x)/e^4 - (6*d^2*x^(2
/3) - 3*d*e*x^(1/3) + 2*e^2)/(e^3*x))*log(c*(d + e/x^(1/3))^n) - (18*d^3*x*
log(d*x^(1/3) + e)^2 + 2*d^3*x*log(x)^2 - 22*d^3*x*log(x) - 66*d^2*e*x^(2/3
) + 15*d*e^2*x^(1/3) - 4*e^3 - 6*(2*d^3*x*log(x) - 11*d^3*x)*log(d*x^(1/3)
+ e))*n^2/(e^3*x))*a*b^2 - 1/108*(54*e*n*(6*d^3*log(d*x^(1/3) + e)/e^4 - 2*
d^3*log(x)/e^4 - (6*d^2*x^(2/3) - 3*d*e*x^(1/3) + 2*e^2)/(e^3*x))*log(c*(d
+ e/x^(1/3))^n)^2 + e*n*((108*d^3*x*log(d*x^(1/3) + e)^3 - 4*d^3*x*log(x)^3
 + 66*d^3*x*log(x)^2 - 510*d^3*x*log(x) - 1530*d^2*e*x^(2/3) + 171*d*e^2*x^
(1/3) - 24*e^3 - 54*(2*d^3*x*log(x) - 11*d^3*x)*log(d*x^(1/3) + e)^2 + 18*(
2*d^3*x*log(x)^2 - 22*d^3*x*log(x) + 85*d^3*x)*log(d*x^(1/3) + e))*n^2/(e^4
*x) - 18*(18*d^3*x*log(d*x^(1/3) + e)^2 + 2*d^3*x*log(x)^2 - 22*d^3*x*log(x
) - 66*d^2*e*x^(2/3) + 15*d*e^2*x^(1/3) - 4*e^3 - 6*(2*d^3*x*log(x) - 11*d^
3*x)*log(d*x^(1/3) + e))*n*log(c*(d + e/x^(1/3))^n)/(e^4*x)))*b^3 - 3*a*b^2
*log(c*(d + e/x^(1/3))^n)^2/x - 3*a^2*b*log(c*(d + e/x^(1/3))^n)/x - a^3/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 846 vs. $2(384) = 768$.

Time = 0.45 (sec) , antiderivative size = 846, normalized size of antiderivative = 1.93

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^2,x, algorithm="giac")
```

```
[Out] -1/36*(36*(3*(d*x^(1/3) + e)*b^3*d^2*n^3/(e^2*x^(1/3)) - 3*(d*x^(1/3) + e)^
2*b^3*d*n^3/(e^2*x^(2/3)) + (d*x^(1/3) + e)^3*b^3*n^3/(e^2*x))*log((d*x^(1/
3) + e)/x^(1/3))^3 - 18*(2*(b^3*n^3 - 3*b^3*n^2*log(c) - 3*a*b^2*n^2)*(d*x^
(1/3) + e)^3/(e^2*x) - 9*(b^3*d*n^3 - 2*b^3*d*n^2*log(c) - 2*a*b^2*d*n^2)*(
d*x^(1/3) + e)^2/(e^2*x^(2/3)) + 18*(b^3*d^2*n^3 - b^3*d^2*n^2*log(c) - a*b
^2*d^2*n^2)*(d*x^(1/3) + e)/(e^2*x^(1/3)))*log((d*x^(1/3) + e)/x^(1/3))^2 +
 6*(2*(2*b^3*n^3 - 6*b^3*n^2*log(c) + 9*b^3*n*log(c)^2 - 6*a*b^2*n^2 + 18*a
*b^2*n*log(c) + 9*a^2*b*n)*(d*x^(1/3) + e)^3/(e^2*x) - 27*(b^3*d*n^3 - 2*b^
3*d*n^2*log(c) + 2*b^3*d*n*log(c)^2 - 2*a*b^2*d*n^2 + 4*a*b^2*d*n*log(c) +
2*a^2*b*d*n)*(d*x^(1/3) + e)^2/(e^2*x^(2/3)) + 54*(2*b^3*d^2*n^3 - 2*b^3*d^
2*n^2*log(c) + b^3*d^2*n*log(c)^2 - 2*a*b^2*d^2*n^2 + 2*a*b^2*d^2*n*log(c)
+ a^2*b*d^2*n)*(d*x^(1/3) + e)/(e^2*x^(1/3)))*log((d*x^(1/3) + e)/x^(1/3))
- 4*(2*b^3*n^3 - 6*b^3*n^2*log(c) + 9*b^3*n*log(c)^2 - 9*b^3*log(c)^3 - 6*a
*b^2*n^2 + 18*a*b^2*n*log(c) - 27*a*b^2*log(c)^2 + 9*a^2*b*n - 27*a^2*b*log
(c) - 9*a^3)*(d*x^(1/3) + e)^3/(e^2*x) + 27*(3*b^3*d*n^3 - 6*b^3*d*n^2*log(
c) + 6*b^3*d*n*log(c)^2 - 4*b^3*d*log(c)^3 - 6*a*b^2*d*n^2 + 12*a*b^2*d*n*l
og(c) - 12*a*b^2*d*log(c)^2 + 6*a^2*b*d*n - 12*a^2*b*d*log(c) - 4*a^3*d)*(d
*x^(1/3) + e)^2/(e^2*x^(2/3)) - 108*(6*b^3*d^2*n^3 - 6*b^3*d^2*n^2*log(c) +
 3*b^3*d^2*n*log(c)^2 - b^3*d^2*log(c)^3 - 6*a*b^2*d^2*n^2 + 6*a*b^2*d^2*n*
log(c) - 3*a*b^2*d^2*log(c)^2 + 3*a^2*b*d^2*n - 3*a^2*b*d^2*log(c) - a^3*d^
2)*(d*x^(1/3) + e)/(e^2*x^(1/3)))/e
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.30

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^2}\,dx$$
$$= \frac{\frac{d\left(3a^3-3a^2bn+2ab^2n^2-\frac{2b^3n^3}{3}\right)}{2e}-\frac{d\left(6a^3-6ab^2n^2+5b^3n^3\right)}{4e}}{x^{2/3}} - \ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)^3\left(\frac{b^3}{x}+\frac{b^3d^3}{e^3}\right)$$
$$-\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^n\right)^2\left(\frac{b^2(3a-bn)}{x}-\frac{\frac{3b^2d(3a-bn)}{2e}-\frac{9ab^2d}{2e}}{x^{2/3}}+\frac{d\left(6ab^2d^2-11b^3d^2n\right)}{2e^3}+\frac{d\left(\frac{3b^2d(3a-bn)}{e}-\right.}{e\,x^{1/3}}\right.$$

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))^3/x^2,x)
```

```
[Out] ((d*(3*a^3 - (2*b^3*n^3)/3 + 2*a*b^2*n^2 - 3*a^2*b*n))/(2*e) - (d*(6*a^3 +
5*b^3*n^3 - 6*a*b^2*n^2))/(4*e))/x^(2/3) - log(c*(d + e/x^(1/3))^n)^3*(b^3/
x + (b^3*d^3)/e^3) - log(c*(d + e/x^(1/3))^n)^2*((b^2*(3*a - b*n))/x - ((3*
b^2*d*(3*a - b*n))/(2*e) - (9*a*b^2*d)/(2*e))/x^(2/3) + (d*(6*a*b^2*d^2 - 1
1*b^3*d^2*n))/(2*e^3) + (d*((3*b^2*d*(3*a - b*n))/e - (9*a*b^2*d)/e))/(e*x^
(1/3))) - (a^3 - (2*b^3*n^3)/9 + (2*a*b^2*n^2)/3 - a^2*b*n)/x - ((d*((d*(3*
a^3 - (2*b^3*n^3)/3 + 2*a*b^2*n^2 - 3*a^2*b*n))/e - (d*(6*a^3 + 5*b^3*n^3 -
 6*a*b^2*n^2))/(2*e)))/e + (b^2*d^2*n^2*(6*a - 11*b*n))/e^2)/x^(1/3) - (log
(c*(d + e/x^(1/3))^n)*(((d*(b*d*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*n) - 3*b*d*e*(
3*a^2 - b^2*n^2)))/e + 6*b^3*d^2*n^2)/(e*x^(1/3)) - (b*d*e*(9*a^2 + 2*b^2*n
^2 - 6*a*b*n) - 3*b*d*e*(3*a^2 - b^2*n^2))/(2*e*x^(2/3)) + (b*e*(9*a^2 + 2*
b^2*n^2 - 6*a*b*n))/(3*x)))/e - (log(d + e/x^(1/3))*(85*b^3*d^3*n^3 - 66*a*
b^2*d^3*n^2 + 18*a^2*b*d^3*n))/(6*e^3)
```

3.507 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 907

$$
\begin{aligned}
&\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^3}\,dx \\
&= \frac{45b^3d^4n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{8e^6}-\frac{20b^3d^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6}+\frac{45b^3d^2n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{64e^6} \\
&\quad -\frac{18b^3dn^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^5}{125e^6}+\frac{b^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{72e^6}+\frac{18ab^2d^5n^2}{e^5\sqrt[3]{x}} \\
&\quad -\frac{18b^3d^5n^3}{e^5\sqrt[3]{x}}+\frac{18b^3d^5n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{e^6} \\
&\quad -\frac{45b^2d^4n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4e^6} \\
&\quad +\frac{20b^2d^3n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^6} \\
&\quad -\frac{45b^2d^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{16e^6} \\
&\quad +\frac{18b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{25e^6} \\
&\quad -\frac{b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{12e^6} \\
&\quad -\frac{9bd^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^6} \\
&\quad +\frac{45bd^4n\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{4e^6} \\
&\quad -\frac{10bd^3n\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^6} \\
&\quad +\frac{45bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{8e^6} \\
&\quad -\frac{9bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{5e^6}
\end{aligned}
$$

```
[Out] 1/72*b^3*n^3*(d+e/x^(1/3))^6/e^6+3*d^5*(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3)
)^n))^3/e^6-15/2*d^4*(d+e/x^(1/3))^2*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^6+10*d
^3*(d+e/x^(1/3))^3*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^6-15/2*d^2*(d+e/x^(1/3))
^4*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^6+3*d*(d+e/x^(1/3))^5*(a+b*ln(c*(d+e/x^(
1/3))^n))^3/e^6-1/2*(d+e/x^(1/3))^6*(a+b*ln(c*(d+e/x^(1/3))^n))^3/e^6+45/8*
b^3*d^4*n^3*(d+e/x^(1/3))^2/e^6-20/9*b^3*d^3*n^3*(d+e/x^(1/3))^3/e^6+45/64*
b^3*d^2*n^3*(d+e/x^(1/3))^4/e^6-18/125*b^3*d*n^3*(d+e/x^(1/3))^5/e^6-18*b^3
*d^5*n^3/e^5/x^(1/3)-1/12*b^2*n^2*(d+e/x^(1/3))^6*(a+b*ln(c*(d+e/x^(1/3))^n
))/e^6+1/4*b*n*(d+e/x^(1/3))^6*(a+b*ln(c*(d+e/x^(1/3))^n))^2/e^6+18*a*b^2*d
^5*n^2/e^5/x^(1/3)-45/16*b^2*d^2*n^2*(d+e/x^(1/3))^4*(a+b*ln(c*(d+e/x^(1/3)
)^n))/e^6+18/25*b^2*d*n^2*(d+e/x^(1/3))^5*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6-9
*b*d^5*n*(d+e/x^(1/3))*(a+b*ln(c*(d+e/x^(1/3))^n))^2/e^6+45/4*b*d^4*n*(d+e/
x^(1/3))^2*(a+b*ln(c*(d+e/x^(1/3))^n))^2/e^6-10*b*d^3*n*(d+e/x^(1/3))^3*(a+
b*ln(c*(d+e/x^(1/3))^n))^2/e^6+45/8*b*d^2*n*(d+e/x^(1/3))^4*(a+b*ln(c*(d+e/
x^(1/3))^n))^2/e^6-9/5*b*d*n*(d+e/x^(1/3))^5*(a+b*ln(c*(d+e/x^(1/3))^n))^2/
e^6+18*b^3*d^5*n^2*(d+e/x^(1/3))*ln(c*(d+e/x^(1/3))^n)/e^6-45/4*b^2*d^4*n^2
*(d+e/x^(1/3))^2*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6+20/3*b^2*d^3*n^2*(d+e/x^(1
/3))^3*(a+b*ln(c*(d+e/x^(1/3))^n))/e^6
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 907, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341\}$

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^3}\,dx$$

$$= \frac{b^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{72e^6} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{2e^6}$$
$$+ \frac{bn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{4e^6}$$
$$- \frac{b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{12e^6}$$
$$- \frac{18b^3dn^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^5}{125e^6} + \frac{3d\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^5}{e^6}$$
$$- \frac{9bdn\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^5}{5e^6}$$
$$+ \frac{18b^2dn^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\left(d+\frac{e}{\sqrt[3]{x}}\right)^5}{25e^6}$$
$$+ \frac{45b^3d^2n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{64e^6} - \frac{15d^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{2e^6}$$
$$+ \frac{45bd^2n\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{8e^6}$$
$$- \frac{45b^2d^2n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{16e^6}$$
$$- \frac{20b^3d^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6} + \frac{10d^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{e^6}$$
$$- \frac{10bd^3n\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{e^6}$$
$$+ \frac{20b^2d^3n^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{3e^6}$$
$$+ \frac{45b^3d^4n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{8e^6} - \frac{15d^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{2e^6}$$
$$+ \frac{45bd^4n\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{4e^6}$$

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x^3,x]
```

```
[Out] (45*b^3*d^4*n^3*(d + e/x^(1/3))^2)/(8*e^6) - (20*b^3*d^3*n^3*(d + e/x^(1/3)
)^3)/(9*e^6) + (45*b^3*d^2*n^3*(d + e/x^(1/3))^4)/(64*e^6) - (18*b^3*d*n^3*
(d + e/x^(1/3))^5)/(125*e^6) + (b^3*n^3*(d + e/x^(1/3))^6)/(72*e^6) + (18*a
*b^2*d^5*n^2)/(e^5*x^(1/3)) - (18*b^3*d^5*n^3)/(e^5*x^(1/3)) + (18*b^3*d^5*
n^2*(d + e/x^(1/3))*Log[c*(d + e/x^(1/3))^n])/e^6 - (45*b^2*d^4*n^2*(d + e/
x^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n]))/(4*e^6) + (20*b^2*d^3*n^2*(d +
 e/x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n]))/(3*e^6) - (45*b^2*d^2*n^2*(
d + e/x^(1/3))^4*(a + b*Log[c*(d + e/x^(1/3))^n]))/(16*e^6) + (18*b^2*d*n^2
*(d + e/x^(1/3))^5*(a + b*Log[c*(d + e/x^(1/3))^n]))/(25*e^6) - (b^2*n^2*(d
 + e/x^(1/3))^6*(a + b*Log[c*(d + e/x^(1/3))^n]))/(12*e^6) - (9*b*d^5*n*(d
+ e/x^(1/3))*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/e^6 + (45*b*d^4*n*(d + e/x
^(1/3))^2*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(4*e^6) - (10*b*d^3*n*(d + e/
x^(1/3))^3*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/e^6 + (45*b*d^2*n*(d + e/x^(
1/3))^4*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(8*e^6) - (9*b*d*n*(d + e/x^(1/
3))^5*(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(5*e^6) + (b*n*(d + e/x^(1/3))^6*
(a + b*Log[c*(d + e/x^(1/3))^n])^2)/(4*e^6) + (3*d^5*(d + e/x^(1/3))*(a + b
*Log[c*(d + e/x^(1/3))^n])^3)/e^6 - (15*d^4*(d + e/x^(1/3))^2*(a + b*Log[c*
(d + e/x^(1/3))^n])^3)/(2*e^6) + (10*d^3*(d + e/x^(1/3))^3*(a + b*Log[c*(d
+ e/x^(1/3))^n])^3)/e^6 - (15*d^2*(d + e/x^(1/3))^4*(a + b*Log[c*(d + e/x^(
1/3))^n])^3)/(2*e^6) + (3*d*(d + e/x^(1/3))^5*(a + b*Log[c*(d + e/x^(1/3))^
n])^3)/e^6 - ((d + e/x^(1/3))^6*(a + b*Log[c*(d + e/x^(1/3))^n])^3)/(2*e^6)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = -\left(3\text{Subst}\left(\int x^5(a+b\log\left(c(d+ex)^n\right))^3\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\text{Subst}\left(\int\left(-\frac{d^5(a+b\log\left(c(d+ex)^n\right))^3}{e^5}+\frac{5d^4(d+ex)\left(a+b\log\left(c(d+ex)^n\right)\right)^3}{e^5}-\frac{10d^3(d+\right.\right.\right.$$

$$= -\frac{3\text{Subst}\left(\int (d+ex)^5 (a+b\log(c(d+ex)^n))^3 \, dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5} + \frac{(15d)\text{Subst}\left(\int (d+ex)^4 (a+b\log(c(d+ex)^n))^3 \, dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5} - \frac{(30d^2)\,\text{Subst}\left(\int (d+ex)^3 (a+b\log(c(d+ex)^n))^3 \, dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5} + \frac{(30d^3)\,\text{Subst}\left(\int (d+ex)^2 (a+b\log(c(d+ex)^n))^3 \, dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5} - \frac{(15d^4)\,\text{Subst}\left(\int (d+ex) (a+b\log(c(d+ex)^n))^3 \, dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5} + \frac{(3d^5)\,\text{Subst}\left(\int (a+b\log(c(d+ex)^n))^3 \, dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5}$$

$$= -\frac{3\text{Subst}\left(\int x^5 (a+b\log(cx^n))^3 \, dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} + \frac{(15d)\text{Subst}\left(\int x^4 (a+b\log(cx^n))^3 \, dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} - \frac{(30d^2)\,\text{Subst}\left(\int x^3 (a+b\log(cx^n))^3 \, dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} + \frac{(30d^3)\,\text{Subst}\left(\int x^2 (a+b\log(cx^n))^3 \, dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} - \frac{(15d^4)\,\text{Subst}\left(\int x (a+b\log(cx^n))^3 \, dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} + \frac{(3d^5)\,\text{Subst}\left(\int (a+b\log(cx^n))^3 \, dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^6}$$

$$
\begin{aligned}
&=\frac{3d^5\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^6} \\
&-\frac{15d^4\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{2e^6} \\
&+\frac{10d^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^6} \\
&-\frac{15d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{2e^6} \\
&+\frac{3d\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^6} \\
&-\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{2e^6} \\
&+\frac{(3bn)\text{Subst}\left(\int x^5(a+b\log(cx^n))^2\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{2e^6} \\
&-\frac{(9bdn)\text{Subst}\left(\int x^4(a+b\log(cx^n))^2\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} \\
&+\frac{(45bd^2n)\,\text{Subst}\left(\int x^3(a+b\log(cx^n))^2\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{2e^6} \\
&-\frac{(30bd^3n)\,\text{Subst}\left(\int x^2(a+b\log(cx^n))^2\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} \\
&+\frac{(45bd^4n)\,\text{Subst}\left(\int x(a+b\log(cx^n))^2\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{2e^6} \\
&-\frac{(9bd^5n)\,\text{Subst}\left(\int (a+b\log(cx^n))^2\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^6}
\end{aligned}
$$

$$= -\frac{9bd^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^6}$$
$$+\frac{45bd^4n\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{4e^6}$$
$$-\frac{10bd^3n\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^6}$$
$$+\frac{45bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{8e^6}$$
$$-\frac{9bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{5e^6}$$
$$+\frac{bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{4e^6}$$
$$+\frac{3d^5\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^6}$$
$$-\frac{15d^4\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{2e^6}$$
$$+\frac{10d^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^6}$$
$$-\frac{15d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{2e^6}$$
$$+\frac{3d\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^6}$$
$$-\frac{\left(d+\frac{e}{\sqrt[3]{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{2e^6}$$
$$-\frac{(b^2n^2)\,\text{Subst}\left(\int x^5(a+b\log(cx^n))\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{2e^6}$$
$$+\frac{(18b^2dn^2)\,\text{Subst}\left(\int x^4(a+b\log(cx^n))\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{5e^6}$$
$$-\frac{(45b^2d^2n^2)\,\text{Subst}\left(\int x^3(a+b\log(cx^n))\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{4e^6}$$
$$(20b^2d^3n^2)\,\text{Subst}\left(\int x^2(a+b\log(cx^n))\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)$$

$$
\begin{aligned}
&= \frac{45b^3d^4n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{8e^6} - \frac{20b^3d^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6} + \frac{45b^3d^2n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{64e^6} \\
&\quad - \frac{18b^3dn^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^5}{125e^6} + \frac{b^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{72e^6} + \frac{18ab^2d^5n^2}{e^5\sqrt[3]{x}} \\
&\quad - \frac{45b^2d^4n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4e^6} \\
&\quad + \frac{20b^2d^3n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^6} \\
&\quad - \frac{45b^2d^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{16e^6} \\
&\quad + \frac{18b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{25e^6} \\
&\quad - \frac{b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{12e^6} \\
&\quad - \frac{9bd^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^6} \\
&\quad + \frac{45bd^4n\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{4e^6} \\
&\quad - \frac{10bd^3n\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^6} \\
&\quad + \frac{45bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{8e^6} \\
&\quad - \frac{9bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{5e^6} \\
&\quad + \frac{bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{4e^6} \\
&\quad + \frac{3d^5\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^6} \\
&\quad - \frac{15d^4\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{2e^6} \\
&\quad 10d^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{45b^3d^4n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}{8e^6} - \frac{20b^3d^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3}{9e^6} + \frac{45b^3d^2n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^4}{64e^6} \\
&\quad - \frac{18b^3dn^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^5}{125e^6} + \frac{b^3n^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^6}{72e^6} + \frac{18ab^2d^5n^2}{e^5\sqrt[3]{x}} \\
&\quad - \frac{18b^3d^5n^3}{e^5\sqrt[3]{x}} + \frac{18b^3d^5n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)}{e^6} \\
&\quad - \frac{45b^2d^4n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{4e^6} \\
&\quad + \frac{20b^2d^3n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{3e^6} \\
&\quad - \frac{45b^2d^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{16e^6} \\
&\quad + \frac{18b^2dn^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{25e^6} \\
&\quad - \frac{b^2n^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)}{12e^6} \\
&\quad - \frac{9bd^5n\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^6} \\
&\quad + \frac{45bd^4n\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{4e^6} \\
&\quad - \frac{10bd^3n\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{e^6} \\
&\quad + \frac{45bd^2n\left(d+\frac{e}{\sqrt[3]{x}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{8e^6} \\
&\quad - \frac{9bdn\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{5e^6} \\
&\quad + \frac{bn\left(d+\frac{e}{\sqrt[3]{x}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^2}{4e^6} \\
&\quad + \frac{3d^5\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{e^6} \\
&\quad \frac{15d^4\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 962, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^3}\,dx$$

$$= \frac{-36000a^3e^6 + 18000a^2be^6n - 6000ab^2e^6n^2 + 1000b^3e^6n^3 - 21600a^2bde^5n\sqrt[3]{x} + 15840ab^2de^5n^2\sqrt[3]{x} - 4368}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^n])^3/x^3,x]
```

```
[Out] (-36000*a^3*e^6 + 18000*a^2*b*e^6*n - 6000*a*b^2*e^6*n^2 + 1000*b^3*e^6*n^3
 - 21600*a^2*b*d*e^5*n*x^(1/3) + 15840*a*b^2*d*e^5*n^2*x^(1/3) - 4368*b^3*d
*e^5*n^3*x^(1/3) + 27000*a^2*b*d^2*e^4*n*x^(2/3) - 33300*a*b^2*d^2*e^4*n^2*
x^(2/3) + 13785*b^3*d^2*e^4*n^3*x^(2/3) - 36000*a^2*b*d^3*e^3*n*x + 68400*a
*b^2*d^3*e^3*n^2*x - 41180*b^3*d^3*e^3*n^3*x + 54000*a^2*b*d^4*e^2*n*x^(4/3
) - 156600*a*b^2*d^4*e^2*n^2*x^(4/3) + 140070*b^3*d^4*e^2*n^3*x^(4/3) - 108
000*a^2*b*d^5*e*n*x^(5/3) + 529200*a*b^2*d^5*e*n^2*x^(5/3) - 809340*b^3*d^5
*e*n^3*x^(5/3) - 72000*b^3*d^6*n^3*x^2*Log[d + e/x^(1/3)]^3 - 36000*b^3*e^6
*Log[c*(d + e/x^(1/3))^n]^3 + 108000*a^2*b*d^6*n*x^2*Log[e + d*x^(1/3)] - 5
29200*a*b^2*d^6*n^2*x^2*Log[e + d*x^(1/3)] + 809340*b^3*d^6*n^3*x^2*Log[e +
 d*x^(1/3)] + 3600*b^2*d^6*n^2*x^2*Log[d + e/x^(1/3)]*(-20*a + 49*b*n - 20*
b*Log[c*(d + e/x^(1/3))^n])*(3*Log[e + d*x^(1/3)] - Log[x]) - 36000*a^2*b*d
^6*n*x^2*Log[x] + 176400*a*b^2*d^6*n^2*x^2*Log[x] - 269780*b^3*d^6*n^3*x^2*
Log[x] + 1800*b^2*d^6*n^2*x^2*Log[d + e/x^(1/3)]^2*(60*a - 147*b*n + 60*b*L
og[c*(d + e/x^(1/3))^n] + 60*b*n*Log[e + d*x^(1/3)] - 20*b*n*Log[x]) + 1800
*b^2*Log[c*(d + e/x^(1/3))^n]^2*(e*(-60*a*e^5 + 10*b*e^5*n - 12*b*d*e^4*n*x
^(1/3) + 15*b*d^2*e^3*n*x^(2/3) - 20*b*d^3*e^2*n*x + 30*b*d^4*e*n*x^(4/3) -
 60*b*d^5*n*x^(5/3)) + 60*b*d^6*n*x^2*Log[e + d*x^(1/3)] - 20*b*d^6*n*x^2*L
og[x]) - 60*b*Log[c*(d + e/x^(1/3))^n]*(1800*a^2*e^6 + b^2*e*n^2*(100*e^5 -
 264*d*e^4*x^(1/3) + 555*d^2*e^3*x^(2/3) - 1140*d^3*e^2*x + 2610*d^4*e*x^(4
/3) - 8820*d^5*x^(5/3)) - 60*a*b*e*n*(10*e^5 - 12*d*e^4*x^(1/3) + 15*d^2*e^
3*x^(2/3) - 20*d^3*e^2*x + 30*d^4*e*x^(4/3) - 60*d^5*x^(5/3)) + 180*b*d^6*n
*(-20*a + 49*b*n)*x^2*Log[e + d*x^(1/3)] + 60*b*d^6*n*(20*a - 49*b*n)*x^2*L
og[x]))/(72000*e^6*x^2)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^{n}\right)\right)^{3}}{x^{3}}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))^n))^3/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 1404, normalized size of antiderivative = 1.55

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^{n}\right)\right)^{3}}{x^{3}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/72000*(1000*b^3*e^6*n^3 - 6000*a*b^2*e^6*n^2 + 18000*a^2*b*e^6*n - 36000*
a^3*e^6 + 36000*(b^3*e^6*x^2 - b^3*e^6)*log(c)^3 + 36000*(b^3*d^6*n^3*x^2 -
 b^3*e^6*n^3)*log((d*x + e*x^(2/3))/x)^3 + 20*(1800*a^3*e^6 + (2059*b^3*d^3
*e^3 - 50*b^3*e^6)*n^3 - 60*(57*a*b^2*d^3*e^3 - 5*a*b^2*e^6)*n^2 + 900*(2*a
^2*b*d^3*e^3 - a^2*b*e^6)*n)*x^2 - 18000*(2*b^3*d^3*e^3*n*x - b^3*e^6*n + 6
*a*b^2*e^6 - (6*a*b^2*e^6 + (2*b^3*d^3*e^3 - b^3*e^6)*n)*x^2)*log(c)^2 - 18
00*(20*b^3*d^3*e^3*n^3*x - 10*b^3*e^6*n^3 + 60*a*b^2*e^6*n^2 + 3*(49*b^3*d^
6*n^3 - 20*a*b^2*d^6*n^2)*x^2 - 60*(b^3*d^6*n^2*x^2 - b^3*e^6*n^2)*log(c) +
 15*(4*b^3*d^5*e*n^3*x - b^3*d^2*e^4*n^3)*x^(2/3) - 6*(5*b^3*d^4*e^2*n^3*x
- 2*b^3*d*e^5*n^3)*x^(1/3))*log((d*x + e*x^(2/3))/x)^2 - 20*(2059*b^3*d^3*e
^3*n^3 - 3420*a*b^2*d^3*e^3*n^2 + 1800*a^2*b*d^3*e^3*n)*x - 1200*(5*b^3*e^6
*n^2 - 30*a*b^2*e^6*n + 90*a^2*b*e^6 - (90*a^2*b*e^6 - (57*b^3*d^3*e^3 - 5*
b^3*e^6)*n^2 + 30*(2*a*b^2*d^3*e^3 - a*b^2*e^6)*n)*x^2 - 3*(19*b^3*d^3*e^3*
n^2 - 20*a*b^2*d^3*e^3*n)*x)*log(c) - 60*(100*b^3*e^6*n^3 - 600*a*b^2*e^6*n
^2 + 1800*a^2*b*e^6*n - (13489*b^3*d^6*n^3 - 8820*a*b^2*d^6*n^2 + 1800*a^2*
b*d^6*n)*x^2 - 1800*(b^3*d^6*n*x^2 - b^3*e^6*n)*log(c)^2 - 60*(19*b^3*d^3*e
^3*n^3 - 20*a*b^2*d^3*e^3*n^2)*x + 60*(20*b^3*d^3*e^3*n^2*x - 10*b^3*e^6*n^
2 + 60*a*b^2*e^6*n + 3*(49*b^3*d^6*n^2 - 20*a*b^2*d^6*n)*x^2)*log(c) + 15*(
37*b^3*d^2*e^4*n^3 - 60*a*b^2*d^2*e^4*n^2 - 12*(49*b^3*d^5*e*n^3 - 20*a*b^2
*d^5*e*n^2)*x + 60*(4*b^3*d^5*e*n^2*x - b^3*d^2*e^4*n^2)*log(c))*x^(2/3) -
6*(44*b^3*d*e^5*n^3 - 120*a*b^2*d*e^5*n^2 - 15*(29*b^3*d^4*e^2*n^3 - 20*a*b
^2*d^4*e^2*n^2)*x + 60*(5*b^3*d^4*e^2*n^2*x - 2*b^3*d*e^5*n^2)*log(c))*x^(1
/3))*log((d*x + e*x^(2/3))/x) + 15*(919*b^3*d^2*e^4*n^3 - 2220*a*b^2*d^2*e^
4*n^2 + 1800*a^2*b*d^2*e^4*n - 1800*(4*b^3*d^5*e*n*x - b^3*d^2*e^4*n)*log(c
)^2 - 4*(13489*b^3*d^5*e*n^3 - 8820*a*b^2*d^5*e*n^2 + 1800*a^2*b*d^5*e*n)*x
```

```
 - 60*(37*b^3*d^2*e^4*n^2 - 60*a*b^2*d^2*e^4*n - 12*(49*b^3*d^5*e*n^2 - 20*
a*b^2*d^5*e*n)*x)*log(c))*x^(2/3) - 6*(728*b^3*d*e^5*n^3 - 2640*a*b^2*d*e^5
*n^2 + 3600*a^2*b*d*e^5*n - 1800*(5*b^3*d^4*e^2*n*x - 2*b^3*d*e^5*n)*log(c)
^2 - 5*(4669*b^3*d^4*e^2*n^3 - 5220*a*b^2*d^4*e^2*n^2 + 1800*a^2*b*d^4*e^2*
n)*x - 60*(44*b^3*d*e^5*n^2 - 120*a*b^2*d*e^5*n - 15*(29*b^3*d^4*e^2*n^2 -
20*a*b^2*d^4*e^2*n)*x)*log(c))*x^(1/3))/(e^6*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**n))**3/x**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 864, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/40*a^2*b*e*n*(60*d^6*log(d*x^(1/3) + e)/e^7 - 20*d^6*log(x)/e^7 - (60*d^5
*x^(5/3) - 30*d^4*e*x^(4/3) + 20*d^3*e^2*x - 15*d^2*e^3*x^(2/3) + 12*d*e^4*
x^(1/3) - 10*e^5)/(e^6*x^2)) + 1/1200*(60*e*n*(60*d^6*log(d*x^(1/3) + e)/e^
7 - 20*d^6*log(x)/e^7 - (60*d^5*x^(5/3) - 30*d^4*e*x^(4/3) + 20*d^3*e^2*x -
 15*d^2*e^3*x^(2/3) + 12*d*e^4*x^(1/3) - 10*e^5)/(e^6*x^2))*log(c*(d + e/x^
(1/3))^n) - (1800*d^6*x^2*log(d*x^(1/3) + e)^2 + 200*d^6*x^2*log(x)^2 - 294
0*d^6*x^2*log(x) - 8820*d^5*e*x^(5/3) + 2610*d^4*e^2*x^(4/3) - 1140*d^3*e^3
*x + 555*d^2*e^4*x^(2/3) - 264*d*e^5*x^(1/3) + 100*e^6 - 60*(20*d^6*x^2*log
(x) - 147*d^6*x^2)*log(d*x^(1/3) + e))*n^2/(e^6*x^2))*a*b^2 + 1/216000*(540
0*e*n*(60*d^6*log(d*x^(1/3) + e)/e^7 - 20*d^6*log(x)/e^7 - (60*d^5*x^(5/3)
- 30*d^4*e*x^(4/3) + 20*d^3*e^2*x - 15*d^2*e^3*x^(2/3) + 12*d*e^4*x^(1/3) -
 10*e^5)/(e^6*x^2))*log(c*(d + e/x^(1/3))^n)^2 + e*n*((108000*d^6*x^2*log(d
*x^(1/3) + e)^3 - 4000*d^6*x^2*log(x)^3 + 88200*d^6*x^2*log(x)^2 - 809340*d
^6*x^2*log(x) - 2428020*d^5*e*x^(5/3) + 420210*d^4*e^2*x^(4/3) - 123540*d^3
*e^3*x + 41355*d^2*e^4*x^(2/3) - 13104*d*e^5*x^(1/3) + 3000*e^6 - 5400*(20*
```

```
d^6*x^2*log(x) - 147*d^6*x^2)*log(d*x^(1/3) + e)^2 + 180*(200*d^6*x^2*log(x
)^2 - 2940*d^6*x^2*log(x) + 13489*d^6*x^2)*log(d*x^(1/3) + e))*n^2/(e^7*x^2
) - 180*(1800*d^6*x^2*log(d*x^(1/3) + e)^2 + 200*d^6*x^2*log(x)^2 - 2940*d^
6*x^2*log(x) - 8820*d^5*e*x^(5/3) + 2610*d^4*e^2*x^(4/3) - 1140*d^3*e^3*x +
 555*d^2*e^4*x^(2/3) - 264*d*e^5*x^(1/3) + 100*e^6 - 60*(20*d^6*x^2*log(x)
- 147*d^6*x^2)*log(d*x^(1/3) + e))*n*log(c*(d + e/x^(1/3))^n)/(e^7*x^2)))*b
^3 - 1/2*b^3*log(c*(d + e/x^(1/3))^n)^3/x^2 - 3/2*a*b^2*log(c*(d + e/x^(1/3
))^n)^2/x^2 - 3/2*a^2*b*log(c*(d + e/x^(1/3))^n)/x^2 - 1/2*a^3/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1747 vs. $2(787) = 1574$.

Time = 0.43 (sec) , antiderivative size = 1747, normalized size of antiderivative = 1.93

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^3}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^n))^3/x^3,x, algorithm="giac")
```

```
[Out] 1/72000*(36000*(6*(d*x^(1/3) + e)*b^3*d^5*n^3/(e^5*x^(1/3)) - 15*(d*x^(1/3)
 + e)^2*b^3*d^4*n^3/(e^5*x^(2/3)) + 20*(d*x^(1/3) + e)^3*b^3*d^3*n^3/(e^5*x
) - 15*(d*x^(1/3) + e)^4*b^3*d^2*n^3/(e^5*x^(4/3)) + 6*(d*x^(1/3) + e)^5*b^
3*d*n^3/(e^5*x^(5/3)) - (d*x^(1/3) + e)^6*b^3*n^3/(e^5*x^2))*log((d*x^(1/3)
 + e)/x^(1/3))^3 + 1800*(10*(b^3*n^3 - 6*b^3*n^2*log(c) - 6*a*b^2*n^2)*(d*x
^(1/3) + e)^6/(e^5*x^2) - 72*(b^3*d*n^3 - 5*b^3*d*n^2*log(c) - 5*a*b^2*d*n^
2)*(d*x^(1/3) + e)^5/(e^5*x^(5/3)) + 225*(b^3*d^2*n^3 - 4*b^3*d^2*n^2*log(c
) - 4*a*b^2*d^2*n^2)*(d*x^(1/3) + e)^4/(e^5*x^(4/3)) - 400*(b^3*d^3*n^3 - 3
*b^3*d^3*n^2*log(c) - 3*a*b^2*d^3*n^2)*(d*x^(1/3) + e)^3/(e^5*x) + 450*(b^3
*d^4*n^3 - 2*b^3*d^4*n^2*log(c) - 2*a*b^2*d^4*n^2)*(d*x^(1/3) + e)^2/(e^5*x
^(2/3)) - 360*(b^3*d^5*n^3 - b^3*d^5*n^2*log(c) - a*b^2*d^5*n^2)*(d*x^(1/3)
 + e)/(e^5*x^(1/3)))*log((d*x^(1/3) + e)/x^(1/3))^2 - 60*(100*(b^3*n^3 - 6*
b^3*n^2*log(c) + 18*b^3*n*log(c)^2 - 6*a*b^2*n^2 + 36*a*b^2*n*log(c) + 18*a
^2*b*n)*(d*x^(1/3) + e)^6/(e^5*x^2) - 432*(2*b^3*d*n^3 - 10*b^3*d*n^2*log(c
) + 25*b^3*d*n*log(c)^2 - 10*a*b^2*d*n^2 + 50*a*b^2*d*n*log(c) + 25*a^2*b*d
*n)*(d*x^(1/3) + e)^5/(e^5*x^(5/3)) + 3375*(b^3*d^2*n^3 - 4*b^3*d^2*n^2*log
(c) + 8*b^3*d^2*n*log(c)^2 - 4*a*b^2*d^2*n^2 + 16*a*b^2*d^2*n*log(c) + 8*a^
2*b*d^2*n)*(d*x^(1/3) + e)^4/(e^5*x^(4/3)) - 4000*(2*b^3*d^3*n^3 - 6*b^3*d^
3*n^2*log(c) + 9*b^3*d^3*n*log(c)^2 - 6*a*b^2*d^3*n^2 + 18*a*b^2*d^3*n*log(
c) + 9*a^2*b*d^3*n)*(d*x^(1/3) + e)^3/(e^5*x) + 13500*(b^3*d^4*n^3 - 2*b^3*
d^4*n^2*log(c) + 2*b^3*d^4*n*log(c)^2 - 2*a*b^2*d^4*n^2 + 4*a*b^2*d^4*n*log
(c) + 2*a^2*b*d^4*n)*(d*x^(1/3) + e)^2/(e^5*x^(2/3)) - 10800*(2*b^3*d^5*n^3
 - 2*b^3*d^5*n^2*log(c) + b^3*d^5*n*log(c)^2 - 2*a*b^2*d^5*n^2 + 2*a*b^2*d^
5*n*log(c) + a^2*b*d^5*n)*(d*x^(1/3) + e)/(e^5*x^(1/3)))*log((d*x^(1/3) + e
```

```
)/x^(1/3)) + 1000*(b^3*n^3 - 6*b^3*n^2*log(c) + 18*b^3*n*log(c)^2 - 36*b^3*
log(c)^3 - 6*a*b^2*n^2 + 36*a*b^2*n*log(c) - 108*a*b^2*log(c)^2 + 18*a^2*b*
n - 108*a^2*b*log(c) - 36*a^3)*(d*x^(1/3) + e)^6/(e^5*x^2) - 1728*(6*b^3*d*
n^3 - 30*b^3*d*n^2*log(c) + 75*b^3*d*n*log(c)^2 - 125*b^3*d*log(c)^3 - 30*a
*b^2*d*n^2 + 150*a*b^2*d*n*log(c) - 375*a*b^2*d*log(c)^2 + 75*a^2*b*d*n - 3
75*a^2*b*d*log(c) - 125*a^3*d)*(d*x^(1/3) + e)^5/(e^5*x^(5/3)) + 16875*(3*b
^3*d^2*n^3 - 12*b^3*d^2*n^2*log(c) + 24*b^3*d^2*n*log(c)^2 - 32*b^3*d^2*log
(c)^3 - 12*a*b^2*d^2*n^2 + 48*a*b^2*d^2*n*log(c) - 96*a*b^2*d^2*log(c)^2 +
24*a^2*b*d^2*n - 96*a^2*b*d^2*log(c) - 32*a^3*d^2)*(d*x^(1/3) + e)^4/(e^5*x
^(4/3)) - 80000*(2*b^3*d^3*n^3 - 6*b^3*d^3*n^2*log(c) + 9*b^3*d^3*n*log(c)^
2 - 9*b^3*d^3*log(c)^3 - 6*a*b^2*d^3*n^2 + 18*a*b^2*d^3*n*log(c) - 27*a*b^2
*d^3*log(c)^2 + 9*a^2*b*d^3*n - 27*a^2*b*d^3*log(c) - 9*a^3*d^3)*(d*x^(1/3)
 + e)^3/(e^5*x) + 135000*(3*b^3*d^4*n^3 - 6*b^3*d^4*n^2*log(c) + 6*b^3*d^4*
n*log(c)^2 - 4*b^3*d^4*log(c)^3 - 6*a*b^2*d^4*n^2 + 12*a*b^2*d^4*n*log(c) -
 12*a*b^2*d^4*log(c)^2 + 6*a^2*b*d^4*n - 12*a^2*b*d^4*log(c) - 4*a^3*d^4)*(
d*x^(1/3) + e)^2/(e^5*x^(2/3)) - 216000*(6*b^3*d^5*n^3 - 6*b^3*d^5*n^2*log(
c) + 3*b^3*d^5*n*log(c)^2 - b^3*d^5*log(c)^3 - 6*a*b^2*d^5*n^2 + 6*a*b^2*d^
5*n*log(c) - 3*a*b^2*d^5*log(c)^2 + 3*a^2*b*d^5*n - 3*a^2*b*d^5*log(c) - a^
3*d^5)*(d*x^(1/3) + e)/(e^5*x^(1/3)))/e
```

Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 992, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^n\right)\right)^3}{x^3}\,dx = \text{Too large to display}$$

```
[In] int((a + b*log(c*(d + e/x^(1/3))^n))^3/x^3,x)
```

```
[Out] (b^3*n^3)/(72*x^2) - (b^3*log(c*(d + e/x^(1/3))^n)^3)/(2*x^2) - a^3/(2*x^2)
 - (3*a*b^2*log(c*(d + e/x^(1/3))^n)^2)/(2*x^2) + (b^3*n*log(c*(d + e/x^(1/
3))^n)^2)/(4*x^2) - (b^3*n^2*log(c*(d + e/x^(1/3))^n))/(12*x^2) - (a*b^2*n^
2)/(12*x^2) + (b^3*d^6*log(c*(d + e/x^(1/3))^n)^3)/(2*e^6) - (3*a^2*b*log(c
*(d + e/x^(1/3))^n))/(2*x^2) + (a^2*b*n)/(4*x^2) + (a*b^2*n*log(c*(d + e/x^
(1/3))^n))/(2*x^2) + (13489*b^3*d^6*n^3*log(d + e/x^(1/3)))/(1200*e^6) - (2
059*b^3*d^3*n^3)/(3600*e^3*x) + (919*b^3*d^2*n^3)/(4800*e^2*x^(4/3)) + (466
9*b^3*d^4*n^3)/(2400*e^4*x^(2/3)) - (13489*b^3*d^5*n^3)/(1200*e^5*x^(1/3))
+ (3*a*b^2*d^6*log(c*(d + e/x^(1/3))^n)^2)/(2*e^6) - (147*b^3*d^6*n*log(c*(
d + e/x^(1/3))^n)^2)/(40*e^6) - (91*b^3*d*n^3)/(1500*e*x^(5/3)) + (3*a^2*b*
d^6*n*log(d + e/x^(1/3)))/(2*e^6) - (3*b^3*d*n*log(c*(d + e/x^(1/3))^n)^2)/
(10*e*x^(5/3)) + (11*b^3*d*n^2*log(c*(d + e/x^(1/3))^n))/(50*e*x^(5/3)) - (
a^2*b*d^3*n)/(2*e^3*x) + (11*a*b^2*d*n^2)/(50*e*x^(5/3)) + (3*a^2*b*d^2*n)/
(8*e^2*x^(4/3)) + (3*a^2*b*d^4*n)/(4*e^4*x^(2/3)) - (3*a^2*b*d^5*n)/(2*e^5*
x^(1/3)) - (147*a*b^2*d^6*n^2*log(d + e/x^(1/3)))/(20*e^6) - (b^3*d^3*n*log
```

```
(c*(d + e/x^(1/3))^n)^2)/(2*e^3*x) + (19*b^3*d^3*n^2*log(c*(d + e/x^(1/3))^
n))/(20*e^3*x) + (3*b^3*d^2*n*log(c*(d + e/x^(1/3))^n)^2)/(8*e^2*x^(4/3)) -
 (37*b^3*d^2*n^2*log(c*(d + e/x^(1/3))^n))/(80*e^2*x^(4/3)) + (3*b^3*d^4*n*
log(c*(d + e/x^(1/3))^n)^2)/(4*e^4*x^(2/3)) - (87*b^3*d^4*n^2*log(c*(d + e/
x^(1/3))^n))/(40*e^4*x^(2/3)) - (3*b^3*d^5*n*log(c*(d + e/x^(1/3))^n)^2)/(2
*e^5*x^(1/3)) + (147*b^3*d^5*n^2*log(c*(d + e/x^(1/3))^n))/(20*e^5*x^(1/3))
 + (19*a*b^2*d^3*n^2)/(20*e^3*x) - (37*a*b^2*d^2*n^2)/(80*e^2*x^(4/3)) - (8
7*a*b^2*d^4*n^2)/(40*e^4*x^(2/3)) + (147*a*b^2*d^5*n^2)/(20*e^5*x^(1/3)) -
(3*a^2*b*d*n)/(10*e*x^(5/3)) - (3*a*b^2*d*n*log(c*(d + e/x^(1/3))^n))/(5*e*
x^(5/3)) - (a*b^2*d^3*n*log(c*(d + e/x^(1/3))^n))/(e^3*x) + (3*a*b^2*d^2*n*
log(c*(d + e/x^(1/3))^n))/(4*e^2*x^(4/3)) + (3*a*b^2*d^4*n*log(c*(d + e/x^(
1/3))^n))/(2*e^4*x^(2/3)) - (3*a*b^2*d^5*n*log(c*(d + e/x^(1/3))^n))/(e^5*x
^(1/3))
```

3.508 $\int x^3 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 143

$$\int x^3 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right) \, dx = \frac{b e^5 n x^{2/3}}{4 d^5}-\frac{b e^4 n x^{4/3}}{8 d^4}+\frac{b e^3 n x^2}{12 d^3}-\frac{b e^2 n x^{8/3}}{16 d^2}+\frac{b e n x^{10/3}}{20 d}-\frac{b e^6 n \log \left(d+\frac{e}{x^{2/3}}\right)}{4 d^6}+\frac{1}{4} x^4 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{b e^6 n \log (x)}{6 d^6}$$

```
[Out] 1/4*b*e^5*n*x^(2/3)/d^5-1/8*b*e^4*n*x^(4/3)/d^4+1/12*b*e^3*n*x^2/d^3-1/16*b
*e^2*n*x^(8/3)/d^2+1/20*b*e*n*x^(10/3)/d-1/4*b*e^6*n*ln(d+e/x^(2/3))/d^6+1/
4*x^4*(a+b*ln(c*(d+e/x^(2/3))^n))-1/6*b*e^6*n*ln(x)/d^6
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 46}

$$\int x^3 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right) \, dx = \frac{1}{4} x^4 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{b e^6 n \log \left(d+\frac{e}{x^{2/3}}\right)}{4 d^6}-\frac{b e^6 n \log (x)}{6 d^6}+\frac{b e^5 n x^{2/3}}{4 d^5}-\frac{b e^4 n x^{4/3}}{8 d^4}+\frac{b e^3 n x^2}{12 d^3}-\frac{b e^2 n x^{8/3}}{16 d^2}+\frac{b e n x^{10/3}}{20 d}$$

```
[In] Int[x^3*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

```
[Out] (b*e^5*n*x^(2/3))/(4*d^5) - (b*e^4*n*x^(4/3))/(8*d^4) + (b*e^3*n*x^2)/(12*d
^3) - (b*e^2*n*x^(8/3))/(16*d^2) + (b*e*n*x^(10/3))/(20*d) - (b*e^6*n*Log[d
```

```
 + e/x^(2/3)])/(4*d^6) + (x^4*(a + b*Log[c*(d + e/x^(2/3))^n]))/4 - (b*e^6*
n*Log[x])/(6*d^6)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(\frac{3}{2}\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^7}\,dx, x, \frac{1}{x^{2/3}}\right)\right)\\
&= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{1}{4}(ben)\text{Subst}\left(\int \frac{1}{x^6(d+ex)}\,dx, x, \frac{1}{x^{2/3}}\right)\\
&= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\\
&\quad -\frac{1}{4}(ben)\text{Subst}\left(\int \left(\frac{1}{dx^6}-\frac{e}{d^2x^5}+\frac{e^2}{d^3x^4}-\frac{e^3}{d^4x^3}+\frac{e^4}{d^5x^2}-\frac{e^5}{d^6x}+\frac{e^6}{d^6(d+ex)}\right)\,dx, x, \frac{1}{x^{2/3}}\right)\\
&= \frac{be^5nx^{2/3}}{4d^5}-\frac{be^4nx^{4/3}}{8d^4}+\frac{be^3nx^2}{12d^3}-\frac{be^2nx^{8/3}}{16d^2}+\frac{benx^{10/3}}{20d}\\
&\quad -\frac{be^6n\log\left(d+\frac{e}{x^{2/3}}\right)}{4d^6}+\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{be^6n\log(x)}{6d^6}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.96

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{ax^4}{4}+\frac{1}{4}bx^4\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+\frac{1}{6}ben\left(\frac{3e^4x^{2/3}}{2d^5}-\frac{3e^3x^{4/3}}{4d^4}+\frac{e^2x^2}{2d^3}-\frac{3ex^{8/3}}{8d^2}+\frac{3x^{10/3}}{10d}-\frac{3e^5\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^6}-\frac{e^5\log(x)}{d^6}\right)$$

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

```
[Out] (a*x^4)/4 + (b*x^4*Log[c*(d + e/x^(2/3))^n])/4 + (b*e*n*((3*e^4*x^(2/3))/(2
*d^5) - (3*e^3*x^(4/3))/(4*d^4) + (e^2*x^2)/(2*d^3) - (3*e*x^(8/3))/(8*d^2)
 + (3*x^(10/3))/(10*d) - (3*e^5*Log[d + e/x^(2/3)])/(2*d^6) - (e^5*Log[x])/
d^6))/6
```

Maple [F]

$$\int x^3\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)dx$$

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n)),x)
```

```
[Out] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.12

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{60\,bd^6x^4\log(c)+60\,ad^6x^4+20\,bd^3e^3nx^2-120\,bd^6n\log\left(x^{\frac{1}{3}}\right)+60\,(bd^6-be^6\cdots}{\cdots}$$

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="fricas")
```

```
[Out] 1/240*(60*b*d^6*x^4*log(c) + 60*a*d^6*x^4 + 20*b*d^3*e^3*n*x^2 - 120*b*d^6*
n*log(x^(1/3)) + 60*(b*d^6 - b*e^6)*n*log(d*x^(2/3) + e) + 60*(b*d^6*n*x^4
- b*d^6*n)*log((d*x + e*x^(1/3))/x) - 15*(b*d^4*e^2*n*x^2 - 4*b*d*e^5*n)*x^
(2/3) + 6*(2*b*d^5*e*n*x^3 - 5*b*d^2*e^4*n*x)*x^(1/3))/d^6
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)^n\right)\right) \, dx = \text{Timed out}$$

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))**n)),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.69

$$\int x^3 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)^n\right)\right) \, dx = \frac{1}{4} b x^4 \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^n\right) + \frac{1}{4} a x^4 - \frac{1}{240} b e n \left(\frac{60 \, e^5 \log \left(d x^{\frac{2}{3}} + e\right)}{d^6} - \frac{12 \, d^4 x^{\frac{10}{3}} - 15 \, d^3 e x^{\frac{8}{3}} + 20 \, d^2 e^2 x^2 - 30 \, d e^3 x^{\frac{4}{3}} + 60 \, e^4 x^{\frac{2}{3}}}{d^5}\right)$$

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="maxima")
[Out] 1/4*b*x^4*log(c*(d + e/x^(2/3))^n) + 1/4*a*x^4 - 1/240*b*e*n*(60*e^5*log(d*
x^(2/3) + e)/d^6 - (12*d^4*x^(10/3) - 15*d^3*e*x^(8/3) + 20*d^2*e^2*x^2 - 3
0*d*e^3*x^(4/3) + 60*e^4*x^(2/3))/d^5)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.73

$$\int x^3 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)^n\right)\right) \, dx = \frac{1}{4} b x^4 \log (c) + \frac{1}{4} a x^4 + \frac{1}{240} \Bigg(60 \, x^4 \log \left(d + \frac{e}{x^{\frac{2}{3}}}\right) - e \Bigg(\frac{60 \, e^5 \log \left(\left|d x^{\frac{2}{3}} + e\right|\right)}{d^6} - \frac{12 \, d^4 x^{\frac{10}{3}} - 15 \, d^3 e x^{\frac{8}{3}} + 20 \, d^2 e^2 x^2 - 30 \, d e^3 x^{\frac{4}{3}} + 60 \, e}{d^5}$$

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="giac")
[Out] 1/4*b*x^4*log(c) + 1/4*a*x^4 + 1/240*(60*x^4*log(d + e/x^(2/3)) - e*(60*e^5
*log(abs(d*x^(2/3) + e))/d^6 - (12*d^4*x^(10/3) - 15*d^3*e*x^(8/3) + 20*d^2
*e^2*x^2 - 30*d*e^3*x^(4/3) + 60*e^4*x^(2/3))/d^5))*b*n
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{x^{10/3}\left(\frac{b\,e\,n}{5\,d}-\frac{b\,e^2\,n}{4\,d^2\,x^{2/3}}-\frac{b\,e^4\,n}{2\,d^4\,x^2}+\frac{b\,e^3\,n}{3\,d^3\,x^{4/3}}+\frac{b\,e^5\,n}{d^5\,x^{8/3}}\right)}{4}+\frac{a\,x^4}{4}+\frac{b\,x^4\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{4}-\frac{b\,e^6\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,x^{2/3}}+1\right)}{2\,d^6}$$

```
[In] int(x^3*(a + b*log(c*(d + e/x^(2/3))^n)),x)
```

```
[Out] (x^(10/3)*((b*e*n)/(5*d) - (b*e^2*n)/(4*d^2*x^(2/3)) - (b*e^4*n)/(2*d^4*x^2
) + (b*e^3*n)/(3*d^3*x^(4/3)) + (b*e^5*n)/(d^5*x^(8/3))))/4 + (a*x^4)/4 + (
b*x^4*log(c*(d + e/x^(2/3))^n))/4 - (b*e^6*n*atanh((2*e)/(d*x^(2/3)) + 1))/
(2*d^6)
```

3.509 $\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=-\frac{2be^4n\sqrt[3]{x}}{3d^4}+\frac{2be^3nx}{9d^3}-\frac{2be^2nx^{5/3}}{15d^2}+\frac{2benx^{7/3}}{21d}+\frac{2be^{9/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3d^{9/2}}+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)$$

```
[Out] -2/3*b*e^4*n*x^(1/3)/d^4+2/9*b*e^3*n*x/d^3-2/15*b*e^2*n*x^(5/3)/d^2+2/21*b*
e*n*x^(7/3)/d+2/3*b*e^(9/2)*n*arctan(x^(1/3)*d^(1/2)/e^(1/2))/d^(9/2)+1/3*x
^3*(a+b*ln(c*(d+e/x^(2/3))^n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2505, 269, 348, 308, 211}

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{2be^{9/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3d^{9/2}}-\frac{2be^4n\sqrt[3]{x}}{3d^4}+\frac{2be^3nx}{9d^3}-\frac{2be^2nx^{5/3}}{15d^2}+\frac{2benx^{7/3}}{21d}$$

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

```
[Out] (-2*b*e^4*n*x^(1/3))/(3*d^4) + (2*b*e^3*n*x)/(9*d^3) - (2*b*e^2*n*x^(5/3))/
(15*d^2) + (2*b*e*n*x^(7/3))/(21*d) + (2*b*e^(9/2)*n*ArcTan[(Sqrt[d]*x^(1/3
))/Sqrt[e]])/(3*d^(9/2)) + (x^3*(a + b*Log[c*(d + e/x^(2/3))^n]))/3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{1}{9}(2ben)\int\frac{x^{4/3}}{d+\frac{e}{x^{2/3}}}\,dx\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{1}{9}(2ben)\int\frac{x^2}{e+dx^{2/3}}\,dx\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{1}{3}(2ben)\text{Subst}\left(\int\frac{x^8}{e+dx^2}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\\
&\quad+\frac{1}{3}(2ben)\text{Subst}\left(\int\left(-\frac{e^3}{d^4}+\frac{e^2x^2}{d^3}-\frac{ex^4}{d^2}+\frac{x^6}{d}+\frac{e^4}{d^4\left(e+dx^2\right)}\right)\,dx,x,\sqrt[3]{x}\right)\\
&= -\frac{2be^4n\sqrt[3]{x}}{3d^4}+\frac{2be^3nx}{9d^3}-\frac{2be^2nx^{5/3}}{15d^2}+\frac{2benx^{7/3}}{21d}\\
&\quad+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{\left(2be^5n\right)\text{Subst}\left(\int\frac{1}{e+dx^2}\,dx,x,\sqrt[3]{x}\right)}{3d^4}
\end{aligned}
$$

$$= -\frac{2be^4n\sqrt[3]{x}}{3d^4} + \frac{2be^3nx}{9d^3} - \frac{2be^2nx^{5/3}}{15d^2} + \frac{2benx^{7/3}}{21d} + \frac{2be^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3d^{9/2}} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.54

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx = \frac{ax^3}{3} + \frac{2benx^{7/3}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},-\frac{e}{dx^{2/3}}\right)}{21d} + \frac{1}{3}bx^3\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

```
[Out] (a*x^3)/3 + (2*b*e*n*x^(7/3)*Hypergeometric2F1[-7/2, 1, -5/2, -(e/(d*x^(2/3
)))])/(21*d) + (b*x^3*Log[c*(d + e/x^(2/3))^n])/3
```

Maple [F]

$$\int x^2\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)dx$$

```
[In] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n)),x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 399, normalized size of antiderivative = 3.30

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx = \left[\frac{105\,bd^4x^3\log\left(c\right)+105\,ad^4x^3-42\,bd^2e^2nx^{\frac{5}{3}}+105\,be^4n\sqrt{-\frac{e}{d}}\log\left(\frac{d^3x^2-2\,d^2ex\sqrt{}}{}\right.}{}\right.$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="fricas")
```

```
[Out] [1/315*(105*b*d^4*x^3*log(c) + 105*a*d^4*x^3 - 42*b*d^2*e^2*n*x^(5/3) + 105
*b*e^4*n*sqrt(-e/d)*log((d^3*x^2 - 2*d^2*e*x*sqrt(-e/d) - e^3 + 2*(d^3*x*sq
rt(-e/d) + d*e^2)*x^(2/3) - 2*(d^2*e*x - d*e^2*sqrt(-e/d))*x^(1/3))/(d^3*x^
2 + e^3)) + 70*b*d*e^3*n*x + 105*b*d^4*n*log(d*x^(2/3) + e) - 210*b*d^4*n*l
og(x^(1/3)) + 105*(b*d^4*n*x^3 - b*d^4*n)*log((d*x + e*x^(1/3))/x) + 30*(b*
d^3*e*n*x^2 - 7*b*e^4*n)*x^(1/3))/d^4, 1/315*(105*b*d^4*x^3*log(c) + 105*a*
d^4*x^3 - 42*b*d^2*e^2*n*x^(5/3) + 210*b*e^4*n*sqrt(e/d)*arctan(d*x^(1/3)*s
qrt(e/d)/e) + 70*b*d*e^3*n*x + 105*b*d^4*n*log(d*x^(2/3) + e) - 210*b*d^4*n
*log(x^(1/3)) + 105*(b*d^4*n*x^3 - b*d^4*n)*log((d*x + e*x^(1/3))/x) + 30*(
b*d^3*e*n*x^2 - 7*b*e^4*n)*x^(1/3))/d^4]
```

Sympy [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))**n)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.85

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{1}{3}\,bx^3\log\left(c\right)+\frac{1}{3}\,ax^3$$
$$+\frac{1}{315}\left(105\,x^3\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)+2\,e\left(\frac{105\,e^4\arctan\left(\frac{dx^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}d^4}+\frac{15\,d^6x^{\frac{7}{3}}-21\,d^5ex^{\frac{5}{3}}+35\,d^4e^2x-105\,d^3e^3x^{\frac{1}{3}}}{d^7}\right)\right)$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*log(c) + 1/3*a*x^3 + 1/315*(105*x^3*log(d + e/x^(2/3)) + 2*e*(105
*e^4*arctan(d*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*d^4) + (15*d^6*x^(7/3) - 21*d^5
*e*x^(5/3) + 35*d^4*e^2*x - 105*d^3*e^3*x^(1/3))/d^7))*b*n
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\int x^2\,\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)\right)\,dx$$

```
[In] int(x^2*(a + b*log(c*(d + e/x^(2/3))^n)),x)
```

```
[Out] int(x^2*(a + b*log(c*(d + e/x^(2/3))^n)), x)
```

3.510 $\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 94

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=-\frac{be^2nx^{2/3}}{2d^2}+\frac{benx^{4/3}}{4d}+\frac{be^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{be^3n\log(x)}{3d^3}$$

```
[Out] -1/2*b*e^2*n*x^(2/3)/d^2+1/4*b*e*n*x^(4/3)/d+1/2*b*e^3*n*ln(d+e/x^(2/3))/d^
3+1/2*x^2*(a+b*ln(c*(d+e/x^(2/3))^n))+1/3*b*e^3*n*ln(x)/d^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2504, 2442, 46}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{be^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}+\frac{be^3n\log(x)}{3d^3}-\frac{be^2nx^{2/3}}{2d^2}+\frac{benx^{4/3}}{4d}$$

```
[In] Int[x*(a + b*Log[c*(d + e/x^(2/3))^n]),x]
```

```
[Out] -1/2*(b*e^2*n*x^(2/3))/d^2 + (b*e*n*x^(4/3))/(4*d) + (b*e^3*n*Log[d + e/x^(
2/3)])/(2*d^3) + (x^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/2 + (b*e^3*n*Log[x]
)/(3*d^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{3}{2}\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x^4}\,dx, x, \frac{1}{x^{2/3}}\right)\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{1}{2}(ben)\text{Subst}\left(\int \frac{1}{x^3(d+ex)}\,dx, x, \frac{1}{x^{2/3}}\right)\\
&= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\\
&\quad -\frac{1}{2}(ben)\text{Subst}\left(\int \left(\frac{1}{dx^3}-\frac{e}{d^2x^2}+\frac{e^2}{d^3x}-\frac{e^3}{d^3(d+ex)}\right)\,dx, x, \frac{1}{x^{2/3}}\right)\\
&= -\frac{be^2nx^{2/3}}{2d^2}+\frac{benx^{4/3}}{4d}+\frac{be^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)+\frac{be^3n\log(x)}{3d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx = \frac{ax^2}{2} + \frac{1}{2}bx^2\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+\frac{1}{3}ben\left(-\frac{3ex^{2/3}}{2d^2}+\frac{3x^{4/3}}{4d}+\frac{3e^2\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}+\frac{e^2\log(x)}{d^3}\right)$$

[In] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^n]),x]

[Out] (a*x^2)/2 + (b*x^2*Log[c*(d + e/x^(2/3))^n])/2 + (b*e*n*((-3*e*x^(2/3))/(2*d^2) + (3*x^(4/3))/(4*d) + (3*e^2*Log[d + e/x^(2/3)])/(2*d^3) + (e^2*Log[x])/d^3))/3

Maple [F]

$$\int x\left(a+b\ln\left(c{\left(d+\frac{e}{x^{\frac{2}{3}}}\right)}^{n}\right)\right)dx$$

[In] int(x*(a+b*ln(c*(d+e/x^(2/3))^n)),x)

[Out] int(x*(a+b*ln(c*(d+e/x^(2/3))^n)),x)

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.20

$$\int x\left(a+b\log\left(c{\left(d+\frac{e}{x^{2/3}}\right)}^{n}\right)\right)dx=\frac{2\,bd^{3}x^{2}\log\left(c\right)+bd^{2}enx^{\frac{4}{3}}+2\,ad^{3}x^{2}-4\,bd^{3}n\log\left(x^{\frac{1}{3}}\right)-2\,bde^{2}nx^{\frac{2}{3}}+2\,(bd^{3}+}{4\,d^{3}}$$

[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="fricas")

[Out] 1/4*(2*b*d^3*x^2*log(c) + b*d^2*e*n*x^(4/3) + 2*a*d^3*x^2 - 4*b*d^3*n*log(x^(1/3)) - 2*b*d*e^2*n*x^(2/3) + 2*(b*d^3 + b*e^3)*n*log(d*x^(2/3) + e) + 2*(b*d^3*n*x^2 - b*d^3*n)*log((d*x + e*x^(1/3))/x))/d^3

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c{\left(d+\frac{e}{x^{2/3}}\right)}^{n}\right)\right)dx=\text{Timed out}$$

[In] integrate(x*(a+b*ln(c*(d+e/x**(2/3))**n)),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.67

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{1}{4}\,ben\left(\frac{2\,e^2\log\left(dx^{\frac{2}{3}}+e\right)}{d^3}+\frac{dx^{\frac{4}{3}}-2\,ex^{\frac{2}{3}}}{d^2}\right)+\frac{1}{2}\,bx^2\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+\frac{1}{2}\,ax^2$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="maxima")
[Out] 1/4*b*e*n*(2*e^2*log(d*x^(2/3) + e)/d^3 + (d*x^(4/3) - 2*e*x^(2/3))/d^2) +
1/2*b*x^2*log(c*(d + e/x^(2/3))^n) + 1/2*a*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{1}{2}\,bx^2\log\left(c\right)+\frac{1}{4}\left(2\,x^2\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)+e\left(\frac{2\,e^2\log\left(\left|dx^{\frac{2}{3}}+e\right|\right)}{d^3}+\frac{dx^{\frac{4}{3}}-2\,ex^{\frac{2}{3}}}{d^2}\right)\right)bn+\frac{1}{2}\,ax^2$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n)),x, algorithm="giac")
[Out] 1/2*b*x^2*log(c) + 1/4*(2*x^2*log(d + e/x^(2/3)) + e*(2*e^2*log(abs(d*x^(2/
3) + e))/d^3 + (d*x^(4/3) - 2*e*x^(2/3))/d^2))*b*n + 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{x^{4/3}\left(\frac{b\,e\,n}{2\,d}-\frac{b\,e^2\,n}{d^2\,x^{2/3}}\right)}{2}+\frac{a\,x^2}{2}+\frac{b\,x^2\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)}{2}+\frac{b\,e^3\,n\,\mathrm{atanh}\left(\frac{2\,e}{d\,x^{2/3}}+1\right)}{d^3}$$

```
[In] int(x*(a + b*log(c*(d + e/x^(2/3))^n)),x)
[Out] (x^(4/3)*((b*e*n)/(2*d) - (b*e^2*n)/(d^2*x^(2/3))))/2 + (a*x^2)/2 + (b*x^2*
log(c*(d + e/x^(2/3))^n))/2 + (b*e^3*n*atanh((2*e)/(d*x^(2/3)) + 1))/d^3
```

3.511 $\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 65

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=\frac{2ben\sqrt[3]{x}}{d}+ax-\frac{2be^{3/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}}+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)$$

```
[Out] 2*b*e*n*x^(1/3)/d+a*x-2*b*e^(3/2)*n*arctan(x^(1/3)*d^(1/2)/e^(1/2))/d^(3/2)
+b*x*ln(c*(d+e/x^(2/3))^n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2498, 269, 249, 327, 211}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=ax-\frac{2be^{3/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}}+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+\frac{2ben\sqrt[3]{x}}{d}$$

```
[In] Int[a + b*Log[c*(d + e/x^(2/3))^n],x]
```

```
[Out] (2*b*e*n*x^(1/3))/d + a*x - (2*b*e^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]
])/d^(3/2) + b*x*Log[c*(d + e/x^(2/3))^n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 249

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Free
Q[{a, b, p}, x] && FractionQ[n]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= ax + b\int \log\left(c\Big(d+\frac{e}{x^{2/3}}\Big)^n\right)\,dx\\
&= ax + bx\log\left(c\Big(d+\frac{e}{x^{2/3}}\Big)^n\right) + \frac{1}{3}(2ben)\int \frac{1}{\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}}\,dx\\
&= ax + bx\log\left(c\Big(d+\frac{e}{x^{2/3}}\Big)^n\right) + \frac{1}{3}(2ben)\int \frac{1}{e+dx^{2/3}}\,dx\\
&= ax + bx\log\left(c\Big(d+\frac{e}{x^{2/3}}\Big)^n\right) + (2ben)\text{Subst}\left(\int \frac{x^2}{e+dx^2}\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{2ben\sqrt[3]{x}}{d} + ax + bx\log\left(c\Big(d+\frac{e}{x^{2/3}}\Big)^n\right) - \frac{(2be^2n)\,\text{Subst}\left(\int \frac{1}{e+dx^2}\,dx, x, \sqrt[3]{x}\right)}{d}\\
&= \frac{2ben\sqrt[3]{x}}{d} + ax - \frac{2be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}} + bx\log\left(c\Big(d+\frac{e}{x^{2/3}}\Big)^n\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx=ax+\frac{2ben\sqrt[3]{x}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{e}{dx^{2/3}}\right)}{d}+bx\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)$$

```
[In] Integrate[a + b*Log[c*(d + e/x^(2/3))^n],x]
[Out] a*x + (2*b*e*n*x^(1/3)*Hypergeometric2F1[-1/2, 1, 1/2, -(e/(d*x^(2/3)))])/d
 + b*x*Log[c*(d + e/x^(2/3))^n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(51) = 102.

Time = 0.59 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.37

method	result
default	$ax+b\left(x\ln\left(c\left(\frac{e+d\,x^{\frac{2}{3}}}{x^{\frac{2}{3}}}\right)^n\right)+\frac{2en\left(\frac{e\arctan\left(\frac{x\,d^2}{e\sqrt{de}}\right)}{d\sqrt{de}}+\frac{3x^{\frac{1}{3}}}{d}-\frac{2e\arctan\left(\frac{d\,x^{\frac{1}{3}}}{\sqrt{de}}\right)}{d\sqrt{de}}+\frac{e\arctan\left(\frac{\sqrt{3}\,\sqrt{d}\,\sqrt{e}-2d\,x^{\frac{1}{3}}}{\sqrt{de}}\right)}{d\sqrt{de}}-\frac{e\arctan\left(\frac{2d\,x}{}\right.}{d}\right.}{3}\right.$
parts	$ax+b\left(x\ln\left(c\left(\frac{e+d\,x^{\frac{2}{3}}}{x^{\frac{2}{3}}}\right)^n\right)+\frac{2en\left(\frac{e\arctan\left(\frac{x\,d^2}{e\sqrt{de}}\right)}{d\sqrt{de}}+\frac{3x^{\frac{1}{3}}}{d}-\frac{2e\arctan\left(\frac{d\,x^{\frac{1}{3}}}{\sqrt{de}}\right)}{d\sqrt{de}}+\frac{e\arctan\left(\frac{\sqrt{3}\,\sqrt{d}\,\sqrt{e}-2d\,x^{\frac{1}{3}}}{\sqrt{de}}\right)}{d\sqrt{de}}-\frac{e\arctan\left(\frac{2d\,x}{}\right.}{d}\right.}{3}\right.$

```
[In] int(a+b*ln(c*(d+e/x^(2/3))^n),x,method=_RETURNVERBOSE)
[Out] a*x+b*(x*ln(c*((e+d*x^(2/3))/x^(2/3))^n)+2/3*e*n*(e/d/(d*e)^(1/2)*arctan(x*
d^2/e/(d*e)^(1/2))+3/d*x^(1/3)-2/d*e/(d*e)^(1/2)*arctan(d*x^(1/3)/(d*e)^(1/
2))+1/d*e/(d*e)^(1/2)*arctan((3^(1/2)*d^(1/2)*e^(1/2)-2*d*x^(1/3))/(d*e)^(1
/2))-1/d*e/(d*e)^(1/2)*arctan((2*d*x^(1/3)+3^(1/2)*d^(1/2)*e^(1/2))/(d*e)^(
1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(51) = 102$.

Time = 0.35 (sec) , antiderivative size = 279, normalized size of antiderivative = 4.29

$$\int \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)^n\right)\right) dx = \left[\frac{ben\sqrt{-\frac{e}{d}} \log \left(\frac{d^3x^2 + 2\,d^2ex\sqrt{-\frac{e}{d}} - e^3 - 2\left(d^3x\sqrt{-\frac{e}{d}} - de^2\right)x^{\frac{2}{3}} - 2\left(d^2ex + de^2\sqrt{-\frac{e}{d}}\right)x^{\frac{1}{3}}}{d^3x^2 + e^3}\right) + bdn\,\mathrm{l}}{} \right.$$

$$-\frac{2\,ben\sqrt{\frac{e}{d}} \arctan\left(\frac{dx^{\frac{1}{3}}\sqrt{\frac{e}{d}}}{e}\right) - bdn\log\left(dx^{\frac{2}{3}} + e\right) - bdx\log(c) + 2\,bdn\log\left(x^{\frac{1}{3}}\right) - 2\,benx^{\frac{1}{3}} - adx - (bdnx -}{d}$$

```
[In] integrate(a+b*log(c*(d+e/x^(2/3))^n),x, algorithm="fricas")
```

```
[Out] [(b*e*n*sqrt(-e/d)*log((d^3*x^2 + 2*d^2*e*x*sqrt(-e/d) - e^3 - 2*(d^3*x*sqr
t(-e/d) - d*e^2)*x^(2/3) - 2*(d^2*e*x + d*e^2*sqrt(-e/d))*x^(1/3))/(d^3*x^2
 + e^3)) + b*d*n*log(d*x^(2/3) + e) + b*d*x*log(c) - 2*b*d*n*log(x^(1/3)) +
 2*b*e*n*x^(1/3) + a*d*x + (b*d*n*x - b*d*n)*log((d*x + e*x^(1/3))/x))/d, -
(2*b*e*n*sqrt(e/d)*arctan(d*x^(1/3)*sqrt(e/d)/e) - b*d*n*log(d*x^(2/3) + e)
 - b*d*x*log(c) + 2*b*d*n*log(x^(1/3)) - 2*b*e*n*x^(1/3) - a*d*x - (b*d*n*x
 - b*d*n)*log((d*x + e*x^(1/3))/x))/d]
```

Sympy [A] (verification not implemented)

Time = 16.43 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \left(a + b\log\left(c\left(d + \frac{e}{x^{2/3}}\right)^n\right)\right) dx = ax + b\left(\frac{2en\left(\frac{3\sqrt[3]{x}}{d} - \frac{3e\,\mathrm{atan}\left(\frac{\sqrt[3]{x}}{\sqrt{\frac{e}{d}}}\right)}{d^2\sqrt{\frac{e}{d}}}\right)}{3} + x\log\left(c\left(d + \frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)$$

```
[In] integrate(a+b*ln(c*(d+e/x**(2/3))**n),x)
```

```
[Out] a*x + b*(2*e*n*(3*x**(1/3)/d - 3*e*atan(x**(1/3)/sqrt(e/d))/(d**2*sqrt(e/d)
))/3 + x*log(c*(d + e/x**(2/3))**n))
```

Maxima [F(-2)]

Exception generated.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(a+b*log(c*(d+e/x^(2/3))^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.60 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx = -\left(\left(2\,e\left(\frac{e\arctan\left(\frac{dx^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}d}-\frac{x^{\frac{1}{3}}}{d}\right)-x\log\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)n-x\log\left(c\right)\right)b+ax$$

```
[In] integrate(a+b*log(c*(d+e/x^(2/3))^n),x, algorithm="giac")
```

```
[Out] -((2*e*(e*arctan(d*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*d) - x^(1/3)/d) - x*log(d
+ e/x^(2/3)))*n - x*log(c))*b + a*x
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\,dx = a\,x+b\,x\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)+\frac{2\,b\,e\,n\,x^{1/3}}{d}-\frac{2\,b\,e^{3/2}\,n\,\mathrm{atan}\left(\frac{\sqrt{d}\,x^{1/3}}{\sqrt{e}}\right)}{d^{3/2}}$$

```
[In] int(a + b*log(c*(d + e/x^(2/3))^n),x)
```

```
[Out] a*x + b*x*log(c*(d + e/x^(2/3))^n) + (2*b*e*n*x^(1/3))/d - (2*b*e^(3/2)*n*a
tan((d^(1/2)*x^(1/3))/e^(1/2)))/d^(3/2)
```

3.512 $\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx = -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(-\frac{e}{dx^{2/3}}\right)-\frac{3}{2}bn\,\text{PolyLog}\left(2,1+\frac{e}{dx^{2/3}}\right)$$

```
[Out] -3/2*(a+b*ln(c*(d+e/x^(2/3))^n))*ln(-e/d/x^(2/3))-3/2*b*n*polylog(2,1+e/d/x
^(2/3))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2441, 2352}

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx = -\frac{3}{2}\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{3}{2}bn\,\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)$$

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])/x,x]
```

```
[Out] (-3*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[-(e/(d*x^(2/3)))])/2 - (3*b*n*Poly
Log[2, 1 + e/(d*x^(2/3))])/2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{3}{2}\text{Subst}\left(\int \frac{a+b\log\left(c(d+ex)^n\right)}{x}\,dx, x, \frac{1}{x^{2/3}}\right)\right)\\
&= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(-\frac{e}{dx^{2/3}}\right)+\frac{1}{2}(3ben)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, \frac{1}{x^{2/3}}\right)\\
&= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(-\frac{e}{dx^{2/3}}\right)-\frac{3}{2}bn\text{Li}_2\left(1+\frac{e}{dx^{2/3}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx = a\log(x) - \frac{3}{2}b\left(\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\log\left(-\frac{e}{dx^{2/3}}\right)+n\,\text{PolyLog}\left(2,\frac{d+\frac{e}{x^{2/3}}}{d}\right)\right)$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])/x,x]
[Out] a*Log[x] - (3*b*(Log[c*(d + e/x^(2/3))^n]*Log[-(e/(d*x^(2/3)))] + n*PolyLog
[2, (d + e/x^(2/3))/d]))/2
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)}{x}dx$$

[In] int((a+b*ln(c*(d+e/x^(2/3))^n))/x,x)

[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))/x,x)

Fricas [F]

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)}{x}\,dx=\int \frac{b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a}{x}\,dx$$

[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x,x, algorithm="fricas")

[Out] integral((b*log(c*((d*x + e*x^(1/3))/x)^n) + a)/x, x)

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)}{x}\,dx=\text{Timed out}$$

[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))/x,x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(44)=88$.

Time = 0.47 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.31

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)}{x}\,dx=-\frac{3}{2}\left(2\log\left(\frac{dx^{\frac{2}{3}}}{e}+1\right)\log\left(x^{\frac{1}{3}}\right)+\mathrm{Li}_2\left(-\frac{dx^{\frac{2}{3}}}{e}\right)\right)bn+\frac{2\,ben\log\left(x\right)^{2}+6\,bdnx^{\frac{2}{3}}\log\left(x\right)+6\,be\log\left(\left(dx^{\frac{2}{3}}+e\right)^{n}\right)\log\left(x\right)-12\,be\log\left(x\right)\log\left(x^{\frac{1}{3}n}\right)-9\,bdnx^{\frac{2}{3}}+6\,(be}{6\,e}$$

[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x,x, algorithm="maxima")

[Out] -3/2*(2*log(d*x^(2/3)/e + 1)*log(x^(1/3)) + dilog(-d*x^(2/3)/e))*b*n + 1/6*(2*b*e*n*log(x)^2 + 6*b*d*n*x^(2/3)*log(x) + 6*b*e*log((d*x^(2/3) + e)^n)*log(x) - 12*b*e*log(x)*log(x^(1/3*n)) - 9*b*d*n*x^(2/3) + 6*(b*e*log(c) + a*e)*log(x) - 3*(2*b*d*n*x*log(x) - 3*b*d*n*x)/x^(1/3))/e

Giac [F]

$$\int \frac{a + b\log\left(c\left(d + \frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx = \int \frac{b\log\left(c\left(d + \frac{e}{x^{\frac{2}{3}}}\right)^n\right) + a}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c\left(d + \frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx = \int \frac{a + b\,\ln\left(c\left(d + \frac{e}{x^{2/3}}\right)^n\right)}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))/x,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))/x, x)
```

3.513 $\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^2}\,dx = \frac{2bn}{3x}-\frac{2bdn}{e\sqrt[3]{x}}-\frac{2bd^{3/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{e^{3/2}}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}$$

```
[Out] 2/3*b*n/x-2*b*d*n/e/x^(1/3)-2*b*d^(3/2)*n*arctan(x^(1/3)*d^(1/2)/e^(1/2))/e
^(3/2)+(-a-b*ln(c*(d+e/x^(2/3))^n))/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2505, 269, 348, 331, 211}

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^2}\,dx = -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}-\frac{2bd^{3/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{e^{3/2}}-\frac{2bdn}{e\sqrt[3]{x}}+\frac{2bn}{3x}$$

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])/x^2,x]
```

```
[Out] (2*b*n)/(3*x) - (2*b*d*n)/(e*x^(1/3)) - (2*b*d^(3/2)*n*ArcTan[(Sqrt[d]*x^(1
/3))/Sqrt[e]])/e^(3/2) - (a + b*Log[c*(d + e/x^(2/3))^n])/x
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}-\frac{1}{3}(2ben)\int\frac{1}{\left(d+\frac{e}{x^{2/3}}\right)x^{8/3}}\,dx\\
&= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}-\frac{1}{3}(2ben)\int\frac{1}{\left(e+dx^{2/3}\right)x^2}\,dx\\
&= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}-(2ben)\text{Subst}\left(\int\frac{1}{x^4\left(e+dx^2\right)}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{2bn}{3x}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}+(2bdn)\text{Subst}\left(\int\frac{1}{x^2\left(e+dx^2\right)}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{2bn}{3x}-\frac{2bdn}{e\sqrt[3]{x}}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x}-\frac{\left(2bd^2n\right)\text{Subst}\left(\int\frac{1}{e+dx^2}\,dx,x,\sqrt[3]{x}\right)}{e}
\end{aligned}
$$

$$= \frac{2bn}{3x} - \frac{2bdn}{e\sqrt[3]{x}} - \frac{2bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{e^{3/2}} - \frac{a + b\log\left(c\left(d + \frac{e}{x^{2/3}}\right)^n\right)}{x}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04

$$\int \frac{a + b\log\left(c\left(d + \frac{e}{x^{2/3}}\right)^n\right)}{x^2}\,dx = -\frac{a}{x} + \frac{2bn}{3x} - \frac{2bdn}{e\sqrt[3]{x}} + \frac{2bd^{3/2}n\arctan\left(\frac{\sqrt{e}}{\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}} - \frac{b\log\left(c\left(d + \frac{e}{x^{2/3}}\right)^n\right)}{x}$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])/x^2,x]
[Out] -(a/x) + (2*b*n)/(3*x) - (2*b*d*n)/(e*x^(1/3)) + (2*b*d^(3/2)*n*ArcTan[Sqrt
[e]/(Sqrt[d]*x^(1/3))])/e^(3/2) - (b*Log[c*(d + e/x^(2/3))^n])/x
```

Maple [F]

$$\int \frac{a + b\ln\left(c\left(d + \frac{e}{x^{\frac{2}{3}}}\right)^n\right)}{x^2}\,dx$$

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))/x^2,x)
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))/x^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.05

$$\int \frac{a + b\log\left(c\left(d + \frac{e}{x^{2/3}}\right)^n\right)}{x^2}\,dx = \left[\frac{3\,bdnx\sqrt{-\frac{d}{e}}\log\left(\frac{d^3x^2 + 2\,de^2x\sqrt{-\frac{d}{e}} - e^3 - 2\left(d^2ex\sqrt{-\frac{d}{e}} - de^2\right)x^{\frac{2}{3}} - 2\left(d^2ex + e^3\sqrt{-\frac{d}{e}}\right)x^{\frac{1}{3}}}{d^3x^2 + e^3}\right)}{3\,ex}\right.$$

$$\left. - \frac{6\,bdnx\sqrt{\frac{d}{e}}\arctan\left(x^{\frac{1}{3}}\sqrt{\frac{d}{e}}\right) + 3\,ben\log\left(\frac{dx + ex^{\frac{1}{3}}}{x}\right) + 6\,bdnx^{\frac{2}{3}} - 2\,ben + 3\,be\log\left(c\right) + 3\,ae}{3\,ex}\right]$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^2,x, algorithm="fricas")
```

```
[Out] [1/3*(3*b*d*n*x*sqrt(-d/e)*log((d^3*x^2 + 2*d*e^2*x*sqrt(-d/e) - e^3 - 2*(d
^2*e*x*sqrt(-d/e) - d*e^2)*x^(2/3) - 2*(d^2*e*x + e^3*sqrt(-d/e))*x^(1/3))/
(d^3*x^2 + e^3)) - 3*b*e*n*log((d*x + e*x^(1/3))/x) - 6*b*d*n*x^(2/3) + 2*b
*e*n - 3*b*e*log(c) - 3*a*e)/(e*x), -1/3*(6*b*d*n*x*sqrt(d/e)*arctan(x^(1/3
)*sqrt(d/e)) + 3*b*e*n*log((d*x + e*x^(1/3))/x) + 6*b*d*n*x^(2/3) - 2*b*e*n
 + 3*b*e*log(c) + 3*a*e)/(e*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)^n\right)}{x^2}\, dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))/x**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)^n\right)}{x^2}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)^n\right)}{x^2}\, dx = -\frac{1}{3} \left(2\, e \left(\frac{3\, d^2 \arctan\left(\frac{d x^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de} e^2} + \frac{3\, d x^{\frac{2}{3}} - e}{e^2 x}\right) + \frac{3\, \log\left(d + \frac{e}{x^{\frac{2}{3}}}\right)}{x}\right) bn - \frac{b \log\left(c\right)}{x} - \frac{a}{x}$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^2,x, algorithm="giac")
```

```
[Out] -1/3*(2*e*(3*d^2*arctan(d*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*e^2) + (3*d*x^(2/3)
 - e)/(e^2*x)) + 3*log(d + e/x^(2/3))/x)*b*n - b*log(c)/x - a/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^2}\,dx = \int \frac{a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^2}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))/x^2,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))/x^2, x)
```

3.514 $\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^3}\,dx = \frac{bn}{6x^2}-\frac{bdn}{4ex^{4/3}}+\frac{bd^2n}{2e^2x^{2/3}} -\frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2e^3}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2}$$

```
[Out] 1/6*b*n/x^2-1/4*b*d*n/e/x^(4/3)+1/2*b*d^2*n/e^2/x^(2/3)-1/2*b*d^3*n*ln(d+e/
x^(2/3))/e^3+1/2*(-a-b*ln(c*(d+e/x^(2/3))^n))/x^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2504, 2442, 45}

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^3}\,dx = -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2} -\frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2e^3}+\frac{bd^2n}{2e^2x^{2/3}}-\frac{bdn}{4ex^{4/3}}+\frac{bn}{6x^2}$$

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])/x^3,x]
```

```
[Out] (b*n)/(6*x^2) - (b*d*n)/(4*e*x^(4/3)) + (b*d^2*n)/(2*e^2*x^(2/3)) - (b*d^3*
n*Log[d + e/x^(2/3)])/(2*e^3) - (a + b*Log[c*(d + e/x^(2/3))^n])/(2*x^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2442

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^n])/(
g*(q + 1))), x] - Dist[b*e*(n/(g*(q + 1))), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{3}{2}\text{Subst}\left(\int x^2(a+b\log\left(c(d+ex)^n\right))\,dx,x,\frac{1}{x^{2/3}}\right)\right)\\
&= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\text{Subst}\left(\int\frac{x^3}{d+ex}\,dx,x,\frac{1}{x^{2/3}}\right)\\
&= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2}+\frac{1}{2}(ben)\text{Subst}\left(\int\left(\frac{d^2}{e^3}-\frac{dx}{e^2}+\frac{x^2}{e}-\frac{d^3}{e^3(d+ex)}\right)dx,x,\frac{1}{x^{2/3}}\right)\\
&= \frac{bn}{6x^2}-\frac{bdn}{4ex^{4/3}}+\frac{bd^2n}{2e^2x^{2/3}}-\frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2e^3}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\begin{aligned}
\int\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^3}\,dx &= -\frac{a}{2x^2}+\frac{bn}{6x^2}-\frac{bdn}{4ex^{4/3}}\\
&\quad+\frac{bd^2n}{2e^2x^{2/3}}-\frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)}{2e^3}-\frac{b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2x^2}
\end{aligned}$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])/x^3,x]
```

```
[Out] -1/2*a/x^2 + (b*n)/(6*x^2) - (b*d*n)/(4*e*x^(4/3)) + (b*d^2*n)/(2*e^2*x^(2/
3)) - (b*d^3*n*Log[d + e/x^(2/3)])/(2*e^3) - (b*Log[c*(d + e/x^(2/3))^n])/(
2*x^2)
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)}{x^{3}}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))/x^3,x)
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)}{x^{3}}\,dx = \frac{6\,bd^{2}enx^{\frac{4}{3}} - 3\,bde^{2}nx^{\frac{2}{3}} + 2\,be^{3}n - 6\,be^{3}\log\left(c\right) - 6\,ae^{3} - 6\left(bd^{3}nx^{2} + be^{3}n\right)}{12\,e^{3}x^{2}}$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^3,x, algorithm="fricas")
[Out] 1/12*(6*b*d^2*e*n*x^(4/3) - 3*b*d*e^2*n*x^(2/3) + 2*b*e^3*n - 6*b*e^3*log(c
) - 6*a*e^3 - 6*(b*d^3*n*x^2 + b*e^3*n)*log((d*x + e*x^(1/3))/x))/(e^3*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)}{x^{3}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))/x**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)}{x^{3}}\,dx = -\frac{1}{12}\,ben\left(\frac{6\,d^{3}\log\left(dx^{\frac{2}{3}}+e\right)}{e^{4}} - \frac{6\,d^{3}\log\left(x^{\frac{2}{3}}\right)}{e^{4}} - \frac{6\,d^{2}x^{\frac{4}{3}} - 3\,dex^{\frac{2}{3}} + 2\,e^{2}}{e^{3}x^{2}}\right) - \frac{b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)}{2\,x^{2}} - \frac{a}{2\,x^{2}}$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^3,x, algorithm="maxima")
```

```
[Out] -1/12*b*e*n*(6*d^3*log(d*x^(2/3) + e)/e^4 - 6*d^3*log(x^(2/3))/e^4 - (6*d^2
*x^(4/3) - 3*d*e*x^(2/3) + 2*e^2)/(e^3*x^2)) - 1/2*b*log(c*(d + e/x^(2/3))^
n)/x^2 - 1/2*a/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.18

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^3}\,dx = \frac{1}{12}\left(e\left(\frac{12\,d^3\log\left(x^{\frac{1}{3}}\right)}{e^4} - \frac{6\,d^3\log\left(\left|dx^{\frac{2}{3}}+e\right|\right)}{e^4} - \frac{11\,d^3x^2 - 6\,d^2ex^{\frac{4}{3}} + 3\,de^2x\right.\right.}{e^4x^2}$$
$$- \frac{b\log\left(c\right)}{2\,x^2} - \frac{a}{2\,x^2}$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^3,x, algorithm="giac")
```

```
[Out] 1/12*(e*(12*d^3*log(x^(1/3))/e^4 - 6*d^3*log(abs(d*x^(2/3) + e))/e^4 - (11*
d^3*x^2 - 6*d^2*e*x^(4/3) + 3*d*e^2*x^(2/3) - 2*e^3)/(e^4*x^2)) - 6*log(d +
 e/x^(2/3))/x^2)*b*n - 1/2*b*log(c)/x^2 - 1/2*a/x^2
```

Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^3}\,dx = \frac{b\,n}{6\,x^2} - \frac{a}{2\,x^2} - \frac{b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2\,x^2} - \frac{b\,d\,n}{4\,e\,x^{4/3}} - \frac{b\,d^3\,n\,\ln\left(d+\frac{e}{x^{2/3}}\right)}{2\,e^3} + \frac{b\,d^2\,n}{2\,e^2\,x^{2/3}}$$

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))/x^3,x)
```

```
[Out] (b*n)/(6*x^2) - a/(2*x^2) - (b*log(c*(d + e/x^(2/3))^n))/(2*x^2) - (b*d*n)/
(4*e*x^(4/3)) - (b*d^3*n*log(d + e/x^(2/3)))/(2*e^3) + (b*d^2*n)/(2*e^2*x^(
2/3))
```

3.515 $\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 132

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx=\frac{2bn}{27x^3}-\frac{2bdn}{21ex^{7/3}}+\frac{2bd^2n}{15e^2x^{5/3}}-\frac{2bd^3n}{9e^3x}+\frac{2bd^4n}{3e^4\sqrt[3]{x}}+\frac{2bd^{9/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{9/2}}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}$$

```
[Out] 2/27*b*n/x^3-2/21*b*d*n/e/x^(7/3)+2/15*b*d^2*n/e^2/x^(5/3)-2/9*b*d^3*n/e^3/
x+2/3*b*d^4*n/e^4/x^(1/3)+2/3*b*d^(9/2)*n*arctan(x^(1/3)*d^(1/2)/e^(1/2))/e
^(9/2)+1/3*(-a-b*ln(c*(d+e/x^(2/3))^n))/x^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2505, 269, 348, 331, 211}

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx=-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}+\frac{2bd^{9/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{9/2}}+\frac{2bd^4n}{3e^4\sqrt[3]{x}}-\frac{2bd^3n}{9e^3x}+\frac{2bd^2n}{15e^2x^{5/3}}-\frac{2bdn}{21ex^{7/3}}+\frac{2bn}{27x^3}$$

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])/x^4,x]
```

```
[Out] (2*b*n)/(27*x^3) - (2*b*d*n)/(21*e*x^(7/3)) + (2*b*d^2*n)/(15*e^2*x^(5/3))
- (2*b*d^3*n)/(9*e^3*x) + (2*b*d^4*n)/(3*e^4*x^(1/3)) + (2*b*d^(9/2)*n*ArcT
```

```
an[(Sqrt[d]*x^(1/3))/Sqrt[e]])/(3*e^(9/2)) - (a + b*Log[c*(d + e/x^(2/3))^n
])/(3*x^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}-\frac{1}{9}(2ben)\int\frac{1}{\left(d+\frac{e}{x^{2/3}}\right)x^{14/3}}\,dx\\
&= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}-\frac{1}{9}(2ben)\int\frac{1}{\left(e+dx^{2/3}\right)x^4}\,dx\\
&= -\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}-\frac{1}{3}(2ben)\text{Subst}\left(\int\frac{1}{x^{10}\left(e+dx^2\right)}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{2bn}{27x^3}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}+\frac{1}{3}(2bdn)\text{Subst}\left(\int\frac{1}{x^8\left(e+dx^2\right)}\,dx,x,\sqrt[3]{x}\right)
\end{aligned}
$$

$$=\frac{2bn}{27x^3}-\frac{2bdn}{21ex^{7/3}}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}-\frac{(2bd^2n)\operatorname{Subst}\left(\int\frac{1}{x^6(e+dx^2)}\,dx,x,\sqrt[3]{x}\right)}{3e}$$

$$=\frac{2bn}{27x^3}-\frac{2bdn}{21ex^{7/3}}+\frac{2bd^2n}{15e^2x^{5/3}}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}+\frac{(2bd^3n)\operatorname{Subst}\left(\int\frac{1}{x^4(e+dx^2)}\,dx,x,\sqrt[3]{x}\right)}{3e^2}$$

$$=\frac{2bn}{27x^3}-\frac{2bdn}{21ex^{7/3}}+\frac{2bd^2n}{15e^2x^{5/3}}-\frac{2bd^3n}{9e^3x}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}-\frac{(2bd^4n)\operatorname{Subst}\left(\int\frac{1}{x^2(e+dx^2)}\,dx,x,\sqrt[3]{x}\right)}{3e^3}$$

$$=\frac{2bn}{27x^3}-\frac{2bdn}{21ex^{7/3}}+\frac{2bd^2n}{15e^2x^{5/3}}-\frac{2bd^3n}{9e^3x}+\frac{2bd^4n}{3e^4\sqrt[3]{x}}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}+\frac{(2bd^5n)\operatorname{Subst}\left(\int\frac{1}{e+dx^2}\,dx,x,\sqrt[3]{x}\right)}{3e^4}$$

$$=\frac{2bn}{27x^3}-\frac{2bdn}{21ex^{7/3}}+\frac{2bd^2n}{15e^2x^{5/3}}-\frac{2bd^3n}{9e^3x}+\frac{2bd^4n}{3e^4\sqrt[3]{x}}+\frac{2bd^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{9/2}}-\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

$$\int\frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx=-\frac{a}{3x^3}-\frac{2}{9}ben\left(-\frac{1}{3ex^3}+\frac{3d}{7e^2x^{7/3}}-\frac{3d^2}{5e^3x^{5/3}}+\frac{d^3}{e^4x}-\frac{3d^4}{e^5\sqrt[3]{x}}+\frac{3d^{9/2}\arctan\left(\frac{\sqrt{e}}{\sqrt{d}\sqrt[3]{x}}\right)}{e^{11/2}}\right)-\frac{b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3x^3}$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (2*b*e*n*(-1/3*1/(e*x^3) + (3*d)/(7*e^2*x^(7/3)) - (3*d^2)/(5*
e^3*x^(5/3)) + d^3/(e^4*x) - (3*d^4)/(e^5*x^(1/3)) + (3*d^(9/2)*ArcTan[Sqrt
[e]/(Sqrt[d]*x^(1/3))])/e^(11/2)))/9 - (b*Log[c*(d + e/x^(2/3))^n])/(3*x^3)
```

Maple [F]

$$\int \frac{a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)}{x^4}\,dx$$

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.57

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx = \left[\frac{315\,bd^4nx^3\sqrt{-\frac{d}{e}}\log\left(\frac{d^3x^2-2\,de^2x\sqrt{-\frac{d}{e}}-e^3+2\,\left(d^2ex\sqrt{-\frac{d}{e}}+de^2\right)x^{\frac{2}{3}}-2\,\left(d^2ex-e^3\sqrt{-\frac{d}{e}}\right)x}{d^3x^2+e^3}\right.}{}\right.$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^4,x, algorithm="fricas")
```

```
[Out] [1/945*(315*b*d^4*n*x^3*sqrt(-d/e)*log((d^3*x^2 - 2*d*e^2*x*sqrt(-d/e) - e^
3 + 2*(d^2*e*x*sqrt(-d/e) + d*e^2)*x^(2/3) - 2*(d^2*e*x - e^3*sqrt(-d/e))*x
^(1/3))/(d^3*x^2 + e^3)) - 210*b*d^3*e*n*x^2 + 126*b*d^2*e^2*n*x^(4/3) - 31
5*b*e^4*n*log((d*x + e*x^(1/3))/x) + 70*b*e^4*n - 315*b*e^4*log(c) - 315*a*
e^4 + 90*(7*b*d^4*n*x^2 - b*d*e^3*n)*x^(2/3))/(e^4*x^3), 1/945*(630*b*d^4*n
*x^3*sqrt(d/e)*arctan(x^(1/3)*sqrt(d/e)) - 210*b*d^3*e*n*x^2 + 126*b*d^2*e^
2*n*x^(4/3) - 315*b*e^4*n*log((d*x + e*x^(1/3))/x) + 70*b*e^4*n - 315*b*e^4
*log(c) - 315*a*e^4 + 90*(7*b*d^4*n*x^2 - b*d*e^3*n)*x^(2/3))/(e^4*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))/x**4,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.84

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx = \frac{1}{945}\left(2\,e\left(\frac{315\,d^5\arctan\left(\frac{dx^{\frac{1}{3}}}{\sqrt{de}}\right)}{\sqrt{de}e^5} + \frac{315\,d^4x^{\frac{8}{3}} - 105\,d^3ex^2 + 63\,d^2e^2x^{\frac{4}{3}} - 45\,d\right.\right.\right.$$
$$-\frac{b\log\left(c\right)}{3\,x^3} - \frac{a}{3\,x^3}$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))/x^4,x, algorithm="giac")
```

```
[Out] 1/945*(2*e*(315*d^5*arctan(d*x^(1/3)/sqrt(d*e))/(sqrt(d*e)*e^5) + (315*d^4*
x^(8/3) - 105*d^3*e*x^2 + 63*d^2*e^2*x^(4/3) - 45*d*e^3*x^(2/3) + 35*e^4)/(
e^5*x^3)) - 315*log(d + e/x^(2/3))/x^3)*b*n - 1/3*b*log(c)/x^3 - 1/3*a/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{x^4}\,dx = \int \frac{a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)}{x^4}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))/x^4,x)
```

```
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))/x^4, x)
```

3.516 $\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 24, antiderivative size = 412

$$\begin{aligned}\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx &= -\frac{77b^2e^5n^2x^{2/3}}{120d^5}\\
&+\frac{47b^2e^4n^2x^{4/3}}{240d^4}-\frac{3b^2e^3n^2x^2}{40d^3}+\frac{b^2e^2n^2x^{8/3}}{40d^2}+\frac{77b^2e^6n^2\log\left(d+\frac{e}{x^{2/3}}\right)}{120d^6}\\
&+\frac{be^5n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^6}-\frac{be^4nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4d^4}\\
&+\frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{6d^3}-\frac{be^2nx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{8d^2}\\
&+\frac{benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{10d}+\frac{be^6n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^6}\\
&+\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{137b^2e^6n^2\log(x)}{180d^6}-\frac{b^2e^6n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{x^{2/3}}}\right)}{2d^6}\end{aligned}$$

```
[Out] -77/120*b^2*e^5*n^2*x^(2/3)/d^5+47/240*b^2*e^4*n^2*x^(4/3)/d^4-3/40*b^2*e^3
*n^2*x^2/d^3+1/40*b^2*e^2*n^2*x^(8/3)/d^2+77/120*b^2*e^6*n^2*ln(d+e/x^(2/3)
)/d^6+1/2*b*e^5*n*(d+e/x^(2/3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^6-1/4
*b*e^4*n*x^(4/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^4+1/6*b*e^3*n*x^2*(a+b*ln(c*
(d+e/x^(2/3))^n))/d^3-1/8*b*e^2*n*x^(8/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^2+1
/10*b*e*n*x^(10/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d+1/2*b*e^6*n*ln(1-d/(d+e/x^
(2/3)))*(a+b*ln(c*(d+e/x^(2/3))^n))/d^6+1/4*x^4*(a+b*ln(c*(d+e/x^(2/3))^n))
^2+137/180*b^2*e^6*n^2*ln(x)/d^6-1/2*b^2*e^6*n^2*polylog(2,d/(d+e/x^(2/3)))
/d^6
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\frac{be^6n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^6}+\frac{be^5nx^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^6}-\frac{be^4nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4d^4}+\frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{6d^3}-\frac{be^2nx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{8d^2}+\frac{benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{10d}+\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-\frac{b^2e^6n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{x^{2/3}}}\right)}{2d^6}+\frac{77b^2e^6n^2\log\left(d+\frac{e}{x^{2/3}}\right)}{120d^6}+\frac{137b^2e^6n^2\log(x)}{180d^6}$$

```
[In] Int[x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (-77*b^2*e^5*n^2*x^(2/3))/(120*d^5) + (47*b^2*e^4*n^2*x^(4/3))/(240*d^4) -
(3*b^2*e^3*n^2*x^2)/(40*d^3) + (b^2*e^2*n^2*x^(8/3))/(40*d^2) + (77*b^2*e^6
*n^2*Log[d + e/x^(2/3)])/(120*d^6) + (b*e^5*n*(d + e/x^(2/3))*x^(2/3)*(a +
b*Log[c*(d + e/x^(2/3))^n]))/(2*d^6) - (b*e^4*n*x^(4/3)*(a + b*Log[c*(d + e
/x^(2/3))^n]))/(4*d^4) + (b*e^3*n*x^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(6*
d^3) - (b*e^2*n*x^(8/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(8*d^2) + (b*e*n*
x^(10/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(10*d) + (b*e^6*n*Log[1 - d/(d +
 e/x^(2/3))]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(2*d^6) + (x^4*(a + b*Log[c*
(d + e/x^(2/3))^n])^2)/4 + (137*b^2*e^6*n^2*Log[x])/(180*d^6) - (b^2*e^6*n^
2*PolyLog[2, d/(d + e/x^(2/3))])/(2*d^6)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
```

```
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = -\left(\frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^7}\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 - \frac{1}{2}(ben)\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^6(d+ex)}\,dx, x, \frac{1}{x^{2/3}}\right)$$

$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 - \frac{1}{2}(bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+\frac{e}{x^{2/3}}\right)$$

$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 - \frac{(bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+\frac{e}{x^{2/3}}\right)}{2d} + \frac{(ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+\frac{e}{x^{2/3}}\right)}{2d}$$

$$= \frac{benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{10d} + \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + \frac{(ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+\frac{e}{x^{2/3}}\right)}{2d^2} - \frac{(be^2n)\,\text{Subst}\left(\int \frac{a}{x}\right.}{}$$

$$= -\frac{be^2nx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{8d^2} + \frac{benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{10d} + \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 - \frac{(be^2n)\,\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+\frac{e}{x^{2/3}}\right)}{2d^3} + \frac{(be^3n)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{b^2e^5n^2x^{2/3}}{10d^5} + \frac{b^2e^4n^2x^{4/3}}{20d^4} - \frac{b^2e^3n^2x^2}{30d^3} + \frac{b^2e^2n^2x^{8/3}}{40d^2} + \frac{b^2e^6n^2\log\left(d+\frac{e}{x^{2/3}}\right)}{10d^6} + \frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{6d^3} - \frac{be^2nx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{8d^2} + \frac{benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{10d} + \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + \frac{b^2e^6n^2\log(x)}{15d^6} + \frac{(be^3n)\operatorname{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+\frac{e}{x^{2/3}}\right)}{2d^4} - \cdots$$

$$= -\frac{9b^2e^5n^2x^{2/3}}{40d^5} + \frac{9b^2e^4n^2x^{4/3}}{80d^4} - \frac{3b^2e^3n^2x^2}{40d^3} + \frac{b^2e^2n^2x^{8/3}}{40d^2} + \frac{9b^2e^6n^2\log\left(d+\frac{e}{x^{2/3}}\right)}{40d^6} - \frac{be^4nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4d^4} + \frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{6d^3} - \frac{be^2nx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{8d^2} + \frac{benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{10d} + \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + \frac{3b^2e^6n^2\log(x)}{20d^6} - \frac{(be^4n)\operatorname{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+\frac{e}{x^{2/3}}\right)}{2d^5} + \cdots$$

$$= -\frac{47b^2e^5n^2x^{2/3}}{120d^5} + \frac{47b^2e^4n^2x^{4/3}}{240d^4} - \frac{3b^2e^3n^2x^2}{40d^3} + \frac{b^2e^2n^2x^{8/3}}{40d^2} + \frac{47b^2e^6n^2\log\left(d+\frac{e}{x^{2/3}}\right)}{120d^6} + \frac{be^5n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^6} - \frac{be^4nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4d^4} + \frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{6d^3} - \frac{be^2nx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{8d^2} + \frac{benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{10d} + \frac{be^6n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^6} + \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + \frac{47b^2e^6n^2\log(x)}{180d^6} + \frac{(b^2e^4n^2)\operatorname{Subst}\left(\int\left(\frac{e^2}{d(d-x)^2}+\frac{e^2}{d^2(d-x)}+\frac{e^2}{d^2x}\right) \cdots\right)}{4d^4}$$

$$
\begin{aligned}
&= -\frac{77b^2e^5n^2x^{2/3}}{120d^5} + \frac{47b^2e^4n^2x^{4/3}}{240d^4} - \frac{3b^2e^3n^2x^2}{40d^3} + \frac{b^2e^2n^2x^{8/3}}{40d^2} \\
&+ \frac{77b^2e^6n^2\log\left(d+\frac{e}{x^{2/3}}\right)}{120d^6} + \frac{be^5n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^6} \\
&- \frac{be^4nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4d^4} + \frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{6d^3} \\
&- \frac{be^2nx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{8d^2} + \frac{benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{10d} \\
&+ \frac{be^6n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^6} \\
&+ \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + \frac{137b^2e^6n^2\log(x)}{180d^6} - \frac{b^2e^6n^2\operatorname{Li}_2\left(\frac{d}{d+\frac{e}{x^{2/3}}}\right)}{2d^6}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 830 vs. $2(412) = 824$.

Time = 0.43 (sec) , antiderivative size = 830, normalized size of antiderivative = 2.01

$$
\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2
$$

$$
+\frac{ben\Big(360ade^4x^{2/3} - 462bde^4nx^{2/3} - 180ad^2e^3x^{4/3} + 141bd^2e^3nx^{4/3} + 120ad^3e^2x^2 - 54bd^3e^2nx^2 - 90ad^4ex}{}
$$

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (x^4*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/4 + (b*e*n*(360*a*d*e^4*x^(2/3) -
462*b*d*e^4*n*x^(2/3) - 180*a*d^2*e^3*x^(4/3) + 141*b*d^2*e^3*n*x^(4/3) + 1
20*a*d^3*e^2*x^2 - 54*b*d^3*e^2*n*x^2 - 90*a*d^4*e*x^(8/3) + 18*b*d^4*e*n*x
^(8/3) + 72*a*d^5*x^(10/3) + 822*b*e^5*n*Log[d + e/x^(2/3)] + 360*b*d*e^4*x
^(2/3)*Log[c*(d + e/x^(2/3))^n] - 180*b*d^2*e^3*x^(4/3)*Log[c*(d + e/x^(2/3
))^n] + 120*b*d^3*e^2*x^2*Log[c*(d + e/x^(2/3))^n] - 90*b*d^4*e*x^(8/3)*Log
[c*(d + e/x^(2/3))^n] + 72*b*d^5*x^(10/3)*Log[c*(d + e/x^(2/3))^n] - 360*a*
e^5*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] - 360*b*e^5*Log[c*(d + e/x^(2/3))^n]*Lo
g[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 180*b*e^5*n*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]
^2 - 360*a*e^5*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] - 360*b*e^5*Log[c*(d + e/x^(
2/3))^n]*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 180*b*e^5*n*Log[Sqrt[e] + Sqrt[-
d]*x^(1/3)]^2 + 360*b*e^5*n*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[1/2 - (Sqrt
[-d]*x^(1/3))/(2*Sqrt[e])] + 360*b*e^5*n*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Lo
g[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 720*b*e^5*n*Log[Sqrt[e] + Sqrt[-d]*
x^(1/3)]*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])] - 720*b*e^5*n*Log[Sqrt[e] - Sqr
t[-d]*x^(1/3)]*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]] + 548*b*e^5*n*Log[x] - 720*b
```

```
*e^5*n*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[e]] + 360*b*e^5*n*PolyLog[2,
1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] + 360*b*e^5*n*PolyLog[2, (1 + (Sqrt[-
d]*x^(1/3))/Sqrt[e])/2] - 720*b*e^5*n*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqr
t[e]]))/(720*d^6)
```

Maple [F]

$$\int x^3\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)^2dx$$

[In] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)

[Out] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)

Fricas [F]

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2x^3\,dx$$

[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="fricas")

[Out] integral(b^2*x^3*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*x^3*log(c*((d*x + e*x^(1/3))/x)^n) + a^2*x^3, x)

Sympy [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

[In] integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))**n))**2,x)

[Out] Timed out

Maxima [F]

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2x^3\,dx$$

[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="maxima")

[Out] 1/4*b^2*x^4*log((d*x^(2/3) + e)^n)^2 - integrate(-1/3*(3*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x^4 + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x

```
^(10/3) + 12*(b^2*d*x^4 + b^2*e*x^(10/3))*log(x^(1/3*n))^2 - (b^2*d*n*x^4 -
 6*(b^2*d*log(c) + a*b*d)*x^4 - 6*(b^2*e*log(c) + a*b*e)*x^(10/3) + 12*(b^2
*d*x^4 + b^2*e*x^(10/3))*log(x^(1/3*n)))*log((d*x^(2/3) + e)^n) - 12*((b^2*
d*log(c) + a*b*d)*x^4 + (b^2*e*log(c) + a*b*e)*x^(10/3))*log(x^(1/3*n)))/(d
*x + e*x^(1/3)), x)
```

Giac [F]

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2x^3\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\int x^3\,\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx$$

```
[In] int(x^3*(a + b*log(c*(d + e/x^(2/3))^n))^2,x)
[Out] int(x^3*(a + b*log(c*(d + e/x^(2/3))^n))^2, x)
```

3.517 $\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 239

$$
\begin{aligned}
\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx &= \frac{b^2e^2n^2x^{2/3}}{2d^2}-\frac{b^2e^3n^2\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3} \\
&- \frac{be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}+\frac{benx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d} \\
&- \frac{be^3n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3} \\
&+ \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-\frac{b^2e^3n^2\log(x)}{d^3}+\frac{b^2e^3n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{x^{2/3}}}\right)}{d^3}
\end{aligned}
$$

```
[Out] 1/2*b^2*e^2*n^2*x^(2/3)/d^2-1/2*b^2*e^3*n^2*ln(d+e/x^(2/3))/d^3-b*e^2*n*(d+
e/x^(2/3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^3+1/2*b*e*n*x^(4/3)*(a+b*l
n(c*(d+e/x^(2/3))^n))/d-b*e^3*n*ln(1-d/(d+e/x^(2/3)))*(a+b*ln(c*(d+e/x^(2/3
))^n))/d^3+1/2*x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^2-b^2*e^3*n^2*ln(x)/d^3+b^2*
e^3*n^2*polylog(2,d/(d+e/x^(2/3)))/d^3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {2504, 2445, 2458, 2389, 2379, 2438, 2351, 31, 2356, 46}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = -\frac{be^3n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3} - \frac{be^2nx^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3} + \frac{benx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d} + \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + \frac{b^2e^3n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{x^{2/3}}}\right)}{d^3} - \frac{b^2e^3n^2\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3} - \frac{b^2e^3n^2\log(x)}{d^3} + b^2e$$

```
[In] Int[x*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (b^2*e^2*n^2*x^(2/3))/(2*d^2) - (b^2*e^3*n^2*Log[d + e/x^(2/3)])/(2*d^3) -
(b*e^2*n*(d + e/x^(2/3))*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/d^3 + (b
*e*n*x^(4/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(2*d) - (b*e^3*n*Log[1 - d/(
d + e/x^(2/3))]*(a + b*Log[c*(d + e/x^(2/3))^n]))/d^3 + (x^2*(a + b*Log[c*(
d + e/x^(2/3))^n])^2)/2 - (b^2*e^3*n^2*Log[x])/d^3 + (b^2*e^3*n^2*PolyLog[2
, d/(d + e/x^(2/3))])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
```

```
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = -\left(\frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^4}\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-(ben)\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x^3(d+ex)}\,dx, x, \frac{1}{x^{2/3}}\right)$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-(bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+\frac{e}{x^{2/3}}\right)$$

$$= \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-\frac{(bn)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx, x, d+\frac{e}{x^{2/3}}\right)}{d}+\frac{(ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+\frac{e}{x^{2/3}}\right)}{d}$$

$$= \frac{benx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{(ben)\text{Subst}\left(\int \frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx, x, d+\frac{e}{x^{2/3}}\right)}{d^2}-\frac{(be^2n)\,\text{Subst}\left(\int \frac{a}{}\right.}{}$$

$$= -\frac{be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}+\frac{benx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d}-\frac{be^3n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-\frac{(b^2en^2)\,\text{Subst}\left(\int \left(\frac{e^2}{d(d-x)^2}+\frac{e^2}{d^2(d-x)}+\frac{e^2}{d^2x}\right)dx, x, d+\frac{e}{x^{2/3}}\right)}{2d}+\cdots$$

$$= \frac{b^2e^2n^2x^{2/3}}{2d^2}-\frac{b^2e^3n^2\log\left(d+\frac{e}{x^{2/3}}\right)}{2d^3}-\frac{be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}+\frac{benx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d}-\frac{be^3n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-\frac{b^2e^3n^2\log(x)}{d^3}+\frac{b^2e^3n^2\text{Li}_2\left(\frac{d}{d+\frac{e}{x^{2/3}}}\right)}{d^3}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 542 vs. $2(239) = 478$.

Time = 0.29 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.27

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = \frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 - \frac{ben\left(6dex^{2/3}(a+b\log(c(d+\frac{e}{x^{2/3}})^n)) - 3d^2x^{4/3}(a+b\log(c(d+\frac{e}{x^{2/3}})^n)) - 6e^2(a+b\log(c(d+\frac{e}{x^{2/3}})^n))\right.}{}$$

[In] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]

[Out] (x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/2 - (b*e*n*(6*d*e*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n]) - 3*d^2*x^(4/3)*(a + b*Log[c*(d + e/x^(2/3))^n]) - 6*e^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] - 6*e^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 2*b*e^2*n*(3*Log[d + e/x^(2/3)] + 2*Log[x]) + b*e*n*(-3*d*x^(2/3) + 3*e*Log[d + e/x^(2/3)] + 2*e*Log[x]) + 3*b*e^2*n*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 2*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 4*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]]) - 4*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[e]] + 2*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])]) + 3*b*e^2*n*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 2*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] - 4*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])]) + 2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 4*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]])))/(6*d^3)

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)^2 dx$$

[In] int(x*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)

[Out] int(x*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)

Fricas [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = \int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2 x\,dx$$

[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="fricas")

[Out] integral(b^2*x*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*x*log(c*((d*x + e*x^(1/3))/x)^n) + a^2*x, x)

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(2/3))**n))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*log((d*x^(2/3) + e)^n)^2 - integrate(-1/3*(3*(b^2*d*log(c)^2 +
2*a*b*d*log(c) + a^2*d)*x^2 + 12*(b^2*d*x^2 + b^2*e*x^(4/3))*log(x^(1/3*n))
^2 + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^(4/3) - 2*(b^2*d*n*x^2 -
 3*(b^2*d*log(c) + a*b*d)*x^2 - 3*(b^2*e*log(c) + a*b*e)*x^(4/3) + 6*(b^2*d
*x^2 + b^2*e*x^(4/3))*log(x^(1/3*n)))*log((d*x^(2/3) + e)^n) - 12*((b^2*d*l
og(c) + a*b*d)*x^2 + (b^2*e*log(c) + a*b*e)*x^(4/3))*log(x^(1/3*n)))/(d*x +
 e*x^(1/3)), x)
```

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\int x\,\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx$$

```
[In] int(x*(a + b*log(c*(d + e/x^(2/3))^n))^2,x)
[Out] int(x*(a + b*log(c*(d + e/x^(2/3))^n))^2, x)
```

3.518 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 95

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx = -\frac{3}{2}\Big(a$$
$$+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\Big)^2\log\left(-\frac{e}{dx^{2/3}}\right)-3bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\text{PolyLog}\left(2,1+\frac{e}{dx^{2/3}}\right)+3b^2n^2\,\text{Poly}$$

```
[Out] -3/2*(a+b*ln(c*(d+e/x^(2/3))^n))^2*ln(-e/d/x^(2/3))-3*b*n*(a+b*ln(c*(d+e/x^
(2/3))^n))*polylog(2,1+e/d/x^(2/3))+3*b^2*n^2*polylog(3,1+e/d/x^(2/3))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2504, 2443, 2481, 2421, 6724}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx = -3bn\,\text{PolyLog}\Big(2,\frac{e}{dx^{2/3}}$$
$$+1\Big)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{3}{2}\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+3b^2n^2\,\text{PolyLog}\left(3,\frac{e}{dx^{2/3}}+1\right.$$

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x,x]
```

```
[Out] (-3*(a + b*Log[c*(d + e/x^(2/3))^n])^2*Log[-(e/(d*x^(2/3)))])/2 - 3*b*n*(a
+ b*Log[c*(d + e/x^(2/3))^n])*PolyLog[2, 1 + e/(d*x^(2/3))] + 3*b^2*n^2*Pol
yLog[3, 1 + e/(d*x^(2/3))]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(\frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x}\,dx, x, \frac{1}{x^{2/3}}\right)\right)\\
&= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\log\left(-\frac{e}{dx^{2/3}}\right)\\
&\quad+(3ben)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))}{d+ex}\,dx, x, \frac{1}{x^{2/3}}\right)
\end{aligned}
$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\log\left(-\frac{e}{dx^{2/3}}\right) + (3bn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}dx,x,d+\frac{e}{x^{2/3}}\right)$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\log\left(-\frac{e}{dx^{2/3}}\right) - 3bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\text{Li}_2\left(1+\frac{e}{dx^{2/3}}\right) + \left(3b^2n^2\right)\text{Subst}\left(\int\frac{\text{Li}_2\left(\frac{x}{d}\right)}{x}dx,x,d+\frac{e}{x^{2/3}}\right)$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\log\left(-\frac{e}{dx^{2/3}}\right) - 3bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\text{Li}_2\left(1+\frac{e}{dx^{2/3}}\right) + 3b^2n^2\text{Li}_3\left(1+\frac{e}{dx^{2/3}}\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 1701, normalized size of antiderivative = 17.91

$$\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}dx = \Big(a$$

$$-bn\log\left(d+\frac{e}{x^{2/3}}\right)+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\Big)^2\log(x)+2bn\left(a-bn\log\left(d+\frac{e}{x^{2/3}}\right)+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\Big(\Big(\text{lo}$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x,x]
```

```
[Out] (a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])^2*Log[x] + 2*b*n*
(a - b*n*Log[d + e/x^(2/3)] + b*Log[c*(d + e/x^(2/3))^n])*((Log[d + e/x^(2/
3)] - Log[1 + e/(d*x^(2/3))])*Log[x] + (3*PolyLog[2, -(e/(d*x^(2/3)))])/2)
+ 3*b^2*n^2*(Log[((-I)*Sqrt[e])/Sqrt[d] + x^(1/3)]^2*Log[((-I)*Sqrt[d]*x^(1
/3))/Sqrt[e]] + 2*Log[((-I)*Sqrt[e])/Sqrt[d] + x^(1/3)]*Log[(I*Sqrt[e])/Sqr
t[d] + x^(1/3)]*Log[((-I)*Sqrt[d]*x^(1/3))/Sqrt[e]] + Log[1 - (I*Sqrt[d]*x^
(1/3))/Sqrt[e]]*(-2*Log[((-I)*Sqrt[e])/Sqrt[d] + x^(1/3)] + Log[1 - (I*Sqrt
[d]*x^(1/3))/Sqrt[e]])*(Log[((-I)*Sqrt[d]*x^(1/3))/Sqrt[e]] - Log[(I*Sqrt[d
]*x^(1/3))/Sqrt[e]]) + Log[(I*Sqrt[e])/Sqrt[d] + x^(1/3)]^2*Log[(I*Sqrt[d]*
x^(1/3))/Sqrt[e]] + 2*Log[(Sqrt[e] - I*Sqrt[d]*x^(1/3))/(Sqrt[e] + I*Sqrt[d
]*x^(1/3))]*Log[1 - (I*Sqrt[d]*x^(1/3))/Sqrt[e]]*(-Log[((-I)*Sqrt[d]*x^(1/3
))/Sqrt[e]] + Log[(I*Sqrt[d]*x^(1/3))/Sqrt[e]]) + Log[(Sqrt[e] - I*Sqrt[d]*
x^(1/3))/(Sqrt[e] + I*Sqrt[d]*x^(1/3))]^2*(Log[(2*Sqrt[e])/(Sqrt[e] + I*Sqr
t[d]*x^(1/3))] + Log[((-I)*Sqrt[d]*x^(1/3))/Sqrt[e]] - Log[(2*x^(1/3))/(((-
I)*Sqrt[e])/Sqrt[d] + x^(1/3))]) + ((-Log[d + e/x^(2/3)] + Log[((-I)*Sqrt[e
])/Sqrt[d] + x^(1/3)] + Log[(I*Sqrt[e])/Sqrt[d] + x^(1/3)] - (2*Log[x])/3)^
```

```
2*Log[x])/3 + (4*Log[x]^3)/81 + 2*Log[(Sqrt[e] - I*Sqrt[d]*x^(1/3))/(Sqrt[e
] + I*Sqrt[d]*x^(1/3))]*(-PolyLog[2, (I*Sqrt[e] + Sqrt[d]*x^(1/3))/(I*Sqrt[
e] - Sqrt[d]*x^(1/3))] + PolyLog[2, (I*Sqrt[e] + Sqrt[d]*x^(1/3))/((-I)*Sqr
t[e] + Sqrt[d]*x^(1/3))]) + 2*Log[(I*Sqrt[e])/Sqrt[d] + x^(1/3)]*PolyLog[2,
 1 - (I*Sqrt[d]*x^(1/3))/Sqrt[e]] + 2*(Log[((-I)*Sqrt[e])/Sqrt[d] + x^(1/3)
] + Log[(Sqrt[e] - I*Sqrt[d]*x^(1/3))/(Sqrt[e] + I*Sqrt[d]*x^(1/3))])*PolyL
og[2, 1 - (I*Sqrt[d]*x^(1/3))/Sqrt[e]] + 2*Log[((-I)*Sqrt[e])/Sqrt[d] + x^(
1/3)]*PolyLog[2, 1 + (I*Sqrt[d]*x^(1/3))/Sqrt[e]] + 2*(Log[(I*Sqrt[e])/Sqrt
[d] + x^(1/3)] - Log[(Sqrt[e] - I*Sqrt[d]*x^(1/3))/(Sqrt[e] + I*Sqrt[d]*x^(
1/3))])*PolyLog[2, 1 + (I*Sqrt[d]*x^(1/3))/Sqrt[e]] + 2*(Log[d + e/x^(2/3)]
 - Log[((-I)*Sqrt[e])/Sqrt[d] + x^(1/3)] - Log[(I*Sqrt[e])/Sqrt[d] + x^(1/3
)] + (2*Log[x])/3)*(((3*Log[((-I)*Sqrt[e])/Sqrt[d] + x^(1/3)] + 3*Log[(I*Sq
rt[e])/Sqrt[d] + x^(1/3)] - 3*Log[1 - (I*Sqrt[d]*x^(1/3))/Sqrt[e]] - 3*Log[
1 + (I*Sqrt[d]*x^(1/3))/Sqrt[e]] - Log[x])*Log[x])/9 - PolyLog[2, ((-I)*Sqr
t[d]*x^(1/3))/Sqrt[e]] - PolyLog[2, (I*Sqrt[d]*x^(1/3))/Sqrt[e]]) + 2*PolyL
og[3, (I*Sqrt[e] + Sqrt[d]*x^(1/3))/(I*Sqrt[e] - Sqrt[d]*x^(1/3))] - 2*Poly
Log[3, (I*Sqrt[e] + Sqrt[d]*x^(1/3))/((-I)*Sqrt[e] + Sqrt[d]*x^(1/3))] - 4*
PolyLog[3, 1 - (I*Sqrt[d]*x^(1/3))/Sqrt[e]] - 4*PolyLog[3, 1 + (I*Sqrt[d]*x
^(1/3))/Sqrt[e]] - (2*((Log[(I*Sqrt[e])/Sqrt[d] + x^(1/3)] - Log[1 - (I*Sqr
t[d]*x^(1/3))/Sqrt[e]])*Log[x]^2 + (Log[((-I)*Sqrt[e])/Sqrt[d] + x^(1/3)] -
 Log[1 + (I*Sqrt[d]*x^(1/3))/Sqrt[e]])*Log[x]^2 - 6*Log[x]*PolyLog[2, ((-I)
*Sqrt[d]*x^(1/3))/Sqrt[e]] - 6*Log[x]*PolyLog[2, (I*Sqrt[d]*x^(1/3))/Sqrt[e
]] + 18*PolyLog[3, ((-I)*Sqrt[d]*x^(1/3))/Sqrt[e]] + 18*PolyLog[3, (I*Sqrt[
d]*x^(1/3))/Sqrt[e]]))/9)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{2}}{x}dx$$

[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x,x)

[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x,x)

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{2}}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{2}}{x}\,dx$$

[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x,x, algorithm="fricas")

[Out] integral((b^2*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*log(c*((d*x + e*x^(1
/3))/x)^n) + a^2)/x, x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2/x,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x,x, algorithm="maxima")
[Out] b^2*log((d*x^(2/3) + e)^n)^2*log(x) - integrate(-1/3*(12*(b^2*d*x + b^2*e*x
^(1/3))*log(x^(1/3*n))^2 + 3*(b^2*d*log(c)^2 + 2*a*b*d*log(c) + a^2*d)*x -
2*(2*b^2*d*n*x*log(x) - 3*(b^2*d*log(c) + a*b*d)*x + 6*(b^2*d*x + b^2*e*x^(
1/3))*log(x^(1/3*n)) - 3*(b^2*e*log(c) + a*b*e)*x^(1/3))*log((d*x^(2/3) + e
)^n) - 12*((b^2*d*log(c) + a*b*d)*x + (b^2*e*log(c) + a*b*e)*x^(1/3))*log(x
^(1/3*n)) + 3*(b^2*e*log(c)^2 + 2*a*b*e*log(c) + a^2*e)*x^(1/3))/(d*x^2 + e
*x^(4/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^2/x,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))^2/x, x)
```

3.519 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 276

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx &= \frac{3b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2}{4e^3}-\frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3}\\ &-\frac{3b^2d^2n^2}{e^2x^{2/3}}+\frac{b^2d^3n^2\log^2\left(d+\frac{e}{x^{2/3}}\right)}{2e^3}+\frac{3bd^2n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^3}\\ &-\frac{3bdn\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2e^3}\\ &+\frac{bn\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^3}\\ &-\frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^3}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2x^2}\end{aligned}$$

```
[Out] 3/4*b^2*d*n^2*(d+e/x^(2/3))^2/e^3-1/9*b^2*n^2*(d+e/x^(2/3))^3/e^3-3*b^2*d^2
*n^2/e^2/x^(2/3)+1/2*b^2*d^3*n^2*ln(d+e/x^(2/3))^2/e^3+3*b*d^2*n*(d+e/x^(2/
3))*(a+b*ln(c*(d+e/x^(2/3))^n))/e^3-3/2*b*d*n*(d+e/x^(2/3))^2*(a+b*ln(c*(d+
e/x^(2/3))^n))/e^3+1/3*b*n*(d+e/x^(2/3))^3*(a+b*ln(c*(d+e/x^(2/3))^n))/e^3-
b*d^3*n*ln(d+e/x^(2/3))*(a+b*ln(c*(d+e/x^(2/3))^n))/e^3-1/2*(a+b*ln(c*(d+e/
x^(2/3))^n))^2/x^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx = -\frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^3} + \frac{3bd^2n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^3} - \frac{3bdn\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2e^3} + \frac{bn\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^3} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2x^2} + \frac{b^2d^3n^2\log^2\left(d+\frac{e}{x^{2/3}}\right)}{2e^3} - \frac{3b^2d^2n^2}{e^2x^{2/3}} + \frac{3b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2}{4e^3} - \frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3}$$

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^3,x]
```

```
[Out] (3*b^2*d*n^2*(d + e/x^(2/3))^2)/(4*e^3) - (b^2*n^2*(d + e/x^(2/3))^3)/(9*e^
3) - (3*b^2*d^2*n^2)/(e^2*x^(2/3)) + (b^2*d^3*n^2*Log[d + e/x^(2/3)]^2)/(2*
e^3) + (3*b*d^2*n*(d + e/x^(2/3))*(a + b*Log[c*(d + e/x^(2/3))^n]))/e^3 - (
3*b*d*n*(d + e/x^(2/3))^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(2*e^3) + (b*n*
(d + e/x^(2/3))^3*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*e^3) - (b*d^3*n*Log[
d + e/x^(2/3)]*(a + b*Log[c*(d + e/x^(2/3))^n]))/e^3 - (a + b*Log[c*(d + e/
x^(2/3))^n])^2/(2*x^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = -\left(\frac{3}{2}\text{Subst}\left(\int x^2(a+b\log\left(c(d+ex)^n\right))^2\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2x^2} + (ben)\text{Subst}\left(\int \frac{x^3(a+b\log\left(c(d+ex)^n\right))}{d+ex}\,dx, x, \frac{1}{x^{2/3}}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2x^2}+(bn)\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^3(a+b\log(cx^n))}{x}\,dx,x,d+\frac{e}{x^{2/3}}\right)$$

$$= \frac{3bd^2n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^3} - \frac{3bdn\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2e^3} + \frac{bn\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^3} - \frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^3} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2x^2} - \left(b^2n^2\right)\text{Subst}\left(\int\frac{18d^2x-9dx^2+2x^3-6d^3\log(x)}{6e^3x}\,dx,x,d+\frac{e}{x^{2/3}}\right)$$

$$= \frac{3bd^2n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^3} - \frac{3bdn\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2e^3} + \frac{bn\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^3} - \frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^3} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2x^2} - \frac{\left(b^2n^2\right)\text{Subst}\left(\int\frac{18d^2x-9dx^2+2x^3-6d^3\log(x)}{x}\,dx,x,d+\frac{e}{x^{2/3}}\right)}{6e^3}$$

$$= \frac{3bd^2n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^3} - \frac{3bdn\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2e^3} + \frac{bn\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^3} - \frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^3} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2x^2} - \frac{\left(b^2n^2\right)\text{Subst}\left(\int\left(18d^2-9dx+2x^2-\frac{6d^3\log(x)}{x}\right)dx,x,d+\frac{e}{x^{2/3}}\right)}{6e^3}$$

$$
\begin{aligned}
&= \frac{3b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2}{4e^3} - \frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3} - \frac{3b^2d^2n^2}{e^2x^{2/3}} \\
&\quad + \frac{3bd^2n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^3} \\
&\quad - \frac{3bdn\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2e^3} \\
&\quad + \frac{bn\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^3} \\
&\quad - \frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^3} \\
&\quad - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2x^2} + \frac{(b^2d^3n^2)\,\mathrm{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, d+\frac{e}{x^{2/3}}\right)}{e^3} \\
&= \frac{3b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2}{4e^3} - \frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3} - \frac{3b^2d^2n^2}{e^2x^{2/3}} \\
&\quad + \frac{b^2d^3n^2\log^2\left(d+\frac{e}{x^{2/3}}\right)}{2e^3} + \frac{3bd^2n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^3} \\
&\quad - \frac{3bdn\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2e^3} \\
&\quad + \frac{bn\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^3} \\
&\quad - \frac{bd^3n\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^3} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2x^2}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 691, normalized size of antiderivative = 2.50

$$
\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx = \frac{-18e^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + bn\Big(9bdnx^{2/3}\left(e\left(e-2dx^{2/3}\right)+2d^2x^{4}\right.}{}
$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^3,x]
```

```
[Out] (-18*e^3*(a + b*Log[c*(d + e/x^(2/3))^n])^2 + b*n*(9*b*d*n*x^(2/3)*(e*(e -
2*d*x^(2/3)) + 2*d^2*x^(4/3)*Log[d + e/x^(2/3)]) - 2*b*n*(e*(2*e^2 - 3*d*e*
x^(2/3) + 6*d^2*x^(4/3)) - 6*d^3*x^2*Log[d + e/x^(2/3)]) + 12*e^3*(a + b*Lo
g[c*(d + e/x^(2/3))^n]) - 18*d*e^2*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n])
 + 36*d^2*x^(4/3)*(e*(a - b*n) + b*(e + d*x^(2/3))*Log[c*(d + e/x^(2/3))^n]
) - 36*d^3*x^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] - Sqrt[-d]*x^(1
/3)] - 36*d^3*x^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] + Sqrt[-d]*x
^(1/3)] - 36*d^3*x^2*((a + b*Log[c*(d + e/x^(2/3))^n])*Log[-(e/(d*x^(2/3)))
```

```
] + b*n*PolyLog[2, 1 + e/(d*x^(2/3))]) + 18*b*d^3*n*x^2*(Log[Sqrt[e] - Sqrt
[-d]*x^(1/3)]*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 2*Log[(1 + (Sqrt[-d]*x^(1/
3))/Sqrt[e])/2] - 4*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]]) - 4*PolyLog[2, 1 - (Sq
rt[-d]*x^(1/3))/Sqrt[e]] + 2*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e]
)]) + 18*b*d^3*n*x^2*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] + Sqrt[-
d]*x^(1/3)] + 2*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] - 4*Log[-((Sqrt[-
d]*x^(1/3))/Sqrt[e])]) + 2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] -
 4*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]])))/(36*e^3*x^2)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{2}}{x^{3}}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.11

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{2}}{x^{3}}\,dx =$$

$$-\frac{4\,b^{2}e^{3}n^{2}+18\,b^{2}e^{3}\log\left(c\right)^{2}-12\,abe^{3}n+18\,a^{2}e^{3}+18\left(b^{2}d^{3}n^{2}x^{2}+b^{2}e^{3}n^{2}\right)\log\left(\frac{dx+ex^{\frac{1}{3}}}{x}\right)^{2}-12\left(b^{2}e^{3}n-3\,abe^{3}\right.}{}$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/36*(4*b^2*e^3*n^2 + 18*b^2*e^3*log(c)^2 - 12*a*b*e^3*n + 18*a^2*e^3 + 18
*(b^2*d^3*n^2*x^2 + b^2*e^3*n^2)*log((d*x + e*x^(1/3))/x)^2 - 12*(b^2*e^3*n
 - 3*a*b*e^3)*log(c) - 6*(6*b^2*d^2*e*n^2*x^(4/3) - 3*b^2*d*e^2*n^2*x^(2/3)
 + 2*b^2*e^3*n^2 - 6*a*b*e^3*n + (11*b^2*d^3*n^2 - 6*a*b*d^3*n)*x^2 - 6*(b^
2*d^3*n*x^2 + b^2*e^3*n)*log(c))*log((d*x + e*x^(1/3))/x) - 3*(5*b^2*d*e^2*
n^2 - 6*b^2*d*e^2*n*log(c) - 6*a*b*d*e^2*n)*x^(2/3) - 6*(6*b^2*d^2*e*n*x*lo
g(c) - (11*b^2*d^2*e*n^2 - 6*a*b*d^2*e*n)*x)*x^(1/3))/(e^3*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2/x**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx=$$
$$-\frac{1}{6}aben\left(\frac{6\,d^3\log\left(dx^{\frac{2}{3}}+e\right)}{e^4}-\frac{6\,d^3\log\left(x^{\frac{2}{3}}\right)}{e^4}-\frac{6\,d^2x^{\frac{4}{3}}-3\,dex^{\frac{2}{3}}+2\,e^2}{e^3x^2}\right)$$
$$-\frac{1}{36}\left(6\,en\left(\frac{6\,d^3\log\left(dx^{\frac{2}{3}}+e\right)}{e^4}-\frac{6\,d^3\log\left(x^{\frac{2}{3}}\right)}{e^4}-\frac{6\,d^2x^{\frac{4}{3}}-3\,dex^{\frac{2}{3}}+2\,e^2}{e^3x^2}\right)\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)-\frac{\left(18\,d^3\right.}{}\right.$$
$$-\frac{b^2\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)^2}{2\,x^2}-\frac{ab\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)}{x^2}-\frac{a^2}{2\,x^2}$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^3,x, algorithm="maxima")
[Out] -1/6*a*b*e*n*(6*d^3*log(d*x^(2/3) + e)/e^4 - 6*d^3*log(x^(2/3))/e^4 - (6*d^
2*x^(4/3) - 3*d*e*x^(2/3) + 2*e^2)/(e^3*x^2)) - 1/36*(6*e*n*(6*d^3*log(d*x^
(2/3) + e)/e^4 - 6*d^3*log(x^(2/3))/e^4 - (6*d^2*x^(4/3) - 3*d*e*x^(2/3) +
2*e^2)/(e^3*x^2))*log(c*(d + e/x^(2/3))^n) - (18*d^3*x^2*log(d*x^(2/3) + e)
^2 + 8*d^3*x^2*log(x)^2 - 44*d^3*x^2*log(x) - 66*d^2*e*x^(4/3) + 15*d*e^2*x
^(2/3) - 4*e^3 - 6*(4*d^3*x^2*log(x) - 11*d^3*x^2)*log(d*x^(2/3) + e))*n^2/
(e^3*x^2))*b^2 - 1/2*b^2*log(c*(d + e/x^(2/3))^n)^2/x^2 - a*b*log(c*(d + e/
x^(2/3))^n)/x^2 - 1/2*a^2/x^2
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^3}\,dx = \frac{\frac{d\left(\frac{3a^2}{2}-a\,b\,n+\frac{b^2n^2}{3}\right)}{2e}-\frac{d\,(3a^2-b^2n^2)}{4e}}{x^{4/3}}$$

$$-\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)^2\left(\frac{b^2}{2x^2}+\frac{b^2d^3}{2e^3}\right)-\frac{\frac{a^2}{2}-\frac{a\,b\,n}{3}+\frac{b^2n^2}{9}}{x^2}-\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\left(\frac{b\,(3a-b\,n)}{3x^2}-\frac{\frac{b\,d\,(3a-b\,n)}{2e}-3}{x^{4/3}}\right.$$

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^2/x^3,x)
```

```
[Out] ((d*((3*a^2)/2 + (b^2*n^2)/3 - a*b*n))/(2*e) - (d*(3*a^2 - b^2*n^2))/(4*e))
/x^(4/3) - log(c*(d + e/x^(2/3))^n)^2*(b^2/(2*x^2) + (b^2*d^3)/(2*e^3)) - (
a^2/2 + (b^2*n^2)/9 - (a*b*n)/3)/x^2 - log(c*(d + e/x^(2/3))^n)*((b*(3*a -
b*n))/(3*x^2) - ((b*d*(3*a - b*n))/(2*e) - (3*a*b*d)/(2*e))/x^(4/3) + (d*((
b*d*(3*a - b*n))/e - (3*a*b*d)/e))/(e*x^(2/3))) - ((d*((d*((3*a^2)/2 + (b^2
*n^2)/3 - a*b*n))/e - (d*(3*a^2 - b^2*n^2))/(2*e)))/e + (b^2*d^2*n^2)/e^2)/
x^(2/3) + (log(d + e/x^(2/3))*(11*b^2*d^3*n^2 - 6*a*b*d^3*n))/(6*e^3)
```

3.520 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 482

$$\begin{aligned}
\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^5}\,dx &= -\frac{15b^2d^4n^2\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^6}+\frac{10b^2d^3n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^6} \\
&\quad -\frac{15b^2d^2n^2\left(d+\frac{e}{x^{2/3}}\right)^4}{32e^6}+\frac{3b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^5}{25e^6}-\frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^6}{72e^6}+\frac{3b^2d^5n^2}{e^5x^{2/3}} \\
&\quad -\frac{b^2d^6n^2\log^2\left(d+\frac{e}{x^{2/3}}\right)}{4e^6}-\frac{3bd^5n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^6} \\
&\quad +\frac{15bd^4n\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4e^6} \\
&\quad -\frac{10bd^3n\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^6} \\
&\quad +\frac{15bd^2n\left(d+\frac{e}{x^{2/3}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{8e^6} \\
&\quad -\frac{3bdn\left(d+\frac{e}{x^{2/3}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{5e^6} \\
&\quad +\frac{bn\left(d+\frac{e}{x^{2/3}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{12e^6} \\
&\quad +\frac{bd^6n\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2e^6}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4x^4}
\end{aligned}$$

```
[Out] -15/8*b^2*d^4*n^2*(d+e/x^(2/3))^2/e^6+10/9*b^2*d^3*n^2*(d+e/x^(2/3))^3/e^6-
15/32*b^2*d^2*n^2*(d+e/x^(2/3))^4/e^6+3/25*b^2*d*n^2*(d+e/x^(2/3))^5/e^6-1/
72*b^2*n^2*(d+e/x^(2/3))^6/e^6+3*b^2*d^5*n^2/e^5/x^(2/3)-1/4*b^2*d^6*n^2*ln
(d+e/x^(2/3))^2/e^6-3*b*d^5*n*(d+e/x^(2/3))*(a+b*ln(c*(d+e/x^(2/3))^n))/e^6
+15/4*b*d^4*n*(d+e/x^(2/3))^2*(a+b*ln(c*(d+e/x^(2/3))^n))/e^6-10/3*b*d^3*n*
```

```
(d+e/x^(2/3))^3*(a+b*ln(c*(d+e/x^(2/3))^n))/e^6+15/8*b*d^2*n*(d+e/x^(2/3))^
4*(a+b*ln(c*(d+e/x^(2/3))^n))/e^6-3/5*b*d*n*(d+e/x^(2/3))^5*(a+b*ln(c*(d+e/
x^(2/3))^n))/e^6+1/12*b*n*(d+e/x^(2/3))^6*(a+b*ln(c*(d+e/x^(2/3))^n))/e^6+1
/2*b*d^6*n*ln(d+e/x^(2/3))*(a+b*ln(c*(d+e/x^(2/3))^n))/e^6-1/4*(a+b*ln(c*(d
+e/x^(2/3))^n))^2/x^4
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2445, 2458, 45, 2372, 12, 14, 2338}

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^5}\,dx = {} & \frac{bd^6n\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2e^6} \\ & -\frac{3bd^5n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^6} \\ & +\frac{15bd^4n\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4e^6} \\ & -\frac{10bd^3n\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^6} \\ & +\frac{15bd^2n\left(d+\frac{e}{x^{2/3}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{8e^6} \\ & -\frac{3bdn\left(d+\frac{e}{x^{2/3}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{5e^6} \\ & +\frac{bn\left(d+\frac{e}{x^{2/3}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{12e^6}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4x^4} \\ & -\frac{b^2d^6n^2\log^2\left(d+\frac{e}{x^{2/3}}\right)}{4e^6}+\frac{3b^2d^5n^2}{e^5x^{2/3}}-\frac{15b^2d^4n^2\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^6}+\frac{10b^2d^3n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^6} \\ & -\frac{15b^2d^2n^2\left(d+\frac{e}{x^{2/3}}\right)^4}{32e^6}+\frac{3b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^5}{25e^6}-\frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^6}{72e^6}\end{aligned}$$

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^5,x]
```

```
[Out] (-15*b^2*d^4*n^2*(d + e/x^(2/3))^2)/(8*e^6) + (10*b^2*d^3*n^2*(d + e/x^(2/3
))^3)/(9*e^6) - (15*b^2*d^2*n^2*(d + e/x^(2/3))^4)/(32*e^6) + (3*b^2*d*n^2*
(d + e/x^(2/3))^5)/(25*e^6) - (b^2*n^2*(d + e/x^(2/3))^6)/(72*e^6) + (3*b^2
*d^5*n^2)/(e^5*x^(2/3)) - (b^2*d^6*n^2*Log[d + e/x^(2/3)]^2)/(4*e^6) - (3*b
*d^5*n*(d + e/x^(2/3))*(a + b*Log[c*(d + e/x^(2/3))^n]))/e^6 + (15*b*d^4*n*
(d + e/x^(2/3))^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(4*e^6) - (10*b*d^3*n*(
d + e/x^(2/3))^3*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*e^6) + (15*b*d^2*n*(d
 + e/x^(2/3))^4*(a + b*Log[c*(d + e/x^(2/3))^n]))/(8*e^6) - (3*b*d*n*(d + e
/x^(2/3))^5*(a + b*Log[c*(d + e/x^(2/3))^n]))/(5*e^6) + (b*n*(d + e/x^(2/3)
)^6*(a + b*Log[c*(d + e/x^(2/3))^n]))/(12*e^6) + (b*d^6*n*Log[d + e/x^(2/3)
```

```
]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(2*e^6) - (a + b*Log[c*(d + e/x^(2/3))^
n])^2/(4*x^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2372

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Dist[a +
b*Log[c*x^n], u, x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; F
reeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q, 1]
 && EqQ[m, -1])
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
```

```
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(\frac{3}{2}\text{Subst}\left(\int x^5(a+b\log\left(c(d+ex)^n\right))^2\,dx,x,\frac{1}{x^{2/3}}\right)\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4x^4}+\frac{1}{2}(ben)\text{Subst}\left(\int\frac{x^6(a+b\log\left(c(d+ex)^n\right))}{d+ex}\,dx,x,\frac{1}{x^{2/3}}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4x^4}+\frac{1}{2}(bn)\text{Subst}\left(\int\frac{\left(-\frac{d}{e}+\frac{x}{e}\right)^6(a+b\log\left(cx^n\right))}{x}\,dx,x,d+\frac{e}{x^{2/3}}\right)\\
&= -\frac{3bd^5n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^6}\\
&\quad+\frac{15bd^4n\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4e^6}\\
&\quad-\frac{10bd^3n\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^6}\\
&\quad+\frac{15bd^2n\left(d+\frac{e}{x^{2/3}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{8e^6}\\
&\quad-\frac{3bdn\left(d+\frac{e}{x^{2/3}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{5e^6}\\
&\quad+\frac{bn\left(d+\frac{e}{x^{2/3}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{12e^6}\\
&\quad+\frac{bd^6n\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2e^6}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4x^4}\\
&\quad-\frac{1}{2}\left(b^2n^2\right)\text{Subst}\left(\int\frac{x(-360d^5+450d^4x-400d^3x^2+225d^2x^3-72dx^4+10x^5)+60d^6\log(x)}{60e^6x}\,dx,\right.
\end{aligned}
$$

$$= -\frac{3bd^5n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^6} + \frac{15bd^4n\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4e^6} - \frac{10bd^3n\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^6} + \frac{15bd^2n\left(d+\frac{e}{x^{2/3}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{8e^6} - \frac{3bdn\left(d+\frac{e}{x^{2/3}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{5e^6} + \frac{bn\left(d+\frac{e}{x^{2/3}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{12e^6} + \frac{bd^6n\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2e^6} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4x^4} - \frac{(b^2n^2)\,\mathrm{Subst}\left(\int \frac{x(-360d^5+450d^4x-400d^3x^2+225d^2x^3-72dx^4+10x^5)+60d^6\log(x)}{x}\,dx, x, d+\frac{e}{x^{2/3}}\right)}{120e^6}$$

$$= -\frac{3bd^5n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^6} + \frac{15bd^4n\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4e^6} - \frac{10bd^3n\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^6} + \frac{15bd^2n\left(d+\frac{e}{x^{2/3}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{8e^6} - \frac{3bdn\left(d+\frac{e}{x^{2/3}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{5e^6} + \frac{bn\left(d+\frac{e}{x^{2/3}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{12e^6} + \frac{bd^6n\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2e^6} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4x^4} - \frac{(b^2n^2)\,\mathrm{Subst}\left(\int \left(-360d^5+450d^4x-400d^3x^2+225d^2x^3-72dx^4+10x^5+\frac{60d^6\log(x)}{x}\right)dx, x, d

$$
\begin{aligned}
&= -\frac{15b^2d^4n^2\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^6} + \frac{10b^2d^3n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^6} \\
&\quad - \frac{15b^2d^2n^2\left(d+\frac{e}{x^{2/3}}\right)^4}{32e^6} + \frac{3b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^5}{25e^6} - \frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^6}{72e^6} \\
&\quad + \frac{3b^2d^5n^2}{e^5x^{2/3}} - \frac{3bd^5n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^6} \\
&\quad + \frac{15bd^4n\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4e^6} \\
&\quad - \frac{10bd^3n\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^6} \\
&\quad + \frac{15bd^2n\left(d+\frac{e}{x^{2/3}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{8e^6} \\
&\quad - \frac{3bdn\left(d+\frac{e}{x^{2/3}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{5e^6} \\
&\quad + \frac{bn\left(d+\frac{e}{x^{2/3}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{12e^6} \\
&\quad + \frac{bd^6n\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2e^6} \\
&\quad - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4x^4} - \frac{(b^2d^6n^2)\,\mathrm{Subst}\left(\int\frac{\log(x)}{x}\,dx, x, d+\frac{e}{x^{2/3}}\right)}{2e^6} \\
&= -\frac{15b^2d^4n^2\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^6} + \frac{10b^2d^3n^2\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^6} - \frac{15b^2d^2n^2\left(d+\frac{e}{x^{2/3}}\right)^4}{32e^6} \\
&\quad + \frac{3b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^5}{25e^6} - \frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^6}{72e^6} + \frac{3b^2d^5n^2}{e^5x^{2/3}} \\
&\quad - \frac{b^2d^6n^2\log^2\left(d+\frac{e}{x^{2/3}}\right)}{4e^6} - \frac{3bd^5n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^6} \\
&\quad + \frac{15bd^4n\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4e^6} \\
&\quad - \frac{10bd^3n\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^6} \\
&\quad + \frac{15bd^2n\left(d+\frac{e}{x^{2/3}}\right)^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{8e^6} \\
&\quad - \frac{3bdn\left(d+\frac{e}{x^{2/3}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{5e^6} \\
&\quad + \frac{bn\left(d+\frac{e}{x^{2/3}}\right)^6\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{12e^6} \\
&\quad + \frac{bd^6n\log\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2e^6} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4x^4}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.54 (sec) , antiderivative size = 988, normalized size of antiderivative = 2.05

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^5}\,dx =$$

$$-\frac{1800\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{bn\left(-600ae^6+100be^6n+720ade^5x^{2/3}-264bde^5nx^{2/3}-900ad^2e^4x^{4/3}+555bd^2e^4nx^{4/3}+1200ad^3e\right.}{}}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^5,x]
```

```
[Out] -1/7200*(1800*(a + b*Log[c*(d + e/x^(2/3))^n])^2 + (b*n*(-600*a*e^6 + 100*b
*e^6*n + 720*a*d*e^5*x^(2/3) - 264*b*d*e^5*n*x^(2/3) - 900*a*d^2*e^4*x^(4/3
) + 555*b*d^2*e^4*n*x^(4/3) + 1200*a*d^3*e^3*x^2 - 1140*b*d^3*e^3*n*x^2 - 1
800*a*d^4*e^2*x^(8/3) + 2610*b*d^4*e^2*n*x^(8/3) + 3600*a*d^5*e*x^(10/3) -
8820*b*d^5*e*n*x^(10/3) + 8820*b*d^6*n*x^4*Log[d + e/x^(2/3)] - 600*b*e^6*L
og[c*(d + e/x^(2/3))^n] + 720*b*d*e^5*x^(2/3)*Log[c*(d + e/x^(2/3))^n] - 90
0*b*d^2*e^4*x^(4/3)*Log[c*(d + e/x^(2/3))^n] + 1200*b*d^3*e^3*x^2*Log[c*(d
+ e/x^(2/3))^n] - 1800*b*d^4*e^2*x^(8/3)*Log[c*(d + e/x^(2/3))^n] + 3600*b*
d^5*e*x^(10/3)*Log[c*(d + e/x^(2/3))^n] - 3600*a*d^6*x^4*Log[Sqrt[e] - Sqrt
[-d]*x^(1/3)] - 3600*b*d^6*x^4*Log[c*(d + e/x^(2/3))^n]*Log[Sqrt[e] - Sqrt[
-d]*x^(1/3)] + 1800*b*d^6*n*x^4*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]^2 - 3600*a*
d^6*x^4*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] - 3600*b*d^6*x^4*Log[c*(d + e/x^(2/
3))^n]*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 1800*b*d^6*n*x^4*Log[Sqrt[e] + Sqr
t[-d]*x^(1/3)]^2 + 3600*b*d^6*n*x^4*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[1/2
 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] + 3600*b*d^6*n*x^4*Log[Sqrt[e] - Sqrt[-d
]*x^(1/3)]*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 3600*b*d^6*x^4*Log[c*(
d + e/x^(2/3))^n]*Log[-(e/(d*x^(2/3)))] - 7200*b*d^6*n*x^4*Log[Sqrt[e] + Sq
rt[-d]*x^(1/3)]*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])] - 7200*b*d^6*n*x^4*Log[S
qrt[e] - Sqrt[-d]*x^(1/3)]*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]] + 2400*a*d^6*x^4
*Log[x] - 3600*b*d^6*n*x^4*PolyLog[2, 1 + e/(d*x^(2/3))] - 7200*b*d^6*n*x^4
*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[e]] + 3600*b*d^6*n*x^4*PolyLog[2, 1
/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] + 3600*b*d^6*n*x^4*PolyLog[2, (1 + (Sq
rt[-d]*x^(1/3))/Sqrt[e])/2] - 7200*b*d^6*n*x^4*PolyLog[2, 1 + (Sqrt[-d]*x^(
1/3))/Sqrt[e]]))/e^6)/x^4
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{2}}{x^{5}}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^5,x)
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^5,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{2}}{x^{5}}\,dx =$$

$$-\frac{100\,b^{2}e^{6}n^{2}+1800\,b^{2}e^{6}\log\left(c\right)^{2}-600\,abe^{6}n+1800\,a^{2}e^{6}-60\,(19\,b^{2}d^{3}e^{3}n^{2}-20\,abd^{3}e^{3}n)x^{2}-1800\,(b^{2}d^{6}n^{2}x^{4}}{}$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^5,x, algorithm="fricas")
[Out] -1/7200*(100*b^2*e^6*n^2 + 1800*b^2*e^6*log(c)^2 - 600*a*b*e^6*n + 1800*a^2
*e^6 - 60*(19*b^2*d^3*e^3*n^2 - 20*a*b*d^3*e^3*n)*x^2 - 1800*(b^2*d^6*n^2*x
^4 - b^2*e^6*n^2)*log((d*x + e*x^(1/3))/x)^2 + 600*(2*b^2*d^3*e^3*n*x^2 - b
^2*e^6*n + 6*a*b*e^6)*log(c) + 60*(20*b^2*d^3*e^3*n^2*x^2 - 10*b^2*e^6*n^2
+ 60*a*b*e^6*n + 3*(49*b^2*d^6*n^2 - 20*a*b*d^6*n)*x^4 - 60*(b^2*d^6*n*x^4
- b^2*e^6*n)*log(c) - 6*(5*b^2*d^4*e^2*n^2*x^2 - 2*b^2*d*e^5*n^2)*x^(2/3) +
 15*(4*b^2*d^5*e*n^2*x^3 - b^2*d^2*e^4*n^2*x)*x^(1/3))*log((d*x + e*x^(1/3)
)/x) - 6*(44*b^2*d*e^5*n^2 - 120*a*b*d*e^5*n - 15*(29*b^2*d^4*e^2*n^2 - 20*
a*b*d^4*e^2*n)*x^2 + 60*(5*b^2*d^4*e^2*n*x^2 - 2*b^2*d*e^5*n)*log(c))*x^(2/
3) - 15*(12*(49*b^2*d^5*e*n^2 - 20*a*b*d^5*e*n)*x^3 - (37*b^2*d^2*e^4*n^2 -
 60*a*b*d^2*e^4*n)*x - 60*(4*b^2*d^5*e*n*x^3 - b^2*d^2*e^4*n*x)*log(c))*x^(
1/3))/(e^6*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{2}}{x^{5}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2/x**5,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^5}\,dx=\frac{1}{120}\,aben\left(\frac{60\,d^6\log\left(dx^{\frac{2}{3}}+e\right)}{e^7}-\frac{60\,d^6\log\left(x^{\frac{2}{3}}\right)}{e^7}-\frac{60\,d^5x^{\frac{10}{3}}-30\,d^4ex^{\frac{8}{3}}}{}\right.$$

$$+\frac{1}{7200}\left(60\,en\left(\frac{60\,d^6\log\left(dx^{\frac{2}{3}}+e\right)}{e^7}-\frac{60\,d^6\log\left(x^{\frac{2}{3}}\right)}{e^7}-\frac{60\,d^5x^{\frac{10}{3}}-30\,d^4ex^{\frac{8}{3}}+20\,d^3e^2x^2-15\,d^2e^3x^{\frac{4}{3}}+12}{e^6x^4}\right.\right.$$

$$-\frac{b^2\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)^2}{4\,x^4}-\frac{ab\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)}{2\,x^4}-\frac{a^2}{4\,x^4}$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/120*a*b*e*n*(60*d^6*log(d*x^(2/3) + e)/e^7 - 60*d^6*log(x^(2/3))/e^7 - (6
0*d^5*x^(10/3) - 30*d^4*e*x^(8/3) + 20*d^3*e^2*x^2 - 15*d^2*e^3*x^(4/3) + 1
2*d*e^4*x^(2/3) - 10*e^5)/(e^6*x^4)) + 1/7200*(60*e*n*(60*d^6*log(d*x^(2/3)
 + e)/e^7 - 60*d^6*log(x^(2/3))/e^7 - (60*d^5*x^(10/3) - 30*d^4*e*x^(8/3) +
 20*d^3*e^2*x^2 - 15*d^2*e^3*x^(4/3) + 12*d*e^4*x^(2/3) - 10*e^5)/(e^6*x^4)
)*log(c*(d + e/x^(2/3))^n) - (1800*d^6*x^4*log(d*x^(2/3) + e)^2 + 800*d^6*x
^4*log(x)^2 - 5880*d^6*x^4*log(x) - 8820*d^5*e*x^(10/3) + 2610*d^4*e^2*x^(8
/3) - 1140*d^3*e^3*x^2 + 555*d^2*e^4*x^(4/3) - 264*d*e^5*x^(2/3) + 100*e^6
- 60*(40*d^6*x^4*log(x) - 147*d^6*x^4)*log(d*x^(2/3) + e))*n^2/(e^6*x^4))*b
^2 - 1/4*b^2*log(c*(d + e/x^(2/3))^n)^2/x^4 - 1/2*a*b*log(c*(d + e/x^(2/3))
^n)/x^4 - 1/4*a^2/x^4
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^5}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2}{x^5}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2/x^5, x)
```

Mupad [B] (verification not implemented)

Time = 2.81 (sec) , antiderivative size = 440, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^5}\,dx = \frac{b^2\,d^6\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)^2}{4\,e^6} - \frac{b^2\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)^2}{4\,x^4} - \frac{b^2\,n^2}{72\,x^4} - \frac{a\,b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2\,x^4} - \frac{a^2}{4\,x^4} + \frac{a\,b\,n}{12\,x^4} + \frac{b^2\,n\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{12\,x^4} - \frac{49\,b^2\,d^6\,n^2\,\ln\left(d+\frac{e}{x^{2/3}}\right)}{40\,e^6} + \frac{19\,b^2\,d^3\,n^2}{120\,e^3\,x^2} - \frac{37\,b^2\,d^2\,n^2}{480\,e^2\,x^{8/3}} - \frac{29\,b^2\,d^4\,n^2}{80\,e^4\,x^{4/3}} + \frac{49\,b^2\,d^5\,n^2}{40\,e^5\,x^{2/3}} + \frac{11\,b^2\,d\,n^2}{300\,e\,x^{10/3}} - \frac{b^2\,d^3\,n\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{6\,e^3\,x^2} + \frac{b^2\,d^2\,n\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{8\,e^2\,x^{8/3}} + \frac{b^2\,d^4\,n\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{4\,e^4\,x^{4/3}} - \frac{b^2\,d^5\,n\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{2\,e^5\,x^{2/3}} - \frac{a\,b\,d\,n}{10\,e\,x^{10/3}} + \frac{a\,b\,d^6\,n\,\ln\left(d+\frac{e}{x^{2/3}}\right)}{2\,e^6} - \frac{b^2\,d\,n\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{10\,e\,x^{10/3}} - \frac{a\,b\,d^3\,n}{6\,e^3\,x^2} + \frac{a\,b\,d^2\,n}{8\,e^2\,x^{8/3}} + \frac{a\,b\,d^4\,n}{4\,e^4\,x^{4/3}} - \frac{a\,b\,d^5\,n}{2\,e^5\,x^{2/3}}$$

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^2/x^5,x)
```

```
[Out] (b^2*d^6*log(c*(d + e/x^(2/3))^n)^2)/(4*e^6) - (b^2*log(c*(d + e/x^(2/3))^n
)^2)/(4*x^4) - (b^2*n^2)/(72*x^4) - (a*b*log(c*(d + e/x^(2/3))^n))/(2*x^4)
- a^2/(4*x^4) + (a*b*n)/(12*x^4) + (b^2*n*log(c*(d + e/x^(2/3))^n))/(12*x^4
) - (49*b^2*d^6*n^2*log(d + e/x^(2/3)))/(40*e^6) + (19*b^2*d^3*n^2)/(120*e^
3*x^2) - (37*b^2*d^2*n^2)/(480*e^2*x^(8/3)) - (29*b^2*d^4*n^2)/(80*e^4*x^(4
/3)) + (49*b^2*d^5*n^2)/(40*e^5*x^(2/3)) + (11*b^2*d*n^2)/(300*e*x^(10/3))
- (b^2*d^3*n*log(c*(d + e/x^(2/3))^n))/(6*e^3*x^2) + (b^2*d^2*n*log(c*(d +
e/x^(2/3))^n))/(8*e^2*x^(8/3)) + (b^2*d^4*n*log(c*(d + e/x^(2/3))^n))/(4*e^
4*x^(4/3)) - (b^2*d^5*n*log(c*(d + e/x^(2/3))^n))/(2*e^5*x^(2/3)) - (a*b*d*
n)/(10*e*x^(10/3)) + (a*b*d^6*n*log(d + e/x^(2/3)))/(2*e^6) - (b^2*d*n*log(
c*(d + e/x^(2/3))^n))/(10*e*x^(10/3)) - (a*b*d^3*n)/(6*e^3*x^2) + (a*b*d^2*
n)/(8*e^2*x^(8/3)) + (a*b*d^4*n)/(4*e^4*x^(4/3)) - (a*b*d^5*n)/(2*e^5*x^(2/
3))
```

3.521 $\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 24, antiderivative size = 490

$$\begin{aligned}\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx &= -\frac{4abe^4n\sqrt[3]{x}}{3d^4}+\frac{568b^2e^4n^2\sqrt[3]{x}}{315d^4}-\frac{32b^2e^3n^2x}{105d^3}\\&+\frac{8b^2e^2n^2x^{5/3}}{105d^2}-\frac{1408b^2e^{9/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{315d^{9/2}}-\frac{4ib^2e^{9/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{3d^{9/2}}\\&+\frac{8b^2e^{9/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{3d^{9/2}}\\&-\frac{4b^2e^4n\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3d^4}+\frac{4be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{9d^3}\\&-\frac{4be^2nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{15d^2}+\frac{4benx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{21d}\\&+\frac{4be^{9/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3d^{9/2}}\\&+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-\frac{4ib^2e^{9/2}n^2\operatorname{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{3d^{9/2}}\end{aligned}$$

```
[Out] -4/3*a*b*e^4*n*x^(1/3)/d^4+568/315*b^2*e^4*n^2*x^(1/3)/d^4-32/105*b^2*e^3*n
^2*x/d^3+8/105*b^2*e^2*n^2*x^(5/3)/d^2-1408/315*b^2*e^(9/2)*n^2*arctan(x^(1
/3)*d^(1/2)/e^(1/2))/d^(9/2)-4/3*I*b^2*e^(9/2)*n^2*arctan(x^(1/3)*d^(1/2)/e
^(1/2))^2/d^(9/2)-4/3*b^2*e^4*n*x^(1/3)*ln(c*(d+e/x^(2/3))^n)/d^4+4/9*b*e^3
*n*x*(a+b*ln(c*(d+e/x^(2/3))^n))/d^3-4/15*b*e^2*n*x^(5/3)*(a+b*ln(c*(d+e/x^
(2/3))^n))/d^2+4/21*b*e*n*x^(7/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d+4/3*b*e^(9/
2)*n*arctan(x^(1/3)*d^(1/2)/e^(1/2))*(a+b*ln(c*(d+e/x^(2/3))^n))/d^(9/2)+1/
```

```
3*x^3*(a+b*ln(c*(d+e/x^(2/3))^n))^2+8/3*b^2*e^(9/2)*n^2*arctan(x^(1/3)*d^(1
/2)/e^(1/2))*ln(2-2*e^(1/2)/(-I*x^(1/3)*d^(1/2)+e^(1/2)))/d^(9/2)-4/3*I*b^2
*e^(9/2)*n^2*polylog(2,-1+2*e^(1/2)/(-I*x^(1/3)*d^(1/2)+e^(1/2)))/d^(9/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$, Rules used = {2508, 2507, 2526, 2498, 269, 211, 2505, 199, 327, 308, 2520, 12, 266, 6820, 5044, 4988, 2497}

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2dx=\frac{4be^{9/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3d^{9/2}}$$
$$+\frac{4be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{9d^3}-\frac{4be^2nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{15d^2}$$
$$+\frac{4benx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{21d}$$
$$+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-\frac{4abe^4n\sqrt[3]{x}}{3d^4}-\frac{4ib^2e^{9/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{3d^{9/2}}-\frac{1408b^2e^{9/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{315d^{9/2}}$$

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (-4*a*b*e^4*n*x^(1/3))/(3*d^4) + (568*b^2*e^4*n^2*x^(1/3))/(315*d^4) - (32*
b^2*e^3*n^2*x)/(105*d^3) + (8*b^2*e^2*n^2*x^(5/3))/(105*d^2) - (1408*b^2*e^
(9/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]])/(315*d^(9/2)) - (((4*I)/3)*b^2
*e^(9/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/d^(9/2) + (8*b^2*e^(9/2)*
n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt
[d]*x^(1/3))])/(3*d^(9/2)) - (4*b^2*e^4*n*x^(1/3)*Log[c*(d + e/x^(2/3))^n])
/(3*d^4) + (4*b*e^3*n*x*(a + b*Log[c*(d + e/x^(2/3))^n]))/(9*d^3) - (4*b*e^
2*n*x^(5/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(15*d^2) + (4*b*e*n*x^(7/3)*(
a + b*Log[c*(d + e/x^(2/3))^n]))/(21*d) + (4*b*e^(9/2)*n*ArcTan[(Sqrt[d]*x^
(1/3))/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*d^(9/2)) + (x^3*(a + b
*Log[c*(d + e/x^(2/3))^n])^2)/3 - (((4*I)/3)*b^2*e^(9/2)*n^2*PolyLog[2, -1
+ (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/d^(9/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
```

```
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2508

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{1}{3}(4ben)\text{Subst}\left(\int\frac{x^6\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)}{d+\frac{e}{x^2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 +\frac{1}{3}(4ben)\text{Subst}\left(\int\left(-\frac{e^3\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)}{d^4}+\frac{e^2x^2\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)}{d^3}-\frac{ex^4\left(a+b\log\right.}{}\right.\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + \frac{(4ben)\text{Subst}\left(\int x^6\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)\,dx, x, \sqrt[3]{x}\right)}{3d} - \frac{(4be^2n)\,\text{Subst}\left(\int x^4\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)\,dx, x, \sqrt[3]{x}\right)}{3d^2} + \frac{(4be^3n)\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)\,dx, x, \sqrt[3]{x}\right)}{3d^3} - \frac{(4be^4n)\,\text{Subst}\left(\int\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)\,dx, x, \sqrt[3]{x}\right)}{3d^4} + \frac{(4be^5n)\,\text{Subst}\left(\int\frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{e+dx^2}\,dx, x, \sqrt[3]{x}\right)}{3d^4}$$

$$= -\frac{4abe^4n\sqrt[3]{x}}{3d^4}+\frac{4be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{9d^3}-\frac{4be^2nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{15d^2} +\frac{4benx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{21d}+\frac{4be^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3d^{9/2}} +\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-\frac{(4b^2e^4n)\,\text{Subst}\left(\int\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\,dx, x, \sqrt[3]{x}\right)}{3d^4}+\frac{(8b^2e^2n^2)\,\text{Sub}}{}$$

$$= -\frac{4abe^4n\sqrt[3]{x}}{3d^4} - \frac{4b^2e^4n\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3d^4} + \frac{4be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{9d^3}$$
$$- \frac{4be^2nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{15d^2} + \frac{4benx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{21d}$$
$$+ \frac{4be^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3d^{9/2}}$$
$$+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + \frac{(8b^2e^2n^2)\,\text{Subst}\left(\int \frac{x^6}{e+dx^2}\,dx, x, \sqrt[3]{x}\right)}{21d} - \frac{(8b^2e^3n^2)\,\text{Subst}\left(\int \frac{x^4}{e+dx^2}\right.}{15d^2}$$

$$= -\frac{4abe^4n\sqrt[3]{x}}{3d^4} + \frac{8b^2e^4n^2\sqrt[3]{x}}{9d^4} - \frac{4b^2e^4n\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3d^4}$$
$$+ \frac{4be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{9d^3} - \frac{4be^2nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{15d^2}$$
$$+ \frac{4benx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{21d}$$
$$+ \frac{4be^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3d^{9/2}}$$
$$+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + \frac{(8b^2e^2n^2)\,\text{Subst}\left(\int \left(\frac{e^2}{d^3} - \frac{ex^2}{d^2} + \frac{x^4}{d} - \frac{e^3}{d^3(e+dx^2)}\right)dx, x, \sqrt[3]{x}\right)}{21d} - \frac{(8}{}$$

$$= -\frac{4abe^4n\sqrt[3]{x}}{3d^4} + \frac{568b^2e^4n^2\sqrt[3]{x}}{315d^4} - \frac{32b^2e^3n^2x}{105d^3} + \frac{8b^2e^2n^2x^{5/3}}{105d^2}$$
$$- \frac{32b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{9d^{9/2}} - \frac{4ib^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{3d^{9/2}}$$
$$- \frac{4b^2e^4n\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3d^4} + \frac{4be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{9d^3}$$
$$- \frac{4be^2nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{15d^2} + \frac{4benx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{21d}$$
$$+ \frac{4be^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3d^{9/2}}$$
$$+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + \frac{(8ib^2e^{9/2}n^2)\,\text{Subst}\left(\int \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{e}}\right)}{x\left(i+\frac{\sqrt{d}x}{\sqrt{e}}\right)}\,dx, x, \sqrt[3]{x}\right)}{3d^{9/2}} - \frac{(8b^2e^5n^2)\,\text{Subst}}{2}$$

$$
\begin{aligned}
&= -\frac{4abe^4n\sqrt[3]{x}}{3d^4} + \frac{568b^2e^4n^2\sqrt[3]{x}}{315d^4} - \frac{32b^2e^3n^2x}{105d^3} + \frac{8b^2e^2n^2x^{5/3}}{105d^2} \\
&\quad - \frac{1408b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{315d^{9/2}} - \frac{4ib^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{3d^{9/2}} \\
&\quad + \frac{8b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{3d^{9/2}} \\
&\quad - \frac{4b^2e^4n\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3d^4} + \frac{4be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{9d^3} \\
&\quad - \frac{4be^2nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{15d^2} + \frac{4benx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{21d} \\
&\quad + \frac{4be^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3d^{9/2}} \\
&\quad + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 - \frac{(8b^2e^4n^2)\operatorname{Subst}\left(\int\frac{\log\left(2-\frac{2}{1-\frac{i\sqrt{d}x}{\sqrt{e}}}\right)}{1+\frac{dx^2}{e}}dx,x,\sqrt[3]{x}\right)}{3d^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4abe^4n\sqrt[3]{x}}{3d^4} + \frac{568b^2e^4n^2\sqrt[3]{x}}{315d^4} - \frac{32b^2e^3n^2x}{105d^3} + \frac{8b^2e^2n^2x^{5/3}}{105d^2} \\
&\quad - \frac{1408b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{315d^{9/2}} - \frac{4ib^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{3d^{9/2}} \\
&\quad + \frac{8b^2e^{9/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{3d^{9/2}} \\
&\quad - \frac{4b^2e^4n\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{3d^4} + \frac{4be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{9d^3} \\
&\quad - \frac{4be^2nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{15d^2} + \frac{4benx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{21d} \\
&\quad + \frac{4be^{9/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3d^{9/2}} \\
&\quad + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 - \frac{4ib^2e^{9/2}n^2\operatorname{Li}_2\left(-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{3d^{9/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.43 (sec) , antiderivative size = 735, normalized size of antiderivative = 1.50

$$\int x^2\Bigg(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\Bigg)^2\,dx=\frac{1}{3}\left(x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-4ben\left(\frac{ae^3\sqrt[3]{x}}{d^4}-\frac{2be^{7/2}n\arctan\left(\frac{\sqrt{e}}{\sqrt{d}\sqrt[3]{x}}\right)}{d^{9/2}}-\right.\right.$$

[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]

[Out] (x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^2 - 4*b*e*n*((a*e^3*x^(1/3))/d^4 - (2*b*e^(7/2)*n*ArcTan[Sqrt[e]/(Sqrt[d]*x^(1/3))])/d^(9/2) - (2*b*e*n*x^(5/3)*Hypergeometric2F1[-5/2, 1, -3/2, -(e/(d*x^(2/3)))])/(35*d^2) + (2*b*e^2*n*x*Hypergeometric2F1[-3/2, 1, -1/2, -(e/(d*x^(2/3)))])/(15*d^3) - (2*b*e^3*n*x^(1/3)*Hypergeometric2F1[-1/2, 1, 1/2, -(e/(d*x^(2/3)))])/(3*d^4) + (b*e^3*x^(1/3)*Log[c*(d + e/x^(2/3))^n])/d^4 - (e^2*x*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*d^3) + (e*x^(5/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(5*d^2) - (x^(7/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(7*d) + (e^(7/2)*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)])/(2*(-d)^(9/2)) - (e^(7/2)*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)])/(2*(-d)^(9/2)) - (b*e^(7/2)*n*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 2*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 4*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]]) - 4*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[e]] + 2*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])]))/(4*(-d)^(9/2)) + (b*e^(7/2)*n*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 2*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] - 4*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])]) + 2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 4*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]]))/(4*(-d)^(9/2))))/3

Maple [F]

$$\int x^2\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)^2dx$$

[In] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)

[Out] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^2,x)

Fricas [F]

$$\int x^2 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = \int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^n\right) + a\right)^2 x^2 dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="fricas")
[Out] integral(b^2*x^2*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*x^2*log(c*((d*x +
 e*x^(1/3))/x)^n) + a^2*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = \text{Timed out}$$

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))**n))**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int x^2 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = \int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^n\right) + a\right)^2 x^2 dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\,dx=\int x^2\,\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)\right)^2\,dx$$

```
[In] int(x^2*(a + b*log(c*(d + e/x^(2/3))^n))^2,x)
[Out] int(x^2*(a + b*log(c*(d + e/x^(2/3))^n))^2, x)
```

3.522 $\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx$

Optimal result

Integrand size = 20, antiderivative size = 309

$$\begin{aligned}\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx &= \frac{4aben\sqrt[3]{x}}{d}+\frac{8b^2e^{3/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}}\\ &+\frac{4ib^2e^{3/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{d^{3/2}}-\frac{8b^2e^{3/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{d^{3/2}}\\ &+\frac{4b^2en\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{d}-\frac{4be^{3/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^{3/2}}\\ &+x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{4ib^2e^{3/2}n^2\operatorname{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{d^{3/2}}\end{aligned}$$

```
[Out] 4*a*b*e*n*x^(1/3)/d+8*b^2*e^(3/2)*n^2*arctan(x^(1/3)*d^(1/2)/e^(1/2))/d^(3/
2)+4*I*b^2*e^(3/2)*n^2*arctan(x^(1/3)*d^(1/2)/e^(1/2))^2/d^(3/2)+4*b^2*e*n*
x^(1/3)*ln(c*(d+e/x^(2/3))^n)/d-4*b*e^(3/2)*n*arctan(x^(1/3)*d^(1/2)/e^(1/2
))*(a+b*ln(c*(d+e/x^(2/3))^n))/d^(3/2)+x*(a+b*ln(c*(d+e/x^(2/3))^n))^2-8*b^
2*e^(3/2)*n^2*arctan(x^(1/3)*d^(1/2)/e^(1/2))*ln(2-2*e^(1/2)/(-I*x^(1/3)*d^
(1/2)+e^(1/2)))/d^(3/2)+4*I*b^2*e^(3/2)*n^2*polylog(2,-1+2*e^(1/2)/(-I*x^(1
/3)*d^(1/2)+e^(1/2)))/d^(3/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {2501, 2507, 2521, 2498, 269, 211, 2520, 12, 266, 6820, 5044, 4988, 2497}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = -\frac{4be^{3/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^{3/2}} +x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{4aben\sqrt[3]{x}}{d}+\frac{4ib^2e^{3/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{d^{3/2}}+\frac{8b^2e^{3/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}}-\frac{8b^2e}{}$$

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] (4*a*b*e*n*x^(1/3))/d + (8*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]
])/d^(3/2) + ((4*I)*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/d^
(3/2) - (8*b^2*e^(3/2)*n^2*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqr
t[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/d^(3/2) + (4*b^2*e*n*x^(1/3)*Log[c*(d
 + e/x^(2/3))^n])/d - (4*b*e^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a +
 b*Log[c*(d + e/x^(2/3))^n]))/d^(3/2) + x*(a + b*Log[c*(d + e/x^(2/3))^n])^
2 + ((4*I)*b^2*e^(3/2)*n^2*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]
*x^(1/3))])/d^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
```

```
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2\,dx, x, \sqrt[3]{x}\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+(4ben)\text{Subst}\left(\int\frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{d+\frac{e}{x^2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 +(4ben)\text{Subst}\left(\int\left(\frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{d}-\frac{e\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)}{d\,(e+dx^2)}\right)\,dx, x, \sqrt[3]{x}\right)$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{(4ben)\text{Subst}\left(\int\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)\,dx, x, \sqrt[3]{x}\right)}{d} - \frac{(4be^2n)\,\text{Subst}\left(\int\frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{e+dx^2}\,dx, x, \sqrt[3]{x}\right)}{d}$$

$$= \frac{4aben\sqrt[3]{x}}{d}-\frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^{3/2}} +x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{(4b^2en)\,\text{Subst}\left(\int\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\,dx, x, \sqrt[3]{x}\right)}{d}-\frac{(8b^2e^3n^2)\,\text{Subst}\left(\int\right.}{}$$

$$=\frac{4aben\sqrt[3]{x}}{d}+\frac{4b^2en\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{d}-\frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^{3/2}}+x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{(8b^2e^2n^2)\,\text{Subst}\left(\int\frac{1}{\left(d+\frac{e}{x^2}\right)x^2}\,dx,x,\sqrt[3]{x}\right)}{d}-\frac{(8b^2e^{5/2}n^2)\,\text{Subst}\left(\int\right.}{d^{3}}$$

$$=\frac{4aben\sqrt[3]{x}}{d}+\frac{4b^2en\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{d}-\frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^{3/2}}+x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{(8b^2e^2n^2)\,\text{Subst}\left(\int\frac{1}{e+dx^2}\,dx,x,\sqrt[3]{x}\right)}{d}-\frac{(8b^2e^{5/2}n^2)\,\text{Subst}\left(\int\frac{\tan^{-1}}{x(e+\right.}{d^{3/2}}$$

$$=\frac{4aben\sqrt[3]{x}}{d}+\frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}}+\frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{d^{3/2}}+\frac{4b^2en\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{d}-\frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^{3/2}}+x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-\frac{(8ib^2e^{3/2}n^2)\,\text{Subst}\left(\int\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{e}}\right)}{x\left(i+\frac{\sqrt{d}x}{\sqrt{e}}\right)}\,dx,x,\sqrt[3]{x}\right)}{d^{3/2}}$$

$$=\frac{4aben\sqrt[3]{x}}{d}+\frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}}+\frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{d^{3/2}}-\frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{d^{3/2}}+\frac{4b^2en\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{d}-\frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^{3/2}}+x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{(8b^2en^2)\,\text{Subst}\left(\int\frac{\log\left(2-\frac{2}{1-\frac{i\sqrt{d}x}{\sqrt{e}}}\right)}{1+\frac{dx^2}{e}}\,dx,x,\sqrt[3]{x}\right)}{d}$$

$$= \frac{4aben\sqrt[3]{x}}{d} + \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{d^{3/2}} + \frac{4ib^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{d^{3/2}}$$
$$- \frac{8b^2e^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{d^{3/2}} + \frac{4b^2en\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{d}$$
$$- \frac{4be^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^{3/2}}$$
$$+ x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 + \frac{4ib^2e^{3/2}n^2\text{Li}_2\left(-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{d^{3/2}}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.69

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2$$
$$+ben\left(\frac{4a\sqrt[3]{x}}{d} - \frac{8b\sqrt{e}n\arctan\left(\frac{\sqrt{e}}{\sqrt{d}\sqrt[3]{x}}\right)}{d^{3/2}} + \frac{4b\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{d} - \frac{2\sqrt{e}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{e}-\right.}{(-d)^{3/2}}\right.$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2,x]
```

```
[Out] x*(a + b*Log[c*(d + e/x^(2/3))^n])^2 + b*e*n*((4*a*x^(1/3))/d - (8*b*Sqrt[e
]*n*ArcTan[Sqrt[e]/(Sqrt[d]*x^(1/3))])/d^(3/2) + (4*b*x^(1/3)*Log[c*(d + e/
x^(2/3))^n])/d - (2*Sqrt[e]*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] -
Sqrt[-d]*x^(1/3)])/(-d)^(3/2) + (2*Sqrt[e]*(a + b*Log[c*(d + e/x^(2/3))^n])
*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)])/(-d)^(3/2) + (b*Sqrt[e]*n*(Log[Sqrt[e] -
Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 2*Log[(1 + (Sqrt[-d]*x
^(1/3))/Sqrt[e])/2] - 4*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]]) - 4*PolyLog[2, 1 -
 (Sqrt[-d]*x^(1/3))/Sqrt[e]] + 2*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqr
t[e])]))/(-d)^(3/2) + (b*d*Sqrt[e]*n*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*(Log[
Sqrt[e] + Sqrt[-d]*x^(1/3)] + 2*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])] -
 4*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])]) + 2*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3
))/Sqrt[e])/2] - 4*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]]))/(-d)^(5/2))
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{2}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^2,x)
```

Fricas [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{2}\,dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*log(c*((d*x + e*x^(1/
3))/x)^n) + a^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = \int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2 dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx = \int \left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2 dx$$

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^2,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))^2, x)
```

3.523 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 361

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx = & -\frac{8b^2n^2}{9x}+\frac{32b^2dn^2}{3e\sqrt[3]{x}} \\ & +\frac{32b^2d^{3/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{3/2}}+\frac{4ib^2d^{3/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{e^{3/2}} \\ & -\frac{8b^2d^{3/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}} \\ & +\frac{4bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x}-\frac{4bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} \\ & -\frac{4bd^{3/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{3/2}} \\ & -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}+\frac{4ib^2d^{3/2}n^2\operatorname{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}}\end{aligned}$$

```
[Out] -8/9*b^2*n^2/x+32/3*b^2*d*n^2/e/x^(1/3)+32/3*b^2*d^(3/2)*n^2*arctan(x^(1/3)
*d^(1/2)/e^(1/2))/e^(3/2)+4*I*b^2*d^(3/2)*n^2*arctan(x^(1/3)*d^(1/2)/e^(1/2
))^2/e^(3/2)+4/3*b*n*(a+b*ln(c*(d+e/x^(2/3))^n))/x-4*b*d*n*(a+b*ln(c*(d+e/x
^(2/3))^n))/e/x^(1/3)-4*b*d^(3/2)*n*arctan(x^(1/3)*d^(1/2)/e^(1/2))*(a+b*ln
(c*(d+e/x^(2/3))^n))/e^(3/2)-(a+b*ln(c*(d+e/x^(2/3))^n))^2/x-8*b^2*d^(3/2)*
n^2*arctan(x^(1/3)*d^(1/2)/e^(1/2))*ln(2-2*e^(1/2)/(-I*x^(1/3)*d^(1/2)+e^(1
/2)))/e^(3/2)+4*I*b^2*d^(3/2)*n^2*polylog(2,-1+2*e^(1/2)/(-I*x^(1/3)*d^(1/2
)+e^(1/2)))/e^(3/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {2508, 2507, 2526, 2505, 269, 331, 211, 2520, 12, 266, 6820, 5044, 4988, 2497}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx = -\frac{4bd^{3/2}n\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{3/2}} - \frac{4bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} + \frac{4bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} + \frac{4ib^2d^{3/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{e^{3/2}} + \frac{32b^2d^{3/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{3/2}} - \frac{8b^2d^{3/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}} + \frac{4ib^2d^{3/2}n^2\operatorname{PolyLog}\left(2,\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}-1\right)}{e^{3/2}} + \frac{32b^2dn^2}{3e\sqrt[3]{x}} - \frac{8b^2n^2}{9x}$$

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^2,x]
```

```
[Out] (-8*b^2*n^2)/(9*x) + (32*b^2*d*n^2)/(3*e*x^(1/3)) + (32*b^2*d^(3/2)*n^2*Arc
Tan[(Sqrt[d]*x^(1/3))/Sqrt[e]])/(3*e^(3/2)) + ((4*I)*b^2*d^(3/2)*n^2*ArcTan
[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/e^(3/2) - (8*b^2*d^(3/2)*n^2*ArcTan[(Sqrt[d]
*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/e^(3
/2) + (4*b*n*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*x) - (4*b*d*n*(a + b*Log[
c*(d + e/x^(2/3))^n]))/(e*x^(1/3)) - (4*b*d^(3/2)*n*ArcTan[(Sqrt[d]*x^(1/3)
)/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/e^(3/2) - (a + b*Log[c*(d + e/
x^(2/3))^n])^2/x + ((4*I)*b^2*d^(3/2)*n^2*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt
[e] - I*Sqrt[d]*x^(1/3))])/e^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2508

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*(x_)^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1)
- 1)*(a + b*Log[c*(d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a,
b, c, d, e, m, p, q}, x] && FractionQ[n]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - (4ben)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{\left(d+\frac{e}{x^2}\right)x^6}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - (4ben)\text{Subst}\left(\int\left(\frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{ex^4} - \frac{d\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)}{e^2x^2} + \frac{d^2\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)}{e^2\left(e+dx^2\right)}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - (4bn)\text{Subst}\left(\int\frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{x^4}\,dx, x, \sqrt[3]{x}\right) + \frac{(4bdn)\text{Subst}\left(\int\frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{x^2}\,dx, x, \sqrt[3]{x}\right)}{e} - \frac{\left(4bd^2n\right)\text{Subst}\left(\int\frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{e+dx^2}\,dx, x, \sqrt[3]{x}\right)}{e}$$

$$= \frac{4bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} - \frac{4bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} - \frac{4bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{3/2}} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - \left(8b^2dn^2\right)\text{Subst}\left(\int\frac{1}{\left(d+\frac{e}{x^2}\right)x^4}\,dx, x, \sqrt[3]{x}\right) - \left(8b^2d^2n^2\right)\text{Subst}\left(\int\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{e}}\right)}{\sqrt{d}\sqrt{e}\left(d+\frac{e}{x^2}\right)x^3}\,dx, x, \sqrt[3]{x}\right.$$

$$= \frac{4bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} - \frac{4bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} - \frac{4bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{3/2}} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - \left(8b^2dn^2\right)\text{Subst}\left(\int\frac{1}{x^2\left(e+dx^2\right)}\,dx, x, \sqrt[3]{x}\right) - \frac{\left(8b^2d^{3/2}n^2\right)\text{Subst}\left(\int\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{e}}\right)}{\left(d+\frac{e}{x^2}\right)x^3}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{e}} + \frac{1}{3}(8$$

$$= -\frac{8b^2n^2}{9x} + \frac{8b^2dn^2}{e\sqrt[3]{x}} + \frac{4bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} - \frac{4bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} - \frac{4bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{3/2}} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - \frac{1}{3}\left(8b^2dn^2\right)\text{Subst}\left(\int\frac{1}{x^2\left(e+dx^2\right)}\,dx, x, \sqrt[3]{x}\right) + \frac{\left(8b^2d^2n^2\right)\text{Subst}\left(\int\frac{1}{e+dx^2}\,dx, x, \sqrt[3]{x}\right)}{e} - \frac{\left(8b^2d^{3/2}n\right.}{}$$

$$= -\frac{8b^2n^2}{9x} + \frac{32b^2dn^2}{3e\sqrt[3]{x}} + \frac{8b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{e^{3/2}} + \frac{4ib^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{e^{3/2}} + \frac{4bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} - \frac{4bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} - \frac{4bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{3/2}} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - \frac{(8ib^2d^{3/2}n^2)\,\text{Subst}\left(\int\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{e}}\right)}{x\left(i+\frac{\sqrt{d}x}{\sqrt{e}}\right)}\,dx,x,\sqrt[3]{x}\right)}{e^{3/2}} + \frac{(8b^2d^2n^2)\,\text{Subst}\left(\int\frac{1}{e+dx^2}\,dx,x,\sqrt[3]{x}\right)}{3e}$$

$$= -\frac{8b^2n^2}{9x} + \frac{32b^2dn^2}{3e\sqrt[3]{x}} + \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{3/2}} + \frac{4ib^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{e^{3/2}} - \frac{8b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}} + \frac{4bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} - \frac{4bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} - \frac{4bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{3/2}} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} + \frac{(8b^2d^2n^2)\,\text{Subst}\left(\int\frac{\log\left(2-\frac{2}{1-\frac{i\sqrt{d}x}{\sqrt{e}}}\right)}{1+\frac{dx^2}{e}}\,dx,x,\sqrt[3]{x}\right)}{e^2}$$

$$= -\frac{8b^2n^2}{9x} + \frac{32b^2dn^2}{3e\sqrt[3]{x}} + \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{3/2}} + \frac{4ib^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{e^{3/2}} - \frac{8b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}} + \frac{4bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} - \frac{4bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} - \frac{4bd^{3/2}n\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{3/2}} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} + \frac{4ib^2d^{3/2}n^2\text{Li}_2\left(-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}}$$

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.69

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx =$$

$$-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2+\frac{bn\left(36ad\sqrt{e}x^{2/3}-72bd\sqrt{e}nx^{2/3}+72bd^{3/2}nx\arctan\left(\frac{\sqrt{e}}{\sqrt{d}\sqrt[3]{x}}\right)+8bn\left(\sqrt{e}(e-3dx^{2/3})+3d^{3/2}x\arctan\left(\right.\right.\right.}{}}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^2/x^2,x]
```

```
[Out] -1/9*(9*(a + b*Log[c*(d + e/x^(2/3))^n])^2 + (b*n*(36*a*d*Sqrt[e]*x^(2/3) -
 72*b*d*Sqrt[e]*n*x^(2/3) + 72*b*d^(3/2)*n*x*ArcTan[Sqrt[e]/(Sqrt[d]*x^(1/3
))] + 8*b*n*(Sqrt[e]*(e - 3*d*x^(2/3)) + 3*d^(3/2)*x*ArcTan[Sqrt[e]/(Sqrt[d
]*x^(1/3))]) + 36*b*d*Sqrt[e]*x^(2/3)*Log[c*(d + e/x^(2/3))^n] - 12*e^(3/2)
*(a + b*Log[c*(d + e/x^(2/3))^n]) + 18*Sqrt[-d]*d*x*(a + b*Log[c*(d + e/x^(
2/3))^n])*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 18*(-d)^(3/2)*x*(a + b*Log[c*(d
 + e/x^(2/3))^n])*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 9*b*(-d)^(3/2)*n*x*(Log
[Sqrt[e] - Sqrt[-d]*x^(1/3)]*(Log[Sqrt[e] - Sqrt[-d]*x^(1/3)] + 2*Log[(1 +
(Sqrt[-d]*x^(1/3))/Sqrt[e])/2] - 4*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]]) - 4*Pol
yLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[e]] + 2*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1
/3))/(2*Sqrt[e])]) + 9*b*Sqrt[-d]*d*n*x*(Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*(L
og[Sqrt[e] + Sqrt[-d]*x^(1/3)] + 2*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])
] - 4*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])]) + 2*PolyLog[2, (1 + (Sqrt[-d]*x^(
1/3))/Sqrt[e])/2] - 4*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3))/Sqrt[e]])))/e^(3/2)
)/x
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)^2}{x^2}\,dx$$

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^2/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^2,x, algorithm="fricas")
[Out] integral((b^2*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 2*a*b*log(c*((d*x + e*x^(1
/3))/x)^n) + a^2)/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**2/x**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^2/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x^2}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^2/x^2,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))^2/x^2, x)
```

3.524 $\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 773

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\frac{71b^3e^5n^3x^{2/3}}{80d^5}$$
$$-\frac{3b^3e^4n^3x^{4/3}}{20d^4}+\frac{b^3e^3n^3x^2}{40d^3}-\frac{71b^3e^6n^3\log\left(d+\frac{e}{x^{2/3}}\right)}{80d^6}$$
$$-\frac{77b^2e^5n^2\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{40d^6}+\frac{47b^2e^4n^2x^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{80d^4}$$
$$-\frac{9b^2e^3n^2x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{40d^3}+\frac{3b^2e^2n^2x^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{40d^2}$$
$$-\frac{77b^2e^6n^2\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{40d^6}$$
$$+\frac{3be^5n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4d^6}-\frac{3be^4nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{8d^4}$$
$$+\frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4d^3}-\frac{3be^2nx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{16d^2}$$
$$+\frac{3benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{20d}+\frac{3be^6n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4d^6}$$
$$+\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3-\frac{3b^2e^6n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(-\frac{e}{dx^{2/3}}\right)}{2d^6}-\frac{15b^3e^6n^3\log(x)}{8d^6}+\frac{77b^3e^6}{}$$

```
[Out] 71/80*b^3*e^5*n^3*x^(2/3)/d^5-3/20*b^3*e^4*n^3*x^(4/3)/d^4+1/40*b^3*e^3*n^3
*x^2/d^3-71/80*b^3*e^6*n^3*ln(d+e/x^(2/3))/d^6-77/40*b^2*e^5*n^2*(d+e/x^(2/
3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^6+47/80*b^2*e^4*n^2*x^(4/3)*(a+b*
```

```
ln(c*(d+e/x^(2/3))^n))/d^4-9/40*b^2*e^3*n^2*x^2*(a+b*ln(c*(d+e/x^(2/3))^n))
/d^3+3/40*b^2*e^2*n^2*x^(8/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^2-77/40*b^2*e^6
*n^2*ln(1-d/(d+e/x^(2/3)))*(a+b*ln(c*(d+e/x^(2/3))^n))/d^6+3/4*b*e^5*n*(d+e
/x^(2/3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d^6-3/8*b*e^4*n*x^(4/3)*(a+
b*ln(c*(d+e/x^(2/3))^n))^2/d^4+1/4*b*e^3*n*x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^
2/d^3-3/16*b*e^2*n*x^(8/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d^2+3/20*b*e*n*x^(
10/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d+3/4*b*e^6*n*ln(1-d/(d+e/x^(2/3)))*(a+
b*ln(c*(d+e/x^(2/3))^n))^2/d^6+1/4*x^4*(a+b*ln(c*(d+e/x^(2/3))^n))^3-3/2*b^
2*e^6*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))*ln(-e/d/x^(2/3))/d^6-15/8*b^3*e^6*n^3
*ln(x)/d^6+77/40*b^3*e^6*n^3*polylog(2,d/(d+e/x^(2/3)))/d^6-3/2*b^2*e^6*n^2
*(a+b*ln(c*(d+e/x^(2/3))^n))*polylog(2,d/(d+e/x^(2/3)))/d^6-3/2*b^3*e^6*n^3
*polylog(2,1+e/d/x^(2/3))/d^6-3/2*b^3*e^6*n^3*polylog(3,d/(d+e/x^(2/3)))/d^
6
```

Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 773, normalized size of antiderivative = 1.00, number of steps used = 62, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules

used = {2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31, 46}

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx =$$
$$-\frac{3b^2e^6n^2\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^6}$$
$$-\frac{77b^2e^6n^2\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{40d^6}$$
$$-\frac{3b^2e^6n^2\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^6}$$
$$-\frac{77b^2e^5n^2x^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{40d^6}$$
$$+\frac{47b^2e^4n^2x^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{80d^4}-\frac{9b^2e^3n^2x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{40d^3}$$
$$+\frac{3b^2e^2n^2x^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{40d^2}+\frac{3be^6n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4d^6}$$
$$+\frac{3be^5nx^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4d^6}$$
$$-\frac{3be^4nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{8d^4}+\frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4d^3}$$
$$-\frac{3be^2nx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{16d^2}+\frac{3benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{20d}$$
$$+\frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3+\frac{77b^3e^6n^3\operatorname{PolyLog}\left(2,\frac{d}{d+\frac{e}{x^{2/3}}}\right)}{40d^6}-\frac{3b^3e^6n^3\operatorname{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)}{2d^6}-\frac{3b^3e^6n^3}{}$$

```
[In] Int[x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
[Out] (71*b^3*e^5*n^3*x^(2/3))/(80*d^5) - (3*b^3*e^4*n^3*x^(4/3))/(20*d^4) + (b^3
*e^3*n^3*x^2)/(40*d^3) - (71*b^3*e^6*n^3*Log[d + e/x^(2/3)])/(80*d^6) - (77
*b^2*e^5*n^2*(d + e/x^(2/3))*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(40*
d^6) + (47*b^2*e^4*n^2*x^(4/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(80*d^4) -
 (9*b^2*e^3*n^2*x^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(40*d^3) + (3*b^2*e^2
*n^2*x^(8/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(40*d^2) - (77*b^2*e^6*n^2*L
og[1 - d/(d + e/x^(2/3))]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(40*d^6) + (3*b
*e^5*n*(d + e/x^(2/3))*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(4*d^6)
- (3*b*e^4*n*x^(4/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(8*d^4) + (b*e^3*n
*x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(4*d^3) - (3*b*e^2*n*x^(8/3)*(a +
b*Log[c*(d + e/x^(2/3))^n])^2)/(16*d^2) + (3*b*e*n*x^(10/3)*(a + b*Log[c*(d
 + e/x^(2/3))^n])^2)/(20*d) + (3*b*e^6*n*Log[1 - d/(d + e/x^(2/3))]*(a + b*
Log[c*(d + e/x^(2/3))^n])^2)/(4*d^6) + (x^4*(a + b*Log[c*(d + e/x^(2/3))^n]
```

```
)^3)/4 - (3*b^2*e^6*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[-(e/(d*x^(2/3)
))])/(2*d^6) - (15*b^3*e^6*n^3*Log[x])/(8*d^6) + (77*b^3*e^6*n^3*PolyLog[2,
 d/(d + e/x^(2/3))])/(40*d^6) - (3*b^2*e^6*n^2*(a + b*Log[c*(d + e/x^(2/3))
^n])*PolyLog[2, d/(d + e/x^(2/3))])/(2*d^6) - (3*b^3*e^6*n^3*PolyLog[2, 1 +
 e/(d*x^(2/3))])/(2*d^6) - (3*b^3*e^6*n^3*PolyLog[3, d/(d + e/x^(2/3))])/(2
*d^6)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
```

```
NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\left(\frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^7}\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{1}{4}(3ben)\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^2}{x^6(d+ex)}\,dx, x, \frac{1}{x^{2/3}}\right)$$

$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{1}{4}(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+\frac{e}{x^{2/3}}\right)$$

$$= \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{(3bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^6}\,dx, x, d+\frac{e}{x^{2/3}}\right)}{4d} + \frac{(3ben)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+\frac{e}{x^{2/3}}\right)}{4d}$$

$$= \frac{3benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{20d} + \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{(3ben)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^5}\,dx, x, d+\frac{e}{x^{2/3}}\right)}{4d^2} - \frac{(3be^2n)\,\text{Subst}}{}$$

$$= -\frac{3be^2nx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{16d^2} + \frac{3benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{20d} + \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{(3be^2n)\,\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^4}\,dx, x, d+\frac{e}{x^{2/3}}\right)}{4d^3} + \frac{(3be^3n)\,\text{Subs}}{}$$

$$= \frac{3b^2e^2n^2x^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{40d^2} + \frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4d^3} - \frac{3be^2nx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{16d^2} + \frac{3benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{20d} + \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{(3be^3n)\,\mathrm{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}dx, x, d+\frac{e}{x^{2/3}}\right)}{4d^4} - \frac{(3be^4n)\,\mathrm{Subst}}{}$$

$$= -\frac{9b^2e^3n^2x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{40d^3} + \frac{3b^2e^2n^2x^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{40d^2} - \frac{3be^4nx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{8d^4} + \frac{be^3nx^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4d^3} - \frac{3be^2nx^{8/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{16d^2} + \frac{3benx^{10/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{20d} + \frac{1}{4}x^4\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{(3be^4n)\,\mathrm{Subst}\left(\int \frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}dx, x, d+\frac{e}{x^{2/3}}\right)}{4d^5} + \frac{(3be^5n)\,\mathrm{Subst}}{}$$

$= \text{Too large to display}$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 18.53 (sec) , antiderivative size = 5557, normalized size of antiderivative = 7.19

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = \text{Result too large to show}$$

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
[Out] Result too large to show
```

Maple [F]

$$\int x^3\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)^3 dx$$

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
[Out] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

Fricas [F]

$$\int x^3 \left(a + b \log\left(c \left(d + \frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = \int \left(b \log\left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^n\right) + a\right)^3 x^3 dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*x^3*log(c*((d*x
 + e*x^(1/3))/x)^n)^2 + 3*a^2*b*x^3*log(c*((d*x + e*x^(1/3))/x)^n) + a^3*x^
3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(a + b \log\left(c \left(d + \frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = \text{Timed out}$$

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))**n))**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x^3 \left(a + b \log\left(c \left(d + \frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = \int \left(b \log\left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^n\right) + a\right)^3 x^3 dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4*log((d*x^(2/3) + e)^n)^3 - integrate(-1/2*(2*(b^3*d*log(c)^3 +
3*a*b^2*d*log(c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x^4 + 2*(b^3*e*log(c)^3 + 3*
a*b^2*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x^(10/3) - 16*(b^3*d*x^4 + b^3
*e*x^(10/3))*log(x^(1/3*n))^3 - (b^3*d*n*x^4 - 6*(b^3*d*log(c) + a*b^2*d)*x
^4 - 6*(b^3*e*log(c) + a*b^2*e)*x^(10/3) + 12*(b^3*d*x^4 + b^3*e*x^(10/3))*
log(x^(1/3*n)))*log((d*x^(2/3) + e)^n)^2 + 24*((b^3*d*log(c) + a*b^2*d)*x^4
 + (b^3*e*log(c) + a*b^2*e)*x^(10/3))*log(x^(1/3*n))^2 + 6*((b^3*d*log(c)^2
 + 2*a*b^2*d*log(c) + a^2*b*d)*x^4 + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a
^2*b*e)*x^(10/3) + 4*(b^3*d*x^4 + b^3*e*x^(10/3))*log(x^(1/3*n))^2 - 4*((b^
3*d*log(c) + a*b^2*d)*x^4 + (b^3*e*log(c) + a*b^2*e)*x^(10/3))*log(x^(1/3*n
)))*log((d*x^(2/3) + e)^n) - 12*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b
*d)*x^4 + (b^3*e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(10/3))*log(x^(1/
3*n)))/(d*x + e*x^(1/3)), x)
```

Giac [F]

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3x^3\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\int x^3\left(a+b\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$$

```
[In] int(x^3*(a + b*log(c*(d + e/x^(2/3))^n))^3,x)
[Out] int(x^3*(a + b*log(c*(d + e/x^(2/3))^n))^3, x)
```

3.525 $\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 451

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\frac{3b^2e^2n^2\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3}$$
$$+\frac{3b^2e^3n^2\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3}$$
$$-\frac{3be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2d^3}+\frac{3benx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4d}$$
$$-\frac{3be^3n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2d^3}$$
$$+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3+\frac{3b^2e^3n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(-\frac{e}{dx^{2/3}}\right)}{d^3}+\frac{b^3e^3n^3\log(x)}{d^3}-\frac{3b^3e^3n^3}{}$$

```
[Out] 3/2*b^2*e^2*n^2*(d+e/x^(2/3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))/d^3+3/2*b
^2*e^3*n^2*ln(1-d/(d+e/x^(2/3)))*(a+b*ln(c*(d+e/x^(2/3))^n))/d^3-3/2*b*e^2*
n*(d+e/x^(2/3))*x^(2/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d^3+3/4*b*e*n*x^(4/3)
*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d-3/2*b*e^3*n*ln(1-d/(d+e/x^(2/3)))*(a+b*ln(
c*(d+e/x^(2/3))^n))^2/d^3+1/2*x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^3+3*b^2*e^3*n
^2*(a+b*ln(c*(d+e/x^(2/3))^n))*ln(-e/d/x^(2/3))/d^3+b^3*e^3*n^3*ln(x)/d^3-3
/2*b^3*e^3*n^3*polylog(2,d/(d+e/x^(2/3)))/d^3+3*b^2*e^3*n^2*(a+b*ln(c*(d+e/
x^(2/3))^n))*polylog(2,d/(d+e/x^(2/3)))/d^3+3*b^3*e^3*n^3*polylog(2,1+e/d/x
^(2/3))/d^3+3*b^3*e^3*n^3*polylog(3,d/(d+e/x^(2/3)))/d^3
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {2504, 2445, 2458, 2389, 2379, 2421, 6724, 2355, 2354, 2438, 2356, 2351, 31}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\frac{3b^2e^3n^2\,\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}+\frac{3b^2e^3n^2\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3}+\frac{3b^2e^3n^2\log\left(-\frac{e}{dx^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{d^3}+\frac{3b^2e^2n^2x^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3}-\frac{3be^3n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2d^3}-\frac{3be^2nx^{2/3}\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2d^3}+\frac{3benx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4d}+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3-\frac{3b^3e^3n^3\,\text{PolyLog}\left(2,\frac{d}{d+\frac{e}{x^{2/3}}}\right)}{2d^3}+\frac{3b^3e^3n^3\,\text{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)}{d^3}+\frac{3b^3e^3n^3\,\text{P}}{}$$

```
[In] Int[x*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (3*b^2*e^2*n^2*(d + e/x^(2/3))*x^(2/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(2
*d^3) + (3*b^2*e^3*n^2*Log[1 - d/(d + e/x^(2/3))]*(a + b*Log[c*(d + e/x^(2/
3))^n]))/(2*d^3) - (3*b*e^2*n*(d + e/x^(2/3))*x^(2/3)*(a + b*Log[c*(d + e/x
^(2/3))^n])^2)/(2*d^3) + (3*b*e*n*x^(4/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^
2)/(4*d) - (3*b*e^3*n*Log[1 - d/(d + e/x^(2/3))]*(a + b*Log[c*(d + e/x^(2/3
))^n])^2)/(2*d^3) + (x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^3)/2 + (3*b^2*e^3
*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[-(e/(d*x^(2/3)))])/d^3 + (b^3*e^3
*n^3*Log[x])/d^3 - (3*b^3*e^3*n^3*PolyLog[2, d/(d + e/x^(2/3))])/(2*d^3) +
(3*b^2*e^3*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*PolyLog[2, d/(d + e/x^(2/3)
)])/d^3 + (3*b^3*e^3*n^3*PolyLog[2, 1 + e/(d*x^(2/3))])/d^3 + (3*b^3*e^3*n^
3*PolyLog[3, d/(d + e/x^(2/3))])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Dist[b*
(n/d), Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 2354

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Dist[b*n*(p/e),
 Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2355

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Dist[b*n*(p/d),
Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n
, p}, x] && GtQ[p, 0]
```

Rule 2356

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Dist[b*n*(p/(e*(q + 1))), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 2379

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Dist[b*n*(p/(d*r)), Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

Rule 2389

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x)
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2445

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[(f + g*x)^(q + 1)*((a + b*Log[c*(d + e*x)^
n])^p/(g*(q + 1))), x] - Dist[b*e*n*(p/(g*(q + 1))), Int[(f + g*x)^(q + 1)*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && Int
egersQ[2*p, 2*q] && ( !IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2458

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\left(\frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x^4}\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3-\frac{1}{2}(3ben)\text{Subst}\left(\int\frac{(a+b\log(c(d+ex)^n))^2}{x^3(d+ex)}\,dx,x,\frac{1}{x^{2/3}}\right)$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3-\frac{1}{2}(3bn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx,x,d+\frac{e}{x^{2/3}}\right)$$

$$=\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3-\frac{(3bn)\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^3}\,dx,x,d+\frac{e}{x^{2/3}}\right)}{2d}$$
$$+\frac{(3ben)\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2}{x\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+\frac{e}{x^{2/3}}\right)}{2d}$$

$$=\frac{3benx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4d}$$
$$+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3+\frac{(3ben)\text{Subst}\left(\int\frac{(a+b\log(cx^n))^2}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+\frac{e}{x^{2/3}}\right)}{2d^2}-\frac{(3be^2n)\,\text{Subst}}{}$$

$$=-\frac{3be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2d^3}$$
$$+\frac{3benx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4d}$$
$$-\frac{3be^3n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2d^3}$$
$$+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3-\frac{(3b^2en^2)\,\text{Subst}\left(\int\frac{a+b\log(cx^n)}{\left(-\frac{d}{e}+\frac{x}{e}\right)^2}\,dx,x,d+\frac{e}{x^{2/3}}\right)}{2d^2}+\frac{(3b^2e^2n^2)\,\text{Sub}}{}$$

$$= \frac{3b^2e^2n^2\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3} + \frac{3b^2e^3n^2\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3} - \frac{3be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2d^3} + \frac{3benx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4d} - \frac{3be^3n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2d^3}$$

$$+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{3b^2e^3n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(-\frac{e}{dx^{2/3}}\right)}{d^3} + \frac{3b^2e^3n^2\left(a+b\right.}{}$$

$$= \frac{3b^2e^2n^2\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3} + \frac{3b^2e^3n^2\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{2d^3} - \frac{3be^2n\left(d+\frac{e}{x^{2/3}}\right)x^{2/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2d^3} + \frac{3benx^{4/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4d} - \frac{3be^3n\log\left(1-\frac{d}{d+\frac{e}{x^{2/3}}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2d^3}$$

$$+\frac{1}{2}x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{3b^2e^3n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(-\frac{e}{dx^{2/3}}\right)}{d^3} + \frac{b^3e^3n^3\log(x)}{d^3}$$

Mathematica [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = \int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx$$

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
[Out] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^n])^3, x]
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{3}dx$$

```
[In] int(x*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

Fricas [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{3}dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{3}x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*x*log(c*((d*x + e
*x^(1/3))/x)^n)^2 + 3*a^2*b*x*log(c*((d*x + e*x^(1/3))/x)^n) + a^3*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{3}dx=\text{Timed out}$$

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(2/3))**n))**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{3}dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{3}x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2*log((d*x^(2/3) + e)^n)^3 - integrate((8*(b^3*d*x^2 + b^3*e*x^(4
/3))*log(x^(1/3*n))^3 - (b^3*d*log(c)^3 + 3*a*b^2*d*log(c)^2 + 3*a^2*b*d*lo
g(c) + a^3*d)*x^2 + (b^3*d*n*x^2 - 3*(b^3*d*log(c) + a*b^2*d)*x^2 - 3*(b^3*
e*log(c) + a*b^2*e)*x^(4/3) + 6*(b^3*d*x^2 + b^3*e*x^(4/3))*log(x^(1/3*n)))
*log((d*x^(2/3) + e)^n)^2 - 12*((b^3*d*log(c) + a*b^2*d)*x^2 + (b^3*e*log(c
) + a*b^2*e)*x^(4/3))*log(x^(1/3*n))^2 - (b^3*e*log(c)^3 + 3*a*b^2*e*log(c)
^2 + 3*a^2*b*e*log(c) + a^3*e)*x^(4/3) - 3*((b^3*d*log(c)^2 + 2*a*b^2*d*log
```

```
(c) + a^2*b*d)*x^2 + 4*(b^3*d*x^2 + b^3*e*x^(4/3))*log(x^(1/3*n))^2 + (b^3*
e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(4/3) - 4*((b^3*d*log(c) + a*b^2
*d)*x^2 + (b^3*e*log(c) + a*b^2*e)*x^(4/3))*log(x^(1/3*n)))*log((d*x^(2/3)
+ e)^n) + 6*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x^2 + (b^3*e*log
(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(4/3))*log(x^(1/3*n)))/(d*x + e*x^(1/
3)), x)
```

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\int x\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$$

```
[In] int(x*(a + b*log(c*(d + e/x^(2/3))^n))^3,x)
[Out] int(x*(a + b*log(c*(d + e/x^(2/3))^n))^3, x)
```

3.526 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 139

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx = -\frac{3}{2}\Big(a +b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\Big)^3\log\left(-\frac{e}{dx^{2/3}}\right)-\frac{9}{2}bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\operatorname{PolyLog}\left(2,1+\frac{e}{dx^{2/3}}\right)+9b^2n^2\Big(a+$$

```
[Out] -3/2*(a+b*ln(c*(d+e/x^(2/3))^n))^3*ln(-e/d/x^(2/3))-9/2*b*n*(a+b*ln(c*(d+e/
x^(2/3))^n))^2*polylog(2,1+e/d/x^(2/3))+9*b^2*n^2*(a+b*ln(c*(d+e/x^(2/3))^n
))*polylog(3,1+e/d/x^(2/3))-9*b^3*n^3*polylog(4,1+e/d/x^(2/3))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2504, 2443, 2481, 2421, 2430, 6724}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx = 9b^2n^2\operatorname{PolyLog}\left(3,\frac{e}{dx^{2/3}} +1\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)-\frac{9}{2}bn\operatorname{PolyLog}\left(2,\frac{e}{dx^{2/3}}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2-\frac{3}{2}\log\left(-\frac{e}{dx^{2/3}}\right)$$

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x,x]
```

```
[Out] (-3*(a + b*Log[c*(d + e/x^(2/3))^n])^3*Log[-(e/(d*x^(2/3)))])/2 - (9*b*n*(a
 + b*Log[c*(d + e/x^(2/3))^n])^2*PolyLog[2, 1 + e/(d*x^(2/3))])/2 + 9*b^2*n
^2*(a + b*Log[c*(d + e/x^(2/3))^n])*PolyLog[3, 1 + e/(d*x^(2/3))] - 9*b^3*n
^3*PolyLog[4, 1 + e/(d*x^(2/3))]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\left(\frac{3}{2}\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^n))^3}{x}\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\log\left(-\frac{e}{dx^{2/3}}\right) + \frac{1}{2}(9ben)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^n))^2}{d+ex}\,dx, x, \frac{1}{x^{2/3}}\right)$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\log\left(-\frac{e}{dx^{2/3}}\right) + \frac{1}{2}(9bn)\text{Subst}\left(\int \frac{(a+b\log(cx^n))^2\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx, x, d+\frac{e}{x^{2/3}}\right)$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\log\left(-\frac{e}{dx^{2/3}}\right) - \frac{9}{2}bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\text{Li}_2\left(1+\frac{e}{dx^{2/3}}\right)+\left(9b^2n^2\right)\text{Subst}\left(\int \frac{(a+b\log(cx^n))\text{Li}_2\left(\frac{x}{d}\right)}{x}\,dx,$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\log\left(-\frac{e}{dx^{2/3}}\right) - \frac{9}{2}bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\text{Li}_2\left(1+\frac{e}{dx^{2/3}}\right)+9b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\text{Li}_3\left(1+\frac{e}{dx^{2/3}}\right.$$

$$= -\frac{3}{2}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\log\left(-\frac{e}{dx^{2/3}}\right) - \frac{9}{2}bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2\text{Li}_2\left(1+\frac{e}{dx^{2/3}}\right)+9b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\text{Li}_3\left(1+\frac{e}{dx^{2/3}}\right.$$

Mathematica [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx = \int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x, x]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{3}}{x}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{3}}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{3}}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x^
(1/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(1/3))/x)^n) + a^3)/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{3}}{x}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3/x,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{3}}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{3}}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x,x, algorithm="maxima")
```

```
[Out] b^3*log((d*x^(2/3) + e)^n)^3*log(x) - integrate((8*(b^3*d*x + b^3*e*x^(1/3)
)*log(x^(1/3*n))^3 + (2*b^3*d*n*x*log(x) - 3*(b^3*d*log(c) + a*b^2*d)*x + 6
*(b^3*d*x + b^3*e*x^(1/3))*log(x^(1/3*n)) - 3*(b^3*e*log(c) + a*b^2*e)*x^(1
/3))*log((d*x^(2/3) + e)^n)^2 - 12*((b^3*d*log(c) + a*b^2*d)*x + (b^3*e*log
(c) + a*b^2*e)*x^(1/3))*log(x^(1/3*n))^2 - (b^3*d*log(c)^3 + 3*a*b^2*d*log(
```

```
c)^2 + 3*a^2*b*d*log(c) + a^3*d)*x - 3*(4*(b^3*d*x + b^3*e*x^(1/3))*log(x^(
1/3*n))^2 + (b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x - 4*((b^3*d*log
(c) + a*b^2*d)*x + (b^3*e*log(c) + a*b^2*e)*x^(1/3))*log(x^(1/3*n)) + (b^3*
e*log(c)^2 + 2*a*b^2*e*log(c) + a^2*b*e)*x^(1/3))*log((d*x^(2/3) + e)^n) +
6*((b^3*d*log(c)^2 + 2*a*b^2*d*log(c) + a^2*b*d)*x + (b^3*e*log(c)^2 + 2*a*
b^2*e*log(c) + a^2*b*e)*x^(1/3))*log(x^(1/3*n)) - (b^3*e*log(c)^3 + 3*a*b^2
*e*log(c)^2 + 3*a^2*b*e*log(c) + a^3*e)*x^(1/3))/(d*x^2 + e*x^(4/3)), x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^3/x,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))^3/x, x)
```

3.527 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 449

$$\begin{aligned}
\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}\,dx = {} & -\frac{9b^3dn^3\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^3}+\frac{b^3n^3\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3} \\
& -\frac{9ab^2d^2n^2}{e^2x^{2/3}}+\frac{9b^3d^2n^3}{e^2x^{2/3}}-\frac{9b^3d^2n^2\left(d+\frac{e}{x^{2/3}}\right)\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{e^3} \\
& +\frac{9b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4e^3} \\
& -\frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^3} \\
& +\frac{9bd^2n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2e^3} \\
& -\frac{9bdn\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4e^3} \\
& +\frac{bn\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2e^3} \\
& -\frac{3d^2\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3} \\
& +\frac{3d\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3} \\
& -\frac{\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3}
\end{aligned}$$

```
[Out] -9/8*b^3*d*n^3*(d+e/x^(2/3))^2/e^3+1/9*b^3*n^3*(d+e/x^(2/3))^3/e^3-9*a*b^2*
d^2*n^2/e^2/x^(2/3)+9*b^3*d^2*n^3/e^2/x^(2/3)-9*b^3*d^2*n^2*(d+e/x^(2/3))*l
```

```
n(c*(d+e/x^(2/3))^n)/e^3+9/4*b^2*d*n^2*(d+e/x^(2/3))^2*(a+b*ln(c*(d+e/x^(2/
3))^n))/e^3-1/3*b^2*n^2*(d+e/x^(2/3))^3*(a+b*ln(c*(d+e/x^(2/3))^n))/e^3+9/2
*b*d^2*n*(d+e/x^(2/3))*(a+b*ln(c*(d+e/x^(2/3))^n))^2/e^3-9/4*b*d*n*(d+e/x^(
2/3))^2*(a+b*ln(c*(d+e/x^(2/3))^n))^2/e^3+1/2*b*n*(d+e/x^(2/3))^3*(a+b*ln(c
*(d+e/x^(2/3))^n))^2/e^3-3/2*d^2*(d+e/x^(2/3))*(a+b*ln(c*(d+e/x^(2/3))^n))^
3/e^3+3/2*d*(d+e/x^(2/3))^2*(a+b*ln(c*(d+e/x^(2/3))^n))^3/e^3-1/2*(d+e/x^(2
/3))^3*(a+b*ln(c*(d+e/x^(2/3))^n))^3/e^3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2504, 2448, 2436, 2333, 2332, 2437, 2342, 2341}

$$\begin{aligned}\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}\,dx=&-\frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^3}\\&+\frac{9b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4e^3}\\&-\frac{9ab^2d^2n^2}{e^2x^{2/3}}+\frac{9bd^2n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2e^3}\\&-\frac{3d^2\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3}\\&+\frac{bn\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2e^3}\\&-\frac{9bdn\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4e^3}\\&-\frac{\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3}\\&+\frac{3d\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3}-\frac{9b^3d^2n^2\left(d+\frac{e}{x^{2/3}}\right)\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{e^3}\\&+\frac{9b^3d^2n^3}{e^2x^{2/3}}+\frac{b^3n^3\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3}-\frac{9b^3dn^3\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^3}\end{aligned}$$

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^3,x]
```

```
[Out] (-9*b^3*d*n^3*(d + e/x^(2/3))^2)/(8*e^3) + (b^3*n^3*(d + e/x^(2/3))^3)/(9*e
^3) - (9*a*b^2*d^2*n^2)/(e^2*x^(2/3)) + (9*b^3*d^2*n^3)/(e^2*x^(2/3)) - (9*
b^3*d^2*n^2*(d + e/x^(2/3))*Log[c*(d + e/x^(2/3))^n])/e^3 + (9*b^2*d*n^2*(d
 + e/x^(2/3))^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(4*e^3) - (b^2*n^2*(d + e
/x^(2/3))^3*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*e^3) + (9*b*d^2*n*(d + e/x
^(2/3))*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(2*e^3) - (9*b*d*n*(d + e/x^(2/
3))^2*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(4*e^3) + (b*n*(d + e/x^(2/3))^3*
```

```
(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(2*e^3) - (3*d^2*(d + e/x^(2/3))*(a + b
*Log[c*(d + e/x^(2/3))^n])^3)/(2*e^3) + (3*d*(d + e/x^(2/3))^2*(a + b*Log[c
*(d + e/x^(2/3))^n])^3)/(2*e^3) - ((d + e/x^(2/3))^3*(a + b*Log[c*(d + e/x^
(2/3))^n])^3)/(2*e^3)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
```

```
d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = -\left(\frac{3}{2}\text{Subst}\left(\int x^2(a+b\log(c(d+ex)^n))^3\,dx, x, \frac{1}{x^{2/3}}\right)\right)$$

$$= -\left(\frac{3}{2}\text{Subst}\left(\int\left(\frac{d^2(a+b\log(c(d+ex)^n))^3}{e^2} - \frac{2d(d+ex)(a+b\log(c(d+ex)^n))^3}{e^2} + \frac{(d+ex)^2(a\right.\right.\right.$$

$$= -\frac{3\text{Subst}\left(\int(d+ex)^2(a+b\log(c(d+ex)^n))^3\,dx, x, \frac{1}{x^{2/3}}\right)}{2e^2} + \frac{(3d)\text{Subst}\left(\int(d+ex)(a+b\log(c(d+ex)^n))^3\,dx, x, \frac{1}{x^{2/3}}\right)}{e^2} - \frac{(3d^2)\,\text{Subst}\left(\int(a+b\log(c(d+ex)^n))^3\,dx, x, \frac{1}{x^{2/3}}\right)}{2e^2}$$

$$= -\frac{3\text{Subst}\left(\int x^2(a+b\log(cx^n))^3\,dx, x, d+\frac{e}{x^{2/3}}\right)}{2e^3} + \frac{(3d)\text{Subst}\left(\int x(a+b\log(cx^n))^3\,dx, x, d+\frac{e}{x^{2/3}}\right)}{e^3} - \frac{(3d^2)\,\text{Subst}\left(\int(a+b\log(cx^n))^3\,dx, x, d+\frac{e}{x^{2/3}}\right)}{2e^3}$$

$$= -\frac{3d^2\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3} + \frac{3d\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3} - \frac{\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3} + \frac{(3bn)\text{Subst}\left(\int x^2(a+b\log(cx^n))^2\,dx, x, d+\frac{e}{x^{2/3}}\right)}{2e^3} - \frac{(9bdn)\text{Subst}\left(\int x(a+b\log(cx^n))^2\,dx, x, d+\frac{e}{x^{2/3}}\right)}{2e^3} + \frac{(9bd^2n)\,\text{Subst}\left(\int(a+b\log(cx^n))^2\,dx, x, d+\frac{e}{x^{2/3}}\right)}{2e^3}$$

$$= \frac{9bd^2n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2e^3} - \frac{9bdn\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4e^3} + \frac{bn\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2e^3} - \frac{3d^2\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3} + \frac{3d\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3} - \frac{\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3} - \frac{(b^2n^2)\operatorname{Subst}\left(\int x^2(a+b\log(cx^n))\,dx,x,d+\frac{e}{x^{2/3}}\right)}{e^3} + \frac{(9b^2dn^2)\operatorname{Subst}\left(\int x(a+b\log(cx^n))\,dx,x,d+\frac{e}{x^{2/3}}\right)}{2e^3} - \frac{(9b^2d^2n^2)\operatorname{Subst}\left(\int (a+b\log(cx^n))\,dx,x,d+\frac{e}{x^{2/3}}\right)}{e^3}$$

$$= -\frac{9b^3dn^3\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^3} + \frac{b^3n^3\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3} - \frac{9ab^2d^2n^2}{e^2x^{2/3}} + \frac{9b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4e^3} - \frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^3} + \frac{9bd^2n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2e^3} - \frac{9bdn\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4e^3} + \frac{bn\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2e^3} - \frac{3d^2\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3} + \frac{3d\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3} - \frac{\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3} - \frac{(9b^3d^2n^2)\operatorname{Subst}\left(\int \log(cx^n)\,dx,x,d+\frac{e}{x^{2/3}}\right)}{e^3}$$

$$
\begin{aligned}
&= -\frac{9b^3dn^3\left(d+\frac{e}{x^{2/3}}\right)^2}{8e^3} + \frac{b^3n^3\left(d+\frac{e}{x^{2/3}}\right)^3}{9e^3} - \frac{9ab^2d^2n^2}{e^2x^{2/3}} \\
&\quad + \frac{9b^3d^2n^3}{e^2x^{2/3}} - \frac{9b^3d^2n^2\left(d+\frac{e}{x^{2/3}}\right)\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{e^3} \\
&\quad + \frac{9b^2dn^2\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{4e^3} \\
&\quad - \frac{b^2n^2\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3e^3} \\
&\quad + \frac{9bd^2n\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2e^3} \\
&\quad - \frac{9bdn\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{4e^3} \\
&\quad + \frac{bn\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{2e^3} \\
&\quad - \frac{3d^2\left(d+\frac{e}{x^{2/3}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3} \\
&\quad + \frac{3d\left(d+\frac{e}{x^{2/3}}\right)^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3} \\
&\quad - \frac{\left(d+\frac{e}{x^{2/3}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{2e^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.54

$$
\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}\,dx = \frac{-36a^3e^3+36a^2be^3n-24ab^2e^3n^2+8b^3e^3n^3-54a^2bde^2nx^{2/3}+90ab^2de}{}
$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^3,x]
```

```
[Out] (-36*a^3*e^3 + 36*a^2*b*e^3*n - 24*a*b^2*e^3*n^2 + 8*b^3*e^3*n^3 - 54*a^2*b
*d*e^2*n*x^(2/3) + 90*a*b^2*d*e^2*n^2*x^(2/3) - 57*b^3*d*e^2*n^3*x^(2/3) +
108*a^2*b*d^2*e*n*x^(4/3) - 396*a*b^2*d^2*e*n^2*x^(4/3) + 510*b^3*d^2*e*n^3
*x^(4/3) + 72*b^3*d^3*n^3*x^2*Log[d + e/x^(2/3)]^3 - 36*b^3*e^3*Log[c*(d +
e/x^(2/3))^n]^3 - 108*a^2*b*d^3*n*x^2*Log[e + d*x^(2/3)] + 396*a*b^2*d^3*n^
2*x^2*Log[e + d*x^(2/3)] - 510*b^3*d^3*n^3*x^2*Log[e + d*x^(2/3)] + 12*b^2*
d^3*n^2*x^2*Log[d + e/x^(2/3)]*(6*a - 11*b*n + 6*b*Log[c*(d + e/x^(2/3))^n]
)*(3*Log[e + d*x^(2/3)] - 2*Log[x]) + 72*a^2*b*d^3*n*x^2*Log[x] - 264*a*b^2
*d^3*n^2*x^2*Log[x] + 340*b^3*d^3*n^3*x^2*Log[x] - 18*b^2*d^3*n^2*x^2*Log[d
 + e/x^(2/3)]^2*(6*a - 11*b*n + 6*b*Log[c*(d + e/x^(2/3))^n] + 6*b*n*Log[e
+ d*x^(2/3)] - 4*b*n*Log[x]) + 18*b^2*Log[c*(d + e/x^(2/3))^n]^2*(e*(-6*a*e
^2 + 2*b*e^2*n - 3*b*d*e*n*x^(2/3) + 6*b*d^2*n*x^(4/3)) - 6*b*d^3*n*x^2*Log
```

```
[e + d*x^(2/3)] + 4*b*d^3*n*x^2*Log[x]) - 6*b*Log[c*(d + e/x^(2/3))^n]*(18*
a^2*e^3 - 6*a*b*e*n*(2*e^2 - 3*d*e*x^(2/3) + 6*d^2*x^(4/3)) + b^2*e*n^2*(4*
e^2 - 15*d*e*x^(2/3) + 66*d^2*x^(4/3)) + 6*b*d^3*n*(6*a - 11*b*n)*x^2*Log[e
 + d*x^(2/3)] + 4*b*d^3*n*(-6*a + 11*b*n)*x^2*Log[x]))/(72*e^3*x^2)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{3}}{x^{3}}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 725, normalized size of antiderivative = 1.61

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{3}}{x^{3}}\,dx = \frac{8\,b^{3}e^{3}n^{3}-36\,b^{3}e^{3}\log\left(c\right)^{3}-24\,ab^{2}e^{3}n^{2}+36\,a^{2}be^{3}n-36\,a^{3}e^{3}-36\,(b^{3}d^{3}n}{}$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/72*(8*b^3*e^3*n^3 - 36*b^3*e^3*log(c)^3 - 24*a*b^2*e^3*n^2 + 36*a^2*b*e^3
*n - 36*a^3*e^3 - 36*(b^3*d^3*n^3*x^2 + b^3*e^3*n^3)*log((d*x + e*x^(1/3))/
x)^3 + 36*(b^3*e^3*n - 3*a*b^2*e^3)*log(c)^2 + 18*(6*b^3*d^2*e*n^3*x^(4/3)
- 3*b^3*d*e^2*n^3*x^(2/3) + 2*b^3*e^3*n^3 - 6*a*b^2*e^3*n^2 + (11*b^3*d^3*n
^3 - 6*a*b^2*d^3*n^2)*x^2 - 6*(b^3*d^3*n^2*x^2 + b^3*e^3*n^2)*log(c))*log((
d*x + e*x^(1/3))/x)^2 - 12*(2*b^3*e^3*n^2 - 6*a*b^2*e^3*n + 9*a^2*b*e^3)*lo
g(c) - 6*(4*b^3*e^3*n^3 - 12*a*b^2*e^3*n^2 + 18*a^2*b*e^3*n + (85*b^3*d^3*n
^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*d^3*n)*x^2 + 18*(b^3*d^3*n*x^2 + b^3*e^3*n
)*log(c)^2 - 6*(2*b^3*e^3*n^2 - 6*a*b^2*e^3*n + (11*b^3*d^3*n^2 - 6*a*b^2*d
^3*n)*x^2)*log(c) - 3*(5*b^3*d*e^2*n^3 - 6*b^3*d*e^2*n^2*log(c) - 6*a*b^2*d
*e^2*n^2)*x^(2/3) - 6*(6*b^3*d^2*e*n^2*x*log(c) - (11*b^3*d^2*e*n^3 - 6*a*b
^2*d^2*e*n^2)*x)*x^(1/3))*log((d*x + e*x^(1/3))/x) - 3*(19*b^3*d*e^2*n^3 +
18*b^3*d*e^2*n*log(c)^2 - 30*a*b^2*d*e^2*n^2 + 18*a^2*b*d*e^2*n - 6*(5*b^3*
d*e^2*n^2 - 6*a*b^2*d*e^2*n)*log(c))*x^(2/3) + 6*(18*b^3*d^2*e*n*x*log(c)^2
 - 6*(11*b^3*d^2*e*n^2 - 6*a*b^2*d^2*e*n)*x*log(c) + (85*b^3*d^2*e*n^3 - 66
*a*b^2*d^2*e*n^2 + 18*a^2*b*d^2*e*n)*x)*x^(1/3))/(e^3*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3/x**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.52

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}\,dx=$$

$$-\frac{1}{4}a^2ben\left(\frac{6\,d^3\log\left(dx^{\frac{2}{3}}+e\right)}{e^4}-\frac{6\,d^3\log\left(x^{\frac{2}{3}}\right)}{e^4}-\frac{6\,d^2x^{\frac{4}{3}}-3\,dex^{\frac{2}{3}}+2\,e^2}{e^3x^2}\right)$$

$$-\frac{1}{12}\left(6\,en\left(\frac{6\,d^3\log\left(dx^{\frac{2}{3}}+e\right)}{e^4}-\frac{6\,d^3\log\left(x^{\frac{2}{3}}\right)}{e^4}-\frac{6\,d^2x^{\frac{4}{3}}-3\,dex^{\frac{2}{3}}+2\,e^2}{e^3x^2}\right)\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)-\frac{\left(18\,d^3\right.}{}\right.$$

$$-\frac{1}{216}\left(54\,en\left(\frac{6\,d^3\log\left(dx^{\frac{2}{3}}+e\right)}{e^4}-\frac{6\,d^3\log\left(x^{\frac{2}{3}}\right)}{e^4}-\frac{6\,d^2x^{\frac{4}{3}}-3\,dex^{\frac{2}{3}}+2\,e^2}{e^3x^2}\right)\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)^2+en\left(\right.\right.$$

$$-\frac{b^3\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)^3}{2\,x^2}-\frac{3\,ab^2\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)^2}{2\,x^2}-\frac{3\,a^2b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)}{2\,x^2}-\frac{a^3}{2\,x^2}$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*a^2*b*e*n*(6*d^3*log(d*x^(2/3) + e)/e^4 - 6*d^3*log(x^(2/3))/e^4 - (6*
d^2*x^(4/3) - 3*d*e*x^(2/3) + 2*e^2)/(e^3*x^2)) - 1/12*(6*e*n*(6*d^3*log(d*
x^(2/3) + e)/e^4 - 6*d^3*log(x^(2/3))/e^4 - (6*d^2*x^(4/3) - 3*d*e*x^(2/3)
+ 2*e^2)/(e^3*x^2))*log(c*(d + e/x^(2/3))^n) - (18*d^3*x^2*log(d*x^(2/3) +
e)^2 + 8*d^3*x^2*log(x)^2 - 44*d^3*x^2*log(x) - 66*d^2*e*x^(4/3) + 15*d*e^2
*x^(2/3) - 4*e^3 - 6*(4*d^3*x^2*log(x) - 11*d^3*x^2)*log(d*x^(2/3) + e))*n^
2/(e^3*x^2))*a*b^2 - 1/216*(54*e*n*(6*d^3*log(d*x^(2/3) + e)/e^4 - 6*d^3*lo
g(x^(2/3))/e^4 - (6*d^2*x^(4/3) - 3*d*e*x^(2/3) + 2*e^2)/(e^3*x^2))*log(c*(
d + e/x^(2/3))^n)^2 + e*n*((108*d^3*x^2*log(d*x^(2/3) + e)^3 - 32*d^3*x^2*l
og(x)^3 + 264*d^3*x^2*log(x)^2 - 1020*d^3*x^2*log(x) - 1530*d^2*e*x^(4/3) +
```

```
171*d*e^2*x^(2/3) - 24*e^3 - 54*(4*d^3*x^2*log(x) - 11*d^3*x^2)*log(d*x^(2
/3) + e)^2 + 18*(8*d^3*x^2*log(x)^2 - 44*d^3*x^2*log(x) + 85*d^3*x^2)*log(d
*x^(2/3) + e))*n^2/(e^4*x^2) - 18*(18*d^3*x^2*log(d*x^(2/3) + e)^2 + 8*d^3*
x^2*log(x)^2 - 44*d^3*x^2*log(x) - 66*d^2*e*x^(4/3) + 15*d*e^2*x^(2/3) - 4*
e^3 - 6*(4*d^3*x^2*log(x) - 11*d^3*x^2)*log(d*x^(2/3) + e))*n*log(c*(d + e/
x^(2/3))^n)/(e^4*x^2)))*b^3 - 1/2*b^3*log(c*(d + e/x^(2/3))^n)^3/x^2 - 3/2*
a*b^2*log(c*(d + e/x^(2/3))^n)^2/x^2 - 3/2*a^2*b*log(c*(d + e/x^(2/3))^n)/x
^2 - 1/2*a^3/x^2
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.29

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^3}\,dx=\frac{\frac{d\left(\frac{3a^3}{2}-\frac{3a^2bn}{2}+ab^2n^2-\frac{b^3n^3}{3}\right)}{2e}-\frac{d\left(6a^3-6ab^2n^2+5b^3n^3\right)}{8e}}{x^{4/3}}$$

$$-\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)^3\left(\frac{b^3}{2x^2}+\frac{b^3d^3}{2e^3}\right)-\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)^2\left(\frac{b^2\left(3a-bn\right)}{2x^2}-\frac{\frac{3b^2d\left(3a-bn\right)}{2e}-\frac{9ab^2d}{2e}}{2x^{4/3}}+\frac{d\left(6ab^2\right.}{}\right.$$

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^3/x^3,x)
```

```
[Out] ((d*((3*a^3)/2 - (b^3*n^3)/3 + a*b^2*n^2 - (3*a^2*b*n)/2))/(2*e) - (d*(6*a^
3 + 5*b^3*n^3 - 6*a*b^2*n^2))/(8*e))/x^(4/3) - log(c*(d + e/x^(2/3))^n)^3*(
b^3/(2*x^2) + (b^3*d^3)/(2*e^3)) - log(c*(d + e/x^(2/3))^n)^2*((b^2*(3*a -
b*n))/(2*x^2) - ((3*b^2*d*(3*a - b*n))/(2*e) - (9*a*b^2*d)/(2*e))/(2*x^(4/3
)) + (d*(6*a*b^2*d^2 - 11*b^3*d^2*n))/(4*e^3) + (d*((6*b^2*d*(3*a - b*n))/e
 - (18*a*b^2*d)/e))/(4*e*x^(2/3))) - ((d*((d*((3*a^3)/2 - (b^3*n^3)/3 + a*b
^2*n^2 - (3*a^2*b*n)/2))/e - (d*(6*a^3 + 5*b^3*n^3 - 6*a*b^2*n^2))/(4*e)))/
e + (b^2*d^2*n^2*(6*a - 11*b*n))/(2*e^2))/x^(2/3) - (a^3/2 - (b^3*n^3)/9 +
(a*b^2*n^2)/3 - (a^2*b*n)/2)/x^2 - (log(c*(d + e/x^(2/3))^n)*(((d*(2*b*d*e*
(9*a^2 + 2*b^2*n^2 - 6*a*b*n) - 6*b*d*e*(3*a^2 - b^2*n^2)))/e + 12*b^3*d^2*
n^2)/(2*e*x^(2/3)) - (2*b*d*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*n) - 6*b*d*e*(3*a^
```

```
2 - b^2*n^2))/(4*e*x^(4/3)) + (b*e*(9*a^2 + 2*b^2*n^2 - 6*a*b*n))/(3*x^2)))
/(2*e) - (log(d + e/x^(2/3))*(85*b^3*d^3*n^3 - 66*a*b^2*d^3*n^2 + 18*a^2*b*
d^3*n))/(12*e^3)
```

3.528 $\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\frac{568ab^2e^4n^2\sqrt[3]{x}}{105d^4}-\frac{16b^3e^4n^3\sqrt[3]{x}}{7d^4}+\frac{16b^3e^3n^3x}{105d^3}+\frac{1376b^3e^{9/2}n^3\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{105d^{9/2}}+\frac{568ib^3e^{9/2}n^3\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{105d^{9/2}}-\frac{1136b^3e^{9/2}n^3\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{105d^{9/2}}+\frac{568b^3e^4n^2\sqrt[3]{x}\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)}{105d^4}-\frac{32b^2e^3n^2x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{35d^3}+\frac{8b^2e^2n^2x^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{35d^2}-\frac{568b^2e^{9/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{105d^{9/2}}-\frac{2be^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d^4}+\frac{2be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{3d^3}-\frac{2be^2nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{5d^2}+\frac{2benx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{7d}$$

$$+\frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3+\frac{4b^2e^{9/2}n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{9/2}}-\frac{2b^3e^{9/2}n^3\log^2\left(\right.}{(-d}$$

```
[Out] 568/105*I*b^3*e^(9/2)*n^3*arctan(x^(1/3)*d^(1/2)/e^(1/2))^2/d^(9/2)+16/105*
b^3*e^3*n^3*x/d^3-16/7*b^3*e^4*n^3*x^(1/3)/d^4+2*b^3*e^(9/2)*n^3*ln(x^(1/3)
*(-d)^(1/2)+e^(1/2))^2/(-d)^(9/2)+8*b^3*e^(9/2)*n^3*polylog(2,1-x^(1/3)*(-d
```

```
)^(1/2)/e^(1/2))/(-d)^(9/2)-4*b^3*e^(9/2)*n^3*polylog(2,1/2-1/2*x^(1/3)*(-d
)^(1/2)/e^(1/2))/(-d)^(9/2)+4*b^3*e^(9/2)*n^3*polylog(2,1/2+1/2*x^(1/3)*(-d
)^(1/2)/e^(1/2))/(-d)^(9/2)-8*b^3*e^(9/2)*n^3*polylog(2,1+x^(1/3)*(-d)^(1/2
)/e^(1/2))/(-d)^(9/2)+2/3*b*e^5*n*Unintegrable((a+b*ln(c*(d+e/x^(2/3))^n))^
2/(e+d*x^(2/3))/x^(2/3),x)/d^4+1376/105*b^3*e^(9/2)*n^3*arctan(x^(1/3)*d^(1
/2)/e^(1/2))/d^(9/2)-2*b^3*e^(9/2)*n^3*ln(-x^(1/3)*(-d)^(1/2)+e^(1/2))^2/(-
d)^(9/2)+2/3*b*e^3*n*x*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d^3-2/5*b*e^2*n*x^(5/3
)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d^2+2/7*b*e*n*x^(7/3)*(a+b*ln(c*(d+e/x^(2/3
))^n))^2/d+4*b^2*e^(9/2)*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))*ln(-x^(1/3)*(-d)^(
1/2)+e^(1/2))/(-d)^(9/2)-4*b^3*e^(9/2)*n^3*ln(1/2+1/2*x^(1/3)*(-d)^(1/2)/e^
(1/2))*ln(-x^(1/3)*(-d)^(1/2)+e^(1/2))/(-d)^(9/2)+8*b^3*e^(9/2)*n^3*ln(x^(1
/3)*(-d)^(1/2)/e^(1/2))*ln(-x^(1/3)*(-d)^(1/2)+e^(1/2))/(-d)^(9/2)-4*b^2*e^
(9/2)*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))*ln(x^(1/3)*(-d)^(1/2)+e^(1/2))/(-d)^(
9/2)+4*b^3*e^(9/2)*n^3*ln(1/2-1/2*x^(1/3)*(-d)^(1/2)/e^(1/2))*ln(x^(1/3)*(-
d)^(1/2)+e^(1/2))/(-d)^(9/2)-8*b^3*e^(9/2)*n^3*ln(-x^(1/3)*(-d)^(1/2)/e^(1/
2))*ln(x^(1/3)*(-d)^(1/2)+e^(1/2))/(-d)^(9/2)-1136/105*b^3*e^(9/2)*n^3*arct
an(x^(1/3)*d^(1/2)/e^(1/2))*ln(2-2*e^(1/2)/(-I*x^(1/3)*d^(1/2)+e^(1/2)))/d^
(9/2)+568/105*I*b^3*e^(9/2)*n^3*polylog(2,-1+2*e^(1/2)/(-I*x^(1/3)*d^(1/2)+
e^(1/2)))/d^(9/2)+1/3*x^3*(a+b*ln(c*(d+e/x^(2/3))^n))^3+568/105*a*b^2*e^4*n
^2*x^(1/3)/d^4+568/105*b^3*e^4*n^2*x^(1/3)*ln(c*(d+e/x^(2/3))^n)/d^4-32/35*
b^2*e^3*n^2*x*(a+b*ln(c*(d+e/x^(2/3))^n))/d^3+8/35*b^2*e^2*n^2*x^(5/3)*(a+b
*ln(c*(d+e/x^(2/3))^n))/d^2-568/105*b^2*e^(9/2)*n^2*arctan(x^(1/3)*d^(1/2)/
e^(1/2))*(a+b*ln(c*(d+e/x^(2/3))^n))/d^(9/2)-2*b*e^4*n*x^(1/3)*(a+b*ln(c*(d
+e/x^(2/3))^n))^2/d^4
```

Rubi [N/A]

Not integrable

Time = 2.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$$

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]

[Out] (568*a*b^2*e^4*n^2*x^(1/3))/(105*d^4) - (16*b^3*e^4*n^3*x^(1/3))/(7*d^4) +
(16*b^3*e^3*n^3*x)/(105*d^3) + (1376*b^3*e^(9/2)*n^3*ArcTan[(Sqrt[d]*x^(1/3
))/Sqrt[e]])/(105*d^(9/2)) + (((568*I)/105)*b^3*e^(9/2)*n^3*ArcTan[(Sqrt[d]
*x^(1/3))/Sqrt[e]]^2)/d^(9/2) - (1136*b^3*e^(9/2)*n^3*ArcTan[(Sqrt[d]*x^(1/
3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/(105*d^(9/
2)) + (568*b^3*e^4*n^2*x^(1/3)*Log[c*(d + e/x^(2/3))^n])/(105*d^4) - (32*b^
2*e^3*n^2*x*(a + b*Log[c*(d + e/x^(2/3))^n]))/(35*d^3) + (8*b^2*e^2*n^2*x^(
5/3)*(a + b*Log[c*(d + e/x^(2/3))^n]))/(35*d^2) - (568*b^2*e^(9/2)*n^2*ArcT
an[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(105*d^(9/2
```

```
)) - (2*b*e^4*n*x^(1/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/d^4 + (2*b*e^3*
n*x*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(3*d^3) - (2*b*e^2*n*x^(5/3)*(a + b
*Log[c*(d + e/x^(2/3))^n])^2)/(5*d^2) + (2*b*e*n*x^(7/3)*(a + b*Log[c*(d +
e/x^(2/3))^n])^2)/(7*d) + (x^3*(a + b*Log[c*(d + e/x^(2/3))^n])^3)/3 + (4*b
^2*e^(9/2)*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e] - Sqrt[-d]*x^(1
/3)])/(-d)^(9/2) - (2*b^3*e^(9/2)*n^3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]^2)/(-
d)^(9/2) - (4*b^2*e^(9/2)*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*Log[Sqrt[e]
+ Sqrt[-d]*x^(1/3)])/(-d)^(9/2) + (2*b^3*e^(9/2)*n^3*Log[Sqrt[e] + Sqrt[-d]
*x^(1/3)]^2)/(-d)^(9/2) + (4*b^3*e^(9/2)*n^3*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)
]*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])])/(-d)^(9/2) - (4*b^3*e^(9/2)*n^
3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(1 + (Sqrt[-d]*x^(1/3))/Sqrt[e])/2])/
(-d)^(9/2) - (8*b^3*e^(9/2)*n^3*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[-((Sqrt
[-d]*x^(1/3))/Sqrt[e])])/(-d)^(9/2) + (8*b^3*e^(9/2)*n^3*Log[Sqrt[e] - Sqrt
[-d]*x^(1/3)]*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]])/(-d)^(9/2) + (((568*I)/105)*
b^3*e^(9/2)*n^3*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])
/d^(9/2) + (8*b^3*e^(9/2)*n^3*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[e]])/(
-d)^(9/2) - (4*b^3*e^(9/2)*n^3*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[
e])])/(-d)^(9/2) + (4*b^3*e^(9/2)*n^3*PolyLog[2, (1 + (Sqrt[-d]*x^(1/3))/Sq
rt[e])/2])/(-d)^(9/2) - (8*b^3*e^(9/2)*n^3*PolyLog[2, 1 + (Sqrt[-d]*x^(1/3)
)/Sqrt[e]])/(-d)^(9/2) + (2*b*e^5*n*Defer[Subst][Defer[Int][(a + b*Log[c*(d
 + e/x^2)^n])^2/(e + d*x^2), x], x, x^(1/3)])/d^4
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^3\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3+(2ben)\text{Subst}\left(\int \frac{x^6\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{d+\frac{e}{x^2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3$$

$$+(2ben)\text{Subst}\left(\int\left(-\frac{e^3\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{d^4}+\frac{e^2x^2\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{d^3}-\frac{ex^4\left(a+b\log\right.}{d}\right.\right.$$

$$= \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{(2ben)\text{Subst}\left(\int x^6\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2\,dx, x, \sqrt[3]{x}\right)}{d} - \frac{(2be^2n)\,\text{Subst}\left(\int x^4\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2\,dx, x, \sqrt[3]{x}\right)}{d^2} + \frac{(2be^3n)\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2\,dx, x, \sqrt[3]{x}\right)}{d^3} - \frac{(2be^4n)\,\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2\,dx, x, \sqrt[3]{x}\right)}{d^4} + \frac{(2be^5n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}\,dx, x, \sqrt[3]{x}\right)}{d^4}$$

$$= -\frac{2be^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d^4} + \frac{2be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{3d^3} - \frac{2be^2nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{5d^2} + \frac{2benx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{7d} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{(2be^5n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}\,dx, x, \sqrt[3]{x}\right)}{d^4} + \frac{(8b^2e^2n^2)\,\text{S}}{}$$

$$= -\frac{2be^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d^4} + \frac{2be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{3d^3} - \frac{2be^2nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{5d^2} + \frac{2benx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{7d} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{(2be^5n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}\,dx, x, \sqrt[3]{x}\right)}{d^4} + \frac{(8b^2e^2n^2)\,\text{S}}{}$$

$$= -\frac{2be^4n\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d^4} + \frac{2be^3nx\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{3d^3} - \frac{2be^2nx^{5/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{5d^2} + \frac{2benx^{7/3}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{7d} + \frac{1}{3}x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{(2be^5n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}\,dx, x, \sqrt[3]{x}\right)}{d^4} + \frac{(8b^2e^2n^2)\,\text{S}}{}$$

$$= \text{Too large to display}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 5975 vs. 2(1278) = 2556.

Time = 23.09 (sec) , antiderivative size = 5975, normalized size of antiderivative = 248.96

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = \text{Result too large to show}$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] Result too large to show
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^{2} \left(a +b \ln \left(c \left(d +\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{3}dx$$

```
[In] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

```
[Out] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^n))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.88

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = \int \left(b \log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{3} x^{2}\, dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*x^2*log(c*((d*x
 + e*x^(1/3))/x)^n)^2 + 3*a^2*b*x^2*log(c*((d*x + e*x^(1/3))/x)^n) + a^3*x^
2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))**n))**3,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx=\int x^2\left(a+b\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx$$

```
[In] int(x^2*(a + b*log(c*(d + e/x^(2/3))^n))^3,x)
[Out] int(x^2*(a + b*log(c*(d + e/x^(2/3))^n))^3, x)
```

3.529 $\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d}$$

$$+x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3+\frac{12b^2e^{3/2}n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}}-\frac{6b^3e^{3/2}n^3\log^2\left(\sqrt{\right.}{(-d)}$$

```
[Out] 6*b*e*n*x^(1/3)*(a+b*ln(c*(d+e/x^(2/3))^n))^2/d+x*(a+b*ln(c*(d+e/x^(2/3))^n
))^3+12*b^2*e^(3/2)*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))*ln(-x^(1/3)*(-d)^(1/2)+
e^(1/2))/(-d)^(3/2)-12*b^3*e^(3/2)*n^3*ln(1/2+1/2*x^(1/3)*(-d)^(1/2)/e^(1/2
))*ln(-x^(1/3)*(-d)^(1/2)+e^(1/2))/(-d)^(3/2)+24*b^3*e^(3/2)*n^3*ln(x^(1/3)
*(-d)^(1/2)/e^(1/2))*ln(-x^(1/3)*(-d)^(1/2)+e^(1/2))/(-d)^(3/2)-6*b^3*e^(3/
2)*n^3*ln(-x^(1/3)*(-d)^(1/2)+e^(1/2))^2/(-d)^(3/2)-12*b^2*e^(3/2)*n^2*(a+b
*ln(c*(d+e/x^(2/3))^n))*ln(x^(1/3)*(-d)^(1/2)+e^(1/2))/(-d)^(3/2)+12*b^3*e^
(3/2)*n^3*ln(1/2-1/2*x^(1/3)*(-d)^(1/2)/e^(1/2))*ln(x^(1/3)*(-d)^(1/2)+e^(1
/2))/(-d)^(3/2)-24*b^3*e^(3/2)*n^3*ln(-x^(1/3)*(-d)^(1/2)/e^(1/2))*ln(x^(1/
3)*(-d)^(1/2)+e^(1/2))/(-d)^(3/2)+6*b^3*e^(3/2)*n^3*ln(x^(1/3)*(-d)^(1/2)+e
^(1/2))^2/(-d)^(3/2)+24*b^3*e^(3/2)*n^3*polylog(2,1-x^(1/3)*(-d)^(1/2)/e^(1
/2))/(-d)^(3/2)-12*b^3*e^(3/2)*n^3*polylog(2,1/2-1/2*x^(1/3)*(-d)^(1/2)/e^(
1/2))/(-d)^(3/2)+12*b^3*e^(3/2)*n^3*polylog(2,1/2+1/2*x^(1/3)*(-d)^(1/2)/e^
(1/2))/(-d)^(3/2)-24*b^3*e^(3/2)*n^3*polylog(2,1+x^(1/3)*(-d)^(1/2)/e^(1/2)
)/(-d)^(3/2)-2*b*e^2*n*Unintegrable((a+b*ln(c*(d+e/x^(2/3))^n))^2/(e+d*x^(2
/3))/x^(2/3),x)/d
```

Rubi [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = \int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx$$

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
```

```
[Out] (6*b*e*n*x^(1/3)*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/d + x*(a + b*Log[c*(d
+ e/x^(2/3))^n])^3 + (12*b^2*e^(3/2)*n^2*(a + b*Log[c*(d + e/x^(2/3))^n])*L
og[Sqrt[e] - Sqrt[-d]*x^(1/3)])/(-d)^(3/2) - (6*b^3*e^(3/2)*n^3*Log[Sqrt[e]
 - Sqrt[-d]*x^(1/3)]^2)/(-d)^(3/2) - (12*b^2*e^(3/2)*n^2*(a + b*Log[c*(d +
e/x^(2/3))^n])*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)])/(-d)^(3/2) + (6*b^3*e^(3/2)
*n^3*Log[Sqrt[e] + Sqrt[-d]*x^(1/3)]^2)/(-d)^(3/2) + (12*b^3*e^(3/2)*n^3*Lo
g[Sqrt[e] + Sqrt[-d]*x^(1/3)]*Log[1/2 - (Sqrt[-d]*x^(1/3))/(2*Sqrt[e])])/(-
d)^(3/2) - (12*b^3*e^(3/2)*n^3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(1 + (Sq
rt[-d]*x^(1/3))/Sqrt[e])/2])/(-d)^(3/2) - (24*b^3*e^(3/2)*n^3*Log[Sqrt[e] +
 Sqrt[-d]*x^(1/3)]*Log[-((Sqrt[-d]*x^(1/3))/Sqrt[e])])/(-d)^(3/2) + (24*b^3
*e^(3/2)*n^3*Log[Sqrt[e] - Sqrt[-d]*x^(1/3)]*Log[(Sqrt[-d]*x^(1/3))/Sqrt[e]
])/(-d)^(3/2) + (24*b^3*e^(3/2)*n^3*PolyLog[2, 1 - (Sqrt[-d]*x^(1/3))/Sqrt[
e]])/(-d)^(3/2) - (12*b^3*e^(3/2)*n^3*PolyLog[2, 1/2 - (Sqrt[-d]*x^(1/3))/(
2*Sqrt[e])])/(-d)^(3/2) + (12*b^3*e^(3/2)*n^3*PolyLog[2, (1 + (Sqrt[-d]*x^(
1/3))/Sqrt[e])/2])/(-d)^(3/2) - (24*b^3*e^(3/2)*n^3*PolyLog[2, 1 + (Sqrt[-d
]*x^(1/3))/Sqrt[e]])/(-d)^(3/2) - (6*b*e^2*n*Defer[Subst][Defer[Int][(a + b
*Log[c*(d + e/x^2)^n])^2/(e + d*x^2), x], x, x^(1/3)])/d
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^3 dx, x, \sqrt[3]{x}\right) \\
&= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3+(6ben)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{d+\frac{e}{x^2}} dx, x, \sqrt[3]{x}\right) \\
&= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 \\
&\quad +(6ben)\text{Subst}\left(\int \left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{d}-\frac{e\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{d\left(e+dx^2\right)}\right) dx, x, \sqrt[3]{x}\right)
\end{aligned}$$

$$= x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 + \frac{(6ben)\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2 dx, x, \sqrt[3]{x}\right)}{d} - \frac{(6be^2n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2} dx, x, \sqrt[3]{x}\right)}{d}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{(6be^2n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2} dx, x, \sqrt[3]{x}\right)}{d} + \frac{(24b^2e^2n^2)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{\left(d+\frac{e}{x^2}\right)x^2} dx, x, \sqrt[3]{x}\right)}{d}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{(6be^2n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2} dx, x, \sqrt[3]{x}\right)}{d} + \frac{(24b^2e^2n^2)\,\text{Subst}\left(\int \left(\frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}x\right)} + \frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}x\right)}\right) dx, x, \sqrt[3]{x}\right)}{d}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 - \frac{(6be^2n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2} dx, x, \sqrt[3]{x}\right)}{d} + \frac{(12b^2e^{3/2}n^2)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{\sqrt{e}-\sqrt{-d}x} dx, x, \sqrt[3]{x}\right)}{d} + \frac{(12b^2e^{3/2}n^2)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{\sqrt{e}+\sqrt{-d}x} dx, x, \sqrt[3]{x}\right)}{d}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3$$
$$+ \frac{12b^2e^{3/2}n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}}$$
$$- \frac{12b^2e^{3/2}n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}}$$
$$- \frac{(6be^2n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}\,dx, x, \sqrt[3]{x}\right)}{d}$$
$$+ \frac{\left(24b^3e^{5/2}n^3\right)\text{Subst}\left(\int \frac{\log\left(\sqrt{e}-\sqrt{-d}x\right)}{\left(d+\frac{e}{x^2}\right)x^3}\,dx, x, \sqrt[3]{x}\right)}{(-d)^{3/2}}$$
$$- \frac{\left(24b^3e^{5/2}n^3\right)\text{Subst}\left(\int \frac{\log\left(\sqrt{e}+\sqrt{-d}x\right)}{\left(d+\frac{e}{x^2}\right)x^3}\,dx, x, \sqrt[3]{x}\right)}{(-d)^{3/2}}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3$$
$$+ \frac{12b^2e^{3/2}n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}}$$
$$- \frac{12b^2e^{3/2}n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}}$$
$$- \frac{(6be^2n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}\,dx, x, \sqrt[3]{x}\right)}{d}$$
$$+ \frac{\left(24b^3e^{5/2}n^3\right)\text{Subst}\left(\int \left(\frac{\log\left(\sqrt{e}-\sqrt{-d}x\right)}{ex} - \frac{dx\log\left(\sqrt{e}-\sqrt{-d}x\right)}{e(e+dx^2)}\right)dx, x, \sqrt[3]{x}\right)}{(-d)^{3/2}}$$
$$- \frac{\left(24b^3e^{5/2}n^3\right)\text{Subst}\left(\int \left(\frac{\log\left(\sqrt{e}+\sqrt{-d}x\right)}{ex} - \frac{dx\log\left(\sqrt{e}+\sqrt{-d}x\right)}{e(e+dx^2)}\right)dx, x, \sqrt[3]{x}\right)}{(-d)^{3/2}}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d}+x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3$$
$$+\frac{12b^2e^{3/2}n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}}$$
$$-\frac{12b^2e^{3/2}n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}}$$
$$-\frac{(6be^2n)\operatorname{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}dx,x,\sqrt[3]{x}\right)}{d}$$
$$+\frac{\left(24b^3e^{3/2}n^3\right)\operatorname{Subst}\left(\int\frac{\log\left(\sqrt{e}-\sqrt{-d}x\right)}{x}dx,x,\sqrt[3]{x}\right)}{(-d)^{3/2}}$$
$$-\frac{\left(24b^3e^{3/2}n^3\right)\operatorname{Subst}\left(\int\frac{\log\left(\sqrt{e}+\sqrt{-d}x\right)}{x}dx,x,\sqrt[3]{x}\right)}{(-d)^{3/2}}$$
$$+\frac{\left(24b^3e^{3/2}n^3\right)\operatorname{Subst}\left(\int\frac{x\log\left(\sqrt{e}-\sqrt{-d}x\right)}{e+dx^2}dx,x,\sqrt[3]{x}\right)}{\sqrt{-d}}$$
$$-\frac{\left(24b^3e^{3/2}n^3\right)\operatorname{Subst}\left(\int\frac{x\log\left(\sqrt{e}+\sqrt{-d}x\right)}{e+dx^2}dx,x,\sqrt[3]{x}\right)}{\sqrt{-d}}$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d}+x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3$$
$$+\frac{12b^2e^{3/2}n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}}$$
$$-\frac{12b^2e^{3/2}n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}}$$
$$-\frac{24b^3e^{3/2}n^3\log\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)\log\left(-\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)}{(-d)^{3/2}}$$
$$+\frac{24b^3e^{3/2}n^3\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)\log\left(\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)}{(-d)^{3/2}}$$
$$-\frac{(6be^2n)\,\text{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}\,dx,x,\sqrt[3]{x}\right)}{d}$$
$$+\frac{\left(24b^3e^{3/2}n^3\right)\text{Subst}\left(\int\left(-\frac{\sqrt{-d}\log\left(\sqrt{e}-\sqrt{-d}x\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{\sqrt{-d}\log\left(\sqrt{e}-\sqrt{-d}x\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx,x,\sqrt[3]{x}\right)}{\sqrt{-d}}$$
$$-\frac{\left(24b^3e^{3/2}n^3\right)\text{Subst}\left(\int\left(-\frac{\sqrt{-d}\log\left(\sqrt{e}+\sqrt{-d}x\right)}{2d\left(\sqrt{e}-\sqrt{-d}x\right)}+\frac{\sqrt{-d}\log\left(\sqrt{e}+\sqrt{-d}x\right)}{2d\left(\sqrt{e}+\sqrt{-d}x\right)}\right)dx,x,\sqrt[3]{x}\right)}{\sqrt{-d}}$$
$$-\frac{\left(24b^3e^{3/2}n^3\right)\text{Subst}\left(\int\frac{\log\left(-\frac{\sqrt{-d}x}{\sqrt{e}}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx,x,\sqrt[3]{x}\right)}{d}$$
$$-\frac{\left(24b^3e^{3/2}n^3\right)\text{Subst}\left(\int\frac{\log\left(\frac{\sqrt{-d}x}{\sqrt{e}}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx,x,\sqrt[3]{x}\right)}{d}$$

$$
\begin{aligned}
&= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d}+x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\\
&+\frac{12b^2e^{3/2}n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}}\\
&-\frac{12b^2e^{3/2}n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}}\\
&-\frac{24b^3e^{3/2}n^3\log\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)\log\left(-\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)}{(-d)^{3/2}}\\
&+\frac{24b^3e^{3/2}n^3\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)\log\left(\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)}{(-d)^{3/2}}+\frac{24b^3e^{3/2}n^3\operatorname{Li}_2\left(1-\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)}{(-d)^{3/2}}\\
&-\frac{24b^3e^{3/2}n^3\operatorname{Li}_2\left(1+\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)}{(-d)^{3/2}}-\frac{(6be^2n)\operatorname{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}\,dx,x,\sqrt[3]{x}\right)}{d}\\
&-\frac{(12b^3e^{3/2}n^3)\operatorname{Subst}\left(\int\frac{\log\left(\sqrt{e}-\sqrt{-d}x\right)}{\sqrt{e}-\sqrt{-d}x}\,dx,x,\sqrt[3]{x}\right)}{d}\\
&+\frac{(12b^3e^{3/2}n^3)\operatorname{Subst}\left(\int\frac{\log\left(\sqrt{e}-\sqrt{-d}x\right)}{\sqrt{e}+\sqrt{-d}x}\,dx,x,\sqrt[3]{x}\right)}{d}\\
&+\frac{(12b^3e^{3/2}n^3)\operatorname{Subst}\left(\int\frac{\log\left(\sqrt{e}+\sqrt{-d}x\right)}{\sqrt{e}-\sqrt{-d}x}\,dx,x,\sqrt[3]{x}\right)}{d}\\
&-\frac{(12b^3e^{3/2}n^3)\operatorname{Subst}\left(\int\frac{\log\left(\sqrt{e}+\sqrt{-d}x\right)}{\sqrt{e}+\sqrt{-d}x}\,dx,x,\sqrt[3]{x}\right)}{d}
\end{aligned}
$$

$$= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d}+x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3$$
$$+\frac{12b^2e^{3/2}n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}}$$
$$-\frac{12b^2e^{3/2}n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}}$$
$$+\frac{12b^3e^{3/2}n^3\log\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)\log\left(\frac{1}{2}-\frac{\sqrt{-d}\sqrt[3]{x}}{2\sqrt{e}}\right)}{(-d)^{3/2}}$$
$$-\frac{12b^3e^{3/2}n^3\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)\log\left(\frac{1}{2}\left(1+\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)\right)}{(-d)^{3/2}}$$
$$-\frac{24b^3e^{3/2}n^3\log\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)\log\left(-\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)}{(-d)^{3/2}}$$
$$+\frac{24b^3e^{3/2}n^3\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)\log\left(\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)}{(-d)^{3/2}}+\frac{24b^3e^{3/2}n^3\text{Li}_2\left(1-\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)}{(-d)^{3/2}}$$
$$-\frac{24b^3e^{3/2}n^3\text{Li}_2\left(1+\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)}{(-d)^{3/2}}-\frac{(6be^2n)\,\text{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}\,dx,x,\sqrt[3]{x}\right)}{d}$$
$$-\frac{(12b^3e^{3/2}n^3)\,\text{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}}$$
$$+\frac{(12b^3e^{3/2}n^3)\,\text{Subst}\left(\int\frac{\log(x)}{x}\,dx,x,\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}}$$
$$+\frac{(12b^3e^{3/2}n^3)\,\text{Subst}\left(\int\frac{\log\left(\frac{\sqrt{e}-\sqrt{-d}x}{2\sqrt{e}}\right)}{\sqrt{e}+\sqrt{-d}x}\,dx,x,\sqrt[3]{x}\right)}{d}$$
$$+\frac{(12b^3e^{3/2}n^3)\,\text{Subst}\left(\int\frac{\log\left(\frac{\sqrt{e}+\sqrt{-d}x}{2\sqrt{e}}\right)}{\sqrt{e}-\sqrt{-d}x}\,dx,x,\sqrt[3]{x}\right)}{d}$$

$$
\begin{aligned}
&= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d}+x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 \\
&+\frac{12b^2e^{3/2}n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}} \\
&-\frac{6b^3e^{3/2}n^3\log^2\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}} \\
&-\frac{12b^2e^{3/2}n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}} \\
&+\frac{6b^3e^{3/2}n^3\log^2\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}} \\
&+\frac{12b^3e^{3/2}n^3\log\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)\log\left(\frac{1}{2}-\frac{\sqrt{-d}\sqrt[3]{x}}{2\sqrt{e}}\right)}{(-d)^{3/2}} \\
&-\frac{12b^3e^{3/2}n^3\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)\log\left(\frac{1}{2}\left(1+\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)\right)}{(-d)^{3/2}} \\
&-\frac{24b^3e^{3/2}n^3\log\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)\log\left(-\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)}{(-d)^{3/2}} \\
&+\frac{24b^3e^{3/2}n^3\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)\log\left(\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)}{(-d)^{3/2}}+\frac{24b^3e^{3/2}n^3\operatorname{Li}_2\left(1-\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)}{(-d)^{3/2}} \\
&-\frac{24b^3e^{3/2}n^3\operatorname{Li}_2\left(1+\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)}{(-d)^{3/2}}-\frac{(6be^2n)\operatorname{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}\,dx,x,\sqrt[3]{x}\right)}{d} \\
&+\frac{\left(12b^3e^{3/2}n^3\right)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{x}{2\sqrt{e}}\right)}{x}\,dx,x,\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}} \\
&-\frac{\left(12b^3e^{3/2}n^3\right)\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{x}{2\sqrt{e}}\right)}{x}\,dx,x,\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{6ben\sqrt[3]{x}\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{d} + x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 \\
&\quad + \frac{12b^2e^{3/2}n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}} \\
&\quad - \frac{6b^3e^{3/2}n^3\log^2\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}} \\
&\quad - \frac{12b^2e^{3/2}n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)\log\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}} \\
&\quad + \frac{6b^3e^{3/2}n^3\log^2\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)}{(-d)^{3/2}} \\
&\quad + \frac{12b^3e^{3/2}n^3\log\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)\log\left(\frac{1}{2}-\frac{\sqrt{-d}\sqrt[3]{x}}{2\sqrt{e}}\right)}{(-d)^{3/2}} \\
&\quad - \frac{12b^3e^{3/2}n^3\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)\log\left(\frac{1}{2}\left(1+\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)\right)}{(-d)^{3/2}} \\
&\quad - \frac{24b^3e^{3/2}n^3\log\left(\sqrt{e}+\sqrt{-d}\sqrt[3]{x}\right)\log\left(-\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)}{(-d)^{3/2}} \\
&\quad + \frac{24b^3e^{3/2}n^3\log\left(\sqrt{e}-\sqrt{-d}\sqrt[3]{x}\right)\log\left(\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)}{(-d)^{3/2}} + \frac{24b^3e^{3/2}n^3\mathrm{Li}_2\left(1-\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)}{(-d)^{3/2}} \\
&\quad - \frac{12b^3e^{3/2}n^3\mathrm{Li}_2\left(\frac{1}{2}-\frac{\sqrt{-d}\sqrt[3]{x}}{2\sqrt{e}}\right)}{(-d)^{3/2}} + \frac{12b^3e^{3/2}n^3\mathrm{Li}_2\left(\frac{1}{2}\left(1+\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)\right)}{(-d)^{3/2}} \\
&\quad - \frac{24b^3e^{3/2}n^3\mathrm{Li}_2\left(1+\frac{\sqrt{-d}\sqrt[3]{x}}{\sqrt{e}}\right)}{(-d)^{3/2}} - \frac{(6be^2n)\,\mathrm{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}\,dx, x, \sqrt[3]{x}\right)}{d}
\end{aligned}
$$

Mathematica [N/A]

Not integrable

Time = 4.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$
\int\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx = \int\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3\,dx
$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3, x]
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(a+b\ln \left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)\right)^{3}dx$$

[In] `int((a+b*ln(c*(d+e/x^(2/3))^n))^3,x)`

[Out] `int((a+b*ln(c*(d+e/x^(2/3))^n))^3,x)`

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 4.00

$$\int \left(a+b\log \left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{3}\,dx=\int \left(b\log \left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^{n}\right)+a\right)^{3}\,dx$$

[In] `integrate((a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="fricas")`

[Out] `integral(b^3*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x^(1/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(1/3))/x)^n) + a^3, x)`

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\log \left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{3}\,dx=\text{Timed out}$$

[In] `integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3,x)`

[Out] `Timed out`

Maxima [F(-2)]

Exception generated.

$$\int \left(a+b\log \left(c\left(d+\frac{e}{x^{2/3}}\right)^{n}\right)\right)^{3}\,dx=\text{Exception raised: ValueError}$$

[In] `integrate((a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="maxima")`

[Out] `Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e`

Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = \int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3 dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx = \int \left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3 dx$$

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^3,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))^3, x)
```

3.530 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}
\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx &= \frac{16b^3n^3}{9x}-\frac{208b^3dn^3}{3e\sqrt[3]{x}} \\
&-\frac{208b^3d^{3/2}n^3\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{3/2}}-\frac{32ib^3d^{3/2}n^3\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{e^{3/2}} \\
&+\frac{64b^3d^{3/2}n^3\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}} \\
&-\frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x}+\frac{32b^2dn^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} \\
&+\frac{32b^2d^{3/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{3/2}} \\
&+\frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}-\frac{6bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e\sqrt[3]{x}} \\
&-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}-\frac{32ib^3d^{3/2}n^3\operatorname{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}} \\
&-\frac{2bd^2n\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{\left(e+dx^{2/3}\right)x^{2/3}},x\right)}{e}
\end{aligned}$$

```
[Out] 16/9*b^3*n^3/x-208/3*b^3*d*n^3/e/x^(1/3)-208/3*b^3*d^(3/2)*n^3*arctan(x^(1/
3)*d^(1/2)/e^(1/2))/e^(3/2)-32*I*b^3*d^(3/2)*n^3*arctan(x^(1/3)*d^(1/2)/e^(
```

```
1/2))^2/e^(3/2)-8/3*b^2*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))/x+32*b^2*d*n^2*(a+b
*ln(c*(d+e/x^(2/3))^n))/e/x^(1/3)+32*b^2*d^(3/2)*n^2*arctan(x^(1/3)*d^(1/2)
/e^(1/2))*(a+b*ln(c*(d+e/x^(2/3))^n))/e^(3/2)+2*b*n*(a+b*ln(c*(d+e/x^(2/3))
^n))^2/x-6*b*d*n*(a+b*ln(c*(d+e/x^(2/3))^n))^2/e/x^(1/3)-(a+b*ln(c*(d+e/x^(
2/3))^n))^3/x+64*b^3*d^(3/2)*n^3*arctan(x^(1/3)*d^(1/2)/e^(1/2))*ln(2-2*e^(
1/2)/(-I*x^(1/3)*d^(1/2)+e^(1/2)))/e^(3/2)-32*I*b^3*d^(3/2)*n^3*polylog(2,-
1+2*e^(1/2)/(-I*x^(1/3)*d^(1/2)+e^(1/2)))/e^(3/2)-2*b*d^2*n*Unintegrable((a
+b*ln(c*(d+e/x^(2/3))^n))^2/(e+d*x^(2/3))/x^(2/3),x)/e
```

Rubi [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^2,x]
```

```
[Out] (16*b^3*n^3)/(9*x) - (208*b^3*d*n^3)/(3*e*x^(1/3)) - (208*b^3*d^(3/2)*n^3*A
rcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]])/(3*e^(3/2)) - ((32*I)*b^3*d^(3/2)*n^3*Arc
Tan[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/e^(3/2) + (64*b^3*d^(3/2)*n^3*ArcTan[(Sqr
t[d]*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/
e^(3/2) - (8*b^2*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(3*x) + (32*b^2*d*n^
2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(e*x^(1/3)) + (32*b^2*d^(3/2)*n^2*ArcTa
n[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/e^(3/2) + (2
*b*n*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/x - (6*b*d*n*(a + b*Log[c*(d + e/x
^(2/3))^n])^2)/(e*x^(1/3)) - (a + b*Log[c*(d + e/x^(2/3))^n])^3/x - ((32*I)
*b^3*d^(3/2)*n^3*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))]
)/e^(3/2) - (6*b*d^2*n*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)^n])
^2/(e + d*x^2), x], x, x^(1/3)])/e
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^3}{x^4}\,dx,x,\sqrt[3]{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}-(6ben)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{\left(d+\frac{e}{x^2}\right)x^6}\,dx,x,\sqrt[3]{x}\right)\\
&= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}\\
&\quad-(6ben)\text{Subst}\left(\int \left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{ex^4}-\frac{d\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e^2x^2}+\frac{d^2\left(a+b\log\left(c\left(d+\right.\right.\right.}{e^2\left(e+dx^2\right)}\right.\right.
\end{aligned}$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x} - (6bn)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{x^4}\,dx, x, \sqrt[3]{x}\right) + \frac{(6bdn)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{x^2}\,dx, x, \sqrt[3]{x}\right)}{e} - \frac{(6bd^2n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}\,dx, x, \sqrt[3]{x}\right)}{e}$$

$$= \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - \frac{6bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e\sqrt[3]{x}} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x} - \frac{(6bd^2n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}\,dx, x, \sqrt[3]{x}\right)}{e} - (24b^2dn^2)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{\left(d+\frac{e}{x^2}\right)x^4}\,dx, x, \sqrt[3]{x}\right) + (8b^2en^2)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)\right.}{\left(d+\frac{e}{x^2}\right)x^6}\right.$$

$$= \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - \frac{6bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e\sqrt[3]{x}} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x} - \frac{(6bd^2n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}\,dx, x, \sqrt[3]{x}\right)}{e} - (24b^2dn^2)\,\text{Subst}\left(\int \left(\frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{ex^2} - \frac{d\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)}{e\left(e+dx^2\right)}\right)dx, x, \sqrt[3]{x}\right) + (8b^2en$$

$$= \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - \frac{6bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e\sqrt[3]{x}} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x} - \frac{(6bd^2n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}\,dx, x, \sqrt[3]{x}\right)}{e} + (8b^2n^2)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{x^4}\,dx, x, \sqrt[3]{x}\right) - \frac{(8b^2dn^2)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{x^2}\,dx, x,\right.}{e}$$

$$= -\frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x}+\frac{32b^2dn^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}}$$
$$+\frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{3/2}}$$
$$+\frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}-\frac{6bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e\sqrt[3]{x}}$$
$$-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}-\frac{(6bd^2n)\,\text{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}\,dx,x,\sqrt[3]{x}\right)}{e}$$
$$+\left(16b^3dn^3\right)\text{Subst}\left(\int\frac{1}{\left(d+\frac{e}{x^2}\right)x^4}\,dx,x,\sqrt[3]{x}\right)+\left(48b^3dn^3\right)\text{Subst}\left(\int\frac{1}{\left(d+\frac{e}{x^2}\right)x^4}\,dx,x,\sqrt[3]{x}\right)+(1$$

$$= -\frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x}+\frac{32b^2dn^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}}$$
$$+\frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{3/2}}$$
$$+\frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x}-\frac{6bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e\sqrt[3]{x}}$$
$$-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x}-\frac{(6bd^2n)\,\text{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}\,dx,x,\sqrt[3]{x}\right)}{e}$$
$$+\left(16b^3dn^3\right)\text{Subst}\left(\int\frac{1}{x^2\left(e+dx^2\right)}\,dx,x,\sqrt[3]{x}\right)+\left(48b^3dn^3\right)\text{Subst}\left(\int\frac{1}{x^2\left(e+dx^2\right)}\,dx,x,\sqrt[3]{x}\right)+-$$

$$= \frac{16b^3n^3}{9x} - \frac{64b^3dn^3}{e\sqrt[3]{x}} - \frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} + \frac{32b^2dn^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} + \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{3/2}} + \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - \frac{6bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e\sqrt[3]{x}} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x} - \frac{(6bd^2n)\,\mathrm{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}\,dx,x,\sqrt[3]{x}\right)}{e} + \frac{1}{3}(16b^3dn^3)\,\mathrm{Subst}\left(\int\frac{1}{x^2(e+dx^2)}\,dx,x,\sqrt[3]{x}\right) - \frac{(16b^3d^2n^3)\,\mathrm{Subst}\left(\int\frac{1}{e+dx^2}\,dx,x,\sqrt[3]{x}\right)}{e} - \frac{(48b^3d^2n$$

$$= \frac{16b^3n^3}{9x} - \frac{208b^3dn^3}{3e\sqrt[3]{x}} - \frac{64b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{e^{3/2}} - \frac{32ib^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{e^{3/2}} - \frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} + \frac{32b^2dn^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}} + \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{3/2}} + \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - \frac{6bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e\sqrt[3]{x}} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x} - \frac{(6bd^2n)\,\mathrm{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}\,dx,x,\sqrt[3]{x}\right)}{e} + \frac{(16ib^3d^{3/2}n^3)\,\mathrm{Subst}\left(\int\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{e}}\right)}{x\left(i+\frac{\sqrt{d}x}{\sqrt{e}}\right)}\,dx,x,\sqrt[3]{x}\right)}{e^{3/2}} + \frac{(48ib^3d^{3/2}n^3)\,\mathrm{Subst}\left(\int\frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{e}}\right)}{x\left(i+\frac{\sqrt{d}x}{\sqrt{e}}\right)}\,dx,x,\sqrt[3]{x}\right)}{e^{3/2}} - \frac{(16b^3d^2n^3)\,\mathrm{Subst}\left(\int\frac{1}{e+dx^2}\,dx,x,\sqrt[3]{x}\right)}{3e}$$

$$= \frac{16b^3n^3}{9x} - \frac{208b^3dn^3}{3e\sqrt[3]{x}} - \frac{208b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{3/2}} - \frac{32ib^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{e^{3/2}}$$
$$+ \frac{64b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}}$$
$$- \frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} + \frac{32b^2dn^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}}$$
$$+ \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{3/2}}$$
$$+ \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - \frac{6bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e\sqrt[3]{x}}$$
$$- \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x} - \frac{(6bd^2n)\,\mathrm{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}\,dx,x,\sqrt[3]{x}\right)}{e}$$
$$- \frac{(16b^3d^2n^3)\,\mathrm{Subst}\left(\int\frac{\log\left(2-\frac{2}{1-\frac{i\sqrt{d}x}{\sqrt{e}}}\right)}{1+\frac{dx^2}{e}}\,dx,x,\sqrt[3]{x}\right)}{e^2}$$
$$- \frac{(48b^3d^2n^3)\,\mathrm{Subst}\left(\int\frac{\log\left(2-\frac{2}{1-\frac{i\sqrt{d}x}{\sqrt{e}}}\right)}{1+\frac{dx^2}{e}}\,dx,x,\sqrt[3]{x}\right)}{e^2}$$

$$= \frac{16b^3n^3}{9x} - \frac{208b^3dn^3}{3e\sqrt[3]{x}} - \frac{208b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{3e^{3/2}} - \frac{32ib^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{e^{3/2}}$$
$$+ \frac{64b^3d^{3/2}n^3\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}}$$
$$- \frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{3x} + \frac{32b^2dn^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e\sqrt[3]{x}}$$
$$+ \frac{32b^2d^{3/2}n^2\tan^{-1}\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{e^{3/2}}$$
$$+ \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{x} - \frac{6bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e\sqrt[3]{x}}$$
$$- \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x} - \frac{32ib^3d^{3/2}n^3\operatorname{Li}_2\left(-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{e^{3/2}}$$
$$- \frac{(6bd^2n)\operatorname{Subst}\left(\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}dx, x, \sqrt[3]{x}\right)}{e}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 5504 vs. 2(483) = 966.

Time = 13.16 (sec) , antiderivative size = 5504, normalized size of antiderivative = 229.33

$$\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^2,x]
[Out] Result too large to show
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int\frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)^3}{x^2}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^2,x)
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.50

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x^
(1/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(1/3))/x)^n) + a^3)/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3/x**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{2/3}}\right)}^n\right)\right)^3}{x^2}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^3/x^2,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))^3/x^2, x)
```

3.531 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx=\frac{16b^3n^3}{729x^3}-\frac{3088b^3dn^3}{27783ex^{7/3}}+\frac{221344b^3d^2n^3}{496125e^2x^{5/3}}-\frac{637984b^3d^3n^3}{297675e^3x}+\frac{3475504b^3d^4n^3}{99225e^4\sqrt[3]{x}}+\frac{3475504b^3d^{9/2}n^3\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)}{99225e^{9/2}}+\frac{4504ib^3d^{9/2}n^3\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)^2}{315e^{9/2}}-\frac{9008b^3d^{9/2}n^3\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\log\left(2-\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{315e^{9/2}}-\frac{8b^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{81x^3}+\frac{128b^2dn^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{441ex^{7/3}}-\frac{1144b^2d^2n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{1575e^2x^{5/3}}+\frac{1984b^2d^3n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{945e^3x}-\frac{4504b^2d^4n^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{315e^4\sqrt[3]{x}}-\frac{4504b^2d^{9/2}n^2\arctan\left(\frac{\sqrt{d}\sqrt[3]{x}}{\sqrt{e}}\right)\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)}{315e^{9/2}}+\frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{9x^3}-\frac{2bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{7ex^{7/3}}+\frac{2bd^2n\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{5e^2x^{5/3}}-\frac{2bd^3n\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{3e^3x}+\frac{2bd^4n\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e^4\sqrt[3]{x}}-\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{3x^3}+\frac{4504ib^3d^{9/2}n^3\,\text{PolyLog}\left(2,-1+\frac{2\sqrt{e}}{\sqrt{e}-i\sqrt{d}\sqrt[3]{x}}\right)}{315e^{9/2}}+\frac{2bd^5n\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{\left(e+dx^{2/3}\right)x^{2/3}},x\right)}{3e^4}$$

```
[Out] 16/729*b^3*n^3/x^3-3088/27783*b^3*d*n^3/e/x^(7/3)+221344/496125*b^3*d^2*n^3
/e^2/x^(5/3)-637984/297675*b^3*d^3*n^3/e^3/x+3475504/99225*b^3*d^4*n^3/e^4/
x^(1/3)+3475504/99225*b^3*d^(9/2)*n^3*arctan(x^(1/3)*d^(1/2)/e^(1/2))/e^(9/
2)+4504/315*I*b^3*d^(9/2)*n^3*polylog(2,-1+2*e^(1/2)/(-I*x^(1/3)*d^(1/2)+e^
```

```
(1/2)))/e^(9/2)-8/81*b^2*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))/x^3+128/441*b^2*d*
n^2*(a+b*ln(c*(d+e/x^(2/3))^n))/e/x^(7/3)-1144/1575*b^2*d^2*n^2*(a+b*ln(c*(
d+e/x^(2/3))^n))/e^2/x^(5/3)+1984/945*b^2*d^3*n^2*(a+b*ln(c*(d+e/x^(2/3))^n
))/e^3/x-4504/315*b^2*d^4*n^2*(a+b*ln(c*(d+e/x^(2/3))^n))/e^4/x^(1/3)-4504/
315*b^2*d^(9/2)*n^2*arctan(x^(1/3)*d^(1/2)/e^(1/2))*(a+b*ln(c*(d+e/x^(2/3))
^n))/e^(9/2)+2/9*b*n*(a+b*ln(c*(d+e/x^(2/3))^n))^2/x^3-2/7*b*d*n*(a+b*ln(c*
(d+e/x^(2/3))^n))^2/e/x^(7/3)+2/5*b*d^2*n*(a+b*ln(c*(d+e/x^(2/3))^n))^2/e^2
/x^(5/3)-2/3*b*d^3*n*(a+b*ln(c*(d+e/x^(2/3))^n))^2/e^3/x+2*b*d^4*n*(a+b*ln(
c*(d+e/x^(2/3))^n))^2/e^4/x^(1/3)-1/3*(a+b*ln(c*(d+e/x^(2/3))^n))^3/x^3-900
8/315*b^3*d^(9/2)*n^3*arctan(x^(1/3)*d^(1/2)/e^(1/2))*ln(2-2*e^(1/2)/(-I*x^
(1/3)*d^(1/2)+e^(1/2)))/e^(9/2)+4504/315*I*b^3*d^(9/2)*n^3*arctan(x^(1/3)*d
^(1/2)/e^(1/2))^2/e^(9/2)+2/3*b*d^5*n*Unintegrable((a+b*ln(c*(d+e/x^(2/3))^
n))^2/(e+d*x^(2/3))/x^(2/3),x)/e^4
```

Rubi [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^4,x]
```

```
[Out] (16*b^3*n^3)/(729*x^3) - (3088*b^3*d*n^3)/(27783*e*x^(7/3)) + (221344*b^3*d
^2*n^3)/(496125*e^2*x^(5/3)) - (637984*b^3*d^3*n^3)/(297675*e^3*x) + (34755
04*b^3*d^4*n^3)/(99225*e^4*x^(1/3)) + (3475504*b^3*d^(9/2)*n^3*ArcTan[(Sqrt
[d]*x^(1/3))/Sqrt[e]])/(99225*e^(9/2)) + (((4504*I)/315)*b^3*d^(9/2)*n^3*Ar
cTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]^2)/e^(9/2) - (9008*b^3*d^(9/2)*n^3*ArcTan[(
Sqrt[d]*x^(1/3))/Sqrt[e]]*Log[2 - (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))
])/(315*e^(9/2)) - (8*b^2*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(81*x^3) +
(128*b^2*d*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(441*e*x^(7/3)) - (1144*b^
2*d^2*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(1575*e^2*x^(5/3)) + (1984*b^2*
d^3*n^2*(a + b*Log[c*(d + e/x^(2/3))^n]))/(945*e^3*x) - (4504*b^2*d^4*n^2*(
a + b*Log[c*(d + e/x^(2/3))^n]))/(315*e^4*x^(1/3)) - (4504*b^2*d^(9/2)*n^2*
ArcTan[(Sqrt[d]*x^(1/3))/Sqrt[e]]*(a + b*Log[c*(d + e/x^(2/3))^n]))/(315*e^
(9/2)) + (2*b*n*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(9*x^3) - (2*b*d*n*(a +
 b*Log[c*(d + e/x^(2/3))^n])^2)/(7*e*x^(7/3)) + (2*b*d^2*n*(a + b*Log[c*(d
+ e/x^(2/3))^n])^2)/(5*e^2*x^(5/3)) - (2*b*d^3*n*(a + b*Log[c*(d + e/x^(2/3
))^n])^2)/(3*e^3*x) + (2*b*d^4*n*(a + b*Log[c*(d + e/x^(2/3))^n])^2)/(e^4*x
^(1/3)) - (a + b*Log[c*(d + e/x^(2/3))^n])^3/(3*x^3) + (((4504*I)/315)*b^3*
d^(9/2)*n^3*PolyLog[2, -1 + (2*Sqrt[e])/(Sqrt[e] - I*Sqrt[d]*x^(1/3))])/e^(
9/2) + (2*b*d^5*n*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)^n])^2/(e
 + d*x^2), x], x, x^(1/3)])/e^4
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^3}{x^{10}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{3x^3} - (2ben)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{\left(d+\frac{e}{x^2}\right)x^{12}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{3x^3} - (2ben)\text{Subst}\left(\int \left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{ex^{10}} - \frac{d\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e^2x^8} + \frac{d^2\left(a+b\log\left(c\left(d+\right.\right.\right.}{e^3x^6}\right.\right.$$

$$= -\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{3x^3} - (2bn)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{x^{10}}\,dx, x, \sqrt[3]{x}\right) - \frac{(2bd^4n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{x^2}\,dx, x, \sqrt[3]{x}\right)}{e^4} + \frac{(2bd^5n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}\,dx, x, \sqrt[3]{x}\right)}{e^4} + \frac{(2bd^3n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{x^4}\,dx, x, \sqrt[3]{x}\right)}{e^3} - \frac{(2bd^2n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{x^6}\,dx, x, \sqrt[3]{x}\right)}{e^2} + \frac{(2bdn)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{x^8}\,dx, x, \sqrt[3]{x}\right)}{e}$$

$$
\begin{aligned}
&= \frac{2bn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{9x^3} - \frac{2bdn\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{7ex^{7/3}} \\
&\quad + \frac{2bd^2n\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{5e^2x^{5/3}} - \frac{2bd^3n\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{3e^3x} \\
&\quad + \frac{2bd^4n\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^2}{e^4\sqrt[3]{x}} - \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{3x^3} \\
&\quad + \frac{(2bd^5n)\,\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)\right)^2}{e+dx^2}\,dx, x, \sqrt[3]{x}\right)}{e^4} \\
&\quad - \frac{1}{7}(8b^2dn^2)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{\left(d+\frac{e}{x^2}\right)x^{10}}\,dx, x, \sqrt[3]{x}\right) + \frac{(8b^2d^4n^2)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{e}{x^2}\right)^n\right)}{\left(d+\frac{e}{x^2}\right)x^4}\,d\right.}{e^3} \\
&= \text{Too large to display}
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 6328 vs. $2(784) = 1568$.

Time = 21.30 (sec) , antiderivative size = 6328, normalized size of antiderivative = 263.67

$$
\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^n])^3/x^4,x]
```

```
[Out] Result too large to show
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$
\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)\right)^3}{x^4}dx
$$

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(2/3))^n))^3/x^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.50

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*log(c*((d*x + e*x^(1/3))/x)^n)^3 + 3*a*b^2*log(c*((d*x + e*x^
(1/3))/x)^n)^2 + 3*a^2*b*log(c*((d*x + e*x^(1/3))/x)^n) + a^3)/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**n))**3/x**4,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^n\right)+a\right)^3}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^n))^3/x^4,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^n) + a)^3/x^4, x)
```

Mupad [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx = \int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^n\right)\right)^3}{x^4}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(2/3))^n))^3/x^4,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^n))^3/x^4, x)
```

3.532 $\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 730

$$
\begin{aligned}
\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx
&= \frac{2^{-2-3p}e^{-\frac{8a}{b}}\Gamma\left(1+p,-\frac{8\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^8e^8} \\
&- \frac{2\;7^{-p}de^{-\frac{7a}{b}}\Gamma\left(1+p,-\frac{7\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^7e^8} \\
&+ \frac{7\;6^{-p}d^2e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^6e^8} \\
&- \frac{14\;5^{-p}d^3e^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^5e^8} \\
&+ \frac{35\;2^{-1-2p}d^4e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^4e^8} \\
&- \frac{14\;3^{-p}d^5e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^3e^8} \\
&+ \frac{7\;2^{-p}d^6e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^2e^8} \\
&- \frac{2d^7e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{ce^8}
\end{aligned}
$$

```
[Out] 2^(-2-3*p)*GAMMA(p+1,-8*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2)
)))^p/c^8/e^8/exp(8*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d*GAMMA(p+1,-
7*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(7^p)/c^7/e^8/ex
```

```
p(7*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)+7*d^2*GAMMA(p+1,-6*(a+b*ln(c*(d
+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(6^p)/c^6/e^8/exp(6*a/b)/(((-a
-b*ln(c*(d+e*x^(1/2))))/b)^p)-14*d^3*GAMMA(p+1,-5*(a+b*ln(c*(d+e*x^(1/2))))
/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(5^p)/c^5/e^8/exp(5*a/b)/(((-a-b*ln(c*(d+e*
x^(1/2))))/b)^p)+35*2^(-1-2*p)*d^4*GAMMA(p+1,-4*(a+b*ln(c*(d+e*x^(1/2))))/b
)*(a+b*ln(c*(d+e*x^(1/2))))^p/c^4/e^8/exp(4*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))
))/b)^p)-14*d^5*GAMMA(p+1,-3*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^
(1/2))))^p/(3^p)/c^3/e^8/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)+7*d^
6*GAMMA(p+1,-2*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(2^
p)/c^2/e^8/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d^7*GAMMA(p+1,(-
a-b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/c/e^8/exp(a/b)/(((-
a-b*ln(c*(d+e*x^(1/2))))/b)^p)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 730, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2504, 2448, 2436, 2336, 2212, 2437, 2346}

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$
$$=\frac{2^{-3p-2}e^{-\frac{8a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{8\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^8e^8}$$
$$-\frac{2d7^{-p}e^{-\frac{7a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{7\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^7e^8}$$
$$+\frac{7d^26^{-p}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^6e^8}$$
$$-\frac{14d^35^{-p}e^{-\frac{5a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^5e^8}$$
$$+\frac{35d^42^{-2p-1}e^{-\frac{4a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^4e^8}$$
$$-\frac{14d^53^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^3e^8}$$
$$+\frac{7d^62^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^2e^8}$$
$$-\frac{2d^7e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)}{ce^8}$$

```
[In] Int[x^3*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (2^(-2 - 3*p)*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Lo
g[c*(d + e*Sqrt[x])])^p)/(c^8*e^8*E^((8*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x
])])/b))^p) - (2*d*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a +
 b*Log[c*(d + e*Sqrt[x])])^p)/(7^p*c^7*e^8*E^((7*a)/b)*(-((a + b*Log[c*(d +
 e*Sqrt[x])])/b))^p) + (7*d^2*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*Sqrt[x])
]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(6^p*c^6*e^8*E^((6*a)/b)*(-((a + b
*Log[c*(d + e*Sqrt[x])])/b))^p) - (14*d^3*Gamma[1 + p, (-5*(a + b*Log[c*(d
+ e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(5^p*c^5*e^8*E^((5*a)/
b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) + (35*2^(-1 - 2*p)*d^4*Gamma[1
+ p, (-4*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^
p)/(c^4*e^8*E^((4*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) - (14*d^5*
Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqr
t[x])])^p)/(3^p*c^3*e^8*E^((3*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p
) + (7*d^6*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c
*(d + e*Sqrt[x])])^p)/(2^p*c^2*e^8*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[
x])])/b))^p) - (2*d^7*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])])/b)]*(a
+ b*Log[c*(d + e*Sqrt[x])])^p)/(c*e^8*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x
])])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^7(a+b\log(c(d+ex)))^p\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(-\frac{d^7(a+b\log(c(d+ex)))^p}{e^7}+\frac{7d^6(d+ex)(a+b\log(c(d+ex)))^p}{e^7}\right.\right.\\
&\quad -\frac{21d^5(d+ex)^2(a+b\log(c(d+ex)))^p}{e^7}+\frac{35d^4(d+ex)^3(a+b\log(c(d+ex)))^p}{e^7}\\
&\quad -\frac{35d^3(d+ex)^4(a+b\log(c(d+ex)))^p}{e^7}+\frac{21d^2(d+ex)^5(a+b\log(c(d+ex)))^p}{e^7}\\
&\quad -\frac{7d(d+ex)^6(a+b\log(c(d+ex)))^p}{e^7}\\
&\quad \left.\left.+\frac{(d+ex)^7(a+b\log(c(d+ex)))^p}{e^7}\right)\,dx, x, \sqrt{x}\right)
\end{aligned}$$

$$= \frac{2\text{Subst}\left(\int (d+ex)^7(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)}{e^7} - \frac{(14d)\text{Subst}\left(\int (d+ex)^6(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)}{e^7} + \frac{(42d^2)\,\text{Subst}\left(\int (d+ex)^5(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)}{e^7} - \frac{(70d^3)\,\text{Subst}\left(\int (d+ex)^4(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)}{e^7} + \frac{(70d^4)\,\text{Subst}\left(\int (d+ex)^3(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)}{e^7} - \frac{(42d^5)\,\text{Subst}\left(\int (d+ex)^2(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)}{e^7} + \frac{(14d^6)\,\text{Subst}\left(\int (d+ex)(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)}{e^7} - \frac{(2d^7)\,\text{Subst}\left(\int (a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)}{e^7}$$

$$= \frac{2\text{Subst}\left(\int x^7(a+b\log(cx))^p\,dx,x,d+e\sqrt{x}\right)}{e^8} - \frac{(14d)\text{Subst}\left(\int x^6(a+b\log(cx))^p\,dx,x,d+e\sqrt{x}\right)}{e^8} + \frac{(42d^2)\,\text{Subst}\left(\int x^5(a+b\log(cx))^p\,dx,x,d+e\sqrt{x}\right)}{e^8} - \frac{(70d^3)\,\text{Subst}\left(\int x^4(a+b\log(cx))^p\,dx,x,d+e\sqrt{x}\right)}{e^8} + \frac{(70d^4)\,\text{Subst}\left(\int x^3(a+b\log(cx))^p\,dx,x,d+e\sqrt{x}\right)}{e^8} - \frac{(42d^5)\,\text{Subst}\left(\int x^2(a+b\log(cx))^p\,dx,x,d+e\sqrt{x}\right)}{e^8} + \frac{(14d^6)\,\text{Subst}\left(\int x(a+b\log(cx))^p\,dx,x,d+e\sqrt{x}\right)}{e^8} - \frac{(2d^7)\,\text{Subst}\left(\int (a+b\log(cx))^p\,dx,x,d+e\sqrt{x}\right)}{e^8}$$

$$
\begin{aligned}
&= \frac{2\text{Subst}\left(\int e^{8x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^8e^8} \\
&\quad - \frac{(14d)\text{Subst}\left(\int e^{7x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^7e^8} \\
&\quad + \frac{(42d^2)\,\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^6e^8} \\
&\quad - \frac{(70d^3)\,\text{Subst}\left(\int e^{5x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^5e^8} \\
&\quad + \frac{(70d^4)\,\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^4e^8} \\
&\quad - \frac{(42d^5)\,\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^3e^8} \\
&\quad + \frac{(14d^6)\,\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^2e^8} \\
&\quad - \frac{(2d^7)\,\text{Subst}\left(\int e^{x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{ce^8}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2^{-2-3p}e^{-\frac{8a}{b}}\Gamma\left(1+p,-\frac{8\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^8e^8} \\
&\quad - \frac{2\ 7^{-p}de^{-\frac{7a}{b}}\Gamma\left(1+p,-\frac{7\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^7e^8} \\
&\quad + \frac{7\ 6^{-p}d^2e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^6e^8} \\
&\quad - \frac{14\ 5^{-p}d^3e^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^5e^8} \\
&\quad + \frac{35\ 2^{-1-2p}d^4e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^4e^8} \\
&\quad - \frac{14\ 3^{-p}d^5e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^3e^8} \\
&\quad + \frac{7\ 2^{-p}d^6e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^2e^8} \\
&\quad - \frac{2d^7e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{ce^8}

Mathematica [F]

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
[Out] Integrate[x^3*(a + b*Log[c*(d + e*Sqrt[x])])^p, x]
```

Maple [F]

$$\int x^3\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)\right)\right)^p dx$$

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
[Out] int(x^3*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

Fricas [F]

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p x^3\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e*sqrt(x) + c*d) + a)^p*x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/2))))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int x^3 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx = \int \left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p x^3\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x^3, x)
```

Giac [F]

$$\int x^3 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx = \int \left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p x^3\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx = \int x^3\,\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)\right)\right)^p\,dx$$

```
[In] int(x^3*(a + b*log(c*(d + e*x^(1/2))))^p,x)
[Out] int(x^3*(a + b*log(c*(d + e*x^(1/2))))^p, x)
```

3.533 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 551

$$
\begin{aligned}
&\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx \\
&=\frac{2^{-p}3^{-1-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^6e^6} \\
&-\frac{2\,5^{-p}de^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^5e^6} \\
&+\frac{5\,4^{-p}d^2e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^4e^6} \\
&-\frac{20\,3^{-1-p}d^3e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^3e^6} \\
&+\frac{5\,2^{-p}d^4e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^2e^6} \\
&-\frac{2d^5e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{ce^6}
\end{aligned}
$$

```
[Out] 3^(-1-p)*GAMMA(p+1,-6*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2)))
)^p/(2^p)/c^6/e^6/exp(6*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d*GAMMA(p
+1,-5*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(5^p)/c^5/e^
6/exp(5*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)+5*d^2*GAMMA(p+1,-4*(a+b*ln(
c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(4^p)/c^4/e^6/exp(4*a/b)/(
((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)-20*3^(-1-p)*d^3*GAMMA(p+1,-3*(a+b*ln(c*(d
+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/c^3/e^6/exp(3*a/b)/(((-a-b*ln(
c*(d+e*x^(1/2))))/b)^p)+5*d^4*GAMMA(p+1,-2*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+
```

```
b*ln(c*(d+e*x^(1/2))))^p/(2^p)/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2)
)))/b)^p)-2*d^5*GAMMA(p+1,(-a-b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1
/2))))^p/c/e^6/exp(a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2504, 2448, 2436, 2336, 2212, 2437, 2346}

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$
$$=\frac{2^{-p}3^{-p-1}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^6e^6}$$
$$-\frac{2d5^{-p}e^{-\frac{5a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^5e^6}$$
$$+\frac{5d^2 4^{-p}e^{-\frac{4a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^4e^6}$$
$$-\frac{20d^3 3^{-p-1}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^3e^6}$$
$$+\frac{5d^4 2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^2e^6}$$
$$-\frac{2d^5e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)}{ce^6}$$

```
[In] Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (3^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[
c*(d + e*Sqrt[x])])^p)/(2^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*Sqrt
[x])])/b))^p) - (2*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a
 + b*Log[c*(d + e*Sqrt[x])])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c*(d
 + e*Sqrt[x])])/b))^p) + (5*d^2*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x
])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(4^p*c^4*e^6*E^((4*a)/b)*(-((a +
 b*Log[c*(d + e*Sqrt[x])])/b))^p) - (20*3^(-1 - p)*d^3*Gamma[1 + p, (-3*(a
+ b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(c^3*e^6*
E^((3*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) + (5*d^4*Gamma[1 + p,
(-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(2
^p*c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p) - (2*d^5*Ga
mma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])])/b)]*(a + b*Log[c*(d + e*Sqrt[x]
)])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int x^5(a+b\log(c(d+ex)))^p\,dx, x, \sqrt{x}\right)$$
$$= 2\text{Subst}\left(\int\left(-\frac{d^5(a+b\log(c(d+ex)))^p}{e^5} + \frac{5d^4(d+ex)(a+b\log(c(d+ex)))^p}{e^5} - \frac{10d^3(d+ex)^2(a+b\log(c(d+ex)))^p}{e^5} + \frac{10d^2(d+ex)^3(a+b\log(c(d+ex)))^p}{e^5} - \frac{5d(d+ex)^4(a+b\log(c(d+ex)))^p}{e^5} + \frac{(d+ex)^5(a+b\log(c(d+ex)))^p}{e^5}\right)dx, x, \sqrt{x}\right)$$
$$= \frac{2\text{Subst}\left(\int(d+ex)^5(a+b\log(c(d+ex)))^p\,dx, x, \sqrt{x}\right)}{e^5} - \frac{(10d)\text{Subst}\left(\int(d+ex)^4(a+b\log(c(d+ex)))^p\,dx, x, \sqrt{x}\right)}{e^5} + \frac{(20d^2)\,\text{Subst}\left(\int(d+ex)^3(a+b\log(c(d+ex)))^p\,dx, x, \sqrt{x}\right)}{e^5} - \frac{(20d^3)\,\text{Subst}\left(\int(d+ex)^2(a+b\log(c(d+ex)))^p\,dx, x, \sqrt{x}\right)}{e^5} + \frac{(10d^4)\,\text{Subst}\left(\int(d+ex)(a+b\log(c(d+ex)))^p\,dx, x, \sqrt{x}\right)}{e^5} - \frac{(2d^5)\,\text{Subst}\left(\int(a+b\log(c(d+ex)))^p\,dx, x, \sqrt{x}\right)}{e^5}$$
$$= \frac{2\text{Subst}\left(\int x^5(a+b\log(cx))^p\,dx, x, d+e\sqrt{x}\right)}{e^6} - \frac{(10d)\text{Subst}\left(\int x^4(a+b\log(cx))^p\,dx, x, d+e\sqrt{x}\right)}{e^6} + \frac{(20d^2)\,\text{Subst}\left(\int x^3(a+b\log(cx))^p\,dx, x, d+e\sqrt{x}\right)}{e^6} - \frac{(20d^3)\,\text{Subst}\left(\int x^2(a+b\log(cx))^p\,dx, x, d+e\sqrt{x}\right)}{e^6} + \frac{(10d^4)\,\text{Subst}\left(\int x(a+b\log(cx))^p\,dx, x, d+e\sqrt{x}\right)}{e^6} - \frac{(2d^5)\,\text{Subst}\left(\int(a+b\log(cx))^p\,dx, x, d+e\sqrt{x}\right)}{e^6}$$

$$= \frac{2\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^6e^6} - \frac{(10d)\text{Subst}\left(\int e^{5x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^5e^6} + \frac{(20d^2)\,\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^4e^6} - \frac{(20d^3)\,\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^3e^6} + \frac{(10d^4)\,\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^2e^6} - \frac{(2d^5)\,\text{Subst}\left(\int e^{x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{ce^6}$$

$$= \frac{2^{-p}3^{-1-p}e^{-\frac{6a}{b}}\Gamma\left(1+p, -\frac{6\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^6e^6} - \frac{2\ 5^{-p}de^{-\frac{5a}{b}}\Gamma\left(1+p, -\frac{5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^5e^6} + \frac{5\ 4^{-p}d^2e^{-\frac{4a}{b}}\Gamma\left(1+p, -\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^4e^6} - \frac{20\ 3^{-1-p}d^3e^{-\frac{3a}{b}}\Gamma\left(1+p, -\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^3e^6} + \frac{5\ 2^{-p}d^4e^{-\frac{2a}{b}}\Gamma\left(1+p, -\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^2e^6} - \frac{2d^5e^{-\frac{a}{b}}\Gamma\left(1+p, -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{ce^6}$$

Mathematica [F]

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx = \int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
[Out] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])])^p, x]
```

Maple [F]

$$\int x^2 \left(a+b\ln\left(c\left(d+e\sqrt{x}\right)\right)\right)^p dx$$

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

Fricas [F]

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p \, dx = \int \left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p x^2 \, dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e*sqrt(x) + c*d) + a)^p*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p \, dx = \text{Timed out}$$

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p \, dx = \int \left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p x^2 \, dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x^2, x)
```

Giac [F]

$$\int x^2\big(a+b\log\big(c\big(d+e\sqrt{x}\big)\big)\big)^p\,dx=\int\big(b\log\big(\big(e\sqrt{x}+d\big)c\big)+a\big)^p x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\big(a+b\log\big(c\big(d+e\sqrt{x}\big)\big)\big)^p\,dx=\int x^2\,\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)\right)\right)^p\,dx$$

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/2))))^p,x)
[Out] int(x^2*(a + b*log(c*(d + e*x^(1/2))))^p, x)
```

3.534 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 360

$$\begin{aligned}
&\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx\\
&=\frac{2^{-1-2p}e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^4e^4}\\
&-\frac{2\ 3^{-p}de^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^3e^4}\\
&+\frac{3\ 2^{-p}d^2e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^2e^4}\\
&-\frac{2d^3e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{ce^4}
\end{aligned}$$

```
[Out] 2^(-1-2*p)*GAMMA(p+1,-4*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2)
)))^p/c^4/e^4/exp(4*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d*GAMMA(p+1,-
3*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(3^p)/c^3/e^4/ex
p(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)+3*d^2*GAMMA(p+1,-2*(a+b*ln(c*(d
+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(2^p)/c^2/e^4/exp(2*a/b)/(((-a
-b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d^3*GAMMA(p+1,(-a-b*ln(c*(d+e*x^(1/2))))/b)
*(a+b*ln(c*(d+e*x^(1/2))))^p/c/e^4/exp(a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)
^p)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2504, 2448, 2436, 2336, 2212, 2437, 2346}

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$
$$=\frac{2^{-2p-1}e^{-\frac{4a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^4e^4}$$
$$-\frac{2d3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^3e^4}$$
$$+\frac{3d^22^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^2e^4}$$
$$-\frac{2d^3e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)}{ce^4}$$

```
[In] Int[x*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (2^(-1 - 2*p)*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Lo
g[c*(d + e*Sqrt[x])])^p)/(c^4*e^4*E^((4*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x
])])/b))^p) - (2*d*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a +
 b*Log[c*(d + e*Sqrt[x])])^p)/(3^p*c^3*e^4*E^((3*a)/b)*(-((a + b*Log[c*(d +
 e*Sqrt[x])])/b))^p) + (3*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])
]))/b]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(2^p*c^2*e^4*E^((2*a)/b)*(-((a + b
*Log[c*(d + e*Sqrt[x])])/b))^p) - (2*d^3*Gamma[1 + p, -((a + b*Log[c*(d + e
*Sqrt[x])])/b)]*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(c*e^4*E^(a/b)*(-((a + b*
Log[c*(d + e*Sqrt[x])])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int x^3(a+b\log(c(d+ex)))^p\,dx, x, \sqrt{x}\right)\\ &= 2\text{Subst}\left(\int \left(-\frac{d^3(a+b\log(c(d+ex)))^p}{e^3}+\frac{3d^2(d+ex)(a+b\log(c(d+ex)))^p}{e^3}\right.\right.\\ &\qquad\left.\left. -\frac{3d(d+ex)^2(a+b\log(c(d+ex)))^p}{e^3}+\frac{(d+ex)^3(a+b\log(c(d+ex)))^p}{e^3}\right)dx, x, \sqrt{x}\right)\end{aligned}$$

$$= \frac{2\text{Subst}\left(\int (d+ex)^3(a+b\log(c(d+ex)))^p\,dx, x, \sqrt{x}\right)}{e^3} - \frac{(6d)\text{Subst}\left(\int (d+ex)^2(a+b\log(c(d+ex)))^p\,dx, x, \sqrt{x}\right)}{e^3} + \frac{(6d^2)\,\text{Subst}\left(\int (d+ex)(a+b\log(c(d+ex)))^p\,dx, x, \sqrt{x}\right)}{e^3} - \frac{(2d^3)\,\text{Subst}\left(\int (a+b\log(c(d+ex)))^p\,dx, x, \sqrt{x}\right)}{e^3}$$

$$= \frac{2\text{Subst}\left(\int x^3(a+b\log(cx))^p\,dx, x, d+e\sqrt{x}\right)}{e^4} - \frac{(6d)\text{Subst}\left(\int x^2(a+b\log(cx))^p\,dx, x, d+e\sqrt{x}\right)}{e^4} + \frac{(6d^2)\,\text{Subst}\left(\int x(a+b\log(cx))^p\,dx, x, d+e\sqrt{x}\right)}{e^4} - \frac{(2d^3)\,\text{Subst}\left(\int (a+b\log(cx))^p\,dx, x, d+e\sqrt{x}\right)}{e^4}$$

$$= \frac{2\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^4e^4} - \frac{(6d)\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^3e^4} + \frac{(6d^2)\,\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^2e^4} - \frac{(2d^3)\,\text{Subst}\left(\int e^{x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{ce^4}$$

$$= \frac{2^{-1-2p}e^{-\frac{4a}{b}}\Gamma\left(1+p, -\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^4e^4} - \frac{2\ 3^{-p}de^{-\frac{3a}{b}}\Gamma\left(1+p, -\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^3e^4} + \frac{3\ 2^{-p}d^2e^{-\frac{2a}{b}}\Gamma\left(1+p, -\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^2e^4} - \frac{2d^3e^{-\frac{a}{b}}\Gamma\left(1+p, -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{ce^4}$$

Mathematica [A] (verified)

Time = 4.04 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.64

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$
$$=\frac{2^{-1-2p}3^{-p}e^{-\frac{4a}{b}}\left(3^p\Gamma\left(1+p,-\frac{4(a+b\log(c(d+e\sqrt{x})))}{b}\right)-2^{1+p}cde^{a/b}\left(2^{1+p}\Gamma\left(1+p,-\frac{3(a+b\log(c(d+e\sqrt{x})))}{b}\right)+3^pcd\right.\right.}{}$$

```
[In] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (2^(-1 - 2*p)*(3^p*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x])]))/b] - 2^
(1 + p)*c*d*E^(a/b)*(2^(1 + p)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x]
)]))/b] + 3^p*c*d*E^(a/b)*(-3*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])
]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])]
)/b)])))*(a + b*Log[c*(d + e*Sqrt[x])])^p)/(3^p*c^4*e^4*E^((4*a)/b)*(-((a +
 b*Log[c*(d + e*Sqrt[x])])/b))^p)
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$$

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

```
[Out] int(x*(a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

Fricas [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^px\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*e*sqrt(x) + c*d) + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/2))))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x, x)
```

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx=\int x\,\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)\right)\right)^p\,dx$$

```
[In] int(x*(a + b*log(c*(d + e*x^(1/2))))^p,x)
[Out] int(x*(a + b*log(c*(d + e*x^(1/2))))^p, x)
```

3.535 $\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 174

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx = \frac{2^{-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^2e^2} - \frac{2de^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{ce^2}$$

```
[Out] GAMMA(p+1,-2*(a+b*ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/(2^p)
/c^2/e^2/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))))/b)^p)-2*d*GAMMA(p+1,(-a-b*
ln(c*(d+e*x^(1/2))))/b)*(a+b*ln(c*(d+e*x^(1/2))))^p/c/e^2/exp(a/b)/(((-a-b*
ln(c*(d+e*x^(1/2))))/b)^p)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2501, 2448, 2436, 2336, 2212, 2437, 2346}

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx = \frac{2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)}{c^2e^2} - \frac{2de^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)}{ce^2}$$

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
```

```
[Out] (Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b]*(a + b*Log[c*(d + e*Sq
rt[x])])^p)/(2^p*c^2*e^2*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^
p) - (2*d*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])])/b)]*(a + b*Log[c*(d
 + e*Sqrt[x])])^p)/(c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)\\
&=2\text{Subst}\left(\int\left(-\frac{d(a+b\log(c(d+ex)))^p}{e}+\frac{(d+ex)(a+b\log(c(d+ex)))^p}{e}\right)\,dx,x,\sqrt{x}\right)\\
&=\frac{2\text{Subst}\left(\int(d+ex)(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)}{e}\\
&\quad-\frac{(2d)\text{Subst}\left(\int(a+b\log(c(d+ex)))^p\,dx,x,\sqrt{x}\right)}{e}\\
&=\frac{2\text{Subst}\left(\int x(a+b\log(cx))^p\,dx,x,d+e\sqrt{x}\right)}{e^2}-\frac{(2d)\text{Subst}\left(\int(a+b\log(cx))^p\,dx,x,d+e\sqrt{x}\right)}{e^2}\\
&=\frac{2\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{c^2e^2}\\
&\quad-\frac{(2d)\text{Subst}\left(\int e^{x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{ce^2}\\
&=\frac{2^{-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^2e^2}\\
&\quad-\frac{2de^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)^{-p}}{ce^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.75

$$
\int\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p\,dx
=\frac{2^{-p}e^{-\frac{2a}{b}}\left(\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)}{b}\right)-2^{1+p}cde^{a/b}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)\right)}{b}\right)\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right.\right.}{c^2e^2}
$$

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p,x]
[Out] ((Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])]))/b] - 2^(1 + p)*c*d*E^(a/
b)*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])])/b)])*(a + b*Log[c*(d + e*S
qrt[x])])^p)/(2^p*c^2*e^2*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])])/b))
^p)
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+e\sqrt{x}\right)\right)\right)^{p} dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/2))))^p,x)
[Out] int((a+b*ln(c*(d+e*x^(1/2))))^p,x)
```

Fricas [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^{p}\, dx = \int \left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^{p} dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e*sqrt(x) + c*d) + a)^p, x)
```

Sympy [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^{p}\, dx = \int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^{p}\, dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))))**p,x)
[Out] Integral((a + b*log(c*(d + e*sqrt(x))))**p, x)
```

Maxima [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^{p}\, dx = \int \left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^{p} dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p, x)
```

Giac [F]

$$\int \left(a + b\log\left(c(d + e\sqrt{x})\right)\right)^p \, dx = \int \left(b\log\left((e\sqrt{x} + d)c\right) + a\right)^p \, dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a + b\log\left(c(d + e\sqrt{x})\right)\right)^p \, dx = \int \left(a + b\,\ln\left(c\left(d + e\sqrt{x}\right)\right)\right)^p \, dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/2))))^p,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))))^p, x)
```

3.536 $\int \frac{(a+b\log(c(d+e\sqrt{x})))^p}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx=\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/2))))^p/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])])^p/x,x]
[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)])^p/x, x], x, Sqrt[x]]
```

Rubi steps

$$\text{integral}=2\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)))^p}{x}\,dx,x,\sqrt{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p/x, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/2))))^p/x,x)
[Out] int((a+b*ln(c*(d+e*x^(1/2))))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x,x, algorithm="fricas")
[Out] integral((b*log(c*e*sqrt(x) + c*d) + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))))**p/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x,x, algorithm="maxima")
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x}\,dx=\int \frac{{\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)\right)\right)}^p}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/2))))^p/x,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))))^p/x, x)
```

3.537 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx=\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/2))))^p/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])])^p/x^2,x]
```

```
[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)])^p/x^3, x], x, Sqrt[x]]
```

Rubi steps

$$\text{integral}=2\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)))^p}{x^3}\,dx,x,\sqrt{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e*Sqrt[x])])^p/x^2, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/2))))^p/x^2,x)
[Out] int((a+b*ln(c*(d+e*x^(1/2))))^p/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log(c*e*sqrt(x) + c*d) + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))))**p/x**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)*c) + a)^p/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)\right)\right)^p}{x^2}\,dx=\int \frac{{\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)\right)\right)}^p}{x^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/2))))^p/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))))^p/x^2, x)
```

3.538 $\int x^3 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 907

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

$$=\frac{2^{-2(1+p)}e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{c^4e^8}$$

$$-\frac{2^{1+p}7^{-p}de^{-\frac{7a}{2b}}\left(d+e\sqrt{x}\right)^7\Gamma\left(1+p,-\frac{7\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c(d+e\right.}{b}\right.}{e^8\left(c\left(d+e\sqrt{x}\right)^2\right)^{7/2}}$$

$$+\frac{7\ 3^{-p}d^2e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{c^3e^8}$$

$$-\frac{7\ 2^{1+p}5^{-p}d^3e^{-\frac{5a}{2b}}\left(d+e\sqrt{x}\right)^5\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c(\right.}{b}\right.}{e^8\left(c\left(d+e\sqrt{x}\right)^2\right)^{5/2}}$$

$$+\frac{35\ 2^{-1-p}d^4e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{c^2e^8}$$

$$-\frac{7\ 2^{1+p}3^{-p}d^5e^{-\frac{3a}{2b}}\left(d+e\sqrt{x}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c(\right.}{b}\right.}{e^8\left(c\left(d+e\sqrt{x}\right)^2\right)^{3/2}}$$

$$+\frac{7d^6e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{ce^8}$$

$$-\frac{2^{1+p}d^7e^{-\frac{a}{2b}}\left(d+e\sqrt{x}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)}{e^8\sqrt{c\left(d+e\sqrt{x}\right)^2}}$$

```
[Out] GAMMA(p+1,-4*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/(
2^(2+2*p))/c^4/e^8/exp(4*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)+7*d^2*GA
MMA(p+1,-3*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/(3^
p)/c^3/e^8/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)+35*2^(-1-p)*d^4*
GAMMA(p+1,-2*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/c
^2/e^8/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)+7*d^6*GAMMA(p+1,(-a-
b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/c/e^8/exp(a/b)/((
(-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)-2^(p+1)*d*GAMMA(p+1,-7/2*(a+b*ln(c*(d+e*
```

```
x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))^7/(7^p)/e^8/exp
(7/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(7/2)-7*
2^(p+1)*d^3*GAMMA(p+1,-5/2*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^
(1/2))^2))^p*(d+e*x^(1/2))^5/(5^p)/e^8/exp(5/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/
2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(5/2)-7*2^(p+1)*d^5*GAMMA(p+1,-3/2*(a+b*l
n(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))^3/(3^p
)/e^8/exp(3/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)
^(3/2)-2^(p+1)*d^7*GAMMA(p+1,1/2*(-a-b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*
(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))/e^8/exp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2
))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 907, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used

$= \{2504, 2448, 2436, 2337, 2212, 2437, 2347\}$

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

$$= \frac{2^{-2(p+1)}e^{-\frac{4a}{b}}\Gamma\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{c^4e^8}$$
$$- \frac{2^{p+1}7^{-p}de^{-\frac{7a}{2b}}\left(d+e\sqrt{x}\right)^7\Gamma\left(p+1,-\frac{7\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\cdots\right.\right.}{b}\cdots\right.}{e^8\left(c\left(d+e\sqrt{x}\right)^2\right)^{7/2}}$$
$$+ \frac{7\;3^{-p}d^2e^{-\frac{3a}{b}}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{c^3e^8}$$
$$- \frac{7\;2^{p+1}5^{-p}d^3e^{-\frac{5a}{2b}}\left(d+e\sqrt{x}\right)^5\Gamma\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\cdots\right.}{b\cdots}\cdots\right.}{e^8\left(c\left(d+e\sqrt{x}\right)^2\right)^{5/2}}$$
$$+ \frac{35\;2^{-p-1}d^4e^{-\frac{2a}{b}}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{c^2e^8}$$
$$- \frac{7\;2^{p+1}3^{-p}d^5e^{-\frac{3a}{2b}}\left(d+e\sqrt{x}\right)^3\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\cdots\right.}{b\cdots}\cdots\right.}{e^8\left(c\left(d+e\sqrt{x}\right)^2\right)^{3/2}}$$
$$+ \frac{7d^6e^{-\frac{a}{b}}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{ce^8}$$
$$- \frac{2^{p+1}d^7e^{-\frac{a}{2b}}\left(d+e\sqrt{x}\right)\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)\cdots}{e^8\sqrt{c\left(d+e\sqrt{x}\right)^2}}$$

```
[In] Int[x^3*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

```
[Out] (Gamma[1 + p, (-4*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a + b*Log[c*(d + e*
Sqrt[x])^2])^p)/(2^(2*(1 + p))*c^4*e^8*E^((4*a)/b)*(-((a + b*Log[c*(d + e*S
qrt[x])^2])/b))^p) - (2^(1 + p)*d*(d + e*Sqrt[x])^7*Gamma[1 + p, (-7*(a + b
*Log[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(7^p
*e^8*E^((7*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(7/2)*(-((a + b*Log[c*(d + e*Sqr
t[x])^2])/b))^p) + (7*d^2*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2])
)/b]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(3^p*c^3*e^8*E^((3*a)/b)*(-((a + b
*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (7*2^(1 + p)*d^3*(d + e*Sqrt[x])^5*Gamm
```

```
a[1 + p, (-5*(a + b*Log[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e*S
qrt[x])^2])^p)/(5^p*e^8*E^((5*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(5/2)*(-((a +
 b*Log[c*(d + e*Sqrt[x])^2])/b))^p) + (35*2^(-1 - p)*d^4*Gamma[1 + p, (-2*(
a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(c^
2*e^8*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (7*2^(1 + p)
*d^5*(d + e*Sqrt[x])^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2]))/(
2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(3^p*e^8*E^((3*a)/(2*b))*(c*(d +
e*Sqrt[x])^2)^(3/2)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) + (7*d^6*Gam
ma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e*Sqrt[x
])^2])^p)/(c*e^8*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (2^(1
 + p)*d^7*(d + e*Sqrt[x])*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e*Sqrt[x])^2]
)/b]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(e^8*E^(a/(2*b))*Sqrt[c*(d + e*Sqr
t[x])^2]*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^7\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt{x}\right)\\
&= 2\text{Subst}\left(\int\left(-\frac{d^7\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^7}+\frac{7d^6(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^7}\right.\right.\\
&\quad-\frac{21d^5(d+ex)^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^7}\\
&\quad+\frac{35d^4(d+ex)^3\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^7}-\frac{35d^3(d+ex)^4\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^7}\\
&\quad+\frac{21d^2(d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^7}-\frac{7d(d+ex)^6\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^7}\\
&\quad\left.\left.+\frac{(d+ex)^7\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^7}\right)\,dx,x,\sqrt{x}\right)
\end{aligned}$$

$$= \frac{2\text{Subst}\left(\int (d+ex)^7 (a+b\log(c(d+ex)^2))^p \, dx, x, \sqrt{x}\right)}{e^7} - \frac{(14d)\text{Subst}\left(\int (d+ex)^6 (a+b\log(c(d+ex)^2))^p \, dx, x, \sqrt{x}\right)}{e^7} + \frac{(42d^2)\,\text{Subst}\left(\int (d+ex)^5 (a+b\log(c(d+ex)^2))^p \, dx, x, \sqrt{x}\right)}{e^7} - \frac{(70d^3)\,\text{Subst}\left(\int (d+ex)^4 (a+b\log(c(d+ex)^2))^p \, dx, x, \sqrt{x}\right)}{e^7} + \frac{(70d^4)\,\text{Subst}\left(\int (d+ex)^3 (a+b\log(c(d+ex)^2))^p \, dx, x, \sqrt{x}\right)}{e^7} - \frac{(42d^5)\,\text{Subst}\left(\int (d+ex)^2 (a+b\log(c(d+ex)^2))^p \, dx, x, \sqrt{x}\right)}{e^7} + \frac{(14d^6)\,\text{Subst}\left(\int (d+ex) (a+b\log(c(d+ex)^2))^p \, dx, x, \sqrt{x}\right)}{e^7} - \frac{(2d^7)\,\text{Subst}\left(\int (a+b\log(c(d+ex)^2))^p \, dx, x, \sqrt{x}\right)}{e^7}$$

$$= \frac{2\text{Subst}\left(\int x^7 (a+b\log(cx^2))^p \, dx, x, d+e\sqrt{x}\right)}{e^8} - \frac{(14d)\text{Subst}\left(\int x^6 (a+b\log(cx^2))^p \, dx, x, d+e\sqrt{x}\right)}{e^8} + \frac{(42d^2)\,\text{Subst}\left(\int x^5 (a+b\log(cx^2))^p \, dx, x, d+e\sqrt{x}\right)}{e^8} - \frac{(70d^3)\,\text{Subst}\left(\int x^4 (a+b\log(cx^2))^p \, dx, x, d+e\sqrt{x}\right)}{e^8} + \frac{(70d^4)\,\text{Subst}\left(\int x^3 (a+b\log(cx^2))^p \, dx, x, d+e\sqrt{x}\right)}{e^8} - \frac{(42d^5)\,\text{Subst}\left(\int x^2 (a+b\log(cx^2))^p \, dx, x, d+e\sqrt{x}\right)}{e^8} + \frac{(14d^6)\,\text{Subst}\left(\int x (a+b\log(cx^2))^p \, dx, x, d+e\sqrt{x}\right)}{e^8} - \frac{(2d^7)\,\text{Subst}\left(\int (a+b\log(cx^2))^p \, dx, x, d+e\sqrt{x}\right)}{e^8}$$

$$
\begin{aligned}
&= \frac{\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{c^4e^8} \\
&\quad + \frac{(21d^2)\,\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{c^3e^8} \\
&\quad + \frac{(35d^4)\,\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{c^2e^8} \\
&\quad + \frac{(7d^6)\,\text{Subst}\left(\int e^{x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{ce^8} \\
&\quad - \frac{\left(7d\left(d+e\sqrt{x}\right)^7\right)\text{Subst}\left(\int e^{7x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{e^8\left(c\left(d+e\sqrt{x}\right)^2\right)^{7/2}} \\
&\quad - \frac{\left(35d^3\left(d+e\sqrt{x}\right)^5\right)\text{Subst}\left(\int e^{5x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{e^8\left(c\left(d+e\sqrt{x}\right)^2\right)^{5/2}} \\
&\quad - \frac{\left(21d^5\left(d+e\sqrt{x}\right)^3\right)\text{Subst}\left(\int e^{3x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{e^8\left(c\left(d+e\sqrt{x}\right)^2\right)^{3/2}} \\
&\quad - \frac{\left(d^7\left(d+e\sqrt{x}\right)\right)\text{Subst}\left(\int e^{x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{e^8\sqrt{c\left(d+e\sqrt{x}\right)^2}}
\end{aligned}
$$

$$=\frac{4^{-1-p}e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{c^4e^8}$$

$$-\frac{2^{1+p}7^{-p}de^{-\frac{7a}{2b}}\left(d+e\sqrt{x}\right)^7\Gamma\left(1+p,-\frac{7\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log}{}\right.}{e^8\left(c\left(d+e\sqrt{x}\right)^2\right)^{7/2}}$$

$$+\frac{7\ 3^{-p}d^2e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)}{c^3e^8}$$

$$-\frac{7\ 2^{1+p}5^{-p}d^3e^{-\frac{5a}{2b}}\left(d+e\sqrt{x}\right)^5\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+}{}\right.}{e^8\left(c\left(d+e\sqrt{x}\right)^2\right)^{5/2}}$$

$$+\frac{35\ 2^{-1-p}d^4e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right.}{b}\right.}{c^2e^8}$$

$$-\frac{7\ 2^{1+p}3^{-p}d^5e^{-\frac{3a}{2b}}\left(d+e\sqrt{x}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+}{}\right.}{e^8\left(c\left(d+e\sqrt{x}\right)^2\right)^{3/2}}$$

$$+\frac{7d^6e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{ce^8}$$

$$-\frac{2^{1+p}d^7e^{-\frac{a}{2b}}\left(d+e\sqrt{x}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\right.\right.}{b}\right.}{e^8\sqrt{c\left(d+e\sqrt{x}\right)^2}}$$

Mathematica [F]

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
[Out] Integrate[x^3*(a + b*Log[c*(d + e*Sqrt[x])^2])^p, x]
```

Maple [F]

$$\int x^3\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p dx$$

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
[Out] int(x^3*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

Fricas [F]

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p x^3\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p*x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/2))**2))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p x^3\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x^3, x)
```

Giac [F]

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^px^3\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int x^3\,\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^2\right)\right)^p\,dx$$

```
[In] int(x^3*(a + b*log(c*(d + e*x^(1/2))^2))^p,x)
[Out] int(x^3*(a + b*log(c*(d + e*x^(1/2))^2))^p, x)
```

3.539 $\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 677

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

$$= \frac{3^{-1-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{c^3e^6}$$

$$- \frac{2^{1+p}5^{-p}de^{-\frac{5a}{2b}}\left(d+e\sqrt{x}\right)^5\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\Big(-\frac{a+b\log\big(c(d+e}{b}}{e^6\left(c\left(d+e\sqrt{x}\right)^2\right)^{5/2}}$$

$$+ \frac{5\ 2^{-p}d^2e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{c^2e^6}$$

$$- \frac{5\ 2^{2+p}3^{-1-p}d^3e^{-\frac{3a}{2b}}\left(d+e\sqrt{x}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\Big(-\frac{a+b\log\big(}{}}{e^6\left(c\left(d+e\sqrt{x}\right)^2\right)^{3/2}}$$

$$+ \frac{5d^4e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{ce^6}$$

$$- \frac{2^{1+p}d^5e^{-\frac{a}{2b}}\left(d+e\sqrt{x}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)}{e^6\sqrt{c\left(d+e\sqrt{x}\right)^2}}$$

```
[Out] 3^(-1-p)*GAMMA(p+1,-3*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2)
)^2))^p/c^3/e^6/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)+5*d^2*GAMMA
```

```
(p+1,-2*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/(2^p)/
c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)+5*d^4*GAMMA(p+1,(-a
-b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/c/e^6/exp(a/b)/(
((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)-2^(p+1)*d*GAMMA(p+1,-5/2*(a+b*ln(c*(d+e
*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))^5/(5^p)/e^6/ex
p(5/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(5/2)-5
*2^(2+p)*3^(-1-p)*d^3*GAMMA(p+1,-3/2*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln
(c*(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))^3/e^6/exp(3/2*a/b)/(((-a-b*ln(c*(d+e*x
^(1/2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(3/2)-2^(p+1)*d^5*GAMMA(p+1,1/2*(-a-b
*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))/e^6/
exp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 677, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2504, 2448, 2436, 2337, 2212, 2437, 2347}

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$
$$=\frac{3^{-p-1}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)}{c^3e^6}$$
$$+\frac{5d^22^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)}{c^2e^6}$$
$$-\frac{d^52^{p+1}e^{-\frac{a}{2b}}\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{2b}\right)}{e^6\sqrt{c\left(d+e\sqrt{x}\right)^2}}$$
$$+\frac{5d^4e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)}{ce^6}$$
$$-\frac{5d^32^{p+2}3^{-p-1}e^{-\frac{3a}{2b}}\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{2b}\right)}{e^6\left(c\left(d+e\sqrt{x}\right)^2\right)^{3/2}}$$
$$-\frac{d2^{p+1}5^{-p}e^{-\frac{5a}{2b}}\left(d+e\sqrt{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{2b}\right)}{e^6\left(c\left(d+e\sqrt{x}\right)^2\right)^{5/2}}$$

```
[In] Int[x^2*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

```
[Out] (3^(-1 - p)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a + b*Lo
g[c*(d + e*Sqrt[x])^2])^p)/(c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*Sqrt
[x])^2])/b))^p) - (2^(1 + p)*d*(d + e*Sqrt[x])^5*Gamma[1 + p, (-5*(a + b*Lo
g[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(5^p*e^
6*E^((5*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(5/2)*(-((a + b*Log[c*(d + e*Sqrt[x
])^2])/b))^p) + (5*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b
]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(2^p*c^2*e^6*E^((2*a)/b)*(-((a + b*Lo
g[c*(d + e*Sqrt[x])^2])/b))^p) - (5*2^(2 + p)*3^(-1 - p)*d^3*(d + e*Sqrt[x]
)^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*
(d + e*Sqrt[x])^2])^p)/(e^6*E^((3*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(3/2)*(-(
(a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) + (5*d^4*Gamma[1 + p, -((a + b*Log[
c*(d + e*Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(c*e^6*E^(a/b
)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (2^(1 + p)*d^5*(d + e*Sqrt[x
])*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e*Sqrt[x])^2])/b]*(a + b*Log[c*(d +
e*Sqrt[x])^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e*Sqrt[x])^2]*(-((a + b*Log[
c*(d + e*Sqrt[x])^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt{x}\right)\\
&= 2\text{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}+\frac{5d^4(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}\right.\right.\\
&\qquad-\frac{10d^3(d+ex)^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}\\
&\qquad+\frac{10d^2(d+ex)^3\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}-\frac{5d(d+ex)^4\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}\\
&\qquad\left.\left.+\frac{(d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}\right)\,dx,x,\sqrt{x}\right)\\
&= \frac{2\text{Subst}\left(\int(d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt{x}\right)}{e^5}\\
&\quad-\frac{(10d)\text{Subst}\left(\int(d+ex)^4\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt{x}\right)}{e^5}\\
&\quad+\frac{(20d^2)\,\text{Subst}\left(\int(d+ex)^3\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt{x}\right)}{e^5}\\
&\quad-\frac{(20d^3)\,\text{Subst}\left(\int(d+ex)^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt{x}\right)}{e^5}\\
&\quad+\frac{(10d^4)\,\text{Subst}\left(\int(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt{x}\right)}{e^5}\\
&\quad-\frac{(2d^5)\,\text{Subst}\left(\int\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt{x}\right)}{e^5}
\end{aligned}
$$

$$= \frac{2\text{Subst}\left(\int x^5(a+b\log(cx^2))^p\,dx, x, d+e\sqrt{x}\right)}{e^6} - \frac{(10d)\text{Subst}\left(\int x^4(a+b\log(cx^2))^p\,dx, x, d+e\sqrt{x}\right)}{e^6} + \frac{(20d^2)\,\text{Subst}\left(\int x^3(a+b\log(cx^2))^p\,dx, x, d+e\sqrt{x}\right)}{e^6} - \frac{(20d^3)\,\text{Subst}\left(\int x^2(a+b\log(cx^2))^p\,dx, x, d+e\sqrt{x}\right)}{e^6} + \frac{(10d^4)\,\text{Subst}\left(\int x(a+b\log(cx^2))^p\,dx, x, d+e\sqrt{x}\right)}{e^6} - \frac{(2d^5)\,\text{Subst}\left(\int (a+b\log(cx^2))^p\,dx, x, d+e\sqrt{x}\right)}{e^6}$$

$$= \frac{\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{c^3e^6} + \frac{(10d^2)\,\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{c^2e^6} + \frac{(5d^4)\,\text{Subst}\left(\int e^{x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{ce^6} - \frac{\left(5d\left(d+e\sqrt{x}\right)^5\right)\text{Subst}\left(\int e^{5x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{e^6\left(c\left(d+e\sqrt{x}\right)^2\right)^{5/2}} - \frac{\left(10d^3\left(d+e\sqrt{x}\right)^3\right)\text{Subst}\left(\int e^{3x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{e^6\left(c\left(d+e\sqrt{x}\right)^2\right)^{3/2}} - \frac{\left(d^5\left(d+e\sqrt{x}\right)\right)\text{Subst}\left(\int e^{x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{e^6\sqrt{c\left(d+e\sqrt{x}\right)^2}}$$

$$= \frac{3^{-1-p} e^{-\frac{3a}{b}} \Gamma\left(1+p, -\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right) \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{c^3 e^6}$$

$$- \frac{2^{1+p} 5^{-p} d e^{-\frac{5a}{2b}} \left(d+e\sqrt{x}\right)^5 \Gamma\left(1+p, -\frac{5\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{2b}\right) \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p \left(-\frac{a+b\log}{}\right.}{e^6 \left(c\left(d+e\sqrt{x}\right)^2\right)^{5/2}}$$

$$+ \frac{5\ 2^{-p} d^2 e^{-\frac{2a}{b}} \Gamma\left(1+p, -\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right) \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)}{c^2 e^6}$$

$$- \frac{5\ 2^{2+p} 3^{-1-p} d^3 e^{-\frac{3a}{2b}} \left(d+e\sqrt{x}\right)^3 \Gamma\left(1+p, -\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{2b}\right) \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p \left(-\right.}{e^6 \left(c\left(d+e\sqrt{x}\right)^2\right)^{3/2}}$$

$$+ \frac{5 d^4 e^{-\frac{a}{b}} \Gamma\left(1+p, -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right) \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{c e^6}$$

$$- \frac{2^{1+p} d^5 e^{-\frac{a}{2b}} \left(d+e\sqrt{x}\right) \Gamma\left(1+p, -\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{2b}\right) \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\right.\right.}{b}\right.}{e^6 \sqrt{c\left(d+e\sqrt{x}\right)^2}}$$

Mathematica [F]

$$\int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\, dx = \int x^2 \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\, dx$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
[Out] Integrate[x^2*(a + b*Log[c*(d + e*Sqrt[x])^2])^p, x]
```

Maple [F]

$$\int x^2 \left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p dx$$

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

Fricas [F]

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^px^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/2))**2))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^px^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x^2, x)
```

Giac [F]

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^px^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int x^2\,\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^2\right)\right)^p\,dx$$

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/2))^2))^p,x)
[Out] int(x^2*(a + b*log(c*(d + e*x^(1/2))^2))^p, x)
```

3.540 $\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 445

$$
\begin{aligned}
&\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx \\
&= \frac{2^{-1-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{c^2e^4} \\
&\quad - \frac{2^{1+p}3^{-p}de^{-\frac{3a}{2b}}\left(d+e\sqrt{x}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{e^4\left(c\left(d+e\sqrt{x}\right)^2\right)^{3/2}} \\
&\quad + \frac{3d^2e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{ce^4} \\
&\quad - \frac{2^{1+p}d^3e^{-\frac{a}{2b}}\left(d+e\sqrt{x}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{e^4\sqrt{c\left(d+e\sqrt{x}\right)^2}}
\end{aligned}
$$

```
[Out] 2^(-1-p)*GAMMA(p+1,-2*(a+b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2)
)^2))^p/c^2/e^4/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)+3*d^2*GAMMA
(p+1,(-a-b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/c/e^4/ex
p(a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)-2^(p+1)*d*GAMMA(p+1,-3/2*(a+b*l
n(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))^3/(3^p
)/e^4/exp(3/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)
^(3/2)-2^(p+1)*d^3*GAMMA(p+1,1/2*(-a-b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*
(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))/e^4/exp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2
))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2504, 2448, 2436, 2337, 2212, 2437, 2347}

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$
$$=\frac{2^{-p-1}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{b}\right)}{c^2e^4}$$
$$-\frac{d^3 2^{p+1}e^{-\frac{a}{2b}}\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{2b}\right)}{e^4\sqrt{c\left(d+e\sqrt{x}\right)^2}}$$
$$+\frac{3d^2e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)}{ce^4}$$
$$-\frac{d2^{p+1}3^{-p}e^{-\frac{3a}{2b}}\left(d+e\sqrt{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{2b}\right)}{e^4\left(c\left(d+e\sqrt{x}\right)^2\right)^{3/2}}$$

```
[In] Int[x*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

```
[Out] (2^(-1 - p)*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*Sqrt[x])^2]))/b]*(a + b*Lo
g[c*(d + e*Sqrt[x])^2])^p)/(c^2*e^4*E^((2*a)/b)*(-((a + b*Log[c*(d + e*Sqrt
[x])^2])/b))^p) - (2^(1 + p)*d*(d + e*Sqrt[x])^3*Gamma[1 + p, (-3*(a + b*Lo
g[c*(d + e*Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(3^p*e^
4*E^((3*a)/(2*b))*(c*(d + e*Sqrt[x])^2)^(3/2)*(-((a + b*Log[c*(d + e*Sqrt[x
])^2])/b))^p) + (3*d^2*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])^2])/b)]*
(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(c*e^4*E^(a/b)*(-((a + b*Log[c*(d + e*S
qrt[x])^2])/b))^p) - (2^(1 + p)*d^3*(d + e*Sqrt[x])*Gamma[1 + p, -1/2*(a +
b*Log[c*(d + e*Sqrt[x])^2])/b]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(e^4*E^(
a/(2*b))*Sqrt[c*(d + e*Sqrt[x])^2]*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^
p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int x^3\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(-\frac{d^3(a+b\log(c(d+ex)^2))^p}{e^3}+\frac{3d^2(d+ex)\left(a+b\log(c(d+ex)^2)\right)^p}{e^3}-\frac{3d(d+ex)^2\left(a+b\log(c(d+ex)^2)\right)^p}{e^3}+\frac{(d+ex)^3\left(a+b\log(c(d+ex)^2)\right)^p}{e^3}\right)dx,x,\sqrt{x}\right)$$

$$= \frac{2\text{Subst}\left(\int (d+ex)^3\left(a+b\log(c(d+ex)^2)\right)^p\,dx,x,\sqrt{x}\right)}{e^3} - \frac{(6d)\text{Subst}\left(\int (d+ex)^2\left(a+b\log(c(d+ex)^2)\right)^p\,dx,x,\sqrt{x}\right)}{e^3} + \frac{(6d^2)\,\text{Subst}\left(\int (d+ex)\left(a+b\log(c(d+ex)^2)\right)^p\,dx,x,\sqrt{x}\right)}{e^3} - \frac{(2d^3)\,\text{Subst}\left(\int \left(a+b\log(c(d+ex)^2)\right)^p\,dx,x,\sqrt{x}\right)}{e^3}$$

$$= \frac{2\text{Subst}\left(\int x^3(a+b\log(cx^2))^p\,dx,x,d+e\sqrt{x}\right)}{e^4} - \frac{(6d)\text{Subst}\left(\int x^2(a+b\log(cx^2))^p\,dx,x,d+e\sqrt{x}\right)}{e^4} + \frac{(6d^2)\,\text{Subst}\left(\int x(a+b\log(cx^2))^p\,dx,x,d+e\sqrt{x}\right)}{e^4} - \frac{(2d^3)\,\text{Subst}\left(\int (a+b\log(cx^2))^p\,dx,x,d+e\sqrt{x}\right)}{e^4}$$

$$= \frac{\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{c^2e^4} + \frac{(3d^2)\,\text{Subst}\left(\int e^{x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{ce^4} - \frac{\left(3d\left(d+e\sqrt{x}\right)^3\right)\text{Subst}\left(\int e^{3x/2}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{e^4\left(c\left(d+e\sqrt{x}\right)^2\right)^{3/2}} - \frac{\left(d^3\left(d+e\sqrt{x}\right)\right)\text{Subst}\left(\int e^{x/2}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{e^4\sqrt{c\left(d+e\sqrt{x}\right)^2}}$$

$$
= \frac{2^{-1-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{2}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{2}\right)\right)^{p}\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^{2}\right)}{b}\right)^{-p}}{c^{2}e^{4}}
- \frac{2^{1+p}3^{-p}de^{-\frac{3a}{2b}}\left(d+e\sqrt{x}\right)^{3}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{2}\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{2}\right)\right)^{p}\left(-\frac{a+b}{}\right.}{e^{4}\left(c\left(d+e\sqrt{x}\right)^{2}\right)^{3/2}}
+ \frac{3d^{2}e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^{2}\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{2}\right)\right)^{p}\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^{2}\right)}{b}\right)^{-p}}{ce^{4}}
- \frac{2^{1+p}d^{3}e^{-\frac{a}{2b}}\left(d+e\sqrt{x}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^{2}\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{2}\right)\right)^{p}\left(-\frac{a+b\log\left(c(d+\right.}{b}\right.}{e^{4}\sqrt{c\left(d+e\sqrt{x}\right)^{2}}}
$$

Mathematica [F]

$$
\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{2}\right)\right)^{p}\,dx=\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{2}\right)\right)^{p}\,dx
$$

```
[In] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e*Sqrt[x])^2])^p, x]
```

Maple [F]

$$
\int x\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^{2}\right)\right)^{p}dx
$$

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
[Out] int(x*(a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

Fricas [F]

$$
\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^{2}\right)\right)^{p}\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)^{2}c\right)+a\right)^{p}x\,dx
$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/2))**2))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^px\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x, x)
```

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^px\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int x\left(a+b\,\ln\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$

```
[In] int(x*(a + b*log(c*(d + e*x^(1/2))^2))^p,x)
[Out] int(x*(a + b*log(c*(d + e*x^(1/2))^2))^p, x)
```

3.541 $\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 213

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$
$$=\frac{e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{ce^2}$$
$$-\frac{2^{1+p}de^{-\frac{a}{2b}}\left(d+e\sqrt{x}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-}}{e^2\sqrt{c\left(d+e\sqrt{x}\right)^2}}$$

```
[Out] GAMMA(p+1,(-a-b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p/c/e
^2/exp(a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)-2^(p+1)*d*GAMMA(p+1,1/2*(-
a-b*ln(c*(d+e*x^(1/2))^2))/b)*(a+b*ln(c*(d+e*x^(1/2))^2))^p*(d+e*x^(1/2))/e
^2/exp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/2))^2))/b)^p)/(c*(d+e*x^(1/2))^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used

$= \{2501, 2448, 2436, 2337, 2212, 2437, 2347\}$

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$
$$= \frac{e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)}{ce^2} - \frac{d2^{p+1}e^{-\frac{a}{2b}}\left(d+e\sqrt{x}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{2b}\right)}{e^2\sqrt{c\left(d+e\sqrt{x}\right)^2}}$$

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
```

```
[Out] (Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e*Sq
rt[x])^2])^p)/(c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p) - (
2^(1 + p)*d*(d + e*Sqrt[x])*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e*Sqrt[x])^
2])/b]*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(e^2*E^(a/(2*b))*Sqrt[c*(d + e*S
qrt[x])^2]*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt{x}\right)\\
&=2\text{Subst}\left(\int\left(-\frac{d(a+b\log(c(d+ex)^2))^p}{e}+\frac{(d+ex)\left(a+b\log(c(d+ex)^2)\right)^p}{e}\right)\,dx,x,\sqrt{x}\right)\\
&=\frac{2\text{Subst}\left(\int(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt{x}\right)}{e}\\
&\quad-\frac{(2d)\text{Subst}\left(\int\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt{x}\right)}{e}\\
&=\frac{2\text{Subst}\left(\int x(a+b\log(cx^2))^p\,dx,x,d+e\sqrt{x}\right)}{e^2}\\
&\quad-\frac{(2d)\text{Subst}\left(\int(a+b\log(cx^2))^p\,dx,x,d+e\sqrt{x}\right)}{e^2}\\
&=\frac{\text{Subst}\left(\int e^x(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{ce^2}\\
&\quad-\frac{\left(d\left(d+e\sqrt{x}\right)\right)\text{Subst}\left(\int e^{x/2}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)}{e^2\sqrt{c\left(d+e\sqrt{x}\right)^2}}
\end{aligned}
$$

$$= \frac{e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)^{-p}}{ce^2}$$
$$-\frac{2^{1+p}de^{-\frac{a}{2b}}\left(d+e\sqrt{x}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right.}{e^2\sqrt{c\left(d+e\sqrt{x}\right)^2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.85

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx$$
$$=\frac{e^{-\frac{a}{b}}\left(\sqrt{c\left(d+e\sqrt{x}\right)^2}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{b}\right)-2^{1+p}cde^{\frac{a}{2b}}\left(d+e\sqrt{x}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)}{2b}\right)\right)}{ce^2\sqrt{c\left(d+e\sqrt{x}\right)^2}}$$

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p,x]
[Out] ((Sqrt[c*(d + e*Sqrt[x])^2]*Gamma[1 + p, -((a + b*Log[c*(d + e*Sqrt[x])^2])
/b)] - 2^(1 + p)*c*d*E^(a/(2*b))*(d + e*Sqrt[x])*Gamma[1 + p, -1/2*(a + b*L
og[c*(d + e*Sqrt[x])^2])/b])*(a + b*Log[c*(d + e*Sqrt[x])^2])^p)/(c*e^2*E^(
a/b)*Sqrt[c*(d + e*Sqrt[x])^2]*(-((a + b*Log[c*(d + e*Sqrt[x])^2])/b))^p)
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
[Out] int((a+b*ln(c*(d+e*x^(1/2))^2))^p,x)
```

Fricas [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int \left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**2))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int \left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p, x)
```

Giac [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int \left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p\,dx=\int {\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^2\right)\right)}^p\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/2))^2))^p,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))^2))^p, x)
```

3.542 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx = \text{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/2))^2))^p/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx = \int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x,x]
[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)^2])^p/x, x], x, Sqrt[x]]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{x}\,dx, x, \sqrt{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^2))^p/x,x)
[Out] int((a+b*ln(c*(d+e*x^(1/2))^2))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.38

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**2))**p/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx = \int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x,x, algorithm="maxima")
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx = \int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,\sqrt{x}\right)}^2\right)\right)^p}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/2))^2))^p/x,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))^2))^p/x, x)
```

3.543 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx=\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/2))^2))^p/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x^2,x]
[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)^2])^p/x^3, x], x, Sqrt[x]]
```

Rubi steps

$$\text{integral}=2\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^2))^p}{x^3}\,dx,x,\sqrt{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e*Sqrt[x])^2])^p/x^2, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/2))^2))^p/x^2,x)
[Out] int((a+b*ln(c*(d+e*x^(1/2))^2))^p/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.38

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x + 2*c*d*e*sqrt(x) + c*d^2) + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/2))**2))**p/x**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p/x^2, x)
```

Giac [N/A]

Not integrable

Time = 1.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(e\sqrt{x}+d\right)^2c\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/2))^2))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log((e*sqrt(x) + d)^2*c) + a)^p/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,\sqrt{x}\right)^2\right)\right)^p}{x^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/2))^2))^p/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(1/2))^2))^p/x^2, x)
```

3.544 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx=\mathrm{Int}\left(x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x*(a+b*ln(c*(d+e/x^(1/2))))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx$$

```
[In] Int[x*(a + b*Log[c*(d + e/Sqrt[x])])^p,x]
[Out] 2*Defer[Subst][Defer[Int][x^3*(a + b*Log[c*(d + e/x)])^p, x], x, Sqrt[x]]
```

Rubi steps

$$\text{integral}=2\mathrm{Subst}\left(\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x}\right)\right)\right)^p\,dx,x,\sqrt{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx$$

```
[In] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])])^p, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx$$

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/2))))^p,x)
[Out] int(x*(a+b*ln(c*(d+e/x^(1/2))))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/2))))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p*x, x)
```

Giac [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p*x, x)
```

Mupad [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx=\int x\left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx$$

```
[In] int(x*(a + b*log(c*(d + e/x^(1/2))))^p,x)
[Out] int(x*(a + b*log(c*(d + e/x^(1/2))))^p, x)
```

3.545 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx=\text{Int}\left(\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/2))))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx=\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx$$

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])])^p,x]
[Out] 2*Defer[Subst][Defer[Int][x*(a + b*Log[c*(d + e/x)])^p, x], x, Sqrt[x]]
```

Rubi steps

$$\text{integral}=2\text{Subst}\left(\int x\left(a+b\log\left(c\left(d+\frac{e}{x}\right)\right)\right)^p\,dx,x,\sqrt{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \left(a+b\log \left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p \,dx=\int \left(a+b\log \left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p \,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \left(a+b\ln \left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^{p}d x$$

```
[In] int((a+b*ln(c*(d+e/x^(1/2))))^p,x)
[Out] int((a+b*ln(c*(d+e/x^(1/2))))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \left(a+b\log \left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p \,dx=\int \left(b\log \left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^{p} \,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\log \left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p \,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx=\int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx=\int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\,dx=\int {\left(a+b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}^p\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/2))))^p,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))))^p, x)
```

3.546 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx=\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/2))))^p/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])])^p/x,x]
[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x)])^p/x, x], x, Sqrt[x]]
```

Rubi steps

$$\text{integral}=2\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x}\right)\right)\right)^p}{x}\,dx,x,\sqrt{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/2))))^p/x,x)
[Out] int((a+b*ln(c*(d+e/x^(1/2))))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))))**p/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/2))))^p/x,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))))^p/x, x)
```

3.547 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 175

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx = -\frac{2^{-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^2e^2} + \frac{2de^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{ce^2}$$

```
[Out] -GAMMA(p+1,-2*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(2^p
)/c^2/e^2/exp(2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)+2*d*GAMMA(p+1,(-a-b
*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/c/e^2/exp(a/b)/(((-a-b
*ln(c*(d+e/x^(1/2))))/b)^p)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used

$= \{2504, 2448, 2436, 2336, 2212, 2437, 2346\}$

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx$$
$$= \frac{2de^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)}{ce^2} - \frac{2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)}{c^2e^2}$$

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^2,x]
```

```
[Out] -((Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/
Sqrt[x])])^p)/(2^p*c^2*e^2*E^((2*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b)
)^p)) + (2*d*Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])])/b)]*(a + b*Log[c
*(d + e/Sqrt[x])])^p)/(c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^
p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(2\text{Subst}\left(\int x(a+b\log(c(d+ex)))^p\,dx,x,\frac{1}{\sqrt{x}}\right)\right)\\
&= -\left(2\text{Subst}\left(\int\left(-\frac{d(a+b\log(c(d+ex)))^p}{e}+\frac{(d+ex)(a+b\log(c(d+ex)))^p}{e}\right)\,dx,x,\frac{1}{\sqrt{x}}\right)\right)\\
&= -\frac{2\text{Subst}\left(\int(d+ex)(a+b\log(c(d+ex)))^p\,dx,x,\frac{1}{\sqrt{x}}\right)}{e}\\
&\quad+\frac{(2d)\text{Subst}\left(\int(a+b\log(c(d+ex)))^p\,dx,x,\frac{1}{\sqrt{x}}\right)}{e}\\
&= -\frac{2\text{Subst}\left(\int x(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^2}+\frac{(2d)\text{Subst}\left(\int(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^2}\\
&= -\frac{2\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^2e^2}\\
&\quad+\frac{(2d)\text{Subst}\left(\int e^{x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{ce^2}
\end{aligned}
$$

$$= -\frac{2^{-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^2e^2} + \frac{2de^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{ce^2}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx = \frac{2^{-p}e^{-\frac{2a}{b}}\left(-\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)+2^{1+p}cde^{a/b}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)\right)\left(a+b\log\left(c\left(d+\right.\right.\right.}{c^2e^2}$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^2,x]
[Out] ((-Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b] + 2^(1 + p)*c*d*E^(a
/b)*Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])])/b)])*(a + b*Log[c*(d + e/
Sqrt[x])])^p)/(2^p*c^2*e^2*E^((2*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b)
)^p)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/2))))^p/x^2,x)
[Out] int((a+b*ln(c*(d+e/x^(1/2))))^p/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a + b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\, dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))))**p/x**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a + b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\, dx = \int \frac{\left(b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)\right) + a\right)^p}{x^2}\, dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{\left(a + b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\, dx = \int \frac{\left(b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)\right) + a\right)^p}{x^2}\, dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a + b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\, dx = \int \frac{\left(a + b\,\ln\left(c\left(d + \frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^2}\, dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/2))))^p/x^2,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))))^p/x^2, x)
```

3.548 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 552

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx = -\frac{2^{-p}3^{-1-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^6e^6}$$
$$+\frac{2\,5^{-p}de^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^5e^6}$$
$$-\frac{5\,4^{-p}d^2e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^4e^6}$$
$$+\frac{20\,3^{-1-p}d^3e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^3e^6}$$
$$-\frac{5\,2^{-p}d^4e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^2e^6}$$
$$+\frac{2d^5e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{ce^6}$$

```
[Out] -3^(-1-p)*GAMMA(p+1,-6*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))
))^p/(2^p)/c^6/e^6/exp(6*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)+2*d*GAMMA(
p+1,-5*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(5^p)/c^5/e
```

```
^6/exp(5*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)-5*d^2*GAMMA(p+1,-4*(a+b*ln
(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(4^p)/c^4/e^6/exp(4*a/b)/
(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)+20*3^(-1-p)*d^3*GAMMA(p+1,-3*(a+b*ln(c*(
d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/c^3/e^6/exp(3*a/b)/(((-a-b*ln
(c*(d+e/x^(1/2))))/b)^p)-5*d^4*GAMMA(p+1,-2*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a
+b*ln(c*(d+e/x^(1/2))))^p/(2^p)/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2
))))/b)^p)+2*d^5*GAMMA(p+1,(-a-b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(
1/2))))^p/c/e^6/exp(a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2504, 2448, 2436, 2336, 2212, 2437, 2346}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx = -\frac{2^{-p}3^{-p-1}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)}{c^6e^6}$$
$$+\frac{2d5^{-p}e^{-\frac{5a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)}{c^5e^6}$$
$$-\frac{5d^24^{-p}e^{-\frac{4a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)}{c^4e^6}$$
$$+\frac{20d^33^{-p-1}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)}{c^3e^6}$$
$$-\frac{5d^42^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)}{c^2e^6}$$
$$+\frac{2d^5e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)}{ce^6}$$

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^4,x]
```

```
[Out] -((3^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Lo
g[c*(d + e/Sqrt[x])])^p)/(2^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e/Sq
rt[x])])/b))^p)) + (2*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqrt[x])]))/b]
*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c
*(d + e/Sqrt[x])])/b))^p) - (5*d^2*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/Sqr
```

```
t[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(4^p*c^4*e^6*E^((4*a)/b)*(-((
a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (20*3^(-1 - p)*d^3*Gamma[1 + p, (-3*
(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(c^3*e
^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) - (5*d^4*Gamma[1 +
p, (-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)
/(2^p*c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (2*d^5
*Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])])/b)]*(a + b*Log[c*(d + e/Sqrt
[x])])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
```

```
d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = -\left(2\text{Subst}\left(\int x^5(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\text{Subst}\left(\int \left(-\frac{d^5(a+b\log(c(d+ex)))^p}{e^5}+\frac{5d^4(d+ex)(a+b\log(c(d+ex)))^p}{e^5}-\frac{10d^3(d+ex)^2(\right.\right.\right.$$

$$= -\frac{2\text{Subst}\left(\int (d+ex)^5(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^5} + \frac{(10d)\text{Subst}\left(\int (d+ex)^4(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^5} - \frac{(20d^2)\,\text{Subst}\left(\int (d+ex)^3(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^5} + \frac{(20d^3)\,\text{Subst}\left(\int (d+ex)^2(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^5} - \frac{(10d^4)\,\text{Subst}\left(\int (d+ex)(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^5} + \frac{(2d^5)\,\text{Subst}\left(\int (a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^5}$$

$$= -\frac{2\text{Subst}\left(\int x^5(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^6} + \frac{(10d)\text{Subst}\left(\int x^4(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^6} - \frac{(20d^2)\,\text{Subst}\left(\int x^3(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^6} + \frac{(20d^3)\,\text{Subst}\left(\int x^2(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^6} - \frac{(10d^4)\,\text{Subst}\left(\int x(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^6} + \frac{(2d^5)\,\text{Subst}\left(\int (a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^6}$$

$$= -\frac{2\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^6e^6} + \frac{(10d)\text{Subst}\left(\int e^{5x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^5e^6} - \frac{(20d^2)\,\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^4e^6} + \frac{(20d^3)\,\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^3e^6} - \frac{(10d^4)\,\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^2e^6} + \frac{(2d^5)\,\text{Subst}\left(\int e^{x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{ce^6}$$

$$= -\frac{2^{-p}3^{-1-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^6e^6}$$
$$+\frac{2\,5^{-p}de^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^5e^6}$$
$$-\frac{5\,4^{-p}d^2e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^4e^6}$$
$$+\frac{20\,3^{-1-p}d^3e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^3e^6}$$
$$-\frac{5\,2^{-p}d^4e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^2e^6}$$
$$+\frac{2d^5e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{ce^6}$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.59

$$\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx$$
$$=\frac{3^{-1-p}20^{-p}e^{-\frac{6a}{b}}\left(-10^p\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)+cde^{a/b}\left(2^{1+2p}3^{1+p}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)-\right.\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^4,x]
```

```
[Out] (3^(-1 - p)*(-(10^p*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/Sqrt[x])]))/b]) +
c*d*E^(a/b)*(2^(1 + 2*p)*3^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqr
t[x])]))/b] + 5^p*c*d*E^(a/b)*(-5*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Log[c*(
d + e/Sqrt[x])]))/b] + 2^p*c*d*E^(a/b)*(5*2^(2 + p)*Gamma[1 + p, (-3*(a + b
*Log[c*(d + e/Sqrt[x])]))/b] + 3^(1 + p)*c*d*E^(a/b)*(-5*Gamma[1 + p, (-2*(
a + b*Log[c*(d + e/Sqrt[x])]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -((a
 + b*Log[c*(d + e/Sqrt[x])])/b)])))))*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(20
^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/2))))^p/x^4,x)
[Out] int((a+b*ln(c*(d+e/x^(1/2))))^p/x^4,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^4,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))))**p/x**4,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^4,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^4, x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^4,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^4}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/2))))^p/x^4,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))))^p/x^4, x)
```

3.549 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 926

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx = -\frac{2^{-p}5^{-1-p}e^{-\frac{10a}{b}}\Gamma\left(1+p,-\frac{10\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^{10}e^{10}}$$
$$+\frac{2\ 9^{-p}de^{-\frac{9a}{b}}\Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^9e^{10}}$$
$$-\frac{9\ 8^{-p}d^2e^{-\frac{8a}{b}}\Gamma\left(1+p,-\frac{8\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^8e^{10}}$$
$$+\frac{24\ 7^{-p}d^3e^{-\frac{7a}{b}}\Gamma\left(1+p,-\frac{7\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^7e^{10}}$$
$$-\frac{7\ 6^{1-p}d^4e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^6e^{10}}$$
$$+\frac{252\ 5^{-1-p}d^5e^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^5e^{10}}$$
$$-\frac{21\ 2^{1-2p}d^6e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^4e^{10}}$$
$$+\frac{8\ 3^{1-p}d^7e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^3e^{10}}$$
$$-\frac{9\ 2^{-p}d^8e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^2e^{10}}$$
$$+\frac{2d^9e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{ce^{10}}$$

```
[Out] -5^(-1-p)*GAMMA(p+1,-10*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2)
)))^p/(2^p)/c^10/e^10/exp(10*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)+2*d*GA
MMA(p+1,-9*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(9^p)/c
^9/e^10/exp(9*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)-9*d^2*GAMMA(p+1,-8*(a
+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(8^p)/c^8/e^10/exp(8
```

```
*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)+24*d^3*GAMMA(p+1,-7*(a+b*ln(c*(d+e
/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(7^p)/c^7/e^10/exp(7*a/b)/(((-a-
b*ln(c*(d+e/x^(1/2))))/b)^p)-7*6^(1-p)*d^4*GAMMA(p+1,-6*(a+b*ln(c*(d+e/x^(1
/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/c^6/e^10/exp(6*a/b)/(((-a-b*ln(c*(d+e
/x^(1/2))))/b)^p)+252*5^(-1-p)*d^5*GAMMA(p+1,-5*(a+b*ln(c*(d+e/x^(1/2))))/b
)*(a+b*ln(c*(d+e/x^(1/2))))^p/c^5/e^10/exp(5*a/b)/(((-a-b*ln(c*(d+e/x^(1/2)
)))/b)^p)-21*2^(1-2*p)*d^6*GAMMA(p+1,-4*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*l
n(c*(d+e/x^(1/2))))^p/c^4/e^10/exp(4*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p
)+8*3^(1-p)*d^7*GAMMA(p+1,-3*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^
(1/2))))^p/c^3/e^10/exp(3*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)-9*d^8*GAM
MA(p+1,-2*(a+b*ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/(2^p)/c^
2/e^10/exp(2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))))/b)^p)+2*d^9*GAMMA(p+1,(-a-b*
ln(c*(d+e/x^(1/2))))/b)*(a+b*ln(c*(d+e/x^(1/2))))^p/c/e^10/exp(a/b)/(((-a-b
*ln(c*(d+e/x^(1/2))))/b)^p)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 926, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used

$= \{2504, 2448, 2436, 2336, 2212, 2437, 2346\}$

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx = -\frac{2^{-p}5^{-p-1}e^{-\frac{10a}{b}}\Gamma\left(p+1,-\frac{10\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^{10}e^{10}}$$
$$+\frac{2\ 9^{-p}de^{-\frac{9a}{b}}\Gamma\left(p+1,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^9e^{10}}$$
$$-\frac{9\ 8^{-p}d^2e^{-\frac{8a}{b}}\Gamma\left(p+1,-\frac{8\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^8e^{10}}$$
$$+\frac{24\ 7^{-p}d^3e^{-\frac{7a}{b}}\Gamma\left(p+1,-\frac{7\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^7e^{10}}$$
$$-\frac{7\ 6^{1-p}d^4e^{-\frac{6a}{b}}\Gamma\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^6e^{10}}$$
$$+\frac{252\ 5^{-p-1}d^5e^{-\frac{5a}{b}}\Gamma\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^5e^{10}}$$
$$-\frac{21\ 2^{1-2p}d^6e^{-\frac{4a}{b}}\Gamma\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^4e^{10}}$$
$$+\frac{8\ 3^{1-p}d^7e^{-\frac{3a}{b}}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^3e^{10}}$$
$$-\frac{9\ 2^{-p}d^8e^{-\frac{2a}{b}}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{c^2e^{10}}$$
$$+\frac{2d^9e^{-\frac{a}{b}}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)}{b}\right)^{-p}}{ce^{10}}$$

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^6,x]
```

```
[Out] -((5^(-1 - p)*Gamma[1 + p, (-10*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*L
og[c*(d + e/Sqrt[x])])^p)/(2^p*c^10*e^10*E^((10*a)/b)*(-((a + b*Log[c*(d +
e/Sqrt[x])])/b))^p)) + (2*d*Gamma[1 + p, (-9*(a + b*Log[c*(d + e/Sqrt[x])])
)/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(9^p*c^9*e^10*E^((9*a)/b)*(-((a + b*
Log[c*(d + e/Sqrt[x])])/b))^p) - (9*d^2*Gamma[1 + p, (-8*(a + b*Log[c*(d +
```

```
e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(8^p*c^8*e^10*E^((8*a)/b
)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (24*d^3*Gamma[1 + p, (-7*(a +
b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(7^p*c^7*e^
10*E^((7*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) - (7*6^(1 - p)*d^4*
Gamma[1 + p, (-6*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqr
t[x])])^p)/(c^6*e^10*E^((6*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) +
 (252*5^(-1 - p)*d^5*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a
 + b*Log[c*(d + e/Sqrt[x])])^p)/(c^5*e^10*E^((5*a)/b)*(-((a + b*Log[c*(d +
e/Sqrt[x])])/b))^p) - (21*2^(1 - 2*p)*d^6*Gamma[1 + p, (-4*(a + b*Log[c*(d
+ e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(c^4*e^10*E^((4*a)/b)*
(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (8*3^(1 - p)*d^7*Gamma[1 + p, (-
3*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(c^3
*e^10*E^((3*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) - (9*d^8*Gamma[1
 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])]))/b]*(a + b*Log[c*(d + e/Sqrt[x])])
^p)/(2^p*c^2*e^10*E^((2*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p) + (2
*d^9*Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])])/b)]*(a + b*Log[c*(d + e/
Sqrt[x])])^p)/(c*e^10*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = -\left(2\text{Subst}\left(\int x^9(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\text{Subst}\left(\int\left(-\frac{d^9(a+b\log(c(d+ex)))^p}{e^9}+\frac{9d^8(d+ex)(a+b\log(c(d+ex)))^p}{e^9}-\frac{36d^7(d+ex)^2(\right.\right.\right.$$

$$= -\frac{2\text{Subst}\left(\int (d+ex)^9(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^9} + \frac{(18d)\text{Subst}\left(\int (d+ex)^8(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^9} - \frac{(72d^2)\,\text{Subst}\left(\int (d+ex)^7(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^9} + \frac{(168d^3)\,\text{Subst}\left(\int (d+ex)^6(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^9} - \frac{(252d^4)\,\text{Subst}\left(\int (d+ex)^5(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^9} + \frac{(252d^5)\,\text{Subst}\left(\int (d+ex)^4(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^9} - \frac{(168d^6)\,\text{Subst}\left(\int (d+ex)^3(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^9} + \frac{(72d^7)\,\text{Subst}\left(\int (d+ex)^2(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^9} - \frac{(18d^8)\,\text{Subst}\left(\int (d+ex)(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^9} + \frac{(2d^9)\,\text{Subst}\left(\int (a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt{x}}\right)}{e^9}$$

$$
\begin{aligned}
&= -\frac{2\text{Subst}\left(\int x^9(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^{10}}\\
&+\frac{(18d)\text{Subst}\left(\int x^8(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^{10}}\\
&-\frac{(72d^2)\,\text{Subst}\left(\int x^7(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^{10}}\\
&+\frac{(168d^3)\,\text{Subst}\left(\int x^6(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^{10}}\\
&-\frac{(252d^4)\,\text{Subst}\left(\int x^5(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^{10}}\\
&+\frac{(252d^5)\,\text{Subst}\left(\int x^4(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^{10}}\\
&-\frac{(168d^6)\,\text{Subst}\left(\int x^3(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^{10}}\\
&+\frac{(72d^7)\,\text{Subst}\left(\int x^2(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^{10}}\\
&-\frac{(18d^8)\,\text{Subst}\left(\int x(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^{10}}\\
&+\frac{(2d^9)\,\text{Subst}\left(\int (a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^{10}}
\end{aligned}
$$

$$= -\frac{2\text{Subst}\left(\int e^{10x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^{10}e^{10}} + \frac{(18d)\text{Subst}\left(\int e^{9x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^9e^{10}} - \frac{(72d^2)\,\text{Subst}\left(\int e^{8x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^8e^{10}} + \frac{(168d^3)\,\text{Subst}\left(\int e^{7x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^7e^{10}} - \frac{(252d^4)\,\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^6e^{10}} + \frac{(252d^5)\,\text{Subst}\left(\int e^{5x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^5e^{10}} - \frac{(168d^6)\,\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^4e^{10}} + \frac{(72d^7)\,\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^3e^{10}} - \frac{(18d^8)\,\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{c^2e^{10}} + \frac{(2d^9)\,\text{Subst}\left(\int e^{x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{ce^{10}}$$

$$= \text{Too large to display}$$

Mathematica [A] (verified)

Time = 3.49 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.57

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx = \frac{5^{-1-p}504^{-p}e^{-\frac{10a}{b}}\left(-252^p\Gamma\left(1+p,-\frac{10\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right)+cde^{a/b}\left(2^{1+3p}5^{1+p}7^p\Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)}{b}\right.\right.\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])])^p/x^6,x]
```

```
[Out] (5^(-1 - p)*(-(252^p*Gamma[1 + p, (-10*(a + b*Log[c*(d + e/Sqrt[x])]))/b])
+ c*d*E^(a/b)*(2^(1 + 3*p)*5^(1 + p)*7^p*Gamma[1 + p, (-9*(a + b*Log[c*(d +
 e/Sqrt[x])]))/b] + c*d*E^(a/b)*(-(7^p*45^(1 + p)*Gamma[1 + p, (-8*(a + b*L
og[c*(d + e/Sqrt[x])]))/b]) + 2^p*c*d*E^(a/b)*(2^(3 + 2*p)*3^(1 + 2*p)*5^(1
```

```
 + p)*Gamma[1 + p, (-7*(a + b*Log[c*(d + e/Sqrt[x])]))/b] + 7^p*c*d*E^(a/b)
*(-7*30^(1 + p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/Sqrt[x])]))/b] + c*d*E
^(a/b)*(7*36^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqrt[x])]))/b] +
3^p*5^(1 + p)*c*d*E^(a/b)*(-14*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Log[c*(d +
 e/Sqrt[x])]))/b] + 2^p*c*d*E^(a/b)*(3*2^(3 + p)*Gamma[1 + p, (-3*(a + b*Lo
g[c*(d + e/Sqrt[x])]))/b] + 3^p*c*d*E^(a/b)*(-9*Gamma[1 + p, (-2*(a + b*Log
[c*(d + e/Sqrt[x])]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -((a + b*Log[
c*(d + e/Sqrt[x])])/b)])))))))))*(a + b*Log[c*(d + e/Sqrt[x])])^p)/(504^p*c
^10*e^10*E^((10*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])])/b))^p)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^{p}}{x^{6}}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/2))))^p/x^6,x)
[Out] int((a+b*ln(c*(d+e/x^(1/2))))^p/x^6,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^{p}}{x^{6}}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^{p}}{x^{6}}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^6,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*sqrt(x))/x) + a)^p/x^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^{p}}{x^{6}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))))**p/x**6,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^6}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^6,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^6, x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)+a\right)^p}{x^6}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))))^p/x^6,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))) + a)^p/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)\right)\right)^p}{x^6}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/2))))^p/x^6,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))))^p/x^6, x)
```

3.550 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx=\text{Int}\left(x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x*(a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx$$

```
[In] Int[x*(a + b*Log[c*(d + e/Sqrt[x])^2])^p,x]
[Out] 2*Defer[Subst][Defer[Int][x^3*(a + b*Log[c*(d + e/x)^2])^p, x], x, Sqrt[x]]
```

Rubi steps

$$\text{integral}=2\text{Subst}\left(\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^2\right)\right)^p\,dx,x,\sqrt{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx$$

```
[In] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^2])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e/Sqrt[x])^2])^p, x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx$$

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
[Out] int(x*(a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/2))**2))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p*x, x)
```

Giac [N/A]

Not integrable

Time = 2.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p*x, x)
```

Mupad [N/A]

Not integrable

Time = 1.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx=\int x\,\left(a+b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx$$

```
[In] int(x*(a + b*log(c*(d + e/x^(1/2))^2))^p,x)
[Out] int(x*(a + b*log(c*(d + e/x^(1/2))^2))^p, x)
```

3.551 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx=\text{Int}\left(\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx=\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx$$

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p,x]
[Out] 2*Defer[Subst][Defer[Int][x*(a + b*Log[c*(d + e/x)^2])^p, x], x, Sqrt[x]]
```

Rubi steps

$$\text{integral}=2\text{Subst}\left(\int x\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^2\right)\right)^p dx,x,\sqrt{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx = \int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
[Out] int((a+b*ln(c*(d+e/x^(1/2))^2))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p dx = \int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx=\int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx=\int \left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx=\int \left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/2))^2))^p,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))^2))^p, x)
```

3.552 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx=\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/2))^2))^p/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x,x]
[Out] 2*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x)^2])^p/x, x], x, Sqrt[x]]
```

Rubi steps

$$\text{integral}=2\operatorname{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^2\right)\right)^p}{x}\,dx,x,\sqrt{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x,x)
[Out] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 2.89 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\ \ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x, x)
```

3.553 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 216

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx = \frac{2^{1+p}de^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)\Gamma\left(1+p,\frac{-a-b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}}{e^2\sqrt{c\left(d+\frac{e}{\sqrt{x}}\right)^2}} - \frac{e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}}{ce^2}$$

```
[Out] -GAMMA(p+1,(-a-b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/c/
e^2/exp(a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)+2^(p+1)*d*GAMMA(p+1,1/2*(
-a-b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p*(d+e/x^(1/2))/
e^2/exp(1/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2))^2)^(
1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2504, 2448, 2436, 2337, 2212, 2437, 2347}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx$$
$$=\frac{d2^{p+1}e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{2b}\right)}{e^2\sqrt{c\left(d+\frac{e}{\sqrt{x}}\right)^2}}$$
$$-\frac{e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)}{ce^2}$$

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^2,x]
```

```
[Out] -((Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e/
Sqrt[x])^2])^p)/(c*e^2*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p))
+ (2^(1 + p)*d*(d + e/Sqrt[x])*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e/Sqrt[x
])^2])/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(e^2*E^(a/(2*b))*Sqrt[c*(d +
e/Sqrt[x])^2]*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(2\text{Subst}\left(\int x\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\frac{1}{\sqrt{x}}\right)\right)\\
&= -\left(2\text{Subst}\left(\int\left(-\frac{d\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e}+\frac{(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e}\right)dx,x,\frac{1}{\sqrt{x}}\right)\right)\\
&= -\frac{2\text{Subst}\left(\int(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\frac{1}{\sqrt{x}}\right)}{e}\\
&\quad+\frac{(2d)\text{Subst}\left(\int\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\frac{1}{\sqrt{x}}\right)}{e}\\
&= -\frac{2\text{Subst}\left(\int x\left(a+b\log\left(cx^2\right)\right)^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^2}\\
&\quad+\frac{(2d)\text{Subst}\left(\int\left(a+b\log\left(cx^2\right)\right)^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^2}
\end{aligned}
$$

$$= -\frac{\text{Subst}\left(\int e^x(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{ce^2} + \frac{\left(d\left(d+\frac{e}{\sqrt{x}}\right)\right)\text{Subst}\left(\int e^{x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{e^2\sqrt{c\left(d+\frac{e}{\sqrt{x}}\right)^2}}$$

$$= -\frac{e^{-\frac{a}{b}}\Gamma\left(1+p, -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}}{ce^2} + \frac{2^{1+p}de^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)\Gamma\left(1+p, -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\right.\right.}{b}\right.}{e^2\sqrt{c\left(d+\frac{e}{\sqrt{x}}\right)^2}}$$

Mathematica [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx = \int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^2, x]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^2,x)
[Out] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p/x**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^2}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^2,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^2, x)
```

3.554 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 676

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx =$$

$$-\frac{3^{-1-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}}{c^3e^6}$$

$$+\frac{2^{1+p}5^{-p}de^{-\frac{5a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)^5\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\right.\right.}{b}\right.}{e^6\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)^{5/2}}$$

$$-\frac{5\ 2^{-p}d^2e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}}{c^2e^6}$$

$$+\frac{5\ 2^{2+p}3^{-1-p}d^3e^{-\frac{3a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log}{}\right.}{e^6\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)^{3/2}}$$

$$-\frac{5d^4e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}}{ce^6}$$

$$+\frac{2^{1+p}d^5e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right.}{e^6\sqrt{c\left(d+\frac{e}{\sqrt{x}}\right)^2}}$$

```
[Out] -3^(-1-p)*GAMMA(p+1,-3*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2
))^2))^p/c^3/e^6/exp(3*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)-5*d^2*GAMM
A(p+1,-2*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/(2^p)
/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)-5*d^4*GAMMA(p+1,(-
a-b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/c/e^6/exp(a/b)/
(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)+2^(p+1)*d*GAMMA(p+1,-5/2*(a+b*ln(c*(d+
e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p*(d+e/x^(1/2))^5/(5^p)/e^6/e
xp(5/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2))^2)^(5/2)+
5*2^(2+p)*3^(-1-p)*d^3*GAMMA(p+1,-3/2*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*l
n(c*(d+e/x^(1/2))^2))^p*(d+e/x^(1/2))^3/e^6/exp(3/2*a/b)/(((-a-b*ln(c*(d+e/
```

```
x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2))^2)^(3/2)+2^(p+1)*d^5*GAMMA(p+1,1/2*(-a-
b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p*(d+e/x^(1/2))/e^6
/exp(1/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2))^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 676, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {2504, 2448, 2436, 2337, 2212, 2437, 2347}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx = -\frac{3^{-p-1}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)}{c^3e^6}$$
$$-\frac{5d^2 2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)}{c^2e^6}$$
$$+\frac{d^5 2^{p+1}e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{2b}\right.}{e^6\sqrt{c\left(d+\frac{e}{\sqrt{x}}\right)^2}}$$
$$-\frac{5d^4e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)}{ce^6}$$
$$+\frac{5d^3 2^{p+2}3^{-p-1}e^{-\frac{3a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(\cdots\right.\right.}{\cdots}\right.}{e^6\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)^{3/2}}$$
$$+\frac{d2^{p+1}5^{-p}e^{-\frac{5a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+\cdots\right.\right.\right.}{2b}\right.}{e^6\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)^{5/2}}$$

```
[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^4,x]
```

```
[Out] -((3^(-1 - p)*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*(a + b*
Log[c*(d + e/Sqrt[x])^2])^p)/(c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e/Sq
rt[x])^2])/b))^p)) + (2^(1 + p)*d*(d + e/Sqrt[x])^5*Gamma[1 + p, (-5*(a + b
*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(5^p
*e^6*E^((5*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(5/2)*(-((a + b*Log[c*(d + e/Sqr
t[x])^2])/b))^p) - (5*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])^2])
)/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(2^p*c^2*e^6*E^((2*a)/b)*(-((a + b
*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (5*2^(2 + p)*3^(-1 - p)*d^3*(d + e/Sqrt
[x])^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log
[c*(d + e/Sqrt[x])^2])^p)/(e^6*E^((3*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(3/2)*
(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) - (5*d^4*Gamma[1 + p, -((a + b*L
og[c*(d + e/Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(c*e^6*E^(
a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (2^(1 + p)*d^5*(d + e/Sqr
t[x])*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e/Sqrt[x])^2])/b]*(a + b*Log[c*(d
 + e/Sqrt[x])^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e/Sqrt[x])^2]*(-((a + b*L
og[c*(d + e/Sqrt[x])^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(2\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\frac{1}{\sqrt{x}}\right)\right)\\
&= -\left(2\text{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}+\frac{5d^4(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}-\frac{10d^3(d+ex\right.\right.\right.\\
&= -\frac{2\text{Subst}\left(\int(d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\frac{1}{\sqrt{x}}\right)}{e^5}\\
&\quad+\frac{(10d)\text{Subst}\left(\int(d+ex)^4\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\frac{1}{\sqrt{x}}\right)}{e^5}\\
&\quad-\frac{\left(20d^2\right)\text{Subst}\left(\int(d+ex)^3\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\frac{1}{\sqrt{x}}\right)}{e^5}\\
&\quad+\frac{\left(20d^3\right)\text{Subst}\left(\int(d+ex)^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\frac{1}{\sqrt{x}}\right)}{e^5}\\
&\quad-\frac{\left(10d^4\right)\text{Subst}\left(\int(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\frac{1}{\sqrt{x}}\right)}{e^5}\\
&\quad+\frac{\left(2d^5\right)\text{Subst}\left(\int\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\frac{1}{\sqrt{x}}\right)}{e^5}
\end{aligned}$$

$$= -\frac{2\text{Subst}\left(\int x^5(a+b\log(cx^2))^p\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6} + \frac{(10d)\text{Subst}\left(\int x^4(a+b\log(cx^2))^p\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6} - \frac{(20d^2)\,\text{Subst}\left(\int x^3(a+b\log(cx^2))^p\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6} + \frac{(20d^3)\,\text{Subst}\left(\int x^2(a+b\log(cx^2))^p\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6} - \frac{(10d^4)\,\text{Subst}\left(\int x(a+b\log(cx^2))^p\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6} + \frac{(2d^5)\,\text{Subst}\left(\int (a+b\log(cx^2))^p\,dx, x, d+\frac{e}{\sqrt{x}}\right)}{e^6}$$

$$= -\frac{\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{c^3e^6} - \frac{(10d^2)\,\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{c^2e^6} - \frac{(5d^4)\,\text{Subst}\left(\int e^{x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{ce^6} + \frac{\left(5d\left(d+\frac{e}{\sqrt{x}}\right)^5\right)\text{Subst}\left(\int e^{5x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{e^6\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)^{5/2}} + \frac{\left(10d^3\left(d+\frac{e}{\sqrt{x}}\right)^3\right)\text{Subst}\left(\int e^{3x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{e^6\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)^{3/2}} + \frac{\left(d^5\left(d+\frac{e}{\sqrt{x}}\right)\right)\text{Subst}\left(\int e^{x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{e^6\sqrt{c\left(d+\frac{e}{\sqrt{x}}\right)^2}}$$

$$= -\frac{3^{-1-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)}{c^3e^6}$$
$$+\frac{2^{1+p}5^{-p}de^{-\frac{5a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)^5\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log}{}\right)}{e^6\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)^{5/2}}$$
$$-\frac{5\ 2^{-p}d^2e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)}{c^2e^6}$$
$$+\frac{5\ 2^{2+p}3^{-1-p}d^3e^{-\frac{3a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\right.}{e^6\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)^{3/2}}$$
$$-\frac{5d^4e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}}{ce^6}$$
$$+\frac{2^{1+p}d^5e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\right.\right.}{b}\right.}{e^6\sqrt{c\left(d+\frac{e}{\sqrt{x}}\right)^2}}$$

Mathematica [F]

$$\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx=\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^4,x]
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^4, x]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^4,x)
[Out] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^4,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^4,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p/x**4,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^4,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^4, x)
```

Giac [F]

$$\int \frac{\left(a + b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx = \int \frac{\left(b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^2\right) + a\right)^p}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^4,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a + b\log\left(c\left(d + \frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx = \int \frac{\left(a + b\,\ln\left(c\left(d + \frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^4}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^4,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^4, x)
```

3.555 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1141

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx =$$

$$-\frac{5^{-1-p}e^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}}{c^5e^{10}}$$

$$+\frac{2^{1+p}9^{-p}de^{-\frac{9a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)^9\Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{}}\right.\right.}{b}\right.}{e^{10}\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)^{9/2}}$$

$$-\frac{9\ 4^{-p}d^2e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}}{c^4e^{10}}$$

$$+\frac{3\ 2^{3+p}7^{-p}d^3e^{-\frac{7a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)^7\Gamma\left(1+p,-\frac{7\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(\right.\right.}{b}\right.}{e^{10}\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)^{7/2}}$$

$$-\frac{14\ 3^{1-p}d^4e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}}{c^3e^{10}}$$

$$+\frac{63\ 2^{2+p}5^{-1-p}d^5e^{-\frac{5a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)^5\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log}{}\right.}{e^{10}\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)^{5/2}}$$

$$-\frac{21\ 2^{1-p}d^6e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}}{c^2e^{10}}$$

$$+\frac{2^{3+p}3^{1-p}d^7e^{-\frac{3a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\right.\right.}{b}\right.}{e^{10}\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)^{3/2}}$$

$$-\frac{9d^8e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}}{ce^{10}}$$

$$+\frac{2^{1+p}d^9e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)}{}$$

```
[Out] -5^(-1-p)*GAMMA(p+1,-5*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2
))^2))^p/c^5/e^10/exp(5*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)-9*d^2*GAM
MA(p+1,-4*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/(4^p
)/c^4/e^10/exp(4*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)-14*3^(1-p)*d^4*G
AMMA(p+1,-3*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/c^
3/e^10/exp(3*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)-21*2^(1-p)*d^6*GAMMA
(p+1,-2*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/c^2/e^
10/exp(2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)-9*d^8*GAMMA(p+1,(-a-b*ln
(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p/c/e^10/exp(a/b)/(((-a
-b*ln(c*(d+e/x^(1/2))^2))/b)^p)+2^(p+1)*d*GAMMA(p+1,-9/2*(a+b*ln(c*(d+e/x^(
1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p*(d+e/x^(1/2))^9/(9^p)/e^10/exp(9
/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2))^2)^(9/2)+3*2^
(3+p)*d^3*GAMMA(p+1,-7/2*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1
/2))^2))^p*(d+e/x^(1/2))^7/(7^p)/e^10/exp(7/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2
))^2))/b)^p)/(c*(d+e/x^(1/2))^2)^(7/2)+63*2^(2+p)*5^(-1-p)*d^5*GAMMA(p+1,-5
/2*(a+b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p*(d+e/x^(1/2
))^5/e^10/exp(5/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2)
)^2)^(5/2)+2^(3+p)*3^(1-p)*d^7*GAMMA(p+1,-3/2*(a+b*ln(c*(d+e/x^(1/2))^2))/b
)*(a+b*ln(c*(d+e/x^(1/2))^2))^p*(d+e/x^(1/2))^3/e^10/exp(3/2*a/b)/(((-a-b*l
n(c*(d+e/x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2))^2)^(3/2)+2^(p+1)*d^9*GAMMA(p+1
,1/2*(-a-b*ln(c*(d+e/x^(1/2))^2))/b)*(a+b*ln(c*(d+e/x^(1/2))^2))^p*(d+e/x^(
1/2))/e^10/exp(1/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/2))^2))/b)^p)/(c*(d+e/x^(1/2
))^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 1141, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used

$= \{2504, 2448, 2436, 2337, 2212, 2437, 2347\}$

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx =$$

$$-\frac{5^{-p-1}e^{-\frac{5a}{b}}\Gamma\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}}{c^5e^{10}}$$

$$+\frac{2^{p+1}9^{-p}de^{-\frac{9a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)^9\Gamma\left(p+1,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\right.\right.}{b}\right.}{e^{10}\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)^{9/2}}$$

$$-\frac{9\ 4^{-p}d^2e^{-\frac{4a}{b}}\Gamma\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}}{c^4e^{10}}$$

$$+\frac{3\ 2^{p+3}7^{-p}d^3e^{-\frac{7a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)^7\Gamma\left(p+1,-\frac{7\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(\right.\right.}{b}\right.}{e^{10}\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)^{7/2}}$$

$$-\frac{14\ 3^{1-p}d^4e^{-\frac{3a}{b}}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}}{c^3e^{10}}$$

$$+\frac{63\ 2^{p+2}5^{-p-1}d^5e^{-\frac{5a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)^5\Gamma\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log}{}\right.}{e^{10}\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)^{5/2}}$$

$$-\frac{21\ 2^{1-p}d^6e^{-\frac{2a}{b}}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}}{c^2e^{10}}$$

$$+\frac{2^{p+3}3^{1-p}d^7e^{-\frac{3a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)^3\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\right.\right.}{b}\right.}{e^{10}\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)^{3/2}}$$

$$-\frac{9d^8e^{-\frac{a}{b}}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)^{-p}}{ce^{10}}$$

$$+\frac{2^{p+1}d^9e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt{x}}\right)\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)}{b}\right)}{e^{10}\sqrt{c\left(d+\frac{e}{\right.}\right)^2}}$$

[In] Int[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^6,x]

[Out] -((5^(-1 - p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(c^5*e^10*E^((5*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p)) + (2^(1 + p)*d*(d + e/Sqrt[x])^9*Gamma[1 + p, (-9*(a + b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(9^p*e^10*E^((9*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(9/2)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) - (9*d^2*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(4^p*c^4*e^10*E^((4*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (3*2^(3 + p)*d^3*(d + e/Sqrt[x])^7*Gamma[1 + p, (-7*(a + b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(7^p*e^10*E^((7*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(7/2)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) - (14*3^(1 - p)*d^4*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(c^3*e^10*E^((3*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (63*2^(2 + p)*5^(-1 - p)*d^5*(d + e/Sqrt[x])^5*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(e^10*E^((5*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(5/2)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) - (21*2^(1 - p)*d^6*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/Sqrt[x])^2]))/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(c^2*e^10*E^((2*a)/b)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (2^(3 + p)*3^(1 - p)*d^7*(d + e/Sqrt[x])^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/Sqrt[x])^2]))/(2*b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(e^10*E^((3*a)/(2*b))*(c*(d + e/Sqrt[x])^2)^(3/2)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) - (9*d^8*Gamma[1 + p, -((a + b*Log[c*(d + e/Sqrt[x])^2])/b)]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(c*e^10*E^(a/b)*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p) + (2^(1 + p)*d^9*(d + e/Sqrt[x])*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e/Sqrt[x])^2])/b]*(a + b*Log[c*(d + e/Sqrt[x])^2])^p)/(e^10*E^(a/(2*b))*Sqrt[c*(d + e/Sqrt[x])^2]*(-((a + b*Log[c*(d + e/Sqrt[x])^2])/b))^p)

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
```

```
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule steps

$$\text{integral} = -\left(2\text{Subst}\left(\int x^9\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\frac{1}{\sqrt{x}}\right)\right)$$

$$= -\left(2\text{Subst}\left(\int\left(-\frac{d^9\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^9}+\frac{9d^8(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^9}-\frac{36d^7(d+ex\right.\right.\right.$$

$$=-\frac{2\text{Subst}\left(\int (d+ex)^9 \left(a+b\log \left(c(d+ex)^2\right)\right)^p \,dx,x,\frac{1}{\sqrt{x}}\right)}{e^9}$$
$$+\frac{(18d)\text{Subst}\left(\int (d+ex)^8 \left(a+b\log \left(c(d+ex)^2\right)\right)^p \,dx,x,\frac{1}{\sqrt{x}}\right)}{e^9}$$
$$-\frac{(72d^2)\,\text{Subst}\left(\int (d+ex)^7 \left(a+b\log \left(c(d+ex)^2\right)\right)^p \,dx,x,\frac{1}{\sqrt{x}}\right)}{e^9}$$
$$+\frac{(168d^3)\,\text{Subst}\left(\int (d+ex)^6 \left(a+b\log \left(c(d+ex)^2\right)\right)^p \,dx,x,\frac{1}{\sqrt{x}}\right)}{e^9}$$
$$-\frac{(252d^4)\,\text{Subst}\left(\int (d+ex)^5 \left(a+b\log \left(c(d+ex)^2\right)\right)^p \,dx,x,\frac{1}{\sqrt{x}}\right)}{e^9}$$
$$+\frac{(252d^5)\,\text{Subst}\left(\int (d+ex)^4 \left(a+b\log \left(c(d+ex)^2\right)\right)^p \,dx,x,\frac{1}{\sqrt{x}}\right)}{e^9}$$
$$-\frac{(168d^6)\,\text{Subst}\left(\int (d+ex)^3 \left(a+b\log \left(c(d+ex)^2\right)\right)^p \,dx,x,\frac{1}{\sqrt{x}}\right)}{e^9}$$
$$+\frac{(72d^7)\,\text{Subst}\left(\int (d+ex)^2 \left(a+b\log \left(c(d+ex)^2\right)\right)^p \,dx,x,\frac{1}{\sqrt{x}}\right)}{e^9}$$
$$-\frac{(18d^8)\,\text{Subst}\left(\int (d+ex) \left(a+b\log \left(c(d+ex)^2\right)\right)^p \,dx,x,\frac{1}{\sqrt{x}}\right)}{e^9}$$
$$+\frac{(2d^9)\,\text{Subst}\left(\int \left(a+b\log \left(c(d+ex)^2\right)\right)^p \,dx,x,\frac{1}{\sqrt{x}}\right)}{e^9}$$

$$
\begin{aligned}
&= -\frac{2\text{Subst}\left(\int x^9(a+b\log(cx^2))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^{10}} \\
&+ \frac{(18d)\text{Subst}\left(\int x^8(a+b\log(cx^2))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^{10}} \\
&- \frac{(72d^2)\,\text{Subst}\left(\int x^7(a+b\log(cx^2))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^{10}} \\
&+ \frac{(168d^3)\,\text{Subst}\left(\int x^6(a+b\log(cx^2))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^{10}} \\
&- \frac{(252d^4)\,\text{Subst}\left(\int x^5(a+b\log(cx^2))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^{10}} \\
&+ \frac{(252d^5)\,\text{Subst}\left(\int x^4(a+b\log(cx^2))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^{10}} \\
&- \frac{(168d^6)\,\text{Subst}\left(\int x^3(a+b\log(cx^2))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^{10}} \\
&+ \frac{(72d^7)\,\text{Subst}\left(\int x^2(a+b\log(cx^2))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^{10}} \\
&- \frac{(18d^8)\,\text{Subst}\left(\int x(a+b\log(cx^2))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^{10}} \\
&+ \frac{(2d^9)\,\text{Subst}\left(\int (a+b\log(cx^2))^p\,dx,x,d+\frac{e}{\sqrt{x}}\right)}{e^{10}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\mathrm{Subst}\left(\int e^{5x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{c^5e^{10}} \\
&- \frac{(36d^2)\,\mathrm{Subst}\left(\int e^{4x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{c^4e^{10}} \\
&- \frac{(126d^4)\,\mathrm{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{c^3e^{10}} \\
&- \frac{(84d^6)\,\mathrm{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{c^2e^{10}} \\
&- \frac{(9d^8)\,\mathrm{Subst}\left(\int e^{x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{ce^{10}} \\
&+ \frac{\left(9d\left(d+\frac{e}{\sqrt{x}}\right)^9\right)\mathrm{Subst}\left(\int e^{9x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{e^{10}\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)^{9/2}} \\
&+ \frac{\left(84d^3\left(d+\frac{e}{\sqrt{x}}\right)^7\right)\mathrm{Subst}\left(\int e^{7x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{e^{10}\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)^{7/2}} \\
&+ \frac{\left(126d^5\left(d+\frac{e}{\sqrt{x}}\right)^5\right)\mathrm{Subst}\left(\int e^{5x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{e^{10}\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)^{5/2}} \\
&+ \frac{\left(36d^7\left(d+\frac{e}{\sqrt{x}}\right)^3\right)\mathrm{Subst}\left(\int e^{3x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{e^{10}\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)^{3/2}} \\
&+ \frac{\left(d^9\left(d+\frac{e}{\sqrt{x}}\right)\right)\mathrm{Subst}\left(\int e^{x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)}{e^{10}\sqrt{c\left(d+\frac{e}{\sqrt{x}}\right)^2}}
\end{aligned}
$$

$= \text{Too large to display}$

Mathematica [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^6,x]
[Out] Integrate[(a + b*Log[c*(d + e/Sqrt[x])^2])^p/x^6, x]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^6,x)
[Out] int((a+b*ln(c*(d+e/x^(1/2))^2))^p/x^6,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^6}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^6,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*e*sqrt(x) + c*e^2)/x) + a)^p/x^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/2))**2))**p/x**6,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^6}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^6,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^6, x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)+a\right)^p}{x^6}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/2))^2))^p/x^6,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/sqrt(x))^2) + a)^p/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx = \int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{\sqrt{x}}\right)^2\right)\right)^p}{x^6}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^6,x)
[Out] int((a + b*log(c*(d + e/x^(1/2))^2))^p/x^6, x)
```

3.556 $\int x^3 \left(a+b \log \left(c \left(d+e \sqrt[3]{x}\right)\right)\right)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 1121

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$$

$$=\frac{3^{-p}4^{-1-p}e^{-\frac{12a}{b}}\Gamma\left(1+p,-\frac{12\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^{12}e^{12}}$$

$$-\frac{3\ 11^{-p}de^{-\frac{11a}{b}}\Gamma\left(1+p,-\frac{11\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^{11}e^{12}}$$

$$+\frac{33\ 2^{-1-p}5^{-p}d^2e^{-\frac{10a}{b}}\Gamma\left(1+p,-\frac{10\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^{10}e^{12}}$$

$$-\frac{55\ 9^{-p}d^3e^{-\frac{9a}{b}}\Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^9e^{12}}$$

$$+\frac{495\ 2^{-2-3p}d^4e^{-\frac{8a}{b}}\Gamma\left(1+p,-\frac{8\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^8e^{12}}$$

$$-\frac{198\ 7^{-p}d^5e^{-\frac{7a}{b}}\Gamma\left(1+p,-\frac{7\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^7e^{12}}$$

$$+\frac{77\ 2^{-p}3^{1-p}d^6e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^6e^{12}}$$

$$-\frac{198\ 5^{-p}d^7e^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^5e^{12}}$$

$$+\frac{495\ 4^{-1-p}d^8e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^4e^{12}}$$

$$-\frac{55\ 3^{-p}d^9e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^3e^{12}}$$

$$+\frac{33\ 2^{-1-p}d^{10}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^2e^{12}}$$

$$3d^{11}e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}$$

```
[Out] 4^(-1-p)*GAMMA(p+1,-12*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))
))^p/(3^p)/c^12/e^12/exp(12*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-3*d*GAM
MA(p+1,-11*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(11^p)/
c^11/e^12/exp(11*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)+33*2^(-1-p)*d^2*GA
MMA(p+1,-10*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(5^p)/
c^10/e^12/exp(10*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-55*d^3*GAMMA(p+1,-
9*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(9^p)/c^9/e^12/e
xp(9*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)+495*2^(-2-3*p)*d^4*GAMMA(p+1,-
8*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c^8/e^12/exp(8*a
/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-198*d^5*GAMMA(p+1,-7*(a+b*ln(c*(d+e*
x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(7^p)/c^7/e^12/exp(7*a/b)/(((-a-b
*ln(c*(d+e*x^(1/3))))/b)^p)+77*3^(1-p)*d^6*GAMMA(p+1,-6*(a+b*ln(c*(d+e*x^(1
/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(2^p)/c^6/e^12/exp(6*a/b)/(((-a-b*ln(
c*(d+e*x^(1/3))))/b)^p)-198*d^7*GAMMA(p+1,-5*(a+b*ln(c*(d+e*x^(1/3))))/b)*(
a+b*ln(c*(d+e*x^(1/3))))^p/(5^p)/c^5/e^12/exp(5*a/b)/(((-a-b*ln(c*(d+e*x^(1
/3))))/b)^p)+495*4^(-1-p)*d^8*GAMMA(p+1,-4*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+
b*ln(c*(d+e*x^(1/3))))^p/c^4/e^12/exp(4*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b
)^p)-55*d^9*GAMMA(p+1,-3*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3
))))^p/(3^p)/c^3/e^12/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)+33*2^(-
1-p)*d^10*GAMMA(p+1,-2*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))
))^p/c^2/e^12/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-3*d^11*GAMMA(p+
1,(-a-b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c/e^12/exp(a/b)
/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 1121, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used

$$= \{2504,\ 2448,\ 2436,\ 2336,\ 2212,\ 2437,\ 2346\}$$

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$$

$$= \frac{3^{-p}4^{-p-1}e^{-\frac{12a}{b}}\Gamma\left(p+1,-\frac{12\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^{12}e^{12}}$$
$$-\frac{3\ 11^{-p}de^{-\frac{11a}{b}}\Gamma\left(p+1,-\frac{11\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^{11}e^{12}}$$
$$+\frac{33\ 2^{-p-1}5^{-p}d^2e^{-\frac{10a}{b}}\Gamma\left(p+1,-\frac{10\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^{10}e^{12}}$$
$$-\frac{55\ 9^{-p}d^3e^{-\frac{9a}{b}}\Gamma\left(p+1,-\frac{9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^9e^{12}}$$
$$+\frac{495\ 2^{-3p-2}d^4e^{-\frac{8a}{b}}\Gamma\left(p+1,-\frac{8\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^8e^{12}}$$
$$-\frac{198\ 7^{-p}d^5e^{-\frac{7a}{b}}\Gamma\left(p+1,-\frac{7\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^7e^{12}}$$
$$+\frac{77\ 2^{-p}3^{1-p}d^6e^{-\frac{6a}{b}}\Gamma\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^6e^{12}}$$
$$-\frac{198\ 5^{-p}d^7e^{-\frac{5a}{b}}\Gamma\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^5e^{12}}$$
$$+\frac{495\ 4^{-p-1}d^8e^{-\frac{4a}{b}}\Gamma\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^4e^{12}}$$
$$-\frac{55\ 3^{-p}d^9e^{-\frac{3a}{b}}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^3e^{12}}$$
$$+\frac{33\ 2^{-p-1}d^{10}e^{-\frac{2a}{b}}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^2e^{12}}$$
$$-\frac{3d^{11}e^{-\frac{a}{b}}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{ce^{12}}$$

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] (4^(-1 - p)*Gamma[1 + p, (-12*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log
[c*(d + e*x^(1/3))])^p)/(3^p*c^12*e^12*E^((12*a)/b)*(-((a + b*Log[c*(d + e*
x^(1/3))])/b))^p) - (3*d*Gamma[1 + p, (-11*(a + b*Log[c*(d + e*x^(1/3))]))/
b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(11^p*c^11*e^12*E^((11*a)/b)*(-((a + b
*Log[c*(d + e*x^(1/3))])/b))^p) + (33*2^(-1 - p)*d^2*Gamma[1 + p, (-10*(a +
 b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(5^p*c^10*
e^12*E^((10*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (55*d^3*Gamma[
1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))]
)^p)/(9^p*c^9*e^12*E^((9*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (
495*2^(-2 - 3*p)*d^4*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a
 + b*Log[c*(d + e*x^(1/3))])^p)/(c^8*e^12*E^((8*a)/b)*(-((a + b*Log[c*(d +
e*x^(1/3))])/b))^p) - (198*d^5*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*x^(1/3)
)]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(7^p*c^7*e^12*E^((7*a)/b)*(-((a +
 b*Log[c*(d + e*x^(1/3))])/b))^p) + (77*3^(1 - p)*d^6*Gamma[1 + p, (-6*(a +
 b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(2^p*c^6*e
^12*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (198*d^7*Gamma[1
 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])
^p)/(5^p*c^5*e^12*E^((5*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (4
95*4^(-1 - p)*d^8*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a +
b*Log[c*(d + e*x^(1/3))])^p)/(c^4*e^12*E^((4*a)/b)*(-((a + b*Log[c*(d + e*x
^(1/3))])/b))^p) - (55*d^9*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))]))
/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(3^p*c^3*e^12*E^((3*a)/b)*(-((a + b*L
og[c*(d + e*x^(1/3))])/b))^p) + (33*2^(-1 - p)*d^10*Gamma[1 + p, (-2*(a + b
*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c^2*e^12*E^
((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (3*d^11*Gamma[1 + p, -
((a + b*Log[c*(d + e*x^(1/3))])/b)]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c*e^
12*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
```

```
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int x^{11}(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)$$

$$= 3\text{Subst}\left(\int \left(-\frac{d^{11}(a+b\log(c(d+ex)))^p}{e^{11}}+\frac{11d^{10}(d+ex)(a+b\log(c(d+ex)))^p}{e^{11}} - \frac{55d^9(d+ex)^2(a+b\log(c(d+ex)))^p}{e^{11}}+\frac{165d^8(d+ex)^3(a+b\log(c(d+ex)))^p}{e^{11}} - \frac{330d^7(d+ex)^4(a+b\log(c(d+ex)))^p}{e^{11}}+\frac{462d^6(d+ex)^5(a+b\log(c(d+ex)))^p}{e^{11}} - \frac{462d^5(d+ex)^6(a+b\log(c(d+ex)))^p}{e^{11}}+\frac{330d^4(d+ex)^7(a+b\log(c(d+ex)))^p}{e^{11}} - \frac{165d^3(d+ex)^8(a+b\log(c(d+ex)))^p}{e^{11}}+\frac{55d^2(d+ex)^9(a+b\log(c(d+ex)))^p}{e^{11}} - \frac{11d(d+ex)^{10}(a+b\log(c(d+ex)))^p}{e^{11}} + \frac{(d+ex)^{11}(a+b\log(c(d+ex)))^p}{e^{11}}\right)dx, x, \sqrt[3]{x}\right)$$

$$= \frac{3\text{Subst}\left(\int (d+ex)^{11}(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^{11}} - \frac{(33d)\text{Subst}\left(\int (d+ex)^{10}(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^{11}} + \frac{(165d^2)\,\text{Subst}\left(\int (d+ex)^9(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^{11}} - \frac{(495d^3)\,\text{Subst}\left(\int (d+ex)^8(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^{11}} + \frac{(990d^4)\,\text{Subst}\left(\int (d+ex)^7(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^{11}} - \frac{(1386d^5)\,\text{Subst}\left(\int (d+ex)^6(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^{11}} + \frac{(1386d^6)\,\text{Subst}\left(\int (d+ex)^5(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^{11}} - \frac{(990d^7)\,\text{Subst}\left(\int (d+ex)^4(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^{11}} + \frac{(495d^8)\,\text{Subst}\left(\int (d+ex)^3(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^{11}} - \frac{(165d^9)\,\text{Subst}\left(\int (d+ex)^2(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^{11}} + \frac{(33d^{10})\,\text{Subst}\left(\int (d+ex)(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^{11}} - \frac{(3d^{11})\,\text{Subst}\left(\int (a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^{11}}$$

$$
\begin{aligned}
&= \frac{3\text{Subst}\left(\int x^{11}(a+b\log(cx))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} \\
&\quad - \frac{(33d)\text{Subst}\left(\int x^{10}(a+b\log(cx))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} \\
&\quad + \frac{(165d^2)\,\text{Subst}\left(\int x^{9}(a+b\log(cx))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} \\
&\quad - \frac{(495d^3)\,\text{Subst}\left(\int x^{8}(a+b\log(cx))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} \\
&\quad + \frac{(990d^4)\,\text{Subst}\left(\int x^{7}(a+b\log(cx))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} \\
&\quad - \frac{(1386d^5)\,\text{Subst}\left(\int x^{6}(a+b\log(cx))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} \\
&\quad + \frac{(1386d^6)\,\text{Subst}\left(\int x^{5}(a+b\log(cx))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} \\
&\quad - \frac{(990d^7)\,\text{Subst}\left(\int x^{4}(a+b\log(cx))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} \\
&\quad + \frac{(495d^8)\,\text{Subst}\left(\int x^{3}(a+b\log(cx))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} \\
&\quad - \frac{(165d^9)\,\text{Subst}\left(\int x^{2}(a+b\log(cx))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} \\
&\quad + \frac{(33d^{10})\,\text{Subst}\left(\int x(a+b\log(cx))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} \\
&\quad - \frac{(3d^{11})\,\text{Subst}\left(\int (a+b\log(cx))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3\mathrm{Subst}\left(\int e^{12x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^{12}e^{12}}\\
&\quad-\frac{(33d)\mathrm{Subst}\left(\int e^{11x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^{11}e^{12}}\\
&\quad+\frac{(165d^2)\,\mathrm{Subst}\left(\int e^{10x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^{10}e^{12}}\\
&\quad-\frac{(495d^3)\,\mathrm{Subst}\left(\int e^{9x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^{9}e^{12}}\\
&\quad+\frac{(990d^4)\,\mathrm{Subst}\left(\int e^{8x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^{8}e^{12}}\\
&\quad-\frac{(1386d^5)\,\mathrm{Subst}\left(\int e^{7x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^{7}e^{12}}\\
&\quad+\frac{(1386d^6)\,\mathrm{Subst}\left(\int e^{6x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^{6}e^{12}}\\
&\quad-\frac{(990d^7)\,\mathrm{Subst}\left(\int e^{5x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^{5}e^{12}}\\
&\quad+\frac{(495d^8)\,\mathrm{Subst}\left(\int e^{4x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^{4}e^{12}}\\
&\quad-\frac{(165d^9)\,\mathrm{Subst}\left(\int e^{3x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^{3}e^{12}}\\
&\quad+\frac{(33d^{10})\,\mathrm{Subst}\left(\int e^{2x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^{2}e^{12}}\\
&\quad-\frac{(3d^{11})\,\mathrm{Subst}\left(\int e^{x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{ce^{12}}\\
&= \text{Too large to display}
\end{aligned}
$$

Mathematica [F]

$$
\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx
$$

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
[Out] Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))])^p, x]
```

Maple [F]

$$\int x^{3}\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)\right)\right)^{p}dx$$

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
[Out] int(x^3*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

Fricas [F]

$$\int x^{3}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^{p}\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}x^{3}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e*x^(1/3) + c*d) + a)^p*x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{3}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^{p}\,dx=\text{Timed out}$$

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/3))))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int x^{3}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^{p}\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}x^{3}\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x^3, x)
```

Giac [F]

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p x^3\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\int x^3\,\left(a+b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)\right)\right)^p\,dx$$

```
[In] int(x^3*(a + b*log(c*(d + e*x^(1/3))))^p,x)
[Out] int(x^3*(a + b*log(c*(d + e*x^(1/3))))^p, x)
```

3.557 $\int x^2 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p \,dx$

Optimal result

Integrand size = 22, antiderivative size = 831

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$$

$$=\frac{3^{-1-2p}e^{-\frac{9a}{b}}\Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^9e^9}$$
$$-\frac{3\,8^{-p}de^{-\frac{8a}{b}}\Gamma\left(1+p,-\frac{8\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^8e^9}$$
$$+\frac{12\,7^{-p}d^2e^{-\frac{7a}{b}}\Gamma\left(1+p,-\frac{7\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^7e^9}$$
$$-\frac{7\,2^{2-p}3^{-p}d^3e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^6e^9}$$
$$+\frac{42\,5^{-p}d^4e^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^5e^9}$$
$$-\frac{21\,2^{1-2p}d^5e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^4e^9}$$
$$+\frac{28\,3^{-p}d^6e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^3e^9}$$
$$-\frac{3\,2^{2-p}d^7e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^2e^9}$$
$$+\frac{3d^8e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{ce^9}$$

```
[Out] 3^(-1-2*p)*GAMMA(p+1,-9*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3)
)))^p/c^9/e^9/exp(9*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-3*d*GAMMA(p+1,-
8*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(8^p)/c^8/e^9/ex
p(8*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)+12*d^2*GAMMA(p+1,-7*(a+b*ln(c*(
```

```
d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(7^p)/c^7/e^9/exp(7*a/b)/(((-
a-b*ln(c*(d+e*x^(1/3))))/b)^p)-7*2^(2-p)*d^3*GAMMA(p+1,-6*(a+b*ln(c*(d+e*x^
(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(3^p)/c^6/e^9/exp(6*a/b)/(((-a-b*ln
(c*(d+e*x^(1/3))))/b)^p)+42*d^4*GAMMA(p+1,-5*(a+b*ln(c*(d+e*x^(1/3))))/b)*(
a+b*ln(c*(d+e*x^(1/3))))^p/(5^p)/c^5/e^9/exp(5*a/b)/(((-a-b*ln(c*(d+e*x^(1/
3))))/b)^p)-21*2^(1-2*p)*d^5*GAMMA(p+1,-4*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b
*ln(c*(d+e*x^(1/3))))^p/c^4/e^9/exp(4*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^
p)+28*d^6*GAMMA(p+1,-3*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))
))^p/(3^p)/c^3/e^9/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-3*2^(2-p)*
d^7*GAMMA(p+1,-2*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c
^2/e^9/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)+3*d^8*GAMMA(p+1,(-a-b*
ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c/e^9/exp(a/b)/(((-a-b*
ln(c*(d+e*x^(1/3))))/b)^p)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 831, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used

$= \{2504, 2448, 2436, 2336, 2212, 2437, 2346\}$

$$\int x^2 \left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)\right)\right)^p \, dx$$

$$= \frac{3^{-2p-1}e^{-\frac{9a}{b}}\Gamma\left(p+1, -\frac{9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^9e^9}$$
$$- \frac{3\ 8^{-p}de^{-\frac{8a}{b}}\Gamma\left(p+1, -\frac{8\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^8e^9}$$
$$+ \frac{12\ 7^{-p}d^2e^{-\frac{7a}{b}}\Gamma\left(p+1, -\frac{7\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^7e^9}$$
$$- \frac{7\ 2^{2-p}3^{-p}d^3e^{-\frac{6a}{b}}\Gamma\left(p+1, -\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^6e^9}$$
$$+ \frac{42\ 5^{-p}d^4e^{-\frac{5a}{b}}\Gamma\left(p+1, -\frac{5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^5e^9}$$
$$- \frac{21\ 2^{1-2p}d^5e^{-\frac{4a}{b}}\Gamma\left(p+1, -\frac{4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^4e^9}$$
$$+ \frac{28\ 3^{-p}d^6e^{-\frac{3a}{b}}\Gamma\left(p+1, -\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^3e^9}$$
$$- \frac{3\ 2^{2-p}d^7e^{-\frac{2a}{b}}\Gamma\left(p+1, -\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^2e^9}$$
$$+ \frac{3d^8e^{-\frac{a}{b}}\Gamma\left(p+1, -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{ce^9}$$

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] (3^(-1 - 2*p)*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Lo
g[c*(d + e*x^(1/3))])^p)/(c^9*e^9*E^((9*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3
))])/b))^p) - (3*d*Gamma[1 + p, (-8*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a +
 b*Log[c*(d + e*x^(1/3))])^p)/(8^p*c^8*e^9*E^((8*a)/b)*(-((a + b*Log[c*(d +
```

```
e*x^(1/3))])/b))^p) + (12*d^2*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*x^(1/3)
)]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(7^p*c^7*e^9*E^((7*a)/b)*(-((a +
b*Log[c*(d + e*x^(1/3))])/b))^p) - (7*2^(2 - p)*d^3*Gamma[1 + p, (-6*(a + b
*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(3^p*c^6*e^9
*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (42*d^4*Gamma[1 + p
, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/
(5^p*c^5*e^9*E^((5*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (21*2^(
1 - 2*p)*d^5*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log
[c*(d + e*x^(1/3))])^p)/(c^4*e^9*E^((4*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3)
)])/b))^p) + (28*d^6*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a
 + b*Log[c*(d + e*x^(1/3))])^p)/(3^p*c^3*e^9*E^((3*a)/b)*(-((a + b*Log[c*(d
 + e*x^(1/3))])/b))^p) - (3*2^(2 - p)*d^7*Gamma[1 + p, (-2*(a + b*Log[c*(d
+ e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c^2*e^9*E^((2*a)/b)*(
-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (3*d^8*Gamma[1 + p, -((a + b*Log[
c*(d + e*x^(1/3))])/b)]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c*e^9*E^(a/b)*(-
((a + b*Log[c*(d + e*x^(1/3))])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^8(a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)\\
&= 3\text{Subst}\left(\int \left(\frac{d^8(a+b\log(c(d+ex)))^p}{e^8}-\frac{8d^7(d+ex)(a+b\log(c(d+ex)))^p}{e^8}\right.\right.\\
&\quad+\frac{28d^6(d+ex)^2(a+b\log(c(d+ex)))^p}{e^8}-\frac{56d^5(d+ex)^3(a+b\log(c(d+ex)))^p}{e^8}\\
&\quad+\frac{70d^4(d+ex)^4(a+b\log(c(d+ex)))^p}{e^8}-\frac{56d^3(d+ex)^5(a+b\log(c(d+ex)))^p}{e^8}\\
&\quad+\frac{28d^2(d+ex)^6(a+b\log(c(d+ex)))^p}{e^8}-\frac{8d(d+ex)^7(a+b\log(c(d+ex)))^p}{e^8}\\
&\quad\left.\left.+\frac{(d+ex)^8(a+b\log(c(d+ex)))^p}{e^8}\right)\,dx,x,\sqrt[3]{x}\right)
\end{aligned}$$

$$
\begin{aligned}
&= \frac{3\text{Subst}\left(\int (d+ex)^8(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^8} \\
&\quad - \frac{(24d)\text{Subst}\left(\int (d+ex)^7(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^8} \\
&\quad + \frac{(84d^2)\,\text{Subst}\left(\int (d+ex)^6(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^8} \\
&\quad - \frac{(168d^3)\,\text{Subst}\left(\int (d+ex)^5(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^8} \\
&\quad + \frac{(210d^4)\,\text{Subst}\left(\int (d+ex)^4(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^8} \\
&\quad - \frac{(168d^5)\,\text{Subst}\left(\int (d+ex)^3(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^8} \\
&\quad + \frac{(84d^6)\,\text{Subst}\left(\int (d+ex)^2(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^8} \\
&\quad - \frac{(24d^7)\,\text{Subst}\left(\int (d+ex)(a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^8} \\
&\quad + \frac{(3d^8)\,\text{Subst}\left(\int (a+b\log(c(d+ex)))^p\,dx, x, \sqrt[3]{x}\right)}{e^8} \\
&= \frac{3\text{Subst}\left(\int x^8(a+b\log(cx))^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} \\
&\quad - \frac{(24d)\text{Subst}\left(\int x^7(a+b\log(cx))^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} \\
&\quad + \frac{(84d^2)\,\text{Subst}\left(\int x^6(a+b\log(cx))^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} \\
&\quad - \frac{(168d^3)\,\text{Subst}\left(\int x^5(a+b\log(cx))^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} \\
&\quad + \frac{(210d^4)\,\text{Subst}\left(\int x^4(a+b\log(cx))^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} \\
&\quad - \frac{(168d^5)\,\text{Subst}\left(\int x^3(a+b\log(cx))^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} \\
&\quad + \frac{(84d^6)\,\text{Subst}\left(\int x^2(a+b\log(cx))^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} \\
&\quad - \frac{(24d^7)\,\text{Subst}\left(\int x(a+b\log(cx))^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} \\
&\quad + \frac{(3d^8)\,\text{Subst}\left(\int (a+b\log(cx))^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^9}
\end{aligned}
$$

$$
\begin{aligned}
= & \frac{3\text{Subst}\left(\int e^{9x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^9e^9} \\
& -\frac{(24d)\text{Subst}\left(\int e^{8x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^8e^9} \\
& +\frac{(84d^2)\,\text{Subst}\left(\int e^{7x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^7e^9} \\
& -\frac{(168d^3)\,\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^6e^9} \\
& +\frac{(210d^4)\,\text{Subst}\left(\int e^{5x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^5e^9} \\
& -\frac{(168d^5)\,\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^4e^9} \\
& +\frac{(84d^6)\,\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^3e^9} \\
& -\frac{(24d^7)\,\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^2e^9} \\
& +\frac{(3d^8)\,\text{Subst}\left(\int e^{x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{ce^9}
\end{aligned}
$$

$$= \frac{3^{-1-2p} e^{-\frac{9a}{b}} \Gamma\left(1+p, -\frac{9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right) \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)}{c^9 e^9}$$

$$- \frac{3\ 8^{-p} d e^{-\frac{8a}{b}} \Gamma\left(1+p, -\frac{8\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right) \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right.}{c^8 e^9}$$

$$+ \frac{12\ 7^{-p} d^2 e^{-\frac{7a}{b}} \Gamma\left(1+p, -\frac{7\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right) \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.}{b}\right.}{c^7 e^9}$$

$$- \frac{7\ 2^{2-p} 3^{-p} d^3 e^{-\frac{6a}{b}} \Gamma\left(1+p, -\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right) \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\right.\right.}{b}\right.}{c^6 e^9}$$

$$+ \frac{42\ 5^{-p} d^4 e^{-\frac{5a}{b}} \Gamma\left(1+p, -\frac{5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right) \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.}{b}\right.}{c^5 e^9}$$

$$- \frac{21\ 2^{1-2p} d^5 e^{-\frac{4a}{b}} \Gamma\left(1+p, -\frac{4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right) \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{}\right.\right.}{b}\right.}{c^4 e^9}$$

$$+ \frac{28\ 3^{-p} d^6 e^{-\frac{3a}{b}} \Gamma\left(1+p, -\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right) \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.}{b}\right.}{c^3 e^9}$$

$$- \frac{3\ 2^{2-p} d^7 e^{-\frac{2a}{b}} \Gamma\left(1+p, -\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right) \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.}{b}\right.}{c^2 e^9}$$

$$+ \frac{3 d^8 e^{-\frac{a}{b}} \Gamma\left(1+p, -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right) \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c e^9}$$

Mathematica [F]

$$\int x^2 \left(a + b \log\left(c\left(d + e\sqrt[3]{x}\right)\right)\right)^p \, dx = \int x^2 \left(a + b \log\left(c\left(d + e\sqrt[3]{x}\right)\right)\right)^p \, dx$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
[Out] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))])^p, x]
```

Maple [F]

$$\int x^2 \left(a + b \ln\left(c\left(d + e\,x^{\frac{1}{3}}\right)\right)\right)^p dx$$

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

Fricas [F]

$$\int x^2 \left(a + b \log\left(c\left(d + e\sqrt[3]{x}\right)\right)\right)^p \, dx = \int \left(b \log\left(\left(ex^{\frac{1}{3}} + d\right)c\right) + a\right)^p x^2 \, dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e*x^(1/3) + c*d) + a)^p*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(a + b \log\left(c\left(d + e\sqrt[3]{x}\right)\right)\right)^p \, dx = \text{Timed out}$$

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int x^2 \left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)\right)\right)^p\,dx = \int \left(b\log\left(\left(ex^{\frac{1}{3}} + d\right)c\right) + a\right)^p x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x^2, x)
```

Giac [F]

$$\int x^2 \left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)\right)\right)^p\,dx = \int \left(b\log\left(\left(ex^{\frac{1}{3}} + d\right)c\right) + a\right)^p x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)\right)\right)^p\,dx = \int x^2 \left(a + b\,\ln\left(c\left(d + e\,x^{1/3}\right)\right)\right)^p\,dx$$

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/3))))^p,x)
[Out] int(x^2*(a + b*log(c*(d + e*x^(1/3))))^p, x)
```

3.558 $\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 553

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$$
$$= \frac{2^{-1-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^6e^6}$$
$$- \frac{3\ 5^{-p}de^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^5e^6}$$
$$+ \frac{15\ 2^{-1-2p}d^2e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^4e^6}$$
$$- \frac{10\ 3^{-p}d^3e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^3e^6}$$
$$+ \frac{15\ 2^{-1-p}d^4e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^2e^6}$$
$$- \frac{3d^5e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{ce^6}$$

```
[Out] 2^(-1-p)*GAMMA(p+1,-6*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3)))
)^p/(3^p)/c^6/e^6/exp(6*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-3*d*GAMMA(p
```

```
+1,-5*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(5^p)/c^5/e^
6/exp(5*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)+15*2^(-1-2*p)*d^2*GAMMA(p+1
,-4*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c^4/e^6/exp(4*
a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-10*d^3*GAMMA(p+1,-3*(a+b*ln(c*(d+e*
x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(3^p)/c^3/e^6/exp(3*a/b)/(((-a-b*
ln(c*(d+e*x^(1/3))))/b)^p)+15*2^(-1-p)*d^4*GAMMA(p+1,-2*(a+b*ln(c*(d+e*x^(1
/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+e*
x^(1/3))))/b)^p)-3*d^5*GAMMA(p+1,(-a-b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d
+e*x^(1/3))))^p/c/e^6/exp(a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2504, 2448, 2436, 2336, 2212, 2437, 2346}

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$$

$$=\frac{2^{-p-1}3^{-p}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)}{c^6e^6}$$
$$-\frac{3d5^{-p}e^{-\frac{5a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)}{c^5e^6}$$
$$+\frac{15d^22^{-2p-1}e^{-\frac{4a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)}{c^4e^6}$$
$$-\frac{10d^33^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)}{c^3e^6}$$
$$+\frac{15d^42^{-p-1}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)}{c^2e^6}$$
$$-\frac{3d^5e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)}{ce^6}$$

```
[In] Int[x*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
[Out] (2^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[
c*(d + e*x^(1/3))])^p)/(3^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(1
```

```
/3))])/b))^p) - (3*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a
 + b*Log[c*(d + e*x^(1/3))])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c*(d
 + e*x^(1/3))])/b))^p) + (15*2^(-1 - 2*p)*d^2*Gamma[1 + p, (-4*(a + b*Log[c
*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(c^4*e^6*E^((4*a)/
b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) - (10*d^3*Gamma[1 + p, (-3*(a +
 b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^(1/3))])^p)/(3^p*c^3*e
^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p) + (15*2^(-1 - p)*d^
4*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x
^(1/3))])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p)
- (3*d^5*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))])/b)]*(a + b*Log[c*(d
+ e*x^(1/3))])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^5(a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)\\
&= 3\text{Subst}\left(\int \left(-\frac{d^5(a+b\log(c(d+ex)))^p}{e^5}+\frac{5d^4(d+ex)(a+b\log(c(d+ex)))^p}{e^5}\right.\right.\\
&\quad -\frac{10d^3(d+ex)^2(a+b\log(c(d+ex)))^p}{e^5}+\frac{10d^2(d+ex)^3(a+b\log(c(d+ex)))^p}{e^5}\\
&\quad -\frac{5d(d+ex)^4(a+b\log(c(d+ex)))^p}{e^5}\\
&\quad \left.\left.+\frac{(d+ex)^5(a+b\log(c(d+ex)))^p}{e^5}\right)\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{3\text{Subst}\left(\int (d+ex)^5(a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)}{e^5}\\
&\quad -\frac{(15d)\text{Subst}\left(\int (d+ex)^4(a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)}{e^5}\\
&\quad +\frac{(30d^2)\,\text{Subst}\left(\int (d+ex)^3(a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)}{e^5}\\
&\quad -\frac{(30d^3)\,\text{Subst}\left(\int (d+ex)^2(a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)}{e^5}\\
&\quad +\frac{(15d^4)\,\text{Subst}\left(\int (d+ex)(a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)}{e^5}\\
&\quad -\frac{(3d^5)\,\text{Subst}\left(\int (a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)}{e^5}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{3\text{Subst}\left(\int x^5(a+b\log(cx))^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^6} \\
&\quad - \frac{(15d)\text{Subst}\left(\int x^4(a+b\log(cx))^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^6} \\
&\quad + \frac{(30d^2)\,\text{Subst}\left(\int x^3(a+b\log(cx))^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^6} \\
&\quad - \frac{(30d^3)\,\text{Subst}\left(\int x^2(a+b\log(cx))^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^6} \\
&\quad + \frac{(15d^4)\,\text{Subst}\left(\int x(a+b\log(cx))^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^6} \\
&\quad - \frac{(3d^5)\,\text{Subst}\left(\int (a+b\log(cx))^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^6} \\
&= \frac{3\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^6e^6} \\
&\quad - \frac{(15d)\text{Subst}\left(\int e^{5x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^5e^6} \\
&\quad + \frac{(30d^2)\,\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^4e^6} \\
&\quad - \frac{(30d^3)\,\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^3e^6} \\
&\quad + \frac{(15d^4)\,\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^2e^6} \\
&\quad - \frac{(3d^5)\,\text{Subst}\left(\int e^{x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{ce^6}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2^{-1-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c(d+e\sqrt[3]{x})\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right.}{c^6e^6} \\
&- \frac{3\ 5^{-p}de^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c(d+e\sqrt[3]{x})\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right.}{c^5e^6} \\
&+ \frac{15\ 2^{-1-2p}d^2e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c(d+e\sqrt[3]{x})\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\right.\right.}{b}\right.}{c^4e^6} \\
&- \frac{10\ 3^{-p}d^3e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c(d+e\sqrt[3]{x})\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.}{b}\right.}{c^3e^6} \\
&+ \frac{15\ 2^{-1-p}d^4e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c(d+e\sqrt[3]{x})\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\right.\right.}{b}\right.}{c^2e^6} \\
&- \frac{3d^5e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)\left(a+b\log\left(c(d+e\sqrt[3]{x})\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{ce^6}
\end{aligned}
$$

Mathematica [F]

$$
\int x\left(a+b\log\left(c(d+e\sqrt[3]{x})\right)\right)^p\,dx = \int x\left(a+b\log\left(c(d+e\sqrt[3]{x})\right)\right)^p\,dx
$$

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))])^p, x]
```

Maple [F]

$$
\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)\right)\right)^p dx
$$

```
[In] int(x*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
[Out] int(x*(a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

Fricas [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e*x^(1/3) + c*d) + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(1/3))))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x, x)
```

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\int x\,\left(a+b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)\right)\right)^p\,dx$$

```
[In] int(x*(a + b*log(c*(d + e*x^(1/3))))^p,x)
[Out] int(x*(a + b*log(c*(d + e*x^(1/3))))^p, x)
```

3.559 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 266

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx = \frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^3e^3} - \frac{3\,2^{-p}de^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^2e^3} + \frac{3d^2e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{ce^3}$$

```
[Out] GAMMA(p+1,-3*(a+b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(3^p)
/c^3/e^3/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)-3*d*GAMMA(p+1,-2*(a+
b*ln(c*(d+e*x^(1/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/(2^p)/c^2/e^3/exp(2*a
/b)/(((-a-b*ln(c*(d+e*x^(1/3))))/b)^p)+3*d^2*GAMMA(p+1,(-a-b*ln(c*(d+e*x^(1
/3))))/b)*(a+b*ln(c*(d+e*x^(1/3))))^p/c/e^3/exp(a/b)/(((-a-b*ln(c*(d+e*x^(1
/3))))/b)^p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2501, 2448, 2436, 2336, 2212, 2437, 2346}

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx$$
$$=\frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)}{c^3e^3}$$
$$-\frac{3d2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)}{c^2e^3}$$
$$+\frac{3d^2e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)}{ce^3}$$

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] (Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*(d + e*x^
(1/3))])^p)/(3^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b))^
p) - (3*d*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b]*(a + b*Log[c*
(d + e*x^(1/3))])^p)/(2^p*c^2*e^3*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3
))])/b))^p) + (3*d^2*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))])/b)]*(a +
 b*Log[c*(d + e*x^(1/3))])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3)
)])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^2(a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)\\
&= 3\text{Subst}\left(\int\left(\frac{d^2(a+b\log(c(d+ex)))^p}{e^2}-\frac{2d(d+ex)(a+b\log(c(d+ex)))^p}{e^2}\right.\right.\\
&\qquad\left.\left.+\frac{(d+ex)^2(a+b\log(c(d+ex)))^p}{e^2}\right)\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{3\text{Subst}\left(\int(d+ex)^2(a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)}{e^2}\\
&\quad-\frac{(6d)\text{Subst}\left(\int(d+ex)(a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)}{e^2}\\
&\quad+\frac{(3d^2)\,\text{Subst}\left(\int(a+b\log(c(d+ex)))^p\,dx,x,\sqrt[3]{x}\right)}{e^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3\text{Subst}\left(\int x^2(a+b\log(cx))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^3} \\
&\quad - \frac{(6d)\text{Subst}\left(\int x(a+b\log(cx))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^3} \\
&\quad + \frac{(3d^2)\,\text{Subst}\left(\int (a+b\log(cx))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^3} \\
&= \frac{3\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^3e^3} \\
&\quad - \frac{(6d)\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{c^2e^3} \\
&\quad + \frac{(3d^2)\,\text{Subst}\left(\int e^{x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{ce^3} \\
&= \frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{c^3e^3} \\
&\quad - \frac{3\ 2^{-p}de^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right.}{c^2e^3} \\
&\quad + \frac{3d^2e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)}{b}\right)^{-p}}{ce^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.65

$$
\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx
$$

$$
= \frac{6^{-p}e^{-\frac{3a}{b}}\left(2^p\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)+3^{1+p}cde^{a/b}\left(-\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)}{b}\right)+2^pc\right.\right.}{c^3e^3}
$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p,x]
```

```
[Out] ((2^p*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 3^(1 + p)*c*d*E
^(a/b)*(-Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))]))/b] + 2^p*c*d*E^(a
/b)*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(1/3))])/b)]))*(a + b*Log[c*(d + e
*x^(1/3))])^p)/(6^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))])/b
))^p)
```

Maple [F]

$$\int \left(a+b\ln \left(c\left(d+e\,x^{\frac{1}{3}}\right)\right)\right)^{p}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/3))))^p,x)
[Out] int((a+b*ln(c*(d+e*x^(1/3))))^p,x)
```

Fricas [F]

$$\int \left(a+b\log \left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^{p}\,dx=\int \left(b\log \left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e*x^(1/3) + c*d) + a)^p, x)
```

Sympy [F]

$$\int \left(a+b\log \left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^{p}\,dx=\int \left(a+b\log \left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^{p}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))))**p,x)
[Out] Integral((a + b*log(c*(d + e*x**(1/3))))**p, x)
```

Maxima [F]

$$\int \left(a+b\log \left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^{p}\,dx=\int \left(b\log \left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p, x)
```

Giac [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\int \left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p\,dx=\int \left(a+b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)\right)\right)^p\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/3))))^p,x)
[Out] int((a + b*log(c*(d + e*x^(1/3))))^p, x)
```

3.560 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx=\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/3))))^p/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))])^p/x,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)))^p}{x}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p/x, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)\right)\right)^{p}}{x}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/3))))^p/x,x)
[Out] int((a+b*ln(c*(d+e*x^(1/3))))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^{p}}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x,x, algorithm="fricas")
[Out] integral((b*log(c*e*x^(1/3) + c*d) + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))))**p/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x,x, algorithm="giac")
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)\right)\right)^p}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/3))))^p/x,x)
[Out] int((a + b*log(c*(d + e*x^(1/3))))^p/x, x)
```

3.561 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx=\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/3))))^p/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))])^p/x^2,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)])^p/x^4, x], x, x^(1/3)]
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)))^p}{x^4}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(1/3))])^p/x^2, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)\right)\right)^p}{x^2}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/3))))^p/x^2,x)
[Out] int((a+b*ln(c*(d+e*x^(1/3))))^p/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log(c*e*x^(1/3) + c*d) + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))))**p/x**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log((e*x^(1/3) + d)*c) + a)^p/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)\right)\right)^p}{x^2}\,dx = \int \frac{\left(a+b\,\ln\left(c\left(d+e\,x^{1/3}\right)\right)\right)^p}{x^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/3))))^p/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(1/3))))^p/x^2, x)
```

3.562 $\int x^3 \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1363

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

$$= \frac{2^{-2-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)}{c^6e^{12}}$$

$$-\frac{3\left(\frac{2}{11}\right)^p de^{-\frac{11a}{2b}}\left(d+e\sqrt[3]{x}\right)^{11}\Gamma\left(1+p,-\frac{11\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log}{}\right.}{e^{12}\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{11/2}}$$

$$+\frac{33\ 5^{-p}d^2e^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right.}{b}\right.}{2c^5e^{12}}$$

$$-\frac{55\left(\frac{2}{9}\right)^p d^3e^{-\frac{9a}{2b}}\left(d+e\sqrt[3]{x}\right)^9\Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log}{}\right.}{e^{12}\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{9/2}}$$

$$+\frac{495\ 2^{-2(1+p)}d^4e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{}\right.\right.}{b}\right.}{c^4e^{12}}$$

$$-\frac{99\ 2^{1+p}7^{-p}d^5e^{-\frac{7a}{2b}}\left(d+e\sqrt[3]{x}\right)^7\Gamma\left(1+p,-\frac{7\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+}{}\right.}{e^{12}\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{7/2}}$$

$$+\frac{77\ 3^{1-p}d^6e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right.}{b}\right.}{c^3e^{12}}$$

$$-\frac{99\ 2^{1+p}5^{-p}d^7e^{-\frac{5a}{2b}}\left(d+e\sqrt[3]{x}\right)^5\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+}{}\right.}{e^{12}\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{5/2}}$$

$$\frac{495\ 2^{-2-p}d^8e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.}{b}\right.}{}$$

```
[Out] 2^(-2-p)*GAMMA(p+1,-6*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3)
)^2))^p/(3^p)/c^6/e^12/exp(6*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-3*(2
/11)^p*d*(d+e*x^(1/3))^11*GAMMA(p+1,-11/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a
+b*ln(c*(d+e*x^(1/3))^2))^p/e^12/exp(11/2*a/b)/(c*(d+e*x^(1/3))^2)^(11/2)/(
((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)+33/2*d^2*GAMMA(p+1,-5*(a+b*ln(c*(d+e*x^
(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(5^p)/c^5/e^12/exp(5*a/b)/(((-a
-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-55*(2/9)^p*d^3*(d+e*x^(1/3))^9*GAMMA(p+1,-9
/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^12/exp(9/
2*a/b)/(c*(d+e*x^(1/3))^2)^(9/2)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)+495*d
^4*GAMMA(p+1,-4*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^
p/(2^(2+2*p))/c^4/e^12/exp(4*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-99*2
^(p+1)*d^5*(d+e*x^(1/3))^7*GAMMA(p+1,-7/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a
+b*ln(c*(d+e*x^(1/3))^2))^p/(7^p)/e^12/exp(7/2*a/b)/(c*(d+e*x^(1/3))^2)^(7/
2)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)+77*3^(1-p)*d^6*GAMMA(p+1,-3*(a+b*ln
(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c^3/e^12/exp(3*a/b)/(
((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-99*2^(p+1)*d^7*(d+e*x^(1/3))^5*GAMMA(p+
1,-5/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(5^p)/e
^12/exp(5/2*a/b)/(c*(d+e*x^(1/3))^2)^(5/2)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b
)^p)+495*2^(-2-p)*d^8*GAMMA(p+1,-2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c
*(d+e*x^(1/3))^2))^p/c^2/e^12/exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^
p)-55*(2/3)^p*d^9*(d+e*x^(1/3))^3*GAMMA(p+1,-3/2*(a+b*ln(c*(d+e*x^(1/3))^2)
)/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^12/exp(3/2*a/b)/(c*(d+e*x^(1/3))^2)^(3
/2)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)+33/2*d^10*GAMMA(p+1,(-a-b*ln(c*(d+
e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c/e^12/exp(a/b)/(((-a-b*ln(
c*(d+e*x^(1/3))^2))/b)^p)-3*2^p*d^11*(d+e*x^(1/3))*GAMMA(p+1,1/2*(-a-b*ln(c
*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^12/exp(1/2*a/b)/(((-a
-b*ln(c*(d+e*x^(1/3))^2))/b)^p)/(c*(d+e*x^(1/3))^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 1363, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used

$= \{2504, 2448, 2436, 2337, 2212, 2437, 2347\}$

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

$$= \frac{2^{-p-2}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)}{c^6e^{12}}$$

$$- \frac{3\left(\frac{2}{11}\right)^p de^{-\frac{11a}{2b}}\left(d+e\sqrt[3]{x}\right)^{11}\Gamma\left(p+1,-\frac{11\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log}{}\right.}{e^{12}\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{11/2}}$$

$$+ \frac{33\ 5^{-p}d^2e^{-\frac{5a}{b}}\Gamma\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right.}{2c^5e^{12}}$$

$$- \frac{55\left(\frac{2}{9}\right)^p d^3e^{-\frac{9a}{2b}}\left(d+e\sqrt[3]{x}\right)^9\Gamma\left(p+1,-\frac{9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log}{}\right.}{e^{12}\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{9/2}}$$

$$+ \frac{495\ 2^{-2(p+1)}d^4e^{-\frac{4a}{b}}\Gamma\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.}{b}\right.}{c^4e^{12}}$$

$$- \frac{99\ 2^{p+1}7^{-p}d^5e^{-\frac{7a}{2b}}\left(d+e\sqrt[3]{x}\right)^7\Gamma\left(p+1,-\frac{7\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+}{}\right.}{e^{12}\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{7/2}}$$

$$+ \frac{77\ 3^{1-p}d^6e^{-\frac{3a}{b}}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right.}{b}\right.}{c^3e^{12}}$$

$$- \frac{99\ 2^{p+1}5^{-p}d^7e^{-\frac{5a}{2b}}\left(d+e\sqrt[3]{x}\right)^5\Gamma\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+}{}\right.}{e^{12}\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{5/2}}$$

$$+ \frac{495\ 2^{-p-2}d^8e^{-\frac{2a}{b}}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.}{b}\right.}{c^2e^{12}}$$

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

```
[Out] (2^(-2 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Lo
g[c*(d + e*x^(1/3))^2])^p)/(3^p*c^6*e^12*E^((6*a)/b)*(-((a + b*Log[c*(d + e
*x^(1/3))^2])/b))^p) - (3*(2/11)^p*d*(d + e*x^(1/3))^11*Gamma[1 + p, (-11*(
a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)
/(e^12*E^((11*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(11/2)*(-((a + b*Log[c*(d + e
*x^(1/3))^2])/b))^p) + (33*d^2*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3)
)^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(2*5^p*c^5*e^12*E^((5*a)/b)*(
-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (55*(2/9)^p*d^3*(d + e*x^(1/3))
^9*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(
d + e*x^(1/3))^2])^p)/(e^12*E^((9*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(9/2)*(-(
(a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (495*d^4*Gamma[1 + p, (-4*(a + b*
Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(2^(2*(1
+ p))*c^4*e^12*E^((4*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (99
*2^(1 + p)*d^5*(d + e*x^(1/3))^7*Gamma[1 + p, (-7*(a + b*Log[c*(d + e*x^(1/
3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(7^p*e^12*E^((7*a)/(2*b
))*(c*(d + e*x^(1/3))^2)^(7/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) +
 (77*3^(1 - p)*d^6*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a
 + b*Log[c*(d + e*x^(1/3))^2])^p)/(c^3*e^12*E^((3*a)/b)*(-((a + b*Log[c*(d
+ e*x^(1/3))^2])/b))^p) - (99*2^(1 + p)*d^7*(d + e*x^(1/3))^5*Gamma[1 + p,
(-5*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2
])^p)/(5^p*e^12*E^((5*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(5/2)*(-((a + b*Log[c
*(d + e*x^(1/3))^2])/b))^p) + (495*2^(-2 - p)*d^8*Gamma[1 + p, (-2*(a + b*L
og[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(c^2*e^12*
E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (55*(2/3)^p*d^9*(d
 + e*x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(
a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^12*E^((3*a)/(2*b))*(c*(d + e*x^(1/3))
^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (33*d^10*Gamma[1 + p
, -((a + b*Log[c*(d + e*x^(1/3))^2])/b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p
)/(2*c*e^12*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (3*2^p*d^1
1*(d + e*x^(1/3))*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e*x^(1/3))^2])/b]*(a
+ b*Log[c*(d + e*x^(1/3))^2])^p)/(e^12*E^(a/(2*b))*Sqrt[c*(d + e*x^(1/3))^2
]*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
```

```
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int x^{11}\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)$$

$$=3\text{Subst}\left(\int\left(-\frac{d^{11}(a+b\log(c(d+ex)^2))^p}{e^{11}}+\frac{11d^{10}(d+ex)\left(a+b\log(c(d+ex)^2)\right)^p}{e^{11}}-\frac{55d^9(d+ex)^2\left(a+b\log(c(d+ex)^2)\right)^p}{e^{11}}+\frac{165d^8(d+ex)^3\left(a+b\log(c(d+ex)^2)\right)^p}{e^{11}}-\frac{330d^7(d+ex)^4\left(a+b\log(c(d+ex)^2)\right)^p}{e^{11}}+\frac{462d^6(d+ex)^5\left(a+b\log(c(d+ex)^2)\right)^p}{e^{11}}-\frac{462d^5(d+ex)^6\left(a+b\log(c(d+ex)^2)\right)^p}{e^{11}}+\frac{330d^4(d+ex)^7\left(a+b\log(c(d+ex)^2)\right)^p}{e^{11}}-\frac{165d^3(d+ex)^8\left(a+b\log(c(d+ex)^2)\right)^p}{e^{11}}+\frac{55d^2(d+ex)^9\left(a+b\log(c(d+ex)^2)\right)^p}{e^{11}}-\frac{11d(d+ex)^{10}\left(a+b\log(c(d+ex)^2)\right)^p}{e^{11}}+\frac{(d+ex)^{11}\left(a+b\log(c(d+ex)^2)\right)^p}{e^{11}}\right)dx,x,\sqrt[3]{x}\right)$$

$$= \frac{3\text{Subst}\left(\int (d+ex)^{11} \left(a+b\log\left(c(d+ex)^2\right)\right)^p\, dx, x, \sqrt[3]{x}\right)}{e^{11}}$$
$$- \frac{(33d)\text{Subst}\left(\int (d+ex)^{10} \left(a+b\log\left(c(d+ex)^2\right)\right)^p\, dx, x, \sqrt[3]{x}\right)}{e^{11}}$$
$$+ \frac{(165d^2)\,\text{Subst}\left(\int (d+ex)^9 \left(a+b\log\left(c(d+ex)^2\right)\right)^p\, dx, x, \sqrt[3]{x}\right)}{e^{11}}$$
$$- \frac{(495d^3)\,\text{Subst}\left(\int (d+ex)^8 \left(a+b\log\left(c(d+ex)^2\right)\right)^p\, dx, x, \sqrt[3]{x}\right)}{e^{11}}$$
$$+ \frac{(990d^4)\,\text{Subst}\left(\int (d+ex)^7 \left(a+b\log\left(c(d+ex)^2\right)\right)^p\, dx, x, \sqrt[3]{x}\right)}{e^{11}}$$
$$- \frac{(1386d^5)\,\text{Subst}\left(\int (d+ex)^6 \left(a+b\log\left(c(d+ex)^2\right)\right)^p\, dx, x, \sqrt[3]{x}\right)}{e^{11}}$$
$$+ \frac{(1386d^6)\,\text{Subst}\left(\int (d+ex)^5 \left(a+b\log\left(c(d+ex)^2\right)\right)^p\, dx, x, \sqrt[3]{x}\right)}{e^{11}}$$
$$- \frac{(990d^7)\,\text{Subst}\left(\int (d+ex)^4 \left(a+b\log\left(c(d+ex)^2\right)\right)^p\, dx, x, \sqrt[3]{x}\right)}{e^{11}}$$
$$+ \frac{(495d^8)\,\text{Subst}\left(\int (d+ex)^3 \left(a+b\log\left(c(d+ex)^2\right)\right)^p\, dx, x, \sqrt[3]{x}\right)}{e^{11}}$$
$$- \frac{(165d^9)\,\text{Subst}\left(\int (d+ex)^2 \left(a+b\log\left(c(d+ex)^2\right)\right)^p\, dx, x, \sqrt[3]{x}\right)}{e^{11}}$$
$$+ \frac{(33d^{10})\,\text{Subst}\left(\int (d+ex) \left(a+b\log\left(c(d+ex)^2\right)\right)^p\, dx, x, \sqrt[3]{x}\right)}{e^{11}}$$
$$- \frac{(3d^{11})\,\text{Subst}\left(\int \left(a+b\log\left(c(d+ex)^2\right)\right)^p\, dx, x, \sqrt[3]{x}\right)}{e^{11}}$$

$$
\begin{aligned}
&= \frac{3\text{Subst}\left(\int x^{11}(a+b\log(cx^2))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} \\
&\quad - \frac{(33d)\text{Subst}\left(\int x^{10}(a+b\log(cx^2))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} \\
&\quad + \frac{(165d^2)\,\text{Subst}\left(\int x^{9}(a+b\log(cx^2))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} \\
&\quad - \frac{(495d^3)\,\text{Subst}\left(\int x^{8}(a+b\log(cx^2))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} \\
&\quad + \frac{(990d^4)\,\text{Subst}\left(\int x^{7}(a+b\log(cx^2))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} \\
&\quad - \frac{(1386d^5)\,\text{Subst}\left(\int x^{6}(a+b\log(cx^2))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} \\
&\quad + \frac{(1386d^6)\,\text{Subst}\left(\int x^{5}(a+b\log(cx^2))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} \\
&\quad - \frac{(990d^7)\,\text{Subst}\left(\int x^{4}(a+b\log(cx^2))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} \\
&\quad + \frac{(495d^8)\,\text{Subst}\left(\int x^{3}(a+b\log(cx^2))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} \\
&\quad - \frac{(165d^9)\,\text{Subst}\left(\int x^{2}(a+b\log(cx^2))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} \\
&\quad + \frac{(33d^{10})\,\text{Subst}\left(\int x(a+b\log(cx^2))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}} \\
&\quad - \frac{(3d^{11})\,\text{Subst}\left(\int (a+b\log(cx^2))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^{12}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3\mathrm{Subst}\left(\int e^{6x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2c^6e^{12}} \\
&\quad + \frac{(165d^2)\,\mathrm{Subst}\left(\int e^{5x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2c^5e^{12}} \\
&\quad + \frac{(495d^4)\,\mathrm{Subst}\left(\int e^{4x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{c^4e^{12}} \\
&\quad + \frac{(693d^6)\,\mathrm{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{c^3e^{12}} \\
&\quad + \frac{(495d^8)\,\mathrm{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2c^2e^{12}} \\
&\quad + \frac{(33d^{10})\,\mathrm{Subst}\left(\int e^{x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2ce^{12}} \\
&\quad - \frac{\left(33d\left(d+e\sqrt[3]{x}\right)^{11}\right)\mathrm{Subst}\left(\int e^{11x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2e^{12}\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{11/2}} \\
&\quad - \frac{\left(495d^3\left(d+e\sqrt[3]{x}\right)^{9}\right)\mathrm{Subst}\left(\int e^{9x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2e^{12}\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{9/2}} \\
&\quad - \frac{\left(693d^5\left(d+e\sqrt[3]{x}\right)^{7}\right)\mathrm{Subst}\left(\int e^{7x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{e^{12}\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{7/2}} \\
&\quad - \frac{\left(495d^7\left(d+e\sqrt[3]{x}\right)^{5}\right)\mathrm{Subst}\left(\int e^{5x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{e^{12}\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{5/2}} \\
&\quad - \frac{\left(165d^9\left(d+e\sqrt[3]{x}\right)^{3}\right)\mathrm{Subst}\left(\int e^{3x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2e^{12}\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{3/2}} \\
&\quad - \frac{\left(3d^{11}\left(d+e\sqrt[3]{x}\right)\right)\mathrm{Subst}\left(\int e^{x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2e^{12}\sqrt{c\left(d+e\sqrt[3]{x}\right)^2}}
\end{aligned}
$$

$= \text{Too large to display}$

Mathematica [F]

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
[Out] Integrate[x^3*(a + b*Log[c*(d + e*x^(1/3))^2])^p, x]
```

Maple [F]

$$\int x^3\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^2\right)\right)^p dx$$

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
[Out] int(x^3*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

Fricas [F]

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p x^3\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p*x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(1/3))**2))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^px^3\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x^3, x)
```

Giac [F]

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^px^3\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int x^3\,\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^2\right)\right)^p\,dx$$

```
[In] int(x^3*(a + b*log(c*(d + e*x^(1/3))^2))^p,x)
[Out] int(x^3*(a + b*log(c*(d + e*x^(1/3))^2))^p, x)
```

3.563 $\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1035

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

$$=\frac{2^p3^{-1-2p}e^{-\frac{9a}{2b}}\left(d+e\sqrt[3]{x}\right)^9\Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(\right.\right.}{}\right.}{e^9\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{9/2}}$$

$$-\frac{3\ 4^{-p}de^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)}{c^4e^9}$$

$$+\frac{3\ 2^{2+p}7^{-p}d^2e^{-\frac{7a}{2b}}\left(d+e\sqrt[3]{x}\right)^7\Gamma\left(1+p,-\frac{7\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\,\mathrm{l}}{}\right.}{e^9\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{7/2}}$$

$$-\frac{28\ 3^{-p}d^3e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right.}{b}\right.}{c^3e^9}$$

$$+\frac{21\ 2^{1+p}5^{-p}d^4e^{-\frac{5a}{2b}}\left(d+e\sqrt[3]{x}\right)^5\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+}{}\right.}{e^9\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{5/2}}$$

$$-\frac{21\ 2^{1-p}d^5e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right.}{b}\right.}{c^2e^9}$$

$$+\frac{7\ 2^{2+p}3^{-p}d^6e^{-\frac{3a}{2b}}\left(d+e\sqrt[3]{x}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\,\mathrm{l}}{}\right.}{e^9\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{3/2}}$$

$$-\frac{12d^7e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}}{ce^9}$$

$$3\ 2^pd^8e^{-\frac{a}{2b}}\left(d+e\sqrt[3]{x}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{\right.\right.}}{b}\right.$$

```
[Out] 2^p*3^(-1-2*p)*(d+e*x^(1/3))^9*GAMMA(p+1,-9/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b
)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^9/exp(9/2*a/b)/(c*(d+e*x^(1/3))^2)^(9/2)/
(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-3*d*GAMMA(p+1,-4*(a+b*ln(c*(d+e*x^(1/3
))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(4^p)/c^4/e^9/exp(4*a/b)/(((-a-b*ln
(c*(d+e*x^(1/3))^2))/b)^p)+3*2^(2+p)*d^2*(d+e*x^(1/3))^7*GAMMA(p+1,-7/2*(a+
b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(7^p)/e^9/exp(7/2
*a/b)/(c*(d+e*x^(1/3))^2)^(7/2)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-28*d^3
*GAMMA(p+1,-3*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/
(3^p)/c^3/e^9/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)+21*2^(p+1)*d^
4*(d+e*x^(1/3))^5*GAMMA(p+1,-5/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(
d+e*x^(1/3))^2))^p/(5^p)/e^9/exp(5/2*a/b)/(c*(d+e*x^(1/3))^2)^(5/2)/(((-a-b
*ln(c*(d+e*x^(1/3))^2))/b)^p)-21*2^(1-p)*d^5*GAMMA(p+1,-2*(a+b*ln(c*(d+e*x^
(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c^2/e^9/exp(2*a/b)/(((-a-b*ln(c
*(d+e*x^(1/3))^2))/b)^p)+7*2^(2+p)*d^6*(d+e*x^(1/3))^3*GAMMA(p+1,-3/2*(a+b*
ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(3^p)/e^9/exp(3/2*a
/b)/(c*(d+e*x^(1/3))^2)^(3/2)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-12*d^7*G
AMMA(p+1,(-a-b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c/e^
9/exp(a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)+3*2^p*d^8*(d+e*x^(1/3))*GAM
MA(p+1,1/2*(-a-b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^
9/exp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)/(c*(d+e*x^(1/3))^2)^(1/
2)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 1035, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used

$$= \{2504, 2448, 2436, 2337, 2212, 2437, 2347\}$$

$$\int x^2 \left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^2\right)\right)^p dx$$

$$= \frac{2^p 3^{-2p-1} e^{-\frac{9a}{2b}} \left(d + e\sqrt[3]{x}\right)^9 \Gamma\left(p+1, -\frac{9\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right) \left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^2\right)\right)^p \left(-\frac{a+b\log\left(c\left(\cdots\right.\right.}{\cdots}\right.}{e^9 \left(c\left(d + e\sqrt[3]{x}\right)^2\right)^{9/2}}$$

$$- \frac{3\ 4^{-p} d e^{-\frac{4a}{b}} \Gamma\left(p+1, -\frac{4\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right) \left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^2\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)\cdots}{c^4 e^9}$$

$$+ \frac{3\ 2^{p+2} 7^{-p} d^2 e^{-\frac{7a}{2b}} \left(d + e\sqrt[3]{x}\right)^7 \Gamma\left(p+1, -\frac{7\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right) \left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^2\right)\right)^p \left(-\frac{a+b\cdots}{\cdots}\right.}{e^9 \left(c\left(d + e\sqrt[3]{x}\right)^2\right)^{7/2}}$$

$$- \frac{28\ 3^{-p} d^3 e^{-\frac{3a}{b}} \Gamma\left(p+1, -\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right) \left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^2\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right.}{b}\right.}{c^3 e^9}$$

$$+ \frac{21\ 2^{p+1} 5^{-p} d^4 e^{-\frac{5a}{2b}} \left(d + e\sqrt[3]{x}\right)^5 \Gamma\left(p+1, -\frac{5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right) \left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^2\right)\right)^p \left(-\frac{a+\cdots}{\cdots}\right.}{e^9 \left(c\left(d + e\sqrt[3]{x}\right)^2\right)^{5/2}}$$

$$- \frac{21\ 2^{1-p} d^5 e^{-\frac{2a}{b}} \Gamma\left(p+1, -\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right) \left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^2\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right.}{b}\right.}{c^2 e^9}$$

$$+ \frac{7\ 2^{p+2} 3^{-p} d^6 e^{-\frac{3a}{2b}} \left(d + e\sqrt[3]{x}\right)^3 \Gamma\left(p+1, -\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right) \left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^2\right)\right)^p \left(-\frac{a+b\cdots}{\cdots}\right.}{e^9 \left(c\left(d + e\sqrt[3]{x}\right)^2\right)^{3/2}}$$

$$- \frac{12 d^7 e^{-\frac{a}{b}} \Gamma\left(p+1, -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right) \left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^2\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}}{c e^9}$$

$$+ \frac{3\ 2^p d^8 e^{-\frac{a}{2b}} \left(d + e\sqrt[3]{x}\right) \Gamma\left(p+1, -\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{2b}\right) \left(a + b\log\left(c\left(d + e\sqrt[3]{x}\right)^2\right)\right)^p \left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{\cdots}\right.\right.}{b}\right.}{e^9 \sqrt{c\left(d + e\sqrt[3]{x}\right)^2}}$$

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

```
[Out] (2^p*3^(-1 - 2*p)*(d + e*x^(1/3))^9*Gamma[1 + p, (-9*(a + b*Log[c*(d + e*x^
(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^9*E^((9*a)/(2*b))
*(c*(d + e*x^(1/3))^2)^(9/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (
3*d*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d +
 e*x^(1/3))^2])^p)/(4^p*c^4*e^9*E^((4*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))
^2])/b))^p) + (3*2^(2 + p)*d^2*(d + e*x^(1/3))^7*Gamma[1 + p, (-7*(a + b*Lo
g[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(7^p*e^
9*E^((7*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(7/2)*(-((a + b*Log[c*(d + e*x^(1/3
))^2])/b))^p) - (28*d^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/
b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(3^p*c^3*e^9*E^((3*a)/b)*(-((a + b*L
og[c*(d + e*x^(1/3))^2])/b))^p) + (21*2^(1 + p)*d^4*(d + e*x^(1/3))^5*Gamma
[1 + p, (-5*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^
(1/3))^2])^p)/(5^p*e^9*E^((5*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(5/2)*(-((a +
b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (21*2^(1 - p)*d^5*Gamma[1 + p, (-2*(a
+ b*Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(c^2*
e^9*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (7*2^(2 + p)*d
^6*(d + e*x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*
b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(3^p*e^9*E^((3*a)/(2*b))*(c*(d + e*
x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (12*d^7*Gamm
a[1 + p, -((a + b*Log[c*(d + e*x^(1/3))^2])/b)]*(a + b*Log[c*(d + e*x^(1/3)
)^2])^p)/(c*e^9*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (3*2^p
*d^8*(d + e*x^(1/3))*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e*x^(1/3))^2])/b]*
(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^9*E^(a/(2*b))*Sqrt[c*(d + e*x^(1/3))
^2]*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^8\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)\\
&= 3\text{Subst}\left(\int\left(\frac{d^8\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^8}-\frac{8d^7(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^8}\right.\right.\\
&\quad+\frac{28d^6(d+ex)^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^8}-\frac{56d^5(d+ex)^3\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^8}\\
&\quad+\frac{70d^4(d+ex)^4\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^8}-\frac{56d^3(d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^8}\\
&\quad+\frac{28d^2(d+ex)^6\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^8}-\frac{8d(d+ex)^7\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^8}\\
&\quad\left.\left.+\frac{(d+ex)^8\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^8}\right)\,dx,x,\sqrt[3]{x}\right)
\end{aligned}$$

$$= \frac{3\text{Subst}\left(\int (d+ex)^8 \left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)}{e^8} - \frac{(24d)\text{Subst}\left(\int (d+ex)^7 \left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)}{e^8} + \frac{(84d^2)\,\text{Subst}\left(\int (d+ex)^6 \left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)}{e^8} - \frac{(168d^3)\,\text{Subst}\left(\int (d+ex)^5 \left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)}{e^8} + \frac{(210d^4)\,\text{Subst}\left(\int (d+ex)^4 \left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)}{e^8} - \frac{(168d^5)\,\text{Subst}\left(\int (d+ex)^3 \left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)}{e^8} + \frac{(84d^6)\,\text{Subst}\left(\int (d+ex)^2 \left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)}{e^8} - \frac{(24d^7)\,\text{Subst}\left(\int (d+ex) \left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)}{e^8} + \frac{(3d^8)\,\text{Subst}\left(\int \left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)}{e^8}$$

$$= \frac{3\text{Subst}\left(\int x^8 \left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} - \frac{(24d)\text{Subst}\left(\int x^7 \left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} + \frac{(84d^2)\,\text{Subst}\left(\int x^6 \left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} - \frac{(168d^3)\,\text{Subst}\left(\int x^5 \left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} + \frac{(210d^4)\,\text{Subst}\left(\int x^4 \left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} - \frac{(168d^5)\,\text{Subst}\left(\int x^3 \left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} + \frac{(84d^6)\,\text{Subst}\left(\int x^2 \left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} - \frac{(24d^7)\,\text{Subst}\left(\int x \left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^9} + \frac{(3d^8)\,\text{Subst}\left(\int \left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+e\sqrt[3]{x}\right)}{e^9}$$

$$
\begin{aligned}
&= -\frac{(12d)\mathrm{Subst}\left(\int e^{4x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{c^4e^9} \\
&\quad - \frac{(84d^3)\,\mathrm{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{c^3e^9} \\
&\quad - \frac{(84d^5)\,\mathrm{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{c^2e^9} \\
&\quad - \frac{(12d^7)\,\mathrm{Subst}\left(\int e^{x}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{ce^9} \\
&\quad + \frac{\left(3\left(d+e\sqrt[3]{x}\right)^9\right)\,\mathrm{Subst}\left(\int e^{9x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2e^9\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{9/2}} \\
&\quad + \frac{\left(42d^2\left(d+e\sqrt[3]{x}\right)^7\right)\,\mathrm{Subst}\left(\int e^{7x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{e^9\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{7/2}} \\
&\quad + \frac{\left(105d^4\left(d+e\sqrt[3]{x}\right)^5\right)\,\mathrm{Subst}\left(\int e^{5x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{e^9\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{5/2}} \\
&\quad + \frac{\left(42d^6\left(d+e\sqrt[3]{x}\right)^3\right)\,\mathrm{Subst}\left(\int e^{3x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{e^9\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{3/2}} \\
&\quad + \frac{\left(3d^8\left(d+e\sqrt[3]{x}\right)\right)\,\mathrm{Subst}\left(\int e^{x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2e^9\sqrt{c\left(d+e\sqrt[3]{x}\right)^2}} \\
&= \text{Too large to display}
\end{aligned}
$$

Mathematica [F]

$$
\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx = \int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx
$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
[Out] Integrate[x^2*(a + b*Log[c*(d + e*x^(1/3))^2])^p, x]
```

Maple [F]

$$\int x^{2} \left(a +b \ln \left(c \left(d +e \,x^{\frac{1}{3}}\right)^{2}\right)\right)^{p}d x$$

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
[Out] int(x^2*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

Fricas [F]

$$\int x^{2}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{2}\right)\right)^{p}\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{2}c\right)+a\right)^{p}x^{2}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{2}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{2}\right)\right)^{p}\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(1/3))**2))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int x^{2}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{2}\right)\right)^{p}\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{2}c\right)+a\right)^{p}x^{2}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x^2, x)
```

Giac [F]

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^px^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int x^2\,\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^2\right)\right)^p\,dx$$

```
[In] int(x^2*(a + b*log(c*(d + e*x^(1/3))^2))^p,x)
[Out] int(x^2*(a + b*log(c*(d + e*x^(1/3))^2))^p, x)
```

3.564 $\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 673

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

$$=\frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}}{2c^3e^6}$$

$$-\frac{3\left(\frac{2}{5}\right)^p de^{-\frac{5a}{2b}}\left(d+e\sqrt[3]{x}\right)^5\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\Bigg(-\frac{a+b\log\Big(c\Big(}{}}{e^6\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{5/2}}$$

$$+\frac{15\ 2^{-1-p}d^2e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\Bigg(-\frac{a+b\log\Big(c\Big(d+e\sqrt[3]{x}\Big)}{b}}{c^2e^6}$$

$$-\frac{5\ 2^{1+p}3^{-p}d^3e^{-\frac{3a}{2b}}\left(d+e\sqrt[3]{x}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\Bigg(-\frac{a+b}{}}{e^6\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{3/2}}$$

$$+\frac{15d^4e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}}{2ce^6}$$

$$-\frac{3\ 2^pd^5e^{-\frac{a}{2b}}\left(d+e\sqrt[3]{x}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\Bigg(-\frac{a+b\log\Big(c\Big(d+e\sqrt[3]{}}{b}}{e^6\sqrt{c\left(d+e\sqrt[3]{x}\right)^2}}$$

```
[Out] 1/2*GAMMA(p+1,-3*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))
^p/(3^p)/c^3/e^6/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-3*(2/5)^p*
d*(d+e*x^(1/3))^5*GAMMA(p+1,-5/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(
d+e*x^(1/3))^2))^p/e^6/exp(5/2*a/b)/(c*(d+e*x^(1/3))^2)^(5/2)/(((-a-b*ln(c*
(d+e*x^(1/3))^2))/b)^p)+15*2^(-1-p)*d^2*GAMMA(p+1,-2*(a+b*ln(c*(d+e*x^(1/3)
)^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+e
*x^(1/3))^2))/b)^p)-5*2^(p+1)*d^3*(d+e*x^(1/3))^3*GAMMA(p+1,-3/2*(a+b*ln(c*
```

```
(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/(3^p)/e^6/exp(3/2*a/b)/(
c*(d+e*x^(1/3))^2)^(3/2)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)+15/2*d^4*GAMM
A(p+1,(-a-b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c/e^6/e
xp(a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)-3*2^p*d^5*(d+e*x^(1/3))*GAMMA(
p+1,1/2*(-a-b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^6/e
xp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2))/b)^p)/(c*(d+e*x^(1/3))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used

$= \{2504, 2448, 2436, 2337, 2212, 2437, 2347\}$

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

$$= \frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)}{2c^3e^6}$$

$$+ \frac{15d^2 2^{-p-1}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.\right.}{b}\right.}{c^2e^6}$$

$$- \frac{3d^5 2^p e^{-\frac{a}{2b}}\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\right.\right.}{2b}\right.}{e^6\sqrt{c\left(d+e\sqrt[3]{x}\right)^2}}$$

$$+ \frac{15d^4e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)}{2ce^6}$$

$$- \frac{5d^3 2^{p+1}3^{-p}e^{-\frac{3a}{2b}}\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\right.}{}\right.}{e^6\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{3/2}}$$

$$- \frac{3d\left(\frac{2}{5}\right)^p e^{-\frac{5a}{2b}}\left(d+e\sqrt[3]{x}\right)^5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{5\left(a+b\log\left(c\right.\right.}{}\right.}{e^6\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{5/2}}$$

```
[In] Int[x*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

```
[Out] (Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/b]*(a + b*Log[c*(d + e*
x^(1/3))^2])^p)/(2*3^p*c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(1/3))^
2])/b))^p) - (3*(2/5)^p*d*(d + e*x^(1/3))^5*Gamma[1 + p, (-5*(a + b*Log[c*(
d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^6*E^((5*a
)/(2*b))*(c*(d + e*x^(1/3))^2)^(5/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b)
)^p) + (15*2^(-1 - p)*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(1/3))^2])
)/b]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log
```

```
[c*(d + e*x^(1/3))^2])/b))^p) - (5*2^(1 + p)*d^3*(d + e*x^(1/3))^3*Gamma[1
+ p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(1/
3))^2])^p)/(3^p*e^6*E^((3*a)/(2*b))*(c*(d + e*x^(1/3))^2)^(3/2)*(-((a + b*L
og[c*(d + e*x^(1/3))^2])/b))^p) + (15*d^4*Gamma[1 + p, -((a + b*Log[c*(d +
e*x^(1/3))^2])/b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(2*c*e^6*E^(a/b)*(-(
(a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (3*2^p*d^5*(d + e*x^(1/3))*Gamma[
1 + p, -1/2*(a + b*Log[c*(d + e*x^(1/3))^2])/b]*(a + b*Log[c*(d + e*x^(1/3)
)^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e*x^(1/3))^2]*(-((a + b*Log[c*(d + e*
x^(1/3))^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
```

```
+ e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)\\
&= 3\text{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}+\frac{5d^4(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}-\frac{10d^3(d+ex)^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}\right.\right.\\
&\quad\left.\left.+\frac{10d^2(d+ex)^3\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}-\frac{5d(d+ex)^4\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}+\frac{(d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}\right)\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{3\text{Subst}\left(\int(d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)}{e^5}\\
&\quad-\frac{(15d)\text{Subst}\left(\int(d+ex)^4\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)}{e^5}\\
&\quad+\frac{\left(30d^2\right)\text{Subst}\left(\int(d+ex)^3\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)}{e^5}\\
&\quad-\frac{\left(30d^3\right)\text{Subst}\left(\int(d+ex)^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)}{e^5}\\
&\quad+\frac{\left(15d^4\right)\text{Subst}\left(\int(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)}{e^5}\\
&\quad-\frac{\left(3d^5\right)\text{Subst}\left(\int\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)}{e^5}
\end{aligned}$$

$$= \frac{3\text{Subst}\left(\int x^5(a+b\log(cx^2))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^6} - \frac{(15d)\text{Subst}\left(\int x^4(a+b\log(cx^2))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^6} + \frac{(30d^2)\,\text{Subst}\left(\int x^3(a+b\log(cx^2))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^6} - \frac{(30d^3)\,\text{Subst}\left(\int x^2(a+b\log(cx^2))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^6} + \frac{(15d^4)\,\text{Subst}\left(\int x(a+b\log(cx^2))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^6} - \frac{(3d^5)\,\text{Subst}\left(\int (a+b\log(cx^2))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^6}$$

$$= \frac{3\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2c^3e^6} + \frac{(15d^2)\,\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{c^2e^6} + \frac{(15d^4)\,\text{Subst}\left(\int e^{x}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2ce^6} - \frac{\left(15d\left(d+e\sqrt[3]{x}\right)^5\right)\text{Subst}\left(\int e^{5x/2}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2e^6\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{5/2}} - \frac{\left(15d^3\left(d+e\sqrt[3]{x}\right)^3\right)\text{Subst}\left(\int e^{3x/2}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{e^6\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{3/2}} - \frac{\left(3d^5\left(d+e\sqrt[3]{x}\right)\right)\text{Subst}\left(\int e^{x/2}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2e^6\sqrt{c\left(d+e\sqrt[3]{x}\right)^2}}$$

$$= \frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right.}{2c^3e^6}$$

$$- \frac{3\left(\frac{2}{5}\right)^p de^{-\frac{5a}{2b}}\left(d+e\sqrt[3]{x}\right)^5\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\right.}{e^6\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{5/2}}$$

$$+ \frac{15\ 2^{-1-p}d^2e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(\right.\right.}{}\right.}{c^2e^6}$$

$$- \frac{5\ 2^{1+p}3^{-p}d^3e^{-\frac{3a}{2b}}\left(d+e\sqrt[3]{x}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{e^6\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{3/2}}$$

$$+ \frac{15d^4e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right.}{2ce^6}$$

$$- \frac{3\ 2^pd^5e^{-\frac{a}{2b}}\left(d+e\sqrt[3]{x}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(\right.}{}\right.}{e^6\sqrt{c\left(d+e\sqrt[3]{x}\right)^2}}$$

Mathematica [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx = \int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e*x^(1/3))^2])^p, x]
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{2}\right)\right)^{p}dx$$

[In] `int(x*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)`

[Out] `int(x*(a+b*ln(c*(d+e*x^(1/3))^2))^p,x)`

Fricas [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{2}\right)\right)^{p}\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{2}c\right)+a\right)^{p}x\,dx$$

[In] `integrate(x*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="fricas")`

[Out] `integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p*x, x)`

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{2}\right)\right)^{p}\,dx=\text{Timed out}$$

[In] `integrate(x*(a+b*ln(c*(d+e*x**(1/3))**2))**p,x)`

[Out] `Timed out`

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^{2}\right)\right)^{p}\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{2}c\right)+a\right)^{p}x\,dx$$

[In] `integrate(x*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="maxima")`

[Out] `integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x, x)`

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int x\left(a+b\,\ln\left(c\left(d+e\,x^{1/3}\right)^2\right)\right)^p\,dx$$

```
[In] int(x*(a + b*log(c*(d + e*x^(1/3))^2))^p,x)
[Out] int(x*(a + b*log(c*(d + e*x^(1/3))^2))^p, x)
```

3.565 $\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 338

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

$$= \frac{\left(\frac{2}{3}\right)^p e^{-\frac{3a}{2b}}\left(d+e\sqrt[3]{x}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.}{b}\right.}{e^3\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{3/2}}$$

$$-\frac{3de^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}}{ce^3}$$

$$+\frac{3\ 2^p d^2 e^{-\frac{a}{2b}}\left(d+e\sqrt[3]{x}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right.\right.}{b}\right.}{e^3\sqrt{c\left(d+e\sqrt[3]{x}\right)^2}}$$

```
[Out] (2/3)^p*(d+e*x^(1/3))^3*GAMMA(p+1,-3/2*(a+b*ln(c*(d+e*x^(1/3))^2))/b)*(a+b*
ln(c*(d+e*x^(1/3))^2))^p/e^3/exp(3/2*a/b)/(c*(d+e*x^(1/3))^2)^(3/2)/(((-a-b
*ln(c*(d+e*x^(1/3))^2))/b)^p)-3*d*GAMMA(p+1,(-a-b*ln(c*(d+e*x^(1/3))^2))/b)
*(a+b*ln(c*(d+e*x^(1/3))^2))^p/c/e^3/exp(a/b)/(((-a-b*ln(c*(d+e*x^(1/3))^2)
)/b)^p)+3*2^p*d^2*(d+e*x^(1/3))*GAMMA(p+1,1/2*(-a-b*ln(c*(d+e*x^(1/3))^2))/
b)*(a+b*ln(c*(d+e*x^(1/3))^2))^p/e^3/exp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(1/3)
)^2))/b)^p)/(c*(d+e*x^(1/3))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2501, 2448, 2436, 2337, 2212, 2437, 2347}

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p dx$$

$$=\frac{3d^2 2^p e^{-\frac{a}{2b}}\left(d+e\sqrt[3]{x}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{2b}\right)}{e^3\sqrt{c\left(d+e\sqrt[3]{x}\right)^2}}$$
$$+\frac{\left(\frac{2}{3}\right)^p e^{-\frac{3a}{2b}}\left(d+e\sqrt[3]{x}\right)^3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)}{e^3\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{3/2}}$$
$$-\frac{3de^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)}{ce^3}$$

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
```

```
[Out] ((2/3)^p*(d + e*x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(1/3))^2]
))/(2*b)]*(a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^3*E^((3*a)/(2*b))*(c*(d +
e*x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) - (3*d*Gamma
[1 + p, -((a + b*Log[c*(d + e*x^(1/3))^2])/b)]*(a + b*Log[c*(d + e*x^(1/3))
^2])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p) + (3*2^p*
d^2*(d + e*x^(1/3))*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e*x^(1/3))^2])/b]*(
a + b*Log[c*(d + e*x^(1/3))^2])^p)/(e^3*E^(a/(2*b))*Sqrt[c*(d + e*x^(1/3))^
2]*(-((a + b*Log[c*(d + e*x^(1/3))^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
```

```
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2501

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Log[c*(
d + e*x^(k*n))^p])^q, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q},
x] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \sqrt[3]{x}\right)\\
&= 3\text{Subst}\left(\int\left(\frac{d^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^2}-\frac{2d(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^2}\right.\right.\\
&\quad\left.\left.+\frac{(d+ex)^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^2}\right)\,dx, x, \sqrt[3]{x}\right)
\end{aligned}$$

$$= \frac{3\text{Subst}\left(\int (d+ex)^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)}{e^2} - \frac{(6d)\text{Subst}\left(\int (d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)}{e^2} + \frac{(3d^2)\,\text{Subst}\left(\int \left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)}{e^2}$$

$$= \frac{3\text{Subst}\left(\int x^2(a+b\log(cx^2))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^3} - \frac{(6d)\text{Subst}\left(\int x(a+b\log(cx^2))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^3} + \frac{(3d^2)\,\text{Subst}\left(\int (a+b\log(cx^2))^p\,dx,x,d+e\sqrt[3]{x}\right)}{e^3}$$

$$= -\frac{(3d)\text{Subst}\left(\int e^x(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{ce^3} + \frac{\left(3\left(d+e\sqrt[3]{x}\right)^3\right)\text{Subst}\left(\int e^{3x/2}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2e^3\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{3/2}} + \frac{\left(3d^2\left(d+e\sqrt[3]{x}\right)\right)\text{Subst}\left(\int e^{x/2}(a+bx)^p\,dx,x,\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2e^3\sqrt{c\left(d+e\sqrt[3]{x}\right)^2}}$$

$$= \frac{\left(\frac{2}{3}\right)^p e^{-\frac{3a}{2b}}\left(d+e\sqrt[3]{x}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log}{}\right.}{e^3\left(c\left(d+e\sqrt[3]{x}\right)^2\right)^{3/2}}$$

$$- \frac{3de^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{b}\right)}{ce^3}$$

$$+ \frac{3\;2^p d^2 e^{-\frac{a}{2b}}\left(d+e\sqrt[3]{x}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\left(-\frac{a+b\log}{}\right.}{e^3\sqrt{c\left(d+e\sqrt[3]{x}\right)^2}}$$

Mathematica [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p, x]
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{2}\right)\right)^{p}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
[Out] int((a+b*ln(c*(d+e*x^(1/3))^2))^p,x)
```

Fricas [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int \left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**2))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int \left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p, x)
```

Giac [F]

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int \left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p\,dx=\int \left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^2\right)\right)^p\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/3))^2))^p,x)
[Out] int((a + b*log(c*(d + e*x^(1/3))^2))^p, x)
```

3.566 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx = \text{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/3))^2))^p/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx = \int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)^2])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^2))^p}{x}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{2}\right)\right)^{p}}{x}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^2))^p/x,x)
[Out] int((a+b*ln(c*(d+e*x^(1/3))^2))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{2}c\right)+a\right)^{p}}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**2))**p/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 2.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x,x, algorithm="giac")
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{1/3}\right)^2\right)\right)^p}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/3))^2))^p/x,x)
[Out] int((a + b*log(c*(d + e*x^(1/3))^2))^p/x, x)
```

3.567 $\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx = \text{Int}\left(\frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(1/3))^2))^p/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx = \int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x^2,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x)^2])^p/x^4, x], x, x^(1/3)]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^2))^p}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(1/3))^2])^p/x^2, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{1}{3}}\right)^{2}\right)\right)^{p}}{x^{2}}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(1/3))^2))^p/x^2,x)
[Out] int((a+b*ln(c*(d+e*x^(1/3))^2))^p/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^{2}c\right)+a\right)^{p}}{x^{2}}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x^(2/3) + 2*c*d*e*x^(1/3) + c*d^2) + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(1/3))**2))**p/x**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p/x^2, x)
```

Giac [N/A]

Not integrable

Time = 2.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{1}{3}}+d\right)^2c\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(1/3))^2))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log((e*x^(1/3) + d)^2*c) + a)^p/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+e\sqrt[3]{x}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{1/3}\right)}^2\right)\right)^p}{x^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(1/3))^2))^p/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(1/3))^2))^p/x^2, x)
```

3.568 $\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 557

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\frac{2^{-2-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\cdots}{\cdots}\right)^{\cdots}}{c^6e^6}$$
$$-\frac{3\ 5^{-p}de^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{2c^5e^6}$$
$$+\frac{15\ 2^{-2(1+p)}d^2e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{c^4e^6}$$
$$-\frac{5\ 3^{-p}d^3e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{c^3e^6}$$
$$+\frac{15\ 2^{-2-p}d^4e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{c^2e^6}$$
$$-\frac{3d^5e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{2ce^6}$$

```
[Out] 2^(-2-p)*GAMMA(p+1,-6*(a+b*ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3)))
)^p/(3^p)/c^6/e^6/exp(6*a/b)/(((-a-b*ln(c*(d+e*x^(2/3))))/b)^p)-3/2*d*GAMMA
(p+1,-5*(a+b*ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/(5^p)/c^5/
e^6/exp(5*a/b)/(((-a-b*ln(c*(d+e*x^(2/3))))/b)^p)+15*d^2*GAMMA(p+1,-4*(a+b*
ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/(2^(2+2*p))/c^4/e^6/exp
(4*a/b)/(((-a-b*ln(c*(d+e*x^(2/3))))/b)^p)-5*d^3*GAMMA(p+1,-3*(a+b*ln(c*(d+
e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/(3^p)/c^3/e^6/exp(3*a/b)/(((-a-
b*ln(c*(d+e*x^(2/3))))/b)^p)+15*2^(-2-p)*d^4*GAMMA(p+1,-2*(a+b*ln(c*(d+e*x^
```

```
(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+
e*x^(2/3))))/b)^p)-3/2*d^5*GAMMA(p+1,(-a-b*ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(
c*(d+e*x^(2/3))))^p/c/e^6/exp(a/b)/(((-a-b*ln(c*(d+e*x^(2/3))))/b)^p)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2504, 2448, 2436, 2336, 2212, 2437, 2346}

$$\int x^3(a+b\log\left(c\left(d+ex^{2/3}\right)\right))^p\,dx=\frac{2^{-p-2}3^{-p}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{6(a+\cdots}{\cdots}\right.}{c^6e^6}$$
$$-\frac{3d5^{-p}e^{-\frac{5a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)}{2c^5e^6}$$
$$+\frac{15d^2 2^{-2(p+1)}e^{-\frac{4a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)}{c^4e^6}$$
$$-\frac{5d^3 3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)}{c^3e^6}$$
$$+\frac{15d^4 2^{-p-2}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)}{c^2e^6}$$
$$-\frac{3d^5e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)}{2ce^6}$$

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

```
[Out] (2^(-2 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b*Log[
c*(d + e*x^(2/3))])^p)/(3^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e*x^(2
/3))])/b))^p) - (3*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a
 + b*Log[c*(d + e*x^(2/3))])^p)/(2*5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c*
(d + e*x^(2/3))])/b))^p) + (15*d^2*Gamma[1 + p, (-4*(a + b*Log[c*(d + e*x^(
2/3))]))/b]*(a + b*Log[c*(d + e*x^(2/3))])^p)/(2^(2*(1 + p))*c^4*e^6*E^((4*
a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^p) - (5*d^3*Gamma[1 + p, (-3*(a
 + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b*Log[c*(d + e*x^(2/3))])^p)/(3^p*c^3
*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^p) + (15*2^(-2 - p)*
d^4*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b*Log[c*(d + e
*x^(2/3))])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^p
) - (3*d^5*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(2/3))])/b)]*(a + b*Log[c*(
d + e*x^(2/3))])^p)/(2*c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b))^
p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
```

```
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3}{2}\text{Subst}\left(\int x^5(a+b\log(c(d+ex)))^p\,dx, x, x^{2/3}\right)\\
&= \frac{3}{2}\text{Subst}\left(\int \left(-\frac{d^5(a+b\log(c(d+ex)))^p}{e^5}+\frac{5d^4(d+ex)(a+b\log(c(d+ex)))^p}{e^5}\right.\right.\\
&\quad -\frac{10d^3(d+ex)^2(a+b\log(c(d+ex)))^p}{e^5}+\frac{10d^2(d+ex)^3(a+b\log(c(d+ex)))^p}{e^5}\\
&\quad -\frac{5d(d+ex)^4(a+b\log(c(d+ex)))^p}{e^5}\\
&\quad \left.\left.+\frac{(d+ex)^5(a+b\log(c(d+ex)))^p}{e^5}\right)\,dx, x, x^{2/3}\right)\\
&= \frac{3\text{Subst}\left(\int (d+ex)^5(a+b\log(c(d+ex)))^p\,dx, x, x^{2/3}\right)}{2e^5}\\
&\quad -\frac{(15d)\text{Subst}\left(\int (d+ex)^4(a+b\log(c(d+ex)))^p\,dx, x, x^{2/3}\right)}{2e^5}\\
&\quad +\frac{(15d^2)\,\text{Subst}\left(\int (d+ex)^3(a+b\log(c(d+ex)))^p\,dx, x, x^{2/3}\right)}{e^5}\\
&\quad -\frac{(15d^3)\,\text{Subst}\left(\int (d+ex)^2(a+b\log(c(d+ex)))^p\,dx, x, x^{2/3}\right)}{e^5}\\
&\quad +\frac{(15d^4)\,\text{Subst}\left(\int (d+ex)(a+b\log(c(d+ex)))^p\,dx, x, x^{2/3}\right)}{2e^5}\\
&\quad -\frac{(3d^5)\,\text{Subst}\left(\int (a+b\log(c(d+ex)))^p\,dx, x, x^{2/3}\right)}{2e^5}\\
&= \frac{3\text{Subst}\left(\int x^5(a+b\log(cx))^p\,dx, x, d+ex^{2/3}\right)}{2e^6}\\
&\quad -\frac{(15d)\text{Subst}\left(\int x^4(a+b\log(cx))^p\,dx, x, d+ex^{2/3}\right)}{2e^6}\\
&\quad +\frac{(15d^2)\,\text{Subst}\left(\int x^3(a+b\log(cx))^p\,dx, x, d+ex^{2/3}\right)}{e^6}\\
&\quad -\frac{(15d^3)\,\text{Subst}\left(\int x^2(a+b\log(cx))^p\,dx, x, d+ex^{2/3}\right)}{e^6}\\
&\quad +\frac{(15d^4)\,\text{Subst}\left(\int x(a+b\log(cx))^p\,dx, x, d+ex^{2/3}\right)}{2e^6}\\
&\quad -\frac{(3d^5)\,\text{Subst}\left(\int (a+b\log(cx))^p\,dx, x, d+ex^{2/3}\right)}{2e^6}
\end{aligned}$$

$$=\frac{3\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx,x,\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{2c^6e^6}-\frac{(15d)\text{Subst}\left(\int e^{5x}(a+bx)^p\,dx,x,\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{2c^5e^6}+\frac{(15d^2)\,\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx,x,\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{c^4e^6}-\frac{(15d^3)\,\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx,x,\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{c^3e^6}+\frac{(15d^4)\,\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx,x,\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{2c^2e^6}-\frac{(3d^5)\,\text{Subst}\left(\int e^{x}(a+bx)^p\,dx,x,\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{2ce^6}$$

$$=\frac{2^{-2-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{c^6e^6}-\frac{3\ 5^{-p}de^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{2c^5e^6}+\frac{15\ 4^{-1-p}d^2e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{c^4e^6}-\frac{5\ 3^{-p}d^3e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{c^3e^6}+\frac{15\ 2^{-2-p}d^4e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{c^2e^6}-\frac{3d^5e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{2ce^6}$$

Mathematica [F]

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$$

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
[Out] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))])^p, x]
```

Maple [F]

$$\int x^3 \left(a + b\ln\left(c\left(d + e\,x^{\frac{2}{3}}\right)\right)\right)^p dx$$

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
[Out] int(x^3*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Fricas [F]

$$\int x^3 (a + b\log(c(d + ex^{2/3})))^p \, dx = \int \left(b\log\left(\left(ex^{\frac{2}{3}} + d\right)c\right) + a\right)^p x^3 \, dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e*x^(2/3) + c*d) + a)^p*x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 (a + b\log(c(d + ex^{2/3})))^p \, dx = \text{Timed out}$$

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int x^3 (a + b\log(c(d + ex^{2/3})))^p \, dx = \int \left(b\log\left(\left(ex^{\frac{2}{3}} + d\right)c\right) + a\right)^p x^3 \, dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x^3, x)
```

Giac [F]

$$\int x^3 \left(a + b\log \left(c\left(d + ex^{2/3}\right)\right)\right)^p \, dx = \int \left(b\log \left(\left(ex^{\frac{2}{3}} + d\right)c\right) + a\right)^p x^3 \, dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b\log \left(c\left(d + ex^{2/3}\right)\right)\right)^p \, dx = \int x^3 \left(a + b \ln \left(c \left(d + e\, x^{2/3}\right)\right)\right)^p dx$$

```
[In] int(x^3*(a + b*log(c*(d + e*x^(2/3))))^p,x)
[Out] int(x^3*(a + b*log(c*(d + e*x^(2/3))))^p, x)
```

3.569 $\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 273

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{2c^3e^3}-\frac{3\,2^{-1-p}de^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{c^2e^3}+\frac{3d^2e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}}{2ce^3}$$

```
[Out] 1/2*GAMMA(p+1,-3*(a+b*ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/(
3^p)/c^3/e^3/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(2/3))))/b)^p)-3*2^(-1-p)*d*GAM
MA(p+1,-2*(a+b*ln(c*(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/c^2/e^3/
exp(2*a/b)/(((-a-b*ln(c*(d+e*x^(2/3))))/b)^p)+3/2*d^2*GAMMA(p+1,(-a-b*ln(c*
(d+e*x^(2/3))))/b)*(a+b*ln(c*(d+e*x^(2/3))))^p/c/e^3/exp(a/b)/(((-a-b*ln(c*
(d+e*x^(2/3))))/b)^p)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used

$= \{2504, 2448, 2436, 2336, 2212, 2437, 2346\}$

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx = \frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3(a+b\log(c(d}{b}\right.}{2c^3e^3}$$
$$-\frac{3d2^{-p-1}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)}{b}\right)}{c^2e^3}$$
$$+\frac{3d^2e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)\right)}{b}\right)}{2ce^3}$$

```
[In] Int[x*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
```

```
[Out] (Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))]))/b]*(a + b*Log[c*(d + e*x^
(2/3))])^p)/(2*3^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))])/b)
)^p) - (3*2^(-1 - p)*d*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(2/3))]))/b]*
(a + b*Log[c*(d + e*x^(2/3))])^p)/(c^2*e^3*E^((2*a)/b)*(-((a + b*Log[c*(d +
 e*x^(2/3))])/b))^p) + (3*d^2*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(2/3))])
/b)]*(a + b*Log[c*(d + e*x^(2/3))])^p)/(2*c*e^3*E^(a/b)*(-((a + b*Log[c*(d
+ e*x^(2/3))])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3}{2}\text{Subst}\left(\int x^2(a+b\log(c(d+ex)))^p\,dx,x,x^{2/3}\right)\\
&= \frac{3}{2}\text{Subst}\left(\int \left(\frac{d^2(a+b\log(c(d+ex)))^p}{e^2}-\frac{2d(d+ex)(a+b\log(c(d+ex)))^p}{e^2}\right.\right.\\
&\qquad\left.\left.+\frac{(d+ex)^2(a+b\log(c(d+ex)))^p}{e^2}\right)\,dx,x,x^{2/3}\right)\\
&= \frac{3\text{Subst}\left(\int (d+ex)^2(a+b\log(c(d+ex)))^p\,dx,x,x^{2/3}\right)}{2e^2}\\
&\quad-\frac{(3d)\text{Subst}\left(\int (d+ex)(a+b\log(c(d+ex)))^p\,dx,x,x^{2/3}\right)}{e^2}\\
&\quad+\frac{(3d^2)\,\text{Subst}\left(\int (a+b\log(c(d+ex)))^p\,dx,x,x^{2/3}\right)}{2e^2}\\
&= \frac{3\text{Subst}\left(\int x^2(a+b\log(cx))^p\,dx,x,d+ex^{2/3}\right)}{2e^3}\\
&\quad-\frac{(3d)\text{Subst}\left(\int x(a+b\log(cx))^p\,dx,x,d+ex^{2/3}\right)}{e^3}\\
&\quad+\frac{(3d^2)\,\text{Subst}\left(\int (a+b\log(cx))^p\,dx,x,d+ex^{2/3}\right)}{2e^3}
\end{aligned}$$

$$= \frac{3\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+ex^{2/3}\right)\right)\right)}{2c^3e^3} - \frac{(3d)\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log\left(c\left(d+ex^{2/3}\right)\right)\right)}{c^2e^3} + \frac{(3d^2)\,\text{Subst}\left(\int e^{x}(a+bx)^p\,dx, x, \log\left(c\left(d+ex^{2/3}\right)\right)\right)}{2ce^3}$$

$$= \frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3(a+b\log(c(d+ex^{2/3})))}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log(c(d+ex^{2/3}))}{b}\right)^{-p}}{2c^3e^3} - \frac{3\;2^{-1-p}de^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2(a+b\log(c(d+ex^{2/3})))}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log(c(d+ex^{2/3}))}{b}\right)^{-p}}{c^2e^3} + \frac{3d^2e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log(c(d+ex^{2/3}))}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\left(-\frac{a+b\log(c(d+ex^{2/3}))}{b}\right)^{-p}}{2ce^3}$$

Mathematica [F]

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx = \int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$$

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))])^p, x]
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)\right)\right)^p dx$$

```
[In] int(x*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
[Out] int(x*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Fricas [F]

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx = \int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e*x^(2/3) + c*d) + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(2/3))))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x, x)
```

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int x\,\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)\right)\right)^p\,dx$$

```
[In] int(x*(a + b*log(c*(d + e*x^(2/3))))^p,x)
[Out] int(x*(a + b*log(c*(d + e*x^(2/3))))^p, x)
```

3.570 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx=\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))))^p/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))])^p/x,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)\right)\right)^p}{x}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)\right)\right)^{p}}{x}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(2/3))))^p/x,x)
[Out] int((a+b*ln(c*(d+e*x^(2/3))))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^{p}}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x,x, algorithm="fricas")
[Out] integral((b*log(c*e*x^(2/3) + c*d) + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))))**p/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)\right)\right)^p}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(2/3))))^p/x,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))))^p/x, x)
```

3.571 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx=\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))))^p/x^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))])^p/x^3,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)])^p/x^7, x], x, x^(1/3)]
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)\right)\right)^p}{x^7}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x^3,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x^3, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)\right)\right)^{p}}{x^{3}}d x$$

```
[In] int((a+b*ln(c*(d+e*x^(2/3))))^p/x^3,x)
[Out] int((a+b*ln(c*(d+e*x^(2/3))))^p/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^3}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^{p}}{x^{3}}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^3,x, algorithm="fricas")
[Out] integral((b*log(c*e*x^(2/3) + c*d) + a)^p/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\log(c(d+ex^{2/3})))^p}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))))**p/x**3,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex^{2/3})))^p}{x^3}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^3,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex^{2/3})))^p}{x^3}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^3,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 1.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(c(d+ex^{2/3})))^p}{x^3}\,dx = \int \frac{\left(a+b\,\ln\left(c\left(d+e\,x^{2/3}\right)\right)\right)^p}{x^3}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(2/3))))^p/x^3,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))))^p/x^3, x)
```

3.572 $\int x^2 \left(a+b \log \left(c \left(d+e x^{2/3}\right)\right)\right)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^2 \left(a+b \log \left(c \left(d+e x^{2/3}\right)\right)\right)^p \, dx=\text{Int}\left(x^2 \left(a+b \log \left(c \left(d+e x^{2/3}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x^2*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2 \left(a+b \log \left(c \left(d+e x^{2/3}\right)\right)\right)^p \, dx=\int x^2 \left(a+b \log \left(c \left(d+e x^{2/3}\right)\right)\right)^p \, dx$$

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^8*(a + b*Log[c*(d + e*x^2)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\text{integral}=3 \text{Subst}\left(\int x^8 \left(a+b \log \left(c \left(d+e x^2\right)\right)\right)^p \, dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))])^p,x]
[Out] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))])^p, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x^{2} {\left(a +b \ln \left(c \left(d +e \,x^{\frac{2}{3}}\right)\right)\right)}^{p}d x$$

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
[Out] int(x^2*(a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^{p}x^{2}\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e*x^(2/3) + c*d) + a)^p*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^px^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^px^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int x^2\,\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)\right)\right)^p\,dx$$

```
[In] int(x^2*(a + b*log(c*(d + e*x^(2/3))))^p,x)
[Out] int(x^2*(a + b*log(c*(d + e*x^(2/3))))^p, x)
```

3.573 $\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\operatorname{Int}\left(\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e*x^2)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\text{integral}=3\operatorname{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+ex^2\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \left(a+b\ln \left(c\left(d+e\,x^{\frac{2}{3}}\right)\right)\right)^{p}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(2/3))))^p,x)
[Out] int((a+b*ln(c*(d+e*x^(2/3))))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\int \left(b\log \left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e*x^(2/3) + c*d) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)\right)\right)^p\, dx = \int \left(b\log \left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p\, dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)\right)\right)^p\, dx = \int \left(b\log \left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p\, dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)\right)\right)^p\, dx = \int {\left(a+b\,\ln \left(c\,\left(d+e\,x^{2/3}\right)\right)\right)}^p \,d x$$

```
[In] int((a + b*log(c*(d + e*x^(2/3))))^p,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))))^p, x)
```

3.574 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx=\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))))^p/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))])^p/x^2,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)])^p/x^4, x], x, x^(1/3)]
```

Rubi steps

$$\text{integral}=3\operatorname{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)\right)\right)^p}{x^4}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))])^p/x^2, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)\right)\right)^{p}}{x^{2}}d x$$

```
[In] int((a+b*ln(c*(d+e*x^(2/3))))^p/x^2,x)
[Out] int((a+b*ln(c*(d+e*x^(2/3))))^p/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log(c*e*x^(2/3) + c*d) + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))))**p/x**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)c\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)*c) + a)^p/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)\right)\right)^p}{x^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(2/3))))^p/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))))^p/x^2, x)
```

3.575 $\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 678

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx =$$

$$-\frac{3\,2^{-1+p}d^5e^{-\frac{a}{2b}}\left(d+ex^{2/3}\right)\Gamma\left(1+p,\frac{-a-b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right.}{e^6\sqrt{c\left(d+ex^{2/3}\right)^2}}$$

$$+\frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}}{4c^3e^6}$$

$$-\frac{3\,2^{-1+p}5^{-p}de^{-\frac{5a}{2b}}\left(d+ex^{2/3}\right)^5\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\right.}{}\right.}{e^6\left(c\left(d+ex^{2/3}\right)^2\right)^{5/2}}$$

$$+\frac{15\,2^{-2-p}d^2e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}}{c^2e^6}$$

$$-\frac{5\left(\frac{2}{3}\right)^pd^3e^{-\frac{3a}{2b}}\left(d+ex^{2/3}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex\right.\right.}{b}\right.}{e^6\left(c\left(d+ex^{2/3}\right)^2\right)^{3/2}}$$

$$+\frac{15d^4e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}}{4ce^6}$$

```
[Out] 1/4*GAMMA(p+1,-3*(a+b*ln(c*(d+e*x^(2/3))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))
^p/(3^p)/c^3/e^6/exp(3*a/b)/(((-a-b*ln(c*(d+e*x^(2/3))^2))/b)^p)-3*2^(-1+p)
```

```
*d*(d+e*x^(2/3))^5*GAMMA(p+1,-5/2*(a+b*ln(c*(d+e*x^(2/3))^2))/b)*(a+b*ln(c*
(d+e*x^(2/3))^2))^p/(5^p)/e^6/exp(5/2*a/b)/(c*(d+e*x^(2/3))^2)^(5/2)/(((-a-
b*ln(c*(d+e*x^(2/3))^2))/b)^p)+15*2^(-2-p)*d^2*GAMMA(p+1,-2*(a+b*ln(c*(d+e*
x^(2/3))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))^p/c^2/e^6/exp(2*a/b)/(((-a-b*ln
(c*(d+e*x^(2/3))^2))/b)^p)-5*(2/3)^p*d^3*(d+e*x^(2/3))^3*GAMMA(p+1,-3/2*(a+
b*ln(c*(d+e*x^(2/3))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))^p/e^6/exp(3/2*a/b)/
(c*(d+e*x^(2/3))^2)^(3/2)/(((-a-b*ln(c*(d+e*x^(2/3))^2))/b)^p)+15/4*d^4*GAM
MA(p+1,(-a-b*ln(c*(d+e*x^(2/3))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))^p/c/e^6/
exp(a/b)/(((-a-b*ln(c*(d+e*x^(2/3))^2))/b)^p)-3*2^(-1+p)*d^5*(d+e*x^(2/3))*
GAMMA(p+1,1/2*(-a-b*ln(c*(d+e*x^(2/3))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))^p
/e^6/exp(1/2*a/b)/(((-a-b*ln(c*(d+e*x^(2/3))^2))/b)^p)/(c*(d+e*x^(2/3))^2)^
(1/2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 675, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2504, 2448, 2436, 2337, 2212, 2437, 2347}

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+\cdots\right.}{}\right)}{4c^3e^6}+\frac{15d^22^{-p-2}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{b}\right)}{c^2e^6}-\frac{3d^52^{p-1}e^{-\frac{a}{2b}}\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\cdots\right.}{2b}\right)}{e^6\sqrt{c\left(d+ex^{2/3}\right)^2}}+\frac{15d^4e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)}{4ce^6}-\frac{5d^3\left(\frac{2}{3}\right)^pe^{-\frac{3a}{2b}}\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+ex^{2}\cdots\right.\right.\right.}{2b}\right)}{e^6\left(c\left(d+ex^{2/3}\right)^2\right)^{3/2}}-\frac{3d2^{p-1}5^{-p}e^{-\frac{5a}{2b}}\left(d+ex^{2/3}\right)^5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+\cdots\right.\right.\right.}{2b}\right)}{e^6\left(c\left(d+ex^{2/3}\right)^2\right)^{5/2}}$$

```
[In] Int[x^3*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

[Out] (Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))^2]))/b]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(4*3^p*c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p) - (3*2^(-1 + p)*d*(d + e*x^(2/3))^5*Gamma[1 + p, (-5*(a + b*Log[c*(d + e*x^(2/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(5^p*e^6*E^((5*a)/(2*b))*(c*(d + e*x^(2/3))^2)^(5/2)*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p) + (15*2^(-2 - p)*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e*x^(2/3))^2]))/b]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p) - (5*(2/3)^p*d^3*(d + e*x^(2/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(e^6*E^((3*a)/(2*b))*(c*(d + e*x^(2/3))^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p) + (15*d^4*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(2/3))^2])/b)]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(4*c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p) - (3*2^(-1 + p)*d^5*(d + e*x^(2/3))*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e*x^(2/3))^2])/b]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e*x^(2/3))^2]*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p)

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3}{2}\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, x^{2/3}\right)\\
&= \frac{3}{2}\text{Subst}\left(\int\left(-\frac{d^5(a+b\log\left(c(d+ex)^2\right))^p}{e^5}+\frac{5d^4(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}\right.\right.\\
&\qquad -\frac{10d^3(d+ex)^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}\\
&\qquad +\frac{10d^2(d+ex)^3\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}-\frac{5d(d+ex)^4\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}\\
&\qquad \left.\left.+\frac{(d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}\right)\,dx, x, x^{2/3}\right)\\
&= \frac{3\text{Subst}\left(\int (d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, x^{2/3}\right)}{2e^5}\\
&\quad -\frac{(15d)\text{Subst}\left(\int (d+ex)^4\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, x^{2/3}\right)}{2e^5}\\
&\quad +\frac{(15d^2)\,\text{Subst}\left(\int (d+ex)^3\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, x^{2/3}\right)}{e^5}\\
&\quad -\frac{(15d^3)\,\text{Subst}\left(\int (d+ex)^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, x^{2/3}\right)}{e^5}\\
&\quad +\frac{(15d^4)\,\text{Subst}\left(\int (d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, x^{2/3}\right)}{2e^5}\\
&\quad -\frac{(3d^5)\,\text{Subst}\left(\int \left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, x^{2/3}\right)}{2e^5}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{3\text{Subst}\left(\int x^5(a+b\log(cx^2))^p\,dx, x, d+ex^{2/3}\right)}{2e^6} \\
&\quad - \frac{(15d)\text{Subst}\left(\int x^4(a+b\log(cx^2))^p\,dx, x, d+ex^{2/3}\right)}{2e^6} \\
&\quad + \frac{(15d^2)\,\text{Subst}\left(\int x^3(a+b\log(cx^2))^p\,dx, x, d+ex^{2/3}\right)}{e^6} \\
&\quad - \frac{(15d^3)\,\text{Subst}\left(\int x^2(a+b\log(cx^2))^p\,dx, x, d+ex^{2/3}\right)}{e^6} \\
&\quad + \frac{(15d^4)\,\text{Subst}\left(\int x(a+b\log(cx^2))^p\,dx, x, d+ex^{2/3}\right)}{2e^6} \\
&\quad - \frac{(3d^5)\,\text{Subst}\left(\int (a+b\log(cx^2))^p\,dx, x, d+ex^{2/3}\right)}{2e^6} \\
&= \frac{3\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{4c^3e^6} \\
&\quad + \frac{(15d^2)\,\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{2c^2e^6} \\
&\quad + \frac{(15d^4)\,\text{Subst}\left(\int e^{x}(a+bx)^p\,dx, x, \log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{4ce^6} \\
&\quad - \frac{\left(15d\left(d+ex^{2/3}\right)^5\right)\text{Subst}\left(\int e^{5x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{4e^6\left(c\left(d+ex^{2/3}\right)^2\right)^{5/2}} \\
&\quad - \frac{\left(15d^3\left(d+ex^{2/3}\right)^3\right)\text{Subst}\left(\int e^{3x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{2e^6\left(c\left(d+ex^{2/3}\right)^2\right)^{3/2}} \\
&\quad - \frac{\left(3d^5\left(d+ex^{2/3}\right)\right)\text{Subst}\left(\int e^{x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{4e^6\sqrt{c\left(d+ex^{2/3}\right)^2}}
\end{aligned}
$$

$$= \frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}}{4c^3e^6}$$

$$- \frac{3\ 2^{-1+p}5^{-p}de^{-\frac{5a}{2b}}\left(d+ex^{2/3}\right)^5\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\right.}{e^6\left(c\left(d+ex^{2/3}\right)^2\right)^{5/2}}$$

$$+ \frac{15\ 2^{-2-p}d^2e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/\right.\right.}}{b}\right.}{c^2e^6}$$

$$- \frac{5\left(\frac{2}{3}\right)^pd^3e^{-\frac{3a}{2b}}\left(d+ex^{2/3}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b}{}\right.}{e^6\left(c\left(d+ex^{2/3}\right)^2\right)^{3/2}}$$

$$+ \frac{15d^4e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}}{4ce^6}$$

$$- \frac{3\ 2^{-1+p}d^5e^{-\frac{a}{2b}}\left(d+ex^{2/3}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log}{}\right.}{e^6\sqrt{c\left(d+ex^{2/3}\right)^2}}$$

Mathematica [F]

$$\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int x^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$$

```
[In] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
[Out] Integrate[x^3*(a + b*Log[c*(d + e*x^(2/3))^2])^p, x]
```

Maple [F]

$$\int x^3\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^2\right)\right)^pdx$$

```
[In] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
[Out] int(x^3*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Fricas [F]

$$\int x^3 \left(a + b \log \left(c \left(d + e x^{2/3}\right)^2\right)\right)^p \, dx = \int \left(b \log \left(\left(e x^{\frac{2}{3}} + d\right)^2 c\right) + a\right)^p x^3 \, dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p*x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(a + b \log \left(c \left(d + e x^{2/3}\right)^2\right)\right)^p \, dx = \text{Timed out}$$

```
[In] integrate(x**3*(a+b*ln(c*(d+e*x**(2/3))**2))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int x^3 \left(a + b \log \left(c \left(d + e x^{2/3}\right)^2\right)\right)^p \, dx = \int \left(b \log \left(\left(e x^{\frac{2}{3}} + d\right)^2 c\right) + a\right)^p x^3 \, dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x^3, x)
```

Giac [F]

$$\int x^3 \left(a + b \log \left(c \left(d + e x^{2/3}\right)^2\right)\right)^p \, dx = \int \left(b \log \left(\left(e x^{\frac{2}{3}} + d\right)^2 c\right) + a\right)^p x^3 \, dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b \log \left(c\left(d + ex^{2/3}\right)^2\right)\right)^p \, dx = \int x^3 \left(a + b \ln \left(c \left(d + e\, x^{2/3}\right)^2\right)\right)^p \, dx$$

```
[In] int(x^3*(a + b*log(c*(d + e*x^(2/3))^2))^p,x)
[Out] int(x^3*(a + b*log(c*(d + e*x^(2/3))^2))^p, x)
```

3.576 $\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 350

$$\int x\Big(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\Big)^p\,dx=\frac{3\ 2^{-1+p}d^2e^{-\frac{a}{2b}}\left(d+ex^{2/3}\right)\Gamma\left(1+p,\frac{-a-b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+ex\right.\right.\right.}{e^3\sqrt{c\left(d+ex^{2/3}\right)^2}}$$
$$+\frac{2^{-1+p}3^{-p}e^{-\frac{3a}{2b}}\left(d+ex^{2/3}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex\right.\right.}{b}\right.}{e^3\left(c\left(d+ex^{2/3}\right)^2\right)^{3/2}}$$
$$-\frac{3de^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}}{2ce^3}$$

```
[Out] 2^(-1+p)*(d+e*x^(2/3))^3*GAMMA(p+1,-3/2*(a+b*ln(c*(d+e*x^(2/3))^2))/b)*(a+b
*ln(c*(d+e*x^(2/3))^2))^p/(3^p)/e^3/exp(3/2*a/b)/(c*(d+e*x^(2/3))^2)^(3/2)/
(((-a-b*ln(c*(d+e*x^(2/3))^2))/b)^p)-3/2*d*GAMMA(p+1,(-a-b*ln(c*(d+e*x^(2/3
))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))^p/c/e^3/exp(a/b)/(((-a-b*ln(c*(d+e*x^
(2/3))^2))/b)^p)+3*2^(-1+p)*d^2*(d+e*x^(2/3))*GAMMA(p+1,1/2*(-a-b*ln(c*(d+e
*x^(2/3))^2))/b)*(a+b*ln(c*(d+e*x^(2/3))^2))^p/e^3/exp(1/2*a/b)/(((-a-b*ln(
c*(d+e*x^(2/3))^2))/b)^p)/(c*(d+e*x^(2/3))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2504, 2448, 2436, 2337, 2212, 2437, 2347}

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\frac{3d^2 2^{p-1}e^{-\frac{a}{2b}}\left(d+ex^{2/3}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}}{e^3\sqrt{c\left(d+ex^{2/3}\right)^2}}+\frac{2^{p-1}3^{-p}e^{-\frac{3a}{2b}}\left(d+ex^{2/3}\right)^3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/\right.\right.\right.}}{2b}\right.}{e^3\left(c\left(d+ex^{2/3}\right)^2\right)^{3/2}}-\frac{3de^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)}{2ce^3}$$

```
[In] Int[x*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

```
[Out] (2^(-1 + p)*(d + e*x^(2/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e*x^(2/3))
^2]))/(2*b)]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(3^p*e^3*E^((3*a)/(2*b))*(
c*(d + e*x^(2/3))^2)^(3/2)*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p) - (3*
d*Gamma[1 + p, -((a + b*Log[c*(d + e*x^(2/3))^2])/b)]*(a + b*Log[c*(d + e*x
^(2/3))^2])^p)/(2*c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p)
+ (3*2^(-1 + p)*d^2*(d + e*x^(2/3))*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e*x
^(2/3))^2])/b]*(a + b*Log[c*(d + e*x^(2/3))^2])^p)/(e^3*E^(a/(2*b))*Sqrt[c*
(d + e*x^(2/3))^2]*(-((a + b*Log[c*(d + e*x^(2/3))^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{3}{2}\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx,x,x^{2/3}\right)\\ &= \frac{3}{2}\text{Subst}\left(\int\left(\frac{d^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^2}-\frac{2d(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^2}\right.\right.\\ &\qquad\left.\left.+\frac{(d+ex)^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^2}\right)\,dx,x,x^{2/3}\right)\end{aligned}$$

$$= \frac{3\text{Subst}\left(\int (d+ex)^2 \left(a+b\log\left(c(d+ex)^2\right)\right)^p \,dx, x, x^{2/3}\right)}{2e^2} - \frac{(3d)\text{Subst}\left(\int (d+ex) \left(a+b\log\left(c(d+ex)^2\right)\right)^p \,dx, x, x^{2/3}\right)}{e^2} + \frac{(3d^2)\,\text{Subst}\left(\int \left(a+b\log\left(c(d+ex)^2\right)\right)^p \,dx, x, x^{2/3}\right)}{2e^2}$$

$$= \frac{3\text{Subst}\left(\int x^2\left(a+b\log\left(cx^2\right)\right)^p \,dx, x, d+ex^{2/3}\right)}{2e^3} - \frac{(3d)\text{Subst}\left(\int x\left(a+b\log\left(cx^2\right)\right)^p \,dx, x, d+ex^{2/3}\right)}{e^3} + \frac{(3d^2)\,\text{Subst}\left(\int \left(a+b\log\left(cx^2\right)\right)^p \,dx, x, d+ex^{2/3}\right)}{2e^3}$$

$$= -\frac{(3d)\text{Subst}\left(\int e^x(a+bx)^p\,dx, x, \log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{2ce^3} + \frac{\left(3\left(d+ex^{2/3}\right)^3\right)\text{Subst}\left(\int e^{3x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{4e^3\left(c\left(d+ex^{2/3}\right)^2\right)^{3/2}} + \frac{\left(3d^2\left(d+ex^{2/3}\right)\right)\text{Subst}\left(\int e^{x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{4e^3\sqrt{c\left(d+ex^{2/3}\right)^2}}$$

$$= \frac{2^{-1+p}3^{-p}e^{-\frac{3a}{2b}}\left(d+ex^{2/3}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log}{}\right.}{e^3\left(c\left(d+ex^{2/3}\right)^2\right)^{3/2}} - \frac{3de^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{b}\right)^{-p}}{2ce^3} + \frac{3\;2^{-1+p}d^2e^{-\frac{a}{2b}}\left(d+ex^{2/3}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\left(-\frac{a+b\log(}{}\right.}{e^3\sqrt{c\left(d+ex^{2/3}\right)^2}}$$

Mathematica [F]

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$$

```
[In] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e*x^(2/3))^2])^p, x]
```

Maple [F]

$$\int x\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^2\right)\right)^p dx$$

```
[In] int(x*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
[Out] int(x*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Fricas [F]

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x*(a+b*ln(c*(d+e*x**(2/3))**2))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p x\,dx$$

[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="maxima")

[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x, x)

Giac [F]

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p x\,dx$$

[In] integrate(x*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="giac")

[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x, x)

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int x\left(a+b\,\ln\left(c\left(d+e\,x^{2/3}\right)^2\right)\right)^p\,dx$$

[In] int(x*(a + b*log(c*(d + e*x^(2/3))^2))^p,x)

[Out] int(x*(a + b*log(c*(d + e*x^(2/3))^2))^p, x)

3.577 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx=\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))^2))^p/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)^2])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\text{integral}=3\operatorname{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^2\right)\right)^p}{x}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{2}\right)\right)^{p}}{x}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x,x)
[Out] int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^{2}c\right)+a\right)^{p}}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**2))**p/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^2\right)\right)^p}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(2/3))^2))^p/x,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^2))^p/x, x)
```

3.578 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx=\mathrm{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^3,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)^2])^p/x^7, x], x, x^(1/3
)]
```

Rubi steps

$$\text{integral}=3\mathrm{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^2\right)\right)^p}{x^7}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^3,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^3, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^2\right)\right)^p}{x^3}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^3,x)
[Out] int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^3,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**2))**p/x**3,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^3,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x^3, x)
```

Giac [N/A]

Not integrable

Time = 2.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx = \int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^3,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^3}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^2\right)\right)^p}{x^3}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(2/3))^2))^p/x^3,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^2))^p/x^3, x)
```

3.579 $\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\text{Int}\left(x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x^2*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$$

```
[In] Int[x^2*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^8*(a + b*Log[c*(d + e*x^2)^2])^p, x], x, x^(1/3
)]
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+ex^2\right)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
[Out] Integrate[x^2*(a + b*Log[c*(d + e*x^(2/3))^2])^p, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^{2} \left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{2}\right)\right)^{p}dx$$

```
[In] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
[Out] int(x^2*(a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(a+b*ln(c*(d+e*x**(2/3))**2))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^px^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x^2, x)
```

Giac [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^px^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int x^2\,\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^2\right)\right)^p\,dx$$

```
[In] int(x^2*(a + b*log(c*(d + e*x^(2/3))^2))^p,x)
[Out] int(x^2*(a + b*log(c*(d + e*x^(2/3))^2))^p, x)
```

3.580 $\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p dx = \mathrm{Int}\left(\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p, x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p dx = \int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p dx$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e*x^2)^2])^p, x], x, x^(1/3
)]
```

Rubi steps

$$\text{integral} = 3\mathrm{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+ex^2\right)^2\right)\right)^p dx, x, \sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^{2}\right)\right)^{p}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
[Out] int((a+b*ln(c*(d+e*x^(2/3))^2))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int \left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**2))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int \left(b\log \left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int \left(b\log \left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log \left(c\left(d+ex^{2/3}\right)^2\right)\right)^p\,dx=\int {\left(a+b\,\ln\left(c\,{\left(d+e\,x^{2/3}\right)}^2\right)\right)}^p\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(2/3))^2))^p,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^2))^p, x)
```

3.581 $\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx=\mathrm{Int}\left(\frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^2,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e*x^2)^2])^p/x^4, x], x, x^(1/3
)]
```

Rubi steps

$$\text{integral}=3\mathrm{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+ex^2\right)^2\right)\right)^p}{x^4}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e*x^(2/3))^2])^p/x^2, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+e\,x^{\frac{2}{3}}\right)^2\right)\right)^p}{x^2}dx$$

```
[In] int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^2,x)
[Out] int((a+b*ln(c*(d+e*x^(2/3))^2))^p/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log(c*e^2*x^(4/3) + 2*c*d*e*x^(2/3) + c*d^2) + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**(2/3))**2))**p/x**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x^2, x)
```

Giac [N/A]

Not integrable

Time = 2.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(\left(ex^{\frac{2}{3}}+d\right)^2c\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^(2/3))^2))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log((e*x^(2/3) + d)^2*c) + a)^p/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+ex^{2/3}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+e\,x^{2/3}\right)^2\right)\right)^p}{x^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^(2/3))^2))^p/x^2,x)
[Out] int((a + b*log(c*(d + e*x^(2/3))^2))^p/x^2, x)
```

3.582 $\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=\operatorname{Int}\left(x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x*(a+b*ln(c*(d+e/x^(1/3))))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$$

```
[In] Int[x*(a + b*Log[c*(d + e/x^(1/3))])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^5*(a + b*Log[c*(d + e/x)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int x^5\left(a+b\log\left(c\left(d+\frac{e}{x}\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$$

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))])^p, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)\right)^p dx$$

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/3))))^p,x)
[Out] int(x*(a+b*ln(c*(d+e/x^(1/3))))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/3))))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p*x, x)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p*x, x)
```

Mupad [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=\int x\,\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{1/3}}\right)\right)\right)^p\,dx$$

```
[In] int(x*(a + b*log(c*(d + e/x^(1/3))))^p,x)
[Out] int(x*(a + b*log(c*(d + e/x^(1/3))))^p, x)
```

3.583 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p dx = \mathrm{Int}\left(\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p, x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/3))))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p dx = \int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p dx$$

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e/x)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x}\right)\right)\right)^p dx, x, \sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \left(a+b\log \left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=\int \left(a+b\log \left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \left(a+b\ln \left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)\right)^{p}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/3))))^p,x)
[Out] int((a+b*ln(c*(d+e/x^(1/3))))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \left(a+b\log \left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=\int \left(b\log \left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^{p}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\log \left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 1.62 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\,dx=\int \left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)\right)\right)^p\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/3))))^p,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))))^p, x)
```

3.584 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx=\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/3))))^p/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))])^p/x,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x)])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x}\right)\right)\right)^p}{x}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)\right)^p}{x}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/3))))^p/x,x)
[Out] int((a+b*ln(c*(d+e/x^(1/3))))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))))**p/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)\right)\right)^p}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/3))))^p/x,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))))^p/x, x)
```

3.585 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 267

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}\,dx = -\frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}}{c^3e^3}$$
$$+\frac{3\ 2^{-p}de^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}}{c^2e^3}$$
$$-\frac{3d^2e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}}{ce^3}$$

```
[Out] -GAMMA(p+1,-3*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(3^p
)/c^3/e^3/exp(3*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)+3*d*GAMMA(p+1,-2*(a
+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(2^p)/c^2/e^3/exp(2*
a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)-3*d^2*GAMMA(p+1,(-a-b*ln(c*(d+e/x^(
1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/c/e^3/exp(a/b)/(((-a-b*ln(c*(d+e/x^(
1/3))))/b)^p)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2504, 2448, 2436, 2336, 2212, 2437, 2346}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}\,dx = -\frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)}{c^3e^3}+\frac{3d2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)}{c^2e^3}-\frac{3d^2e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{ce^3}$$

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))])^p/x^2,x]
```

```
[Out] -((Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/
x^(1/3))])^p)/(3^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b)
)^p)) + (3*d*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log
[c*(d + e/x^(1/3))])^p)/(2^p*c^2*e^3*E^((2*a)/b)*(-((a + b*Log[c*(d + e/x^(
1/3))])/b))^p) - (3*d^2*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))])/b)]*(
a + b*Log[c*(d + e/x^(1/3))])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1
/3))])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
```

```
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = -\left(3\text{Subst}\left(\int x^2(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\text{Subst}\left(\int \left(\frac{d^2(a+b\log(c(d+ex)))^p}{e^2} - \frac{2d(d+ex)(a+b\log(c(d+ex)))^p}{e^2} + \frac{(d+ex)^2(a+b\right.\right.\right.$$

$$= -\frac{3\text{Subst}\left(\int (d+ex)^2(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^2} + \frac{(6d)\text{Subst}\left(\int (d+ex)(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^2} - \frac{(3d^2)\,\text{Subst}\left(\int (a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^2}$$

$$= -\frac{3\text{Subst}\left(\int x^2(a+b\log(cx))^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} + \frac{(6d)\text{Subst}\left(\int x(a+b\log(cx))^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} - \frac{(3d^2)\,\text{Subst}\left(\int (a+b\log(cx))^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}$$

$$= -\frac{3\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^3e^3} + \frac{(6d)\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^2e^3} - \frac{(3d^2)\,\text{Subst}\left(\int e^{x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{ce^3}$$

$$= -\frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p, -\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right.}{c^3e^3} + \frac{3\ 2^{-p}de^{-\frac{2a}{b}}\Gamma\left(1+p, -\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right.}{c^2e^3} - \frac{3d^2e^{-\frac{a}{b}}\Gamma\left(1+p, -\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}}{ce^3}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.66

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}\,dx = -\frac{6^{-p}e^{-\frac{3a}{b}}\left(2^p\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)+3^{1+p}cde^{a/b}\left(-\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\right.\right.}{c^3}$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x^2,x]
```

```
[Out] -(((2^p*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 3^(1 + p)*c*d
*E^(a/b)*(-Gamma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 2^p*c*d*E^
(a/b)*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))])/b)]))*(a + b*Log[c*(d +
 e/x^(1/3))])^p)/(6^p*c^3*e^3*E^((3*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])
/b))^p))
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)\right)^p}{x^2}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/3))))^p/x^2,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))))^p/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))))**p/x**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^2}\,dx = \int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)\right)\right)^p}{x^2}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/3))))^p/x^2,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))))^p/x^2, x)
```

3.586 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 554

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^3}\,dx = -\frac{2^{-1-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{c^6e^6}$$
$$+\frac{3\ 5^{-p}de^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}}{c^5e^6}$$
$$-\frac{15\ 2^{-1-2p}d^2e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{c^4e^6}$$
$$+\frac{10\ 3^{-p}d^3e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{c^3e^6}$$
$$-\frac{15\ 2^{-1-p}d^4e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{c^2e^6}$$
$$+\frac{3d^5e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}}{ce^6}$$

```
[Out] -2^(-1-p)*GAMMA(p+1,-6*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))
))^p/(3^p)/c^6/e^6/exp(6*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)+3*d*GAMMA(
p+1,-5*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(5^p)/c^5/e
^6/exp(5*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)-15*2^(-1-2*p)*d^2*GAMMA(p+
1,-4*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/c^4/e^6/exp(4
*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)+10*d^3*GAMMA(p+1,-3*(a+b*ln(c*(d+e
/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(3^p)/c^3/e^6/exp(3*a/b)/(((-a-b
*ln(c*(d+e/x^(1/3))))/b)^p)-15*2^(-1-p)*d^4*GAMMA(p+1,-2*(a+b*ln(c*(d+e/x^(
1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+e
```

```
/x^(1/3))))/b)^p)+3*d^5*GAMMA(p+1,(-a-b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(
d+e/x^(1/3))))^p/c/e^6/exp(a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2504, 2448, 2436, 2336, 2212, 2437, 2346}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^3}\,dx =$$

$$-\frac{2^{-p-1}3^{-p}e^{-\frac{6a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)}{c^6e^6}$$

$$+\frac{3d5^{-p}e^{-\frac{5a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)}{c^5e^6}$$

$$-\frac{15d^22^{-2p-1}e^{-\frac{4a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)}{c^4e^6}$$

$$+\frac{10d^33^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)}{c^3e^6}$$

$$-\frac{15d^42^{-p-1}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)}{c^2e^6}$$

$$+\frac{3d^5e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{ce^6}$$

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))])^p/x^3,x]
```

```
[Out] -((2^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Lo
g[c*(d + e/x^(1/3))])^p)/(3^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e/x^
```

```
(1/3))])/b))^p)) + (3*d*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/x^(1/3))]))/b]
*(a + b*Log[c*(d + e/x^(1/3))])^p)/(5^p*c^5*e^6*E^((5*a)/b)*(-((a + b*Log[c
*(d + e/x^(1/3))])/b))^p) - (15*2^(-1 - 2*p)*d^2*Gamma[1 + p, (-4*(a + b*Lo
g[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(c^4*e^6*E^((4*
a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) + (10*d^3*Gamma[1 + p, (-3*(
a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(3^p*c^
3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) - (15*2^(-1 - p)
*d^4*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d +
e/x^(1/3))])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^
p) + (3*d^5*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))])/b)]*(a + b*Log[c*
(d + e/x^(1/3))])^p)/(c*e^6*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p
)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = -\left(3\text{Subst}\left(\int x^5(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\text{Subst}\left(\int \left(-\frac{d^5(a+b\log(c(d+ex)))^p}{e^5}+\frac{5d^4(d+ex)(a+b\log(c(d+ex)))^p}{e^5}-\frac{10d^3(d+ex)^2}{}\right.\right.\right.$$

$$= -\frac{3\text{Subst}\left(\int (d+ex)^5(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5} + \frac{(15d)\text{Subst}\left(\int (d+ex)^4(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5} - \frac{(30d^2)\,\text{Subst}\left(\int (d+ex)^3(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5} + \frac{(30d^3)\,\text{Subst}\left(\int (d+ex)^2(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5} - \frac{(15d^4)\,\text{Subst}\left(\int (d+ex)(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5} + \frac{(3d^5)\,\text{Subst}\left(\int (a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5}$$

$$= -\frac{3\text{Subst}\left(\int x^5(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} + \frac{(15d)\text{Subst}\left(\int x^4(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} - \frac{(30d^2)\,\text{Subst}\left(\int x^3(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} + \frac{(30d^3)\,\text{Subst}\left(\int x^2(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} - \frac{(15d^4)\,\text{Subst}\left(\int x(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} + \frac{(3d^5)\,\text{Subst}\left(\int (a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^6}$$

$$= -\frac{3\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^6e^6} + \frac{(15d)\text{Subst}\left(\int e^{5x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^5e^6} - \frac{(30d^2)\,\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^4e^6} + \frac{(30d^3)\,\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^3e^6} - \frac{(15d^4)\,\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^2e^6} + \frac{(3d^5)\,\text{Subst}\left(\int e^{x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{ce^6}$$

$$= -\frac{2^{-1-p}3^{-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d\right.\right.}{b}\right.}{c^6e^6}$$

$$+\frac{3\ 5^{-p}de^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{}}\right.\right.}{b}\right.}{c^5e^6}$$

$$-\frac{15\ 2^{-1-2p}d^2e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\right.}{}\right.}{c^4e^6}$$

$$+\frac{10\ 3^{-p}d^3e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\right.\right.}{b}\right.}{c^3e^6}$$

$$-\frac{15\ 2^{-1-p}d^4e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\right.}{}\right.}{c^2e^6}$$

$$+\frac{3d^5e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{ce^6}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.59

$$\int\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^3}\,dx$$

$$=\frac{2^{-1-2p}15^{-p}e^{-\frac{6a}{b}}\left(-10^p\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)+cde^{a/b}\left(2^{1+2p}3^{1+p}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\right.\right.\right.}{b}\right.\right.\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x^3,x]
```

```
[Out] (2^(-1 - 2*p)*(-(10^p*Gamma[1 + p, (-6*(a + b*Log[c*(d + e/x^(1/3))]))/b])
+ c*d*E^(a/b)*(2^(1 + 2*p)*3^(1 + p)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/x
^(1/3))]))/b] + 5^p*c*d*E^(a/b)*(-5*3^(1 + p)*Gamma[1 + p, (-4*(a + b*Log[c
*(d + e/x^(1/3))]))/b] + 2^p*c*d*E^(a/b)*(5*2^(2 + p)*Gamma[1 + p, (-3*(a +
 b*Log[c*(d + e/x^(1/3))]))/b] + 3^(1 + p)*c*d*E^(a/b)*(-5*Gamma[1 + p, (-2
*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 2^(1 + p)*c*d*E^(a/b)*Gamma[1 + p, -(
(a + b*Log[c*(d + e/x^(1/3))])/b)])))))*(a + b*Log[c*(d + e/x^(1/3))])^p)/(
15^p*c^6*e^6*E^((6*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)\right)^{p}}{x^{3}}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/3))))^p/x^3,x)
[Out] int((a+b*ln(c*(d+e/x^(1/3))))^p/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^{p}}{x^{3}}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^{p}}{x^{3}}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^3,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^{p}}{x^{3}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))))**p/x**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a + b\log\left(c\left(d + \frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^3}\,dx = \int \frac{\left(b\log\left(c\left(d + \frac{e}{x^{\frac{1}{3}}}\right)\right) + a\right)^p}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^3,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{\left(a + b\log\left(c\left(d + \frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^3}\,dx = \int \frac{\left(b\log\left(c\left(d + \frac{e}{x^{\frac{1}{3}}}\right)\right) + a\right)^p}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^3,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a + b\log\left(c\left(d + \frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^3}\,dx = \int \frac{\left(a + b\,\ln\left(c\left(d + \frac{e}{x^{1/3}}\right)\right)\right)^p}{x^3}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/3))))^p/x^3,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))))^p/x^3, x)
```

3.587 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 832

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}\,dx =$$

$$-\frac{3^{-1-2p}e^{-\frac{9a}{b}}\Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{c^9e^9}$$

$$+\frac{3\ 8^{-p}de^{-\frac{8a}{b}}\Gamma\left(1+p,-\frac{8\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{c^8e^9}$$

$$-\frac{12\ 7^{-p}d^2e^{-\frac{7a}{b}}\Gamma\left(1+p,-\frac{7\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{c^7e^9}$$

$$+\frac{7\ 2^{2-p}3^{-p}d^3e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{c^6e^9}$$

$$-\frac{42\ 5^{-p}d^4e^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{c^5e^9}$$

$$+\frac{21\ 2^{1-2p}d^5e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{c^4e^9}$$

$$-\frac{28\ 3^{-p}d^6e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{c^3e^9}$$

$$+\frac{3\ 2^{2-p}d^7e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{c^2e^9}$$

$$-\frac{3d^8e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}}{ce^9}$$

```
[Out] -3^(-1-2*p)*GAMMA(p+1,-9*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3
))))^p/c^9/e^9/exp(9*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)+3*d*GAMMA(p+1,
-8*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(8^p)/c^8/e^9/e
xp(8*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)-12*d^2*GAMMA(p+1,-7*(a+b*ln(c*
(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(7^p)/c^7/e^9/exp(7*a/b)/(((
-a-b*ln(c*(d+e/x^(1/3))))/b)^p)+7*2^(2-p)*d^3*GAMMA(p+1,-6*(a+b*ln(c*(d+e/x
^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/(3^p)/c^6/e^9/exp(6*a/b)/(((-a-b*l
n(c*(d+e/x^(1/3))))/b)^p)-42*d^4*GAMMA(p+1,-5*(a+b*ln(c*(d+e/x^(1/3))))/b)*
(a+b*ln(c*(d+e/x^(1/3))))^p/(5^p)/c^5/e^9/exp(5*a/b)/(((-a-b*ln(c*(d+e/x^(1
/3))))/b)^p)+21*2^(1-2*p)*d^5*GAMMA(p+1,-4*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+
b*ln(c*(d+e/x^(1/3))))^p/c^4/e^9/exp(4*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)
^p)-28*d^6*GAMMA(p+1,-3*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3)
)))^p/(3^p)/c^3/e^9/exp(3*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)+3*2^(2-p)
*d^7*GAMMA(p+1,-2*(a+b*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/
c^2/e^9/exp(2*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))))/b)^p)-3*d^8*GAMMA(p+1,(-a-b
*ln(c*(d+e/x^(1/3))))/b)*(a+b*ln(c*(d+e/x^(1/3))))^p/c/e^9/exp(a/b)/(((-a-b
*ln(c*(d+e/x^(1/3))))/b)^p)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 832, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used

$= \{2504, 2448, 2436, 2336, 2212, 2437, 2346\}$

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}\,dx =$$

$$-\frac{3^{-2p-1}e^{-\frac{9a}{b}}\Gamma\left(p+1,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{c^9e^9}$$

$$+\frac{3\ 8^{-p}de^{-\frac{8a}{b}}\Gamma\left(p+1,-\frac{8\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{c^8e^9}$$

$$-\frac{12\ 7^{-p}d^2e^{-\frac{7a}{b}}\Gamma\left(p+1,-\frac{7\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{c^7e^9}$$

$$+\frac{7\ 2^{2-p}3^{-p}d^3e^{-\frac{6a}{b}}\Gamma\left(p+1,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{c^6e^9}$$

$$-\frac{42\ 5^{-p}d^4e^{-\frac{5a}{b}}\Gamma\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{c^5e^9}$$

$$+\frac{21\ 2^{1-2p}d^5e^{-\frac{4a}{b}}\Gamma\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{c^4e^9}$$

$$-\frac{28\ 3^{-p}d^6e^{-\frac{3a}{b}}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{c^3e^9}$$

$$+\frac{3\ 2^{2-p}d^7e^{-\frac{2a}{b}}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{c^2e^9}$$

$$-\frac{3d^8e^{-\frac{a}{b}}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)^{-p}}{ce^9}$$

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))])^p/x^4,x]
```

```
[Out] -((3^(-1 - 2*p)*Gamma[1 + p, (-9*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*
Log[c*(d + e/x^(1/3))])^p)/(c^9*e^9*E^((9*a)/b)*(-((a + b*Log[c*(d + e/x^(1
/3))])/b))^p)) + (3*d*Gamma[1 + p, (-8*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(
a + b*Log[c*(d + e/x^(1/3))])^p)/(8^p*c^8*e^9*E^((8*a)/b)*(-((a + b*Log[c*(
d + e/x^(1/3))])/b))^p) - (12*d^2*Gamma[1 + p, (-7*(a + b*Log[c*(d + e/x^(1
/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(7^p*c^7*e^9*E^((7*a)/b)*(-((a
 + b*Log[c*(d + e/x^(1/3))])/b))^p) + (7*2^(2 - p)*d^3*Gamma[1 + p, (-6*(a
+ b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(3^p*c^6*
e^9*E^((6*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) - (42*d^4*Gamma[1
+ p, (-5*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^
p)/(5^p*c^5*e^9*E^((5*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) + (21*
2^(1 - 2*p)*d^5*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/x^(1/3))]))/b]*(a + b*
Log[c*(d + e/x^(1/3))])^p)/(c^4*e^9*E^((4*a)/b)*(-((a + b*Log[c*(d + e/x^(1
/3))])/b))^p) - (28*d^6*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))]))/b]
*(a + b*Log[c*(d + e/x^(1/3))])^p)/(3^p*c^3*e^9*E^((3*a)/b)*(-((a + b*Log[c
*(d + e/x^(1/3))])/b))^p) + (3*2^(2 - p)*d^7*Gamma[1 + p, (-2*(a + b*Log[c*
(d + e/x^(1/3))]))/b]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(c^2*e^9*E^((2*a)/b
)*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p) - (3*d^8*Gamma[1 + p, -((a + b*L
og[c*(d + e/x^(1/3))])/b)]*(a + b*Log[c*(d + e/x^(1/3))])^p)/(c*e^9*E^(a/b)
*(-((a + b*Log[c*(d + e/x^(1/3))])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
```

```
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = -\left(3\text{Subst}\left(\int x^8(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\text{Subst}\left(\int \left(\frac{d^8(a+b\log(c(d+ex)))^p}{e^8} - \frac{8d^7(d+ex)(a+b\log(c(d+ex)))^p}{e^8} + \frac{28d^6(d+ex)^2(\right.\right.\right.$$

$$= -\frac{3\text{Subst}\left(\int (d+ex)^8(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^8} + \frac{(24d)\text{Subst}\left(\int (d+ex)^7(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^8} - \frac{(84d^2)\,\text{Subst}\left(\int (d+ex)^6(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^8} + \frac{(168d^3)\,\text{Subst}\left(\int (d+ex)^5(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^8} - \frac{(210d^4)\,\text{Subst}\left(\int (d+ex)^4(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^8} + \frac{(168d^5)\,\text{Subst}\left(\int (d+ex)^3(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^8} - \frac{(84d^6)\,\text{Subst}\left(\int (d+ex)^2(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^8} + \frac{(24d^7)\,\text{Subst}\left(\int (d+ex)(a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^8} - \frac{(3d^8)\,\text{Subst}\left(\int (a+b\log(c(d+ex)))^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^8}$$

$$
\begin{aligned}
&= -\frac{3\text{Subst}\left(\int x^8(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^9}\\
&+\frac{(24d)\text{Subst}\left(\int x^7(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^9}\\
&-\frac{(84d^2)\,\text{Subst}\left(\int x^6(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^9}\\
&+\frac{(168d^3)\,\text{Subst}\left(\int x^5(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^9}\\
&-\frac{(210d^4)\,\text{Subst}\left(\int x^4(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^9}\\
&+\frac{(168d^5)\,\text{Subst}\left(\int x^3(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^9}\\
&-\frac{(84d^6)\,\text{Subst}\left(\int x^2(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^9}\\
&+\frac{(24d^7)\,\text{Subst}\left(\int x(a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^9}\\
&-\frac{(3d^8)\,\text{Subst}\left(\int (a+b\log(cx))^p\,dx,x,d+\frac{e}{\sqrt[3]{x}}\right)}{e^9}
\end{aligned}
$$

$$= -\frac{3\text{Subst}\left(\int e^{9x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^9e^9}$$
$$+\frac{(24d)\text{Subst}\left(\int e^{8x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^8e^9}$$
$$-\frac{(84d^2)\,\text{Subst}\left(\int e^{7x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^7e^9}$$
$$+\frac{(168d^3)\,\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^6e^9}$$
$$-\frac{(210d^4)\,\text{Subst}\left(\int e^{5x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^5e^9}$$
$$+\frac{(168d^5)\,\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^4e^9}$$
$$-\frac{(84d^6)\,\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^3e^9}$$
$$+\frac{(24d^7)\,\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{c^2e^9}$$
$$-\frac{(3d^8)\,\text{Subst}\left(\int e^{x}(a+bx)^p\,dx,x,\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{ce^9}$$

$$=$$

$$-\frac{3^{-1-2p}e^{-\frac{9a}{b}}\Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right.}{c^9e^9}$$

$$+\frac{3\ 8^{-p}de^{-\frac{8a}{b}}\Gamma\left(1+p,-\frac{8\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right.}{c^8e^9}$$

$$-\frac{12\ 7^{-p}d^2e^{-\frac{7a}{b}}\Gamma\left(1+p,-\frac{7\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right.}{c^7e^9}$$

$$+\frac{7\ 2^{2-p}3^{-p}d^3e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\right.}{}\right.}{c^6e^9}$$

$$-\frac{42\ 5^{-p}d^4e^{-\frac{5a}{b}}\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\right.\right.}{b}\right.}{c^5e^9}$$

$$+\frac{21\ 2^{1-2p}d^5e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\right.}{b}\right.}{c^4e^9}$$

$$-\frac{28\ 3^{-p}d^6e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\right.\right.}{b}\right.}{c^3e^9}$$

$$+\frac{3\ 2^{2-p}d^7e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\right.\right.}{b}\right.}{c^2e^9}$$

$$-\frac{3d^8e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)}{b}\right)}{ce^9}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 502, normalized size of antiderivative = 0.60

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}\,dx =$$

$$-\frac{3^{-1-2p}280^{-p}e^{-\frac{9a}{b}}\left(280^p\Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right)-9^{1+p}35^pcde^{a/b}\Gamma\left(1+p,-\frac{8\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)}{b}\right.\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))])^p/x^4,x]
```

```
[Out] -((3^(-1 - 2*p)*(280^p*Gamma[1 + p, (-9*(a + b*Log[c*(d + e/x^(1/3))]))/b]
- 9^(1 + p)*35^p*c*d*E^(a/b)*Gamma[1 + p, (-8*(a + b*Log[c*(d + e/x^(1/3))]
))/b] + 2^(2 + 3*p)*5^p*9^(1 + p)*c^2*d^2*E^((2*a)/b)*Gamma[1 + p, (-7*(a +
 b*Log[c*(d + e/x^(1/3))]))/b] - 5^p*84^(1 + p)*c^3*d^3*E^((3*a)/b)*Gamma[1
 + p, (-6*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 2^(1 + 3*p)*63^(1 + p)*c^4*d
^4*E^((4*a)/b)*Gamma[1 + p, (-5*(a + b*Log[c*(d + e/x^(1/3))]))/b] - 5^p*12
6^(1 + p)*c^5*d^5*E^((5*a)/b)*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/x^(1/3))
]))/b] + 2^(2 + 3*p)*5^p*21^(1 + p)*c^6*d^6*E^((6*a)/b)*Gamma[1 + p, (-3*(a
 + b*Log[c*(d + e/x^(1/3))]))/b] - 35^p*36^(1 + p)*c^7*d^7*E^((7*a)/b)*Gamm
a[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))]))/b] + 9^(1 + p)*280^p*c^8*d^8*E
^((8*a)/b)*Gamma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))])/b)])*(a + b*Log[c*
(d + e/x^(1/3))])^p)/(280^p*c^9*e^9*E^((9*a)/b)*(-((a + b*Log[c*(d + e/x^(1
/3))])/b))^p))
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)\right)^p}{x^4}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/3))))^p/x^4,x)
```

```
[Out] int((a+b*ln(c*(d+e/x^(1/3))))^p/x^4,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^4,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*x^(2/3))/x) + a)^p/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))))**p/x**4,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^4,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^4, x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)\right)+a\right)^p}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))))^p/x^4,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))) + a)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\right)^p}{x^4}\,dx=\int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)\right)\right)^p}{x^4}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/3))))^p/x^4,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))))^p/x^4, x)
```

3.588 $\int x \left(a+b \log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x \left(a+b \log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p \, dx=\text{Int}\left(x \left(a+b \log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x*(a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x \left(a+b \log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p \, dx=\int x \left(a+b \log \left(c \left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p \, dx$$

```
[In] Int[x*(a + b*Log[c*(d + e/x^(1/3))^2])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^5*(a + b*Log[c*(d + e/x)^2])^p, x], x, x^(1/3)]
```

Rubi steps

$$\text{integral}=3 \text{Subst}\left(\int x^5 \left(a+b \log \left(c \left(d+\frac{e}{x}\right)^2\right)\right)^p \, dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p dx$$

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^2])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e/x^(1/3))^2])^p, x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)\right)^p dx$$

```
[In] int(x*(a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
[Out] int(x*(a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(1/3))**2))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p*x, x)
```

Giac [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p*x, x)
```

Mupad [N/A]

Not integrable

Time = 1.74 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx=\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^2\right)\right)^p\,dx$$

```
[In] int(x*(a + b*log(c*(d + e/x^(1/3))^2))^p,x)
[Out] int(x*(a + b*log(c*(d + e/x^(1/3))^2))^p, x)
```

3.589 $\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx=\text{Int}\left(\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx=\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx$$

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e/x)^2])^p, x], x, x^(1/3)]
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p dx = \int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)\right)^p dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
[Out] int((a+b*ln(c*(d+e/x^(1/3))^2))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p dx = \int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx = \int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx = \int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 1.69 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\,dx = \int \left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^2\right)\right)^p\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/3))^2))^p,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))^2))^p, x)
```

3.590 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx=\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(1/3))^2))^p/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x)^2])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^2\right)\right)^p}{x}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)\right)^p}{x}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x,x)
[Out] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 1.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^2\right)\right)^p}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x, x)
```

3.591 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 342

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx = -\frac{3\,2^p d^2 e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)\Gamma\left(1+p,\frac{-a-b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}}{e^3\sqrt{c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}} - \frac{\left(\frac{2}{3}\right)^p e^{-\frac{3a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}}{e^3\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{3/2}} + \frac{3de^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}}{ce^3}$$

```
[Out] -(2/3)^p*(d+e/x^(1/3))^3*GAMMA(p+1,-3/2*(a+b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b
```

```
*ln(c*(d+e/x^(1/3))^2))^p/e^3/exp(3/2*a/b)/(c*(d+e/x^(1/3))^2)^(3/2)/(((-a-
b*ln(c*(d+e/x^(1/3))^2))/b)^p)+3*d*GAMMA(p+1,(-a-b*ln(c*(d+e/x^(1/3))^2))/b
)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/c/e^3/exp(a/b)/(((-a-b*ln(c*(d+e/x^(1/3))^2
))/b)^p)-3*2^p*d^2*(d+e/x^(1/3))*GAMMA(p+1,1/2*(-a-b*ln(c*(d+e/x^(1/3))^2))
/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/e^3/exp(1/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/3
))^2))/b)^p)/(c*(d+e/x^(1/3))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2504, 2448, 2436, 2337, 2212, 2437, 2347}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx =$$

$$-\frac{3d^2 2^p e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{2b}\right)}{e^3\sqrt{c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}}$$

$$-\frac{\left(\frac{2}{3}\right)^p e^{-\frac{3a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)}{e^3\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{3/2}}$$

$$+\frac{3de^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)}{ce^3}$$

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^2,x]
```

```
[Out] -(((2/3)^p*(d + e/x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^
2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^3*E^((3*a)/(2*b))*(c*(d
+ e/x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p)) + (3*d*Ga
mma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))^2])/b)]*(a + b*Log[c*(d + e/x^(1/
3))^2])^p)/(c*e^3*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (3*2
^p*d^2*(d + e/x^(1/3))*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e/x^(1/3))^2])/b
```

```
]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^3*E^(a/(2*b))*Sqrt[c*(d + e/x^(1/3
))^2]*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
```

```
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = -\left(3\text{Subst}\left(\int x^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\text{Subst}\left(\int\left(\frac{d^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^2} - \frac{2d(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^2} + \frac{(d+ex)^2\left(a+\right.}{}\right.\right.\right.$$

$$= -\frac{3\text{Subst}\left(\int (d+ex)^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^2} + \frac{(6d)\text{Subst}\left(\int (d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^2} - \frac{(3d^2)\,\text{Subst}\left(\int \left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^2}$$

$$= -\frac{3\text{Subst}\left(\int x^2\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} + \frac{(6d)\text{Subst}\left(\int x\left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^3} - \frac{(3d^2)\,\text{Subst}\left(\int \left(a+b\log\left(cx^2\right)\right)^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^3}$$

$$= \frac{(3d)\text{Subst}\left(\int e^x(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{ce^3} - \frac{\left(3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\right)\text{Subst}\left(\int e^{3x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2e^3\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{3/2}} - \frac{\left(3d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int e^{x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2e^3\sqrt{c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}}$$

$$= -\frac{\left(\frac{2}{3}\right)^p e^{-\frac{3a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^3 \Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p \left(-\right.}{e^3\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{3/2}}$$
$$+\frac{3de^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right.}{ce^3}$$
$$-\frac{3\ 2^p d^2 e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+}{}\right.}{e^3\sqrt{c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}}$$

Mathematica [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx = \int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^2, x]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)\right)^p}{x^2}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^2,x)
[Out] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p/x^2,
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p/x**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^2\right)\right)^p}{x^2}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^2,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^2, x)
```

3.592 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 673

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx =$$

$$-\frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)}{2c^3e^6}$$

$$+\frac{3\left(\frac{2}{5}\right)^p de^{-\frac{5a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log}{}\right.}{e^6\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{5/2}}$$

$$-\frac{15\;2^{-1-p}d^2e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right.\right.}{b}\right.}{c^2e^6}$$

$$+\frac{5\;2^{1+p}3^{-p}d^3e^{-\frac{3a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\right.}{e^6\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{3/2}}$$

$$-\frac{15d^4e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}}{2ce^6}$$

$$+\frac{3\;2^pd^5e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d\right.\right.}{b}\right.}{e^6\sqrt{c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}}$$

```
[Out] -1/2*GAMMA(p+1,-3*(a+b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2)
)^p/(3^p)/c^3/e^6/exp(3*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)+3*(2/5)^p
*d*(d+e/x^(1/3))^5*GAMMA(p+1,-5/2*(a+b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*
(d+e/x^(1/3))^2))^p/e^6/exp(5/2*a/b)/(c*(d+e/x^(1/3))^2)^(5/2)/(((-a-b*ln(c
*(d+e/x^(1/3))^2))/b)^p)-15*2^(-1-p)*d^2*GAMMA(p+1,-2*(a+b*ln(c*(d+e/x^(1/3
))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/c^2/e^6/exp(2*a/b)/(((-a-b*ln(c*(d+
e/x^(1/3))^2))/b)^p)+5*2^(p+1)*d^3*(d+e/x^(1/3))^3*GAMMA(p+1,-3/2*(a+b*ln(c
*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/(3^p)/e^6/exp(3/2*a/b)/
(c*(d+e/x^(1/3))^2)^(3/2)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)-15/2*d^4*GAM
MA(p+1,(-a-b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/c/e^6/
exp(a/b)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)+3*2^p*d^5*(d+e/x^(1/3))*GAMMA
(p+1,1/2*(-a-b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/e^6/
exp(1/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)/(c*(d+e/x^(1/3))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used

$= \{2504, 2448, 2436, 2337, 2212, 2437, 2347\}$

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx =$$

$$-\frac{3^{-p}e^{-\frac{3a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{b}\right.}{2c^3e^6}$$

$$-\frac{15d^2 2^{-p-1}e^{-\frac{2a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right.\right.\right.}{b}\right.}{c^2e^6}$$

$$+\frac{3d^5 2^p e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(\right.\right.}{}\right.}{e^6\sqrt{c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}}$$

$$-\frac{15d^4e^{-\frac{a}{b}}\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)}{2ce^6}$$

$$+\frac{5d^3 2^{p+1}3^{-p}e^{-\frac{3a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3\left(\right.}{}\right.}{e^6\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{3/2}}$$

$$+\frac{3d\left(\frac{2}{5}\right)^p e^{-\frac{5a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{5\left(a+b\right.}{}\right.}{e^6\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{5/2}}$$

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^3,x]
```

```
[Out] -1/2*(Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^2]))/b]*(a + b*Log[c*(d
 + e/x^(1/3))^2])^p)/(3^p*c^3*e^6*E^((3*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3
))^2])/b))^p) + (3*(2/5)^p*d*(d + e/x^(1/3))^5*Gamma[1 + p, (-5*(a + b*Log[
c*(d + e/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^6*E^((
5*a)/(2*b))*(c*(d + e/x^(1/3))^2)^(5/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])
/b))^p) - (15*2^(-1 - p)*d^2*Gamma[1 + p, (-2*(a + b*Log[c*(d + e/x^(1/3))^
2]))/b]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(c^2*e^6*E^((2*a)/b)*(-((a + b*
Log[c*(d + e/x^(1/3))^2])/b))^p) + (5*2^(1 + p)*d^3*(d + e/x^(1/3))^3*Gamma
[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^
(1/3))^2])^p)/(3^p*e^6*E^((3*a)/(2*b))*(c*(d + e/x^(1/3))^2)^(3/2)*(-((a +
b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (15*d^4*Gamma[1 + p, -((a + b*Log[c*(d
 + e/x^(1/3))^2])/b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(2*c*e^6*E^(a/b)*
(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) + (3*2^p*d^5*(d + e/x^(1/3))*Gam
ma[1 + p, -1/2*(a + b*Log[c*(d + e/x^(1/3))^2])/b]*(a + b*Log[c*(d + e/x^(1
/3))^2])^p)/(e^6*E^(a/(2*b))*Sqrt[c*(d + e/x^(1/3))^2]*(-((a + b*Log[c*(d +
 e/x^(1/3))^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
```

```
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = -\left(3\text{Subst}\left(\int x^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\text{Subst}\left(\int\left(-\frac{d^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}+\frac{5d^4(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^5}-\frac{10d^3(d+e\right.\right.\right.$$

$$= -\frac{3\text{Subst}\left(\int(d+ex)^5\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5}$$
$$+\frac{(15d)\text{Subst}\left(\int(d+ex)^4\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5}$$
$$-\frac{(30d^2)\,\text{Subst}\left(\int(d+ex)^3\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5}$$
$$+\frac{(30d^3)\,\text{Subst}\left(\int(d+ex)^2\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5}$$
$$-\frac{(15d^4)\,\text{Subst}\left(\int(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5}$$
$$+\frac{(3d^5)\,\text{Subst}\left(\int\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^5}$$

$$= -\frac{3\text{Subst}\left(\int x^5(a+b\log(cx^2))^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} + \frac{(15d)\text{Subst}\left(\int x^4(a+b\log(cx^2))^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} - \frac{(30d^2)\,\text{Subst}\left(\int x^3(a+b\log(cx^2))^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} + \frac{(30d^3)\,\text{Subst}\left(\int x^2(a+b\log(cx^2))^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} - \frac{(15d^4)\,\text{Subst}\left(\int x(a+b\log(cx^2))^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^6} + \frac{(3d^5)\,\text{Subst}\left(\int (a+b\log(cx^2))^p\,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^6}$$

$$= -\frac{3\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2c^3e^6}$$

$$-\frac{(15d^2)\,\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{c^2e^6}$$

$$-\frac{(15d^4)\,\text{Subst}\left(\int e^{x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2ce^6}$$

$$+\frac{\left(15d\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\right)\text{Subst}\left(\int e^{5x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2e^6\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{5/2}}$$

$$+\frac{\left(15d^3\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\right)\text{Subst}\left(\int e^{3x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{e^6\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{3/2}}$$

$$+\frac{\left(3d^5\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int e^{x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2e^6\sqrt{c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}}$$

$$= -\frac{3^{-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right.\right.}{b}\right.}{2c^3e^6}$$

$$+\frac{3\left(\frac{2}{5}\right)^p de^{-\frac{5a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(\right.}{e^6\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{5/2}}$$

$$-\frac{15\ 2^{-1-p}d^2e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(\right.}{}\right.}{c^2e^6}$$

$$+\frac{5\ 2^{1+p}3^{-p}d^3e^{-\frac{3a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right.}{e^6\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{3/2}}$$

$$-\frac{15d^4e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right.}{b}\right.}{2ce^6}$$

$$+\frac{3\ 2^p d^5e^{-\frac{a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)\Gamma\left(1+p,-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log}{}\right.}{e^6\sqrt{c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}}$$

Mathematica [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^3,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^3, x]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)\right)^p}{x^3}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^3,x)
[Out] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^3,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p/x^3,
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p/x**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^3,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^3,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^3}\,dx = \int \frac{\left(a+b\,\ln\left(c\,{\left(d+\frac{e}{x^{1/3}}\right)}^2\right)\right)^p}{x^3}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^3,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^3, x)
```

3.593 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1036

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx =$$

$$-\frac{2^p 3^{-1-2p} e^{-\frac{9a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^9 \Gamma\left(1+p,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log}{\cdots}\right.}{e^9\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{9/2}}$$

$$+\frac{3\ 4^{-p} d e^{-\frac{4a}{b}} \Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right.}{c^4 e^9}$$

$$-\frac{3\ 2^{2+p} 7^{-p} d^2 e^{-\frac{7a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^7 \Gamma\left(1+p,-\frac{7\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a}{\cdots}\right.}{e^9\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{7/2}}$$

$$+\frac{28\ 3^{-p} d^3 e^{-\frac{3a}{b}} \Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right.}{b}\right.}{c^3 e^9}$$

$$-\frac{21\ 2^{1+p} 5^{-p} d^4 e^{-\frac{5a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^5 \Gamma\left(1+p,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\right.}{e^9\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{5/2}}$$

$$+\frac{21\ 2^{1-p} d^5 e^{-\frac{2a}{b}} \Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right.\right.}{b}\right.}{c^2 e^9}$$

$$-\frac{7\ 2^{2+p} 3^{-p} d^6 e^{-\frac{3a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^3 \Gamma\left(1+p,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a}{\cdots}\right.}{\left(\cdots\right)^{3/2}}$$

```
[Out] -2^p*3^(-1-2*p)*(d+e/x^(1/3))^9*GAMMA(p+1,-9/2*(a+b*ln(c*(d+e/x^(1/3))^2))/
b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/e^9/exp(9/2*a/b)/(c*(d+e/x^(1/3))^2)^(9/2)
/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)+3*d*GAMMA(p+1,-4*(a+b*ln(c*(d+e/x^(1/
3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/(4^p)/c^4/e^9/exp(4*a/b)/(((-a-b*l
n(c*(d+e/x^(1/3))^2))/b)^p)-3*2^(2+p)*d^2*(d+e/x^(1/3))^7*GAMMA(p+1,-7/2*(a
+b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/(7^p)/e^9/exp(7/
2*a/b)/(c*(d+e/x^(1/3))^2)^(7/2)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)+28*d^
3*GAMMA(p+1,-3*(a+b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p
/(3^p)/c^3/e^9/exp(3*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)-21*2^(p+1)*d
^4*(d+e/x^(1/3))^5*GAMMA(p+1,-5/2*(a+b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*
(d+e/x^(1/3))^2))^p/(5^p)/e^9/exp(5/2*a/b)/(c*(d+e/x^(1/3))^2)^(5/2)/(((-a-
b*ln(c*(d+e/x^(1/3))^2))/b)^p)+21*2^(1-p)*d^5*GAMMA(p+1,-2*(a+b*ln(c*(d+e/x
^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/c^2/e^9/exp(2*a/b)/(((-a-b*ln(
c*(d+e/x^(1/3))^2))/b)^p)-7*2^(2+p)*d^6*(d+e/x^(1/3))^3*GAMMA(p+1,-3/2*(a+b
*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/(3^p)/e^9/exp(3/2*
a/b)/(c*(d+e/x^(1/3))^2)^(3/2)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)+12*d^7*
GAMMA(p+1,(-a-b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/c/e
^9/exp(a/b)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)-3*2^p*d^8*(d+e/x^(1/3))*GA
MMA(p+1,1/2*(-a-b*ln(c*(d+e/x^(1/3))^2))/b)*(a+b*ln(c*(d+e/x^(1/3))^2))^p/e
^9/exp(1/2*a/b)/(((-a-b*ln(c*(d+e/x^(1/3))^2))/b)^p)/(c*(d+e/x^(1/3))^2)^(1
/2)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 1036, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used

$= \{2504, 2448, 2436, 2337, 2212, 2437, 2347\}$

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx =$$

$$-\frac{2^p 3^{-2p-1} e^{-\frac{9a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^9 \Gamma\left(p+1,-\frac{9\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log}{}\right.}{e^9\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{9/2}}$$

$$+\frac{3\ 4^{-p} d e^{-\frac{4a}{b}} \Gamma\left(p+1,-\frac{4\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)}{b}\right.}{c^4 e^9}$$

$$-\frac{3\ 2^{p+2} 7^{-p} d^2 e^{-\frac{7a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^7 \Gamma\left(p+1,-\frac{7\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\right.}{e^9\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{7/2}}$$

$$+\frac{28\ 3^{-p} d^3 e^{-\frac{3a}{b}} \Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right.}{b}\right.}{c^3 e^9}$$

$$-\frac{21\ 2^{p+1} 5^{-p} d^4 e^{-\frac{5a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^5 \Gamma\left(p+1,-\frac{5\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\right.}{e^9\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{5/2}}$$

$$+\frac{21\ 2^{1-p} d^5 e^{-\frac{2a}{b}} \Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\frac{a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)\right.}{b}\right.}{c^2 e^9}$$

$$-\frac{7\ 2^{p+2} 3^{-p} d^6 e^{-\frac{3a}{2b}}\left(d+\frac{e}{\sqrt[3]{x}}\right)^3 \Gamma\left(p+1,-\frac{3\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2b}\right)\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p\left(-\right.}{e^9\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{3/2}}$$

```
[In] Int[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^4,x]
```

```
[Out] -((2^p*3^(-1 - 2*p)*(d + e/x^(1/3))^9*Gamma[1 + p, (-9*(a + b*Log[c*(d + e/
x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^9*E^((9*a)/(2*b
))*(c*(d + e/x^(1/3))^2)^(9/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p))
+ (3*d*Gamma[1 + p, (-4*(a + b*Log[c*(d + e/x^(1/3))^2]))/b]*(a + b*Log[c*(
d + e/x^(1/3))^2])^p)/(4^p*c^4*e^9*E^((4*a)/b)*(-((a + b*Log[c*(d + e/x^(1/
3))^2])/b))^p) - (3*2^(2 + p)*d^2*(d + e/x^(1/3))^7*Gamma[1 + p, (-7*(a + b
*Log[c*(d + e/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(7^p
*e^9*E^((7*a)/(2*b))*(c*(d + e/x^(1/3))^2)^(7/2)*(-((a + b*Log[c*(d + e/x^(
1/3))^2])/b))^p) + (28*d^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^2]
))/b]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(3^p*c^3*e^9*E^((3*a)/b)*(-((a +
b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (21*2^(1 + p)*d^4*(d + e/x^(1/3))^5*Ga
mma[1 + p, (-5*(a + b*Log[c*(d + e/x^(1/3))^2]))/(2*b)]*(a + b*Log[c*(d + e
/x^(1/3))^2])^p)/(5^p*e^9*E^((5*a)/(2*b))*(c*(d + e/x^(1/3))^2)^(5/2)*(-((a
 + b*Log[c*(d + e/x^(1/3))^2])/b))^p) + (21*2^(1 - p)*d^5*Gamma[1 + p, (-2*
(a + b*Log[c*(d + e/x^(1/3))^2]))/b]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(c
^2*e^9*E^((2*a)/b)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (7*2^(2 + p
)*d^6*(d + e/x^(1/3))^3*Gamma[1 + p, (-3*(a + b*Log[c*(d + e/x^(1/3))^2]))/
(2*b)]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(3^p*e^9*E^((3*a)/(2*b))*(c*(d +
 e/x^(1/3))^2)^(3/2)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) + (12*d^7*G
amma[1 + p, -((a + b*Log[c*(d + e/x^(1/3))^2])/b)]*(a + b*Log[c*(d + e/x^(1
/3))^2])^p)/(c*e^9*E^(a/b)*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p) - (3*
2^p*d^8*(d + e/x^(1/3))*Gamma[1 + p, -1/2*(a + b*Log[c*(d + e/x^(1/3))^2])/
b]*(a + b*Log[c*(d + e/x^(1/3))^2])^p)/(e^9*E^(a/(2*b))*Sqrt[c*(d + e/x^(1/
3))^2]*(-((a + b*Log[c*(d + e/x^(1/3))^2])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2437

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[(f*(x/d))^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2448

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
 + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
 d*g, 0] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\text{integral} = -\left(3\text{Subst}\left(\int x^8\left(a+b\log\left(c(d+ex)^2\right)\right)^p\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)$$

$$= -\left(3\text{Subst}\left(\int\left(\frac{d^8\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^8}-\frac{8d^7(d+ex)\left(a+b\log\left(c(d+ex)^2\right)\right)^p}{e^8}+\frac{28d^6(d+ex)^2\right.\right.\right.$$

$$= -\frac{3\text{Subst}\left(\int (d+ex)^8 \left(a+b\log\left(c(d+ex)^2\right)\right)^p \, dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^8} + \frac{(24d)\text{Subst}\left(\int (d+ex)^7 \left(a+b\log\left(c(d+ex)^2\right)\right)^p \, dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^8} - \frac{(84d^2)\,\text{Subst}\left(\int (d+ex)^6 \left(a+b\log\left(c(d+ex)^2\right)\right)^p \, dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^8} + \frac{(168d^3)\,\text{Subst}\left(\int (d+ex)^5 \left(a+b\log\left(c(d+ex)^2\right)\right)^p \, dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^8} - \frac{(210d^4)\,\text{Subst}\left(\int (d+ex)^4 \left(a+b\log\left(c(d+ex)^2\right)\right)^p \, dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^8} + \frac{(168d^5)\,\text{Subst}\left(\int (d+ex)^3 \left(a+b\log\left(c(d+ex)^2\right)\right)^p \, dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^8} - \frac{(84d^6)\,\text{Subst}\left(\int (d+ex)^2 \left(a+b\log\left(c(d+ex)^2\right)\right)^p \, dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^8} + \frac{(24d^7)\,\text{Subst}\left(\int (d+ex) \left(a+b\log\left(c(d+ex)^2\right)\right)^p \, dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^8} - \frac{(3d^8)\,\text{Subst}\left(\int \left(a+b\log\left(c(d+ex)^2\right)\right)^p \, dx, x, \frac{1}{\sqrt[3]{x}}\right)}{e^8}$$

$$
\begin{aligned}
&= -\frac{3\text{Subst}\left(\int x^8 (a+b\log(cx^2))^p \,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^9} \\
&+ \frac{(24d)\text{Subst}\left(\int x^7 (a+b\log(cx^2))^p \,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^9} \\
&- \frac{(84d^2)\,\text{Subst}\left(\int x^6 (a+b\log(cx^2))^p \,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^9} \\
&+ \frac{(168d^3)\,\text{Subst}\left(\int x^5 (a+b\log(cx^2))^p \,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^9} \\
&- \frac{(210d^4)\,\text{Subst}\left(\int x^4 (a+b\log(cx^2))^p \,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^9} \\
&+ \frac{(168d^5)\,\text{Subst}\left(\int x^3 (a+b\log(cx^2))^p \,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^9} \\
&- \frac{(84d^6)\,\text{Subst}\left(\int x^2 (a+b\log(cx^2))^p \,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^9} \\
&+ \frac{(24d^7)\,\text{Subst}\left(\int x (a+b\log(cx^2))^p \,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^9} \\
&- \frac{(3d^8)\,\text{Subst}\left(\int (a+b\log(cx^2))^p \,dx, x, d+\frac{e}{\sqrt[3]{x}}\right)}{e^9}
\end{aligned}
$$

$$= \frac{(12d)\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{c^4e^9} + \frac{(84d^3)\,\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{c^3e^9} + \frac{(84d^5)\,\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{c^2e^9} + \frac{(12d^7)\,\text{Subst}\left(\int e^{x}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{ce^9} - \frac{\left(3\left(d+\frac{e}{\sqrt[3]{x}}\right)^9\right)\text{Subst}\left(\int e^{9x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2e^9\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{9/2}} - \frac{\left(42d^2\left(d+\frac{e}{\sqrt[3]{x}}\right)^7\right)\text{Subst}\left(\int e^{7x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{e^9\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{7/2}} - \frac{\left(105d^4\left(d+\frac{e}{\sqrt[3]{x}}\right)^5\right)\text{Subst}\left(\int e^{5x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{e^9\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{5/2}} - \frac{\left(42d^6\left(d+\frac{e}{\sqrt[3]{x}}\right)^3\right)\text{Subst}\left(\int e^{3x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{e^9\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)^{3/2}} - \frac{\left(3d^8\left(d+\frac{e}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int e^{x/2}(a+bx)^p\,dx, x, \log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)}{2e^9\sqrt{c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2}}$$

$= \text{Too large to display}$

Mathematica [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^4,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(1/3))^2])^p/x^4, x]
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)\right)^p}{x^4}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^4,x)
[Out] int((a+b*ln(c*(d+e/x^(1/3))^2))^p/x^4,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^4,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x + 2*c*d*e*x^(2/3) + c*e^2*x^(1/3))/x) + a)^p/x^4,
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(1/3))**2))**p/x**4,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^4,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^4, x)
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{1}{3}}}\right)^2\right)+a\right)^p}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(1/3))^2))^p/x^4,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(1/3))^2) + a)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{\sqrt[3]{x}}\right)^2\right)\right)^p}{x^4}\,dx = \int \frac{\left(a+b\,\ln\left(c\left(d+\frac{e}{x^{1/3}}\right)^2\right)\right)^p}{x^4}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^4,x)
[Out] int((a + b*log(c*(d + e/x^(1/3))^2))^p/x^4, x)
```

3.594 $\int x^3 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\mathrm{Int}\left(x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x^3*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

```
[In] Int[x^3*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^11*(a + b*Log[c*(d + e/x^2)])^p, x], x, x^(1/3)
]
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int x^{11}\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^3 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p \, dx=\int x^3 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p \, dx$$

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
[Out] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))])^p, x]
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x^{3} \left(a+b \ln \left(c \left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)\right)^{p}d x$$

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
[Out] int(x^3*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int x^3 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p \, dx=\int \left(b \log \left(c \left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^{p} x^{3} \, dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p*x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(a+b \log \left(c \left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p \, dx=\text{Timed out}$$

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^3 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)\right)\right)^p dx = \int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)\right) + a\right)^p x^3 \, dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^3 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)\right)\right)^p dx = \int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)\right) + a\right)^p x^3 \, dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x^3, x)
```

Mupad [N/A]

Not integrable

Time = 1.74 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^3 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)\right)\right)^p dx = \int x^3 \left(a + b \ln \left(c \left(d + \frac{e}{x^{2/3}}\right)\right)\right)^p dx$$

```
[In] int(x^3*(a + b*log(c*(d + e/x^(2/3))))^p,x)
[Out] int(x^3*(a + b*log(c*(d + e/x^(2/3))))^p, x)
```

3.595 $\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\text{Int}\left(x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x^2*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int x^2 \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^8*(a + b*Log[c*(d + e/x^2)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
[Out] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))])^p, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x^2\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)\right)^p dx$$

```
[In] int(x^2*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
[Out] int(x^2*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p*x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^2 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)\right)\right)^p dx = \int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)\right) + a\right)^p x^2 \, dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^2 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)\right)\right)^p dx = \int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)\right) + a\right)^p x^2 \, dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^2 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)\right)\right)^p dx = \int x^2 \left(a + b \ln \left(c \left(d + \frac{e}{x^{2/3}}\right)\right)\right)^p dx$$

```
[In] int(x^2*(a + b*log(c*(d + e/x^(2/3))))^p,x)
[Out] int(x^2*(a + b*log(c*(d + e/x^(2/3))))^p, x)
```

3.596 $\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\operatorname{Int}\left(x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

```
[In] Int[x*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^5*(a + b*Log[c*(d + e/x^2)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int x^5\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))])^p, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)\right)^p dx$$

```
[In] int(x*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
[Out] int(x*(a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p*x, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(2/3))))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x, x)
```

Giac [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p*x, x)
```

Mupad [N/A]

Not integrable

Time = 1.62 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int x\,\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

```
[In] int(x*(a + b*log(c*(d + e/x^(2/3))))^p,x)
[Out] int(x*(a + b*log(c*(d + e/x^(2/3))))^p, x)
```

3.597 $\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\text{Int}\left(\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))])^p,x]
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e/x^2)])^p, x], x, x^(1/3)]
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)\right)^p dx$$

```
[In] int((a+b*ln(c*(d+e/x^(2/3))))^p,x)
[Out] int((a+b*ln(c*(d+e/x^(2/3))))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx=\int \left(a+b\,\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(2/3))))^p,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))))^p, x)
```

3.598 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x} dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx=\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(2/3))))^p/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))])^p/x,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)\right)\right)^p}{x}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p/x, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)\right)^{p}}{x}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(2/3))))^p/x,x)
[Out] int((a+b*ln(c*(d+e/x^(2/3))))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^{p}}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))))**p/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(2/3))))^p/x,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))))^p/x, x)
```

3.599 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx=\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(2/3))))^p/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))])^p/x^2,x]
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)])^p/x^4, x], x, x^(1/3)]
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)\right)\right)^p}{x^4}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))])^p/x^2, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)\right)^p}{x^2}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(2/3))))^p/x^2,x)
[Out] int((a+b*ln(c*(d+e/x^(2/3))))^p/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log((c*d*x + c*e*x^(1/3))/x) + a)^p/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))))**p/x**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x^2,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))) + a)^p/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx = \int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)\right)\right)^p}{x^2}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(2/3))))^p/x^2,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))))^p/x^2, x)
```

3.600 $\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\operatorname{Int}\left(x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x^3*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

```
[In] Int[x^3*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^11*(a + b*Log[c*(d + e/x^2)^2])^p, x], x, x^(1/
3)]
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int x^{11}\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

```
[In] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
[Out] Integrate[x^3*(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^3\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)\right)^p dx$$

```
[In] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
[Out] int(x^3*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int x^3\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p x^3\,dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p*x
^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)^2\right)\right)^p dx = \text{Timed out}$$

```
[In] integrate(x**3*(a+b*ln(c*(d+e/x**(2/3))**2))**p,x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int x^3 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)^2\right)\right)^p dx = \text{Timed out}$$

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="maxima")
[Out] Timed out
```

Giac [N/A]

Not integrable

Time = 2.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^3 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)^2\right)\right)^p dx = \int \left(b \log \left(c \left(d + \frac{e}{x^{\frac{2}{3}}}\right)^2\right) + a\right)^p x^3 dx$$

```
[In] integrate(x^3*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x^3, x)
```

Mupad [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^3 \left(a + b \log \left(c \left(d + \frac{e}{x^{2/3}}\right)^2\right)\right)^p dx = \int x^3 \left(a + b \ln \left(c \left(d + \frac{e}{x^{2/3}}\right)^2\right)\right)^p dx$$

```
[In] int(x^3*(a + b*log(c*(d + e/x^(2/3))^2))^p,x)
[Out] int(x^3*(a + b*log(c*(d + e/x^(2/3))^2))^p, x)
```

3.601 $\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\mathrm{Int}\left(x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x^2*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

```
[In] Int[x^2*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^8*(a + b*Log[c*(d + e/x^2)^2])^p, x], x, x^(1/3
)]
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int x^8\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

```
[In] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
[Out] Integrate[x^2*(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^2\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)\right)^p dx$$

```
[In] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
[Out] int(x^2*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p*x
^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x**2*(a+b*ln(c*(d+e/x**(2/3))**2))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x^2, x)
```

Giac [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int x^2\left(a+b\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

```
[In] int(x^2*(a + b*log(c*(d + e/x^(2/3))^2))^p,x)
[Out] int(x^2*(a + b*log(c*(d + e/x^(2/3))^2))^p, x)
```

3.602 $\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\text{Int}\left(x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable(x*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

```
[In] Int[x*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^5*(a + b*Log[c*(d + e/x^2)^2])^p, x], x, x^(1/3
)]
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int x^5\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^2\right)\right)^p\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

```
[In] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
[Out] Integrate[x*(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)\right)^p dx$$

```
[In] int(x*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
[Out] int(x*(a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p*x
, x)
```

Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x*(a+b*ln(c*(d+e/x**(2/3))**2))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x, x)
```

Giac [N/A]

Not integrable

Time = 2.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p*x, x)
```

Mupad [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int x\left(a+b\ln\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

```
[In] int(x*(a + b*log(c*(d + e/x^(2/3))^2))^p,x)
[Out] int(x*(a + b*log(c*(d + e/x^(2/3))^2))^p, x)
```

3.603 $\int \left(a+b\log \left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \left(a+b\log \left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx=\text{Int}\left(\left(a+b\log \left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\log \left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx=\int \left(a+b\log \left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p dx$$

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][x^2*(a + b*Log[c*(d + e/x^2)^2])^p, x], x, x^(1/3
)]
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int x^2\left(a+b\log \left(c\left(d+\frac{e}{x^2}\right)^2\right)\right)^p dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)\right)^p dx$$

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
[Out] int((a+b*ln(c*(d+e/x^(2/3))^2))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p,
x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**2))**p,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 1.78 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int \left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx=\int \left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(2/3))^2))^p,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^2))^p, x)
```

3.604 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx=\text{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(2/3))^2))^p/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)^2])^p/x, x], x, x^(1/3)]
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^2\right)\right)^p}{x}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)\right)^p}{x}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^2))^p/x,x)
[Out] int((a+b*ln(c*(d+e/x^(2/3))^2))^p/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p/x
, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e/x**(2/3))**2))**p/x,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p/x, x)
```

Giac [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(2/3))^2))^p/x,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^2))^p/x, x)
```

3.605 $\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx=\operatorname{Int}\left(\frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/x^(2/3))^2))^p/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x^2,x]
```

```
[Out] 3*Defer[Subst][Defer[Int][(a + b*Log[c*(d + e/x^2)^2])^p/x^4, x], x, x^(1/3
)]
```

Rubi steps

$$\text{integral}=3\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^2}\right)^2\right)\right)^p}{x^4}\,dx,x,\sqrt[3]{x}\right)$$

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x^2,x]
[Out] Integrate[(a + b*Log[c*(d + e/x^(2/3))^2])^p/x^2, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\ln\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)\right)^p}{x^2}dx$$

```
[In] int((a+b*ln(c*(d+e/x^(2/3))^2))^p/x^2,x)
[Out] int((a+b*ln(c*(d+e/x^(2/3))^2))^p/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log((c*d^2*x^2 + 2*c*d*e*x^(4/3) + c*e^2*x^(2/3))/x^2) + a)^p/x
^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx = \text{Timed out}$$

[In] integrate((a+b*ln(c*(d+e/x**(2/3))**2))**p/x**2,x)

[Out] Timed out

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x^2,x, algorithm="maxima")

[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p/x^2, x)

Giac [N/A]

Not integrable

Time = 2.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx = \int \frac{\left(b\log\left(c\left(d+\frac{e}{x^{\frac{2}{3}}}\right)^2\right)+a\right)^p}{x^2}\,dx$$

[In] integrate((a+b*log(c*(d+e/x^(2/3))^2))^p/x^2,x, algorithm="giac")

[Out] integrate((b*log(c*(d + e/x^(2/3))^2) + a)^p/x^2, x)

Mupad [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\left(d+\frac{e}{x^{2/3}}\right)^2\right)\right)^p}{x^2}\,dx$$

```
[In] int((a + b*log(c*(d + e/x^(2/3))^2))^p/x^2,x)
[Out] int((a + b*log(c*(d + e/x^(2/3))^2))^p/x^2, x)
```

3.606 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{hx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 631

$$
\begin{aligned}
&\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{hx}}\,dx \\
&= \frac{2af\sqrt{hx}}{h}-\frac{8bfp\sqrt{hx}}{h}-\frac{8bgp(hx)^{3/2}}{9h^2}-\frac{2\sqrt{2}b\sqrt[4]{d}fp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}\sqrt{h}} \\
&\quad-\frac{2\sqrt{2}bd^{3/4}gp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}} \\
&\quad+\frac{2\sqrt{2}b\sqrt[4]{d}fp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}\sqrt{h}}+\frac{2\sqrt{2}bd^{3/4}gp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}} \\
&\quad+\frac{2bf\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{h}+\frac{2g(hx)^{3/2}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2} \\
&\quad-\frac{\sqrt{2}b\sqrt[4]{d}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}\sqrt{h}} \\
&\quad+\frac{\sqrt{2}bd^{3/4}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}\sqrt{h}} \\
&\quad+\frac{\sqrt{2}b\sqrt[4]{d}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}\sqrt{h}} \\
&\quad-\frac{\sqrt{2}bd^{3/4}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}\sqrt{h}}
\end{aligned}
$$

```
[Out] -8/9*b*g*p*(h*x)^(3/2)/h^2+2/3*g*(h*x)^(3/2)*(a+b*ln(c*(e*x^2+d)^p))/h^2-2*
b*d^(1/4)*f*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)
/e^(1/4)/h^(1/2)-2/3*b*d^(3/4)*g*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(
1/4)/h^(1/2))*2^(1/2)/e^(3/4)/h^(1/2)+2*b*d^(1/4)*f*p*arctan(1+e^(1/4)*2^(1
/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(1/4)/h^(1/2)+2/3*b*d^(3/4)*g*p*
arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(3/4)/h^(1/
2)-b*d^(1/4)*f*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/
2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(1/2)+1/3*b*d^(3/4)*g*p*ln(d^(1/2)*h^(1/2
)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(3/4)/h^
(1/2)+b*d^(1/4)*f*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^
(1/2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(1/2)-1/3*b*d^(3/4)*g*p*ln(d^(1/2)*h^(
1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(3/4)
/h^(1/2)+2*a*f*(h*x)^(1/2)/h-8*b*f*p*(h*x)^(1/2)/h+2*b*f*ln(c*(e*x^2+d)^p)*
(h*x)^(1/2)/h
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules used = {2517, 2521, 2498, 327, 217, 1179, 642, 1176, 631, 210, 2505, 303}

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx$$

$$= \frac{2g(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2}+\frac{2af\sqrt{hx}}{h}-\frac{2\sqrt{2}bd^{3/4}gp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}}$$
$$+\frac{2\sqrt{2}bd^{3/4}gp\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{3e^{3/4}\sqrt{h}}-\frac{2\sqrt{2}b\sqrt[4]{d}fp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}\sqrt{h}}$$
$$+\frac{2\sqrt{2}b\sqrt[4]{d}fp\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{\sqrt[4]{e}\sqrt{h}}+\frac{2bf\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h}$$
$$+\frac{\sqrt{2}bd^{3/4}gp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}\right)}{3e^{3/4}\sqrt{h}}$$
$$-\frac{\sqrt{2}bd^{3/4}gp\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}\right)}{3e^{3/4}\sqrt{h}}$$
$$-\frac{\sqrt{2}b\sqrt[4]{d}fp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}\right)}{\sqrt[4]{e}\sqrt{h}}$$
$$+\frac{\sqrt{2}b\sqrt[4]{d}fp\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}\right)}{\sqrt[4]{e}\sqrt{h}}-\frac{8bfp\sqrt{hx}}{h}-\frac{8bgp(hx)^{3/2}}{9h^2}$$

```
[In] Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h*x],x]
```

```
[Out] (2*a*f*Sqrt[h*x])/h - (8*b*f*p*Sqrt[h*x])/h - (8*b*g*p*(h*x)^(3/2))/(9*h^2)
 - (2*Sqrt[2]*b*d^(1/4)*f*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)
*Sqrt[h])])/(e^(1/4)*Sqrt[h]) - (2*Sqrt[2]*b*d^(3/4)*g*p*ArcTan[1 - (Sqrt[2
]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3/4)*Sqrt[h]) + (2*Sqrt[2]*b
*d^(1/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^
(1/4)*Sqrt[h]) + (2*Sqrt[2]*b*d^(3/4)*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[
h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3/4)*Sqrt[h]) + (2*b*f*Sqrt[h*x]*Log[c*(d +
 e*x^2)^p])/h + (2*g*(h*x)^(3/2)*(a + b*Log[c*(d + e*x^2)^p]))/(3*h^2) - (S
qrt[2]*b*d^(1/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1
/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*Sqrt[h]) + (Sqrt[2]*b*d^(3/4)*g*p*Log[Sqrt
[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^
(3/4)*Sqrt[h]) + (Sqrt[2]*b*d^(1/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[
h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*Sqrt[h]) - (Sqrt[2]*b*d
^(3/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[h*x]])/(3*e^(3/4)*Sqrt[h])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
```

```
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \left(f+\frac{gx^2}{h}\right)\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\text{Subst}\left(\int \left(f\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)+\frac{gx^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h}\right)dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{(2g)\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx, x, \sqrt{hx}\right)}{h^2} + \frac{(2f)\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2af\sqrt{hx}}{h} + \frac{2g(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2} + \frac{(2bf)\text{Subst}\left(\int \log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)dx, x, \sqrt{hx}\right)}{h} - \frac{(8begp)\text{Subst}\left(\int \frac{x^6}{d+\frac{ex^4}{h^2}}dx, x, \sqrt{hx}\right)}{3h^4}$$

$$= \frac{2af\sqrt{hx}}{h} - \frac{8bgp(hx)^{3/2}}{9h^2} + \frac{2bf\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h} + \frac{2g(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2} - \frac{(8befp)\text{Subst}\left(\int \frac{x^4}{d+\frac{ex^4}{h^2}}dx, x, \sqrt{hx}\right)}{h^3} + \frac{(8bdgp)\text{Subst}\left(\int \frac{x^2}{d+\frac{ex^4}{h^2}}dx, x, \sqrt{hx}\right)}{3h^2}$$

$$
\begin{aligned}
&= \frac{2af\sqrt{hx}}{h} - \frac{8bfp\sqrt{hx}}{h} - \frac{8bgp(hx)^{3/2}}{9h^2} + \frac{2bf\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h} \\
&\quad + \frac{2g(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2} - \frac{(4bdgp)\text{Subst}\left(\int \frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{3\sqrt{e}h^2} \\
&\quad + \frac{(4bdgp)\text{Subst}\left(\int \frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{3\sqrt{e}h^2} + \frac{(8bdfp)\text{Subst}\left(\int \frac{1}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2af\sqrt{hx}}{h} - \frac{8bfp\sqrt{hx}}{h} - \frac{8bgp(hx)^{3/2}}{9h^2} + \frac{2bf\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h} \\
&\quad + \frac{2g(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2} \\
&\quad + \frac{(2bdgp)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{3e} \\
&\quad + \frac{(2bdgp)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{3e} \\
&\quad + \frac{\left(4b\sqrt{d}fp\right)\text{Subst}\left(\int \frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^2} \\
&\quad + \frac{\left(4b\sqrt{d}fp\right)\text{Subst}\left(\int \frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^2} \\
&\quad + \frac{\left(\sqrt{2}bd^{3/4}gp\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{3e^{3/4}\sqrt{h}} \\
&\quad + \frac{\left(\sqrt{2}bd^{3/4}gp\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{3e^{3/4}\sqrt{h}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2af\sqrt{hx}}{h} - \frac{8bfp\sqrt{hx}}{h} - \frac{8bgp(hx)^{3/2}}{9h^2} + \frac{2bf\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h} \\
&+ \frac{2g(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2} \\
&+ \frac{\sqrt{2}bd^{3/4}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}\sqrt{h}} \\
&- \frac{\sqrt{2}bd^{3/4}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}\sqrt{h}} \\
&+ \frac{\left(2b\sqrt{d}fp\right)\mathrm{Subst}\left(\int\frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx,x,\sqrt{hx}\right)}{\sqrt{e}} \\
&+ \frac{\left(2b\sqrt{d}fp\right)\mathrm{Subst}\left(\int\frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx,x,\sqrt{hx}\right)}{\sqrt{e}} \\
&- \frac{\left(\sqrt{2}b\sqrt[4]{d}fp\right)\mathrm{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx,x,\sqrt{hx}\right)}{\sqrt[4]{e}\sqrt{h}} \\
&- \frac{\left(\sqrt{2}b\sqrt[4]{d}fp\right)\mathrm{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx,x,\sqrt{hx}\right)}{\sqrt[4]{e}\sqrt{h}} \\
&+ \frac{(2\sqrt{2}bd^{3/4}gp)\,\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}} \\
&- \frac{(2\sqrt{2}bd^{3/4}gp)\,\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2af\sqrt{hx}}{h} - \frac{8bfp\sqrt{hx}}{h} - \frac{8bgp(hx)^{3/2}}{9h^2} - \frac{2\sqrt{2}bd^{3/4}gp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}} \\
&+ \frac{2\sqrt{2}bd^{3/4}gp\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}} \\
&+ \frac{2bf\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h} + \frac{2g(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2} \\
&- \frac{\sqrt{2}b\sqrt[4]{d}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}\sqrt{h}} \\
&+ \frac{\sqrt{2}bd^{3/4}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}\sqrt{h}} \\
&+ \frac{\sqrt{2}b\sqrt[4]{d}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}\sqrt{h}} \\
&- \frac{\sqrt{2}bd^{3/4}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}\sqrt{h}} \\
&+ \frac{\left(2\sqrt{2}b\sqrt[4]{d}fp\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}\sqrt{h}} \\
&- \frac{\left(2\sqrt{2}b\sqrt[4]{d}fp\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}\sqrt{h}}
\end{aligned}
$$

$$= \frac{2af\sqrt{hx}}{h} - \frac{8bfp\sqrt{hx}}{h} - \frac{8bgp(hx)^{3/2}}{9h^2} - \frac{2\sqrt{2}b\sqrt[4]{d}fp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}\sqrt{h}}$$
$$- \frac{2\sqrt{2}bd^{3/4}gp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}} + \frac{2\sqrt{2}b\sqrt[4]{d}fp\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}\sqrt{h}}$$
$$+ \frac{2\sqrt{2}bd^{3/4}gp\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}} + \frac{2bf\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h}$$
$$+ \frac{2g(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2}$$
$$- \frac{\sqrt{2}b\sqrt[4]{d}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}\sqrt{h}}$$
$$+ \frac{\sqrt{2}bd^{3/4}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}\sqrt{h}}$$
$$+ \frac{\sqrt{2}b\sqrt[4]{d}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}\sqrt{h}}$$
$$- \frac{\sqrt{2}bd^{3/4}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}\sqrt{h}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.59

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx$$
$$= \frac{2\sqrt{x}\left(af\sqrt{x}-4bfp\sqrt{x}+\frac{1}{3}agx^{3/2}-\frac{4}{9}bgpx^{3/2}-\frac{\sqrt{2}b\sqrt[4]{d}fp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)}{\sqrt[4]{e}}+\frac{\sqrt{2}b\sqrt[4]{d}fp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)}{\sqrt[4]{e}}\right.}{}$$

```
[In] Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h*x],x]
```

```
[Out] (2*Sqrt[x]*(a*f*Sqrt[x] - 4*b*f*p*Sqrt[x] + (a*g*x^(3/2))/3 - (4*b*g*p*x^(3
/2))/9 - (Sqrt[2]*b*d^(1/4)*f*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4
)])/e^(1/4) + (Sqrt[2]*b*d^(1/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d
^(1/4)])/e^(1/4) - (2*b*(-d)^(3/4)*g*p*ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)]
)/(3*e^(3/4)) + (2*b*(-d)^(3/4)*g*p*ArcTanh[(e^(1/4)*Sqrt[x])/(-d)^(1/4)])/
(3*e^(3/4)) - (b*d^(1/4)*f*p*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x]
+ Sqrt[e]*x])/(Sqrt[2]*e^(1/4)) + (b*d^(1/4)*f*p*Log[Sqrt[d] + Sqrt[2]*d^(1
```

```
/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x])/(Sqrt[2]*e^(1/4)) + b*f*Sqrt[x]*Log[c*(d
+ e*x^2)^p] + (b*g*x^(3/2)*Log[c*(d + e*x^2)^p])/3))/Sqrt[h*x]
```

Maple [F]

$$\int \frac{(gx+f)\left(a+b\ln\left(c(e\,x^{2}+d)^{p}\right)\right)}{\sqrt{hx}}dx$$

```
[In] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2),x)
```

```
[Out] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1196 vs. $2(441) = 882$.

Time = 0.34 (sec) , antiderivative size = 1196, normalized size of antiderivative = 1.90

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="fricas
")
```

```
[Out] -2/9*(3*h*sqrt(-(6*b^2*d*f*g*p^2 + e*h*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2
*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)))/(e*h))*log(-32*(81*b^3*e^2*f^4 -
b^3*d^2*g^4)*sqrt(h*x)*p^3 + 32*(e^2*g*h^2*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4
*d^2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)) + 3*(9*b^2*e^2*f^3 - b^2*d*e*f
*g^2)*h*p^2)*sqrt(-(6*b^2*d*f*g*p^2 + e*h*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*
d^2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)))/(e*h))) - 3*h*sqrt(-(6*b^2*d*f
*g*p^2 + e*h*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 + b^4*d^3*g^4)*
p^4/(e^3*h^2)))/(e*h))*log(-32*(81*b^3*e^2*f^4 - b^3*d^2*g^4)*sqrt(h*x)*p^3
 - 32*(e^2*g*h^2*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 + b^4*d^3*g
^4)*p^4/(e^3*h^2)) + 3*(9*b^2*e^2*f^3 - b^2*d*e*f*g^2)*h*p^2)*sqrt(-(6*b^2*
d*f*g*p^2 + e*h*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 + b^4*d^3*g^
4)*p^4/(e^3*h^2)))/(e*h))) - 3*h*sqrt(-(6*b^2*d*f*g*p^2 - e*h*sqrt(-(81*b^4
*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)))/(e*h))*log
(-32*(81*b^3*e^2*f^4 - b^3*d^2*g^4)*sqrt(h*x)*p^3 + 32*(e^2*g*h^2*sqrt(-(81
*b^4*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)) - 3*(9*
b^2*e^2*f^3 - b^2*d*e*f*g^2)*h*p^2)*sqrt(-(6*b^2*d*f*g*p^2 - e*h*sqrt(-(81*
b^4*d*e^2*f^4 - 18*b^4*d^2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)))/(e*h)))
 + 3*h*sqrt(-(6*b^2*d*f*g*p^2 - e*h*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2*e*
f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)))/(e*h))*log(-32*(81*b^3*e^2*f^4 - b^3
*d^2*g^4)*sqrt(h*x)*p^3 - 32*(e^2*g*h^2*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^
2*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)) - 3*(9*b^2*e^2*f^3 - b^2*d*e*f*g^
2)*h*p^2)*sqrt(-(6*b^2*d*f*g*p^2 - e*h*sqrt(-(81*b^4*d*e^2*f^4 - 18*b^4*d^2
```

```
*e*f^2*g^2 + b^4*d^3*g^4)*p^4/(e^3*h^2)))/(e*h))) + (36*b*f*p - 9*a*f + (4*
b*g*p - 3*a*g)*x - 3*(b*g*p*x + 3*b*f*p)*log(e*x^2 + d) - 3*(b*g*x + 3*b*f)
*log(c))*sqrt(h*x))/h
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(1/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 754, normalized size of antiderivative = 1.19

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx$$

$$= \frac{2\,bgx^2\log\left((ex^2+d)^p c\right)}{3\,\sqrt{hx}} + \frac{2\,agx^2}{3\,\sqrt{hx}} + \frac{2\,\sqrt{hx}bf\log\left((ex^2+d)^p c\right)}{h}$$

$$- \frac{\left(\frac{8\,\sqrt{hx}h^2}{e} - \frac{\left(\frac{\sqrt{2}h^4\log\left(\sqrt{e}hx+\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{\left(dh^2\right)^{\frac{3}{4}}e^{\frac{1}{4}}} - \frac{\sqrt{2}h^4\log\left(\sqrt{e}hx-\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{\left(dh^2\right)^{\frac{3}{4}}e^{\frac{1}{4}}} + \frac{\sqrt{2}h^3\log\left(-\frac{\sqrt{2}\sqrt{-\sqrt{d}\sqrt{e}h}+\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\frac{1}{4}}\cdots}{\sqrt{2}\sqrt{-\sqrt{d}\sqrt{e}h}-\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\frac{1}{4}}\cdots}\cdots\right.}{\sqrt{-\sqrt{d}\sqrt{e}h}\sqrt{d}}\cdots}{e}\cdots}{h^3\cdots}$$

$$+ \frac{2\,\sqrt{hx}af}{h}$$

$$- \frac{\left(\frac{3\,dh^4\left(\frac{\sqrt{2}\log\left(\sqrt{e}hx+\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{\left(dh^2\right)^{\frac{1}{4}}e^{\frac{3}{4}}} - \frac{\sqrt{2}\log\left(\sqrt{e}hx-\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{\left(dh^2\right)^{\frac{1}{4}}e^{\frac{3}{4}}} - \frac{\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{-\sqrt{d}\sqrt{e}h}+\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\frac{1}{4}}-2\,\sqrt{hx}\sqrt{e}}{\sqrt{2}\sqrt{-\sqrt{d}\sqrt{e}h}-\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\frac{1}{4}}+2\,\sqrt{hx}\sqrt{e}}\right)}{\sqrt{-\sqrt{d}\sqrt{e}h}\sqrt{e}} - \cdots\right.}{e}\cdots}{9\,h^4\cdots}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="maxima
")
```

```
[Out] 2/3*b*g*x^2*log((e*x^2 + d)^p*c)/sqrt(h*x) + 2/3*a*g*x^2/sqrt(h*x) + 2*sqrt
(h*x)*b*f*log((e*x^2 + d)^p*c)/h - (8*sqrt(h*x)*h^2/e - (sqrt(2)*h^4*log(sq
rt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(
3/4)*e^(1/4)) - sqrt(2)*h^4*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*
x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) + sqrt(2)*h^3*log(-(sqrt(2)
*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqr
t(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2
*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(d)) + sqrt(2)*h^3*log(-
(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(
h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(
1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(d)))*d/e)*b*e*f
*p/h^3 + 2*sqrt(h*x)*a*f/h - 1/9*(3*d*h^4*(sqrt(2)*log(sqrt(e)*h*x + sqrt(2
)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sq
rt(2)*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h
)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h)
+ sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(
d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt
(-sqrt(d)*sqrt(e)*h)*sqrt(e)) - sqrt(2)*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)
*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-s
qrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(
sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)))/e + 8*(h*x)^(3/2)*h^2/e)*b*e*g*p/h^4
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 556, normalized size of antiderivative = 0.88

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx$$

$$= \frac{6\sqrt{hx}bgx\log(c) + 9\left(e\left(\frac{2\sqrt{2}(de^3h^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{dh^2}{e}\right)^{\frac{1}{4}}+2\sqrt{hx}\right)}{2\left(\frac{dh^2}{e}\right)^{\frac{1}{4}}}\right)}{e^2} + \frac{2\sqrt{2}(de^3h^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{dh^2}{e}\right)^{\frac{1}{4}}-2\sqrt{hx}\right)}{2\left(\frac{dh^2}{e}\right)^{\frac{1}{4}}}\right)}{e^2}\right.\right.}{}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/9*(6*sqrt(h*x)*b*g*x*log(c) + 9*(e*(2*sqrt(2)*(d*e^3*h^2)^(1/4)*arctan(1/
2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) + 2*sqrt(h*x))/(d*h^2/e)^(1/4))/e^2 + 2*
sqrt(2)*(d*e^3*h^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) - 2*
sqrt(h*x))/(d*h^2/e)^(1/4))/e^2 + sqrt(2)*(d*e^3*h^2)^(1/4)*log(h*x + sqrt(
2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/e^2 - sqrt(2)*(d*e^3*h^2)^(1/
4)*log(h*x - sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/e^2 - 8*sqr
t(h*x)/e) + 2*sqrt(h*x)*log(e*x^2 + d))*b*f*p + 6*sqrt(h*x)*a*g*x + 18*sqrt
(h*x)*b*f*log(c) + (6*sqrt(h*x)*h*x*log(e*x^2 + d) - (8*sqrt(h*x)*h*x/e - 6
*sqrt(2)*(d*e^3*h^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) + 2*
sqrt(h*x))/(d*h^2/e)^(1/4))/e^4 - 6*sqrt(2)*(d*e^3*h^2)^(3/4)*arctan(-1/2*s
qrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) - 2*sqrt(h*x))/(d*h^2/e)^(1/4))/e^4 + 3*sqr
t(2)*(d*e^3*h^2)^(3/4)*log(h*x + sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d
*h^2/e))/e^4 - 3*sqrt(2)*(d*e^3*h^2)^(3/4)*log(h*x - sqrt(2)*(d*h^2/e)^(1/4
)*sqrt(h*x) + sqrt(d*h^2/e))/e^4)*e)*b*g*p/h + 18*sqrt(h*x)*a*f)/h
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx = \int \frac{(f+g\,x)\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{\sqrt{h\,x}}\,dx$$

```
[In] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(1/2),x)
[Out] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(1/2), x)
```

3.607 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 603

$$
\begin{aligned}
\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{3/2}}\,dx &= \frac{2ag\sqrt{hx}}{h^2}-\frac{8bgp\sqrt{hx}}{h^2} \\
&- \frac{2\sqrt{2}b\sqrt[4]{e}fp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}}-\frac{2\sqrt{2}b\sqrt[4]{d}gp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{3/2}} \\
&+ \frac{2\sqrt{2}b\sqrt[4]{e}fp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}}+\frac{2\sqrt{2}b\sqrt[4]{d}gp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{3/2}} \\
&+ \frac{2bg\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{h^2}-\frac{2f\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h\sqrt{hx}} \\
&+ \frac{\sqrt{2}b\sqrt[4]{e}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}} \\
&- \frac{\sqrt{2}b\sqrt[4]{d}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}h^{3/2}} \\
&- \frac{\sqrt{2}b\sqrt[4]{e}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}} \\
&+ \frac{\sqrt{2}b\sqrt[4]{d}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}h^{3/2}}
\end{aligned}
$$

```
[Out] -2*b*e^(1/4)*f*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1
/2)/d^(1/4)/h^(3/2)-2*b*d^(1/4)*g*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^
(1/4)/h^(1/2))*2^(1/2)/e^(1/4)/h^(3/2)+2*b*e^(1/4)*f*p*arctan(1+e^(1/4)*2^(
```

```
1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(1/4)/h^(3/2)+2*b*d^(1/4)*g*p*a
rctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(1/4)/h^(3/2
)+b*e^(1/4)*f*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2
)*(h*x)^(1/2))*2^(1/2)/d^(1/4)/h^(3/2)-b*d^(1/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e
^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(3/2)
-b*e^(1/4)*f*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)
*(h*x)^(1/2))*2^(1/2)/d^(1/4)/h^(3/2)+b*d^(1/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e^
(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(3/2)-
2*f*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(1/2)+2*a*g*(h*x)^(1/2)/h^2-8*b*g*p*(h*
x)^(1/2)/h^2+2*b*g*ln(c*(e*x^2+d)^p)*(h*x)^(1/2)/h^2
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {2517, 2526, 2498, 327, 217, 1179, 642, 1176, 631, 210, 2505, 303}

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = -\frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h\sqrt{hx}} + \frac{2ag\sqrt{hx}}{h^2} - \frac{2\sqrt{2}b\sqrt[4]{e}fp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}} + \frac{2\sqrt{2}b\sqrt[4]{e}fp\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{\sqrt[4]{d}h^{3/2}} - \frac{2\sqrt{2}b\sqrt[4]{d}gp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{3/2}} + \frac{2\sqrt{2}b\sqrt[4]{d}gp\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{\sqrt[4]{e}h^{3/2}} + \frac{2bg\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h^2} + \frac{\sqrt{2}b\sqrt[4]{e}fp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x\right)}{\sqrt[4]{d}h^{3/2}} - \frac{\sqrt{2}b\sqrt[4]{e}fp\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x\right)}{\sqrt[4]{d}h^{3/2}} - \frac{\sqrt{2}b\sqrt[4]{d}gp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x\right)}{\sqrt[4]{e}h^{3/2}} + \frac{\sqrt{2}b\sqrt[4]{d}gp\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x\right)}{\sqrt[4]{e}h^{3/2}} - \frac{8bgp\sqrt{hx}}{h^2}$$

```
[In] Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(3/2),x]
```

```
[Out] (2*a*g*Sqrt[h*x])/h^2 - (8*b*g*p*Sqrt[h*x])/h^2 - (2*Sqrt[2]*b*e^(1/4)*f*p*
ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(3/2)
```

```
) - (2*Sqrt[2]*b*d^(1/4)*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4
)*Sqrt[h])])/(e^(1/4)*h^(3/2)) + (2*Sqrt[2]*b*e^(1/4)*f*p*ArcTan[1 + (Sqrt[
2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(3/2)) + (2*Sqrt[2]*b*
d^(1/4)*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(
1/4)*h^(3/2)) + (2*b*g*Sqrt[h*x]*Log[c*(d + e*x^2)^p])/h^2 - (2*f*(a + b*Lo
g[c*(d + e*x^2)^p]))/(h*Sqrt[h*x]) + (Sqrt[2]*b*e^(1/4)*f*p*Log[Sqrt[d]*Sqr
t[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*h^(
3/2)) - (Sqrt[2]*b*d^(1/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sq
rt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*h^(3/2)) - (Sqrt[2]*b*e^(1/4)*f*
p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*
x]])/(d^(1/4)*h^(3/2)) + (Sqrt[2]*b*d^(1/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[
e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*h^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^2}\,dx, x, \sqrt{hx}\right)}{h}\\
&= \frac{2\text{Subst}\left(\int \left(\frac{g\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h}+\frac{f\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^2}\right)\,dx, x, \sqrt{hx}\right)}{h}\\
&= \frac{(2g)\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)\,dx, x, \sqrt{hx}\right)}{h^2}\\
&\quad+\frac{(2f)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^2}\,dx, x, \sqrt{hx}\right)}{h}\\
&= \frac{2ag\sqrt{hx}}{h^2}-\frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h\sqrt{hx}}\\
&\quad+\frac{(2bg)\text{Subst}\left(\int \log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\,dx, x, \sqrt{hx}\right)}{h^2}\\
&\quad+\frac{(8befp)\text{Subst}\left(\int \frac{x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^3}\\
&= \frac{2ag\sqrt{hx}}{h^2}+\frac{2bg\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h^2}-\frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h\sqrt{hx}}\\
&\quad-\frac{(8begp)\text{Subst}\left(\int \frac{x^4}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^4}-\frac{\left(4b\sqrt{e}fp\right)\text{Subst}\left(\int \frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^3}\\
&\quad+\frac{\left(4b\sqrt{e}fp\right)\text{Subst}\left(\int \frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^3}
\end{aligned}$$

$$= \frac{2ag\sqrt{hx}}{h^2} - \frac{8bgp\sqrt{hx}}{h^2} + \frac{2bg\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h^2}$$

$$- \frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h\sqrt{hx}} + \frac{(8bdgp)\text{Subst}\left(\int \frac{1}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^2}$$

$$+ \frac{\left(\sqrt{2}b\sqrt[4]{e}fp\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}}$$

$$+ \frac{\left(\sqrt{2}b\sqrt[4]{e}fp\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}}$$

$$+ \frac{(2bfp)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{h}$$

$$+ \frac{(2bfp)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2ag\sqrt{hx}}{h^2} - \frac{8bgp\sqrt{hx}}{h^2} + \frac{2bg\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h^2} - \frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h\sqrt{hx}}$$
$$+ \frac{\sqrt{2}b\sqrt[4]{e}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}}$$
$$- \frac{\sqrt{2}b\sqrt[4]{e}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}}$$
$$+ \frac{\left(4b\sqrt{d}gp\right)\text{Subst}\left(\int\frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^3}$$
$$+ \frac{\left(4b\sqrt{d}gp\right)\text{Subst}\left(\int\frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^3}$$
$$+ \frac{\left(2\sqrt{2}b\sqrt[4]{e}fp\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}}$$
$$- \frac{\left(2\sqrt{2}b\sqrt[4]{e}fp\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}}$$

$$= \frac{2ag\sqrt{hx}}{h^2} - \frac{8bgp\sqrt{hx}}{h^2} - \frac{2\sqrt{2}b\sqrt[4]{e}fp\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}}$$
$$+ \frac{2\sqrt{2}b\sqrt[4]{e}fp\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}}$$
$$+ \frac{2bg\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h^2} - \frac{2f\left(a + b\log\left(c(d+ex^2)^p\right)\right)}{h\sqrt{hx}}$$
$$+ \frac{\sqrt{2}b\sqrt[4]{e}fp\log\left(\sqrt{d}\sqrt{h} + \sqrt{e}\sqrt{h}x - \sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}}$$
$$- \frac{\sqrt{2}b\sqrt[4]{e}fp\log\left(\sqrt{d}\sqrt{h} + \sqrt{e}\sqrt{h}x + \sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}}$$
$$- \frac{\left(\sqrt{2}b\sqrt[4]{d}gp\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}} + 2x}{-\frac{\sqrt{d}h}{\sqrt{e}} - \frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}} - x^2}dx, x, \sqrt{hx}\right)}{\sqrt[4]{e}h^{3/2}}$$
$$- \frac{\left(\sqrt{2}b\sqrt[4]{d}gp\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}} - 2x}{-\frac{\sqrt{d}h}{\sqrt{e}} + \frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}} - x^2}dx, x, \sqrt{hx}\right)}{\sqrt[4]{e}h^{3/2}}$$
$$+ \frac{\left(2b\sqrt{d}gp\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}} - \frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}} + x^2}dx, x, \sqrt{hx}\right)}{\sqrt{e}h}$$
$$+ \frac{\left(2b\sqrt{d}gp\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}} + \frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}} + x^2}dx, x, \sqrt{hx}\right)}{\sqrt{e}h}$$

$$= \frac{2ag\sqrt{hx}}{h^2} - \frac{8bgp\sqrt{hx}}{h^2} - \frac{2\sqrt{2}b\sqrt[4]{e}fp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}}$$
$$+ \frac{2\sqrt{2}b\sqrt[4]{e}fp\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}}$$
$$+ \frac{2bg\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h^2} - \frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h\sqrt{hx}}$$
$$+ \frac{\sqrt{2}b\sqrt[4]{e}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}}$$
$$- \frac{\sqrt{2}b\sqrt[4]{d}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}h^{3/2}}$$
$$- \frac{\sqrt{2}b\sqrt[4]{e}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}}$$
$$+ \frac{\sqrt{2}b\sqrt[4]{d}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}h^{3/2}}$$
$$+ \frac{\left(2\sqrt{2}b\sqrt[4]{d}gp\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{3/2}}$$
$$- \frac{\left(2\sqrt{2}b\sqrt[4]{d}gp\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{3/2}}$$

$$= \frac{2ag\sqrt{hx}}{h^2} - \frac{8bgp\sqrt{hx}}{h^2} - \frac{2\sqrt{2}b\sqrt[4]{e}fp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}}$$
$$- \frac{2\sqrt{2}b\sqrt[4]{d}gp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{3/2}}$$
$$+ \frac{2\sqrt{2}b\sqrt[4]{e}fp\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}} + \frac{2\sqrt{2}b\sqrt[4]{d}gp\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{3/2}}$$
$$+ \frac{2bg\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h^2} - \frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h\sqrt{hx}}$$
$$+ \frac{\sqrt{2}b\sqrt[4]{e}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}}$$
$$- \frac{\sqrt{2}b\sqrt[4]{d}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}h^{3/2}}$$
$$- \frac{\sqrt{2}b\sqrt[4]{e}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}}$$
$$+ \frac{\sqrt{2}b\sqrt[4]{d}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}h^{3/2}}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.55

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \frac{2x^{3/2}\left(ag\sqrt{x}-4bgp\sqrt{x}-\frac{\sqrt{2}b\sqrt[4]{d}gp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)}{\sqrt[4]{e}}+\frac{\sqrt{2}b\sqrt[4]{d}gp\,\mathrm{a}}{}\right.}{}$$

```
[In] Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(3/2),x]
```

```
[Out] (2*x^(3/2)*(a*g*Sqrt[x] - 4*b*g*p*Sqrt[x] - (Sqrt[2]*b*d^(1/4)*g*p*ArcTan[1
 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)])/e^(1/4) + (Sqrt[2]*b*d^(1/4)*g*p*Arc
Tan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)])/e^(1/4) + (2*b*e^(1/4)*f*p*(Arc
Tan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + ArcTanh[(d*e^(1/4)*Sqrt[x])/(-d)^(5/4)]
))/(-d)^(1/4) - (b*d^(1/4)*g*p*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x
] + Sqrt[e]*x])/(Sqrt[2]*e^(1/4)) + (b*d^(1/4)*g*p*Log[Sqrt[d] + Sqrt[2]*d^
(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x])/(Sqrt[2]*e^(1/4)) + b*g*Sqrt[x]*Log[c*(
d + e*x^2)^p] - (f*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[x]))/(h*x)^(3/2)
```

Maple [F]

$$\int \frac{(gx+f)\left(a+b\ln\left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{3}{2}}}dx$$

```
[In] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2),x)
[Out] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. $2(427) = 854$.

Time = 0.37 (sec) , antiderivative size = 1162, normalized size of antiderivative = 1.93

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \frac{2\left(h^2x\sqrt{-\frac{2b^2fgp^2+h^3\sqrt{-\frac{\left(b^4e^2f^4-2b^4def^2g^2+b^4d^2g^4\right)p^4}{deh^6}}}{h^3}}\log\left(-32\left(b^3e^2f\right.\right.\right.}{}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="fricas
")
[Out] 2*(h^2*x*sqrt(-(2*b^2*f*g*p^2 + h^3*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2
+ b^4*d^2*g^4)*p^4/(d*e*h^6)))/h^3)*log(-32*(b^3*e^2*f^4 - b^3*d^2*g^4)*sqr
t(h*x)*p^3 + 32*(d*e*f*h^5*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*d^2
*g^4)*p^4/(d*e*h^6)) - (b^2*d*e*f^2*g - b^2*d^2*g^3)*h^2*p^2)*sqrt(-(2*b^2*
f*g*p^2 + h^3*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*
e*h^6)))/h^3)) - h^2*x*sqrt(-(2*b^2*f*g*p^2 + h^3*sqrt(-(b^4*e^2*f^4 - 2*b^
4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)))/h^3)*log(-32*(b^3*e^2*f^4 - b^
3*d^2*g^4)*sqrt(h*x)*p^3 - 32*(d*e*f*h^5*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2
*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)) - (b^2*d*e*f^2*g - b^2*d^2*g^3)*h^2*p^2)
*sqrt(-(2*b^2*f*g*p^2 + h^3*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*d^
2*g^4)*p^4/(d*e*h^6)))/h^3)) - h^2*x*sqrt(-(2*b^2*f*g*p^2 - h^3*sqrt(-(b^4*
e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)))/h^3)*log(-32*(b^
3*e^2*f^4 - b^3*d^2*g^4)*sqrt(h*x)*p^3 + 32*(d*e*f*h^5*sqrt(-(b^4*e^2*f^4 -
 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)) + (b^2*d*e*f^2*g - b^2*d^2
*g^3)*h^2*p^2)*sqrt(-(2*b^2*f*g*p^2 - h^3*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^
2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)))/h^3)) + h^2*x*sqrt(-(2*b^2*f*g*p^2 - h
^3*sqrt(-(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)))/h^
3)*log(-32*(b^3*e^2*f^4 - b^3*d^2*g^4)*sqrt(h*x)*p^3 - 32*(d*e*f*h^5*sqrt(-
(b^4*e^2*f^4 - 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)) + (b^2*d*e*f
^2*g - b^2*d^2*g^3)*h^2*p^2)*sqrt(-(2*b^2*f*g*p^2 - h^3*sqrt(-(b^4*e^2*f^4
- 2*b^4*d*e*f^2*g^2 + b^4*d^2*g^4)*p^4/(d*e*h^6)))/h^3)) - (a*f + (4*b*g*p
- a*g)*x - (b*g*p*x - b*f*p)*log(e*x^2 + d) - (b*g*x - b*f)*log(c))*sqrt(h*
x))/(h^2*x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(3/2),x)

[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 731, normalized size of antiderivative = 1.21

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx =$$

$$-\frac{befp\left(\frac{\sqrt{2}\log\left(\sqrt{e}hx+\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{1}{4}}e^{\frac{3}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{e}hx-\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{1}{4}}e^{\frac{3}{4}}}-\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{-\sqrt{d}\sqrt{e}h}+\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}-2\sqrt{h}}{\sqrt{2}\sqrt{-\sqrt{d}\sqrt{e}h}-\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}+2\sqrt{h}}\right.}{\sqrt{-\sqrt{d}\sqrt{e}h}\sqrt{e}}\right.}{h}$$

$$+\frac{2bgx^2\log\left((ex^2+d)^pc\right)}{(hx)^{\frac{3}{2}}}+\frac{2agx^2}{(hx)^{\frac{3}{2}}}-\frac{2bf\log\left((ex^2+d)^pc\right)}{\sqrt{hx}h}$$

$$-\frac{\left(\frac{8\sqrt{hx}h^2}{e}-\frac{\left(\frac{\sqrt{2}h^4\log\left(\sqrt{e}hx+\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{3}{4}}e^{\frac{1}{4}}}-\frac{\sqrt{2}h^4\log\left(\sqrt{e}hx-\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{3}{4}}e^{\frac{1}{4}}}+\frac{\sqrt{2}h^3\log\left(-\frac{\sqrt{2}\sqrt{-\sqrt{d}\sqrt{e}h}+\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}-2\sqrt{}}{\sqrt{2}\sqrt{-\sqrt{d}\sqrt{e}h}-\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}+2\sqrt{}}\right.}{\sqrt{-\sqrt{d}\sqrt{e}h}\sqrt{d}}\right.}{e}\right.}{h^4}$$

$$-\frac{2af}{\sqrt{hx}h}$$

[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="maxima
")

[Out] -b*e*f*p*(sqrt(2)*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4)
 + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(sqrt(e)*h*x - sqrt(2)*(
d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(

```
2)*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) -
 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(
1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)) - s
qrt(2)*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/
4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^
2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e))
)/h + 2*b*g*x^2*log((e*x^2 + d)^p*c)/(h*x)^(3/2) + 2*a*g*x^2/(h*x)^(3/2) -
2*b*f*log((e*x^2 + d)^p*c)/(sqrt(h*x)*h) - (8*sqrt(h*x)*h^2/e - (sqrt(2)*h^
4*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((
d*h^2)^(3/4)*e^(1/4)) - sqrt(2)*h^4*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)
*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) + sqrt(2)*h^3*log(-
(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(
h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(
1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(d)) + sqrt(2)*h
^3*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) -
 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(
1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(d)))*d/
e)*b*e*g*p/h^4 - 2*a*f/(sqrt(h*x)*h)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 448, normalized size of antiderivative = 0.74

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx =$$

$$-\frac{2\left(\frac{bfp}{\sqrt{hx}}-\frac{\sqrt{hx}bgp}{h}\right)\log\left(eh^2x^2+dh^2\right)-\frac{2\left(bfp\log\left(h^2\right)-bf\log\left(c\right)-af\right)}{\sqrt{hx}}+\frac{2\left(bgp\log\left(h^2\right)+4\,bgp-bg\log\left(c\right)-ag\right)\sqrt{hx}}{h}-\frac{2\left(\sqrt{2}(de^3\right.}{}}{}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="giac")
```

```
[Out] -(2*(b*f*p/sqrt(h*x) - sqrt(h*x)*b*g*p/h)*log(e*h^2*x^2 + d*h^2) - 2*(b*f*p
*log(h^2) - b*f*log(c) - a*f)/sqrt(h*x) + 2*(b*g*p*log(h^2) + 4*b*g*p - b*g
*log(c) - a*g)*sqrt(h*x)/h - 2*(sqrt(2)*(d*e^3*h^2)^(1/4)*b*d*e*g*h*p + sqr
t(2)*(d*e^3*h^2)^(3/4)*b*f*p)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) +
 2*sqrt(h*x))/(d*h^2/e)^(1/4))/(d*e^2*h^2) - 2*(sqrt(2)*(d*e^3*h^2)^(1/4)*b
*d*e*g*h*p + sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*p)*arctan(-1/2*sqrt(2)*(sqrt(2)*
(d*h^2/e)^(1/4) - 2*sqrt(h*x))/(d*h^2/e)^(1/4))/(d*e^2*h^2) - (sqrt(2)*(d*e
^3*h^2)^(1/4)*b*d*e*g*h*p - sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*p)*log(h*x + sqrt
(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/(d*e^2*h^2) + (sqrt(2)*(d*e^
3*h^2)^(1/4)*b*d*e*g*h*p - sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*p)*log(h*x - sqrt(
2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/(d*e^2*h^2))/h
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \int \frac{(f+g\,x)\,\left(a+b\,\ln\left(c\,(e\,x^2+d)^p\right)\right)}{(h\,x)^{3/2}}\,dx$$

```
[In] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(3/2),x)
[Out] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(3/2), x)
```

3.608 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 588

$$\begin{aligned}
&\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \\
&-\frac{2\sqrt{2}be^{3/4}fp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}}-\frac{2\sqrt{2}b\sqrt[4]{e}gp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{5/2}} \\
&+\frac{2\sqrt{2}be^{3/4}fp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}}+\frac{2\sqrt{2}b\sqrt[4]{e}gp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{5/2}} \\
&-\frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h(hx)^{3/2}}-\frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^2\sqrt{hx}} \\
&-\frac{\sqrt{2}be^{3/4}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{5/2}} \\
&+\frac{\sqrt{2}b\sqrt[4]{e}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{5/2}} \\
&+\frac{\sqrt{2}be^{3/4}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{5/2}} \\
&-\frac{\sqrt{2}b\sqrt[4]{e}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{5/2}}
\end{aligned}$$

```
[Out] -2/3*f*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(3/2)-2/3*b*e^(3/4)*f*p*arctan(1-e^(
1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(3/4)/h^(5/2)-2*b*e^(1/
4)*g*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(1/4
```

```
)/h^(5/2)+2/3*b*e^(3/4)*f*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^
(1/2))*2^(1/2)/d^(3/4)/h^(5/2)+2*b*e^(1/4)*g*p*arctan(1+e^(1/4)*2^(1/2)*(h*
x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(1/4)/h^(5/2)-1/3*b*e^(3/4)*f*p*ln(d^(1
/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/
d^(3/4)/h^(5/2)+b*e^(1/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*
e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(1/4)/h^(5/2)+1/3*b*e^(3/4)*f*p*ln(d
^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/
2)/d^(3/4)/h^(5/2)-b*e^(1/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/
4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(1/4)/h^(5/2)-2*g*(a+b*ln(c*(e*x^
2+d)^p))/h^2/(h*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2517, 2526, 2505, 217, 1179, 642, 1176, 631, 210, 303}

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx =$$
$$-\frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h(hx)^{3/2}}-\frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^2\sqrt{hx}}$$
$$-\frac{2\sqrt{2}be^{3/4}fp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}}+\frac{2\sqrt{2}be^{3/4}fp\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{3d^{3/4}h^{5/2}}$$
$$-\frac{2\sqrt{2}b\sqrt[4]{e}gp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{5/2}}+\frac{2\sqrt{2}b\sqrt[4]{e}gp\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{\sqrt[4]{d}h^{5/2}}$$
$$-\frac{\sqrt{2}be^{3/4}fp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x\right)}{3d^{3/4}h^{5/2}}$$
$$+\frac{\sqrt{2}be^{3/4}fp\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x\right)}{3d^{3/4}h^{5/2}}$$
$$+\frac{\sqrt{2}b\sqrt[4]{e}gp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x\right)}{\sqrt[4]{d}h^{5/2}}$$
$$-\frac{\sqrt{2}b\sqrt[4]{e}gp\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x\right)}{\sqrt[4]{d}h^{5/2}}$$

```
[In] Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[2]*b*e^(3/4)*f*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*S
qrt[h])])/(3*d^(3/4)*h^(5/2)) - (2*Sqrt[2]*b*e^(1/4)*g*p*ArcTan[1 - (Sqrt[2
]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(5/2)) + (2*Sqrt[2]*b*e
```

```
^(3/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^
(3/4)*h^(5/2)) + (2*Sqrt[2]*b*e^(1/4)*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[
h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(5/2)) - (2*f*(a + b*Log[c*(d + e*x^2)
^p]))/(3*h*(h*x)^(3/2)) - (2*g*(a + b*Log[c*(d + e*x^2)^p]))/(h^2*Sqrt[h*x]
) - (Sqrt[2]*b*e^(3/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2
]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(5/2)) + (Sqrt[2]*b*e^(1/4)*g*p*
Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]
])/(d^(1/4)*h^(5/2)) + (Sqrt[2]*b*e^(3/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]
*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(5/2)) - (Sqr
t[2]*b*e^(1/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4
)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*h^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^4}\,dx, x, \sqrt{hx}\right)}{h}\\
&= \frac{2\text{Subst}\left(\int \left(\frac{f\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^4}+\frac{g\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{hx^2}\right)\,dx, x, \sqrt{hx}\right)}{h}
\end{aligned}$$

$$= \frac{(2g)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^2}\,dx, x, \sqrt{hx}\right)}{h^2} + \frac{(2f)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^4}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= -\frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h(hx)^{3/2}} - \frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^2\sqrt{hx}} + \frac{(8begp)\text{Subst}\left(\int \frac{x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^4} + \frac{(8befp)\text{Subst}\left(\int \frac{1}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{3h^3}$$

$$= -\frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h(hx)^{3/2}} - \frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^2\sqrt{hx}} + \frac{(4befp)\text{Subst}\left(\int \frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{3\sqrt{d}h^4} + \frac{(4befp)\text{Subst}\left(\int \frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{3\sqrt{d}h^4} - \frac{\left(4b\sqrt{e}gp\right)\text{Subst}\left(\int \frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^4} + \frac{\left(4b\sqrt{e}gp\right)\text{Subst}\left(\int \frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^4}$$

$$
\begin{aligned}
&= -\frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h(hx)^{3/2}} - \frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^2\sqrt{hx}} \\
&- \frac{\left(\sqrt{2}be^{3/4}fp\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{3d^{3/4}h^{5/2}} \\
&- \frac{\left(\sqrt{2}be^{3/4}fp\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{3d^{3/4}h^{5/2}} \\
&+ \frac{\left(\sqrt{2}b\sqrt[4]{e}gp\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{\sqrt[4]{d}h^{5/2}} \\
&+ \frac{\left(\sqrt{2}b\sqrt[4]{e}gp\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{\sqrt[4]{d}h^{5/2}} \\
&+ \frac{\left(2b\sqrt{e}fp\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{3\sqrt{d}h^2} \\
&+ \frac{\left(2b\sqrt{e}fp\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{3\sqrt{d}h^2} \\
&+ \frac{(2bgp)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{h^2} \\
&+ \frac{(2bgp)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{h^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h(hx)^{3/2}} - \frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^2\sqrt{hx}} \\
&\quad - \frac{\sqrt{2}be^{3/4}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{5/2}} \\
&\quad + \frac{\sqrt{2}b\sqrt[4]{e}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{5/2}} \\
&\quad + \frac{\sqrt{2}be^{3/4}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{5/2}} \\
&\quad - \frac{\sqrt{2}b\sqrt[4]{e}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{5/2}} \\
&\quad + \frac{\left(2\sqrt{2}be^{3/4}fp\right)\operatorname{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}} \\
&\quad - \frac{\left(2\sqrt{2}be^{3/4}fp\right)\operatorname{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}} \\
&\quad + \frac{\left(2\sqrt{2}b\sqrt[4]{e}gp\right)\operatorname{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{5/2}} \\
&\quad - \frac{\left(2\sqrt{2}b\sqrt[4]{e}gp\right)\operatorname{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{5/2}}
\end{aligned}
$$

$$=-\frac{2\sqrt{2}be^{3/4}fp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}}-\frac{2\sqrt{2}b\sqrt[4]{e}gp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{5/2}}$$
$$+\frac{2\sqrt{2}be^{3/4}fp\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}}+\frac{2\sqrt{2}b\sqrt[4]{e}gp\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{5/2}}$$
$$-\frac{2f(a+b\log(c(d+ex^2)^p))}{3h(hx)^{3/2}}-\frac{2g(a+b\log(c(d+ex^2)^p))}{h^2\sqrt{hx}}$$
$$-\frac{\sqrt{2}be^{3/4}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{5/2}}$$
$$+\frac{\sqrt{2}b\sqrt[4]{e}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{5/2}}$$
$$+\frac{\sqrt{2}be^{3/4}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{5/2}}$$
$$-\frac{\sqrt{2}b\sqrt[4]{e}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{5/2}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.46

$$\int\frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx=\frac{2x^{5/2}\left(\frac{2b\sqrt[4]{e}gp\left(\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{-d}}\right)+\operatorname{arctanh}\left(\frac{d\sqrt[4]{e}\sqrt{x}}{(-d)^{5/4}}\right)\right)}{\sqrt[4]{-d}}-\frac{be^{3/4}fp\left(2\arctan\left(\right.\right.}{}\right.}{}$$

```
[In] Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(5/2),x]
```

```
[Out] (2*x^(5/2)*((2*b*e^(1/4)*g*p*(ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + ArcTan
h[(d*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (b*e^(3/4)*f*p*(2*ArcTan[1
 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[
x])/d^(1/4)] + Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] -
 Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/(3*Sqrt[2]*d^
(3/4)) - (f*(a + b*Log[c*(d + e*x^2)^p]))/(3*x^(3/2)) - (g*(a + b*Log[c*(d
+ e*x^2)^p]))/Sqrt[x]))/(h*x)^(5/2)
```

Maple [F]

$$\int \frac{(gx+f)\left(a+b\ln\left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{5}{2}}}dx$$

```
[In] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(5/2),x)
[Out] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1236 vs. $2(406) = 812$.

Time = 0.40 (sec) , antiderivative size = 1236, normalized size of antiderivative = 2.10

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="fricas
")
[Out] -2/3*(h^3*x^2*sqrt(-(6*b^2*e*f*g*p^2 + d*h^5*sqrt(-(b^4*e^3*f^4 - 18*b^4*d*
e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))*log(-32*(b^3*e^3*
f^4 - 81*b^3*d^2*e*g^4)*sqrt(h*x)*p^3 + 32*(3*d^3*g*h^8*sqrt(-(b^4*e^3*f^4
- 18*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)) + (b^2*d*e^2*f^3
 - 9*b^2*d^2*e*f*g^2)*h^3*p^2)*sqrt(-(6*b^2*e*f*g*p^2 + d*h^5*sqrt(-(b^4*e^
3*f^4 - 18*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5)))
 - h^3*x^2*sqrt(-(6*b^2*e*f*g*p^2 + d*h^5*sqrt(-(b^4*e^3*f^4 - 18*b^4*d*e^2
*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))*log(-32*(b^3*e^3*f^4
 - 81*b^3*d^2*e*g^4)*sqrt(h*x)*p^3 - 32*(3*d^3*g*h^8*sqrt(-(b^4*e^3*f^4 - 1
8*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)) + (b^2*d*e^2*f^3 -
9*b^2*d^2*e*f*g^2)*h^3*p^2)*sqrt(-(6*b^2*e*f*g*p^2 + d*h^5*sqrt(-(b^4*e^3*f
^4 - 18*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))) -
h^3*x^2*sqrt(-(6*b^2*e*f*g*p^2 - d*h^5*sqrt(-(b^4*e^3*f^4 - 18*b^4*d*e^2*f^
2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))*log(-32*(b^3*e^3*f^4 -
81*b^3*d^2*e*g^4)*sqrt(h*x)*p^3 + 32*(3*d^3*g*h^8*sqrt(-(b^4*e^3*f^4 - 18*b
^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)) - (b^2*d*e^2*f^3 - 9*b
^2*d^2*e*f*g^2)*h^3*p^2)*sqrt(-(6*b^2*e*f*g*p^2 - d*h^5*sqrt(-(b^4*e^3*f^4
- 18*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))) + h^3
*x^2*sqrt(-(6*b^2*e*f*g*p^2 - d*h^5*sqrt(-(b^4*e^3*f^4 - 18*b^4*d*e^2*f^2*g
^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))*log(-32*(b^3*e^3*f^4 - 81*
b^3*d^2*e*g^4)*sqrt(h*x)*p^3 - 32*(3*d^3*g*h^8*sqrt(-(b^4*e^3*f^4 - 18*b^4*
d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)) - (b^2*d*e^2*f^3 - 9*b^2*
d^2*e*f*g^2)*h^3*p^2)*sqrt(-(6*b^2*e*f*g*p^2 - d*h^5*sqrt(-(b^4*e^3*f^4 - 1
8*b^4*d*e^2*f^2*g^2 + 81*b^4*d^2*e*g^4)*p^4/(d^3*h^10)))/(d*h^5))) + (3*a*g
*x + a*f + (3*b*g*p*x + b*f*p)*log(e*x^2 + d) + (3*b*g*x + b*f)*log(c))*sqr
t(h*x))/(h^3*x^2)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(5/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.20

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx =$$

$$-\frac{begp\left(\frac{\sqrt{2}\log\left(\sqrt{e}hx+\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{1}{4}}e^{\frac{3}{4}}} - \frac{\sqrt{2}\log\left(\sqrt{e}hx-\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{1}{4}}e^{\frac{3}{4}}} - \frac{\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{-\sqrt{d}\sqrt{e}h}+\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}-2\sqrt{h}\cdots}{\sqrt{2}\sqrt{-\sqrt{d}\sqrt{e}h}-\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}+2\sqrt{h}\cdots}\cdots\right.}{\sqrt{-\sqrt{d}\sqrt{e}h}\sqrt{e}}\cdots\right.}{h^2}$$

$$-\frac{2\,bgx^2\log\left((ex^2+d)^pc\right)}{(hx)^{\frac{5}{2}}}$$

$$+\frac{\left(\frac{\sqrt{2}h^2\log\left(\sqrt{e}hx+\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{3}{4}}e^{\frac{1}{4}}} - \frac{\sqrt{2}h^2\log\left(\sqrt{e}hx-\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{3}{4}}e^{\frac{1}{4}}} + \frac{\sqrt{2}h\log\left(-\frac{\sqrt{2}\sqrt{-\sqrt{d}\sqrt{e}h}+\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}-2\sqrt{h}\cdots}{\sqrt{2}\sqrt{-\sqrt{d}\sqrt{e}h}-\sqrt{2}(dh^2)^{\frac{1}{4}}e^{\frac{1}{4}}+2\sqrt{h}\cdots}\cdots\right.}{\sqrt{-\sqrt{d}\sqrt{e}h}\sqrt{d}}\cdots\right.}{3\,h^3}$$

$$-\frac{2\,agx^2}{(hx)^{\frac{5}{2}}} - \frac{2\,bf\log\left((ex^2+d)^pc\right)}{3\,(hx)^{\frac{3}{2}}\,h} - \frac{2\,af}{3\,(hx)^{\frac{3}{2}}\,h}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="maxima
")
```

```
[Out] -b*e*g*p*(sqrt(2)*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4)
 + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(sqrt(e)*h*x - sqrt(2)*(
d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(
2)*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) -
 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(
1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)) - s
qrt(2)*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/
```

```
4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^
2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e))
)/h^2 - 2*b*g*x^2*log((e*x^2 + d)^p*c)/(h*x)^(5/2) + 1/3*(sqrt(2)*h^2*log(s
qrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^
(3/4)*e^(1/4)) - sqrt(2)*h^2*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h
*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) + sqrt(2)*h*log(-(sqrt(2)*
sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt
(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*
sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(d)) + sqrt(2)*h*log(-(sq
rt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x
)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4
) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(d)))*b*e*f*p/h^3 -
 2*a*g*x^2/(h*x)^(5/2) - 2/3*b*f*log((e*x^2 + d)^p*c)/((h*x)^(3/2)*h) - 2/3
*a*f/((h*x)^(3/2)*h)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 471, normalized size of antiderivative = 0.80

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx =$$

$$-\frac{\frac{2\left(3\,bgh^2px+bfh^2p\right)\log\left(eh^2x^2+dh^2\right)}{\sqrt{hx}hx}-\frac{2\left(3\,bgh^2px\log\left(h^2\right)+bfh^2p\log\left(h^2\right)-3\,bgh^2x\log\left(c\right)-3\,agh^2x-bfh^2\log\left(c\right)-afh^2\right)}{\sqrt{hx}hx}-\frac{2\left(\sqrt{2}\left(de^3h^2\right)^{\frac{1}{4}}\right.}{}}{}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(2*(3*b*g*h^2*p*x + b*f*h^2*p)*log(e*h^2*x^2 + d*h^2)/(sqrt(h*x)*h*x)
- 2*(3*b*g*h^2*p*x*log(h^2) + b*f*h^2*p*log(h^2) - 3*b*g*h^2*x*log(c) - 3*a
*g*h^2*x - b*f*h^2*log(c) - a*f*h^2)/(sqrt(h*x)*h*x) - 2*(sqrt(2)*(d*e^3*h^
2)^(1/4)*b*e^2*f*h*p + 3*sqrt(2)*(d*e^3*h^2)^(3/4)*b*g*p)*arctan(1/2*sqrt(2
)*(sqrt(2)*(d*h^2/e)^(1/4) + 2*sqrt(h*x))/(d*h^2/e)^(1/4))/(d*e^2*h) - 2*(s
qrt(2)*(d*e^3*h^2)^(1/4)*b*e^2*f*h*p + 3*sqrt(2)*(d*e^3*h^2)^(3/4)*b*g*p)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) - 2*sqrt(h*x))/(d*h^2/e)^(1/4))
/(d*e^2*h) - (sqrt(2)*(d*e^3*h^2)^(1/4)*b*e^2*f*h*p - 3*sqrt(2)*(d*e^3*h^2)
^(3/4)*b*g*p)*log(h*x + sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/
(d*e^2*h) + (sqrt(2)*(d*e^3*h^2)^(1/4)*b*e^2*f*h*p - 3*sqrt(2)*(d*e^3*h^2)^
(3/4)*b*g*p)*log(h*x - sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/(
d*e^2*h))/h^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \int \frac{(f+g\,x)\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{5/2}}\,dx$$

```
[In] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(5/2),x)
[Out] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(5/2), x)
```

3.609 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{7/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 620

$$
\begin{aligned}
\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx &= -\frac{8befp}{5dh^3\sqrt{hx}} \\
&+ \frac{2\sqrt{2}be^{5/4}fp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{7/2}} - \frac{2\sqrt{2}be^{3/4}gp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}} \\
&- \frac{2\sqrt{2}be^{5/4}fp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{7/2}} + \frac{2\sqrt{2}be^{3/4}gp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}} \\
&- \frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h(hx)^{5/2}} - \frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2(hx)^{3/2}} \\
&- \frac{\sqrt{2}be^{5/4}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{7/2}} \\
&- \frac{\sqrt{2}be^{3/4}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{7/2}} \\
&+ \frac{\sqrt{2}be^{5/4}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{7/2}} \\
&+ \frac{\sqrt{2}be^{3/4}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{7/2}}
\end{aligned}
$$

```
[Out] -2/5*f*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(5/2)-2/3*g*(a+b*ln(c*(e*x^2+d)^p))/
h^2/(h*x)^(3/2)+2/5*b*e^(5/4)*f*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1
/4)/h^(1/2))*2^(1/2)/d^(5/4)/h^(7/2)-2/3*b*e^(3/4)*g*p*arctan(1-e^(1/4)*2^(
1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(3/4)/h^(7/2)-2/5*b*e^(5/4)*f*p
```

```
*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(5/4)/h^(7
/2)+2/3*b*e^(3/4)*g*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))
*2^(1/2)/d^(3/4)/h^(7/2)-1/5*b*e^(5/4)*f*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(
1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(5/4)/h^(7/2)-1/3*b*e^(
3/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)
^(1/2))*2^(1/2)/d^(3/4)/h^(7/2)+1/5*b*e^(5/4)*f*p*ln(d^(1/2)*h^(1/2)+x*e^(1
/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(5/4)/h^(7/2)+1/
3*b*e^(3/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2
)*(h*x)^(1/2))*2^(1/2)/d^(3/4)/h^(7/2)-8/5*b*e*f*p/d/h^3/(h*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {2517, 2526, 2505, 331, 303, 1176, 631, 210, 1179, 642, 217}

$$\begin{aligned}
&\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \\
&-\frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h(hx)^{5/2}}-\frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2(hx)^{3/2}} \\
&+\frac{2\sqrt{2}be^{5/4}fp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{7/2}}-\frac{2\sqrt{2}be^{5/4}fp\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{5d^{5/4}h^{7/2}} \\
&-\frac{2\sqrt{2}be^{3/4}gp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}}+\frac{2\sqrt{2}be^{3/4}gp\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{3d^{3/4}h^{7/2}} \\
&-\frac{\sqrt{2}be^{5/4}fp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x\right)}{5d^{5/4}h^{7/2}} \\
&+\frac{\sqrt{2}be^{5/4}fp\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x\right)}{5d^{5/4}h^{7/2}} \\
&-\frac{\sqrt{2}be^{3/4}gp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x\right)}{3d^{3/4}h^{7/2}} \\
&+\frac{\sqrt{2}be^{3/4}gp\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x\right)}{3d^{3/4}h^{7/2}}-\frac{8befp}{5dh^3\sqrt{hx}}
\end{aligned}$$

```
[In] Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(7/2),x]
```

```
[Out] (-8*b*e*f*p)/(5*d*h^3*Sqrt[h*x]) + (2*Sqrt[2]*b*e^(5/4)*f*p*ArcTan[1 - (Sqr
t[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(7/2)) - (2*Sqrt[2
]*b*e^(3/4)*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/
(3*d^(3/4)*h^(7/2)) - (2*Sqrt[2]*b*e^(5/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*
Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(7/2)) + (2*Sqrt[2]*b*e^(3/4)*g
```

```
*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4)*h^
(7/2)) - (2*f*(a + b*Log[c*(d + e*x^2)^p]))/(5*h*(h*x)^(5/2)) - (2*g*(a + b
*Log[c*(d + e*x^2)^p]))/(3*h^2*(h*x)^(3/2)) - (Sqrt[2]*b*e^(5/4)*f*p*Log[Sq
rt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*
d^(5/4)*h^(7/2)) - (Sqrt[2]*b*e^(3/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqr
t[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(7/2)) + (Sqrt[2]
*b*e^(5/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^
(1/4)*Sqrt[h*x]])/(5*d^(5/4)*h^(7/2)) + (Sqrt[2]*b*e^(3/4)*g*p*Log[Sqrt[d]*
Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4
)*h^(7/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^6}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{f\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^6} + \frac{g\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{hx^4}\right)\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{(2g)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^4}\,dx, x, \sqrt{hx}\right)}{h^2} + \frac{(2f)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^6}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= -\frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h(hx)^{5/2}} - \frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2(hx)^{3/2}} + \frac{(8begp)\text{Subst}\left(\int \frac{1}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{3h^4} + \frac{(8befp)\text{Subst}\left(\int \frac{1}{x^2\left(d+\frac{ex^4}{h^2}\right)}\,dx, x, \sqrt{hx}\right)}{5h^3}$$

$$= -\frac{8befp}{5dh^3\sqrt{hx}} - \frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h(hx)^{5/2}} - \frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2(hx)^{3/2}} - \frac{(8be^2fp)\,\text{Subst}\left(\int \frac{x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{5dh^5} + \frac{(4begp)\text{Subst}\left(\int \frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{3\sqrt{d}h^5} + \frac{(4begp)\text{Subst}\left(\int \frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{3\sqrt{d}h^5}$$

$$= -\frac{8befp}{5dh^3\sqrt{hx}} - \frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h(hx)^{5/2}} - \frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2(hx)^{3/2}}$$
$$+ \frac{\left(4be^{3/2}fp\right)\text{Subst}\left(\int \frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{5dh^5}$$
$$- \frac{\left(4be^{3/2}fp\right)\text{Subst}\left(\int \frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{5dh^5}$$
$$- \frac{\left(\sqrt{2}be^{3/4}gp\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{3d^{3/4}h^{7/2}}$$
$$- \frac{\left(\sqrt{2}be^{3/4}gp\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{3d^{3/4}h^{7/2}}$$
$$+ \frac{\left(2b\sqrt{e}gp\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{3\sqrt{d}h^3}$$
$$+ \frac{\left(2b\sqrt{e}gp\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{3\sqrt{d}h^3}$$

$$= -\frac{8befp}{5dh^3\sqrt{hx}} - \frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h(hx)^{5/2}} - \frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2(hx)^{3/2}}$$
$$- \frac{\sqrt{2}be^{3/4}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{7/2}}$$
$$+ \frac{\sqrt{2}be^{3/4}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{7/2}}$$
$$- \frac{\left(\sqrt{2}be^{5/4}fp\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx,x,\sqrt{hx}\right)}{5d^{5/4}h^{7/2}}$$
$$- \frac{\left(\sqrt{2}be^{5/4}fp\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx,x,\sqrt{hx}\right)}{5d^{5/4}h^{7/2}}$$
$$+ \frac{\left(2\sqrt{2}be^{3/4}gp\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}}$$
$$- \frac{\left(2\sqrt{2}be^{3/4}gp\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}}$$
$$- \frac{(2befp)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx,x,\sqrt{hx}\right)}{5dh^3}$$
$$- \frac{(2befp)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx,x,\sqrt{hx}\right)}{5dh^3}$$

$$
\begin{aligned}
&= -\frac{8befp}{5dh^3\sqrt{hx}} - \frac{2\sqrt{2}be^{3/4}gp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}} \\
&+ \frac{2\sqrt{2}be^{3/4}gp\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}} \\
&- \frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h(hx)^{5/2}} - \frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2(hx)^{3/2}} \\
&- \frac{\sqrt{2}be^{5/4}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{7/2}} \\
&- \frac{\sqrt{2}be^{3/4}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{7/2}} \\
&+ \frac{\sqrt{2}be^{5/4}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{7/2}} \\
&+ \frac{\sqrt{2}be^{3/4}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{7/2}} \\
&- \frac{\left(2\sqrt{2}be^{5/4}fp\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{7/2}} \\
&+ \frac{\left(2\sqrt{2}be^{5/4}fp\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{7/2}}
\end{aligned}
$$

$$=-\frac{8befp}{5dh^3\sqrt{hx}}+\frac{2\sqrt{2}be^{5/4}fp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{7/2}}$$
$$-\frac{2\sqrt{2}be^{3/4}gp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}}$$
$$-\frac{2\sqrt{2}be^{5/4}fp\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{7/2}}+\frac{2\sqrt{2}be^{3/4}gp\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}}$$
$$-\frac{2f(a+b\log(c(d+ex^2)^p))}{5h(hx)^{5/2}}-\frac{2g(a+b\log(c(d+ex^2)^p))}{3h^2(hx)^{3/2}}$$
$$-\frac{\sqrt{2}be^{5/4}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{7/2}}$$
$$-\frac{\sqrt{2}be^{3/4}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{7/2}}$$
$$+\frac{\sqrt{2}be^{5/4}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{7/2}}$$
$$+\frac{\sqrt{2}be^{3/4}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{7/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.38

$$\int\frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx=\frac{x\left(-24befpx^2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\frac{ex^2}{d}\right)-5\sqrt{2}b\sqrt[4]{d}e\right.}{}$$

```
[In] Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(7/2),x]
```

```
[Out] (x*(-24*b*e*f*p*x^2*Hypergeometric2F1[-1/4, 1, 3/4, -((e*x^2)/d)] - 5*Sqrt[
2]*b*d^(1/4)*e^(3/4)*g*p*x^(5/2)*(2*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^
(1/4)] - 2*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Log[Sqrt[d] - Sq
rt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] - Log[Sqrt[d] + Sqrt[2]*d^(1/4)*
e^(1/4)*Sqrt[x] + Sqrt[e]*x]) - 6*d*f*(a + b*Log[c*(d + e*x^2)^p]) - 10*d*g
*x*(a + b*Log[c*(d + e*x^2)^p])))/(15*d*(h*x)^(7/2))
```

Maple [F]

$$\int \frac{(gx+f)\left(a+b\ln\left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{7}{2}}}dx$$

[In] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(7/2),x)

[Out] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(7/2),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1348 vs. $2(424) = 848$.

Time = 0.38 (sec) , antiderivative size = 1348, normalized size of antiderivative = 2.17

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="fricas
")

[Out] 2/15*(d*h^4*x^3*sqrt((d^2*h^7*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2
 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5*h^14)) + 30*b^2*e^2*f*g*p^2)/(d^2*h^7))*lo
g(-32*(81*b^3*e^4*f^4 - 625*b^3*d^2*e^2*g^4)*sqrt(h*x)*p^3 + 32*(3*d^4*f*h^
11*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4
/(d^5*h^14)) - 5*(9*b^2*d^2*e^2*f^2*g - 25*b^2*d^3*e*g^3)*h^4*p^2)*sqrt((d^
2*h^7*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*
p^4/(d^5*h^14)) + 30*b^2*e^2*f*g*p^2)/(d^2*h^7))) - d*h^4*x^3*sqrt((d^2*h^7
*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4/(
d^5*h^14)) + 30*b^2*e^2*f*g*p^2)/(d^2*h^7))*log(-32*(81*b^3*e^4*f^4 - 625*b
^3*d^2*e^2*g^4)*sqrt(h*x)*p^3 - 32*(3*d^4*f*h^11*sqrt(-(81*b^4*e^5*f^4 - 45
0*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5*h^14)) - 5*(9*b^2*d^2*e
^2*f^2*g - 25*b^2*d^3*e*g^3)*h^4*p^2)*sqrt((d^2*h^7*sqrt(-(81*b^4*e^5*f^4 -
 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5*h^14)) + 30*b^2*e^2*
f*g*p^2)/(d^2*h^7))) - d*h^4*x^3*sqrt(-(d^2*h^7*sqrt(-(81*b^4*e^5*f^4 - 450
*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5*h^14)) - 30*b^2*e^2*f*g*
p^2)/(d^2*h^7))*log(-32*(81*b^3*e^4*f^4 - 625*b^3*d^2*e^2*g^4)*sqrt(h*x)*p^
3 + 32*(3*d^4*f*h^11*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^
4*d^2*e^3*g^4)*p^4/(d^5*h^14)) + 5*(9*b^2*d^2*e^2*f^2*g - 25*b^2*d^3*e*g^3)
*h^4*p^2)*sqrt(-(d^2*h^7*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 62
5*b^4*d^2*e^3*g^4)*p^4/(d^5*h^14)) - 30*b^2*e^2*f*g*p^2)/(d^2*h^7))) + d*h^
4*x^3*sqrt(-(d^2*h^7*sqrt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^
4*d^2*e^3*g^4)*p^4/(d^5*h^14)) - 30*b^2*e^2*f*g*p^2)/(d^2*h^7))*log(-32*(81
*b^3*e^4*f^4 - 625*b^3*d^2*e^2*g^4)*sqrt(h*x)*p^3 - 32*(3*d^4*f*h^11*sqrt(-
(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5*h^1
4)) + 5*(9*b^2*d^2*e^2*f^2*g - 25*b^2*d^3*e*g^3)*h^4*p^2)*sqrt(-(d^2*h^7*sq
```

```
rt(-(81*b^4*e^5*f^4 - 450*b^4*d*e^4*f^2*g^2 + 625*b^4*d^2*e^3*g^4)*p^4/(d^5
*h^14)) - 30*b^2*e^2*f*g*p^2)/(d^2*h^7))) - (12*b*e*f*p*x^2 + 5*a*d*g*x + 3
*a*d*f + (5*b*d*g*p*x + 3*b*d*f*p)*log(e*x^2 + d) + (5*b*d*g*x + 3*b*d*f)*l
og(c))*sqrt(h*x))/(d*h^4*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(7/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.17

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \frac{befp\left(\frac{e\left(\frac{\sqrt{2}\log\left(\sqrt{e}hx+\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{\left(dh^2\right)^{\frac{1}{4}}e^{\frac{3}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{e}hx-\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}\right.}{\left(dh^2\right)^{\frac{1}{4}}e^{\frac{3}{4}}}\right.}{}\right.}{} - \frac{2\,bgx^2\log\left(\left(ex^2+d\right)^pc\right)}{3\,(hx)^{\frac{7}{2}}} + \frac{\left(\frac{\sqrt{2}h^2\log\left(\sqrt{e}hx+\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{3}{4}}e^{\frac{1}{4}}}-\frac{\sqrt{2}h^2\log\left(\sqrt{e}hx-\sqrt{2}(dh^2)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{(dh^2)^{\frac{3}{4}}e^{\frac{1}{4}}}+\frac{\sqrt{2}h\log\left(-\frac{\sqrt{2}\sqrt{-\sqrt{d}\sqrt{e}h}+\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\frac{1}{4}}-2\sqrt{}}{\sqrt{2}\sqrt{-\sqrt{d}\sqrt{e}h}-\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\frac{1}{4}}+2\sqrt{}}\right.}{\sqrt{-\sqrt{d}\sqrt{e}h}\sqrt{d}}\right.}{3\,h^4} - \frac{2\,agx^2}{3\,(hx)^{\frac{7}{2}}} - \frac{2\,bf\log\left(\left(ex^2+d\right)^pc\right)}{5\,(hx)^{\frac{5}{2}}\,h} - \frac{2\,af}{5\,(hx)^{\frac{5}{2}}\,h}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="maxima
")
```

```
[Out] 1/5*b*e*f*p*(e*(sqrt(2)*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e
^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(sqrt(e)*h*x - sqr
t(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) -
 sqrt(2)*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(
1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*
h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e
)) - sqrt(2)*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)
*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)
*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sq
rt(e)))/d - 8/(sqrt(h*x)*d))/h^3 - 2/3*b*g*x^2*log((e*x^2 + d)^p*c)/(h*x)^(
7/2) + 1/3*(sqrt(2)*h^2*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e
^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) - sqrt(2)*h^2*log(sqrt(e)*h*x -
 sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4
)) + sqrt(2)*h*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/
4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(
2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*
sqrt(d)) + sqrt(2)*h*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^
2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) +
 sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(
e)*h)*sqrt(d)))*b*e*g*p/h^4 - 2/3*a*g*x^2/(h*x)^(7/2) - 2/5*b*f*log((e*x^2
+ d)^p*c)/((h*x)^(5/2)*h) - 2/5*a*f/((h*x)^(5/2)*h)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 494, normalized size of antiderivative = 0.80

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \frac{2\left(5\sqrt{2}\left(de^3h^2\right)^{\frac{1}{4}}bdeghp-3\sqrt{2}\left(de^3h^2\right)^{\frac{3}{4}}bfp\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{dh^2}{e}\right)^{\frac{1}{4}}+2\sqrt{hx}\right)}{2\left(\frac{dh^2}{e}\right)^{\frac{1}{4}}}\right)}{d^2eh} +$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(2*(5*sqrt(2)*(d*e^3*h^2)^(1/4)*b*d*e*g*h*p - 3*sqrt(2)*(d*e^3*h^2)^(3
/4)*b*f*p)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) + 2*sqrt(h*x))/(d*h^
2/e)^(1/4))/(d^2*e*h) + 2*(5*sqrt(2)*(d*e^3*h^2)^(1/4)*b*d*e*g*h*p - 3*sqrt
(2)*(d*e^3*h^2)^(3/4)*b*f*p)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) -
 2*sqrt(h*x))/(d*h^2/e)^(1/4))/(d^2*e*h) + (5*sqrt(2)*(d*e^3*h^2)^(1/4)*b*d
*e*g*h*p + 3*sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*p)*log(h*x + sqrt(2)*(d*h^2/e)^(
1/4)*sqrt(h*x) + sqrt(d*h^2/e))/(d^2*e*h) - (5*sqrt(2)*(d*e^3*h^2)^(1/4)*b*
d*e*g*h*p + 3*sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*p)*log(h*x - sqrt(2)*(d*h^2/e)^
(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/(d^2*e*h) - 2*(5*b*g*h^3*p*x + 3*b*f*h^3*p
)*log(e*h^2*x^2 + d*h^2)/(sqrt(h*x)*h^2*x^2) - 2*(12*b*e*f*h^3*p*x^2 - 5*b*
```

```
d*g*h^3*p*x*log(h^2) - 3*b*d*f*h^3*p*log(h^2) + 5*b*d*g*h^3*x*log(c) + 5*a*
d*g*h^3*x + 3*b*d*f*h^3*log(c) + 3*a*d*f*h^3)/(sqrt(h*x)*d*h^2*x^2))/h^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx=\int \frac{(f+g\,x)\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{7/2}} \,dx$$

```
[In] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(7/2),x)
[Out] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(7/2), x)
```

3.610 $\int \frac{(f+gx)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{9/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 641

$$
\begin{aligned}
\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = & -\frac{8befp}{21dh^3(hx)^{3/2}}-\frac{8begp}{5dh^4\sqrt{hx}} \\
& +\frac{2\sqrt{2}be^{7/4}fp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{7d^{7/4}h^{9/2}}+\frac{2\sqrt{2}be^{5/4}gp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{9/2}} \\
& -\frac{2\sqrt{2}be^{7/4}fp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{7d^{7/4}h^{9/2}}-\frac{2\sqrt{2}be^{5/4}gp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{9/2}} \\
& -\frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{7h(hx)^{7/2}}-\frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h^2(hx)^{5/2}} \\
& +\frac{\sqrt{2}be^{7/4}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{7d^{7/4}h^{9/2}} \\
& -\frac{\sqrt{2}be^{5/4}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{9/2}} \\
& -\frac{\sqrt{2}be^{7/4}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{7d^{7/4}h^{9/2}} \\
& +\frac{\sqrt{2}be^{5/4}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{9/2}}
\end{aligned}
$$

```
[Out] -8/21*b*e*f*p/d/h^3/(h*x)^(3/2)-2/7*f*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(7/2)
-2/5*g*(a+b*ln(c*(e*x^2+d)^p))/h^2/(h*x)^(5/2)+2/7*b*e^(7/4)*f*p*arctan(1-e
^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(7/4)/h^(9/2)+2/5*b*e
^(5/4)*g*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^
```

```
(5/4)/h^(9/2)-2/7*b*e^(7/4)*f*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4
)/h^(1/2))*2^(1/2)/d^(7/4)/h^(9/2)-2/5*b*e^(5/4)*g*p*arctan(1+e^(1/4)*2^(1/
2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(5/4)/h^(9/2)+1/7*b*e^(7/4)*f*p*l
n(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^
(1/2)/d^(7/4)/h^(9/2)-1/5*b*e^(5/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2
)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(5/4)/h^(9/2)-1/7*b*e^(7/4
)*f*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1
/2))*2^(1/2)/d^(7/4)/h^(9/2)+1/5*b*e^(5/4)*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)
*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(5/4)/h^(9/2)-8/5*b
*e*g*p/d/h^4/(h*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {2517, 2526, 2505, 331, 217, 1179, 642, 1176, 631, 210, 303}

$$\begin{aligned}
&\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = \\
&-\frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{7h(hx)^{7/2}}-\frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h^2(hx)^{5/2}} \\
&+\frac{2\sqrt{2}be^{7/4}fp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{7d^{7/4}h^{9/2}}-\frac{2\sqrt{2}be^{7/4}fp\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{7d^{7/4}h^{9/2}} \\
&+\frac{2\sqrt{2}be^{5/4}gp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{9/2}}-\frac{2\sqrt{2}be^{5/4}gp\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{5d^{5/4}h^{9/2}} \\
&+\frac{\sqrt{2}be^{7/4}fp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x\right)}{7d^{7/4}h^{9/2}} \\
&-\frac{\sqrt{2}be^{7/4}fp\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x\right)}{7d^{7/4}h^{9/2}} \\
&-\frac{\sqrt{2}be^{5/4}gp\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x\right)}{5d^{5/4}h^{9/2}} \\
&+\frac{\sqrt{2}be^{5/4}gp\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x\right)}{5d^{5/4}h^{9/2}}-\frac{8befp}{21dh^3(hx)^{3/2}}-\frac{8begp}{5dh^4\sqrt{hx}}
\end{aligned}$$

```
[In] Int[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(9/2),x]
```

```
[Out] (-8*b*e*f*p)/(21*d*h^3*(h*x)^(3/2)) - (8*b*e*g*p)/(5*d*h^4*Sqrt[h*x]) + (2*
Sqrt[2]*b*e^(7/4)*f*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[
h])])/(7*d^(7/4)*h^(9/2)) + (2*Sqrt[2]*b*e^(5/4)*g*p*ArcTan[1 - (Sqrt[2]*e^
(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(9/2)) - (2*Sqrt[2]*b*e^(
```

```
7/4)*f*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(7*d^(7
/4)*h^(9/2)) - (2*Sqrt[2]*b*e^(5/4)*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*
x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(9/2)) - (2*f*(a + b*Log[c*(d + e*x^2)
^p]))/(7*h*(h*x)^(7/2)) - (2*g*(a + b*Log[c*(d + e*x^2)^p]))/(5*h^2*(h*x)^(
5/2)) + (Sqrt[2]*b*e^(7/4)*f*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sq
rt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(7*d^(7/4)*h^(9/2)) - (Sqrt[2]*b*e^(5/4)*
g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[
h*x]])/(5*d^(5/4)*h^(9/2)) - (Sqrt[2]*b*e^(7/4)*f*p*Log[Sqrt[d]*Sqrt[h] + S
qrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(7*d^(7/4)*h^(9/2))
+ (Sqrt[2]*b*e^(5/4)*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*
d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*d^(5/4)*h^(9/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^8}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{f\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^8} + \frac{g\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{hx^6}\right)\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{(2g)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^6}\,dx, x, \sqrt{hx}\right)}{h^2} + \frac{(2f)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^8}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= -\frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{7h(hx)^{7/2}} - \frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h^2(hx)^{5/2}} + \frac{(8begp)\text{Subst}\left(\int \frac{1}{x^2\left(d+\frac{ex^4}{h^2}\right)}\,dx, x, \sqrt{hx}\right)}{5h^4} + \frac{(8befp)\text{Subst}\left(\int \frac{1}{x^4\left(d+\frac{ex^4}{h^2}\right)}\,dx, x, \sqrt{hx}\right)}{7h^3}$$

$$= -\frac{8befp}{21dh^3(hx)^{3/2}} - \frac{8begp}{5dh^4\sqrt{hx}} - \frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{7h(hx)^{7/2}} - \frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h^2(hx)^{5/2}} - \frac{(8be^2gp)\,\text{Subst}\left(\int \frac{x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{5dh^6} - \frac{(8be^2fp)\,\text{Subst}\left(\int \frac{1}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{7dh^5}$$

$$= -\frac{8befp}{21dh^3(hx)^{3/2}} - \frac{8begp}{5dh^4\sqrt{hx}} - \frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{7h(hx)^{7/2}} - \frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h^2(hx)^{5/2}} - \frac{(4be^2fp)\,\text{Subst}\left(\int \frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{7d^{3/2}h^6} - \frac{(4be^2fp)\,\text{Subst}\left(\int \frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{7d^{3/2}h^6} + \frac{(4be^{3/2}gp)\,\text{Subst}\left(\int \frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{5dh^6} - \frac{(4be^{3/2}gp)\,\text{Subst}\left(\int \frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{5dh^6}$$

$$= -\frac{8befp}{21dh^3(hx)^{3/2}} - \frac{8begp}{5dh^4\sqrt{hx}} - \frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{7h(hx)^{7/2}} - \frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h^2(hx)^{5/2}}$$
$$+ \frac{\left(\sqrt{2}be^{7/4}fp\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}dx, x, \sqrt{hx}\right)}{7d^{7/4}h^{9/2}}$$
$$+ \frac{\left(\sqrt{2}be^{7/4}fp\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}dx, x, \sqrt{hx}\right)}{7d^{7/4}h^{9/2}}$$
$$- \frac{\left(\sqrt{2}be^{5/4}gp\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}dx, x, \sqrt{hx}\right)}{5d^{5/4}h^{9/2}}$$
$$- \frac{\left(\sqrt{2}be^{5/4}gp\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}dx, x, \sqrt{hx}\right)}{5d^{5/4}h^{9/2}}$$
$$- \frac{\left(2be^{3/2}fp\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}dx, x, \sqrt{hx}\right)}{7d^{3/2}h^4}$$
$$- \frac{\left(2be^{3/2}fp\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}dx, x, \sqrt{hx}\right)}{7d^{3/2}h^4}$$
$$- \frac{(2begp)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}dx, x, \sqrt{hx}\right)}{5dh^4}$$
$$- \frac{(2begp)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}dx, x, \sqrt{hx}\right)}{5dh^4}$$

$$
\begin{aligned}
&= -\frac{8befp}{21dh^3(hx)^{3/2}} - \frac{8begp}{5dh^4\sqrt{hx}} - \frac{2f\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{7h(hx)^{7/2}} \\
&\quad - \frac{2g\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h^2(hx)^{5/2}} \\
&\quad + \frac{\sqrt{2}be^{7/4}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{7d^{7/4}h^{9/2}} \\
&\quad - \frac{\sqrt{2}be^{5/4}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{9/2}} \\
&\quad - \frac{\sqrt{2}be^{7/4}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{7d^{7/4}h^{9/2}} \\
&\quad + \frac{\sqrt{2}be^{5/4}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{9/2}} \\
&\quad - \frac{\left(2\sqrt{2}be^{7/4}fp\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{7d^{7/4}h^{9/2}} \\
&\quad + \frac{\left(2\sqrt{2}be^{7/4}fp\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{7d^{7/4}h^{9/2}} \\
&\quad - \frac{\left(2\sqrt{2}be^{5/4}gp\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{9/2}} \\
&\quad + \frac{\left(2\sqrt{2}be^{5/4}gp\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{9/2}}
\end{aligned}
$$

$$= -\frac{8befp}{21dh^3(hx)^{3/2}} - \frac{8begp}{5dh^4\sqrt{hx}} + \frac{2\sqrt{2}be^{7/4}fp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{7d^{7/4}h^{9/2}}$$
$$+ \frac{2\sqrt{2}be^{5/4}gp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{9/2}}$$
$$- \frac{2\sqrt{2}be^{7/4}fp\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{7d^{7/4}h^{9/2}} - \frac{2\sqrt{2}be^{5/4}gp\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{9/2}}$$
$$- \frac{2f(a+b\log(c(d+ex^2)^p))}{7h(hx)^{7/2}} - \frac{2g(a+b\log(c(d+ex^2)^p))}{5h^2(hx)^{5/2}}$$
$$+ \frac{\sqrt{2}be^{7/4}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{7d^{7/4}h^{9/2}}$$
$$- \frac{\sqrt{2}be^{5/4}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{9/2}}$$
$$- \frac{\sqrt{2}be^{7/4}fp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{7d^{7/4}h^{9/2}}$$
$$+ \frac{\sqrt{2}be^{5/4}gp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{9/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.16

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx =$$
$$-\frac{2\sqrt{hx}\left(20befpx^2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},1,\frac{1}{4},-\frac{ex^2}{d}\right)+84begpx^3\,\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\frac{ex^2}{d}\right)\right.}{105dh^5x^4}$$

```
[In] Integrate[((f + g*x)*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(9/2),x]
```

```
[Out] (-2*Sqrt[h*x]*(20*b*e*f*p*x^2*Hypergeometric2F1[-3/4, 1, 1/4, -((e*x^2)/d)]
 + 84*b*e*g*p*x^3*Hypergeometric2F1[-1/4, 1, 3/4, -((e*x^2)/d)] + 3*d*(5*f
+ 7*g*x)*(a + b*Log[c*(d + e*x^2)^p])))/(105*d*h^5*x^4)
```

Maple [F]

$$\int \frac{(gx+f)\left(a+b\ln\left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{9}{2}}}dx$$

[In] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(9/2),x)

[Out] int((g*x+f)*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(9/2),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1369 vs. $2(441) = 882$.

Time = 0.41 (sec) , antiderivative size = 1369, normalized size of antiderivative = 2.14

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = \text{Too large to display}$$

[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="fricas
")

[Out] 2/105*(3*d*h^5*x^4*sqrt(-(d^3*h^9*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f
^2*g^2 + 2401*b^4*d^2*e^5*g^4)*p^4/(d^7*h^18)) + 70*b^2*e^3*f*g*p^2)/(d^3*h
^9))*log(-32*(625*b^3*e^6*f^4 - 2401*b^3*d^2*e^4*g^4)*sqrt(h*x)*p^3 + 32*(7
*d^6*g*h^14*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*
e^5*g^4)*p^4/(d^7*h^18)) + 5*(25*b^2*d^2*e^4*f^3 - 49*b^2*d^3*e^3*f*g^2)*h^
5*p^2)*sqrt(-(d^3*h^9*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 240
1*b^4*d^2*e^5*g^4)*p^4/(d^7*h^18)) + 70*b^2*e^3*f*g*p^2)/(d^3*h^9))) - 3*d*
h^5*x^4*sqrt(-(d^3*h^9*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 24
01*b^4*d^2*e^5*g^4)*p^4/(d^7*h^18)) + 70*b^2*e^3*f*g*p^2)/(d^3*h^9))*log(-3
2*(625*b^3*e^6*f^4 - 2401*b^3*d^2*e^4*g^4)*sqrt(h*x)*p^3 - 32*(7*d^6*g*h^14
*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e^5*g^4)*p^
4/(d^7*h^18)) + 5*(25*b^2*d^2*e^4*f^3 - 49*b^2*d^3*e^3*f*g^2)*h^5*p^2)*sqrt
(-(d^3*h^9*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e
^5*g^4)*p^4/(d^7*h^18)) + 70*b^2*e^3*f*g*p^2)/(d^3*h^9))) - 3*d*h^5*x^4*sqr
t((d^3*h^9*sqrt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e
^5*g^4)*p^4/(d^7*h^18)) - 70*b^2*e^3*f*g*p^2)/(d^3*h^9))*log(-32*(625*b^3*e
^6*f^4 - 2401*b^3*d^2*e^4*g^4)*sqrt(h*x)*p^3 + 32*(7*d^6*g*h^14*sqrt(-(625*
b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e^5*g^4)*p^4/(d^7*h^18)
) - 5*(25*b^2*d^2*e^4*f^3 - 49*b^2*d^3*e^3*f*g^2)*h^5*p^2)*sqrt((d^3*h^9*sq
rt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e^5*g^4)*p^4/(
d^7*h^18)) - 70*b^2*e^3*f*g*p^2)/(d^3*h^9))) + 3*d*h^5*x^4*sqrt((d^3*h^9*sq
rt(-(625*b^4*e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e^5*g^4)*p^4/(
d^7*h^18)) - 70*b^2*e^3*f*g*p^2)/(d^3*h^9))*log(-32*(625*b^3*e^6*f^4 - 2401
*b^3*d^2*e^4*g^4)*sqrt(h*x)*p^3 - 32*(7*d^6*g*h^14*sqrt(-(625*b^4*e^7*f^4 -
 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e^5*g^4)*p^4/(d^7*h^18)) - 5*(25*b^2

```
*d^2*e^4*f^3 - 49*b^2*d^3*e^3*f*g^2)*h^5*p^2)*sqrt((d^3*h^9*sqrt(-(625*b^4*
e^7*f^4 - 2450*b^4*d*e^6*f^2*g^2 + 2401*b^4*d^2*e^5*g^4)*p^4/(d^7*h^18)) -
70*b^2*e^3*f*g*p^2)/(d^3*h^9))) - (84*b*e*g*p*x^3 + 20*b*e*f*p*x^2 + 21*a*d
*g*x + 15*a*d*f + 3*(7*b*d*g*p*x + 5*b*d*f*p)*log(e*x^2 + d) + 3*(7*b*d*g*x
 + 5*b*d*f)*log(c))*sqrt(h*x))/(d*h^5*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(9/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 739, normalized size of antiderivative = 1.15

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx =$$

$$-\frac{befp\left(\frac{3\left(\frac{\sqrt{2}e^{\frac{3}{4}}\log\left(\sqrt{e}hx+\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{\left(dh^2\right)^{\frac{3}{4}}}-\frac{\sqrt{2}e^{\frac{3}{4}}\log\left(\sqrt{e}hx-\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{\left(dh^2\right)^{\frac{3}{4}}}+\frac{\sqrt{2}e\log\left(-\frac{\sqrt{2}\sqrt{-\sqrt{d}\sqrt{e}h}+\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\frac{1}{4}}-2\sqrt{hx}\cdots}{\sqrt{2}\sqrt{-\sqrt{d}\sqrt{e}h}-\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\frac{1}{4}}+2\sqrt{hx}\cdots}\right)}{\sqrt{-\sqrt{d}\sqrt{e}h}\sqrt{d}h}\cdots}{d}\cdots\right)}{21h^3}$$

$$+\frac{begp\left(\frac{e\left(\frac{\sqrt{2}\log\left(\sqrt{e}hx+\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{\left(dh^2\right)^{\frac{1}{4}}e^{\frac{3}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{e}hx-\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}\sqrt{hx}e^{\frac{1}{4}}+\sqrt{d}h\right)}{\left(dh^2\right)^{\frac{1}{4}}e^{\frac{3}{4}}}-\frac{\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{-\sqrt{d}\sqrt{e}h}+\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\frac{1}{4}}-2\sqrt{hx}\sqrt{e}}{\sqrt{2}\sqrt{-\sqrt{d}\sqrt{e}h}-\sqrt{2}\left(dh^2\right)^{\frac{1}{4}}e^{\frac{1}{4}}+2\sqrt{hx}\sqrt{e}}\right)}{\sqrt{-\sqrt{d}\sqrt{e}h}\sqrt{e}}-\cdots\right)}{d}\cdots\right)}{5h^4}$$

$$-\frac{2bgx^2\log\left(\left(ex^2+d\right)^pc\right)}{5\left(hx\right)^{\frac{9}{2}}}-\frac{2agx^2}{5\left(hx\right)^{\frac{9}{2}}}-\frac{2bf\log\left(\left(ex^2+d\right)^pc\right)}{7\left(hx\right)^{\frac{7}{2}}h}-\frac{2af}{7\left(hx\right)^{\frac{7}{2}}h}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="maxima
")
```

```
[Out] -1/21*b*e*f*p*(3*(sqrt(2)*e^(3/4)*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*s
qrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) - sqrt(2)*e^(3/4)*log(sqrt(e)*h
*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) + s
qrt(2)*e*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(
1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*
h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(d
)*h) + sqrt(2)*e*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(
1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqr
t(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h
)*sqrt(d)*h))/d + 8/((h*x)^(3/2)*d))/h^3 + 1/5*b*e*g*p*(e*(sqrt(2)*log(sqrt
(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/
4)*e^(3/4)) - sqrt(2)*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(
1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(-(sqrt(2)*sqrt(-sqr
t(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqr
t(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)
*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)) - sqrt(2)*log(-(sqrt(2)*sqrt(
-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/
(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(
h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)))/d - 8/(sqrt(h*x)*d))/h^4
 - 2/5*b*g*x^2*log((e*x^2 + d)^p*c)/(h*x)^(9/2) - 2/5*a*g*x^2/(h*x)^(9/2) -
 2/7*b*f*log((e*x^2 + d)^p*c)/((h*x)^(7/2)*h) - 2/7*a*f/((h*x)^(7/2)*h)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 511, normalized size of antiderivative = 0.80

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = -\frac{\frac{6\left(5\sqrt{2}(de^3h^2)^{\frac{1}{4}}be^2fhp+7\sqrt{2}(de^3h^2)^{\frac{3}{4}}bgp\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{dh^2}{e}\right)^{\frac{1}{4}}+2\sqrt{hx}\right)}{2\left(\frac{dh^2}{e}\right)^{\frac{1}{4}}}\right)}{d^2eh}+\frac{6\left(5\sqrt{2}(de^3h^2)^{\frac{1}{4}}be^2fhp+7\sqrt{2}(de^3h^2)^{\frac{3}{4}}bgp\right)\arctan\left(-\frac{\sqrt{}}{}\right.}{d^2eh}}{}$$

```
[In] integrate((g*x+f)*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(6*(5*sqrt(2)*(d*e^3*h^2)^(1/4)*b*e^2*f*h*p + 7*sqrt(2)*(d*e^3*h^2)^
(3/4)*b*g*p)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) + 2*sqrt(h*x))/(d*
h^2/e)^(1/4))/(d^2*e*h) + 6*(5*sqrt(2)*(d*e^3*h^2)^(1/4)*b*e^2*f*h*p + 7*sq
rt(2)*(d*e^3*h^2)^(3/4)*b*g*p)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4)
 - 2*sqrt(h*x))/(d*h^2/e)^(1/4))/(d^2*e*h) + 3*(5*sqrt(2)*(d*e^3*h^2)^(1/4)
```

```
*b*e^2*f*h*p - 7*sqrt(2)*(d*e^3*h^2)^(3/4)*b*g*p)*log(h*x + sqrt(2)*(d*h^2/
e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/(d^2*e*h) - 3*(5*sqrt(2)*(d*e^3*h^2)^(1
/4)*b*e^2*f*h*p - 7*sqrt(2)*(d*e^3*h^2)^(3/4)*b*g*p)*log(h*x - sqrt(2)*(d*h
^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/(d^2*e*h) + 6*(7*b*g*h^4*p*x + 5*b*f
*h^4*p)*log(e*h^2*x^2 + d*h^2)/(sqrt(h*x)*h^3*x^3) + 2*(84*b*e*g*h^4*p*x^3
+ 20*b*e*f*h^4*p*x^2 - 21*b*d*g*h^4*p*x*log(h^2) - 15*b*d*f*h^4*p*log(h^2)
+ 21*b*d*g*h^4*x*log(c) + 21*a*d*g*h^4*x + 15*b*d*f*h^4*log(c) + 15*a*d*f*h
^4)/(sqrt(h*x)*d*h^3*x^3))/h^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx=\int \frac{(f+g\,x)\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{9/2}}\,dx$$

```
[In] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(9/2),x)
[Out] int(((f + g*x)*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(9/2), x)
```

3.611 $\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{hx}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 1002

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx$$
$$= \frac{2af^2\sqrt{hx}}{h}-\frac{8bf^2p\sqrt{hx}}{h}+\frac{8bdg^2p\sqrt{hx}}{5eh}-\frac{16bfgp(hx)^{3/2}}{9h^2}$$
$$-\frac{8bg^2p(hx)^{5/2}}{25h^3}-\frac{2\sqrt{2}b\sqrt[4]{d}f^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}\sqrt{h}}$$
$$-\frac{4\sqrt{2}bd^{3/4}fgp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}}+\frac{2\sqrt{2}bd^{5/4}g^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5e^{5/4}\sqrt{h}}$$
$$+\frac{2\sqrt{2}b\sqrt[4]{d}f^2p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}\sqrt{h}}+\frac{4\sqrt{2}bd^{3/4}fgp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}}$$
$$-\frac{2\sqrt{2}bd^{5/4}g^2p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5e^{5/4}\sqrt{h}}+\frac{2bf^2\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h}$$
$$+\frac{4fg(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2}+\frac{2g^2(hx)^{5/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h^3}$$
$$-\frac{\sqrt{2}b\sqrt[4]{d}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}\sqrt{h}}$$
$$+\frac{2\sqrt{2}bd^{3/4}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}\sqrt{h}}$$
$$+\frac{\sqrt{2}bd^{5/4}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5e^{5/4}\sqrt{h}}$$
$$+\frac{\sqrt{2}b\sqrt[4]{d}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}\sqrt{h}}$$
$$-\frac{2\sqrt{2}bd^{3/4}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}\sqrt{h}}$$
$$-\frac{\sqrt{2}bd^{5/4}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5e^{5/4}\sqrt{h}}$$

```
[Out] -16/9*b*f*g*p*(h*x)^(3/2)/h^2-8/25*b*g^2*p*(h*x)^(5/2)/h^3+4/3*f*g*(h*x)^(3
/2)*(a+b*ln(c*(e*x^2+d)^p))/h^2+2/5*g^2*(h*x)^(5/2)*(a+b*ln(c*(e*x^2+d)^p))
/h^3-2*b*d^(1/4)*f^2*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2)
)*2^(1/2)/e^(1/4)/h^(1/2)-4/3*b*d^(3/4)*f*g*p*arctan(1-e^(1/4)*2^(1/2)*(h*x
```

```
)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(3/4)/h^(1/2)+2/5*b*d^(5/4)*g^2*p*arctan
(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(5/4)/h^(1/2)+2*b
*d^(1/4)*f^2*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2
)/e^(1/4)/h^(1/2)+4/3*b*d^(3/4)*f*g*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/
d^(1/4)/h^(1/2))*2^(1/2)/e^(3/4)/h^(1/2)-2/5*b*d^(5/4)*g^2*p*arctan(1+e^(1/
4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(5/4)/h^(1/2)-b*d^(1/4)*f
^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/
2))*2^(1/2)/e^(1/4)/h^(1/2)+2/3*b*d^(3/4)*f*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2
)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(3/4)/h^(1/2)+1/5*
b*d^(5/4)*g^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2
)*(h*x)^(1/2))*2^(1/2)/e^(5/4)/h^(1/2)+b*d^(1/4)*f^2*p*ln(d^(1/2)*h^(1/2)+x
*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(1/
2)-2/3*b*d^(3/4)*f*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)
*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(3/4)/h^(1/2)-1/5*b*d^(5/4)*g^2*p*ln(d^(1/2
)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^
(5/4)/h^(1/2)+2*a*f^2*(h*x)^(1/2)/h-8*b*f^2*p*(h*x)^(1/2)/h+8/5*b*d*g^2*p*(
h*x)^(1/2)/e/h+2*b*f^2*ln(c*(e*x^2+d)^p)*(h*x)^(1/2)/h
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 1002, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules

used = {2517, 2521, 2498, 327, 217, 1179, 642, 1176, 631, 210, 2505, 303, 308}

$$\begin{aligned}
&\int \frac{(f+gx)^2 \left(a+b\log \left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx \\
&= -\frac{8bg^2p(hx)^{5/2}}{25h^3}+\frac{2g^2\left(a+b\log \left(c(ex^2+d)^p\right)\right)(hx)^{5/2}}{5h^3} \\
&\quad -\frac{16bfgp(hx)^{3/2}}{9h^2}+\frac{4fg\left(a+b\log \left(c(ex^2+d)^p\right)\right)(hx)^{3/2}}{3h^2}-\frac{8bf^2p\sqrt{hx}}{h} \\
&\quad +\frac{8bdg^2p\sqrt{hx}}{5eh}+\frac{2bf^2\log \left(c(ex^2+d)^p\right)\sqrt{hx}}{h}+\frac{2af^2\sqrt{hx}}{h} \\
&\quad -\frac{2\sqrt{2}b\sqrt[4]{d}f^2p\arctan \left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}\sqrt{h}}+\frac{2\sqrt{2}bd^{5/4}g^2p\arctan \left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5e^{5/4}\sqrt{h}} \\
&\quad -\frac{4\sqrt{2}bd^{3/4}fgp\arctan \left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}}+\frac{2\sqrt{2}b\sqrt[4]{d}f^2p\arctan \left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{\sqrt[4]{e}\sqrt{h}} \\
&\quad -\frac{2\sqrt{2}bd^{5/4}g^2p\arctan \left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{5e^{5/4}\sqrt{h}}+\frac{4\sqrt{2}bd^{3/4}fgp\arctan \left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{3e^{3/4}\sqrt{h}} \\
&\quad -\frac{\sqrt{2}b\sqrt[4]{d}f^2p\log \left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}\sqrt{h}} \\
&\quad +\frac{\sqrt{2}bd^{5/4}g^2p\log \left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5e^{5/4}\sqrt{h}} \\
&\quad +\frac{2\sqrt{2}bd^{3/4}fgp\log \left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}\sqrt{h}} \\
&\quad +\frac{\sqrt{2}b\sqrt[4]{d}f^2p\log \left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}\sqrt{h}} \\
&\quad -\frac{\sqrt{2}bd^{5/4}g^2p\log \left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5e^{5/4}\sqrt{h}} \\
&\quad -\frac{2\sqrt{2}bd^{3/4}fgp\log \left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}\sqrt{h}}
\end{aligned}$$

```
[In] Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h*x],x]
```

```
[Out] (2*a*f^2*Sqrt[h*x])/h - (8*b*f^2*p*Sqrt[h*x])/h + (8*b*d*g^2*p*Sqrt[h*x])/(
5*e*h) - (16*b*f*g*p*(h*x)^(3/2))/(9*h^2) - (8*b*g^2*p*(h*x)^(5/2))/(25*h^3
) - (2*Sqrt[2]*b*d^(1/4)*f^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1
/4)*Sqrt[h])])/(e^(1/4)*Sqrt[h]) - (4*Sqrt[2]*b*d^(3/4)*f*g*p*ArcTan[1 - (S
qrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3/4)*Sqrt[h]) + (2*Sqrt
```

```
[2]*b*d^(5/4)*g^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h]
)])/(5*e^(5/4)*Sqrt[h]) + (2*Sqrt[2]*b*d^(1/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e^
(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(1/4)*Sqrt[h]) + (4*Sqrt[2]*b*d^(3/
4)*f*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3
/4)*Sqrt[h]) - (2*Sqrt[2]*b*d^(5/4)*g^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[
h*x])/(d^(1/4)*Sqrt[h])])/(5*e^(5/4)*Sqrt[h]) + (2*b*f^2*Sqrt[h*x]*Log[c*(d
 + e*x^2)^p])/h + (4*f*g*(h*x)^(3/2)*(a + b*Log[c*(d + e*x^2)^p]))/(3*h^2)
+ (2*g^2*(h*x)^(5/2)*(a + b*Log[c*(d + e*x^2)^p]))/(5*h^3) - (Sqrt[2]*b*d^(
1/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[h*x]])/(e^(1/4)*Sqrt[h]) + (2*Sqrt[2]*b*d^(3/4)*f*g*p*Log[Sqrt[d]*Sq
rt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^(3/4)*
Sqrt[h]) + (Sqrt[2]*b*d^(5/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x
 - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*e^(5/4)*Sqrt[h]) + (Sqrt[2]*b*d^(
1/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[h*x]])/(e^(1/4)*Sqrt[h]) - (2*Sqrt[2]*b*d^(3/4)*f*g*p*Log[Sqrt[d]*Sq
rt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^(3/4)*
Sqrt[h]) - (Sqrt[2]*b*d^(5/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x
 + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*e^(5/4)*Sqrt[h])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
```

```
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2521

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \left(f+\frac{gx^2}{h}\right)^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx,x,\sqrt{hx}\right)}{h}$$

$$= \frac{2\text{Subst}\left(\int \left(f^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)+\frac{2fgx^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h}+\frac{g^2x^4\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h^2}\right)dx,\right.}{h}$$

$$= \frac{(2g^2)\,\text{Subst}\left(\int x^4\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx,x,\sqrt{hx}\right)}{h^3} + \frac{(4fg)\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx,x,\sqrt{hx}\right)}{h^2} + \frac{(2f^2)\,\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx,x,\sqrt{hx}\right)}{h}$$

$$= \frac{2af^2\sqrt{hx}}{h}+\frac{4fg(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2} + \frac{2g^2(hx)^{5/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h^3} + \frac{(2bf^2)\,\text{Subst}\left(\int \log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)dx,x,\sqrt{hx}\right)}{h} - \frac{(8beg^2p)\,\text{Subst}\left(\int \frac{x^8}{d+\frac{ex^4}{h^2}}\,dx,x,\sqrt{hx}\right)}{5h^5} - \frac{(16befgp)\text{Subst}\left(\int \frac{x^6}{d+\frac{ex^4}{h^2}}\,dx,x,\sqrt{hx}\right)}{3h^4}$$

$$
\begin{aligned}
&= \frac{2af^2\sqrt{hx}}{h} - \frac{16bfgp(hx)^{3/2}}{9h^2} + \frac{2bf^2\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h} \\
&+ \frac{4fg(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2} + \frac{2g^2(hx)^{5/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h^3} \\
&- \frac{(8beg^2p)\,\mathrm{Subst}\left(\int\left(-\frac{dh^4}{e^2}+\frac{h^2x^4}{e}+\frac{d^2h^4}{e^2\left(d+\frac{ex^4}{h^2}\right)}\right)dx, x, \sqrt{hx}\right)}{5h^5} \\
&- \frac{(8bef^2p)\,\mathrm{Subst}\left(\int\frac{x^4}{d+\frac{ex^4}{h^2}}dx, x, \sqrt{hx}\right)}{h^3} + \frac{(16bdfgp)\mathrm{Subst}\left(\int\frac{x^2}{d+\frac{ex^4}{h^2}}dx, x, \sqrt{hx}\right)}{3h^2} \\
&= \frac{2af^2\sqrt{hx}}{h} - \frac{8bf^2p\sqrt{hx}}{h} + \frac{8bdg^2p\sqrt{hx}}{5eh} - \frac{16bfgp(hx)^{3/2}}{9h^2} - \frac{8bg^2p(hx)^{5/2}}{25h^3} \\
&+ \frac{2bf^2\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h} + \frac{4fg(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2} \\
&+ \frac{2g^2(hx)^{5/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h^3} - \frac{(8bdfgp)\mathrm{Subst}\left(\int\frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}dx, x, \sqrt{hx}\right)}{3\sqrt{e}h^2} \\
&+ \frac{(8bdfgp)\mathrm{Subst}\left(\int\frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}dx, x, \sqrt{hx}\right)}{3\sqrt{e}h^2} \\
&+ \frac{(8bdf^2p)\,\mathrm{Subst}\left(\int\frac{1}{d+\frac{ex^4}{h^2}}dx, x, \sqrt{hx}\right)}{h} - \frac{(8bd^2g^2p)\,\mathrm{Subst}\left(\int\frac{1}{d+\frac{ex^4}{h^2}}dx, x, \sqrt{hx}\right)}{5eh}
\end{aligned}
$$

$$= \frac{2af^2\sqrt{hx}}{h} - \frac{8bf^2p\sqrt{hx}}{h} + \frac{8bdg^2p\sqrt{hx}}{5eh} - \frac{16bfgp(hx)^{3/2}}{9h^2} - \frac{8bg^2p(hx)^{5/2}}{25h^3} + \frac{2bf^2\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h} + \frac{4fg(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2} + \frac{2g^2(hx)^{5/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h^3} + \frac{(4bdfgp)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}} - \frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}} + x^2}\,dx, x, \sqrt{hx}\right)}{3e} + \frac{(4bdfgp)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}} + \frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}} + x^2}\,dx, x, \sqrt{hx}\right)}{3e} + \frac{\left(4b\sqrt{d}f^2p\right)\text{Subst}\left(\int \frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^2} + \frac{\left(4b\sqrt{d}f^2p\right)\text{Subst}\left(\int \frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^2} - \frac{\left(4bd^{3/2}g^2p\right)\text{Subst}\left(\int \frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{5eh^2} - \frac{\left(4bd^{3/2}g^2p\right)\text{Subst}\left(\int \frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{5eh^2} + \frac{\left(2\sqrt{2}bd^{3/4}fgp\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}h}{\sqrt{e}} - \frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}} - x^2}\,dx, x, \sqrt{hx}\right)}{3e^{3/4}\sqrt{h}} + \frac{\left(2\sqrt{2}bd^{3/4}fgp\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}h}{\sqrt{e}} + \frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}} - x^2}\,dx, x, \sqrt{hx}\right)}{3e^{3/4}\sqrt{h}}$$

$$= \frac{2af^2\sqrt{hx}}{h} - \frac{8bf^2p\sqrt{hx}}{h} + \frac{8bdg^2p\sqrt{hx}}{5eh} - \frac{16bfgp(hx)^{3/2}}{9h^2}$$
$$- \frac{8bg^2p(hx)^{5/2}}{25h^3} + \frac{2bf^2\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h}$$
$$+ \frac{4fg(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2} + \frac{2g^2(hx)^{5/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h^3}$$
$$+ \frac{2\sqrt{2}bd^{3/4}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}\sqrt{h}}$$
$$- \frac{2\sqrt{2}bd^{3/4}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}\sqrt{h}}$$
$$+ \frac{\left(2b\sqrt{d}f^2p\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{\sqrt{e}}$$
$$+ \frac{\left(2b\sqrt{d}f^2p\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{\sqrt{e}}$$
$$- \frac{\left(2bd^{3/2}g^2p\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{5e^{3/2}}$$
$$- \frac{\left(2bd^{3/2}g^2p\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{5e^{3/2}}$$
$$- \frac{\left(\sqrt{2}b\sqrt[4]{d}f^2p\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{\sqrt[4]{e}\sqrt{h}}$$
$$- \frac{\left(\sqrt{2}b\sqrt[4]{d}f^2p\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{\sqrt[4]{e}\sqrt{h}}$$
$$+ \frac{\left(4\sqrt{2}bd^{3/4}fgp\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}}$$
$$- \frac{\left(4\sqrt{2}bd^{3/4}fgp\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}}$$

$$= \frac{2af^2\sqrt{hx}}{h} - \frac{8bf^2p\sqrt{hx}}{h} + \frac{8bdg^2p\sqrt{hx}}{5eh} - \frac{16bfgp(hx)^{3/2}}{9h^2} - \frac{8bg^2p(hx)^{5/2}}{25h^3} - \frac{4\sqrt{2}bd^{3/4}fgp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}} + \frac{4\sqrt{2}bd^{3/4}fgp\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}} + \frac{2bf^2\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{h} + \frac{4fg(hx)^{3/2}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2} + \frac{2g^2(hx)^{5/2}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h^3} - \frac{\sqrt{2}b\sqrt[4]{d}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}\sqrt{h}} + \frac{2\sqrt{2}bd^{3/4}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}\sqrt{h}} + \frac{\sqrt{2}bd^{5/4}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5e^{5/4}\sqrt{h}} + \frac{\sqrt{2}b\sqrt[4]{d}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}\sqrt{h}} - \frac{2\sqrt{2}bd^{3/4}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}\sqrt{h}} - \frac{\sqrt{2}bd^{5/4}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5e^{5/4}\sqrt{h}} + \frac{\left(2\sqrt{2}b\sqrt[4]{d}f^2p\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}\sqrt{h}} - \frac{\left(2\sqrt{2}b\sqrt[4]{d}f^2p\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}\sqrt{h}} - \frac{\left(2\sqrt{2}bd^{5/4}g^2p\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5e^{5/4}\sqrt{h}} + \frac{\left(2\sqrt{2}bd^{5/4}g^2p\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5e^{5/4}\sqrt{h}}$$

$$= \frac{2af^2\sqrt{hx}}{h} - \frac{8bf^2p\sqrt{hx}}{h} + \frac{8bdg^2p\sqrt{hx}}{5eh} - \frac{16bfgp(hx)^{3/2}}{9h^2}$$
$$- \frac{8bg^2p(hx)^{5/2}}{25h^3} - \frac{2\sqrt{2}b\sqrt[4]{d}f^2p\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}\sqrt{h}}$$
$$- \frac{4\sqrt{2}bd^{3/4}fgp\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}} + \frac{2\sqrt{2}bd^{5/4}g^2p\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5e^{5/4}\sqrt{h}}$$
$$+ \frac{2\sqrt{2}b\sqrt[4]{d}f^2p\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}\sqrt{h}} + \frac{4\sqrt{2}bd^{3/4}fgp\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}\sqrt{h}}$$
$$- \frac{2\sqrt{2}bd^{5/4}g^2p\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5e^{5/4}\sqrt{h}} + \frac{2bf^2\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h}$$
$$+ \frac{4fg(hx)^{3/2}\left(a + b\log\left(c(d+ex^2)^p\right)\right)}{3h^2} + \frac{2g^2(hx)^{5/2}\left(a + b\log\left(c(d+ex^2)^p\right)\right)}{5h^3}$$
$$- \frac{\sqrt{2}b\sqrt[4]{d}f^2p\log\left(\sqrt{d}\sqrt{h} + \sqrt{e}\sqrt{h}x - \sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}\sqrt{h}}$$
$$+ \frac{2\sqrt{2}bd^{3/4}fgp\log\left(\sqrt{d}\sqrt{h} + \sqrt{e}\sqrt{h}x - \sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}\sqrt{h}}$$
$$+ \frac{\sqrt{2}bd^{5/4}g^2p\log\left(\sqrt{d}\sqrt{h} + \sqrt{e}\sqrt{h}x - \sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5e^{5/4}\sqrt{h}}$$
$$+ \frac{\sqrt{2}b\sqrt[4]{d}f^2p\log\left(\sqrt{d}\sqrt{h} + \sqrt{e}\sqrt{h}x + \sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}\sqrt{h}}$$
$$- \frac{2\sqrt{2}bd^{3/4}fgp\log\left(\sqrt{d}\sqrt{h} + \sqrt{e}\sqrt{h}x + \sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}\sqrt{h}}$$
$$- \frac{\sqrt{2}bd^{5/4}g^2p\log\left(\sqrt{d}\sqrt{h} + \sqrt{e}\sqrt{h}x + \sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5e^{5/4}\sqrt{h}}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 628, normalized size of antiderivative = 0.63

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx$$

$$= \frac{2\sqrt{x}\left(af^2\sqrt{x}-4bf^2p\sqrt{x}+\frac{2}{3}afgx^{3/2}-\frac{8}{9}bfgpx^{3/2}-\frac{\sqrt{2}b\sqrt[4]{d}f^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)}{\sqrt[4]{e}}+\frac{\sqrt{2}b\sqrt[4]{d}f^2p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)}{\sqrt[4]{e}}\right.}{}$$

[In] Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/Sqrt[h*x],x]

[Out] (2*Sqrt[x]*(a*f^2*Sqrt[x] - 4*b*f^2*p*Sqrt[x] + (2*a*f*g*x^(3/2))/3 - (8*b*f*g*p*x^(3/2))/9 - (Sqrt[2]*b*d^(1/4)*f^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)])/e^(1/4) + (Sqrt[2]*b*d^(1/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)])/e^(1/4) - (4*b*(-d)^(3/4)*f*g*p*ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)])/(3*e^(3/4)) + (4*b*(-d)^(3/4)*f*g*p*ArcTanh[(e^(1/4)*Sqrt[x])/(-d)^(1/4)])/(3*e^(3/4)) - (b*d^(1/4)*f^2*p*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x])/(Sqrt[2]*e^(1/4)) + (b*d^(1/4)*f^2*p*Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x])/(Sqrt[2]*e^(1/4)) - (b*g^2*p*(-40*d*e^(1/4)*Sqrt[x] + 8*e^(5/4)*x^(5/2) - 10*Sqrt[2]*d^(5/4)*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + 10*Sqrt[2]*d^(5/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 5*Sqrt[2]*d^(5/4)*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] + 5*Sqrt[2]*d^(5/4)*Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/(50*e^(5/4)) + b*f^2*Sqrt[x]*Log[c*(d + e*x^2)^p] + (2*b*f*g*x^(3/2)*Log[c*(d + e*x^2)^p])/3 + (g^2*x^(5/2)*(a + b*Log[c*(d + e*x^2)^p]))/5))/Sqrt[h*x]

Maple [F]

$$\int \frac{(gx+f)^2 \left(a+b\ln\left(c(e\,x^2+d)^p\right)\right)}{\sqrt{hx}}dx$$

[In] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2),x)

[Out] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2178 vs. 2(708) = 1416.

Time = 0.43 (sec) , antiderivative size = 2178, normalized size of antiderivative = 2.17

$$\int \frac{(f+gx)^2 \left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="fric
as")
```

```
[Out] 2/225*(15*e*h*sqrt(-(e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f
^6*g^2 + 40150*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^
8)*p^4/(e^5*h^2)) + 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h))*log(
16*(50625*b^3*e^4*f^8 - 40500*b^3*d*e^3*f^6*g^2 + 2150*b^3*d^2*e^2*f^4*g^4
- 1620*b^3*d^3*e*f^2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 + 16*(10*e^4*f*g*h
^2*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2 + 40150*b^4*d^3*e
^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4/(e^5*h^2)) + 3*(1
125*b^2*e^4*f^6 - 1175*b^2*d*e^3*f^4*g^2 + 235*b^2*d^2*e^2*f^2*g^4 - 9*b^2*
d^3*e*g^6)*h*p^2)*sqrt(-(e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e
^3*f^6*g^2 + 40150*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^
5*g^8)*p^4/(e^5*h^2)) + 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h)))
 - 15*e*h*sqrt(-(e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g
^2 + 40150*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p
^4/(e^5*h^2)) + 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h))*log(16*(
50625*b^3*e^4*f^8 - 40500*b^3*d*e^3*f^6*g^2 + 2150*b^3*d^2*e^2*f^4*g^4 - 16
20*b^3*d^3*e*f^2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 - 16*(10*e^4*f*g*h^2*s
qrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2 + 40150*b^4*d^3*e^2*f
^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4/(e^5*h^2)) + 3*(1125*
b^2*e^4*f^6 - 1175*b^2*d*e^3*f^4*g^2 + 235*b^2*d^2*e^2*f^2*g^4 - 9*b^2*d^3*
e*g^6)*h*p^2)*sqrt(-(e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f
^6*g^2 + 40150*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^
8)*p^4/(e^5*h^2)) + 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h))) - 1
5*e*h*sqrt((e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2 +
40150*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4/(e
^5*h^2)) - 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h))*log(16*(50625
```

```
*b^3*e^4*f^8 - 40500*b^3*d*e^3*f^6*g^2 + 2150*b^3*d^2*e^2*f^4*g^4 - 1620*b^
3*d^3*e*f^2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 + 16*(10*e^4*f*g*h^2*sqrt(-
(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2 + 40150*b^4*d^3*e^2*f^4*g^
4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4/(e^5*h^2)) - 3*(1125*b^2*e
^4*f^6 - 1175*b^2*d*e^3*f^4*g^2 + 235*b^2*d^2*e^2*f^2*g^4 - 9*b^2*d^3*e*g^6
)*h*p^2)*sqrt((e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2
 + 40150*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4
/(e^5*h^2)) - 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h))) + 15*e*h*
sqrt((e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2 + 40150*
b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4/(e^5*h^2
)) - 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h))*log(16*(50625*b^3*e
^4*f^8 - 40500*b^3*d*e^3*f^6*g^2 + 2150*b^3*d^2*e^2*f^4*g^4 - 1620*b^3*d^3*
e*f^2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 - 16*(10*e^4*f*g*h^2*sqrt(-(50625
*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2 + 40150*b^4*d^3*e^2*f^4*g^4 - 34
20*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4/(e^5*h^2)) - 3*(1125*b^2*e^4*f^6
 - 1175*b^2*d*e^3*f^4*g^2 + 235*b^2*d^2*e^2*f^2*g^4 - 9*b^2*d^3*e*g^6)*h*p^
2)*sqrt((e^2*h*sqrt(-(50625*b^4*d*e^4*f^8 - 85500*b^4*d^2*e^3*f^6*g^2 + 401
50*b^4*d^3*e^2*f^4*g^4 - 3420*b^4*d^4*e*f^2*g^6 + 81*b^4*d^5*g^8)*p^4/(e^5*
h^2)) - 60*(5*b^2*d*e*f^3*g - b^2*d^2*f*g^3)*p^2)/(e^2*h))) + (225*a*e*f^2
- 9*(4*b*e*g^2*p - 5*a*e*g^2)*x^2 - 180*(5*b*e*f^2 - b*d*g^2)*p - 50*(4*b*e
*f*g*p - 3*a*e*f*g)*x + 15*(3*b*e*g^2*p*x^2 + 10*b*e*f*g*p*x + 15*b*e*f^2*p
)*log(e*x^2 + d) + 15*(3*b*e*g^2*x^2 + 10*b*e*f*g*x + 15*b*e*f^2)*log(c))*s
qrt(h*x))/(e*h)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(1/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 1165, normalized size of antiderivative = 1.16

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 2/5*b*g^2*x^3*log((e*x^2 + d)^p*c)/sqrt(h*x) + 2/5*a*g^2*x^3/sqrt(h*x) + 4/
3*b*f*g*x^2*log((e*x^2 + d)^p*c)/sqrt(h*x) + 4/3*a*f*g*x^2/sqrt(h*x) + 2*sq
rt(h*x)*b*f^2*log((e*x^2 + d)^p*c)/h - (8*sqrt(h*x)*h^2/e - (sqrt(2)*h^4*lo
g(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^
2)^(3/4)*e^(1/4)) - sqrt(2)*h^4*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqr
t(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) + sqrt(2)*h^3*log(-(sqr
t(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)
*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4)
 + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(d)) + sqrt(2)*h^3*l
og(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*s
qrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)
*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(d)))*d/e)*b
*e*f^2*p/h^3 + 2*sqrt(h*x)*a*f^2/h - 2/9*(3*d*h^4*(sqrt(2)*log(sqrt(e)*h*x
+ sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/
4)) - sqrt(2)*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + s
qrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(-(sqrt(2)*sqrt(-sqrt(d)*sqr
t(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqr
t(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)
))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)) - sqrt(2)*log(-(sqrt(2)*sqrt(-sqrt(d)
*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)
*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqr
t(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)))/e + 8*(h*x)^(3/2)*h^2/e)*b*e*f*g
*p/h^4 - 1/25*b*(5*(sqrt(2)*h^6*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqr
t(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) - sqrt(2)*h^6*log(sqrt(
e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4
)*e^(1/4)) + sqrt(2)*h^5*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(
d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*
h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*s
qrt(e)*h)*sqrt(d)) + sqrt(2)*h^5*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - s
qrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*
sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-s
qrt(d)*sqrt(e)*h)*sqrt(d)))*d^2/e^2 + 8*((h*x)^(5/2)*e*h^2 - 5*sqrt(h*x)*d*
h^4)/e^2)*e*g^2*p/h^5
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 885, normalized size of antiderivative = 0.88

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2),x, algorithm="giac
")
```

[Out] 1/225*(90*sqrt(h*x)*b*g^2*x^2*log(c) + 90*sqrt(h*x)*a*g^2*x^2 + 300*sqrt(h*x)*b*f*g*x*log(c) + 225*(e*(2*sqrt(2)*(d*e^3*h^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) + 2*sqrt(h*x))/(d*h^2/e)^(1/4))/e^2 + 2*sqrt(2)*(d*e^3*h^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) - 2*sqrt(h*x))/(d*h^2/e)^(1/4))/e^2 + sqrt(2)*(d*e^3*h^2)^(1/4)*log(h*x + sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/e^2 - sqrt(2)*(d*e^3*h^2)^(1/4)*log(h*x - sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/e^2 - 8*sqrt(h*x)/e) + 2*sqrt(h*x)*log(e*x^2 + d))*b*f^2*p + 9*(10*sqrt(h*x)*x^2*log(e*x^2 + d) - e*(10*sqrt(2)*(d*e^3*h^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) + 2*sqrt(h*x))/(d*h^2/e)^(1/4))/e^3 + 10*sqrt(2)*(d*e^3*h^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) - 2*sqrt(h*x))/(d*h^2/e)^(1/4))/e^3 + 5*sqrt(2)*(d*e^3*h^2)^(1/4)*d*log(h*x + sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/e^3 - 5*sqrt(2)*(d*e^3*h^2)^(1/4)*d*log(h*x - sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/e^3 + 8*(sqrt(h*x)*e^4*h^10*x^2 - 5*sqrt(h*x)*d*e^3*h^10)/(e^5*h^10)))*b*g^2*p + 300*sqrt(h*x)*a*f*g*x + 450*sqrt(h*x)*b*f^2*log(c) + 50*(6*sqrt(h*x)*h*x*log(e*x^2 + d) - (8*sqrt(h*x)*h*x/e - 6*sqrt(2)*(d*e^3*h^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) + 2*sqrt(h*x))/(d*h^2/e)^(1/4))/e^4 - 6*sqrt(2)*(d*e^3*h^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) - 2*sqrt(h*x))/(d*h^2/e)^(1/4))/e^4 + 3*sqrt(2)*(d*e^3*h^2)^(3/4)*log(h*x + sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/e^4 - 3*sqrt(2)*(d*e^3*h^2)^(3/4)*log(h*x - sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/e^4)*e)*b*f*g*p/h + 450*sqrt(h*x)*a*f^2)/h

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{hx}}\,dx=\int \frac{(f+g\,x)^2\,\left(a+b\,\ln\left(c\,{(e\,x^2+d)}^p\right)\right)}{\sqrt{h\,x}}\,dx$$

[In] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(1/2),x)

[Out] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(1/2), x)

3.612 $\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 949

$$\int \frac{(f+gx)^2 \left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \frac{4afg\sqrt{hx}}{h^2}-\frac{16bfgp\sqrt{hx}}{h^2}$$
$$-\frac{8bg^2p(hx)^{3/2}}{9h^3}-\frac{2\sqrt{2}b\sqrt[4]{e}f^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}}$$
$$-\frac{4\sqrt{2}b\sqrt[4]{d}fgp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{3/2}}$$
$$-\frac{2\sqrt{2}bd^{3/4}g^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}h^{3/2}}$$
$$+\frac{2\sqrt{2}b\sqrt[4]{e}f^2p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}}$$
$$+\frac{4\sqrt{2}b\sqrt[4]{d}fgp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{3/2}}$$
$$+\frac{2\sqrt{2}bd^{3/4}g^2p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}h^{3/2}}+\frac{4bfg\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h^2}$$
$$-\frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h\sqrt{hx}}+\frac{2g^2(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^3}$$
$$+\frac{\sqrt{2}b\sqrt[4]{e}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}}$$
$$-\frac{2\sqrt{2}b\sqrt[4]{d}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}h^{3/2}}$$
$$+\frac{\sqrt{2}bd^{3/4}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}h^{3/2}}$$
$$-\frac{\sqrt{2}b\sqrt[4]{e}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}}$$
$$+\frac{2\sqrt{2}b\sqrt[4]{d}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}h^{3/2}}$$
$$-\frac{\sqrt{2}bd^{3/4}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}h^{3/2}}$$

```
[Out] -8/9*b*g^2*p*(h*x)^(3/2)/h^3+2/3*g^2*(h*x)^(3/2)*(a+b*ln(c*(e*x^2+d)^p))/h^
3-2*b*e^(1/4)*f^2*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2
^(1/2)/d^(1/4)/h^(3/2)-4*b*d^(1/4)*f*g*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/
2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(1/4)/h^(3/2)-2/3*b*d^(3/4)*g^2*p*arctan(1-e^
(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(3/4)/h^(3/2)+2*b*e^(1
/4)*f^2*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(
1/4)/h^(3/2)+4*b*d^(1/4)*f*g*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)
/h^(1/2))*2^(1/2)/e^(1/4)/h^(3/2)+2/3*b*d^(3/4)*g^2*p*arctan(1+e^(1/4)*2^(1
/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/e^(3/4)/h^(3/2)+b*e^(1/4)*f^2*p*ln
(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(
1/2)/d^(1/4)/h^(3/2)-2*b*d^(1/4)*f*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)
-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(3/2)+1/3*b*d^(3/4)
*g^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(
1/2))*2^(1/2)/e^(3/4)/h^(3/2)-b*e^(1/4)*f^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*
h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(1/4)/h^(3/2)+2*b*d^
(1/4)*f*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h
*x)^(1/2))*2^(1/2)/e^(1/4)/h^(3/2)-1/3*b*d^(3/4)*g^2*p*ln(d^(1/2)*h^(1/2)+x
*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(3/4)/h^(3/
2)-2*f^2*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(1/2)+4*a*f*g*(h*x)^(1/2)/h^2-16*b
*f*g*p*(h*x)^(1/2)/h^2+4*b*f*g*ln(c*(e*x^2+d)^p)*(h*x)^(1/2)/h^2
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 949, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules

used = {2517, 2526, 2498, 327, 217, 1179, 642, 1176, 631, 210, 2505, 303}

$$
\begin{aligned}
&\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \\
&-\frac{2\sqrt{2}b\sqrt[4]{e}p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f^2}{\sqrt[4]{d}h^{3/2}} \\
&+\frac{2\sqrt{2}b\sqrt[4]{e}p\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f^2}{\sqrt[4]{d}h^{3/2}}-\frac{2\left(a+b\log\left(c(ex^2+d)^p\right)\right)f^2}{h\sqrt{hx}} \\
&+\frac{\sqrt{2}b\sqrt[4]{e}p\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)f^2}{\sqrt[4]{d}h^{3/2}} \\
&-\frac{\sqrt{2}b\sqrt[4]{e}p\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)f^2}{\sqrt[4]{d}h^{3/2}} \\
&-\frac{4\sqrt{2}b\sqrt[4]{d}gp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f}{\sqrt[4]{e}h^{3/2}} \\
&+\frac{4\sqrt{2}b\sqrt[4]{d}gp\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f}{\sqrt[4]{e}h^{3/2}}+\frac{4bg\sqrt{hx}\log\left(c(ex^2+d)^p\right)f}{h^2} \\
&-\frac{2\sqrt{2}b\sqrt[4]{d}gp\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)f}{\sqrt[4]{e}h^{3/2}} \\
&+\frac{2\sqrt{2}b\sqrt[4]{d}gp\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)f}{\sqrt[4]{e}h^{3/2}}-\frac{16bgp\sqrt{hx}f}{h^2} \\
&+\frac{4ag\sqrt{hx}f}{h^2}-\frac{8bg^2p(hx)^{3/2}}{9h^3}-\frac{2\sqrt{2}bd^{3/4}g^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}h^{3/2}} \\
&+\frac{2\sqrt{2}bd^{3/4}g^2p\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{3e^{3/4}h^{3/2}} \\
&+\frac{2g^2(hx)^{3/2}\left(a+b\log\left(c(ex^2+d)^p\right)\right)}{3h^3} \\
&+\frac{\sqrt{2}bd^{3/4}g^2p\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}h^{3/2}} \\
&-\frac{\sqrt{2}bd^{3/4}g^2p\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}h^{3/2}}
\end{aligned}
$$

```
[In] Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(3/2),x]
```

```
[Out] (4*a*f*g*Sqrt[h*x])/h^2 - (16*b*f*g*p*Sqrt[h*x])/h^2 - (8*b*g^2*p*(h*x)^(3/
2))/(9*h^3) - (2*Sqrt[2]*b*e^(1/4)*f^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h
*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(3/2)) - (4*Sqrt[2]*b*d^(1/4)*f*g*p*Arc
Tan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(1/4)*h^(3/2)) -
 (2*Sqrt[2]*b*d^(3/4)*g^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)
*Sqrt[h])])/(3*e^(3/4)*h^(3/2)) + (2*Sqrt[2]*b*e^(1/4)*f^2*p*ArcTan[1 + (Sq
rt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(3/2)) + (4*Sqrt[2]
*b*d^(1/4)*f*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])
/(e^(1/4)*h^(3/2)) + (2*Sqrt[2]*b*d^(3/4)*g^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)
*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*e^(3/4)*h^(3/2)) + (4*b*f*g*Sqrt[h*x]*Lo
g[c*(d + e*x^2)^p])/h^2 - (2*f^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*Sqrt[h*x]
) + (2*g^2*(h*x)^(3/2)*(a + b*Log[c*(d + e*x^2)^p]))/(3*h^3) + (Sqrt[2]*b*e
^(1/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1
/4)*Sqrt[h*x]])/(d^(1/4)*h^(3/2)) - (2*Sqrt[2]*b*d^(1/4)*f*g*p*Log[Sqrt[d]*
Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*
h^(3/2)) + (Sqrt[2]*b*d^(3/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x
 - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^(3/4)*h^(3/2)) - (Sqrt[2]*b*e^(
1/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[h*x]])/(d^(1/4)*h^(3/2)) + (2*Sqrt[2]*b*d^(1/4)*f*g*p*Log[Sqrt[d]*Sq
rt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*h^
(3/2)) - (Sqrt[2]*b*d^(3/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x +
 Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*e^(3/4)*h^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^2}\,dx,x,\sqrt{hx}\right)}{h}\\
&= \frac{2\text{Subst}\left(\int \left(\frac{2fg\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h}+\frac{f^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^2}+\frac{g^2x^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h^2}\right)\,dx,x,\sqrt{hx}\right)}{h}\\
&= \frac{(2g^2)\,\text{Subst}\left(\int x^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)\,dx,x,\sqrt{hx}\right)}{h^3}\\
&\quad+\frac{(4fg)\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)\,dx,x,\sqrt{hx}\right)}{h^2}\\
&\quad+\frac{(2f^2)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^2}\,dx,x,\sqrt{hx}\right)}{h}\\
&= \frac{4afg\sqrt{hx}}{h^2}-\frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h\sqrt{hx}}+\frac{2g^2(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^3}\\
&\quad+\frac{(4bfg)\text{Subst}\left(\int \log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\,dx,x,\sqrt{hx}\right)}{h^2}\\
&\quad-\frac{(8beg^2p)\,\text{Subst}\left(\int \frac{x^6}{d+\frac{ex^4}{h^2}}\,dx,x,\sqrt{hx}\right)}{3h^5}+\frac{(8bef^2p)\,\text{Subst}\left(\int \frac{x^2}{d+\frac{ex^4}{h^2}}\,dx,x,\sqrt{hx}\right)}{h^3}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{4afg\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} + \frac{4bfg\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h^2} \\
&\quad - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h\sqrt{hx}} + \frac{2g^2(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^3} \\
&\quad - \frac{(16befgp)\text{Subst}\left(\int \frac{x^4}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^4} \\
&\quad - \frac{\left(4b\sqrt{e}f^2p\right)\text{Subst}\left(\int \frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^3} \\
&\quad + \frac{\left(4b\sqrt{e}f^2p\right)\text{Subst}\left(\int \frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^3} \\
&\quad + \frac{\left(8bdg^2p\right)\text{Subst}\left(\int \frac{x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{3h^3}
\end{aligned}
$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{16bfgp\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} + \frac{4bfg\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h^2} - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h\sqrt{hx}} + \frac{2g^2(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^3} - \frac{(4bdg^2p)\,\text{Subst}\left(\int \frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{3\sqrt{e}h^3} + \frac{(4bdg^2p)\,\text{Subst}\left(\int \frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{3\sqrt{e}h^3} + \frac{(16bdfgp)\text{Subst}\left(\int \frac{1}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^2} + \frac{\left(\sqrt{2}b\sqrt[4]{e}f^2p\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}} + \frac{\left(\sqrt{2}b\sqrt[4]{e}f^2p\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}} + \frac{(2bf^2p)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{h} + \frac{(2bf^2p)\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{h}$$

$$
\begin{aligned}
&= \frac{4afg\sqrt{hx}}{h^2} - \frac{16bfgp\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} + \frac{4bfg\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h^2} \\
&\quad - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h\sqrt{hx}} + \frac{2g^2(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^3} \\
&\quad + \frac{\sqrt{2}b\sqrt[4]{e}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}} \\
&\quad - \frac{\sqrt{2}b\sqrt[4]{e}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}} \\
&\quad + \frac{\left(8b\sqrt{d}fgp\right)\text{Subst}\left(\int \frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^3} \\
&\quad + \frac{\left(8b\sqrt{d}fgp\right)\text{Subst}\left(\int \frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^3} \\
&\quad + \frac{\left(2\sqrt{2}b\sqrt[4]{e}f^2p\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}} \\
&\quad - \frac{\left(2\sqrt{2}b\sqrt[4]{e}f^2p\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}} \\
&\quad + \frac{\left(\sqrt{2}bd^{3/4}g^2p\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{3e^{3/4}h^{3/2}} \\
&\quad + \frac{\left(\sqrt{2}bd^{3/4}g^2p\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{3e^{3/4}h^{3/2}} \\
&\quad + \frac{\left(2bdg^2p\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{3eh} \\
&\quad + \frac{\left(2bdg^2p\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{3eh}
\end{aligned}
$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{16bfgp\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} - \frac{2\sqrt{2}b\sqrt[4]{e}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}}$$

$$+ \frac{2\sqrt{2}b\sqrt[4]{e}f^2p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}} + \frac{4bfg\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h^2}$$

$$- \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h\sqrt{hx}} + \frac{2g^2(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^3}$$

$$+ \frac{\sqrt{2}b\sqrt[4]{e}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}}$$

$$+ \frac{\sqrt{2}bd^{3/4}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}h^{3/2}}$$

$$- \frac{\sqrt{2}b\sqrt[4]{e}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}}$$

$$- \frac{\sqrt{2}bd^{3/4}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}h^{3/2}}$$

$$- \frac{\left(2\sqrt{2}b\sqrt[4]{d}fgp\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx,x,\sqrt{hx}\right)}{\sqrt[4]{e}h^{3/2}}$$

$$- \frac{\left(2\sqrt{2}b\sqrt[4]{d}fgp\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx,x,\sqrt{hx}\right)}{\sqrt[4]{e}h^{3/2}}$$

$$+ \frac{\left(2\sqrt{2}bd^{3/4}g^2p\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}h^{3/2}}$$

$$- \frac{\left(2\sqrt{2}bd^{3/4}g^2p\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}h^{3/2}}$$

$$+ \frac{\left(4b\sqrt{d}fgp\right)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx,x,\sqrt{hx}\right)}{\sqrt{e}h}$$

$$+ \frac{\left(4b\sqrt{d}fgp\right)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx,x,\sqrt{hx}\right)}{\sqrt{e}h}$$

$$= \frac{4afg\sqrt{hx}}{h^2} - \frac{16bfgp\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} - \frac{2\sqrt{2}b\sqrt[4]{e}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}}$$
$$- \frac{2\sqrt{2}bd^{3/4}g^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}h^{3/2}} + \frac{2\sqrt{2}b\sqrt[4]{e}f^2p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}}$$
$$+ \frac{2\sqrt{2}bd^{3/4}g^2p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}h^{3/2}} + \frac{4bfg\sqrt{hx}\log\left(c\left(d+ex^2\right)^p\right)}{h^2}$$
$$- \frac{2f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h\sqrt{hx}} + \frac{2g^2(hx)^{3/2}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^3}$$
$$+ \frac{\sqrt{2}b\sqrt[4]{e}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}}$$
$$- \frac{2\sqrt{2}b\sqrt[4]{d}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}h^{3/2}}$$
$$+ \frac{\sqrt{2}bd^{3/4}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}h^{3/2}}$$
$$- \frac{\sqrt{2}b\sqrt[4]{e}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}}$$
$$+ \frac{2\sqrt{2}b\sqrt[4]{d}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}h^{3/2}}$$
$$- \frac{\sqrt{2}bd^{3/4}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}h^{3/2}}$$
$$+ \frac{\left(4\sqrt{2}b\sqrt[4]{d}fgp\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{3/2}}$$
$$- \frac{\left(4\sqrt{2}b\sqrt[4]{d}fgp\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{3/2}}$$

$$
\begin{aligned}
&= \frac{4afg\sqrt{hx}}{h^2} - \frac{16bfgp\sqrt{hx}}{h^2} - \frac{8bg^2p(hx)^{3/2}}{9h^3} - \frac{2\sqrt{2}b\sqrt[4]{e}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}} \\
&\quad - \frac{4\sqrt{2}b\sqrt[4]{d}fgp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{3/2}} - \frac{2\sqrt{2}bd^{3/4}g^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}h^{3/2}} \\
&\quad + \frac{2\sqrt{2}b\sqrt[4]{e}f^2p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{3/2}} + \frac{4\sqrt{2}b\sqrt[4]{d}fgp\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{3/2}} \\
&\quad + \frac{2\sqrt{2}bd^{3/4}g^2p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3e^{3/4}h^{3/2}} + \frac{4bfg\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h^2} \\
&\quad - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h\sqrt{hx}} + \frac{2g^2(hx)^{3/2}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^3} \\
&\quad + \frac{\sqrt{2}b\sqrt[4]{e}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}} \\
&\quad - \frac{2\sqrt{2}b\sqrt[4]{d}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}h^{3/2}} \\
&\quad + \frac{\sqrt{2}bd^{3/4}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}h^{3/2}} \\
&\quad - \frac{\sqrt{2}b\sqrt[4]{e}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{3/2}} \\
&\quad + \frac{2\sqrt{2}b\sqrt[4]{d}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}h^{3/2}} \\
&\quad - \frac{\sqrt{2}bd^{3/4}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3e^{3/4}h^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 478, normalized size of antiderivative = 0.50

$$
\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \frac{2x^{3/2}\left(2afg\sqrt{x}-8bfgp\sqrt{x}+\frac{1}{3}ag^2x^{3/2}-\frac{4}{9}bg^2px^{3/2}-\frac{2\sqrt{2}b\sqrt[4]{d}f\cdots}{\cdots}\right.}{\cdots}
$$

```
[In] Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(3/2),x]
```

```
[Out] (2*x^(3/2)*(2*a*f*g*Sqrt[x] - 8*b*f*g*p*Sqrt[x] + (a*g^2*x^(3/2))/3 - (4*b*
g^2*p*x^(3/2))/9 - (2*Sqrt[2]*b*d^(1/4)*f*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*S
qrt[x])/d^(1/4)])/e^(1/4) + (2*Sqrt[2]*b*d^(1/4)*f*g*p*ArcTan[1 + (Sqrt[2]*
e^(1/4)*Sqrt[x])/d^(1/4)])/e^(1/4) - (2*b*(-d)^(3/4)*g^2*p*ArcTan[(e^(1/4)*
Sqrt[x])/(-d)^(1/4)])/(3*e^(3/4)) + (2*b*(-d)^(3/4)*g^2*p*ArcTanh[(e^(1/4)*
Sqrt[x])/(-d)^(1/4)])/(3*e^(3/4)) + (2*b*e^(1/4)*f^2*p*(ArcTan[(e^(1/4)*Sqr
t[x])/(-d)^(1/4)] + ArcTanh[(d*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) -
(Sqrt[2]*b*d^(1/4)*f*g*p*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sq
rt[e]*x])/e^(1/4) + (Sqrt[2]*b*d^(1/4)*f*g*p*Log[Sqrt[d] + Sqrt[2]*d^(1/4)*
e^(1/4)*Sqrt[x] + Sqrt[e]*x])/e^(1/4) + 2*b*f*g*Sqrt[x]*Log[c*(d + e*x^2)^p
] + (b*g^2*x^(3/2)*Log[c*(d + e*x^2)^p])/3 - (f^2*(a + b*Log[c*(d + e*x^2)^
p]))/Sqrt[x]))/(h*x)^(3/2)
```

Maple [F]

$$\int \frac{(gx+f)^2 \left(a+b\ln \left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{3}{2}}}dx$$

```
[In] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2),x)
```

```
[Out] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2118 vs. 2(673) = 1346.

Time = 0.46 (sec) , antiderivative size = 2118, normalized size of antiderivative = 2.23

$$\int \frac{(f+gx)^2 \left(a+b\log \left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="fric
as")
```

```
[Out] -2/9*(3*h^2*x*sqrt(-(e*h^3*sqrt(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 +
918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^
6)) + 12*(3*b^2*e*f^3*g + b^2*d*f*g^3)*p^2)/(e*h^3))*log(32*(81*b^3*e^4*f^8
 + 108*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 12*b^3*d^3*e*f^2*g^6
+ b^3*d^4*g^8)*sqrt(h*x)*p^3 + 32*((3*d*e^3*f^2 + d^2*e^2*g^2)*h^5*sqrt(-(8
1*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^
3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) - 6*(9*b^2*d*e^3*f^5*g - 30*b^2
*d^2*e^2*f^3*g^3 + b^2*d^3*e*f*g^5)*h^2*p^2)*sqrt(-(e*h^3*sqrt(-(81*b^4*e^4
*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^3*e*f^2*g
^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) + 12*(3*b^2*e*f^3*g + b^2*d*f*g^3)*p^2)/
(e*h^3))) - 3*h^2*x*sqrt(-(e*h^3*sqrt(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*
g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*
```

```
e^3*h^6)) + 12*(3*b^2*e*f^3*g + b^2*d*f*g^3)*p^2)/(e*h^3))*log(32*(81*b^3*e
^4*f^8 + 108*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 12*b^3*d^3*e*f^
2*g^6 + b^3*d^4*g^8)*sqrt(h*x)*p^3 - 32*((3*d*e^3*f^2 + d^2*e^2*g^2)*h^5*sq
rt(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*
b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) - 6*(9*b^2*d*e^3*f^5*g -
30*b^2*d^2*e^2*f^3*g^3 + b^2*d^3*e*f*g^5)*h^2*p^2)*sqrt(-(e*h^3*sqrt(-(81*b
^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^3*e
*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) + 12*(3*b^2*e*f^3*g + b^2*d*f*g^3)
*p^2)/(e*h^3))) - 3*h^2*x*sqrt((e*h^3*sqrt(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3
*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^
4/(d*e^3*h^6)) - 12*(3*b^2*e*f^3*g + b^2*d*f*g^3)*p^2)/(e*h^3))*log(32*(81*
b^3*e^4*f^8 + 108*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 12*b^3*d^3
*e*f^2*g^6 + b^3*d^4*g^8)*sqrt(h*x)*p^3 + 32*((3*d*e^3*f^2 + d^2*e^2*g^2)*h
^5*sqrt(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4
- 60*b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) + 6*(9*b^2*d*e^3*f^5
*g - 30*b^2*d^2*e^2*f^3*g^3 + b^2*d^3*e*f*g^5)*h^2*p^2)*sqrt((e*h^3*sqrt(-(
81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d
^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) - 12*(3*b^2*e*f^3*g + b^2*d*f*
g^3)*p^2)/(e*h^3))) + 3*h^2*x*sqrt((e*h^3*sqrt(-(81*b^4*e^4*f^8 - 540*b^4*d
*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8
)*p^4/(d*e^3*h^6)) - 12*(3*b^2*e*f^3*g + b^2*d*f*g^3)*p^2)/(e*h^3))*log(32*
(81*b^3*e^4*f^8 + 108*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 12*b^3
*d^3*e*f^2*g^6 + b^3*d^4*g^8)*sqrt(h*x)*p^3 - 32*((3*d*e^3*f^2 + d^2*e^2*g^
2)*h^5*sqrt(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*
g^4 - 60*b^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) + 6*(9*b^2*d*e^3
*f^5*g - 30*b^2*d^2*e^2*f^3*g^3 + b^2*d^3*e*f*g^5)*h^2*p^2)*sqrt((e*h^3*sqr
t(-(81*b^4*e^4*f^8 - 540*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 60*b
^4*d^3*e*f^2*g^6 + b^4*d^4*g^8)*p^4/(d*e^3*h^6)) - 12*(3*b^2*e*f^3*g + b^2*
d*f*g^3)*p^2)/(e*h^3))) + (9*a*f^2 + (4*b*g^2*p - 3*a*g^2)*x^2 + 18*(4*b*f*
g*p - a*f*g)*x - 3*(b*g^2*p*x^2 + 6*b*f*g*p*x - 3*b*f^2*p)*log(e*x^2 + d) -
 3*(b*g^2*x^2 + 6*b*f*g*x - 3*b*f^2)*log(c))*sqrt(h*x))/(h^2*x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(3/2),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 1119, normalized size of antiderivative = 1.18

$$\int \frac{(f+gx)^2 \left(a+b\log \left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\, dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] 2/3*b*g^2*x^3*log((e*x^2 + d)^p*c)/(h*x)^(3/2) - b*e*f^2*p*(sqrt(2)*log(sqr
t(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1
/4)*e^(3/4)) - sqrt(2)*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^
(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(-(sqrt(2)*sqrt(-sq
rt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sq
rt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x
)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)) - sqrt(2)*log(-(sqrt(2)*sqrt
(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))
/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt
(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)))/h + 2/3*a*g^2*x^3/(h*x)
^(3/2) + 4*b*f*g*x^2*log((e*x^2 + d)^p*c)/(h*x)^(3/2) + 4*a*f*g*x^2/(h*x)^(
3/2) - 2*b*f^2*log((e*x^2 + d)^p*c)/(sqrt(h*x)*h) - 2*(8*sqrt(h*x)*h^2/e -
(sqrt(2)*h^4*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sq
rt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) - sqrt(2)*h^4*log(sqrt(e)*h*x - sqrt(2)*(d
*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) + sqrt(2
)*h^3*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4
) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2
)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(d))
+ sqrt(2)*h^3*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4
)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2
)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*s
qrt(d)))*d/e)*b*e*f*g*p/h^4 - 2*a*f^2/(sqrt(h*x)*h) - 1/9*(3*d*h^4*(sqrt(2)
*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d
*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt
(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(-(sqrt(2)*
sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt
(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*
sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)) - sqrt(2)*log(-(sqrt
(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*
sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4)
+ 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)))/e + 8*(h*x)^(3/
2)*h^2/e)*b*e*g^2*p/h^5
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 577, normalized size of antiderivative = 0.61

$$\int \frac{(f+gx)^2 \left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \frac{6\left(\frac{\sqrt{hx}bg^2px}{h} - \frac{3bf^2p}{\sqrt{hx}} + \frac{6\sqrt{hx}bfgp}{h}\right)\log\left(eh^2x^2+dh^2\right) - \frac{2\left(3bg^2p\log(h\right.}{}}{}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2),x, algorithm="giac
")
```

```
[Out] 1/9*(6*(sqrt(h*x)*b*g^2*p*x/h - 3*b*f^2*p/sqrt(h*x) + 6*sqrt(h*x)*b*f*g*p/h
)*log(e*h^2*x^2 + d*h^2) - 2*(3*b*g^2*p*log(h^2) + 4*b*g^2*p - 3*b*g^2*log(
c) - 3*a*g^2)*sqrt(h*x)*x/h + 6*(6*sqrt(2)*b*d*e^2*f*g*h*p + 3*sqrt(2)*sqrt
(d*e)*b*e^2*f^2*h*p + sqrt(2)*sqrt(d*e)*b*d*e*g^2*h*p)*arctan(1/2*sqrt(2)*(
sqrt(2)*(d*h^2/e)^(1/4) + 2*sqrt(h*x))/(d*h^2/e)^(1/4))/(d*e^3*h^2)^(3/4) +
 6*(6*sqrt(2)*b*d*e^2*f*g*h*p + 3*sqrt(2)*sqrt(d*e)*b*e^2*f^2*h*p + sqrt(2)
*sqrt(d*e)*b*d*e*g^2*h*p)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) - 2*
sqrt(h*x))/(d*h^2/e)^(1/4))/(d*e^3*h^2)^(3/4) + 3*(6*sqrt(2)*b*d*e^2*f*g*h*
p + 3*sqrt(2)*sqrt(d*e)*b*e^2*f^2*h*p + sqrt(2)*sqrt(d*e)*b*d*e*g^2*h*p)*lo
g(h*x + sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/(d*e^3*h^2)^(3/4
) - 3*(6*sqrt(2)*b*d*e^2*f*g*h*p - 3*sqrt(2)*sqrt(d*e)*b*e^2*f^2*h*p - sqrt
(2)*sqrt(d*e)*b*d*e*g^2*h*p)*log(h*x - sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) +
sqrt(d*h^2/e))/(d*e^3*h^2)^(3/4) + 18*(b*f^2*p*log(h^2) - b*f^2*log(c) - a*
f^2)/sqrt(h*x) - 36*(b*f*g*p*log(h^2) + 4*b*f*g*p - b*f*g*log(c) - a*f*g)*s
qrt(h*x)/h)/h
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2 \left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{3/2}}\,dx = \int \frac{{\left(f+g\,x\right)}^2\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{3/2}}\,dx$$

```
[In] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(3/2),x)
```

```
[Out] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(3/2), x)
```

3.613 $\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 932

$$
\begin{aligned}
\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx &= \frac{2ag^2\sqrt{hx}}{h^3} \\
&- \frac{8bg^2p\sqrt{hx}}{h^3} - \frac{2\sqrt{2}be^{3/4}f^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}} \\
&- \frac{4\sqrt{2}b\sqrt[4]{e}fgp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{5/2}} \\
&- \frac{2\sqrt{2}b\sqrt[4]{d}g^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{5/2}} \\
&+ \frac{2\sqrt{2}be^{3/4}f^2p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}} \\
&+ \frac{4\sqrt{2}b\sqrt[4]{e}fgp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{5/2}} \\
&+ \frac{2\sqrt{2}b\sqrt[4]{d}g^2p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{5/2}} + \frac{2bg^2\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h^3} \\
&- \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h(hx)^{3/2}} - \frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^2\sqrt{hx}} \\
&- \frac{\sqrt{2}be^{3/4}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{5/2}} \\
&+ \frac{2\sqrt{2}b\sqrt[4]{e}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{5/2}} \\
&- \frac{\sqrt{2}b\sqrt[4]{d}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}h^{5/2}} \\
&+ \frac{\sqrt{2}be^{3/4}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{5/2}} \\
&- \frac{2\sqrt{2}b\sqrt[4]{e}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{5/2}} \\
&+ \frac{\sqrt{2}b\sqrt[4]{d}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}h^{5/2}}
\end{aligned}
$$

```
[Out] -2/3*f^2*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(3/2)-2/3*b*e^(3/4)*f^2*p*arctan(1
-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(3/4)/h^(5/2)-4*b*e
^(1/4)*f*g*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/
d^(1/4)/h^(5/2)-2*b*d^(1/4)*g^2*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1
/4)/h^(1/2))*2^(1/2)/e^(1/4)/h^(5/2)+2/3*b*e^(3/4)*f^2*p*arctan(1+e^(1/4)*2
^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(3/4)/h^(5/2)+4*b*e^(1/4)*f*g
*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(1/4)/h^
(5/2)+2*b*d^(1/4)*g^2*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2
))*2^(1/2)/e^(1/4)/h^(5/2)-1/3*b*e^(3/4)*f^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)
*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(3/4)/h^(5/2)+2*b*e
^(1/4)*f*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(
h*x)^(1/2))*2^(1/2)/d^(1/4)/h^(5/2)-b*d^(1/4)*g^2*p*ln(d^(1/2)*h^(1/2)+x*e^
(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(5/2)+
1/3*b*e^(3/4)*f^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^
(1/2)*(h*x)^(1/2))*2^(1/2)/d^(3/4)/h^(5/2)-2*b*e^(1/4)*f*g*p*ln(d^(1/2)*h^(
1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(1/4)
/h^(5/2)+b*d^(1/4)*g^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/
4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/e^(1/4)/h^(5/2)-4*f*g*(a+b*ln(c*(e*x^2+d)^p
))/h^2/(h*x)^(1/2)+2*a*g^2*(h*x)^(1/2)/h^3-8*b*g^2*p*(h*x)^(1/2)/h^3+2*b*g^
2*ln(c*(e*x^2+d)^p)*(h*x)^(1/2)/h^3
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 932, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules

used = {2517, 2526, 2498, 327, 217, 1179, 642, 1176, 631, 210, 2505, 303}

$$\int \frac{(f+gx)^2 \left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx =$$

$$-\frac{2\sqrt{2}be^{3/4}p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f^2}{3d^{3/4}h^{5/2}} + \frac{2\sqrt{2}be^{3/4}p\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f^2}{3d^{3/4}h^{5/2}} - \frac{2\left(a+b\log\left(c(ex^2+d)^p\right)\right)f^2}{3h(hx)^{3/2}} - \frac{\sqrt{2}be^{3/4}p\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)f^2}{3d^{3/4}h^{5/2}} + \frac{\sqrt{2}be^{3/4}p\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)f^2}{3d^{3/4}h^{5/2}} - \frac{4\sqrt{2}b\sqrt[4]{e}gp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f}{\sqrt[4]{d}h^{5/2}} + \frac{4\sqrt{2}b\sqrt[4]{e}gp\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f}{\sqrt[4]{d}h^{5/2}} - \frac{4g\left(a+b\log\left(c(ex^2+d)^p\right)\right)f}{h^2\sqrt{hx}} + \frac{2\sqrt{2}b\sqrt[4]{e}gp\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)f}{\sqrt[4]{d}h^{5/2}} - \frac{2\sqrt{2}b\sqrt[4]{e}gp\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)f}{\sqrt[4]{d}h^{5/2}} - \frac{2\sqrt{2}b\sqrt[4]{d}g^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{5/2}} + \frac{2\sqrt{2}b\sqrt[4]{d}g^2p\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{\sqrt[4]{e}h^{5/2}} + \frac{2bg^2\sqrt{hx}\log\left(c(ex^2+d)^p\right)}{h^3} - \frac{\sqrt{2}b\sqrt[4]{d}g^2p\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}h^{5/2}} + \frac{\sqrt{2}b\sqrt[4]{d}g^2p\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}h^{5/2}} - \frac{8bg^2p\sqrt{hx}}{h^3} + \frac{2ag^2\sqrt{hx}}{h^3}$$

```
[In] Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(5/2),x]
```

```
[Out] (2*a*g^2*Sqrt[h*x])/h^3 - (8*b*g^2*p*Sqrt[h*x])/h^3 - (2*Sqrt[2]*b*e^(3/4)*
f^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4)
*h^(5/2)) - (4*Sqrt[2]*b*e^(1/4)*f*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x
])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(5/2)) - (2*Sqrt[2]*b*d^(1/4)*g^2*p*ArcTa
n[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(e^(1/4)*h^(5/2)) + (
2*Sqrt[2]*b*e^(3/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*S
qrt[h])])/(3*d^(3/4)*h^(5/2)) + (4*Sqrt[2]*b*e^(1/4)*f*g*p*ArcTan[1 + (Sqrt
[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(5/2)) + (2*Sqrt[2]*b
*d^(1/4)*g^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(
e^(1/4)*h^(5/2)) + (2*b*g^2*Sqrt[h*x]*Log[c*(d + e*x^2)^p])/h^3 - (2*f^2*(a
 + b*Log[c*(d + e*x^2)^p]))/(3*h*(h*x)^(3/2)) - (4*f*g*(a + b*Log[c*(d + e*
x^2)^p]))/(h^2*Sqrt[h*x]) - (Sqrt[2]*b*e^(3/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] +
Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(5/2))
 + (2*Sqrt[2]*b*e^(1/4)*f*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqr
t[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*h^(5/2)) - (Sqrt[2]*b*d^(1/4)*g^2
*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h
*x]])/(e^(1/4)*h^(5/2)) + (Sqrt[2]*b*e^(3/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sq
rt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(5/2)) -
 (2*Sqrt[2]*b*e^(1/4)*f*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[
2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*h^(5/2)) + (Sqrt[2]*b*d^(1/4)*g^2*p
*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x
]])/(e^(1/4)*h^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^4}\,dx,x,\sqrt{hx}\right)}{h}$$

$$=\frac{2\text{Subst}\left(\int\left(\frac{g^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h^2}+\frac{f^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^4}+\frac{2fg\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{hx^2}\right)dx,x,\sqrt{hx}\right)}{h}$$

$$=\frac{(2g^2)\,\text{Subst}\left(\int\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx,x,\sqrt{hx}\right)}{h^3}$$
$$+\frac{(4fg)\text{Subst}\left(\int\frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^2}\,dx,x,\sqrt{hx}\right)}{h^2}$$
$$+\frac{(2f^2)\,\text{Subst}\left(\int\frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^4}\,dx,x,\sqrt{hx}\right)}{h}$$

$$=\frac{2ag^2\sqrt{hx}}{h^3}-\frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h(hx)^{3/2}}-\frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^2\sqrt{hx}}$$
$$+\frac{(2bg^2)\,\text{Subst}\left(\int\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)dx,x,\sqrt{hx}\right)}{h^3}$$
$$+\frac{(16befgp)\text{Subst}\left(\int\frac{x^2}{d+\frac{ex^4}{h^2}}\,dx,x,\sqrt{hx}\right)}{h^4}+\frac{(8bef^2p)\,\text{Subst}\left(\int\frac{1}{d+\frac{ex^4}{h^2}}\,dx,x,\sqrt{hx}\right)}{3h^3}$$

$$
\begin{aligned}
&= \frac{2ag^2\sqrt{hx}}{h^3} + \frac{2bg^2\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h^3} - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h(hx)^{3/2}} \\
&\quad - \frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^2\sqrt{hx}} - \frac{(8beg^2p)\,\mathrm{Subst}\left(\int \frac{x^4}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^5} \\
&\quad + \frac{(4bef^2p)\,\mathrm{Subst}\left(\int \frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{3\sqrt{d}h^4} \\
&\quad + \frac{(4bef^2p)\,\mathrm{Subst}\left(\int \frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{3\sqrt{d}h^4} \\
&\quad - \frac{(8b\sqrt{e}fgp)\,\mathrm{Subst}\left(\int \frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^4} \\
&\quad + \frac{(8b\sqrt{e}fgp)\,\mathrm{Subst}\left(\int \frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^4}
\end{aligned}
$$

$$
= \frac{2ag^2\sqrt{hx}}{h^3} - \frac{8bg^2p\sqrt{hx}}{h^3} + \frac{2bg^2\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h^3} - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h(hx)^{3/2}}
$$

$$
- \frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^2\sqrt{hx}} + \frac{(8bdg^2p)\,\mathrm{Subst}\left(\int \frac{1}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^3}
$$

$$
- \frac{\left(\sqrt{2}be^{3/4}f^2p\right)\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{3d^{3/4}h^{5/2}}
$$

$$
- \frac{\left(\sqrt{2}be^{3/4}f^2p\right)\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{3d^{3/4}h^{5/2}}
$$

$$
+ \frac{\left(2\sqrt{2}b\sqrt[4]{e}fgp\right)\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{\sqrt[4]{d}h^{5/2}}
$$

$$
+ \frac{\left(2\sqrt{2}b\sqrt[4]{e}fgp\right)\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{\sqrt[4]{d}h^{5/2}}
$$

$$
+ \frac{\left(2b\sqrt{e}f^2p\right)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{3\sqrt{d}h^2}
$$

$$
+ \frac{\left(2b\sqrt{e}f^2p\right)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{3\sqrt{d}h^2}
$$

$$
+ \frac{(4bfgp)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{h^2}
$$

$$
+ \frac{(4bfgp)\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{h^2}
$$

$$
\begin{aligned}
&= \frac{2ag^2\sqrt{hx}}{h^3} - \frac{8bg^2p\sqrt{hx}}{h^3} + \frac{2bg^2\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h^3} \\
&\quad - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h(hx)^{3/2}} - \frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^2\sqrt{hx}} \\
&\quad - \frac{\sqrt{2}be^{3/4}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{5/2}} \\
&\quad + \frac{2\sqrt{2}b\sqrt[4]{e}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{5/2}} \\
&\quad + \frac{\sqrt{2}be^{3/4}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{5/2}} \\
&\quad - \frac{2\sqrt{2}b\sqrt[4]{e}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{5/2}} \\
&\quad + \frac{\left(4b\sqrt{d}g^2p\right)\text{Subst}\left(\int\frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx,x,\sqrt{hx}\right)}{h^4} \\
&\quad + \frac{\left(4b\sqrt{d}g^2p\right)\text{Subst}\left(\int\frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx,x,\sqrt{hx}\right)}{h^4} \\
&\quad + \frac{\left(2\sqrt{2}be^{3/4}f^2p\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}} \\
&\quad - \frac{\left(2\sqrt{2}be^{3/4}f^2p\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}} \\
&\quad + \frac{\left(4\sqrt{2}b\sqrt[4]{e}fgp\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{5/2}} \\
&\quad - \frac{\left(4\sqrt{2}b\sqrt[4]{e}fgp\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{5/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2ag^2\sqrt{hx}}{h^3} - \frac{8bg^2p\sqrt{hx}}{h^3} - \frac{2\sqrt{2}be^{3/4}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}} \\
&- \frac{4\sqrt{2}b\sqrt[4]{e}fgp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{5/2}} + \frac{2\sqrt{2}be^{3/4}f^2p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}} \\
&+ \frac{4\sqrt{2}b\sqrt[4]{e}fgp\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{5/2}} + \frac{2bg^2\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h^3} \\
&- \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h(hx)^{3/2}} - \frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^2\sqrt{hx}} \\
&- \frac{\sqrt{2}be^{3/4}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{5/2}} \\
&+ \frac{2\sqrt{2}b\sqrt[4]{e}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{5/2}} \\
&+ \frac{\sqrt{2}be^{3/4}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{5/2}} \\
&- \frac{2\sqrt{2}b\sqrt[4]{e}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{5/2}} \\
&- \frac{\left(\sqrt{2}b\sqrt[4]{d}g^2p\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx,x,\sqrt{hx}\right)}{\sqrt[4]{e}h^{5/2}} \\
&- \frac{\left(\sqrt{2}b\sqrt[4]{d}g^2p\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx,x,\sqrt{hx}\right)}{\sqrt[4]{e}h^{5/2}} \\
&+ \frac{\left(2b\sqrt{d}g^2p\right)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx,x,\sqrt{hx}\right)}{\sqrt{e}h^2} \\
&+ \frac{\left(2b\sqrt{d}g^2p\right)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx,x,\sqrt{hx}\right)}{\sqrt{e}h^2}
\end{aligned}
$$

$$= \frac{2ag^2\sqrt{hx}}{h^3} - \frac{8bg^2p\sqrt{hx}}{h^3} - \frac{2\sqrt{2}be^{3/4}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}}$$
$$- \frac{4\sqrt{2}b\sqrt[4]{e}fgp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{5/2}} + \frac{2\sqrt{2}be^{3/4}f^2p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}}$$
$$+ \frac{4\sqrt{2}b\sqrt[4]{e}fgp\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{5/2}} + \frac{2bg^2\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h^3}$$
$$- \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h(hx)^{3/2}} - \frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^2\sqrt{hx}}$$
$$- \frac{\sqrt{2}be^{3/4}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{5/2}}$$
$$+ \frac{2\sqrt{2}b\sqrt[4]{e}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{5/2}}$$
$$- \frac{\sqrt{2}b\sqrt[4]{d}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}h^{5/2}}$$
$$+ \frac{\sqrt{2}be^{3/4}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{5/2}}$$
$$- \frac{2\sqrt{2}b\sqrt[4]{e}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{5/2}}$$
$$+ \frac{\sqrt{2}b\sqrt[4]{d}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}h^{5/2}}$$
$$+ \frac{\left(2\sqrt{2}b\sqrt[4]{d}g^2p\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{5/2}}$$
$$- \frac{\left(2\sqrt{2}b\sqrt[4]{d}g^2p\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{5/2}}$$

$$
\begin{aligned}
&= \frac{2ag^2\sqrt{hx}}{h^3} - \frac{8bg^2p\sqrt{hx}}{h^3} - \frac{2\sqrt{2}be^{3/4}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}} \\
&- \frac{4\sqrt{2}b\sqrt[4]{e}fgp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{5/2}} - \frac{2\sqrt{2}b\sqrt[4]{d}g^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{5/2}} \\
&+ \frac{2\sqrt{2}be^{3/4}f^2p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{5/2}} + \frac{4\sqrt{2}b\sqrt[4]{e}fgp\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{5/2}} \\
&+ \frac{2\sqrt{2}b\sqrt[4]{d}g^2p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}h^{5/2}} + \frac{2bg^2\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{h^3} \\
&- \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h(hx)^{3/2}} - \frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^2\sqrt{hx}} \\
&- \frac{\sqrt{2}be^{3/4}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{5/2}} \\
&+ \frac{2\sqrt{2}b\sqrt[4]{e}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{5/2}} \\
&- \frac{\sqrt{2}b\sqrt[4]{d}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}h^{5/2}} \\
&+ \frac{\sqrt{2}be^{3/4}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{5/2}} \\
&- \frac{2\sqrt{2}b\sqrt[4]{e}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{5/2}} \\
&+ \frac{\sqrt{2}b\sqrt[4]{d}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}h^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 527, normalized size of antiderivative = 0.57

$$
\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \frac{2x^{5/2}\left(ag^2\sqrt{x}-4bg^2p\sqrt{x}-\frac{\sqrt{2}b\sqrt[4]{d}g^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)}{\sqrt[4]{e}}+\frac{\sqrt{2}b\sqrt[4]{}\cdots}{\cdots}\right.}{\cdots}
$$

```
[In] Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(5/2),x]
```

```
[Out] (2*x^(5/2)*(a*g^2*Sqrt[x] - 4*b*g^2*p*Sqrt[x] - (Sqrt[2]*b*d^(1/4)*g^2*p*Ar
cTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)])/e^(1/4) + (Sqrt[2]*b*d^(1/4)*g
^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)])/e^(1/4) + (4*b*e^(1/4)*
f*g*p*(ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + ArcTanh[(d*e^(1/4)*Sqrt[x])/(
-d)^(5/4)]))/(-d)^(1/4) - (b*d^(1/4)*g^2*p*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^
(1/4)*Sqrt[x] + Sqrt[e]*x])/(Sqrt[2]*e^(1/4)) - (b*e^(3/4)*f^2*p*(2*ArcTan[
1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt
[x])/d^(1/4)] + Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]
- Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/(3*Sqrt[2]*d
^(3/4)) + (b*d^(1/4)*g^2*p*Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] +
Sqrt[e]*x])/(Sqrt[2]*e^(1/4)) + b*g^2*Sqrt[x]*Log[c*(d + e*x^2)^p] - (f^2*(
a + b*Log[c*(d + e*x^2)^p]))/(3*x^(3/2)) - (2*f*g*(a + b*Log[c*(d + e*x^2)^
p]))/Sqrt[x]))/(h*x)^(5/2)
```

Maple [F]

$$\int \frac{(gx+f)^2 \left(a+b\ln \left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{5}{2}}}dx$$

```
[In] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(5/2),x)
```

```
[Out] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2112 vs. $2(660) = 1320$.

Time = 0.44 (sec) , antiderivative size = 2112, normalized size of antiderivative = 2.27

$$\int \frac{(f+gx)^2 \left(a+b\log \left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="fric
as")
```

```
[Out] 2/3*(h^3*x^2*sqrt(-(d*h^5*sqrt(-(b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b
^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^1
0)) + 12*(b^2*e*f^3*g + 3*b^2*d*f*g^3)*p^2)/(d*h^5))*log(16*(b^3*e^4*f^8 +
12*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 108*b^3*d^3*e*f^2*g^6 + 8
1*b^3*d^4*g^8)*sqrt(h*x)*p^3 + 16*(6*d^3*e*f*g*h^8*sqrt(-(b^4*e^4*f^8 - 60*
b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^6 + 81*b^
4*d^4*g^8)*p^4/(d^3*e*h^10)) + (b^2*d*e^3*f^6 - 27*b^2*d^2*e^2*f^4*g^2 - 81
*b^2*d^3*e*f^2*g^4 + 27*b^2*d^4*g^6)*h^3*p^2)*sqrt(-(d*h^5*sqrt(-(b^4*e^4*f
^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^6
 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) + 12*(b^2*e*f^3*g + 3*b^2*d*f*g^3)*p^2
)/(d*h^5))) - h^3*x^2*sqrt(-(d*h^5*sqrt(-(b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^
```

```
2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(
d^3*e*h^10)) + 12*(b^2*e*f^3*g + 3*b^2*d*f*g^3)*p^2)/(d*h^5))*log(16*(b^3*e
^4*f^8 + 12*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 108*b^3*d^3*e*f^
2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 - 16*(6*d^3*e*f*g*h^8*sqrt(-(b^4*e^4*
f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^
6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) + (b^2*d*e^3*f^6 - 27*b^2*d^2*e^2*f^4
*g^2 - 81*b^2*d^3*e*f^2*g^4 + 27*b^2*d^4*g^6)*h^3*p^2)*sqrt(-(d*h^5*sqrt(-(
b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*
e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) + 12*(b^2*e*f^3*g + 3*b^2*d*f
*g^3)*p^2)/(d*h^5))) - h^3*x^2*sqrt((d*h^5*sqrt(-(b^4*e^4*f^8 - 60*b^4*d*e^
3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^6 + 81*b^4*d^4*g^
8)*p^4/(d^3*e*h^10)) - 12*(b^2*e*f^3*g + 3*b^2*d*f*g^3)*p^2)/(d*h^5))*log(1
6*(b^3*e^4*f^8 + 12*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 108*b^3*
d^3*e*f^2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 + 16*(6*d^3*e*f*g*h^8*sqrt(-(
b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*
e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) - (b^2*d*e^3*f^6 - 27*b^2*d^2
*e^2*f^4*g^2 - 81*b^2*d^3*e*f^2*g^4 + 27*b^2*d^4*g^6)*h^3*p^2)*sqrt((d*h^5*
sqrt(-(b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b
^4*d^3*e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) - 12*(b^2*e*f^3*g + 3*
b^2*d*f*g^3)*p^2)/(d*h^5))) + h^3*x^2*sqrt((d*h^5*sqrt(-(b^4*e^4*f^8 - 60*b
^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b^4*d^3*e*f^2*g^6 + 81*b^4
*d^4*g^8)*p^4/(d^3*e*h^10)) - 12*(b^2*e*f^3*g + 3*b^2*d*f*g^3)*p^2)/(d*h^5)
)*log(16*(b^3*e^4*f^8 + 12*b^3*d*e^3*f^6*g^2 - 1242*b^3*d^2*e^2*f^4*g^4 + 1
08*b^3*d^3*e*f^2*g^6 + 81*b^3*d^4*g^8)*sqrt(h*x)*p^3 - 16*(6*d^3*e*f*g*h^8*
sqrt(-(b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4 - 540*b
^4*d^3*e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) - (b^2*d*e^3*f^6 - 27*
b^2*d^2*e^2*f^4*g^2 - 81*b^2*d^3*e*f^2*g^4 + 27*b^2*d^4*g^6)*h^3*p^2)*sqrt(
(d*h^5*sqrt(-(b^4*e^4*f^8 - 60*b^4*d*e^3*f^6*g^2 + 918*b^4*d^2*e^2*f^4*g^4
- 540*b^4*d^3*e*f^2*g^6 + 81*b^4*d^4*g^8)*p^4/(d^3*e*h^10)) - 12*(b^2*e*f^3
*g + 3*b^2*d*f*g^3)*p^2)/(d*h^5))) - (6*a*f*g*x + a*f^2 + 3*(4*b*g^2*p - a*
g^2)*x^2 - (3*b*g^2*p*x^2 - 6*b*f*g*p*x - b*f^2*p)*log(e*x^2 + d) - (3*b*g^
2*x^2 - 6*b*f*g*x - b*f^2)*log(c))*sqrt(h*x))/(h^3*x^2)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(5/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 1102, normalized size of antiderivative = 1.18

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] 2*b*g^2*x^3*log((e*x^2 + d)^p*c)/(h*x)^(5/2) - 2*b*e*f*g*p*(sqrt(2)*log(sqr
t(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1
/4)*e^(3/4)) - sqrt(2)*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^
(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(-(sqrt(2)*sqrt(-sq
rt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sq
rt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x
)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)) - sqrt(2)*log(-(sqrt(2)*sqrt
(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))
/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt
(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)))/h^2 + 2*a*g^2*x^3/(h*x)
^(5/2) - 4*b*f*g*x^2*log((e*x^2 + d)^p*c)/(h*x)^(5/2) + 1/3*(sqrt(2)*h^2*lo
g(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^
2)^(3/4)*e^(1/4)) - sqrt(2)*h^2*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqr
t(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) + sqrt(2)*h*log(-(sqrt(
2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*s
qrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) +
 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(d)) + sqrt(2)*h*log(-
(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(
h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(
1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(d)))*b*e*f^2*p/
h^3 - 4*a*f*g*x^2/(h*x)^(5/2) - 2/3*b*f^2*log((e*x^2 + d)^p*c)/((h*x)^(3/2)
*h) - (8*sqrt(h*x)*h^2/e - (sqrt(2)*h^4*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(
1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) - sqrt(2)*h^4*l
og(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h
^2)^(3/4)*e^(1/4)) + sqrt(2)*h^3*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + s
qrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*
sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-s
qrt(d)*sqrt(e)*h)*sqrt(d)) + sqrt(2)*h^3*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e
)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-
sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/
(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(d)))*d/e)*b*e*g^2*p/h^5 - 2/3*a*f^2/((h*x)^(
3/2)*h)
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 641, normalized size of antiderivative = 0.69

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \frac{2\left(3\sqrt{hx}bg^2p-\frac{6\,bfgh^2px+bf^2h^2p}{\sqrt{hx}hx}\right)\log\left(eh^2x^2+dh^2\right)-6\left(bg^2p\log\left(\right.\right.}{}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(5/2),x, algorithm="giac
")
```

```
[Out] 1/3*(2*(3*sqrt(h*x)*b*g^2*p - (6*b*f*g*h^2*p*x + b*f^2*h^2*p)/(sqrt(h*x)*h*
x))*log(e*h^2*x^2 + d*h^2) - 6*(b*g^2*p*log(h^2) + 4*b*g^2*p - b*g^2*log(c)
 - a*g^2)*sqrt(h*x) + 2*(6*b*f*g*h^2*p*x*log(h^2) + b*f^2*h^2*p*log(h^2) -
6*b*f*g*h^2*x*log(c) - 6*a*f*g*h^2*x - b*f^2*h^2*log(c) - a*f^2*h^2)/(sqrt(
h*x)*h*x) + 2*(sqrt(2)*(d*e^3*h^2)^(1/4)*b*e^2*f^2*h*p + 3*sqrt(2)*(d*e^3*h
^2)^(1/4)*b*d*e*g^2*h*p + 6*sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*g*p)*arctan(1/2*s
qrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) + 2*sqrt(h*x))/(d*h^2/e)^(1/4))/(d*e^2*h) +
 2*(sqrt(2)*(d*e^3*h^2)^(1/4)*b*e^2*f^2*h*p + 3*sqrt(2)*(d*e^3*h^2)^(1/4)*b
*d*e*g^2*h*p + 6*sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*g*p)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(d*h^2/e)^(1/4) - 2*sqrt(h*x))/(d*h^2/e)^(1/4))/(d*e^2*h) + (sqrt(2)*
(d*e^3*h^2)^(1/4)*b*e^2*f^2*h*p + 3*sqrt(2)*(d*e^3*h^2)^(1/4)*b*d*e*g^2*h*p
 - 6*sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*g*p)*log(h*x + sqrt(2)*(d*h^2/e)^(1/4)*s
qrt(h*x) + sqrt(d*h^2/e))/(d*e^2*h) - (sqrt(2)*(d*e^3*h^2)^(1/4)*b*e^2*f^2*
h*p + 3*sqrt(2)*(d*e^3*h^2)^(1/4)*b*d*e*g^2*h*p - 6*sqrt(2)*(d*e^3*h^2)^(3/
4)*b*f*g*p)*log(h*x - sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/(d
*e^2*h))/h^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{5/2}}\,dx = \int \frac{\left(f+g\,x\right)^2\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{5/2}}\,dx$$

```
[In] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(5/2),x)
```

```
[Out] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(5/2), x)
```

3.614 $\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{7/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 935

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = -\frac{8bef^2p}{5dh^3\sqrt{hx}}+\frac{2\sqrt{2}be^{5/4}f^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{7/2}} - \frac{4\sqrt{2}be^{3/4}fgp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}} - \frac{2\sqrt{2}b\sqrt[4]{e}g^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{7/2}} - \frac{2\sqrt{2}be^{5/4}f^2p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{7/2}} + \frac{4\sqrt{2}be^{3/4}fgp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}} + \frac{2\sqrt{2}b\sqrt[4]{e}g^2p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{7/2}} - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h(hx)^{5/2}} - \frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2(hx)^{3/2}} - \frac{2g^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^3\sqrt{hx}} - \frac{\sqrt{2}be^{5/4}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{7/2}} - \frac{2\sqrt{2}be^{3/4}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{7/2}} + \frac{\sqrt{2}b\sqrt[4]{e}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{7/2}} + \frac{\sqrt{2}be^{5/4}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{7/2}} + \frac{2\sqrt{2}be^{3/4}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{7/2}} - \frac{\sqrt{2}b\sqrt[4]{e}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{7/2}}$$

```
[Out] -2/5*f^2*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(5/2)-4/3*f*g*(a+b*ln(c*(e*x^2+d)^
p))/h^2/(h*x)^(3/2)+2/5*b*e^(5/4)*f^2*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2
)/d^(1/4)/h^(1/2))*2^(1/2)/d^(5/4)/h^(7/2)-4/3*b*e^(3/4)*f*g*p*arctan(1-e^(
1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(3/4)/h^(7/2)-2*b*e^(1/
4)*g^2*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(1
/4)/h^(7/2)-2/5*b*e^(5/4)*f^2*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4
)/h^(1/2))*2^(1/2)/d^(5/4)/h^(7/2)+4/3*b*e^(3/4)*f*g*p*arctan(1+e^(1/4)*2^(
1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(3/4)/h^(7/2)+2*b*e^(1/4)*g^2*p
*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(1/4)/h^(7
/2)-1/5*b*e^(5/4)*f^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4
)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(5/4)/h^(7/2)-2/3*b*e^(3/4)*f*g*p*ln(d^(1/
2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d
^(3/4)/h^(7/2)+b*e^(1/4)*g^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)
*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(1/4)/h^(7/2)+1/5*b*e^(5/4)*f^2*p*l
n(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^
(1/2)/d^(5/4)/h^(7/2)+2/3*b*e^(3/4)*f*g*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1
/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(3/4)/h^(7/2)-b*e^(1/4)*
g^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1
/2))*2^(1/2)/d^(1/4)/h^(7/2)-8/5*b*e*f^2*p/d/h^3/(h*x)^(1/2)-2*g^2*(a+b*ln(
c*(e*x^2+d)^p))/h^3/(h*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 935, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules

used = {2517, 2526, 2505, 331, 303, 1176, 631, 210, 1179, 642, 217}

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \frac{2\sqrt{2}be^{5/4}p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f^2}{5d^{5/4}h^{7/2}}$$
$$-\frac{2\sqrt{2}be^{5/4}p\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f^2}{5d^{5/4}h^{7/2}}-\frac{2\left(a+b\log\left(c(ex^2+d)^p\right)\right)f^2}{5h(hx)^{5/2}}$$
$$-\frac{\sqrt{2}be^{5/4}p\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)f^2}{5d^{5/4}h^{7/2}}$$
$$+\frac{\sqrt{2}be^{5/4}p\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)f^2}{5d^{5/4}h^{7/2}}-\frac{8bepf^2}{5dh^3\sqrt{hx}}$$
$$-\frac{4\sqrt{2}be^{3/4}gp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f}{3d^{3/4}h^{7/2}}+\frac{4\sqrt{2}be^{3/4}gp\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f}{3d^{3/4}h^{7/2}}$$
$$-\frac{4g\left(a+b\log\left(c(ex^2+d)^p\right)\right)f}{3h^2(hx)^{3/2}}-\frac{2\sqrt{2}be^{3/4}gp\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)f}{3d^{3/4}h^{7/2}}$$
$$+\frac{2\sqrt{2}be^{3/4}gp\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)f}{3d^{3/4}h^{7/2}}$$
$$-\frac{2\sqrt{2}b\sqrt[4]{e}g^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{7/2}}+\frac{2\sqrt{2}b\sqrt[4]{e}g^2p\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{\sqrt[4]{d}h^{7/2}}$$
$$-\frac{2g^2\left(a+b\log\left(c(ex^2+d)^p\right)\right)}{h^3\sqrt{hx}}+\frac{\sqrt{2}b\sqrt[4]{e}g^2p\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{7/2}}$$
$$-\frac{\sqrt{2}b\sqrt[4]{e}g^2p\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{7/2}}$$

```
[In] Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(7/2),x]
```

```
[Out] (-8*b*e*f^2*p)/(5*d*h^3*Sqrt[h*x]) + (2*Sqrt[2]*b*e^(5/4)*f^2*p*ArcTan[1 -
(Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(7/2)) - (4*Sq
rt[2]*b*e^(3/4)*f*g*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[
h])])/(3*d^(3/4)*h^(7/2)) - (2*Sqrt[2]*b*e^(1/4)*g^2*p*ArcTan[1 - (Sqrt[2]*
e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(7/2)) - (2*Sqrt[2]*b*e^(
5/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^
(5/4)*h^(7/2)) + (4*Sqrt[2]*b*e^(3/4)*f*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqr
t[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4)*h^(7/2)) + (2*Sqrt[2]*b*e^(1/4)*g^2*
p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*h^(7/
2)) - (2*f^2*(a + b*Log[c*(d + e*x^2)^p]))/(5*h*(h*x)^(5/2)) - (4*f*g*(a +
b*Log[c*(d + e*x^2)^p]))/(3*h^2*(h*x)^(3/2)) - (2*g^2*(a + b*Log[c*(d + e*x
```

```
^2)^p]))/(h^3*Sqrt[h*x]) - (Sqrt[2]*b*e^(5/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + S
qrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*d^(5/4)*h^(7/2))
- (2*Sqrt[2]*b*e^(3/4)*f*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt
[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(7/2)) + (Sqrt[2]*b*e^(1/4)*g^
2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[
h*x]])/(d^(1/4)*h^(7/2)) + (Sqrt[2]*b*e^(5/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + S
qrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(5*d^(5/4)*h^(7/2))
+ (2*Sqrt[2]*b*e^(3/4)*f*g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt
[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(7/2)) - (Sqrt[2]*b*e^(1/4)*g^
2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[
h*x]])/(d^(1/4)*h^(7/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^6}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{f^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^6} + \frac{2fg\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{hx^4} + \frac{g^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h^2x^2}\right)\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{(2g^2)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^2}\,dx, x, \sqrt{hx}\right)}{h^3} + \frac{(4fg)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^4}\,dx, x, \sqrt{hx}\right)}{h^2} + \frac{(2f^2)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^6}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= -\frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h(hx)^{5/2}} - \frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2(hx)^{3/2}} - \frac{2g^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^3\sqrt{hx}} + \frac{(8beg^2p)\,\text{Subst}\left(\int \frac{x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^5} + \frac{(16befgp)\text{Subst}\left(\int \frac{1}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{3h^4} + \frac{(8bef^2p)\,\text{Subst}\left(\int \frac{1}{x^2\left(d+\frac{ex^4}{h^2}\right)}\,dx, x, \sqrt{hx}\right)}{5h^3}$$

$$= -\frac{8bef^2p}{5dh^3\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h(hx)^{5/2}} - \frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2(hx)^{3/2}}$$

$$- \frac{2g^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^3\sqrt{hx}} - \frac{(8be^2f^2p)\,\mathrm{Subst}\left(\int \frac{x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{5dh^5}$$

$$+ \frac{(8befgp)\mathrm{Subst}\left(\int \frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{3\sqrt{d}h^5}$$

$$+ \frac{(8befgp)\mathrm{Subst}\left(\int \frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{3\sqrt{d}h^5}$$

$$- \frac{\left(4b\sqrt{e}g^2p\right)\,\mathrm{Subst}\left(\int \frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^5}$$

$$+ \frac{\left(4b\sqrt{e}g^2p\right)\,\mathrm{Subst}\left(\int \frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{h^5}$$

$$
\begin{aligned}
&= -\frac{8bef^2p}{5dh^3\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h(hx)^{5/2}} - \frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2(hx)^{3/2}} \\
&\quad - \frac{2g^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^3\sqrt{hx}} + \frac{\left(4be^{3/2}f^2p\right)\text{Subst}\left(\int \frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{5dh^5} \\
&\quad - \frac{\left(4be^{3/2}f^2p\right)\text{Subst}\left(\int \frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{5dh^5} \\
&\quad - \frac{\left(2\sqrt{2}be^{3/4}fgp\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{3d^{3/4}h^{7/2}} \\
&\quad - \frac{\left(2\sqrt{2}be^{3/4}fgp\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{3d^{3/4}h^{7/2}} \\
&\quad + \frac{\left(\sqrt{2}b\sqrt[4]{e}g^2p\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{\sqrt[4]{d}h^{7/2}} \\
&\quad + \frac{\left(\sqrt{2}b\sqrt[4]{e}g^2p\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{\sqrt[4]{d}h^{7/2}} \\
&\quad + \frac{\left(4b\sqrt{e}fgp\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{3\sqrt{d}h^3} \\
&\quad + \frac{\left(4b\sqrt{e}fgp\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{3\sqrt{d}h^3} \\
&\quad + \frac{\left(2bg^2p\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{h^3} \\
&\quad + \frac{\left(2bg^2p\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{h^3}
\end{aligned}
$$

$$= -\frac{8bef^2p}{5dh^3\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h(hx)^{5/2}} - \frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2(hx)^{3/2}} - \frac{2g^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^3\sqrt{hx}} - \frac{2\sqrt{2}be^{3/4}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{7/2}} + \frac{\sqrt{2}b\sqrt[4]{e}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{7/2}} + \frac{2\sqrt{2}be^{3/4}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{7/2}} - \frac{\sqrt{2}b\sqrt[4]{e}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{7/2}} - \frac{\left(\sqrt{2}be^{5/4}f^2p\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx,x,\sqrt{hx}\right)}{5d^{5/4}h^{7/2}} - \frac{\left(\sqrt{2}be^{5/4}f^2p\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx,x,\sqrt{hx}\right)}{5d^{5/4}h^{7/2}} + \frac{\left(4\sqrt{2}be^{3/4}fgp\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}} - \frac{\left(4\sqrt{2}be^{3/4}fgp\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}} + \frac{\left(2\sqrt{2}b\sqrt[4]{e}g^2p\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{7/2}} - \frac{\left(2\sqrt{2}b\sqrt[4]{e}g^2p\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{7/2}} - \frac{\left(2bef^2p\right)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx,x,\sqrt{hx}\right)}{5dh^3} - \frac{\left(2bef^2p\right)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx,x,\sqrt{hx}\right)}{5dh^3}$$

$$= -\frac{8bef^2p}{5dh^3\sqrt{hx}} - \frac{4\sqrt{2}be^{3/4}fgp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}}$$

$$- \frac{2\sqrt{2}b\sqrt[4]{e}g^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{7/2}} + \frac{4\sqrt{2}be^{3/4}fgp\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}}$$

$$+ \frac{2\sqrt{2}b\sqrt[4]{e}g^2p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{7/2}} - \frac{2f^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{5h(hx)^{5/2}}$$

$$- \frac{4fg\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{3h^2(hx)^{3/2}} - \frac{2g^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{h^3\sqrt{hx}}$$

$$- \frac{\sqrt{2}be^{5/4}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{7/2}}$$

$$- \frac{2\sqrt{2}be^{3/4}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{7/2}}$$

$$+ \frac{\sqrt{2}b\sqrt[4]{e}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{7/2}}$$

$$+ \frac{\sqrt{2}be^{5/4}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{7/2}}$$

$$+ \frac{2\sqrt{2}be^{3/4}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{7/2}}$$

$$- \frac{\sqrt{2}b\sqrt[4]{e}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{7/2}}$$

$$- \frac{\left(2\sqrt{2}be^{5/4}f^2p\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{7/2}}$$

$$+ \frac{\left(2\sqrt{2}be^{5/4}f^2p\right)\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{7/2}}$$

$$
\begin{aligned}
&= -\frac{8bef^2p}{5dh^3\sqrt{hx}} + \frac{2\sqrt{2}be^{5/4}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{7/2}} \\
&\quad - \frac{4\sqrt{2}be^{3/4}fgp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}} - \frac{2\sqrt{2}b\sqrt[4]{e}g^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{7/2}} \\
&\quad - \frac{2\sqrt{2}be^{5/4}f^2p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{7/2}} + \frac{4\sqrt{2}be^{3/4}fgp\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{7/2}} \\
&\quad + \frac{2\sqrt{2}b\sqrt[4]{e}g^2p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}h^{7/2}} - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h(hx)^{5/2}} \\
&\quad - \frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^2(hx)^{3/2}} - \frac{2g^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{h^3\sqrt{hx}} \\
&\quad - \frac{\sqrt{2}be^{5/4}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{7/2}} \\
&\quad - \frac{2\sqrt{2}be^{3/4}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{7/2}} \\
&\quad + \frac{\sqrt{2}b\sqrt[4]{e}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{7/2}} \\
&\quad + \frac{\sqrt{2}be^{5/4}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{7/2}} \\
&\quad + \frac{2\sqrt{2}be^{3/4}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{7/2}} \\
&\quad - \frac{\sqrt{2}b\sqrt[4]{e}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}h^{7/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.64 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.36

$$
\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \frac{2x^{7/2}\left(\frac{2b\sqrt[4]{e}g^2p\left(\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{-d}}\right)+\operatorname{arctanh}\left(\frac{d\sqrt[4]{e}\sqrt{x}}{(-d)^{5/4}}\right)\right)}{\sqrt[4]{-d}} - \frac{4bef^2p\,\text{Hyperge}\cdots}{\cdots}\right.}{\cdots}
$$

```
[In] Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(7/2),x]
```

```
[Out] (2*x^(7/2)*((2*b*e^(1/4)*g^2*p*(ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + ArcT
anh[(d*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (4*b*e*f^2*p*Hypergeomet
ric2F1[-1/4, 1, 3/4, -((e*x^2)/d)])/(5*d*Sqrt[x]) - (Sqrt[2]*b*e^(3/4)*f*g*
p*(2*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*
e^(1/4)*Sqrt[x])/d^(1/4)] + Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] +
 Sqrt[e]*x] - Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x]))/
(3*d^(3/4)) - (f^2*(a + b*Log[c*(d + e*x^2)^p]))/(5*x^(5/2)) - (2*f*g*(a +
b*Log[c*(d + e*x^2)^p]))/(3*x^(3/2)) - (g^2*(a + b*Log[c*(d + e*x^2)^p]))/S
qrt[x]))/(h*x)^(7/2)
```

Maple [F]

$$\int \frac{(gx+f)^2 \left(a+b\ln\left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{7}{2}}}dx$$

```
[In] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(7/2),x)
```

```
[Out] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(7/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2205 vs. $2(653) = 1306$.

Time = 0.46 (sec) , antiderivative size = 2205, normalized size of antiderivative = 2.36

$$\int \frac{(f+gx)^2 \left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="fric
as")
```

```
[Out] -2/15*(d*h^4*x^3*sqrt((d^2*h^7*sqrt(-(81*b^4*e^5*f^8 - 3420*b^4*d*e^4*f^6*g
^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e^2*f^2*g^6 + 50625*b^4*d^4*
e*g^8)*p^4/(d^5*h^14)) + 60*(b^2*e^2*f^3*g - 5*b^2*d*e*f*g^3)*p^2)/(d^2*h^7
))*log(32*(81*b^3*e^5*f^8 - 1620*b^3*d*e^4*f^6*g^2 + 2150*b^3*d^2*e^3*f^4*g
^4 - 40500*b^3*d^3*e^2*f^2*g^6 + 50625*b^3*d^4*e*g^8)*sqrt(h*x)*p^3 + 32*(3
*(d^4*e*f^2 - 5*d^5*g^2)*h^11*sqrt(-(81*b^4*e^5*f^8 - 3420*b^4*d*e^4*f^6*g^
2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e^2*f^2*g^6 + 50625*b^4*d^4*e
*g^8)*p^4/(d^5*h^14)) - 10*(9*b^2*d^2*e^3*f^5*g - 190*b^2*d^3*e^2*f^3*g^3 +
 225*b^2*d^4*e*f*g^5)*h^4*p^2)*sqrt((d^2*h^7*sqrt(-(81*b^4*e^5*f^8 - 3420*b
^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e^2*f^2*g^6 +
50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) + 60*(b^2*e^2*f^3*g - 5*b^2*d*e*f*g^3)
*p^2)/(d^2*h^7))) - d*h^4*x^3*sqrt((d^2*h^7*sqrt(-(81*b^4*e^5*f^8 - 3420*b^
4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e^2*f^2*g^6 + 5
0625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) + 60*(b^2*e^2*f^3*g - 5*b^2*d*e*f*g^3)*
p^2)/(d^2*h^7))*log(32*(81*b^3*e^5*f^8 - 1620*b^3*d*e^4*f^6*g^2 + 2150*b^3*
```

```
d^2*e^3*f^4*g^4 - 40500*b^3*d^3*e^2*f^2*g^6 + 50625*b^3*d^4*e*g^8)*sqrt(h*x
)*p^3 - 32*(3*(d^4*e*f^2 - 5*d^5*g^2)*h^11*sqrt(-(81*b^4*e^5*f^8 - 3420*b^4
*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e^2*f^2*g^6 + 50
625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) - 10*(9*b^2*d^2*e^3*f^5*g - 190*b^2*d^3*
e^2*f^3*g^3 + 225*b^2*d^4*e*f*g^5)*h^4*p^2)*sqrt((d^2*h^7*sqrt(-(81*b^4*e^5
*f^8 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e
^2*f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) + 60*(b^2*e^2*f^3*g - 5*b
^2*d*e*f*g^3)*p^2)/(d^2*h^7))) - d*h^4*x^3*sqrt(-(d^2*h^7*sqrt(-(81*b^4*e^5
*f^8 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e
^2*f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) - 60*(b^2*e^2*f^3*g - 5*b
^2*d*e*f*g^3)*p^2)/(d^2*h^7))*log(32*(81*b^3*e^5*f^8 - 1620*b^3*d*e^4*f^6*g
^2 + 2150*b^3*d^2*e^3*f^4*g^4 - 40500*b^3*d^3*e^2*f^2*g^6 + 50625*b^3*d^4*e
*g^8)*sqrt(h*x)*p^3 + 32*(3*(d^4*e*f^2 - 5*d^5*g^2)*h^11*sqrt(-(81*b^4*e^5*
f^8 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 85500*b^4*d^3*e^
2*f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) + 10*(9*b^2*d^2*e^3*f^5*g
- 190*b^2*d^3*e^2*f^3*g^3 + 225*b^2*d^4*e*f*g^5)*h^4*p^2)*sqrt(-(d^2*h^7*sq
rt(-(81*b^4*e^5*f^8 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 -
85500*b^4*d^3*e^2*f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) - 60*(b^2*
e^2*f^3*g - 5*b^2*d*e*f*g^3)*p^2)/(d^2*h^7))) + d*h^4*x^3*sqrt(-(d^2*h^7*sq
rt(-(81*b^4*e^5*f^8 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 -
85500*b^4*d^3*e^2*f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) - 60*(b^2*
e^2*f^3*g - 5*b^2*d*e*f*g^3)*p^2)/(d^2*h^7))*log(32*(81*b^3*e^5*f^8 - 1620*
b^3*d*e^4*f^6*g^2 + 2150*b^3*d^2*e^3*f^4*g^4 - 40500*b^3*d^3*e^2*f^2*g^6 +
50625*b^3*d^4*e*g^8)*sqrt(h*x)*p^3 - 32*(3*(d^4*e*f^2 - 5*d^5*g^2)*h^11*sqr
t(-(81*b^4*e^5*f^8 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2*e^3*f^4*g^4 - 8
5500*b^4*d^3*e^2*f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^14)) + 10*(9*b^2
*d^2*e^3*f^5*g - 190*b^2*d^3*e^2*f^3*g^3 + 225*b^2*d^4*e*f*g^5)*h^4*p^2)*sq
rt(-(d^2*h^7*sqrt(-(81*b^4*e^5*f^8 - 3420*b^4*d*e^4*f^6*g^2 + 40150*b^4*d^2
*e^3*f^4*g^4 - 85500*b^4*d^3*e^2*f^2*g^6 + 50625*b^4*d^4*e*g^8)*p^4/(d^5*h^
14)) - 60*(b^2*e^2*f^3*g - 5*b^2*d*e*f*g^3)*p^2)/(d^2*h^7))) + (10*a*d*f*g*
x + 3*a*d*f^2 + 3*(4*b*e*f^2*p + 5*a*d*g^2)*x^2 + (15*b*d*g^2*p*x^2 + 10*b*
d*f*g*p*x + 3*b*d*f^2*p)*log(e*x^2 + d) + (15*b*d*g^2*x^2 + 10*b*d*f*g*x +
3*b*d*f^2)*log(c))*sqrt(h*x))/(d*h^4*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2 \left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 1088, normalized size of antiderivative = 1.16

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="maxi
ma")
```

```
[Out] -2*b*g^2*x^3*log((e*x^2 + d)^p*c)/(h*x)^(7/2) + 1/5*b*e*f^2*p*(e*(sqrt(2)*l
og(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h
^2)^(1/4)*e^(3/4)) - sqrt(2)*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h
*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(-(sqrt(2)*sq
rt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e
))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sq
rt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)) - sqrt(2)*log(-(sqrt(2
)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sq
rt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) +
2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(e)))/d - 8/(sqrt(h*x)*
d))/h^3 - b*e*g^2*p*(sqrt(2)*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h
*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(sqrt(e)*h*x
- sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/
4)) - sqrt(2)*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4
)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2
)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*s
qrt(e)) - sqrt(2)*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^
(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sq
rt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*
h)*sqrt(e)))/h^3 - 2*a*g^2*x^3/(h*x)^(7/2) - 4/3*b*f*g*x^2*log((e*x^2 + d)^
p*c)/(h*x)^(7/2) + 2/3*(sqrt(2)*h^2*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)
*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) - sqrt(2)*h^2*log(s
qrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^
(3/4)*e^(1/4)) + sqrt(2)*h*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)
*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e
)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)
*sqrt(e)*h)*sqrt(d)) + sqrt(2)*h*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - s
qrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*
sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-s
qrt(d)*sqrt(e)*h)*sqrt(d)))*b*e*f*g*p/h^4 - 4/3*a*f*g*x^2/(h*x)^(7/2) - 2/5
*b*f^2*log((e*x^2 + d)^p*c)/((h*x)^(5/2)*h) - 2/5*a*f^2/((h*x)^(5/2)*h)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 675, normalized size of antiderivative = 0.72

$$\int \frac{(f+gx)^2 \left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx =$$

$$-\frac{\frac{2\left(15\,bg^2h^3px^2+10\,bfgh^3px+3\,bf^2h^3p\right)\log\left(eh^2x^2+dh^2\right)}{\sqrt{hx}h^2x^2}-\frac{2\left(10\,\sqrt{2}\left(de^3h^2\right)^{\frac{1}{4}}bde^2fghp-3\,\sqrt{2}\left(de^3h^2\right)^{\frac{3}{4}}bef^2p+15\,\sqrt{2}\left(de^3h^2\right)^{\frac{3}{4}}bdg^2p\right)\arctan\left(\right.}{d^2e^2h}}{}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(7/2),x, algorithm="giac
")
```

```
[Out] -1/15*(2*(15*b*g^2*h^3*p*x^2 + 10*b*f*g*h^3*p*x + 3*b*f^2*h^3*p)*log(e*h^2*
x^2 + d*h^2)/(sqrt(h*x)*h^2*x^2) - 2*(10*sqrt(2)*(d*e^3*h^2)^(1/4)*b*d*e^2*
f*g*h*p - 3*sqrt(2)*(d*e^3*h^2)^(3/4)*b*e*f^2*p + 15*sqrt(2)*(d*e^3*h^2)^(3
/4)*b*d*g^2*p)*arctan(1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) + 2*sqrt(h*x))/(
d*h^2/e)^(1/4))/(d^2*e^2*h) - 2*(10*sqrt(2)*(d*e^3*h^2)^(1/4)*b*d*e^2*f*g*h
*p - 3*sqrt(2)*(d*e^3*h^2)^(3/4)*b*e*f^2*p + 15*sqrt(2)*(d*e^3*h^2)^(3/4)*b
*d*g^2*p)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) - 2*sqrt(h*x))/(d*h^
2/e)^(1/4))/(d^2*e^2*h) - (10*sqrt(2)*(d*e^3*h^2)^(1/4)*b*d*e^2*f*g*h*p + 3
*sqrt(2)*(d*e^3*h^2)^(3/4)*b*e*f^2*p - 15*sqrt(2)*(d*e^3*h^2)^(3/4)*b*d*g^2
*p)*log(h*x + sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/(d^2*e^2*h
) + (10*sqrt(2)*(d*e^3*h^2)^(1/4)*b*d*e^2*f*g*h*p + 3*sqrt(2)*(d*e^3*h^2)^(
3/4)*b*e*f^2*p - 15*sqrt(2)*(d*e^3*h^2)^(3/4)*b*d*g^2*p)*log(h*x - sqrt(2)*
(d*h^2/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/(d^2*e^2*h) - 2*(15*b*d*g^2*h^3*
p*x^2*log(h^2) - 12*b*e*f^2*h^3*p*x^2 + 10*b*d*f*g*h^3*p*x*log(h^2) - 15*b*
d*g^2*h^3*x^2*log(c) - 15*a*d*g^2*h^3*x^2 + 3*b*d*f^2*h^3*p*log(h^2) - 10*b
*d*f*g*h^3*x*log(c) - 10*a*d*f*g*h^3*x - 3*b*d*f^2*h^3*log(c) - 3*a*d*f^2*h
^3)/(sqrt(h*x)*d*h^2*x^2))/h^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2 \left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{7/2}}\,dx = \int \frac{\left(f+g\,x\right)^2\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{{\left(h\,x\right)}^{7/2}} \,dx$$

```
[In] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(7/2),x)
```

```
[Out] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(7/2), x)
```

3.615 $\int \frac{(f+gx)^2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{(hx)^{9/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 968

$$
\begin{aligned}
&\int \frac{(f+gx)^2 \left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = -\frac{8bef^2p}{21dh^3(hx)^{3/2}} \\
&-\frac{16befgp}{5dh^4\sqrt{hx}}+\frac{2\sqrt{2}be^{7/4}f^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{7d^{7/4}h^{9/2}} \\
&+\frac{4\sqrt{2}be^{5/4}fgp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{9/2}} \\
&-\frac{2\sqrt{2}be^{3/4}g^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{9/2}} \\
&-\frac{2\sqrt{2}be^{7/4}f^2p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{7d^{7/4}h^{9/2}} \\
&-\frac{4\sqrt{2}be^{5/4}fgp\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{9/2}} \\
&+\frac{2\sqrt{2}be^{3/4}g^2p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{9/2}}-\frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{7h(hx)^{7/2}} \\
&-\frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h^2(hx)^{5/2}}-\frac{2g^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^3(hx)^{3/2}} \\
&+\frac{\sqrt{2}be^{7/4}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{7d^{7/4}h^{9/2}} \\
&-\frac{2\sqrt{2}be^{5/4}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{9/2}} \\
&-\frac{\sqrt{2}be^{3/4}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{9/2}} \\
&-\frac{\sqrt{2}be^{7/4}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{7d^{7/4}h^{9/2}} \\
&+\frac{2\sqrt{2}be^{5/4}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{9/2}} \\
&+\frac{\sqrt{2}be^{3/4}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{9/2}}
\end{aligned}
$$

```
[Out] -8/21*b*e*f^2*p/d/h^3/(h*x)^(3/2)-2/7*f^2*(a+b*ln(c*(e*x^2+d)^p))/h/(h*x)^(
7/2)-4/5*f*g*(a+b*ln(c*(e*x^2+d)^p))/h^2/(h*x)^(5/2)-2/3*g^2*(a+b*ln(c*(e*x
```

```
^2+d)^p))/h^3/(h*x)^(3/2)+2/7*b*e^(7/4)*f^2*p*arctan(1-e^(1/4)*2^(1/2)*(h*x
)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(7/4)/h^(9/2)+4/5*b*e^(5/4)*f*g*p*arctan
(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(5/4)/h^(9/2)-2/3
*b*e^(3/4)*g^2*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1
/2)/d^(3/4)/h^(9/2)-2/7*b*e^(7/4)*f^2*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2
)/d^(1/4)/h^(1/2))*2^(1/2)/d^(7/4)/h^(9/2)-4/5*b*e^(5/4)*f*g*p*arctan(1+e^(
1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^(5/4)/h^(9/2)+2/3*b*e^(
3/4)*g^2*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2)/d^
(3/4)/h^(9/2)+1/7*b*e^(7/4)*f^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1
/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(7/4)/h^(9/2)-2/5*b*e^(5/4)*f*g*
p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))
*2^(1/2)/d^(5/4)/h^(9/2)-1/3*b*e^(3/4)*g^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h
^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(3/4)/h^(9/2)-1/7*b*e
^(7/4)*f^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(
h*x)^(1/2))*2^(1/2)/d^(7/4)/h^(9/2)+2/5*b*e^(5/4)*f*g*p*ln(d^(1/2)*h^(1/2)+
x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(5/4)/h^(9
/2)+1/3*b*e^(3/4)*g^2*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4
)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(3/4)/h^(9/2)-16/5*b*e*f*g*p/d/h^4/(h*x)^(
1/2)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 968, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules

used = {2517, 2526, 2505, 331, 217, 1179, 642, 1176, 631, 210, 303}

$$\int \frac{(f+gx)^2 \left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = \frac{2\sqrt{2}be^{7/4}p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f^2}{7d^{7/4}h^{9/2}}$$
$$-\frac{2\sqrt{2}be^{7/4}p\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f^2}{7d^{7/4}h^{9/2}}-\frac{2\left(a+b\log\left(c(ex^2+d)^p\right)\right)f^2}{7h(hx)^{7/2}}$$
$$+\frac{\sqrt{2}be^{7/4}p\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)f^2}{7d^{7/4}h^{9/2}}$$
$$-\frac{\sqrt{2}be^{7/4}p\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)f^2}{7d^{7/4}h^{9/2}}-\frac{8bepf^2}{21dh^3(hx)^{3/2}}$$
$$+\frac{4\sqrt{2}be^{5/4}gp\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)f}{5d^{5/4}h^{9/2}}-\frac{4\sqrt{2}be^{5/4}gp\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)f}{5d^{5/4}h^{9/2}}$$
$$-\frac{4g\left(a+b\log\left(c(ex^2+d)^p\right)\right)f}{5h^2(hx)^{5/2}}-\frac{2\sqrt{2}be^{5/4}gp\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)f}{5d^{5/4}h^{9/2}}$$
$$+\frac{2\sqrt{2}be^{5/4}gp\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)f}{5d^{5/4}h^{9/2}}-\frac{16begpf}{5dh^4\sqrt{hx}}$$
$$-\frac{2\sqrt{2}be^{3/4}g^2p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{9/2}}+\frac{2\sqrt{2}be^{3/4}g^2p\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{3d^{3/4}h^{9/2}}$$
$$-\frac{2g^2\left(a+b\log\left(c(ex^2+d)^p\right)\right)}{3h^3(hx)^{3/2}}-\frac{\sqrt{2}be^{3/4}g^2p\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{9/2}}$$
$$+\frac{\sqrt{2}be^{3/4}g^2p\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{9/2}}$$

```
[In] Int[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(9/2),x]
```

```
[Out] (-8*b*e*f^2*p)/(21*d*h^3*(h*x)^(3/2)) - (16*b*e*f*g*p)/(5*d*h^4*Sqrt[h*x])
+ (2*Sqrt[2]*b*e^(7/4)*f^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4
)*Sqrt[h])])/(7*d^(7/4)*h^(9/2)) + (4*Sqrt[2]*b*e^(5/4)*f*g*p*ArcTan[1 - (S
qrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^(5/4)*h^(9/2)) - (2*Sqrt
[2]*b*e^(3/4)*g^2*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h]
)])/(3*d^(3/4)*h^(9/2)) - (2*Sqrt[2]*b*e^(7/4)*f^2*p*ArcTan[1 + (Sqrt[2]*e^
(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(7*d^(7/4)*h^(9/2)) - (4*Sqrt[2]*b*e^(
5/4)*f*g*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(5*d^
(5/4)*h^(9/2)) + (2*Sqrt[2]*b*e^(3/4)*g^2*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqr
t[h*x])/(d^(1/4)*Sqrt[h])])/(3*d^(3/4)*h^(9/2)) - (2*f^2*(a + b*Log[c*(d +
e*x^2)^p]))/(7*h*(h*x)^(7/2)) - (4*f*g*(a + b*Log[c*(d + e*x^2)^p]))/(5*h^2
```

```
*(h*x)^(5/2)) - (2*g^2*(a + b*Log[c*(d + e*x^2)^p]))/(3*h^3*(h*x)^(3/2)) +
(Sqrt[2]*b*e^(7/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*
d^(1/4)*e^(1/4)*Sqrt[h*x]])/(7*d^(7/4)*h^(9/2)) - (2*Sqrt[2]*b*e^(5/4)*f*g*
p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*
x]])/(5*d^(5/4)*h^(9/2)) - (Sqrt[2]*b*e^(3/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] + S
qrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(9/2))
- (Sqrt[2]*b*e^(7/4)*f^2*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2
]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(7*d^(7/4)*h^(9/2)) + (2*Sqrt[2]*b*e^(5/4)*f*
g*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[
h*x]])/(5*d^(5/4)*h^(9/2)) + (Sqrt[2]*b*e^(3/4)*g^2*p*Log[Sqrt[d]*Sqrt[h] +
 Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(3*d^(3/4)*h^(9/2)
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{\left(f+\frac{gx^2}{h}\right)^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^8}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{f^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{x^8} + \frac{2fg\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{hx^6} + \frac{g^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{h^2x^4}\right)dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{(2g^2)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^4}\,dx, x, \sqrt{hx}\right)}{h^3} + \frac{(4fg)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^6}\,dx, x, \sqrt{hx}\right)}{h^2} + \frac{(2f^2)\,\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^8}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= -\frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{7h(hx)^{7/2}} - \frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h^2(hx)^{5/2}} - \frac{2g^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^3(hx)^{3/2}} + \frac{(8beg^2p)\,\text{Subst}\left(\int \frac{1}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{3h^5} + \frac{(16befgp)\text{Subst}\left(\int \frac{1}{x^2\left(d+\frac{ex^4}{h^2}\right)}\,dx, x, \sqrt{hx}\right)}{5h^4} + \frac{(8bef^2p)\,\text{Subst}\left(\int \frac{1}{x^4\left(d+\frac{ex^4}{h^2}\right)}\,dx, x, \sqrt{hx}\right)}{7h^3}$$

$$
\begin{aligned}
&= -\frac{8bef^2p}{21dh^3(hx)^{3/2}} - \frac{16befgp}{5dh^4\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{7h(hx)^{7/2}} \\
&\quad - \frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h^2(hx)^{5/2}} - \frac{2g^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^3(hx)^{3/2}} \\
&\quad - \frac{(16be^2fgp)\,\mathrm{Subst}\left(\int \frac{x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{5dh^6} \\
&\quad + \frac{(4beg^2p)\,\mathrm{Subst}\left(\int \frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{3\sqrt{d}h^6} \\
&\quad + \frac{(4beg^2p)\,\mathrm{Subst}\left(\int \frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{3\sqrt{d}h^6} \\
&\quad - \frac{(8be^2f^2p)\,\mathrm{Subst}\left(\int \frac{1}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{7dh^5}
\end{aligned}
$$

$$= -\frac{8bef^2p}{21dh^3(hx)^{3/2}} - \frac{16befgp}{5dh^4\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{7h(hx)^{7/2}}$$

$$- \frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h^2(hx)^{5/2}} - \frac{2g^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^3(hx)^{3/2}}$$

$$- \frac{(4be^2f^2p)\,\mathrm{Subst}\left(\int \frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{7d^{3/2}h^6}$$

$$- \frac{(4be^2f^2p)\,\mathrm{Subst}\left(\int \frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{7d^{3/2}h^6}$$

$$+ \frac{(8be^{3/2}fgp)\,\mathrm{Subst}\left(\int \frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{5dh^6}$$

$$- \frac{(8be^{3/2}fgp)\,\mathrm{Subst}\left(\int \frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{5dh^6}$$

$$- \frac{(\sqrt{2}be^{3/4}g^2p)\,\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{3d^{3/4}h^{9/2}}$$

$$- \frac{(\sqrt{2}be^{3/4}g^2p)\,\mathrm{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{3d^{3/4}h^{9/2}}$$

$$+ \frac{(2b\sqrt{e}g^2p)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{3\sqrt{d}h^4}$$

$$+ \frac{(2b\sqrt{e}g^2p)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{3\sqrt{d}h^4}$$

$$
\begin{aligned}
&= -\frac{8bef^2p}{21dh^3(hx)^{3/2}} - \frac{16befgp}{5dh^4\sqrt{hx}} - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{7h(hx)^{7/2}} \\
&- \frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h^2(hx)^{5/2}} - \frac{2g^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^3(hx)^{3/2}} \\
&- \frac{\sqrt{2}be^{3/4}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{9/2}} \\
&+ \frac{\sqrt{2}be^{3/4}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{9/2}} \\
&+ \frac{\left(\sqrt{2}be^{7/4}f^2p\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{dh}}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{hx}}{\sqrt[4]{e}}-x^2}\,dx,x,\sqrt{hx}\right)}{7d^{7/4}h^{9/2}} \\
&+ \frac{\left(\sqrt{2}be^{7/4}f^2p\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{dh}}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{hx}}{\sqrt[4]{e}}-x^2}\,dx,x,\sqrt{hx}\right)}{7d^{7/4}h^{9/2}} \\
&- \frac{\left(2\sqrt{2}be^{5/4}fgp\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{dh}}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{hx}}{\sqrt[4]{e}}-x^2}\,dx,x,\sqrt{hx}\right)}{5d^{5/4}h^{9/2}} \\
&- \frac{\left(2\sqrt{2}be^{5/4}fgp\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{dh}}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{hx}}{\sqrt[4]{e}}-x^2}\,dx,x,\sqrt{hx}\right)}{5d^{5/4}h^{9/2}} \\
&+ \frac{\left(2\sqrt{2}be^{3/4}g^2p\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{9/2}} \\
&- \frac{\left(2\sqrt{2}be^{3/4}g^2p\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{9/2}} \\
&- \frac{\left(2be^{3/2}f^2p\right)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{dh}}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{hx}}{\sqrt[4]{e}}+x^2}\,dx,x,\sqrt{hx}\right)}{7d^{3/2}h^4} \\
&- \frac{\left(2be^{3/2}f^2p\right)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{dh}}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{hx}}{\sqrt[4]{e}}+x^2}\,dx,x,\sqrt{hx}\right)}{7d^{3/2}h^4} \\
&- \frac{(4befgp)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{dh}}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{hx}}{\sqrt[4]{e}}+x^2}\,dx,x,\sqrt{hx}\right)}{5dh^4}
\end{aligned}
$$

$$= -\frac{8bef^2p}{21dh^3(hx)^{3/2}} - \frac{16befgp}{5dh^4\sqrt{hx}} - \frac{2\sqrt{2}be^{3/4}g^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{9/2}}$$
$$+ \frac{2\sqrt{2}be^{3/4}g^2p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{9/2}} - \frac{2f^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{7h(hx)^{7/2}}$$
$$- \frac{4fg\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{5h^2(hx)^{5/2}} - \frac{2g^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{3h^3(hx)^{3/2}}$$
$$+ \frac{\sqrt{2}be^{7/4}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{7d^{7/4}h^{9/2}}$$
$$- \frac{2\sqrt{2}be^{5/4}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{9/2}}$$
$$- \frac{\sqrt{2}be^{3/4}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{9/2}}$$
$$- \frac{\sqrt{2}be^{7/4}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{7d^{7/4}h^{9/2}}$$
$$+ \frac{2\sqrt{2}be^{5/4}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{9/2}}$$
$$+ \frac{\sqrt{2}be^{3/4}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{9/2}}$$
$$- \frac{\left(2\sqrt{2}be^{7/4}f^2p\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{7d^{7/4}h^{9/2}}$$
$$+ \frac{\left(2\sqrt{2}be^{7/4}f^2p\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{7d^{7/4}h^{9/2}}$$
$$- \frac{\left(4\sqrt{2}be^{5/4}fgp\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{9/2}}$$
$$+ \frac{\left(4\sqrt{2}be^{5/4}fgp\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{9/2}}$$

$$= -\frac{8bef^2p}{21dh^3(hx)^{3/2}} - \frac{16befgp}{5dh^4\sqrt{hx}} + \frac{2\sqrt{2}be^{7/4}f^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{7d^{7/4}h^{9/2}}$$
$$+ \frac{4\sqrt{2}be^{5/4}fgp\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{9/2}} - \frac{2\sqrt{2}be^{3/4}g^2p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{9/2}}$$
$$- \frac{2\sqrt{2}be^{7/4}f^2p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{7d^{7/4}h^{9/2}} - \frac{4\sqrt{2}be^{5/4}fgp\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{5d^{5/4}h^{9/2}}$$
$$+ \frac{2\sqrt{2}be^{3/4}g^2p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{3d^{3/4}h^{9/2}} - \frac{2f^2(a+b\log(c(d+ex^2)^p))}{7h(hx)^{7/2}}$$
$$- \frac{4fg(a+b\log(c(d+ex^2)^p))}{5h^2(hx)^{5/2}} - \frac{2g^2(a+b\log(c(d+ex^2)^p))}{3h^3(hx)^{3/2}}$$
$$+ \frac{\sqrt{2}be^{7/4}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{7d^{7/4}h^{9/2}}$$
$$- \frac{2\sqrt{2}be^{5/4}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{9/2}}$$
$$- \frac{\sqrt{2}be^{3/4}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{9/2}}$$
$$- \frac{\sqrt{2}be^{7/4}f^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{7d^{7/4}h^{9/2}}$$
$$+ \frac{2\sqrt{2}be^{5/4}fgp\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{5d^{5/4}h^{9/2}}$$
$$+ \frac{\sqrt{2}be^{3/4}g^2p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{3d^{3/4}h^{9/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.30

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = \frac{x\left(-40bef^2px^2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},1,\frac{1}{4},-\frac{ex^2}{d}\right)-336befg\right.}{}$$

```
[In] Integrate[((f + g*x)^2*(a + b*Log[c*(d + e*x^2)^p]))/(h*x)^(9/2),x]
```

```
[Out] (x*(-40*b*e*f^2*p*x^2*Hypergeometric2F1[-3/4, 1, 1/4, -((e*x^2)/d)] - 336*b
*e*f*g*p*x^3*Hypergeometric2F1[-1/4, 1, 3/4, -((e*x^2)/d)] - 35*Sqrt[2]*b*d
```

```
^(1/4)*e^(3/4)*g^2*p*x^(7/2)*(2*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4
)] - 2*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] + Log[Sqrt[d] - Sqrt[2
]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] - Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1
/4)*Sqrt[x] + Sqrt[e]*x]) - 30*d*f^2*(a + b*Log[c*(d + e*x^2)^p]) - 84*d*f*
g*x*(a + b*Log[c*(d + e*x^2)^p]) - 70*d*g^2*x^2*(a + b*Log[c*(d + e*x^2)^p]
)))/(105*d*(h*x)^(9/2))
```

Maple [F]

$$\int \frac{(gx+f)^2 \left(a+b\ln \left(c(e\,x^2+d)^p\right)\right)}{(hx)^{\frac{9}{2}}}dx$$

```
[In] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(9/2),x)
```

```
[Out] int((g*x+f)^2*(a+b*ln(c*(e*x^2+d)^p))/(h*x)^(9/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2283 vs. $2(672) = 1344$.

Time = 0.43 (sec) , antiderivative size = 2283, normalized size of antiderivative = 2.36

$$\int \frac{(f+gx)^2 \left(a+b\log \left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="fric
as")
```

```
[Out] -2/105*(d*h^5*x^4*sqrt(-(d^3*h^9*sqrt(-(50625*b^4*e^7*f^8 - 1266300*b^4*d*e
^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 15
00625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18)) + 420*(3*b^2*e^3*f^3*g - 7*b^2*d*e^2
*f*g^3)*p^2)/(d^3*h^9))*log(16*(50625*b^3*e^6*f^8 - 472500*b^3*d*e^5*f^6*g^
2 - 1457946*b^3*d^2*e^4*f^4*g^4 - 2572500*b^3*d^3*e^3*f^2*g^6 + 1500625*b^3
*d^4*e^2*g^8)*sqrt(h*x)*p^3 + 16*(42*d^6*f*g*h^14*sqrt(-(50625*b^4*e^7*f^8
- 1266300*b^4*d*e^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3
*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18)) + 5*(675*b^2*d^2*e^
4*f^6 - 10017*b^2*d^3*e^3*f^4*g^2 + 23373*b^2*d^4*e^2*f^2*g^4 - 8575*b^2*d^
5*e*g^6)*h^5*p^2)*sqrt(-(d^3*h^9*sqrt(-(50625*b^4*e^7*f^8 - 1266300*b^4*d*e
^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 15
00625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18)) + 420*(3*b^2*e^3*f^3*g - 7*b^2*d*e^2
*f*g^3)*p^2)/(d^3*h^9))) - d*h^5*x^4*sqrt(-(d^3*h^9*sqrt(-(50625*b^4*e^7*f^
8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d
^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18)) + 420*(3*b^2*e^3*
f^3*g - 7*b^2*d*e^2*f*g^3)*p^2)/(d^3*h^9))*log(16*(50625*b^3*e^6*f^8 - 4725
00*b^3*d*e^5*f^6*g^2 - 1457946*b^3*d^2*e^4*f^4*g^4 - 2572500*b^3*d^3*e^3*f^
2*g^6 + 1500625*b^3*d^4*e^2*g^8)*sqrt(h*x)*p^3 - 16*(42*d^6*f*g*h^14*sqrt(-
```

```
(50625*b^4*e^7*f^8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^
4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18))
+ 5*(675*b^2*d^2*e^4*f^6 - 10017*b^2*d^3*e^3*f^4*g^2 + 23373*b^2*d^4*e^2*f^
2*g^4 - 8575*b^2*d^5*e*g^6)*h^5*p^2)*sqrt(-(d^3*h^9*sqrt(-(50625*b^4*e^7*f^
8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d
^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18)) + 420*(3*b^2*e^3*
f^3*g - 7*b^2*d*e^2*f*g^3)*p^2)/(d^3*h^9))) - d*h^5*x^4*sqrt((d^3*h^9*sqrt(
-(50625*b^4*e^7*f^8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g
^4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18))
 - 420*(3*b^2*e^3*f^3*g - 7*b^2*d*e^2*f*g^3)*p^2)/(d^3*h^9))*log(16*(50625*
b^3*e^6*f^8 - 472500*b^3*d*e^5*f^6*g^2 - 1457946*b^3*d^2*e^4*f^4*g^4 - 2572
500*b^3*d^3*e^3*f^2*g^6 + 1500625*b^3*d^4*e^2*g^8)*sqrt(h*x)*p^3 + 16*(42*d
^6*f*g*h^14*sqrt(-(50625*b^4*e^7*f^8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*
b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8
)*p^4/(d^7*h^18)) - 5*(675*b^2*d^2*e^4*f^6 - 10017*b^2*d^3*e^3*f^4*g^2 + 23
373*b^2*d^4*e^2*f^2*g^4 - 8575*b^2*d^5*e*g^6)*h^5*p^2)*sqrt((d^3*h^9*sqrt(-
(50625*b^4*e^7*f^8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^
4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18))
- 420*(3*b^2*e^3*f^3*g - 7*b^2*d*e^2*f*g^3)*p^2)/(d^3*h^9))) + d*h^5*x^4*sq
rt((d^3*h^9*sqrt(-(50625*b^4*e^7*f^8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*
b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8
)*p^4/(d^7*h^18)) - 420*(3*b^2*e^3*f^3*g - 7*b^2*d*e^2*f*g^3)*p^2)/(d^3*h^9
))*log(16*(50625*b^3*e^6*f^8 - 472500*b^3*d*e^5*f^6*g^2 - 1457946*b^3*d^2*e
^4*f^4*g^4 - 2572500*b^3*d^3*e^3*f^2*g^6 + 1500625*b^3*d^4*e^2*g^8)*sqrt(h*
x)*p^3 - 16*(42*d^6*f*g*h^14*sqrt(-(50625*b^4*e^7*f^8 - 1266300*b^4*d*e^6*f
^6*g^2 + 8469846*b^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 150062
5*b^4*d^4*e^3*g^8)*p^4/(d^7*h^18)) - 5*(675*b^2*d^2*e^4*f^6 - 10017*b^2*d^3
*e^3*f^4*g^2 + 23373*b^2*d^4*e^2*f^2*g^4 - 8575*b^2*d^5*e*g^6)*h^5*p^2)*sqr
t((d^3*h^9*sqrt(-(50625*b^4*e^7*f^8 - 1266300*b^4*d*e^6*f^6*g^2 + 8469846*b
^4*d^2*e^5*f^4*g^4 - 6894300*b^4*d^3*e^4*f^2*g^6 + 1500625*b^4*d^4*e^3*g^8)
*p^4/(d^7*h^18)) - 420*(3*b^2*e^3*f^3*g - 7*b^2*d*e^2*f*g^3)*p^2)/(d^3*h^9)
)) + (168*b*e*f*g*p*x^3 + 42*a*d*f*g*x + 15*a*d*f^2 + 5*(4*b*e*f^2*p + 7*a*
d*g^2)*x^2 + (35*b*d*g^2*p*x^2 + 42*b*d*f*g*p*x + 15*b*d*f^2*p)*log(e*x^2 +
 d) + (35*b*d*g^2*x^2 + 42*b*d*f*g*x + 15*b*d*f^2)*log(c))*sqrt(h*x))/(d*h^
5*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((g*x+f)**2*(a+b*ln(c*(e*x**2+d)**p))/(h*x)**(9/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 1110, normalized size of antiderivative = 1.15

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx=\text{Too large to display}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="maxi
ma")
```

```
[Out] -1/21*b*e*f^2*p*(3*(sqrt(2)*e^(3/4)*log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)
*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) - sqrt(2)*e^(3/4)*log(sqrt(e)
*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/(d*h^2)^(3/4) +
 sqrt(2)*e*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e
^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(
d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt
(d)*h) + sqrt(2)*e*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)
^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + s
qrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)
*h)*sqrt(d)*h))/d + 8/((h*x)^(3/2)*d))/h^3 - 2/3*b*g^2*x^3*log((e*x^2 + d)^
p*c)/(h*x)^(9/2) + 2/5*b*e*f*g*p*(e*(sqrt(2)*log(sqrt(e)*h*x + sqrt(2)*(d*h
^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(1/4)*e^(3/4)) - sqrt(2)*
log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*
h^2)^(1/4)*e^(3/4)) - sqrt(2)*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt
(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqr
t(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt
(d)*sqrt(e)*h)*sqrt(e)) - sqrt(2)*log(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) -
sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)
*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-
sqrt(d)*sqrt(e)*h)*sqrt(e)))/d - 8/(sqrt(h*x)*d))/h^4 - 2/3*a*g^2*x^3/(h*x)
^(9/2) - 4/5*b*f*g*x^2*log((e*x^2 + d)^p*c)/(h*x)^(9/2) + 1/3*(sqrt(2)*h^2*
log(sqrt(e)*h*x + sqrt(2)*(d*h^2)^(1/4)*sqrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*
h^2)^(3/4)*e^(1/4)) - sqrt(2)*h^2*log(sqrt(e)*h*x - sqrt(2)*(d*h^2)^(1/4)*s
qrt(h*x)*e^(1/4) + sqrt(d)*h)/((d*h^2)^(3/4)*e^(1/4)) + sqrt(2)*h*log(-(sqr
t(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqrt(h*x)
*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4)
 + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(d)) + sqrt(2)*h*log
(-(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) - sqrt(2)*(d*h^2)^(1/4)*e^(1/4) - 2*sqr
t(h*x)*sqrt(e))/(sqrt(2)*sqrt(-sqrt(d)*sqrt(e)*h) + sqrt(2)*(d*h^2)^(1/4)*e
^(1/4) + 2*sqrt(h*x)*sqrt(e)))/(sqrt(-sqrt(d)*sqrt(e)*h)*sqrt(d)))*b*e*g^2*
p/h^5 - 4/5*a*f*g*x^2/(h*x)^(9/2) - 2/7*b*f^2*log((e*x^2 + d)^p*c)/((h*x)^(
7/2)*h) - 2/7*a*f^2/((h*x)^(7/2)*h)
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 688, normalized size of antiderivative = 0.71

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = -\frac{\frac{2\left(15\sqrt{2}(de^3h^2)^{\frac{1}{4}}be^2f^2hp-35\sqrt{2}(de^3h^2)^{\frac{1}{4}}bdeg^2hp+42\sqrt{2}(de^3h^2)^{\frac{3}{4}}bfgp\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{dh^2}{e}\right)^{\frac{1}{4}}+2\sqrt{hx}\right)}{2\left(\frac{dh^2}{e}\right)^{\frac{1}{4}}}\right)}{d^2eh}+\frac{2\left(15\sqrt{2}(de^3h^2)^{\frac{1}{4}}be^2f^2hp-\right.}{}}{}$$

```
[In] integrate((g*x+f)^2*(a+b*log(c*(e*x^2+d)^p))/(h*x)^(9/2),x, algorithm="giac
")
```

```
[Out] -1/105*(2*(15*sqrt(2)*(d*e^3*h^2)^(1/4)*b*e^2*f^2*h*p - 35*sqrt(2)*(d*e^3*h
^2)^(1/4)*b*d*e*g^2*h*p + 42*sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*g*p)*arctan(1/2*
sqrt(2)*(sqrt(2)*(d*h^2/e)^(1/4) + 2*sqrt(h*x))/(d*h^2/e)^(1/4))/(d^2*e*h)
+ 2*(15*sqrt(2)*(d*e^3*h^2)^(1/4)*b*e^2*f^2*h*p - 35*sqrt(2)*(d*e^3*h^2)^(1
/4)*b*d*e*g^2*h*p + 42*sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*g*p)*arctan(-1/2*sqrt(
2)*(sqrt(2)*(d*h^2/e)^(1/4) - 2*sqrt(h*x))/(d*h^2/e)^(1/4))/(d^2*e*h) + (15
*sqrt(2)*(d*e^3*h^2)^(1/4)*b*e^2*f^2*h*p - 35*sqrt(2)*(d*e^3*h^2)^(1/4)*b*d
*e*g^2*h*p - 42*sqrt(2)*(d*e^3*h^2)^(3/4)*b*f*g*p)*log(h*x + sqrt(2)*(d*h^2
/e)^(1/4)*sqrt(h*x) + sqrt(d*h^2/e))/(d^2*e*h) - (15*sqrt(2)*(d*e^3*h^2)^(1
/4)*b*e^2*f^2*h*p - 35*sqrt(2)*(d*e^3*h^2)^(1/4)*b*d*e*g^2*h*p - 42*sqrt(2)
*(d*e^3*h^2)^(3/4)*b*f*g*p)*log(h*x - sqrt(2)*(d*h^2/e)^(1/4)*sqrt(h*x) + s
qrt(d*h^2/e))/(d^2*e*h) + 2*(35*b*g^2*h^4*p*x^2 + 42*b*f*g*h^4*p*x + 15*b*f
^2*h^4*p)*log(e*h^2*x^2 + d*h^2)/(sqrt(h*x)*h^3*x^3) + 2*(168*b*e*f*g*h^4*p
*x^3 - 35*b*d*g^2*h^4*p*x^2*log(h^2) + 20*b*e*f^2*h^4*p*x^2 - 42*b*d*f*g*h^
4*p*x*log(h^2) + 35*b*d*g^2*h^4*x^2*log(c) + 35*a*d*g^2*h^4*x^2 - 15*b*d*f^
2*h^4*p*log(h^2) + 42*b*d*f*g*h^4*x*log(c) + 42*a*d*f*g*h^4*x + 15*b*d*f^2*
h^4*log(c) + 15*a*d*f^2*h^4)/(sqrt(h*x)*d*h^3*x^3))/h^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{(hx)^{9/2}}\,dx = \int \frac{(f+g\,x)^2\,\left(a+b\,\ln\left(c\,(e\,x^2+d)^p\right)\right)}{(h\,x)^{9/2}}\,dx$$

```
[In] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(9/2),x)
```

```
[Out] int(((f + g*x)^2*(a + b*log(c*(d + e*x^2)^p)))/(h*x)^(9/2), x)
```

3.616 $\int \frac{\sqrt{hx}\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{f+gx}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 1680

$$\int \frac{\sqrt{hx}(a+b\log(c(d+ex^2)^p))}{f+gx}\,dx$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{8bp\sqrt{hx}}{g} - \frac{2\sqrt{2}b\sqrt[4]{d}\sqrt{h}p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}g}$$

$$+ \frac{2\sqrt{2}b\sqrt[4]{d}\sqrt{h}p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}g} + \frac{2b\sqrt{hx}\log(c(d+ex^2)^p)}{g}$$

$$- \frac{2\sqrt{f}\sqrt{h}\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)(a+b\log(c(d+ex^2)^p))}{g^{3/2}}$$

$$- \frac{\sqrt{2}b\sqrt[4]{d}\sqrt{h}p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}g}$$

$$+ \frac{\sqrt{2}b\sqrt[4]{d}\sqrt{h}p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}g}$$

$$- \frac{8b\sqrt{f}\sqrt{h}p\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{h}}{\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}}\right)}{g^{3/2}}$$

$$+ \frac{2b\sqrt{f}\sqrt{h}p\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\sqrt{h}\left(\sqrt[4]{-d}\sqrt{-h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(\sqrt[4]{-d}\sqrt{g}\sqrt{-h}-i\sqrt[4]{e}\sqrt{f}\sqrt{h}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{g^{3/2}}$$

$$+ \frac{2b\sqrt{f}\sqrt{h}p\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[4]{-d}\sqrt{h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(i\sqrt[4]{e}\sqrt{f}-\sqrt[4]{-d}\sqrt{g}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{g^{3/2}}$$

$$+ \frac{2b\sqrt{f}\sqrt{h}p\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\sqrt{h}\left(\sqrt[4]{-d}\sqrt{-h}+\sqrt[4]{e}\sqrt{hx}\right)}{\left(\sqrt[4]{-d}\sqrt{g}\sqrt{-h}+i\sqrt[4]{e}\sqrt{f}\sqrt{h}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{g^{3/2}}$$

$$+ \frac{2b\sqrt{f}\sqrt{h}p\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[4]{-d}\sqrt{h}+\sqrt[4]{e}\sqrt{hx}\right)}{\left(i\sqrt[4]{e}\sqrt{f}+\sqrt[4]{-d}\sqrt{g}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{g^{3/2}}$$

$$+ \frac{4ib\sqrt{f}\sqrt{h}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{h}}{\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}}\right)}{g^{3/2}}$$

$$- \frac{ib\sqrt{f}\sqrt{h}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\sqrt{h}\left(\sqrt[4]{-d}\sqrt{-h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(\sqrt[4]{-d}\sqrt{g}\sqrt{-h}-i\sqrt[4]{e}\sqrt{f}\sqrt{h}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{g^{3/2}}$$

$$- \frac{ib\sqrt{f}\sqrt{h}p\,\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[4]{-d}\sqrt{h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(i\sqrt[4]{e}\sqrt{f}-\sqrt[4]{-d}\sqrt{g}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{g^{3/2}}$$

```
[Out] -2*b*d^(1/4)*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2
)*h^(1/2)/e^(1/4)/g+2*b*d^(1/4)*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1
/4)/h^(1/2))*2^(1/2)*h^(1/2)/e^(1/4)/g-b*d^(1/4)*p*ln(d^(1/2)*h^(1/2)+x*e^(
1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)*h^(1/2)/e^(1/4)/g
+b*d^(1/4)*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(
h*x)^(1/2))*2^(1/2)*h^(1/2)/e^(1/4)/g-2*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/
h^(1/2))*(a+b*ln(c*(e*x^2+d)^p))*f^(1/2)*h^(1/2)/g^(3/2)-8*b*p*arctan(g^(1/
2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*h^(1/2)/(f^(1/2)*h^(1/2)-I*g^(
1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2)+2*b*p*arctan(g^(1/2)*(h*x)^(1/2)
/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)-e^(1/
4)*(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)-I*e^(1/4)*f^(1/2)*h^(1/2))/(
f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2)+2*b*p*arcta
n(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(-2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^
(1/2)-e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)-(-d)^(1/4)*g^(1/2))/(f^(1/2)*
h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2)+2*b*p*arctan(g^(1/2
)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h
)^(1/2)+e^(1/4)*(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)+I*e^(1/4)*f^(1/
2)*h^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2
)+2*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*((
-d)^(1/4)*h^(1/2)+e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)+(-d)^(1/4)*g^(1/2
))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2)-I*b*p*p
olylog(2,1-2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)+e^(1/4)*(h*x)^(
1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)+I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h^
(1/2)-I*g^(1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2)-I*b*p*polylog(2,1+2*f
^(1/2)*g^(1/2)*((-d)^(1/4)*h^(1/2)-e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)-
(-d)^(1/4)*g^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2
)/g^(3/2)-I*b*p*polylog(2,1-2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^(1/2)+e^(1/4)*(
h*x)^(1/2))/(I*e^(1/4)*f^(1/2)+(-d)^(1/4)*g^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/
2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2)-I*b*p*polylog(2,1-2*f^(1/2)*g^(1/2
)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)-e^(1/4)*(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(
-h)^(1/2)-I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)
))*f^(1/2)*h^(1/2)/g^(3/2)+4*I*b*p*polylog(2,1-2*f^(1/2)*h^(1/2)/(f^(1/2)*h
^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*f^(1/2)*h^(1/2)/g^(3/2)+2*a*(h*x)^(1/2)/g-8*
b*p*(h*x)^(1/2)/g+2*b*ln(c*(e*x^2+d)^p)*(h*x)^(1/2)/g
```

Rubi [A] (verified)

Time = 2.28 (sec) , antiderivative size = 1680, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.645$, Rules used = {2517, 2526, 2498, 327, 217, 1179, 642, 1176, 631, 210, 211, 2520, 12, 266, 6857,

5048, 4966, 2449, 2352, 2497}

$$\int \frac{\sqrt{hx}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{f+gx}\,dx$$

$$= \frac{2\sqrt{hx}a}{g} - \frac{2\sqrt{2}b\sqrt[4]{d}\sqrt{h}p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}g} + \frac{2\sqrt{2}b\sqrt[4]{d}\sqrt{h}p\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{\sqrt[4]{e}g}$$
$$+ \frac{2b\sqrt{hx}\log\left(c(ex^2+d)^p\right)}{g} - \frac{2\sqrt{f}\sqrt{h}\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c(ex^2+d)^p\right)\right)}{g^{3/2}}$$
$$- \frac{\sqrt{2}b\sqrt[4]{d}\sqrt{h}p\log\left(\sqrt{e}\sqrt{hx}+\sqrt{d}\sqrt{h}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}g}$$
$$+ \frac{\sqrt{2}b\sqrt[4]{d}\sqrt{h}p\log\left(\sqrt{e}\sqrt{hx}+\sqrt{d}\sqrt{h}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}g}$$
$$- \frac{8b\sqrt{f}\sqrt{h}p\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{h}}{\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}}\right)}{g^{3/2}}$$
$$+ \frac{2b\sqrt{f}\sqrt{h}p\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\sqrt{h}\left(\sqrt[4]{-d}\sqrt{-h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(\sqrt[4]{-d}\sqrt{g}\sqrt{-h}-i\sqrt[4]{e}\sqrt{f}\sqrt{h}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{g^{3/2}}$$
$$+ \frac{2b\sqrt{f}\sqrt{h}p\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[4]{-d}\sqrt{h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(i\sqrt[4]{e}\sqrt{f}-\sqrt[4]{-d}\sqrt{g}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{g^{3/2}}$$
$$+ \frac{2b\sqrt{f}\sqrt{h}p\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\sqrt{h}\left(\sqrt[4]{-d}\sqrt{-h}+\sqrt[4]{e}\sqrt{hx}\right)}{\left(\sqrt[4]{-d}\sqrt{g}\sqrt{-h}+i\sqrt[4]{e}\sqrt{f}\sqrt{h}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{g^{3/2}}$$
$$+ \frac{2b\sqrt{f}\sqrt{h}p\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[4]{-d}\sqrt{h}+\sqrt[4]{e}\sqrt{hx}\right)}{\left(i\sqrt[4]{e}\sqrt{f}+\sqrt[4]{-d}\sqrt{g}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{g^{3/2}}$$
$$+ \frac{4ib\sqrt{f}\sqrt{h}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{h}}{\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}}\right)}{g^{3/2}}$$
$$- \frac{ib\sqrt{f}\sqrt{h}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\sqrt{h}\left(\sqrt[4]{-d}\sqrt{-h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(\sqrt[4]{-d}\sqrt{g}\sqrt{-h}-i\sqrt[4]{e}\sqrt{f}\sqrt{h}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{g^{3/2}}$$
$$- \frac{ib\sqrt{f}\sqrt{h}p\,\text{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[4]{-d}\sqrt{h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(i\sqrt[4]{e}\sqrt{f}-\sqrt[4]{-d}\sqrt{g}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}+1\right)}{g^{3/2}}$$
$$- \frac{ib\sqrt{f}\sqrt{h}p\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\sqrt{h}\left(\sqrt[4]{-d}\sqrt{-h}+\sqrt[4]{e}\sqrt{hx}\right)}{\left(\sqrt[4]{-d}\sqrt{g}\sqrt{-h}+i\sqrt[4]{e}\sqrt{f}\sqrt{h}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{g^{3/2}}$$

```
[In] Int[(Sqrt[h*x]*(a + b*Log[c*(d + e*x^2)^p]))/(f + g*x),x]
```

```
[Out] (2*a*Sqrt[h*x])/g - (8*b*p*Sqrt[h*x])/g - (2*Sqrt[2]*b*d^(1/4)*Sqrt[h]*p*Ar
cTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])]])/(e^(1/4)*g) + (2*S
qrt[2]*b*d^(1/4)*Sqrt[h]*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*
Sqrt[h])]])/(e^(1/4)*g) + (2*b*Sqrt[h*x]*Log[c*(d + e*x^2)^p])/g - (2*Sqrt[f
]*Sqrt[h]*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*(a + b*Log[c*(d + e
*x^2)^p]))/g^(3/2) - (Sqrt[2]*b*d^(1/4)*Sqrt[h]*p*Log[Sqrt[d]*Sqrt[h] + Sqr
t[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*g) + (Sqrt[2]
*b*d^(1/4)*Sqrt[h]*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1
/4)*e^(1/4)*Sqrt[h*x]])/(e^(1/4)*g) - (8*b*Sqrt[f]*Sqrt[h]*p*ArcTan[(Sqrt[g
]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[h])/(Sqrt[f]*Sqrt[h] -
I*Sqrt[g]*Sqrt[h*x])]])/g^(3/2) + (2*b*Sqrt[f]*Sqrt[h]*p*ArcTan[(Sqrt[g]*Sqr
t[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[
-h] - e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]
*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))]])/g^(3/2) + (2*b*Sqrt[f]
*Sqrt[h]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(-2*Sqrt[f]*Sq
rt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] - (-d)^
(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))]])/g^(3/2) + (2*b*Sq
rt[f]*Sqrt[h]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f
]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*S
qrt[g]*Sqrt[-h] + I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*S
qrt[h*x]))]])/g^(3/2) + (2*b*Sqrt[f]*Sqrt[h]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(S
qrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[
h*x]))/((I*e^(1/4)*Sqrt[f] + (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[
g]*Sqrt[h*x]))]])/g^(3/2) + ((4*I)*b*Sqrt[f]*Sqrt[h]*p*PolyLog[2, 1 - (2*Sqr
t[f]*Sqrt[h])/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x])]])/g^(3/2) - (I*b*Sqrt
[f]*Sqrt[h]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h
] - e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*S
qrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))]])/g^(3/2) - (I*b*Sqrt[f]*S
qrt[h]*p*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/4)*Sq
rt[h*x]))/((I*e^(1/4)*Sqrt[f] - (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sq
rt[g]*Sqrt[h*x]))]])/g^(3/2) - (I*b*Sqrt[f]*Sqrt[h]*p*PolyLog[2, 1 - (2*Sqrt
[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)
*Sqrt[g]*Sqrt[-h] + I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]
*Sqrt[h*x]))]])/g^(3/2) - (I*b*Sqrt[f]*Sqrt[h]*p*PolyLog[2, 1 - (2*Sqrt[f]*S
qrt[g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] + (-d)
^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))]])/g^(3/2)
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 210</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
```

```
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
```

```
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \frac{x^2\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{f+\frac{gx^2}{h}}\,dx, x, \sqrt{hx}\right)}{h}\\
&= \frac{2\text{Subst}\left(\int \left(\frac{h\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{g}-\frac{fh\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{g\left(f+\frac{gx^2}{h}\right)}\right)dx, x, \sqrt{hx}\right)}{h}\\
&= \frac{2\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)dx, x, \sqrt{hx}\right)}{g}\\
&\quad-\frac{(2f)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{f+\frac{gx^2}{h}}\,dx, x, \sqrt{hx}\right)}{g}
\end{aligned}$$

$$=\frac{2a\sqrt{hx}}{g}-\frac{2\sqrt{f}\sqrt{h}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{g^{3/2}}$$
$$+\frac{(2b)\text{Subst}\left(\int\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)dx,x,\sqrt{hx}\right)}{g}$$
$$+\frac{(8befp)\text{Subst}\left(\int\frac{\sqrt{h}x^3\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{\sqrt{f}\sqrt{g}\left(d+\frac{ex^4}{h^2}\right)}dx,x,\sqrt{hx}\right)}{gh^2}$$

$$=\frac{2a\sqrt{hx}}{g}+\frac{2b\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{g}-\frac{2\sqrt{f}\sqrt{h}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{g^{3/2}}$$
$$-\frac{(8bep)\text{Subst}\left(\int\frac{x^4}{d+\frac{ex^4}{h^2}}dx,x,\sqrt{hx}\right)}{gh^2}+\frac{\left(8be\sqrt{f}p\right)\text{Subst}\left(\int\frac{x^3\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{d+\frac{ex^4}{h^2}}dx,x,\sqrt{hx}\right)}{g^{3/2}h^{3/2}}$$

$$=\frac{2a\sqrt{hx}}{g}-\frac{8bp\sqrt{hx}}{g}+\frac{2b\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{g}$$
$$-\frac{2\sqrt{f}\sqrt{h}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{g^{3/2}}$$
$$+\frac{(8bdp)\text{Subst}\left(\int\frac{1}{d+\frac{ex^4}{h^2}}dx,x,\sqrt{hx}\right)}{g}$$
$$+\frac{\left(8be\sqrt{f}p\right)\text{Subst}\left(\int\left(\frac{h^2x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{2\left(-\sqrt{-d}\sqrt{e}h+ex^2\right)}+\frac{h^2x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{2\left(\sqrt{-d}\sqrt{e}h+ex^2\right)}\right)dx,x,\sqrt{hx}\right)}{g^{3/2}h^{3/2}}$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{8bp\sqrt{hx}}{g} + \frac{2b\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{g} - \frac{2\sqrt{f}\sqrt{h}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{g^{3/2}} + \frac{\left(4b\sqrt{d}p\right)\text{Subst}\left(\int\frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{gh} + \frac{\left(4b\sqrt{d}p\right)\text{Subst}\left(\int\frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{gh} + \frac{\left(4be\sqrt{f}\sqrt{h}p\right)\text{Subst}\left(\int\frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{-\sqrt{-d}\sqrt{e}h+ex^2}\,dx, x, \sqrt{hx}\right)}{g^{3/2}} + \frac{\left(4be\sqrt{f}\sqrt{h}p\right)\text{Subst}\left(\int\frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{\sqrt{-d}\sqrt{e}h+ex^2}\,dx, x, \sqrt{hx}\right)}{g^{3/2}}$$

$$= \frac{2a\sqrt{hx}}{g} - \frac{8bp\sqrt{hx}}{g} + \frac{2b\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{g}$$
$$- \frac{2\sqrt{f}\sqrt{h}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{g^{3/2}}$$
$$+ \frac{\left(4be\sqrt{f}\sqrt{hp}\right)\text{Subst}\left(\int\left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{2e^{3/4}\left(\sqrt[4]{-d}\sqrt{-h}-\sqrt[4]{e}x\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{2e^{3/4}\left(\sqrt[4]{-d}\sqrt{-h}+\sqrt[4]{e}x\right)}\right)dx, x, \sqrt{hx}\right)}{g^{3/2}}$$
$$+ \frac{\left(4be\sqrt{f}\sqrt{hp}\right)\text{Subst}\left(\int\left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{2e^{3/4}\left(\sqrt[4]{-d}\sqrt{h}-\sqrt[4]{e}x\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{2e^{3/4}\left(\sqrt[4]{-d}\sqrt{h}+\sqrt[4]{e}x\right)}\right)dx, x, \sqrt{hx}\right)}{g^{3/2}}$$
$$- \frac{\left(\sqrt{2}b\sqrt[4]{d}\sqrt{hp}\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}dx, x, \sqrt{hx}\right)}{\sqrt[4]{e}g}$$
$$- \frac{\left(\sqrt{2}b\sqrt[4]{d}\sqrt{hp}\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}dx, x, \sqrt{hx}\right)}{\sqrt[4]{e}g}$$
$$+ \frac{\left(2b\sqrt{d}hp\right)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}dx, x, \sqrt{hx}\right)}{\sqrt{e}g}$$
$$+ \frac{\left(2b\sqrt{d}hp\right)\text{Subst}\left(\int\frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}dx, x, \sqrt{hx}\right)}{\sqrt{e}g}$$

$$
\begin{aligned}
&= \frac{2a\sqrt{hx}}{g} - \frac{8bp\sqrt{hx}}{g} + \frac{2b\sqrt{hx}\log\left(c(d+ex^2)^p\right)}{g} \\
&- \frac{2\sqrt{f}\sqrt{h}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{g^{3/2}} \\
&- \frac{\sqrt{2}b\sqrt[4]{d}\sqrt{h}p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}g} \\
&+ \frac{\sqrt{2}b\sqrt[4]{d}\sqrt{h}p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{hx}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{e}g} \\
&- \frac{\left(2b\sqrt[4]{e}\sqrt{f}\sqrt{h}p\right)\text{Subst}\left(\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{\sqrt[4]{-d}\sqrt{-h}-\sqrt[4]{e}x}\,dx, x, \sqrt{hx}\right)}{g^{3/2}} \\
&- \frac{\left(2b\sqrt[4]{e}\sqrt{f}\sqrt{h}p\right)\text{Subst}\left(\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{\sqrt[4]{-d}\sqrt{h}-\sqrt[4]{e}x}\,dx, x, \sqrt{hx}\right)}{g^{3/2}} \\
&+ \frac{\left(2b\sqrt[4]{e}\sqrt{f}\sqrt{h}p\right)\text{Subst}\left(\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{\sqrt[4]{-d}\sqrt{-h}+\sqrt[4]{e}x}\,dx, x, \sqrt{hx}\right)}{g^{3/2}} \\
&+ \frac{\left(2b\sqrt[4]{e}\sqrt{f}\sqrt{h}p\right)\text{Subst}\left(\int \frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{\sqrt[4]{-d}\sqrt{h}+\sqrt[4]{e}x}\,dx, x, \sqrt{hx}\right)}{g^{3/2}} \\
&+ \frac{\left(2\sqrt{2}b\sqrt[4]{d}\sqrt{h}p\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}g} \\
&- \frac{\left(2\sqrt{2}b\sqrt[4]{d}\sqrt{h}p\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{e}g} \\
&= \text{Too large to display}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 1506, normalized size of antiderivative = 0.90

$$
\int \frac{\sqrt{hx}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{f+gx}\,dx
$$

$$
= \frac{\sqrt{hx}\left(2a\sqrt{g}\sqrt{x} - 8b\sqrt{g}p\sqrt{x} - \frac{2\sqrt{2}b\sqrt[4]{d}\sqrt{g}p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)}{\sqrt[4]{e}} + \frac{2\sqrt{2}b\sqrt[4]{d}\sqrt{g}p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)}{\sqrt[4]{e}} - \frac{\sqrt{2}b\sqrt[4]{d}\sqrt{g}p\,\text{lo}
$$

[In] Integrate[(Sqrt[h*x]*(a + b*Log[c*(d + e*x^2)^p]))/(f + g*x),x]

[Out] (Sqrt[h*x]*(2*a*Sqrt[g]*Sqrt[x] - 8*b*Sqrt[g]*p*Sqrt[x] - (2*Sqrt[2]*b*d^(1/4)*Sqrt[g]*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)])/e^(1/4) + (2*Sqrt[2]*b*d^(1/4)*Sqrt[g]*p*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)])/e^(1/4) - (Sqrt[2]*b*d^(1/4)*Sqrt[g]*p*Log[Sqrt[d] - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x])/e^(1/4) + (Sqrt[2]*b*d^(1/4)*Sqrt[g]*p*Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x])/e^(1/4) + 2*b*Sqrt[g]*Sqrt[x]*Log[c*(d + e*x^2)^p] + Sqrt[-f]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]]*(a + b*Log[c*(d + e*x^2)^p]) - Sqrt[-f]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]]*(a + b*Log[c*(d + e*x^2)^p]) - b*Sqrt[-f]*p*(Log[(Sqrt[g]*((-d)^(1/4) - e^(1/4)*Sqrt[x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/4)*Sqrt[x]))/(I*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*(I*(-d)^(1/4) + e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] - (-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]) + b*Sqrt[-f]*p*(Log[(Sqrt[g]*((-d)^(1/4) - e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) - I*e^(1/4)*Sqrt[x]))/(I*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/4)*Sqrt[x]))/((-I)*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + e^(1/4)*Sqrt[x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] - (-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])])))/(g^(3/2)*Sqrt[x])

Maple [F]

$$\int \frac{\sqrt{hx}\,\left(a+b\ln\left(c(e\,x^{2}+d)^{p}\right)\right)}{gx+f}dx$$

[In] int((h*x)^(1/2)*(a+b*ln(c*(e*x^2+d)^p))/(g*x+f),x)

[Out] int((h*x)^(1/2)*(a+b*ln(c*(e*x^2+d)^p))/(g*x+f),x)

Fricas [F]

$$\int \frac{\sqrt{hx}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{f+gx}\,dx = \int \frac{\sqrt{hx}\left(b\log\left((ex^2+d)^pc\right)+a\right)}{gx+f}\,dx$$

```
[In] integrate((h*x)^(1/2)*(a+b*log(c*(e*x^2+d)^p))/(g*x+f),x, algorithm="fricas
")
[Out] integral((sqrt(h*x)*b*log((e*x^2 + d)^p*c) + sqrt(h*x)*a)/(g*x + f), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{hx}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{f+gx}\,dx = \text{Timed out}$$

```
[In] integrate((h*x)**(1/2)*(a+b*ln(c*(e*x**2+d)**p))/(g*x+f),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{hx}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{f+gx}\,dx = \int \frac{\sqrt{hx}\left(b\log\left((ex^2+d)^pc\right)+a\right)}{gx+f}\,dx$$

```
[In] integrate((h*x)^(1/2)*(a+b*log(c*(e*x^2+d)^p))/(g*x+f),x, algorithm="maxima
")
[Out] b*integrate((sqrt(h)*sqrt(x)*log((e*x^2 + d)^p) + sqrt(h)*sqrt(x)*log(c))/(
g*x + f), x) - 2*(f*h^2*arctan(sqrt(h*x)*g/sqrt(f*g*h))/(sqrt(f*g*h)*g) - s
qrt(h*x)*h/g)*a/h
```

Giac [F]

$$\int \frac{\sqrt{hx}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{f+gx}\,dx = \int \frac{\sqrt{hx}\left(b\log\left((ex^2+d)^pc\right)+a\right)}{gx+f}\,dx$$

```
[In] integrate((h*x)^(1/2)*(a+b*log(c*(e*x^2+d)^p))/(g*x+f),x, algorithm="giac")
[Out] integrate(sqrt(h*x)*(b*log((e*x^2 + d)^p*c) + a)/(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{hx}\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{f+gx}\,dx=\int \frac{\sqrt{h\,x}\,\left(a+b\,\ln\left(c\,{\left(e\,x^2+d\right)}^p\right)\right)}{f+g\,x}\,dx$$

```
[In] int(((h*x)^(1/2)*(a + b*log(c*(d + e*x^2)^p)))/(f + g*x),x)
[Out] int(((h*x)^(1/2)*(a + b*log(c*(d + e*x^2)^p)))/(f + g*x), x)
```

3.617 $\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{\sqrt{hx}(f+gx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 1361

$$\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{\sqrt{hx}(f+gx)}\,dx$$

$$= \frac{2\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} + \frac{8bp\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{h}}{\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}$$

$$- \frac{2bp\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\sqrt{h}\left(\sqrt[4]{-d}\sqrt{-h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(\sqrt[4]{-d}\sqrt{g}\sqrt{-h}-i\sqrt[4]{e}\sqrt{f}\sqrt{h}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}$$

$$- \frac{2bp\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[4]{-d}\sqrt{h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(i\sqrt[4]{e}\sqrt{f}-\sqrt[4]{-d}\sqrt{g}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}$$

$$- \frac{2bp\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\sqrt{h}\left(\sqrt[4]{-d}\sqrt{-h}+\sqrt[4]{e}\sqrt{hx}\right)}{\left(\sqrt[4]{-d}\sqrt{g}\sqrt{-h}+i\sqrt[4]{e}\sqrt{f}\sqrt{h}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}$$

$$- \frac{2bp\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[4]{-d}\sqrt{h}+\sqrt[4]{e}\sqrt{hx}\right)}{\left(i\sqrt[4]{e}\sqrt{f}+\sqrt[4]{-d}\sqrt{g}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}$$

$$- \frac{4ibp\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{h}}{\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}$$

$$+ \frac{ibp\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\sqrt{h}\left(\sqrt[4]{-d}\sqrt{-h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(\sqrt[4]{-d}\sqrt{g}\sqrt{-h}-i\sqrt[4]{e}\sqrt{f}\sqrt{h}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}$$

$$+ \frac{ibp\,\text{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[4]{-d}\sqrt{h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(i\sqrt[4]{e}\sqrt{f}-\sqrt[4]{-d}\sqrt{g}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}$$

$$+ \frac{ibp\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\sqrt{h}\left(\sqrt[4]{-d}\sqrt{-h}+\sqrt[4]{e}\sqrt{hx}\right)}{\left(\sqrt[4]{-d}\sqrt{g}\sqrt{-h}+i\sqrt[4]{e}\sqrt{f}\sqrt{h}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}$$

$$+ \frac{ibp\,\text{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[4]{-d}\sqrt{h}+\sqrt[4]{e}\sqrt{hx}\right)}{\left(i\sqrt[4]{e}\sqrt{f}+\sqrt[4]{-d}\sqrt{g}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}$$

```
[Out] 2*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*(a+b*ln(c*(e*x^2+d)^p))/f^(1/
2)/g^(1/2)/h^(1/2)+8*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f
^(1/2)*h^(1/2)/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(1/2)/g^(1/2)/h^(
1/2)-2*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)
*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)-e^(1/4)*(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-
h)^(1/2)-I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2))
)/f^(1/2)/g^(1/2)/h^(1/2)-2*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))
*ln(-2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^(1/2)-e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*
f^(1/2)-(-d)^(1/4)*g^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(1/2
)/g^(1/2)/h^(1/2)-2*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^
(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)+e^(1/4)*(h*x)^(1/2))/((-d)^(1/
4)*g^(1/2)*(-h)^(1/2)+I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)
*(h*x)^(1/2)))/f^(1/2)/g^(1/2)/h^(1/2)-2*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(
1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^(1/2)+e^(1/4)*(h*x)^(1/2))
/(I*e^(1/4)*f^(1/2)+(-d)^(1/4)*g^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1
/2)))/f^(1/2)/g^(1/2)/h^(1/2)-4*I*b*p*polylog(2,1-2*f^(1/2)*h^(1/2)/(f^(1/2
)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(1/2)/g^(1/2)/h^(1/2)+I*b*p*polylog(2,1
-2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)-e^(1/4)*(h*x)^(1/2))/((-d
)^(1/4)*g^(1/2)*(-h)^(1/2)-I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h^(1/2)-I*g^
(1/2)*(h*x)^(1/2)))/f^(1/2)/g^(1/2)/h^(1/2)+I*b*p*polylog(2,1+2*f^(1/2)*g^(
1/2)*((-d)^(1/4)*h^(1/2)-e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)-(-d)^(1/4)
*g^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(1/2)/g^(1/2)/h^(1/2)+
I*b*p*polylog(2,1-2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)+e^(1/4)*
(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)+I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(
1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(1/2)/g^(1/2)/h^(1/2)+I*b*p*polylog(
2,1-2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^(1/2)+e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f
^(1/2)+(-d)^(1/4)*g^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))/f^(1/2)
/g^(1/2)/h^(1/2)
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 1361, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules

used = {2517, 211, 2520, 12, 266, 6857, 5048, 4966, 2449, 2352, 2497}

$$
\begin{aligned}
&\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{\sqrt{hx}(f+gx)}\,dx \\
&= \frac{2\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c(ex^2+d)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} + \frac{8bp\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{h}}{\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} \\
&- \frac{2bp\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\sqrt{h}\left(\sqrt[4]{-d}\sqrt{-h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(\sqrt[4]{-d}\sqrt{g}\sqrt{-h}-i\sqrt[4]{e}\sqrt{f}\sqrt{h}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} \\
&- \frac{2bp\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[4]{-d}\sqrt{h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(i\sqrt[4]{e}\sqrt{f}-\sqrt[4]{-d}\sqrt{g}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} \\
&- \frac{2bp\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\sqrt{h}\left(\sqrt[4]{-d}\sqrt{-h}+\sqrt[4]{e}\sqrt{hx}\right)}{\left(\sqrt[4]{-d}\sqrt{g}\sqrt{-h}+i\sqrt[4]{e}\sqrt{f}\sqrt{h}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} \\
&- \frac{2bp\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[4]{-d}\sqrt{h}+\sqrt[4]{e}\sqrt{hx}\right)}{\left(i\sqrt[4]{e}\sqrt{f}+\sqrt[4]{-d}\sqrt{g}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} \\
&- \frac{4ibp\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{h}}{\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} \\
&+ \frac{ibp\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\sqrt{h}\left(\sqrt[4]{-d}\sqrt{-h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(\sqrt[4]{-d}\sqrt{g}\sqrt{-h}-i\sqrt[4]{e}\sqrt{f}\sqrt{h}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} \\
&+ \frac{ibp\,\mathrm{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[4]{-d}\sqrt{h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(i\sqrt[4]{e}\sqrt{f}-\sqrt[4]{-d}\sqrt{g}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}+1\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} \\
&+ \frac{ibp\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\sqrt{h}\left(\sqrt[4]{-d}\sqrt{-h}+\sqrt[4]{e}\sqrt{hx}\right)}{\left(\sqrt[4]{-d}\sqrt{g}\sqrt{-h}+i\sqrt[4]{e}\sqrt{f}\sqrt{h}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} \\
&+ \frac{ibp\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[4]{-d}\sqrt{h}+\sqrt[4]{e}\sqrt{hx}\right)}{\left(i\sqrt[4]{e}\sqrt{f}+\sqrt[4]{-d}\sqrt{g}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}
\end{aligned}
$$

```
[In] Int[(a + b*Log[c*(d + e*x^2)^p])/(Sqrt[h*x]*(f + g*x)),x]
```

```
[Out] (2*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*(a + b*Log[c*(d + e*x^2)^p
]))/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (8*b*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*
Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[h])/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x])])
/(Sqrt[f]*Sqrt[g]*Sqrt[h]) - (2*b*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqr
t[h])]*Log[(2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] - e^(1/4)*Sqrt[h
*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*S
qrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) - (2*b*p*ArcTan[
(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(-2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*
Sqrt[h] - e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] - (-d)^(1/4)*Sqrt[g])*(Sq
rt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) - (2*b*p*
ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*Sqrt[h
]*((-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h]
+ I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sq
rt[f]*Sqrt[g]*Sqrt[h]) - (2*b*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h]
)]*Log[(2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(
1/4)*Sqrt[f] + (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))
])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) - ((4*I)*b*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[h]
)/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x])])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (I*
b*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] - e^(1/4
)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*Sqrt[h])*(S
qrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (I*b*p
*PolyLog[2, 1 + (2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/4)*Sqrt[h*x])
)/((I*e^(1/4)*Sqrt[f] - (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sq
rt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (I*b*p*PolyLog[2, 1 - (2*Sqrt[f]*Sq
rt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[
g]*Sqrt[-h] + I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[
h*x]))])/(Sqrt[f]*Sqrt[g]*Sqrt[h]) + (I*b*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[
g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] + (-d)^(1/
4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(Sqrt[f]*Sqrt[g]*Sqr
t[h])
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 211</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

<u>Rule 266</u>

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
```

```
/; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{f+\frac{gx^2}{h}}\,dx, x, \sqrt{hx}\right)}{h}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} - \frac{(8bep)\text{Subst}\left(\int \frac{\sqrt{h}x^3\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{\sqrt{f}\sqrt{g}\left(d+\frac{ex^4}{h^2}\right)}\,dx, x, \sqrt{hx}\right)}{h^3}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} - \frac{(8bep)\text{Subst}\left(\int \frac{x^3\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{\sqrt{f}\sqrt{g}h^{5/2}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} - \frac{(8bep)\text{Subst}\left(\int \left(\frac{h^2x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{2\left(-\sqrt{-d}\sqrt{e}h+ex^2\right)} + \frac{h^2x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{2\left(\sqrt{-d}\sqrt{e}h+ex^2\right)}\right)dx, x, \sqrt{hx}\right)}{\sqrt{f}\sqrt{g}h^{5/2}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} - \frac{(4bep)\text{Subst}\left(\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{-\sqrt{-d}\sqrt{e}h+ex^2}\,dx, x, \sqrt{hx}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} - \frac{(4bep)\text{Subst}\left(\int \frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{\sqrt{-d}\sqrt{e}h+ex^2}\,dx, x, \sqrt{hx}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} - \frac{(4bep)\text{Subst}\left(\int\left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{2e^{3/4}\left(\sqrt[4]{-d}\sqrt{-h}-\sqrt[4]{e}x\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{2e^{3/4}\left(\sqrt[4]{-d}\sqrt{-h}+\sqrt[4]{e}x\right)}\right)dx,x,\sqrt{hx}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} - \frac{(4bep)\text{Subst}\left(\int\left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{2e^{3/4}\left(\sqrt[4]{-d}\sqrt{h}-\sqrt[4]{e}x\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{2e^{3/4}\left(\sqrt[4]{-d}\sqrt{h}+\sqrt[4]{e}x\right)}\right)dx,x,\sqrt{hx}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} + \frac{\left(2b\sqrt[4]{e}p\right)\text{Subst}\left(\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{\sqrt[4]{-d}\sqrt{-h}-\sqrt[4]{e}x}dx,x,\sqrt{hx}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} + \frac{\left(2b\sqrt[4]{e}p\right)\text{Subst}\left(\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{\sqrt[4]{-d}\sqrt{h}-\sqrt[4]{e}x}dx,x,\sqrt{hx}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} - \frac{\left(2b\sqrt[4]{e}p\right)\text{Subst}\left(\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{\sqrt[4]{-d}\sqrt{-h}+\sqrt[4]{e}x}dx,x,\sqrt{hx}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}} - \frac{\left(2b\sqrt[4]{e}p\right)\text{Subst}\left(\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{\sqrt[4]{-d}\sqrt{h}+\sqrt[4]{e}x}dx,x,\sqrt{hx}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}}$$

$$= \text{Too large to display}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 1297, normalized size of antiderivative = 0.95

$$\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{\sqrt{hx}(f+gx)}\,dx$$

$$= \frac{\sqrt{x}\left(a\log\left(\sqrt{-f}-\sqrt{g}\sqrt{x}\right)-bp\log\left(\frac{\sqrt{g}\left(\sqrt[4]{-d}-\sqrt[4]{e}\sqrt{x}\right)}{-\sqrt[4]{e}\sqrt{-f}+\sqrt[4]{-d}\sqrt{g}}\right)\log\left(\sqrt{-f}-\sqrt{g}\sqrt{x}\right)-bp\log\left(\frac{\sqrt{g}\left(\sqrt[4]{-d}+i\sqrt[4]{e}\sqrt{}\right.}{i\sqrt[4]{e}\sqrt{-f}+\sqrt[4]{-d}\right.}\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^2)^p])/(Sqrt[h*x]*(f + g*x)),x]
```

[Out] (Sqrt[x]*(a*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] - b*p*Log[(Sqrt[g]*((-d)^(1/4) - e^(1/4)*Sqrt[x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] - b*p*Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/4)*Sqrt[x]))/(I*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] - b*p*Log[(Sqrt[g]*(I*(-d)^(1/4) + e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] - b*p*Log[(Sqrt[g]*((-d)^(1/4) + e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] - a*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + b*p*Log[(Sqrt[g]*((-d)^(1/4) - e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + b*p*Log[(Sqrt[g]*((-d)^(1/4) - I*e^(1/4)*Sqrt[x]))/(I*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + b*p*Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/4)*Sqrt[x]))/((-I)*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + b*p*Log[(Sqrt[g]*((-d)^(1/4) + e^(1/4)*Sqrt[x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + b*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]]*Log[c*(d + e*x^2)^p] - b*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]]*Log[c*(d + e*x^2)^p] - b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] - (-d)^(1/4)*Sqrt[g])] - b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])] - b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])] - b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])] + b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] - (-d)^(1/4)*Sqrt[g])] + b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])] + b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])] + b*p*PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]))/(Sqrt[-f]*Sqrt[g]*Sqrt[h*x])

Maple [F]

$$\int \frac{a + b\ln\left(c(e\,x^2 + d)^p\right)}{\sqrt{hx}\,(gx + f)}dx$$

[In] int((a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x)

[Out] int((a+b*ln(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x)

Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{\sqrt{hx}(f+gx)}\,dx = \int \frac{b\log\left((ex^2+d)^p c\right)+a}{(gx+f)\sqrt{hx}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x, algorithm="fricas
")
```

```
[Out] integral((sqrt(h*x)*b*log((e*x^2 + d)^p*c) + sqrt(h*x)*a)/(g*h*x^2 + f*h*x)
, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{\sqrt{hx}(f+gx)}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x**2+d)**p))/(h*x)**(1/2)/(g*x+f),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{\sqrt{hx}(f+gx)}\,dx = \int \frac{b\log\left((ex^2+d)^p c\right)+a}{(gx+f)\sqrt{hx}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x, algorithm="maxima
")
```

```
[Out] b*integrate((sqrt(h)*log((e*x^2 + d)^p) + sqrt(h)*log(c))/(g*h*x^(3/2) + f*
h*sqrt(x)), x) + 2*a*arctan(sqrt(h*x)*g/sqrt(f*g*h))/sqrt(f*g*h)
```

Giac [F]

$$\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{\sqrt{hx}(f+gx)}\,dx = \int \frac{b\log\left((ex^2+d)^p c\right)+a}{(gx+f)\sqrt{hx}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(1/2)/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^2 + d)^p*c) + a)/((g*x + f)*sqrt(h*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d + ex^2)^p\right)}{\sqrt{hx}(f + gx)}\,dx = \int \frac{a + b\,\ln\left(c\,(e\,x^2 + d)^p\right)}{(f + g\,x)\,\sqrt{h\,x}}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^2)^p))/((f + g*x)*(h*x)^(1/2)),x)
[Out] int((a + b*log(c*(d + e*x^2)^p))/((f + g*x)*(h*x)^(1/2)), x)
```

3.618 $\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{(hx)^{3/2}(f+gx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 1659

$$\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{(hx)^{3/2}(f+gx)}\,dx = -\frac{2\sqrt{2}b\sqrt[4]{e}p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}fh^{3/2}}$$
$$+\frac{2\sqrt{2}b\sqrt[4]{e}p\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}fh^{3/2}}-\frac{2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{fh\sqrt{hx}}$$
$$-\frac{2\sqrt{g}\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{f^{3/2}h^{3/2}}$$
$$+\frac{\sqrt{2}b\sqrt[4]{e}p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}fh^{3/2}}$$
$$-\frac{\sqrt{2}b\sqrt[4]{e}p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}fh^{3/2}}$$
$$-\frac{8b\sqrt{g}p\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{h}}{\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}}\right)}{f^{3/2}h^{3/2}}$$
$$+\frac{2b\sqrt{g}p\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\sqrt{h}\left(\sqrt[4]{-d}\sqrt{-h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(\sqrt[4]{-d}\sqrt{g}\sqrt{-h}-i\sqrt[4]{e}\sqrt{f}\sqrt{h}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{f^{3/2}h^{3/2}}$$
$$+\frac{2b\sqrt{g}p\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[4]{-d}\sqrt{h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(i\sqrt[4]{e}\sqrt{f}-\sqrt[4]{-d}\sqrt{g}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{f^{3/2}h^{3/2}}$$
$$+\frac{2b\sqrt{g}p\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\sqrt{h}\left(\sqrt[4]{-d}\sqrt{-h}+\sqrt[4]{e}\sqrt{hx}\right)}{\left(\sqrt[4]{-d}\sqrt{g}\sqrt{-h}+i\sqrt[4]{e}\sqrt{f}\sqrt{h}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{f^{3/2}h^{3/2}}$$
$$+\frac{2b\sqrt{g}p\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[4]{-d}\sqrt{h}+\sqrt[4]{e}\sqrt{hx}\right)}{\left(i\sqrt[4]{e}\sqrt{f}+\sqrt[4]{-d}\sqrt{g}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{f^{3/2}h^{3/2}}$$
$$+\frac{4ib\sqrt{g}p\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{h}}{\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}}\right)}{f^{3/2}h^{3/2}}$$
$$-\frac{ib\sqrt{g}p\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\sqrt{h}\left(\sqrt[4]{-d}\sqrt{-h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(\sqrt[4]{-d}\sqrt{g}\sqrt{-h}-i\sqrt[4]{e}\sqrt{f}\sqrt{h}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{f^{3/2}h^{3/2}}$$
$$-\frac{ib\sqrt{g}p\operatorname{PolyLog}\left(2,1+\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[4]{-d}\sqrt{h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(i\sqrt[4]{e}\sqrt{f}-\sqrt[4]{-d}\sqrt{g}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{f^{3/2}h^{3/2}}$$
$$ib\sqrt{g}p\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\sqrt{h}\left(\sqrt[4]{-d}\sqrt{-h}+\sqrt[4]{e}\sqrt{hx}\right)}{\ldots}\right)$$

```
[Out] -2*b*e^(1/4)*p*arctan(1-e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1/4)/h^(1/2))*2^(1/2
)/d^(1/4)/f/h^(3/2)+2*b*e^(1/4)*p*arctan(1+e^(1/4)*2^(1/2)*(h*x)^(1/2)/d^(1
/4)/h^(1/2))*2^(1/2)/d^(1/4)/f/h^(3/2)+b*e^(1/4)*p*ln(d^(1/2)*h^(1/2)+x*e^(
1/2)*h^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(h*x)^(1/2))*2^(1/2)/d^(1/4)/f/h^(3/2)
-b*e^(1/4)*p*ln(d^(1/2)*h^(1/2)+x*e^(1/2)*h^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(
h*x)^(1/2))*2^(1/2)/d^(1/4)/f/h^(3/2)-2*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/
h^(1/2))*(a+b*ln(c*(e*x^2+d)^p))*g^(1/2)/f^(3/2)/h^(3/2)-8*b*p*arctan(g^(1/
2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*h^(1/2)/(f^(1/2)*h^(1/2)-I*g^(
1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2)+2*b*p*arctan(g^(1/2)*(h*x)^(1/2)
/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)-e^(1/
4)*(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)-I*e^(1/4)*f^(1/2)*h^(1/2))/(
f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2)+2*b*p*arcta
n(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(-2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^
(1/2)-e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)-(-d)^(1/4)*g^(1/2))/(f^(1/2)*
h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2)+2*b*p*arctan(g^(1/2
)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h
)^(1/2)+e^(1/4)*(h*x)^(1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)+I*e^(1/4)*f^(1/
2)*h^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2
)+2*b*p*arctan(g^(1/2)*(h*x)^(1/2)/f^(1/2)/h^(1/2))*ln(2*f^(1/2)*g^(1/2)*((
-d)^(1/4)*h^(1/2)+e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)+(-d)^(1/4)*g^(1/2
))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2)+4*I*b*p
*polylog(2,1-2*f^(1/2)*h^(1/2)/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*g^(
1/2)/f^(3/2)/h^(3/2)-I*b*p*polylog(2,1-2*f^(1/2)*g^(1/2)*((-d)^(1/4)*h^(1/2
)+e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)+(-d)^(1/4)*g^(1/2))/(f^(1/2)*h^(1
/2)-I*g^(1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2)-I*b*p*polylog(2,1-2*f^(
1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)+e^(1/4)*(h*x)^(1/2))/((-d)^(1/4
)*g^(1/2)*(-h)^(1/2)+I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*
(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2)-I*b*p*polylog(2,1+2*f^(1/2)*g^(1/2)*(
(-d)^(1/4)*h^(1/2)-e^(1/4)*(h*x)^(1/2))/(I*e^(1/4)*f^(1/2)-(-d)^(1/4)*g^(1/
2))/(f^(1/2)*h^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2)-I*b*p*
polylog(2,1-2*f^(1/2)*g^(1/2)*h^(1/2)*((-d)^(1/4)*(-h)^(1/2)-e^(1/4)*(h*x)^
(1/2))/((-d)^(1/4)*g^(1/2)*(-h)^(1/2)-I*e^(1/4)*f^(1/2)*h^(1/2))/(f^(1/2)*h
^(1/2)-I*g^(1/2)*(h*x)^(1/2)))*g^(1/2)/f^(3/2)/h^(3/2)-2*(a+b*ln(c*(e*x^2+d
)^p))/f/h/(h*x)^(1/2)
```

Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 1659, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.613$, Rules used = {2517, 2526, 2505, 303, 1176, 631, 210, 1179, 642, 211, 2520, 12, 266, 6857, 5048,

4966, 2449, 2352, 2497}

$$
\begin{aligned}
\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{(hx)^{3/2}(f+gx)}\,dx = & -\frac{2\sqrt{2}b\sqrt[4]{e}p\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}fh^{3/2}} \\
& + \frac{2\sqrt{2}b\sqrt[4]{e}p\arctan\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}+1\right)}{\sqrt[4]{d}fh^{3/2}} \\
& - \frac{2\sqrt{g}\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c(ex^2+d)^p\right)\right)}{f^{3/2}h^{3/2}} \\
& - \frac{2\left(a+b\log\left(c(ex^2+d)^p\right)\right)}{fh\sqrt{hx}} \\
& + \frac{\sqrt{2}b\sqrt[4]{e}p\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}fh^{3/2}} \\
& - \frac{\sqrt{2}b\sqrt[4]{e}p\log\left(\sqrt{e}\sqrt{h}x+\sqrt{d}\sqrt{h}+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}fh^{3/2}} \\
& - \frac{8b\sqrt{g}p\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{h}}{\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}}\right)}{f^{3/2}h^{3/2}} \\
& + \frac{2b\sqrt{g}p\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\sqrt{h}\left(\sqrt[4]{-d}\sqrt{-h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(\sqrt[4]{-d}\sqrt{g}\sqrt{-h}-i\sqrt[4]{e}\sqrt{f}\sqrt{h}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{f^{3/2}h^{3/2}} \\
& + \frac{2b\sqrt{g}p\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(-\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[4]{-d}\sqrt{h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(i\sqrt[4]{e}\sqrt{f}-\sqrt[4]{-d}\sqrt{g}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{f^{3/2}h^{3/2}} \\
& + \frac{2b\sqrt{g}p\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\sqrt{h}\left(\sqrt[4]{-d}\sqrt{-h}+\sqrt[4]{e}\sqrt{hx}\right)}{\left(\sqrt[4]{-d}\sqrt{g}\sqrt{-h}+i\sqrt[4]{e}\sqrt{f}\sqrt{h}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{f^{3/2}h^{3/2}} \\
& + \frac{2b\sqrt{g}p\arctan\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\log\left(\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[4]{-d}\sqrt{h}+\sqrt[4]{e}\sqrt{hx}\right)}{\left(i\sqrt[4]{e}\sqrt{f}+\sqrt[4]{-d}\sqrt{g}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{f^{3/2}h^{3/2}} \\
& + \frac{4ib\sqrt{g}p\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{h}}{\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}}\right)}{f^{3/2}h^{3/2}} \\
& - \frac{ib\sqrt{g}p\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{f}\sqrt{g}\sqrt{h}\left(\sqrt[4]{-d}\sqrt{-h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(\sqrt[4]{-d}\sqrt{g}\sqrt{-h}-i\sqrt[4]{e}\sqrt{f}\sqrt{h}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}\right)}{f^{3/2}h^{3/2}} \\
& - \frac{ib\sqrt{g}p\,\mathrm{PolyLog}\left(2,\frac{2\sqrt{f}\sqrt{g}\left(\sqrt[4]{-d}\sqrt{h}-\sqrt[4]{e}\sqrt{hx}\right)}{\left(i\sqrt[4]{e}\sqrt{f}-\sqrt[4]{-d}\sqrt{g}\right)\left(\sqrt{f}\sqrt{h}-i\sqrt{g}\sqrt{hx}\right)}+1\right)}{f^{3/2}h^{3/2}}
\end{aligned}
$$

```
[In] Int[(a + b*Log[c*(d + e*x^2)^p])/((h*x)^(3/2)*(f + g*x)),x]
```

```
[Out] (-2*Sqrt[2]*b*e^(1/4)*p*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[h*x])/(d^(1/4)*Sqr
t[h])])/(d^(1/4)*f*h^(3/2)) + (2*Sqrt[2]*b*e^(1/4)*p*ArcTan[1 + (Sqrt[2]*e^
(1/4)*Sqrt[h*x])/(d^(1/4)*Sqrt[h])])/(d^(1/4)*f*h^(3/2)) - (2*(a + b*Log[c*
(d + e*x^2)^p]))/(f*h*Sqrt[h*x]) - (2*Sqrt[g]*ArcTan[(Sqrt[g]*Sqrt[h*x])/(S
qrt[f]*Sqrt[h])]*(a + b*Log[c*(d + e*x^2)^p]))/(f^(3/2)*h^(3/2)) + (Sqrt[2]
*b*e^(1/4)*p*Log[Sqrt[d]*Sqrt[h] + Sqrt[e]*Sqrt[h]*x - Sqrt[2]*d^(1/4)*e^(1
/4)*Sqrt[h*x]])/(d^(1/4)*f*h^(3/2)) - (Sqrt[2]*b*e^(1/4)*p*Log[Sqrt[d]*Sqrt
[h] + Sqrt[e]*Sqrt[h]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[h*x]])/(d^(1/4)*f*h^
(3/2)) - (8*b*Sqrt[g]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(
2*Sqrt[f]*Sqrt[h])/(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x])])/(f^(3/2)*h^(3/
2)) + (2*b*Sqrt[g]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*S
qrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] - e^(1/4)*Sqrt[h*x]))/(((-d)^(1
/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt
[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2)) + (2*b*Sqrt[g]*p*ArcTan[(Sqrt[g]*Sqrt[h
*x])/(Sqrt[f]*Sqrt[h])]*Log[(-2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/
4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] - (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] -
 I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2)) + (2*b*Sqrt[g]*p*ArcTan[(Sqrt[g]
*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*S
qrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] + I*e^(1/4)*Sqr
t[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2))
+ (2*b*Sqrt[g]*p*ArcTan[(Sqrt[g]*Sqrt[h*x])/(Sqrt[f]*Sqrt[h])]*Log[(2*Sqrt[
f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] +
(-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(
3/2)) + ((4*I)*b*Sqrt[g]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[h])/(Sqrt[f]*Sqrt
[h] - I*Sqrt[g]*Sqrt[h*x])])/(f^(3/2)*h^(3/2)) - (I*b*Sqrt[g]*p*PolyLog[2,
1 - (2*Sqrt[f]*Sqrt[g]*Sqrt[h]*((-d)^(1/4)*Sqrt[-h] - e^(1/4)*Sqrt[h*x]))/(
((-d)^(1/4)*Sqrt[g]*Sqrt[-h] - I*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h]
- I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2)) - (I*b*Sqrt[g]*p*PolyLog[2, 1 +
 (2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*Sqrt[h] - e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*S
qrt[f] - (-d)^(1/4)*Sqrt[g])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^
(3/2)*h^(3/2)) - (I*b*Sqrt[g]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*Sqrt[h]*(
(-d)^(1/4)*Sqrt[-h] + e^(1/4)*Sqrt[h*x]))/(((-d)^(1/4)*Sqrt[g]*Sqrt[-h] + I
*e^(1/4)*Sqrt[f]*Sqrt[h])*(Sqrt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/
2)*h^(3/2)) - (I*b*Sqrt[g]*p*PolyLog[2, 1 - (2*Sqrt[f]*Sqrt[g]*((-d)^(1/4)*
Sqrt[h] + e^(1/4)*Sqrt[h*x]))/((I*e^(1/4)*Sqrt[f] + (-d)^(1/4)*Sqrt[g])*(Sq
rt[f]*Sqrt[h] - I*Sqrt[g]*Sqrt[h*x]))])/(f^(3/2)*h^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
```

```
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2517

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(p_.)]*(b_.))^(q_.)*((h_.)
*(x_))^(m_)*((f_.) + (g_.)*(x_))^(r_.), x_Symbol] :> With[{k = Denominator[
m]}, Dist[k/h, Subst[Int[x^(k*(m + 1) - 1)*(f + g*(x^k/h))^r*(a + b*Log[c*(
d + e*(x^(k*n)/h^n))^p])^q, x], x, (h*x)^(1/k)], x]] /; FreeQ[{a, b, c, d,
e, f, g, h, p, r}, x] && FractionQ[m] && IntegerQ[n] && IntegerQ[r]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
```

```
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^2\left(f+\frac{gx^2}{h}\right)}\,dx, x, \sqrt{hx}\right)}{h}\\
&= \frac{2\text{Subst}\left(\int \left(\frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{fx^2}-\frac{g\left(a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)\right)}{f(fh+gx^2)}\right)dx, x, \sqrt{hx}\right)}{h}\\
&= \frac{2\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{x^2}\,dx, x, \sqrt{hx}\right)}{fh}-\frac{(2g)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{ex^4}{h^2}\right)^p\right)}{fh+gx^2}\,dx, x, \sqrt{hx}\right)}{fh}\\
&= -\frac{2\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{fh\sqrt{hx}}-\frac{2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c(d+ex^2)^p\right)\right)}{f^{3/2}h^{3/2}}\\
&\quad+\frac{(8bep)\text{Subst}\left(\int \frac{x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{fh^3}+\frac{(8begp)\text{Subst}\left(\int \frac{x^3\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{\sqrt{f}\sqrt{g}\sqrt{h}\left(d+\frac{ex^4}{h^2}\right)}\,dx, x, \sqrt{hx}\right)}{fh^3}
\end{aligned}
$$

$$
= -\frac{2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{fh\sqrt{hx}} - \frac{2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{f^{3/2}h^{3/2}}
+ \frac{\left(8be\sqrt{g}p\right)\text{Subst}\left(\int \frac{x^3\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{f^{3/2}h^{7/2}}
- \frac{\left(4b\sqrt{e}p\right)\text{Subst}\left(\int \frac{\sqrt{d}h-\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{fh^3}
+ \frac{\left(4b\sqrt{e}p\right)\text{Subst}\left(\int \frac{\sqrt{d}h+\sqrt{e}x^2}{d+\frac{ex^4}{h^2}}\,dx, x, \sqrt{hx}\right)}{fh^3}
$$

$$
= -\frac{2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{fh\sqrt{hx}} - \frac{2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{f^{3/2}h^{3/2}}
+ \frac{\left(8be\sqrt{g}p\right)\text{Subst}\left(\int \left(\frac{h^2x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{2\left(-\sqrt{-d}\sqrt{e}h+ex^2\right)} + \frac{h^2x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{2\left(\sqrt{-d}\sqrt{e}h+ex^2\right)}\right)dx, x, \sqrt{hx}\right)}{f^{3/2}h^{7/2}}
+ \frac{\left(\sqrt{2}b\sqrt[4]{e}p\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}+2x}{-\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{\sqrt[4]{d}fh^{3/2}}
+ \frac{\left(\sqrt{2}b\sqrt[4]{e}p\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}}{\sqrt[4]{e}}-2x}{-\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}-x^2}\,dx, x, \sqrt{hx}\right)}{\sqrt[4]{d}fh^{3/2}}
+ \frac{(2bp)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{fh}
+ \frac{(2bp)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}h}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{h}x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{hx}\right)}{fh}
$$

$$= -\frac{2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{fh\sqrt{hx}} - \frac{2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{f^{3/2}h^{3/2}} + \frac{\sqrt{2}b\sqrt[4]{e}p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}fh^{3/2}} - \frac{\sqrt{2}b\sqrt[4]{e}p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}fh^{3/2}} + \frac{\left(2\sqrt{2}b\sqrt[4]{e}p\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}fh^{3/2}} - \frac{\left(2\sqrt{2}b\sqrt[4]{e}p\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}fh^{3/2}} + \frac{\left(4be\sqrt{g}p\right)\text{Subst}\left(\int\frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{-\sqrt{-d}\sqrt{e}h+ex^2}\,dx,x,\sqrt{hx}\right)}{f^{3/2}h^{3/2}} + \frac{\left(4be\sqrt{g}p\right)\text{Subst}\left(\int\frac{x\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{\sqrt{-d}\sqrt{e}h+ex^2}\,dx,x,\sqrt{hx}\right)}{f^{3/2}h^{3/2}}$$

$$= -\frac{2\sqrt{2}b\sqrt[4]{e}p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}fh^{3/2}} + \frac{2\sqrt{2}b\sqrt[4]{e}p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}fh^{3/2}} - \frac{2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{fh\sqrt{hx}} - \frac{2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{f^{3/2}h^{3/2}} + \frac{\sqrt{2}b\sqrt[4]{e}p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}fh^{3/2}} - \frac{\sqrt{2}b\sqrt[4]{e}p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}fh^{3/2}} + \frac{\left(4be\sqrt{g}p\right)\text{Subst}\left(\int\left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{2e^{3/4}\left(\sqrt[4]{-d}\sqrt{-h}-\sqrt[4]{e}x\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{2e^{3/4}\left(\sqrt[4]{-d}\sqrt{-h}+\sqrt[4]{e}x\right)}\right)dx,x,\sqrt{hx}\right)}{f^{3/2}h^{3/2}} + \frac{\left(4be\sqrt{g}p\right)\text{Subst}\left(\int\left(-\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{2e^{3/4}\left(\sqrt[4]{-d}\sqrt{h}-\sqrt[4]{e}x\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{2e^{3/4}\left(\sqrt[4]{-d}\sqrt{h}+\sqrt[4]{e}x\right)}\right)dx,x,\sqrt{hx}\right)}{f^{3/2}h^{3/2}}$$

$$
\begin{aligned}
&= -\frac{2\sqrt{2}b\sqrt[4]{e}p\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}fh^{3/2}}+\frac{2\sqrt{2}b\sqrt[4]{e}p\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{e}\sqrt{hx}}{\sqrt[4]{d}\sqrt{h}}\right)}{\sqrt[4]{d}fh^{3/2}} \\
&\quad -\frac{2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{fh\sqrt{hx}}-\frac{2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{hx}}{\sqrt{f}\sqrt{h}}\right)\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{f^{3/2}h^{3/2}} \\
&\quad +\frac{\sqrt{2}b\sqrt[4]{e}p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}fh^{3/2}} \\
&\quad -\frac{\sqrt{2}b\sqrt[4]{e}p\log\left(\sqrt{d}\sqrt{h}+\sqrt{e}\sqrt{h}x+\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{hx}\right)}{\sqrt[4]{d}fh^{3/2}} \\
&\quad -\frac{\left(2b\sqrt[4]{e}\sqrt{g}p\right)\text{Subst}\left(\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{\sqrt[4]{-d}\sqrt{-h}-\sqrt[4]{e}x}\,dx,x,\sqrt{hx}\right)}{f^{3/2}h^{3/2}} \\
&\quad -\frac{\left(2b\sqrt[4]{e}\sqrt{g}p\right)\text{Subst}\left(\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{\sqrt[4]{-d}\sqrt{h}-\sqrt[4]{e}x}\,dx,x,\sqrt{hx}\right)}{f^{3/2}h^{3/2}} \\
&\quad +\frac{\left(2b\sqrt[4]{e}\sqrt{g}p\right)\text{Subst}\left(\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{\sqrt[4]{-d}\sqrt{-h}+\sqrt[4]{e}x}\,dx,x,\sqrt{hx}\right)}{f^{3/2}h^{3/2}} \\
&\quad +\frac{\left(2b\sqrt[4]{e}\sqrt{g}p\right)\text{Subst}\left(\int\frac{\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}\sqrt{h}}\right)}{\sqrt[4]{-d}\sqrt{h}+\sqrt[4]{e}x}\,dx,x,\sqrt{hx}\right)}{f^{3/2}h^{3/2}} \\
&= \text{Too large to display}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 1336, normalized size of antiderivative = 0.81

$$
\int\frac{a+b\log\left(c\left(d+ex^2\right)^p\right)}{(hx)^{3/2}(f+gx)}\,dx=\frac{x^{3/2}\left(\frac{4b\sqrt[4]{e}p\left(\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{-d}}\right)+\text{arctanh}\left(\frac{d\sqrt[4]{e}\sqrt{x}}{(-d)^{5/4}}\right)\right)}{\sqrt[4]{-d}}-\frac{2\left(a+b\log\left(c\left(d+ex^2\right)^p\right)\right)}{\sqrt{x}}+\frac{f\sqrt{g}\,\text{lo}\cdots}{\cdots}\right.}{\cdots}
$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^2)^p])/((h*x)^(3/2)*(f + g*x)),x]
```

```
[Out] (x^(3/2)*((4*b*e^(1/4)*p*(ArcTan[(e^(1/4)*Sqrt[x])/(-d)^(1/4)] + ArcTanh[(d
*e^(1/4)*Sqrt[x])/(-d)^(5/4)]))/(-d)^(1/4) - (2*(a + b*Log[c*(d + e*x^2)^p]
))/Sqrt[x] + (f*Sqrt[g]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]]*(a + b*Log[c*(d + e
*x^2)^p]))/(-f)^(3/2) + (Sqrt[g]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]]*(a + b*Log
```

[c*(d + e*x^2)^p]))/Sqrt[-f] + (b*Sqrt[g]*p*(Log[(Sqrt[g]*((-d)^(1/4) - e^(1/4)*Sqrt[x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/4)*Sqrt[x]))/(I*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*(I*(-d)^(1/4) + e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] - Sqrt[g]*Sqrt[x]] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] - (-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] - Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]))/Sqrt[-f] + (b*f*Sqrt[g]*p*(Log[(Sqrt[g]*((-d)^(1/4) - e^(1/4)*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) - I*e^(1/4)*Sqrt[x]))/(I*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + I*e^(1/4)*Sqrt[x]))/((-I)*e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + Log[(Sqrt[g]*((-d)^(1/4) + e^(1/4)*Sqrt[x]))/(-(e^(1/4)*Sqrt[-f]) + (-d)^(1/4)*Sqrt[g])]*Log[Sqrt[-f] + Sqrt[g]*Sqrt[x]] + PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] - (-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] - I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + I*(-d)^(1/4)*Sqrt[g])] + PolyLog[2, (e^(1/4)*(Sqrt[-f] + Sqrt[g]*Sqrt[x]))/(e^(1/4)*Sqrt[-f] + (-d)^(1/4)*Sqrt[g])]))/(-f)^(3/2)))/(f*(h*x)^(3/2))

Maple [F]

$$\int \frac{a+b\ln\left(c(e\,x^2+d)^p\right)}{(hx)^{\frac{3}{2}}\,(gx+f)}dx$$

[In] int((a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x)

[Out] int((a+b*ln(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x)

Fricas [F]

$$\int \frac{a+b\log\left(c(d+ex^2)^p\right)}{(hx)^{3/2}(f+gx)}\,dx=\int \frac{b\log\left((ex^2+d)^pc\right)+a}{(gx+f)\,(hx)^{\frac{3}{2}}}\,dx$$

[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x, algorithm="fricas")

[Out] integral((sqrt(h*x)*b*log((e*x^2 + d)^p*c) + sqrt(h*x)*a)/(g*h^2*x^3 + f*h^2*x^2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log \left(c(d+ex^2)^p\right)}{(hx)^{3/2}(f+gx)}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*(e*x**2+d)**p))/(h*x)**(3/2)/(g*x+f),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+b\log \left(c(d+ex^2)^p\right)}{(hx)^{3/2}(f+gx)}\,dx = \int \frac{b\log \left((ex^2+d)^p c\right)+a}{(gx+f)\,(hx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x, algorithm="maxima
")
[Out] b*integrate((sqrt(h)*log((e*x^2 + d)^p) + sqrt(h)*log(c))/(g*h^2*x^(5/2) +
f*h^2*x^(3/2)), x) - 2*a*(g*arctan(sqrt(h*x)*g/sqrt(f*g*h))/(sqrt(f*g*h)*f)
 + 1/(sqrt(h*x)*f))/h
```

Giac [F]

$$\int \frac{a+b\log \left(c(d+ex^2)^p\right)}{(hx)^{3/2}(f+gx)}\,dx = \int \frac{b\log \left((ex^2+d)^p c\right)+a}{(gx+f)\,(hx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*log(c*(e*x^2+d)^p))/(h*x)^(3/2)/(g*x+f),x, algorithm="giac")
[Out] integrate((b*log((e*x^2 + d)^p*c) + a)/((g*x + f)*(h*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log \left(c(d+ex^2)^p\right)}{(hx)^{3/2}(f+gx)}\,dx = \int \frac{a+b\,\ln \left(c\,{\left(e\,x^2+d\right)}^p\right)}{\left(f+g\,x\right)\,{\left(h\,x\right)}^{3/2}}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^2)^p))/((f + g*x)*(h*x)^(3/2)),x)
[Out] int((a + b*log(c*(d + e*x^2)^p))/((f + g*x)*(h*x)^(3/2)), x)
```

3.619 $\int \frac{\log(fx^p)\log(1+ex^m)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 33

$$\int \frac{\log(fx^p)\log(1+ex^m)}{x}\,dx = -\frac{\log(fx^p)\operatorname{PolyLog}(2,-ex^m)}{m}+\frac{p\operatorname{PolyLog}(3,-ex^m)}{m^2}$$

```
[Out] -ln(f*x^p)*polylog(2,-e*x^m)/m+p*polylog(3,-e*x^m)/m^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2421, 6724}

$$\int \frac{\log(fx^p)\log(1+ex^m)}{x}\,dx = \frac{p\operatorname{PolyLog}(3,-ex^m)}{m^2}-\frac{\operatorname{PolyLog}(2,-ex^m)\log(fx^p)}{m}$$

```
[In] Int[(Log[f*x^p]*Log[1 + e*x^m])/x,x]
```

```
[Out] -((Log[f*x^p]*PolyLog[2, -(e*x^m)])/m) + (p*PolyLog[3, -(e*x^m)])/m^2
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\log\left(fx^p\right)\text{Li}_2(-ex^m)}{m}+\frac{p\int\frac{\text{Li}_2(-ex^m)}{x}\,dx}{m}\\
&= -\frac{\log\left(fx^p\right)\text{Li}_2(-ex^m)}{m}+\frac{p\text{Li}_3(-ex^m)}{m^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$
\int\frac{\log\left(fx^p\right)\log\left(1+ex^m\right)}{x}\,dx=-\frac{\log\left(fx^p\right)\text{PolyLog}\left(2,-ex^m\right)}{m}+\frac{p\,\text{PolyLog}\left(3,-ex^m\right)}{m^2}
$$

```
[In] Integrate[(Log[f*x^p]*Log[1 + e*x^m])/x,x]
[Out] -((Log[f*x^p]*PolyLog[2, -(e*x^m)])/m) + (p*PolyLog[3, -(e*x^m)])/m^2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.54 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.48

method	result
risch	$-\frac{p\ln(x)\,\text{Li}_2(-e\,x^m)}{m}+\frac{p\ \text{Li}_3(-e\,x^m)}{m^2}-\frac{(\ln(x^p)-p\ln(x))\,\text{dilog}(1+e\,x^m)}{m}-\frac{\left(-\frac{i\pi\ \text{csgn}(if)\,\text{csgn}(ix^p)\,\text{csgn}(if\,x^p)}{2}+\frac{i\pi\ \text{csgn}(if)\,\text{csgn}(}{2}\right.}{}$

```
[In] int(ln(f*x^p)*ln(1+e*x^m)/x,x,method=_RETURNVERBOSE)
[Out] -p/m*ln(x)*polylog(2,-e*x^m)+p*polylog(3,-e*x^m)/m^2-1/m*(ln(x^p)-p*ln(x))*
dilog(1+e*x^m)-(-1/2*I*Pi*csgn(I*f)*csgn(I*x^p)*csgn(I*f*x^p)+1/2*I*Pi*csgn
(I*f)*csgn(I*f*x^p)^2+1/2*I*Pi*csgn(I*x^p)*csgn(I*f*x^p)^2-1/2*I*Pi*csgn(I*
f*x^p)^3+ln(f))/m*dilog(1+e*x^m)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{\log (fx^p) \log (1+ex^m)}{x} \, dx = -\frac{(mp\log (x) + m\log (f))\mathrm{Li}_2(-ex^m) - p\mathrm{polylog}(3, -ex^m)}{m^2}$$

```
[In] integrate(log(f*x^p)*log(1+e*x^m)/x,x, algorithm="fricas")
[Out] -((m*p*log(x) + m*log(f))*dilog(-e*x^m) - p*polylog(3, -e*x^m))/m^2
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log (fx^p) \log (1+ex^m)}{x} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(ln(f*x**p)*ln(1+e*x**m)/x,x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{\log (fx^p) \log (1+ex^m)}{x} \, dx = \int \frac{\log (ex^m + 1) \log (fx^p)}{x} \, dx$$

```
[In] integrate(log(f*x^p)*log(1+e*x^m)/x,x, algorithm="maxima")
[Out] -1/2*(p*log(x)^2 - 2*log(f)*log(x) - 2*log(x)*log(x^p))*log(e*x^m + 1) - in
tegrate(1/2*(2*e*m*x^m*log(x)*log(x^p) - (e*m*p*log(x)^2 - 2*e*m*log(f)*log
(x))*x^m)/(e*x*x^m + x), x)
```

Giac [F]

$$\int \frac{\log (fx^p) \log (1+ex^m)}{x} \, dx = \int \frac{\log (ex^m + 1) \log (fx^p)}{x} \, dx$$

```
[In] integrate(log(f*x^p)*log(1+e*x^m)/x,x, algorithm="giac")
[Out] integrate(log(e*x^m + 1)*log(f*x^p)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(fx^p)\log(1+ex^m)}{x}\,dx = \int \frac{\ln\left(f\,x^p\right)\,\ln\left(e\,x^m+1\right)}{x}\,dx$$

```
[In] int((log(f*x^p)*log(e*x^m + 1))/x,x)
[Out] int((log(f*x^p)*log(e*x^m + 1))/x, x)
```

3.620 $\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx = \frac{\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{em}+\frac{2p\log(fx^p)\operatorname{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{em^2}-\frac{2p^2\operatorname{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{em^3}$$

```
[Out] ln(f*x^p)^2*ln(1+e*x^m/d)/e/m+2*p*ln(f*x^p)*polylog(2,-e*x^m/d)/e/m^2-2*p^2
*polylog(3,-e*x^m/d)/e/m^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2375, 2421, 6724}

$$\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx = \frac{2p\log(fx^p)\operatorname{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{em^2}+\frac{\log^2(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{em}-\frac{2p^2\operatorname{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{em^3}$$

```
[In] Int[(x^(-1 + m)*Log[f*x^p]^2)/(d + e*x^m),x]
```

```
[Out] (Log[f*x^p]^2*Log[1 + (e*x^m)/d])/(e*m) + (2*p*Log[f*x^p]*PolyLog[2, -((e*x
^m)/d)])/(e*m^2) - (2*p^2*PolyLog[3, -((e*x^m)/d)])/(e*m^3)
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
```

```
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{em} - \frac{(2p)\int \frac{\log(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{x}\,dx}{em} \\
&= \frac{\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{em} + \frac{2p\log(fx^p)\,\mathrm{Li}_2\left(-\frac{ex^m}{d}\right)}{em^2} - \frac{(2p^2)\int \frac{\mathrm{Li}_2\left(-\frac{ex^m}{d}\right)}{x}\,dx}{em^2} \\
&= \frac{\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{em} + \frac{2p\log(fx^p)\,\mathrm{Li}_2\left(-\frac{ex^m}{d}\right)}{em^2} - \frac{2p^2\mathrm{Li}_3\left(-\frac{ex^m}{d}\right)}{em^3}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 220 vs. 2(75) = 150.

Time = 0.18 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.93

$$
\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx
$$

$$
= \frac{p^2\log^3(x) + 3p\log^2(x)\left(-p\log(x)+\log(fx^p)\right) + 3\log(x)\left(-p\log(x)+\log(fx^p)\right)^2 - \frac{3(-p\log(x)+\log(fx^p))^2(\log}{m}
$$

```
[In] Integrate[(x^(-1 + m)*Log[f*x^p]^2)/(d + e*x^m),x]
```

```
[Out] (p^2*Log[x]^3 + 3*p*Log[x]^2*(-(p*Log[x]) + Log[f*x^p]) + 3*Log[x]*(-(p*Log
[x]) + Log[f*x^p])^2 - (3*(-(p*Log[x]) + Log[f*x^p])^2*(Log[x^m] - Log[d*m*
(d + e*x^m)]))/m - (6*p*(-(p*Log[x]) + Log[f*x^p])*((m^2*Log[x]^2)/2 + (-(m
*Log[x]) + Log[-((e*x^m)/d)])*Log[d + e*x^m] + PolyLog[2, 1 + (e*x^m)/d]))/
m^2 + (3*p^2*(m^2*Log[x]^2*Log[1 + d/(e*x^m)] - 2*m*Log[x]*PolyLog[2, -(d/(
e*x^m))] - 2*PolyLog[3, -(d/(e*x^m))]))/m^3)/(3*e)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.42 (sec) , antiderivative size = 496, normalized size of antiderivative = 6.61

method	result
risch	$\frac{\ln(d+e\,x^m)\ln(x)^2 p^2}{me}-\frac{2\ln(d+e\,x^m)\ln(x)\ln(x^p)p}{me}+\frac{\ln(d+e\,x^m)\ln(x^p)^2}{me}+\frac{p^2\ln(x)^2\ln\left(1+\frac{e\,x^m}{d}\right)}{me}+\frac{2p^2\ln(x)\,\mathrm{Li}_2\left(-\frac{e\,x^m}{d}\right)}{m^2 e}-$

```
[In] int(x^(m-1)*ln(f*x^p)^2/(d+e*x^m),x,method=_RETURNVERBOSE)
```

```
[Out] 1/m*ln(d+e*x^m)/e*ln(x)^2*p^2-2/m*ln(d+e*x^m)/e*ln(x)*ln(x^p)*p+1/m*ln(d+e*
x^m)/e*ln(x^p)^2+1/m*p^2/e*ln(x)^2*ln(1+e*x^m/d)+2/m^2*p^2/e*ln(x)*polylog(
2,-e*x^m/d)-2*p^2*polylog(3,-e*x^m/d)/e/m^3-2/m^2*p^2*dilog((d+e*x^m)/d)/e*
ln(x)+2/m^2*p*dilog((d+e*x^m)/d)/e*ln(x^p)-2/m*p^2*ln(x)^2*ln((d+e*x^m)/d)/
e+2/m*p*ln(x)*ln((d+e*x^m)/d)/e*ln(x^p)+(-I*Pi*csgn(I*f)*csgn(I*x^p)*csgn(I
*f*x^p)+I*Pi*csgn(I*f)*csgn(I*f*x^p)^2+I*Pi*csgn(I*x^p)*csgn(I*f*x^p)^2-I*P
i*csgn(I*f*x^p)^3+2*ln(f))*(1/m*(ln(x^p)-p*ln(x))*ln(d+e*x^m)/e+1/m^2*p*dil
og((d+e*x^m)/d)/e+1/m*p*ln(x)*ln((d+e*x^m)/d)/e)+1/4*(-I*Pi*csgn(I*f)*csgn(
I*x^p)*csgn(I*f*x^p)+I*Pi*csgn(I*f)*csgn(I*f*x^p)^2+I*Pi*csgn(I*x^p)*csgn(I
*f*x^p)^2-I*Pi*csgn(I*f*x^p)^3+2*ln(f))^2/m*ln(d+e*x^m)/e
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.40

$$\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx$$
$$=\frac{m^2\log(ex^m+d)\log(f)^2-2\,p^2\mathrm{polylog}\left(3,-\frac{ex^m}{d}\right)+2\,(mp^2\log(x)+mp\log(f))\mathrm{Li}_2\left(-\frac{ex^m+d}{d}+1\right)+\left(m^2p^2\right.}{em^3}$$

```
[In] integrate(x^(-1+m)*log(f*x^p)^2/(d+e*x^m),x, algorithm="fricas")
```

```
[Out] (m^2*log(e*x^m + d)*log(f)^2 - 2*p^2*polylog(3, -e*x^m/d) + 2*(m*p^2*log(x)
 + m*p*log(f))*dilog(-(e*x^m + d)/d + 1) + (m^2*p^2*log(x)^2 + 2*m^2*p*log(
f)*log(x))*log((e*x^m + d)/d))/(e*m^3)
```

Sympy [F]

$$\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx = \int \frac{x^{m-1}\log(fx^p)^2}{d+ex^m}\,dx$$

```
[In] integrate(x**(-1+m)*ln(f*x**p)**2/(d+e*x**m),x)
[Out] Integral(x**(m - 1)*log(f*x**p)**2/(d + e*x**m), x)
```

Maxima [F]

$$\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx = \int \frac{x^{m-1}\log(fx^p)^2}{ex^m+d}\,dx$$

```
[In] integrate(x^(-1+m)*log(f*x^p)^2/(d+e*x^m),x, algorithm="maxima")
[Out] integrate(x^(m - 1)*log(f*x^p)^2/(e*x^m + d), x)
```

Giac [F]

$$\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx = \int \frac{x^{m-1}\log(fx^p)^2}{ex^m+d}\,dx$$

```
[In] integrate(x^(-1+m)*log(f*x^p)^2/(d+e*x^m),x, algorithm="giac")
[Out] integrate(x^(m - 1)*log(f*x^p)^2/(e*x^m + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx = \int \frac{x^{m-1}\ln(f\,x^p)^2}{d+e\,x^m}\,dx$$

```
[In] int((x^(m - 1)*log(f*x^p)^2)/(d + e*x^m),x)
[Out] int((x^(m - 1)*log(f*x^p)^2)/(d + e*x^m), x)
```

3.621 $\int \frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 161

$$\int \frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx = \frac{\log^4(fx^p)(a+b\log(c(d+ex^m)^n))}{4p} - \frac{bn\log^4(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{4p} - \frac{bn\log^3(fx^p)\,\text{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m} + \frac{3bnp\log^2(fx^p)\,\text{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2} - \frac{6bnp^2\log(fx^p)\,\text{PolyLog}\left(4,-\frac{ex^m}{d}\right)}{m^3} + \frac{6bnp^3\,\text{PolyLog}\left(5,-\frac{ex^m}{d}\right)}{m^4}$$

```
[Out] 1/4*ln(f*x^p)^4*(a+b*ln(c*(d+e*x^m)^n))/p-1/4*b*n*ln(f*x^p)^4*ln(1+e*x^m/d)
/p-b*n*ln(f*x^p)^3*polylog(2,-e*x^m/d)/m+3*b*n*p*ln(f*x^p)^2*polylog(3,-e*x
^m/d)/m^2-6*b*n*p^2*ln(f*x^p)*polylog(4,-e*x^m/d)/m^3+6*b*n*p^3*polylog(5,-
e*x^m/d)/m^4
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2531, 2375, 2421, 2430, 6724}

$$\int \frac{\log^3(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{x}\,dx = \frac{\log^4(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{4p} - \frac{6bnp^2\log(fx^p)\operatorname{PolyLog}\left(4,-\frac{ex^m}{d}\right)}{m^3} + \frac{3bnp\log^2(fx^p)\operatorname{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2} - \frac{bn\log^3(fx^p)\operatorname{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m} - \frac{bn\log^4(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{4p} + \frac{6bnp^3\operatorname{PolyLog}\left(5,-\frac{ex^m}{d}\right)}{m^4}$$

```
[In] Int[(Log[f*x^p]^3*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

```
[Out] (Log[f*x^p]^4*(a + b*Log[c*(d + e*x^m)^n]))/(4*p) - (b*n*Log[f*x^p]^4*Log[1
 + (e*x^m)/d])/(4*p) - (b*n*Log[f*x^p]^3*PolyLog[2, -((e*x^m)/d)])/m + (3*b
*n*p*Log[f*x^p]^2*PolyLog[3, -((e*x^m)/d)])/m^2 - (6*b*n*p^2*Log[f*x^p]*Pol
yLog[4, -((e*x^m)/d)])/m^3 + (6*b*n*p^3*PolyLog[5, -((e*x^m)/d)])/m^4
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
```

```
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2531

```
Int[(Log[(f_.)*(x_)^(q_.)]^(m_.)*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_)
)^(p_.)]*(b_.)))/(x_), x_Symbol] :> Simp[Log[f*x^q]^(m + 1)*((a + b*Log[c*(
d + e*x^n)^p])/(q*(m + 1))), x] - Dist[b*e*n*(p/(q*(m + 1))), Int[x^(n - 1)
*(Log[f*x^q]^(m + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n,
 p, q}, x] && NeQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\log^4(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{4p}-\frac{(bemn)\int\frac{x^{-1+m}\log^4(fx^p)}{d+ex^m}\,dx}{4p}\\
&= \frac{\log^4(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{4p}-\frac{bn\log^4(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{4p}\\
&\quad+(bn)\int\frac{\log^3(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{x}\,dx\\
&= \frac{\log^4(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{4p}-\frac{bn\log^4(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{4p}\\
&\quad-\frac{bn\log^3(fx^p)\operatorname{Li}_2\left(-\frac{ex^m}{d}\right)}{m}+\frac{(3bnp)\int\frac{\log^2(fx^p)\operatorname{Li}_2\left(-\frac{ex^m}{d}\right)}{x}\,dx}{m}\\
&= \frac{\log^4(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{4p}\\
&\quad-\frac{bn\log^4(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{4p}-\frac{bn\log^3(fx^p)\operatorname{Li}_2\left(-\frac{ex^m}{d}\right)}{m}\\
&\quad+\frac{3bnp\log^2(fx^p)\operatorname{Li}_3\left(-\frac{ex^m}{d}\right)}{m^2}-\frac{(6bnp^2)\int\frac{\log(fx^p)\operatorname{Li}_3\left(-\frac{ex^m}{d}\right)}{x}\,dx}{m^2}\\
&= \frac{\log^4(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{4p}-\frac{bn\log^4(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{4p}-\frac{bn\log^3(fx^p)\operatorname{Li}_2\left(-\frac{ex^m}{d}\right)}{m}\\
&\quad+\frac{3bnp\log^2(fx^p)\operatorname{Li}_3\left(-\frac{ex^m}{d}\right)}{m^2}-\frac{6bnp^2\log(fx^p)\operatorname{Li}_4\left(-\frac{ex^m}{d}\right)}{m^3}+\frac{(6bnp^3)\int\frac{\operatorname{Li}_4\left(-\frac{ex^m}{d}\right)}{x}\,dx}{m^3}
\end{aligned}
$$

$$= \frac{\log^4(fx^p)(a+b\log(c(d+ex^m)^n))}{4p} - \frac{bn\log^4(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{4p} - \frac{bn\log^3(fx^p)\operatorname{Li}_2\left(-\frac{ex^m}{d}\right)}{m} + \frac{3bnp\log^2(fx^p)\operatorname{Li}_3\left(-\frac{ex^m}{d}\right)}{m^2} - \frac{6bnp^2\log(fx^p)\operatorname{Li}_4\left(-\frac{ex^m}{d}\right)}{m^3} + \frac{6bnp^3\operatorname{Li}_5\left(-\frac{ex^m}{d}\right)}{m^4}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 659 vs. $2(161) = 322$.

Time = 0.20 (sec) , antiderivative size = 659, normalized size of antiderivative = 4.09

$$\int \frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx$$
$$= -\frac{3}{10}bmnp^3\log^5(x) + \frac{3}{4}bmnp^2\log^4(x)\log(fx^p) - \frac{1}{2}bmnp\log^3(x)\log^2(fx^p) + \frac{a\log^4(fx^p)}{4p} - \frac{3}{4}bnp^3\log^4(x)\log\left(1+\frac{dx^{-m}}{e}\right) + 2bnp^2\log^3(x)\log(fx^p)\log\left(1+\frac{dx^{-m}}{e}\right) - \frac{3}{2}bnp\log^2(x)\log^2(fx^p)\log\left(1+\frac{dx^{-m}}{e}\right) + bnp^3\log^4(x)\log(d+ex^m) - \frac{bnp^3\log^3(x)\log\left(-\frac{ex^m}{d}\right)\log(d+ex^m)}{m} - 3bnp^2\log^3(x)\log(fx^p)\log(d+ex^m) + \frac{3bnp^2\log^2(x)\log\left(-\frac{ex^m}{d}\right)\log(fx^p)\log(d+ex^m)}{m} + 3bnp\log^2(x)\log^2(fx^p)\log(d+ex^m) - \frac{3bnp\log(x)\log\left(-\frac{ex^m}{d}\right)\log^2(fx^p)\log(d+ex^m)}{m} - bn\log(x)\log^3(fx^p)\log(d+ex^m) + \frac{bn\log\left(-\frac{ex^m}{d}\right)\log^3(fx^p)\log(d+ex^m)}{m} - \frac{1}{4}bp^3\log^4(x)\log(c(d+ex^m)^n) + bp^2\log^3(x)\log(fx^p)\log(c(d+ex^m)^n) - \frac{3}{2}bp\log^2(x)\log^2(fx^p)\log(c(d+ex^m)^n) + b\log(x)\log^3(fx^p)\log(c(d+ex^m)^n) + \frac{bnp\log(x)\left(p^2\log^2(x)-3p\log(x)\log(fx^p)+3\log^2(fx^p)\right)\operatorname{PolyLog}\left(2,-\frac{dx^{-m}}{e}\right)}{m} - \frac{bn(p\log(x)-\log(fx^p))^3\operatorname{PolyLog}\left(2,1+\frac{ex^m}{d}\right)}{m} + \frac{3bnp\log^2(fx^p)\operatorname{PolyLog}\left(3,-\frac{dx^{-m}}{e}\right)}{m^2} + \frac{6bnp^2\log(fx^p)\operatorname{PolyLog}\left(4,-\frac{dx^{-m}}{e}\right)}{m^3} + \frac{6bnp^3\operatorname{PolyLog}\left(5,-\frac{dx^{-m}}{e}\right)}{m^4}$$

```
[In] Integrate[(Log[f*x^p]^3*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

```
[Out] (-3*b*m*n*p^3*Log[x]^5)/10 + (3*b*m*n*p^2*Log[x]^4*Log[f*x^p])/4 - (b*m*n*p
*Log[x]^3*Log[f*x^p]^2)/2 + (a*Log[f*x^p]^4)/(4*p) - (3*b*n*p^3*Log[x]^4*Lo
g[1 + d/(e*x^m)])/4 + 2*b*n*p^2*Log[x]^3*Log[f*x^p]*Log[1 + d/(e*x^m)] - (3
```

```
*b*n*p*Log[x]^2*Log[f*x^p]^2*Log[1 + d/(e*x^m)])/2 + b*n*p^3*Log[x]^4*Log[d
 + e*x^m] - (b*n*p^3*Log[x]^3*Log[-((e*x^m)/d)]*Log[d + e*x^m])/m - 3*b*n*p
^2*Log[x]^3*Log[f*x^p]*Log[d + e*x^m] + (3*b*n*p^2*Log[x]^2*Log[-((e*x^m)/d
)]*Log[f*x^p]*Log[d + e*x^m])/m + 3*b*n*p*Log[x]^2*Log[f*x^p]^2*Log[d + e*x
^m] - (3*b*n*p*Log[x]*Log[-((e*x^m)/d)]*Log[f*x^p]^2*Log[d + e*x^m])/m - b*
n*Log[x]*Log[f*x^p]^3*Log[d + e*x^m] + (b*n*Log[-((e*x^m)/d)]*Log[f*x^p]^3*
Log[d + e*x^m])/m - (b*p^3*Log[x]^4*Log[c*(d + e*x^m)^n])/4 + b*p^2*Log[x]^
3*Log[f*x^p]*Log[c*(d + e*x^m)^n] - (3*b*p*Log[x]^2*Log[f*x^p]^2*Log[c*(d +
 e*x^m)^n])/2 + b*Log[x]*Log[f*x^p]^3*Log[c*(d + e*x^m)^n] + (b*n*p*Log[x]*
(p^2*Log[x]^2 - 3*p*Log[x]*Log[f*x^p] + 3*Log[f*x^p]^2)*PolyLog[2, -(d/(e*x
^m))])/m - (b*n*(p*Log[x] - Log[f*x^p])^3*PolyLog[2, 1 + (e*x^m)/d])/m + (3
*b*n*p*Log[f*x^p]^2*PolyLog[3, -(d/(e*x^m))])/m^2 + (6*b*n*p^2*Log[f*x^p]*P
olyLog[4, -(d/(e*x^m))])/m^3 + (6*b*n*p^3*PolyLog[5, -(d/(e*x^m))])/m^4
```

Maple [F]

$$\int \frac{\ln(f\,x^p)^3\,(a+b\ln(c(d+e\,x^m)^n))}{x}dx$$

[In] `int(ln(f*x^p)^3*(a+b*ln(c*(d+e*x^m)^n))/x,x)`

[Out] `int(ln(f*x^p)^3*(a+b*ln(c*(d+e*x^m)^n))/x,x)`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(156) = 312$.

Time = 0.38 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.59

$$\int \frac{\log^3(fx^p)\,(a+b\log(c(d+ex^m)^n))}{x}\,dx$$
$$= \frac{24\,bnp^3\mathrm{polylog}\left(5,-\frac{ex^m}{d}\right) + 4\,(bm^4\log(c)+am^4)\log(f)^3\log(x) + 6\,(bm^4p\log(c)+am^4p)\log(f)^2\log(x)^2}{}$$

[In] `integrate(log(f*x^p)^3*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="fricas")`

```
[Out] 1/4*(24*b*n*p^3*polylog(5, -e*x^m/d) + 4*(b*m^4*log(c) + a*m^4)*log(f)^3*lo
g(x) + 6*(b*m^4*p*log(c) + a*m^4*p)*log(f)^2*log(x)^2 + 4*(b*m^4*p^2*log(c)
 + a*m^4*p^2)*log(f)*log(x)^3 + (b*m^4*p^3*log(c) + a*m^4*p^3)*log(x)^4 - 4
*(b*m^3*n*p^3*log(x)^3 + 3*b*m^3*n*p^2*log(f)*log(x)^2 + 3*b*m^3*n*p*log(f)
^2*log(x) + b*m^3*n*log(f)^3)*dilog(-(e*x^m + d)/d + 1) + (b*m^4*n*p^3*log(
x)^4 + 4*b*m^4*n*p^2*log(f)*log(x)^3 + 6*b*m^4*n*p*log(f)^2*log(x)^2 + 4*b*
m^4*n*log(f)^3*log(x))*log(e*x^m + d) - (b*m^4*n*p^3*log(x)^4 + 4*b*m^4*n*p
^2*log(f)*log(x)^3 + 6*b*m^4*n*p*log(f)^2*log(x)^2 + 4*b*m^4*n*log(f)^3*log
(x))*log((e*x^m + d)/d) - 24*(b*m*n*p^3*log(x) + b*m*n*p^2*log(f))*polylog(
4, -e*x^m/d) + 12*(b*m^2*n*p^3*log(x)^2 + 2*b*m^2*n*p^2*log(f)*log(x) + b*m
^2*n*p*log(f)^2)*polylog(3, -e*x^m/d))/m^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\log^3(fx^p)\,(a+b\log(c(d+ex^m)^n))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(ln(f*x**p)**3*(a+b*ln(c*(d+e*x**m)**n))/x,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\log^3(fx^p)\,(a+b\log(c(d+ex^m)^n))}{x}\,dx = \int \frac{(b\log((ex^m+d)^n c)+a)\log(fx^p)^3}{x}\,dx$$

```
[In] integrate(log(f*x^p)^3*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="maxima")
[Out] -1/4*(b*p^3*log(x)^4 - 4*b*p^2*log(f)*log(x)^3 + 6*b*p*log(f)^2*log(x)^2 -
4*b*log(f)^3*log(x) - 4*b*log(x)*log(x^p)^3 + 6*(b*p*log(x)^2 - 2*b*log(f)*
log(x))*log(x^p)^2 - 4*(b*p^2*log(x)^3 - 3*b*p*log(f)*log(x)^2 + 3*b*log(f)
^2*log(x))*log(x^p))*log((e*x^m + d)^n) - integrate(-1/4*(4*b*d*log(c)*log(
f)^3 + 4*a*d*log(f)^3 + 4*(b*d*log(c) + a*d - (b*e*m*n*log(x) - b*e*log(c)
- a*e)*x^m)*log(x^p)^3 + 6*(2*b*d*log(c)*log(f) + 2*a*d*log(f) + (b*e*m*n*p
*log(x)^2 - 2*b*e*m*n*log(f)*log(x) + 2*b*e*log(c)*log(f) + 2*a*e*log(f))*x
^m)*log(x^p)^2 + (b*e*m*n*p^3*log(x)^4 - 4*b*e*m*n*p^2*log(f)*log(x)^3 + 6*
b*e*m*n*p*log(f)^2*log(x)^2 - 4*b*e*m*n*log(f)^3*log(x) + 4*b*e*log(c)*log(
f)^3 + 4*a*e*log(f)^3)*x^m + 4*(3*b*d*log(c)*log(f)^2 + 3*a*d*log(f)^2 - (b
*e*m*n*p^2*log(x)^3 - 3*b*e*m*n*p*log(f)*log(x)^2 + 3*b*e*m*n*log(f)^2*log(
x) - 3*b*e*log(c)*log(f)^2 - 3*a*e*log(f)^2)*x^m)*log(x^p))/(e*x*x^m + d*x)
, x)
```

Giac [F]

$$\int \frac{\log^3(fx^p)\,(a+b\log(c(d+ex^m)^n))}{x}\,dx = \int \frac{(b\log((ex^m+d)^n c)+a)\log(fx^p)^3}{x}\,dx$$

```
[In] integrate(log(f*x^p)^3*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="giac")
[Out] integrate((b*log((e*x^m + d)^n*c) + a)*log(f*x^p)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^3(fx^p)\,(a+b\log(c(d+ex^m)^n))}{x}\,dx=\int \frac{\ln\left(f\,x^p\right)^3\,\left(a+b\,\ln\left(c\,{\left(d+e\,x^m\right)}^n\right)\right)}{x}\,dx$$

```
[In] int((log(f*x^p)^3*(a + b*log(c*(d + e*x^m)^n)))/x,x)
[Out] int((log(f*x^p)^3*(a + b*log(c*(d + e*x^m)^n)))/x, x)
```

3.622 $\int \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 132

$$\int \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx = \frac{\log^3(fx^p)(a+b\log(c(d+ex^m)^n))}{3p} - \frac{bn\log^3(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{3p} - \frac{bn\log^2(fx^p)\operatorname{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m} + \frac{2bnp\log(fx^p)\operatorname{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2} - \frac{2bnp^2\operatorname{PolyLog}\left(4,-\frac{ex^m}{d}\right)}{m^3}$$

```
[Out] 1/3*ln(f*x^p)^3*(a+b*ln(c*(d+e*x^m)^n))/p-1/3*b*n*ln(f*x^p)^3*ln(1+e*x^m/d)
/p-b*n*ln(f*x^p)^2*polylog(2,-e*x^m/d)/m+2*b*n*p*ln(f*x^p)*polylog(3,-e*x^m
/d)/m^2-2*b*n*p^2*polylog(4,-e*x^m/d)/m^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used

$= \{2531, 2375, 2421, 2430, 6724\}$

$$\int \frac{\log^2(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{x}\,dx = \frac{\log^3(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{3p} + \frac{2bnp\log(fx^p)\operatorname{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2} - \frac{bn\log^2(fx^p)\operatorname{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m} - \frac{bn\log^3(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{3p} - \frac{2bnp^2\operatorname{PolyLog}\left(4,-\frac{ex^m}{d}\right)}{m^3}$$

```
[In] Int[(Log[f*x^p]^2*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

```
[Out] (Log[f*x^p]^3*(a + b*Log[c*(d + e*x^m)^n]))/(3*p) - (b*n*Log[f*x^p]^3*Log[1
 + (e*x^m)/d])/(3*p) - (b*n*Log[f*x^p]^2*PolyLog[2, -((e*x^m)/d)])/m + (2*b
*n*p*Log[f*x^p]*PolyLog[3, -((e*x^m)/d)])/m^2 - (2*b*n*p^2*PolyLog[4, -((e*
x^m)/d)])/m^3
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2531

```
Int[(Log[(f_.)*(x_)^(q_.)]^(m_.)*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_)
)^(p_.)]*(b_.)))/(x_), x_Symbol] :> Simp[Log[f*x^q]^(m + 1)*((a + b*Log[c*(
```

```
d + e*x^n)^p])/(q*(m + 1))), x] - Dist[b*e*n*(p/(q*(m + 1))), Int[x^(n - 1)
*(Log[f*x^q]^(m + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n,
 p, q}, x] && NeQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\log^3(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{3p} - \frac{(bemn)\int \frac{x^{-1+m}\log^3(fx^p)}{d+ex^m}\,dx}{3p} \\
&= \frac{\log^3(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{3p} - \frac{bn\log^3(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{3p} \\
&\quad + (bn)\int \frac{\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{x}\,dx \\
&= \frac{\log^3(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{3p} - \frac{bn\log^3(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{3p} \\
&\quad - \frac{bn\log^2(fx^p)\,\mathrm{Li}_2\left(-\frac{ex^m}{d}\right)}{m} + \frac{(2bnp)\int \frac{\log(fx^p)\mathrm{Li}_2\left(-\frac{ex^m}{d}\right)}{x}\,dx}{m} \\
&= \frac{\log^3(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{3p} - \frac{bn\log^3(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{3p} \\
&\quad - \frac{bn\log^2(fx^p)\,\mathrm{Li}_2\left(-\frac{ex^m}{d}\right)}{m} + \frac{2bnp\log(fx^p)\,\mathrm{Li}_3\left(-\frac{ex^m}{d}\right)}{m^2} - \frac{(2bnp^2)\int \frac{\mathrm{Li}_3\left(-\frac{ex^m}{d}\right)}{x}\,dx}{m^2} \\
&= \frac{\log^3(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{3p} - \frac{bn\log^3(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{3p} \\
&\quad - \frac{bn\log^2(fx^p)\,\mathrm{Li}_2\left(-\frac{ex^m}{d}\right)}{m} + \frac{2bnp\log(fx^p)\,\mathrm{Li}_3\left(-\frac{ex^m}{d}\right)}{m^2} - \frac{2bnp^2\mathrm{Li}_4\left(-\frac{ex^m}{d}\right)}{m^3}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 456 vs. $2(132) = 264$.

Time = 0.16 (sec) , antiderivative size = 456, normalized size of antiderivative = 3.45

$$
\begin{aligned}
&\int \frac{\log^2(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{x}\,dx \\
&= \frac{1}{4}bmnp^2\log^4(x)-\frac{1}{3}bmnp\log^3(x)\log(fx^p) \\
&\quad+\frac{a\log^3(fx^p)}{3p}+\frac{2}{3}bnp^2\log^3(x)\log\left(1+\frac{dx^{-m}}{e}\right) \\
&\quad-bnp\log^2(x)\log(fx^p)\log\left(1+\frac{dx^{-m}}{e}\right)-bnp^2\log^3(x)\log(d+ex^m) \\
&\quad+\frac{bnp^2\log^2(x)\log\left(-\frac{ex^m}{d}\right)\log(d+ex^m)}{m}+2bnp\log^2(x)\log(fx^p)\log(d+ex^m) \\
&\quad-\frac{2bnp\log(x)\log\left(-\frac{ex^m}{d}\right)\log(fx^p)\log(d+ex^m)}{m}-bn\log(x)\log^2(fx^p)\log(d+ex^m) \\
&\quad+\frac{bn\log\left(-\frac{ex^m}{d}\right)\log^2(fx^p)\log(d+ex^m)}{m}+\frac{1}{3}bp^2\log^3(x)\log\left(c(d+ex^m)^n\right) \\
&\quad-bp\log^2(x)\log(fx^p)\log\left(c(d+ex^m)^n\right)+b\log(x)\log^2(fx^p)\log\left(c(d+ex^m)^n\right) \\
&\quad-\frac{bnp\log(x)\left(p\log(x)-2\log(fx^p)\right)\operatorname{PolyLog}\left(2,-\frac{dx^{-m}}{e}\right)}{m} \\
&\quad+\frac{bn\left(-p\log(x)+\log(fx^p)\right)^2\operatorname{PolyLog}\left(2,1+\frac{ex^m}{d}\right)}{m} \\
&\quad+\frac{2bnp\log(fx^p)\operatorname{PolyLog}\left(3,-\frac{dx^{-m}}{e}\right)}{m^2}+\frac{2bnp^2\operatorname{PolyLog}\left(4,-\frac{dx^{-m}}{e}\right)}{m^3}
\end{aligned}
$$

```
[In] Integrate[(Log[f*x^p]^2*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

```
[Out] (b*m*n*p^2*Log[x]^4)/4 - (b*m*n*p*Log[x]^3*Log[f*x^p])/3 + (a*Log[f*x^p]^3)
/(3*p) + (2*b*n*p^2*Log[x]^3*Log[1 + d/(e*x^m)])/3 - b*n*p*Log[x]^2*Log[f*x
^p]*Log[1 + d/(e*x^m)] - b*n*p^2*Log[x]^3*Log[d + e*x^m] + (b*n*p^2*Log[x]^
2*Log[-((e*x^m)/d)]*Log[d + e*x^m])/m + 2*b*n*p*Log[x]^2*Log[f*x^p]*Log[d +
 e*x^m] - (2*b*n*p*Log[x]*Log[-((e*x^m)/d)]*Log[f*x^p]*Log[d + e*x^m])/m -
b*n*Log[x]*Log[f*x^p]^2*Log[d + e*x^m] + (b*n*Log[-((e*x^m)/d)]*Log[f*x^p]^
2*Log[d + e*x^m])/m + (b*p^2*Log[x]^3*Log[c*(d + e*x^m)^n])/3 - b*p*Log[x]^
2*Log[f*x^p]*Log[c*(d + e*x^m)^n] + b*Log[x]*Log[f*x^p]^2*Log[c*(d + e*x^m)
^n] - (b*n*p*Log[x]*(p*Log[x] - 2*Log[f*x^p])*PolyLog[2, -(d/(e*x^m))])/m +
 (b*n*(-(p*Log[x]) + Log[f*x^p])^2*PolyLog[2, 1 + (e*x^m)/d])/m + (2*b*n*p*
Log[f*x^p]*PolyLog[3, -(d/(e*x^m))])/m^2 + (2*b*n*p^2*PolyLog[4, -(d/(e*x^m
))])/m^3
```

Maple [F]

$$\int \frac{\ln (f\,x^{p})^{2} \left(a+b\ln \left(c \left(d+e\,x^{m}\right)^{n}\right)\right)}{x}d x$$

[In] int(ln(f*x^p)^2*(a+b*ln(c*(d+e*x^m)^n))/x,x)

[Out] int(ln(f*x^p)^2*(a+b*ln(c*(d+e*x^m)^n))/x,x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(127) = 254$.

Time = 0.32 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.13

$$\int \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx = -\frac{6\,bnp^2\mathrm{polylog}\left(4,-\frac{ex^m}{d}\right) - 3\,(bm^3\log(c)+am^3)\log(f)^2\log(x) - 3\,(bm^3p\log(c)+am^3p)\log(f)\log(x}{}$$

[In] integrate(log(f*x^p)^2*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="fricas")

[Out] -1/3*(6*b*n*p^2*polylog(4, -e*x^m/d) - 3*(b*m^3*log(c) + a*m^3)*log(f)^2*log(x) - 3*(b*m^3*p*log(c) + a*m^3*p)*log(f)*log(x)^2 - (b*m^3*p^2*log(c) + a*m^3*p^2)*log(x)^3 + 3*(b*m^2*n*p^2*log(x)^2 + 2*b*m^2*n*p*log(f)*log(x) + b*m^2*n*log(f)^2)*dilog(-(e*x^m + d)/d + 1) - (b*m^3*n*p^2*log(x)^3 + 3*b*m^3*n*p*log(f)*log(x)^2 + 3*b*m^3*n*log(f)^2*log(x))*log(e*x^m + d) + (b*m^3*n*p^2*log(x)^3 + 3*b*m^3*n*p*log(f)*log(x)^2 + 3*b*m^3*n*log(f)^2*log(x))*log((e*x^m + d)/d) - 6*(b*m*n*p^2*log(x) + b*m*n*p*log(f))*polylog(3, -e*x^m/d))/m^3

Sympy [F]

$$\int \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx = \int \frac{(a+b\log(c(d+ex^m)^n))\log(fx^p)^2}{x}\,dx$$

[In] integrate(ln(f*x**p)**2*(a+b*ln(c*(d+e*x**m)**n))/x,x)

[Out] Integral((a + b*log(c*(d + e*x**m)**n))*log(f*x**p)**2/x, x)

Maxima [F]

$$\int \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx = \int \frac{(b\log((ex^m+d)^n c)+a)\log(fx^p)^2}{x}\,dx$$

[In] integrate(log(f*x^p)^2*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="maxima")

[Out] 1/3*(b*p^2*log(x)^3 - 3*b*p*log(f)*log(x)^2 + 3*b*log(f)^2*log(x) + 3*b*log(x)*log(x^p)^2 - 3*(b*p*log(x)^2 - 2*b*log(f)*log(x))*log(x^p))*log((e*x^m + d)^n) - integrate(-1/3*(3*b*d*log(c)*log(f)^2 + 3*a*d*log(f)^2 + 3*(b*d*log(c) + a*d - (b*e*m*n*log(x) - b*e*log(c) - a*e)*x^m)*log(x^p)^2 - (b*e*m*n*p^2*log(x)^3 - 3*b*e*m*n*p*log(f)*log(x)^2 + 3*b*e*m*n*log(f)^2*log(x) - 3*b*e*log(c)*log(f)^2 - 3*a*e*log(f)^2)*x^m + 3*(2*b*d*log(c)*log(f) + 2*a*d*log(f) + (b*e*m*n*p*log(x)^2 - 2*b*e*m*n*log(f)*log(x) + 2*b*e*log(c)*log(f) + 2*a*e*log(f))*x^m)*log(x^p))/(e*x*x^m + d*x), x)

Giac [F]

$$\int \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx = \int \frac{(b\log((ex^m+d)^n c)+a)\log(fx^p)^2}{x}\,dx$$

[In] integrate(log(f*x^p)^2*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="giac")

[Out] integrate((b*log((e*x^m + d)^n*c) + a)*log(f*x^p)^2/x, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx = \int \frac{\ln(f\,x^p)^2\,(a+b\,\ln(c\,(d+e\,x^m)^n))}{x}\,dx$$

[In] int((log(f*x^p)^2*(a + b*log(c*(d + e*x^m)^n)))/x,x)

[Out] int((log(f*x^p)^2*(a + b*log(c*(d + e*x^m)^n)))/x, x)

3.623 $\int \frac{\log(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 102

$$\int \frac{\log(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx = \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{2p} - \frac{bn\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{2p} - \frac{bn\log(fx^p)\operatorname{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m} + \frac{bnp\operatorname{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}$$

```
[Out] 1/2*ln(f*x^p)^2*(a+b*ln(c*(d+e*x^m)^n))/p-1/2*b*n*ln(f*x^p)^2*ln(1+e*x^m/d)
/p-b*n*ln(f*x^p)*polylog(2,-e*x^m/d)/m+b*n*p*polylog(3,-e*x^m/d)/m^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2531, 2375, 2421, 6724}

$$\int \frac{\log(fx^p)(a+b\log(c(d+ex^m)^n))}{x}\,dx = \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{2p} - \frac{bn\log(fx^p)\operatorname{PolyLog}\left(2,-\frac{ex^m}{d}\right)}{m} - \frac{bn\log^2(fx^p)\log\left(\frac{ex^m}{d}+1\right)}{2p} + \frac{bnp\operatorname{PolyLog}\left(3,-\frac{ex^m}{d}\right)}{m^2}$$

```
[In] Int[(Log[f*x^p]*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

```
[Out] (Log[f*x^p]^2*(a + b*Log[c*(d + e*x^m)^n]))/(2*p) - (b*n*Log[f*x^p]^2*Log[1
 + (e*x^m)/d])/(2*p) - (b*n*Log[f*x^p]*PolyLog[2, -((e*x^m)/d)])/m + (b*n*p
*PolyLog[3, -((e*x^m)/d)])/m^2
```

Rule 2375

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_)
+ (e_.)*(x_)^(r_)), x_Symbol] :> Simp[f^m*Log[1 + e*(x^r/d)]*((a + b*Log[c*
x^n])^p/(e*r)), x] - Dist[b*f^m*n*(p/(e*r)), Int[Log[1 + e*(x^r/d)]*((a + b
*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, r}, x] &&
 EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || GtQ[f, 0]) && NeQ[r, n]
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2531

```
Int[(Log[(f_.)*(x_)^(q_.)]^(m_.)*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_)
)^(p_.)]*(b_.)))/(x_), x_Symbol] :> Simp[Log[f*x^q]^(m + 1)*((a + b*Log[c*(
d + e*x^n)^p])/(q*(m + 1))), x] - Dist[b*e*n*(p/(q*(m + 1))), Int[x^(n - 1)
*(Log[f*x^q]^(m + 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n,
 p, q}, x] && NeQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{2p} - \frac{(bemn)\int\frac{x^{-1+m}\log^2(fx^p)}{d+ex^m}\,dx}{2p} \\
&= \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{2p} - \frac{bn\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{2p} \\
&\quad + (bn)\int\frac{\log(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{x}\,dx
\end{aligned}$$

$$= \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{2p} - \frac{bn\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{2p} - \frac{bn\log(fx^p)\operatorname{Li}_2\left(-\frac{ex^m}{d}\right)}{m} + \frac{(bnp)\int\frac{\operatorname{Li}_2\left(-\frac{ex^m}{d}\right)}{x}dx}{m}$$

$$= \frac{\log^2(fx^p)(a+b\log(c(d+ex^m)^n))}{2p} - \frac{bn\log^2(fx^p)\log\left(1+\frac{ex^m}{d}\right)}{2p} - \frac{bn\log(fx^p)\operatorname{Li}_2\left(-\frac{ex^m}{d}\right)}{m} + \frac{bnp\operatorname{Li}_3\left(-\frac{ex^m}{d}\right)}{m^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 265 vs. $2(102) = 204$.

Time = 0.15 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.60

$$\int\frac{\log(fx^p)(a+b\log(c(d+ex^m)^n))}{x}dx$$

$$= -\frac{1}{6}bmnp\log^3(x) + \frac{a\log^2(fx^p)}{2p} - \frac{1}{2}bnp\log^2(x)\log\left(1+\frac{dx^{-m}}{e}\right) + bnp\log^2(x)\log(d+ex^m) - \frac{bnp\log(x)\log\left(-\frac{ex^m}{d}\right)\log(d+ex^m)}{m} - bn\log(x)\log(fx^p)\log(d+ex^m) + \frac{bn\log\left(-\frac{ex^m}{d}\right)\log(fx^p)\log(d+ex^m)}{m} - \frac{1}{2}bp\log^2(x)\log(c(d+ex^m)^n) + b\log(x)\log(fx^p)\log(c(d+ex^m)^n) + \frac{bnp\log(x)\operatorname{PolyLog}\left(2,-\frac{dx^{-m}}{e}\right)}{m} - \frac{bn(p\log(x)-\log(fx^p))\operatorname{PolyLog}\left(2,1+\frac{ex^m}{d}\right)}{m} + \frac{bnp\operatorname{PolyLog}\left(3,-\frac{dx^{-m}}{e}\right)}{m^2}$$

```
[In] Integrate[(Log[f*x^p]*(a + b*Log[c*(d + e*x^m)^n]))/x,x]
```

```
[Out] -1/6*(b*m*n*p*Log[x]^3) + (a*Log[f*x^p]^2)/(2*p) - (b*n*p*Log[x]^2*Log[1 +
d/(e*x^m)])/2 + b*n*p*Log[x]^2*Log[d + e*x^m] - (b*n*p*Log[x]*Log[-((e*x^m)
/d)]*Log[d + e*x^m])/m - b*n*Log[x]*Log[f*x^p]*Log[d + e*x^m] + (b*n*Log[-(
(e*x^m)/d)]*Log[f*x^p]*Log[d + e*x^m])/m - (b*p*Log[x]^2*Log[c*(d + e*x^m)^
n])/2 + b*Log[x]*Log[f*x^p]*Log[c*(d + e*x^m)^n] + (b*n*p*Log[x]*PolyLog[2,
 -(d/(e*x^m))])/m - (b*n*(p*Log[x] - Log[f*x^p])*PolyLog[2, 1 + (e*x^m)/d])
/m + (b*n*p*PolyLog[3, -(d/(e*x^m))])/m^2
```

Maple [F]

$$\int \frac{\ln (f\, x^{p}) (a+b\ln (c(d+e\, x^{m})^{n}))}{x}dx$$

[In] int(ln(f*x^p)*(a+b*ln(c*(d+e*x^m)^n))/x,x)

[Out] int(ln(f*x^p)*(a+b*ln(c*(d+e*x^m)^n))/x,x)

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.58

$$\int \frac{\log (fx^p) (a+b\log (c(d+ex^m)^n))}{x}\,dx = \frac{2\,bnp\text{polylog}\left(3,-\frac{ex^m}{d}\right)+2\,(bm^2\log (c)+am^2)\log (f)\log (x)+(bm^2p\log (c)+am^2p)\log (x)^2-2\,(bmnp\,\mathrm{l}}{}$$

[In] integrate(log(f*x^p)*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="fricas")

[Out] 1/2*(2*b*n*p*polylog(3, -e*x^m/d) + 2*(b*m^2*log(c) + a*m^2)*log(f)*log(x) + (b*m^2*p*log(c) + a*m^2*p)*log(x)^2 - 2*(b*m*n*p*log(x) + b*m*n*log(f))*dilog(-(e*x^m + d)/d + 1) + (b*m^2*n*p*log(x)^2 + 2*b*m^2*n*log(f)*log(x))*log(e*x^m + d) - (b*m^2*n*p*log(x)^2 + 2*b*m^2*n*log(f)*log(x))*log((e*x^m + d)/d))/m^2

Sympy [F(-2)]

Exception generated.

$$\int \frac{\log (fx^p) (a+b\log (c(d+ex^m)^n))}{x}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(ln(f*x**p)*(a+b*ln(c*(d+e*x**m)**n))/x,x)

[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo

Maxima [F]

$$\int \frac{\log(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{x}\,dx = \int \frac{(b\log\left((ex^m+d)^n c\right)+a)\log(fx^p)}{x}\,dx$$

```
[In] integrate(log(f*x^p)*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="maxima")
```

```
[Out] -1/2*(b*p*log(x)^2 - 2*b*log(f)*log(x) - 2*b*log(x)*log(x^p))*log((e*x^m +
d)^n) - integrate(-1/2*(2*b*d*log(c)*log(f) + 2*a*d*log(f) + (b*e*m*n*p*log
(x)^2 - 2*b*e*m*n*log(f)*log(x) + 2*b*e*log(c)*log(f) + 2*a*e*log(f))*x^m +
 2*(b*d*log(c) + a*d - (b*e*m*n*log(x) - b*e*log(c) - a*e)*x^m)*log(x^p))/(
e*x*x^m + d*x), x)
```

Giac [F]

$$\int \frac{\log(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{x}\,dx = \int \frac{(b\log\left((ex^m+d)^n c\right)+a)\log(fx^p)}{x}\,dx$$

```
[In] integrate(log(f*x^p)*(a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^m + d)^n*c) + a)*log(f*x^p)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log(fx^p)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{x}\,dx = \int \frac{\ln(f\,x^p)\,\left(a+b\,\ln\left(c\,{(d+e\,x^m)}^n\right)\right)}{x}\,dx$$

```
[In] int((log(f*x^p)*(a + b*log(c*(d + e*x^m)^n)))/x,x)
```

```
[Out] int((log(f*x^p)*(a + b*log(c*(d + e*x^m)^n)))/x, x)
```

3.624 $\int \frac{a+b\log(c(d+ex^m)^n)}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x}\,dx = \frac{\log\left(-\frac{ex^m}{d}\right)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{m}+\frac{bn\,\text{PolyLog}\left(2,1+\frac{ex^m}{d}\right)}{m}$$

```
[Out] ln(-e*x^m/d)*(a+b*ln(c*(d+e*x^m)^n))/m+b*n*polylog(2,1+e*x^m/d)/m
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2504, 2441, 2352}

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x}\,dx = \frac{\log\left(-\frac{ex^m}{d}\right)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{m}+\frac{bn\,\text{PolyLog}\left(2,\frac{ex^m}{d}+1\right)}{m}$$

```
[In] Int[(a + b*Log[c*(d + e*x^m)^n])/x,x]
```

```
[Out] (Log[-((e*x^m)/d)]*(a + b*Log[c*(d + e*x^m)^n]))/m + (b*n*PolyLog[2, 1 + (e
*x^m)/d])/m
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{a+b\log(c(d+ex)^n)}{x}\,dx, x, x^m\right)}{m}\\
&= \frac{\log\left(-\frac{ex^m}{d}\right)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{m} - \frac{(ben)\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, x^m\right)}{m}\\
&= \frac{\log\left(-\frac{ex^m}{d}\right)\left(a+b\log\left(c(d+ex^m)^n\right)\right)}{m} + \frac{bn\text{Li}_2\left(1+\frac{ex^m}{d}\right)}{m}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x}\,dx = a\log(x) + \frac{b\left(\log\left(-\frac{ex^m}{d}\right)\log\left(c(d+ex^m)^n\right)+n\,\text{PolyLog}\left(2,\frac{d+ex^m}{d}\right)\right)}{m}$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^m)^n])/x,x]
```

```
[Out] a*Log[x] + (b*(Log[-((e*x^m)/d)]*Log[c*(d + e*x^m)^n] + n*PolyLog[2, (d + e
*x^m)/d]))/m
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.14 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.67

method	result
risch	$b\ln(x)\ln\left((d+e\,x^m)^n\right)+\left(\frac{ib\pi\ \mathrm{csgn}(i(d+e\,x^m)^n)\mathrm{csgn}(ic(d+e\,x^m)^n)^2}{2}-\frac{ib\pi\ \mathrm{csgn}(i(d+e\,x^m)^n)\ \mathrm{csgn}(ic(d+e\,x^m)^n)\ \mathrm{csgn}(i\right.$

```
[In] int((a+b*ln(c*(d+e*x^m)^n))/x,x,method=_RETURNVERBOSE)
```

```
[Out] b*ln(x)*ln((d+e*x^m)^n)+(1/2*I*b*Pi*csgn(I*(d+e*x^m)^n)*csgn(I*c*(d+e*x^m)^
n)^2-1/2*I*b*Pi*csgn(I*(d+e*x^m)^n)*csgn(I*c*(d+e*x^m)^n)*csgn(I*c)-1/2*I*b
*Pi*csgn(I*c*(d+e*x^m)^n)^3+1/2*I*b*Pi*csgn(I*c*(d+e*x^m)^n)^2*csgn(I*c)+b*
ln(c)+a)*ln(x)-b*n/m*dilog((d+e*x^m)/d)-b*n*ln(x)*ln((d+e*x^m)/d)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.41

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x}\,dx = \frac{bmn\log\left(ex^m+d\right)\log\left(x\right)-bmn\log\left(x\right)\log\left(\frac{ex^m+d}{d}\right)-bn\mathrm{Li}_2\left(-\frac{ex^m+d}{d}+1\right)+\left(bm\log\left(c\right)+am\right)\log\left(x\right)}{m}$$

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="fricas")
```

```
[Out] (b*m*n*log(e*x^m + d)*log(x) - b*m*n*log(x)*log((e*x^m + d)/d) - b*n*dilog(
-(e*x^m + d)/d + 1) + (b*m*log(c) + a*m)*log(x))/m
```

Sympy [F]

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x}\,dx = \int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**m)**n))/x,x)
```

```
[Out] Integral((a + b*log(c*(d + e*x**m)**n))/x, x)
```

Maxima [F]

$$\int \frac{a + b\log\left(c(d + ex^m)^n\right)}{x}\,dx = \int \frac{b\log\left((ex^m + d)^n c\right) + a}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="maxima")
```

```
[Out] 1/2*(2*d*m*n*integrate(log(x)/(e*x*x^m + d*x), x) - m*n*log(x)^2 + 2*log((e
*x^m + d)^n)*log(x) + 2*log(c)*log(x))*b + a*log(x)
```

Giac [F]

$$\int \frac{a + b\log\left(c(d + ex^m)^n\right)}{x}\,dx = \int \frac{b\log\left((ex^m + d)^n c\right) + a}{x}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^m + d)^n*c) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\log\left(c(d + ex^m)^n\right)}{x}\,dx = \int \frac{a + b\,\ln\left(c\,(d + e\,x^m)^n\right)}{x}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^m)^n))/x,x)
```

```
[Out] int((a + b*log(c*(d + e*x^m)^n))/x, x)
```

3.625 $\int \frac{a+b\log(c(d+ex^m)^n)}{x\log(fx^p)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log\left(fx^p\right)}\,dx=\frac{a\log\left(\log\left(fx^p\right)\right)}{p}+b\text{Int}\left(\frac{\log\left(c(d+ex^m)^n\right)}{x\log\left(fx^p\right)},x\right)$$

```
[Out] a*ln(ln(f*x^p))/p+b*Unintegrable(ln(c*(d+e*x^m)^n)/x/ln(f*x^p),x)
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log\left(fx^p\right)}\,dx=\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log\left(fx^p\right)}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]),x]
```

```
[Out] (a*Log[Log[f*x^p]])/p + b*Defer[Int][Log[c*(d + e*x^m)^n]/(x*Log[f*x^p]), x
]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(\frac{a}{x\log\left(fx^p\right)}+\frac{b\log\left(c(d+ex^m)^n\right)}{x\log\left(fx^p\right)}\right)\,dx\\&=a\int \frac{1}{x\log\left(fx^p\right)}\,dx+b\int \frac{\log\left(c(d+ex^m)^n\right)}{x\log\left(fx^p\right)}\,dx\end{aligned}$$

$$= b\int \frac{\log \left(c(d+ex^m)^n\right)}{x\log \left(fx^p\right)}\,dx + \frac{a\text{Subst}\left(\int \frac{1}{x}\,dx, x, \log \left(fx^p\right)\right)}{p}$$

$$= \frac{a\log \left(\log \left(fx^p\right)\right)}{p} + b\int \frac{\log \left(c(d+ex^m)^n\right)}{x\log \left(fx^p\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log \left(c(d+ex^m)^n\right)}{x\log \left(fx^p\right)}\,dx = \int \frac{a+b\log \left(c(d+ex^m)^n\right)}{x\log \left(fx^p\right)}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]),x]
[Out] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]), x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{a+b\ln \left(c(d+e\,x^m)^n\right)}{x\ln \left(f\,x^p\right)}dx$$

```
[In] int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p),x)
[Out] int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p),x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log \left(c(d+ex^m)^n\right)}{x\log \left(fx^p\right)}\,dx = \int \frac{b\log \left((ex^m+d)^n c\right)+a}{x\log \left(fx^p\right)}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p),x, algorithm="fricas")
[Out] integral((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)), x)
```

Sympy [N/A]

Not integrable

Time = 53.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{a+b\log \left(c(d+ex^m)^n\right)}{x\log \left(fx^p\right)}\,dx=\int \frac{a+b\log \left(c(d+ex^m)^n\right)}{x\log \left(fx^p\right)}\,dx$$

```
[In] integrate((a+b*ln(c*(d+e*x**m)**n))/x/ln(f*x**p),x)
[Out] Integral((a + b*log(c*(d + e*x**m)**n))/(x*log(f*x**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.57

$$\int \frac{a+b\log \left(c(d+ex^m)^n\right)}{x\log \left(fx^p\right)}\,dx=\int \frac{b\log \left((ex^m+d)^nc\right)+a}{x\log \left(fx^p\right)}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p),x, algorithm="maxima")
[Out] b*integrate((log((e*x^m + d)^n) + log(c))/(x*log(f) + x*log(x^p)), x) + a*l
og(log(f*x^p))/p
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log \left(c(d+ex^m)^n\right)}{x\log \left(fx^p\right)}\,dx=\int \frac{b\log \left((ex^m+d)^nc\right)+a}{x\log \left(fx^p\right)}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p),x, algorithm="giac")
[Out] integrate((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)), x)
```

Mupad [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log\left(fx^p\right)}\,dx = \int \frac{a+b\,\ln\left(c\,(d+e\,x^m)^n\right)}{x\,\ln\left(f\,x^p\right)}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)),x)
[Out] int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)), x)
```

3.626 $\int \frac{a+b\log(c(d+ex^m)^n)}{x\log^2(fx^p)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^2(fx^p)}\,dx=-\frac{a+b\log\left(c(d+ex^m)^n\right)}{p\log(fx^p)}+\frac{bemn\text{Int}\left(\frac{x^{-1+m}}{(d+ex^m)\log(fx^p)},x\right)}{p}$$

```
[Out] (-a-b*ln(c*(d+e*x^m)^n))/p/ln(f*x^p)+b*e*m*n*Unintegrable(x^(-1+m)/(d+e*x^m
)/ln(f*x^p),x)/p
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^2(fx^p)}\,dx=\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^2(fx^p)}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^2),x]
```

```
[Out] -((a + b*Log[c*(d + e*x^m)^n])/(p*Log[f*x^p])) + (b*e*m*n*Defer[Int][x^(-1
+ m)/((d + e*x^m)*Log[f*x^p]), x])/p
```

Rubi steps

$$\text{integral}=-\frac{a+b\log\left(c(d+ex^m)^n\right)}{p\log(fx^p)}+\frac{(bemn)\int \frac{x^{-1+m}}{(d+ex^m)\log(fx^p)}\,dx}{p}$$

Mathematica [N/A]

Not integrable

Time = 2.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^2\left(fx^p\right)}\,dx=\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^2\left(fx^p\right)}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^2),x]
[Out] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^2), x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{a+b\ln\left(c(d+e\,x^m)^n\right)}{x\ln\left(f\,x^p\right)^2}dx$$

```
[In] int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p)^2,x)
[Out] int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^2\left(fx^p\right)}\,dx=\int \frac{b\log\left((ex^m+d)^nc\right)+a}{x\log\left(fx^p\right)^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^2,x, algorithm="fricas")
[Out] integral((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^2\left(fx^p\right)}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**m)**n))/x/ln(f*x**p)**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.25

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^2\left(fx^p\right)}\,dx = \int \frac{b\log\left((ex^m+d)^n c\right)+a}{x\log\left(fx^p\right)^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^2,x, algorithm="maxima")
```

```
[Out] (e*m*n*integrate(x^m/(e*p*x*x^m*log(f) + d*p*x*log(f) + (e*p*x*x^m + d*p*x)
*log(x^p)), x) - (log((e*x^m + d)^n) + log(c))/(p*log(f) + p*log(x^p)))*b -
 a/(p*log(f*x^p))
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^2\left(fx^p\right)}\,dx = \int \frac{b\log\left((ex^m+d)^n c\right)+a}{x\log\left(fx^p\right)^2}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^2,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.80 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^2\left(fx^p\right)}\,dx = \int \frac{a+b\,\ln\left(c\,{\left(d+e\,x^m\right)}^n\right)}{x\,{\ln\left(f\,x^p\right)}^2}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)^2),x)
```

```
[Out] int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)^2), x)
```

3.627 $\int \frac{a+b\log(c(d+ex^m)^n)}{x\log^3(fx^p)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^3(fx^p)}\,dx = -\frac{a+b\log\left(c(d+ex^m)^n\right)}{2p\log^2(fx^p)} + \frac{bemn\text{Int}\left(\frac{x^{-1+m}}{(d+ex^m)\log^2(fx^p)},x\right)}{2p}$$

```
[Out] 1/2*(-a-b*ln(c*(d+e*x^m)^n))/p/ln(f*x^p)^2+1/2*b*e*m*n*Unintegrable(x^(-1+m
)/(d+e*x^m)/ln(f*x^p)^2,x)/p
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^3(fx^p)}\,dx = \int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^3(fx^p)}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^3),x]
```

```
[Out] -1/2*(a + b*Log[c*(d + e*x^m)^n])/(p*Log[f*x^p]^2) + (b*e*m*n*Defer[Int][x^
(-1 + m)/((d + e*x^m)*Log[f*x^p]^2), x])/(2*p)
```

Rubi steps

$$\text{integral} = -\frac{a+b\log\left(c(d+ex^m)^n\right)}{2p\log^2(fx^p)} + \frac{(bemn)\int \frac{x^{-1+m}}{(d+ex^m)\log^2(fx^p)}\,dx}{2p}$$

Mathematica [N/A]

Not integrable

Time = 8.83 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^3\left(fx^p\right)}\,dx=\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^3\left(fx^p\right)}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^3),x]
[Out] Integrate[(a + b*Log[c*(d + e*x^m)^n])/(x*Log[f*x^p]^3), x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{a+b\ln\left(c(d+e\,x^m)^n\right)}{x\ln\left(f\,x^p\right)^3}dx$$

```
[In] int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p)^3,x)
[Out] int((a+b*ln(c*(d+e*x^m)^n))/x/ln(f*x^p)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^3\left(fx^p\right)}\,dx=\int \frac{b\log\left((ex^m+d)^nc\right)+a}{x\log\left(fx^p\right)^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^3,x, algorithm="fricas")
[Out] integral((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^3\left(fx^p\right)}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*ln(c*(d+e*x**m)**n))/x/ln(f*x**p)**3,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 238, normalized size of antiderivative = 8.50

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^3\left(fx^p\right)}\,dx=\int \frac{b\log\left((ex^m+d)^n c\right)+a}{x\log\left(fx^p\right)^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*d*e*m^2*n*integrate(1/2*x^m/(e^2*p^2*x*x^(2*m)*log(f) + 2*d*e*p^2*x*
x^m*log(f) + d^2*p^2*x*log(f) + (e^2*p^2*x*x^(2*m) + 2*d*e*p^2*x*x^m + d^2*
p^2*x)*log(x^p)), x) - (e*m*n*x^m*log(x^p) + d*p*log(c) + (e*m*n*log(f) + e
*p*log(c))*x^m + (e*p*x^m + d*p)*log((e*x^m + d)^n))/(e*p^2*x^m*log(f)^2 +
d*p^2*log(f)^2 + (e*p^2*x^m + d*p^2)*log(x^p)^2 + 2*(e*p^2*x^m*log(f) + d*p
^2*log(f))*log(x^p)))*b - 1/2*a/(p*log(f*x^p)^2)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^3\left(fx^p\right)}\,dx=\int \frac{b\log\left((ex^m+d)^n c\right)+a}{x\log\left(fx^p\right)^3}\,dx$$

```
[In] integrate((a+b*log(c*(d+e*x^m)^n))/x/log(f*x^p)^3,x, algorithm="giac")
```

```
[Out] integrate((b*log((e*x^m + d)^n*c) + a)/(x*log(f*x^p)^3), x)
```

Mupad [N/A]

Not integrable

Time = 2.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{a+b\log\left(c(d+ex^m)^n\right)}{x\log^3\left(fx^p\right)}\,dx=\int \frac{a+b\,\ln\left(c\,{\left(d+e\,x^m\right)}^n\right)}{x\,{\ln\left(f\,x^p\right)}^3}\,dx$$

```
[In] int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)^3),x)
```

```
[Out] int((a + b*log(c*(d + e*x^m)^n))/(x*log(f*x^p)^3), x)
```

3.628 $\int \log \left(c(d+e(f+gx)^p)^q\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 76

$$\int \log \left(c(d+e(f+gx)^p)^q\right) \, dx = -\frac{epq(f+gx)^{1+p} \text{Hypergeometric2F1}\left(1,1+\frac{1}{p},2+\frac{1}{p},-\frac{e(f+gx)^p}{d}\right)}{dg(1+p)} + \frac{(f+gx) \log \left(c(d+e(f+gx)^p)^q\right)}{g}$$

```
[Out] -e*p*q*(g*x+f)^(p+1)*hypergeom([1, 1+1/p],[2+1/p],-e*(g*x+f)^p/d)/d/g/(p+1)
+(g*x+f)*ln(c*(d+e*(g*x+f)^p)^q)/g
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2533, 2498, 371}

$$\int \log \left(c(d+e(f+gx)^p)^q\right) \, dx = \frac{(f+gx) \log \left(c(d+e(f+gx)^p)^q\right)}{g} - \frac{epq(f+gx)^{p+1} \text{Hypergeometric2F1}\left(1,1+\frac{1}{p},2+\frac{1}{p},-\frac{e(f+gx)^p}{d}\right)}{dg(p+1)}$$

```
[In] Int[Log[c*(d + e*(f + g*x)^p)^q],x]
```

```
[Out] -((e*p*q*(f + g*x)^(1 + p)*Hypergeometric2F1[1, 1 + p^(-1), 2 + p^(-1), -((
e*(f + g*x)^p)/d)])/(d*g*(1 + p))) + ((f + g*x)*Log[c*(d + e*(f + g*x)^p)^q
])/g
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2533

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \log \left(c(d+ex^p)^q\right)\,dx,x,f+gx\right)}{g}\\ &= \frac{(f+gx)\log \left(c(d+e(f+gx)^p)^q\right)}{g}-\frac{(epq)\text{Subst}\left(\int \frac{x^p}{d+ex^p}\,dx,x,f+gx\right)}{g}\\ &= -\frac{epq(f+gx)^{1+p}\,{}_2F_1\left(1,1+\frac{1}{p};2+\frac{1}{p};-\frac{e(f+gx)^p}{d}\right)}{dg(1+p)}+\frac{(f+gx)\log \left(c(d+e(f+gx)^p)^q\right)}{g}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.86

$$\int \log \left(c(d+e(f+gx)^p)^q\right)\,dx = -pqx + \frac{pq(f+gx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{p},1+\frac{1}{p},-\frac{e(f+gx)^p}{d}\right)}{g} + \frac{(f+gx)\log \left(c(d+e(f+gx)^p)^q\right)}{g}$$

```
[In] Integrate[Log[c*(d + e*(f + g*x)^p)^q],x]
[Out] -(p*q*x) + (p*q*(f + g*x)*Hypergeometric2F1[1, p^(-1), 1 + p^(-1), -((e*(f
+ g*x)^p)/d)])/g + ((f + g*x)*Log[c*(d + e*(f + g*x)^p)^q])/g
```

Maple [F]

$$\int \ln \left(c(d + e(gx + f)^p)^q\right) dx$$

```
[In] int(ln(c*(d+e*(g*x+f)^p)^q),x)
[Out] int(ln(c*(d+e*(g*x+f)^p)^q),x)
```

Fricas [F]

$$\int \log \left(c(d + e(f + gx)^p)^q\right) \, dx = \int \log \left(((gx + f)^p e + d)^q c\right) \, dx$$

```
[In] integrate(log(c*(d+e*(g*x+f)^p)^q),x, algorithm="fricas")
[Out] integral(log(((g*x + f)^p*e + d)^q*c), x)
```

Sympy [F]

$$\int \log \left(c(d + e(f + gx)^p)^q\right) \, dx = \int \log \left(c(d + e(f + gx)^p)^q\right) dx$$

```
[In] integrate(ln(c*(d+e*(g*x+f)**p)**q),x)
[Out] Integral(log(c*(d + e*(f + g*x)**p)**q), x)
```

Maxima [F]

$$\int \log \left(c(d + e(f + gx)^p)^q\right) \, dx = \int \log \left(((gx + f)^p e + d)^q c\right) \, dx$$

```
[In] integrate(log(c*(d+e*(g*x+f)^p)^q),x, algorithm="maxima")
[Out] d*g*p*q*integrate(x/(d*g*x + (e*g*x + e*f)*(g*x + f)^p + d*f), x) + (f*p*q*
log(g*x + f) + g*x*log(((g*x + f)^p*e + d)^q) - (g*p*q - g*log(c))*x)/g
```

Giac [F]

$$\int \log \left(c(d+e(f+gx)^p)^q\right) \, dx = \int \log \left(((gx+f)^p e+d)^q c\right) \, dx$$

```
[In] integrate(log(c*(d+e*(g*x+f)^p)^q),x, algorithm="giac")
[Out] integrate(log(((g*x + f)^p*e + d)^q*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log \left(c(d+e(f+gx)^p)^q\right) \, dx = \int \ln \left(c\,\left(d+e\,{\left(f+g\,x\right)}^p\right)^q\right) \, dx$$

```
[In] int(log(c*(d + e*(f + g*x)^p)^q),x)
[Out] int(log(c*(d + e*(f + g*x)^p)^q), x)
```

3.629 $\int \log \left(c\left(d+e(f+gx)^3\right)^q\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 169

$$\int \log \left(c\left(d+e(f+gx)^3\right)^q\right)\,dx = -3qx-\frac{\sqrt{3}\sqrt[3]{d}q\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}(f+gx)}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt[3]{e}g}+\frac{\sqrt[3]{d}q\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+gx)\right)}{\sqrt[3]{e}g}-\frac{\sqrt[3]{d}q\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}(f+gx)^2\right)}{2\sqrt[3]{e}g}+\frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)}{g}$$

```
[Out] -3*q*x+d^(1/3)*q*ln(d^(1/3)+e^(1/3)*(g*x+f))/e^(1/3)/g-1/2*d^(1/3)*q*ln(d^(
2/3)-d^(1/3)*e^(1/3)*(g*x+f)+e^(2/3)*(g*x+f)^2)/e^(1/3)/g+(g*x+f)*ln(c*(d+e
*(g*x+f)^3)^q)/g-d^(1/3)*q*arctan(1/3*(d^(1/3)-2*e^(1/3)*(g*x+f))/d^(1/3)*3
^(1/2))*3^(1/2)/e^(1/3)/g
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used

$= \{2533, 2498, 327, 206, 31, 648, 631, 210, 642\}$

$$\int \log \left(c\left(d+e(f+gx)^3\right)^q\right)\,dx = -\frac{\sqrt{3}\sqrt[3]{d}q\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}(f+gx)}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt[3]{e}g} + \frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)}{g} - \frac{\sqrt[3]{d}q\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}(f+gx)^2\right)}{2\sqrt[3]{e}g} + \frac{\sqrt[3]{d}q\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+gx)\right)}{\sqrt[3]{e}g} - 3qx$$

```
[In] Int[Log[c*(d + e*(f + g*x)^3)^q],x]
```

```
[Out] -3*q*x - (Sqrt[3]*d^(1/3)*q*ArcTan[(d^(1/3) - 2*e^(1/3)*(f + g*x))/(Sqrt[3]
*d^(1/3))])/(e^(1/3)*g) + (d^(1/3)*q*Log[d^(1/3) + e^(1/3)*(f + g*x)])/(e^(
1/3)*g) - (d^(1/3)*q*Log[d^(2/3) - d^(1/3)*e^(1/3)*(f + g*x) + e^(2/3)*(f +
 g*x)^2])/(2*e^(1/3)*g) + ((f + g*x)*Log[c*(d + e*(f + g*x)^3)^q])/g
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2533

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \log\left(c(d+ex^3)^q\right)\,dx, x, f+gx\right)}{g}\\
&= \frac{(f+gx)\log\left(c(d+e(f+gx)^3)^q\right)}{g} - \frac{(3eq)\text{Subst}\left(\int \frac{x^3}{d+ex^3}\,dx, x, f+gx\right)}{g}\\
&= -3qx + \frac{(f+gx)\log\left(c(d+e(f+gx)^3)^q\right)}{g} + \frac{(3dq)\text{Subst}\left(\int \frac{1}{d+ex^3}\,dx, x, f+gx\right)}{g}
\end{aligned}$$

$$=-3qx+\frac{(f+gx)\log\left(c(d+e(f+gx)^3)^q\right)}{g}+\frac{\left(\sqrt[3]{d}q\right)\text{Subst}\left(\int\frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx,x,f+gx\right)}{g}+\frac{\left(\sqrt[3]{d}q\right)\text{Subst}\left(\int\frac{2\sqrt[3]{d}-\sqrt[3]{e}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx,x,f+gx\right)}{g}$$

$$=-3qx+\frac{\sqrt[3]{d}q\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+gx)\right)}{\sqrt[3]{e}g}+\frac{(f+gx)\log\left(c(d+e(f+gx)^3)^q\right)}{g}+\frac{\left(3d^{2/3}q\right)\text{Subst}\left(\int\frac{1}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx,x,f+gx\right)}{2g}-\frac{\left(\sqrt[3]{d}q\right)\text{Subst}\left(\int\frac{-\sqrt[3]{d}\sqrt[3]{e}+2e^{2/3}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx,x,f+gx\right)}{2\sqrt[3]{e}g}$$

$$=-3qx+\frac{\sqrt[3]{d}q\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+gx)\right)}{\sqrt[3]{e}g}-\frac{\sqrt[3]{d}q\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}(f+gx)^2\right)}{2\sqrt[3]{e}g}+\frac{(f+gx)\log\left(c(d+e(f+gx)^3)^q\right)}{g}+\frac{\left(3\sqrt[3]{d}q\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{e}(f+gx)}{\sqrt[3]{d}}\right)}{\sqrt[3]{e}g}$$

$$=-3qx-\frac{\sqrt{3}\sqrt[3]{d}q\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}(f+gx)}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{e}g}+\frac{\sqrt[3]{d}q\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+gx)\right)}{\sqrt[3]{e}g}-\frac{\sqrt[3]{d}q\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}(f+gx)^2\right)}{2\sqrt[3]{e}g}+\frac{(f+gx)\log\left(c(d+e(f+gx)^3)^q\right)}{g}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.87

$$\int \log \left(c\left(d+e(f+gx)^3\right)^q\right)\,dx=-3qx+\frac{\sqrt[3]{d}q\left(2\sqrt{3}\arctan\left(\frac{-\sqrt[3]{d}+2\sqrt[3]{e}(f+gx)}{\sqrt{3}\sqrt[3]{d}}\right)+2\log\left(\sqrt[3]{d}+\sqrt[3]{e}(f+gx)\right)-\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}(\right.\right.}{2\sqrt[3]{e}g}+\frac{(f+gx)\log\left(c\left(d+e(f+gx)^3\right)^q\right)}{g}$$

```
[In] Integrate[Log[c*(d + e*(f + g*x)^3)^q],x]
```

```
[Out] -3*q*x + (d^(1/3)*q*(2*Sqrt[3]*ArcTan[(-d^(1/3) + 2*e^(1/3)*(f + g*x))/(Sqr
t[3]*d^(1/3))] + 2*Log[d^(1/3) + e^(1/3)*(f + g*x)] - Log[d^(2/3) - d^(1/3)
*e^(1/3)*(f + g*x) + e^(2/3)*(f + g*x)^2]))/(2*e^(1/3)*g) + ((f + g*x)*Log[
c*(d + e*(f + g*x)^3)^q])/g
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.46 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.78

method	result
parts	$\ln\left(c\left(d+e(gx+f)^3\right)^q\right)x-3geq\left(\frac{x}{ge}+\frac{\sum_{\_R=\mathrm{RootOf}\left(g^3e\_Z^3+3efg^2\_Z^2+3ef^2g\_Z+ef^3+d\right)}\frac{\left(-\_R^2efg^2-2\_Ref^2\right.}{g^2\_R^2+2f}}{3e^2g^2}\right.$
default	$\ln\left(c\left(eg^3x^3+3efg^2x^2+3ef^2gx+ef^3+d\right)^q\right)x-3geq\left(\frac{x}{ge}+\frac{\sum_{\_R=\mathrm{RootOf}\left(g^3e\_Z^3+3efg^2\_Z^2+3ef^2g\_Z+e\right.}}{}\right.$
risch	$x\ln\left(\left(d+e(gx+f)^3\right)^q\right)+\frac{i\,\mathrm{csgn}\left(ic\left(d+e(gx+f)^3\right)^q\right)^2\mathrm{csgn}\left(i\left(d+e(gx+f)^3\right)^q\right)x\pi}{2}-\frac{i\pi x\,\mathrm{csgn}\left(i\left(d+e(gx+f)^3\right)^q\right)\mathrm{csgn}\left(\right.}{2}$

```
[In] int(ln(c*(d+e*(g*x+f)^3)^q),x,method=_RETURNVERBOSE)
```

```
[Out] ln(c*(d+e*(g*x+f)^3)^q)*x-3*g*e*q*(1/g/e*x+1/3/e^2/g^2*sum((-_R^2*e*f*g^2-2
*_R*e*f^2*g-e*f^3-d)/(_R^2*g^2+2*_R*f*g+f^2)*ln(x-_R),_R=RootOf(_Z^3*e*g^3+
```

```
3*_Z^2*e*f*g^2+3*_Z*e*f^2*g+e*f^3+d)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 1134, normalized size of antiderivative = 6.71

$$\int \log \left(c\left(d+e(f+gx)^3\right)^q\right)\,dx = \text{Too large to display}$$

```
[In] integrate(log(c*(d+e*(g*x+f)^3)^q),x, algorithm="fricas")
```

```
[Out] 1/4*(4*g*q*x*log(e*g^3*x^3 + 3*e*f*g^2*x^2 + 3*e*f^2*g*x + e*f^3 + d) - 12*
g*q*x - 2*((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/
(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*g*log(q*x - 1/2*(-1/2*f^3*q^3/g^3
 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) +
1) + f*q/g) + 4*g*x*log(c) + (((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*
(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*g + 6*f*q + s
qrt(3)*g*sqrt(-(((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d
*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*g^2 + 4*((-1/2*f^3*q^3/g^
3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) +
 1) - 2*f*q/g)*f*g*q + 4*f^2*q^2)/g^2))*log(2*g*q*x + 1/2*((-1/2*f^3*q^3/g^
3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) +
 1) - 2*f*q/g)*g + 3*f*q + 1/2*sqrt(3)*g*sqrt(-(((-1/2*f^3*q^3/g^3 + 1/2*d*
q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*
q/g)^2*g^2 + 4*((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*
q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*f*g*q + 4*f^2*q^2)/g^2)) + (
((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(
1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*g + 6*f*q - sqrt(3)*g*sqrt(-(((-1/2*f^3*q^3
/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3
) + 1) - 2*f*q/g)^2*g^2 + 4*((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e
*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*f*g*q + 4*f^2*q
^2)/g^2))*log(2*g*q*x + 1/2*((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e
*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*g + 3*f*q - 1/2
*sqrt(3)*g*sqrt(-(((-1/2*f^3*q^3/g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 +
 d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*g^2 + 4*((-1/2*f^3*q^3/
g^3 + 1/2*d*q^3/(e*g^3) + 1/2*(e*f^3*q^3 + d*q^3)/(e*g^3))^(1/3)*(I*sqrt(3)
 + 1) - 2*f*q/g)*f*g*q + 4*f^2*q^2)/g^2)))/g
```

Sympy [F(-1)]

Timed out.

$$\int \log \left(c\left(d+e(f+gx)^3\right)^q\right)\,dx=\text{Timed out}$$

```
[In] integrate(ln(c*(d+e*(g*x+f)**3)**q),x)
[Out] Timed out
```

Maxima [F]

$$\int \log \left(c\left(d+e(f+gx)^3\right)^q\right)\,dx=\int \log \left(\left((gx+f)^3e+d\right)^q c\right)\,dx$$

```
[In] integrate(log(c*(d+e*(g*x+f)^3)^q),x, algorithm="maxima")
[Out] -(3*q - log(c))*x + 3*q*integrate((e*f*g^2*x^2 + 2*e*f^2*g*x + e*f^3 + d)/(
e*g^3*x^3 + 3*e*f*g^2*x^2 + 3*e*f^2*g*x + e*f^3 + d), x) + x*log((e*g^3*x^3
 + 3*e*f*g^2*x^2 + 3*e*f^2*g*x + e*f^3 + d)^q)
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.54

$$\int \log \left(c\left(d+e(f+gx)^3\right)^q\right)\,dx=qx\log \left(eg^3x^3+3\,efg^2x^2+3\,ef^2gx+ef^3+d\right) - (3\,q-\log (c))x+\frac{fq\log \left(|eg^3x^3+3\,efg^2x^2+3\,ef^2gx+ef^3+d|\right)}{g} + \frac{2\,\sqrt{3}(de^2g^6q^3)^{\frac{1}{3}}\arctan \left(-\frac{egx+ef+(de^2)^{\frac{1}{3}}}{\sqrt{3}egx+\sqrt{3}ef-\sqrt{3}(de^2)^{\frac{1}{3}}}\right)-(de^2g^6q^3)^{\frac{1}{3}}\log \left(4\left(\sqrt{3}egx+\sqrt{3}ef-\sqrt{3}(de^2)^{\frac{1}{3}}\right)^2+4\right.}{2\,eg^3}$$

```
[In] integrate(log(c*(d+e*(g*x+f)^3)^q),x, algorithm="giac")
[Out] q*x*log(e*g^3*x^3 + 3*e*f*g^2*x^2 + 3*e*f^2*g*x + e*f^3 + d) - (3*q - log(c
))*x + f*q*log(abs(e*g^3*x^3 + 3*e*f*g^2*x^2 + 3*e*f^2*g*x + e*f^3 + d))/g
+ 1/2*(2*sqrt(3)*(d*e^2*g^6*q^3)^(1/3)*arctan(-(e*g*x + e*f + (d*e^2)^(1/3)
)/(sqrt(3)*e*g*x + sqrt(3)*e*f - sqrt(3)*(d*e^2)^(1/3))) - (d*e^2*g^6*q^3)^
(1/3)*log(4*(sqrt(3)*e*g*x + sqrt(3)*e*f - sqrt(3)*(d*e^2)^(1/3))^2 + 4*(e*
g*x + e*f + (d*e^2)^(1/3))^2) + 2*(d*e^2*g^6*q^3)^(1/3)*log(abs(e*g*x + e*f
 + (d*e^2)^(1/3))))/(e*g^3)
```

Mupad [B] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.14

$$\int \log \left(c\left(d+e(f+gx)^3\right)^q\right) \, dx = x \ln \left(c \left(d+e \left(f+g \, x\right)^3\right)^q\right) - \left(\sum_{k=1}^{3} \ln \left(d \, e^2 \, g^5 \left(\mathrm{root}\left(b^3 \, e \, g^3+3 \, b^2 \, e \, f \, g^2 \, q+3 \, b \, e \, f^2 \, g \, q^2+e \, f^3 \, q^3+d \, q^3, b, k\right) \, g+f \, q\right) \left(\mathrm{root}\left(b^3 \, e \, g^3+3 \, b^2 \, e \, f \, g^2 \, q+3 \, b \, e \, f^2 \, g \, q^2+e \, f^3 \, q^3+d \, q^3, b, k\right)\right)\right) - 3 \, q \, x$$

```
[In] int(log(c*(d + e*(f + g*x)^3)^q),x)
```

```
[Out] x*log(c*(d + e*(f + g*x)^3)^q) - symsum(log(9*d*e^2*g^5*(root(b^3*e*g^3 + 3
*b^2*e*f*g^2*q + 3*b*e*f^2*g*q^2 + e*f^3*q^3 + d*q^3, b, k)*g + f*q)*(root(
b^3*e*g^3 + 3*b^2*e*f*g^2*q + 3*b*e*f^2*g*q^2 + e*f^3*q^3 + d*q^3, b, k) -
q*x))*root(b^3*e*g^3 + 3*b^2*e*f*g^2*q + 3*b*e*f^2*g*q^2 + e*f^3*q^3 + d*q^
3, b, k), k, 1, 3) - 3*q*x
```

3.630 $\int \log \left(c\left(d+e(f+gx)^2\right)^q\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 63

$$\int \log \left(c\left(d+e(f+gx)^2\right)^q\right)\,dx=-2qx+\frac{2\sqrt{d}q\arctan\left(\frac{\sqrt{e}(f+gx)}{\sqrt{d}}\right)}{\sqrt{e}g}+\frac{(f+gx)\log\left(c\left(d+e(f+gx)^2\right)^q\right)}{g}$$

```
[Out] -2*q*x+(g*x+f)*ln(c*(d+e*(g*x+f)^2)^q)/g+2*q*arctan((g*x+f)*e^(1/2)/d^(1/2)
)*d^(1/2)/g/e^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2533, 2498, 327, 211}

$$\int \log \left(c\left(d+e(f+gx)^2\right)^q\right)\,dx=\frac{2\sqrt{d}q\arctan\left(\frac{\sqrt{e}(f+gx)}{\sqrt{d}}\right)}{\sqrt{e}g}+\frac{(f+gx)\log\left(c\left(d+e(f+gx)^2\right)^q\right)}{g}-2qx$$

```
[In] Int[Log[c*(d + e*(f + g*x)^2)^q],x]
```

```
[Out] -2*q*x + (2*Sqrt[d]*q*ArcTan[(Sqrt[e]*(f + g*x))/Sqrt[d]])/(Sqrt[e]*g) + ((
f + g*x)*Log[c*(d + e*(f + g*x)^2)^q])/g
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2533

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \log\left(c(d+ex^2)^q\right)\,dx,x,f+gx\right)}{g}\\
&= \frac{(f+gx)\log\left(c(d+e(f+gx)^2)^q\right)}{g}-\frac{(2eq)\text{Subst}\left(\int \frac{x^2}{d+ex^2}\,dx,x,f+gx\right)}{g}\\
&= -2qx+\frac{(f+gx)\log\left(c(d+e(f+gx)^2)^q\right)}{g}+\frac{(2dq)\text{Subst}\left(\int \frac{1}{d+ex^2}\,dx,x,f+gx\right)}{g}\\
&= -2qx+\frac{2\sqrt{d}q\tan^{-1}\left(\frac{\sqrt{e}(f+gx)}{\sqrt{d}}\right)}{\sqrt{e}g}+\frac{(f+gx)\log\left(c(d+e(f+gx)^2)^q\right)}{g}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \log \left(c\left(d+e(f+gx)^2\right)^q\right)\,dx=-2qx+\frac{2\sqrt{d}q\arctan\left(\frac{\sqrt{e}(f+gx)}{\sqrt{d}}\right)}{\sqrt{e}g}+\frac{(f+gx)\log\left(c\left(d+e(f+gx)^2\right)^q\right)}{g}$$

[In] Integrate[Log[c*(d + e*(f + g*x)^2)^q],x]

[Out] -2*q*x + (2*Sqrt[d]*q*ArcTan[(Sqrt[e]*(f + g*x))/Sqrt[d]])/(Sqrt[e]*g) + ((f + g*x)*Log[c*(d + e*(f + g*x)^2)^q])/g

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.67

method	result
parts	$\ln\left(c\left(d+e(gx+f)^2\right)^q\right)x-2qeg\left(\frac{x}{ge}+\frac{-\frac{f\ln\left(e\,g^2x^2+2efgx+e\,f^2+d\right)}{2g}-\frac{d\arctan\left(\frac{2e\,g^2x+2efg}{2\sqrt{de}\,g}\right)}{\sqrt{de}\,g}}{eg}\right)$
default	$\ln\left(c(e\,g^2x^2+2efgx+e\,f^2+d)^q\right)x-2qeg\left(\frac{x}{ge}+\frac{-\frac{f\ln\left(e\,g^2x^2+2efgx+e\,f^2+d\right)}{2g}-\frac{d\arctan\left(\frac{2e\,g^2x+2efg}{2\sqrt{de}\,g}\right)}{\sqrt{de}\,g}}{eg}\right)$
risch	$x\ln\left(\left(d+e(gx+f)^2\right)^q\right)+\frac{i\mathrm{csgn}\left(ic\left(d+e(gx+f)^2\right)^q\right)^2\mathrm{csgn}\left(i\left(d+e(gx+f)^2\right)^q\right)x\pi}{2}-\frac{i\pi x\ \mathrm{csgn}\left(i\left(d+e(gx+f)^2\right)^q\right)\mathrm{csgn}(}{2}$

[In] int(ln(c*(d+e*(g*x+f)^2)^q),x,method=_RETURNVERBOSE)

[Out] ln(c*(d+e*(g*x+f)^2)^q)*x-2*q*e*g*(1/g/e*x+1/e/g*(-1/2*f/g*ln(e*g^2*x^2+2*e*f*g*x+e*f^2+d)-d/(d*e)^(1/2)/g*arctan(1/2*(2*e*g^2*x+2*e*f*g)/(d*e)^(1/2)/g)))

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 206, normalized size of antiderivative = 3.27

$$\int \log \left(c\left(d+e(f+gx)^2\right)^q\right)\,dx$$
$$=\left[-\frac{2\,gqx-gx\log\left(c\right)-q\sqrt{-\frac{d}{e}}\log\left(\frac{eg^2x^2+2\,efgx+ef^2+2\,(egx+ef)\sqrt{-\frac{d}{e}}-d}{eg^2x^2+2\,efgx+ef^2+d}\right)-(gqx+fq)\log\left(eg^2x^2+2\,efgx+\right.}{g}\right.$$
$$\left.-\frac{2\,gqx-gx\log\left(c\right)-2\,q\sqrt{\frac{d}{e}}\arctan\left(\frac{(egx+ef)\sqrt{\frac{d}{e}}}{d}\right)-(gqx+fq)\log\left(eg^2x^2+2\,efgx+ef^2+d\right)}{g}\right]$$

```
[In] integrate(log(c*(d+e*(g*x+f)^2)^q),x, algorithm="fricas")
```

```
[Out] [-(2*g*q*x - g*x*log(c) - q*sqrt(-d/e)*log((e*g^2*x^2 + 2*e*f*g*x + e*f^2 +
 2*(e*g*x + e*f)*sqrt(-d/e) - d)/(e*g^2*x^2 + 2*e*f*g*x + e*f^2 + d)) - (g*
q*x + f*q)*log(e*g^2*x^2 + 2*e*f*g*x + e*f^2 + d))/g, -(2*g*q*x - g*x*log(c
) - 2*q*sqrt(d/e)*arctan((e*g*x + e*f)*sqrt(d/e)/d) - (g*q*x + f*q)*log(e*g
^2*x^2 + 2*e*f*g*x + e*f^2 + d))/g]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(58) = 116$.

Time = 95.18 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.73

$$\int \log \left(c\left(d+e(f+gx)^2\right)^q\right)\,dx$$
$$=\begin{cases} x\log\left(0^q c\right) \\ x\log\left(cd^q\right) \\ x\log\left(c\left(d+ef^2\right)^q\right) \\ \frac{f\log\left(c\left(ef^2+2efgx+eg^2x^2\right)^q\right)}{g}-2qx+x\log\left(c\left(ef^2+2efgx+eg^2x^2\right)^q\right) \\ \frac{2dq\log\left(\frac{f}{g}+x-\frac{\sqrt{-de}}{eg}\right)}{g\sqrt{-de}}-\frac{d\log\left(c\left(d+ef^2+2efgx+eg^2x^2\right)^q\right)}{g\sqrt{-de}}+\frac{f\log\left(c\left(d+ef^2+2efgx+eg^2x^2\right)^q\right)}{g}-2qx+x\log\left(c\left(d+ef^2+2e\right.\right. \end{cases}$$

```
[In] integrate(ln(c*(d+e*(g*x+f)**2)**q),x)
```

```
[Out] Piecewise((x*log(0**q*c), Eq(d, 0) & Eq(e, 0) & Eq(g, 0)), (x*log(c*d**q),
Eq(e, 0)), (x*log(c*(d + e*f**2)**q), Eq(g, 0)), (f*log(c*(e*f**2 + 2*e*f*g
```

```
*x + e*g**2*x**2)**q)/g - 2*q*x + x*log(c*(e*f**2 + 2*e*f*g*x + e*g**2*x**2
)**q), Eq(d, 0)), (2*d*q*log(f/g + x - sqrt(-d*e)/(e*g))/(g*sqrt(-d*e)) - d
*log(c*(d + e*f**2 + 2*e*f*g*x + e*g**2*x**2)**q)/(g*sqrt(-d*e)) + f*log(c*
(d + e*f**2 + 2*e*f*g*x + e*g**2*x**2)**q)/g - 2*q*x + x*log(c*(d + e*f**2
+ 2*e*f*g*x + e*g**2*x**2)**q), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \log \left(c\left(d+e(f+gx)^2\right)^q\right)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(c*(d+e*(g*x+f)^2)^q),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.46

$$\int \log \left(c\left(d+e(f+gx)^2\right)^q\right)\,dx = qx\log\left(eg^2x^2+2\,efgx+ef^2+d\right)-(2\,q-\log\left(c\right))x + \frac{fq\log\left(eg^2x^2+2\,efgx+ef^2+d\right)}{g} + \frac{2\,dq\arctan\left(\frac{egx+ef}{\sqrt{de}}\right)}{\sqrt{de}g}$$

```
[In] integrate(log(c*(d+e*(g*x+f)^2)^q),x, algorithm="giac")
```

```
[Out] q*x*log(e*g^2*x^2 + 2*e*f*g*x + e*f^2 + d) - (2*q - log(c))*x + f*q*log(e*g
^2*x^2 + 2*e*f*g*x + e*f^2 + d)/g + 2*d*q*arctan((e*g*x + e*f)/sqrt(d*e))/(
sqrt(d*e)*g)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.30

$$\int \log \left(c\big(d+e(f+gx)^2\big)^q\right)\,dx = x\,\ln\left(c\,\left(d+e\,{\left(f+g\,x\right)}^2\right)^q\right)-2\,q\,x+\frac{f\,q\,\ln\left(e\,f^2+2\,e\,f\,g\,x+e\,g^2\,x^2+d\right)}{g}+\frac{2\,\sqrt{d}\,q\,\mathrm{atan}\left(\frac{\sqrt{e}\,f}{\sqrt{d}}+\frac{\sqrt{e}\,g\,x}{\sqrt{d}}\right)}{\sqrt{e}\,g}$$

```
[In] int(log(c*(d + e*(f + g*x)^2)^q),x)
```

```
[Out] x*log(c*(d + e*(f + g*x)^2)^q) - 2*q*x + (f*q*log(d + e*f^2 + e*g^2*x^2 + 2
*e*f*g*x))/g + (2*d^(1/2)*q*atan((e^(1/2)*f)/d^(1/2) + (e^(1/2)*g*x)/d^(1/2
)))/(e^(1/2)*g)
```

3.631 $\int \log \left(c(d+e(f+gx))^q\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 35

$$\int \log \left(c(d+e(f+gx))^q\right) \, dx = -qx+\frac{(d+ef+egx)\log \left(c(d+e(f+gx))^q\right)}{eg}$$

```
[Out] -q*x+(e*g*x+e*f+d)*ln(c*(d+e*(g*x+f))^q)/e/g
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2494, 2436, 2332}

$$\int \log \left(c(d+e(f+gx))^q\right) \, dx = \frac{(d+ef+egx)\log \left(c(d+e(f+gx))^q\right)}{eg}-qx$$

```
[In] Int[Log[c*(d + e*(f + g*x))^q],x]
```

```
[Out] -(q*x) + ((d + e*f + e*g*x)*Log[c*(d + e*(f + g*x))^q])/(e*g)
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2436

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2494

```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \log\left(c(d+ef+egx)^q\right)\,dx\\
&= \frac{\text{Subst}\left(\int \log\left(cx^q\right)\,dx, x, d+ef+egx\right)}{eg}\\
&= -qx + \frac{(d+ef+egx)\log\left(c(d+e(f+gx))^q\right)}{eg}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.34

$$\int \log\left(c(d+e(f+gx))^q\right)\,dx = -qx + \frac{dq\log(d+ef+egx)}{eg} + \frac{(f+gx)\log\left(c(d+e(f+gx))^q\right)}{g}$$

```
[In] Integrate[Log[c*(d + e*(f + g*x))^q],x]
[Out] -(q*x) + (d*q*Log[d + e*f + e*g*x])/(e*g) + ((f + g*x)*Log[c*(d + e*(f + g*
x))^q])/g
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.34

method	result
norman	$x\ln\left(c\,\mathrm{e}^{q\ln(d+(gx+f)e)}\right)+\frac{q(ef+d)\ln(d+(gx+f)e)}{eg}-qx$
default	$\ln\left(c(egx+ef+d)^q\right)x-qeg\left(\frac{x}{ge}+\frac{(-ef-d)\ln(egx+ef+d)}{e^2g^2}\right)$
parts	$\ln\left(c(d+(gx+f)\,e)^q\right)x-qeg\left(\frac{x}{ge}+\frac{(-ef-d)\ln(egx+ef+d)}{e^2g^2}\right)$
parallelrisch	$\frac{2\ln(egx+ef+d)efq+x\ln(c(egx+ef+d)^q)eg-geqx+2\ln(egx+ef+d)dq-\ln(c(egx+ef+d)^q)ef+efq-d\ln(c(egx+ef+d)^q)+d}{eg}$
risch	$x\ln\left((egx+ef+d)^q\right)+\frac{i\pi x\ \mathrm{csgn}(i(egx+ef+d)^q)\,\mathrm{csgn}(ic(egx+ef+d)^q)^2}{2}-\frac{i\pi x\ \mathrm{csgn}(i(egx+ef+d)^q)\,\mathrm{csgn}(ic(egx}{2}$

```
[In] int(ln(c*(d+(g*x+f)*e)^q),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(c*exp(q*ln(d+(g*x+f)*e)))+q*(e*f+d)/e/g*ln(d+(g*x+f)*e)-q*x
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \log \left(c(d+e(f+gx))^q\right)\,dx = -\frac{egqx - egx\log(c) - (egqx + (ef+d)q)\log(egx + ef + d)}{eg}$$

```
[In] integrate(log(c*(d+e*(g*x+f))^q),x, algorithm="fricas")
```

```
[Out] -(e*g*q*x - e*g*x*log(c) - (e*g*q*x + (e*f + d)*q)*log(e*g*x + e*f + d))/(e
*g)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(29) = 58.

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.29

$$\int \log \left(c(d+e(f+gx))^q\right)\,dx = \begin{cases} x\log(cd^q) & \text{for } e=0 \wedge (e=0 \vee g=0) \\ x\log\left(c(d+ef)^q\right) & \text{for } g=0 \\ \frac{d\log(c(d+ef+egx)^q)}{eg} + \frac{f\log(c(d+ef+egx)^q)}{g} - qx + x\log\left(c(d+ef+egx)^q\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(c*(d+e*(g*x+f))**q),x)
```

```
[Out] Piecewise((x*log(c*d**q), Eq(e, 0) & (Eq(e, 0) | Eq(g, 0))), (x*log(c*(d +
e*f)**q), Eq(g, 0)), (d*log(c*(d + e*f + e*g*x)**q)/(e*g) + f*log(c*(d + e*
f + e*g*x)**q)/g - q*x + x*log(c*(d + e*f + e*g*x)**q), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.54

$$\int \log \left(c(d+e(f+gx))^q\right)\,dx = -egq\left(\frac{x}{eg} - \frac{(ef+d)\log(egx+ef+d)}{e^2g^2}\right) + x\log\left(((gx+f)e+d)^q c\right)$$

```
[In] integrate(log(c*(d+e*(g*x+f))^q),x, algorithm="maxima")
```

```
[Out] -e*g*q*(x/(e*g) - (e*f + d)*log(e*g*x + e*f + d)/(e^2*g^2)) + x*log(((g*x +
 f)*e + d)^q*c)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.83

$$\int \log \left(c(d+e(f+gx))^q\right)\,dx = \frac{(egx+ef+d)q\log(egx+ef+d)}{eg} - \frac{(egx+ef+d)q}{eg} + \frac{(egx+ef+d)\log(c)}{eg}$$

```
[In] integrate(log(c*(d+e*(g*x+f))^q),x, algorithm="giac")
[Out] (e*g*x + e*f + d)*q*log(e*g*x + e*f + d)/(e*g) - (e*g*x + e*f + d)*q/(e*g)
+ (e*g*x + e*f + d)*log(c)/(e*g)
```

Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \log \left(c(d+e(f+gx))^q\right)\,dx = x\,\ln\left(c\,(d+e\,(f+g\,x))^q\right) - q\,x + \frac{\ln(d+e\,f+e\,g\,x)\,(d\,q+e\,f\,q)}{e\,g}$$

```
[In] int(log(c*(d + e*(f + g*x))^q),x)
[Out] x*log(c*(d + e*(f + g*x))^q) - q*x + (log(d + e*f + e*g*x)*(d*q + e*f*q))/(
e*g)
```

3.632 $\int \log \left(c \left(d+\frac{e}{f+gx}\right)^q\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int \log \left(c \left(d+\frac{e}{f+gx}\right)^q\right) \, dx=\frac{(f+gx) \log \left(c \left(d+\frac{e}{f+gx}\right)^q\right)}{g}+\frac{eq \log(e+d(f+gx))}{dg}$$

```
[Out] (g*x+f)*ln(c*(d+e/(g*x+f))^q)/g+e*q*ln(e+d*(g*x+f))/d/g
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2533, 2498, 269, 31}

$$\int \log \left(c \left(d+\frac{e}{f+gx}\right)^q\right) \, dx=\frac{(f+gx) \log \left(c \left(d+\frac{e}{f+gx}\right)^q\right)}{g}+\frac{eq \log(d(f+gx)+e)}{dg}$$

```
[In] Int[Log[c*(d + e/(f + g*x))^q],x]
```

```
[Out] ((f + g*x)*Log[c*(d + e/(f + g*x))^q])/g + (e*q*Log[e + d*(f + g*x)])/(d*g)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2533

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \log\left(c\left(d+\frac{e}{x}\right)^q\right)\,dx,x,f+gx\right)}{g}\\
&= \frac{(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{g}+\frac{(eq)\text{Subst}\left(\int \frac{1}{\left(d+\frac{e}{x}\right)x}\,dx,x,f+gx\right)}{g}\\
&= \frac{(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{g}+\frac{(eq)\text{Subst}\left(\int \frac{1}{e+dx}\,dx,x,f+gx\right)}{g}\\
&= \frac{(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{g}+\frac{eq\log(e+d(f+gx))}{dg}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

$$\int \log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)\,dx = \frac{-dfq\log(f+gx)+(e+df)q\log(e+df+dgx)+dgx\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{dg}$$

```
[In] Integrate[Log[c*(d + e/(f + g*x))^q],x]
```

```
[Out] (-(d*f*q*Log[f + g*x]) + (e + d*f)*q*Log[e + d*f + d*g*x] + d*g*x*Log[c*(d
+ e/(f + g*x))^q])/(d*g)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

method	result	size
parts	$\ln\left(c\left(d+\frac{e}{gx+f}\right)^{q}\right)x+qeg\left(-\frac{f\ln(gx+f)}{g^{2}e}+\frac{(df+e)\ln(dgx+df+e)}{e\,g^{2}d}\right)$	65
default	$\ln\left(c\left(\frac{dgx+df+e}{gx+f}\right)^{q}\right)x+qeg\left(-\frac{f\ln(gx+f)}{g^{2}e}+\frac{(df+e)\ln(dgx+df+e)}{e\,g^{2}d}\right)$	71
parallelrisch	$-\frac{-x\ln\left(c\left(\frac{dgx+df+e}{gx+f}\right)^{q}\right)d\,g^{2}q-\ln(gx+f)eg\,q^{2}-\ln\left(c\left(\frac{dgx+df+e}{gx+f}\right)^{q}\right)dfgq-\ln\left(c\left(\frac{dgx+df+e}{gx+f}\right)^{q}\right)egq}{d\,g^{2}q}$	111

```
[In] int(ln(c*(d+e/(g*x+f))^q),x,method=_RETURNVERBOSE)
[Out] ln(c*(d+e/(g*x+f))^q)*x+q*e*g*(-f/g^2/e*ln(g*x+f)+(d*f+e)/e/g^2/d*ln(d*g*x+
d*f+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int\log\left(c\left(d+\frac{e}{f+gx}\right)^{q}\right)dx$$
$$=\frac{dgqx\log\left(\frac{dgx+df+e}{gx+f}\right)-dfq\log(gx+f)+dgx\log(c)+(df+e)q\log(dgx+df+e)}{dg}$$

```
[In] integrate(log(c*(d+e/(g*x+f))^q),x, algorithm="fricas")
[Out] (d*g*q*x*log((d*g*x + d*f + e)/(g*x + f)) - d*f*q*log(g*x + f) + d*g*x*log(
c) + (d*f + e)*q*log(d*g*x + d*f + e))/(d*g)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(36) = 72$.

Time = 0.78 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.27

$$\int \log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)\,dx = \begin{cases} x\log\left(c\left(\frac{e}{f}\right)^q\right) & \text{for } d=0\wedge g=0 \\ \frac{f\log\left(c\left(\frac{e}{f+gx}\right)^q\right)}{g}+qx+x\log\left(c\left(\frac{e}{f+gx}\right)^q\right) & \text{for } d=0 \\ x\log\left(c\left(d+\frac{e}{f}\right)^q\right) & \text{for } g=0 \\ \frac{f\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)}{g}+x\log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)+\frac{eq\log(df+dgx+e)}{dg} & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(c*(d+e/(g*x+f))**q),x)
[Out] Piecewise((x*log(c*(e/f)**q), Eq(d, 0) & Eq(g, 0)), (f*log(c*(e/(f + g*x))*
*q)/g + q*x + x*log(c*(e/(f + g*x))**q), Eq(d, 0)), (x*log(c*(d + e/f)**q),
 Eq(g, 0)), (f*log(c*(d + e/(f + g*x))**q)/g + x*log(c*(d + e/(f + g*x))**q
) + e*q*log(d*f + d*g*x + e)/(d*g), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)\,dx = -egq\left(\frac{f\log(gx+f)}{eg^2}-\frac{(df+e)\log(dgx+df+e)}{deg^2}\right) + x\log\left(c\left(d+\frac{e}{gx+f}\right)^q\right)$$

```
[In] integrate(log(c*(d+e/(g*x+f))^q),x, algorithm="maxima")
[Out] -e*g*q*(f*log(g*x + f)/(e*g^2) - (d*f + e)*log(d*g*x + d*f + e)/(d*e*g^2))
+ x*log(c*(d + e/(g*x + f))^q)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(45) = 90$.

Time = 0.35 (sec) , antiderivative size = 171, normalized size of antiderivative = 3.80

$$\int \log\left(c\left(d+\frac{e}{f+gx}\right)^q\right)\,dx = \left(\frac{e^2q\log\left(\frac{dgx+df+e}{gx+f}\right)}{dg^2-\frac{(dgx+df+e)g^2}{gx+f}}+\frac{e^2\log(c)}{dg^2-\frac{(dgx+df+e)g^2}{gx+f}}+\frac{e^2q\log\left(-d+\frac{dgx+df+e}{gx+f}\right)}{dg^2}-\frac{e^2q\log\left(\frac{dgx+df+e}{gx+f}\right)}{dg^2}\right)\left(\frac{dfg}{e^2}-\frac{(df}{}\right.$$

```
[In] integrate(log(c*(d+e/(g*x+f))^q),x, algorithm="giac")
```

```
[Out] (e^2*q*log((d*g*x + d*f + e)/(g*x + f))/(d*g^2 - (d*g*x + d*f + e)*g^2/(g*x
 + f)) + e^2*log(c)/(d*g^2 - (d*g*x + d*f + e)*g^2/(g*x + f)) + e^2*q*log(-
d + (d*g*x + d*f + e)/(g*x + f))/(d*g^2) - e^2*q*log((d*g*x + d*f + e)/(g*x
 + f))/(d*g^2))*(d*f*g/e^2 - (d*f + e)*g/e^2)
```

Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.49

$$\int \log \left(c\left(d+\frac{e}{f+gx}\right)^q\right) dx = x \ln \left(c \left(d+\frac{e}{f+g\,x}\right)^q\right)-\frac{f\,q\,\ln \left(f+g\,x\right)}{g}+\frac{f\,q\,\ln \left(e+d\,f+d\,g\,x\right)}{g}+\frac{e\,q\,\ln \left(e+d\,f+d\,g\,x\right)}{d\,g}$$

```
[In] int(log(c*(d + e/(f + g*x))^q),x)
```

```
[Out] x*log(c*(d + e/(f + g*x))^q) - (f*q*log(f + g*x))/g + (f*q*log(e + d*f + d*
g*x))/g + (e*q*log(e + d*f + d*g*x))/(d*g)
```

3.633 $\int \log \left(c \left(d+\frac{e}{(f+gx)^2}\right)^q\right) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 59

$$\int \log \left(c \left(d+\frac{e}{(f+gx)^2}\right)^q\right) \, dx=\frac{2\sqrt{e}q \arctan \left(\frac{\sqrt{d}(f+gx)}{\sqrt{e}}\right)}{\sqrt{d}g}+\frac{(f+gx) \log \left(c \left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}$$

```
[Out] (g*x+f)*ln(c*(d+e/(g*x+f)^2)^q)/g+2*q*arctan((g*x+f)*d^(1/2)/e^(1/2))*e^(1/
2)/g/d^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2533, 2498, 269, 211}

$$\int \log \left(c \left(d+\frac{e}{(f+gx)^2}\right)^q\right) \, dx=\frac{2\sqrt{e}q \arctan \left(\frac{\sqrt{d}(f+gx)}{\sqrt{e}}\right)}{\sqrt{d}g}+\frac{(f+gx) \log \left(c \left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}$$

```
[In] Int[Log[c*(d + e/(f + g*x)^2)^q],x]
```

```
[Out] (2*Sqrt[e]*q*ArcTan[(Sqrt[d]*(f + g*x))/Sqrt[e]])/(Sqrt[d]*g) + ((f + g*x)*
Log[c*(d + e/(f + g*x)^2)^q])/g
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2533

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \log\left(c\left(d+\frac{e}{x^2}\right)^q\right)\,dx, x, f+gx\right)}{g}\\
&= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}+\frac{(2eq)\text{Subst}\left(\int \frac{1}{\left(d+\frac{e}{x^2}\right)x^2}\,dx, x, f+gx\right)}{g}\\
&= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}+\frac{(2eq)\text{Subst}\left(\int \frac{1}{e+dx^2}\,dx, x, f+gx\right)}{g}\\
&= \frac{2\sqrt{e}q\tan^{-1}\left(\frac{\sqrt{d}(f+gx)}{\sqrt{e}}\right)}{\sqrt{d}g}+\frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)}{g}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.34

$$
\int \log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)\,dx = \frac{\frac{2\sqrt{e}q\arctan\left(\frac{\sqrt{d}(f+gx)}{\sqrt{e}}\right)}{\sqrt{d}}-2fq\log(f+gx)+gx\log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)+fq\log\left(e+d(f+gx)^2\right)}{g}
$$

```
[In] Integrate[Log[c*(d + e/(f + g*x)^2)^q],x]
```

```
[Out] ((2*Sqrt[e]*q*ArcTan[(Sqrt[d]*(f + g*x))/Sqrt[e]])/Sqrt[d] - 2*f*q*Log[f +
g*x] + g*x*Log[c*(d + e/(f + g*x)^2)^q] + f*q*Log[e + d*(f + g*x)^2])/g
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(51) = 102.

Time = 0.93 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.88

method	result	si
parts	$\ln\left(c\left(d+\frac{e}{(gx+f)^{2}}\right)^{q}\right)x+2qeg\left(-\frac{f\ln(gx+f)}{g^{2}e}+\frac{\frac{f\ln\left(d\,g^{2}x^{2}+2dfgx+d\,f^{2}+e\right)}{2g}+\frac{e\arctan\left(\frac{2d\,g^{2}x+2dfg}{2\sqrt{de}\,g}\right)}{\sqrt{de}\,g}}{eg}\right)$	11
default	$\ln\left(c\left(\frac{d\,g^{2}x^{2}+2dfgx+d\,f^{2}+e}{(gx+f)^{2}}\right)^{q}\right)x+2qeg\left(-\frac{f\ln(gx+f)}{g^{2}e}+\frac{\frac{f\ln\left(d\,g^{2}x^{2}+2dfgx+d\,f^{2}+e\right)}{2g}+\frac{e\arctan\left(\frac{2d\,g^{2}x+2dfg}{2\sqrt{de}\,g}\right)}{\sqrt{de}\,g}}{eg}\right)$	12

```
[In] int(ln(c*(d+e/(g*x+f)^2)^q),x,method=_RETURNVERBOSE)
```

```
[Out] ln(c*(d+e/(g*x+f)^2)^q)*x+2*q*e*g*(-f/g^2/e*ln(g*x+f)+1/e/g*(1/2*f/g*ln(d*g
^2*x^2+2*d*f*g*x+d*f^2+e)+e/(d*e)^(1/2)/g*arctan(1/2*(2*d*g^2*x+2*d*f*g)/(d
*e)^(1/2)/g)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(51) = 102.

Time = 0.34 (sec) , antiderivative size = 287, normalized size of antiderivative = 4.86

$$\int \log\left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)dx$$
$$=\left[\frac{gqx\log\left(\frac{dg^2x^2+2\,dfgx+df^2+e}{g^2x^2+2\,fgx+f^2}\right)+fq\log\left(dg^2x^2+2\,dfgx+df^2+e\right)-2\,fq\log\left(gx+f\right)+gx\log\left(c\right)+q\sqrt{-}}{g}\right.$$

```
[In] integrate(log(c*(d+e/(g*x+f)^2)^q),x, algorithm="fricas")
```

```
[Out] [(g*q*x*log((d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e)/(g^2*x^2 + 2*f*g*x + f^2))
+ f*q*log(d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e) - 2*f*q*log(g*x + f) + g*x*log
(c) + q*sqrt(-e/d)*log((d*g^2*x^2 + 2*d*f*g*x + d*f^2 + 2*(d*g*x + d*f)*sqr
t(-e/d) - e)/(d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e)))/g, (g*q*x*log((d*g^2*x^2
 + 2*d*f*g*x + d*f^2 + e)/(g^2*x^2 + 2*f*g*x + f^2)) + f*q*log(d*g^2*x^2 +
2*d*f*g*x + d*f^2 + e) - 2*f*q*log(g*x + f) + g*x*log(c) + 2*q*sqrt(e/d)*ar
ctan((d*g*x + d*f)*sqrt(e/d)/e))/g]
```

Sympy [F(-1)]

Timed out.

$$\int \log \left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right) dx = \text{Timed out}$$

```
[In] integrate(ln(c*(d+e/(g*x+f)**2)**q),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \log \left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right) dx = \text{Exception raised: ValueError}$$

```
[In] integrate(log(c*(d+e/(g*x+f)^2)^q),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(51) = 102.

Time = 0.53 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.36

$$\begin{aligned}\int \log \left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right) dx &= deg^4q\left(\frac{f\log(dg^2x^2+2\,dfgx+df^2+e)}{deg^5} - \frac{2\,f\log(|gx+f|)}{deg^5} + \frac{2\arctan\left(\frac{dgx+df}{\sqrt{de}}\right)}{\sqrt{de}dg^5}\right)\\ &\quad + qx\log\left(dg^2x^2+2\,dfgx+df^2+e\right) - qx\log\left(g^2x^2+2\,fgx+f^2\right) + x\log(c)\end{aligned}$$

```
[In] integrate(log(c*(d+e/(g*x+f)^2)^q),x, algorithm="giac")
```

```
[Out] d*e*g^4*q*(f*log(d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e)/(d*e*g^5) - 2*f*log(abs
(g*x + f))/(d*e*g^5) + 2*arctan((d*g*x + d*f)/sqrt(d*e))/(sqrt(d*e)*d*g^5))
 + q*x*log(d*g^2*x^2 + 2*d*f*g*x + d*f^2 + e) - q*x*log(g^2*x^2 + 2*f*g*x +
 f^2) + x*log(c)
```

Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.76

$$\int \log \left(c\left(d+\frac{e}{(f+gx)^2}\right)^q\right)\,dx$$
$$= x\,\ln\left(c\,{\left(d+\frac{e}{{\left(f+g\,x\right)}^2}\right)}^q\right)-\frac{2\,f\,q\,\ln\left(f+g\,x\right)}{g}$$
$$+\frac{\ln\left(e\,\sqrt{-d\,e}-3\,d\,f^2\,\sqrt{-d\,e}+4\,d\,e\,f+d\,e\,g\,x-3\,d\,f\,g\,x\,\sqrt{-d\,e}\right)\,\left(q\,\sqrt{-d\,e}+d\,f\,q\right)}{d\,g}$$
$$-\frac{\ln\left(3\,d\,f^2\,\sqrt{-d\,e}-e\,\sqrt{-d\,e}+4\,d\,e\,f+d\,e\,g\,x+3\,d\,f\,g\,x\,\sqrt{-d\,e}\right)\,\left(q\,\sqrt{-d\,e}-d\,f\,q\right)}{d\,g}$$

```
[In] int(log(c*(d + e/(f + g*x)^2)^q),x)
```

```
[Out] x*log(c*(d + e/(f + g*x)^2)^q) - (2*f*q*log(f + g*x))/g + (log(e*(-d*e)^(1/
2) - 3*d*f^2*(-d*e)^(1/2) + 4*d*e*f + d*e*g*x - 3*d*f*g*x*(-d*e)^(1/2))*(q*
(-d*e)^(1/2) + d*f*q))/(d*g) - (log(3*d*f^2*(-d*e)^(1/2) - e*(-d*e)^(1/2) +
 4*d*e*f + d*e*g*x + 3*d*f*g*x*(-d*e)^(1/2))*(q*(-d*e)^(1/2) - d*f*q))/(d*g
)
```

3.634 $\int \log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right) dx$

Optimal result

Integrand size = 16, antiderivative size = 165

$$\int \log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right) dx = -\frac{\sqrt{3}\sqrt[3]{e}q\arctan\left(\frac{\sqrt[3]{e}-2\sqrt[3]{d}(f+gx)}{\sqrt{3}\sqrt[3]{e}}\right)}{\sqrt[3]{d}g} + \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g} + \frac{\sqrt[3]{e}q\log\left(\sqrt[3]{e}+\sqrt[3]{d}(f+gx)\right)}{\sqrt[3]{d}g} - \frac{\sqrt[3]{e}q\log\left(e^{2/3}-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+d^{2/3}(f+gx)^2\right)}{2\sqrt[3]{d}g}$$

```
[Out] (g*x+f)*ln(c*(d+e/(g*x+f)^3)^q)/g+e^(1/3)*q*ln(e^(1/3)+d^(1/3)*(g*x+f))/d^(
1/3)/g-1/2*e^(1/3)*q*ln(e^(2/3)-d^(1/3)*e^(1/3)*(g*x+f)+d^(2/3)*(g*x+f)^2)/
d^(1/3)/g-e^(1/3)*q*arctan(1/3*(e^(1/3)-2*d^(1/3)*(g*x+f))/e^(1/3)*3^(1/2))
*3^(1/2)/d^(1/3)/g
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {2533, 2498, 269, 206, 31, 648, 631, 210, 642}

$$\int \log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)dx = -\frac{\sqrt{3}\sqrt[3]{e}q\arctan\left(\frac{\sqrt[3]{e}-2\sqrt[3]{d}(f+gx)}{\sqrt{3}\sqrt[3]{e}}\right)}{\sqrt[3]{d}g} + \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g} - \frac{\sqrt[3]{e}q\log\left(d^{2/3}(f+gx)^2-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+e^{2/3}\right)}{2\sqrt[3]{d}g} + \frac{\sqrt[3]{e}q\log\left(\sqrt[3]{d}(f+gx)+\sqrt[3]{e}\right)}{\sqrt[3]{d}g}$$

```
[In] Int[Log[c*(d + e/(f + g*x)^3)^q],x]
```

```
[Out] -((Sqrt[3]*e^(1/3)*q*ArcTan[(e^(1/3) - 2*d^(1/3)*(f + g*x))/(Sqrt[3]*e^(1/3
))]])/(d^(1/3)*g)) + ((f + g*x)*Log[c*(d + e/(f + g*x)^3)^q])/g + (e^(1/3)*q
*Log[e^(1/3) + d^(1/3)*(f + g*x)])/(d^(1/3)*g) - (e^(1/3)*q*Log[e^(2/3) - d
^(1/3)*e^(1/3)*(f + g*x) + d^(2/3)*(f + g*x)^2])/(2*d^(1/3)*g)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2533

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \log\left(c\left(d+\frac{e}{x^3}\right)^q\right)\,dx, x, f+gx\right)}{g}\\ &= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g}+\frac{(3eq)\text{Subst}\left(\int \frac{1}{\left(d+\frac{e}{x^3}\right)x^3}\,dx, x, f+gx\right)}{g}\\ &= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g}+\frac{(3eq)\text{Subst}\left(\int \frac{1}{e+dx^3}\,dx, x, f+gx\right)}{g}\end{aligned}$$

$$= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g} + \frac{(\sqrt[3]{e}q)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{e}+\sqrt[3]{d}x}\,dx, x, f+gx\right)}{g}$$
$$+ \frac{(\sqrt[3]{e}q)\,\mathrm{Subst}\left(\int \frac{2\sqrt[3]{e}-\sqrt[3]{d}x}{e^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}x^2}\,dx, x, f+gx\right)}{g}$$

$$= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g} + \frac{\sqrt[3]{e}q\log\left(\sqrt[3]{e}+\sqrt[3]{d}(f+gx)\right)}{\sqrt[3]{d}g}$$
$$- \frac{(\sqrt[3]{e}q)\,\mathrm{Subst}\left(\int \frac{-\sqrt[3]{d}\sqrt[3]{e}+2d^{2/3}x}{e^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}x^2}\,dx, x, f+gx\right)}{2\sqrt[3]{d}g}$$
$$+ \frac{(3e^{2/3}q)\,\mathrm{Subst}\left(\int \frac{1}{e^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}x^2}\,dx, x, f+gx\right)}{2g}$$

$$= \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g} + \frac{\sqrt[3]{e}q\log\left(\sqrt[3]{e}+\sqrt[3]{d}(f+gx)\right)}{\sqrt[3]{d}g}$$
$$- \frac{\sqrt[3]{e}q\log\left(e^{2/3}-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+d^{2/3}(f+gx)^2\right)}{2\sqrt[3]{d}g}$$
$$+ \frac{(3\sqrt[3]{e}q)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}(f+gx)}{\sqrt[3]{e}}\right)}{\sqrt[3]{d}g}$$

$$= -\frac{\sqrt{3}\sqrt[3]{e}q\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}(f+gx)}{\sqrt[3]{e}}}{\sqrt{3}}\right)}{\sqrt[3]{d}g}$$
$$+ \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g} + \frac{\sqrt[3]{e}q\log\left(\sqrt[3]{e}+\sqrt[3]{d}(f+gx)\right)}{\sqrt[3]{d}g}$$
$$- \frac{\sqrt[3]{e}q\log\left(e^{2/3}-\sqrt[3]{d}\sqrt[3]{e}(f+gx)+d^{2/3}(f+gx)^2\right)}{2\sqrt[3]{d}g}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.40

$$\int \log \left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right) dx = -\frac{3eq \operatorname{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-\frac{e}{d(f+gx)^3}\right)}{2dg(f+gx)^2} + \frac{(f+gx)\log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)}{g}$$

```
[In] Integrate[Log[c*(d + e/(f + g*x)^3)^q],x]
[Out] (-3*e*q*Hypergeometric2F1[2/3, 1, 5/3, -(e/(d*(f + g*x)^3))])/(2*d*g*(f + g
*x)^2) + ((f + g*x)*Log[c*(d + e/(f + g*x)^3)^q])/g
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.96 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.84

method	result
parts	$\ln\left(c\left(d+\frac{e}{(gx+f)^3}\right)^q\right)x+3qeg\left(\frac{\sum_{\_R=\operatorname{RootOf}\left(d\,g^3\_Z^3+3d\,g^2f\,\_Z^2+3d\,f^2g\,\_Z+d\,f^3+e\right)}\frac{\left(\_R^2df\,g^2+2\_R d\,f^2g+d\,f^3+e\right)\ln}{g^2\_R^2+2fg\_R+f^2}}{3d\,g^2e}\right.$
default	$\ln\left(c\left(\frac{d\,g^3x^3+3df\,g^2x^2+3d\,f^2gx+d\,f^3+e}{(gx+f)^3}\right)^q\right)x+3qeg\left(\frac{\sum_{\_R=\operatorname{RootOf}\left(d\,g^3\_Z^3+3d\,g^2f\,\_Z^2+3d\,f^2g\,\_Z+d\,f^3+e\right)}\frac{\left(\_R^2df\,g^2+2\right.}{g^2}}{3d\,g^2e}\right.$

```
[In] int(ln(c*(d+e/(g*x+f)^3)^q),x,method=_RETURNVERBOSE)
[Out] ln(c*(d+e/(g*x+f)^3)^q)*x+3*q*e*g*(1/3/d/g^2*sum((_R^2*d*f*g^2+2*_R*d*f^2*g
+d*f^3+e)/(_R^2*g^2+2*_R*f*g+f^2)*ln(x-_R),_R=RootOf(_Z^3*d*g^3+3*_Z^2*d*f*
g^2+3*_Z*d*f^2*g+d*f^3+e))/e-f/g^2/e*ln(g*x+f))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 1169, normalized size of antiderivative = 7.08

$$\int \log\left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right)\,dx = \text{Too large to display}$$

```
[In] integrate(log(c*(d+e/(g*x+f)^3)^q),x, algorithm="fricas")
```

```
[Out] 1/4*(4*g*q*x*log((d*g^3*x^3 + 3*d*f*g^2*x^2 + 3*d*f^2*g*x + d*f^3 + e)/(g^3
*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3)) - 12*f*q*log(g*x + f) - 2*((-1/2*f^3
*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sq
rt(3) + 1) - 2*f*q/g)*g*log(q*x - 1/2*(-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3)
 + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) + f*q/g) + 4*g*x*
log(c) + (((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/
(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*g + 6*f*q + sqrt(3)*g*sqrt(-(((-1
/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)
*(I*sqrt(3) + 1) - 2*f*q/g)^2*g^2 + 4*((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3
) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*f*g*q
 + 4*f^2*q^2)/g^2))*log(2*g*q*x + 1/2*((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3
) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*g + 3
*f*q + 1/2*sqrt(3)*g*sqrt(-(((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d
*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*g^2 + 4*((-1/
2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*
(I*sqrt(3) + 1) - 2*f*q/g)*f*g*q + 4*f^2*q^2)/g^2)) + (((-1/2*f^3*q^3/g^3 +
 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1)
 - 2*f*q/g)*g + 6*f*q - sqrt(3)*g*sqrt(-(((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*
g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*
g^2 + 4*((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d
*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*f*g*q + 4*f^2*q^2)/g^2))*log(2*g*q*
x + 1/2*((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d
*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*g + 3*f*q - 1/2*sqrt(3)*g*sqrt(-(((
-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/
3)*(I*sqrt(3) + 1) - 2*f*q/g)^2*g^2 + 4*((-1/2*f^3*q^3/g^3 + 1/2*e*q^3/(d*g
^3) + 1/2*(d*f^3*q^3 + e*q^3)/(d*g^3))^(1/3)*(I*sqrt(3) + 1) - 2*f*q/g)*f*g
*q + 4*f^2*q^2)/g^2)))/g
```

Sympy [F(-1)]

Timed out.

$$\int \log \left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right) dx = \text{Timed out}$$

```
[In] integrate(ln(c*(d+e/(g*x+f)**3)**q),x)
[Out] Timed out
```

Maxima [F]

$$\int \log \left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right) dx = \int \log \left(c\left(d+\frac{e}{(gx+f)^3}\right)^q\right) dx$$

```
[In] integrate(log(c*(d+e/(g*x+f)^3)^q),x, algorithm="maxima")
[Out] 3*q*integrate((d*f*g^2*x^2 + 2*d*f^2*g*x + d*f^3 + e)/(d*g^3*x^3 + 3*d*f*g^
2*x^2 + 3*d*f^2*g*x + d*f^3 + e), x) - (3*f*q*log(g*x + f) - g*x*log((d*g^3
*x^3 + 3*d*f*g^2*x^2 + 3*d*f^2*g*x + d*f^3 + e)^q) + 3*g*x*log((g*x + f)^q)
 - g*x*log(c))/g
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(132) = 264.

Time = 0.74 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.93

$$\int \log \left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right) dx$$
$$= \frac{1}{2} deg^5 q \left(\frac{2 f \log \left(|dg^3x^3+3 dfg^2x^2+3 df^2gx+df^3+e|\right)}{deg^6} - \frac{6 f \log \left(|gx+f|\right)}{deg^6} + \frac{2 \sqrt{3}(d^5e^4g^{21})^{\frac{1}{3}} \arctan \left(-\right.}{}\right.$$
$$+ qx \log \left(dg^3x^3+3 dfg^2x^2+3 df^2gx+df^3+e\right)$$
$$- qx \log \left(g^3x^3+3 fg^2x^2+3 f^2gx+f^3\right) + x \log (c)$$

```
[In] integrate(log(c*(d+e/(g*x+f)^3)^q),x, algorithm="giac")
[Out] 1/2*d*e*g^5*q*(2*f*log(abs(d*g^3*x^3 + 3*d*f*g^2*x^2 + 3*d*f^2*g*x + d*f^3
+ e))/(d*e*g^6) - 6*f*log(abs(g*x + f))/(d*e*g^6) + (2*sqrt(3)*(d^5*e^4*g^2
1)^(1/3)*arctan(-(d*g*x + d*f + (d^2*e)^(1/3))/(sqrt(3)*d*g*x + sqrt(3)*d*f
 - sqrt(3)*(d^2*e)^(1/3))) - (d^5*e^4*g^21)^(1/3)*log(4*(sqrt(3)*d*g*x + sq
rt(3)*d*f - sqrt(3)*(d^2*e)^(1/3))^2 + 4*(d*g*x + d*f + (d^2*e)^(1/3))^2) +
 2*(d^5*e^4*g^21)^(1/3)*log(abs(d*g*x + d*f + (d^2*e)^(1/3))))/(d^3*e^2*g^1
3)) + q*x*log(d*g^3*x^3 + 3*d*f*g^2*x^2 + 3*d*f^2*g*x + d*f^3 + e) - q*x*lo
g(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3) + x*log(c)
```

Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 499, normalized size of antiderivative = 3.02

$$\int \log \left(c\left(d+\frac{e}{(f+gx)^3}\right)^q\right) dx = x \ln \left(c \left(d+\frac{e}{(f+g\,x)^3}\right)^q\right) - \left(\sum_{k=1}^{3} \ln \left(-d^2\,e^2\,g^{11}\,\left(3\,e\,q^3\,x+\mathrm{root}\left(d\,g^3\,z^3+3\,d\,f\,g^2\,q\,z^2+3\,d\,f^2\,g\,q^2\,z+d\,f^3\,q^3+e\,q^3,z,k\right)\,e\,q^2+\mathrm{root}\left(d\,g^3\,z^3 + 3\,d\,f\,g^2\,q\,z^2 + 3\,d\,f^2\,g\,q^2\,z + d\,f^3\,q^3 + e\,q^3, z, k\right)\right) - \frac{3\,f\,q\,\ln\left(f+g\,x\right)}{g}$$

```
[In] int(log(c*(d + e/(f + g*x)^3)^q),x)
```

```
[Out] x*log(c*(d + e/(f + g*x)^3)^q) - symsum(log(-9*d^2*e^2*g^11*(3*e*q^3*x + ro
ot(d*g^3*z^3 + 3*d*f*g^2*q*z^2 + 3*d*f^2*g*q^2*z + d*f^3*q^3 + e*q^3, z, k)
*e*q^2 + 4*root(d*g^3*z^3 + 3*d*f*g^2*q*z^2 + 3*d*f^2*g*q^2*z + d*f^3*q^3 +
 e*q^3, z, k)^3*d*f*g^2 + 4*root(d*g^3*z^3 + 3*d*f*g^2*q*z^2 + 3*d*f^2*g*q^
2*z + d*f^3*q^3 + e*q^3, z, k)*d*f^3*q^2 + 4*root(d*g^3*z^3 + 3*d*f*g^2*q*z
^2 + 3*d*f^2*g*q^2*z + d*f^3*q^3 + e*q^3, z, k)^3*d*g^3*x + 8*root(d*g^3*z^
3 + 3*d*f*g^2*q*z^2 + 3*d*f^2*g*q^2*z + d*f^3*q^3 + e*q^3, z, k)^2*d*f^2*g*
q + 4*root(d*g^3*z^3 + 3*d*f*g^2*q*z^2 + 3*d*f^2*g*q^2*z + d*f^3*q^3 + e*q^
3, z, k)*d*f^2*g*q^2*x + 8*root(d*g^3*z^3 + 3*d*f*g^2*q*z^2 + 3*d*f^2*g*q^2
*z + d*f^3*q^3 + e*q^3, z, k)^2*d*f*g^2*q*x))*root(d*g^3*z^3 + 3*d*f*g^2*q*
z^2 + 3*d*f^2*g*q^2*z + d*f^3*q^3 + e*q^3, z, k), k, 1, 3) - (3*f*q*log(f +
 g*x))/g
```

3.635 $\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n\,dx=\operatorname{Int}\left(\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n,x\right)$$

```
[Out] Unintegrable((a+b*ln(c*(d+e/(g*x+f))^p))^n,x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n\,dx=\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n\,dx$$

```
[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^n,x]
[Out] Defer[Int][(a + b*Log[c*(d + e/(f + g*x))^p])^n, x]
```

Rubi steps

$$\text{integral}=\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n dx=\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^n,x]
[Out] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^n, x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^p\right)\right)^n dx$$

```
[In] int((a+b*ln(c*(d+e/(g*x+f))^p))^n,x)
[Out] int((a+b*ln(c*(d+e/(g*x+f))^p))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n dx=\int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^n dx$$

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^n,x, algorithm="fricas")
[Out] integral((b*log(c*((d*g*x + d*f + e)/(g*x + f))^p) + a)^n, x)
```

Sympy [N/A]

Not integrable

Time = 18.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n dx=\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n dx$$

```
[In] integrate((a+b*ln(c*(d+e/(g*x+f))**p))**n,x)
[Out] Integral((a + b*log(c*(d + e/(f + g*x))**p))**n, x)
```

Maxima [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n dx = \int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^n dx$$

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^n,x, algorithm="maxima")
[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^n, x)
```

Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n dx = \int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^n dx$$

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^n,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^n, x)
```

Mupad [N/A]

Not integrable

Time = 2.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^n dx = \int \left(a+b\,\ln\left(c\,\left(d+\frac{e}{f+g\,x}\right)^p\right)\right)^n dx$$

```
[In] int((a + b*log(c*(d + e/(f + g*x))^p))^n,x)
[Out] int((a + b*log(c*(d + e/(f + g*x))^p))^n, x)
```

3.636 $\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4 dx$

Optimal result

Integrand size = 22, antiderivative size = 221

$$
\begin{aligned}
\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4 dx
&= -\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} \\
&\quad + \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{dg} \\
&\quad - \frac{12b^2ep^2\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2 \operatorname{PolyLog}\left(2,1+\frac{e}{d(f+gx)}\right)}{dg} \\
&\quad + \frac{24b^3ep^3\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right) \operatorname{PolyLog}\left(3,1+\frac{e}{d(f+gx)}\right)}{dg} \\
&\quad - \frac{24b^4ep^4 \operatorname{PolyLog}\left(4,1+\frac{e}{d(f+gx)}\right)}{dg}
\end{aligned}
$$

```
[Out] -4*b*e*p*ln(-e/d/(g*x+f))*(a+b*ln(c*(d+e/(g*x+f))^p))^3/d/g+(e+d*(g*x+f))*(
a+b*ln(c*(d+e/(g*x+f))^p))^4/d/g-12*b^2*e*p^2*(a+b*ln(c*(d+e/(g*x+f))^p))^2
*polylog(2,1+e/d/(g*x+f))/d/g+24*b^3*e*p^3*(a+b*ln(c*(d+e/(g*x+f))^p))*poly
log(3,1+e/d/(g*x+f))/d/g-24*b^4*e*p^4*polylog(4,1+e/d/(g*x+f))/d/g
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2533, 2499, 2504, 2443, 2481, 2421, 2430, 6724}

$$\begin{aligned}
&\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4 dx \\
&= \frac{24b^3ep^3\operatorname{PolyLog}\left(3,\frac{e}{d(f+gx)}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg} \\
&\quad -\frac{12b^2ep^2\operatorname{PolyLog}\left(2,\frac{e}{d(f+gx)}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} \\
&\quad -\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} \\
&\quad +\frac{(d(f+gx)+e)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{dg}-\frac{24b^4ep^4\operatorname{PolyLog}\left(4,\frac{e}{d(f+gx)}+1\right)}{dg}
\end{aligned}$$

```
[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^4,x]
```

```
[Out] (-4*b*e*p*Log[-(e/(d*(f + g*x)))]*(a + b*Log[c*(d + e/(f + g*x))^p])^3)/(d*
g) + ((e + d*(f + g*x))*(a + b*Log[c*(d + e/(f + g*x))^p])^4)/(d*g) - (12*b
^2*e*p^2*(a + b*Log[c*(d + e/(f + g*x))^p])^2*PolyLog[2, 1 + e/(d*(f + g*x)
)])/(d*g) + (24*b^3*e*p^3*(a + b*Log[c*(d + e/(f + g*x))^p])*PolyLog[3, 1 +
 e/(d*(f + g*x))])/(d*g) - (24*b^4*e*p^4*PolyLog[4, 1 + e/(d*(f + g*x))])/(
d*g)
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2430

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Dist[b*n*(p/q), Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p - 1
)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2499

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[(e + d*x)*((a + b*Log[c*(d + e/x)^p])^q/d), x] + Dist[b*e*p*(q/d), In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2533

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(a + b\log\left(c\left(d + \frac{e}{x}\right)^p\right)\right)^4 \, dx, x, f + gx\right)}{g}$$

$$= \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{dg} + \frac{(4bep)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^p\right)\right)^3}{x}\,dx, x, f+gx\right)}{dg}$$

$$= \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{dg} - \frac{(4bep)\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^p))^3}{x}\,dx, x, \frac{1}{f+gx}\right)}{dg}$$

$$= -\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{dg} + \frac{(12b^2e^2p^2)\,\text{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^p))^2}{d+ex}\,dx, x, \frac{1}{f+gx}\right)}{dg}$$

$$= -\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{dg} + \frac{(12b^2ep^2)\,\text{Subst}\left(\int \frac{(a+b\log(cx^p))^2\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx, x, d+\frac{e}{f+gx}\right)}{dg}$$

$$= -\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{dg} - \frac{12b^2ep^2\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2\text{Li}_2\left(\frac{d+\frac{e}{f+gx}}{d}\right)}{dg} + \frac{(24b^3ep^3)\,\text{Subst}\left(\int \frac{(a+b\log(cx^p))\text{Li}_2\left(\frac{x}{d}\right)}{x}\,dx, x, d+\frac{e}{f+gx}\right)}{dg}$$

$$= -\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{dg} - \frac{12b^2ep^2\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2\operatorname{Li}_2\left(\frac{d+\frac{e}{f+gx}}{d}\right)}{dg} + \frac{24b^3ep^3\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)\operatorname{Li}_3\left(\frac{d+\frac{e}{f+gx}}{d}\right)}{dg} - \frac{(24b^4ep^4)\operatorname{Subst}\left(\int\frac{\operatorname{Li}_3\left(\frac{x}{d}\right)}{x}\,dx, x, d+\frac{e}{f+gx}\right)}{dg}$$

$$= -\frac{4bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4}{dg} - \frac{12b^2ep^2\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2\operatorname{Li}_2\left(\frac{d+\frac{e}{f+gx}}{d}\right)}{dg} + \frac{24b^3ep^3\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)\operatorname{Li}_3\left(\frac{d+\frac{e}{f+gx}}{d}\right)}{dg} - \frac{24b^4ep^4\operatorname{Li}_4\left(\frac{d+\frac{e}{f+gx}}{d}\right)}{dg}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 743 vs. $2(221) = 442$.

Time = 0.92 (sec) , antiderivative size = 743, normalized size of antiderivative = 3.36

$$\int\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4 dx$$

$$= \frac{-4bp\left(df\log(f+gx)-(e+df)\log(e+df+dgx)-dgx\log\left(\frac{e+df+dgx}{f+gx}\right)\right)\left(a-bp\log\left(d+\frac{e}{f+gx}\right)+b\log\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^4,x]
```

```
[Out] (-4*b*p*(d*f*Log[f + g*x] - (e + d*f)*Log[e + d*f + d*g*x] - d*g*x*Log[(e +
 d*f + d*g*x)/(f + g*x)])*(a - b*p*Log[d + e/(f + g*x)] + b*Log[c*(d + e/(f
 + g*x))^p])^3 + d*g*x*(a - b*p*Log[d + e/(f + g*x)] + b*Log[c*(d + e/(f +
g*x))^p])^4 - 6*b^2*p^2*(a - b*p*Log[d + e/(f + g*x)] + b*Log[c*(d + e/(f +
```

```
 g*x))^p])^2*(d*f*Log[-(e/(d*f + d*g*x))]^2 + 2*d*f*Log[-(e/(d*f + d*g*x))]
*Log[(e + d*f + d*g*x)/e] - 2*d*f*Log[-(e/(d*f + d*g*x))]*Log[(e + d*f + d*
g*x)/(f + g*x)] - 2*(e + d*f)*Log[e + d*f + d*g*x]*Log[(e + d*f + d*g*x)/(f
 + g*x)] - d*g*x*Log[(e + d*f + d*g*x)/(f + g*x)]^2 - 2*d*f*PolyLog[2, -((d
*(f + g*x))/e)] - (e + d*f)*((2*Log[-((d*(f + g*x))/e)] - Log[e + d*f + d*g
*x])*Log[e + d*f + d*g*x] + 2*PolyLog[2, (e + d*f + d*g*x)/e])) + 4*b^3*p^3
*(a - b*p*Log[d + e/(f + g*x)] + b*Log[c*(d + e/(f + g*x))^p])*(Log[d + e/(
f + g*x)]^2*(-3*e*Log[-(e/(d*f + d*g*x))] + (e + d*f + d*g*x)*Log[d + e/(f
+ g*x)]) - 6*e*Log[d + e/(f + g*x)]*PolyLog[2, 1 + e/(d*f + d*g*x)] + 6*e*P
olyLog[3, 1 + e/(d*f + d*g*x)]) - b^4*p^4*(4*e*Log[-(e/(d*f + d*g*x))]*Log[
d + e/(f + g*x)]^3 - e*Log[d + e/(f + g*x)]^4 - d*f*Log[d + e/(f + g*x)]^4
- d*g*x*Log[d + e/(f + g*x)]^4 + 12*e*Log[d + e/(f + g*x)]^2*PolyLog[2, 1 +
 e/(d*f + d*g*x)] - 24*e*Log[d + e/(f + g*x)]*PolyLog[3, 1 + e/(d*f + d*g*x
)] + 24*e*PolyLog[4, 1 + e/(d*f + d*g*x)]))/(d*g)
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)\right)^{4}dx$$

[In] `int((a+b*ln(c*(d+e/(g*x+f))^p))^4,x)`

[Out] `int((a+b*ln(c*(d+e/(g*x+f))^p))^4,x)`

Fricas [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)^{4}\,dx=\int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a\right)^{4}\,dx$$

[In] `integrate((a+b*log(c*(d+e/(g*x+f))^p))^4,x, algorithm="fricas")`

[Out]
```
integral(b^4*log(c*((d*g*x + d*f + e)/(g*x + f))^p)^4 + 4*a*b^3*log(c*((d*g
*x + d*f + e)/(g*x + f))^p)^3 + 6*a^2*b^2*log(c*((d*g*x + d*f + e)/(g*x + f
))^p)^2 + 4*a^3*b*log(c*((d*g*x + d*f + e)/(g*x + f))^p) + a^4, x)
```

Sympy [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4 dx = \int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4 dx$$

```
[In] integrate((a+b*ln(c*(d+e/(g*x+f))**p))**4,x)
```

```
[Out] Integral((a + b*log(c*(d + e/(f + g*x))**p))**4, x)
```

Maxima [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4 dx = \int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^4 dx$$

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^4,x, algorithm="maxima")
```

```
[Out] -4*a^3*b*e*g*p*(f*log(g*x + f)/(e*g^2) - (d*f + e)*log(d*g*x + d*f + e)/(d*
e*g^2)) + 4*a^3*b*x*log(c*(d + e/(g*x + f))^p) + a^4*x + (b^4*d*g*x*log((d*
g*x + d*f + e)^p)^4 - 4*(b^4*d*f*p*log(g*x + f) + b^4*d*g*x*log((g*x + f)^p
) - (d*f*p + e*p)*b^4*log(d*g*x + d*f + e) - (b^4*d*g*log(c) + a*b^3*d*g)*x
)*log((d*g*x + d*f + e)^p)^3)/(d*g) + integrate(((d*f + e)*b^4*log(c)^4 + 4
*(d*f + e)*a*b^3*log(c)^3 + 6*(d*f + e)*a^2*b^2*log(c)^2 + (b^4*d*g*x + (d*
f + e)*b^4)*log((g*x + f)^p)^4 - 4*((d*f + e)*b^4*log(c) + (d*f + e)*a*b^3
+ (b^4*d*g*log(c) + a*b^3*d*g)*x)*log((g*x + f)^p)^3 + 6*(2*b^4*d*f*p^2*log
(g*x + f) + (d*f + e)*b^4*log(c)^2 - 2*(d*f*p^2 + e*p^2)*b^4*log(d*g*x + d*
f + e) + 2*(d*f + e)*a*b^3*log(c) + (d*f + e)*a^2*b^2 + (b^4*d*g*x + (d*f +
 e)*b^4)*log((g*x + f)^p)^2 + (a^2*b^2*d*g - 2*(d*g*p - d*g*log(c))*a*b^3 -
 (2*d*g*p*log(c) - d*g*log(c)^2)*b^4)*x - 2*((d*f + e)*b^4*log(c) + (d*f +
e)*a*b^3 + (a*b^3*d*g - (d*g*p - d*g*log(c))*b^4)*x)*log((g*x + f)^p))*log(
(d*g*x + d*f + e)^p)^2 + 6*((d*f + e)*b^4*log(c)^2 + 2*(d*f + e)*a*b^3*log(
c) + (d*f + e)*a^2*b^2 + (b^4*d*g*log(c)^2 + 2*a*b^3*d*g*log(c) + a^2*b^2*d
*g)*x)*log((g*x + f)^p)^2 + (b^4*d*g*log(c)^4 + 4*a*b^3*d*g*log(c)^3 + 6*a^
2*b^2*d*g*log(c)^2)*x + 4*((d*f + e)*b^4*log(c)^3 + 3*(d*f + e)*a*b^3*log(c
)^2 + 3*(d*f + e)*a^2*b^2*log(c) - (b^4*d*g*x + (d*f + e)*b^4)*log((g*x + f
)^p)^3 + 3*((d*f + e)*b^4*log(c) + (d*f + e)*a*b^3 + (b^4*d*g*log(c) + a*b^
3*d*g)*x)*log((g*x + f)^p)^2 + (b^4*d*g*log(c)^3 + 3*a*b^3*d*g*log(c)^2 + 3
*a^2*b^2*d*g*log(c))*x - 3*((d*f + e)*b^4*log(c)^2 + 2*(d*f + e)*a*b^3*log(
c) + (d*f + e)*a^2*b^2 + (b^4*d*g*log(c)^2 + 2*a*b^3*d*g*log(c) + a^2*b^2*d
*g)*x)*log((g*x + f)^p))*log((d*g*x + d*f + e)^p) - 4*((d*f + e)*b^4*log(c)
^3 + 3*(d*f + e)*a*b^3*log(c)^2 + 3*(d*f + e)*a^2*b^2*log(c) + (b^4*d*g*log
(c)^3 + 3*a*b^3*d*g*log(c)^2 + 3*a^2*b^2*d*g*log(c))*x)*log((g*x + f)^p))/(
d*g*x + d*f + e), x)
```

Giac [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4 dx = \int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^4 dx$$

[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^4,x, algorithm="giac")

[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^4, x)

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^4 dx = \int \left(a+b\,\ln\left(c\left(d+\frac{e}{f+g\,x}\right)^p\right)\right)^4 dx$$

[In] int((a + b*log(c*(d + e/(f + g*x))^p))^4,x)

[Out] int((a + b*log(c*(d + e/(f + g*x))^p))^4, x)

3.637 $\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3 dx$

Optimal result

Integrand size = 22, antiderivative size = 168

$$\begin{aligned}
&\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3 dx \\
&= -\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} \\
&\quad+\frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} \\
&\quad-\frac{6b^2ep^2\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)\operatorname{PolyLog}\left(2,1+\frac{e}{d(f+gx)}\right)}{dg} \\
&\quad+\frac{6b^3ep^3\operatorname{PolyLog}\left(3,1+\frac{e}{d(f+gx)}\right)}{dg}
\end{aligned}$$

```
[Out] -3*b*e*p*ln(-e/d/(g*x+f))*(a+b*ln(c*(d+e/(g*x+f))^p))^2/d/g+(e+d*(g*x+f))*(
a+b*ln(c*(d+e/(g*x+f))^p))^3/d/g-6*b^2*e*p^2*(a+b*ln(c*(d+e/(g*x+f))^p))*po
lylog(2,1+e/d/(g*x+f))/d/g+6*b^3*e*p^3*polylog(3,1+e/d/(g*x+f))/d/g
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2533, 2499, 2504, 2443, 2481, 2421, 6724}

$$\begin{aligned}
&\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3dx\\
&=-\frac{6b^2ep^2\operatorname{PolyLog}\left(2,\frac{e}{d(f+gx)}+1\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg}\\
&\quad-\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg}\\
&\quad+\frac{(d(f+gx)+e)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg}+\frac{6b^3ep^3\operatorname{PolyLog}\left(3,\frac{e}{d(f+gx)}+1\right)}{dg}
\end{aligned}$$

```
[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^3,x]
```

```
[Out] (-3*b*e*p*Log[-(e/(d*(f + g*x)))]*(a + b*Log[c*(d + e/(f + g*x))^p])^2)/(d*
g) + ((e + d*(f + g*x))*(a + b*Log[c*(d + e/(f + g*x))^p])^3)/(d*g) - (6*b^
2*e*p^2*(a + b*Log[c*(d + e/(f + g*x))^p])*PolyLog[2, 1 + e/(d*(f + g*x))])
/(d*g) + (6*b^3*e*p^3*PolyLog[3, 1 + e/(d*(f + g*x))])/(d*g)
```

Rule 2421

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Dist[b*n*(p/m), Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c*
x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0
] && EqQ[d*e, 1]
```

Rule 2443

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Dist[b*e*n*(p/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]*
((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2481

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2499

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[(e + d*x)*((a + b*Log[c*(d + e/x)^p])^q/d), x] + Dist[b*e*p*(q/d), In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2533

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{e}{x}\right)^p\right)\right)^3\,dx,x,f+gx\right)}{g}\\
&= \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg}\\
&\quad+\frac{(3bep)\text{Subst}\left(\int \frac{\left(a+b\log\left(c\left(d+\frac{e}{x}\right)^p\right)\right)^2}{x}\,dx,x,f+gx\right)}{dg}\\
&= \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg}-\frac{(3bep)\text{Subst}\left(\int \frac{(a+b\log(c(d+ex)^p))^2}{x}\,dx,x,\frac{1}{f+gx}\right)}{dg}
\end{aligned}
$$

$$= -\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} + \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(6b^2e^2p^2)\operatorname{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)(a+b\log(c(d+ex)^p))}{d+ex}\,dx, x, \frac{1}{f+gx}\right)}{dg}$$

$$= -\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} + \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} + \frac{(6b^2ep^2)\operatorname{Subst}\left(\int \frac{(a+b\log(cx^p))\log\left(-\frac{e\left(-\frac{d}{e}+\frac{x}{e}\right)}{d}\right)}{x}\,dx, x, d+\frac{e}{f+gx}\right)}{dg}$$

$$= -\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} + \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} - \frac{6b^2ep^2\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)\operatorname{Li}_2\left(\frac{d+\frac{e}{f+gx}}{d}\right)}{dg} + \frac{(6b^3ep^3)\operatorname{Subst}\left(\int \frac{\operatorname{Li}_2\left(\frac{x}{d}\right)}{x}\,dx, x, d+\frac{e}{f+gx}\right)}{dg}$$

$$= -\frac{3bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} + \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3}{dg} - \frac{6b^2ep^2\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)\operatorname{Li}_2\left(\frac{d+\frac{e}{f+gx}}{d}\right)}{dg} + \frac{6b^3ep^3\operatorname{Li}_3\left(\frac{d+\frac{e}{f+gx}}{d}\right)}{dg}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 441 vs. $2(168) = 336$.

Time = 0.43 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.62

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3 dx$$
$$= \frac{3bdp(f+gx)\log\left(d+\frac{e}{f+gx}\right)\left(a-bp\log\left(d+\frac{e}{f+gx}\right)+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2+d(f+gx)\left(a-bp\log\left(\right.\right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^3,x]
```

```
[Out] (3*b*d*p*(f + g*x)*Log[d + e/(f + g*x)]*(a - b*p*Log[d + e/(f + g*x)] + b*L
og[c*(d + e/(f + g*x))^p])^2 + d*(f + g*x)*(a - b*p*Log[d + e/(f + g*x)] +
b*Log[c*(d + e/(f + g*x))^p])^3 + 3*b*e*p*(a - b*p*Log[d + e/(f + g*x)] + b
*Log[c*(d + e/(f + g*x))^p])^2*Log[e + d*(f + g*x)] + 3*b^2*p^2*(a - b*p*Lo
g[d + e/(f + g*x)] + b*Log[c*(d + e/(f + g*x))^p])*(d*(f + g*x)*Log[d + e/(
f + g*x)]^2 + e*(Log[e/d + f + g*x]^2 + 2*(Log[f + g*x] - Log[e/d + f + g*x
] + Log[d + e/(f + g*x)])*Log[e + d*(f + g*x)] - 2*(Log[f + g*x]*Log[1 + (d
*(f + g*x))/e] + PolyLog[2, -((d*(f + g*x))/e)]))) + b^3*p^3*(Log[d + e/(f
+ g*x)]^2*(-3*e*Log[-(e/(d*f + d*g*x))] + (e + d*f + d*g*x)*Log[d + e/(f +
g*x)]) - 6*e*Log[d + e/(f + g*x)]*PolyLog[2, 1 + e/(d*f + d*g*x)] + 6*e*Pol
yLog[3, 1 + e/(d*f + d*g*x)]))/(d*g)
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^p\right)\right)^3 dx$$

```
[In] int((a+b*ln(c*(d+e/(g*x+f))^p))^3,x)
```

```
[Out] int((a+b*ln(c*(d+e/(g*x+f))^p))^3,x)
```

Fricas [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3 dx = \int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^3 dx$$

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*log(c*((d*g*x + d*f + e)/(g*x + f))^p)^3 + 3*a*b^2*log(c*((d*g
*x + d*f + e)/(g*x + f))^p)^2 + 3*a^2*b*log(c*((d*g*x + d*f + e)/(g*x + f))
^p) + a^3, x)
```

Sympy [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3 dx = \int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3 dx$$

```
[In] integrate((a+b*ln(c*(d+e/(g*x+f))**p))**3,x)
```

```
[Out] Integral((a + b*log(c*(d + e/(f + g*x))**p))**3, x)
```

Maxima [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3 dx = \int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^3 dx$$

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^3,x, algorithm="maxima")
```

```
[Out] -3*a^2*b*e*g*p*(f*log(g*x + f)/(e*g^2) - (d*f + e)*log(d*g*x + d*f + e)/(d*
e*g^2)) + 3*a^2*b*x*log(c*(d + e/(g*x + f))^p) + a^3*x + (b^3*d*g*x*log((d*
g*x + d*f + e)^p)^3 - 3*(b^3*d*f*p*log(g*x + f) + b^3*d*g*x*log((g*x + f)^p
) - (d*f*p + e*p)*b^3*log(d*g*x + d*f + e) - (b^3*d*g*log(c) + a*b^2*d*g)*x
)*log((d*g*x + d*f + e)^p)^2)/(d*g) + integrate(((d*f + e)*b^3*log(c)^3 + 3
*(d*f + e)*a*b^2*log(c)^2 - (b^3*d*g*x + (d*f + e)*b^3)*log((g*x + f)^p)^3
+ 3*((d*f + e)*b^3*log(c) + (d*f + e)*a*b^2 + (b^3*d*g*log(c) + a*b^2*d*g)*
x)*log((g*x + f)^p)^2 + (b^3*d*g*log(c)^3 + 3*a*b^2*d*g*log(c)^2)*x + 3*(2*
b^3*d*f*p^2*log(g*x + f) + (d*f + e)*b^3*log(c)^2 - 2*(d*f*p^2 + e*p^2)*b^3
*log(d*g*x + d*f + e) + 2*(d*f + e)*a*b^2*log(c) + (b^3*d*g*x + (d*f + e)*b
^3)*log((g*x + f)^p)^2 - (2*(d*g*p - d*g*log(c))*a*b^2 + (2*d*g*p*log(c) -
d*g*log(c)^2)*b^3)*x - 2*((d*f + e)*b^3*log(c) + (d*f + e)*a*b^2 + (a*b^2*d
*g - (d*g*p - d*g*log(c))*b^3)*x)*log((g*x + f)^p))*log((d*g*x + d*f + e)^p
) - 3*((d*f + e)*b^3*log(c)^2 + 2*(d*f + e)*a*b^2*log(c) + (b^3*d*g*log(c)^
2 + 2*a*b^2*d*g*log(c))*x)*log((g*x + f)^p))/(d*g*x + d*f + e), x)
```

Giac [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3 dx = \int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^3 dx$$

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^3\,dx=\int \left(a+b\,\ln\left(c\,\left(d+\frac{e}{f+g\,x}\right)^p\right)\right)^3\,dx$$

```
[In] int((a + b*log(c*(d + e/(f + g*x))^p))^3,x)
[Out] int((a + b*log(c*(d + e/(f + g*x))^p))^3, x)
```

3.638 $\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2 dx$

Optimal result

Integrand size = 22, antiderivative size = 115

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2 dx = -\frac{2bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg} + \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} - \frac{2b^2ep^2\operatorname{PolyLog}\left(2,1+\frac{e}{d(f+gx)}\right)}{dg}$$

```
[Out] -2*b*e*p*ln(-e/d/(g*x+f))*(a+b*ln(c*(d+e/(g*x+f))^p))/d/g+(e+d*(g*x+f))*(a+
b*ln(c*(d+e/(g*x+f))^p))^2/d/g-2*b^2*e*p^2*polylog(2,1+e/d/(g*x+f))/d/g
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2533, 2499, 2504, 2441, 2352}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2 dx = -\frac{2bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg} + \frac{(d(f+gx)+e)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} - \frac{2b^2ep^2\operatorname{PolyLog}\left(2,\frac{e}{d(f+gx)}+1\right)}{dg}$$

```
[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^2,x]
```

```
[Out] (-2*b*e*p*Log[-(e/(d*(f + g*x)))]*(a + b*Log[c*(d + e/(f + g*x))^p]))/(d*g)
 + ((e + d*(f + g*x))*(a + b*Log[c*(d + e/(f + g*x))^p])^2)/(d*g) - (2*b^2*
e*p^2*PolyLog[2, 1 + e/(d*(f + g*x))])/(d*g)
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2441

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Dist[b*e*(n/g), Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2499

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)/(x_))^(p_.)]*(b_.))^(q_), x_Symbol] :>
 Simp[(e + d*x)*((a + b*Log[c*(d + e/x)^p])^q/d), x] + Dist[b*e*p*(q/d), In
t[(a + b*Log[c*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && IGtQ[q, 0]
```

Rule 2504

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2533

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \left(a+b\log\left(c\left(d+\frac{e}{x}\right)^p\right)\right)^2\,dx, x, f+gx\right)}{g}$$

$$= \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} + \frac{(2bep)\text{Subst}\left(\int \frac{a+b\log\left(c\left(d+\frac{e}{x}\right)^p\right)}{x}\,dx, x, f+gx\right)}{dg}$$

$$= \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} - \frac{(2bep)\mathrm{Subst}\left(\int \frac{a+b\log(c(d+ex)^p)}{x}\,dx, x, \frac{1}{f+gx}\right)}{dg}$$

$$= -\frac{2bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg} + \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} + \frac{(2b^2e^2p^2)\,\mathrm{Subst}\left(\int \frac{\log\left(-\frac{ex}{d}\right)}{d+ex}\,dx, x, \frac{1}{f+gx}\right)}{dg}$$

$$= -\frac{2bep\log\left(-\frac{e}{d(f+gx)}\right)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)}{dg} + \frac{(e+d(f+gx))\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}{dg} - \frac{2b^2ep^2\mathrm{Li}_2\left(1+\frac{e}{d(f+gx)}\right)}{dg}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.90

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2 dx = x\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2 - \frac{bp\left(2df\log(f+gx)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right) - 2(e+df)\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)\log(e+d(f+gx)) \right.}{}$$

```
[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^2,x]
```

```
[Out] x*(a + b*Log[c*(d + e/(f + g*x))^p])^2 - (b*p*(2*d*f*Log[f + g*x]*(a + b*Lo
g[c*(d + e/(f + g*x))^p]) - 2*(e + d*f)*(a + b*Log[c*(d + e/(f + g*x))^p])*
Log[e + d*(f + g*x)] + b*d*f*p*(Log[f + g*x]*(Log[f + g*x] - 2*Log[(e + d*f
 + d*g*x)/e]) - 2*PolyLog[2, -((d*(f + g*x))/e)]) - b*(e + d*f)*p*((2*Log[-
((d*(f + g*x))/e)] - Log[e + d*f + d*g*x])*Log[e + d*(f + g*x)] + 2*PolyLog
[2, (e + d*f + d*g*x)/e])))/(d*g)
```

Maple [F]

$$\int \left(a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)\right)^{2}dx$$

```
[In] int((a+b*ln(c*(d+e/(g*x+f))^p))^2,x)
[Out] int((a+b*ln(c*(d+e/(g*x+f))^p))^2,x)
```

Fricas [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)^{2}dx=\int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a\right)^{2}dx$$

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="fricas")
[Out] integral(b^2*log(c*((d*g*x + d*f + e)/(g*x + f))^p)^2 + 2*a*b*log(c*((d*g*x
 + d*f + e)/(g*x + f))^p) + a^2, x)
```

Sympy [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)^{2}dx=\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)^{2}dx$$

```
[In] integrate((a+b*ln(c*(d+e/(g*x+f))**p))**2,x)
[Out] Integral((a + b*log(c*(d + e/(f + g*x))**p))**2, x)
```

Maxima [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^{p}\right)\right)^{2}dx=\int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^{p}\right)+a\right)^{2}dx$$

```
[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="maxima")
[Out] -2*a*b*e*g*p*(f*log(g*x + f)/(e*g^2) - (d*f + e)*log(d*g*x + d*f + e)/(d*e*
g^2)) + 2*a*b*x*log(c*(d + e/(g*x + f))^p) + a^2*x + b^2*((d*g*x*log((d*g*x
 + d*f + e)^p)^2 + d*g*x*log((g*x + f)^p)^2 - (d*f*p^2 + e*p^2)*log(d*g*x +
 d*f + e)^2 + 2*(d*f*p^2 + e*p^2)*log(d*g*x + d*f + e)*log(g*x + f) - 2*(d*
f*p*log(g*x + f) + d*g*x*log((g*x + f)^p) - d*g*x*log(c) - (d*f*p + e*p)*lo
g(d*g*x + d*f + e))*log((d*g*x + d*f + e)^p) + 2*(d*f*p*log(g*x + f) - d*g*
x*log(c) - (d*f*p + e*p)*log(d*g*x + d*f + e))*log((g*x + f)^p))/(d*g) - in
tegrate(-(d*g^2*x^2*log(c)^2 + (d*f^2 + e*f)*log(c)^2 + (2*e*g*p*log(c) + (
2*d*f*g + e*g)*log(c)^2)*x - 2*(d*f^2*p^2 + 2*e*f*p^2 + (d*f*g*p^2 + e*g*p^
2)*x)*log(g*x + f))/(d*g^2*x^2 + d*f^2 + e*f + (2*d*f*g + e*g)*x), x))
```

Giac [F]

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2\,dx=\int \left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^2\,dx$$

[In] integrate((a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="giac")

[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2\,dx=\int \left(a+b\,\ln\left(c\,\left(d+\frac{e}{f+g\,x}\right)^p\right)\right)^2\,dx$$

[In] int((a + b*log(c*(d + e/(f + g*x))^p))^2,x)

[Out] int((a + b*log(c*(d + e/(f + g*x))^p))^2, x)

3.639 $\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)\,dx=ax+\frac{b(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g}+\frac{bep\log(e+d(f+gx))}{dg}$$

```
[Out] a*x+b*(g*x+f)*ln(c*(d+e/(g*x+f))^p)/g+b*e*p*ln(e+d*(g*x+f))/d/g
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2533, 2498, 269, 31}

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)\,dx=ax+\frac{b(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g}+\frac{bep\log(d(f+gx)+e)}{dg}$$

```
[In] Int[a + b*Log[c*(d + e/(f + g*x))^p],x]
```

```
[Out] a*x + (b*(f + g*x)*Log[c*(d + e/(f + g*x))^p])/g + (b*e*p*Log[e + d*(f + g*
x)])/(d*g)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2533

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*((f_.) + (g_.)*(x_))^(n_))^(p_.)]*(b_.
))^(q_.), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[c*(d + e*x^n)^p])^q,
x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IGtQ[q, 0]
&& (EqQ[q, 1] || IntegerQ[n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= ax + b\int \log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\,dx\\
&= ax + \frac{b\text{Subst}\left(\int \log\left(c\left(d+\frac{e}{x}\right)^p\right)\,dx, x, f+gx\right)}{g}\\
&= ax + \frac{b(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g} + \frac{(bep)\text{Subst}\left(\int \frac{1}{\left(d+\frac{e}{x}\right)x}\,dx, x, f+gx\right)}{g}\\
&= ax + \frac{b(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g} + \frac{(bep)\text{Subst}\left(\int \frac{1}{e+dx}\,dx, x, f+gx\right)}{g}\\
&= ax + \frac{b(f+gx)\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g} + \frac{bep\log(e+d(f+gx))}{dg}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)dx = ax-begp\left(\frac{f\log(f+gx)}{eg^2}-\frac{(e+df)\log(e+df+dgx)}{deg^2}\right)+bx\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)$$

```
[In] Integrate[a + b*Log[c*(d + e/(f + g*x))^p],x]
[Out] a*x - b*e*g*p*((f*Log[f + g*x])/(e*g^2) - ((e + d*f)*Log[e + d*f + d*g*x])/
(d*e*g^2)) + b*x*Log[c*(d + e/(f + g*x))^p]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.54

method	result	size
default	$ax+b\left(\ln\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)x+egp\left(-\frac{f\ln(gx+f)}{g^2e}+\frac{(df+e)\ln(dgx+df+e)}{e\,g^2d}\right)\right)$	77
parts	$ax+b\left(\ln\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)x+egp\left(-\frac{f\ln(gx+f)}{g^2e}+\frac{(df+e)\ln(dgx+df+e)}{e\,g^2d}\right)\right)$	77
parallelrisch	$-\frac{b\left(-x\ln\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)d\,g^2p-\ln(gx+f)eg\,p^2-\ln\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)dfgp-\ln\left(c\left(\frac{dgx+df+e}{gx+f}\right)^p\right)egp\right)}{d\,g^2p}+ax$	116

```
[In] int(a+b*ln(c*(d+e/(g*x+f))^p),x,method=_RETURNVERBOSE)
[Out] a*x+b*(ln(c*((d*g*x+d*f+e)/(g*x+f))^p)*x+e*g*p*(-f/g^2/e*ln(g*x+f)+(d*f+e)/
e/g^2/d*ln(d*g*x+d*f+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.52

$$\int \left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)dx = \frac{bdgpx\log\left(\frac{dgx+df+e}{gx+f}\right)-bdfp\log(gx+f)+bdgx\log(c)+adgx+(bdf+be)p\log(dgx+df+e)}{dg}$$

```
[In] integrate(a+b*log(c*(d+e/(g*x+f))^p),x, algorithm="fricas")
```

[Out] (b*d*g*p*x*log((d*g*x + d*f + e)/(g*x + f)) - b*d*f*p*log(g*x + f) + b*d*g*x*log(c) + a*d*g*x + (b*d*f + b*e)*p*log(d*g*x + d*f + e))/(d*g)

Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.14

$$\int\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)dx = ax + b\left(\begin{cases} x\log\left(c\left(\frac{e}{f}\right)^p\right) & \text{for } d=0\wedge g=0 \\ \frac{f\log\left(c\left(\frac{e}{f+gx}\right)^p\right)}{g}+px+x\log\left(c\left(\frac{e}{f+gx}\right)^p\right) & \text{for } d=0 \\ x\log\left(c\left(d+\frac{e}{f}\right)^p\right) & \text{for } g=0 \\ \frac{f\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}{g}+x\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)+\frac{ep\log(df+dgx+e)}{dg} & \text{otherwise} \end{cases}\right)$$

[In] integrate(a+b*ln(c*(d+e/(g*x+f))**p),x)

[Out] a*x + b*Piecewise((x*log(c*(e/f)**p), Eq(d, 0) & Eq(g, 0)), (f*log(c*(e/(f + g*x))**p)/g + p*x + x*log(c*(e/(f + g*x))**p), Eq(d, 0)), (x*log(c*(d + e/f)**p), Eq(g, 0)), (f*log(c*(d + e/(f + g*x))**p)/g + x*log(c*(d + e/(f + g*x))**p) + e*p*log(d*f + d*g*x + e)/(d*g), True))

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

$$\int\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)dx = -begp\left(\frac{f\log(gx+f)}{eg^2}-\frac{(df+e)\log(dgx+df+e)}{deg^2}\right)+bx\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+ax$$

[In] integrate(a+b*log(c*(d+e/(g*x+f))^p),x, algorithm="maxima")

[Out] -b*e*g*p*(f*log(g*x + f)/(e*g^2) - (d*f + e)*log(d*g*x + d*f + e)/(d*e*g^2)) + b*x*log(c*(d + e/(g*x + f))^p) + a*x

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(50) = 100$.

Time = 0.32 (sec) , antiderivative size = 176, normalized size of antiderivative = 3.52

$$\int \left(a + b \log \left(c \left(d + \frac{e}{f + gx}\right)^p\right)\right) dx$$
$$= \left(\frac{e^2 p \log\left(\frac{dgx+df+e}{gx+f}\right)}{dg^2 - \frac{(dgx+df+e)g^2}{gx+f}} + \frac{e^2 \log(c)}{dg^2 - \frac{(dgx+df+e)g^2}{gx+f}} + \frac{e^2 p \log\left(-d + \frac{dgx+df+e}{gx+f}\right)}{dg^2} - \frac{e^2 p \log\left(\frac{dgx+df+e}{gx+f}\right)}{dg^2}\right) b \left(\frac{dfg}{e^2} - \frac{(d\right.$$
$$+ ax$$

```
[In] integrate(a+b*log(c*(d+e/(g*x+f))^p),x, algorithm="giac")
```

```
[Out] (e^2*p*log((d*g*x + d*f + e)/(g*x + f))/(d*g^2 - (d*g*x + d*f + e)*g^2/(g*x
 + f)) + e^2*log(c)/(d*g^2 - (d*g*x + d*f + e)*g^2/(g*x + f)) + e^2*p*log(-
d + (d*g*x + d*f + e)/(g*x + f))/(d*g^2) - e^2*p*log((d*g*x + d*f + e)/(g*x
 + f))/(d*g^2))*b*(d*f*g/e^2 - (d*f + e)*g/e^2) + a*x
```

Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

$$\int \left(a+b \log \left(c \left(d+\frac{e}{f+gx}\right)^p\right)\right) dx = a\,x+b\,x\,\ln\left(c\left(d+\frac{e}{f+g\,x}\right)^p\right)-\frac{b\,f\,p\,\ln(f+g\,x)}{g} + \frac{b\,p\,\ln(e+d\,f+d\,g\,x)\,(e+d\,f)}{d\,g}$$

```
[In] int(a + b*log(c*(d + e/(f + g*x))^p),x)
```

```
[Out] a*x + b*x*log(c*(d + e/(f + g*x))^p) - (b*f*p*log(f + g*x))/g + (b*p*log(e
+ d*f + d*g*x)*(e + d*f))/(d*g)
```

3.640 $\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx=\text{Int}\left(\frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)},x\right)$$

```
[Out] Unintegrable(1/(a+b*ln(c*(d+e/(g*x+f))^p)),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx=\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^(-1),x]
```

```
[Out] Defer[Int][(a + b*Log[c*(d + e/(f + g*x))^p])^(-1), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx=\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^(-1),x]
[Out] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^(-1), x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^p\right)}dx$$

```
[In] int(1/(a+b*ln(c*(d+e/(g*x+f))^p)),x)
[Out] int(1/(a+b*ln(c*(d+e/(g*x+f))^p)),x)
```

Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx=\int \frac{1}{b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p)),x, algorithm="fricas")
[Out] integral(1/(b*log(c*((d*g*x + d*f + e)/(g*x + f))^p) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx=\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(d+e/(g*x+f))**p)),x)
[Out] Integral(1/(a + b*log(c*(d + e/(f + g*x))**p)), x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx=\int \frac{1}{b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p)),x, algorithm="maxima")
[Out] integrate(1/(b*log(c*(d + e/(g*x + f))^p) + a), x)
```

Giac [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx=\int \frac{1}{b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p)),x, algorithm="giac")
[Out] integrate(1/(b*log(c*(d + e/(g*x + f))^p) + a), x)
```

Mupad [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)}\,dx=\int \frac{1}{a+b\,\ln\left(c\left(d+\frac{e}{f+g\,x}\right)^p\right)}\,dx$$

```
[In] int(1/(a + b*log(c*(d + e/(f + g*x))^p)),x)
[Out] int(1/(a + b*log(c*(d + e/(f + g*x))^p)), x)
```

3.641 $\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(a+b*ln(c*(d+e/(g*x+f))^p))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx = \int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx$$

```
[In] Int[(a + b*Log[c*(d + e/(f + g*x))^p])^(-2),x]
```

```
[Out] Defer[Int][(a + b*Log[c*(d + e/(f + g*x))^p])^(-2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx$$

```
[In] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^(-2),x]
[Out] Integrate[(a + b*Log[c*(d + e/(f + g*x))^p])^(-2), x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\ln\left(c\left(d+\frac{e}{gx+f}\right)^p\right)\right)^2}dx$$

```
[In] int(1/(a+b*ln(c*(d+e/(g*x+f))^p))^2,x)
[Out] int(1/(a+b*ln(c*(d+e/(g*x+f))^p))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.82

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx=\int \frac{1}{\left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*log(c*((d*g*x + d*f + e)/(g*x + f))^p)^2 + 2*a*b*log(c*((d*
g*x + d*f + e)/(g*x + f))^p) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 3.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx$$

```
[In] integrate(1/(a+b*ln(c*(d+e/(g*x+f))**p))**2,x)
[Out] Integral((a + b*log(c*(d + e/(f + g*x))**p))**(-2), x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 148, normalized size of antiderivative = 6.73

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx=\int \frac{1}{\left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="maxima")
[Out] (d*g^2*x^2 + d*f^2 + e*f + (2*d*f*g + e*g)*x)/(b^2*e*g*p*log((d*g*x + d*f +
 e)^p) - b^2*e*g*p*log((g*x + f)^p) + b^2*e*g*p*log(c) + a*b*e*g*p) - integ
rate((2*d*g*x + 2*d*f + e)/(b^2*e*p*log((d*g*x + d*f + e)^p) - b^2*e*p*log(
(g*x + f)^p) + b^2*e*p*log(c) + a*b*e*p), x)
```

Giac [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx=\int \frac{1}{\left(b\log\left(c\left(d+\frac{e}{gx+f}\right)^p\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(a+b*log(c*(d+e/(g*x+f))^p))^2,x, algorithm="giac")
[Out] integrate((b*log(c*(d + e/(g*x + f))^p) + a)^(-2), x)
```

Mupad [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a+b\log\left(c\left(d+\frac{e}{f+gx}\right)^p\right)\right)^2}\,dx=\int \frac{1}{\left(a+b\,\ln\left(c\left(d+\frac{e}{f+g\,x}\right)^p\right)\right)^2}\,dx$$

```
[In] int(1/(a + b*log(c*(d + e/(f + g*x))^p))^2,x)
[Out] int(1/(a + b*log(c*(d + e/(f + g*x))^p))^2, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well")
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_coun
        fi;
    fi;
else #ExpnType(result) > ExpnType(optimal)
    if debug then
        print("ExpnType(result) > ExpnType(optimal)");
    fi;
    return "C",cat("Result contains higher order function than in optimal. Order ",
                    convert(ExpnType_result,string)," vs. order ",
                    convert(ExpnType_optimal,string),".");
fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```